Evaluation of FEM modelling for stress propagation under pressure wheel of corn planter

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Abstract: Seeds need a certain range of pressure in the soil bed to germinate and grow ideally. Usually pressure from machinery wheels applies more pressure and prevents seed ideal germination. A finite element model (FEM) was developed to investigate stress propagation in the soil. The pressure wheel of corn planter with 4 km/h speed was chosen to analyze the stress in a sandy-loamy soil. A real corn planter tire was modeled with its mechanical characteristics and imported into ABAQUS/Explicit environment. Frictional contact (based on Mohr-coulomb theory) was used for the soil-tire interaction. The soil was considered as an elastic-perfectly plastic material. Drucker-Prager model was used for soil behavior in plastic region. To evaluate the stress under pressure wheel, FEM results were compared with the Boussinesq theoretical model. On both models, soil stresses decrease with soil depth increasing from zero depth on soil surface to 0.2 m depth. On FEM, stress distribution varied between 47.8 to 8.1 kPa in depth of 0.01 to 0.2 m. FEM and Boussinesq models showed high correlation with each other (R2=95). Our results indicate that the stress under pressure wheels can be properly predicted by using FEM, allowing the pressure simulation to reduce the negative impacts on seed germination and crop yield.

Keywords: stress distribution, soil, tire, ABAQUS/Explicit, simulation

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1 Introduction

Soil compaction is a physical form of soil degradation that changes soil structure, limits water movement and air infiltration, and reduces root penetration (Nawaz et al., 2013). Soil compaction in cropping systems is largely caused by machinery traffic applying stresses greater than the soil bearing capacity (Hamza and Anderson, 2005). Compaction reduces the penetration of roots in the soil by increasing soil cone index, bulk density and reduction in soil porosity (Botta, 2007). The most important factors in the process of artificial compaction of agricultural soils are soil type, soil moisture content, intensity of external load, contact surface area between the soil and tire or track, contact surface shape, and number of passes (Biris et al., 2003).

Knowing the soil response under the action of agricultural machinery is important since optimization of the soil pressure allows the reduction of the effects of surface compaction and also the depth of compaction (Ungureanu et al., 2015). As agricultural soils are not a homogeneous, isotropic and ideal elastic material, the mathematical modeling of stress propagation phenomena is complex (Biris et al., 2007).

The transmission of stress within a soil due to agricultural machinery is of major importance due to soil ability in undergoing deformation due to applied stress, resulting in changes in the soil functions. Knowledge of stress transmission is required to understand the relationships between soil stress due to mechanical loading and changes in soil pore functioning, and to develop predictive models and decision support tools that can help land users prevent soil compaction (Keller et al., 2014).

Advances in computer technology have led to faster processing units and greater memory, reducing the cost of

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complicated analysis (Susila and Hryciw., 2003). At present, one of the most advanced methodologies for modeling the phenomenon of stresses propagation in agricultural soil is the FEM, which is a numerical method for obtaining approximate solutions of ordinary and partial differential equations (Biris et al. 2007). The FEM can easily implement any type of constitutive model, solve problems with difficult geometries, and provide solutions with a high degree of accuracy. For cone penetration modeling, the FEM has many advantages: 1) Soil stiffness and compressibility can easily be modeled, 2) Initial stresses may be prescribed, 3) Increase in stress during the penetration could be determined accurately, 4) Failure modes do not have to be assumed, 5) Both equilibrium equations and yield criterion are satisfied, and, 6) Various constitutive models could be utilized (Susila and Hryciw., 2003).

The theoretical basis of the pseudo-analytical models is the theory of Boussinesq (1885), which describes the stress distribution in a homogeneous, linear elastic, isotropic, and semi-infinite solid mass due to a force being applied at a point on the surface of that mass. These models use a small number of parameters and have been successfully evaluated in field conditions under a wide range of soil and water conditions (De fossez et al., 2002). Keller et al. (2014) evaluated the transmission of vertical soil stress under agricultural tires using theoretical Boussinesq model and FEM method. They measured and simulated soil stress under defined loads. Stress in the soil profile at 0.3, 0.5 and 0.7 m depth was measured. FEM simulations showed the transmission of vertical stresses in a layered soil is not appreciably different from that seen in a homogeneous soil unless very high differences in soil stiffness are considered, with good correlations between Boussinesq model and FEM method.

Gonzalez Cueto et al. (2016) used FEM method in ABAQUS/STANDARD software to model the soil pressures distribution caused by a tire on a Rhodic Ferralsol soil. During the simulation, the tire rotates and moves to a constant speed of 1 m/s. An Extended Drucker Prager constitutive law was used to represent the soil properties. The tire load and the inflation pressure changed the shape of the vertical pressures distribution on the surface of a hard dry soil, but these variables did not affect the distribution of vertical stresses in a soft wet soil or below a depth of 0.15 m. Nankali et al. (2012) evaluated the stress distribution in soft soil under different axial loads and inflation pressures. A two-dimensional finite element model was developed to determine soil behaviour. The Drucker-Prager model was considered for the soil behavior. The maximum contact stress of tire with ground for 15 kN axle load and tire inflation pressure of 150 kPa was equal to 98.6 kPa. Also, the maximum distributed stress was found on the tire side wall.

Hambleton Drescher and (2008)compared predictions of deformation and horizontal (drag) force resulting from three- and two-dimensional numerical simulation of a torque-free (towed) wheel operating on ductile material. The FEM code ABAQUS/Explicit was used to simulate soil deformation. The wheel was simulated as a rigid body and soil behavior and its yield criterion were considered as elastic/perfectly plastic. In particular, Hambleton and Drescher (2008) observed that steady-state penetration is constant over a range of applied vertical forces in the two-dimensional (2D) analysis, whereas steady-state penetration is an increasing function of vertical force for narrow wheels simulated in three dimension (3D) (for narrow wheel). The result showed that two-dimensional simulation cannot predict the wheel penetration accurately, while 3D modeling showed the soil deformation more precisely (Hambleton and Drescher, 2008).

In the field applications of corn planter, the desired depth for seed growth is 4 to 6.5 cm (Sirvastava et al., 2006) and pressure wheel is a corn planter component that provides an optimal amount of compression for seed bed (In Planting Depth for Corn Seed) to allow the seed to germinate and grow best. In this study, pressure wheel moving behavior was modeled by dynamical three-dimensional finite element analysis.

The objectives of this study are: 1) determination of the stress distribution under the pressure wheel in various depths, and 2) evaluation and comparison of the amount of stresses with FEM and Boussinesq theoretical model.

2 Materials and methods

2.1 Finite element modeling in ABAQUS software

In the FEM, real or continuous media objects as solid, liquid and gas are divided into smaller units called elements. These components are considered to be connected together in common certain areas, called nodes. ABAQUS software package (version 6.13; Dassault Systemes, France, Paris) has two main solutions code named ABAQUS/CAE and ABAQUS/Explicit. Explicit solution method requires low storage and can analyze very large three-dimensional models with moderate storage needs. This property is one of the major advantages of this method (Zienkiewicz, 2000).

The explicit solution method was chosen in this study due the soil large deformation. Because of symmetric plate's existence of pressure wheel, soil volume to reduce computation and simulation time, and reduced space required for storing the data, only half of the wheel pressure and its underneath soil was modeled (Chiroux et al., 2005).

2.2 Model attributes

2.2.1 Soil description

In this study, the soil was considered as a cube with $3.2 \text{ m} \text{ length} \times 0.8 \text{ m} \text{ width} \times 1.8 \text{ m} \text{ depth}$. Pressure wheel moves along the length of the soil block middle. Soil volume was meshed with C3D8R type elements representing an eight-node cubic element so that each node has only three degrees of freedom transitional motion in line with the coordinate axes. These types of elements are using reduced integration method to reduce computation time (ABAQUS Doc., 2012). Also, to optimize the model and for optimal use of available hardware resources, the soil was divided into several

grids. The area where the wheel movement occurred has approximately 0.73 m long $\times 0.1$ m wide $\times 0.2$ cm deep, containing most of the mesh. With increasing distance from the wheel, the mesh density reduces. The smallest element length of 0.01 m was considered and a total of 30,000 elements for soil blocks were created.

2.2.2 Wheel description

The study used a pressure wheel of a corn planter with 0.46 m diameter $\times 0.17$ m section width. Most of the wheel was made with metal rim and only one layer of rubber with 0.02 m thickness covered the rim. Therefore, the pressure wheel was considered rigid wheel because Young's modulus of the pressure wheel is higher than the Young's modulus of the soil (Hambleton, 2009). To simulate the rigid behavior of the pressure wheel, R3D4 element type was used. This type of element is a cubic element with four nodes at the corners of a cube (ABAOUS Doc., 2012). A total of 1,851 rigid elements were created in pressure wheel. These types of elements need to have a reference point of loading or obtaining data from. Thus, in rigid wheel center (on symmetry plane), the reference point was defined so the boundary conditions could be applied to the wheel.

2.3 Model mechanical properties

2.3.1 Soil properties

The sandy-loamy soil used in this study was collected at Tabriz University, Khalaat-Poushan, Iran. Analysis of particle aggregation showed that the soil contains 70% sand, 20% loam and 10% silt. Mechanical properties required for soil modeling are: density (ρ), Young's modulus (E), Poisson's ratio (ν), angle of internal friction (φ), soil-metal friction angle (δ) and the dilation angle for plastic flow (ψ) (Tekeste, 2006). Table 1 shows the mechanical properties of the soil. Adhesion values (c) and angle of internal friction (φ) were obtained from the direct shear test. In ABAQUS/Explicit code, the Mohr - Coulomb yield condition cannot be used directly and was estimated by modified Drucker-Prager yield condition with a corresponding associated or non-associated flow potential (Hambleton, 2009).

Angle of So friction	il-metal	Percent Moisture	of	Angle o Dilation	of	Angle of In friction	nternal	Poisson's ratio	Young's kPa	modulus	Density kg/m ³
	26 [°]	10		0	C		33 [°]	0.3		800	1650

Table 1 Mechanical properties of soil in Khalaat-Poushan site

Adhesion (d) and internal friction angle (β) for a criterion of Drucker - Prager are calculated by Equations1 and 2. These parameters differs from adhesion (c) and angle of internal friction (φ) of Mohr - Coulomb yield criterion (ABAQUS Doc., 2012). The necessary relations for converting c and φ parameters in Mohr - Coulomb yield criterion to their equivalent parameters (*d* and β) in criterion of Drucker – Prager are given in Equations 1 and 2. The dilation angle in a sandy-loamy soil for Drucker-Prager yield criterion was considered zero (Susila and Hryciw, 2003).

$$d = \frac{6c\cos\varphi}{3-\sin\varphi} \tag{1}$$

$$\tan\beta = \frac{6\sin\varphi}{3-\sin\varphi} \tag{2}$$

2.3.2 Wheel properties

For simulating the pressure wheel working condition, a 32.5 -kg mass was applied into the reference point of the rigid wheel. The applied weight to the reference point was half of the weight that mentioned in the catalogue for corn planter unit and its enclosures to symmetry in the model. According to the pressure definition, stress results obtained from FEM will not be doubled. Figure 1 describes how the wheel sits on soil and its relevant mesh.



Figure 1 Moving direction, mesh size and wheel placement on the Soil in ABAQUS software.

2.4 Boundary conditions

Proper simulation of pressure wheel and soil interaction requires applying appropriate boundary conditions based on type of problem.

2.4.1 Soil boundary conditions

Soil bottom surface was fully constrained in all directions. Perpendicular planes to X-axis and Z-axis are constrained in the horizontal direction along the X and Z axis, respectively. Soil upper face movement was free in all directions (Mootaz et al., 2003).

2.4.2 Wheel boundary conditions

Gravity constraint was imposed to the rigid wheel in the negative direction of the Y-axis and wheel can freely move in this direction. The wheel could move freely in Z direction due to the existence of linear speed, but the wheel movement is constrained in X direction.

2.5 Soil and tire contact

Interaction between two surfaces has led to tension only if contact is defined between them. To simulate the contact between the wheel and the soil, wheel surface and a top surface of soil block were selected and contact created between them through the "contact pair" method. This method allowed the soil surface and wheel to come in contact but not to cross each other (Chiroux et al., 2005). A dry friction coefficient equal to 0.53 and the penalty method was used to control the pressure and friction between the soil and pressure wheel (Hambleton, 2009).

2.6 Loading and model analysis

Model analysis was done in two time steps. In the first step (6 seconds), gravity load increased gradually from zero to 9.81 m/s^2 and was applied to the entire model. The aim of gradually applying the weight force was to prevent sudden application of the wheel weight on the soil and the resulting stress caused by the wheel

weight on the soil. The sudden stress wave inside the soil interferes with the model's original data. In the second time step (0.649 seconds), the linear speed of 4 km/h was applied to the reference point of the wheel so that the wheel can rotate for about half rounds (about 0.72 m). The data used to draw stress diagrams were obtained from elements in the middle of the movement path, in which the tire has reached its steady state. Also, the software was set to take data from the soil environment every 0.06 seconds. The aim of this selection was to reduce the amount of storage space and the computation time. Von Misses stress, stress and strain tensor elements and reaction forces were chosen for data acquiring (Chiroux et al., 2005, Mootaz et al., 2003).

2.7 Stress calculation method using Boussinesq model

Surface area of the pressure wheel with soil can be theoretically considered as a rectangle that curves at the corners (Biris et al., 2007; Mohsenimanesh et al., 2009). Boussinesq model could be used to find the stress values under uniformly applied load on a rectangular surface at the depth of Y (Boussinesq, 1885). If q is a uniformly applied load into a rectangular surface with 2B and 2L sizes (Figure 2), to find the stress at the depth Y (point A in Figure 2), the rectangle can be divided into four parts and after stress calculation of each part. The stress distribution for rectangular quadrant is obtained using Equations 3 and 4 (Helwany, 2007). Also m, n and I_2 are dimensionless in Equation 4. Total stress is the sum of the stress components acting over the surface element (dA = dxdz) in the contact area. In this analysis, q was estimated 28.9 kPa (The uniform load q is expressed in tire weight per unit contact area (contact area between soil and tire)), and 2B and 2L (width and length of contact) were considered 0.127 and 0.173 m, respectively, in accordance with field measurements.



Figure 2 Uniform load q acting on the quarter of a rectangle and also how to making element components for finding vertical stress.

$$\Delta \sigma_{y} = \int d\sigma = \int_{z=0}^{B} \int_{x=0}^{L} \frac{3qy^{3}(dxdz)}{2\pi (x^{2} + y^{2} + z^{2})^{\frac{5}{2}}} = qI_{2}$$
(3)

$$I_{2} = \frac{1}{4\pi} \left(\frac{2mn\sqrt{m^{2} + n^{2} + 1}}{m^{2} + n^{2} + m^{2}n^{2} + 1} \cdot \frac{m^{2} + n^{2} + 2}{m^{2} + n^{2} + 1} + \tan^{-1} \frac{2mn\sqrt{m^{2} + n^{2} + 1}}{m^{2} + n^{2} - m^{2}n^{2} + 1} \right)$$
(4)
where $\Rightarrow m = B / Y, n = L / Y$

3 Results and discussion

3.1 Results of the finite element method

Plants must overcome the resistance force along y direction for more favorable germination and growth. The analysis was concentrated on the soil vertical stress (σ_Y). Vertical stress of the soil decreases from depth zero (ground level) to a depth of 20 cm in the plane perpendicular to the moving direction (Figure 3).



Figure 3 Decrement of vertical stress values form 47.8 kPa in the soil surface to 17.2 kPa at 11cm soil depth in the plane perpendicular to the moving direction (FEM simulation in the ABAQUS software).

For a uniform loading, the soil stresses decrease with increasing depth (Figure 3). As was predictable, stresses were greater in loading point (wheel contact area with the ground). The soil type, soil moisture content, the intensity of applied forces, tire inflation pressure, tire-soil contact area, shape of contact area and number of passes influence the soil compaction (Biris et al., 2007). In this study, the wheel was considered rigid; inflation pressure has no effect on soil stresses (Keller et al., 2007). The compaction increases by increasing the number of tire passes on soil (Eliasson, 2005), but considering that the pressure wheel pass on soil for the first time, so the soil was not initially compacted (plowed soil), therefore the vertical stresses in the soil will have lower values.

Due to weight on the tires, sinkages are created in soil with machines passing in agricultural field. Sinkage causes two main impacts: compaction and displacement. Soil compaction results from normal forces on the soil. Soil displacement occurs when the soil is pushed horizontally (Liu et al., 2010). In the present study, soil sinkage was greater in the beginning of the move rather than the continue way (Figure 4). At the beginning of the analysis, the wheel sinkage is static for 6 seconds and causes more compaction of the soil underneath. With the wheel movement, there is less opportunity for soil compaction and therefore the compression is reduced. Erbach et al (1995) also found that increasing the speed of the device at the farm from 1 to 25 km/h reduces soil compaction and sinkage.



Figure 4 Soil sinkage trace and magnitude after crossing the pressure wheel with 4 km/h velocity in the ABAQUS software (volume strains).

Empirical findings of the researchers such as Keller et al (2007) showed that the soil sinkage reduced from surface with more depth and also in this study, variation of the finite element results agreed with the experimental findings in previous research (Figure 4).

3.2 Comparison of finite element method with Boussinesq model

Figure 5 shows that the results of the two models are correlated; as expected the correlation was higher in the depth greater than 5 cm. On both models, stresses are decreasing with increasing depth. The reasons for this phenomenon are: 1) distance from the soil surface and 2) applied force of the tire. In a uniform vertical force and tire constant speed, usually force and internal stresses reduce exponentially with depth increasing (Abou-Zeid, 2003, Keller et al, 2014) which this variation is estimated accurately by the two models (Figure 5).



Figure 5 Comparison of vertical stress obtained from the finite element and Boussinesq model in different depths.

The predicted value of FEM and Boussinesq model presented a coefficient of determination $(R^2) = 0.95$ for depths one to 20 cm, indicating these two models were able to estimates stress distribution under pressure wheel (Figure 6).



Figure 6 Accuracy of two models for estimation of stress distribution under pressure wheel.

Because of tire rigidity and its curved shape from the tire center to its walls, therefore applied forces of the tire on soil surface act like the point force in center of tire. This concentrated force in center of tire creates more stresses in surface depths rather than other depths. But Boussinesq model considered the applied force on a rectangular area uniformly, so the estimated values of Boussinesq model for the stress on surface area are less, rather than the finite element model. As shown in Figure 5, the difference is more pronounced in depth 0 to 5 cm between the two models. In the finite element model, the effects of point force are uniform with depth increasing and the results of both models are closer together.

There are many mechanical models to calculate stresses. A number of models to determine the stresses in deep need more parameters to calculate; which should be calculated for the soil and tire. Boussinesq model needs input parameters (soil depth, length and width of the tire contact surface with the soil). According to Boussinesq model, soil is considered homogenous volume, isotropic and with linear elastic behavior; so soil structure is more simplified (Keller and Arvidsson, 2004). This simple model (Boussinesq model) could predict better results. The actual results of the stress is not measured experimentally, but the agreement between the finite element and Boussinesq models shows that these models are able to predict the range of the actual vertical stress which was applied to the seed. Consequently, the actual stresses will be around the estimates of two models.

4 Conclusions

Soil stresses produced by the wheel of corn planter have maximum value at soil surface and decreases by increasing soil depth. The comparison of the FEM and Boussinesq models showed a high coefficient of determination. Boussinesq model has simple calculation procedure and can be used by anyone while FEM needs mathematics knowledge and custom software. The results of this article can be used to investigate the relation of useful pressure around seed and working speed of corn planter.

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References

ABAQUS Version 6.12.1. 2012. Abaqus Theory Manual. ABAQUS, Inc. USA

- Abou-Zeid, A. 2003. Distributed soil displacement and pressure associated with surface loading. ASABE, Annual Meeting; paper number 031024.
- Biriş S. Şt., E. Maican, v G. Paraschi, t V. Vladu, and S. Bungescu. 2007. A model for prediction for prediction of stress state in soil below agricultural types using the finite element method. *Upb Scientific Bulletin*, 69(4):19-28.
- Biriş S. Şt., M. Prunău, V. Vlăduţ, and u S. T. Bungesc. 2003. Study of agricultural soil artificial compaction using the finite element method, Scientific Bulletin of the, Politechnica" University of Timişoara, *Transactions on Mechanics*, Tom 48(62): 41-50.
- Bolton M. D. 1986. The strength and dilatancy of sands. *Geotechnique*, 36(1): 65-78.
- Botta G., O. Pozzolo, M. Bomben, H. Rosatto, D. Rivero, M. Ressia, M. Tourn, E. Soza, and J. Vazquez. 2007. Traffic alternatives in harvest of soybean (Glycine max L.): effect on yields and soil under direct sowing system. *Soil & Tillage Research*, 96(1-2): 145–154.
- Boussinesq J. 1885. Applying potentials to the study of equilibrium and motion of elastic solids, Paris, pp 30.
- Chiroux R. C., Jr. W. A. Foster, C. E. Johnson, S. A. Shoop, and R. L. Raper. 2005. Three-dimensional finite element analysis of soil interaction with a rigid wheel. *Applied Mathematics and Computation*, 162(2):707–722.
- Defossez, P, and G. Richard. 2002. Models of soil compaction due to traffic and their evaluation. *Soil & Tillage Research*, 67(1): 41–64.
- Eliasson, L. 2005. Effects of Forwarder Tire Pressure on Rut Formation and Soil Compaction. *Silva Fennica*, 39(4): 549– 557.
- Erbach, D., D. Wiering, and M. Wermager. 1995. Equipment travel speed effect on soil compaction. American Society of Agricultural Engineers Meetings Papers.
- Gonz ález, C. O., C.E. Iglesias Coronel, E.L., Bravo, C.A. Recarey Morfa, and M.H. Su árez. 2016. Modelling in FEM the soil pressures distribution caused by a tire on a Rhodic Ferralsol soil, *Journal of Terramechanics*. 63(1): 61-67.
- Hambleton, J. P, and A. Drescher. 2008. Modeling wheel-induced rutting in soils: indentation. *Journal of Terramechanics*, 45(6):201–11.
- Hambleton, J.P, and A. Drescher. 2009. On modeling a rolling wheel in the presence of plastic deformation as a three-or two-dimensional process. *International Journal of Mechanical Sciences*, 51(11-12): 846–855.
- Hamza, M.A., and W.K. Anderson. 2005. Soil compaction in cropping systems. A review of the nature, causes and possible solutions. *Soil & Tillage Research*, 82(2): 121–145.

- Helwany, S. 2007. Applied Soil Mechanics with ABAQUS applications. John Wiley & Sons, Inc., Hoboken, New Jersey, ISBN: 978-0-471-79107-2.
- Keller, T., and J. Arvidsson. 2004. Technical solutions to reduce the risk of subsoil compaction: effects of dual wheels, tandem wheels and tire inflation pressure on stress propagation in soil. *Soil & Tillage Research*, 79(2): 171–205.
- Keller, T., M. Berli, S. Ruiz, M. Lamandé, J. Arvidsson, P. Schjønning, and A.P.S. Selvadurai. 2014. Transmission of vertical soil stress under agricultural tires: Comparing measurements with simulations, *Soil and Tillage Research*, 140(1): 106-117.
- Keller, T., P. De fossez, P. Weisskopf, J. Arvidsson, and G. Richard. 2007. SoilFlex: A model for prediction of soil stresses and soil compaction due to agricultural field traffic including a synthesis of analytical approaches. *Soil & Tillage Research*, 93(2):391–411.
- Liu, K., s P. Ayer, H. Howard, A. Anderson. 2010. Lateral slide sinkage tests for a tire and a track shoe. *Journal of Terramechanics*, 47(6): 407–414.
- McKyes, E. 1985. Soil cutting and tillage. *Developments in Agricultural Science*, Amsterdam, Netherlands: Elsevier, volume 7, first edition: pages 217.
- Mohsenimanesh, A., S. M. Ward, and M. D. Gilchrist. 2009. Stress analysis of a multi-laminated tractor tire using non-linear 3D finite element analysis. *Materials and Design*, 30(4): 1124–1132.
- Mootaz, A., R. Hamilton, andJ. T. Boyle. 2003. 3D Dynamic analysis of soil-tool interaction using the finite element method. *Journal of Terramechanics*, 40 (1): 51–62.
- Nankali, N., M. Namjoo, and M. R. Maleki. 2012. Stress Analysis of Tractor Tire Interacting with Soil using 2D Finite Element Method. Advanced Design and Manufacturing Technology, 5(3): 107-111.
- Nawaz, M., G. Bourrie and F. Trolard. 2013. Soil compaction impact and modelling. A review. Agronomy for Sustainable Development, 33 (2): 291–309.
- Srivastava, A. K., C. E. Goering, and R. P. Rohrbach. 2006. Engineering principles of agricultural machines. 2nd Edition. American Society of Agricultural and Biological Engineers Publication. USA.
- Susila, E. and R. D. Hryciw. 2003. Large displacement FEM modelling of the cone penetration test (CPT) in normally consolidated sand. *International Journal for Numerical and Analytical Methods in Geomechanics*, 27(7): 585-602.
- Tekeste, M. Z. 2006. Site-Specific characterization Modeling and Spatial Analysis of Sub-soil compaction (Hardpan) for precision agricultural on southeastern US soils, Ph.D. Dissertation, University of Georgia, Athens.

Ungureanu, N., Şt. Croitoru, S. Biriş, Gh. Voicu, V. Vlăduţ, K. Ç. Selvi, S. Boruz, E. Marin, M. Matache, D. Manea, G. Constantin, and M. Ionescu. 2015. Agricultural Soil Compaction Under the Action of Agricultural Machinery. Symposium "Actual Tasks on Agricultural Engineering", Opatija, Croatia.

Zienkiewicz, O. C., Taylor. 2000. The Finite Element Method, Butterworth-Heinemann, Fifth edition.