CRYSTAL-LIKE GEOMETRIC MODELING

A Thesis

by

ERIC LANDRENEAU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2005

Major Subject: Computer Science

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Approved as to style and content by:

John Keyser (Chair of Committee)	Ergun Akleman (Member)
Glen Williams (Member)	Valerie E. Taylor (Head of Department)

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ABSTRACT

Crystal-Like Geometric Modeling. (May 2005)

Eric Landreneau, B.S., Texas A&M University

Chair of Advisory Committee: Dr. John Keyser

Crystals are natural phenomena that exhibit high degrees of order, symmetry, and recursion. They naturally form interesting and inspiring geometric shapes.

This thesis provides geometric modeling techniques for creating shapes with crystal-like geometry. The tiered extrusion method, along with a face grouping technique, simplifies the creation of complex, intricate faceted shapes. In combination with remeshing, these methods provide the capability to generate geometric shapes exhibiting planar faces, symmetry, and fractal geometry.

The techniques have also been implemented in software, as a proof of concept. They are used in an interactive geometric modeling system, in which users can use these techniques to create crystal-like shapes.

The crystal-like modeling operations are shown to successfully create beautiful geometric shapes. The methods improve upon traditional modeling capabilities, providing an easier way to create crystal-like geometric shapes.

To my family and friends

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CHAPTER I

INTRODUCTION

In the world of interactive geometric modeling, there's always a push for quicker, easier methods to generate complex and detailed models. The focus of this thesis is developing a method for interactively creating complex crystal-like shapes.

This thesis provides descriptions of what can make a shape appear to be crystal-like. Crystalline shapes are described as having facets, symmetry, and fractal geometry; they exhibit a highly ordered shape. The background section describes these properties, as well as discussing the motivation and inspiration for the work.

New methods were developed which provide an easier way to model crystal-like shapes than traditional tools provide. The first of these new methods is extended extrusions. Traditional modeling extrusions have difficulty in modeling many crystal-like shapes. Therefore, it was necessary to develop new extrusions that provide improved capabilities for creation of crystal-like models. The extrusions implement a tier system, which provides a simple and easy-to-use interface through which the user can sculpt the extrusions.

The second modeling method, which is somewhat related to the first, is the use of face groups for extrusions. Faces of a model are assigned face groups that allow an extrusion to be repeated across all faces in a group. This helps to improve the symmetry of an object, and can result in more crystal-like shapes.

Third, combining face grouping with model remeshing provides an easy way to create complex fractal shapes. As a model is subdivided in a similar fashion across all faces,

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This thesis follows the style and format of *IEEE Transactions on Visualization and Computer Graphics.*

face groups are also assigned similarly across the subdivided faces, causing subsequent extrusions to give the shape a fractal quality.

These methods have been implemented in an interactive modeling environment. This system lets the user make a crystal-like shape as he sees fit. Using the tool, a user can create beautiful, interesting crystalline shapes more easily, and faster, than is possible with traditional modeling.

Several crystal-like shapes were created with the modeling tools, and further insights were gathered through the process of creating them. There appears to be many opportunities for further research on the creation of crystal-like shapes.

This thesis describes the work by first giving background on crystals and fractals. The methodology section presents the techniques. This is followed by a user interface section describing how these techniques were implemented in software. Lastly, the results of using these techniques are shown in a series of rendered images, and are followed by concluding thoughts.

CHAPTER II

BACKGROUND

Before quantifying what makes an object "crystal-like", it is important to know the fundamentals of natural crystals. Fractals are often seen in natural objects[13], especially crystals, so it is helpful to know a bit about fractals as well.

II.1. Crystals

Minerals are homogeneous, inorganic substances with narrowly defined chemical and physical properties. Their atoms often have a specific, ordered atomic structure. Often, minerals form geometric shapes with planar faces. Although some amorphous minerals may actually have an internal crystal structure, it is these geometric crystals that will be the focus of this thesis.

Modeling of crystal formation can be approached from a near-atomic level, such as in Kim's ice crystal work[10]. However, this thesis focuses on geometric modeling of large scale crystal-like shapes, such as are seen in gemstones or geodes.

Crystals have influenced humanity throughout history, through currency, trade, architecture, engineering, and art. There are several reasons for this, but one is due to the unique shapes formed by crystals. Usually, objects in nature have round edges, or random and chaotic shapes. This contrasts with manmade structures, which predominantly follow engineering principles and consist of straight lines, carefully measured angles, and well-ordered shapes.

This dichotomy of the natural and artificial world has played an important part in how people perceive the world around them. Man brings order and structure to the world, where nature brings entropy and chaos. This general idea can be summed up with an old saying,

"There are no straight lines in Nature."

What is important is that crystals are one of the most distinct exceptions to this saying. Crystals have straight lines, are highly ordered, and appear as if they were carefully constructed artificially. They are natural objects that exhibit manmade properties, and this makes them fascinating to a great many people[5]. Unlike some natural processes that may be extremely important to scientists, but ultimately boring to the average layman, crystals prove to be an interesting phenomenon to experienced mathematicians and the nonscholarly alike.

II.1.1. Properties of Crystals

It is important to note that the purpose of this thesis is *not* to create true crystals. It is certainly important to know ways in which crystals can be classified, as these groupings are based on properties common to all crystals.

However, geometric modeling of shapes that exactly fit these classifications is outside of the scope of this thesis. What is important is that the crystal-like geometric modeling presented in this thesis can produce shapes that are aesthetically similar to crystals. Observing how crystals are classified leads to a better understanding of what can make an object "crystal-like."

There are two common methods to categorize crystals. The first is to group them by lattice systems. There are seven of these:

1 Cubic/Isometric – Forms cubes, octahedra, dodecahedra. Have 3 orthogonal axes of symmetry.

2 Tetragonal – Although similar to cubic, they are longer along one axis than another, and form prisms.

3 Orthorhombic – Rhombic prisms and dipyramids. All three axes are unequal.

4 Hexagonal – Six-sided prisms, with hexagonal cross-sections. They have 4 axes of symmetry, 3 of which are coplanar.

5 Trigonal – Similar to hexagonal, but with a triangular cross-section.

6 Triclinic – Three axes, with unequal length and oblique intersection angles. Forms unusual, asymmetric shapes.

7 Monoclinic – Similar to tetragonal, but skewed. Can form prisms or double pyramids. Three axes, two of which are coplanar, obliquely intersect, and are orthogonal to the third axis.

Another method of classification is by properties (both chemical and physical). These are:

1 Covalent – All atoms in the crystal have covalent bonds between each other. Effectively forms one huge molecule, with an extremely high melting point.

2 Metallic – Metal atoms sit on lattice 'sites', with the outer electrons moving freely in the lattice. High melting points, and dense.

3 Ionic – Made by ionic bonds. Hard, with somewhat high melting points.

4 Molecular – Made of molecules held together by noncovalent forces, such as van der Waals forces. Low melting points and soft.

Despite these categories, some crystals are difficult to classify, and may not strictly belong to a group. In this way, despite having a well-ordered and simple internal structure, crystals often exhibit unexpected complexity.

II.2. Fractal Geometry

Benoit Mandelbrot states[13] that there is an underlying mathematics to the complexity of natural objects. One example of this fractal geometry can commonly be seen in crystals and gemstones. The atomic structure of these minerals lends itself to a recursive structure.

A simple example of this recursive property is seen in the calcite crystal. As calcite is fractured and re-fractured, it always retains the same skewed rectangular prism shape. These smaller pieces each look similar in form to the original piece, but the original piece can also be created by the many smaller pieces. This property of crystals causes them to form highly ordered, intricate shapes.

This property of crystals exhibits the essence of what defines a fractal. That is, a fractal can be described as a pattern that is repeated at ever decreasing intervals or iterations. In the case of crystals and crystal-like objects, these are geometric shapes that are seen at different scale levels, such as a small cube atop a larger cube which is atop an even larger one. No matter the iteration count, the shape is made of successively smaller cubes, down to the last iteration.

There has already been a great deal of work done on fractals, even in geometric modeling. John Hart has explored the raytracing of volumetric fractals [7]. Other work has been done on generating fractal terrain [14][17], and fractal generated plants [18][15]. The study of fractals is continuously expanding, and there remain a myriad of undiscovered fractals.

One major reason for this work on fractals goes back to Mandelbrot's statement. Since there is an inherent fractal essence in the natural world, it stands to reason that fractals could be used to help simulate the natural world.

Although fractals are useful, a major problem with them is that they are generally procedural. Most of the time, a fractal is automatically generated by a computer program, with little actual input from the user. Most of the time, if a user wants to change a fractal,

he needs to change the generating formula itself. Even then, the actual fractal generation process is generally automatic.

What I present here, in Chapter III, is a method for generating fractal like geometry. However, rather than being static and unchangeable, the user has control over the shape of the fractal at every stage of iteration. Instead of a rigid fractal adhering to only one or a handful of formulae, the subdivision method can be changed completely, the shape can be geometrically altered after, or even during some of the steps. The user has a great degree of control—much greater than most fractal generation methods.

This is a step forward in giving interactive modeling control over a fractal system. Previously, most or all of the adjustment had to be performed when the fractal was completed; in many cases, this means that the fractals are restricted to a narrow range of shapes that adhere to the generation formula. The looser fractal rules in this system allow for much more creativity.

II.3. Crystal-Like Shapes

The discussion of crystals and fractals brings up one of the main points of the thesis. What is it exactly that makes a shape "crystal-like?" While the term is fairly subjective and open to interpretation, there are some quantifiable properties of crystals that can be used to describe something that is "crystal-like."

Crystals often adhere to strict dihedral angles, due to their cleavage planes. In addition, the cleavage planes cause crystals to be naturally faceted. They also often form shapes exhibiting many planes and axes of symmetry. In fact, the lattice system of crystals, mentioned earlier, relies on these axes of symmetry to classify crystals.

In addition, as mentioned in the previous section, crystals naturally form fractal shapes due to their internal structure. These properties together form a list of some properties of a

"crystal-like" shape:

1 Faceted Faces/Dihedral Angles

2 Symmetry

3 Fractal Geometry

This list is by no means an official or standard crystal-like property list. It merely provides the inspiration for crystal-like modeling techniques. In other words, a modeling technique that can easily create shapes with faceted faces, specific dihedral angles, some symmetry, and some fractal geometry, can be said to make crystal-like shapes. This is the goal of this work, to provide such a modeling technique.

CHAPTER III

METHODOLOGY

III.1. Faceted Faces/Dihedral Angles

In current modeling tools, a commonly used tool is the extrusion. Starting from a cube, almost any type of polygonal model can be created by using many repeated extrusions. In fact, this technique is so simple that most CG artists know it as "cube modeling," and often use it due to its versatility. Most other polygonal modeling techniques make plenty of use of face extrusions.

However, extrusions are not without their limitations. One of the major limitations is that they are generally implemented as simple cubical extrusions. This requires a little explanation of exactly what a cubical extrusion is.

III.1.1. Traditional Cubical Extrusions

A cubical extrusion is so named because it is an operation that, when performed on a regular square face with the proper settings, can generate a cube from that face. Fig. 1 shows the extrusion in action.

On faces with higher vertex counts, the cubical extrusion will create a prism, with the shape of the face forming the sides of the extrusion. For example, an extruded hexagon will form a hexagonal prism.

The first limitation of having purely cubical extrusions is topology. When the extrusion is performed, the resulting face has each of its vertices connected to each of the original face's vertices. This means that the sides of the extrusion will always become quadrilaterals.
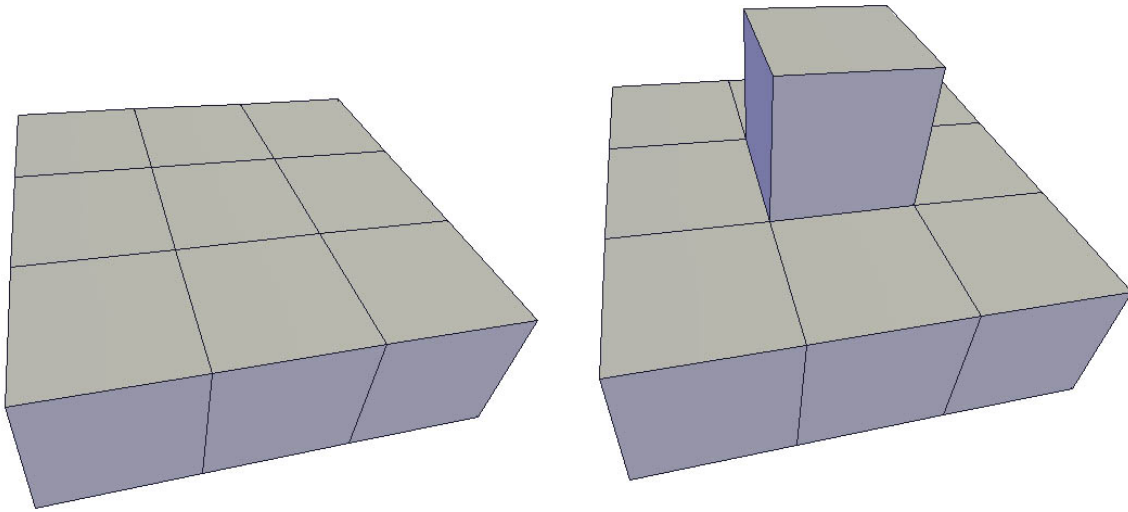
Fig. 1. Traditional cubical extrusion. The face is duplicated and connected to the original face vertices.

Quadrilaterals work very well for polygonal modeling. However, being limited to *only* quadrilaterals is obviously not a good thing; in addition, the quadrilaterals generated by extrusion are usually rather rectangular in shape, making even rhombi difficult to create.

With crystal-like shapes, these shortcomings of extrusions become painfully evident. Plenty of crystal-like shapes have non-quadrilateral faces and rhombic faces. A solution to this is to extend extrusions to other configurations beyond simple cubical extrusions.

III.1.2. *Extended Extrusions*

We began exploring several extrusions[11] beyond the typical cubical extrusions used today. The path we took was to look at other platonic solids beyond cubes, and see if extrusion could be applied to them in some way that is similar to cubical extrusion.

A brief recap of what the platonic solids are is in order first, though. Platonic solids, described by Plato in his *Timaeus*[16], are some of the simplest, if not the simplest, of
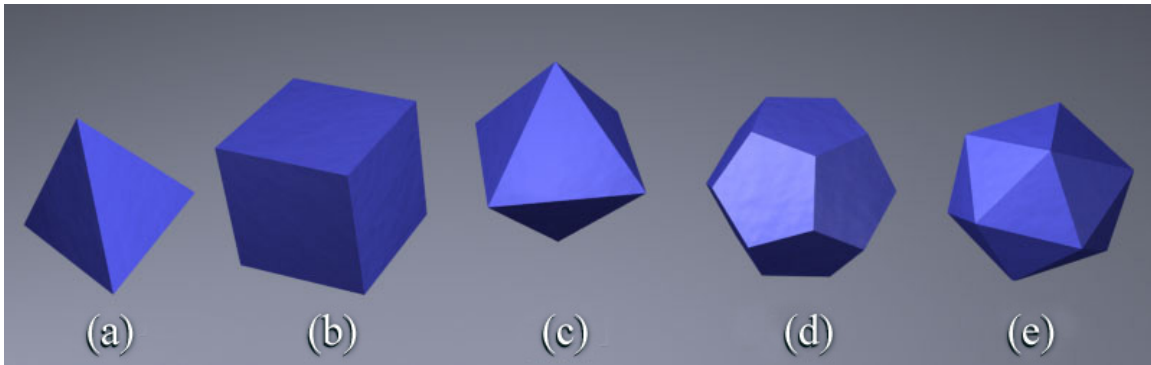
Fig. 2. The five Platonic Solids. (a) Tetrahedron. (b) Cube. (c) Octahedron. (d) Dodecahe-
dron. (e) Icosahedron.

polyhedra. A platonic solid is a convex polyhedron composed of convex, congruent, regular
polygons. There are five such solids, seen in Fig. 2.

1 Tetrahedron – A four-sided shape composed of four equilateral triangles.

2 Cube – A six-sided shape made of squares.

3 Octahedron – An eight-sided shape made of triangles.

4 Dodecahedron – A twelve-sided shape composed of pentagons.

5 Icosahedron – A twenty-sided shape made of twenty triangles.

Using these, and extrapolating from the cubical extrusion, a new set of extrusions
emerged. In the same way that a cube can be formed by a cubical extrusion on a square, a
tetrahedron can be formed from a tetrahedral extrusion on a triangle, a dodecahedron can
be formed from a dodecahedral extrusion on a pentagon, and so on. Of these extrusions,
cubical extrusion is one of the easiest to implement. Simply duplicate the extruding face,
connect each vertex to the corresponding old vertex while deleting the old face, and there
you have it – a cubical extrusion. The dodecahedral and icosahedral extrusions are much
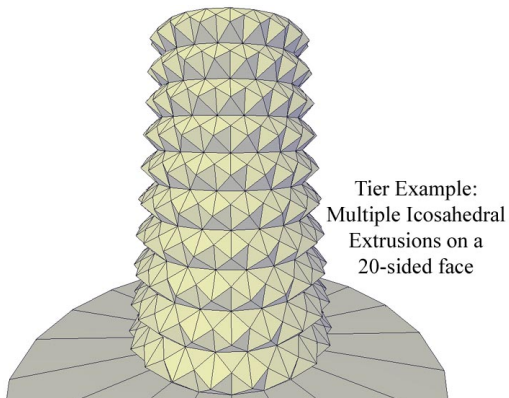
Fig. 3. Example of a tiered extrusion on a 20-sided polygon.

more complex, though, and require a new way to describe an extrusion's topology. Therefore, I use a general extrusion methodology by which many extrusions, including those of all the platonic solids, can be performed. It is a "tier" system, in which vertices are grouped into tiers, based on how close they are, topologically, to the extruding face.

In addition, this tier system also introduces a concept of an angular extrusion. Typical cubical extrusions merely offset the duplicated face according to the face's normal, as there are no vertices to deal with in-between, just edges. If the face needs to be resized, it can be rescaled and it will reshape the entire extrusion. With the tiered system, instead of scaling the resulting face, a dihedral angle is given by the user, as it is simpler than determining arbitrary or strange ratios for how the "middle" vertices of an extrusion should move.

III.1.3. The Tier System

The tier system works as follows. First, classify vertices by their connection to the original face and to each other. A vertex with an edge connecting to an original face vertex would be in tier one, for example. Vertices that connect to tier one vertices would be tier two, et cetera. In most extrusions done in my work, each tier has the same number of vertices as the
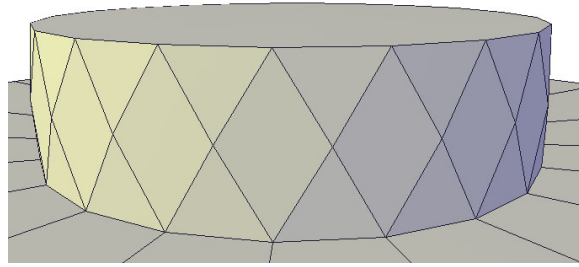
Fig. 4. Tiered extrusions exhibit a tiled, regular structure.

original face. There are some exceptions, like the small rhombicuboctahedron (described later). In addition, tiers can be connected to other tiers in a variety of ways. For example, in some extrusions, tier-one vertices connect to tier-three vertices.

An important aspect of the tier system is that the tier vertices are defined in relation to each original face vertex and the edges touching it; the vertices are independent of the overall shape of the face being extruded. This means that the tier system works for any polygon, regardless of the polygon's vertex count or shape. This can be seen in Fig. 3, which illustrates an icosahedral extrusion on a 20-sided polygon.

Another abstract way to describe this is that the tiers make a pattern that is tileable. Each new edge and vertex in the polygon causes another iteration of the "tile". Since it is causing a repeatable, topologically identical pattern to be repeated, it doesn't matter which vertex is the first face vertex. The vertices of the extrusion will always be connected in the same manner. This tiling can be clearly seen in Fig. 4.

Once these tiers are determined and connected, based on the topology of the particular extrusion, the extrusion needs to be shaped by moving the vertices around. This is done on an individual basis per extrusion, but in general there are some common ground rules:

- Dihedral Angle – Begin with a user-input dihedral angle. This angle specifies what

the angle between the original face and each adjacent face will be. Generally this is obtained by getting the cross product of an edge's vector representation with the face's normal, and using the resulting vector along with the face normal as an x-y basis system for finding the plane containing the adjacent face.

- Edge Length – Take the average of edge lengths of the original face, for use as a common edge length for the extrusion. This is automatically calculated.

- Tier Height – A ratio of the common edge length to be applied to a tier in the extrusion. The first tier height, for example, would be applied to the edge connecting the original vertices with the first tier.

Using these ground rules, extrusions can be constructed in a straightforward manner. The tiers and angles provide a tangible user control that can be interactively adjusted easily. These extrusions can also be applied to any type of polygon, as several figures illustrate.

III.2.   Platonic Solid Extrusions

III.2.1.   Tetrahedral Extrusion

The tetrahedral extrusion is the simplest of the Platonic extrusions. A tetrahedral extrusion, when applied to a regular triangle, can generate a regular tetrahedron shape. This extrusion is unique in that it only has one tier, and that tier only has one vertex. Fig. 5 illustrates this extrusion.

To create the topology of the tetrahedral extrusion, first a new vertex is created. Each original vertex of the extruding face then connects an edge to this new vertex. The dihedral angle is used to find a plane intersecting each edge, then the vertex is moved along the plane from the midpoint of the original edge.
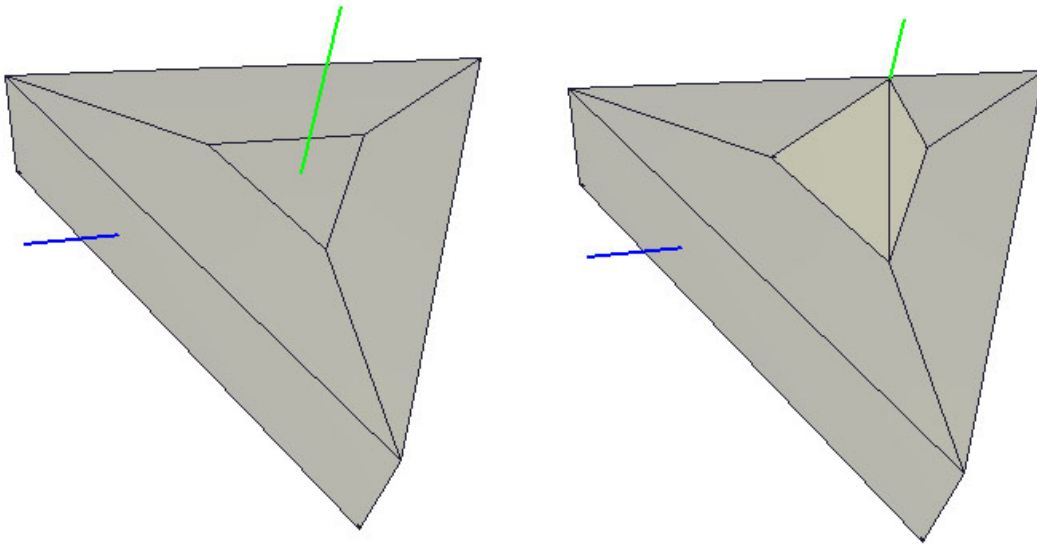
Fig. 5. Tetrahedral Extrusion. A regular triangle can extrude into a regular tetrahedron.

The extrusion is so simple that it can cause some problems with the dihedral angles, as only certain polygons can generate a tetrahedral extrusion which has identical dihedral angles at every edge. Therefore, after the position of the first tier vertex for each edge is calculated using the dihedral angles and tier lengths, all the positions are averaged and the centroid is the final point for the single vertex.

III.2.2. *Cubical Extrusion*

The cubical extrusion, when applied to a square, can generate a cube. Although the angle setting is different, it is in most other aspects identical to extrusions performed by traditional modeling tools. Fig. 6 shows this extrusion being applied to a square.

The topology of the cubical extrusion is fairly simple. The original face is duplicated, and each original vertex is connected to its duplicate on the new face.
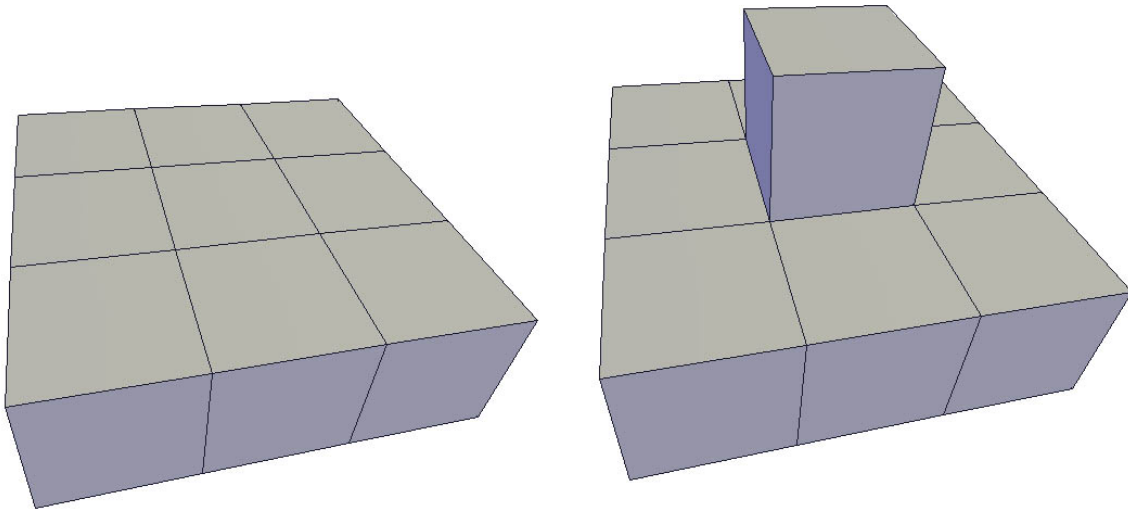
Fig. 6. Cubical Extrusion. A regular square can extrude into a regular cube.

For movement of the vertices, every vertex on the new face is considered to be in the first tier. Instead of only finding the plane intersecting one edge, the planes intersecting both edges adjacent to each original vertex are found. The first tier vertex is then moved along the intersection of these two planes.

III.2.3. Octahedral Extrusion

An octahedral extrusion, when applied to a regular triangle, can generate a regular octahedron. It is similar to the cubical extrusion, with only one major difference.

In the topology of the octahedral extrusion, first a cubical extrusion shape is generated. In a second step, a new edge is placed between an original face vertex and the next vertex on the first tier. This will generate triangles instead of the quadrilaterals formed by the cubical extrusion. An example of this extrusion can be seen in Fig. 7.

For actually moving the vertices, it's important to note that each vertex corresponds to
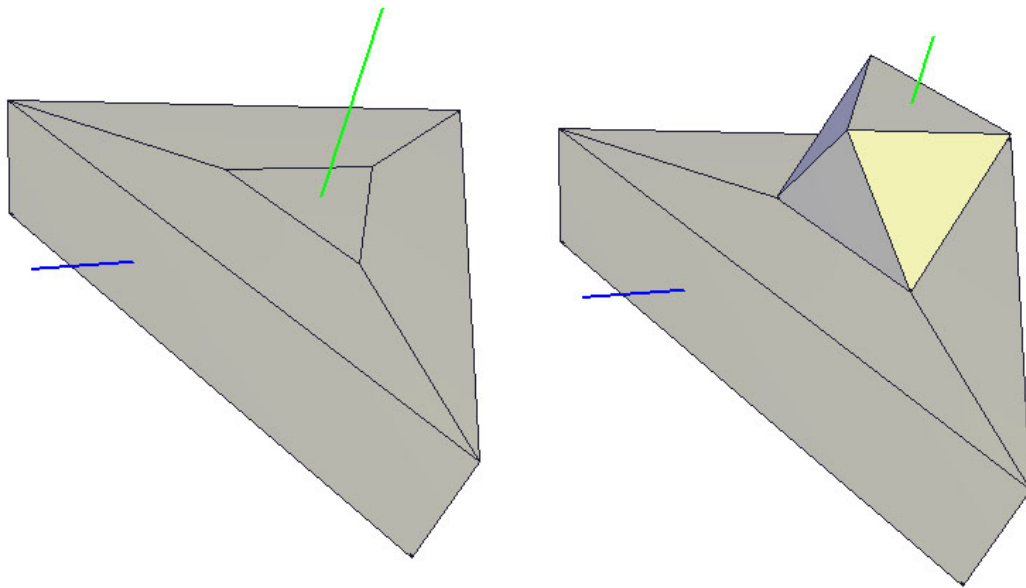
Fig. 7. Octahedral Extrusion. A regular triangle can extrude into a regular octahedron.
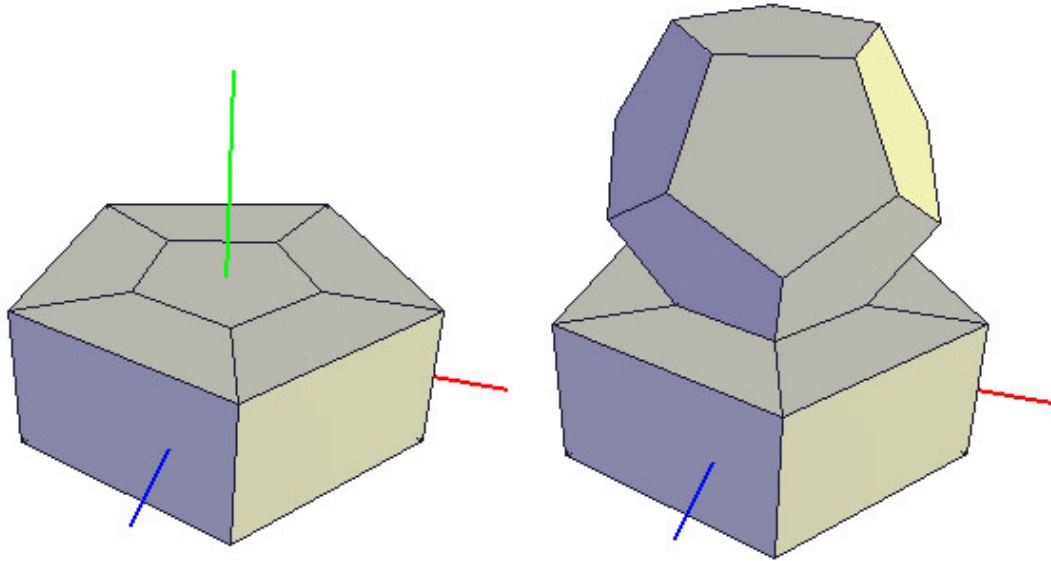
Fig. 8. Dodecahedral Extrusion. A regular pentagon can extrude into a regular dodecahe-
  dron.

the midpoint of an original edge that connects the two vertices it is connected to. The plane

is found which intersects that edge, and the vertex is moved along that plane, starting at the

midpoint of the edge.

III.2.4. *Dodecahedral Extrusion*

The dodecahedral extrusion, when applied to a regular pentagon, can generate a regular

dodecahedron. Fig. 8 shows this extrusion in action. This is where the extrusion process

gets complicated, and the tier system really becomes useful. In fact, the dodecahedral

extrusion was the primary motivation for developing the tier system.

Viewing one face as the face to be extruded, and its opposite face as the extruded, or
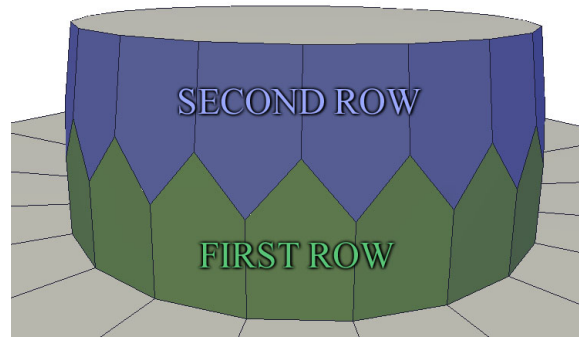
Fig. 9. Pentagonal Rows. The dodecahedral extrusion creates two rows of pentagons that are interleaved with each other.

duplicate face, I observed that there were two rows of pentagons connecting them (Fig. 9) . This observation led to the generalized description of a dodecahedral extrusion.

First, the bottom of the extrusion is the source face. Connected to that face is a row of pentagons, each of which shares a bottom edge with the face. A second row of pentagons is staggered on top of these, each of these sharing an edge with the "top" face.

This led to the tier classification of the vertices. After the original vertices, the first tier vertices are the middle vertices of the first pentagon row, and the bottom vertices of the second row. The second tier vertices are the tips of the first pentagon row, and the middle vertices of the second pentagon row. The third tier vertices make up the extruded or duplicate face at the top. Fig. 10 should help explain the tiers better:

For the topology, the original vertices each connect an edge to the first tier vertices. Each first tier vertex connects to the two closest second tier vertices. Each second tier vertex, which is already connected to two first tiers, connects one edge to a final third tier vertex. The third tier vertices connect together into a ring, forming the top face of the extrusion.

For positioning the vertices, the extrusion begins similarly to the cubical extrusion.
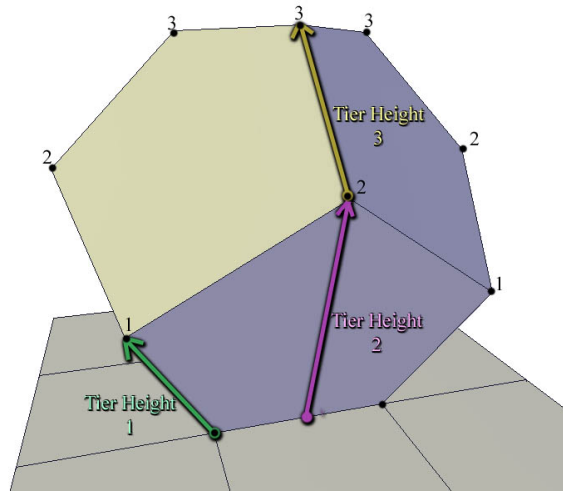
Fig. 10. Tier Illustration. Vertices are numbered by their tier. Arrows denote tier heights, and how vertices are moved outward from the original face.

Planes are determined which pass through each original edge, and the intersection of the planes at vertices are used to determine the vectors the first tier vertices move along. The second tier vertices move similarly to the octahedral extrusion; they move out from the center of each original edge along the plane that intersects that edge.

The third tier is a bit tricky. To keep the faces all planar, the vertices in the first, second, and third tiers need to be coplanar with each other. This is solved by finding the planes containing second and first tier vertices. For the second tier vertex attached to the third tier vertex, find the triangle formed by the next second tier vertex with the next first tier vertex, and the previous second tier vertex with the previous first tier vertex. When these "next" and "previous" planes are found, the third tier vertex will lie along the line formed by the intersection of the two planes.

The dodecahedral extrusion is important because it's the first of these that deals with higher vertex-count faces. Most polygonal modeling can handle triangles and quadrilaterals
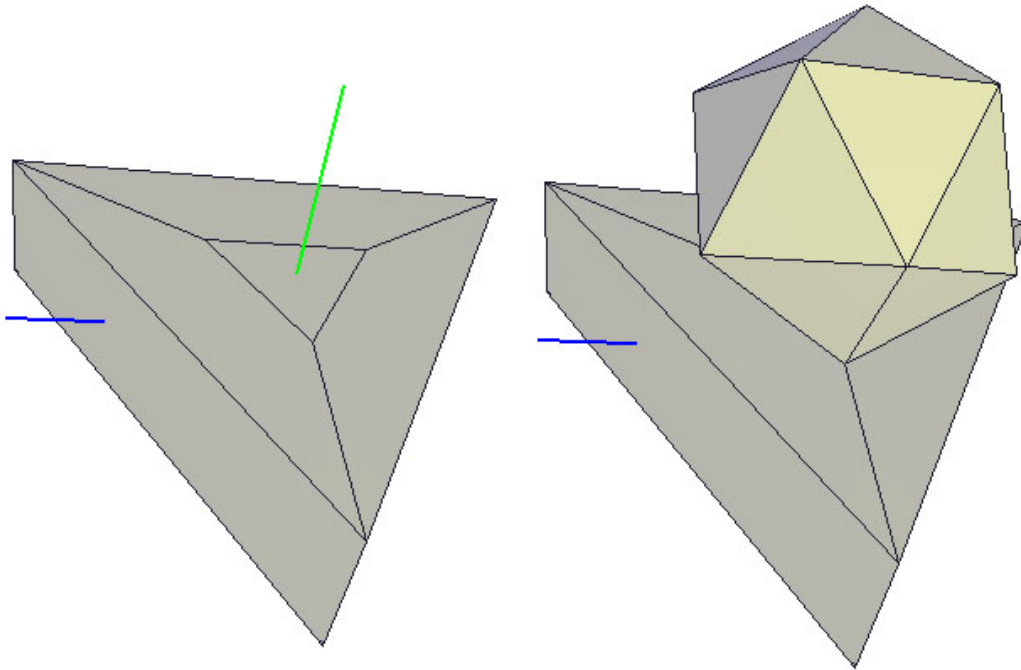
Fig. 11. Icosahedral Extrusion. A regular triangle can extrude into a regular icosahedron.

fairly easily. Handling planar pentagons is more difficult, and this extrusion helps what would otherwise require painstaking geometry tweaking.

The strengths of using a tier system also show well in a dodecahedral extrusion. It is one thing to create a cylindrical shape out of planar rectangles; it is quite another to create a cylindrical shape from planar pentagons. Also, notice how the nature of the tiers causes a repeating, tiled pattern that I mentioned earlier. This repeating pattern could possibly be used in the future for geometric handles, like that presented by Akleman and Srinivasan[3], to connect two similar faces together.

III.2.5.   Icosahedral Extrusion

The icosahedral extrusion, when applied to a regular triangle, can generate a regular icosahedron. In some ways, it is simpler than the dodecahedral extrusion, as it is composed of triangles, and therefore the faces are automatically planar. The icosahedral extrusion is illustrated in Fig. 11.

Like the dodecahedral extrusion, the icosahedral extrusion is three-tiered. The first tier vertices connect to two original vertices, two second tier vertices, and one third tier vertex. The second tier vertices are each connected to one corresponding original vertex, the two closest first tier vertices, and the two closest third tier vertices. Each third tier vertex connects to one first tier vertex, and two second tier vertices.

The positioning of the vertices begins similar to the octahedral extrusion. The planes intersecting each original edge are found, and the first tier vertices are moved out from the midpoint of the original edge. The second tier vertices are much more tricky. First, the dihedral angle is used to find a plane intersecting the edge from the first tier vertex to the original vertex. These planes are found on both sides of the second tier vertex, and it lies along their intersection. The third tier edges are found a similar way, with the dihedral angle plane being found that intersects the edge between a second tier vertex and first tier vertex, then moving the third tier vertex along the intersection line that passes through the first tier vertex.

III.3.   Archimedean Solid Extrusions

Archimedean solids are another fundamental group of basic solids. Archimedean solids are shapes that consist of nonintersecting, regular, convex polygons of two or more different types. In addition, the faces are arranged the same way around each vertex, with all edge lengths identical.
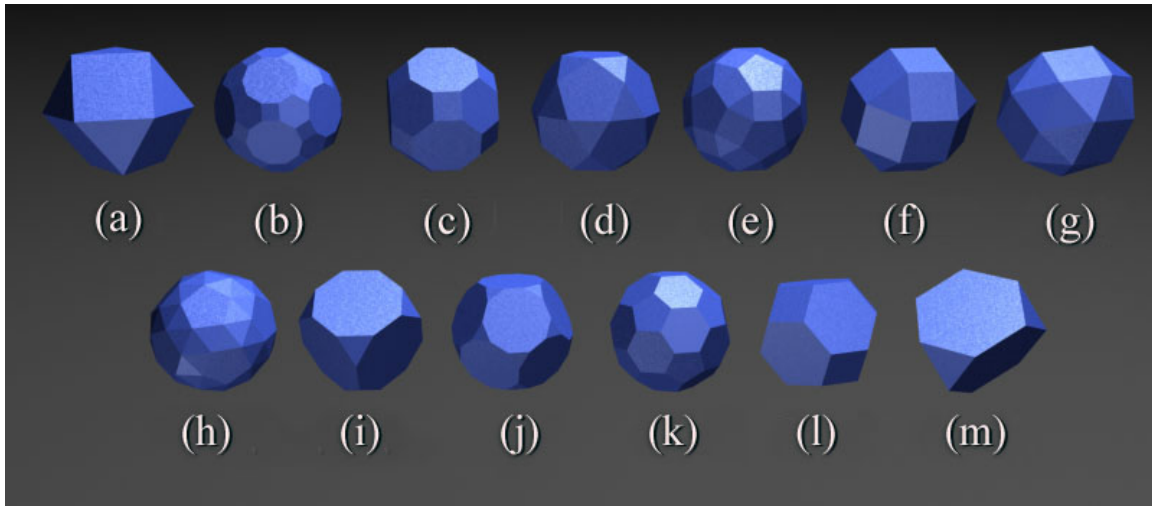
Fig. 12. The Archimedean Solids. (a) Cuboctahedron. (b) Great Rhombicosidodecahedron. (c) Great Rhombicuboctahedron. (d) Icosidodecahedron. (e) Small Rhombicosido-decahedron. (f) Small Rhombicuboctahedron. (g) Snub Cube. (h) Snub Dodecahe-dron. (i) Truncated Cube. (j) Truncated Dodecahedron. (k) Truncated Icosahedron. (l) Truncated Octahedron. (m) Truncated Tetrahedron.

There are 13 Archimedean Solids, seen in Fig. 12:

1 Cuboctahedron – 14 sides. Consists of squares and triangles.

2 Great Rhombicosidodecahedron – 62 sides. Consists of dodecagons, hexagons, and squares. Resembles a truncated icosidodecahedron, as described by Kepler[9].

3 Great Rhombicuboctahedron – 26 sides. Octagons, hexagons, and squares. Looks like a truncated cuboctahedron, again as described by Kepler[9].

4 Icosidodecahedron – 32 sides. Pentagons and triangles.

5 Small Rhombicosidodecahedron – 62 sides. Pentagons, triangles, and squares. This shape can be generated by performing a Doo-Sabin remeshing[6] on a dodecahedron.

6 Small Rhombicuboctahedron – 26 sides. Triangles and squares. Can be generated by performing a Doo-Sabin remeshing on a cube.

7 Snub Cube – 38 sides. Made of squares and triangles. The triangles form a triangle strip that separates the square faces.

8 Snub Dodecahedron – 92 sides. Made of pentagons in triangles, and works similar to the snub cube.

9 Truncated Cube – 14 sides. Made of octagons and triangles. Can be created by cutting off, or truncating, the vertices of a cube.

10 Truncated Dodecahedron – 32 sides. Made of decagons and triangles.

11 Truncated Icosahedron – 32 sides. Made of hexagons and pentagons

12 Truncated Octahedron – 14 sides. Made of hexagons and squares.

13 Truncated Tetrahedron – 8 sides. Made of hexagons and triangles.

Archimedean solids are the next step in complexity beyond Platonic solids. Both groups of solids are well known, fundamental solids[20]. In geometric modeling tools, they are often known as "primitives," and are the building blocks by which many polygonal models are begun.

Beyond being more complex than Platonic solids, Archimedean solids also contain patterns that are difficult to create by using Platonic solid extrusions. Although repeated Platonic extrusions can create more complex shapes, the cuboctahedron, as an example, cannot be created using Platonic extrusions. Changing any Platonic extrusion into a cuboctahedral shape, or really any Archimedean shape, requires a great deal of edge subdivision and vertex manipulation. Therefore, having Archimedean extrusions in addition to Platonic extrusions helps simplify the modeling process.

Two of the Archimedean extrusions were developed, as proof that further shapes beyond the Platonic solids can be done. Although it will be reiterated in the conclusion, an important discovery is that the tier system has some limitations, including that it can't handle the truncated Archimedean solids well.

The truncation limitation comes from the fact that some of the faces have vertex pairs instead of isolated, repeatable vertices. A way to visualize it is this: picture an infinite sided polygon being extruded. At each vertex of this polygon, a group of multiple new vertices is built, unique to that original vertex. The group at each successive vertex of the original polygon is identical in topology to the previous group.

This was one intention of the tier system. Providing a topologic description that repeats at every vertex means that it does not matter how many vertices a face has or which vertex the extrusion begins to build its geometry at; since every vertex's tier is equivalent to every other vertex and tier, this creates an order independent extrusion.

The truncated cube unfortunately breaks this paradigm. If the base face of the extrusion is chosen to be an octagon on the truncated cube, then the face contains topologically equivalent vertex *pairs*, instead of single vertices. Fig. 13 circles the pairs for a visual explanation:

This limitation meant that some extrusions did not work cleanly enough to develop currently. They may be explored in future work, as will be explained in the conclusion.

Regardless of this problem, two of the many Archimedean extrusions without this problem are presented next, the Cuboctahedral and Small Rhombicuboctahedral extrusions.

III.3.1. Cuboctahedral Extrusion

The cuboctahedral extrusion, when applied to a square, can generate a regular cuboctahedron. It is similar to the octahedral extrusion for the first tier. Fig. 14 shows a cuboctahedral
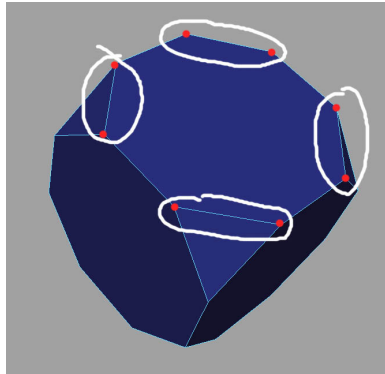
Fig. 13. The "paired vertices" problem in the truncated cube. The tier system requires all vertices of the extrusion (not the original face) to be topologically similar. In this case, the pairs are similar to other pairs, but individual vertices are not.
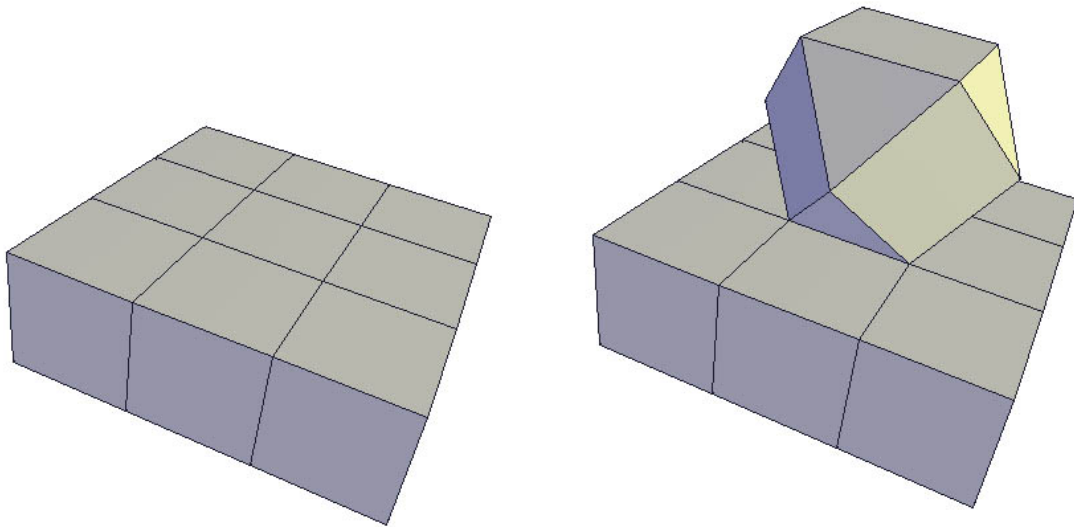
Fig. 14. Cuboctahedral Extrusion. A regular square can extrude into a regular cuboctahedron.
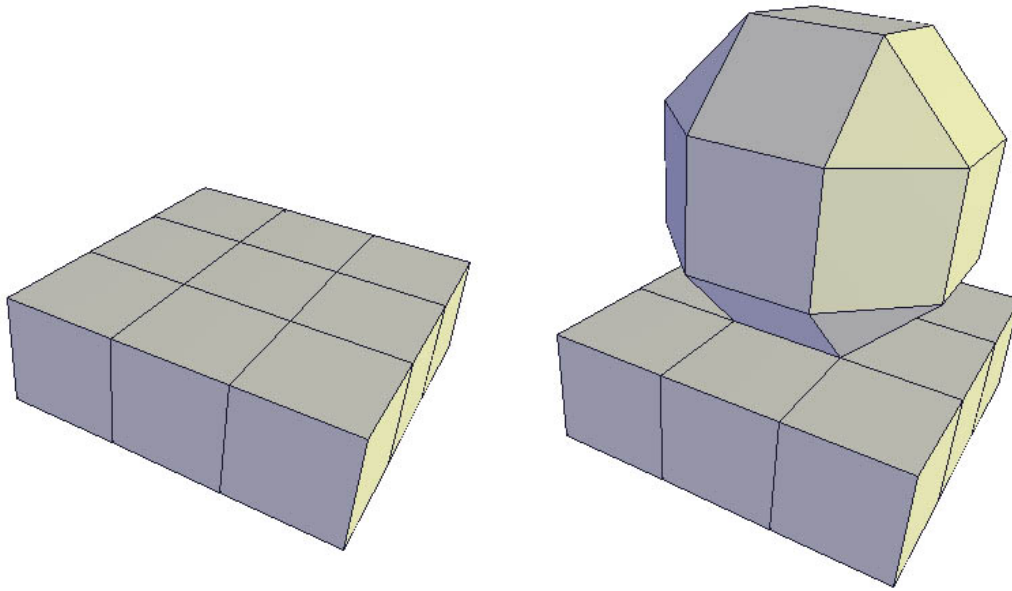
Fig. 15. Small Rhombicuboctahedral Extrusion. A regular square can extrude into a regular small rhombicuboctahedron.

extrusion being applied to a square.

The cuboctahedral extrusion has two tiers. The first tier vertices each connect to the nearest two original vertices, and the nearest two second-tier vertices. It can be thought of as two octahedral extrusions stacked on top of each other, with some edges removed.

Positioning of the vertices is handled in the following way. The first tier vertices are moved out along the dihedral planes, identical to how the octahedral extrusion performs. The second tier vertices depend on a plane formed by its corresponding original vertex, and two adjacent first-tier vertices. By moving the second tier vertex along this plane, the planarity of the resulting quadrilateral is preserved.

III.3.2. Small Rhombicuboctahedral Extrusion

A small rhombicuboctahedral extrusion, in addition to being difficult to pronounce, can create a regular small rhombicuboctahedron when performed on a square. An alternate observation is that the small rhombicuboctahedron looks like a beveled box. The small rhombicuboctahedral extrusion is illustrated in Fig. 15.

The tiers on a small rhombicuboctahedral extrusion are more complicated than in previous extrusions. The first and second tiers consist of pairs of vertices, rather than singular vertices. If the face being extruded is a pentagon, for example, the third tier would contain five vertices, and the first and second tiers would contain ten vertices each.

Each first tier vertex connects to the previous and next first tier vertex, an original face vertex, and a second tier vertex. Each second tier vertex connects to a first tier vertex, a third tier vertex, and a previous and following second tier vertex.

Movement of these vertices is tricky, as each original vertex is connected to a pair of first tier vertices. Basically, the dihedral plane is first determined, and a vertex from each tier one pair is moved along the plane intersecting the original edge. Put another way, each original face edge has two corresponding vertices that move out from its endpoints. This process is repeated, next with the second tier vertices moving along a dihedral plane passing through the first tier vertices, and the third tier vertices moving on a dihedral plane passing through the second tier vertices.

Fig. 16 illustrates the tiers more clearly, with each pair of first and second tier vertices being classified into right/left vertices.

III.3.3. Extrusions Summarized

Although there are certainly more extrusions that could be developed, these extrusions provide modeling potential beyond that of the old cubical extrusion method. Presumably,
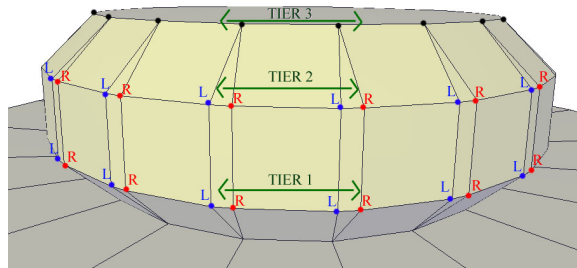
Fig. 16. Tier pairs in the small rhombicuboctahedral extrusion. Each pair has a left(L) and right(R) vertex. For a n-vertex polygon being extruded, there will be n left and n right vertices in each tier. This results in each tier having 2n vertices.

many other solid classifications beyond Platonic and Archimedean solids are possible to use as extrusions. These extrusions just provide a simple and robust method for generating faceted shapes.

III.4. Symmetry

Faceted, angular faces of crystal-like shapes provided the inspiration for the extended extrusions just mentioned. In a similar way, crystal-like shapes often exhibit interesting and beautiful symmetry. This provided an impetus for finding a geometric modeling technique which provided more symmetry in shapes than traditional tools.

While considering exactly how to do this, I remembered an old geometric modeling method to create a symmetric object. The trick involves performing successive extrusions and transformations to multiple, separate faces simultaneously. Since the same operation is performed on two different faces, in a way the geometry added at once face is "duplicated" at the other face. For example in Fig. 17,selecting four sides of a cube, then extruding those sides will create a radially symmetric, four-sided star.
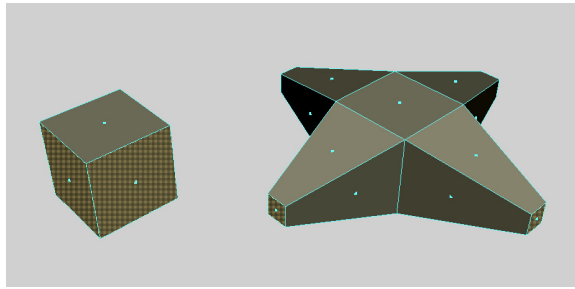
Fig. 17. Extruding four sides of a cube will create a four-way symmetric star shape.

III.5. Face Grouping

Building upon the concept of multiple face extrusions, the idea of face groups was developed. When an extrusion is performed on a face, the extrusion is also performed on every other face that is a part of the face group.

When the extrusion is performed, new face groups are created for the resulting faces. Since the extrusions are supposed to be similar, an incrementing process is used, with each face group's number id.

The process works as follows: when a face group is created, it has a group id number. This number is unique to each group, and begins at 1. When a face is extruded, new face groups are added to the list, in ascending order. So if a cubical extrusion adds five new faces, five new face groups will be created. If previously the largest group id in the face group list was id 10, the new groups will be created with id's of 11, 12, 13, 14, and 15. The first of the new groups is assigned to the "top" face of the extrusion, with the rest assigned to faces in order, beginning at the start vertex. Although the ordering of the faces varies based on the type of extrusion, the order will always be similar for same-vertex-count faces with the same starting vertices.

This schema can also be applied to remeshing operations. Because the remeshing is
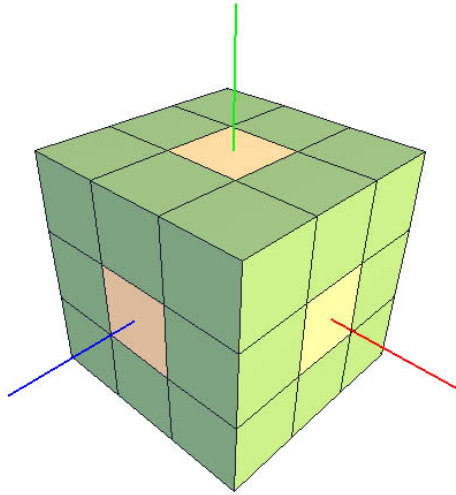
Fig. 18. Checkerboard remeshing of a cube, with face groups being set automatically. Center faces belong to group 1, and all other faces belong to group 2.

global, the remeshing can use absolute values for groups. For example, the center faces in a checkerboard remeshing can be set to group id 1, and the remaining faces will be set to group 2. An example of this can be seen in Fig. 18, where a cube has been remeshed by the checkerboard method.

Once the extrusion's faces are given groups, in this case groups 11-15, the other faces sharing the original face's group are also extruded. Each of them is assigned the same groups. This keeps a relationship between the extrusions – in a way, preserving their history; since they were extruded from faces sharing a face group, the extruded faces should also share face groups.

The exact explanation of why an operation like this can generate symmetric faces is a bit convoluted. Hestenes presents a detailed system of 32 lattice groups and 230 space groups to describe three-dimensional symmetry in his work[8]. Finding the relation between these grouped extrusions and Hestenes's work could provide inspiration for future
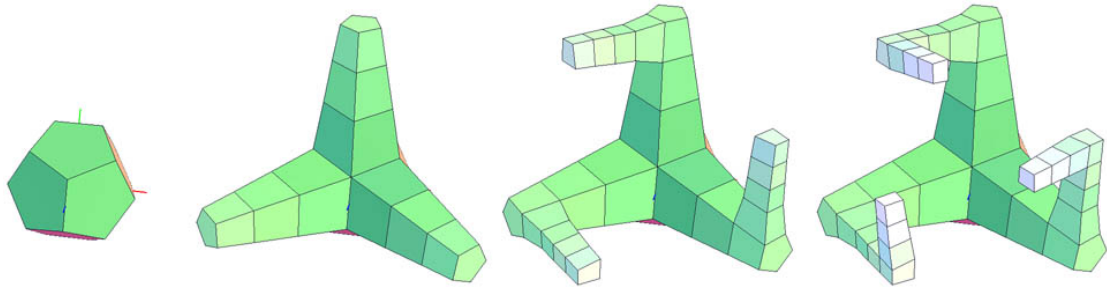
Fig. 19. Example of face grouping causing symmetry. Multiple repeated extrusions create a shape that exhibits radial symmetry.

research. What is important, though, is that the face group extrusion does create some amount of symmetry in an object. Fig. 19 shows this symmetry in action.

In Fig. 19, the faces share a face group, as well as a common vertex; the vertex is "first" for each of the faces, giving each face a similar orientation. When extrusions are performed, the resulting shapes become radially symmetric about that shared vertex. What is happening is that the nature of symmetry is being taken advantage of by positioning identical faces in a ring around the vertex. A tasty analogy is taking a single slice of pie, and repeating it around a central point until you have a whole, radially symmetric pie.

This symmetry isn't restricted to faces with a common vertex; for example, two triangles sharing an edge can also exhibit this symmetry. An important observation is that these are localized symmetry operations – wherever a group of faces share a vertex, this type of symmetry can be obtained. This is beyond the capabilities of traditional geometric modelers, which only mirror an entire object or duplicate it about an axis. This face grouping allows an object to exhibit symmetry across several different groups of extrusions simultaneously.

CHAPTER IV

USER INTERFACE

This work was implemented in Topmod^{Tm}, a geometric modeler developed by Ergun Akleman, Vinod Srinivasan, and a few other people. Topmod^{Tm} uses doubly linked face lists [1], allowing for robust and simple modification of vertices, edges, and faces.

The Topmod^{Tm} implementation is a proof of concept that shows the extrusions and face groupings work well in an actual geometric modeling system. Although this concept could conceivably be implemented in any robust modeling system, doing so in Topmod^{Tm} was facilitated due to my having full access to the source code for the program.

Topmod^{Tm} uses FLTK, the Fast Light ToolKit, for its interface. FLTK provides not only OpenGL windowing support for visualizing 3d objects, but also provides rollers, sliders, and other graphical interface items to make the program easy to control. A visual example of the interface can be seen in Fig. 20.

Objects are viewed and manipulated in the main perspective window. Modeling operations are selected from a menu to the left of the window. Although Topmod^{Tm} implements many geometric modeling techniques, I will focus on the ones relevant to this paper.

IV.1. Extrusions

Extrusions are selected through the menu on the left side of the Topmod^{Tm} window, seen in Fig. 21. When an extrusion is selected, slider bars appear for adjusting the angle of the elevation, its tier lengths, and the number of iterations of the extrusion.

By iterated extrusions, this means that an extrusion may be performed several times in a row. The successive extrusions are performed on the previous extrusion's "top face." In the case that an extrusion does not have an obvious top face, for example the tetrahedral
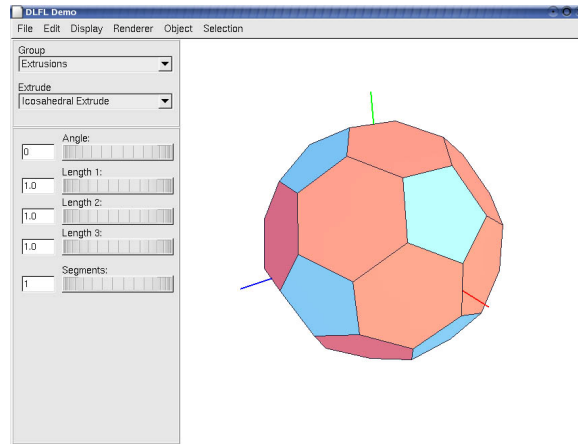
Fig. 20. A screenshot of the Topmod^{Tm} interface. The menu system is located on the left and top, and the main interaction viewport is on the right.
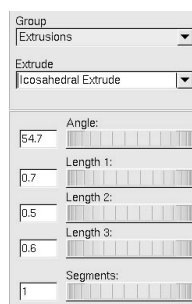

Fig. 21. The extrusion menu, with controls to adjust tier heights and the extrusion angle, as well as choose which extrusion to use.
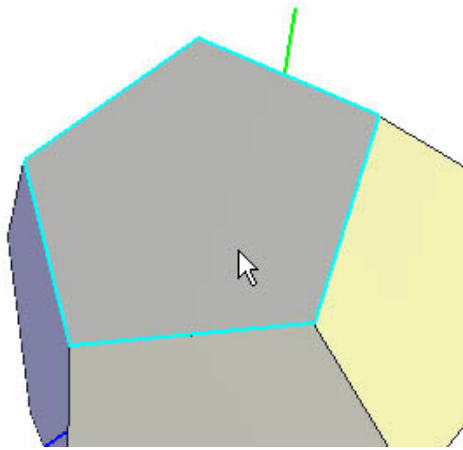
Fig. 22. Selecting a face highlights that face with a blue outline.

extrusion, this control will not be used.

These settings provide extrusion abilities beyond most other modelers. Most geometric modeling tools provide settings for the extrusion distance, rotation, and subdivision of the extrusion. These controls work fairly well for a cubical extrusion, which can be segmented by slicing the extrusion with planes. For more complicated extrusions, such as the dodecahedral extrusion, slicing the extrusion would have an unpredictable effect. Therefore, creation of a segmented extrusion is done by repeating the extrusion automatically.

The tier length and angle settings also replace the distance/rotation/scaling settings, as these are automatically handled due to the way the tier system works. For example, increasing the dihedral angle can have the same effect that scaling would in traditional extrusions.

The extrusions are performed simply by clicking on a face. The model instantly updates with the new geometry. Holding down the mouse button will highlight a face, seen in Fig. 22; letting go of the button initiates the extrusion. This helps in selecting faces that may be small or at odd angles, which would otherwise make them difficult to select.
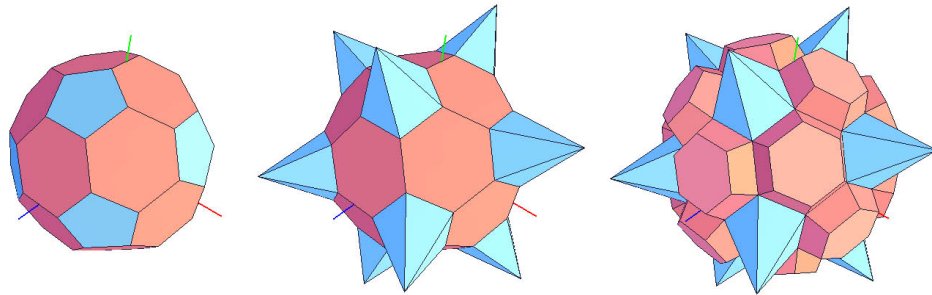
Fig. 23. Colors correspond to face groups. Performing a group extrude will extrude all other faces sharing the same group/color.

IV.2. Remeshing

Remeshings are selected through the same menu system as extrusions. They are global operations, so instead of clicking on the object itself, there is a button that performs the remeshing when clicked.

IV.3. Face Groups

Face groups are assigned like materials to individual polygons. They are randomly assigned colors, giving the user a visual indicator of which groups are which – faces that share the same color also share the same group id. Fig. 23 illustrates this; the blue faces are grouped with each other, and the red faces are grouped.

IV.4. Importing/Exporting Models

Topmod^{Tm} uses FLTK's file loader/saver, which implements a standard GUI for browsing directories to load or save files. Files can be saved to an OBJ format, which is read by any standard 3d modeling software. This makes it easy to export a model for rendering or for

performing operations that Topmod^{Tm} wasn't designed for.

Of note is that when Topmod^{Tm} loads a colored OBJ file, it automatically finds common colors and assigns face groups to these colors. This means that a model can be "prepared" for group extrusions by coloring its faces in an outside program, or in a text editor. The proprietary dlfl file format also saves group data, so that saving and reloading a file doesn't lose the face groups.

Topmod^{Tm} also saves old copies of models in memory, for user-friendly undo/redo operations.

CHAPTER V

RESULTS

This section covers the results of the work. The crystal-like modeling techniques proved a versatile tool, as the figures illustrate.

In Fig. 24, the cuboctahedral (A_1) and small rhombicuboctahedral (A_2) extrusions were performed on a simple rectangular solid. These images show the profound and interesting effect that changing the extrusion angle has on the resulting iterations. The extrusion iteration was set above "1" to exaggerate the effect of the extrusion.

Note that each of the three iterations in each image required one mouse click on a face. Modeling these shapes without these techniques would require much more work, such as multiple extrusions, edge removals and subdivisions, and moving of faces to try to form planar facets. Certainly, a single mouse click is much easier and faster.

Fig. 25 shows the interesting effects that can result from changing the tier lengths on a small rhombicuboctahedral extrusion. These shapes especially evoke a nice architectural theme that is commonly seen in tall buildings; almost all the angles in these shapes are 45 degrees, due to the extrusion angle setting.

Fig. 26 shows that a technique of extruding using face groups, combined with model remeshing, can easily create a fractal shape. These shapes were each created with ten or fewer operations. In each, a repeated pattern of remesh/extrude/remesh/extrude created the recursive shape.

Shapes (a) and (b) in Fig. 26 are particularly interesting because they resemble two well-known fractals, the Menger Sponge and the Sierpinski Tetrahedron. Although there are some minor differences in the structure, the visual similarity is striking.

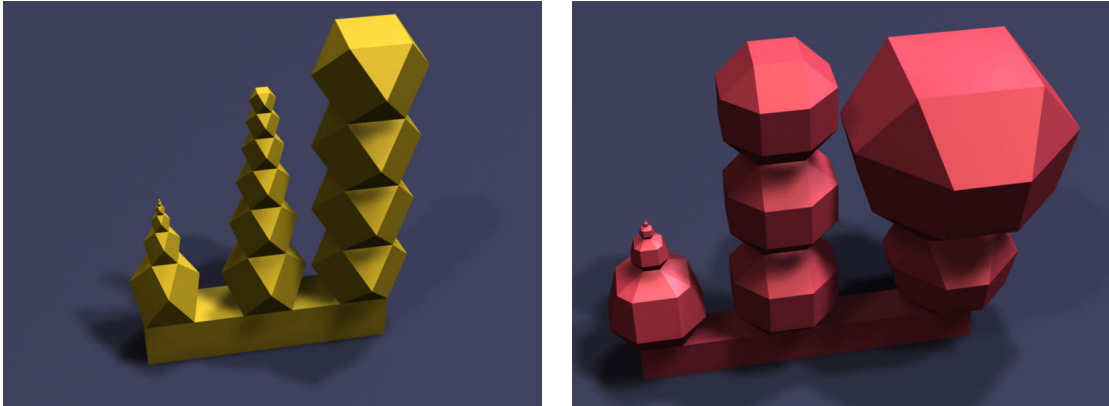In addition, most fractals are procedurally generated according to a strict set of rules.

A_1 A_2

Fig. 24. Cuboctahedral (A_1) and Small Rhombicuboctahedral (A_2) extrusions. Changing only the angle of the extrusion creates varying shapes.
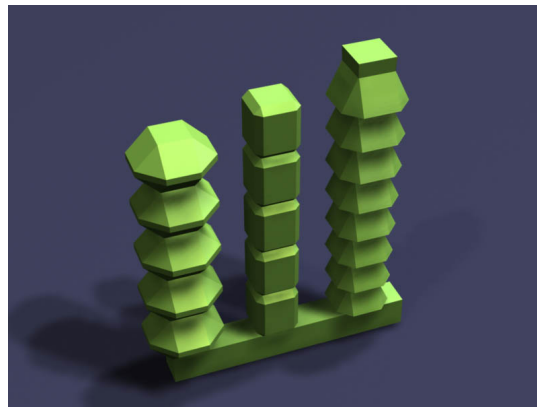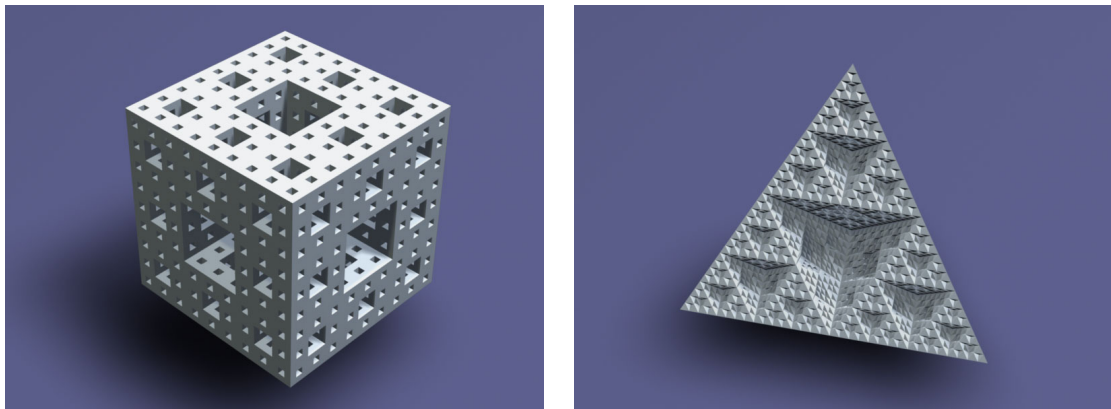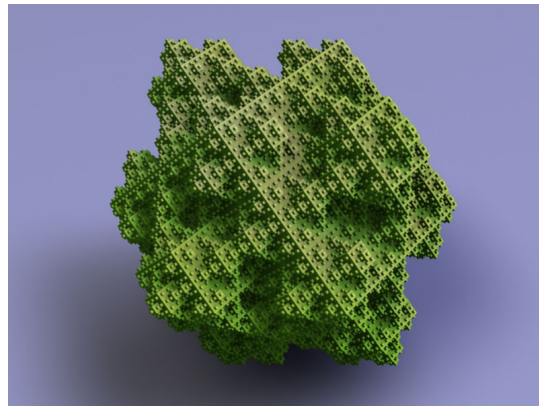

Fig. 25. Changing the tier lengths on a small rhombicuboctahedral extrusion creates a variety of shapes.

(a)

(b)

(c)

Fig. 26. (a) Pseudo-Menger Sponge, (b) Pseudo-Sierpinski Tetrahedron, (c) cube fractal. These were created by alternating remeshing with face regrouping, and grouped extrusions.
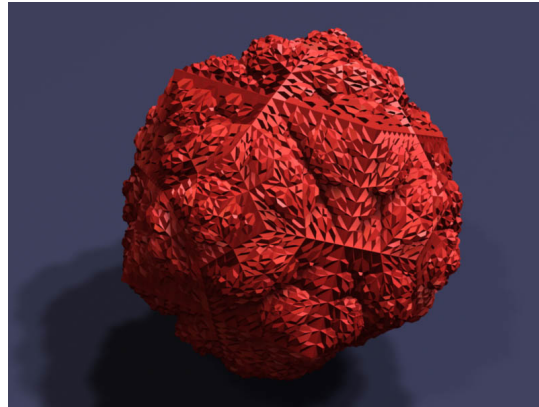
Fig. 27. Interesting underlying symmetry is exhibited after multiple grouped extrusions are performed.

Since the process presented here is interactive, the user can change the rules on the fly as he sees fit. This turns the fractal from a static shape that forms completely on its own, to a malleable shape that can be subtly or drastically altered in form.

Furthermore, although these shapes are created each using one remeshing and extrusion type, the extrusions and remeshings can be interchanged, forming hybrid shapes that would never be possible with traditional fractal generation techniques.

Fig. 27 shows some beautiful symmetry exhibited as a result of extruding face groups. The resulting shape is convoluted and chaotic, yet has an underlying symmetry throughout the shape. The shape is chaotic and ordered at the same time.

Further images are shown in Fig. 28, illustrating how the modeling techniques can produce interesting, crystal-like shapes.

V.1. Analysis

The creation of the aforemention shapes illustrates one of the most important points of this thesis. Although traditional modeling techniques could technically be used to create
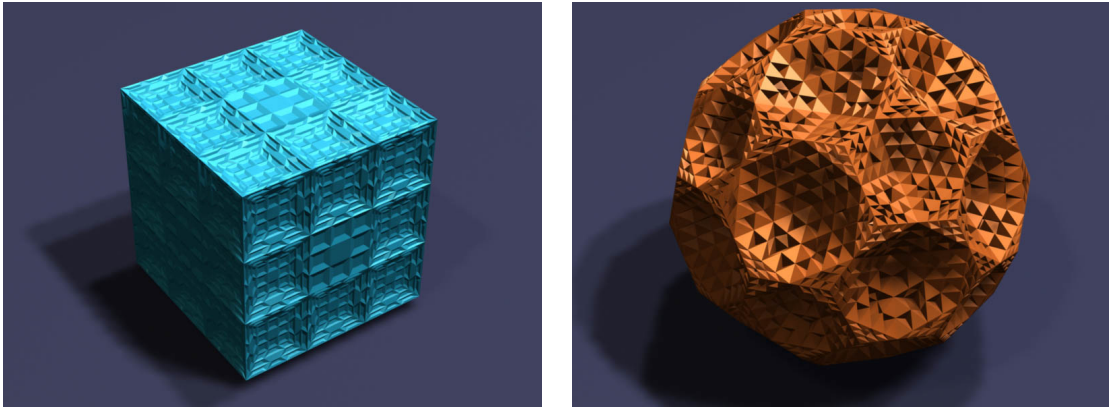
Fig. 28. More unusual shapes can be easily and quickly created.

these shapes, doing so would be incredibly time consuming. Many of the shapes would take such a tedious, time consuming process to model. In contrast, the presented crystal-like modeling methods produced these geometric shapes easily and quickly. None of the models in these images required more than ten operations in total.

A good example of this is the pseudo-Sierpinski tetrahedron, (b) in Fig. 26. This shape is rife with extruded triangles. To create such a shape in a traditional modeler, the user would first have to subdivide each edge by hand, create new edges, and extrude the center faces. The number of operations increases geometrically for each successive iteration of the process; by the fourth or fifth iteration, the user would be dealing with hundreds or even thousands of triangles.

Contrast this with the crystal-like geometric modeling techniques, which only require two button presses for each iteration, regardless of complexity. Since the technique is the hybrid of a procedural modeling technique with an explicit modeling technique, it gives the user ease of use while eliminating the need to click thousands of faces.

Finally, these images show that the thesis was successful in creating shapes that are aesthetically crystalline. Furthermore, the techniques developed to this end have proved

easy and simple to use, leaving the user to spend more effort on creativity, and less time fighting with awkward modeling tools.

CHAPTER VI

CONCLUSION

In this thesis, I set out to provide interactive geometric modeling techniques for creating crystal-like shapes. In this, the thesis was fairly successful. The extrusions, in conjunction with remeshing and face groups, can be used to easily create shapes that exhibit facets, symmetry, and have a fractal nature.

Using these techniques, I developed a proof of concept program that allow a user to interactively model an object. The implementation illustrates, literally, how the methods can create objects that would not be easily modeled by traditional modeling tools. This work should aid in the search for interesting, beautiful, crystal-like shapes that have not been seen before.

VI.1.   Future Work

Although useful, this crystal-like modeling technique is not perfect. There are still plenty of further extrusions to be explored. In addition, there is a problem with the tiered system in relation to some of the Archimedean solids.

In particular, the truncated shapes, such as the truncated dodecahedron, have faces in which every vertex will not have an identical tier system. This means that the extrusion would be limited to faces with an even number of sides, as vertices would need to be paired up to make a repeatable tier.

Also of note is that the lattice system configurations of real crystals have not been followed. It would be interesting to see new modeling techniques that created shapes which fit into the crystal categories. Doing so would require a much more complex system for dealing with symmetry, such as described by Hestenes[8]. In all likelihood, the extrusion and

tier system idea could possibly be extended to a "crystal-growth" type of modeling system, where faces could be pushed in and pulled out, with the model automatically adhering to the cleavage planes. Possibly face groups, used in combination with this, might adhere to the symmetry axes.

Overall, crystals and crystal-like shapes haven't seen a great deal of geometric modeling research focus. This is somewhat of a surprise, as crystals have been studied for hundreds or even thousands of years, and are a popular topic for many people inside and out of academia. Hopefully, much work can be done to improve crystal-like modeling techniques in the future, so that new and interesting shapes can be discovered, studied, and used.

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VITA

Eric Landreneau
6203 Trellick Ct
Bryan, TX 77802
eland@tamu.edu

Education
M.S. in Computer Science Texas A&M University, May 2005
B.S. in Computer Science Texas A&M University, December 2001