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The Stability Interval of the Set of Linear System

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***Abstract*—The article considers the problem of stability of interval-defined linear systems based on the Hurwitz and Lienard-Shipar interval criteria. Krylov, Leverier, and Leverier-Danilevsky algorithms are implemented for automated construction and analysis of the interval characteristic polynomial. The interval mathematics library was used while developing the software. The stability of the dynamic system described by linear ordinary differential equations is determined and based on the properties of the eigenvalues of the interval characteristic polynomial. On the basis of numerical calculations, the authors compare several methods of constructing the characteristic polynomial. The developed software that implements the introduced interval arithmetic operations can be used in the study of dynamic properties of automatic control systems, energy, economic and other non-linear systems.**

***Keywords*—automatic control system, stability, matrix, minor, characteristic polynomial, Hurwitz criterion, Lienard-Shipard criterion of interval mathematics, Lyapunov function**

I. Introduction

IN general mechanics research, many new directions have recently emerged that related to the possibilities of analyzing dynamic systems through the use of computer technology. This made it possible to solve the following problems relevant from the point of view of applications: development of software for automated design of automatic control systems [1, 2], computer verification of analysis and synthesis of systems with the required properties [3, 4], analytical solutions to some applied problems based on the application of methods and software of computer algebra [5, 6].

The concept of stability plays an important role in the analysis of dynamic systems. Stability is usually understood as the property of a system or a state to persist with small changes in the initial States, external influences, system parameters, etc. The Foundations of the theory of stability were laid By L. Euler.

The modern theory of stability is based on Lyapunov's definition of motion stability as the most General one, which determined both the scope and content of the problems covered by the modern theory of stability, and the development of qualitative methods for studying differential equations in relation to solving these problems.

Mathematical analysis of real mechanical systems gives some error. This is due to the fact that in reality the parameters of a mechanical system cannot be set with greater accuracy. Any error, for example, in mass, in the size of links, etc. affects the nature of the system's movement, its stability, and other important dynamic characteristics. As you know, the stability criterion is divided into two large groups: algebraic and frequency. The algebraic criteria are based on the analysis of the characteristic equation of the matrix itself. Such criteria are, for example, the well-known Raus-Hurwitz stability criteria. Frequency criteria allow you to determine the stability of the system using analytical or experimental research of the frequency characteristics of the elements of this system. The Mikhailov and Nyquist criteria are popular in technical applications for this group. However, all these criteria do not take into account the fact that these physical parameters are measured with some error. A strict analytical method for investigating the stability of nonlinear systems is, as mentioned above, the direct (second) Lyapunov method.

II. Methods

A. M. Lyapunov proposed a number of General sufficient conditions for the stability and instability of undisturbed motion. He brought the issue of sustainability to the issue of stability of equilibrium and existence of functions, called Lyapunov function.

An autonomous system of differential equations is considered in this form:

$$\frac{dx_i}{dt} = X_i(x_1, \dots, x_n), \qquad i = 1, \dots, n \tag{1}$$

The Lyapunov stability theorem States that if for this system there exists in some domain a sign-defined function $\dot{V}$, whose time derivative is a sign-constant function of the sign opposite to the sign of the function, then the equilibrium position is stable in the Lyapunov sense.

This method, called by N. G. Chetaev the Lyapunov [7] direct method, is a powerful rigorous analytical method for analyzing various dynamic properties of nonlinear systems of a very different nature and form of description. A significant difficulty limiting the use of this method is the lack of algorithms for constructing Lyapunov functions. In order to overcome this difficulty, expand the scope of the application and increase the efficiency of the method for complex systems, N. G. Chetaev suggested using several Lyapunov functions. This method is called the Lyapunov vector function method.

The Lyapunov function method has found application to a number of problems in mechanics, physics, engineering, control theory, stability analysis, and other dynamic properties of automatic control systems, energy, economic, and other nonlinear systems. [8]

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In practice, when applying this method to specific problems, the main difficulty was in constructing a Lyapunov function or functional that met the conditions of a particular theorem.

From the works of V. M. Matrosov [9] and R. Bellman [10], where the idea of combining the methods of differential inequalities of Chaplygin, Vazhevsky's theorem, the concept of using a set of several Lyapunov functions was formulated, and the history of the modern method of comparison with Lyapunov vector functions began.

Here, along with the original system, an auxiliary system is introduced, which is called a comparison system and is described by an ordinary finite-dimensional differential equation. Using an analog of the Lyapunov requirements and the quasi-monotonicity (Vazhevsky`s) conditions, it is proved that the semi-stability of comparison systems entails the stability of the original system.

The principle of comparison with the Lyapunov vector function gave the Lyapunov method a "second wind", causing a wave of publications in the world. To date, hundreds of comparison theorems with the Lyapunov vector function have been obtained for various dynamic properties of nonlinear differential equations. The comparison method was the first rigorous and universal method for analyzing various properties of various systems, in fact, regardless of their complexity, nature, and form of mathematical description, especially in the dynamics of systems and control theory. It is fundamental in determining algorithms for the derivation of formulations and proofs of comparison theorems with the Lyapunov vector function, which opens a new direction in the field of artificial intelligence, called the algorithmization approach to the derivation of theorems. It is implemented programmatically [11] and with the help of developed programs on electronic computers, more than 300 theorems are obtained, which are new or generalizations and modifications of known ones.

The problem of absolute stability of the equilibrium position of non-linear automatic control systems was first posed by A. I. Lurie and V. N. Postnikov. Based on the method of Lyapunov functions by A. I. Lurie obtained a quadratic system of equations, based on which you can judge the absolute stability of the system under study.

Another approach was proposed by V. M. Popov [12]. In contrast to other methods, the Popov method sets the condition of absolute stability using the frequency response of the linear part of the system. He also developed a new method for studying the stability of nonlinear systems, in particular the first frequency criterion of absolute stability for systems with a lagging argument [13].

Rezvan V. [14] studies are devoted to the absolute stability of systems with a lagging argument. he obtained a frequency criterion of absolute stability for a special class of systems.

In the works of V. I. Rumyantsev and A. S. Oziraner [15], V. I. Vorotnikov [16], stability criteria for dynamical systems with respect to a part of variables are obtained.

In Kazakhstan, many scientists ' works are devoted to the problems of sustainability. It is worth noting the research of A. O. Zhautykov [17] (on the stability of countable systems of differential equations), A. K. Bedelbaev [18], B. Zh.Maigarin [19] (on absolute stability), S. A. Aisagaliev [20] (on constructive methods for studying absolute stability).

The result of this development is a modern theory covering various aspects of stability similar to the above, based on the concept of a dynamical system that uses qualitative behavior of solutions of differential equations and, more generally, topological and functional-analytical methods for investigating the solution of operator equations.

One of the specifics of using computers for solving problems is the distortion of the result due to rounding errors and amplification due to the limited bit depth of the computer. Ignoring this problem can lead to incorrect results being interpreted as objectively correct and used in further calculations. For example, control calculations of notoriously stable order systems showed that due to the accumulation of rounding errors in the construction of the characteristic polynomial, the subsequent use of root synthesis methods led to completely incorrect results. Stable systems with certain combinations of parameters were treated as unstable, and Vice versa unstable systems were considered as stable. Doubling the bit grid only eliminates this flaw to a certain extent, but it slows down the machine four times and overloads the memory.

In real systems, physical parameters (attributes) are measured with some error. To account for these features, we can use a new direction of computational mathematics – interval analysis, the main idea of which is to replace arithmetic operations and real functions over real numbers with interval operations and functions that transform intervals containing these numbers.

The first publication devoted to interval analysis was made by R. E. Moore in 1966 [21]. Shokin Yu. I. [22] in 1981, the basics and methods of interval analysis were systematically described. Then, in 1982, a textbook was published by Nazarenko T. I., Marchenko L. V. [23] on interval methods, and in 1986-a monograph of the overview plan by Kalmykov S. A., Shokin Yu. I., Yuldashev Z. Kh. [24]. These papers systematically set out the basics and methods of interval analysis. Interval arithmetic was given in full, and along with the "classical" one considers a number of its modifications and generalizations. Interval methods for solving linear algebra problems, run-through methods for solving differential equations, and methods for solving systems of nonlinear equations are considered. In [25], an overview of the state of interval mathematics is given.

Interval analysis is a relatively new area of computational mathematics that is widely used to study the properties of mechanical systems. One of the main requirements for the quality of such systems is the requirement of stability. The use of interval analysis in solving the problem of stability of the dynamics of mechanical systems allows us to obtain a criterion of guaranteed stability. But when using interval mathematics, researchers have difficulty solving cumbersome interval equations, and these solutions are "super-sufficient", which in practice is a strict limitation.

Often, when solving various problems, it is necessary to calculate the coefficients of the characteristic polynomials of the system matrix, but accurate calculation requires very high costs, especially for high-order interval matrices. Therefore, intervals containing coefficients of interval characteristic polynomials of the interval matrix are calculated. In [26], several calculation methods are proposed and the width of the resulting intervals is analyzed.

In [27], we propose an analytical method for constructing Hurwitz matrices of interval polynomials that accompany a given Hurwitz characteristic polynomial based on a vector-matrix record of Viet's formulas. we solve the problem of constructing the boundaries of possible variations of the Hurwitz characteristic polynomial coefficients that do not lead to the loss of the Hurwitz property by this polynomial.

One of the problems is how to investigate the positive definiteness of interval matrices. In [28], one of the algorithms for analyzing the positive definiteness of an interval family of symmetric (*NxN*) matrices is described. The algorithm is reduced to checking the positive definiteness of 2^{N-1} matrices constructed at the ends of the change intervals of elements of the studied interval family.

Solving practical problems brings new methods to the theory of interval analysis. for example, [29-31] proposes a method for improving the accuracy of the interval estimation of the calculated values of one of the main indicators of the quality of electric energy-the voltage deviation from the nominal value. The method is based on the representation of the voltage value at the terminals of the electric receiver and its nominal value by means of fuzzy triangular numbers. It is shown that the form of the symmetric membership function of a fuzzy number does not affect the value of the correction of the calculated values of permissible voltage deviations. In [32-34], the direct and inverse problem of chemical kinetics for first-order reactions is considered as interval problems for processing experimental data. The authors proposed and tested elementary interval algorithms for their solution using examples.

Interval analysis is widely used in the study of systems with parametrically indeterminate objects. In this case, the parametric uncertainty is defined as the interval to which the exact values of the object's parameters belong. In [35], a simple sufficient criterion for robust stability of systems with lag is formulated and proved. The characteristic equations of these systems are the sum of conversions of a fixed polynomial, an interval polynomial, and a lag element.

In [36], linear non-stationary control systems with periodic interval constraints on the elements of the system matrix are considered. Sufficient conditions for robust stability of such systems are established based on the comparison method with the Lyapunov vector function of a special type. It is shown that for some additional restrictions, the obtained conditions are not only sufficient, but also necessary. The results are generalized to the case of linear controlled systems with polyhedral periodic constraints.

III. STATEMENT OF PROBLEM

The article deals with the problem of stability of a linearized model of the form [7]:

$$\dot{x} = \mathbf{A}x, \tag{2}$$

where parameters characterizing mechanical parameters (such as weight, metric characteristics, inertia, etc.) are set using the coefficients of the matrix A. It is assumed that the matrix A is interval, i.e. its elements are interval numbers.

For the system (2) in the case when the elements of the matrix A are "point" numbers, stability criteria are developed, expressed in terms of the elements of the matrix A – the Raus-Hurwitz criterion, etc. [37]. However, this does not take into account the fact that these physical parameters are measured with some error. Researchers often forget that the coefficients and roots of the characteristic polynomial can be very sensitive to small errors of matrix elements when making a conclusion about the stability of the system (2) [38].

Interval analysis makes it possible to automatically take into account errors in the input data set and errors caused by machine rounding.

The use of interval analysis in solving the problem of stability of the dynamics of mechanical systems allows us to obtain a criterion of guaranteed stability.

We introduce the notation: $\tilde{\mathbf{A}} = \{\tilde{a}_{ij}\}$ - a point matrix whose elements belong to the corresponding intervals a_{ij} of the interval matrix ***A***, or $\tilde{a}_{ij} \in a_{ij}$.

Axiom. An interval matrix ***A*** has the property if all point matrices have this property $\tilde{\mathbf{A}}$.

It is known that to determine the stability of the interval matrix ***A*** , it is sufficient to determine the stability of 2^{n^2} point matrices $\tilde{\mathbf{A}}$ made up of various combinations of upper and lower bounds of elements of the original interval matrix.

However, this approach is not constructive, since already at n=3 it is necessary to determine the stability of 512 points matrices and at n=4, respectively, 65536 points matrices.

Definition. A system (1) with an interval matrix ***A*** whose elements have a normal distribution is called interval asymptotically stable by Lyapunov, if for any interval solution:

$$x(t) = [\bar{x}(t) - \varepsilon_x(t), \bar{x}(t) + \varepsilon_x(t)] = \{\bar{x}(t), \varepsilon_x(t)\}|_{t\in[0,\infty)} \tag{3}$$

true statement:

for any $\varepsilon>0$ and $t_0\in[0,\infty)$ there exists such $\delta=\delta(\varepsilon,t_0)$ that for all solutions $x=x(t)$ satisfying the condition $\|x(t_0)\|<\delta$, the inequality $\|x(t)\|<\varepsilon$ is true, if $t\in[t_0,\infty)$;

for any $t_0\in[0,\infty)$, there exists such $\lambda=\lambda(t_0)$, that all solutions satisfying $x=x(t)$, the condition $\|x(t_0)\|<\lambda$ have the property:

$$\lim_{t\to\infty}\|\bar{x}(t)\| = 0 \cdot \tag{4}$$

As is known, to determine the stability of a point matrix, the properties of its eigenvalues are analyzed [39]. Similarly, to determine the stability of the interval matrix, a characteristic polynomial with interval coefficients is constructed:

$$\phi_A = \det(\lambda E - \mathbf{A}) = p_n\lambda^n + p_{n-1}\lambda^{n-1} + \ldots + p_0, \tag{5}$$

where $p_i, i = 0,...,n$ - interval numbers.

Definition. An interval characteristic polynomial (5) is called stable if the interval composed of the real parts of the interval eigenvalues does not contain 0 and is completely in the negative region.

Necessary stability condition: all coefficients of the characteristic polynomial (5) must be in the positive domain and not contain 0: for

linearized model of the form [7]:

$$p_i = [\bar{p}_i - \varepsilon_i^p, \bar{p}_i - \varepsilon_i^p], \quad i = 0,...,n \tag{6}$$

must be performed:

$$0 \notin p_i, \bar{p}_i - \varepsilon_i^p > 0, \quad i = 0,...,n \tag{7}$$

Let's make a Hurwitz matrix:

$$M=\begin{bmatrix} p_1 & p_0 & 0 & 0 & \cdots & 0 \\ p_3 & p_2 & p_1 & p_0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{2n-1} & p_{2n-2} & p_{2n-3} & p_{2n-4} & \cdots & p_n \end{bmatrix} \quad (8)$$

where it is accepted p_j=0 if j <0 and j>n. Denote by Δ_1, Δ_2,..., Δ_n main diagonal minors of the matrix M:

$$\Delta_1 = p_1, \quad \Delta_2 = \begin{vmatrix} p_1 & p_0 \\ p_3 & p_2 \end{vmatrix}, \quad \ldots \quad \Delta_n = |M| = p_n\Delta_{n-1} \quad (9)$$

which in turn are interval numbers.

The interval stability criterion Hurwitz: in order that

that Re{ λ_j (A)}<0, j=1,...,n it is necessary and sufficient that the main diagonal minors Δ_1, Δ_2, ..., Δ_n of the matrix M are in the right half-plane, so $\Delta_j \in (0,\infty)$,j=(1,n).

Lienard-Shipard interval stability criterion: If all the interval coefficients of p_0, p_1 ,..., p_n the characteristic polynomial (5) are in the positive domain, so $0 \notin p_i$, $p_i - \varepsilon_i^p > 0$, i=0,...,n, then Re{ λ_j (A)}<0, j=1,...,n it is necessary and sufficient for the main diagonal minors Δ_1,Δ_2, ..., Δn of the matrix M to meet the conditions:

$$\Delta_3 \in (0,\infty), \Delta_5 \in (0,\infty), \ \Delta_7 \in (0,\infty), \ldots \quad (10)$$

or:

$$\Delta_2 \in (0,\infty), \Delta_4 \in (0,\infty), \ \Delta_6 \in (0,\infty), \ldots \quad (11)$$

For the characteristic equation: :

$$a_0\lambda^n + a_1\lambda^{n-1} + \ldots + a_n = 0, \quad (12)$$

let's make the table of Routh (matrix of dimension (n+1)m, where m=[n/2]+2):

$$M=\begin{bmatrix} c_{11}=a_0 & c_{12}=a_2 & \cdots \\ c_{21}=a_1 & c_{22}=a_{31} & \cdots \\ c_{31} & c_{32} & \cdots \\ c_{41} & c_{42} & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix} \quad (13)$$

the first line contains the coefficients of the original characteristic equation with even indexes, and the second line contains odd indexes. Elements of the remaining rows are calculated recursively using the formula:

$$\begin{aligned} c_{1k} &= \begin{cases} a_{2(k-1)} & if \quad 2(k-1) \le n \\ 0 & if \quad 2(k-1) > n \end{cases} \\ c_{2k} &= \begin{cases} a_{2(k-1)} & if \quad 2(k-1) \le n \\ 0 & if \quad 2(k-1) > n \end{cases} \\ c_{ik} &= c_{i-2,k+1} - d_i c_{i-1,k+1} \\ d_i &= c_{i-2,1} / c_{i-1,1} \end{aligned} \quad (14)$$

where i=3, ..., n+1 and k=1, ..., m.

Raus interval stability criterion: for the stability of the system, it is necessary and sufficient that the coefficients of the first column of the matrix C are in the right half-plane, so:

$$c_{1j} \in (0,\infty), \ j = 1,\ldots,n+1. \quad (15)$$

IV. MAIN RESULTS

For automated construction of an interval characteristic polynomial, Krylov [40], Leverier [41], and Leverier-Danilevsky [40, 41] algorithms are implemented, based on both classical and introduced interval mathematics.

The above criteria for interval stability are also implemented in the developed package of interval calculations.

The following example demonstrates how to use the software package:

Let's consider the matrix:

$$\mathbf{A}=\begin{bmatrix} -4 & c_1 & 0 & 0 \\ c_2 & -3 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (13)$$

For C1=6 and C2=2, the matrix (13) is unstable. Reducing slightly C1 or C2 will get stability.

Assume that the matrix A is an interval matrix in which the parameters C1 and C2 are intervals.

For C1=(5.87,5.871) and C2=(1.999, 2.001), the coefficients of the characteristic polynomial are calculated:

$$(1.0,1.0)\lambda^4 + (10.0,10.0)\lambda^3 + (23.25,23.20)\lambda^2 + \\ + (14.71,14.84)\lambda + (0.02,1.01) \quad (14)$$

and the values of the determinants of the major minors of the Raus - Hurwitz matrix constructed on the basis of the polynomial (14)

$$\begin{aligned} \Delta_1 &= (10.0,10.0) > 0, \\ \Delta_2 &= (232.52, 232.65) > 0, \\ \Delta_3 &= (3420.30, 3453.66) > 0, \\ \Delta_4 &= (82.43, 3494.34) > 0. \end{aligned} \quad (15)$$

For C1 = (5.88,5.881) and C2 = (1.999, 2.001), the coefficients of the characteristic polynomial are calculated:

$$(1.0,1.0)\lambda^4 + (10.0,10.0)\lambda^3 + (23.25,23.20)\lambda^2 + \\ + (14.65,14.78)\lambda + (-0.01,0.97) \quad (16)$$

and the values of the determinants of the major minors of the Raus - Hurwitz matrix constructed on the basis of the polynomial (16):

$$\begin{aligned} \Delta_1 &= (10.0,10.0) > 0, \\ \Delta_2 &= (232.52, 232.65) > 0, \\ \Delta_3 &= (3403.40, 3436.76) > 0, \\ \Delta_4 &= (-57.54, 3342.68) < 0. \end{aligned} \quad (17)$$

An interval matrix of the third order is considered [26]:

$$\mathbf{A}=\begin{bmatrix} [1,2] & [0,1] & [-9,-4] \\ [-4,-2] & [1,1] & [2,2] \\ [-1,2] & [3,4] & [0,1] \end{bmatrix} \quad (18)$$

for which the values of the coefficients of the interval characteristic polynomial (ICP) obtained using various methods are given:

by the Leverrier method:

p_1=[-4,-2], p_2=[-18,23], p_3=[-262.3,99.7];

Leverrier method using "exact" calculation of the product of matrices

p_1=[-4,-2], p_2=[-16,23], p_3=[-235.7,91.7];

by the Fadeev method

p_1=[-4,-2], p_2=[-16.5,22.5], p_3=[-242.3,58.3].

Fadeev method using "exact" calculation of the product of matrices

p_1=[-4,-2], p_2=[-14,22.5], p_3=[-182.3,52.3];

by the method of principal minors

p_1=[-4,-2], p_2=[-16,21], p_3=[-165,-2];

the principal minor method using the "exact" calculation of all minors and the matrix determinant

p_1=[-4,-2], p_2=[-14,21], p_3=[-129,-6];

true values of the ICP coefficients

p_1=[-4,-2], p_2=[-14,21], p_3=[-129,-6].

For the "point" of the matrix]:

$$A = \begin{bmatrix} 1.5 & 0.5 & -6.5 \\ -3.0 & 1.0 & 2.0 \\ 0.5 & 3.5 & 0.5 \end{bmatrix} \tag{19}$$

the elements of which represent the midpoints of the corresponding intervals of the interval matrix (18), implemented in the Fortran programming language, the same results were obtained by Leverrier, Fadeev, Danilevsky, and Krylov methods:

$$\mathrm{p}_0 = 1;\ \mathrm{p}_1 = -3.0;\ \mathrm{p}_2 = 0.5;\ p_3 = -63. \tag{20}$$

When using the developed software that implements the entered interval arithmetic operations, the following results are obtained for the interval matrix (18):

by the Fadeev method:

$$p = \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} [1.000,1.000] \\ [-3.707,-2.293] \\ [-6.655,7.655] \\ [-6.655,7.655] \end{pmatrix} = \begin{pmatrix} (1.000,0.000) \\ (-3.000,0.707) \\ (0.500,7.155) \\ (-63.000,23.349) \end{pmatrix},$$

by the Leverrier method

$$p = \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} [1.000,1.000] \\ [-3.707,-2.293] \\ [-6.754,7.754] \\ [-90.811,-35.189] \end{pmatrix} = \begin{pmatrix} (1.000,0.000) \\ (-3.000,0.707) \\ (0.500,7.254) \\ (-63.000,27.811) \end{pmatrix},$$

by the Danilevsky method

$$p = \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} [1.000,1.000] \\ [-4.742,-1.258] \\ [-10.157,11.157] \\ [-110.358,-15.642] \end{pmatrix} = \begin{pmatrix} (1.000,0.000) \\ (-3.000,1.742) \\ (0.500,10.657) \\ (-63.000,47.358) \end{pmatrix},$$

by the Krylov method

$$p = \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} [1.000,1.000] \\ [-6.182,0.182] \\ [-17.686,18.686] \\ [-224.283,98.283] \end{pmatrix} = \begin{pmatrix} (1.000,0.000) \\ (-3.000,3.182) \\ (0.500,18.186) \\ (-63.000,161.282) \end{pmatrix},$$

An interval matrix of the fifth order is considered [26]:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 4 & 7 \\ -1 & -2 & 0 & 2 & 0 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 2 & -3 & 0 & 2 \\ -1 & 1 & -5 & 0 & [1,2] \end{pmatrix} \tag{21}$$

for which the values of the ICP coefficients obtained using various methods are given:

by the Leverrier method

p_1=[0,1], p_2=[-1,1], p_3=[37.3,51.7], p_4=[-162.5,-40.6], p_5=[-331.8,285.3];

by the Fadeev method

p_1=[0,1], p_2=[-2.5,2], p_3=[25.2,64.8], p_4=[-244.3,34.2], p_5=[-995.9,975.6];

by the method of principal minors

p_1=[0,1], p_2=[-2,2], p_3=[36,53], p_4=[-131,-84], p_5=[-32,34]

the principal minor method using the “exact” calculation of all minors and the matrix determinant

p_1=[0,1], p_2=[-2,2], p_3=[38,51], p_4=[-123,-92]; p_5=[-32,34];

true values of ICP coefficients

p_1=[0,1], p_2=[-1,1], p_3=[40,49], p_4=[-113,-102], p_5=[-8,10].

For the "point" of the matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 & 4 & 7 \\ -1 & -2 & 0 & 2 & 0 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 2 & -3 & 0 & 2 \\ -1 & 1 & -5 & 0 & 1.5 \end{pmatrix} \tag{22}$$

the elements of which represent the midpoints of the corresponding intervals of the interval matrix (21) using Leverrier and Fadeev methods, the same results are obtained:

p_0=1; p_1=0.5; p_2=0.0; p_3=44,5; p_4=-107.5; p_5=1.0

When using the developed software that implements the entered interval arithmetic operations, the following results are obtained for the interval matrix (21):

by the Fadeev method

$$p = \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{pmatrix} = \begin{pmatrix} [1.000,1.000] \\ [0.000,1.000] \\ [-0.952,0.952] \\ [40.570,48.430] \\ [-118.746,-96.254] \\ [-35.171,37.171] \end{pmatrix} = \begin{pmatrix} (1.000,0.000) \\ (0.500,0.500) \\ (0.000,0.952) \\ (44.500,3.930) \\ (-107.500,11.246) \\ (1.000,36.171) \end{pmatrix},$$

by the Leverrier method

$$p = \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{pmatrix} = \begin{pmatrix} [1.000,1.000] \\ [0.000,1.000] \\ [-0.952,0.952] \\ [40.570,48.430] \\ [-118.746,-96.254] \\ [-35.171,37.171] \end{pmatrix} = \begin{pmatrix} (1.000,0.000) \\ (0.500,0.500) \\ (0.000,0.952) \\ (44.500,3.930) \\ (-107.500,11.246) \\ (1.000,36.171) \end{pmatrix},$$

Automatic frequency adjustment system for a heterodyne receiver.

In heterodyne receivers [42-43], to ensure high-quality sound, a system of automatic frequency tuning (APF) is introduced (figure 1).

Automatic frequency adjustment system for a heterodyne receiver.

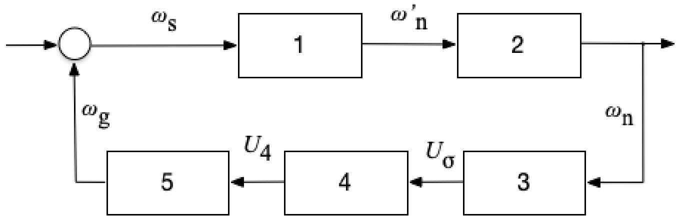

Fig. 1. A system of automatic frequency tuning (APF).

The operation of each of the devices 1-5 is described by the following ratios recorded for frequency deviations:

1) mixer: $\delta\omega_s - \delta\omega_g = \delta\omega'_n$;
2) intermediate frequency amplifier (UPF):
$T_1 \cdot d/dt(\delta\omega_n) + \delta\omega_n = \delta\omega'_n$;
3) the discriminator: $U_\sigma = K_\sigma \cdot \delta\omega_n$

4) amplifier: $T_2 \cdot d/dt(U_y) + U_y = U_\sigma$
5) control element of the heterodyne:
$T_2 \cdot d/dt(U_y) + U_y = U_\sigma$

For exact values, T_2 =0.3; T_2 = 0.2; T_3 = 0.1 using the Raus criterion, we get stability at the gain . However, if the parameter values change by 1 percent, the system becomes unstable. The interval criterion provides stability at an interval gain K_σ=[10.6, 10.96] = (10.78, 0.107).

V. Conclusion

Software for constructing a characteristic polynomial using several methods has been developed. Several examples are considered for numerical analysis.

From numerical examples, we can conclude that the smallest range of coefficients of the characteristic polynomial is provided by the Fadeev and Leverrier algorithms. In this case, the midpoint of the coefficient intervals of the characteristic polynomial coincides with the coefficients of the characteristic polynomial of the "point" matrix, whose elements represent the midpoints of the corresponding intervals of the original interval matrix. This property is not provided both when using classical interval arithmetic and when completely iterating over all point matrices.

References

[1] Y. Y. Aleksankin, A. E. Brzhozovsky, V. A. Zhdanov and others, "Automateddesign of automatic control systems," ed. V. V. Solodovnikov, Moscow: Mashinostroenie, 1990, pp. 1-332.
[2] A. A. Voronov and I. A. Orourke, "Analysis and optimal synthesis of computer control systems," Moscow: Nauka, 1984, pp. 1–344.
[3] V. E. Balnokin and P. I. Chinaev, "Analysis and synthesis of automatic control systems on a computer. Algorithms and programs: Reference," Moscow: Radio and communications, 1991, pp 1–256.
[4] P. D. Krutko, A. I. Maximov and L. M. Skvortsov, "Algorithms and programs for designing automatic systems," Moscow: Radio and communications, 1988, 1–306.
[5] G. Davenport, I. Sira and E. Tournier, "Computer algebra," Moscow: Mir, 1991, pp. 1-352.
[6] D. M. Klimov, V. M. Rudenko, "Methods of computer algebra in problems of mechanics," Moscow: Nauka, 1989, pp. 1–215.
[7] N. G. Chetayev, "Stability of motion," Moscow: GITL, 1955, pp. 1–207.
[8] V. I. Zubov, "Dynamics of managed systems," High School, 1982, pp. 1–286.
[9] V. M. Matrosov, "On the theory of motion stability," Applied mathematics and mechanics, no 6, pp. 992-1002, 1962.
[10] R. Bellman, "Vector Lyapunov function," J. Soc. indastr. appl. math., vol. 1., no 1, pp. 32-34, 1962.
[11] V. M. Matrosov and S. N. Vasilyev, "Comparison principle for derivation of theorems in mathematical system theory," International conference on artificial intelligence. Moscow: USSR, 1975, pp. 25-34.
[12] V. M. Popov, "On the absolute stability of non-linear automatic control systems," Automatics and telemechanics, vol. XXII, no. 8., pp. 50-59, 1961.
[13] V. M. Popov, "Hyper-Stability of automatic systems," Moscow: Nauka, 1970, pp. 1–456.
[14] V. Rezvan, "Absolute stability of automatic systems with delay," Moscow: Nauka, 1983, pp. 1–360.
[15] V. V. Rumyantsev and A. S. Oziraner "Stability and stabilization of motion in relation to a part of variables," Moscow: Nauka, 1987, pp. 1–256.
[16] V. I. Vorotnikov, "Stability of dynamic systems in relation to some variables," Moscow: Nauka, 1991, pp. 1–288.
[17] K. G. Valeev and O. A. Zhautykov, "Infinite systems of differential equations," Alma-ATA: Nauka, 1974, pp. 1–415.
[18] A. K. Bedelbaev, "Stability of nonlinear automatic control systems," Almaty: ed. anKazSSR, 1960, pp. 1–163.
[19] B. J. Magarin, "The Stability and quality of non-linear automatic control systems," Almaty: Science of The Kazakh SSR, 1980. pp. 1–316.
[20] S. A. Aisagaliev, "Analysis and synthesis of autonomous nonlinear automatic control systems," Almaty: Science of The Kazakh SSR, 1980, pp. 1–244.
[21] R. E. Moor, "Interval analysis," New Jersey: Prentice-Hall, 1966, pp. 1-245.
[22] Y. I. Shokin, "Interval analyze," Novosibirsk: Science, 1986, pp. 1–224.
[23] T. I. Nazarenko and L. V. Marchenko, "Introduction to interval methods of computational mathematics, " Irkutsk: Publishing house of Irkutsk University, 1982, pp. 1–108.
[24] S. A. Kalmykov, Y. I. Shokin and Z. H. Yuldashev, "Methods of interval analyze. – Novosibirsk: Science, 1986. – 224 p.
[25] Yu. M. Gusev, V. N. Efanov, V. G. Krymsky and V. Yu. Rutkovsky, "Analysis and synthesis of linear interval dynamic systems (state of the problem)," RAN. Technical cybernetics, no. 1, 1991, pp. 3-30.
[26] E. M. Smagina, A. N. Moiseev and S. P. Moiseeva, "Methods for calculating the IHP coefficients of interval matrices," Computational technologies, vol. 2, no.1. 1997, pp. 52-61.
[27] V. A. Pochukaev and I. M. Svetlov, "Analytical method of constructing Hurwitz interval polynomials," Automatics and telemechanics, no. 2, 1996, pp. 89-100.
[28] N. A. Bobylev, S. V. Emelyanov and S. K. Korovin, "On positive definiteness of interval families of symmetric matrices," Automatics and telemechanics, no. 8, 2000, pp. 5-10.
[29] S. B. Partushev, "Improving the accuracy of interval estimates of voltage deviations in General-purpose electrical networks," Computational technology, no. 1, 1997, pp. 45-51.
[30] I. V. Svyd, A. I. Obod, G. E. Zavolodko, I. M. Melnychuk, W. Wójcik, S. Orazalieva and G. Ziyatbekova, "Assessment of information support quality by “friend or foe” identification systems," Przeglad Elektrotechniczny, vol. 95, no. 4, 2019, pp. 127-131.

[31] T. Zh. Mazakov, Sh. A. Jomartova, T. S. Shormanov, G. Z. Ziyatbekova, B. S. Amirkhanov and P. Kisala, "The image processing algorithms for biometric identification by fingerprints," News of the national academy of sciences of the Republic of Kazakhstan. Series of Geology and Technical Sciences, vol. 1, no 439. 2020, pp. 14-22.
[32] V. M. Belov, V. A. Sukhanov, E. V. Lagutina, "Interval approach for solving problems of kinetics of simple chemical reactions," Technol, no. 1, 1997, pp. 10-18.
[33] A. Kydyrbekova, M. Othman, O. Mamyrbayev, A. Akhmediyarova and Z. Bagashar, "Identification and authentication of user voice using DNN features and i-vector," Cogent Engineering, vol. 7, 2020, pp. 1-22.
[34] I. Nurdaulet, M. Talgat, M. Orken, G. Ziyatbekova, "Application of fuzzy and interval analysis to the study of the prediction and control model of the epidemiologic situation," Journal of Theoretical and Applied Information Technology, Pakistan, vol. 96, no. 14, 2018, pp. 4358-4368.
[35] V. N., Podlesny and V. G. Rubanov, "A simple frequency criterion for robust stability of a class of linear interval dynamic systems with delay," Automatics and telemechanics, no. 9, 1996, pp. 131-139.
[36] A. P. Molchanov and M. V. Morozov, "Sufficient conditions for robust stability 5f linear non-stationary control systems with periodic interval restrictions," Automatics and telemechanics, no. 1, 1997, pp. 100-107.
[37] A. M. Letov, "Stability of nonlinear control systems, " Moscow: Fizmatgiz, 1962, pp. 1–312.
[38] N. S. Bakhvalov, "Numerical methods," Moscow: Nauka, 1973. pp. 1–632.
[39] A. I. Lurie , "Some nonlinear problems of the automatic control theory," Moscow: GITL, 1951, pp. 1–216.
[40] K. I. Babenko, "Fundamentals of numerical analysis," Moscow: Nauka, 1986. pp. 1–744.
[41] B. P. Demidovich, I. A. Maron and E. Z. Shuvalova, "Numerical methods of analysis. Approximation of functions, differential and integral equations," Moscow: Nauka, 1967, pp. 1–368 p.
[42] V. N. Afanasiev, V. B. Kolmanovsky and V. R. Nosov, "Mathematical theory of designing control systems," Moscow: Higher. SHK., 1989, pp. 1–447.
[43] G.A. Amirkhanova, A. I. Golikov and Yu.G. Evtushenko, "On an inverse linear programming problem," Proceedings of the Steklov Institute of Mathematics, vol. 295. no. 1, 2016, pp. S21-S27.