Math History Study Abroad Program:
Learning Math History in a Cultural Context

Donna Pierce
Whitworth University, Spokane, WA

Donna Pierce (Ph.D., Washington State University) has been on the faculty of Whitworth University since 2001. Her primary academic interest areas are in math history, number theory, cryptography, algebra and ethics. She loves advising and mentoring students, especially with regard to vocation and career planning. Believing in the value of experiencing other cultures she travels whenever she can, and has led a math history study abroad program to Europe.

1. Introduction

In January 2011 fifteen Whitworth University mathematics students and I, their professor, traveled through Europe to study the history of mathematics. The goal was to gain an understanding of how mathematical ideas have developed over time; how social, cultural and historical factors have influenced the development of mathematics and conversely, how mathematics contributed to society and human culture.Over a course of three weeks we traveled to three countries and over a half dozen cities, viewing the tools, papers and workbooks of these mathematicians, seeing their engineering and artistic creations, and learning from local experts as they guided us through science museums and archives. The experience helped us to understand the broad roles mathematics and mathematicians played in the world around them.

The journey actually began several months earlier during a Fall Prep Course. Readings from the textbook, *Mathematics in Historical Context*, by Jeff Suzuki provided a good foundation for understanding how mathematics is an integral part of culture. The book gives a broad overview of the history of mathematics from the Ancient World (Prehistory, Egypt and Mesopotamia) to post-WWII. Though we concentrated mainly on the last seven centuries, we also studied some Greek, Egyptian, Babylonian and Islamic mathematics, in anticipation of seeing some of the artifacts from these civilizations in museums during our travels. Movies about Roman engineering and biographical films on Galileo, Newton and Einstein helped us to both visualize what we would see, but also what the world must have looked like when these mathematicians were making their important contributions. Each student chose a particular mathematician to research. The students wrote a term paper describing their mathematician in terms of historical and cultural context in which he/she lived, as well as their contribution to mathematics. Each of these mathematicians was tied to a particular site we would be visiting during our travels. When we were at that site in Europe, the student would then give an oral presentation regarding their mathematician to the rest of our group. This gave the students the opportunity to learn in-depth about a mathematician both through their library research and then through actually seeing where that mathematician lived and worked.

2. Rome: Engineering, Art, and Politics

Our travels abroad began in Rome where one is surrounded with evidence of the Roman engineering that played such a large role in Rome's ability to grow as an empire. Water was abundant in Rome thanks to an aqueduct system designed using arches, a slight grade and gravitational pull. Eleven aqueducts provided Rome with enough water to sustain a population of one million people. The aqueducts emptied into three holding tanks: one for public drinking fountains, one for public baths and one reserved for the emperor and wealthy Romans who had their own running water. The abundance of water made keeping clean easier; it is said that Romans felt superior because they were cleaner than other cultures.

(a) Roman fountain

(b) Santa Maria degil Angeli, central hall of the former Baths of Diocletian

Figure 1: Abundance of water in Rome

Using his access to water, Nero built a huge lake in the center of Rome as part of his palace complex to demonstrate his wealth and power. After Nero's death, the Emperor Vespasian drained the massive lake Nero had built (for his personal pleasure) and replaced it with the Coliseum built for the people of Rome. Even though we had watched a film about the building of the Coliseum, we were still in wonder when we actually saw the ingenuity and intricacies of its design which included assigned seating, an awning/weather protection system, and layers of subfloor with elevators that allowed wild animals to seemingly spring out of nowhere onto the amphitheater floor. Next to the Coliseum was the Roman Forum, which for centuries had been the center of Roman public life.

The Forum was where commerce, the administration of justice, and religious activities of Rome were conducted. It was the site of triumphal processions and a venue for public speeches. Students marveled at the size and grandeur of the buildings in the Roman Forum, just as the captives of Rome must have as they were paraded through the Forum as part of a Roman victory celebration. Visiting the Forum and Coliseum gave us a real sense of the grandeur and power of Rome and the role engineering played in making it possible. We would continue to see the influence of Roman architecture and engineering in the other cities we visited, especially Florence and the sites in England.

We spent four days in Rome. We learned about life in an ancient commercial city through our visit to the ruins of Ostia Antica, a Roman port town, where we could walk through the streets

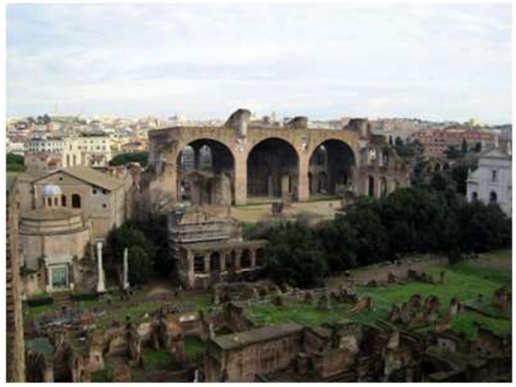
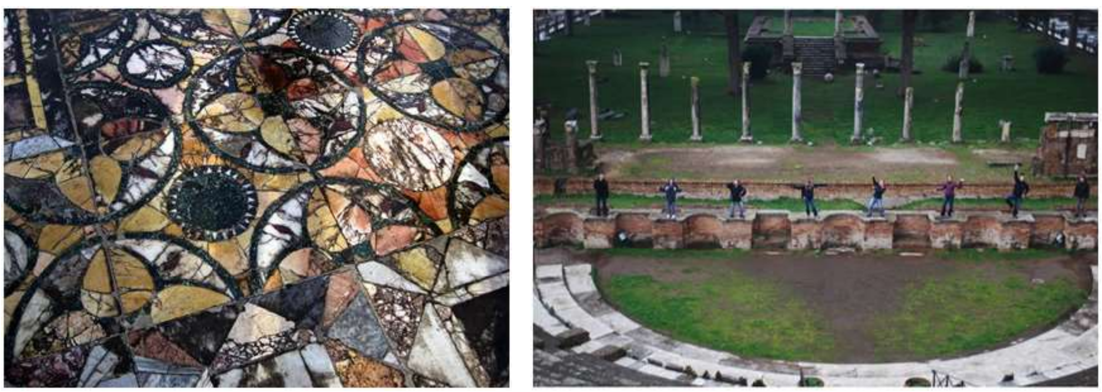


Figure 2: Roman forum

of the city, see the design of amphitheaters and temples, public latrines, burial grounds, homes and marketplaces. Mathematical patterns were reflected in the geometric designs of many of the floor tiles as well as the layout of the city.



(a) Ostia Antica floor

(b) Amphitheater in Ostia Antica

Figure 3: Ruins of Ostia Antica

Mathematics played a large role in the aesthetics of design of plazas (such as Michelangelo's Plaza) and architecture, especially in the construction of the domes of the Pantheon and St. Peter's. The engineers, architects and artists were mathematicians as well mathematics was intricately woven into the everyday lives of the people. Sometimes mathematics was used in the construction of art and sometimes the art itself reflected the important role that mathematics played in the culture. For example, Raphael's painting, *The School of Athens*, in the Vatican Museum in Rome, portrays an idealized community of intellectuals from the entire classical world.Among them in the picture are Pythagoras with his perfect numbers, and Euclid demonstrating some geometric proposition with a pair of compasses upon a slate. Raphael's vision of Humanism pays tribute in *The School of Athens* to the importance of mathematics in human learning and understanding.

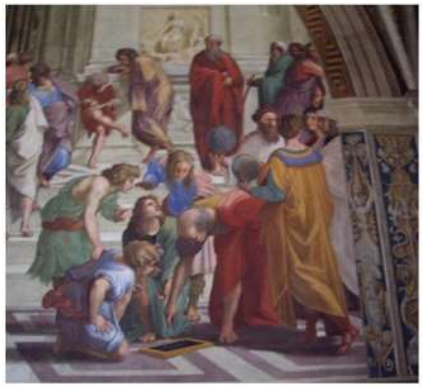


Figure 4: Euclid demonstrating a mathematical proposition in Raphael's *School of Athens*

3. Florence: Renaissance Art and Science

From Rome we journeyed to Florence, the heart of the Italian Renaissance, home to Galileo, Leonardo DaVinci, Michelangelo and many other Renaissance artists and scientists. Florence provided the opportunity to learn about these key figures in the context of where they lived, worshiped and did some of their greatest work. Mathematics played a major role in architecture and art during the Renaissance, especially with the invention of mathematical linear perspective. This innovation is credited to Brunelleschi, sculptor, architect and artisan-engineer. We saw many examples of linear perspective applied to art and architecture in the church of Santa Maria Novella including Masaccio's 1428 fresco of the Trinity, which is credited as the first painting in the history of art to use perfect linear mathematical perspective.

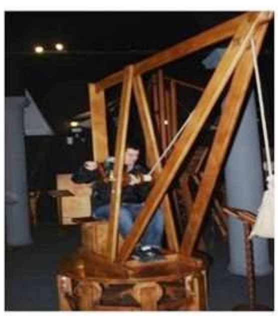
(a) Masaccio's *Trinity* (b) Brunelleschi's dome (c) Giotto's Tower relief

Figure 5: Sample of Florence art

Brunelleschi is also considered the father of Renaissance architecture with its emphasis on symmetry, proportion, geometry and the regularity of parts. Two examples of Brunelleschi's architectural achievements that we saw in Florence were the church of San Lorenzo with its Corinthian columns and geometric balance and harmony, and the dome of Florence's cathedral. This cathedral, known as the Duomo,was begun in 1296 to the design of Cambio who had envisioned a dome for the church even though neither he, nor anyone in Italy at that time had any idea of how to construct it. But the Duomo's early planners believed that by the time building of the cathedral had progressed to the point that the dome was to be built, God would provide a man with the mathematical skill to build such a dome. That man was

Brunelleschi and the students had read the story of the building of this dome in Ross King's book, *Brunelleschi's Dome*, during their prep course. Now they had a chance to climb the largest masonry dome ever built and view the unique herringbone pattern construction of the bricks that allowed it to be built without a support system during construction. Across the street from the Duomo is Giotto's Bell Tower with allegorical reliefs depicting astronomy, medicine, the building art, weaving, navigation, geometry and arithmetic. These decorations recount the destiny of man, from his creation to his dominating the world by learning technology. One relief depicts Gionitus, the mythical inventor of Astronomy, observing the height of celestial bodies using a quadrant.

The Italian region of Tuscany, which includes Pisa and Florence, was Galileo's home, and his work is celebrated and memorialized in the Museo Galileo in Florence. A science guide led us through the museum, explaining how Galileo's instruments worked (the museum houses Galileo's original instruments), how science and mathematics led to the Scientific Revolution and changed the way people viewed and understood the universe. Barometers, telescopes, quadrants, chemical flasks, an all-in-one laboratory table, were just some of the exhibits our guide showed to us. We came away with an appreciation of the variety and extensiveness of the scientific explosion that was taking place across Europe in the 17th and 18th centuries. A day trip out to Pisa gave us the chance to see the Tower of Pisa where Galileo supposedly conducted his experiments on gravity and the chandelier in the Duomo of Pisa where he began to understand pendulum motion.



(a) Crane

(b) Machine gun

(c) Flying machine

Figure 6: Models of Leonardo da Vinci's inventions

Art, architecture, science, inventions, and mathematics were closely intertwined in the Italian Renaissance and no one reflected this collaboration of disciplines more so than Leonardo Da Vinci, the archetypical Renaissance man. At the Leonardo Museum in Florence we had an opportunity to try out some of Da Vinci's inventions. Many of these inventions were very practical work machines. Many were war machines and reflected Da Vinci's understanding that rich rulers were willing to support mathematicians and inventors if these inventions served their military interests. Yet, other of Leonardo's inventions also reflected his visionary and somewhat fanciful designs for flying machines, floats for walking on water and even diving suits for underwater exploration.

4. London: British Royal Society and the Scientific Revolution

The Scientific Revolution may have begun in Italy with Galileo's astronomical observations that supported the Copernican heliocentric universe, but it really took off in England in the scientific community of the British Royal Society, early promoters of the scientific method. A visit to the Royal Society headquarters in London, with a guided tour by the head librarian there, introduced the students to this influential group of individuals who had changed the way science was done: from talk and conjecture to verification by experiments. We saw original scientific papers written by Royal Society members, including Newton's Principia (accompanied by Newton's drawings and cartoon sketches. Other Royal Society items on display were Boyle's air pump, Newton's telescope and Davy's mining lantern.

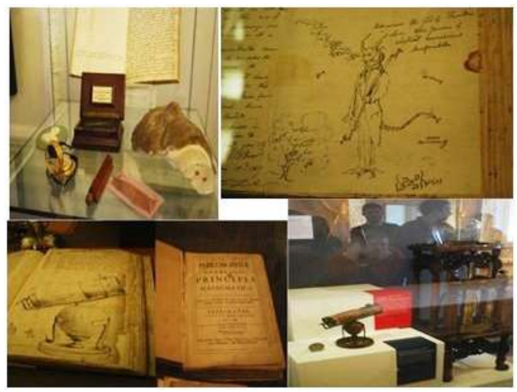


Figure 7: Newton artifacts at the British Royal Society in London

Many members of the Royal Society had worked at Oxford as a part of a scientific research group there. A science tour of Oxford along with a visit to its Museum of Science provided many examples of their work. As part of a workshop at the Museum of Science the students were taught how to use an astrolabe and got an appreciation for how people made sense of their universe. The students also saw how scientific instruments were used to tell time and determine navigational position, and how the development and improved design of these instruments allowed for a better understanding of how the universe worked.

Oxford also provided an opportunity to see one of Christopher Wren's earliest applications of mathematics to architecture in his design of the Sheldonian Theatre. More examples of the influence of Royal Society members in the history of 17th and 18th century science, mathematics and navigation were seen at the Royal Observatory and Maritime Museum in Greenwich and the Science Museum in London. St. Paul's Cathedral in London, Wren's greatest architectural achievement, was full of mathematical elements, from its geometrical staircase to the different mathematical curves that determine each of its three domes. The outer dome is spherical in shape, appealing from a distance but also tapping into the idea of the church representing the spherical shape of the cosmos. The innermost dome, seen from inside, is based on a catenary curve, which serves the dual purpose of an optical illusion—it draws the eye upward, making the dome seem higher than it is, but also provides a real structural advantage as a upside down catenary arch supports its own weight with pure compression and no bending. Finally, middle dome is conical, based on the curve $y = x^3$, which Wren and his fellow BRS friend, Robert Hooke, thought to be the perfect shape for a dome.

Figure 8: The three domes of St. Paul's Cathedral in London

The insides of Wren's churches also reflect the changing worldview of the Reformation with regard to the role of the church building. Wren designed his churches to reflect his view that a congregation should be able to see and hear all that went on in the church. His churches are airy, with windows containing clear glass, so people could have light to read the Scriptures. He also often brought the altars in his churches forward towards the congregation so they could see and hear the liturgy better.

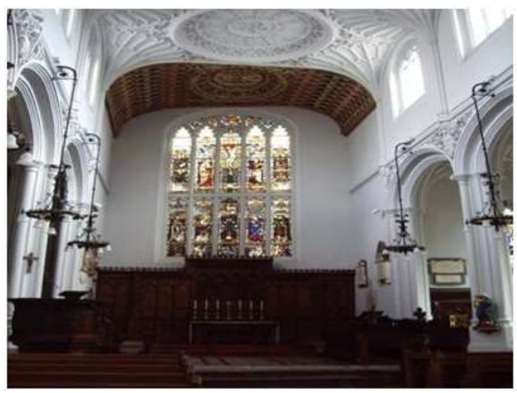


Figure 9: Wren church in London

London provided opportunities to view mathematical artifacts from many different periods of history. At the British Museum in London we were given a behind the scenes viewing of the Rhind Papyrus, which is too delicate to be put on display in the museum galleries. At this museum we also saw examples of Greek architecture, Babylonian mathematical texts, and clocks (important for telling time in a city as well as for navigation) from various centuries. There was also a whole wing devoted to the 18th century Enlightenment period, containing exhibits of mathematical and scientific discoveries of this period.

5. Berlin and Göttingen: Mathematics Centers

Berlin and Göttingen became centers of European mathematics in the 18th and 19th century and Euler, Gauss, Leibniz, Cantor and Riemann were some of the mathematicians that the students learned about during their time in these two cities. One outcome of studying mathematics in a historical context was a deeper awareness of the interplay between mathematics and politics. We saw this in Rome with the way Roman engineering and architecture reflected the worldviews of their rulers. Florence brought further examples with Leonardo da Vinci's war machines, and Galileo's political and religious problems. The members of the Royal Society were caught up in the politics of the English Civil War and the society itself was formed under the Restoration of Charles II. However, the students were surprised to learn that Euler himself got involved in politics, especially during the turmoil during the occupation of Berlin by Russian troops during the Seven Years War. Thus when some of his horses were "borrowed" from his Charlottenburg estate by Russian troops, Euler complained to Frederick of Prussia, at whose request Euler had come to Berlin. Euler demanded compensation, and apparently was generously rewarded. As informal director of the Berlin Academy, Euler assumed many of its administrative duties, which ranged from juggling budgets to overseeing greenhouses. These, and other interesting facts about Euler's daily life, were revealed in some of papers in the Euler's archives, which were translated for us by the academy professor of history and science at the Berlin Academy of Sciences.

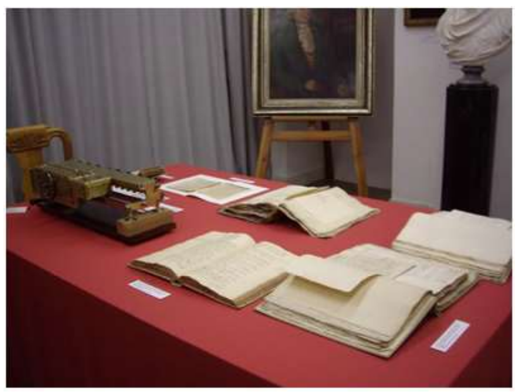
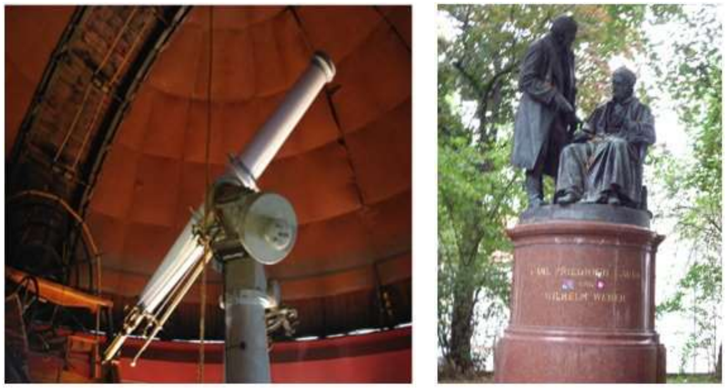


Figure 10: Euler's papers and Leibniz's calculating machine (Berlin Academy of Sciences)

Our visit to the Berlin Academy of Sciences was followed by a day trip out to Göttingen. The students learned about the mathematicians who lived and worked in both these cities and the reasons behind the rivalries between these two centers of mathematics. Markers with the names of famous mathematicians and scientists were on buildings all over Göttingen, denoting the mathematician that had lived or worked there. Göttingen has a small university town feel to it, and Gauss had enjoyed the opportunity to mix with colleagues from variety of disciplines, or keep his distance, as he pleased. He spent much of his time in his observatory, and we had an opportunity to tour this observatory and to see where Gauss did his observations, including the chair in which he sat and the telescope he used. A statue of Gauss and Weber in Göttingen commemorates their collaboration on the invention of the electromagnetic telegraph (1833). They put the system into operation to link their places of work. The statue shows the scientists talking about their joint work with Gauss holding a wire (not preserved) in his right hand, its coil at his feet, while Weber's left hand rests on the telegraphic transmitter. The Göttingen locals like to think of Weber as saying: "Go on Carl, let me sit, too."

Euler's life in Berlin, where he was involved in all aspects of life around him, including household accounts and everyday housekeeping needs of the Berlin Academy, contrasted with Gauss's "quieter" life in Göttingen, where he could choose the degree of his involvement in matters



(a) Gauss's telescope (b) Weber and Gauss statue

Figure 11: Treasures of Göttingen

outside his mathematical interests. Students saw that they were two very different personalities, but each a pillar in German mathematics.

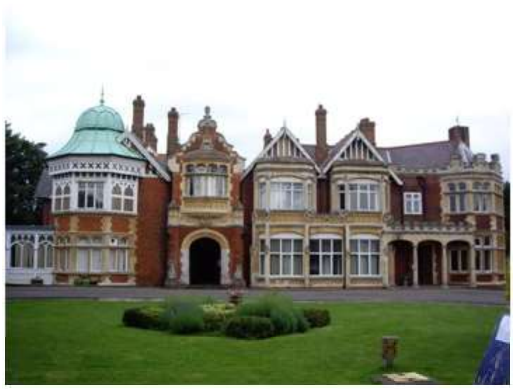
6. WWII Changes the Mathematical Landscape

In the 20th century politics again played an integral role in the mathematical landscape. The persecution of the Jews in the 1930s and 1940's resulted in the migration of many prominent Jewish mathematicians from Germany to the United States. Many of them were applied mathematicians and a new era of applied mathematics resulted in the U.S. Those Jewish mathematicians who remained in Germany lost their jobs, and some, their lives. Readings that students did about this persecution and its ensuing hardships took on new meaning when they visited the Jewish Museum in Berlin and when they saw the many memorials around the city, testifying "Never again."

(a) Jewish museum (b) Holocaust memorial

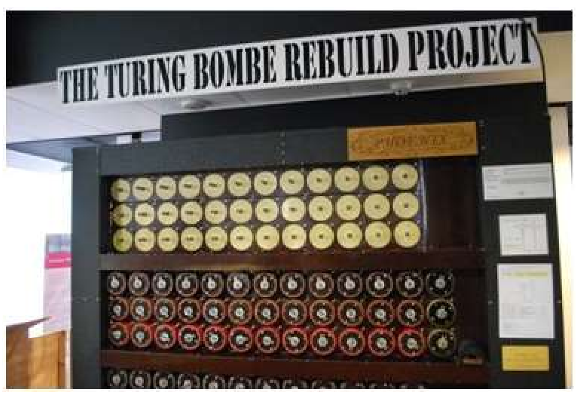
Figure 12: Jewish museum and holocaust memorial (Berlin)

While many mathematicians were being displaced in Germany during the 1930s and 1940s, in England, mathematicians, along with linguists, classicists and "anyone good with puzzles" were being recruited to the British code-breaking facility in Bletchley Park. A Bletchley Park Educational Staff member gave us a tour of the park including the rooms where Alan Turing's Bombe machines were busy working on deciphering the Enigma code. The mathematics behind the Enigma code was explained and students had an opportunity to use a real Enigma machine to encrypt and decrypt a message. Turing, himself often the outcast during his school years, found a home in Bletchley Park, where he thrived on the intellectual camaraderie he found there. His ability to think outside the box, challenging previous assumptions about machine learning, led to his invention of the Bombe and the Turing Machine. A computer science museum on the grounds of Bletchley Park commemorates his, and other computer scientists' work and contains many examples of earliest computers. For the computer science students on the trip, this was a highlight!



(a) Main building

(b) Enigma encryption



(c) Turing bombe

Figure 13: Bletchley Park: England's WWII Cryptography Centre

7. Students' Work and Reflections

In conjunction with our travels, students had daily reading assignments, which gave further background to the time periods, and sites we would be seeing. Many of these readings were original source readings, writings of the mathematicians whose legacies we would be viewing. As we visited the site corresponding to a student's biographical research, that student would give an oral presentation about his/her mathematician. Hearing about the mathematician and then seeing where they lived and instruments they used, contributed to a fuller appreciation of their lives and mathematical contributions. Students reflected on their experiences through daily journaling. They shared about their experiences with family and friends back home through a blog set up on our university website. Each student was assigned a day to blog and was expected to give not only an account of that day's activities, but a historical context to the mathematics encountered. These blogs had an active following back home and generated interest among students back on campus to become part of the next Math History Study Abroad program. You can read the students' blogs and hear about our adventures from their perspectives at http://ma396.blogspot.com/.

When we returned to Whitworth I asked the students to reflect on how their worldview had changed as a result of their experiences abroad. Their comments reflected their deeper appreciation for history and culture. Typical student comments were: "Traveling for this study abroad trip opened my eyes to different cultures and brought the mathematicians to life. As we traveled we also learned a lot about the places we were traveling that helped me to see history from a different perspective;" "I was amazed at how much history is in other cultures. It was cool to walk around a corner and see ruins or an ancient statue or a huge cathedral. There is a lot more to the world than our (American) history and my perspective and worldview;" "Study abroad really opened my eyes to different cultures and how other people's lives are. I got to experience new food, new languages, and met new people. I have a new profound respect for other's cultures." Along with their increased understanding of the historical roots of mathematics the students gained an appreciation of other cultures. All the students expressed a desire to travel abroad again, to continue to grow in their appreciation of the history and worldview perspectives of different cultures. This was as much of a goal for trip as the mathematical component, since it is only by understanding and appreciation a culture that you can appreciate the mathematics that came out of it.

8. Conclusion

Our experiences on this Math History Study Abroad Program confirmed that math history is best learned in a cultural context and that on site experiences add to understanding and appreciation of mathematicians' world and worldviews. A math history study abroad program has benefits for all math, computer science and science students. Future teachers would especially benefit. It makes mathematics "come alive"; imparts the view of mathematics as a continually developing human activity occurring within the framework of the surrounding culture. A Math History Study Abroad program can change the way your students view mathematics. Consider instituting such a program at your university.