

Developing Sub-wavelength Sound Absorber Based on Coiled Up Tube Resonator

Iwan Prasetiyo^{*}, Elsa Nalita Wongso & Joko Sarwono

Engineering Physics, Faculty of Industrial Technology, Institut Teknologi Bandung Jalan Ganesa 10, Bandung 40132, Indonesia *E-mail: i.prasetiyo@fti.itb.ac.id

Abstract. Sub-wavelength sound absorbers are attractive for dealing with noise control at low-frequency (long-wavelength) sounds. To be efficient in absorbing the sound energy, resonator based absorbers are preferable over fibrous porous ones. In this paper, a coiling up space approach is introduced to a tube resonator system in order to realize a sub-wavelength absorber structure. In this way, the air channel of the tube resonator is a coplanar coiled up channel rather than a straight channel as found in conventional tube resonators. The effect of the geometrical properties of the aperture and the air channel were studied further to look at their relationship to impedance mismatch, which coiling up systems typically suffer from. It was found that the proposed approach could realize a sub-wavelength absorber system up to 1/32 wavelength of peak sound absorption. Selection of the shape and dimensions of the aperture must be done with great care as indicated by the measurement results. Moreover, the behavior of the coiled up tube resonator deviates from that of the straight tube as the reflection factor is increased, although the target resonance frequency is close to the target. It was also found that a squared aperture shape as well as increasing the cavity thickness is useful to deal with impedance mismatch.

Keywords: coiling up system; coplanar; sound absorber; sub-wavelength system; tube resonator.

1 Introduction

Sub-wavelength structures are emerging in noise-control elements, including for use as sound absorbers. These absorbers are preferable for a vast variety of applications at a low frequency regime since the absorber dimensions are no longer required to be comparable with the sound wavelength. The coiling up space approach is a technique that has brought interesting acoustic properties to metamaterials, such as a high refractive index, double negativity, and near zero index [1-4]. This technique has been introduced in macro systems such as horns [5]. Alternatively, sub-wavelength structures can be realized using a sub-wavelength resonator as a membrane [6,7]; Helmholtz resonators [8,9]; microperforated panels [10,11]; or the concept of slow sound propagation [12,13].

Received December 17th, 2018, Revised March 11th, 2019, Accepted for publication April 1st, 2019. Copyright ©2019 Published by ITB Journal Publisher, ISSN: 2337-5779, DOI: 10.5614/j.eng.technol.sci.2019.51.3.2 Some analyses on coiling system sound absorption behavior have been proposed by providing analytical models and experimental validation with particular coiling shapes, for example a circular box [14], a circular spiral [15], a labyrinth [16], multi-coiling structures [17], and a meta-atom resonator [18]. The results of these studies indicate that the total thickness of the absorber system could be reduced. Although the results of the analytical models and the experimental ones were in good agreement, these works did not sufficiently discuss the challenge of impedance matching between space-coiling structure and background medium; most of them were aimed at getting perfect absorption performance of the sub-wavelength structure, where the reflection coefficients are expected to be zero. Hence, all incoming sound energy is totally absorbed. Other previous studies have investigated the inclusion of radiation impedance in a coiling up system prediction model [18] and distribution of the visco-thermal layer of a curvature resonator [19].

In the present paper, the issue of impedance mismatch is revisited and discussed by exploring the geometrical effect of the coiling system on the sound absorption coefficient at associated resonance frequencies. Variation of geometrical properties is introduced for aperture size, aperture shape, total effective length of the coiling channel, and air cavity thickness. It was expected that the selected geometrical variations could contribute to the sub-wavelength structure design procedure. Moreover, sound absorption prediction formulae based on a conventional tube resonator are provided and used for comparison with the experimental results.

2 Theoretical Framework

2.1 Sub-wavelength Absorber

An absorber with thickness L smaller than the peak absorption quarter wavelength $\lambda/4$ is called a sub-wavelength absorber. This can be realized by several approaches: (1) a space coiling system [14-16]; (2) the use of a membrane [6,7] or a Helmholtz resonator (HR) [8,9]; and (3) employing slow sound waves by local resonators [12,13,20]. The space coiling system is beneficial to lengthening the propagation path, which is substantially longer than the physical dimensions of the structure.

The use of a membrane or HR is applicable for sub-wavelength structures as either one can introduce modified dynamic density and bulk density so that the sound speed can be determined further to fall into the sub-wavelength regime. The slow sound propagation concept applies the same principles but they are achieved by using local resonators to get a strong dispersion that allows sound propagation at a different phase velocity.

All these mechanisms have their own benefits as well as drawbacks. The membrane system is sensitive to the pre-tension value in achieving a proper resonant frequency, whereas a HR needs a specific viscous property or an exact Q-factor value to attain optimal absorption performance. Meanwhile, slow sound propagation requires a more complex structure of the local resonator arrangement in order to get strong dispersive characteristics [12]. For the case of the coiling up system, it is promising to reduce the geometric structure, which is relatively easy to realize by means of a 3D printer. However, it faces the challenge of impedance mismatch with the background medium or acoustic impedance. Hence, it is of importance to find the proper parameters that enable the coiled up system to have the impedance as close as possible to the characteristic acoustic impedance.

2.2 Absorption of Tube Resonator

The basic dimensions and geometrical properties of the coiling up channel adopted throughout this paper are shown in Figure 1. Sound hard boundaries are assumed to exist between the air and the solid materials of the channel. The resonance frequency of the tube resonator, analogous to an open-closed straight channel, is given by Eq. (1) as follows:

$$f_m = (2m - 1)c/4L$$
(1)

where *m* is a positive integer (m=1,2,3,...) indicating the mode number, L=l+t is the length of the coiled up channel, where *l* is equivalent to a straight channel length, *t* is the panel thickness, and the radius, *a*, is 50 mm. Meanwhile, the sound speed in channel *c* is defined as c_0/α_{∞} with c_0 phase velocity of free space and factor α_{∞} due to the change in bulk modulus and mass density in the channel. A typical value of α_{∞} is 0.908 up to 1.005, while the proper value can be obtained by comparison of numerical simulations and measurements for particular constructions [16]. Considering these values, the sound speed inside the channel can be greater or lower than that of the free space. In this paper, the value α_{∞} was determined by considering the measurement results.

The incident sound wave can be totally absorbed by the surface only if its surface impedance is equal to that of air. In this situation, the critical loss, β_m , depends on w/a, where w is the width of the aperture channel and a is the

channel-to-channel distance or the length side of the panel area in singlechannel cases, which yields

$$\beta_m = 2w / \left[\pi (2m-1)a \right] \tag{2}$$

For the case of a circular panel area, radius a is defined as $a = \sqrt{\pi} r_w$ with r_w as the radius of the circle. We set $r_w = 50$ mm for all cases under consideration. For low frequencies, the first mode, m = 1, is considered so that β_m is reduced to

$$\beta = 2w/\pi a \tag{3}$$

Other than that, the critical loss can be approximated by Eq. (4), as proposed in [20]:

$$\beta = \beta_0 + 2g(p+q)/(p \times q) \tag{4}$$

where p = q = w for a squared aperture and $\beta_0 = 2 \times 10^{-5}$ is the sound loss due to the shear viscosity of air itself (or dynamic viscosity) and $g = 2 \times 10^{-4}$ m accounts for the friction between the air molecules and the channel walls.



Figure 1 General outline of coiling up system and its associated notation used in the calculations: (a) front view of the coiled up channel; (b) side view of the coiled up channel; (c) aperture part of the coiled up channel; (d) a sample with an embedded coiled up channel and its default parameter values.

The normalized surface impedance of a coiled up tube resonator can generally be defined as in Eq. (5) [4]:

$$Z_{s} = \left(\frac{w}{a}\right) \frac{1 - e^{j2k_{c}L}}{1 + e^{j2k_{c}L}}$$
(5)

where $k_c = k\alpha_{\infty}(1+j\beta)$ is a complex wavenumber. From this, the reflection coefficient can be written as

$$R = (Z_{s} - 1) / (Z_{s} + 1)$$
(6)

The normal sound absorption coefficient α_n is thus obtained as follows:

$$\alpha_n = 1 - \left| R \right|^2 \tag{7}$$

3 Experiment and Simulation

3.1 Experimental Setup

In general, the geometrical property of coiling structure l was nominally equal to 205 mm and t was set to 3.4 mm so that L is equal to 208.4 mm. The values of w and a were nominally set to 4.85 mm and $50\sqrt{\pi}$ mm respectively, while the cavity thickness D' was 9.7 mm, unless otherwise stated. For frequencies below 500 Hz, these dimensions allow the resonator to be considered as being acoustically compact ($\lambda >> a$). This also means that a single mode approximation (m = 1) should be sufficient to predict its absorption coefficients. The total thickness of all samples was 18.1 mm, including an additional 5-mm solid part for having a sufficient transmission loss; it is expected that the sound is reduced only by the absorption mechanism and not by any other mechanism. We set $\alpha_{\infty} = 0.95$ for the prediction calculation as a result of fitting to the resonant frequency of the first mode of the measurement data. For analysis purposes, the dimensions were then varied to further look at the absorption behavior. The samples were fabricated by polylactic acid (PLA) filaments using a 3D printer and the end of the channel was in rigid wall condition (see Figure 2).

The absorption coefficients of the samples were measured using an impedance tube according to ISO 10534-2 [21]. A schematic diagram of the sound absorption measurement is shown in Figure 3. The speaker generates white noise, which is considered as a plane wave travelling in a tube with a diameter of 10 cm covering the frequency range between 64 Hz and 1.6 kHz. These plane wave conditions hold as the length of the wave at the highest frequency is

longer than the lateral tube diameter in order to avoid cross-section modes. To get the phase correction factor of the two microphones, the microphones were swapped during the measurements.



Figure 2 Sample produced with a 3D printer.



Figure 3 Schematic diagram of sound absorption measurement using an impedance tube.

3.2 Results and Discussions

3.2.1 Effect of Aperture Dimension

To investigate the impact of ratio w/a on the sound absorption coefficient characteristic, the aperture dimension w was varied by a factor of 2 to the original dimension, while the radius of a was kept the same. This resulted in the w dimension to be nominally 2.425 mm, 4.85 mm and 9.70 mm respectively.

Figure 4 presents a comparison of the sound absorption coefficients obtained by the prediction model and the measured ones for different w/a ratios. Theoretically, the main impact of a difference in w/a is on the absorption bandwidth and the absorption amplitude, indicated by solid curves. This kind of tendency was also observed from the measurement results, except for the case of w = 9.70 mm.

The measurement results show that an increase in aperture dimensions does not lead to a higher amplitude as suggested by the prediction results. Moreover, the resonant frequency associated with the absorption peak is shifted toward a lower frequency for greater aperture areas. For example, the resonant frequency of the tube resonator with w = 9.70 mm was 208 Hz, whereas that of the tube resonator with w = 4.85 mm was 408 Hz, i.e. almost half. This suggests that the coplanar coiling system resonance resembles a Helmholtz resonator (HR) rather than a tube resonator (TR) when the aperture dimension increases, i.e. increasing the total mass because of a 'neck' dimension change. However, this should be discussed based on further investigation results. Limiting the analysis to the underlying theoretical framework in Section 2.2, the resonance frequency of the coiling up system is dictated by $\exp(2k_cL)$ (see Eq. (5)). Hence, k_c is the most relevant parameter responsible for the case of w = 9.70 mm, where the sound speed in the channel is slower than that in free space, causing a resonant frequency of w = 9.70 mm, which is lower than in the other two cases. Lower or higher sound speed in the channel has also been suggested by Liu Liu, *et al.* [16] and [22].



Figure 4 Comparison of normal sound absorption coefficients between prediction results and measured ones for different apertures: (a) $w = 0.5 \times w_0$, (b) $w = w_0 = 4.85 \text{ mm}$, (c) $w = 2 \times w_0$ (— prediction; --- measurement).

Figure 4(a) shows a large discrepancy between the prediction result and the measurement result around 1 kHz. Mode coupling of sound filed inside the tube and the vibrating elements in the impedance tube system may cause this, e.g. loudspeaker displacement absorption due to a low absorption characteristic of the tube resonator ($\alpha_n < 0.2$). Another possibility is related to a non-smooth surface scattering absorption due to imperfect sample fabrication. Hence, the discrepancy around 1 kHz is not related to resonator absorption.

Increasing the perforation ratio can improve Z_s to close to $\rho_0 c_0$, as indicated by Eq. (5), so that the sound absorption coefficients increase. This can be observed from Figure 5. By introducing a double aperture in the system, the perforation ratio increases so that Z_s can better match that of the background medium. Hence, the sound absorption becomes around 0.6 or 0.2 higher than with a single aperture.



Figure 5 Sound absorption coefficient comparison between single and double aperture.

3.2.2 Effect of Aperture Shape

To further investigate the properties of the aperture in relation to the impedance mismatch behavior, the aperture shape was varied while keeping the area the same. For this, 23.5225 mm² was realized by a 2.425 mm \times 9.70 mm aperture and a 4.85 mm \times 4.85 mm aperture while the rest of parameters were kept the same.

Figure 6 presents a comparison of the sound absorption coefficient characteristics. It is clear that the shape of the aperture affects the sound

absorption coefficients. The sound absorption coefficients of the squared aperture were higher than those of the rectangle aperture although both had the same area. These measurement results confirm this tendency, although the absorption coefficients for the case of a rectangle aperture were much lower than the predicted ones. This is attributable to the shape, where the concentration of dissipative energy/critical loss is affected by the geometrical properties [23]. Hence, a squared aperture is still beneficial for the proposed absorber system.



Figure 6 Effect of aperture shape on sound absorption characteristics.

3.2.3 Effect of Air Cavity Length

The length of the air cavity channel was varied for the same cross section dimensions (4.85 mm x 4.85 mm) with panel thickness t equals 3.4 mm. For this, the effective length of channel l was set to 102.5 mm, 205 mm and 410 mm respectively. The results are shown in Figure 7. It was found that the resonant frequency is shifted to a higher frequency for a shorter channel length and vice versa, as expected following Eq. (1). However, the sound absorption coefficient of the measurement results at the resonance frequency was not the same. For the case of 102.5 mm, the sound absorption coefficient was around 0.24 at the resonance frequency. A similar result was found for the 410 mm long channel. The highest absorption coefficient was shown in the case of the 205 mm long channel. It is interesting that such a situation did not arise with a straight tube, where the absorption coefficient should be maintained the same for different channel lengths due to similar energy intensification at a quarter wavelength. Apparently, the introduction of a coiling up system causes the resistance to increase when the coplanar channel is lengthened so that the reflection factor

increases. This condition causes impedance mismatch. Impedance mismatch issues have also been reported in [2,3,24,25].

The absorption bandwidth is also different, where a shorter channel length leads to a wider absorption bandwidth. Theoretically, it was found that the absorption bandwidth of each case evaluated at $\alpha_n = 0.1$ was around 560 Hz, 280 Hz and 140 Hz, respectively. Hence, the shorter channel lengths had a wider absorption bandwidth. The measurements results indicate the same tendency. In the case of the 102.5-mm channel length, an absorption bandwidth of 478 Hz was present. Likewise, an absorption bandwidth of 166 Hz was shown in the case of the 410-mm channel length. An anomaly case was observed in the case of the 205-mm channel length, where the absorption bandwidth was narrower than with a channel length of 410 mm. This should be related to a deviation of the resistive part of the surface impedance.



Figure 7 Effect of air coiled cavity length on normal absorption: (a) l = 102.5 mm (b) l = 205 mm; (c) l = 410 mm (— prediction; --- measurement).

3.2.4 Effect of Air Cavity Thickness

It is interesting to vary the depth of channel in a particular direction so that the channel cavity thickness D' is no longer equal to the double aperture dimension. In this case, D' at 4.85 mm, 9.70 mm, 19.40 mm and 38.80 mm was considered, while the effective channel length l was kept the same, as shown in Figure 8. The idea behind this was to let the incoming waves propagate through the straight channel sufficiently before encountering a sudden turning channel due to coplanar coiling up by which the reflected wave intensity increases.

The results show that the cavity thickness increased the peak of sound absorption as the resonance frequency increased, as shown in Figure 8. It was found that sound absorption coefficients of 0.62 at 380 Hz and 0.78 at 306 Hz were present for the cases of D' = 19.40 mm and D' = 38.80 mm, respectively, while the rest of the cases had lower sound absorption coefficients. A greater sound absorption peak means that the matching impedance of the system to the background medium is better, where $Z_s/\rho_0 c_0$ approaches to 1. Moreover, the resonance frequency shifts to a lower frequency at greater cavity thickness due to the increase of the total of mass of the straight channel just before the coiled channel. A change in cavity thickness D' is useful for dealing with impedance mismatch issues, but this should be applied carefully, particularly when dealing with target sound resonance frequencies at which a high sound absorption is expected. The results also suggest that the system proposed here is capable of absorbing sound frequencies as small as 1/20 up to 1/32 sound wavelength. This indicates that the proposed system meets the sub-wavelength structure absorber criteria, although thinner structures have been found in other metasurfaces, which can be as small as 1/133 sound wavelength [6,26,27] or even smaller, up to 1/233 [14].



Figure 8 Sound absorption comparison for different cavity thicknesses.

Table 1 lists some important findings from this study, particularly recapping our approach of dealing with the impedance mismatch of coiling up spaces. This can be a guide to selecting the coiling up parameters properly.

Table 1 Summary of important findings related to dealing with impedance mismatch.

| Parameter | Finding |
|----------------------|--|
| Number of apertures | Multiple apertures are more effective. Changing the perforation ratio can be useful to have a better impedance match |
| Aperture shape | Square |
| Air cavity thickness | Greater is better but the sub-wavelength structure target needs to be considered |

4 Conclusions

A coiling up space was introduced to a tube resonator system for acquiring a sub-wavelength sound absorber structure. It was found that the proposed approach could realize a sub-wavelength system up to 1/32 wavelength of peak sound absorption. It was also found that selection of the aperture dimensions and shape must be done with great care or a proper perforation ratio should be introduced to alleviate impedance mismatch implications. Moreover, the behavior of the coiled up tube resonator deviates from a straight tube, where the reflection factor is increased although the target resonance frequency can be kept close to the target. The selection of perforation ratio, squared aperture shape and increasing the cavity thickness is useful in dealing with impedance mismatch. It should be noted that the approach of increasing the cavity thickness should be implemented with care as the target resonance frequency can shift to lower or higher frequencies. Future work can be devoted to improving the analytical model's accuracy and a multi-coiled up channel with a different desired resonance frequency in order to have broadband thin absorbers.

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