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Portfolio Insurance Strategies under Volatile Market

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***Abstract*— Portfolio insurance strategies are designed to enable investors to limit downside risk while at the same time to gain profits from rising market. Among that, constant proportion portfolio insurance strategy (CPPI) and option-based portfolio insurance strategy (OBPI) are two typical strategies in portfolio insurance strategies. With the popularity of the portfolio insurance strategies, portfolio optimization problem receives plenty of publicity. Each investor has their own preference for return and risk, investment activities should follow a utility function of return and risk. Therefore, portfolio optimization problem can be modeled as expected utility maximization problems. It is well-known that in the Black-Scholes model, these strategies can be implemented as the optimal solution by forcing an exogenously given guarantee to maximize the expected utility of investors with constant relative risk aversion (CRRA) function. In this research, we combine CRRA utility maximization with the stylized strategies and bring these results together. In particular, we focus on the volatile market and consider the market is under the Constant Elasticity of Variance (CEV) model. In addition, we discuss the advantages and disadvantages of CPPI and OBPI strategies under the distribution of terminal wealth process and utility value in CEV model.**

Keywords: Portfolio insurance strategy, portfolio optimization, CPPI, OBPI, CRRA utility function, volatile market, CEV model.

I. INTRODUCTION

Portfolio insurance strategies are designed to enable investors to limit downside risk while at the same time to gain profits from rising market. Among that, constant proportion portfolio insurance strategy (CPPI) and option-based portfolio insurance strategy (OBPI) are two typical strategies in portfolio insurance strategies. With the popularity of portfolio insurance, more and more people begin to consider the portfolio optimization problem. The problem of investment is that investors allocate their money between investment and consumption reasonably, i.e. to choose the optimal investment strategy to maximize the expected utility of the terminal wealth process. In general, we regard the optimization problem as Merton (1971) problem. It is widely believed that in the Black-Scholes model and a constant relative risk aversion (CRRA) utility function, the trading rule of utility maximization problem is constant mix strategy (CM), i.e. the changes of stock price are always constant and there will be no jumps or discontinuities. The trading rule is very different from the portfolio insurance. If the price of the risky asset falls, the asset exposure will be decreased. Technically speaking, we could achieve CPPI and OBPI strategies as the optimal solution of a modified utility maximization problem based on an exogenously given guarantee. We refer to the literature of Merton (1969、1971、1992), Black and Scholes (1973), Browne (1999), Longin (2001), El Karoui et al. (2005). In particular, Balder and Mahayni (2009) consider a modified optimization problem. The modifications which are imposed on the unconstraint optimization problem give interesting modifications for the payoffs.

In the study of portfolio problem, many assume that the stock price follows a Geometric Brownian motion. For example, Browne (1997) focus on the financial market which only has one kind of risk asset and the price of risk asset is geometric Brownian motion. Jones (1984), Bardhan and Chao (1995) and Guo and Xu (2005) add the discrete jump process to the geometric Brownian motion model. However, Hobson and Rogers (1998) point out that in the real financial market, the volatility of risk assets is randomness. After that, some scholars begin to consider the risk asset model under different stochastic volatility. Hull and While (1987), Heston (1993) and Jonsson and Sircar (2002) use the mean regression model and adding the stochastic jump process. In particular, Cox and Ross (1976) propose a model called Constant Elasticity of Variance (CEV) model. CEV model is an extension of geometric Brownian motion (GBM) model. This model can explain the inclination of stochastic volatility in the real market, and it is convenient to analyze the effect of volatility skew on investors' decision-making.

The following paper is based on study of the paper of Balder and Mahayni (2009) ‘How good are portfolio insurance strategies?’ We apply the optimal strategies of each expected utility maximization given in Balder and Mahayni (2009) for the Black-Scholes model to the CEV model and check their applicability under the model risk. To start with, we elaborated these three optimization problems which imply constant mix, CPPI and OBPI strategies as optimal. We use the well-known results of the optimization problems to explain the main differences of the mechanism of these three strategies. Then, we compare the terminal payoffs of these three strategies. Especially, we focus the volatile market and consider the Constant Elasticity of Variance (CEV) model and apply Euler-Maruyama method to CEV model in order to simulate the sample path of the stock price process. In the case of the CEV model, the optimal strategies of portfolio insurance are not obtained mathematically. As a result, we use the optimal strategies for the Black-Scholes model and check their applicability in the CEV model. In addition, we consider the distribution of terminal wealth process and utility value under the volatile market.

II. METHODS

A. Theoretical method

We will throughout the thesis assume that there exists a probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T^*],} P)$ where Ω is the sample space. Now, we consider two assets. The risk-free bond B evolves according to

$$dB_t = B_t r dt \quad \text{where } B_0 = b. \tag{1}$$

The dynamics of the market value of the risky asset S, a stock or benchmark index, is given by a geometric Brownian motion

$$dS_t = S_t(\mu dt + \sigma dW_t) \ , \ S_0 = s. \tag{2}$$

Let π_t denotes the proportion of the portfolio value at time t which is invested in the risky asset S. Let $V = (V_t)_{o\le t\le T}$ denote the portfolio value process related to the strategy π, the dynamics of V are given by

$$dV_t(\pi) = V_t(\pi_t \frac{dS_t}{S_t} + (1-\pi_t)\frac{dB_t}{B_t}), \tag{3}$$

The relevant optimization problem is given by

$$\sup_{\pi\in\Pi} \mathbb{E}_p = [u(V_T(\pi))] \tag{4}$$

From Balder and Mahayni (2009), we recall the well-known optimization problems which justify three basics strategies

$$\Pi^{\text{CM}} = \{\pi \in \Pi \mid \pi_t = m,\ m \ge 0\}. \tag{5}$$

$$\Pi^{\text{CPPI}} = \left\{\pi \in \Pi \mid \pi_t = m\frac{V_t - e^{-r(T-t)}G_T}{V_t},\ m \ge 0\right\}. \tag{6}$$

$$\Pi^{\text{OBPI}} = \left\{ \begin{array}{c} \pi \in \Pi \mid \pi_t = \frac{\Delta_t S_t}{V_t}, \Delta_t = \frac{\partial}{\partial S_t}\mathbb{E}_{p*}\left[e^{-r(T-t)}(h(S_T) - G_T)^+ | \mathcal{F}_t]\right], \\ h \in C^2,\ \left(\frac{\partial P^*}{\partial P}\right)_T = e^{-\frac{1}{2}(\frac{\mu-r}{\sigma})^2 T - \frac{\mu-r}{\sigma}W_T} \\ \end{array} \right\}. \tag{7}$$

where G_T is the present value of the guarantee.

In the set-up of the Black-Scholes model, it is possible to impose an exogenously given guarantee on the problem of maximizing the expected utility of an investor with a CRRA utility function. From Balder and Mahayni (2009), the optimal payoffs which are suited to the three strategy classes are summarized as follows

$$V_{T,CM}{}^* = \phi(V_0{}^{CM}, m^*) S_T^{m*} \tag{8}$$

$$V_{T,CPPI}{}^* = G_T + \frac{V_0 - e^{-rT}G_T}{V_0} V_{T,CM}{}^* \tag{9}$$

$$V_{T,OBPI}{}^* = \frac{\tilde{V}_0}{V_0}\ V_{T,CM}{}^* + \left[G_T - \frac{\tilde{V}_0}{V_0}\ V_{T,CM}{}^*\right]^+ \tag{10}$$

where set $\phi(x, y\,) := x\left(\frac{1}{S_0}\right)^y e^{(1-y)\left(r+\frac{1}{2}y\sigma^2\right)T}$

and $V_T^{CM} = V_0^{CM} e^{\left(m(\mu-r)+r-\frac{1}{2}m^2\sigma^2\right)T - m\left(\mu-\frac{1}{2}\sigma^2\right)T} \left(\frac{S_T}{S_0}\right)^m$.

$m^* = \frac{\mu-r}{\gamma\sigma^2}$ represents the Merton investment quote and V_0 is the initial investment. For more details, see Balder an Mahayni (2009).

Volatility is a very significant factor that influences the selection of investors in portfolio insurance strategies. As a consequence, we introduce a model called Constant Elasticity of Variance (CEV) model. The standard CEV model assumes that share price S_t of risky asset evolves according to the stochastic differential equation

$$dS_t = \mu S_t dt + \delta S_t{}^{\frac{\beta}{2}} dW_t,\ (t > 0\,) \tag{11}$$

where μ is an expected instantaneous rate of return. μ and δ are constants with initial condition W_0 =0. β is a positive constant and W_t is a Brownian motion. When β =2, the CEV model is the same as the Black-Scholes model with $\sigma = \delta$.

B. Numerical method

In order to get the approximate numerical solution of a stochastic differential equation (SDE), the solution can be approximated by using the Euler-Maruyama Method. The Euler-Maruyama method is demonstrated on the following stochastic differential equation:

$$dS_t = \mu S_t dt + \delta S_t{}^{\frac{\beta}{2}} dW_t,\ (t > 0\,) \tag{12}$$

Applying the Euler-Maruyama Method to simulate the CEV Model gives the following discrete stock price relationship

$$S_t = S_{t-1} + \mu S_{t-1}\Delta t + \delta (S_{t-1})^{\frac{\beta}{2}} \Delta W_t \qquad (13)$$

When β>2 in the CEV model, the solution of the SDE has the possibility to explode to infinity before maturity. In our simulations, we avoid the scenario that numerical solutions explode for β>2.

III. RESULTS

We provide some numerical examples to illustrate the dynamic behavior of the optimal investment strategy with CRRA utility function. We assume the volatile market is under the CEV model and the optimization problems is still obtained by Black-Sholes model. In these examples, we consider three investment horizons: Short-term investment ($T = 1$ year), Mid-term investment ($T = 5$ years) and Long-term investment ($T = 10$ years). Here, we take the distribution of terminal wealth process and utility value in $T = 5$ years as examples. For more details and examples, please refer to the thesis.

The basic parameters: $S_0 = 1$, $\sigma = 0.15$, $r = 0.03$, $\mu = 0.085$, $V_0 = 1$, $\tilde{V}_0 = 0.56$, $\gamma = 0.8$, $m = m^* = 2.037$ and $G_T = 1$. The results of the comparison of the terminal wealth are given by

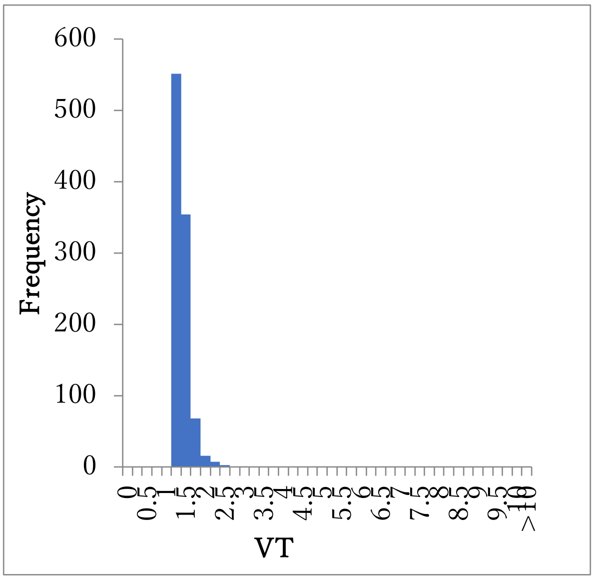

Fig 1. The distribution of terminal wealth V_T for CPPI with $\beta = 1.5$ and investment horizon $T = 5$ years.

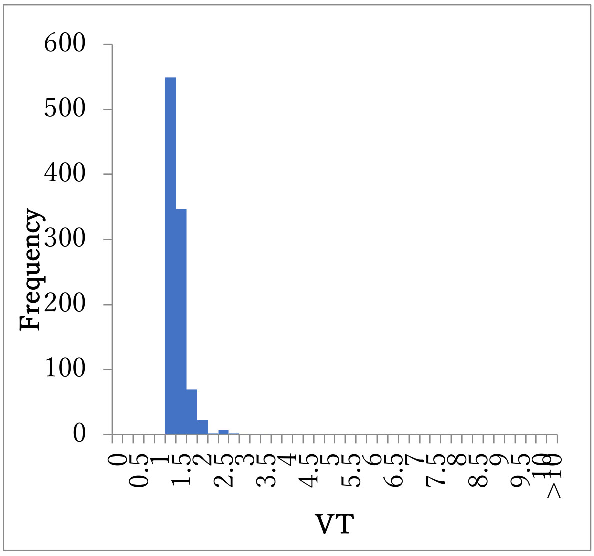

Fig 2. The distribution of terminal wealth V_T for CPPI with $\beta = 2$ and investment horizon $T = 5$ years.

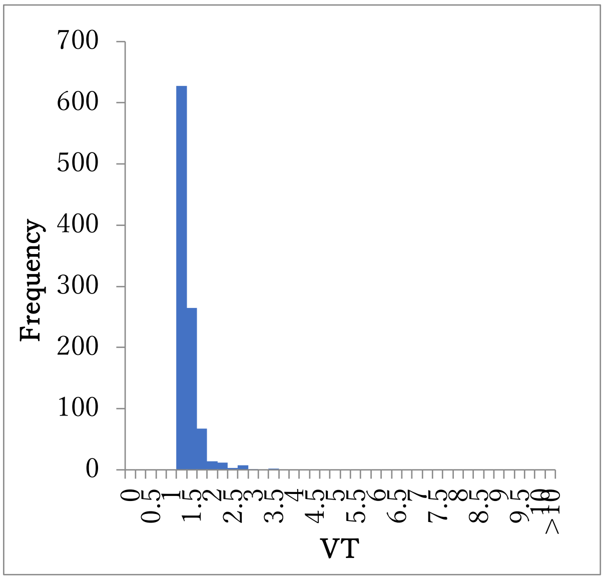

Fig 3. The distribution of terminal wealth V_T for CPPI with $\beta = 2.5$ and investment horizon $T = 5$ years.

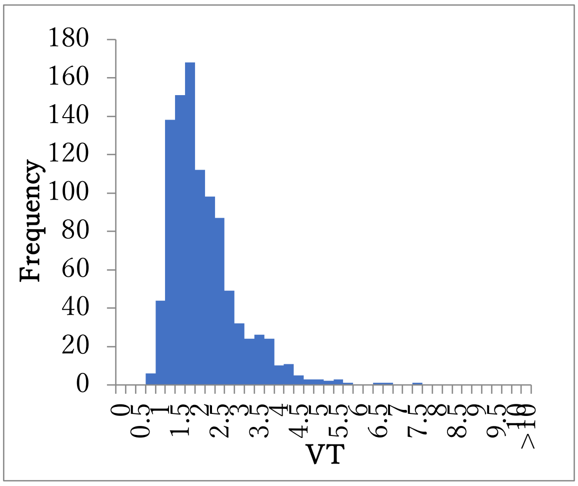

Fig 4. The distribution of terminal wealth V_T *for OBPI with* $\beta = 1.5$ *and investment horizon* $T = 5$ *years*

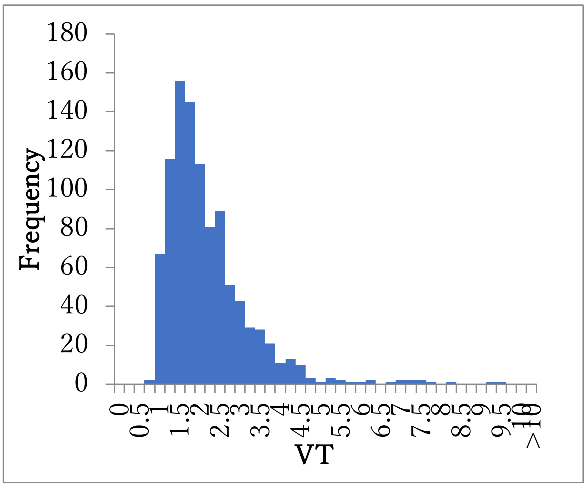

Fig 5. The distribution of terminal wealth V_T *for OBPI with* $\beta = 2$ *and investment horizon* $T = 5$ *years.*

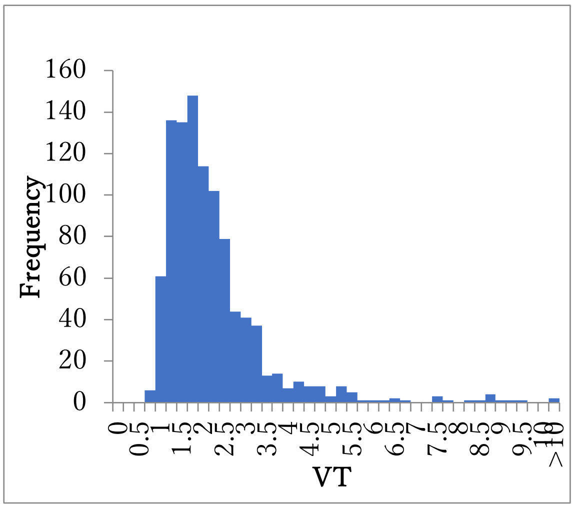

Fig 6. The distribution of terminal wealth V_T *for CPPI with* $\beta = 2.5$ *and investment horizon* $T = 5$ *years.*

When the market fluctuates, the terminal wealth for CPPI strategy is not changed so much. It reflects that CPPI strategy is relatively stable and the accompanying risk is relatively small. OBPI strategy is different from CPPI strategy. On the one hand, the impact of terminal wealth for OBPI strategy is relatively large under the volatile market. On the other hand, in the turbulent market, OBPI is more likely to obtain high returns but also take risks.

Then, we calculate the rankings frequency of terminal wealth w.r.t CM, CPPI and OBPI strategy for each scenario. "1" represents the best, "3" is the worst. The results are given in Table 1.

Table 1. The rankings frequency of terminal wealth w.r.t CM, CPPI and OBPI strategy for each scenario when $\beta = 2.5$*. The investment horizon is T=5 years.*

Rank	CM	CPPI	OBPI
1	362	108	530
2	284	259	457
3	354	633	13

In terms of the frequency with rankings of terminal wealth, OBPI gives a better result than CPPI. Even under a high volatility, OBPI approach always gives a higher return than CPPI.

Besides, we consider the frequency of terminal wealth for three strategies in different range. The results are given as follows

Table 2. The frequency of terminal wealth w.r.t CM, CPPI and OBPI strategy for each range when $\beta = 2.5$*. The investment horizon is T=5 years.*

V_T	CM	CPPI	OBPI
<1	285	0	67
<0.75	158	0	6
<0.5	52	0	0
<0.25	6	0	0

It reflects that OBPI approach is risky and more likely to lose money. In other words, CPPI approach is more stable than OBPI.

In the meanwhile, the comparison of the utility value in shown in Table 3.

Table 3. Utility value of terminal wealth w.r.t CM, CPPI and OBPI strategy with varying parameter β. The investment horizon is T=5 years.

β	CM	CPPI	OBPI
1.5	5.61277042	3.7286536	5.64541546
2	5.61461423	3.7281509	5.67788783
2.5	5.53486282	3.67579598	5.68242807

The utility value of CPPI is decreased when market is volatiles. To compare with CPPI strategy, OBPI gives a better result in utility value under the volatile market. When market is volatiles, the utility value of OBPI strategy is increased. CPPI strategy is more vulnerable to market volatility. The characteristics are more obvious in long-term investment.

It is worth noticing that the utility value of CPPI strategy is always smaller that the OBPI strategy. In particular, the reduction of CPPI strategy in the expected utility can be interpreted as the utility loss arising from the guarantee component $G_T > 0$.

IV. DISCUSSION

The following paper is based on study of the paper of Balder and Mahayni (2009) 'How good are portfolio insurance strategies?' We recall the well-known optimization problems which imply constant mix, CPPI and OBPI strategies as optimal solution. On this basis, we consider the volatile market and introduce another model called the Constant Elasticity of Variance (CEV) model. Then we use Euler-Maruyama method to approximate numerical solution of a stochastic differential equation (SDE). In the case of the CEV model, the optimal strategies of portfolio insurance are not obtained mathematically. As a result, we use the optimal strategies for the Black-Scholes model and check their applicability in the CEV model. Through numerical experiments, those strategies are still effective for the CEV model even if we have model risk. We assume the volatile market is under the CEV model and the optimization problems is still obtained by Black-Sholes model. Then we observe the distribution of terminal wealth w.r.t CPPI and OBPI for different volatile case and different investment horizons. Especially, we calculate the frequency of rankings and utility value of CPPI and OBPI approach. All of these results help us better understand the advantages and disadvantages of CPPI and OBPI under the volatile market from multiple angles. OBPI approach gives a better result than CPPI approach with respect to a utility function which favors the CM strategy. However, if the market is volatility, OBPI approach faces greater risks and has more possibility to lose money than CPPI strategy. In the contrast, CPPI strategy is relatively robust with lower payoffs.

V. CONCLUSION

In this research, we recall the well-known modified optimization problems to help us to better understand the operation mechanisms of typical portfolio insurance strategies: CPPI and OBPI. We combine CRRA utility maximization with the stylized strategies and bring these results together. In terms of optimal payoffs, both payoffs for CPPI and OBPI are higher that CM for low terminal asset price. OBPI approach gives a better result that CPPI approach with respect to a CRRA utility function which favors the CM strategy with portfolio weight m^*. As a result, investors can buy and hold more CM strategies in the case of the OBPI approach.

In the meanwhile, we focus the volatile market and consider the market is under the Constant Elasticity of Variance (CEV) model in different investment horizon and resulting payoffs. In the case of the CEV model, the optimal strategies of portfolio insurance are not obtained mathematically. As a result, we use the optimal strategies for the Black-Scholes model and check their applicability in the CEV model. Through numerical experiments, those strategies are still effective for the CEV model even if we have model risk. The study of the whole distribution associated with payoffs and utility value imply that OBPI approach gives a better result than CPPI approach with respect to a utility function which favors the CM strategy. This is a major advantage in OBPI approach. However, the kinked payoffs-profile of OBPI approach shows that the OBPI strategy is greatly affected by market volatility. If the market is volatility, OBPI approach faces greater risks and has more possibility to lose money than CPPI strategy. The characteristic is more obvious with the investment horizon increased. It means that investors should pay more attention to the risks when consider the OBPI strategy. To compare with OBPI strategy for different investment horizon, the payoffs of CPPI approach is less affected by the market fluctuations. It turns out that the CPPI strategy is relatively robust with lower payoffs.

VI. FUTURE WORK

Firstly, we consider the maximization problems suited to the CEV model. Secondly, we think about the loss rates with strategies due to the utility problem. Thirdly, if the portfolio insurance strategies are impeded by market frictions, it is necessary to consider utility loss caused by trading restrictions in the sense of discrete-time trading and transaction costs.

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References

[1] M. Rubinstein and H. E. Leland, "Replicating Options with Positions in Stock and Cash," Financial Analysts Journal, Vol. 37, No. 4, 1981, pp. 63-72.

[2] Grossman, S. and Villa, J. Portfolio Insurance in Complete Markets: A Note, Journal of Business 62, 1989, 473–476.

[3] Black, F. and Jones, R. Simplifying Portfolio Insurance, The Journal of Portfolio Management 14, 1987, 48–51.

[4] Leland, H. and Rubinstein, M. The Evolution of Portfolio Insurance, in: D.L. Luskin, ed., Portfolio Insurance: A guide to Dynamic Hedging, Wiley, 1976.

[5] Brennan, Michael J, Schwartz, Eduardo S. The pricing of equity-linked life insurance policies with an asset value guarantee[J]. Journal of Financial Economics, 1976, 3(3):195-213.

[6] Cox, J. and Huang, C.-F. Optimal Consumption and Portfolio Policies when the Asset Price follows a Diffusion Process, Journal of Economic Theory 49, 1989, 33–83.

[7] Brennan, M. and Schwartz, E. Portfolio Insurance and Financial Market Equilibrium, Journal of Business 62, 1989, 455–472.

[8] Basak, S. A General Equilibrium Model of Portfolio Insurance, Review of Financial Studies 8(4), 1995, 1059–1090.

[9] Grossman, S. and Zhou, J. Equilibrium Analysis of Portfolio Insurance, Journal of Finance 51, 1996, 1379–1403.

[10] Longin F, Solnik B. Extreme Correlation of International Equity Markets[J]. Journal of Finance, 2001, 56(2):649-676.

[11] Basak, S. A Comparative Study of Portfolio Insurance, The Journal of Economic Dynamics and Control 26(7-8), 2002, 1217–1241.

[12] Jean-Philippe, Barque. Constitutive overexpression of immunoidentical forms of PCP-induced Euglena gracilis CYP-450[J]. Biochemical & Biophysical Research Communications, 2002.

[13] Merton, R. Optimal Consumption and Portfolio Rules in a Continuous Time Model, Journal of Economic Theory 3, 1971, 373–413.

[14] Black F, Sholes M. The pricing of options and corporate liabilities[J]. Journal of political economy, 1973, 81:637-654.

[15] Browne, S. Beating a Moving Target: Optimal Portfolio Strategies for Outperforming a Stochastic Benchmark, Finance and Stochastics 3, 1999, 275–294.

[16] Karoui N E, Jeanblanc M, Lacoste V. Optimal portfolio management with American capital guarantee[J]. Journal of Economic Dynamics & Control, 2005, 29(3): p.449-468.

[17] Mahayni, Antje, Balder, Sven. How good are portfolio insurance strategies[J]. Alternative investments & strategies, 2009.

[18] Browne S. Survival and Growth with Liability: Optimal Portfolio Strategies in Continuous Time[J]. Mathematics of Operations Research, 1997, 22(2): 468-493.

[19] Jones E P. Option arbitrage and strategy with large price changes[J]. J. Financial Economic, 1984, 13:91-113.

[20] Guo W, Xu C. Optimal portfolio selection when stock prices follow a jump-diffusion process[J]. 2005.

[21] Hobson D G, Rogers L C G. Complete models with stochastic volatility[J]. Mathematical Finance, 1998, 8: 27-48.

[22] Hull J, While A. The pricing of options on assets with stochastic volatilities[J]. The Journal of Finance, 1987, 42: 281-300.

[23] Cox J C, Ross S A. The valuation of options for alternative stochastic processes[J]. Journal of Financial Economics, 1976, 3(1):145-166.

[24] Bertrand, P. and Prigent, J.-L. (2003), Portfolio Insurance Strategies: A Comparison of Standard Methods When the Volatility of the Stoch is Stochastic, International Journal of Business 8(4), 15–31.

[25] Shapiro B A. Value-at-Risk Based Risk Management: Optimal Policies and Asset Prices[J]. The Review of Financial Studies, 2001, 14(2):371-405.

[26] Merton R C. Option pricing when underlying stock returns are discontinuous[J]. 1976, 3(1-2):0-144.

[27] Bardhan I, Chao X . Martingale Analysis for Assets with Discontinuous Returns[J]. Mathematics of Operations Research, 1995, 20(1):243-256.

[28] Heston S. A closed form solution for options with stochastic volatility with applications to bond and currency option[J]. Review of Financial Studies, 1993, 6:327-343.

[29] Cox J C, The constant elasticity of variance option pricing model[J]. J Portfolio Manage, 1996, 22:16-17.

[30] Gao J . Optimal investment strategy for annuity contracts under the constant elasticity of variance (CEV) model[J]. Insurance Mathematics and Economics, 2009, 45(1):9-18.

[31] Zhang, P. G., Exotic Options–A Guide to Second Generation Options, World Scientific, 1998.