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On Retention of Eventual Stability of Perturbed Impulsive Differential Systems

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Abstract. In this paper, a system of non nonlinear differential equations with impulse effect at fixed time moments is considered and criteria for retention of uniform eventual stability of its perturbed impulsive differential systems under vanishing perturbations are established. Sufficient conditions are obtained by using piecewise continuous Lyapunov functions. An example is also worked out to illustrate the results.

Keywords: *eventual stability; impulsive differential systems; Lyapunov function; uniform eventual stability; perturbed differential systems.*

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1 Introduction

During evolution, processes are generally subjected to short-term perturbations that are caused by external intrusions. Very often the duration of these effects is negligible in comparison with the whole duration of the process. Therefore it is natural to assume that these perturbations act instantaneously in the form of impulses. Thus impulsive differential systems present a more natural framework for mathematical modeling of real-world phenomena than ordinary differential equations. A large number of mathematicians have shown their interest in the study of the qualitative properties, especially stability, in the mathematical theory on impulsive differential systems. Significant development has been made during the past 3 decades [1-7].

There are many perturbation and adaptive control problems, where the point in question may not be an equilibrium (invariant) point but eventually stable sets that are asymptotically invariant, which enables us to consider Lyapunov stability as a special case of eventual stabilities. Referring Hamamoto, Molyneuxand Papanikolaou in [8], let us consider the stability of a ship. From a practical point of view, if a ship remains in an upright position, it is called stable. However, since the environmental forces acting on it as well as the

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ship's disposition with respect to the sea will change over time, the determination of a safe minimum amount of stability, i.e. a stability criterion, becomes necessary. Following the insightful concept of stability, it is natural to consider the ship to stable if for any combination of ship and surrounding environment the amplitude of the ship remains smaller than a pre-determined safe value. The mathematical counterparts of this definition are eventual stability and boundedness. For the problems arisen in these situations, a new notion of eventual stability has been introduced and discussed for ordinary differential equations by Lakshmikantham [9] and Lakshmikantham, Leela & Martynyukin [3]. Sufficient conditions for eventual stability and Ψ-eventual stability of impulsive differential systems have been obtained by Zhang & Jitao in [5] and Sood & Srivastava in [6] respectively.

As far as stability of perturbed differential systems is concerned, various results for different types of perturbations have been established in the literature. Stability of nonlinear systems without impulses under constantly acting perturbations has been studied by Liu & Sivasundram [10], S.G. Pandit [11], Cantarelli & Zappala [12], Sheldon & Gordon [13] and Andreev & Zappala [14]. Results for eventual stability of impulsive differential systems with fixed time impulses were established by Soliman [15] but with the perturbations considered bounded functions. The comparison principle along with the Lyapunov method was used to obtain the desired results. Uniform eventual stability of impulsive differential systems with non-fixed moments of impulses, having bounded perturbations, has also been established by Kulev [16].

In this research sufficient conditions for retention of uniform eventual stability of perturbed impulsive differential systems have been established. An impulsive differential system with fixed moments of impulses and its perturbed system under vanishing perturbations were studied. Piecewise continuous, auxiliary functions, analogous to Lyapunov functions, were used to obtain the results. This paper is organized as follows: in Section 2, some preliminary notes and definitions, to be used in the paper, are introduced. In Section 3, two theorems for uniform-eventual stability of an impulsive differential system and its perturbed system are proved. One example is given in support of our theoretical results in section 4.

2 Preliminary Notes And Definitions

Let R^n denote n-dimensional Euclidean space with norm $\|.\|$

Let Ω be a domain in R^n containing the origin. Consider the system of differential equations with impulses

$$\left.\begin{aligned} x' &= f(t, x), & t &\neq t_i \\ \Delta x &= I_i(x), & t &= t_i \end{aligned}\right\} \tag{1}$$

where, $i \in N,\ t \in R^+,\ I_i(x) = x(t_i^+) - x(t_i)\ x \in \Omega,\ f: R^+ \times \Omega \to R^n,\ I_i: \Omega \to R^n$ and $0 < t_0 < t_1 < t_2 < \ldots\ldots\ldots\ldots.\infty$.

Let $t_0 \in R^+,\ x_0 \in \Omega$ such that $x(t, t_{0,}\ x_0)$ is the solution of the Eq. (1), satisfying the initial conditions $x(t_0 + 0, t_{0,}\ x_0) = x_0$. The solution $x(t)$ of Eq. (1) are piecewise continuous functions with points of discontinuities of the first type in which they are left continuous, i.e. at moment t_i when the integral curve of the solution $(x(t), y(t))$ meets the hyper planes $\sigma_k : t = t_k$ and the following relations are satisfied:

$$x(t_i - 0) = x(t_i),\ x(t_i + 0) = x(\mathrm{t}_i) + I_i\,(x(t_i)) \ .$$

For practical applications, it is important to have constructive verifiable conditions for retention of system stability. Therefore, to formulate them for eventual stability, together with Eq. (1) we also consider the 'perturbed' system

$$\left.\begin{aligned} x' &= f(t, x) + R(t, x), & t &\neq t_i \\ \Delta x &= I_i(x) + R_i(x), & t &= t_i \end{aligned}\right\} \tag{2}$$

where $R(t, x): R^+ \times \Omega \to R^n$, $R_i\left(x\right): \Omega \to \Omega$ are continuous functions satisfying the Lipschitz conditions in x such that $R\left(t, 0\right) = R_i(0) = 0\ \left(i \in N\right)$.

By analogy with Gladilina in [7], we refer to the perturbing actions vanishing if the following conditions are satisfied:

$$\underset{i \to \infty}{Lim}\, R\left(t, x\right) = 0 \tag{3}$$

$$\underset{i \to \infty}{Lim}\, R_i(x) = 0 \tag{4}$$

Definition 1. Let the sets K be defined as

$$K = \left\{ w \in C\left(R^+, R^+\right) : \text{ strictly increasing and } w\left(0\right) = 0 \right\}$$

We use class V_0 of piecewise continuous functions that are analogue to Lyapunov functions.

Definition 2. We say that function $V : R^+ \times \Omega \to R^n$ belongs to class V_0 if the following conditions hold:

Function V is continuous in $(t_{k-1}, t_k\,] \times \Omega \;\; \forall x \in \Omega$.

V_0 is locally Lipschitzian with respect to x in each $(t_{k-1}, t_k\,] \times \Omega$.

$V(t,0) = 0$ for $t \in R^+$.

Let $V \in V_0$. For $(t, x) \in (t_{k-1}, t_k\,] \times \Omega$, we define the right hand derivative

$$D^+_{(1)} V\left(t, x\right) = \underset{h \to 0^+}{Lim\,Sup} \frac{1}{h}[V\left(t + h, x + h\, f\left(t, x\right)\right) - V(t, x)]$$.

Definition 3. The set $x = 0$ of Eq. (1) is said to be

(i) eventually stable if for all $\varepsilon > 0$, for all $t_0 \in R^+$, there exists $\tau = \tau(\varepsilon) > 0$ and $\delta = \delta(t_0, \varepsilon) > 0$ for all $x_0 \in R^n$ such that $\|x_0\| < \delta$ implies $\|x(t, t_0, x_0)\| < \varepsilon$, $t \geq t_0 \geq \tau(\varepsilon)$.

(ii) uniformly eventually stable if $\delta = \delta(\varepsilon)$ i.e. δ is independent of t_0 .

Definition 4. We say that conditions (A) hold if the following conditions are satisfied:

(A1) The functions $f(t,x)$ is continuous in $R^+ \times \Omega$.

(A2) $f(t,x)$ is locally Lipschitz continuous w. r. t. x in each $R^+ \times \Omega$.

(A3) There exists a constant $L > 0$ such that $\| f\left(t,x\right)\| \leq L < \infty$ for $\left(t,x\right) \in R^+ \times \Omega$.

(A4) $I_k \in C\left[\Omega,\Omega\right], I_k\left(0\right) = 0 : k = 1,2,3........$

3 Main Results

In this section, we prove the necessary conditions for uniform eventual stability of the set $x = 0$ of impulsive differential systems with fixed time impulses. The sufficient conditions for retention of uniform eventual stability of $x = 0$ of IDS (1) under vanishing perturbations have also been obtained.

Theorem 1. Assume that

(H1) Conditions (A) hold.

(H2) There exist functions $V \in V_0,\, a,b \in K$ $V \in V_0,\, a,b \in K$ such that $a(\| x(t)\|) \le V(t,x) \le b(\| x(t)\|)$, $(t,x) \in R^+ \times \Omega$.

(H3) $V\left(t+0, x+I_k\left(x\right)\right) \le V\left(t,x\right)$ $x \in \Omega, t = t_k, k = 1,2,3........$

(H4) $D^+_{(1)}V\left(t,x\right) \le p(\mathrm{t})\, q\left(t, x(\mathrm{t})\right)$ $t \ne t_k : k = 1,2,3,.............$

where $p:[t_0,\infty) \to R$ is integrable and $q:[t_0,\infty)\times R^n \to R$.

(H5) There exists a number $\Gamma > 0$ such that $\left|q\left(t,x\right)\right| \le \Gamma$ for $\left(t,x\right) \in R^+ \times \Omega$.

(H6) $\int_{t_0}^{\infty} |p(\mathrm{s})|\, ds < \infty$.

Then, the set $x = 0$ is a uniformly eventually stable set of Eq. (1).

Proof. Let $\varepsilon > 0$ be given and choose $\delta = \delta(\varepsilon) < b^{-1}\left(\frac{1}{2}a\left(\varepsilon\right)\right) : 0 < \delta < \varepsilon$

Let the number $\tau = \tau\left(\varepsilon\right) > 0$ be chosen so that

$$\int_{t}^{\infty} |p(\mathrm{s})|\, ds < \frac{b(\delta)}{\Gamma},\ t \ge \tau . \tag{5}$$

(This is possible because of condition (H6)).

Let $t_0 \ge \tau$ and $x\left(t\right) = x\left(t, t_0, x_0\right)$ be the solution of Eq. (1).

From hypothesis (H4) and (H5)

$$\int_{t_0}^{t} D^+_{(1)}V\left(s, x\left(s\right)\right) ds \le \int_{t_0}^{t} p\left(s\right) q\left(s, x\left(s\right)\right) ds \le \int_{t_0}^{t} \Gamma\left|p\left(s\right)\right| ds < \int_{t_0}^{\infty} \Gamma\left|p\left(s\right)\right| ds$$

$$< \Gamma\frac{b\left(\delta\right)}{\Gamma} = b\left(\delta\right) \quad \forall\, t \ge t_0 . \tag{6}$$

Without loss of generality, let $t_{k+l} < t < t_{k+l+1}$ where t_k is the moment in which the integral curve $\left(t, x\left(t, t_0, x_0\right)\right)$ meets hyper plane σ_k.

Now we have

$$\int_{t_0}^{t} D^+_{(1)}V\left(s,x\left(s\right)\right)ds \le \int_{t_0}^{t_1} D^+_{(1)}V\left(s,x\left(s\right)\right)ds + \sum_{j=2}^{k+l} \int_{t_{j-1}}^{t_j} D^+_{(1)}V\left(s,x\left(s\right)\right)ds$$

$$+\int_{t_{k+l}}^{t} D^+_{(1)}V\left(s,x\left(s\right)\right)ds = V\left(t_1,x\left(t_1\right)\right)-V\left(t_0+0,x\left(t_0\right)\right)+$$

$$\sum_{j=2}^{k+l}\left\{V\left(t_j,x(t_j)\right)-V\left(t_{j-1}+0,x(t_{j-1}+0)\right)\right\}+V\left(t,x\left(t\right)\right)-V\left(t_{k+l}+0,x\left(t_{k+l}+0\right)\right)$$

$$=V(t_1,x(t_1))-V(t_0+0,\ x(t_0))+V(t_2,x(t_2))$$

$$-V(t_1+0,\ x(t_1+0))+V(t_3,x(t_3))-V(t_2+0,\ x(t_2+0))+\ldots\ldots\ldots\ldots\ldots$$

$$+V(t_{k+l},x(t_{k+l}))-V(t_{k+l-1}+0,\ x(t_{k+l-1}+0))+V(t,x(t))-V(t_{k+l}+0,\ x(t_{k+l}+0))$$

$$\ge V(t,x(t))-V(t_0+0,\ x(t_0))\quad \text{(using (H3))} \tag{7}$$

From (H2), Eq. (5), Eq. (6) and Eq. (7),

$$a(\|x(t)\|)\le V\left(t,x\left(t\right)\right)\le V\left(t_0+0,x\left(t_0+0\right)\right)+\int_{t_0}^{t} D^+_{(1)}V\left(s,x\left(s\right)\right)ds \le b\left(\delta\right)+b\left(\delta\right)$$

$$<\frac{1}{2}a\left(\varepsilon\right)+\frac{1}{2}a\left(\varepsilon\right)=a\left(\varepsilon\right) \text{ for } t\ge t_0\ge\tau(\varepsilon).$$

Thus for all $\varepsilon>0$, for all $t_0\in R^+$, there exists $\tau=\tau(\varepsilon)>0$ and $\delta=\delta(\varepsilon)>0$ such that $\|x_0\|<\delta$ implies $\|x(t,t_0,x_0)\|<\varepsilon$, $t\ge t_0\ge\tau(\varepsilon)$ for all $x_0\in\Omega$.

Hence $x=0$ is a uniformly eventually stable set of Eq. (1).

For practical applications, it is important to have constructive verifiable conditions for retention of system stability. To formulate them, together with Eq. (1) we also consider the perturbed Eq. (2) along with vanishing perturbations Eq. (3) and Eq. (4) .

Theorem 2. Let for Eq. (1), function $V\in V_0$ satisfying the conditions of Theorem 1, be a locally Lipschitzian w.r.t. x in $\left(\tau_{k-1},\tau_k\right)\times\Omega$ with Lipschitz constant p_k such that $\left|p_k\right|\le P$ for all $k\in N$ for some constant P. If there exist $a_i\ge 0$ such that $\|R_i(x)\|\le a_i$, $x\in\Omega$ and the series $\sum_{i=1}^{\infty}a_i$ converges, then the set $x=0$ for Eq. (2) is uniformly eventually stable.

Proof. First we take into consideration that the partial derivatives are constrained and estimate the derivative of V along the solution of Eq. (2) for $t \neq t_i$. As $V(t,x)$ is taken Lipschitzian, therefore using the analogy by Lakshmikanthamin [9], we get

$$D^{+}_{(2)}V(t,x) = \underset{s\to 0^{+}}{Lim\,Sup}\frac{1}{s}\left\{V\left(t+s, x+s\left[f(t,x)+R(t,x)\right]\right)-V(t,x)\right\}$$

$$\leq D^{+}_{(1)}V(t,x) + P\left\|R(t,x)\right\|$$

Now magnitude of jump

$$\Delta V_{(2)}(t_i,x) = V\left(t_i+0, x+I_i(x)+R_i(x)\right)-V(t_i,x)$$

$$= V\left(t_i+0, x+I_i(x)+R_i(x)\right)-V\left(t_i+0, x+I_i(x)\right)+V\left(t_i+0, x+I_i(x)\right)-V(t_i,x)$$

$$\leq V\left(t_i+0, x+I_i(x)+R_i(x)\right)-V\left(t_i+0, x+I_i(x)\right) \leq P\left\|x+I_i(x)+R_i(x)-x-I_i(x)\right\|$$

$$= P\left\|R_i(x)\right\|.$$

Now we demonstrate that the set $x = 0$ is uniformly eventually stable.

Let $\varepsilon > 0$ and t_0 be a sufficiently large initial time. We prove that there exists a $\delta = \delta(\varepsilon) > 0$, $\tau = \tau(\varepsilon) > 0$ such that for any solution $x(t,t_0,x_0)$ of Eq. (2),

$$\left\|x_0\right\| < \delta \Rightarrow \left\|x(t,t_0,x_0)\right\| < \varepsilon, t \geq t_0 \geq \tau(\varepsilon).$$

Let $\delta < b^{-1}\left(\frac{1}{6}a(\varepsilon)\right)$ and $\tau = \tau(\varepsilon) > 0$ be chosen so that $\int_t^\infty \left|p(s)\right| ds < \frac{b(\delta)}{\Gamma}$ for $t \geq \tau$.

Further, let $\left\|R(t,x)\right\| \leq \lambda = \frac{\Gamma}{P}\left\|p(t)\right\|$ for $t \geq \tau$ (this is possible because of Eq. (3)).

Let $t_{k+l} < t < t_{k+l+1}$, where t_k is the moment in which the integral curve $\left(t, x(t,t_0,x_0)\right)$ meets hyper plane σ_k

$$\int_{t_0}^{t} D^+_{(2)}V\left(s,x(s)\right)ds \le \int_{t_0}^{t_1} D^+_{(2)}V\left(s,x(s)\right)ds + \sum_{j=2}^{k+l}\int_{t_{j-1}}^{t_j} D^+_{(2)}V\left(s,x(s)\right)ds +$$

$$\int_{t_{k+l}}^{t} D^+_{(2)}V\left(s,x(s)\right)ds \le \int_{t_0}^{t_1}\left\{D^+_{(1)}V\left(s,x(s)\right) + P\left\|R(s,x)\right\|\right\}ds +$$

$$\sum_{j=2}^{k+l}\int_{t_{j-1}}^{t_j}\left\{D^+_{(1)}V\left(s,x(s)\right) + P\left\|R(s,x)\right\|\right\}ds + \int_{t_{k+l}}^{t}\left\{D^+_{(1)}V\left(s,x(s)\right) + P\left\|R(s,x)\right\|\right\}ds$$

$$= \left\{\int_{t_0}^{t_1} D^+_{(1)}V\left(s,x(s)\right)ds + \sum_{j=2}^{k+l}\int_{t_{j-1}}^{t_j} D^+_{(1)}V\left(s,x(s)\right)ds + \int_{t_{k+l}}^{t} D^+_{(1)}V\left(s,x(s)\right)ds\right\}$$

$$+\left\{\int_{t_0}^{t_1} P\left\|R(s,x)\right\|ds + \sum_{j=2}^{k+l}\int_{t_{j-1}}^{t_j} P\left\|R(s,x)\right\|ds + \int_{t_{k+l}}^{t} P\left\|R(s,x)\right\|ds\right\}$$

$$= \int_{t_0}^{t} D^+_{(1)}V\left(s,x(s)\right)ds + \int_{t_0}^{t} P\left\|R(s,x)\right\|ds$$

$$\le \int_{t_0}^{t} D^+_{(1)}V\left(s,x(s)\right)ds + P\int_{t_0}^{\infty}\left\|R(s,x)\right\|ds \le \int_{t_0}^{t} D^+_{(1)}V\left(s,x(s)\right)ds + P\int_{t_0}^{\infty}\frac{\Gamma}{P}\left|p(t)\right|ds$$

$$\le \int_{t_0}^{t} D^+_{(1)}V\left(s,x(s)\right)ds + \Gamma\frac{b(\delta)}{\Gamma} < b(\delta) + b(\delta) = 2b(\delta) < \frac{a(\varepsilon)}{3}$$

Now we demand that $\sum_{t_i \ge t_0}\left\|R_i(x)\right\| < \frac{a(\varepsilon)}{2P}$ be simultaneously satisfied for $\|x\| < \delta$ which is feasible by virtue of the uniform convergence of the series. Therefore, $\sum_{t_0<t_i<t}\Delta V_{(2)} \le \sum_{t_0<t_i<t} P\left\|R_i(x)\right\| < \frac{a(\varepsilon)}{2}$ and hence by condition (H2),

$$a(\|x(t)\|) \le V(t,x) = V(t_0+0,x) + \int_{t_0}^{t} D^+_{(2)}V\left(s,x(s)\right)ds + \sum_{t_0<t_i<t}\Delta V_{(2)}$$

$$< \frac{a(\varepsilon)}{6} + \frac{a(\varepsilon)}{3} + \frac{a(\varepsilon)}{2} = a(\varepsilon)$$

Thus for all $t \ge t_0 \ge \tau(\varepsilon)$, $\|x_0\| < \delta \Rightarrow \left\|x(t,t_0,x_0)\right\| < \varepsilon$.

Hence $x = 0$ is a uniformly eventually stable set of the perturbed Eq. (2).

4 Example

Consider the system

$$\left.\begin{aligned} x' &= p(t)x(t-r(t)) && t \neq t_i \\ \Delta x &= I_i(x) = -x + \frac{x}{2}\sqrt{1-x^2 Sin^2 t} && t = t_i \end{aligned}\right\} \tag{8}$$

and corresponding perturbed system

$$\left.\begin{aligned} x' &= p(t)x(t-r(t)) + p(t)e^{-t} && t \neq t_i \\ \Delta x &= I_i(x) + R_i(x) = -x + \frac{x}{2}\sqrt{1-x^2 Sin^2 t} + \frac{x^2}{1+t^2} && t = t_i \end{aligned}\right\} \tag{9}$$

where $\Omega = \{x : x \in R, |x| < 1\}$ $0 < r(t) < r', x \in R,$ $p(t) \in C(R^+, R)$ such that $|p(t)| < \alpha$.

Let us further assume that,

$x(s) \leq x(t)$ for $t - r' \leq s \leq t$ and $q(t,x) = 2 \ \forall (t,x) \in R^+ \times \Omega$.

Define the following functions $V(t,x) = |x|$.

Then for $t \geq 0,\ t \neq t_i$ we have,

$$D^+_{(8)}V(t,x(t)) = \operatorname{sgn} x(t)\left[p(t)x(t-r(t)) + p(t)e^{-t}\right] = \left|p(t)x(t-r(t)) + p(t)e^{-t}\right|$$
$$\leq |p(t)|\left|x(t-r(t)) + e^{-t}\right| \leq |p(t)|\left[|x(t-r(t))| + |e^{-t}|\right] \leq |p(t)|\left[|x(t)| + |e^{-t}|\right]$$

$$\leq |p(t)|\left[1 + |x(t)|\right] < |p(t)|[1+1] = 2p(t)$$

Let $a, b \in K$ such that $a(t) = \dfrac{t^2}{2}$, $b(t) = 2t$, so that

$$a(|x|) = \frac{|x|^2}{2} \leq |x| \leq 2|x| = b(|x|)$$

Now $V(t+0, x + I_i(x)) = |x + I_i(x)| = \left|\dfrac{x}{2}\sqrt{1-x^2 Sin^2 t}\right| \leq \left|\dfrac{x}{2}\right| \leq V(t,x)$ for $x \in \Omega, t = t_i,\ i = 1,2,3........$

Thus all the conditions of Theorem 1 are satisfied and hence $x=0$ is a uniformly eventually stable set of Eq. (8).

$V(t,x)=|x|$ is Lipschitzian w.r.t. x in $(t_{k-1},t_k)\times\Omega$ with Lipschitz constant 1.

Now if we consider Eq. (9), with the functions $R(t,x)=p(t)e^{-t}$ and $R_i(x)=\dfrac{x^2}{1+t_i^2}$, we see that

$|R_i(x)|=\left|\dfrac{x^2}{1+t_i^2}\right|\le\dfrac{1}{1+t_i^2}=a_i$ and $\sum_{i=1}^{\infty}a_i=\sum_{i=1}^{\infty}\dfrac{1}{1+t_i^2}\approx\sum_{i=1}^{\infty}\dfrac{1}{t_i^2}$ which is a convergent series. Also $\underset{t\to\infty}{Lim}\,R(t,x)=\underset{t\to\infty}{Lim}\,p(t)e^{-t}=0$

Thus all the conditions of Theorem 2 hold and hence the set $x=0$ is a uniformly eventually stable set of the perturbed IDE Eq. (9).

5 Conclusion

Sufficient conditions for retention of uniform eventual stability of impulsive differential systems in the presence of vanishing perturbations are given. Our results show that although the system may not be stable in the sense of Lyapunov, even then it can be eventually stable. In our example $x=0$ is not stable in the sense of Lyapunov because it is not equilibrium for Eq. (8) but we have proved its uniform eventual stability.

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