COMPUTATIONAL EVALUATION OF A NOVEL APPROACH TO PROCESS PLANNING FOR CIRCUIT CARD ASSEMBLY ON DUAL HEAD PLACEMENT MACHINES

A Thesis

by

NILANJAN DUTTA CHOWDHURY

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2004

Major Subject: Industrial Engineering

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ABSTRACT

Computational Evaluation of a
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Chair of Advisory Committee: Dr. Wilbert E. Wilhelm

Dual head placement machines are commonly used in industry for placing components on circuit cards with great speed and accuracy. This thesis evaluates a novel approach for prescribing process plans for circuit card assembly on dual head placement machines. Process planning involves assigning component types to heads and to feeder slots associated with each head and prescribing appropriate sequences of picking, placing and nozzle-changing steps. The approach decomposes these decisions into four inter-related problems: P1, P2, P3 and P4. This thesis reviews this approach; presents a new heuristic to address P1; a method to facilitate P2 and P3 solutions; a method to control nozzle changes in P4; tests approaches to P1, P2, P3 and P4; and presents a thorough analysis of computational results to evaluate the efficacy of the approach which aims to balance workloads on machine heads to maximize assembly line throughput.

ACKNOWLEDGEMENTS

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It is to be noted that the copyright for the control of this thesis will be held by the journals that publish : Wilhelm, W. E. and Damodaran, P. (2004) (Optimizing Placement Operations on Dual-Head Placement Machines); Wilhelm, W. E. and Arambula, I. (2004) (A Column Generation Approach to Optimizing Picking Operations on Dual-Head Placement Machines); Wilhelm, W. E., Gott, J., Khotekar, N. and Rao, B.V. (2004) (Process Planning for CC Assembly on Dual Head Placement Machines).

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CHAPTER I

INTRODUCTION

With the advent of Large Scale Integration (LSI) and Very Large Scale Integration (VLSI) Technologies in electronics, Surface Mount Technology (SMT) (which involves placing components on the surface of a circuit card (CC)) has virtually displaced the Through-Hole Technology (which involves components being placed on one side of a CC and their pins, which pass through holes on the CC, soldered at the other end). A greater level of automation has been achieved, leading to a more efficient assembly process. CCs can now be assembled at speeds that were previously unthinkable and with a greater degree of accuracy.

A typical SMT assembly line comprises a screen printer that applies solder paste on the CC, placement or pick and place machines to place components on the CC , reflow ovens to melt the solder paste to adhere components to the CC, and inspection stations to inspect the CC after assembly. Placement machines present the bottleneck in a SMT line; hence, it is critical to design a competitive process plan for these machines, balancing workloads on machine heads to maximize the throughput of the line.

This thesis follows the style and model of *IIE Transactions*.

The Dual Head Placement Machine (DHPM) is a particular type of SMT Machine used for assembling large and/or odd shaped components with a great degree of accuracy. Process plans for DHPMs must account for a number of intricate details and a gamut of practical considerations.

Section 1.1 describes the DHPM and related devices in detail. Section 1.2 discusses picking, placing and nozzle changing operations along with certain practical considerations related to these operations.

1.1 Description of the DHPM and related devices

Figure 1 (from Gott and Wilhelm (1999)) depicts the top view of a typical DHPM:

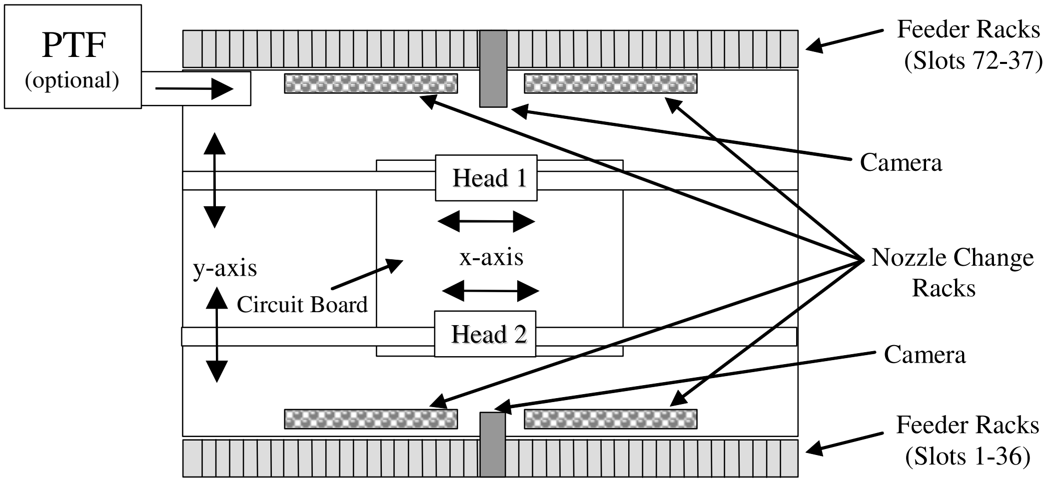

Figure 1. A typical DHPM

The DHPM consists of two heads that can move along the x and y-axes simultaneously, but independently. Heads are distinguished by the nozzle types as well as the lighting and the resolution capabilities of the cameras assigned to them. Each head has a set of four spindles, each of which holds a nozzle that uses vacuum to pick a component. Spindles can be rotated around the z-axis simultaneously, by a common drive motor to achieve proper component orientations. Each head can pick components from a set of two racks, each having 32 feeder slots. Components are affixed to tapes; these tapes are wound about the feeders and the feeders inserted into the slots in the racks. A camera to view component alignment and orientation is mounted between the two racks.

A component type (CT) consists of identical components which require similar slot widths, orientations and nozzle types and hence are wound together on the same tape reel. Individual components in a CT vary only in their respective placement locations on the CC.

Two nozzle change racks are also associated with each head. These consist of nozzle pads, which hold different nozzle types. Operation managers pre-assign nozzle types to heads. Nozzles are picked up from the pads by spindles using vacuum.

1.2 The DHPM picking, placing and nozzle changing operations

A picking step involves the head moving to selected feeder slots, and spindles picking (up to) four components. Then the components are positioned individually for viewing by the camera. Once a component is picked, the feeder advances the tape and a peeler blade removes the tape seal holding the next component in the feeder so that it may be picked next. After a pick has been accomplished, depending on whether a component needs to be placed in the same orientation with which it was picked or not, the spindles may be rotated (around the z-axis) by 0, 90, 180 or 270 degrees. Rotation is fast and the head may rotate spindles while moving to make the next pick.

A component type picking combination (CTPC) is a group of (up to) four CTs that are picked by a head in a picking step. Wilhelm and Arambula (2001) identified five different picks that may be combined to form a CTPC: gang picks, no-move picks, multiple picks, eclectic picks and no-picks.

The DHPM controller requires components to be placed in the same order in which they are picked. The placing step time is the sum of times required by the head to move from the camera to the first placement location and place each picked component at its pre-specified x-y location.

A nozzle changing step involves a head moving from the last placement location to the nozzle changing rack (if a nozzle change is required), moving along the rack to deposit and grasp nozzles and moving from the last nozzle pickup pad to the position on the feeder rack where the first pick will be made for the next CTPC.

Chapter II presents a comprehensive literature review, including a review of the approach that this thesis evaluates (Chapters I and II borrow freely from presentations in Wilhelm and Arambula(2001), Wilhelm and Damodaran (2004) and Wilhelm, Gott, Khotekar and Rao (2004)); Chapter III presents the analytical contributions of this thesis; Chapter IV discusses the experimental design and the computational evaluation, and, presents the conclusions drawn from the tests and outlines some of the future research that may be undertaken in this field.

CHAPTER II

LITERATURE REVIEW

This chapter presents a comprehensive literature review of process planning for SMT machines. A variety of placement machines are available and each presents different sets of restrictions and considerations for process planning. Ayob et al. (2002) provided an extensive survey of the work related to optimizating placement machine operations. According to their survey, placement machines can be classified into dual delivery placement machines (Ahmadi et al. (1988), Chan and Mercer (1989)), multi station placement machines (Wang et al. (1999)), turret style placement machines (Wilhelm and Kiatchai (2003)), multi head placement machines (Hong et al. (2000), Burke et al. (2001)) and sequential pick and place machines (Kumar and Li (1995)). The DHPM (Gott and Wilhelm (1999)) studied in this thesis is structurally different from other types and, to the best of the author's knowledge, has only been studied by Wilhelm, Arambula and Chowdhury (2004), Wilhelm, Chowdhury and Damodaran (2004) and Wilhelm, Gott, Khotekar and Rao (2004).

McGinnis et al. (1992) developed a general framework for process planning in CC assembly. They classified placement machines as sequential or concurrent, depending on the way they operate, but, identified a set of common operations performed by all placement machines.

Crama, Flippo, Van de klundert and Spieksma (1996) and Crama, Flippo, Van de klundert and Spieksma (1997) noted that process planning must prescribe the following decisions: (1) Partition CCs into groups or families, assign families to different lines and determine the order of assembly within these groups (2) Assign sets of CTs to each machine. (3) Assign CTs to feeder slots on each particular machine (the feeder assignment problem) (4) Determine the sequence of component placement at each machine for each CC (5) Determine an appropriate retireval plan if a CT is assigned to more than one feeder. Decisions 2-4 are pertinent to the approach reviewed in this thesis.

The dual delivery machine (the DYNAPERT MPS500) studied by Ahmadi, Grotzinger and Johnson (1988), had two heads, which operated independently of each other, mounted on a single arm. The arm could move only in the y-direction and the machine had two component carriers that could pick components from ski vibratory feeders or reels and move in the x-direction to fixed positions to pick. Their study addressed the problem of determining the number of feeders to be assigned as well as the assignment to carriers and the assignment of tools (or nozzles). Their mixed integer programming (MIP) model minimized the idle time that resulted from operational imbalances, excess rotations of heads, and nozzle changes. In a related work, Ahmadi et al. (1995) devloped heuristics to address the feeder positioning problem (assigning feeders to feeder slots) associated with the DYNAPERT MPS500. Ahmadi and Kouvelis (1994) also solved the staging problem (allocation of components to feeder carriers and nozzles used to pick components) associated with the the dual-delivery placement machine- DYNAPERT MPS500 using a Lagrangian Relaxation based branch-and-bound procedure.

Chan and Mercer (1989) studied the Universal Machine with two heads mounted on the same beam and an xy table that positioned the board for placement. They modeled the chip insertion problem as a traveling salesman problem. Wang et al. (1999) developed a Genetic Algorithm to optimize feeder assignment for the Fuji QP-122 multi station placement machine. Wilhelm and Kiatchai (2003) studied tandem turret-type placement machines. They proposed a set of heuristics to address inter-related decisions of allocating CTs to a placement machine, assigning each CT to a feeder slot and sequencing of placements. Hong et al. (2000) developed a biological immune algorithm based on the human immune system with the goal of minimizing CC assembly times on multi-spindle single head machines.

Most algorithm developers have approached process planning by applying heuristics or integer programming techniques to solve decomposed problems. Grotzinger (1992) used a linear MIP formulation to solve the feeder assignment problem associated with concurrent machines. Crama, Flippo, Van de Klundert and Spieksma (1996) solved the problem of retrieving CTs assigned to more than one feeder slot by devising a polynomial time algorithm.

This thesis evaluates an approach that was devised by Wilhelm (1999), decomposes these decisions into four related problems: P1, P2, P3 and P4. P1 assigns CTs to heads and to feeder slots for each head. P2 prescribes component type picking combinations (CTPCs) to minimize picking time. P3 prescribes individual components for each placing step to minimize placement time. P4 prescribes a sequence of picking, placing and nozzle-changing steps that minimizes the time between placement and picking steps. The goal of P1- P4 is to balance workloads assigned to heads. The workload assigned to a head h, W_h, is the total time the head is involved in picking, placing and nozzle-changing for a particular CC; the cycle time for the line is defined as $\max_{h \epsilon H}\{W_h\}$ (where H is the set of DHPM heads in the line) and the workload imbalance (as a percentage) is defined as:

$$100(\max_{h\,\epsilon H}\{W_h\} - \overline{W})/\overline{W}\text{, where } \quad \overline{W} = (1/|H|)\sum_{h\,\epsilon H}\{W_h\},$$

Sections 2.1, 2.2, 2.3 and 2.4 review the approaches P1, P2, P3 and P4 respectively, that were at hand when this thesis research began.

2.1 P1

P1 assigns component types (CTs) to slots on feeder racks, indirectly attempting to maximize the number of gang picks, minimize the number of nozzle changes, and balance the workload assigned to all heads. Two approaches were developed to address P1: H1 (Wilhelm (2001b)), H2 (Khotekar (2001)). It is, of course, not possible to determine exactly how many gang picks will result until P2 is solved, the time it will take to place components until P2 and P3 are solved, or how many nozzle changes will be needed or what the workload balance will be until P2, P3, and P4 are solved. Thus, P1 works with indirect measures to initiate a CT assignment to feeder slots.

The inputs to P1 are the kinematic parameters that define head movements; cameras; and lighting; nozzles assigned to each head; the number of CTs to a CC type; the number of components in each CT; head restrictions (if any); and, for each CT, feeder width, nozzle type and orientation required. P1 assigns each CT to a particular, head, machine, rack and feeder slot (*fhmr)* combination.

The following sub-sections review heuristics H1 and H2, which address P1.

2.1.1 Heuristic H1

H1 (Wilhelm (2001b)) forms groups of CTs based on similarity of nozzle requirements, orientations, and widths and assigns groups sequentially, each to the least loaded rack in the attempt to promote efficient operations by maximizing the number of gang picks. H1 is a list processing heuristic of low-order polynomial time complexity.

H1 involves three steps: the first checks feasibility, the second sorts CTs according to selected charecteristics, and the third assigns CT groups to feeder slots in the sorted order.

The feasibility checking step assures that, after reserving enough slots to accommodate CTs with head restrictions, enough slots remain to accommodate CTs with no head restrictions.

The sorting step sorts CTs according to width, nozzle type (within width), orientation (within nozzle type) and frequency (within nozzle type). This procedure then defines each CT group as CTs with same nozzle, feeder width and orientation requirements. Finally, the assignment step allocates the sorted list of CT groups to feeder slots, seeking to maximize gang picks. At each step, the assignment procedure determines the least loaded rack based on a surrogate measure of workload and then assigns a group of CTs from the top of the list of sorted CTs to this least loaded rack. Assignment starts from the first empty slot closest to the camera to ensure a low measure of surrogate workload. If the number of CTs in a group exceeds the number of slots available (which is determined by another procedure), the group is split, the assignment step fillas all availabe slots, and the rest of the CTs in the group are resorted into the list of unassigned CTs to be allocated in another iteration.

2.1.2 Heuristic H2

Heuristic H2 (Khotekar (2001)) forms groups of CTs as does H1 but it also forms super groups based on CT nozzle similarity and assigns CT super groups to the least loaded rack at each step, with the goals of maximizing gang picking and minimizing the number of nozzle changes. The first step in H2 checks feasibility. The second step sorts CTs according to the nozzle type, feeder width (within nozzle type) and orientation (within each nozzle-width category). CT groups are then formed and sorted. Finally, the sorting procedure forms super groups, each with the same nozzle requirement. The formation of super groups promotes efficiency by seeking to minimize the number of nozzle changes, maximize gang picks and ordering assignments (those with greater widths further away from the camera) relative to impact on the surrogate workload. The assignment procedure is similar to the one used by H1 but it assigns super groups to the least loaded rack at each step with the goal of minimizing the number of nozzle changes (as well as maximizing the number of gang picks).

2.2 P2

P2 prescribes CTPCs to minimize picking times. A CTPC is a group of (up to) four CTs that are picked by a head on a picking step. The definition of a CTPC includes the CT picked by each spindle and the order in which the CTs are picked. Wilhelm and Arambula (2001) identified five different types of picks, which may be combined to form CTPCs. Wilhelm (2001c) modeled the P2 problem as an integer set covering program with the objective of minimizing total picking time (which includes the time for the head movement along feeder racks while picking, the time to pick all components and display them at the camera).

The objective sought to minimize the total picking time for all CTPCs. The first set of inequalities ensured all components in each CT will be picked and the next set of inequalities imposed non-negativity, upper-bound (the lowest frequency of all CTs in the CTPC) and integer requirements. The integer value of a decision variable prescribes the number of times the associated CTPC is used to pick.

To prescribe an optimal integral solution, Wilhelm and Arambula (2001) devised a Type-II (Wilhelm (2001a)) column generation approach to solve the linear relaxation of the problem at each node in the branch and bound tree, using specially constructed sub-problems that are constrained shortest path problems (CSPPs) to generate columns defining CTPCs. Sub-problem networks represented the order in which the spindles picked in a CTPC, so there were 4! sub-problem networks. In each of these networks, there were 4 layers, each representing a spindle picking order and 16 columns of nodes representing a feeder slot in head, machine, rack (*hmr*) combination. Arcs in the network were labeled with appropriate reduced costs that depended on the time required to pick components (including the time to rotate the components if proper orientation was required) and on the time required to view and display the components.

To solve each sub-problem, a shortest path was found from the dummy start node to the dummy end node, subject to the resource constraints posed by the spindles. Each sub-problem network was expanded to allow the CSPP to be solved as a SPP on the expanded network using a method described by Wilhelm (2003). At each iteration, the current values of dual variables in the restricted master problem (RMP) were used to update the reduced costs on the arcs. For each network, the shortest path found would be an improving column if it had a negative reduced cost and the column with the most negative reduced cost in all the sub-problem networks would be the entering column for that iteration. The linear relaxation of the model was solved at each node in the branch and bound tree, branching on the most fractional variable.

2.3 P3

P3 (Wilhelm and Damodaran (2001)) minimizes the total time to place all the components and prescribes the individual components that are to be placed on each step, based on the CTPCs prescribed in P2. Wilhelm (2001d) formulated P3 as a binary set covering problem.

The objective function sought to minimize the total placing time for all placing steps. The first set of inequalities ensured that all components were placed (a set covering formulation was used as a relaxation of the set partitioning formulation). The second set of inequalities ensured that P3 uses each CTPC the number of times prescribed by P2.

Wilhelm and Damodaran (2001) devised a type-II (Wilhelm (2001a)) column generation approach to solve the linear relaxation of the problem at each node in the B&B tree, using specially constructed sub-problems, which are CSPPs, to generate columns defining individual components placed on each placing step. Each of the sub-problem networks represented a CTPC prescribed by P2. Each network had levels determined by CTs in the CTPC and the ordering of the levels was determined by the sequence in which the CTs were picked. Each node in a level corresponded to an individual component comprising the CT. Arcs were labeled with the time for the placement operation of that particular component (in the first layer this would be the time taken for head to move from the camera to the first placing position and place that component), an appropriate reduced cost and amount of resource (i.e., time) required by the operation.

The CSPP was solved using a pseudo-polynomial time algorithm based on the work of Wilhelm, Damodaran and Li (2003), which used dynamic programming to construct an expanded network on which the CSPP was solved as a SPP in polynomial time. Each path through a sub-problem network represented a column and the shortest path through each network represented an improving column; the column with the minimum reduced cost would be entered into the basis in each iteration.

At each branch and bound node, the variable with the largest fractional part was branched on and two child nodes were created at which the variable was fixed to 0 and 1, respectively. At the left child node (i.e., at which the variable is fixed to 0), any column in the basis that includes that variable must be removed. The expanded network must be modified to ensure that other no path through the network uses the variable set to 0. Algorithm Bypass, which is described in Section 3.2, was devised for that purpose.

2.4 P4

P4 uses the CTPCs prescribed by P2, the set of placing steps prescribed by P3, and minimizes inter-round times, the time taken between successive rounds for the head to move from the location at which the last component is placed on a placing step, to the feeder location of the first component to be picked in the next picking step. This includes the time for the head to move from the last placement to the nozzle change rack (if nozzle changing is required), the time taken to change nozzles, and the time to move from the nozzle change rack to the feeder location of the first component to be picked on the next picking step .

The solution to P4 involves:

1. Devising a scheme for nozzle changes and a method to calculate the times required to change nozzles under different scenarios. (This is discussed in the next chapter)
2. Formulating an asymmetric traveling salesman problem (ATSP) with nodes representing picking, placing, and nozzle-changing steps and arcs modeling specific segments of inter round time between relevant pairs of nodes.
3. Solving the ATSP using a Fortran code developed by Carpaneto et al. (1990)

The solution to a TSP gives the total workload (i.e., time required by a head to place all components assigned to both of its racks, including picking, placing and inter-round times). Times assigned to all heads must then be compared to measure the workload balance achieved by a solution prescribed by P1, P2, P3, and P4.

CHAPTER III

ANALYTICAL CONTRIBUTIONS

The first contribution of this thesis involved a laborious debugging of individual computer programs written by students to solve P1 (i.e., H1 and H2), P2, P3 and P4. The second contribution involved analytical contribution to enhance approaches of the four problems including developing a new heuristic for P1, implementing a means of branching in the special case in which a fractional (integer) variable is fixed to zero in P2 and P3 and devising a scheme for nozzle changes, in P4. The third contribution of this work was to assure appropriate interfaces were devised so that these programs communicate effectively with each other. Fourth, efficient means were developed to run this suite of programs and tabulate results. This chapter deals with the analytical contributions (the second set of contributions). Section 3.1 discusses the development of H3 to address P1, section 3.2 discusses the bypass algorithm for P2 and P3 and finally section 3.3, discusses a nozzle-change strategy for P4.

The fifth type of contribution, which was the primary contribution of this thesis research, was to design and perform experiments to evaluate this approach with regards to process planning for DHPMs. As discussed in Chapter IV, this research devised a set of practical test cases, designed an appropriate experiment to evaluate the efficacy of the approach, tabulated test results, analyzed the results, and, drew conclusions about the efficacy of the approach.

3.1 Heuristic H3 to address P1

Heuristic H3 (Chowdhury (2004)) divides the number of CTs by the number of racks to balance the number of CTs assigned to each head, forms groups and super groups as does H2 and assigns individual CTs in a group until a rack is full or until the desired number of CTs have been assigned to the rack. H3 seeks to minimize the number of nozzle changes, maximize gang picking and minimize the workload imbalance among the heads.

H3 numbers racks sequentially; for example, racks numbered 0 and 1 are associated with head 1 on machine1 and racks numbered 2 and 3 are associated with head 2 and so on. The first step uses the sorting procedure developed by Khotekar (2001) to sort groups and super groups of CTs. The assignment procedure, then, uses the sorted list, assigning CTs sequentially to racks, checking at each step if the rack limit ŋ (ratio of the number of CTs to the number of racks) has been reached. The sequential assignments of sorted CTs are made to slots closest to the camera. If the ŋ limit is reached for a rack, the next assignment is made to the next rack, considering the rack numbering specified above. After each assignment, the list of filled slots for the particular h, m, r combination is updated. In case ŋ is an odd number, the lower bound $\lfloor \eta \rfloor$ is allocated to each rack and to allocate the remaining CTs, the racks are again surveyed and these CTs are assigned to empty slots in racks, next to CTs with similar nozzle or orientation requirements. The assignment, like H1 and H2, promotes gang picking by assigning CT groups together if possible, seeking to minimize nozzle changes by assigning as many CTs of a super group as possible to the same rack and, most importantly, maintaining a balance of CTs allocated to heads. Since the time spent placing components dominates picking and nozzle changing times, it is expected that balancing the number of CTs assigned to heads will typically lead to better workload balances.

3.2 Algorithm Bypass for P2 and P3

In the special case of P2 and P3, in the general case that branching fixes a variable to the value zero, the expanded network must be modified to eliminate any column in the basis that includes the associated path in the the expanded network, which must be updated to assure that this path will not be prescribed as optimal. This thesis research devised a technique, Algorithm Bypass (Wilhelm, Arambula and Chowdhury (2004)) to implement this restriction. Some additional notation has to be defined before detailing the technique:

l	index set of levels associated with the sub-problem network (l= 1, 2…$l_{\max}$)
Π_p	index set of arcs on path p, which is restricted from being optimal
γ_p	index set of nodes in levels 1- $l_{\max}$ on path p, which is restricted from being optimal, $\gamma_p = \{ v_l : l = 1,\ldots, l_{\max} \}$, where node v_l is in level l for (l= 1, 2,…… $l_{\max}$) and $1 \leq l_{\max} \leq 4$, depending on the sub-problem network.

Γ_p	index set of nodes on path p, which is to be eliminated, $\Gamma_p = \{Sta, \gamma_p, En\}$ (*Sta* and *En* are the starting and ending dummy nodes of the network respectively)
I_v	index set of nodes from which arcs that point to node υ emanate
$\lvert I_v \rvert$	in-degree of node υ
O_v	index set of nodes to which arcs that emanate from node υ point
$\lvert O_v \rvert$	out-degree of node υ
q_a	reduced cost associated with arc a
$\sum_{a \in \Pi^*} q_a$	the reduced cost associated with the non-basic column (i.e., feasible solution defined by the path from node *Sta* to *En*)

Algorithm Bypass :

1. If $\lvert I_v \rvert = \lvert O_v \rvert = 1$ for all $\upsilon \in \gamma_p$, set $q_{(v_{l\max,},En)} = Big\_M$
2. Else, find $l^* = \arg\min\{l: \lvert I_{v_l} \rvert > 1, l = 2, \ldots.. l_{max}-1\}$ and set $q_{(v_{l^*-1}, v_{l^*})} = Big\_M$

3. If $\lvert O_{v_l} \rvert > 1$ for any $l = l^*, \ldots\ldots l_{max}-l$, augment arc (v_{l^*-1}, v) where $\upsilon \in O_{v_l} \backslash \{ \upsilon_{l+1} \}$ (for $l = l^*, \ldots\ldots, l_{max}-1$) and set $q_{(v_{l^*-1}, v_{l^*})}$ = sum of reduced costs associatd with arcs on bypassed path $(v_{l^*-1}, v_{l^*} \ldots. v_l, v)$.

The Algorithm has to ensure that the path, which is restricted from being optimal, cannot be prescribed as optimal while allowing other feasible paths that share any subset of arcs on the restricted path.

In step 1, there is no common arc between Π_p (arcs on the path to be restricted- *p*) and any other path in the network so a very high cost (a *Big_M* cost) is assigned to the last arc on path *p* so that it cannot be prescribed optimal (owing to its high cost).

If one or more arcs are common between *p* and other paths), it has to be ensured that those arcs must not be excluded.

In Step 2, path Π_p is traversed, starting with the dummy node *Sta* and the first node v_{l*} with $| I_v| > 1$ is identified. If the out-degree of node v_{l*} is one, then, since the arc pointing to v_{l*} from the node in the above layer along Π_p ((v_{l*-1}, v_{l*})) is unique to path *p*, assigning a *Big_M* cost to this arc would prevent path *p* from being prescribed as optimal.

In the case that there is more than one arc emanating from v_{l*}, it must be ensured that excluding the arc (v_{l*-1}, v_{l*}) will not exclude other feasible paths sharing this arc. In step

3, an arc is augmented from node v_{l*-1} to each node v in levels $l= l*+1,\ldots l_{max}$ that were originally reached by edges (v_{l*-1}, v_{l*}). Thus, path ($v_{l*-1}, v_{l*},\ldots\ldots, v$) is bypassed by a single arc. This bypassing arc is assigned a reduced cost equal to the sum of reduced costs of arcs on the path ($v_{l*-1}, v_{l*},\ldots\ldots, v$). This augmented network is used at the child node in the branch and bound tree to prescribe the path with the minimum reduced cost that excludes the path set to zero by branching. Algorithm Bypass is facilitated by the structure of the expanded networks. At descendant nodes in the B&B tree, the reduced cost is updated on each arc, the *Big_M* cost is retained on each arc designated by the the Algorithm Bypass. Wilhelm (2004) detailed a proof of the Algorithm Bypass.

3.3 A greedy heuristic for nozzle changing in P4

Each type of nozzle provided to a head is assigned a specific pad(s) where it is stored on the nozzle change rack. Inputs to the heuristic include the number of nozzle types, number of nozzles of each type and the specific pad on which nozzles of each type are stored. At each iteration, the heuristic determines the last placement location on the CC of the last step, assesses the present nozzle configuration on the spindles and the nozzle configuration desired, then determines which pads will be left empty, computes the number of nozzle changes involved and, finally, determines the first pick point for the next CTPC.

The heuristic finds the appropriate nozzle changing strategy by using one of two strategies that requires the least time to change nozzles. In the first strategy, the head deposits nozzles, moving from left to right on the nozzle change rack, then graps needed nozzle types as it moves from right to left. Each deposit is made on the leftmost pad that can store the given nozzle type. Each nozzle is picked up from the nearest pad that holds nozzles of the given type, starting with the neighbor nearest to the last deposit pad. After all the required nozzles have been picked up by the head, it moves from the last pickup point to the feeder rack slot from which the first pick will be made in the next CTPC.

The second strategy deposits nozzles moving from right to left on the nozzle change rack and then grasps needed nozzle types as it moves to the left to right.

Finally, the total nozzle change time, including time to move from last placement location on the CC to the first nozzle deposit pad; time to move along the nozzle change rack depositing, then picking nozzles; time to deposit/ pickup nozzles; and time to move from the last nozzle pickup pad to the feeder rack slot from which the first pick will be made in the next CTPC, is compared for strategies 1 and 2 and the strategy that gives the shorter total time is prescribed.

CHAPTER IV

COMPUTATIONAL EVALUATION AND CONCLUSIONS

This chapter discusses the factors used to evaluate the approach, the generation of test instances and the computational results for each of the decomposed problems. Section 4.1 discusses the results for P2, section 4.2 describes the results for P3, and finally, section 4.3 presents the results for P4 (relative to the P1 heuristics). All tests were performed on a Pentium II PC (with 400 MHz and 128 MB RAM). Specific languages and softwares used in different problems are discussed in the separate sections describing them.

4.1 P2 computational results

This section describes the experiments used to evaluate the efficacy of the column generation approach of Wilhelm and Arambula (2001) in solving P2. The programs were coded in C in the Watcom-C editor and all tests were performed interfacing with MINTO 3.0a and CPLEX 4.0. Certain details have not been divulged due to a non-disclosure agreement with the industrial collaborator. Sub-sections describe the factors used to evaluate the approach, the generated test instances, and computational results.

4.1.1 Experimental design

The experimental design assigns levels to each of the factors to evaluate its effect on a range of performances. Factor 1 has three levels corresponding to the three heuristics H1 (level 1), H2 (level 2) and H3 (level 3) (described in chapter III) by which CTs are assigned to the feeder slots on each DHPM, to evaluate the sensitivity of workload balances that result from three different but related strategies. Factor 2 specifies the number of DHPMs: (level 1) 1 and (level 2) 2. Factor 3 defines the number of CTs and width of each. Two levels were selected: (level 1) 32 CTs, each requiring 2 slots and (level 2) 64 CTs, each requiring 1 slot. This fills all slots on a single DHPM, but when two DHPMs are used (level 2 of factor 2) this set of CTs fills only half of the available slots. Factor 4 designates the number of components of each CT: (level 1) 10 components and (level 2) a number generated from a discrete uniform distribution on [5, 15]. Factor 5 provides either (level 1) two or (level 2) four types of nozzles to each head, with four copies of each. Level 1 assigns a nozzle type to each CT using DU[1, 2]; and level 2, DU[1, 4], to require a wider variety of nozzle types. Factor 6 assigns an orientation to each CT. The levels of this factor comprised two empirical distributions selected to study requirements of different types. These factors were selected to represent industrial applications and to provide a realistic set of data.

Factors and Levels are as follows:

1. Assignment of CTs to DHPMs to slots on each machine: H1 or H2 or H3

2. Number of DHPMs: 1or 2
3. Number of CTs : 32 CTs, each 2 slots wide or 64 CTs, each 1 slot wide
4. Number of components of each: CT 10 or DU[5,15]
5. Nozzle type assigned to each CT: DU[1, 2] or DU[1, 4]
6. Orientation angle assigned to each CT:

Degrees	Probability distribution
0, 90, 270 or 180	0.4, 0.3, 0.2, 0.1
0, 90, 270 or 180	0.25, 0.25, 0.25, 0.25

A unique combination of levels of all factors characterizes each of the 96 test instances. A case is defined as a set of instances with a similar level of a factor (e.g. instances with a single machine). For each test instance, the number of CTs, the number of components in each CT, the nozzle type and the orientation required for each CT is randomly generated. The P1 heuristic assigns CTs to DHPMs and to feeder slots on each DHPM. Instances with a single DHPM required solution of 4 rack problems while those with two DHPMs required solution of 8 rack problems, leading to a total of 576 problems in P2 and P3. In contrast, P4 solves a problem for each head to prescribe a solution to each instance.

4.1.2 Test results

Tables 3, 4 and 5 record test results associated with H1, H2 and H3, respectively. Tables 3, 4 and 5 give overall measures of performance; columns 1-7 describe the instance and columns 8-12 summarize test results. A P2 problem is solved for each head, machine, rack combination separately, but, to conserve space, the tables give composite results for all rack problems (4 racks for 1 DHPM and 8 racks for 2 DHPMs, representing levels 1 and 2 of factor 2). The acronyms that head the columns of Tables 3, 4 and 5 are defined below in table 1 and for tables 6,7 and 8 in table 2:

Table 1. Acronyms of columns for tables 3, 4 and 5

Column	Acronym	Description
1	Instance #	Instance number
2	F1 H#	Factor 1: heuristic number (i.e., H1, H2, H3)
3	F2 #M	Factor 2: number of DHPMs
4	F3 #CT	Factor 3: number of CTs (i.e., 32 or 64)
5	F4 # C/CT	Factor 4: number of components per CT
6	F5 #NT	Factor 5: nozzle type assignment
7	Theta	Factor 6: CT orientation
8	Total RT	Total run time to prescribe optimal solutions to all rack problems
9	Max RT	Maximum run time to solve any rack problem

Table 1

(contd.)

Column	Acronym	Description
10	#SP Solved	Number of sub-problems solved
11	#Improv cols	Number of improving columns generated
12	#Entrd cols	Number of columns entered
13	#B&B Nodes	Number of branch and bound nodes required to optimize all rack problems
12	Total RT	Total run time to prescribe optimal solutions to all rack problems
13	Max RT	Maximum run time to solve any rack problem

Tables 6, 7 and 8 provide detailed measures associated with individual rack problems; their columns are headed by the following acronyms:

Table 2. Acronyms of columns for tables 6,7 and 8

Column	Acronym	Description
1	Instance #	# Instance number
2	Rack #	Rack number
3	*%GAP*	$\%GAP = 100(Z_{IP}^{*} - Z_{LP}^{*}) / Z_{LP}^{*}$
4	#NodesSP	Number of nodes in all sub-problems networks
5	#ArcsSP	Number of arcs in all sub-problem networks
6	#NodesEXP	Number of nodes in all expanded networks
7	#ArcsEXP	Number of arcs in all expanded networks
8	SPs #	Number of sub-problems (and CTPCs)

Table 3. Summary of results for P2 using Heuristic H1

Instance # 1	F1 H# 2	F2 #M 3	F3 #CT 4	F4 #NT 5	F5 #C/CT 6	F6 Theta 7	Total RT 8	Max RT 9	#SP Solved 10	#Improv Cols 11	#Entrd Cols 12	#B&B Nodes 13
1	1	1	32	1	10	1	2	0.55	1320	1051	83	6
2	1	1	32	1	10	2	4.66	1.31	1368	1019	85	4
3	1	1	32	2	10	1	2.47	0.93	648	363	55	9
4	1	1	32	2	10	2	4.66	1.3	1368	1091	85	4
5	1	1	32	1	[5,15]	1	9.84	3.41	2664	2011	123	4
6	1	1	32	1	[5,15]	2	9.84	3.41	2664	2011	123	16
7	1	1	32	2	[5,15]	1	5.78	1.89	1608	1107	95	11
8	1	1	32	2	[5,15]	2	9.34	3.76	2472	1695	113	17
9	1	1	64	1	10	1	6.11	1.83	1656	1541	129	4
10	1	1	64	1	10	2	8.74	2.59	2376	1827	159	4
11	1	1	64	2	10	1	4.59	1.29	2544	2059	166	4
12	1	1	64	2	10	2	7.75	2.44	2064	1847	146	6
13	1	1	64	1	[5,15]	1	13.54	5.51	3360	2482	187	20
14	1	1	64	1	[5,15]	2	16.82	6.21	3984	2704	204	11
15	1	1	64	2	[5,15]	1	12.39	4.97	3048	2494	180	11
16	1	1	64	2	[5,15]	2	12.64	6.31	3432	2463	177	24
17	1	2	32	1	10	1	4.46	1.15	1084	801	73	28
18	1	2	32	1	10	2	3.22	0.61	656	347	56	10
19	1	2	32	2	10	1	4.05	0.68	660	294	64	14
20	1	2	32	2	10	2	3.37	0.63	540	320	54	18
21	1	2	32	1	[5,15]	1	9.59	5.73	2308	1679	104	32
22	1	2	32	1	[5,15]	2	3.71	0.77	704	476	58	24
23	1	2	32	2	[5,15]	1	1.33	0.25	654	410	64	28
24	1	2	32	2	[5,15]	2	1.25	0.2	678	405	61	25
25	1	2	64	1	10	1	2.28	0.52	1326	731	119	24
26	1	2	64	1	10	2	1.99	0.55	1248	678	108	24
27	1	2	64	2	10	1	2.11	0.51	1344	814	115	24
28	1	2	64	2	10	2	1.81	0.27	1224	803	107	23
29	1	2	64	1	[5,15]	1	2.31	0.59	1350	938	114	22
30	1	2	64	1	[5,15]	2	2.39	0.46	1488	1009	120	22
31	1	2	64	2	[5,15]	1	2.21	0.37	1368	1043	118	21
32	1	2	64	2	[5,15]	2	2.37	0.44	1512	1044	119	24

Table 4. Summary of results for P2 using Heuristic H2

Instance #	F1 H#	F2 #M	F3 #CT	F4 #NT	F5 #C/CT	F6 Theta	Total RT	Max RT	#SP Solved	#Improv Cols	#Entrd Cols	#B&B Nodes
1	2	3	4	5	6	7	8	9	10	11	12	13
33	1	1	32	1	10	1	1.94	0.61	1272	914	81	4
34	1	1	32	1	10	2	1.57	0.58	1104	901	74	4
35	1	1	32	2	10	1	1.87	0.5	1200	924	78	4
36	1	1	32	2	10	2	2.07	0.61	1296	1077	82	4
37	1	1	32	1	[5,15]	1	2.35	0.72	1416	1078	87	17
38	1	1	32	1	[5,15]	2	2.29	0.79	1440	1043	85	21
39	1	1	32	2	[5,15]	1	2.23	0.75	1248	951	78	4
40	1	1	32	2	[5,15]	2	2.41	1.7	3024	1941	130	25
41	1	1	64	1	10	1	1.89	1.75	3192	2445	193	8
42	1	1	64	1	10	2	2.25	1.75	2832	2382	178	19
43	1	1	64	2	10	1	1.4	0.84	2496	2173	164	6
44	1	1	64	2	10	2	2.32	1.07	2856	2109	179	13
45	1	1	64	1	[5,15]	1	1.91	1.46	2616	1983	169	20
46	1	1	64	1	[5,15]	2	2.22	1.64	2520	2081	160	25
47	1	1	64	2	[5,15]	1	2.52	1.95	2664	1686	169	23
48	1	1	64	2	[5,15]	2	0.97	0.43	504	504	81	23
49	1	2	32	1	10	1	0.97	0.16	504	504	81	8
50	1	2	32	1	10	2	0.21	0.16	120	30	10	8
51	1	2	32	2	10	1	0.95	0.16	490	113	50	8
52	1	2	32	2	10	2	0.92	0.13	456	110	50	10
53	1	2	32	1	[5,15]	1	0.64	0.17	311	93	26	9
54	1	2	32	1	[5,15]	2	1.11	0.31	702	194	56	26
55	1	2	32	2	[5,15]	1	1.13	0.28	533	146	53	23
56	1	2	32	2	[5,15]	2	1.15	0.28	592	161	58	9
57	1	2	64	1	10	1	1.99	0.47	1134	724	106	24
58	1	2	64	1	10	2	1.92	0.44	1176	762	101	22
59	1	2	64	2	10	1	2.03	0.46	1272	754	109	24
60	1	2	64	2	10	2	1.91	0.32	1248	756	104	10
61	1	2	64	1	[5,15]	1	2.04	0.45	1182	812	108	27
62	1	2	64	1	[5,15]	2	1.2	0.17	864	483	73	9
63	1	2	64	2	[5,15]	1	2.05	0.34	1224	968	123	12
64	1	2	64	2	[5,15]	2	2.67	0.37	1536	1167	125	19

Table 5. Summary of results for P2 using Heuristic H3

Instance # 1	F1 H# 2	F2 #M 3	F3 #CT 4	F4 #NT 5	F5 #C/CT 6	F6 Theta 7	Total RT 8	Max RT 9	#SP Solved 10	#Improv Cols 11	#Entrd Cols 12	#B&B Nodes 13
65	1	1	32	1	10	1	1.89	0.49	1272	942	81	6
66	1	1	32	1	10	2	2.11	0.61	1416	1067	86	5
67	1	1	32	2	10	1	1.85	0.58	1200	1020	78	7
68	1	1	32	2	10	2	1.82	0.53	1200	986	77	12
69	1	1	32	1	[5,15]	1	3.55	1.74	2592	2009	116	32
70	1	1	32	1	[5,15]	2	4.91	3.13	3744	2791	141	54
71	1	1	32	2	[5,15]	1	2.16	0.69	1512	1254	87	8
72	1	1	32	2	[5,15]	2	5.12	1.96	3600	2572	151	49
73	1	1	64	1	10	1	4.47	1.48	2664	2307	171	3
74	1	1	64	1	10	2	4.14	1.46	2304	1980	156	4
75	1	1	64	2	10	1	5.05	1.61	2736	2400	174	14
76	1	1	64	2	10	2	7.12	2.39	4056	3281	219	17
77	1	1	64	1	[5,15]	1	5.49	1.6	3168	2783	189	7
78	1	1	64	1	[5,15]	2	42.1	37.6	17136	13243	543	233
79	1	1	64	2	[5,15]	1	4.92	1.23	2592	1740	164	19
80	1	1	64	2	[5,15]	2	0.97	0.43	504	504	81	4
81	1	2	32	1	10	1	0.95	0.22	648	389	51	19
82	1	2	32	1	10	2	0.9	0.2	648	340	51	18
83	1	2	32	2	10	1	0.95	0.16	696	404	53	16
84	1	2	32	2	10	2	1.13	0.2	816	434	58	21
85	1	2	32	1	[5,15]	1	1.13	0.2	792	541	57	21
86	1	2	32	1	[5,15]	2	1.14	0.17	816	554	58	18
87	1	2	32	2	[5,15]	1	1.02	0.14	768	576	56	14
88	1	2	32	2	[5,15]	2	1.1	0.16	792	500	57	16
89	1	2	64	1	10	1	1.85	0.3	1248	710	108	6
90	1	2	64	1	10	2	1.64	0.22	1152	638	104	12
91	1	2	64	2	10	1	1.49	0.22	1032	619	99	10
92	1	2	64	2	10	2	2.04	0.32	1320	783	111	12
93	1	2	64	1	[5,15]	1	2.43	0.39	1464	1012	117	22
94	1	2	64	1	[5,15]	2	2.59	0.37	1656	1017	125	17
95	1	2	64	2	[5,15]	1	2.31	0.42	1512	1023	119	20
96	1	2	64	2	[5,15]	2	2.04	0.31	1344	895	112	12

Column 3 gives the *%GAP* for the rack problem, where Z_{LP}^{*} is the value of the optimal solution to the linear relaxation and Z_{IP}^{*} is the value of the optimal integer solution.

Tables 3, 4 and 5 report the effect of each factor on the run time. Factor 1, the heuristic used to assign CTs to feeder slots, has a substantial effect on run time. H2 prescribes CT assignments that lead to lower run times for picking operations than those prescribed by H1 or H3 (1.58 seconds in H2 versus 5.61 seconds in H1 and 3.28 seconds in H3). However, the degree to which a heuristic balances workloads on heads is a better measure of its performance, so P2 run times do not indicate that H2 is preferred over H1 or H3.

Factor 2, the number of DHPMs has a substantial effect on the run time. For all three heuristics, level 2 (i.e., 2 DHPMs) results in lower P2 run times than level 1. Since the same number of CTs are distributed across four racks in the level 1 case (one DHPM) and eight in the level 2 case, the two DHPM case deals with fewer CTs on each rack and hence, results in smaller run times on each rack and smaller run times overall (even though they involve more rack problems).

Run times increases with factor 3, the number of CTs, since the number of sub-problem decision variables increases with an increase in the number of CTs. Factor 4, the number of components in each CT, also shows a substantial effect on run time. Most instances that employ level 1 of the factor 4, which assigns 10 components to each CT, do not require no picks (because the number of CTs on each rack is a multiple of 4) and promote gang picks. However, level 2, which assigns a random number of components from a discrete uniform distribution [5, 15] to each CT, may require no picks, which tend to require longer run times for P2 to identify an optimal set of CTPCs.

Factor 5, nozzle type assignment to each CT, which primarily affects the nozzle-changing time, does not show a consistent effect on run time (it is not relevant to picking operations).

Finally, factor 6, assignment of an orientation to each CT, affects the efficiency of picking operations because only CTs requiring the same orientation may be gang picked. Level 1, a decreasing empirical probability distribution, assigns an orientation of 0° to 40% of the CTs (thus facilitating gang picking of CTs with an orientation of 0°) while level 2 assigns an orientation of 0° to only 25% of the CTs. Overall, however, level 2 describes greater similarity among orientations, promoting gang picking, which results in fewer CTPCs and, hence, lower run times.

Figures 2-5 graphically depict run time relationships. Figure 2 shows run time as a function of the instance number as well as the number of CTs. This figure shows that the heuristics exhibit comparable run times, although H2 requires slightly less run time because it entails solution of fewer sub-problems and smaller branch and bound trees.

Figures 3, 4 and 5 show run times versus the number of sub-problems solved for H1, H2 and H3, respectively. In all cases, run time increases (approximately) linearly with the number of sub-problems solved. A few instances (13, 14, 15, 16 and 78) require relatively large numbers of sub-problems (3360, 3984, 3048, 3432 and 17136, respectively) to be solved. A few instances (70, 78) require relatively large branch and bound trees (54 and 233 nodes respectively), leading to relatively large run times.

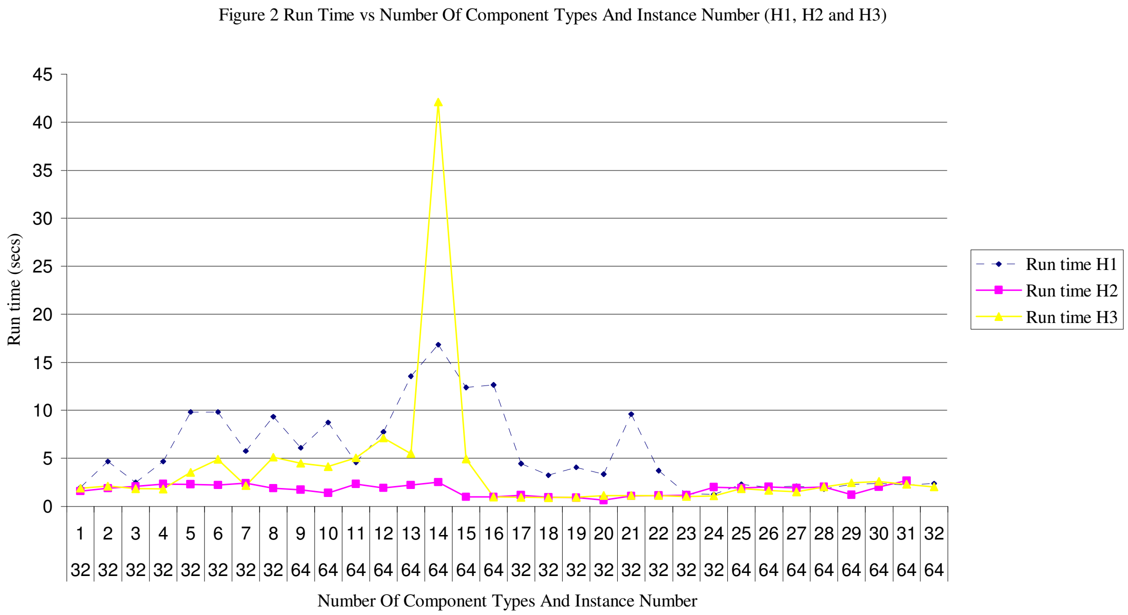

Fig. 2. Run time vs number of component types and instance number (H1, H2 and H3).

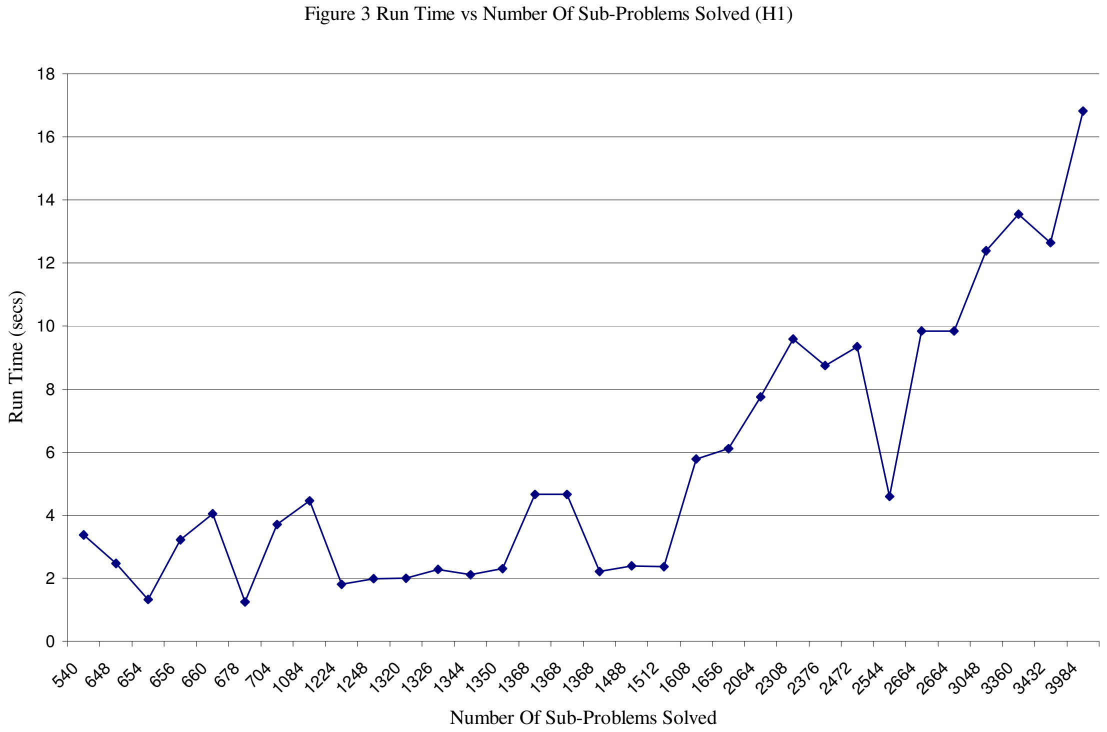

Fig. 3. Run time vs number of sub-problems solved (H1).

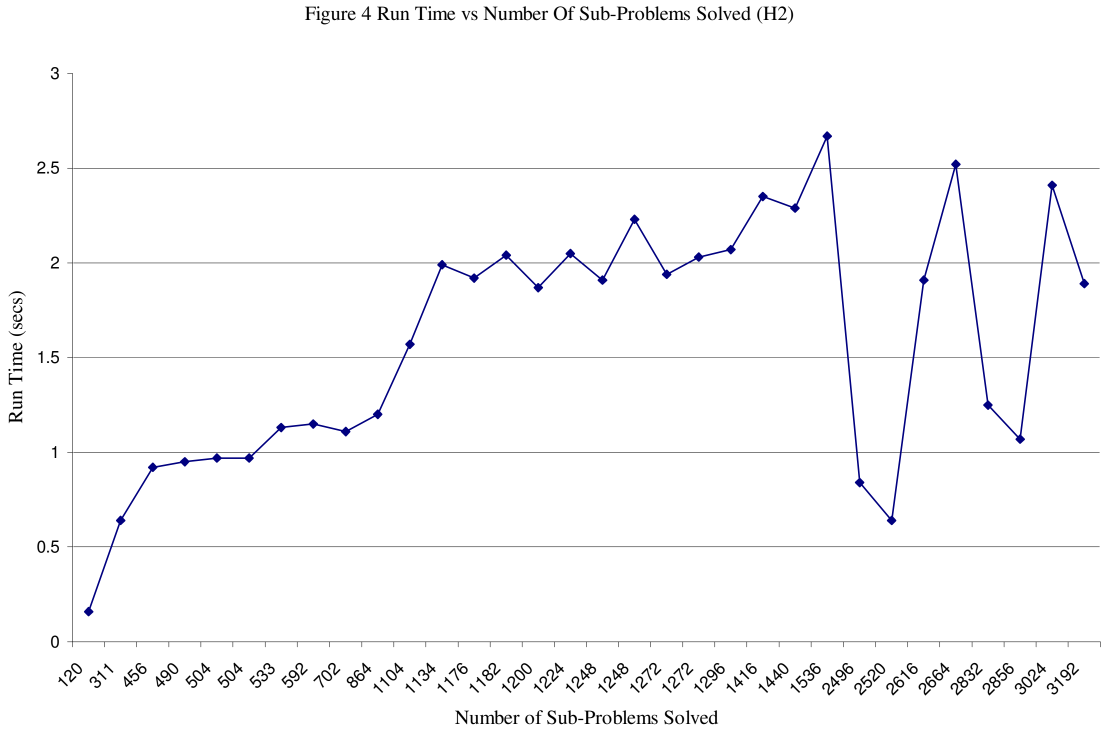

Fig. 4. Run time vs number of sub-problems solved (H2).

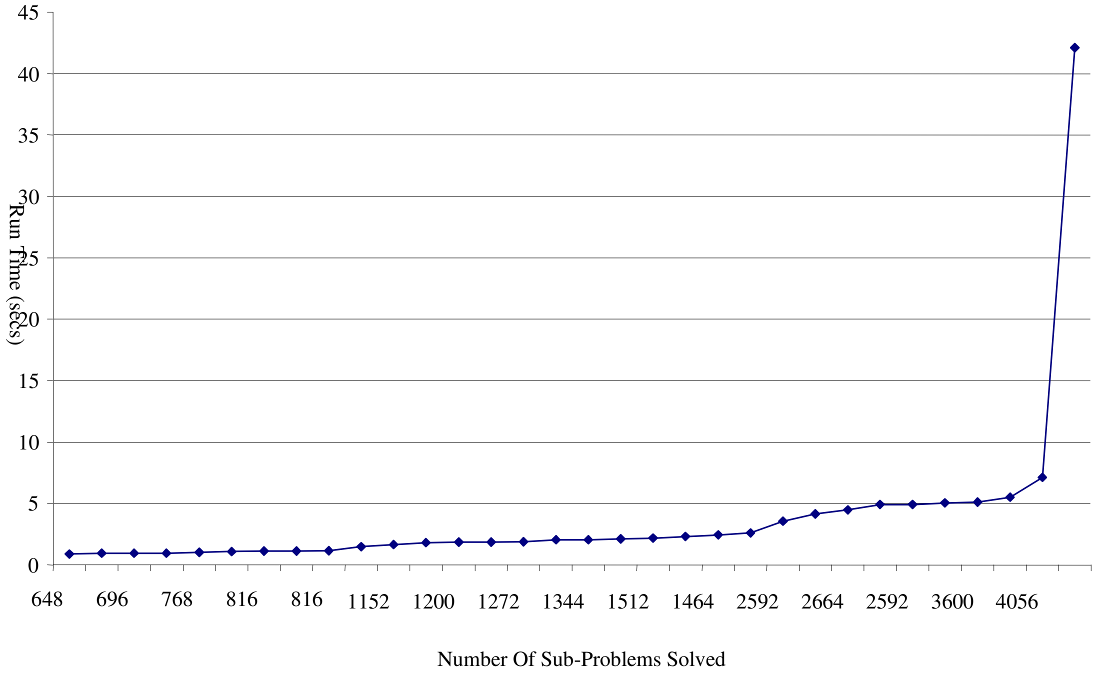

Fig. 5. Run time vs number of sub-problems solved (H3).

4.1.3 Statistical analysis

MINITAB v 13.1 was used to analyze the factorial designs, first for each heuristic individually and then all three together, as a means of identifying factors that most markedly affect run time.

MINITAB's estimated effects and coefficients tables shows that, for all H1, H2 and H3 (i.e., factor 2, the number of machines), had the greatest effect on run time. Factor 3, factor 4 and the interaction term of factors 2 and 3 also had marked effects on run time. The effect term for factor 2 was negative indicating, that level 2 (2 DHPMs) had lower run times than level 1 (1 DHPM). For H1, H2 and H3 taken together, the analysis showed that Factor 1 (i.e., the heuristic used to assign CTs to feeder slots) had the strongest effect in determining run times.

4.1.4 Overall performance measures

Column 9 of tables 3, 4 and 5 shows that a particular rack problem often takes more run time than do other rack problems associated with an instance. Since H1, H2 and H3 assign CTs to promote gang picking, leftover CTs, which are not compatible with gang picking, might be assigned together on one rack so that more time is required to identify an optimal set of CTPCs. Columns 10-12 of tables 3, 4 and 5 show that the column generation scheme typically solves a large number of sub-problems for each improving column it identifies and enters only a portion of them into solution. Maintaining a column pool to manage improving columns could improve the implementation.

Column 13 of tables 3, 4 and 5 show that (with a few exceptions) relatively few branch and bound nodes are required to solve each instance. Of the 192 rack problems associated with each heuristic, 92 for H1, 130 for H2 and 99 for H3 rack problems solved at the root node (racks with *%GAP* =0 in tables 4, 5 and 6). This would indicate that the integer optimal solution is found frequently at the root node and, hence, indicates that the model used is “tight” and promotes effectiveness.

Table 6. Results for individual rack problems using heuristic H1

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
1	2	3	4	5	6	7	8	9	10	11	12
1	0	0.0	0	100	0	0	0	768	2592	788	2439
	1	0.0	0	100	0	0	0	768	2592	1286	3195
	2	0.0	0	83.333	16.667	0	0	768	2592	1030	2729
	3	5.6	0	65.217	14.493	1.449	18.84	768	2592	788	2439
2	0	0.0	0	50	50	0	0	768	2592	1030	2729
	1	0.0	0	66.667	33.333	0	0	768	2592	1116	2860
	2	0.0	0	66.667	33.333	0	0	768	2592	1061	2869
	3	0.0	0	100	0	0	0	768	2592	1286	3195
3	0	0.0	0	53.333	13.333	6.667	26.67	530	1528	696	1574
	1	4.4	0	73.913	0	8.696	17.39	740	2676	1067	2849
	2	3.6	0	51.19	29.762	7.143	11.9	932	4084	2165	5369
	3	0.0	0	33.333	66.667	0	0	740	2676	1050	2862
4	0	0.0	0	50	50	0	0	768	2592	1371	3227
	1	0.0	0	83.333	16.667	0	0	768	2592	1614	3570
	2	0.0	0	66.667	33.333	0	0	768	2592	1854	3803
	3	0.0	0	83.333	16.667	0	0	768	2592	1696	3721
5	0	0.0	0	92.982	0	0	7.018	768	2592	788	2439
	1	0.0	0	87.5	0	2.083	10.42	768	2592	1286	3195
	2	0.0	0	74.667	12	4	9.333	768	2592	1030	2729
	3	0.0	0	68.116	20.29	4.348	7.246	768	2592	788	2439
6	0	0.2	0	45	36.667	0	18.33	768	2592	1061	2869
	1	0.0	0	53.03	28.788	0	18.18	768	2592	1116	2860
	2	1.3	0	53.03	28.788	0	18.18	768	2592	1030	2729
	3	4.7	0	74.074	0	0	25.93	768	2592	1286	3195
7	0	0.0	0	48.611	29.167	0	22.22	768	2592	1202	3015
	1	0.7	0	85.417	0	0	14.58	768	2592	1202	3015
	2	1.2	0	58.974	33.333	0	7.692	768	2592	1524	3435
	3	0.7	0	51.667	38.333	1.667	8.333	768	2592	1061	2869
8	0	0.1	0	40.909	39.394	6.061	13.64	768	2592	1734	3736
	1	0.0	0	40.909	39.394	6.061	13.64	768	2592	2033	4037
	2	10.7	4.55	60.606	18.182	1.515	15.15	768	2592	1371	3227
	3	0.0	0	27.778	57.407	0	14.81	768	2592	1392	3293
9	0	0.0	0	83.333	16.667	0	0	1392	8094	1862	8226
	1	0.0	0	83.333	16.667	0	0	1392	8094	2016	8828
	2	0.0	0	83.333	16.667	0	0	1392	8094	1953	8435
	3	0.0	0	100	0	0	0	1392	8094	1831	8445
10	0	0.0	0	66.667	33.333	0	0	1392	8094	2176	9156
	1	0.0	0	83.333	16.667	0	0	1392	8094	2017	8637
	2	0.0	0	100	0	0	0	1392	8094	2176	9156
	3	0.0	0	75	25	0	0	1392	8094	2248	9444
11	0	0.0	0	66.667	33.333	0	0	1392	8094	2572	9618
	1	0.0	0	66.667	33.333	0	0	1392	8094	2703	1001
	2	0.0	0	66.667	33.333	0	0	1392	8094	3018	1030
	3	0.0	0	75	25	0	0	1392	8094	2514	9570
12	0	0.0	0	83.333	16.667	0	0	1392	8094	3549	1155
	1	2.0	0	62.121	18.182	1.515	18.18	1392	8094	3243	1076
	2	0.0	0	83.333	16.667	0	0	1392	8094	3216	1088

Table 6
(contd.)

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
	3	0.0	0	75	25	0	0	1392	8094	2921	1044
13	0	0.0	0	75	25	0	0	1392	8094	1831	8445
	1	0.0	0	75	25	0	0	1392	8094	2016	8828
	2	0.1	0	48.611	41.667	1.389	8.333	1392	8094	1862	8226
	3	0.1	0	62.162	18.018	1.802	18.02	1392	8094	1953	8435
14	0	0.0	0	80.702	0	7.895	11.4	1392	8094	2176	9156
	1	0.0	0	80.702	0	7.895	11.4	1392	8094	2017	8637
	2	0.0	0	58.696	29.71	0	11.59	1392	8094	2176	9156
	3	0.9	0	58.696	29.71	0	11.59	1392	8094	2248	9444
15	0	0.0	0	31.852	48.148	0	20	1392	8094	2830	1026
	1	0.0	0	66.667	14.141	0	19.19	1392	8094	3189	1088
	2	0.0	0	66.667	14.141	0	19.19	1392	8094	2674	9980
	3	0.9	0	37.607	47.863	3.419	11.11	1392	8094	2550	9730
16	0	0.0	0	37.607	47.863	3.419	11.11	1392	8094	3549	1155
	1	0.0	4.5	58.559	26.126	0	10.81	1392	8094	3612	1183
	2	0.0	4.5	58.559	26.126	0	10.81	1392	8094	2625	9761
	3	0.0	0	57.778	22.963	2.222	17.04	1392	8094	2406	9328
17	0	0.0	0	100	0	0	0	768	2592	788	2439
	1	0.0	14.3	47.619	0	14.29	23.81	442	978	462	929
	2	0.0	0	0	0	100	0	4	3	0	24
	3	0.0	0	0	0	100	0	4	3	0	24
	4	0.0	0	0	0	100	0	4	3	0	24
	5	0.0	0	100	0	0	0	768	2592	788	2439
	6	0.0	0	0	0	100	0	4	3	0	24
	7	0.0	0	100	0	0	0	768	2592	788	2439
18	0	0.0	0	0	0	100	0	4	3	0	24
	1	0.0	0	0	0	100	0	15	21	16	48
	2	0.0	0	83.333	0	0	16.67	628	1862	648	1744
	3	27.4	14.3	47.619	0	14.29	23.81	442	978	462	929
	4	0.0	0	83.333	0	0	16.67	628	1862	648	1744
	5	0.0	0	0	0	100	0	4	3	0	24
	6	0.0	0	83.333	0	0	16.67	708	2292	728	2150
	7	0.0	0	66.667	0	11.11	22.22	538	1408	558	1322
19	0	15.7	0	0	0	100	0	74	135	79	162
	1	15.7	0	0	0	100	0	74	135	79	162
	2	0.0	0	0	0	100	0	15	21	17	49
	3	0.0	0	66.667	0	11.11	22.22	538	1408	558	1322
	4	0.0	0	66.667	16.667	0	16.67	708	2292	970	2440
	5	0.0	0	66.667	16.667	0	16.67	442	978	657	1164
	6	15.7	0	0	0	100	0	74	135	79	162
	7	0.0	0	66.667	0	11.11	22.22	538	1408	558	1322
20	0	7.8	0	0	0	100	0	74	135	100	183
	1	0.0	0	66.667	0	11.11	22.22	538	1408	831	1673
	2	27.3	14.3	47.619	0	14.29	23.81	442	978	657	1164
	3	15.7	0	0	0	100	0	74	135	79	162
	4	0.0	0	66.667	0	11.11	22.22	538	1408	558	1322
	5	0.0	0	66.667	22.222	11.11	0	538	1408	800	1612
	6	15.6	0	0	0	100	0	74	135	79	162
	7	27.3	14.3	47.619	0	14.29	23.81	442	978	462	929

Table 6
(contd.)

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
21	0	0.2	0	92.982	0	0	7.018	768	2592	788	2439
	1	1.7	0	66.667	0	18.18	15.15	442	978	462	929
	2	0.0	0	0	0	100	0	4	3	0	24
	3	0.0	0	0	0	100	0	4	3	0	24
	4	0.0	0	0	0	100	0	4	3	0	24
	5	0.0	0	98.485	0	0	1.515	768	2592	788	2439
	6	0.0	0	0	0	100	0	4	3	0	24
	7	0.6	0	85.185	0	0	14.81	768	2592	788	2439
22	0	4.2	0	77.083	0	4.167	18.75	628	1862	648	1744
	1	16.5	0	0	0	100	0	15	21	16	48
	2	0.0	0	0	0	100	0	4	3	0	24
	3	1.6	0	66.667	0	18.18	15.15	442	978	462	929
	4	0.0	0	0	0	100	0	4	3	0	24
	5	0.0	0	78.788	0	10.61	10.61	628	1862	648	1744
	6	36.3	17.8	62.22	0	20	0	538	1408	558	1322
	7	3.1	0	80	0	10	10	708	2292	728	2150
23	0	29.6	0	0	0	100	0	74	135	79	162
	1	20.0	0	0	0	100	0	74	135	79	162
	2	12.9	4	60	16	5.333	14.67	708	2292	970	2440
	3	11.8	0	0	0	100	0	15	21	17	49
	4	15.8	6.67	75.556	0	8.889	8.889	538	1408	558	1322
	5	0.0	6.67	75.556	0	8.889	8.889	442	978	657	1164
	6	33.7	0	0	0	100	0	74	135	79	162
	7	4.0	0	66.667	0	19.44	13.89	538	1408	558	1322
24	0	33.9	17.6	41.176	27.451	13.73	0	538	1408	831	1673
	1	0.0	0	63.889	16.667	11.11	8.333	538	1408	800	1612
	2	16.5	7.14	54.762	28.571	7.143	2.381	442	978	657	1164
	3	20.0	0	0	0	100	0	74	135	79	162
	4	11.8	0	0	0	100	0	15	21	17	49
	5	15.8	6.67	75.556	0	8.889	8.889	538	1408	558	1322
	6	14.9	0	0	0	100	0	74	135	79	162
	7	13.2	6.67	62.222	0	4.444	26.67	538	1408	831	1673
25	0	32.2	14.3	23.81	23.81	14.29	23.81	370	771	288	584
	1	0.0	6.9	68.966	0	12.64	11.49	932	4084	930	3630
	2	16.5	6.9	68.966	0	12.64	11.49	454	1126	421	935
	3	1.8	0	85.714	0	4.762	9.524	1112	5612	1110	5054
	4	11.8	14.3	23.81	23.81	14.29	23.81	370	771	288	584
	5	15.8	0	0	0	100	0	14	19	14	45
	6	14.9	0	83.333	0	0	16.67	1272	7080	1270	6446
	7	13.2	0	85.714	0	4.762	9.524	1192	6324	1190	5728
26	0	32.2	0	73.913	0	8.696	17.39	740	2676	738	2340
	1	0.0	0	53.333	13.333	6.667	26.67	530	1528	497	1279
	2	16.5	0	73.913	0	8.696	17.39	740	2676	738	2340
	3	20.0	0	80	0	6.667	13.33	836	3344	834	2949
	4	11.8	6.9	68.966	0	12.64	11.49	932	4084	930	3630
	5	15.8	14.3	23.81	23.81	14.29	23.81	370	771	288	584
	6	14.8	0	79.798	0	8.081	12.12	1022	4818	1020	4312

Table 6
(contd.)

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
	7	13.2	0	73.913	0	8.696	17.39	740	2676	738	2340
27	0	1.6	0	17.544	42.105	10.53	29.82	530	1528	696	1574
	1	0.0	10.5	17.544	35.088	10.53	26.32	530	1528	628	1472
	2	16.5	0	55.556	22.222	0	22.22	836	3344	1112	3337
	3	20.0	0	79.798	0	8.081	12.12	1022	4818	1492	5156
	4	11.8	0	66.667	11.111	0	22.22	1022	4818	1492	5108
	5	15.8	0	66.667	11.111	0	22.22	740	2676	1016	2728
	6	14.9	0	53.333	13.333	6.667	26.67	530	1528	497	1279
	7	13.2	0	66.667	0	6.667	26.67	644	2080	642	1803
28	0	0.0	0	44.444	33.333	0	22.22	740	2676	993	2763
	1	0.0	0	48.611	18.056	0	33.33	740	2676	993	2763
	2	16.5	0	66.667	0	6.667	26.67	644	2080	863	2116
	3	20.0	0	44.444	27.778	11.11	16.67	740	2676	993	2763
	4	11.8	0	66.667	0	6.667	26.67	644	2080	642	1803
	5	15.8	0	50.575	22.989	12.64	13.79	932	4084	1319	4289
	6	14.9	0	66.667	0	6.667	26.67	644	2080	896	2177
	7	13.2	0	80	0	6.667	13.33	836	3344	1158	3513
29	0	0.0	0	81.481	0	5.556	12.96	1112	5612	1110	5054
	1	0.0	3.33	82.222	0	8.889	5.556	932	4084	930	3630
	2	16.5	7.14	33.333	14.286	30.95	14.29	370	771	288	584
	3	20.0	7.14	33.333	14.286	30.95	14.29	454	1126	421	935
	4	11.8	0	21.429	21.429	30.95	26.19	370	771	288	584
	5	15.8	0	0	0	100	0	14	19	14	45
	6	14.9	2.56	76.068	0	12.82	8.547	1272	7080	1270	6446
	7	14.9	0	82.828	0	7.071	10.1	1192	6324	1190	5728
30	0	0.0	0	85	0	0	15	740	2676	738	2340
	1	0.0	0	85	0	0	15	530	1528	497	1279
	2	16.5	0	73.016	0	4.762	22.22	740	2676	738	2340
	3	20.0	8	74.667	0	2.667	14.67	836	3344	834	2949
	4	11.8	17.8	62.22	0	20	0	370	771	288	584
	5	15.8	5.88	79.412	0	7.843	6.863	932	4084	930	3630
	6	14.9	0	81.111	0	3.333	15.56	1022	4818	1020	4312
	7	13.2	4.55	72.727	0	7.576	15.15	740	2676	738	2340
31	0	0.0	4.55	72.727	0	7.576	15.15	530	1528	696	1574
	1	0.0	11.8	74.51	0	8.824	4.902	1022	4818	1492	5108
	2	16.5	4.35	34.783	27.536	4.348	28.99	836	3344	1112	3337
	3	20.0	9.38	71.875	0	10.42	8.333	1022	4818	1492	5156
	4	11.8	9.38	71.875	0	10.42	8.333	740	2676	1016	2728
	5	15.8	4	64	0	10.67	21.33	644	2080	642	1803
	6	14.9	5.26	54.386	0	29.82	10.53	530	1528	497	1279
	7	13.2	5.26	54.386	0	29.82	10.53	530	1528	628	1472
32	0	8.2	3.45	52.874	26.437	11.49	5.747	932	4084	1319	4289
	1	0.0	75	4.1667	12.5	0	8.333	740	2676	993	2763
	2	16.5	0	28.986	46.377	8.696	15.94	740	2676	993	2763
	3	20.0	4.76	60.317	3.1746	7.937	23.81	644	2080	863	2116
	4	11.8	4.76	60.317	3.1746	7.937	23.81	644	2080	896	2177
	5	15.8	4.55	60.606	3.0303	12.12	19.7	644	2080	642	1803
	6	14.9	10.3	56.322	19.54	5.747	8.046	836	3344	1158	3513
	7	13.2	0	84.058	0	1.449	14.49	740	2676	993	2763

Table 7. Results for individual rack problems using heuristic H2

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
1	2	3	4	5	6	7	8	9	10	11	12
33	0	0.0	0	33.333	66.667	0	0	768	2592	788	2439
	1	0.0	0	66.667	33.333	0	0	768	2592	1286	3195
	2	0.0	0	66.667	33.333	0	0	768	2592	1030	2729
	3	0.0	0	50	50	0	0	768	2592	788	2439
34	0	0.0	0	50	50	0	0	768	2592	1030	2729
	1	0.0	0	50	50	0	0	768	2592	1116	2860
	2	0.0	0	50	50	0	0	768	2592	1061	2869
	3	0.0	0	33.333	66.667	0	0	768	2592	1286	3195
35	0	0.0	0	33.333	66.667	0	0	530	1528	696	1574
	1	0.0	0	33.333	66.667	0	0	740	2676	1067	2849
	2	0.0	0	50	50	0	0	932	4084	2165	5369
	3	0.0	0	50	50	0	0	740	2676	1050	2862
36	0	0.0	0	33.333	66.667	0	0	768	2592	1371	3227
	1	0.0	0	33.333	66.667	0	0	768	2592	1614	3570
	2	0.0	0	50	50	0	0	768	2592	1854	3803
	3	0.0	0	50	50	0	0	768	2592	1696	3721
37	0	3.7	0	55	40	3.333	1.667	768	2592	788	2439
	1	0.0	0	52.632	40.351	0	7.018	768	2592	1286	3195
	2	7.5	0	30.556	40.278	9.722	19.44	768	2592	1030	2729
	3	0.3	0	49.275	43.478	2.899	4.348	768	2592	788	2439
38	0	6.6	0	19.444	72.222	5.556	2.778	768	2592	1061	2869
	1	0.0	0	31.944	48.611	2.778	16.67	768	2592	1116	2860
	2	0.3	0	31.579	50.877	5.263	12.28	768	2592	1030	2729
	3	37.1	13.6	22.727	48.485	15.15	0	768	2592	1286	3195
39	0	0.0	13.6	22.727	48.485	15.15	0	768	2592	1202	3015
	1	0.0	13.6	22.727	48.485	15.15	0	768	2592	1202	3015
	2	0.0	0	50	29.63	16.67	3.704	768	2592	1524	3435
	3	0.0	0	56.14	33.333	0	10.53	768	2592	1061	2869
40	0	10.6	0	36.667	28.333	8.333	26.67	768	2592	1734	3736
	1	0.0	0	36.667	28.333	8.333	26.67	768	2592	2033	4037
	2	1.1	0	18.519	53.086	13.58	14.81	768	2592	1371	3227
	3	4.0	0	34.615	43.59	15.38	6.41	768	2592	1392	3293
41	0	0.0	0	58.333	41.667	0	0	1392	8094	1862	8226
	1	0.0	0	34.921	47.619	0	17.46	1392	8094	2016	8828
	2	0.7	0	24.242	62.121	1.515	12.12	1392	8094	1953	8435
	3	0.0	0	41.667	50	0	8.333	1392	8094	1831	8445
42	0	0.0	0	41.667	50	0	8.333	1392	8094	2176	9156
	1	0.0	0	33.333	66.667	0	0	1392	8094	2017	8637
	2	3.5	0	29.63	42.222	1.481	26.67	1392	8094	2176	9156
	3	4.7	2.27	21.212	56.818	0	19.7	1392	8094	2248	9444
43	0	0.0	0	33.333	66.667	0	0	1392	8094	2572	9618
	1	0.0	0	33.333	66.667	0	0	1392	8094	2703	1001
	2	0.0	0	33.333	66.667	0	0	1392	8094	3018	1030
	3	0.2	0	25.397	58.73	0	15.87	1392	8094	2514	9570

Table 7
(contd.)

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
44	0	0.0	0	58.333	41.667	0	0	1392	8094	3549	1155
	1	0.7	0	38.211	47.967	2.439	11.38	1392	8094	3243	1076
	2	0.8	0	18.605	67.442	1.55	12.4	1392	8094	3216	1088
	3	0.0	0	31.746	58.73	1.587	7.937	1392	8094	2921	1044
45	0	1.6	0	42.636	43.411	0	13.95	1392	8094	1831	8445
	1	0.0	0	42.636	43.411	0	13.95	1392	8094	2016	8828
	2	0.0	0	19.82	56.757	0	23.42	1392	8094	1862	8226
	3	10.1	2.27	25	50	3.03	19.7	1392	8094	1953	8435
46	0	0.1	0	31.532	47.748	0	20.72	1392	8094	2176	9156
	1	0.0	15.2	18.18	51.51	0	15.15	1392	8094	2017	8637
	2	1.9	0	27.132	55.039	0	17.83	1392	8094	2176	9156
	3	1.8	0	23.577	53.659	0.813	21.95	1392	8094	2248	9444
47	0	0.2	0	26.389	53.472	1.389	18.75	1392	8094	2830	1026
	1	0.0	6.84	37.6	37.6	4.27	13.68	1392	8094	3189	1088
	2	3.8	0	23.188	54.348	0.725	21.74	1392	8094	2674	9980
	3	0.0	0	23.188	54.348	0.725	21.74	1392	8094	2550	9730
48	0	0.0	3.7	31.48	53.7	0	11.11	1392	8094	3549	1155
	1	0.0	11.1	37.037	40.74	0	11.11	1392	8094	3612	1183
	2	0.0	16.7	16.67	50	0	16..67	1392	8094	2625	9761
	3	0.0	7.4	39.5	39.5	0	13.58	1392	8094	2406	9328
49	0	0.0	0	16.667	58.333	8.333	16.67	768	2592	788	2439
	1	0.0	0	0	0	100	0	442	978	462	929
	2	0.0	0	0	0	100	0	4	3	0	24
	3	0.0	0	0	0	100	0	4	3	0	24
	4	0.0	0	0	0	100	0	4	3	0	24
	5	0.0	0	0	0	100	0	768	2592	788	2439
	6	0.0	0	33.333	33.333	0	33.33	4	3	0	24
	7	0.0	0	0	0	100	0	768	2592	788	2439
50	0	0.0	0	0	0	100	0	4	3	0	24
	1	0.0	0	0	0	100	0	15	21	16	48
	2	0.0	0	0	0	100	0	628	1862	648	1744
	3	0.0	0	0	0	100	0	442	978	462	929
	4	0.0	0	33.333	33.333	0	33.33	628	1862	648	1744
	5	0.0	0	0	0	100	0	4	3	0	24
	6	0.0	0	0	0	100	0	708	2292	728	2150
	7	0.0	0	16.667	58.333	8.333	16.67	538	1408	558	1322
51	0	0.0	0	0	88.889	0	11.11	74	135	79	162
	1	0.0	0	0	0	100	0	74	135	79	162
	2	0.0	0	33.333	33.333	0	33.33	15	21	17	49
	3	0.0	0	33.333	33.333	0	33.33	538	1408	558	1322
	4	0.0	0	0	88.889	0	11.11	708	2292	970	2440
	5	0.0	0	0	0	100	0	442	978	657	1164
	6	0.0	0	0	0	100	0	74	135	79	162
	7	0.0	0	0	0	100	0	538	1408	558	1322
52	0	0.0	0	33.333	33.333	0	33.33	74	135	100	183
	1	0.0	0	0	0	100	0	538	1408	831	1673
	2	0.0	0	22.222	66.667	0	11.11	442	978	657	1164

Table 7
(contd.)

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
	3	0.0	0	22.222	66.667	0	11.11	74	135	79	162
	4	0.0	0	0	0	100	0	538	1408	558	1322
	5	0.0	0	0	0	100	0	538	1408	800	1612
	6	0.0	0	38.095	28.571	0	33.33	74	135	79	162
	7	9.5	0	0	0	100	0	442	978	462	929
53	0	0.0	0	0	0	100	0	768	2592	788	2439
	1	0.0	0	0	0	100	0	442	978	462	929
	2	0.0	0	0	0	100	0	4	3	0	24
	3	0.0	0	0	0	100	0	4	3	0	24
	4	0.0	0	0	0	100	0	4	3	0	24
	5	0.0	0	10.256	66.667	0	23.08	768	2592	788	2439
	6	2.6	0	10.256	66.667	0	23.08	4	3	0	24
	7	0.0	0	0	0	100	0	768	2592	788	2439
54	0	0.0	4.35	11.594	50.725	7.246	26.09	628	1862	648	1744
	1	9.9	5.56	22.222	40.741	0	31.48	15	21	16	48
	2	7.3	0	25	47.222	5.556	22.22	4	3	0	24
	3	0.0	0	0	0	0	0	442	978	462	929
	4	0.0	7.69	0	69.231	0	23.08	4	3	0	24
	5	22.9	0	0	0	100	0	628	1862	648	1744
	6	0.0	9.52	0	68.254	0	22.22	538	1408	558	1322
	7	16.4	0	0	0	100	0	708	2292	728	2150
55	0	0.0	0	0	0	100	0	74	135	79	162
	1	0.0	0	0	0	100	0	74	135	79	162
	2	0.0	9.09	6.0606	57.576	0	27.27	708	2292	970	2440
	3	9.4	10.5	12.281	59.649	1.754	15.79	15	21	17	49
	4	19.0	0	0	0	100	0	538	1408	558	1322
	5	0.0	0	0	0	100	0	442	978	657	1164
	6	10.0	0	0	0	100	0	74	135	79	162
	7	11.0	4.17	13.889	50	6.944	25	538	1408	558	1322
56	0	0.0	4.17	13.889	50	6.944	25	538	1408	831	1673
	1	0.0	0	0	0	100	0	538	1408	800	1612
	2	0.0	0	0	0	100	0	442	978	657	1164
	3	0.0	0	0	0	100	0	74	135	79	162
	4	0.0	0	0	75.758	0	24.24	15	21	17	49
	5	0.0	0	0	0	100	0	538	1408	558	1322
	6	0.0	5.26	22.807	45.614	3.509	22.81	74	135	79	162
	7	8.7	13.3	26.667	31.111	0	28.89	538	1408	831	1673
57	0	0.0	0	36.036	41.441	11.71	10.81	370	771	288	584
	1	1.3	0	33.333	50	0	16.67	932	4084	930	3630
	2	0.0	0	0	0	100	0	454	1126	421	935
	3	0.0	14.3	47.619	0	14.29	23.81	1112	5612	1110	5054
	4	32.9	0	32.353	49.02	7.843	10.78	370	771	288	584
	5	3.5	6.9	22.989	45.977	12.64	11.49	14	19	14	45
	6	13.9	14.3	47.619	0	14.29	23.81	1272	7080	1270	6446
	7	32.9	0	16.667	50	0	33.33	1192	6324	1190	5728
58	0	0.0	0	0	58.333	13.89	27.78	740	2676	738	2340
	1	9.7	0	0	58.333	13.89	27.78	530	1528	497	1279
	2	0.0	14.3	23.81	23.81	14.29	23.81	740	2676	738	2340

Table 7
(contd.)

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
	3	32.2	0	20.833	45.833	4.167	29.17	836	3344	834	2949
	4	10.6	0	20.833	45.833	4.167	29.17	932	4084	930	3630
	5	0.0	10.5	17.544	35.088	19.3	17.54	370	771	288	584
	6	20.0	10.5	17.544	35.088	19.3	17.54	1022	4818	1020	4312
	7	0.0	0	13.889	44.444	13.89	27.78	740	2676	738	2340
59	0	0.0	0	33.333	33.333	16.67	16.67	530	1528	696	1574
	1	0.0	0	21.053	38.596	10.53	29.82	530	1528	628	1472
	2	0.0	0	43.478	30.435	8.696	17.39	836	3344	1112	3337
	3	4.4	3.23	32.258	35.484	16.13	12.9	1022	4818	1492	5156
	4	12.0	0	33.333	44.444	0	22.22	1022	4818	1492	5108
	5	0.0	10.5	35.088	17.544	10.53	26.32	740	2676	1016	2728
	6	20.6	10.5	17.544	35.088	19.3	17.54	530	1528	497	1279
	7	21.0	0	40	40	0	20	644	2080	642	1803
60	0	0.0	0	26.667	53.333	0	20	740	2676	993	2763
	1	0.0	0	33.333	33.333	16.67	16.67	740	2676	993	2763
	2	0.0	0	33.333	33.333	16.67	16.67	644	2080	863	2116
	3	0.0	0	16.667	50	8.333	25	740	2676	993	2763
	4	0.0	0	16.667	50	8.333	25	644	2080	642	1803
	5	0.0	8.33	27.778	27.778	15.28	20.83	932	4084	1319	4289
	6	14.7	0	40	40	0	20	644	2080	896	2177
	7	0.0	0	30.303	15.152	21.21	33.33	836	3344	1158	3513
61	0	0.0	0	44.444	31.481	5.556	18.52	1112	5612	1110	5054
	1	5.7	3.79	88.618	3.794	0	3.794	932	4084	930	3630
	2	0.0	0	0	0	100	0	370	771	288	584
	3	15.7	8.33	0	55.556	8.333	27.78	454	1126	421	935
	4	16.1	13.3	13.333	44.444	2.222	26.67	370	771	288	584
	5	33.2	3.33	25.556	43.333	8.889	18.89	14	19	14	45
	6	14.0	3.45	22.989	35.632	16.09	21.84	1272	7080	1270	6446
	7	9.7	13.3	0	46.667	22.22	17.78	1192	6324	1190	5728
62	0	0.0	13.3	0	46.667	22.22	17.78	740	2676	738	2340
	1	0.0	20	46.67	13.33	0	20	530	1528	497	1279
	2	39.5	16.7	33.33	33.33	0	16.66	740	2676	738	2340
	3	0.0	21.4	19.048	19.048	21.43	19.05	836	3344	834	2949
	4	0.0	21.4	19.048	19.048	21.43	19.05	370	771	288	584
	5	0.0	21.4	19.048	19.048	21.43	19.05	932	4084	930	3630
	6	0.0	21.4	19.048	19.048	21.43	19.05	1022	4818	1020	4312
	7	0.0	21.4	19.048	19.048	21.43	19.05	740	2676	738	2340
63	0	0.0	0	33.333	49.383	3.704	13.58	530	1528	696	1574
	1	0.2	0	33.333	49.383	3.704	13.58	1022	4818	1492	5108
	2	0.0	0	33.333	49.383	3.704	13.58	836	3344	1112	3337
	3	0.0	0	33.333	49.383	3.704	13.58	1022	4818	1492	5156
	4	0.0	0	20	43.333	6.667	30	740	2676	1016	2728
	5	1.9	12.1	42.42	24.24	0	21.21	644	2080	642	1803
	6	0.0	12.1	42.42	24.24	0	21.21	530	1528	497	1279

Table 7
(contd.)

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
	7	0.0	0	56.322	32.184	4.598	6.897	530	1528	628	1472
64	0	0.0	0	36.842	31.579	21.05	10.53	932	4084	1319	4289
	1	6.0	0	36.842	31.579	21.05	10.53	740	2676	993	2763
	2	0.0	13.3	32.222	25.556	5.556	23.33	740	2676	993	2763
	3	26.1	13.3	32.222	25.556	5.556	23.33	644	2080	863	2116
	4	0.0	13.3	32.222	25.556	5.556	23.33	644	2080	896	2177
	5	0.0	7.14	25	30.952	20.24	16.67	644	2080	642	1803
	6	13.4	7.14	25	30.952	20.24	16.67	836	3344	1158	3513
	7	0.0	0	40	43.333	0	16.67	740	2676	993	2763

Table 8. Results for individual rack problems using heuristic H3

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
1	2	3	4	5	6	7	8	9	10	11	12
65	0	10.96	0	63.889	16.667	2.778	16.667	768	2592	788	2439
	1	0.00	0	33.333	66.667	0	0	768	2592	1180	3043
	2	0.00	0	100	0	0	0	768	2592	788	2439
	3	0.00	0	100	0	0	0	768	2592	788	2439
66	0	0.00	0	66.667	33.333	0	0	768	2592	788	2439
	1	0.00	0	66.667	33.333	0	0	768	2592	1180	3043
	2	2.64	0	83.333	0	0	16.667	768	2592	788	2439
	3	0.00	0	66.667	33.333	0	0	768	2592	788	2439
67	0	10.79	0	16.667	63.889	5.556	13.889	768	2592	930	2669
	1	0.00	0	66.667	33.333	0	0	768	2592	1180	3043
	2	0.00	0	50	50	0	0	768	2592	1061	2869
	3	0.00	0	83.333	16.667	0	0	768	2592	788	2439
68	0	0.09	0	22.222	65.079	3.175	9.5238	768	2592	930	2669
	1	0.00	0	83.333	16.667	0	0	768	2592	1180	3043
	2	0.00	0	83.333	16.667	0	0	768	2592	1061	2869

Table 8
(contd.)

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
	3	13.25	4.17	45.833	31.944	2.778	15.278	768	2592	788	2439
69	0	0.13	0	60	21.667	5	13.333	768	2592	788	2439
	1	8.91	4.35	30.435	43.478	8.696	13.043	768	2592	1180	3043
	2	0.00	0	97.101	0	1.449	1.4493	768	2592	788	2439
	3	0.56	0	85.185	0	0	14.815	768	2592	788	2439
70	0	0.00	0	60.317	20.635	1.587	17.46	768	2592	788	2439
	1	0.00	0	41.27	47.619	6.349	4.7619	768	2592	1180	3043
	2	0.00	0	93.939	0	0	6.0606	768	2592	788	2439
	3	0.00	0	76.812	14.493	4.348	4.3478	768	2592	788	2439
71	0	0.21	0	58.333	21.667	0	20	768	2592	930	2669
	1	0.00	0	66.667	23.81	1.587	7.9365	768	2592	1180	3043
	2	0.00	22.6	40.86	25.806	5.376	5.3763	768	2592	1061	2869
	3	0.00	0	77.273	6.0606	1.515	15.152	768	2592	788	2439
72	0	0.00	0	56.667	28.333	0	15	768	2592	930	2669
	1	0.00	0	87.302	0	0	12.698	768	2592	1180	3043
	2	0.00	0	44.444	38.889	4.167	12.5	768	2592	1061	2869
	3	0.00	0	53.623	24.638	8.696	13.043	768	2592	788	2439
73	0	0.00	0	73.016	12.698	0	14.286	1392	8094	1390	7430
	1	0.00	0	58.333	41.667	0	0	1392	8094	1640	8070
	2	0.00	0	58.333	41.667	0	0	1392	8094	1390	7430
	3	0.00	0	83.333	16.667	0	0	1392	8094	1390	7430
74	0	0.00	0	100	0	0	0	1392	8094	1390	7430
	1	0.00	0	58.333	41.667	0	0	1392	8094	1640	8070
	2	0.00	0	75	25	0	0	1392	8094	1390	7430
	3	0.00	0	100	0	0	0	1392	8094	1390	7430
75	0	0.00	0	50	50	0	0	1392	8094	1867	8605
	1	0.00	0	66.667	33.333	0	0	1392	8094	1640	8070
	2	0.22	0	58.333	41.667	0	0	1392	8094	1622	7986
	3	2.96	0	48.837	24.806	2.326	24.031	1392	8094	1390	7430
76	0	0.00	0	41.667	58.333	0	0	1392	8094	1867	8605
	1	0.00	0	58.333	41.667	0	0	1392	8094	1640	8070
	2	0.22	0	36.508	41.27	0	22.222	1392	8094	1622	7986
	3	2.96	0	37.121	41.667	2.273	18.939	1392	8094	1390	7430
77	0	0.00	0	65.217	14.493	2.899	17.391	1392	8094	1390	7430
	1	0.00	0	65.217	14.493	2.899	17.391	1392	8094	1640	8070
	2	0.00	0	57.778	13.333	0	28.889	1392	8094	1390	7430
	3	0.00	0	74.603	15.079	2.381	7.9365	1392	8094	1390	7430
78	0	0.00	0	85.088	0	0	14.912	1392	8094	1390	7430
	1	0.00	0	85.088	0	0	14.912	1392	8094	1640	8070
	2	0.00	0	54.762	21.429	0	23.81	1392	8094	1390	7430
	3	0.00	0	92.857	0	0	7.1429	1392	8094	1390	7430
79	0	0.00	4.88	35.772	34.959	3.252	21.138	1392	8094	1867	8605
	1	0.00	4.88	35.772	34.959	3.252	21.138	1392	8094	1640	8070
	2	0.00	4.88	35.772	34.959	3.252	21.138	1392	8094	1622	7986
	3	0.00	4.88	35.772	34.959	3.252	21.138	1392	8094	1390	7430
80	0	0.00	0.0	3.7	31.48	53.7	11.11	1392	8094	4760	1440
	1	0.00	0.0	11.1	37.037	40.74	11.11	1392	8094	4747	1406

Table 8
(contd.)

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
	2	0.00	0.0	16.67	16.67	50	16.67	1392	8094	5241	1568
	3	0.00	0.0	7.4	39.5	39.5	13.58	1392	8094	5096	1483
81	0	0.00	0	28.571	28.571	30.95	11.905	442	978	462	929
	1	0.00	14.3	47.619	0	14.29	23.81	442	978	462	929
	2	0.22	14.3	47.619	0	14.29	23.81	442	978	462	929
	3	2.96	0	22.222	44.444	33.33	0	442	978	657	1164
	4	0.00	14.3	47.619	0	14.29	23.81	442	978	462	929
	5	0.00	14.3	47.619	0	14.29	23.81	442	978	462	929
	6	0.22	14.3	47.619	0	14.29	23.81	442	978	462	929
	7	2.96	14.3	47.619	0	14.29	23.81	442	978	462	929
82	0	2.80	14.3	47.619	0	14.29	23.81	442	978	462	929
	1	37.63	14.3	47.619	0	14.29	23.81	442	978	462	929
	2	0.00	14.3	47.619	0	14.29	23.81	442	978	462	929
	3	0.00	0	38.889	27.778	11.11	22.222	442	978	657	1164
	4	27.34	14.3	47.619	0	14.29	23.81	442	978	462	929
	5	27.34	14.3	47.619	0	14.29	23.81	442	978	462	929
	6	27.34	14.3	47.619	0	14.29	23.81	442	978	462	929
	7	27.34	14.3	47.619	0	14.29	23.81	442	978	462	929
83	0	27.34	0	28.571	28.571	30.95	11.905	442	978	462	929
	1	0.00	14.3	47.619	0	14.29	23.81	442	978	600	1108
	2	0.00	14.3	47.619	0	14.29	23.81	442	978	462	929
	3	1.00	0	44.444	22.222	33.33	0	442	978	657	1164
	4	37.63	0	44.444	22.222	33.33	0	442	978	462	929
	5	27.34	14.3	23.81	23.81	14.29	23.81	442	978	646	1162
	6	27.34	14.3	47.619	0	14.29	23.81	442	978	462	929
	7	27.34	14.3	47.619	0	14.29	23.81	442	978	462	929
84	0	2.73	0	25.641	41.026	5.128	28.205	442	978	462	929
	1	27.34	14.3	47.619	0	14.29	23.81	442	978	600	1108
	2	37.63	14.3	47.619	0	14.29	23.81	442	978	462	929
	3	0.00	0	38.889	27.778	11.11	22.222	442	978	657	1164
	4	2.00	14.3	47.619	0	14.29	23.81	442	978	462	929
	5	65.33	0	27.778	38.889	11.11	22.222	442	978	646	1162
	6	13.87	0	25.641	41.026	5.128	28.205	442	978	462	929
	7	31.02	0	25.641	41.026	5.128	28.205	442	978	462	929
85	0	0.00	18.2	30.303	39.394	0	12.121	442	978	462	929
	1	0.00	6.25	60.417	0	16.67	16.667	442	978	462	929
	2	0.00	16.7	33.333	22.222	13.89	13.889	442	978	462	929
	3	26.49	0	22.222	35.185	42.59	0	442	978	657	1164
	4	12.64	12.5	58.333	0	12.5	16.667	442	978	462	929
	5	23.76	5.88	50.98	0	17.65	25.49	442	978	462	929
	6	0.00	10	60	0	13.33	16.667	442	978	462	929
	7	3.00	0	64.583	0	16.67	18.75	442	978	462	929
86	0	39.45	25	33.333	33.333	0	8.3333	442	978	462	929
	1	31.02	14.3	52.381	0	9.524	23.81	442	978	462	929
	2	0.00	16.7	33.333	22.222	13.89	13.889	442	978	462	929
	3	5.53	16.7	33.333	22.222	13.89	13.889	442	978	657	1164
	4	17.61	0	62.745	0	19.61	17.647	442	978	462	929

Table 8
(contd.)

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
	5	18.80	6.67	62.222	0	13.33	17.778	442	978	462	929
	6	0.00	0	15.152	33.333	36.36	15.152	442	978	462	929
	7	0.00	0	64.706	0	15.69	19.608	442	978	462	929
87	0	0.00	0	46.154	33.333	10.26	10.256	442	978	462	929
	1	0.00	0	66.667	0	18.18	15.152	442	978	600	1108
	2	0.00	0	61.538	0	23.08	15.385	442	978	462	929
	3	4.00	0	61.538	0	23.08	15.385	442	978	657	1164
	4	7.21	0	61.538	0	23.08	15.385	442	978	462	929
	5	0.00	17.6	19.608	33.333	3.922	25.49	442	978	646	1162
	6	33.59	5.88	60.784	0	15.69	17.647	442	978	462	929
	7	13.41	5.88	60.784	0	15.69	17.647	442	978	462	929
88	0	0.00	8.33	13.889	50	16.67	11.111	442	978	462	929
	1	0.00	0	66.667	0	18.18	15.152	442	978	600	1108
	2	17.32	0	61.538	0	23.08	15.385	442	978	462	929
	3	1.62	7.14	61.905	0	16.67	14.286	442	978	657	1164
	4	7.21	7.14	61.905	0	16.67	14.286	442	978	462	929
	5	5.00	7.14	61.905	0	16.67	14.286	442	978	646	1162
	6	0.00	7.14	50	30.952	11.9	0	442	978	462	929
	7	18.80	7.14	50	30.952	11.9	0	442	978	462	929
89	0	0.00	0	55.556	16.667	11.11	16.667	740	2676	738	2340
	1	0.00	8.33	41.667	13.889	22.22	13.889	740	2676	738	2340
	2	2.93	8.33	41.667	13.889	22.22	13.889	740	2676	738	2340
	3	15.90	0	59.42	14.493	8.696	17.391	740	2676	971	2749
	4	0.00	0	59.42	14.493	8.696	17.391	740	2676	738	2340
	5	4.56	0	73.913	0	8.696	17.391	740	2676	738	2340
	6	0.00	0	73.913	0	8.696	17.391	740	2676	738	2340
	7	6.00	0	73.913	0	8.696	17.391	740	2676	738	2340
90	0	4.36	0	73.913	0	8.696	17.391	740	2676	738	2340
	1	0.00	0	73.913	0	8.696	17.391	740	2676	738	2340
	2	4.36	0	33.333	66.667	0	0	740	2676	738	2340
	3	4.36	0	59.42	14.493	8.696	17.391	740	2676	971	2749
	4	0.00	0	73.913	0	8.696	17.391	740	2676	738	2340
	5	4.36	0	41.667	16.667	13.89	27.778	740	2676	738	2340
	6	4.56	0	73.913	0	8.696	17.391	740	2676	738	2340
	7	11.14	0	73.913	0	8.696	17.391	740	2676	738	2340
91	0	7.00	0	43.056	23.611	0	33.333	740	2676	738	2340
	1	4.56	0	58.333	13.889	11.11	16.667	740	2676	1050	2862
	2	0.00	0	58.333	13.889	11.11	16.667	740	2676	738	2340
	3	3.44	0	59.42	14.493	8.696	17.391	740	2676	971	2749
	4	3.38	0	66.667	33.333	0	0	740	2676	738	2340
	5	0.00	0	58.333	13.889	11.11	16.667	740	2676	936	2680
	6	4.36	0	58.333	13.889	11.11	16.667	740	2676	738	2340
	7	0.00	0	58.333	13.889	11.11	16.667	740	2676	738	2340
92	0	3.50	0	41.667	16.667	13.89	27.778	740	2676	738	2340
	1	0.00	0	47.222	19.444	0	33.333	740	2676	1050	2862
	2	8.00	0	47.222	19.444	0	33.333	740	2676	738	2340
	3	0.00	0	47.222	19.444	0	33.333	740	2676	971	2749

Table 8
(contd.)

Instance Number	Rack	%Gap	%NP	%GP	%NMP	%MP	%EP	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP
	4	0.00	0	55.556	16.667	11.11	16.667	740	2676	738	2340
	5	10.02	0	55.556	16.667	11.11	16.667	740	2676	936	2680
	6	1.80	0	55.556	16.667	11.11	16.667	740	2676	738	2340
	7	0.00	8.33	27.778	27.778	22.22	13.889	740	2676	738	2340
93	0	2.93	8.33	27.778	27.778	22.22	13.889	740	2676	738	2340
	1	0.00	0	77.778	0	4.938	17.284	740	2676	738	2340
	2	18.56	0	73.684	0	5.263	21.053	740	2676	738	2340
	3	9.00	0	36.364	33.333	15.15	15.152	740	2676	971	2749
	4	0.00	0	36.364	33.333	15.15	15.152	740	2676	738	2340
	5	14.85	4.35	72.464	0	5.797	17.391	740	2676	738	2340
	6	10.10	0	22.727	46.97	10.61	19.697	740	2676	738	2340
	7	7.81	0	78.205	0	5.128	16.667	740	2676	738	2340
94	0	0.00	0	73.016	0	4.762	22.222	740	2676	738	2340
	1	2.79	0	85	0	0	15	740	2676	738	2340
	2	0.00	0	85	0	0	15	740	2676	738	2340
	3	0.00	0	78.261	11.594	5.797	4.3478	740	2676	971	2749
	4	0.00	0	78.261	11.594	5.797	4.3478	740	2676	738	2340
	5	10.00	0	78.261	11.594	5.797	4.3478	740	2676	738	2340
	6	31.98	7.41	72.84	0	8.642	11.111	740	2676	738	2340
	7	16.76	4.55	72.727	0	7.576	15.152	740	2676	738	2340
95	0	24.95	13.6	25.758	39.394	7.576	13.636	740	2676	738	2340
	1	3.14	0	63.492	15.873	9.524	11.111	740	2676	1050	2862
	2	3.67	0	56.79	22.222	6.173	14.815	740	2676	738	2340
	3	0.00	45.5	26.263	18.182	5.051	5.0505	740	2676	971	2749
	4	5.45	0	71.429	15.873	3.175	9.5238	740	2676	738	2340
	5	0.00	0	38.596	36.842	0	24.561	740	2676	936	2680
	6	11.00	0	38.596	36.842	0	24.561	740	2676	738	2340
	7	23.52	7.69	43.59	34.615	7.692	6.4103	740	2676	738	2340
96	0	0.00	4.76	23.81	55.556	7.937	7.9365	740	2676	738	2340
	1	12.26	4.76	23.81	55.556	7.937	7.9365	740	2676	1050	2862
	2	0.00	15.4	43.59	20.513	3.846	16.667	740	2676	738	2340
	3	28.22	15.4	43.59	20.513	3.846	16.667	740	2676	971	2749
	4	0.00	15.4	43.59	20.513	3.846	16.667	740	2676	738	2340
	5	6.46	0	19.697	66.667	0	13.636	740	2676	936	2680
	6	0.00	0	19.697	66.667	0	13.636	740	2676	738	2340
	7	12.35	4.35	36.232	36.232	4.348	18.841	740	2676	738	2340

Columns 4-8 in tables 6, 7 and 8 give the percentage of no-picks, gang picks, no-move picks, multiple picks and eclectic picks, prescribed by P2, which average 3%, 44%, 22%, 18% and 13% respectively respectively. The high percentage of gang picks prescribed is desired since they are the most efficient form of picks. All heuristics are designed to promote gang-picking. The low percentage of eclectic picks is also as desired since they are the least efficient.

Columns 9-12 of tables 6, 7 and 8 describe sub-problem networks. Run time increases with the size of the sub-problem. The number of nodes and arcs also increase with the number of CTs assigned to a rack. The number of nodes (arcs) in an expanded network (defined in section 3.2) may be less than, equal to, or greater than the number of nodes (arcs) in the original sub-problem network, depending on feasibility requirements. Some rack problems associated with a few instances (e.g. 17-24 and 49-56) employ only multiple picks. In these rack problems, heuristics H1 and H2 assign a single CT to a rack (these instances involve 2 DHPMs and 32 CTs). A CTPC comprising multiple picks is most efficient for these rack problems. These rack problems lead to a single sub-problem as shown in columns 9-12 of the table 1 and 2. H3 does not lead to this situation because it assigns a balanced number of CTs to each head and, hence, it never assigns a single CT to a particular rack.

4.2 P3 computational results

This section discusses the results from the column generation approach employed to optimize P3. All computer programs were coded in C in the Watcom-C editor and all tests performed interfacing with MINTO 3.0a and CPLEX 4.0. Certain details have not been divulged due to a non-disclosure agreement with the industrial collaborator. The optimal solution prescribed for P2 is input to P3, including the CTPCs and the number of times each is to be used. The experimental design used to evaluate P3 is the same as the design used for P2 and is described in section 4.1.1 above. Sub-sections below describe the generated test instances and computational results.

4.2.1 Test results

Tables 11, 12 and 13 record test results associated with H1, H2 and H3, respectively. Tables 11, 12 and 13 give overall measures of performance; columns 1-7 describe the instance and columns 8-12 summarize test results. A P3 problem is solved for each head, machine, rack combination separately, but, to conserve space, the tables give composite results for all rack problems (4 racks for 1 DHPM and 8 racks for 2 DHPMs, representing levels 1 and 2 of factor 2). The acronyms that head the columns of Tables 11, 12 and 13 are defined below in table 9 and for 14, 15 and 16 in table 10:

Table 9. Acronyms of columns for tables 11, 12 and 13

Column	Acronym	Description
1	Instance #	Instance number
2	F1 H#	Factor 1: heuristic number (i.e., H1, H2, H3)
3	F2 #M	Factor 2: number of DHPMs
4	F3 #CT	Factor 3: number of CTs (i.e., 32 or 64)
5	F4# C/CT	Factor 4: number of components per CT
6	F5 #NT	Factor 5: nozzle type assignment
7	Theta	Factor 6: CT orientation
8	#SP Solved	Number of sub-problems solved
9	#Prom Cols	Number of improving columns generated
10	#Entrd Cols	Number of improving columns entered into the master problem

Table 9

(contd.)

Column	Acronym	Description
11	#B&B Nodes	Number of branch and bound nodes required to optimize all rack problems
12	Total RT	Total run time to prescribe optimal solutions to all rack problems
13	Max RT	Maximum run time to solve any rack problem

The run time reported in column 12 does not include the (negligible) time required to expand the sub-problem networks, a one-time process. Tables 14, 15 and 16 provide detailed measures associated with individual rack problems; and columns are headed by the following acronyms:

Table 10. Acronyms of columns for table 14, 15 and 16

Column	Acronym	Description
1	Instance #	# Instance number
2	Rack #	Rack number

Table 10

(contd.)

Column	Acronym	Description
3	*%GAP*	$\%GAP = 100(Z_{IP}^{*} - Z_{LP}^{*})/Z_{LP}^{*}$
4	#Nodes SP	Number of nodes in all sub-problems networks
5	#Arcs SP	Number of arcs in all sub-problem networks
6	#Nodes EXP	Number of nodes in all expanded networks
7	#Arcs EXP	Number of arcs in all expanded networks
8	SPs #	Number of sub-problems (and CTPCs)

Column 3 gives the *%GAP* for the rack problem, where Z_{LP}^{*} is the value of the optimal solution to the linear relaxation and Z_{IP}^{*} is the value of the optimal integer solution.

Summary measures in Tables 11, 12 and 13 highlight the effect of each factor on run time. The two levels of each factor are compared by adding the run times for instances that involve each level. Factor 1 (heuristic H1, H2 or H3) shows a substantial effect on run time. H2 leads to P3 problems that can be solved in less run time (16.9 seconds vs

19.5 seconds for H1 and 39 seconds for H3 on an average). This means that H2 yields less challenging P3 instances than H1 and H3. This is not to say that H2 is preferred because the heuristics must be judged relative to how well they balance workloads on heads- an issue that will be covered in the section on P4 results. In assigning CTs to feeder slots, H1, H2 and H3 place different emphasis on such attributes as nozzle-type requirement, orientation, and CT width. The approach, thus, shows great robustness in being able to solve instances resulting from using different logic used to assign CTs to feeder slots.

Table 11. Summary of results for P3 using Heuristic H1

Instance # 1	F1 H# 2	F2 #M 3	F3 #CT 4	F4 #NT 5	F5 #C/CT 6	F6 Theta 7	#SP Solved 8	#Prom Cols 9	#Entrd Cols 10	#B&B Nodes 11	Total RT 12	Max RT 13
1	1	1	32	1	10	1	671	519	309	96	38.98	25.48
2	1	1	32	1	10	2	198	186	95	4	11.11	3.34
3	1	1	32	2	10	1	225	212	99	4	10.57	3.34
4	1	1	32	2	10	2	186	169	89	4	10.11	2.8
5	1	1	32	1	[5,15]	1	3039	2996	514	528	115.68	78
6	1	1	32	1	[5,15]	2	244	243	30	4	10.79	3.19
7	1	1	32	2	[5,15]	1	1312	1305	211	236	56.76	47.93
8	1	1	32	2	[5,15]	2	240	240	34	4	11.19	3.19
9	1	1	64	1	10	1	224	224	46	4	11.95	3.19
10	1	1	64	1	10	2	224	224	52	4	11.66	2.93
11	1	1	64	2	10	1	223	220	49	4	11.39	2.96
12	1	1	64	2	10	2	224	220	52	4	11.22	2.93
13	1	1	64	1	[5,15]	1	262	262	29	4	3.91	3.18
14	1	1	64	1	[5,15]	2	262	262	28	4	3.91	3.18
15	1	1	64	2	[5,15]	1	209	209	30	3	5.18	3.21
16	1	1	64	2	[5,15]	2	284	284	30	4	6.57	3.35
17	1	2	32	1	10	1	280	271	159	73	13.48	3.51
18	1	2	32	1	10	2	416	351	220	64	10.67	2.85

Table 11
(contd.)

Instance #	F1 H#	F2 #M	F3 #CT	F4 #NT	F5 #C/CT	F6 Theta	#SP Solved	#Prom Cols	#Entrd Cols	#B&B Nodes	Total RT	Max RT
19	1	2	32	2	10	1	393	350	191	121	6.4	2.33
20	1	2	32	2	10	2	1237	1089	543	199	18.76	10.54
21	1	2	32	1	[5,15]	1	492	460	157	162	11.37	4.28
22	1	2	32	1	[5,15]	2	698	662	149	195	24.16	13.33
23	1	2	32	2	[5,15]	1	600	591	862	100	14.67	13.33
24	1	2	32	2	[5,15]	2	1847	1809	198	532	45.57	42.16
25	1	2	64	1	10	1	594	579	191	40	3.77	1.53
26	1	2	64	1	10	2	1895	1820	396	380	40.37	33.13
27	1	2	64	2	10	1	688	662	200	96	13.37	6.09
28	1	2	64	2	10	2	1269	1233	290	208	30.27	22.19
29	1	2	64	1	[5,15]	1	830	793	245	151	11.09	2.2
30	1	2	64	1	[5,15]	2	1927	1898	408	343	35.71	31.33
31	1	2	64	2	[5,15]	1	757	745	155	88	4.59	2.94
32	1	2	64	2	[5,15]	2	726	685	134	128	7.96	2.14

Table 12. Summary of results for P3 using Heuristic H2

Instance # 1	F1 H# 2	F2 #M 3	F3 #CT 4	F4 #C/CT 5	F5 #NT 6	F6 Theta 7	#SP Solved 8	#Prom Cols 9	#Entrd Cols 10	#B&B Nodes 11	RT Secs 12	RT Max 13
33	2	1	32	10	1	1	168	162	80	4	4.76	1.53
34	2	1	32	10	1	2	168	162	97	4	4.76	1.74
35	2	1	32	10	2	1	206	195	99	4	5.75	1.47
36	2	1	32	10	2	2	378	335	150	74	11.2	5.83
37	2	1	32	[5,15]	1	1	389	346	133	74	11.6	5.83
38	2	1	32	[5,15]	1	2	271	253	80	32	9.22	3.8
39	2	1	32	[5,15]	2	1	284	277	74	24	8.42	2.96
40	2	1	32	[5,15]	2	2	763	755	149	116	26.82	14.72
41	2	1	64	10	1	1	389	359	65	74	10.25	5.83
42	2	1	64	10	1	2	387	355	69	74	10.53	5.83
43	2	1	64	10	2	1	234	234	36	4	6.34	1.64
44	2	1	64	10	2	2	234	234	36	4	5.97	1.57
45	2	1	64	[5,15]	1	1	429	414	71	56	16.47	9.59

Table 12
(contd.)

Instance #	F1 H#	F2 #M	F3 #CT	F4 #NT	F5 #C/CT	F6 Theta	#SP Solved	#Prom Cols	#Entrd Cols	#B&B Nodes	Total RT	Max RT
46	2	1	64	[5,15]	1	2	760	733	112	100	25.9	12.6
7	2	1	64	[5,15]	2	1	261	261	30	4	7.47	2.71
48	2	1	64	[5,15]	2	2	269	268	38	4	7.27	2.71
49	2	2	32	10	1	1	530	657	150	125	11	3.51
50	2	2	32	10	1	2	1750	1397	824	165	10.85	3.38
51	2	2	32	10	2	1	2794	2021	975	196	38.00	15.61
52	2	2	32	10	2	2	2788	2021	133	196	38.00	15.55
53	2	2	32	[5,15]	1	1	676	644	238	233	14.4	7.58
54	2	2	32	[5,15]	1	2	645	569	235	84	18.25	7.1
55	2	2	32	[5,15]	2	1	3333	2537	1127	1599	47.3	16.1
56	2	2	32	[5,15]	2	2	945	933	297	211	22.95	9.6
57	2	2	64	10	1	1	940	920	274	195	23.43	9.6
58	2	2	64	10	1	2	1474	1430	309	171	36.82	25.28
59	2	2	64	10	2	1	524	517	120	24	14.53	3.44
60	2	2	64	10	2	2	470	436	161	8	12.63	2.5
61	2	2	64	[5,15]	1	1	510	489	109	16	13.91	2.26
62	2	2	64	[5,15]	1	2	569	561	137	46	15.98	3.72
63	2	2	64	[5,15]	2	1	1639	1612	476	670	42.41	23.86
64	2	2	64	[5,15]	2	2	890	885	193	128	7.96	2.14

Table 13. Summary of results for P3 using Heuristic H3

Instance #	F1 H#	F2 #M	F3 #CT	F4 #NT	F5 #C/CT	F6 Theta	#SP Solved	#Prom Cols	#Entrd Cols	#B&B Nodes	Total RT	Max RT
1	2	3	4	5	6	7	8	9	10	11	12	13
65	1	1	32	1	10	1	168	162	84	4	4.76	1.53
66	1	1	32	1	10	2	174	154	87	4	10.54	4.03
67	1	1	32	2	10	1	176	300	88	68	16.21	8.69
68	1	1	32	2	10	2	224	207	112	4	10.32	2.91

Table 13
(contd.)

Instance #	F1 H#	F2 #M	F3 #CT	F4 #NT	F5 #C/CT	F6 Theta	#SP Solved	#Prom Cols	#Entrd Cols	#B&B Nodes	Total RT	Max RT
69	1	1	32	1	[5,15]	1	219	219	73	4	9.97	2.81
70	1	1	32	1	[5,15]	2	230	230	56	4	9.36	2.77
71	1	1	32	2	[5,15]	1	200	376	80	44	18.61	10.88
72	1	1	32	2	[5,15]	2	196	372	154	44	18.83	10.88
73	1	1	64	1	10	1	232	230	50	4	10.18	2.6
74	1	1	64	1	10	2	224	221	56	4	10.17	2.56
75	1	1	64	2	10	1	224	218	56	4	10.1	2.53
76	1	1	64	2	10	2	224	224	56	4	10.17	2.55
77	1	1	64	1	[5,15]	1	240	240	56	4	10.74	3.12
78	1	1	64	1	[5,15]	2	275	275	23	4	11.91	3.16
79	1	1	64	2	[5,15]	1	263	263	30	4	11.67	3.74
80	1	1	64	2	[5,15]	2	251	251	23	4	10.21	3.06
81	1	2	32	1	10	1	121	113	113	8	6.23	1.09
82	1	2	32	1	10	2	107	99	99	8	5.68	0.76
83	1	2	32	2	10	1	130	122	122	8	6.46	1.2
84	1	2	32	2	10	2	124	116	116	8	6.28	1.17
85	1	2	32	1	[5,15]	1	194	176	176	43	9.4	2.06
86	1	2	32	1	[5,15]	2	253	224	224	49	10.82	3.32
87	1	2	32	2	[5,15]	1	173	157	157	20	8.29	2
88	1	2	32	2	[5,15]	2	131	123	123	10	7.22	1.18
89	1	2	64	1	10	1	3155	2755	1763	803	159.89	89.8
90	1	2	64	1	10	2	1535	1242	768	78	54.99	18.49
91	1	2	64	2	10	1	4274	3640	2007	1736	359	39.93
92	1	2	64	2	10	2	2748	2304	1224	820	140.49	48.74
93	1	2	64	1	[5,15]	1	4651	3899	1395	145	107.92	35.75
94	1	2	64	1	[5,15]	2	720	621	347	37	25.24	4.66
95	1	2	64	2	[5,15]	1	3208	2949	1158	598	82.84	30.24
96	1	2	64	2	[5,15]	2	2215	1654	900	128	82.72	25.36

Level 2 of factor 2, number of DHPMs, has a more marked effect on run time than level 1 has for all three heuristics. The reason is that 2 DHPMs involve solving more rack problems, increasing the overall run time. There are some exceptions, however. For example, instances 13-16 each involve a large number of CTPCs (sub-problems) but have very low run times because the *%GAP* is very small for instances with 1 DHPM (i.e., level 1). Instances 24, 26, 28, and 30 each involve fewer CTPCs but have longer run times because the *%GAP* is typically large for at least one rack problem associated with each instance that involves 2 DHPMs (i.e., level 2), reflecting the fact that H1 assigned more CTs to that rack resulting in higher run time for it.

Levels 1 and 2 of factor 3, number of CTs, have the same effect on run time for H1 and H2. This is somewhat counterintuitive because one would expect a larger number of CTs to require more CTPCs and, hence, require a higher run time. This result may be affected by the fact that many instances involving 1 DHPM (e.g., 2, 4, 8-16, 33-35, 43-44, 47, 48 and 65) run quickly because each of the rack problems solve at the root node. However, for H3, the number of CTs has a significant effect on run time, especially in the case of two DHPMs. Instances 89-96 have exceptionally high run times because, in these instances, 64 CTs are assigned to two DHPMs and there are more CTPCs per rack, leading to longer run times.

Tables 11, 12 and 13 show that the two levels of factors 4, 5, and 6 have the same effect on run time when either H2 or H3 is used. However, the two levels have significantly different effects when H1 is used, again because the heuristics use different logic to assign CTs to feeder slots. Level 2 of factor 4, number of components per CT, has a much more pronounced effect on run time than level 1 does (when H1 is used). The reason for this is that, for level 2, P2 may prescribe more CTPCs, increasing the number of sub-problems and, thus, run time. A larger number of components has both positive and negative influences. On the negative side, more components require more decisions, increasing run time. On the positive side, more components provide more opportunities to select good combinations for each placing step. These two influences underlie results but it is difficult to distinguish (a priori) when one will dominate the other. Level 1 of factor 5, nozzle type assigned to each CT, has a somewhat stronger influence on run time than level 2 does (when H1 is used). Problem P3, by itself, appears to provide no obvious reason for this difference, which results from the logic that H1 and H2 use to assign CTs to feeder slots and leading to the resulting differences in the nature of P3 instances. It is expected, however, that factor 5 would have a significant effect on P4. Factor 6, orientation requirement, does not have a significant influence on run time, although H1 takes somewhat longer to solve level 1 instances.

It is also noted that, although not a factor, the number of CTPCs prescribed by P2 has a substantial effect on run time. Each CTPC results in a P3 sub-problem so more CTPCs, increase run time to solve the larger number of sub-problems. For example, instances 17, 18, 21, 23, 24, 81-88 each involve several rack problems for which P2 prescribes only 1 CTPC. For these instances, P3 uses a multiple pick of four components (prescribed by P2) on each of the picking steps. Instances 8-16 each entail more CTPCs but require low run times because the optimal solution to each rack problem is prescribed at the root node (column 11 records that 4 branch and bound nodes were used, one for each rack problem).

Overall, run times required to optimize P3 instances are rather small. This suggests that it is relatively easy to identify a good combination of individual components for each placing step.

4.2.2 Statistical analysis

MINITAB 13.1 was used to conduct a factorial design analysis relative to H1, H2 and H3, taken individually as well as together with the goal of identifying which factors and their interactions affect response (i.e., run time) the most.

In all three experiments, the effect term for factor 2 was much larger than the effect term for other factors. This confirms that factor 2, the number of DHPMs, has the most substantial effect on run time. Analysis of the experiment affirms that H3 and H1 entail longer run times than H2 does. All three experiments led to consistent conclusions regarding: (a) Factor 3, number of CTs, does not have a substantial effect on run time for H1 and H2 but it has a substantial effect for H3 (because of the similarity in the assignment procedures for H1 and H2 and their marked difference with that of H3) (b) For H1 and H2 the interaction between factors 2 and 3 has a substantial effect on run time, especially for instances that involve levels 1 and 2 (or 2 and 1) for factors 2 and 3, respectively; (c)factor 4, number of components per CT, has a relatively high influence on run time and level 2 has a more marked effect than level 1 does; and (d) factors 5 and 6 do not show substantial effects on run times. This more formal statistical analysis reinforces the preliminary analysis related above.

4.2.3 Overall performance measures

Columns 8-10 of Tables 11, 12 and 13 demonstrate the performance of the column generation process employed to solve P3. The most striking result is that the number of improving columns is almost as large as the number of sub-problems solved. This results because it is nearly always possible to select a set of individual components that form an improving column (i.e., a placing step). Fewer columns enter, however, because only one column is entered per iteration. On the last iteration, which detects an optimal solution, all sub-problems are solved but no improving column is identified. Column 8 does not count this last round in reporting the number of sub-problems solved. As a result, columns 8 and 9 report the same number for several instances (e.g., 9, 10, 13, 14, 15, 70, 76-80).

Run time increases with the number of sub-problems and the number of branch and bound nodes, as expected. Finally, it is noted that the maximum run time for the set of rack problems associated with an instance typically dominates the run time for that set of problems.

Tables 14, 15 and 16 highlight measures for the rack problems associated with each instance. *%GAP* is quite small for most rack problems, indicating that the model is tight. However, a few rack problems involve substantial gaps. *%GAP* distinguishes the impacts of H1, H2 and H3. H1, H2 and H3 all lead to about the same number of sub-problems but rack problems associated with H2 have smaller gaps than do those associated with H1 and H3.

Table 14. Results for individual rack problems using heuristic H1

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
1	2	3	4	5	6	7	8
1	0	11.035	84	620	645	4314	2
	1	0.035	84	620	626	4194	2
	2	0.148	84	620	629	4284	2
	3	5.606	344	1970	1990	1260	7
2	0	0.000	104	620	623	4154	2
	1	0.000	104	620	631	4155	2
	2	0.000	104	620	636	4254	2
	3	0.000	104	620	629	4224	2
3	0	0.000	136	730	739	4226	3
	1	0.000	104	620	631	4155	2
	2	0.000	104	620	636	4254	2
	3	0.000	104	620	638	4214	2
4	0	0.000	104	620	636	4224	2
	1	0.000	104	620	617	4094	2
	2	0.000	104	620	627	4185	2
	3	0.000	104	620	615	3994	2
5	0	10.895	172	1112	1127	8185	4
	1	5.369	254	1407	1434	7998	6
	2	0.000	428	3165	3149	2539	7
	3	0.000	322	1924	1900	1307	6
6	0	0.000	431	2978	2964	2239	8
	1	0.000	458	2938	2936	2123	8
	2	0.000	428	3165	3149	2539	7
	3	0.000	273	1778	1802	1146	6
7	0	0.000	331	2281	2239	1739	6

Table 14
(contd.)

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
	1	0.000	458	2938	2936	2123	8
	2	0.000	428	3165	3149	2539	7
	3	0.982	305	1705	1697	1100	6
8	0	0.000	454	2982	2970	2101	8
	1	0.000	458	2938	2936	2123	8
	2	0.000	428	3165	3149	2539	7
	3	0.000	195	1381	1377	9925	4
9	0	0.000	208	1240	1269	8408	4
	1	0.000	208	1240	1261	8368	4
	2	0.000	428	3165	3149	2539	7
	3	0.000	208	1240	1270	8408	4
10	0	0.000	208	1240	1260	8369	4
	1	0.000	208	1240	1278	8518	4
	2	0.000	208	1240	1263	8379	4
	3	0.000	208	1240	1268	8448	4
11	0	0.000	208	1240	1255	8370	4
	1	0.000	208	1240	1266	8418	4
	2	0.000	208	1240	1260	8408	4
	3	0.000	250	1450	1469	9500	5
12	0	0.000	208	1240	1268	8458	4
	1	0.000	208	1240	1266	8418	4
	2	0.000	208	1240	1242	8250	4
	3	0.000	208	1240	1253	8359	4
13	0	0.000	208	1240	1268	8458	4
	1	0.000	642	3679	3727	2292	13
	2	0.000	829	6349	6379	5314	13
	3	0.000	592	3278	3295	2003	13
14	0	0.000	208	1240	1268	8458	4
	1	0.000	797	5629	5672	4181	14
	2	0.000	829	6349	6379	5314	13
	3	0.000	592	3278	3295	2003	13
15	0	0.000	208	1240	1268	8458	4
	1	0.000	506	2668	2700	1466	12
	2	0.000	797	5315	5307	4076	13
	3	0.000	790	5380	5376	3993	14
16	0	0.000	208	1240	1268	8458	4
	1	0.000	506	2668	2700	1466	12
	2	0.000	675	4576	4621	3425	12
	3	0.000	958	5719	5748	4158	17
17	0	11.035	104	620	643	4294	2
	1	11.035	104	620	643	4294	2
	2	0.077	42	310	320	2132	1
	3	0.077	42	310	320	2142	1
	4	0.077	42	310	306	2032	1
	5	0.000	84	620	646	4324	2
	6	0.000	42	310	306	2052	1
	7	0.000	84	620	636	4244	2
18	0	0.077	42	310	320	2132	1
	1	0.000	104	620	643	4294	2

Table 14
(contd.)

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
	2	0.077	74	520	540	3264	2
	3	27.351	42	310	320	2142	1
	4	0.000	94	520	522	3065	2
	5	0.077	42	310	306	2032	1
	6	0.000	94	520	529	3175	2
	7	0.000	94	520	521	3164	2
19	0	15.657	66	330	337	1236	3
	1	15.657	119	18	14	3000	3
	2	0.000	74	520	540	3264	2
	3	0.000	94	520	534	3234	2
	4	0.000	94	520	517	3184	2
	5	0.000	42	310	306	2032	1
	6	15.657	94	520	529	3175	2
	7	0.000	94	520	522	3175	2
20	0	0.000	66	330	337	1236	3
	1	25.458	156	910	940	6108	3
	2	0.000	74	520	540	3264	2
	3	0.000	94	520	534	3234	2
	4	0.000	94	520	534	3234	2
	5	0.006	84	620	624	4104	2
	6	0.000	94	520	529	3175	2
	7	0.000	94	520	522	3175	2
21	0	12.2	172	1112	1127	8185	4
	1	9.2	113	868	884	5884	3
	2	21.4	54	520	534	4630	1
	3	19.1	58	602	604	5630	1
	4	20.3	62	690	693	6947	1
	5	0.0	212	1464	1467	1119	4
	6	11.8	54	520	534	4630	1
	7	0.0	309	2130	2126	1476	6
22	0	1.900	182	1267	1251	8841	4
	1	0.000	113	868	884	5884	3
	2	0.000	54	520	534	4630	1
	3	9.156	113	868	884	5884	3
	4	0.000	62	690	693	6947	1
	5	8.600	212	1576	1552	1224	4
	6	0.000	54	520	534	4630	1
	7	2.900	280	2045	2042	1675	5
23	0	0.0	26	121	122	4780	1
	1	0.0	113	868	884	5884	3
	2	0.0	373	2704	2714	2176	6
	3	0.0	113	868	884	5884	3
	4	6.6900	222	1559	1548	1139	4
	5	0.0	45	293	291	1763	1
	6	0.0	34	253	250	1263	1
	7	0.0	280	2045	2042	1675	5
24	0	2.200	26	121	122	4780	1
	1	4.200	113	868	884	5884	3
	2	0.000	373	2704	2714	2176	6

Table 14
(contd.)

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
	3	0.000	113	868	884	5884	3
	4	0.000	222	1559	1548	1139	4
	5	3.200	45	293	291	1763	1
	6	0.000	34	253	250	1263	1
	7	2.400	280	2045	2042	1675	5
25	0	6.400	26	121	122	4780	1
	1	0.000	113	868	884	5884	3
	2	0.000	198	1140	1164	7438	4
	3	0.000	198	1140	1164	7438	4
	4	0.000	198	1140	1164	7438	4
	5	0.000	198	1140	1164	7438	4
	6	0.000	198	1140	1160	7408	4
	7	0.000	198	1140	1167	7469	4
26	0	2.330	250	1450	1472	9461	5
	1	0.000	136	730	741	4267	3
	2	0.000	250	1450	1448	9212	5
	3	0.000	146	830	840	5306	3
	4	4.800	198	1140	1164	7438	4
	5	6.200	198	1140	1164	7438	4
	6	0.000	354	2070	2139	1382	7
	7	0.000	250	1450	1484	9560	5
27	0	5.300	188	1040	1053	6329	4
	1	0.000	136	730	741	4267	3
	2	3.700	146	830	845	5267	3
	3	0.000	354	2070	2101	1368	7
	4	0.000	146	830	822	5276	3
	5	0.000	198	1140	1164	7438	4
	6	0.000	136	730	734	4249	3
	7	0.000	146	830	845	5326	3
28	0	0.000	146	830	855	5376	3
	1	0.000	198	1140	1144	7258	4
	2	0.000	146	830	847	5316	3
	3	0.000	188	1040	1055	6348	4
	4	0.000	146	830	831	5148	3
	5	5.500	250	1450	1459	9312	5
	6	0.000	136	730	734	4249	3
	7	0.000	146	830	845	5326	3
29	0	1.400	146	830	855	5376	3
	1	2.300	198	1140	1144	7258	4
	2	0.000	146	830	847	5316	3
	3	0.000	188	1040	1055	6348	4
	4	1.200	192	1555	1540	1352	3
	5	2.100	250	1450	1459	9312	5
	6	0.000	136	730	734	4249	3
	7	0.000	146	830	845	5326	3
30	0	2.200	278	1974	1989	1361	6
	1	0.000	198	1140	1144	7258	4
	2	0.000	359	2458	2475	1814	7
	3	7.200	188	1040	1055	6348	4

Table 14
(contd.)

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
	4	6.600	192	1555	1540	1352	3
	5	3.400	250	1450	1459	9312	5
	6	0.000	136	730	734	4249	3
	7	2.400	146	830	845	5326	3
31	0	0.000	278	1974	1989	1361	6
	1	0.000	331	2253	2258	1707	6
	2	0.000	433	2359	2402	1615	8
	3	0.000	314	2089	2088	1504	6
	4	4.100	238	1499	1504	1109	4
	5	0.000	222	1772	1744	1334	4
	6	0.000	254	1793	1820	1263	5
	7	6.500	201	1261	1244	9176	4
32	0	0.000	278	1974	1989	1361	6
	1	2.400	331	2253	2258	1707	6
	2	0.000	307	2028	2025	1405	6
	3	2.200	314	2089	2088	1504	6
	4	0.000	238	1499	1504	1109	4
	5	4.200	222	1772	1744	1334	4
	6	0.000	254	1793	1820	1263	5
	7	0.000	348	2587	2556	1866	7

Table 15. Results for individual rack problems using heuristic H2

Instance Number 1	Rack 2	Gap 3	#Nodes SP 4	#Arcs SP 5	#Nodes EXP 6	#Arcs EXP 7	#SPs 8
33	0	0.0	104	620	625	4144	2
	1	0.0	104	620	628	4204	2
	2	0.0	104	620	628	4125	2
	3	0.0	104	620	635	4204	2
34	0	0.0	104	620	636	4314	2
	1	0.0	104	620	628	4204	2
	2	0.0	104	620	628	4125	2
	3	0.0	104	620	627	4214	2
35	0	0.0	104	620	624	4115	2
	1	0.0	104	620	629	4106	2
	2	0.0	104	620	632	4173	2
	3	0.0	104	620	637	4235	2
36	0	0.0	104	620	625	4175	2
	1	0.0	104	620	642	4234	2
	2	0.0	104	620	631	4185	2

Table 15
(contd.)

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
	3	1.2	156	930	922	6128	3
37	0	0.0	373	2246	2263	1575	7
	1	0.0	104	620	642	4234	2
	2	0.0	104	620	631	4185	2
	3	1.2	156	930	922	6128	3
38	0	0.0	373	2246	2263	1575	7
	1	1.2	106	717	719	5275	2
	2	0.0	282	1877	1865	1404	5
	3	0.0	204	1242	1252	8438	4
39	0	0.0	373	2246	2263	1575	7
	1	0.0	474	3286	3360	2610	8
	2	0.0	94	455	470	2520	2
	3	0.0	147	891	903	6035	3
40	0	0.0	142	803	813	4580	3
	1	0.0	392	2401	2405	1586	8
	2	2.4	314	2151	2196	1746	5
	3	3.0	283	1746	1786	1255	5
41	0	0.0	208	1240	1264	8369	4
	1	0.0	490	2800	2851	1789	10
	2	0.0	438	2490	2512	1581	9
	3	3.0	283	1746	1786	1255	5
42	0	0.0	208	1240	1264	8369	4
	1	0.0	260	1550	1578	1039	5
	2	0.0	552	3210	3280	2116	11
	3	3.0	283	1746	1786	1255	5
43	0	0.0	208	1240	1262	8467	4
	1	0.0	208	1240	1260	8410	4
	2	0.0	552	3210	3280	2116	11
	3	0.0	510	3000	3063	2003	10
44	0	0.0	260	1550	1577	1053	5
	1	0.0	208	1240	1260	8410	4
	2	0.0	490	2800	2820	1770	10
	3	0.0	344	1970	1983	1261	7
45	0	0.0	260	1550	1577	1053	5
	1	0.0	414	2621	2655	1971	7
	2	0.0	490	2800	2820	1770	10
	3	3.0	171	1045	1058	7661	3
46	0	0.0	260	1550	1577	1053	5
	1	0.0	315	2196	2168	1831	5
	2	2.4	490	2800	2820	1770	10
	3	0.0	331	2025	2047	1380	6
47	0	0.0	856	5925	5881	4389	15
	1	0.0	315	2196	2168	1831	5
	2	0.0	490	2800	2820	1770	10
	3	0.0	331	2025	2047	1380	6
48	0	0.0	856	5925	5881	4389	15
	1	0.0	315	2196	2168	1831	5
	2	0.0	490	2800	2820	1770	10
	3	0.0	331	2025	2047	1380	6

Table 15
(contd.)

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
49	0	0.0	856	5925	5881	4389	15
	1	0.0	315	2196	2168	1831	5
	2	0.0	490	2800	2820	1770	10
	3	3.7	42	310	321	2152	1
	4	3.7	42	310	321	2152	1
	5	0.0	42	310	322	2142	1
	6	0.0	84	620	624	4084	2
	7	0.0	42	310	320	2152	1
50	0	2.4	104	620	638	4274	2
	1	0.0	156	930	946	6278	3
	2	1.6	42	310	326	2172	1
	3	7.8	84	620	636	4264	2
	4	0.0	42	310	321	2152	1
	5	10.2	42	310	322	2142	1
	6	2.4	42	310	321	2142	1
	7	1.2	96	630	635	3326	3
51	0	20.1	126	930	961	6386	3
	1	0.0	126	630	640	3286	3
	2	0.0	42	310	326	2172	1
	3	0.0	126	930	941	6226	3
	4	12.1	126	930	948	6356	3
	5	0.0	44	220	212	224	2
	6	0.1	42	310	321	2142	1
	7	0.0	96	630	635	3326	3
52	0	13.1	126	930	966	6456	3
	1	4.0	96	630	627	3226	3
	2	32.6	126	930	946	6336	3
	3	0.0	84	620	641	4254	2
	4	0.0	126	930	948	6356	3
	5	0.0	44	220	212	2240	2
	6	0.0	42	310	321	2142	1
	7	0.0	96	630	639	3266	3
53	0	2.6	126	930	966	6456	3
	1	4.2	50	444	456	3662	1
	2	0.0	211	1118	1140	7352	4
	3	1.3	50	444	448	3590	1
	4	2.4	30	154	154	737	1
	5	1.8	169	1079	1058	7634	3
	6	8.6	177	981	977	6781	3
	7	9.4	50	444	428	3434	1
54	0	0.0	305	1793	1785	1306	5
	1	0.0	221	1508	1512	1126	4
	2	1.2	214	1345	1352	8926	4
	3	2.6	34	200	207	1106	1
	4	0.0	97	499	504	2779	2
	5	5.8	169	1079	1058	7634	3
	6	0.0	245	1670	1663	1270	4
	7	8.6	30	154	154	737	1
55	0	0.0	91	497	502	3092	2

Table 15
(contd.)

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
	1	3.2	82	503	514	2495	3
	2	6.9	153	981	993	7316	3
	3	4.1	190	1485	1491	1252	3
	4	0.0	97	499	504	2779	2
	5	13.2	103	704	710	4195	3
	6	4.5	32	26	32	32	3
	7	2.1	30	154	154	7370	1
56	0	1.6	150	726	736	4504	3
	1	2.9	82	442	449	2032	3
	2	3.2	153	981	993	7316	3
	3	7.8	190	1485	1491	1252	3
	4	0.0	97	499	504	2779	2
	5	0.0	468	2790	2834	1881	9
	6	0.0	245	1670	1663	1270	4
	7	0.0	206	1537	1535	1339	3
57	0	0.0	208	1240	1270	8469	4
	1	14.4	82	442	449	2032	3
	2	1.2	153	981	993	7316	3
	3	2.6	156	930	947	6386	3
	4	0.0	364	2170	2222	1470	7
	5	0.0	468	2790	2834	1881	9
	6	0.0	156	930	963	6396	3
	7	12.2	156	930	945	6306	3
58	0	0.0	104	620	629	4144	2
	1	2.5	82	442	449	2032	3
	2	11.6	153	981	993	7316	3
	3	0.0	364	2170	2195	1467	7
	4	0.0	364	2170	2222	1470	7
	5	0.0	104	620	634	4205	2
	6	3.6	364	2170	2218	1472	7
	7	0.0	104	620	634	4254	2
59	0	0.0	208	1240	1261	8379	4
	1	0.0	208	1240	1259	8409	4
	2	0.0	104	620	629	4164	2
	3	0.0	416	2480	2524	1685	8
	4	0.0	312	1860	1905	1272	6
	5	0.0	208	1240	1270	8438	4
	6	0.0	260	1550	1547	1032	5
	7	4.8	208	1240	1279	8478	4
60	0	0.0	146	830	836	5236	3
	1	0.0	208	1240	1268	8498	4
	2	0.0	312	1860	1908	1268	6
	3	0.0	146	830	842	5267	3
	4	0.0	104	620	636	4234	2
	5	0.0	104	620	638	4275	2
	6	0.0	104	620	623	4164	2
	7	0.0	208	1240	1275	8428	4
61	0	0.0	465	2880	2868	1988	9
	1	0.0	550	3853	3887	3063	9

Table 15
(contd.)

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
	2	0.0	312	1860	1908	1268	6
	3	0.0	146	830	842	5267	3
	4	0.0	104	620	636	4234	2
	5	0.0	414	2621	2658	1971	7
	6	0.0	423	2414	2419	1550	9
	7	4.8	172	1130	1130	8648	3
62	0	0.0	216	1170	1147	7574	4
	1	0.0	550	3853	3887	3063	9
	2	0.0	401	2603	2600	1780	8
	3	0.0	146	830	842	5267	3
	4	0.0	98	512	530	3232	2
	5	0.0	226	1448	1418	1061	4
	6	0.0	317	2561	2552	1982	6
	7	5.2	172	1130	1130	8648	3
63	0	0.0	320	1933	1897	1305	6
	1	0.0	317	2590	2567	2220	5
	2	0.0	401	2603	2600	1780	8
	3	2.8	133	641	645	3647	3
	4	0.0	298	1854	1873	1333	5
	5	0.0	569	3629	3666	2604	10
	6	6.7	161	927	930	6080	3
	7	0.0	367	2877	2858	2379	6
64	0	0.0	148	736	751	4460	3
	1	7.8	317	2590	2567	2220	5
	2	0.0	401	2603	2600	1780	8
	3	8.4	169	1277	1275	9944	3
	4	0.0	230	1495	1473	1072	4
	5	3.2	569	3629	3666	2604	10
	6	0.0	161	927	930	6080	3
	7	0.0	270	1540	1540	1038	5

Table 16. Results for individual rack problems using heuristic H3

Instance Number 1	Rack 2	Gap 3	#Nodes SP 4	#Arcs SP 5	#Nodes EXP 6	#Arcs EXP 7	#SPs 8
65	0	0.0	104	620	626	4194	2
	1	0.0	104	620	610	4115	2
	2	0.0	104	620	629	4115	2
	3	0.0	104	620	634	4225	2
66	0	0.0	104	620	646	4284	2
	1	0.0	104	620	636	4214	2
	2	0.0	104	620	625	4085	2
	3	0.0	104	620	632	4246	2
67	0	0.0	104	620	627	4185	2
	1	0.0	104	620	637	4215	2
	2	0.0	104	620	615	4094	2
	3	0.0	104	620	635	4194	2
68	0	0.0	104	620	631	4224	2
	1	0.0	104	620	636	4195	2
	2	0.0	104	620	627	4234	2
	3	0.0	104	620	623	4064	2
69	0	0.0	104	620	631	4224	2
	1	0.0	104	620	636	4195	2
	2	0.0	288	2118	2106	1661	5
	3	0.0	309	2130	2126	1476	6
70	0	0.0	549	3585	3593	2606	10
	1	0.0	104	620	636	4195	2
	2	0.0	394	2723	2704	2024	7
	3	0.0	299	2025	2020	1613	5
71	0	0.0	549	3585	3593	2606	10
	1	0.0	425	2858	2849	1992	8
	2	2.3	237	1502	1509	1180	4
	3	0.0	237	1622	1614	1227	4
72	0	0.0	549	3585	3593	2606	10
	1	0.0	324	2322	2317	1724	6
	2	4.3	237	1502	1509	1180	4
	3	0.0	237	1622	1614	1227	4
73	0	0.0	416	2480	2512	1668	8
	1	0.0	208	1240	1257	8379	4
	2	0.0	208	1240	1264	8388	4
	3	0.0	208	1240	1265	8399	4
74	0	0.0	208	1240	1254	8260	4
	1	0.0	208	1240	1262	8430	4
	2	0.0	208	1240	1253	8379	4
	3	0.0	208	1240	1270	8478	4
75	0	0.0	208	1240	1246	8319	4
	1	0.0	208	1240	1259	8329	4
	2	0.0	208	1240	1255	8300	4
	3	0.0	208	1240	1257	8300	4
76	0	0.0	208	1240	1257	8312	4
	1	0.0	208	1240	1256	8468	4
	2	0.0	208	1240	1269	8458	4

Table 16
(contd.)

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
	3	0.0	208	1240	1260	8419	4
77	0	0.0	208	1240	1257	8312	4
	1	0.0	208	1240	1256	8468	4
	2	0.0	701	4626	4621	3451	12
	3	0.0	208	1240	1260	8419	4
78	0	0.0	621	4561	4590	3315	12
	1	0.0	208	1240	1256	8468	4
	2	0.0	692	4542	4588	3361	12
	3	0.0	832	5640	5629	4100	15
79	0	0.0	621	4561	4590	3315	12
	1	0.0	208	1240	1256	8468	4
	2	0.0	338	2099	2127	1392	7
	3	0.0	746	5300	5342	4272	12
80	0	0.0	621	4561	4590	3315	12
	1	0.0	208	1240	1256	8468	4
	2	0.0	338	2099	2127	1392	7
	3	0.0	538	3240	3267	2227	10
81	0	0.0	42	310	313	2072	1
	1	0.0	42	310	309	2032	1
	2	0.0	42	310	304	1992	1
	3	0.0	42	310	300	2062	1
	4	0.0	42	310	324	2152	1
	5	0.0	42	310	308	2072	1
	6	0.0	42	310	318	2092	1
	7	0.0	42	310	313	2042	1
82	0	0.0	42	310	323	2152	1
	1	0.0	42	310	323	2132	1
	2	0.0	42	310	305	1992	1
	3	0.0	42	310	318	2142	1
	4	0.0	42	310	303	1952	1
	5	0.0	42	310	324	2152	1
	6	0.0	42	310	315	2072	1
	7	0.0	42	310	309	2042	1
83	0	0.0	42	310	314	2082	1
	1	0.0	42	310	315	2122	1
	2	0.0	42	310	324	2172	1
	3	0.0	42	310	316	2102	1
	4	0.0	42	310	317	2142	1
	5	0.0	42	310	299	1982	1
	6	0.0	42	310	318	2102	1
	7	0.0	42	310	314	2062	1
84	0	0.0	42	310	318	2122	1
	1	0.0	42	310	315	2122	1
	2	0.0	42	310	324	2172	1
	3	0.0	42	310	314	2052	1
	4	0.0	42	310	320	2142	1
	5	0.0	42	310	308	2092	1
	6	0.0	42	310	311	2012	1
	7	0.0	42	310	316	2062	1

Table 16
(contd.)

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
85	0	3.2	79	559	573	3937	1
	1	0.0	45	369	368	2659	1
	2	1.6	89	673	681	5054	1
	3	4.2	50	397	394	2814	1
	4	0.0	41	294	293	1876	1
	5	0.0	54	520	517	4578	1
	6	0.0	28	137	141	6040	1
	7	0.0	46	379	378	2737	1
86	0	0.0	42	346	350	2342	1
	1	0.0	42	346	350	2342	1
	2	2.1	89	673	681	5054	1
	3	6.5	99	927	929	7426	1
	4	0.0	50	460	463	3780	1
	5	0.0	39	256	260	1570	1
	6	11.2	86	628	626	4622	1
	7	0.0	51	481	481	3995	1
87	0	0.0	39	186	187	8680	1
	1	0.0	37	252	258	1503	1
	2	4.3	38	260	262	1572	1
	3	0.0	48	357	353	2380	1
	4	3.1	84	608	612	4382	1
	5	0.0	47	351	359	2543	1
	6	0.0	47	402	406	3007	1
	7	0.0	44	285	287	1701	1
88	0	0.0	39	214	216	1329	1
	1	0.0	37	252	258	1503	1
	2	2.2	38	260	262	1572	1
	3	0.0	48	357	353	2380	1
	4	0.0	33	198	204	8720	1
	5	0.0	50	425	430	3412	1
	6	0.0	45	354	355	2345	1
	7	0.0	44	285	287	1701	1
89	0	0.0	84	620	632	4254	2
	1	1.2	84	620	634	4144	1
	2	6.5	84	620	628	4214	1
	3	5.6	84	620	632	4234	1
	4	0.0	364	2170	2222	1470	7
	5	5.2	84	620	630	4164	1
	6	3.1	84	620	640	4254	1
	7	6.7	84	620	636	4274	1
90	0	0.0	84	620	618	4054	1
	1	0.0	84	620	630	4214	1
	2	0.0	84	620	636	4254	1
	3	11.3	84	620	636	4264	1
	4	1.1	364	2170	2222	1470	7
	5	12.2	84	620	626	4204	1
	6	16.7	84	620	640	4284	1
	7	0.0	104	620	634	4254	2
91	0	0.0	84	620	629	4184	1

Table 16
(contd.)

Instance Number	Rack	Gap	#Nodes SP	#Arcs SP	#Nodes EXP	#Arcs EXP	#SPs
	1	2.3	84	620	620	4154	1
	2	2.1	126	930	948	6246	1
	3	0.0	84	620	635	4264	1
	4	3.2	84	620	625	4164	1
	5	3.1	84	620	635	4194	1
	6	2.1	84	620	629	4124	1
	7	17.2	126	930	953	6346	1
92	0	3.3	84	620	639	4234	1
	1	0.0	84	620	628	4194	1
	2	2.5	84	620	621	4214	1
	3	0.0	126	930	953	6356	1
	4	3.1	84	620	632	4214	1
	5	6.5	126	930	953	6396	1
	6	2.4	126	930	944	6276	1
	7	4.3	84	620	636	4244	1
93	0	0.0	465	2880	2868	1988	9
	1	13.2	100	934	924	7376	1
	2	12.2	72	488	499	2876	1
	3	9.1	181	1416	1407	1058	1
	4	8.2	176	1382	1389	9404	1
	5	3.2	85	654	648	4390	1
	6	6.3	123	882	891	5827	1
	7	4.1	91	764	770	5632	1
94	0	1.1	80	611	618	3929	1
	1	2.6	77	561	564	3466	1
	2	2.8	75	498	502	3028	1
	3	12.1	88	707	712	4901	1
	4	0.0	98	512	530	3232	2
	5	0.0	79	606	607	3764	1
	6	8.1	94	835	824	6349	1
	7	5.1	78	575	592	3760	1
95	0	5.2	126	882	884	6250	1
	1	4.6	75	522	523	3145	1
	2	6.5	143	1356	1360	1063	1
	3	11.2	85	623	632	4246	1
	4	0.0	77	500	504	2958	1
	5	0.0	71	489	496	2788	1
	6	3.1	161	927	930	6080	3
	7	2.2	94	746	758	5487	1
96	0	6.1	78	531	534	3227	1
	1	0.0	75	574	575	3318	1
	2	7.6	84	649	663	4550	1
	3	5.6	127	923	917	6425	1
	4	0.0	76	549	553	3263	1
	5	0.0	78	492	496	3117	1
	6	2.1	161	927	930	6080	3
	7	1.6	83	656	662	4591	1

Columns 4 and 5 list the number of nodes and arcs in each sub-problem and columns 6 and 7 show the number of nodes and arcs in the expanded networks. Column 8 lists the number of sub-problems associated with each instance – one for each CTPC prescribed by P2. Instances 13-16 have appreciably more sub-problems with larger sub-problem networks because they represent cases in which P2 prescribes more CTPCs. On the other hand, for H1 and H2, P2 prescribed only one CTPC for a number of rack problems that involve 2 DHPMs, so each such rack problem required just one sub-problem.

Figure 6 compares the run times for H1, H2 and H3 relative to the instance number and the number of CTs. H3 run times can be seen to be somewhat larger, on average. Instances 5, 89 and 91 require an exceptional amount of run times because they require a large number of sub-problems to be solved (3039, 3155 and 4240 respectively) and a large number of branch and bound nodes (528, 803 and 1736, respectively).

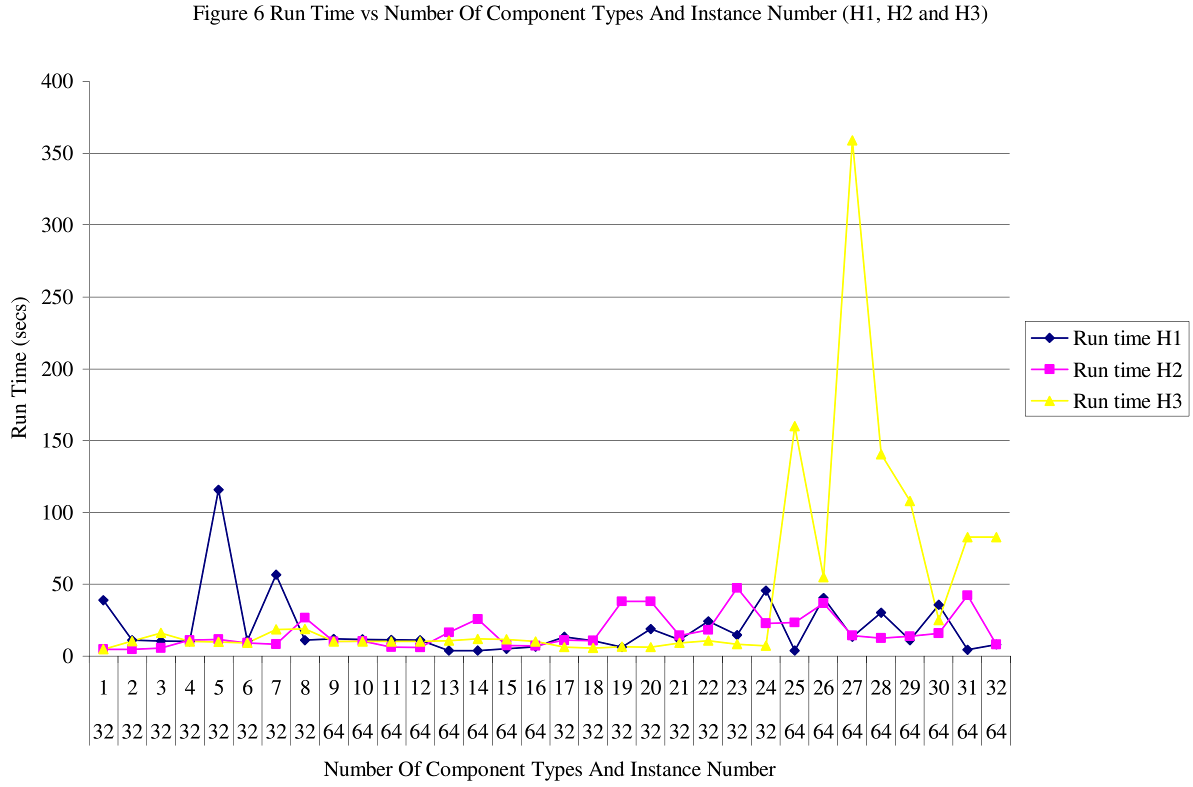

Fig. 6. Run time vs number of component types and instance number (H1, H2 and H3).

4.3 P4 computational results

This section presents P4 computational results relative to the three heuristics (H1, H2 and H3) used in P1 to allocate components to feeder slots. The tests were written in C program in the Watcom C++ editor and interfaced with the Fortran code of Carpaneto, Dell' Amico and Toth (1990), which solves the asymmetric traveling salesman problem to optimality. Sub-section 4.3.1 describes the experimental design, test instances and test results. Sub-section 4.3.2 discusses the overall performance by the heuristics; sub-sections 4.3.3, 4.3.4 and 4.3.5 present performance measures that result from applying heuristics H1, H2 and H3, respectively, and, finally, sub-section 4.3.6 compares the three heuristics.

4.3.1 Experimental design and test instances

The experimental design addresses the same six factors used to test P2 and P3. For details please refer to section 4.1. A set of 96 test instances is characterized by a unique selection of levels, for each of the factors. Each instance is solved by prescribing solutions to P1, P2, P3 and P4 in sequence, using the heuristics described in section 3.3 to solve P1, the approach of Wilhelm and Arambula (2001) to optimize P2, the approach of Wilhelm and Damodaran (2001) to optimize P3, and the approach of Wilhelm, Gott Khotekar and Rao (2004) to optimize P4 respectively.

The P4 problem is solved for each head on each machine (P2 and P3 solve individual rack problems for each head). Tables 18, 19 and 20 give performance measures that result from using heuristics H1, H2 and H3, respectively, to allocate CTs to feeder slots. Columns 1-3 describe the instance and columns 4-9 give test results. Column 1 gives the instance number, which varies from 1-32. Columns 2 and 3 specify the rack numbers associated with each head. In case of a single machine (level 1 of factor 2), racks 0 and 1 are associated with head one; and racks 2 and 3 with head two. In the case of two machines, racks 4 and 5 are associated with head one on the second machine; and racks 6 and 7, with head two. Column 4, 5 and 6 give the percentage of total time that each head spends performing picking, placing and nozzle changing steps. Column 7 gives the number of nozzle changes prescribed for each head. Finally, column 8 provides a measure of the imbalance of workloads assigned to heads on all machines (i.e., one or two machines).

The acronyms that head columns of table 18, 19 and 20 are defined below in table 17:

Table 17. Acronyms of columns for tables 18, 19 and 20

Column	Acronym	Description
1	Instance #	Instance number
2 and 3	Rack#	Rack number
4	%Pick	The percentage of time spent in picking relative to total cycle time
5	%Place	The percentage of time spent in placing relative to total cycle time

Table 17

(contd.)

Column	Acronym	Description
6	%Nchange	The percentage of time spent in nozzle changing relative to total cycle time
7	#NChanges	Total number of nozzle changes
8	IM	The percentage imbalance in workload between the heads defined as: $100(\max_{h \in H}\{W_h\} - (1/\|H\|)*\sum_{h \in H}\{W_h\})/(1/\|H\|)*\sum_{h \in H}\{W_h\}$

Table 18. Summary of results of P4 from using heuristic H1

Instance#	Rack#	Rack#	%Pick	%Place	%Nchange	#Nchanges	IM
1	0	1	20.9	67.4	11.8	4	
1	2	3	25.5	61.6	12.8	0	16.9
2	0	1	20.5	69.2	10.3	3	
2	2	3	21.2	64.8	14.0	4	14.7
3	0	1	11.0	82.0	7.1	0	
3	2	3	13.5	71.0	15.5	1	15.4
4	0	1	24.5	63.1	12.3	15	
4	2	3	19.6	65.9	14.5	7	6.3
5	0	1	22.0	62.0	16.0	0	
5	2	3	27.6	59.6	12.8	1	21.4
6	0	1	24.8	63.3	11.9	7	
6	2	3	25.1	67.7	7.2	4	7.8
7	0	1	26.5	59.4	14.2	18	
7	2	3	19.9	63.7	16.4	2	12.1
8	0	1	23.7	59.8	16.4	24	
8	2	3	22.1	61.0	16.9	4	8.1
9	0	1	21.1	67.6	11.4	6	
9	2	3	20.9	67.4	11.8	4	14.5
10	0	1	25.5	61.6	12.8	0	
10	2	3	20.5	69.2	10.3	3	16.9
11	0	1	21.2	64.8	14.0	4	

Table 18
(contd.)

Instance#	Rack#	Rack#	%Pick	%Place	%Nchange	#Nchanges	IM
11	2	3	7.5	87.7	4.8	0	29.2
12	0	1	13.5	71.0	15.5	1	
12	2	3	24.5	63.1	12.3	15	30.4
13	0	1	19.6	65.9	14.5	7	
13	2	3	22.0	62.0	16.0	0	19.8
14	0	1	27.6	59.6	12.8	1	
14	2	3	24.8	63.3	11.9	7	8.7
15	0	1	25.2	55.9	18.9	4	
15	2	3	26.5	59.4	14.2	18	10.0
16	0	1	19.9	63.7	16.4	2	
16	2	3	23.7	59.8	16.4	24	21.0
17	0	1	7.5	86.7	5.8	4	
17	2	3	21.1	67.6	11.4	6	10.4
17	4	5	17.4	65.5	17.1	2	
17	6	7	21.1	72.1	6.9	3	7.3
18	0	1	19.1	67.8	13.1	7	
18	2	3	21.1	68.9	10.0	9	1.3
18	4	5	23.0	63.3	13.7	20	
18	6	7	19.0	67.7	13.3	19	28.5
19	0	1	18.5	66.5	15.0	19	
19	2	3	23.9	60.0	16.0	10	30.4
19	4	5	15.1	41.4	43.5	0	
19	6	7	24.0	56.8	19.1	2	31.1
20	0	1	9.3	83.2	7.5	14	
20	2	3	11.6	83.1	5.3	12	8.1
20	4	5	11.4	83.8	4.8	4	
20	6	7	4.9	93.6	1.5	5	12.2
21	0	1	6.4	91.6	2.0	6	
21	2	3	11.9	87.0	1.1	8	22.0
21	4	5	8.7	83.0	8.3	17	
21	6	7	6.9	88.7	4.4	7	5.1
22	0	1	11.1	84.8	4.1	10	
22	2	3	6.8	89.9	3.3	12	11.5
22	4	5	23.3	60.5	16.2	10	
22	6	7	18.5	68.4	13.1	15	22.5
23	0	1	18.3	67.6	14.1	12	
23	2	3	11.3	77.5	11.2	2	8.4
23	4	5	21.1	62.2	16.7	0	
23	6	7	35.2	52.7	12.1	4	56.5
24	0	1	29.2	65.5	5.3	0	
24	2	3	20.6	67.1	12.4	0	14.2
24	4	5	37.7	54.9	7.5	0	
24	6	7	19.9	73.7	6.4	0	11.3

Table 18
(contd.)

Instance#	Rack#	Rack#	%Pick	%Place	%Nchange	#Nchanges	IM
25	0	1	27.9	63.0	9.1	0	
25	2	3	28.2	61.0	10.8	0	4.0
25	4	5	23.6	61.8	14.5	0	
25	6	7	28.3	58.9	12.8	0	15.0
26	0	1	25.9	62.3	11.8	4	
26	2	3	21.5	63.2	15.3	0	10.9
26	4	5	38.2	49.9	12.0	4	
26	6	7	27.3	69.6	3.1	0	42.4
27	0	1	26.3	57.4	16.4	0	
27	2	3	23.0	61.6	15.4	0	56.3
27	4	5	36.2	53.2	10.6	0	
27	6	7	23.9	59.1	17.0	0	61.0
28	0	1	14.2	74.8	11.1	0	
28	2	3	15.6	81.5	2.9	0	24.5
28	4	5	17.8	66.7	15.5	0	
28	6	7	20.9	67.4	11.7	0	0.8
29	0	1	20.1	68.9	11.0	0	
29	2	3	22.3	66.5	11.2	0	44.0
29	4	5	20.0	69.3	10.7	0	
29	6	7	32.2	55.3	12.5	2	69.2
30	0	1	17.1	69.0	13.8	9	
30	2	3	18.3	69.5	12.2	9	45.9
30	4	5	18.9	68.4	12.7	0	
30	6	7	21.9	69.2	8.9	4	33.2
31	0	1	17.5	68.5	14.0	7	
31	2	3	21.5	71.1	7.4	4	2.9
31	4	5	31.5	55.3	13.2	0	
31	6	7	16.7	71.7	11.6	0	16.8
32	0	1	25.0	63.3	11.7	0	
32	2	3	5.0	93.1	1.8	0	14.1
32	4	5	2.6	96.6	0.7	0	
32	6	7	21.8	67.2	11.1	7	22.7

Table 19. Summary of results of P4 from using heuristic H2

Instance#	Rack#	Rack#	%Pick	%Place	%Nchange	#Nchanges	IM
33	0	1	27.4	60.5	12.0	6	
33	2	3	26.9	56.1	17.0	2	3.0
34	0	1	25.1	57.2	17.6	4	
34	2	3	30.5	52.8	16.8	10	5.0
35	0	1	11.0	77.6	11.5	66	
35	2	3	9.3	77.3	13.5	13	3.3
36	0	1	4.0	83.6	12.5	92	
36	2	3	3.3	83.7	13.0	34	6.2
37	0	1	21.5	66.2	12.3	5	
37	2	3	23.4	63.5	13.2	10	4.8
38	0	1	26.4	59.9	13.7	8	
38	2	3	25.2	58.8	16.0	5	0.9
39	0	1	24.0	60.5	15.5	6	
39	2	3	32.0	60.9	7.1	2	18.3
40	0	1	17.6	34.4	47.9	3	
40	2	3	16.6	35.4	48.0	2	2.4
41	0	1	22.4	64.3	13.3	12	
41	2	3	22.9	66.2	10.9	11	3.1
42	0	1	4.3	82.5	13.2	10	
42	2	3	3.7	95.1	1.3	1	3.2
43	0	1	21.2	67.7	11.1	25	
43	2	3	21.0	66.1	13.0	9	31.5
44	0	1	18.7	71.4	9.9	7	
44	2	3	22.9	64.2	12.9	25	32.1
45	0	1	26.1	59.1	14.8	20	
45	2	3	26.6	58.5	14.9	16	2.8
46	0	1	18.3	73.5	8.2	0	
46	2	3	20.3	68.3	11.5	0	14.5
47	0	1	29.5	63.6	6.8	4	
47	2	3	29.3	66.2	4.6	2	4.7
48	0	1	27.9	60.5	11.6	6	
48	2	3	29.2	63.1	7.6	8	8.1
49	0	1	21.0	62.3	16.7	5	
49	2	3	25.9	57.2	17.0	7	12.5
49	4	5	17.1	81.1	1.8	0	
49	6	7	17.5	78.9	3.7	0	19.4
50	0	1	29.8	68.6	1.6	0	
50	2	3	27.7	59.7	12.6	8	10.5
50	4	5	27.1	60.1	12.7	12	
50	6	7	25.8	61.5	12.6	0	1.3
51	0	1	14.8	83.0	2.2	4	
51	2	3	15.5	79.7	4.8	8	7.7

Table 19
(contd.)

Instance#	Rack#	Rack#	%Pick	%Place	%Nchange	#Nchanges	IM
51	4	5	15.3	81.2	3.4	5	
51	6	7	13.4	83.0	3.6	6	4.6
52	0	1	29.3	69.9	0.8	0	
52	2	3	17.3	81.0	1.7	2	12.8
52	4	5	16.3	82.3	1.4	3	
52	6	7	14.5	84.2	1.3	1	3.0
53	0	1	20.2	70.5	9.3	4	
53	2	3	15.3	78.8	6.0	2	8.1
53	4	5	20.0	66.1	13.8	4	
53	6	7	24.5	71.8	3.7	0	12.5
54	0	1	41.8	26.5	31.7	0	
54	2	3	23.8	52.9	23.3	0	23.4
54	4	5	28.8	59.0	12.2	0	
54	6	7	24.4	70.0	5.6	0	32.8
55	0	1	27.1	69.6	3.4	6	
55	2	3	34.6	55.3	10.1	0	48.3
55	4	5	10.9	82.9	6.2	3	
55	6	7	22.7	65.0	12.3	4	36.9
56	0	1	8.7	86.0	5.3	2	
56	2	3	11.3	83.3	5.4	4	3.5
56	4	5	14.0	77.9	8.1	2	
56	6	7	21.1	60.8	18.1	4	2.3
57	0	1	13.4	77.6	8.9	2	
57	2	3	9.3	81.6	9.0	2	7.6
57	4	5	16.7	64.7	18.6	0	
57	6	7	15.9	63.9	20.3	0	6.9
58	0	1	17.1	69.5	13.4	1	
58	2	3	16.0	70.6	13.4	2	5.6
58	4	5	25.5	65.7	8.8	2	
58	6	7	25.6	64.6	9.8	5	4.2
59	0	1	26.4	68.1	5.5	2	
59	2	3	28.7	68.0	3.3	1	0.5
59	4	5	33.1	62.3	4.6	0	
59	6	7	34.6	59.9	5.5	1	1.1
60	0	1	2.6	87.9	9.5	3	
60	2	3	2.4	84.8	12.8	57	31.1
60	4	5	40.1	54.9	5.0	0	
60	6	7	39.7	54.8	5.5	1	6.1
61	0	1	18.4	71.9	9.7	12	
61	2	3	19.3	66.9	13.8	16	3.1
61	4	5	20.1	71.2	8.7	4	
61	6	7	21.6	68.2	10.2	3	4.3
62	0	1	38.0	50.1	11.9	8	

Table 19
(contd.)

Instance#	Rack#	Rack#	%Pick	%Place	%Nchange	#Nchanges	IM
62	2	3	24.8	65.0	10.3	4	27.4
62	4	5	23.8	63.8	12.4	7	
62	6	7	23.3	64.4	12.3	8	3.7
63	0	1	17.1	76.0	6.9	0	
63	2	3	20.2	69.3	10.4	5	48.9
63	4	5	32.7	57.8	9.5	17	
63	6	7	27.7	60.5	11.8	9	26.7
64	0	1	20.1	77.3	2.7	7	
64	2	3	23.8	73.5	2.7	5	5.5
64	4	5	16.9	73.6	9.5	7	
64	6	7	7.2	79.8	13.0	9	39.6

Table 20. Summary of results of P4 from using heuristic H3

Instance#	Rack#	Rack#	%Pick	%Place	%NChange	#Nchanges	IM
65	0	1	27.4	60.5	12.0	6	3.0
65	2	3	26.9	56.1	17.0	2	
66	0	1	22.8	69.1	8.1	3	4.0
66	2	3	24.2	61.4	14.4	0	
67	0	1	25.6	62.5	11.9	7	5.1
67	2	3	19.5	67.3	13.2	6	
68	0	1	25.8	63.5	10.7	7	8.8
68	2	3	23.0	61.6	15.4	6	
69	0	1	33.5	61.7	4.8	0	21.6
69	2	3	22.9	63.0	14.0	0	
70	0	1	25.1	59.9	14.9	0	5.0
70	2	3	31.3	54.6	14.1	8	
71	0	1	22.2	65.7	12.1	2	0.6
71	2	3	20.5	68.2	11.3	4	
72	0	1	24.0	60.5	15.5	2	18.3

Table 20
(contd.)

Instance#	Rack#	Rack#	%Pick	%Place	%Nchange	#Nchanges	IM
72	2	3	32.0	60.9	7.1	5	
73	0	1	30.6	64.9	4.5	3	4.2
73	2	3	30.6	67.5	1.9	2	
74	0	1	20.4	71.5	8.1	1	3.7
74	2	3	20.2	67.1	12.7	0	
75	0	1	22.6	68.6	8.9	8	4.3
75	2	3	23.8	62.6	13.5	5	
76	0	1	23.6	67.1	9.3	6	7.7
76	2	3	27.3	58.7	14.0	7	
77	0	1	29.0	65.6	5.4	4	2.3
77	2	3	28.6	67.4	4.0	0	
78	0	1	26.0	60.9	13.0	0	2.4
78	2	3	25.1	63.3	11.6	2	
79	0	1	29.9	59.8	10.3	4	5.0
79	2	3	28.4	56.8	14.9	6	
80	0	1	28.4	67.9	3.7	6	2.5
80	2	3	28.8	69.1	2.2	8	
81	0	1	19.8	62.6	17.6	0	4.7
81	2	3	19.6	65.4	15.0	3	
81	4	5	17.2	78.6	4.3	0	0.8
81	6	7	16.9	63.2	19.9	0	
82	0	1	22.3	69.2	8.5	0	3.5
82	2	3	18.2	68.7	13.0	3	
82	4	5	21.6	69.3	9.0	0	5.4
82	6	7	17.8	67.2	15.0	0	
83	0	1	19.0	70.7	10.3	1	2.4
83	2	3	19.1	64.7	16.3	3	
83	4	5	18.9	73.5	7.6	2	2.2
83	6	7	18.9	69.2	11.9	0	
84	0	1	22.2	69.4	8.4	1	0.5
84	2	3	18.8	69.0	12.2	3	
84	4	5	21.4	69.0	9.5	2	12.2
84	6	7	20.8	66.4	12.8	0	
85	0	1	21.7	60.9	17.4	0	11.2
85	2	3	20.0	65.8	14.1	3	
85	4	5	22.5	71.3	6.2	0	10.6
85	6	7	15.3	64.7	20.0	0	
86	0	1	26.0	69.5	4.6	0	17.6
86	2	3	20.1	64.2	15.8	5	
86	4	5	19.3	72.1	8.7	0	1.7
86	6	7	16.5	65.1	18.3	0	
87	0	1	19.8	68.8	11.4	1	0.3
87	2	3	19.4	68.6	12.0	3	

Table 20
(contd.)

Instance#	Rack#	Rack#	%Pick	%Place	%Nchange	#Nchanges	IM
87	4	5	22.6	71.0	6.4	2	10.4
87	6	7	19.0	69.5	11.5	0	
88	0	1	21.3	70.2	8.6	1	4.5
88	2	3	18.3	67.8	13.9	3	
88	4	5	21.1	69.0	9.9	2	0.9
88	6	7	21.5	65.7	12.8	0	
89	0	1	21.6	73.4	5.0	0	4.1
89	2	3	16.3	66.5	17.2	3	
89	4	5	17.9	69.1	13.0	1	8.3
89	6	7	15.2	67.4	17.4	0	
90	0	1	24.1	65.7	10.2	0	0.9
90	2	3	14.1	68.5	17.4	1	
90	4	5	20.9	66.9	12.1	0	13.5
90	6	7	16.7	71.6	11.7	0	
91	0	1	17.3	71.9	10.8	3	16.3
91	2	3	17.9	69.1	13.0	1	
91	4	5	15.9	72.1	11.9	1	14.2
91	6	7	19.2	68.3	12.5	0	
92	0	1	17.0	75.3	7.7	3	5.7
92	2	3	19.4	62.7	17.8	2	
92	4	5	19.5	72.0	8.5	1	5.7
92	6	7	19.3	62.3	18.3	0	
93	0	1	17.0	65.5	17.6	0	2.0
93	2	3	16.8	67.0	16.2	7	
93	4	5	20.3	63.1	16.6	0	3.7
93	6	7	14.2	68.8	17.0	0	
94	0	1	16.8	66.1	17.1	0	8.9
94	2	3	16.2	68.3	15.5	3	
94	4	5	23.4	70.0	6.6	0	2.6
94	6	7	14.3	73.8	11.9	0	
95	0	1	22.0	68.8	9.2	2	2.7
95	2	3	17.9	69.2	12.8	1	
95	4	5	16.5	65.0	18.5	2	21.1
95	6	7	15.7	66.4	17.8	0	
96	0	1	18.4	72.9	8.7	2	3.7
96	2	3	19.1	64.1	16.9	9	
96	4	5	19.3	69.7	11.0	1	4.3
96	6	7	23.9	70.4	5.8	0	

4.3.2 Analysis of overall performance

Column 5 in tables 18, 19 and 20, shows that placement time dominates picking time (column 4) and nozzle change time (column 6). Placement time includes the time taken for the head to move from the camera to the first placement location and place that component, as well as the times to move from one placement position to the next and to place each component. Due to the high degree of precision that is required while placing, the head must move slowly during placement, making placing steps slower than the picking and nozzle changing steps. These columns also show that nozzle change steps involve a smaller portion of cycle time than picking and placing steps. The developed heuristics contribute to this desirable result by seeking fewer nozzle changes. Also, nozzle change motions are much faster than pick or place motions, leading to a smaller portion of cycle time being used for changing nozzles.

4.3.3 Analysis of H1

Table 18 shows that imbalance is higher in the case involving two DHPMs because these instances require workloads to be balanced on four heads. This shows that, given a larger number of racks, H1 may allocate many CTs to one rack and few CTs to another, causing a workload imbalance. This imbalance results because the logic in H1 tries to maintain group formations and does not consider how many CTs are allocated to each rack. H1 achieves a better workload balance in the case of a single DHPM than in the case with two DHPMs, because it requires workloads to be balanced on only two heads. For the single machine case, the imbalance is higher in instances with 64 CTs (18.8% on an average) than in instances with 32 CTs (12.8% on an average). This difference is largely due to placement times rather than the number of CTs that have been allocated to each head, because all racks are fully loaded. In the case of two machines, H1 results in larger imbalances with 64 CTs because more CTs are spread over a larger number of racks.

The cycle time for the line is determined by the maximum of the workloads assigned to the heads. Please note that in figure 7, y-axis values, which denote the workloads or actual cycle time of the machines, have not been specified due to a non-disclosure agreement with our industrial collaborator. From Figure 7, the graph for H1 workloads shows that in instances with 32 CTs on 2 machines (instances 17-24), the workloads are the highest compared to other instances. Since H1 can result in a large variability in the number of CTs assigned to heads in these instances, the head to which H1 assigns the largest number of CTs, has the largest workload. In the case of a single machine, all racks are completely full and the largest workloads are typically determined by placement times; however in a few cases, the nozzle change times or the picking times (owing to P2 prescribing more CTPCs) determine the largest workload

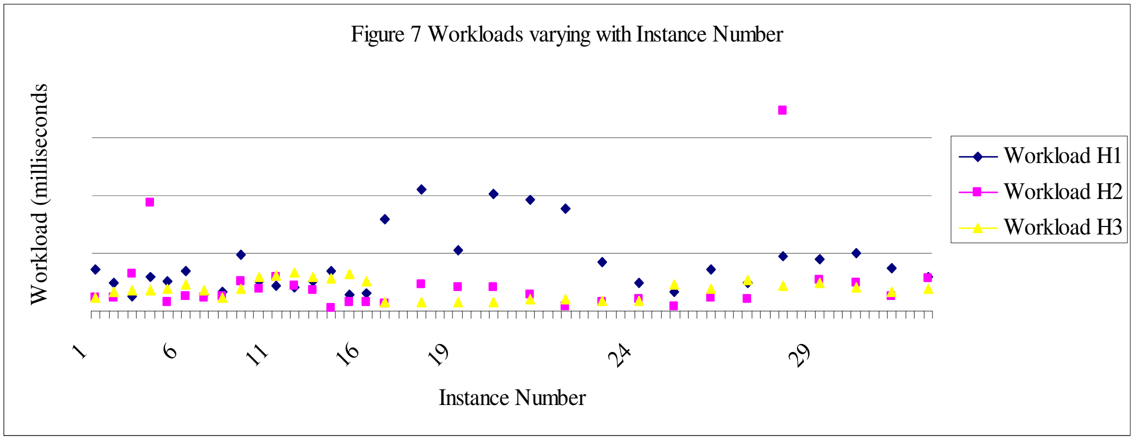

Fig. 7. Workloads varying with instance number.

The number of CTPCs prescribed by P2 determines the number of nozzle changes; and instances with more CTPCs require more nozzle changes (instances 8, 16 and 18).

4.3.4 Analysis of H2

Table 19 shows that H2 results in larger imbalances in the case of two machines compared with the case of involving a single machine. This results because H2 attempts to maximize gang picks and minimize nozzle changes by assigning CT super groups to racks without considering the number of CTs assigned to each rack. In the case of a single machine, H2 again results in imbalances that are higher with 64 CTs (14.26% on an average) than with 32 CTs (9% on an average).

The graph for H2 in Figure 7 shows that cycle times are higher for the case of two DHPMs, both with 32 and 64 CTs. This results because H2, like H1, attempts to maximize gang picks and minimize nozzle changes without considering the number of CTs assigned to heads.

The case of two DHPMs requires fewer nozzle changes than does the case of one machine. H2 gives this result because P2 usually prescribes more CTPCs in the case of one machine than the case involving two machines as it must consider more options to optimize picking operations and more CTs are assigned to each rack (on average) leading to a higher likelihood of forming CTPCs. In the case of two machines, P1 assigns fewer CTs to individual racks so that P2 can find a few CTPCs that optimize picking operations.

4.3.5 Analysis of H3

H3 works to balance the number of CTs on each head so that workloads can be expected to be better balanced whether one or two machines are involved. Placing operations typically require more time than picking and nozzle change operations. Thus, balancing the number of CTs assigned to headstands tends to balance the number of individual components as well as placement times, resulting in balanced workloads. Table 20 confirms this expectation, showing that workload imbalances for the case of two machines (6% on average), is similar to that with a single machine (6.1% average). In the case of a single machine, the workload imbalance is slightly higher with 32 CTs than with 64 CTs; and this is due primarily to the difference in placement times that result from component locations being allocated randomly on the CC. In the case of two machines, workload imbalances for instances with 32 CTs and 64 CTs are similar because H3 tends to balance the workload and the randomness of component locations on the CC have less influence in determining workloads.

Since H3 seeks to balance the number of CTs on each head, instances in which fewer heads must assemble more CTs can be expected to have a higher cycle time. The H3 graph in Figure 7 shows that the cycle times are highest for instances involving 64 CTs on a single machine and lowest for those with 32 CTs on two machines.

H3 results in fewer nozzle changes since it balances the number of CTs on heads and P2 prescribes fewer CTPCs for each rack.

4.3.6 Comparison of workload balances resulting from H1, H2 and H3

The graph of workloads resulting from H2, shows that, apart from a few outliers (instances 5 and 28), H2 results in better workload balances than H1. This result can be expected because super groups used by H2 reduce nozzle changing times as well as picking times. H3 balances the number of CTs assigned to each rack so that, for the instances in which racks are not completely filled (17-32 in the 2 DHPM case), H3 performs much better than H1 or H2. In instances for which all racks are completely full (e.g., the case of a single DHPM), the workload imbalance is low and any imbalance is primarily due to the differences in random locations of components on the CC.

H1, H2 and H3 result in average imbalances of 20%, 12% and 6% respectively. H3 gives the best performance by seeking to balance the number of CTs assigned to each head.

The ultimate goal of process planning is to balance workloads assigned to heads on DHPMs. Heuristics that assign CTs to feeder slots cannot work with direct measures of workload balance that results from their logic because those measures are available only after P2, P3 and P4 are also solved. However, heuristics that resolve P1 can contribute to workload balance, indirectly, by attempting to minimize gang picking, minimize the number of nozzle changes and balance the number of CTs assigned to each head to balance picking times, nozzle changing times and placing times respectively.

4.4 Conclusions and future research

This thesis evaluated a novel approach for CC assembly on DHPMs. It makes research contributions by developing a new heuristic for solving problem P1, the bypass algorithm to permit branching in P2 and P3 and a nozzle change strategy. The approach reflected relevant, practical considerations and provided a solution method to solve instances effectively. Additionally, an interface was created so that these programs could interact effectively. In this thesis, tests were developed to establish computational benchmarks for the approach. It was observed that the approach was able to solve problems of practical size and scope in run times that would promote competitive assembly operations.

For future research, the succesful results promote the use of this approach in prescribing process plans for other types of placements machines as well. Also, it may be intresting to model platform tray feeder operations, develop column generation approaches to solve problem P1 and develop improved nozzle changing strategies. Our research continues in these directions.

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