The Structures of the Actual World.

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ABSTRACT

Scripture teaches that God has a plan *for* the universe. Given creation *ex nihilo*, the universe is *nothing*—nothing but God's acting in a multifaceted, coordinated way according to his composite plan, which we refer to as *the actual world*. In order for it to *function* as a plan, the actual world must have a *temporal* structure, a *representational* structure, and a *proto-causal structure*. This paper presents a formal model of these three structures of the actual world.

Scripture teaches that God has a plan *for* the universe. Given creation *ex nihilo*, the universe is *nothing*—nothing but God's acting in a multifaceted, coordinated way according to his composite plan, which we refer to as *the actual world*.[1] In order for it to *function* as a plan, the actual world must have a *temporal* structure, a *representational* structure, and a *proto-causal structure*. This paper presents a formal model of these three structures of the actual world.

Treating the actual world as God's plan is neither new nor widely-accepted by Christian philosophers. Leibniz, of course, is credited with first proposing the idea back in the 17th century, but his view differs from our view developed in this paper.[2] Among contemporary theistic philosophers and theologians there seems to be an implicit preference for Alvin Plantinga's modal realism, which is a metaphysics of modality according to which the actual world is a maximal, temporally-invariant *state of affairs*.[3] Our view differs from this, too. As we show in this paper and others, recovering the actual world

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[1] We are grateful for the helpful comments we have received on this paper from Bill Eppright, Jonathan Zderad, and Brad Sickler.

[2] While the actual world *functions* as God's plan according to Leibniz, what it *is* is a set of mutually-compossible, complete individual concepts. See Mates (1968). An interesting question is the logical relationship between Leibniz's view and the one presented here. According to what is presented in this paper, a set of divine commitments relating to the transition and co-existence of world states within a discrete 3+1 dimensional grid produce, when enacted, what appear to us to be objects, properties and relations. Tracing the history of such an object with all of its relations yields (we think) an object in a Leibniz actual world.

[3] However, Alexander Pruss has recently argued that Plantinga's platonic view lacks several desiderata and that his Aristotelian/Leibnizian view of modality is superior to both of these even though admits that the details of his view have yet to be worked out. See Alexander R. Pruss, *Actuality, Possibility, and Worlds* (New

as God's plan for the universe provides a theologically-faithful, logically-consistent, theoretically-useful, and comprehensive metaphysics of modality, mathematics, and science. (It is worth mentioning here that our view of the actual world precludes the incoherence that plagues set-theoretic constructions of platonic entities such as Roberts Adams' maximal propositions and Alvin Plantinga's "book" on the actual world.[4] In the end, they must be emended so as to avoid the iterative conception of a set and to preclude there being a power set of the "world-story" *S* or "book on a world" *S*.[5] As far as we know, no one (until now) has revised these set-theoretic accounts to avoid this problem. Our view treats the actual world as a complex relational structure, some of whose parts are themselves mereological sums defined predicatively, so that global self-reference is eliminated.[6])

Three preliminary issues.

Before we present the formal model, clarification of three interconnecting issues may prove helpful. The first is about *representations* and *models*, the second is our view of *events*, and the third is about *time*. All descriptions and models are representations, but not all representations are models, and no models are descriptions. Recognizing these differences is crucial to understanding what we are trying to do. As Anjan Chakravarty explains, while there is a *symmetric similarity* relation between the described and the description, there is an *asymmetric intentional* relation between the thing represented (an object, event, etc.) and the representation (*n*). That is, the communicating agent may intend a thing to represent something even though the representation may bear very little resemblance to the thing represented.[7] My point is that a model need not be isomorphic to the represented in every detail. Whatever isomorphism there may be, it is usually at

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York, New York: The Continuum International Publishing Group, 2011): 205.

[4] Adams acknowledges the "threat of paradox" and hopes that a formulation of a "true-story" account of actuality may be found. See Adams (1974): 229. Bringford (1985) shows the incoherence.

[5] Menzel (1986): 72. Bringsjord (1985).

[6] A *predicative* definition of a mathematical object (i.e., the *definiens*) does not include terms whose reference is the object to be defined (i.e., the *definiendum*), nor does it quantify over a domain of which the object is a member, or over sets of things of which the object being defined is a member. A *predicative* definition of a collection refers only to objects that exist independently of the collection.

[7] Chakravarty (2007): 70-73.

the structural level. This is the key notion in *Ontic Structural Realism* (OSR) in the philosophy of science.[8] It is the view that scientific theories (mathematical models) do not inform us about the *nature* of what is modeled, but rather its *structure*. Accordingly, the model of the actual world being constructed in this paper is not intended as a description of the actual world, but rather only a representation of its structures. Consider figure 1.0 below.

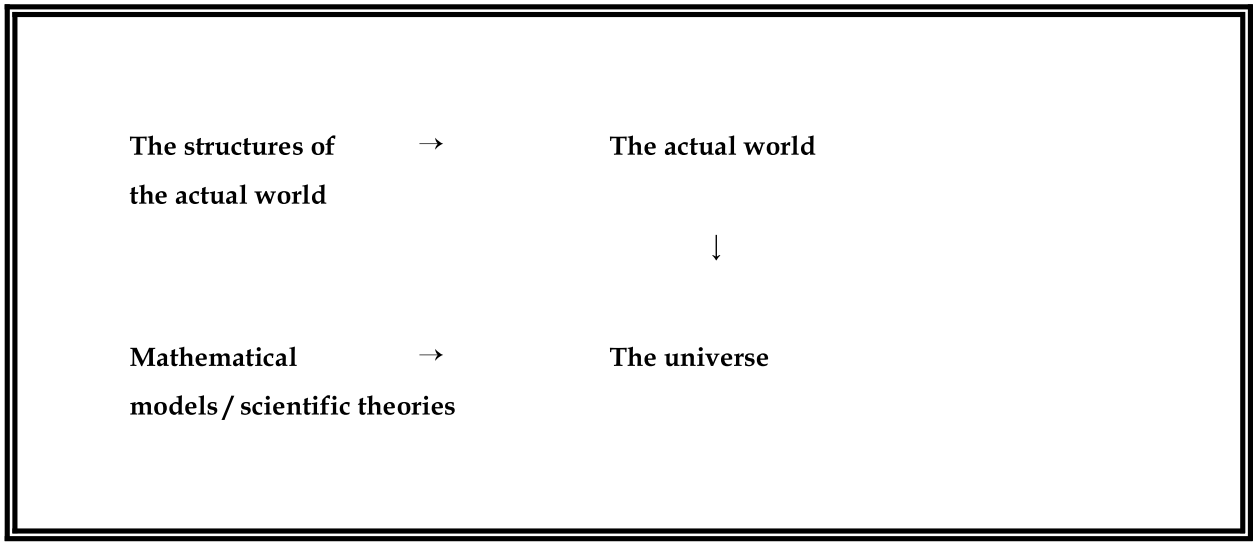

Figure 1.0

The model proposed here lies in the upper left quadrant. It is not the actual world itself (upper right quadrant), which is God's plan for the universe. Nor does it mathematically model the universe (lower quadrants).

event

If the actual world is indeed a divine representation *for* the universe, then some view the nature of events must be assumed. As they are *perceived*—an *event* is a change in an object or a physical system over a duration. As we *conceive* them for the purposes of this paper, an event is the mutual manifestation of disposition partners over a duration. However, we hold that an event—*objectively considered*—is God's

[8] Ladyman and Ross (1998): 130.

compositionally-conferring existence over a sequence of Planck moments, creating Planck regions of the universe according to plan. (A *Planck moment*, according to quantum physics, is the smallest, physically-possible duration, 10^{-43} seconds. A *process* is a sequence of events standing in some causal relation; a *situation* or a *state* of a physical system is the composite of all the invariant features of a complex process. As God *enacts* his plan we perceive some of these processes as various objects, having various dispositional properties, standing in various relations and infer from the manifestation of these properties various laws of nature.[9]) This, then, is a metaphysical account of a basic concept in science. What an event is is not a scientific question.

Even so, the model of the structures of the actual world being proposed in this paper cannot ignore science. If the actual world is indeed a divine representation *for* the universe, then some view of the *structure* of the physical world must also be assumed. We assume that *The base units of space and time are given by Planck-scale physics* (PL), and that *The neo-Lorentzian physical interpretation of the mathematical formalism of the Special Theory of Relativity is correct* (NL). With these in mind consider—as Carlo Rovelli claims—that, while the present knowledge of the elementary dynamical laws of physics are given by Quantum Field Theory (QFT), the Standard Model of Particle Physics (SM) and General Relativity (GR), yet, taken together, they comprise an inconsistent set. GR entails that space-time is not a fixed metric background as assumed in SR, but rather a *dynamical field*. However, Quantum Mechanics (QM) entails that all dynamical fields are quantized. Rovelli concludes that

> What Newton called "space" and Minkowski called "spacetime," is [. . .] nothing but a dynamical object – the gravitational field . . . [which has a] quantized, discrete structure at the Planck scale. [. . .] [10]

A theory of *Quantum Gravity* seeks a synthesis of GR and QM by providing an account of this "quantized, discrete" structure of space-time at the Planck scale. The main competing theories of QG are black hole thermodynamics, string theory, loop quantum gravity. Other, possibly complementary, theories that are noted in the literature are non-commutative geometry and causal sets.[11] Of these, the *Causal Set Hypothesis*

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[9] Events and so-called concrete states of affairs (as objects, property and/or relations) are *processes.* See Smolin (2001).

[10] Rovelli (2009): 8,9.

[11] The *Holographic Principle* may be the key to unifying these. For an introduction accessible to non-specialists, see Smolin (2001).

(CSH) is assumed here. It holds that space-time at the Planck scale is a locally-finite, partially-ordered set of elementary events.[12] These elementary events are the members of causal sets. The *Causal Set Hypothesis* (CSH) is a mathematical model of the dynamics of sequential growth.[13] "Becoming" is a real process due to the continuing "birth" of new elements of the causal set.[14] Also, William Lane Craig has argued from our second assumption (NL) for the reality of absolute simultaneity.[15] Putting these together, (CSH) together with (NL) and (PL) permits us to hypothesize that, for each Planck moment, there is a single, total state of the universe. It follows, first, that the universe is a 3+1 discrete manifold.[16] It also follows that if a representation *for* the universe has an order—an order, that is, of discrete total states—then absolute simultaneity exists first at this ontologically basic level. As such God's plan is truly *the privileged* frame of reference. The remaining issue, is to introduce a *model* of the structures of God's plan, which we call the actual world. So now, taking the actual world to be God's plan *for* the universe, the question before us is this:

> What must the structures of the *actual world* be if (1) the universe is a discrete 3+1 *discrete* manifold at the Planck scale and if (2) the *absolute simultaneity* consistent with the neo-Lorentzian physical interpretation of *Special Relativity* is the case?

In short, the problem is to model a representation *for* the universe over time. Here is an analogy. In addressing the question of how the apparent temporal sequence of the physical world should be represented mathematically, Tim Maudlin thinks that "some intrinsically time-directed representational medium . . ." such as music might be useful.[17] We agree, music is a heuristically fruitful analogy. Since the world states comprising the actual world are God's plans, we may think of the totality of these plans on the analogy of a

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[12] Kinematics. See Dowker 2011,2 .

[13] Dynamics. See Dowker 2011,2.

[14] See Craig (2001) on the importance of the notion of "becoming" for Christian theology.

[15] Noting that both Poincaré and Lorentz acknowledge the logical possibility of absolute simultaneity, Craig notes that mathematical models *of* the universe and its causal structure, such as Minkowski space-time, lack the concepts or resources for a notion of absolute simultaneity by virtue of their epistemological commitment to logical empiricism. See Craig (2008).

[16] See Ostoma and Trushyk (1999).

[17] Maudlin (2007): 135-142.

complex musical score—perhaps a symphony, for example. Three features of music are pertinent. The first is that it is measured in terms of a sequence of "beats". Second, notes being sounded from different instruments stand in differing relations to those being sounded from other instruments. This second point itself suggests a third. There is a sequence of sounds issuing from each instrument. To reiterate, the actual world as God's plan for the universe is a *like* a musical score, having these three structures. If so, the mass-energy distribution constituting physical space and time is nothing other than God's acting according to plan and perceived by us as manifesting dispositional properties and described by laws of nature.

To put all of this another way, scripture entitles us to hold that, given creation *ex nihilo* each *realized total state* of the universe is a matter of God's acting (i.e., willing, speaking)—God's acting at that moment, in a multifaceted, coordinated way, according to his plan. Thus, it is not that the "heavens declare the glory of God" in a way analogous, say, to the way Joshua Bell's performance of Chopin's *Nocturne* in C-sharp minor reflects Chopin and Bell. Rather, Joshua Bell's performance's reflecting Chopin and Bell is the analogue! It can only be a weak analogue to the heavens' declaring the glory of God. Furthermore, for creation, God is both composer and performer. The actual world is the composition. The actual world is a complete *representation for* the universe over time.[18] It has a temporal structure, a representational structure and a proto-causal structure. The model we are constructing is, therefore, a *model* of a *God's representation for the universe* and that representation shares the structural features of a piece of music.

time

This construal, provides a way to synthesize McTaggart's two views of time referred to as the "A-theory" and the "B-theory".[19] The A-theory (sometimes called the *presentist*, *dynamic* or *tensed* view of time) holds that the apparent distinction between the past, the present, and the future is objectively real, though only the present moment is real.[20] The B-theory (sometimes called the *eternalist*, *static*, or *tenseless* view of time) holds that all times and their contents are equally real and stand in a *earlier-than* relation to each other. The model proposed here treats "times" as real entities in so far as they are *frames for* physical events. Both the plan and the plan-realized are real. They are not merely mental constructs. Like sheet music to a

[18] There is the further question of whether the plan is infinite or only potentially infinite. Moreover, there are at least two construals, given a deterministic assumption, and two construals, given libertarian freedom and indeterminism. These are not addressed in this paper.

[19] McTaggart (1980).

[20] Some presentists hold that both the past and present are real.

musician: on the page, the music is B-theoretical; as performed the music is A-theoretical. Like a play to its director: the script is B-theoretical; as performed: A-theoretical. Like a recording to a music lover: on the recording B-theoretical; as played, A-theoretical. Physical time is God's sequential acting and is A-theoretical. However, though God's being constitutes metaphysical time[21], God's plan is B-theoretical. Therefore, times and durations are real as segments of God's plan. Therefore, modeling the structures of the actual world cannot be a merely constructing a *set* or a *conjunction* of states of affairs. It is like modeling music, say a symphony. Let us now consider the structures of the actual world.

The Temporal Structure

Since the actual world is a representation *for* the universe over time, it will have a temporal structure. Typically, the construction of a formal model involves three choices, choices as to which kinds of temporal *individuals* to use (e.g., points, periods or events), which kinds of *relations* that hold between individuals (e.g., precedence, inclusion, overlap, etc.), and which kinds of *conditions* should be incorporated so as to adequately model what is to be modeled.[22] One might think of the actual world as a book with blank pages. Since creation is assumed to have a temporal beginning, the book has a first page but no last page. The content of "pages" are *world states*. Let us pause here to clarify this notion.

world states

A *world state* is the representational content *for* Planck cells and regions. There are several conceptually possible types. Let us suppose that an ***atomic*** world state represents the content of a *Planck cell* (a 3-dimensional irregular hodon) at a *Planck moment* (a chronon). A representation of the content of the entire world at a Planck moment is a ***total*** world state. It *represents* a discrete, irregular "cube" of simultaneity which is the entire universe at a Planck moment. A ***composite*** world state is any combination of atomic world states without a regional or temporal gap. An example is the composite world state picked out as *Lincoln's giving the Gettysburg Address*. Such *composite* world states are *like* the standard view of propositions or states of affairs, but are *not* conceptually identical to them. Unlike the standard view of propositions, they are not the *content* of a person's thought. No person could think a *complete* representation.

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[21] DeWeese (2004:252). God's temporal mode of being is best characterized as *omni-temporality*, namely God is *metaphysically temporal* and exists necessarily so that (1) (in this sense) God exists at all times and is everlasting and (2) God's existence in metaphysical time grounds causality, and therefore physical time.

[22] See van Benthem (1984): 1.

The informational content of a person's belief cannot be as detailed as a world-state, which is a *complete* representation. Propositions are *abbreviated* representations; the content of occurrent propositional attitudes. Propositions represent either particular world states *per se* (not yet "actualized"), world states achieved ("actualized"), or a quantifications over world states (i.e., they may represent all, some, most, world states.)

With the concept of a *world state* in mind, let us return to the notion of temporal structure. We suggested that the temporal structure of the actual world could be conceived as a "book with blank pages." The content of such "pages" are *world states*. Alternatively, the temporal structure may be conceptualized as a blank film tape, which is infinitely long in one "direction." An "exposed frame" on the film tape is a world state. On these analogies we have *pages* and the *contents* of pages and *frames* of film and *contents* of frames. The difference is crucial. On our model, neither concrete events nor the so-called "states of affairs" are the ontological components of the temporal structure we are attempting to model, nor are the more common abstract points or instants (and intervals of them). Rather, the primary ontological components are *world states*, which are *plans.* Plans are temporally framed or demarcated by *discrete frames for time*. They are thereby temporally-located relative to other plans. Events (according to the model being developed) are plans achieved. In sum, the model constructed here takes *discrete frames for time* as its individuals and *precedence* as the basic relation. The temporal structure of the actual world is an infinite, strict linear order of discrete total *frames* for time. Let us formalize this.

Definition. Let an ***a-frame*** be a real, minimal *frame for* time.

What is meant by, 'minimal', is that an *a-frame* demarcates a "snapshot" of the universe at a Planck moment (i.e., 10^{-43} seconds). It is not the content of the "snapshot", nor is it the universe itself. It is not a segment of physical time itself, because it has no *physical* content. Moreover, though it *has* representational content, an *a-frame* is nothing but the frame. An *a-frame* demarcates a temporal "slice" of the mass-energy distribution or a neo-Lorentzian "cube" of simultaneity. Each a-frame's representational content is a total world-state. Since a *total* world state is a finite composite of world states[23] and a *world state* is a state of affairs to be achieved, *a-frames* demarcate *plans*, in so far as they are segments of an overall *plan*. The overall plan could be understood as a complete course of action.[24]

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[23] A "tensed universal description" describes this composite state. See Yandell (1999: 308).

[24] Thus, this is related to an event ontology, simply because it is a sequence of plans and realized plans are events. Concrete states of affairs are derivative. This is consistent with Smolin (2001).

There are two approaches to representing and reasoning about time and temporal relations: the *first-order approach* (initiated by Russell (1903) and continued by Quine (1960)) and the *modal approach* (initiated by Prior (1955) (1967)). The choice of approaches may be merely a matter of a theorist's preference, but some have argued cogently for the superiority of the former.[25] As Meyer (2009) acknowledges, "There are temporal claims that resist regimentation in terms of tense operators, but which are easily accounted for by rival theories that make use of quantification over times [. . .]."[26] However, just as quantifying over worlds implicitly treats them as real entities, quantifying over *frames for time* takes them as real entities. The quantification is intended to objectual, because the objects are assumed to be real *frames* for physical time. This means that something should be said regarding what "times" are supposed to be. In this model, "times' are, first of all, *a-frames* and convex sequences of them. They frame segments of the temporal structure of the actual world. Physical "times" (i.e., time *per se*) just are demarcated periods of God's realizing his plans. Therefore, even though the approach taken here is similar to treating times as platonic abstract objects or as events, *a-frames* are clearly different.

Definition. An ***a-frame structure*** is an ordered couple $\langle \boldsymbol{A}, \prec \rangle$ where A is an infinite set of a-frames (denoted by lower-case, italicized letters x, y and z) and where $\prec$ is the binary relation *precedence* ('$x \prec y$' means "x precedes y") . The binary relation $\prec$ satisfies the conditions IRREF, TRANS, DET, INF, and DISC defined below.[27] These conditions are treated axiomatically, permitting further deductions regarding the structure of the actual world. The first condition that $\prec$ should model is that no *a-frame* precedes itself, which means that $\prec$ is *irreflexive*.

IRREF $\forall x \sim (x \prec x)$ (irreflexive)

Secondly, if one *a-frame* precedes another, and that one a third, then the first precedes the third. In other words, $\prec$ is *transitive*.

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[25] See van Benthem (1983) and Mellor (1981).

[26] Meyer (2009): 201.

[27] Compare to van Benthem (1983): 12-18.

TRANS $\forall x \forall y \forall z ((x \prec y \wedge y \prec z) \rightarrow (x \prec z))$ (transitive)

IRREF and TRANS are intended to construct $\langle A, \prec \rangle$ so as to reflect time as having (or being) a constant rate of "flow" in one "direction" only. These conditions entail asymmetry, i.e., if one *a-frame* precedes another, the second does not precede the first: $\forall x \forall y (x \prec y \rightarrow \sim y \prec x)$. Thus, $\prec$ is *asymmetric*.

Thm1. $\vdash_{FL} \forall x \forall y (x \prec y \rightarrow \sim y \prec x)$ **ASYM**
Proof. Suppose $x \prec y$. Now, for *RAA* suppose $y \prec x$. It follows that $x \prec x$, contradicting IRREF.

To ensure that $\langle A, \prec \rangle$ does not "branch", we stipulate that any two a-frames are either identical or one precedes the other. There are no other alternatives. Thus, $\prec$ is *determinate* and *linear*.

DET $\forall x \forall y (x = y \vee x \prec y \vee y \prec x)$ (determinate)

By imposing conditions IRREF, TRANS, and DET on relation $\prec$, $\langle A, \prec \rangle$ is a strict linear order (i.e., a chain). It is "strict" because of IRREF. It is "linear" (or "total") because DET rules out branching, since all elements are involved and there are no alternatives and because IRREF rules out circularity.

God's *plan* has a first moment, but no last. We capture both of these features by stipulating that one a-frame has no predecessor and every a-frame has a successor.

INF $\exists x \sim \exists y (y \prec x) \wedge \forall x \exists y (x \prec y)$ (infinite)

Thus, INF ensures that $\langle A, \prec \rangle$ has a first moment, but no last moment.

We want to model a representation for time as a sequence of discrete entities.

DISC[28] $\forall x \forall y (x \prec y \rightarrow \exists z ((x \prec z) \wedge \sim \exists u (x \prec u \wedge u \prec z)) \wedge$ (discrete)
$\forall x \forall y (x \prec y \rightarrow \exists z ((z \prec y) \wedge \sim \exists u (z \prec u \wedge u \prec y))$.

[28] Same as van Benthem's DISC, (1983): 18.

To recapitulate, the *temporal* (or sequential) structure of the actual world is an *a-frame structure*, which is an ordered couple $\langle A, \prec \rangle$, where A is an infinite set of a-frames and where $\prec$ is the binary *precedence* relation defined on D so that '$x \prec y$' means "x precedes y". Thus, $\langle A, \prec \rangle$ is an infinite, strict, linear order of discrete entities, which we call, '*a-frames*'. Since *a-frames* demarcate Planck moments, they must be subliminal. Hence, the model is in keeping with our sense of the smooth continuity of time even though it is discrete.[29] Also, since, by definition, every *a-frame* demarcates a Planck moment, $\langle A, \prec \rangle$ is *homogenous*: the pattern of time is invariant. In sum, the *temporal* structure of the actual world is an *a-frame structure* $\langle A, \prec \rangle$, satisfying the five axiom conditions, IRREF, TRANS, DET, INF , and DISC, where each *a-frame* demarcates a discrete moment of the entire universe.

As was said earlier, if a representation *for* the universe is ordered sequence of "discrete total *frames for* time", then absolute simultaneity exists first at this ontologically basic level. And now if this representation is God's plan, then God's plan is truly *the privileged* frame of reference. Moreover, if what we have so far is accurate, then the quantum states of the gravitational field are nothing but God's acting according to plan.

Summary

In sum, the temporal structure of the actual world is an ***a-frame structure***, which is an ordered couple $\langle \boldsymbol{A}, \prec \rangle$ where A is an infinite set of a-frames (denoted by lower-case, italicized letters x, y and z) and where $\prec$ is the binary relation *precedence* ('$x \prec y$' means "x precedes y") . The binary relation $\prec$ satisfies the conditions IRREF, TRANS, DET, INF, and DISC .

The representational structure

Taking stock, the actual world is the infinite history of the universe. It is a totality of discrete frames *for* time with representational content, called "total world states". The *temporal structure* of the actual world

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[29] See Chew (1985) who proposed that "multiple emission and absorption of soft photons in a discrete quantum world (implicate order) generates the continuous Cartesian-Newtonian-Einsteinian space-time world of localizable objects and conscious observers with measuring rods and clocks (explicate order)." "[. . .] explicate order together with space-time is an approximation emerging from complex but coherent collections of "gentle" quantum events—the emission and absorption of soft photons." (59) "The complexity responsible for objective reality within an apparently-continuous space-time we propose to associate with multitudes of coherent low-energy electromagnetic quanta." (59)

is characterized by a precedence relation satisfying certain conditions. Fully-stated, the *temporal structure* of the actual world—God's plan for the universe—is an infinite, strict linear-order of *a-frames*, which when filled with content are called *world states*, which are representations *for* events.

Distinct events vary in duration. This fact underlies the seven types of *temporal* relations between any *two* events. Representations *for* those events will mirror these seven types of relations. Some are *disjoint*. One may *overlap* another and one may *meet* another. One may *begin simultaneously* with another and one may *end simultaneously* with another. One may occur *entirely within the duration* of another or both may *occur simultaneously*. These various relations are the *representational structure* of the actual world.

Three issues must be emphasized and clarified before we proceed. First, the model being proposed is not a model of *time* in any of the familiar senses. Standard tense logic involves an ontology of times which stand in some temporal relation. However, the temporal structure offered in the previous section is a model of time in which *a-frames* are the fundamental ontological entities, as opposed to the more standard points, periods or events. So, although the model presented in *this* section *resembles* a model of time in terms of period, interval or an event structure, it is a model of the *structure of representations*.

Second, since *world states* are representations *for* physical reality, they cannot stand in temporal relations *per se*. Only *realized* world states, which are events and states, stand in temporal relations. Hence, strictly speaking, world states stand in *proto-temporal* relations. To put this another way, since a *world state* (i.e., a theistic plan) is the representational content *for* an event or process, then while events and processes have *temporal duration* (are *in* time), world states have *representational duration* (are *for* time). Intervals of a-frames *demarcate* the *temporal duration* of events and processes (which again are realized plans or world states). But the *representational duration* of every world state *is* an interval of *a-frames* or interval of the *temporal* structure $\langle A, \prec \rangle$.

Third, there are no "vague" concepts or inaccurate perceptions in the actual world.[30] The reason is that the actual world is understood to be God's plan for the universe. God is not apprehending an already existing object.[31] However, since each world state is a complete in every detail, no human could cognitively grasp a world state. Propositions, which are the information content of declarative sentences, can only represent them. Propositions are abbreviated representations of world states. Thus, what is needed is a *model* of the *structure of representations* constituting the actual world.

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[30] See Kamp (1979).

[31] This underscores the difference between *propositions* and *world states* mentioned earlier.

Definition. Let a ***duration*** $\boldsymbol{\delta_n}$ ***of a world state*** be a non-empty convex subset of $\langle A, \prec \rangle$, which has (1) a temporal location and (2) an absolute value, which is its "length" in *a-frames*.

Thus, first of all, each *a-frame* correlates to exactly one natural number. (They may be set in a one-to-one correspondence.) The set of *a-frames A* is an ω-sequence satisfying the axioms of PA^2 (2nd-order Peano Arithmetic). Moreover, each *a-frame* has a "name" or "place", which is some natural number for every member of the set $\mathbb{N}$. Second, since *a-frames* are real, durations as intervals of *a-frames* are real; independent of human conception. Third, every world state correlates to a temporally-invariant proposition. The representational structure is a mereological sum. The representational structure of the actual world is thus "mirrored" by a structure of propositions.[32]

Definition. Let a structure $\boldsymbol{A}$ be ***generated from*** $\boldsymbol{B}$ if

1. $\boldsymbol{A}$ and $\boldsymbol{B}$ have the same signature, $\sigma(\boldsymbol{A}) = \sigma(\boldsymbol{B})$, that is, the meaning of the relation symbols of $\boldsymbol{A}$ are the same as those of $\boldsymbol{B}$,
2. The *domain* of $\boldsymbol{A}$ is constructed from the domain of $\boldsymbol{B}$, and
3. The *interpretation* of the relation symbols of $\boldsymbol{A}$ are the same as those for $\boldsymbol{B}$.

Definition. The ***duration structure*** $\langle \boldsymbol{D}, \prec \rangle$ is generated from $\langle \boldsymbol{A}, \prec \rangle$, where

(i) $\boldsymbol{D}$ is the set of all subsets $\boldsymbol{\delta_n}$ of $\langle A, \prec \rangle$,

(ii) $\prec$ is a ***precedence*** relation on D defined as follows:

$$\boldsymbol{\delta_n} \prec \boldsymbol{\delta_m} \leftrightarrow \forall x \in \delta_n, \forall y \in \delta_m \; x \prec y.$$

Definition. ***inclusion***

1. $\boldsymbol{\delta_n \sqsupset x}$ A duration δ_n ***includes*** **an** ***a-frame*** **x** iff x is a member of δ_n.

$$\delta_n \sqsupset x \leftrightarrow x \in \delta_n$$

2. $\boldsymbol{\delta_n \sqsupset \delta_m}$ A duration δ_n ***includes*** another duration $\boldsymbol{\delta_m}$ iff every *a-frame* that is a member δ_m of

[32] See Sider (2006).

is also a member of δ_n.

$\delta_n \sqsupset \delta_m \leftrightarrow \forall y\ (y \in \delta_m \rightarrow y \in \delta_n)$

Since *durations* demarcate and locate events and concrete states of affairs, they must be "uninterrupted" sequence of *a-frames*. This feature is expressed formally by the notion of *convexity*.

Definition. ***convexity*** A *duration* δ_n is a convex set iff for any two *a-frames* x,y, there is a third, z, such that, if x precedes z and z precedes y, then δ_n includes z.

CONV δ_n iff $\forall x \forall y\ ((\delta_n \sqsupset x \wedge \delta_n \sqsupset y \wedge \exists z \in \langle A, \prec \rangle\ (x \prec z \wedge z \prec y)) \rightarrow \delta_n \sqsupset z)$.

Most, but not all, durations are have first and last members. When they do, we say they are *bounded.*

Definition. ***boundedness*** A duration δ_n is ***bounded by a-frames x and y***, B(x δ_n y) iff every other *a-frame*, z, that is included in δ_n is preceded by x and precedes y.

B(x δ_n y) iff

(i) $x = y \wedge \delta_n \sqsupset x \wedge \delta_n \sqsupset y \wedge \sim \exists z\ \delta_n \sqsupset z$, or

(ii) $(x \neq y \wedge \delta_n \sqsupset x \wedge \delta_n \sqsupset y \wedge \forall z\ (\delta_n \sqsupset z \rightarrow (x \prec z \wedge z \prec y))$

Definition. ***identity*** Any two durations are identical iff each includes all of the *a-frames* included by the other.

$\delta_n = \delta_m$ iff $\forall x\ (\delta_n \sqsupset x \leftrightarrow \delta_m \sqsupset x))$

The aim now is to model the representational structure of the actual world. This involves accounting for the relevant *features* of the durations of world states and for the seven types of *relations* that may obtain be world states and thus between the durations that demarcate and locate them.

AXIOMS.[33]

Since the actual world is a representation *for* events and concrete states of affairs, the first thing to ensure is that the model does not permit the inference that there are temporal gaps within some durations.

A1 **CONV**(δ) Every duration is a convex subset of $\langle A,\prec\rangle$.

$\forall\delta_n\forall x\forall y\ ((\delta_n \sqsupset x \wedge \delta_n \sqsupset y \wedge \exists z\in\langle A,\prec\rangle\ (x\prec z \wedge z\prec y)) \rightarrow \delta_n \sqsupset z)$.

Some states of affairs are perpetually sustained so that they are unbounded into the future.

We need also to ensure that non-perpetual durations (except single *a-frames*) have distinct first and last elements.

A2 **BOUND**(δ) Every finite duration δ_n of D is bounded.

$\forall\delta_n\exists x\exists y\ B(x\ \delta_n\ y)$

Thm 1. ***D*** is partitioned into two subsets: the set of all *finite* subsets and the set of all *unbounded* subsets.

A3 **DET**(δ) Every two distinct *a-frames* determine a duration.

$\forall x\forall y\ (x \neq y \rightarrow \exists\delta_n\ B(x\ \delta_n\ y))$

A4 **CONT**(δ) For every two durations having some *a-frame* in common, there is a duration containing all and only these *a-frames.*

$\forall\delta_n\ \forall\delta_m\ \forall x((\delta_n \sqsupset x \wedge \delta_m \sqsupset x) \rightarrow \exists\delta_k\ \forall y\ (\ \delta_k \sqsupset x \leftrightarrow (\delta_n \sqsupset y \vee \delta_m \sqsupset y))$.

Thm 2. Every *a-frame x* is included some *duration* $\boldsymbol{\delta_n}$,

i.e., $\forall x\in\langle A,\prec\rangle\ \exists\delta_n\ (\delta_n \sqsupset x)$.

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[33] This axiomatic formulation has been influenced by van Benthem, (1983), (1984), Bochman (1990), and Allen (1984). For any two distinct *a-frames*, there is a duration containing a third distinct *a-frame* containing exactly one of the pair.

$\forall x\forall y(x \neq y \rightarrow \exists\delta_n\ \exists z(\delta_n \sqsupset z \wedge (\delta_n \sqsupset x \leftrightarrow \delta_n \not\sqsupset y)))$.

Thm 3. Every duration δ_n is a strict linear order of *a-frames*,

i.e., $\forall x \forall y \forall z \in \delta_n (\sim (x \prec x) \wedge ((x \prec y \wedge y \prec z) \rightarrow (x \prec z)) \wedge (x=y \vee x \prec y \vee y \prec x))$.

There are seven types of relations between any *two* world states. Some are *disjoint*. One *overlaps* another and one *meets* another. One begins at the same time as another (*co-starts*) and one ends at the same time as another (*co-finishes*). One occurs entirely within the duration of another (*during*) or at the same time as the other (*simultaneous*). This is graphically represented in *Figure 2.0*, where the letters, *i, j, k, l, m,* and *n* represent durations.

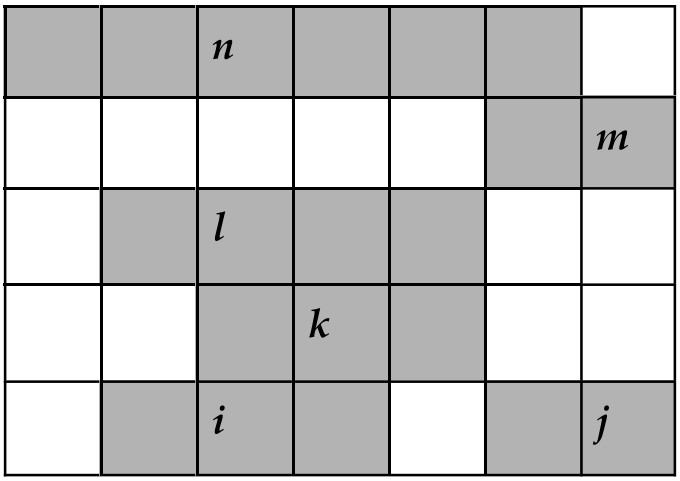

Figure 2.0

A5 ***overlaps*** **(δ_i, δ_k)** $\delta_i \circ \delta_k \leftrightarrow ((\exists x (\delta_i \sqsupset x \wedge \delta_k \sqsupset x)) \wedge (\exists z \, \delta_i \sqsupset z \wedge \sim \delta_k \sqsupset z) \wedge (\exists u \, \delta_k \sqsupset u \wedge \sim \delta_i \sqsupset u)$.

A6 ***meets*** **(δ_k, δ_j)** $\delta_k \mid \delta_j \leftrightarrow \delta_k \prec \delta_j \wedge \sim \exists z (\delta_k \prec z \wedge z \prec \delta_j))$.

A7 ***disjoint*** **(δ_i, δ_j)** δ_i **DIS** $\delta_j \leftrightarrow \delta_i \prec \delta_j \wedge \exists z (\delta_i \prec z \wedge z \prec \delta_j))$.

A8 ***co-starts*** **(δ_i, δ_l)** δ_i **CS** $\delta_l \leftrightarrow \exists x (B(x \, \delta_i \, y) \wedge B(x \, \delta_l \, z))$.

A9 ***co-finishes*** **(δ_k, δ_l)** δ_k **CF** $\delta_l \leftrightarrow \exists z\,(B(x\ \delta_k\ z) \wedge B(y\ \delta_l\ z))$.

A10 ***during*** **(δ_i, δ_n)** δ_i **DUR** $\delta_n \leftrightarrow \forall x\,(\delta_i \sqsupset x \rightarrow \delta_n \sqsupset x) \wedge \exists y\ \exists z(y \prec \delta_i \wedge \delta_i \prec z \wedge \delta_i \sqsupset y \wedge \delta_i \sqsupset z)$.

A11 ***simultaneous*** **(δ_j, δ_m)** δ_j **SIM** $\delta_m \leftrightarrow \delta_j = \delta_m$.

Thm 4 Every finite duration meets and succeeds another.

Summary

In sum, the representational structure of the actual world is a *representational duration structure* $\langle D, \prec \rangle$ generated from the temporal structure $\langle A, \prec \rangle$, where D is the set of all subsets δ_n of $\langle A, \prec \rangle$, and $\prec$ is a *precedence* relation defined on D such that $\delta_n \prec \delta_m \leftrightarrow \forall x \in \delta_n, \forall y \in \delta_m\ x \prec y$. The set D satisfies the four axiom conditions, CONV, BOUND, DET, and CONT. Every temporal relation between events is represented.[34]

The proto-causal structure

Some *world states* stand in a *proto-causal* relationship so that *when* they are physically realized, these become *events* which stand in a *causal* relationship. To put this another way, if *causality* a relation between events (so understood), then a *causal process* is a sequence of such events. Whenever two events occur simultaneously, neither one causes, or is caused by, the other. Therefore, a causal process does *not* include any event that stands in a simultaneity relation with any of its constituents. Furthermore, one event *can* precede another distinct event and yet not stand in a causal relation to the latter. But not every sequence of atomic world states represents a causal process. Therefore, these two types of sequences must be distinguished, which means that the proto-causal structure of the universe must be correlated to a relational substructure of $\langle D, \prec \rangle$. We say "correlated to" because what stand in proto-causal relations are those world states that *have* representational duration, but not temporal duration.

[34] While $\langle A, \prec \rangle$ is a topological structure, $\langle D, \prec \rangle$ is not.

A. A. Robb (1914:371) showed that "the elements of time form a system in conical order."[35] He was able to duplicate the geometric structure of Minkowski Spacetime on the basis of a *before* relation (and conditions) among points. Malament (1977) and Winnie (1977) *take* Robb's *conical* order to be a causal order. The *Causal Set Hypothesis* is an advancement on these views. As we said earlier, it holds that space-time at the Planck scale is a locally-finite, partially-ordered set of discrete elementary events.[36] These elementary events are the members of causal sets. A *causal set* is a sequence of elementary events that stand in a causal relation. Therefore, a *proto-causal set* is a sequence of world states that stand in a proto-causal relation. Causal sets constitute a mathematical model of the dynamics of sequential growth and change at the local level.

Given (CSH), the proto-causal structure of the actual world $\langle C, \prec \rangle$ is a *proto-causal set structure,* where C is the set of all proto-causal sets $_{cN}$ and $\prec$ is a *precedence* relation defined on C. The binary relation $\prec$ satisfies the three axiom conditions TRANS, NCIRC and FIN below:[37]

TRANS	$\forall x \forall y \forall z \in C ((x \prec y \wedge y \prec z) \rightarrow (x \prec z))$	(transitivity)
NCIRC	$\forall x \forall y \forall z \in C ((x \prec y \wedge y \prec z) \rightarrow (x = z))$	(non-circularity)
FIN	$\forall x \forall z \in C$, the set $\{y: x \prec y \prec z\}$ is finite	(finiteness)

How does this *proto-causal* structure relate to the *temporal* and *representational* structures of the actual world? The proto-causal structure of the universe must be correlated to a relational substructure of $\langle D, \prec \rangle$. Therefore, causal structure is not identical to, but depends on (related to) temporal structure. God's plan has *temporal structure*. God's acting according to plan is the observed *causal structure* of space-time. *Precedence* is the basic relation in the temporal structure of the actual world, but *causal precedence* (i.e., the Minkowski relation of "possible causal precedence") is the basic relation for the causal structure of the actual world.

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[35] Robb (1914): 371. It may be that William R. Hamilton (1805-1865) deserves the credit for the first mathematical model and metaphysical interpretation of time and causality.

[36] "The causal set hypothesis is that in the deep quantum regime of very small scales, spacetime is no longer described by a metric on a differentiable manifold, but by a causal set. Just as ordinary matter appears smooth and continuous to us, but is really made of atoms and molecules, so, it is proposed, spacetime appears continuous at large scales but is really a causal set and the continuum spacetime of our experience is just an approximation to the discrete order." Fay Dowker. *Contemporary Physics*, Vol. 47, No. 1, January–February (2006): 5.

[37] Dowker (2006): 4.

Minkowski spacetime and the Causal Set Hypothesis of QG are mathematical models of the universe. They should be—and aim to be—consistent with other physical theories. An account of the actual world has subtle, but crucially different aims. It should be consistent with physical theory—not *as* a physical theory—but as a metaphysical theory. The consistency of a metaphysical theory with a physical theory is a matter of explanatory power.[38] To what extent does the metaphysical theory account for physically-unaccountable features of the physical theory? However, even a theory of the causal nature of time must accept that some issues cannot be accounted for physically. As Robb put almost 100 years ago,

> Though space may be analyzable in terms of time relations, yet these remain in their ultimate nature as mysterious as ever ; and though events occur in time, yet any logical theory of time itself must always imply the Unchangeable.
>
> Thus may I conclude in the words of Carlyle :
>
> > 'Know of a truth that only the Time-shadows have perished, or are perishable ; that the real Being of whatever was, and whatever is, and whatever will be, *is* even now and forever. '

Concluding observations:

With this model of the structure of the actual world we have an ontological grounding of a logically-consistent theory of modality, the basis for an ontology of science and a modal-structuralist philosophy of mathematics,[39] in addition to the basis for a *world-inclusion semantics* for systems of formal logic—all of which are rigorously rooted in the biblical doctrine of God, whose acting is the dynamic underlying reality.

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[38] Maudlin, e.g., takes laws of nature as ontological primitives.

[39] See Walter Schultz, Towards a Realist Modal Structuralism: A Christian Philosophy of Mathematics *Philosophia Christi*, 12.1 (2010): 139-154.