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A SIMPLE METHOD TO CALCULATE THE OSCILLATING LIFT ON A CIRCULAR CYLINDER IN POTENTIAL FLOW

Harijono Djojodihardjo*)

R I N G K A S A N

Suatu model aliran potensial sederhana diuraikan untuk menghitung gaya angkat yang berosilasi pada silinder lingkaran. Pada dasarnya, gaya angkat yang berosilasi ini disebabkan karena adanya vortex yang dilepaskan oleh silinder dari titik pemisahan. Kekuatan vortex dan lintasannya dihitung dengan menggunakan syarat bahwa gaya pada vortex dan bidang vortex yang menumbuhkannya harus sama dengan nol. Vortex ini terputus dari silinder pada waktu kekuatannya mencapai maksimum, dan vortex yang lain mulai dilepaskan pada sisi silinder yang lain.

Hasil perhitungan menunjukkan harga gaya angkat maksimum yang mendekati harga yang diperoleh secara eksperimentil. Demikian pula, untuk aliran subkritis, diperoleh hasil perhitungan bilangan Strouhal yang mendekati harga yang diperoleh secara eksperimen.

A B S T R A C T

A simple potential flow model is presented to calculate the oscillating lift on a circular cylinder. In essence, the time dependent lift acting on a cylinder is due to the presence of a vortex shed off the se-

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*) Mechanical Engineering Department.

paration point on the cylinder. The vortex strength and its trajectory is predicted employing the condition of zero force on the vortex and its feeding vortex sheet. The vortex breaks away from the cylinder and is convected by the fluid after the vortex strength reaches a maximum value, and another vortex is shed at the other side of the cylinder.

Oscillating lift with the right order of magnitude of available experimental results was predicted. For subcritical flow, good agreement is obtained between the predicted Strouhal number and that found experimentally.

LIST OF SYMBOL

a	-	radius of the cylinder
C_L	-	lift coefficient
C_D	-	drag coefficient
D	-	drag force
i	-	$\sqrt{-1}$
L	-	lift force
Re	-	Reynolds number
S	-	Strouhal number
t	-	time
U	-	free stream velocity
$\hat{U}$	-	von Karman's vortex velocity
V	-	velocity
V_o	-	velocity of point vortex at z_o
u_o	-	x component of V_o
v_o	-	y component of V_o
W	-	complex potential
w	-	V/U
x	-	longitudinal coordinate
y	-	lateral coordinate

z	-	x + iy
Γ	-	vortex strength
φ	-	angular position of point vortex
γ	-	dimensionless vortex strength
λ	-	ratio between lateral to longitudinal distances of von Karman vortices
ρ	-	distance between z_o and z_1 expressed in units of a
θ	-	angular position of separation point

I. Introduction

The oscillating lift acting on a circular cylinder has been a subject of continued interest, especially in the light of increased effort to understand and overcome wind excited vibration in the field of industrial aerodynamics. Several experimental investigations were performed by many investigators, among others Roshko (4,5), Fung (3), and Bishop and Hassan (2). Recently, Alexander (1) has proposed a simple theory to calculate the oscillating lift acting on a circular cylinder. The present article presents an analytical model somewhat similar to that introduced by Alexander, but introduces additional conditions based on physical considerations. Alexander has preassumed the vortex trajectory, and also assumed that the vortex shed off the cylinder breaks away from the cylinder or joins the vortex street at the instant when the lift force on the cylinder attained its maximum value, and hence suddenly the force on the cylinder due to the vortex vanishes. In the present theory, the above assumptions are removed, and instead, the vortex trajectory is computed employing the condition of zero force on the vortex and its feeding vortex sheet. The vortex breaks away from the cylinder and is convected by the fluid after its strength reaches a maximum value.

The present model is capable of predicting the oscillating lift within the right order of magnitude of available experimental results. However, to some extent, most experimental results are affected by three dimensional effects.

II. Analytical Model

Consider the flow around a circular cylinder, and assume that initially a simple vortex is shed off at one of the separation points on the cylinder, and then convected downstream.

In this process, the vortex grows in strength. The feeding points - or separation points - will be defined from experimental evidence, or they may be arbitrarily defined and verified later. It is assumed that as soon as the vortex attains its maximum strength, then it stops growing in strength and breaks away from its feeding vortex sheet and starts to be convected downstream at von Karman's velocity and at constant strength. At the same time, another vortex is shed off on the other separation point. This assumption is based on the premise that the vortex cannot decrease in strength, or unroll, and if such tendency seems to occur, the vortex will break away from its feeding vortex sheet and another vortex of opposite sense is generated. The initial path of the vortex shed off the cylinder is governed by the condition of zero force on the vortex and its feeding vortex sheet. The strength of the vortex at any instant is governed by the condition that the velocity at the separation point should always be equal to zero.

a. Governing equations:

Figure 1 illustrates the analytical model of the flow about a cylinder at subcritical Reynolds number. At some instant after shedding, the vortex shed assumes a position z_o and strength Γ. The complex potential at any position, $z = x + iy$ for this situation is given by:

$$W(Z) = -U(z + \frac{a^2}{z}) - \frac{i\Gamma}{2\pi} \ln (z - z_o) + \frac{i\Gamma}{2\pi} \ln (\frac{a^2}{z} - \bar{z}_o) \qquad (1)$$

where all symbols are defined in the "List of symbols". The velocity field at any point z(x,y) in the flow is given by:

$$V = - \frac{dW}{dz}$$

$$= U(1 - \frac{a^2}{z^2}) + \frac{i\Gamma}{2\pi a} (\frac{a}{z - z_o} + \frac{a}{z} \frac{a^2}{a^2 - \bar{z}_o z}) \qquad (2)$$

which has the u and v components described by:

$$u = U - \frac{Ua^2(x^2 - y^2)}{(x^2 + y^2)^2} + \frac{\Gamma}{2\pi} \frac{y - y_o}{(x - x_o)^2 + (y - y_o)^2}$$

$$- \frac{a^2\Gamma}{2(x^2 + y^2)} \frac{\{y(xx_o + yy_o - a^2) + x(yx_o - xy_o)\}}{\{(xx_o + yy_o - a^2)^2 + (yx_o - xy_o)^2\}} \qquad (3)$$

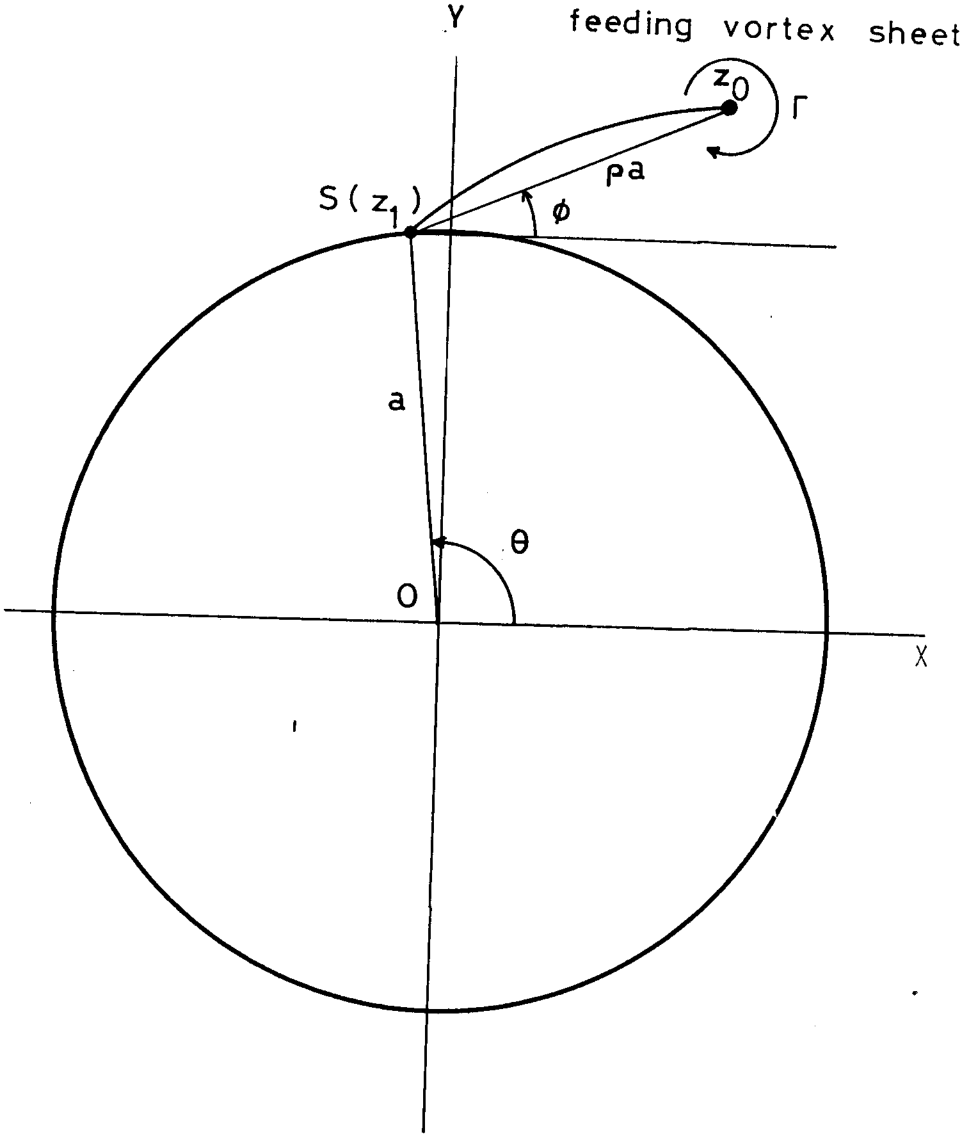

Figure 1. Analytical Model

$$v = -\frac{2Ua^2xy}{(x^2+y^2)^2} - \frac{\Gamma}{2\pi}\frac{x - x_o}{(x-x_o)^2 + (y-y_o)^2}$$
$$+ \frac{a^2\Gamma}{2(x^2+y^2)}\frac{\{x(xx_o + yy_o - a^2) - y(yx_o - xy_o)\}}{\{(xx_o + yy_o - a^2)^2 + (yx_o - xy_o)^2\}} \quad (4)$$

Using Blasius theorem for the forces acting on the cylinder, we obtain:

$$D - iL = \tfrac{1}{2}\, i\rho \oint_c \left(\frac{dW}{dz}\right)^2 dz \quad (5)$$

where the integration is carried out along a closed path around the cylinder, and D and L are the time dependent drag and lift forces acting on the cylinder, respectively. Hence:

$$D = \frac{a^2 x_o \rho \Gamma}{(x_o^2 + y_o^2)^2 (x_o^2 + y_o^2 - a^2)} \times$$
$$\left\{\frac{\Gamma}{2\pi}(x_o^2 + y_o^2) - 2y_oU(x_o^2 + y_o^2 - a^2)\right\} \quad (6)$$

$$L = \frac{a^2 \rho \Gamma}{(x_o^2 + y_o^2)^2 (x_o^2 + y_o^2 - a^2)} \times$$
$$\left\{y_o \frac{\Gamma}{2\pi}(x_o^2 + y_o^2) - U(x_o^2 - y_o^2)(x_o^2 + y_o^2 - a^2)\right\} \quad (7)$$

Since: $C_D = \dfrac{D}{\rho U^2 a}$

and $C_L = \dfrac{L}{\rho U^2 a}$

then:
$$C_D = \frac{a x_o \Gamma}{U^2 (x_o^2 + y_o^2)^2 (x_o^2 + y_o^2 - a^2)} \times$$
$$\left\{\frac{\Gamma}{2\pi}(x_o^2 + y_o^2) - 2y_oU(x_o^2 + y_o^2 - a^2)\right\} \quad (8)$$

$$C_L = \frac{a\Gamma}{U^2(x_o^2 + y_o^2)^2(x_o^2 + y_o^2 - a^2)} \times$$

$$\{y_o \frac{\Gamma}{2\pi} (x_o^2 + y_o^2) + U(x_o^2 - y_o^2)(x_o^2 + y_o^2 - a_o^2)\} \quad (9)$$

b. *The motion of the vortex*

The motion of the vortex is governed by the condition that the total force acting on the feeding vortex sheet - which is assumed to have vanishing strength - and the point vortex is zero. Mathematically there is a branch line between z_o and z_1, the separation point, and the velocity potential ϕ is discontinuous across this branch line. The discontinuity in ϕ is equal to Γ, the strength of the vortex. Therefore there is a discontinuity in pressure across this branch line with a magnitude of $-\rho\dot{\Gamma}$, and the total force acting on this vortex sheet is equal to

$$- i\rho\dot{\Gamma}(z_o - z_1)$$

where z_1 is the location of the separation point.

The force acting on the concentrated vortex is equal to its circulation times, $-\rho i$ times the relative velocity between the fluid and the vortex at the point vortex, $V_o - \dot{z}_o$, hence has a magnitude of $i\rho\Gamma(V_o - \dot{z}_o)$. To make the net force on the concentrated vortex and its feeding vortex sheet vanish, we must have:

$$- i\rho\dot{\Gamma}(z_o - z_1) + i\rho\Gamma(V_o - \dot{z}_o) = 0 \quad (10a)$$

or

$$\dot{z}_o + (z_o - z_1) \frac{\dot{\Gamma}}{\Gamma} = V_o \quad (10)$$

where V_o has components u_o and v_o parallel to the x and y axes, respectively. From equations (3) and (4) and employing a limit process, we obtain:

$$u_o = U - Ua^2 \frac{x_o^2 - y_o^2}{(x_o^2 + y_o^2)^2} - \frac{\Gamma}{2\pi} \frac{a^2 y_o}{(x_o^2 + y_o^2)(x_o^2 + y_o^2 - a^2)} \quad (11)$$

and

$$v_o = - \frac{2Ua^2 x_o y_o}{(x_o^2 + y_o^2)^2} + \frac{\Gamma}{2\pi} \frac{a^2 x_o}{(x_o^2 + y_o^2)(x_o^2 + y_o^2 - a^2)} \quad (12)$$

The trajectory of the point vortex is a function of time, and is described by:

$$x_o = x_o(t) \qquad y_o = y_o(t) \tag{13}$$

furthermore:

$$u_o = u_o(t)$$

and

$$v_o = v_o(t) \tag{14}$$

From equation (10) we obtain:

$$\frac{dx_o}{dt} + \frac{x_o - x_1}{\Gamma} \frac{d}{dt} \Gamma = u_o \tag{15}$$

$$\frac{dy_o}{dt} + \frac{y_o - y_1}{\Gamma} \frac{d}{dt} \Gamma = v_o \tag{16}$$

If Γ at any instant is known, equations (15) and (16) can be employed to predict the vortex trajectory in a progressive fashion, since the initial position of the vortex shed is known, i.e. at the separation point.

c. Non-dimensionalization

For further computation, it is convenient to write all the pertinent equations in dimensionless forms. For this purpose, we define:

$\gamma = \frac{\Gamma}{2\pi U a}$ - dimensionless vortex strength

$w = \frac{V}{U}$ - dimensionless velocity

After some arrangement, equation (2) can be written as:

$$w = (1 - \frac{a^2}{z^2}) + i\gamma[\frac{a}{a - z_o} + \frac{a^3}{z(a^2 - z_o z)}] \tag{2a}$$

d. Calculation of vortex strength

To calculate the strength of the vortex at any instant along its path, the condition that the velocity at the separation point should be equal to zero is imposed.

From equation (2a), and employing a limit process, for $z = z_1$ is obtained:

$$\gamma = \frac{\Gamma}{2\pi Ua} = -\frac{i}{a}\,\frac{a^2 - z_1^2}{\dfrac{z_1^2}{z_1 - z_o} + \dfrac{z_1 a^2}{a^2 - \bar{z}_o z_1}}$$

$$= -\frac{i}{a}\,\frac{(a^2 - z_1^2)(z_1 - z_o)(a^2 - z_o\bar{z}_1)}{2z_1^2a^2 - z_1^3 z_o - z_o z_1 a^2} \tag{17}$$

Referring to figure 1, then we can write

$$z_o - z_1 = a\rho e^{i\phi} \tag{18}$$

and

$$z_1 = ae^{i\theta} \tag{19}$$

There is obtained

$$z_o = a(e^{i\theta} + \rho e^{i\phi}) \tag{20}$$

and

$$\bar{z}_o = a(e^{-i\theta} + \rho e^{-i\phi}) \tag{21}$$

Substituting these values into equation (17) we obtain:

$$\gamma = -\frac{1}{a}\,\frac{(a^2 - a^2e^{2i\theta})(-a\rho e^{i\phi})[a^2 - a^2(1 + \rho e^{i(e-\theta)})]}{2a^4e^{2i\theta} - a^4e^{3i\theta}(e^{-i\theta} + \rho e^{-i\phi}) - a^4e^{i\theta}(e^{i\theta} + \rho e^{i\theta})} \tag{22}$$

and after some algebra, equation (22) can be reduced to:

$$\gamma = \rho \sin\theta \sec(\theta - \phi) \tag{23}$$

which is always real and positive, since from physical reasons,

$$\theta - \frac{\pi}{2} < \phi < \theta + \frac{\pi}{2}$$

e. *Direction of the initial vortex shedding*

To predict the direction of the initial vortex shedding, consider the flow situation shortly after a vortex is shed from the separation point z_1. Substituting expression (18) into equation (2a), we obtain:

$$v = 1 - \frac{1}{(e^{i\theta} + e^{i\phi})^2} - \frac{i\gamma}{\rho}\,\frac{1}{(e^{i\theta} + \rho e^{i\phi})(1 + 2\cos(\theta-\phi))} \qquad (24)$$

Substituting expression (23) into equation (24), we obtain:

$$w = 1 - \frac{1}{(e^{i\theta} + \rho e^{i\phi})^2} - \frac{i\sin\theta\sec(\theta-\phi)}{(e^{i\theta} + \rho e^{i\phi})(1 + 2\cos(\theta-\phi))} \qquad (25)$$

Taylor series expansion of w with respect to ρ yields:

$$w = w_{\rho=0} + \rho\left(\frac{\partial w}{\partial \rho}\right)_{\rho=0} + \ldots\ldots. \qquad (26)$$

Omitting terms of order ρ^2 and higher, and after some algebra, equations (25) and (26) give:

$$w = 1 - e^{-2i\theta} - ie^{-i\theta}\,\frac{\sin\theta\sec(\theta-\phi)}{1 + 2\cos(\theta-\phi)}$$

$$+ e^{i\phi-2i\theta}\left[2e^{-i\theta} + \frac{i\sin\theta\sec(\theta-\phi)}{1 + 2\cos(\theta-\phi)}\right] \qquad (27)$$

Since the velocity at the separation point should be equal to zero, we obtain an additional condition that w = 0 for $\rho \to 0$. This condition yields:

$$1 - e^{-2i\theta} - ie^{-i\theta}\,\frac{\sin\theta\sec(\theta-\phi)}{1 + 2\cos(\theta-\phi)} = 0 \qquad (28a)$$

Two relations between ϕ and θ result from equation 28a, but only one relation is valid, i.e.:

$$\phi = \theta - 0.677 \qquad (28)$$

Equation (28) determines the direction of the initial vortex shedding.

f. Frequency of vortex shedding

At some distance downstream, the vortex being shed off the cylinder stops growing, breaks away from the feeding vortex sheet and joins the rows of von Karman vortices. This situation is assumed to take place when the vortex attained its maximum strength. At the same time, another vortex of opposite sense starts to be shed off at the other separation point, growing and travelling downstream in a similar fashion as the former vortex. Hence vortices are shed off alternately at either separation point, and give rise to time dependent and oscillating lift and drag forces on the cylinder.

To calculate the frequency of vortex shedding, which in dimensionless form is characterized by the Strouhal number, we impose the condition that the vortices which join the von Karman vortex street attain the well known von Karman's vortex velocity. Hence:

$$u = U - \frac{\Gamma}{2l} \tanh \frac{2\pi y}{l} = \hat{U} \tag{29a}$$

or

$$\frac{u}{U} = 1 - \gamma \frac{\pi a}{l} \tanh \frac{2\pi y}{l} \tag{29}$$

where: l - the distance between two successive vortices
y - the lateral distance of the vortices from the symmetry axis
$\hat{U}$ - von Karman's vortex velocity

When the vortex attains its maximum strength, then $\dot{\gamma} = 0$, and from equation (10) we obtain:

$$\dot{z}_o = V_o \tag{30}$$

where V_o is the local fluid velocity. This situation implies that the vortex is then convected by the fluid, the fluid velocity being now identified as von Karman's vortex velocity. Hence:

$$u_o = \hat{U} \tag{31}$$

and $v_o = 0$ (which should be verified later).

$\hat{U}$ is the computed u component of the vortex velocity at the instant when the vortex attains its maximum strength.

Equation (29) can be reduced further into the following form:

$$\tanh \alpha = \frac{\beta}{\alpha} \tag{32}$$

where:

$$\alpha = \frac{2\pi y}{1} \tag{33}$$

$$\beta = \frac{2(1 - \hat{U}/U)}{\gamma a/y} \tag{34}$$

Equation (34) can be solved graphically for calculating 1. The Strouhal number can then be obtained from

$$S = \frac{nD}{U} = \frac{2\hat{U}a}{U\ 1} \tag{35}$$

III. Calculation of vortex trajectory

Equations (15), (16) and (23) comprise the set of equations that determine the strength and motion of the vortex. The initial condition is defined by assigning the location of the separation point, i.e. the initial position of the vortex, and computation of the direction of the initial vortex shedding, which is given by equation (28). To compute the vortex trajectory, equation (15) and (16) are integrated numerically, while the computation progresses in a step by step fashion. Integrating equation (15) and (16) over the time increment Δt, and using linear approximation, we obtain:

$$x_o + (x_o - x_1)\ \ln\gamma - \ln\gamma .\ x_o = u_o .\ \Delta t \tag{36}$$

$$y_o + (y_o - y_1)\ \ln\gamma - \ln\gamma .\ y_o = v_o .\ \Delta t \tag{37}$$

where all variables have been rendered dimensionless through division with appropriate characteristic quantities.

The velocity components u_o and v_o of the vortex (from equation (3a) and (3b) is given by:

$$u_o = 1 - \frac{x_o^2 - y_o^2}{(x_o^2 - y_o^2)^2} - \gamma \frac{y_o}{(x_o^2 + y_o^2)(x_o^2 + y_o^2 - 1)} \tag{38}$$

$$v_o = - \frac{2x_o y_o}{(x_o^2 + y_o^2)^2} + \gamma \frac{y_o}{(x_o^2 + y_o^2)(x_o^2 + y_o^2 - 1)} \tag{39}$$

Again, all variables in equations (38) and (39) are dimensionless. The computation algorithm is shown in figure 2. The lift and drag forces acting on the cylinder due to the vortex throughout its trajectory is computed from equations (6) and (7).

IV. Results and Discussions

Since the present theory does not explicitly incorporate viscosity, the locations of the separation points cannot be predicted, and recourse should be made to experimental evidence in order to determine these locations and then proceed with the calculation of the vortex trajectory.

Experimental results (6) indicate that the separation points, i.e. the point at which the boundary layer leaves the cylinder, occur at about $\theta_s = 95°$ for $Re < 1.06 \times 10^6$ and between 50° to 80° for $1.08 \times 10^6 < Re < 2.12 \times 10^6$.

Based on these results, some values of θ are assumed, and the vortex trajectory, lift and drag coefficients, vortex strength, vortex velocity, λ — the ratio between lateral separation and longitudinal separation of Karman vortices — and Strouhal number S are computed following the procedure outlined previously. The results are tabulated in Table 1. Some values obtained experimentally and by Alexander are also shown in Table 1 for comparison.

For the case in which the separation point is located on the upstream side of the cylinder, say $\theta_s = 1.74$ radian, a maximum lift coefficient of 1.55 was obtained. Experimental results indicate considerable scatter, and time dependent lift as high as 1.3 has been shown by Drescher (cited in reference 1) and Macovsky (cited in reference 11) and Bearman (13). It should be remarked that most experimental data are affected to some degree by three dimensional effects, which result in lower measured lift coefficients than predicted two-dimensional values. Recent results obtained by Jordan and Fromm (12) employing numerical solution of the equations governing time-dependent, viscous, incompressible flow past a cylinder indicate peak to peak amplitude of the lift oscillation of 1.5 for Re = 400 and 1.9 for Re = 1000. Their results are an adequate description of the flow past a circular cylinder for Re 400, but are not entirely valid for Re > 1000.

The Strouhal number for this case was found to be 0.159, which is within the region of experimental results. Relf and Simmons (8) obtained values of S between 1.9 to 2.4 for Reynolds number between 8×10^5 to 3×10^5, and $S = 2.4 \div 3.1$ for Re $4.0 \times 10^5 \div 10^6$, while recent experiment conducted by Bearman (9) showed that $S = 1.8 \div 2.3$ for $Re = 10^5 \div 3.8 \times 10^5$. For supercritical region, i.e. for Re 4×10^5, Bearman

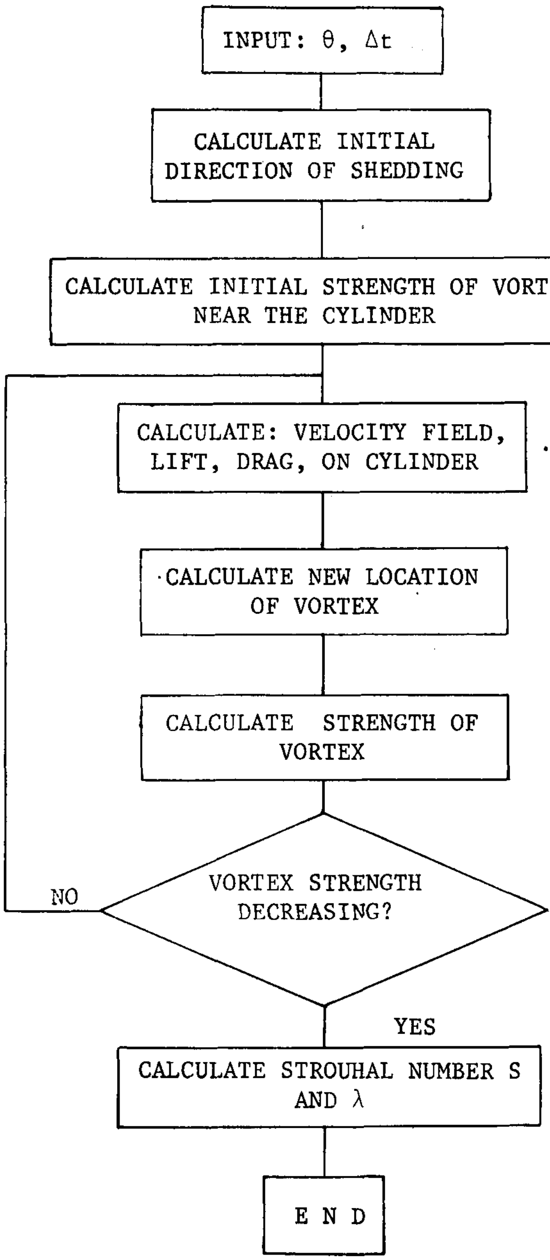

Figure 2: Computation Algorithm

found that S = 0.44 - 0.46, while Delany and Sorensen (10) found that S = 0.34 - 0.48 for Re 10^6. In addition, the present method results in the value of λ of 0.288, which is in good agreement with experimental result 0.2806 reported in reference 7.

For Reynolds number greater than 10^6, the separation point occurs further downstream, i.e. at about 50° - 80°. Assuming separation points located between these values, there results maximum lift coefficient as high as 2.73 for $\theta_s = 1.0$ radian, while $C_L = 1.93$ for $\theta_s = 0.79$ radian. Experimental results indicate that S increases to about 0.24 before sudden reduction of drag coefficient occurs, and increases further to 0.48 at supercritical region. The present method is able to predict the tendency of S to increase if the flow changes from subcritical to supercritical region, i.e. if the separation point moves downstream, but indicate lower values of S at both cases. Clearly the additional increase in S should be due to turbulence, which is not taken into account in the present theory. At supercritical region, λ varies between 0.22 to 0.3 for θ_s varying between 0.79 to 1.00.

The time dependent drag obtained by the present method will be zero only when $v_o = 0$, i.e. when the vortex joins von Karman vortex street. Since in general the vortex trajectory is not parallel to the free stream direction, the time dependent drag is non zero. Careful examination of equation (8) reveals that depending on the value of γ and y_o (lateral position of the vortex), the time dependent drag coefficient may assume positive or negative value. Relatively large value of the time dependent drag C_D is obtained when the vortex is close to the cylinder, and C_D decreases rapidly in inverse propotion to x_o as x_o increases.

The result indicates discrepancy in the lift coefficient in the order of 40% for forward separation point. Relatively better agreement is shown for act separation point, i.e. within 6%. These results also clearly indicate that the presence of small vertical velocity component of the vortex may give rise to significant changes in the values of time dependent drag and vortex strength, in particular for forward separation point. Further evaluation of the theory should be accompanied by comparison with reliable experimental results, which merits further study.

Work is in progress to investigate the problem of stability and convergence of the numerical computational scheme. However, several computation performed with Δt varying from 0.01, 0.02 and 0.01 has indicated reasonable converging behaviour of the solution, in particular for the calculation of γ_o, $C_{l_{max}}$, y_o, x_o, λ and S, as shown in table 1. The variation of

Table 1: Various Computation Results

0	Δt	u_o	v_o	Γ_{max}	$C_{L_{max}}$	$C_{D_{max}}$	y_o	x_o	1	λ	S
1.74	0.03	0.404	0.001	1.049	3.95	0.07	1.265	0.214	5.05	0.501	0.16
1.74	0.02	0.57	0.02	1.041	2.80	-0.146	1.308	0.230	6.45	0.406	0.177
1.74	0.01	0.762	-0.04	1.024	1.55	-0.340	1.384	0.231	9.6	0.288	0.159
1.00	0.03	0.545	0.022	1.034	3.15	0.299	0.867	1.234	5.40	0.320	0.202
1.00	0.02	0.574	0.001	1.018	2.73	0.379	0.883	1.251	5.65	0.312	0.202
0.79	0.03	0.705	0.005	1.021	2.55	0.55	0.704	1.709	6.4	0.22	0.22
0.79	0.02	0.697	0.008	1.013	2.29	0.335	0.695	1.692	6.3	0.22	0.221
0.79	0.01	0.695	0.008	1.007	1.93	0.426	0.694	1.684	6.2	0.22	0.224
0.79*		0.5	0		2.3	0		1.224	3.7	0.382	0.27
					1.3++					** 0.2806	0.18 0.23+
						0.23#					

* From Alexander; Alexander determines S1 = 1 and $S\eta$ = 0.191. where $y_o = \eta$. If S is taken to be 0.27, then η = 0.707, and hence 1 = 3.7, λ = 0.382; if S is taken to be 0.2, then η = 0.955, 1 = 5.0, λ = 0.28.

** From Prandtl & Tietjens.

\+ From P.W. Bearman, at $10^5 < Re < 3.8 \times 10^5$.

++ From Drescher (cited by Alexander).

\# Peak to peak amplitude, from reference 12, for Re = 1.000.

time dependent lift coefficients for $\theta = 1.74$ and $\theta = 0.79$ are shown in figure 3.

The present simple model is in no way a subsitute for viscous flow analysis, but has been shown to be useful in estimating the time dependent lift on a cylinder and in identifying factors contributing to the oscillating lift and drag, and the increase of Strouhal number.

V. *Acknowledgement*

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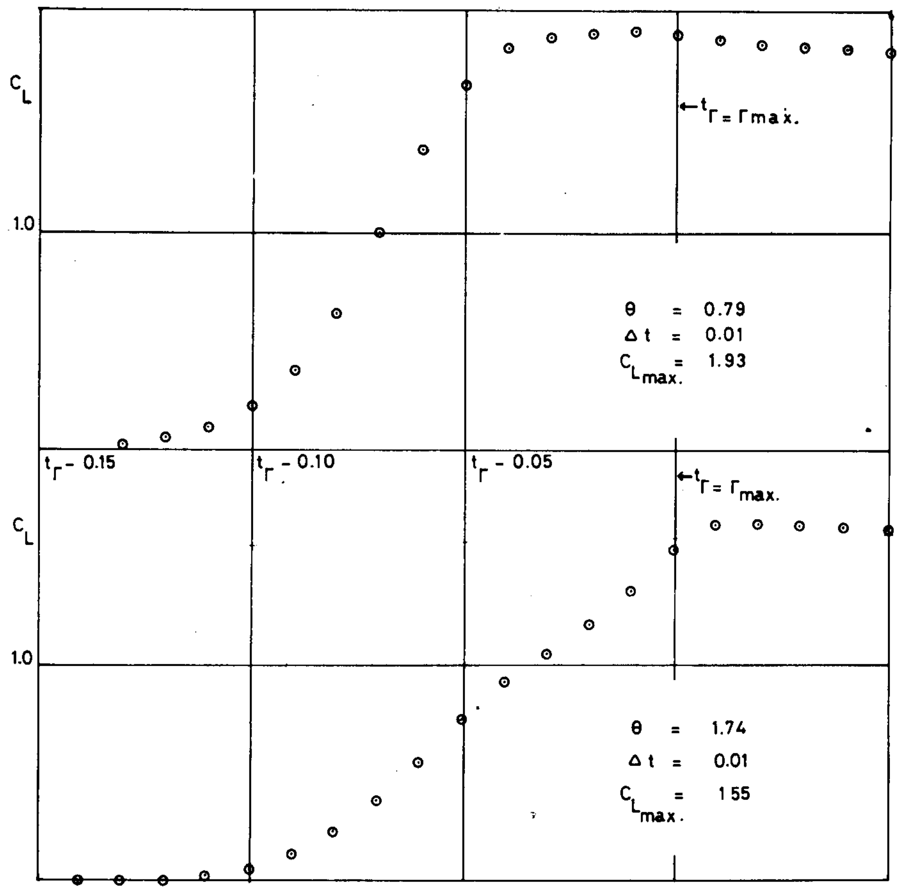

FIGURE 3 : DEVELOPMENT OF TIME-DEPENDENT LIFT DUE TO A SINGLE VORTEX SHED OFF THE CYLINDER.

Figure 3. Development of time-dependent lift due to a single vortex shed off the cylinder

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