Bending and Deformation of Sandwich Panels Due to Localized Pressure

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Abstract. Bending and deformation of sandwich panels due to localized pressure were analyzed using both Rayleigh-Ritz and finite element methods. The faces were made of laminated composite plates, while the core was a honeycomb material. Carbon fiber and glass fiber reinforced plastics were used for composite plate faces. In the case of Rayleigh-Ritz method, first the total energy of the system was calculated and then taking the variations of the total energy, the sandwich panel deflections could be computed. The deflections were assumed by means of Fourier series. A finite element code NASTRAN was exploited extensively in the finite element method. 3-dimensional 8-node brick elements were used to model sandwich panels, for both the faces sheets and the core. The results were then compared to each other and in general they are in good agreements. Dimple phenomena were found in these cases. It shows that localized pressure on sandwich structures will produce dimple on the pressurize region with little effects on the rest of the structures.

Keywords: *sandwich panels; dimple phenomena; localized pressure; finite element methods; composite plate.*

1 Introduction

Sandwich structure consists of a thick core that is placed in between two thin, high strength, high stiffness faces. The core is made from a low density, low stiffness material such as foam and honeycomb. Whilst the face is usually made of high stiffness materials such as steel, aluminum or fiber reinforced composite materials. The resulting sandwich structure is light but at the same time stiff in bending direction. Hence, the structures are usually used in aircraft, ships and nowadays even found in buildings and bridges. Figure 1 shows a typical sandwich structure with honeycomb core.

Since sandwich structure is usually thick and consists of a weak core, the usual Kirchoff-Love assumption regarding bending and deformation of thin plates could not be used to analyze sandwich panels. Kirchoff-Love disregards normal and shear strains in the thickness direction. Since sandwich structure is thick

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and contains a weak core, these two strains should be included in the analysis. A weak core also means that sandwich structure is susceptible to concentrated load, such as impact load, which will produce a local deformation known as dimple. This local deformation is an interesting phenomenon since most of the concentrated load is absorbed locally, with little influence to the other part of the structures. In this paper, bending and deformation of sandwich panels due to concentrated loading will be analyzed using both analytical and finite element methods. The results will be compared each other.

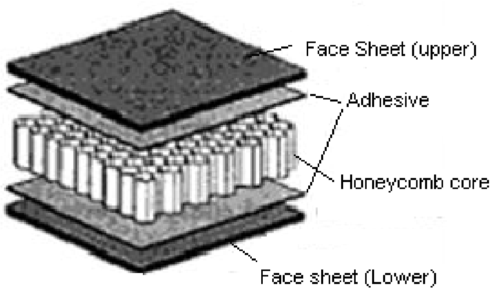


Figure 1 Typical sandwich structures with honeycomb core.

Several authors have studied sandwich structures. Allen [1] and Plantema [2] had produced monographs on the subject, but mostly dealt with buckling and cylindrical bending. Pagano [3] analyzed three-layered sandwich beam and his results were considered to be exact. The results of Pagano were often used as benchmarks for other solutions, such as finite element results. Meyer-Piening [4] developed a model to analyze bending of sandwich beams due to symmetrical loads with consideration of overhanging ends (cantilever beams). He developed further [5] to include sandwich plate's analysis regarding bending, instability and natural frequency. Anderson [6] developed 3-dimensional model to study the deformation of sandwich plate under impact loading. The faces were made of orthotropic materials.

The core is usually made of a honeycomb structure. The structure is relatively stiff in the thickness direction but very weak in the plane direction. Therefore, it is known as an anti-plane core [1]. B.K. Hadi and F.L. Matthews [7] developed a model to calculate the total energy of sandwich panels due to external loads by assuming the core as an anti-plane core. The faces were made of laminated composite plates that were anisotropic plates in nature. They used the model to calculate wrinkling and overall buckling of anisotropic sandwich panels due to compressive in-plane forces. The results were compared to experimental results by Pearce and Webber [8] and also Webber, et. al [9] and they are in good agreements.

In this paper, the methodology developed previously [7] is used to compute deformation and stresses of sandwich panels due to localized pressure. The results are then compared by finite element results.

2 Total Energy of the System

Figure 2 shows the problem. It shows sandwich panels with laminated composite plate faces and an anti-plane core. The transverse load is localized in the center of the panel. In order to solve the problems using Rayleigh-Ritz method, it is necessary to calculate the total energy of the system.

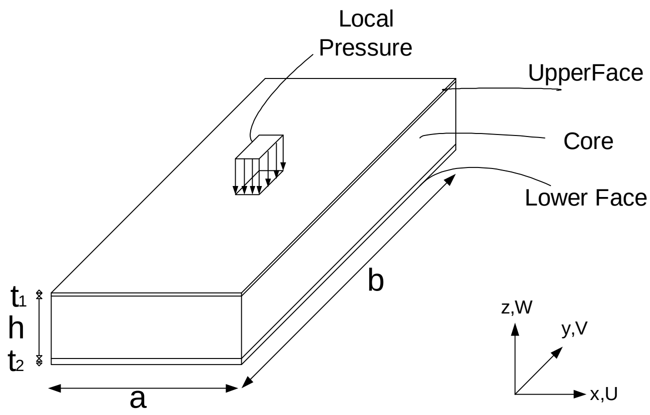


Figure 2 Definition of the problem.

The equations below are taken from [7]. Readers should refer to that publication for detailed analysis of these equations.

2.1 Internal Energy Contribution of the Faces

The strain energy of the faces due to bending and membrane strain is:

$$U_{fi} = \frac{1}{2}\int_0^a\int_0^b \left[\{\varepsilon_{oi}\}^T [A]\{\varepsilon_{oi}\} + 2\{\varepsilon_{oi}\}^T [B]\{\varepsilon_{oi}\} + \{\kappa_i\}^T [D]\{\kappa_i\}\right] \mathrm{d}x\, \mathrm{d}y \tag{1}$$

where

$$\{\varepsilon_{oi}\} = \begin{Bmatrix} \partial U_i / \partial x \\ \partial V_i / \partial y \\ \partial U_i / \partial y + \partial V_i / \partial x \end{Bmatrix}$$ is membrane strain in the middle of the plate faces,

and

$\{\kappa_i\} = \begin{Bmatrix} -\partial^2 W_i / \partial x^2 \\ -\partial^2 W_i / \partial y^2 \\ -2\partial^2 W_i / \partial x \partial y \end{Bmatrix}$ is curvature of the middle of the plate faces, and

i = 1,2 are upper and lower faces respectively. U_1, V_1, W_1 and U_2, V_2, W_2 are displacements of upper and lower faces respectively. [A], [B] and [D] are standards extensional, couple and bending stiffness matrices of laminated composite plate faces [10].

2.2 Internal Energy Contribution of the Faces

We assumed an anti-plane core, in which the stiffness in the x and y direction (plane direction) is neglected. Therefore, the core strain energies are represented by energies due to shear strains and normal strains in the thickness direction only.

Energy due to shear strains is calculated by using equation below:

$$U_{C1} = \left(\frac{h}{2}\right)\int_0^a\int_0^b \left(G_x \gamma_{xy}^2 + G_y \gamma_{yz}^2\right) \mathrm{d}x\, \mathrm{d}y \tag{2}$$

while the energy due to normal strains are given as follows:

$$U_{C2} = \frac{h^3}{24E_z}\int_0^a\int_0^b \left\{G_x \frac{\partial \gamma_x}{\partial x} + G_y \frac{\partial \gamma_y}{\partial y}\right\}^2 \mathrm{d}x\mathrm{d}y + \frac{2E_z}{h}\int_0^a\int_0^b \left(W_1 - W_2\right)^2 \mathrm{d}x\mathrm{d}y \tag{3}$$

where G_x and G_y are shear modulus of the core in the x-z and y-z planes respectively and γ_x and γ_y are shear strains in the x-z and y-z planes respectively. While h is core thickness.

During bending of the panels, displacements' continuity in the interfaces between the core and the upper and lower faces should be maintained. The continuity produces strain energy in the bonded joint between faces and core that can be given as follows:

$$U_a = \int_0^a\int_0^b \left(\lambda_x \phi_x + \lambda_y \phi_y\right) \mathrm{d}x\mathrm{d}y \tag{4}$$

where:

$$\phi_x = (U_1 - U_2) + h_1 \frac{\partial W_1}{\partial x} + h_2 \frac{\partial W_2}{\partial x} + \frac{h^3}{12E_z}\left(G_x \frac{\partial^2 \gamma_x}{\partial x^2} + G_y \frac{\partial^2 \gamma_y}{\partial x \partial y}\right) - h\gamma_x$$

$$\phi_y = (V_1 - V_2) + h_1 \frac{\partial W_1}{\partial y} + h_2 \frac{\partial W_2}{\partial y} + \frac{h^3}{12E_z}\left(G_x \frac{\partial^2 \gamma_x}{\partial x \partial y} + G_y \frac{\partial^2 \gamma_y}{\partial y^2}\right) - h\gamma_y$$

and

$h_1 = \dfrac{h+t_1}{2}$; $h_2 = \dfrac{h+t_2}{2}$. While t_1 and t_2 are upper and lower face thicknesses respectively; and h is core thickness. λ_x and λ_y are Lagrange Multipliers which are arbitrary functions of x and y.

2.3 External Energy Due to Transverse Load

The external load is a localized constant pressure in the shaded area as given in Figure 1. The load produces external energy that is given by:

$$V_e = \int_{a1}^{a2}\int_{b1}^{b2} [P_z . W_1]\, \mathrm{d}x\mathrm{d}y \tag{5}$$

where P_z is a constant pressure over localized area on the upper face.

2.4 Total Energy of the System

Finally, the total energy of the system is given by:

$$\Pi = U_{f1} + U_{f2} + U_{C1} + U_{C2} + U_a + V_e \tag{6}$$

In order to solve Eq. 6, it is necessary to assume deflection functions of the panel. For the case of simply supported plate on all sides, the deflection functions could be given in the form of Fourier series as follows:

$$U_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}^{(1)} \cos\alpha_m x \sin\beta_n y,$$

$$U_2 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}^{(2)} \cos\alpha_m x \sin\beta_n y,$$

$$V_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}^{(3)} \sin\alpha_m x \cos\beta_n y,$$

$$V_2 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}^{(4)} \sin\alpha_m x \cos\beta_n y,$$

$$W_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}^{(5)} \sin\alpha_m x \sin\beta_n y,$$

$$W_2 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}^{(6)} \sin\alpha_m x \sin\beta_n y, \tag{7}$$

$$\gamma_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}^{(7)} \cos\alpha_m x \sin\beta_n y,$$

$$\gamma_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}^{(8)} \sin\alpha_m x \cos\beta_n y,$$

$$\lambda_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}^{(9)} \cos\alpha_m x \sin\beta_n y,$$

$$\lambda_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}^{(10)} \sin\alpha_m x \cos\beta_n y$$

and

$$\alpha_m = \frac{m\pi}{a} \;;\; \beta_n = \frac{n\pi}{b}$$

Ritz method requires that the total energy of the system should be stationery. Inserting Eq. (7) into Eq. (6), and fulfilling Ritz condition as follows,

$$\frac{\partial\Pi}{\partial C_{mn}^{(1)}} = 0, \quad \frac{\partial\Pi}{\partial C_{mn}^{(2)}} = 0, \quad \frac{\partial\Pi}{\partial C_{mn}^{(3)}} = 0, \quad \frac{\partial\Pi}{\partial C_{mn}^{(4)}} = 0, \quad \frac{\partial\Pi}{\partial C_{mn}^{(5)}} = 0, \tag{8}$$

$$\frac{\partial\Pi}{\partial C_{mn}^{(6)}} = 0, \quad \frac{\partial\Pi}{\partial C_{mn}^{(7)}} = 0, \quad \frac{\partial\Pi}{\partial C_{mn}^{(8)}} = 0, \quad \frac{\partial\Pi}{\partial C_{mn}^{(9)}} = 0, \quad \frac{\partial\Pi}{\partial C_{mn}^{(10)}} = 0,$$

we have 10 simultaneous equations in $C_{mn}^{(i)}$, $i = 1,2,......,10$. The Fourier series were expanded until the deformation values are stable, i.e. they don't differ significantly with the increasing Fourier series. In this paper, *m* and *n* were expanded up to 50 terms, in order to get satisfactorily results.

3 Finite Element Analysis

A quarter and a full of sandwich panel were analyzed using 3-dimensional 8-node brick elements (CHEXA elements). Both the faces and the core were modeled using these elements. A single layer of brick elements was used to model the faces, while the numbers of bricklayer for the core depended on the core thickness. Nodal displacements' continuity between the faces and the core should be maintained. A finite element code NASTRAN was used extensively during the analysis. The boundary conditions were simply supports in all sides.

4 Results and Discussions

4.1 Orthotropic Faces

The panel dimension and properties are shown in Figure 2 and Table 1. Loading area is ranging from x = 47.5 mm through 52.5 mm and from y = 90 mm through 110 mm. The pressure is applied on the upper face with a loading pressure of – 1 MPa. The material properties are taken from [5]. As seen in Table 1, the seemingly very low stiffness of the core was intended to exploit the possibility of having a dimple for these cases.

Table 1 Panel Dimension and Properties.

	Dimension	
	a = 100 mm ; b = 200 mm ; Overall thickness = 12 mm	
	Face	
	Upper face thickness = 0.1 mm ; Lower face thickness = 0.5 mm	
E_1 = 70000 MPa	G_{12} = 26000 MPa	ν_{12} = 0.3
E_2 = 71000 MPa	G_{13} = 26000 MPa	ν_{13} = 0.3
E_3 = 69000 MPa	G_{23} = 26000 MPa	ν_{23} = 0.3
	Core	
	Thickness : 11.4 mm	
E = 3 MPa	G = 1 MPa	υ = 0.25

Since the face is an orthotropic material, then the panel is symmetric. Therefore, in the finite element analysis, only a quarter of the panel was analyzed. The maximum deformation occurred in the center of the panel on the upper face, and the value is given in the Table 2.

Table 2 Maximum Deformation of the Sandwich Panel due to Pressurized Loading.

Deformation (mm)	**Meyer-Piening [5]**	**Present Results**	
		Rayleigh-Ritz	**Finite Element Analysis**
Upper face	-3.78	-3.842	-3.775
Lower face	-2.14	-2.156	-2.145

Table 2 shows that a good agreement exists between Rayleigh-Ritz and finite element analysis and comparable with the results of [5]. The Rayleigh-Ritz produces slightly higher deformations, since it used anti-plane core assumptions, whereas both Meyer-Piening [5] and finite element analysis used a full three dimensional model. Nevertheless, the Rayleigh-Ritz method was used due to its simplicity. Figure 3 shows deformation pattern for this loading.

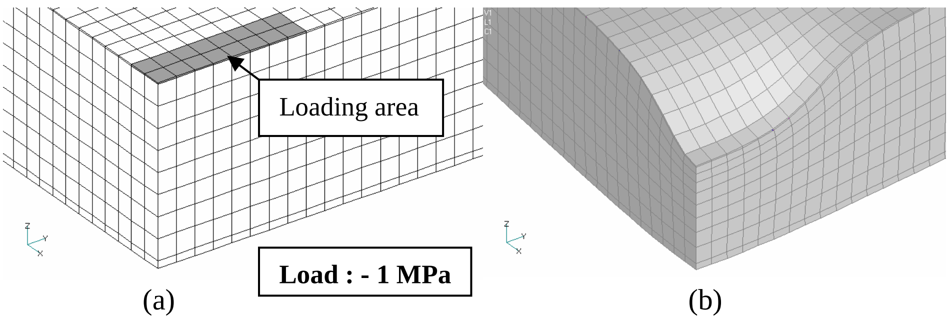


Figure 3 Deformation of orthotropic sandwich panel due to constant pressure. (a) undeformed state, (b) deformed state.

Figure 3 shows that local deformation in the vicinity of the pressure occurs, thus dimple phenomenon occurs. Figure 4 shows deformation distribution in the x direction. It shows that local deformation due to localized pressure exists in the upper face, whereas more distributed deformation occurs in the lower face. Both figures show that local pressure in the sandwich panels will act locally that will not be possible if analyzed using standard Kirchoff-Love assumption. By disregarding the shear and normal stiffness in the core thickness direction, Kirchoff-Love by definition, will produce the same deformations for the upper and lower faces. Therefore, shear and normal strains of the core in the thickness direction should be taken into account when analyzing sandwich structures in order to produce dimple phenomenon as shown in the above results.

The stress distribution could also be calculated. Table 3 shows comparison of the stresses in the faces using Rayleigh-Ritz (R-R) and finite element analysis (FEA).

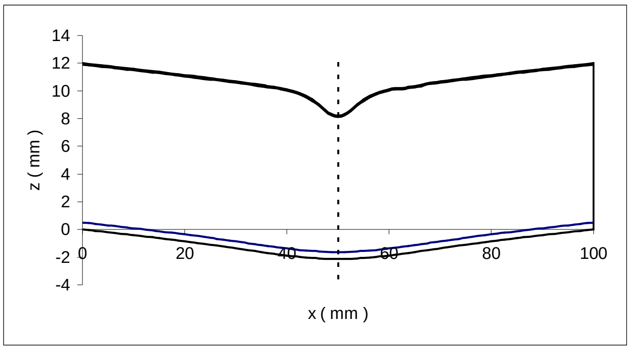


Figure 4 Deformation distribution in the x-direction.

Table 3 Stresses in the Faces due to Pressure Load.

		σ_x (MPa)		σ_y (MPa)	
		R-R	**FEA**	**R-R**	**FEA**
Upper face	Top Bottom	-624 580	-604.6 281	-241 211	-256.7 163.5
Lower face	Top Bottom	-138 146	-106.4 170	-121 127	-62.76 98.88

Unlike deformation results, there are some differences in the stresses results between R-R and FEA. These due to the difficulties in determining stresses in a point in the finite element method, since the stresses are given in the elements, not in point, while R-R could accurately calculate stresses in each point.

4.2 Monoclinic Faces

We also studied the behavior of sandwich panels with anisotropic faces made of typical carbon and glass fiber reinforced plastic (CFRP and GFRP). The monoclinic nature of the faces is achieved by orienting fiber direction away from *x*-axis. Figure 5 shows the sandwich panel with off-axis fiber direction. The material properties of the carbon and glass fiber reinforced plastics are given in Table 4 and 5.

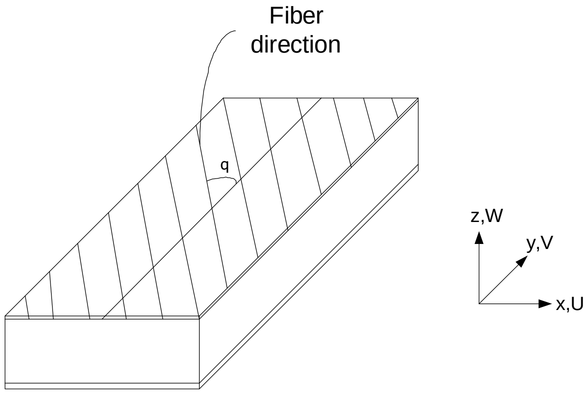


Figure 5 Sandwich panel with off-axis fiber direction for upper and lower faces.

Table 4 Material Properties of Sandwich Structures with Typical CFRP Faces.

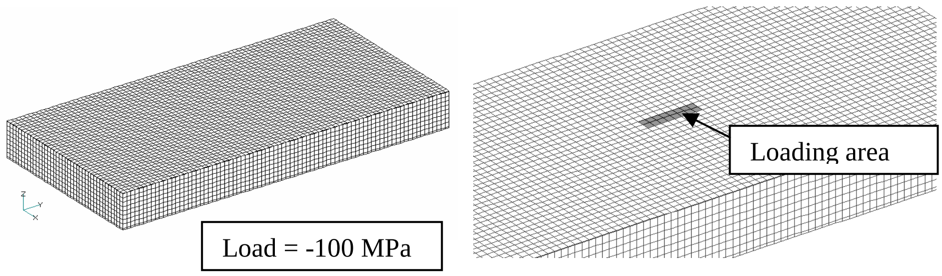
Face		
E_1 = 157889.9 MPa	G_{12} = 5957.07 MPa	v_{12} = 0.32
E_2 = 9583.713 MPa	G_{13} = 5957.07 MPa	v_{13} = 0.48
E_3 = 9583.713 MPa	G_{23} = 2537.271 MPa	v_{23} = 0.32
Core		
E = 103.6282 MPa	G = 39.857 MPa	v = 0.3

Table 5 Material Properties of Sandwich Structures with Typical GFRP faces.

Face (ScotchPly 100 /glass epoxy)		
E_1 = 38600 MPa	G_{12} = 4140 MPa	v_{12} = 0.26
E_2 = 8270 MPa	G_{13} = 4140 MPa	v_{13} = 0.26
E_3 = 8270 MPa	G_{23} = 3281.746 MPa	v_{23} = 0.26
Core (Polyurethane)		
E = 7.9 MPa	G = 3.0385 MPa	v = 0.3

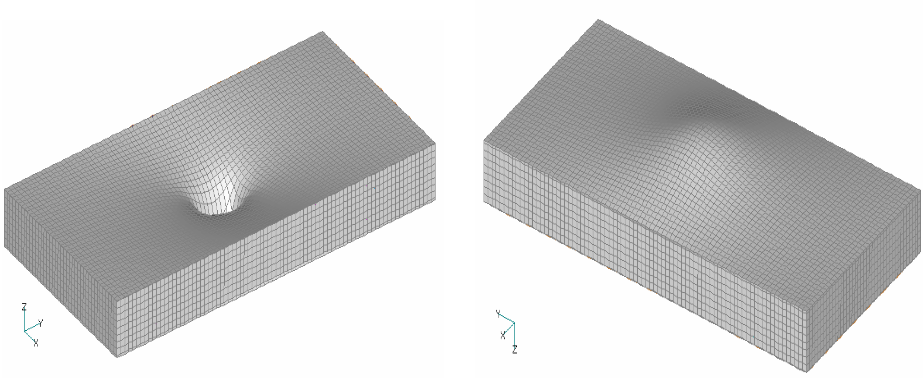
Since these sandwich structures contain monoclinic faces, there is no symmetrical plane. Therefore, in the finite element analysis, the whole panels should be analyzed. In this case, it will not be enough to analyze only a quarter of the panels, as it was the case for sandwich panels with orthotropic faces.

Figure 6 shows a 3-dimensional model for finite element analysis of the sandwich panels, while Figure 7 shows the deformation results for the sandwich panels showing the upper and lower panels.



(a) (b)

Figure 6 Three-dimensional finite element model for full sandwich panels.



(a) (b)

Figure 7 Deformation results of anisotropic sandwich panels due to pressurized loading (a) top view, (b) bottom view.

Figure 7 shows that local deformation occurs extensively in the upper face and not so extensive in the lower face. The rest of the structures do not deform as deep as the loading area. Once again it shows that weak core contributes to this phenomenon.

The deformation value is given in Table 6, for both sandwich panels with CFRP and GFRP faces. Table 6 shows that sandwich panels with 0^o fiber direction have the smallest deformation, thus the stiffest. Sandwich panels with 90^o fiber direction have the greatest deformation. It shows that orienting fiber direction away from the main direction of the panels will produce higher deformations, thus weaker structures. Table 6 also shows that the deformation in the upper face is 3 – 4 times greater than the lower face. It shows that dimple phenomena occur on these cases.

Table 6 Deformation of Sandwich Panels with Typical CFRP and GFRP Faces.

		Deformation (mm)				
		Fiber direction				
		0^o	30^o	45^o	60^o	90^o
CFRP	**Upper face**	-4.435	-4.567	-4.829	-5.122	-5.354
	Lower face	-1.219	-1.559	-1.590	-1.626	-1.654
GFRP	**Upper face**	-5.005	-5.134	-5.266	-5.385	-5.463
	Lower face	-1.764	-1.793	-1.813	-1.826	-1.829

5 Conclusion

The analysis concludes that there were good agreements between the results of sandwich plate deformations due to localized pressure using Rayleigh-Ritz and 3-D finite element methods.

The Rayleigh-Ritz analysis used principle of stationery energy, which also included shear, and normal deformation of the core in the thickness direction. The displacement continuity between faces and core is maintained using Lagrange Multipliers. The finite element analysis used 3-dimensional brick elements (CHEXA) in the NASTRAN code. The agreements between the results of both analyses are very good, especially in the deformation values. The analysis also shows that sandwich panels under localized pressure will produce dimple phenomenon that will not occur in the case of thin plate.

Nomenclature

$[A]$, $[B]$, $[D]$	:	Extensional, coupling and bending stiffness matrices for upper and lower faces.
$\{\varepsilon_{oi}\},\{\kappa_i\}, i=1,2$	:	Membrane strains and curvature of the middle plate of the upper and lower faces.
$U_i, V_i, W_i,\ i=1,2$	:	Displacements in the *x, y* and *z* directions for upper and lower faces.
G_x, G_y	:	Core shear modulus in the *x*-*z* and *y*-*z* planes.

E_z	:	Core modulus of elasticity in the *z* (thickness) directions.
γ_x, γ_y	:	Core shear strains in the *x*-*z* and *y*-*z* planes.
λ_x, λ_y	:	Lagrange Multipliers
P_z	:	Pressure loading
t_1, t_2	:	Thickness of the upper and lower faces
h	:	Core thickness
a, b	:	Sandwich panels width and length
$a1, a2$	:	Coordinates in *x*-direction between which the pressure occurs.
$b1, b2$	:	Coordinates in *y*-direction between which the pressure occurs.

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