Optimization Model for an Airline Crew Rostering Problem: Case of Garuda Indonesia

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Abstract. This paper discusses the cockpit crew rostering problem at Garuda Indonesia, taking into account a number of internal cockpit crew labor regulations. These internal labor regulations are in general more restrictive at Garuda Indonesia than at other airlines, so that modeling the cockpit crew rostering problem for Garuda Indonesia is challenging. We have derived mathematical expressions for the cockpit crew labor regulations and some technical matters. We model a non-linear integer programming for the rostering problem, using the average relative deviation of total flight time to the ideal flight time as the objective function. The optimization model have been tested for all classes of cockpit crews of Garuda Indonesia, using a simulated annealing method for solving the problem. We obtained satisfactory rosters for all crew members in a short amount of computing time. This shows that the optimization problem is well-defined.

Keywords: *crew pairing; crew labor regulation; crew rostering; crew scheduling; optimization; simulated annealing.*

1 Introduction

We have considered a problem faced by Garuda Indonesia, the flag carrier of Indonesia, in expanding their business. Although global economic conditions are still unfavorable, the market for the airline industry in Indonesia is still expanding. Garuda Indonesia has responded to this conducive environment by expanding its fleet, increasing flight frequencies on specific routes and opening several new routes for both domestic and international flights.

A larger flight network leads to a more complex crew rostering problem. Rostering means assigning each cockpit crew pairing,[1] which is an element of

[1] A pairing is a sequence of flights starting and ending at the cockpit crew base.

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the minimum set of pairings,[2] to a group of cockpit crews in such a way that no labor regulations are violated when the cockpit crew runs their assignments. Over time, cockpit crew rostering in the airline industry has become a slightly cumbersome process, being carried out semi-manually by experienced staff members. This approach is surely inefficient and furthermore we may find unfair assignments to cockpit crew. A roster can give some cockpit crews a very tight schedule, while others get a much less tight schedule. The rostering needs to be carried out in such a way that it results in a relatively equal workload for each crew member in an efficient way.

The crew rostering problem at Garuda Indonesia is extra complex because of a number of internal crew labor regulations on the top of crew labor regulations issued by international, regional and national airline regulators. These internal regulations represent the wish of the cockpit crews to avoid exhausting assignments. It is clear that they are more restrictive than the other regulations, otherwise it would not have been necessary to issue them.

In our study we have developed an optimization model for the cockpit crew rostering problem at Garuda Indonesia. We have derived mathematical expressions for the cockpit crew labor regulations and a number of technical matters.[3] Some are corrected expressions for similar constraints proposed by Lucic and Teodorovic [1]. Using the expressions from [1] may lead to errors in counting the working time of the cockpit crews (see fourth paragraph of Section 3 for details) that result in incompact rosters. Moreover, in the case of a rostering problem where the number of cockpit crews is very restricted, the use of the expressions from [1] leads to an infeasible problem. The expressions become the constraints of the optimization model. We have used the average relative deviation of the monthly flight time from the average ideal monthly flight time as the objective function. We think that minimization of this objective function, while respecting all other constraints, will result in a roster in which the workloads of the cockpit crews are relatively equal.

An enormous amount of papers on airline crew rostering problems has been published. Studies by Chang [2], Gamache, *et al.* [3], Kovari [4], Lederer and Nambimadom [5], Medard and Sawhney [6], Vance, *et al.* [7], Yan and Tu [8] are concerned with the modeling of airline crew rostering problems. Many solution methods for crew rostering problems have been proposed, for example in AhmadBeygi, *et al.* [9], Barnhart, *et al.* [10], Deng and Lin [11], Goumopoulos and Housos [12], Gamache [13], Hoffman and Padberg, [14].

[2] The minimum set of pairings covers all flights in the schedule and is constructed by solving the crew-pairing problem.

[3] The presence of internal labor regulations creates more constraints in the present optimization model than there are in cockpit crew rostering models from other studies.

Like we have done in this paper, most of them construct the roster in two stages: first solve the crew-pairing problem and then solve the crew rostering problem. A new approach where the crew pairing problem and the crew rostering problem are solved simultaneously has been proposed in Souai and Teghem [15].

We still rely on the two-stage approach because we have developed an efficient algorithm for constructing the minimum set of crew pairings in a short amount of time. The second reason is, as stated before, that Garuda Indonesia has a number of internal crew labor regulations which are very restricting so that deriving mathematical expressions for these regulations is very challenging. The derivation of the proper mathematical expressions (and the optimization problem as a whole) is the subject of this paper. We need to ensure that the mathematical expressions result in a satisfactory roster before we proceed to the process of simultaneously constructing crew pairings and crew roster.

We can apply the simulated annealing method as proposed in [1] for solving the optimization. An initial feasible solution for the simulated annealing method can be obtained by applying a pairing-by-pairing algorithm that mimics the pilot-by-pilot method from [1], whereby we exchange the position of the pairings and the cockpit crews from the pilot-by-pilot method. We have applied the pairing-by-pairing method and the simulated annealing method to all classes of cockpit crews of Garuda Indonesia and obtained satisfactory rosters in a short amount of time. We checked that all cockpit crew labor regulations were satisfied, so that we can say that our optimization model is well-defined and ready for application in day-to-day operation by the airline.

This paper is organized as follows. After this introduction, in Section 2, we present the labor regulations whose mathematical expressions are part of the constraints of the optimization problem. The mathematical model will be presented in Section 3. In Section 4 we present the pairing-by-pairing method that was used for determining feasible pairings. In Section 5, we present the implementation of the optimization model to a class of cockpit crew, including the numerical results. This paper ends with the conclusion in Section 6. The nomenclature is given in Section 7.

2 Cockpit Crew Labor Regulations

Altogether, the cockpit crew regulations of Garuda Indonesia can be classified as the regulations for one day, two days, three days, seven days, one month, three months, and one year of working time; regulations for two consecutive pairings run by the same crew; and regulations for medical examinations, training, and annual leave. The regulations for one day and two days of working

time have been considered in the construction of crew pairings. In the following we only consider the remaining regulations, for which the mathematical expressions are presented in Section 3.

a. Flight time

If an airline has a number of aircrafts with different licenses for flying them, we have a number of classes of crews, consisting of the crews with the same license. The maximum cumulative flight time during a time period depends on the class of crew. For example, for one class of crew the maximum cumulative flight time can be 24 hours for every consecutive three calendar days, 30 hours for every seven consecutive days, 110 hours for every calendar month, 300 hours for every three calendar months, and 1050 hours for one calendar year.

b. Days off

(i) One day off will be given to a cockpit crew following six consecutive days of assignment. Every cockpit crew has a number of days off after assignment to a pairing which takes the cockpit crew 'away from base' (AFB). The amount of days off is described in Table 1.

Table 1 Number of days off according to length of AFB.

Length of AFB (days)	Minimum Days Off (days)	Length of AFB (days)	Minimum Days Off (days)
5	2	13 - 15	6
6 – 8	3	16 - 18	7
9 – 10	4	19 - 20	8
11 – 12	5		

(ii) One or more days off will be given to a cockpit crew after attending a training, course, seminar or activity other than flight duty. The amount of days off depends on the length of the activity, as described in Table 2.

Table 2 Number of days off according to length of activity.

Length of Activity (days)	Days Off (days)
3 - 6	1
7 - 14	2
15 - 22	3
23 - 30	5
> 30	7

(iii) Every cockpit crew has a minimum of (in total) eight days off in one calendar month.

c. Rest Period

A minimum rest period of nine hours will be given to a crew member who has finished performing an assignment prior to the next assignment. This rest period is calculated from 180 minutes after the end of the last assignment. In total, this type of rest period takes 15 hours and is called 'lay over time'. A minimum rest period of 18 hours will also be given if during a 72-hour period a cockpit crew has reached 24 flight hours or more.

These regulations are some of the internal regulations we have mentioned before.

d. Annual Leave

Every cockpit crew is entitled to annual leave, whereby its length depends on the aggregate working period at the airline.

e. Training

Every cockpit crew has to attend training twice in every calendar year. The length of the training is five days in one calendar year.

f. Medical Examination

Every cockpit crew has to attend a one-day medical examination once in every six calendar months.

3 Mathematical Model

In this section, we derive an optimization model for the cockpit crew rostering problem at Garuda Indonesia. The constraints of the optimization model are a representation of the labor regulations that have been described in the previous section and some technical constraints. We assume that the number of cockpit crews is sufficient for running the flight schedule. Later in this section we discuss how to choose the objective function that represents a measure of fairness for all cockpit crews.

We assume that each cockpit crew member always works with the same group of cockpit crew members. We also assume that each cockpit crew member in the same group has the same schedule for training, medical examination, and annual leave. Relaxation of these assumptions will not change the optimization model, but in the more general model we have to consider individual cockpit crew members instead of groups of cockpit crew members. For brevity, we call a group of cockpit crew members as a *cockpit crew*.

Given the minimum set of cockpit crew pairings and the set of cockpit crews, with cardinality n and m, respectively, we assume that there are DM days in the month to consider. We define the decision variable $x_{i,j}$ which has value 1 if

pairing i is served by cockpit crew j, and 0 otherwise. We define some parameters, listed in Section 7 (Nomenclature). For readability, we also define the following functions of some constraints.

For $i = 1, \dots, n$ and $j = 1, \dots, m$, define

$$nt_i\,(j) = t_i - \sum_{i_1=i+1}^{n} x_{i_1,j}\ p_{i,i_1}. \tag{1}$$

The value of $nt_i\,(j)$ counts the number of working days that cockpit crew j takes if it is assigned to pairing i, excluding the (possible) overlapping working day due to assignment to another pairing. As an illustration, suppose that cockpit crew j is assigned to pairing 1, which starts on day 1 and ends on day 2; and pairing 4, which starts on day 2 and ends on day 5. Both pairings overlap on day 2. We get $nt_i\,(j) = 2 - 1 = 1$. The number of working days on day 2 is calculated in $nt_4\,(j)$ where its value can be established once we know the other pairings assigned to cockpit crew j. With this function, we can avoid the double counting-error regarding working day 2 that we found in [1]. This parameter will be used later for calculating the (net) number of working days of the cockpit crews.

In order to obtain a feasible roster, the following constraints must be satisfied.

a. Each pairing must be served by exactly one cockpit crew

   For $i = 1, \dots, n$

$$\sum_{j=1}^{m} x_{i,j}\ = 1. \tag{2}$$

b. No-overlap constraint between two pairings assigned to one cockpit crew

   For $i = 1, \dots, (n-1)$ and $j = 1, \dots, m$,

$$x_{\mathrm{i,j}} \sum_{i_1=i+1}^{n} x_{i_1,j}\ s_{i,i_1} = 0. \tag{3}$$

c. For each cockpit crew, the accumulative flight time during a certain time period may not exceed the maximum accumulative flight time

   For $j = 1, \dots, m$ and $d = 1, \dots, DM$,

$$\sum_{i=1}^{n} x_{i,j}\ \sum_{l=d}^{d+6}\ ft_{i,l}\ \leq M_1. \tag{4}$$

$$\sum_{i=1}^{n} ft_i x_{i,j} \leq M_2. \quad (5)$$

$$\sum_{i=1}^{n} ft_i x_{i,j} \leq M_2. \quad (6)$$

$$\sum_{i=1}^{n} ft_i x_{i,j} \leq M_4 - aft_{j,y}. \quad (7)$$

Constraint (4) expresses the flight time limitation on seven consecutive days, constraint (5) expresses the monthly flight time limitation, while (6) and (7) ensure that the assignment in the month under consideration will not violate flight time limitations in three consecutive calendar months and in one calendar year, respectively.

d. One day off after an assignment in the last six consecutive working days

For $j = 1, \dots, m$, and $d = 1, \dots, DM - 6$,

$$\sum_{i=1}^{n} x_{i,j} \sum_{l=d}^{d+6} q_{i,l} \; min\{nt_i(j), p + 6 - (l-1)\} \leq 6. \quad (8)$$

This expression is a correction of a similar constraint in [1], which for the Garuda Indonesia case will be given by:

$$\sum_{i=1}^{n} x_{i,j} \sum_{l=d}^{d+6} q_{i,l} \; \mathrm{t_j} \leq 6. \quad (9)$$

We use $min\{nt_i(j), p + 6 - (l-1)\}$ instead of t_j, for the following two reasons. Firstly, the use of t_j will lead to a double-counting error in the calculating process of the cockpit crews' working days. This can occur on a certain day when a cockpit crew is assigned to two different pairings on that same day, as illustrated in the example right after the definition of $nt_i(j)$. Secondly, we need to use the minimum of $nt_i(j)$ and the number of days to go to the end of the time period under consideration as the net number of working days cockpit crew j takes if it carries out pairing i.

e. Minimum of eight days off in one month

For $j = 1, \dots, m$,

$$\sum_{i=1}^{n} x_{i,j} \, nt_i(j) \leq DM - 8. \quad (10)$$

Note that the function $nt_i(j)$ is used in the expression above for the reason explained in (d).

f. 18 hours of rest time after a minimum of 24 flying hours in three consecutive days

For $j = 1, \dots, m$ and $d = 1, \dots, DM - 2$,

$$\left(\sum_{i=1}^{n} x_{i,j} \sum_{l=d}^{d+2} ft_{i,l} - 24\right)\left(\sum_{i=1}^{n} \sum_{i_1 \in J_d}^{n} x_{i_1,j}(q_{i,(d+3)} - 1)(w_{i,i_1} - 1)\right) \geq 0, \tag{11}$$

where J_d is the set of pairings that starts on day d + 3 or later. For $j = 1, \dots, m$, the first term of the left-hand side of (11) has a non-negative value if during three consecutive days starting on day *d*, the cockpit crew *j* each have an accumulative flight time of more than 24 hours. If this is the case, we have to make sure that they take a minimum of 18 hours rest time. Since the maximal flight time in one day is nine hours, they had a maximum of 18 hours flight time on days 1 and 2, therefore on the third day the cockpit crews have flown at least six hours. Taking the minimum 18 hours of rest time means that the next pairing assigned to them will start on the fourth day. This is why in the second term of the left-hand side of (11) we only consider the pairings that start on the fourth day, which is denoted by J_d.

g. Each cockpit crew gets l_{d_1} days off after assignment to a long AFB pairing

For $\boldsymbol{i = 1, \dots, n}$, and $\boldsymbol{j = 1, \dots, m}$

$$x_{i,j} to_{i,d_1} \sum_{l=1}^{DM} q_{i,l} \sum_{i_1=i+1}^{n} \sum_{l_1=d_1}^{d_1+l_{d_1}-1} q_{i,l+l_1} x_{i_1,j} = 0, \tag{12}$$

where d_1 is the length of the AFB pairing. Its value and the length of the days off following the pairing are described in Table 1.

$$x_{i,j} to_{i,d_1} \sum_{l=1}^{DM} q_{i,l} \sum_{l_1=1}^{d_1+l_{d_1}-1} me_{j,l+l_1} = 0, \tag{13}$$

$$x_{i,j} to_{i,d_1} \sum_{l=1}^{DM} q_{i,l} \sum_{l_1=1}^{d_1+l_{d_1}-1} tr_{j,l+l_1} = 0. \tag{14}$$

h. Each cockpit crew gets L_{d_2} day-off after attending training, courses, seminars or activities other than flight duty

For $\boldsymbol{j = 1, \dots, m, l = 1, \dots, DM}$,

$$\left(\sum_{l_{1=l}}^{l+d_2-1}\left(me_{j,l} + tr_{j,l}\right) - d_2 + 1\right)\left(\sum_{i=1}^{n}\sum_{l_1=1}^{L_{d_2}} x_{i,j}\, q_{i,l+d_2+l_1}\right) \leq 0, \quad (15)$$

where d_2 is the length of the activity.

i. No assignment on days of training

For $i = 1, \dots, n$ and $j = 1, \dots, m$,

$$\sum_{l=1}^{DM} x_{i,j}\, tr_{j,l} c_{i,l} = 0. \quad (16)$$

j. No assignment on days of medical examination

For $i = 1, \dots, n$ and $j = 1, \dots, m$,

$$\sum_{l=1}^{DM} x_{i,j}\, me_{j,l} c_{i,l} = 0. \quad (17)$$

k. No assignment on days of annual leave

For $i = 1, \dots, n$ and $j = 1, \dots, m$,

$$\sum_{l=1}^{DM} x_{i,j}\, al_{j,l} c_{i,l} = 0. \quad (18)$$

The input parameters $tr_{j,l}, me_{j,l}$ and $al_{j,l}$, for $j = 1, \dots, m$, must satisfy the following conditions:

$$\sum_{l=1}^{DM} me_{j,l} \leq me_j - ame_j, \quad (19)$$

$$\sum_{l=1}^{DM} tr_{j,l} \leq tr_j - atr_j, \quad (20)$$

$$\sum_{l=1}^{DM} al_{j,l} \leq al_j - aal_j, \quad (21)$$

$$\sum_{l=1}^{DM} me_{j,l} + tr_{j,l} + al_{j,l} = 1. \quad (22)$$

Satisfaction of constraints (19), (20), and (21) means that a cockpit crew's planned medical examination, training and annual leave do not violate regulations. Constraint (22) ensures that in one day a cockpit crew can only be at a training or a medical examination or on annual leave. We can check the satisfaction of these constraints before we proceed to solve the optimization model that will be presented in the following, to that ensure we have proper input for the optimization model.

The ideal average monthly flight time of each cockpit crew is

$$W = \frac{\sum_{i=1}^{n} ft_i}{m}. \tag{23}$$

The average relative deviation of the monthly flight time from the ideal average flight time is then given by

$$D(\mathbf{x}) = \frac{\sum_{j=1}^{m} \left| \left(\sum_{i=1}^{n} ft_i x_{ij} - W \right) \right|}{m}, \tag{24}$$

where $\mathbf{x}$ is a vector with elements $x_{i,j},\ i = 1, \ldots, n, j = 1, \ldots, m$

The crew rostering problem can then be written as

minimize $D(\mathbf{x})$

subject to (2) - (22),

$$x_{i,j} = 0 \text{ or } 1,\ i = 1, \ldots, n,\ j = 1, \ldots, m \tag{25}$$

where $D(\mathbf{x})$ is given in (24). Note that the deviation $D(\mathbf{x})$ is only one of many measures of fairness among cockpit crews. We can choose other measures, such as the deviations proposed in [1], as the objective function without loss of generality. We can also choose the deviation and the crews' cost as the objective functions, so that the optimization model becomes a multi-objective optimization model.

4 Solution Method

The optimization problem (25) can be solved with several solution methods. We have used simulated annealing, because we think that heuristic methods are more efficient compared to exact methods and because simulated annealing is very easy to apply to our optimization problem.

Each iteration of the simulated annealing algorithm has to produce a feasible roster. For constructing a feasible roster, we use the pairing-by-pairing method as described in the following. This algorithm mimics the pilot-by-pilot method in [1], whereby we exchange the position of the pairings and the cockpit crews from the pilot-by-pilot method. We think the pairing-by-pairing method can be used to determine a feasible roster more efficiently compared to the pilot-by-pilot method if the set of cockpit crew pairings is sorted with respect to the departure times from the earliest to the latest. We try to assign the first pairing to a certain cockpit crew that is available, then we try to assign the second pairing to another cockpit crew, and so on. With this approach, the sequence of pairings that is assigned to a certain cockpit crew will be a sequence of pairings sorted with respect to departure time. Therefore, when attempting to assign another pairing to the candidate cockpit crew, checking satisfaction of a number of constraints related to labor regulations on a certain number of consecutive days does not have to happen from the first day of the month, but only from a specific day prior to the last pairing that has been assigned to the candidate cockpit crew. The efficiency of our method comes from this process.

The pairing-by-pairing method comprises of the following steps:

Algorithm Pairing-by-Pairing

Step 1. $k = 1$.
Step 2. Assign pairing k to a specific cockpit crew randomly chosen from the list of cockpit crew except the cockpit crew assigned to pairing $k - 1$, such that all constraints are satisfied.
Step 3. $k = k + 1$.
Step 4. If $k > n$, stop. Otherwise go to Step 2.

We can refer to [1] for a brief explanation of the application of simulated annealing for the optimization of rostering problems.

5 Numerical Results

We have tested the optimization problem for all classes of cockpit crews of Garuda Indonesia and obtained satisfactory rosters in a short amount of time. Here, we present the results of the application of our model to an Airbus 330 cockpit crew for the winter 2010 schedule. The rostering of this class of cockpit crew is challenging since most of the pairings are multi-day pairings. In the instant month, there are 287 cockpit crew pairings, one of them is a six-day pairing in which the cockpit crews assigned to such a pairing must have three days off after the assignment, and 21 of them are five-day pairings in which the

cockpit crews assigned to such a pairing must have two days off after the assignment.

The total flight time of all pairings is 3012.5 hours. There are 43 cockpit crews to carry out all pairings. The ideal average monthly flight time is 70.05 hours, which is less than the maximum accumulative flight time for one month.

We are given the cockpit crew's plans for medical examination, training, annual leave and (mandatory) days off as described in Table 3, 4, and 5.

Table 3 Cockpit crew's plans for medical examination.

Crew Number	Start Day of Medical Examination	Length of Medical Examination
8	11	1
12	16	1
16	23	1
20	31	1
26	9	1
33	17	1

Table 4 Cockpit crew's plan for training.

Crew Number	Start Day of Training	Length of Training
3	23	2
16	5	3
29	23	2
39	5	3

Table 5 Cockpit crew's plans for annual leave.

Crew Number	Start Day of Annual Leave	Length of Anual Leave
1	8	1
2	26	1
3	29	1
4	19	1
5	12	1

The total flight time and days off in the instant month are shown in Table 6 and day-to-day assignments are shown in Figure 1.

Crew Number	Day																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1				39	39	39	39	AL	AL	AL	117	117	117		164	164	164	164			213		239	239	239	239	239				
2				40	40		70	70	70			127	127	151	151	151		183				235	235	235		AL	AL				
3				43	43	43	43		96	96	121	121	121		156	156			199	199	199		T	T	269	269	269		AL	AL	
4	1				53	53	53	89	89	89		132	132	132	165	165	165		AL	AL	AL	AL	245	245	265	265					
5		23	23	23				84	84			AL	AL	AL		171	171	171	171	171			237	237	267	267	267				
6		26	26	26	26	26			100	100	100	100	100					190	190	190		233	233	250	250	250					
7			34	34	34	34				105	105	105		149	149	149	149	149				234		258	258	258					
8		17			54	54	54	54	95	95	ME	129	129	129	129		179	179	179	212	212	212		253	253						
9		15	15		57	57	57		93	93	93		136	136	136		180	180	180	180		227	227	259	259	259	259				
10			29	29	29	66	66	66		109	109	109	138	138			172	172	172	172		224	224	224	224	224					
11	8	8	8		50	50	74	74	74		115	115	143	143	159	159		188	188	207	207	207		251	251		284	284			
12	5	5	5			59	59		91		122	122	122			ME			194	194	194		247	247	247		286	286	286	286	286
13		13	13	13	13			78	97	97	97		140	140	140	140	140				216	216	216		264	264	287	287	287	287	
14	3	3	3	3	3				102	102	102	102	102				175	175	175		222	222	222	222		280	280	280	280	280	280
15	2	2			49	49		82	82	82	123	123				169	169	169		204	204	230	230	230		275	275				
16		25	25	25	T	T	T		98	98	98			147	147		174	174			214	214	ME	254	254	254					
17			27			61	61	61	101	101		134	134			167	167	189	189	209	209		240	240	240		285	285	285	285	
18		18	18		48	48		83	99	99	99		142	142			177	177	201	201	201		242	242	242	273	273	273			
19			28	28		62	62	62	103	108	108			146	146	146		184	184		219	219	219	219	219						
20			35	35	35			80	80	111	111	111		144	157	157	157			211	211	211	211	211			281	281	281		ME
21	7	7	7		56	56	56		92	92	118	118	118	118				186	186	186		226	226	226	226		282	282	282	282	
22	9	9				60	60	79	79	79		133	133	133	133			187	187	187	187		243	243	243	243					
23				42	42	42	72	72	72		124	124	124	124		170	170	170				232	232	232	271	271	271				
24			36	36	36	36		87	87	110	110	110	110		158	158	158	158	158			229	229	252	252	252	252				
25		22	22			63	75	75	75		116	116	116		154	154	154	154		210	210	210		257	257	278	278	278			
26	6	6		44	44				555		114	125	125		161	161	161		197	197		231	231	231	263	263				
27				37	55	55	55	55				126	126	126	160	160	160				220	220	220	220	220					
28	12	12	12	12		65	65	65	65	65			137	137	137		178	178	178	206			249	249	249	249	249				
29	10	10		46	46	46	69	69	69			131	131	131	162	162			200	200	200	200	T	T	266	266	266				
30			30	30	30		71	71		107	107	130	130	130	130		182	182	182	182		228	228	261	261	261	261				
31		24	24	24			67	81	81	81	119	119		150	150	150				205	205		246	246	246						
32			31	31	58	58	58	58		106	106	106	139	139	139				203	203	203	203	241	241		277	277	277			
33			32	32	52	52	68	68		112	112	112	112				ME		195	195	195		244	244	262	262	262				
34		20	20	20	20			77	77	77	77	77				168	168		192	208	208	208		256	256	279	279	279			
35	4	4	4	45	45		76	76	76	76		128	128	128			173	173			218	218	218	218		272	272				
36				41	41	41		86	86	86	86	135	135		163	163	163	185	185	185		225	225		270	270	270	270				
37					38			85	85	85				145	145	145			198	198	221	221	221		268	268	283	283	283			
38		21	21			64	64	64		113	113		141	141	141				196	196	196	196		255	255	255					
39		19	19	19	T	T	T		90	90	90	90		148	148	148		191	191	191		223	223	223	223	223					
40	11	11	11	47	47	47		88	88	88	88	88			166	166	166	166	193	193		236	236	236	236	276	276				
41		14			51	51	51		94	94	94	94		152	152	152	152		202	202	202			238		274	274	274	274	274	
42		16	16	16	16	16				104	120	120			155		176	176	176		217	217	248	248	248						
43			33	33	33		73	73	73	73	73			153	153		181	181	181		215	215	215	260	260	260					

ME : Medical examination T: Training AL: Annual leave : Five -days pairing : Six -days pairing

Figure 1 Day-to-day assignment.

We can see in Figure 1 that every cockpit crew that is assigned to a five-day pairing (pairings no. 3, 16, 23, 65, 73, 77, 88, 100,102, 140, 149, 158, 171, 211, 219, 220, 223, 224, 239, 249, 274, 288) has two days off after the assignment. In the next month, two days off must be given to cockpit crew 11 and three days off must be given to cockpit crew 14 after assignment to pairings 286 and 280, respectively. These mandatory days off must be considered as input for the next month's rostering.

Table 6 The total flight time and days off.

Crew Number	Total Flying Time	Total Days Off	Crew Number	Total Flying Time	Total Days Off
1	45,3	11	23	80,3	12
2	59,9	15	24	54,5	10
3	73,3	10	25	77,6	10
4	52,3	10	26	49,7	13
5	53,2	13	27	66,4	15
6	54,1	13	28	64,6	10
7	67,4	14	29	79,5	9
8	66,1	11	30	75,0	10
9	91,9	10	31	77,0	14
10	60,0	14	32	80,6	10
11	78,4	8	33	58,4	12
12	84,1	10	34	71,6	11
13	80,0	9	35	68,5	11
14	48,2	8	36	77,4	10
15	78,5	12	37	78,9	13
16	61,0	12	38	76,7	14
17	78,3	10	39	52,0	10
18	88,5	10	40	70,7	8
19	72,9	13	41	92,9	9
20	68,4	10	42	58,7	14
21	80,5	8	43	83,3	12
22	74,4	13			

If we use the expressions in [1] for constraints (8) and (9), instant testing becomes infeasible. This means that the schedule cannot be carried out by 43 cockpit crews. This proves that the mathematical expressions (8) and (9) are correct and with these expressions we can obtain compact rosters.

6 Conclusion

In this paper, an optimization model for the cockpit crew rostering problem at Garuda Indonesia was presented. The optimization problem has a large number

of constraints that represent international, regional, national and internal cockpit crew labor regulations. Some of the constraints have a non-linear form, so that the optimization problem is a large-size non-linear binary problem that needs an appropriate solution method. We have applied a simulated annealing method for solving the problem, where feasible rosters are constructed by applying a random method called pairing-by-pairing. The satisfactory roster presented in Figure 1 was obtained in a short amount of computing time, although we realize that some techniques can be added in order to make the pairing-by-pairing method run more efficiently. For example, making a list of constraints from the hardest to the easiest (in satisfying), and performing the pairing-by-pairing method in such a way that in assigning a cockpit crew pairing to a candidate cockpit crew we check satisfaction of the hardest constraint first, then if it is satisfied we proceed to checking satisfaction of the next constraint in the list, and so on.

Nomenclature

For $j = 1, \dots, m$,

$al_{j,l}$ = $\begin{cases} 1, \text{if cockpit crew } j \text{ starts annual leave on day } l, \\ 0, \text{otherwise} \end{cases}$

aal_j = The accumulative number of days of annual leave that have been taken by cockpit crew j

$aft_{j,2}$ = The accumulative flight time of cockpit crew j during the two previous months

$aft_{j,y}$ = The accumulative flight time of cockpit crew j during the current year

ame_j = The accumulative days of medical examination that have been taken by cockpit crew j

atr_j = The accumulative days of training that have been taken by cockpit crew j

$c_{i,l}$ = $\begin{cases} 1, \text{if pairing } i \text{ covers on day } l, \\ 0, \text{otherwise} \end{cases}$

ft_i = The total flight time of pairing i

$ft_{i,l}$ = The net flight time of pairing i on its lth day

$me_{j,l}$ = $\begin{cases} 1, \text{if cockpit crew } j \text{ starts medical examination on day } l, \\ 0, \text{otherwise} \end{cases}$

M_1 = The maximum accumulative flight time in seven consecutive days

M_2 = The maximum accumulative flight time in one calendar month

M_3 = The maximum accumulative flight time in three consecutive calendar months

M_4 = The maximum accumulative flight time in one calendar year

p_{i_1,i_2} = $\begin{cases}1, \text{if pairing } i_1 \text{ and } i_2 \text{ have a common working day,}\\0, \text{otherwise}\end{cases}$

$q_{i,l}$ = $\begin{cases}1, \text{if pairing } i \text{ is started on day } l,\\0, \text{otherwise}\end{cases}$

s_{i_1,i_2} = $\begin{cases}1, \text{if pairing } i_2 \text{ starts earlier than the end of pairing } i_1 \text{ plus}\\ \quad \text{the lay over time,}\\0, \text{otherwise}\end{cases}$

t_i = The number of days covered by pairing i

$tr_{j,l}$ = $\begin{cases}1, \text{if cockpit crew } j \text{ starts training on day } l,\\0, \text{otherwise}\end{cases}$

$to_{i,d}$ = $\begin{cases}1, \text{if pairing } i \text{ covers } d \text{ working days}\\0, \text{otherwise}\end{cases}$

s_{i_1,i_2} = $\begin{cases}1, \text{if pairing } i_2 \text{ starts earlier than the end of pairing } i_1 \text{ plus}\\ \quad \text{the lay over time,}\\0, \text{otherwise}\end{cases}$

w_{i_1,i_2} = $\begin{cases}1, \text{if the time difference between the end of pairing } i_1 \text{ and}\\ \quad \text{the beginning of pairing } i_2 \text{ at least 18 hours,}\\0, \text{otherwise}\end{cases}$

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