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A Finite Element Computer Model of the Captive Column

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A FINITE ELEMENT COMPUTER MODEL
OF THE CAPTIVE COLUMN

By

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This Project Report submitted by Craig P. Kipp in partial fulfillment of the requirements for the Degree of Master of Engineering from the University of North Dakota is hereby approved by the Faculty Advisor and the Department Chairman under whom the work has been done.

J. Peter Sadler — Oct. 21, 1981

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ABSTRACT

This report describes the computerized mathematical modeling of a composite structural assemblage referred to as a "captive column". The captive column is a potentially useful structural member (beam, column, or torsion member) which exhibits a high strength-to-weight ratio. The captive column consists of three basic components: a lightweight core section, the principal load bearing elements referred to as caps, and a filamentous wrap, helically wound around the other two members. Together the three elements act as an integral unit and can be constructed in multigeometrical cross sections and diverse lengths.

A linearly elastic finite element computer model was developed to analyze the structural behavior of captive columns under static bending loads. On this model the captive column core ribs were represented by a combination of orthotropic plane stress elements and beam elements. Beam elements were also utilized for modeling the caps, while truss elements represented the wrap strands. Typical computer model sizes of the columns included 60 nodes and 213 elements for the triangular cross section and 105 nodes and 404 elements for the square cross section.

A total of ten experimental test specimens, all 28 inches long, were constructed for the purpose of verifying the computer model. The specimens were loaded as simply supported beams while the applied load, deflections under the load, and core strain 3.5 inches on either side of the load were recorded. These experimental results were then compared with the computer model results. These results are as follows.

The computer model deflections at the midspan of the column, under a concentrated load, were 10 to 12 percent less than the actual experimentally measured deflections. Furthermore, for the captive columns with steel caps, the computer model core stresses, at a point 3.5 inches on either side of the midspan load, differ by no more than 20 percent from the experimentally measured core stresses. For the captive columns with fiberglass caps, the computer model cores stresses differ by 95 percent and 74 percent for the algebraically smallest principal stress and less than 8 percent for the other, larger, principal stress. Principal directions of the two dimensional stress element differed by no more than 11 percent for the steel capped captive column and from 2 to 28 percent for the fiberglass capped captive column.

In conclusion, initial verification has been obtained for a finite element model of the captive column structural composite. Additionally, preliminary design procedures have been outlined for specifying the cap, wrap, and core of the captive column for specific loading applications.

NOMENCLATURE

A_c = Cross sectional area of one captive column cap (in^2)

A_{cr} = Cross sectional area of one rib of the captive column core (in^2)

BE_{cap} = Modulus of elasticity of the cap beam elements (psi)

BE_{core} = Modulus of elasticity of the core beam elements (psi)

D = Horizontal distance between caps (in)

E_η = Modulus of elasticity in the η direction of the core. Direction parallel to the caps (psi)

E_s = Modulus of elasticity in the s direction of the core. Direction radially outward from the center of core, perpendicular to caps (psi)

E_T = Modulus of elasticity in the T direction of the core. The thickness direction (psi)

$(EI)_{eq}$ = The calculated EI equivalent of the core and cap (in^4)
$(EI)_{eq} = E_{cap}I_{cap} + E_{core}I_{core}$

F = Load applied to the captive column (lb)

F_c = Force in the caps (lb)

$G_{\eta s}$ = Modulus of rigidity (of the core) in the η - s direction (psi)

h = Length of one rib in the core (in)

I_c = Moment of inertia of one cap about its centrodial axis (in^4)

I_{cap} = Moment of inertia of all the caps on a captive column with respect to the column's centrodial axis (in^4)

I_{core} = Moment of inertia of the entire core with respect to the column's centrodial axis (in^4)

I_x = Moment of inertia of one rib, about its centrodial axis in the x-direction (in^4)

I'_x = Moment of inertia of two ribs, about the x centrodial axis of the column (in^4)

I_y = Moment of inertia of one rib, about its centrodial axis in the y-direction (in^4)

L = Length of the captive column (in)

L_1 = The distance squared from the column's centrodial axis to the center of the farthest cap for the triangular cross section (in^2)

L_2 = The distance squared from the column's centrodial axis to the center of either of the two closer caps for the triangular cross section (in^2)

L_3 = The distance squared from the column's centrodial axis to either of the four caps for the square cross section (in^2)

L_4 = The distance squared from the column's centrodial axis to the centrodial axis of one of the four ribs in the square cross section (in^4)

M = Moment

N = Number of caps above the neutral axis

PSE_{core} = Modulus of elasticity of the plane stress core elements (psi)

r = Radius of the cap (in)

TE_{wrap} = Modulus of elasticity of the truss wrap element (psi)

W = Width of the core ribs (in)

δ = Deflection of the top cap or caps at the midspan of the captive column (in)

ε_p = Maximum principal strain $(\frac{in}{in})$

ε_q = Minimum principal strain $(\frac{in}{in})$

ε_1 = Strain from strain gauge one of the strain rosette $(\frac{in}{in})$

ε_2 = Strain from strain gauge two of the strain rosette $(\frac{in}{in})$

ε_3 = Strain from strain gauge three of the strain rosette $(\frac{in}{in})$

ν = Poisson's ratio

$\nu_{\eta s}$ = Poisson's ratio in the η-s direction of the core

$\nu_{\eta T}$ = Poisson's ratio in the η-T direction of the core

ν_{sT} = Poisson's ratio in the s-T direction of the core

σ_p = Maximum principal stress (psi)

σ_q = Minimum principal stress (psi)

ϕ = Orientation of the axis of the maximum normal stress, measured from strain gauge one in the direction of strain gauge three

CHAPTER 1

INTRODUCTION

The Captive Column is a high strength, lightweight structural composite made up of three components; namely, a lightweight core section, the principal load bearing elements referred to as caps, and a filamentous wrap, helically wound around the other two members. Together, the three elements act as an integral unit and can be constructed in multi-geometrical cross sections and diverse lengths. Materials such as fiberglass, steel, and wire rope are used for the caps; balsa wood, aluminum, and plexiglass for the core, Kevlar (a Dupont product), other synthetic fibers and metallic strands for the wrap (A detailed description of the captive column is presented in Chapter 2).

Potential applications of the captive column include transmission towers, bridges, pilings, light poles, and essentially any application where a typical structural column or beam is used [1]. The alluring feature of the captive column stems from its high strength (or stiffness) to weight ratio. However, such additional assets as portability, a wide selection of materials, and the potential for local production contribute to the overall optimism surrounding the captive column's marketability.

Exhibited in Table 1 is the advantageous stiffness to weight characteristic of the captive column. By selecting beams with the same flexural rigidity (modulus of elasticity times moment of inertia), or nearly the same, as that which was experimentally determined for a 5.875 inch square cross section captive column, a weight comparison, for beams exhibiting

TABLE 1

STIFFNESS TO WEIGHT COMPARISON OF THE CAPTIVE COLUMN
WITH STANDARD STRUCTURAL MEMBERS

Beam	Size (In)	E (10^6 Lb/In^2)	I (In^4)	E·I (10^6 Lb-In^2)	Weight (Lb/Ft)	Weight Ratio
Captive Column:						
3/8" Fiberglass Caps	5.875" Square			17.1	.73	+ 1.0
3/8" Balsa Core	9' Long					
60^{o} Wrap						
I-Beams:						
A) Glass Reinforced Polyester [2]	4" x 4" x 1/2"	2.3	7.94	18.2	2.12	+ 2.9
B) Aluminum	3" x 1-1/2" x 1/4"	10.3	1.75	18.0	1.66	+ 2.27
C) Structural Steel	2" x 2" x 1/8"	30.0	.496	14.8	2.25	+ 3.5
Channels:						
A) Glass Reinforced Polyester [2]	5" x 1-3/8" x 1/4"	2.3	5.78	13.2	1.32	+ 1.8
B) Aluminum	4" x 1-1/16" x 1/8"	10.3	1.55	15.9	.84	+ 1.15
C) Structural Steel	3" x 13/16" x 1/8"	30.0	.637	19.0	1.79	+ 2.45

The flexural rigidity of the captive column was determined experimentally ($EI = \frac{FL^3}{48\delta}$), while the flexural rigidity of the I-Beams and channels were taken from reference [2] similar midspan deflections, can be made. Notice that the captive column is typically two to three times lighter than comparable structural beams of equal stiffness.

Unlike typical structural members (steel beams, trusses, and reinforced concrete), design information for the captive column is, unfortunately, nonexistent. Established beam theory, although potentially applicable, has not been verified, modified, or in any way related to the captive column [3]. Additionally, analysis of the captive column is complicated by: 1) the uncertainty surrounding the interrelationships of the three elements -- cap, core, and wrap, and 2) the large number of construction variables intrinsic in the concept. These variables include the angle of wrap, each elements size and material, column geometry, adhesives, loading patterns, and, of course, construction techniques. Understandably, this void between a potentially useful product and adequate design information creates a wide chasm between the captive column portent and a latent commercial market.

These multifarious design variables suggest the use of modern computer based techniques to analyze the captive column. Therefore, the objective of this research effort, as described in this paper, is to develop a finite element computer model, applicable to the captive column which, eventually, can be used for the analysis of existing captive column designs and for the determination of possible new improved designs.

Specifically, three long range objectives were defined for the computer based analytical development:

1. Determine the validity of finite element computer techniques in predicting captive column structural performance.

2. Evaluate how material properties and geometries influence structural performance.

3. Use the computer program for design optimization.

This report concentrates on the first objective; where load deflection and core stress comparisons between the computer model and the experimental results are highlighted.

Two finite element computer models are developed. One for a triangular cross section captive column and the other for a square cross section captive column. The models have 60 and 105 nodal points, respectively. These nodal points define the size and shape of the computer model. They are connected with specific element types that determine the characteristics of the mathematical paradigm. Thus, the model mathematically represents the actual physical column in terms of geometry, element size, and material properties.

In addition to the finite element computer model, a laboratory experimental program was developed and undertaken. The results from this laboratory testing, plus the computer model and theoretical beam theory deflections, are compared and analyzed. These laboratory tests clearly supported the computer model development, and assisted, through observation and experience, in refining captive column construction techniques.

It should be noted that all of the work presented here involves simply supported beams (captive columns) loaded at the midspan. These computer models do not simulate column buckling. According to the models, an axially loaded column would simply deflect according to $\delta = FL/AE$. Typically, in the case of the captive column, the caps, with a much larger AE value, are the components which govern in tension and compression. Therefore, further analytical development must be done before captive columns, loaded as columns, can be modeled in buckling.

The remainder of this report deals with the development of the captive column finite element computer model; and the procedure for laboratory testing of the captive columns, as well as the data acquired from these tests. Chapter 6, Results, compares deflections and stresses obtained from the computer model, the laboratory tests, and classical beam theory calculations. Chapter 7 deals with the conclusions gained from this research and also recommendations for improvements in the computer model.

CHAPTER 2

THE CAPTIVE COLUMN

In this chapter the captive column concept is described, briefly presenting the principles involved, while also detailing the specific captive column members; the cap, core, and wrap.

One of the simplest captive column geometry is the triangular cross section shown in Figures 1 through 3, an equilateral triangle with a high strength cap at the apex of each angle. The caps are held in position by an internal core which prevents inward buckling. To prevent outward and lateral buckling of the caps, the entire column is wrapped by a tension-only filament. Thus, the captive column concept comprises at least three caps, fully constrained, preventing movement relative to each other; hence, the name "captive column".

Captive Column Concept

The basic principles behind the captive column are, naturally, the same as those behind any other beam or column. Increases in either the moment of inertia or the modulus of elasticity increase the load carrying capacity of a column or beam, both in axial compression and bending. The more material placed at greater distances from the neutral axis, the larger the moment of inertia. A well known example of this concept is the structural I-beam. Thus, in the case of the captive column, by placing a relatively high modulus of elasticity material (caps) as far away as possible from the neutral axis, without sacrificing structural integrity, the load carrying capacity of the captive column is increased. The column's weight per lineal foot can then be minimized by selecting

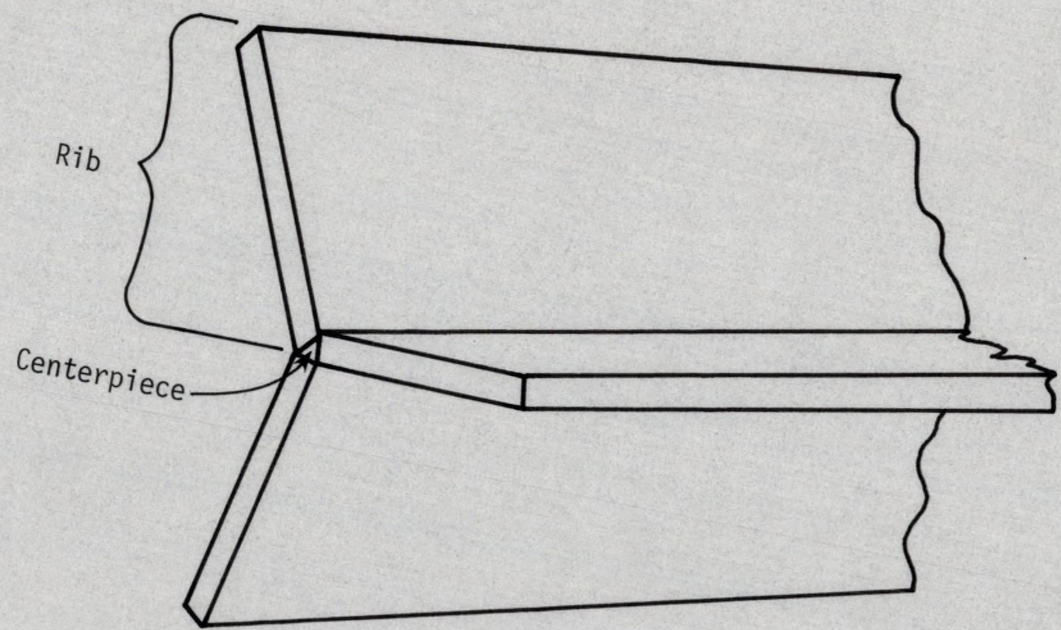
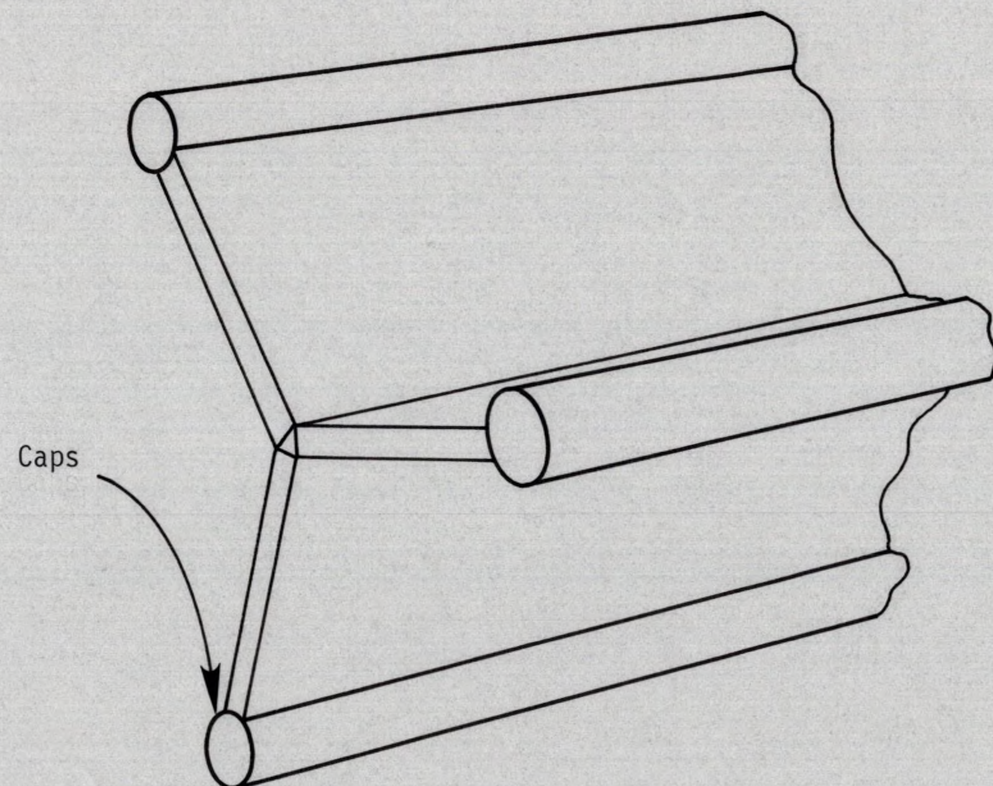


FIGURE 1 - TRIANGULAR CAPTIVE COLUMN CORE



<u>FIGURE 2</u> - CAPTIVE COLUMN CAPS

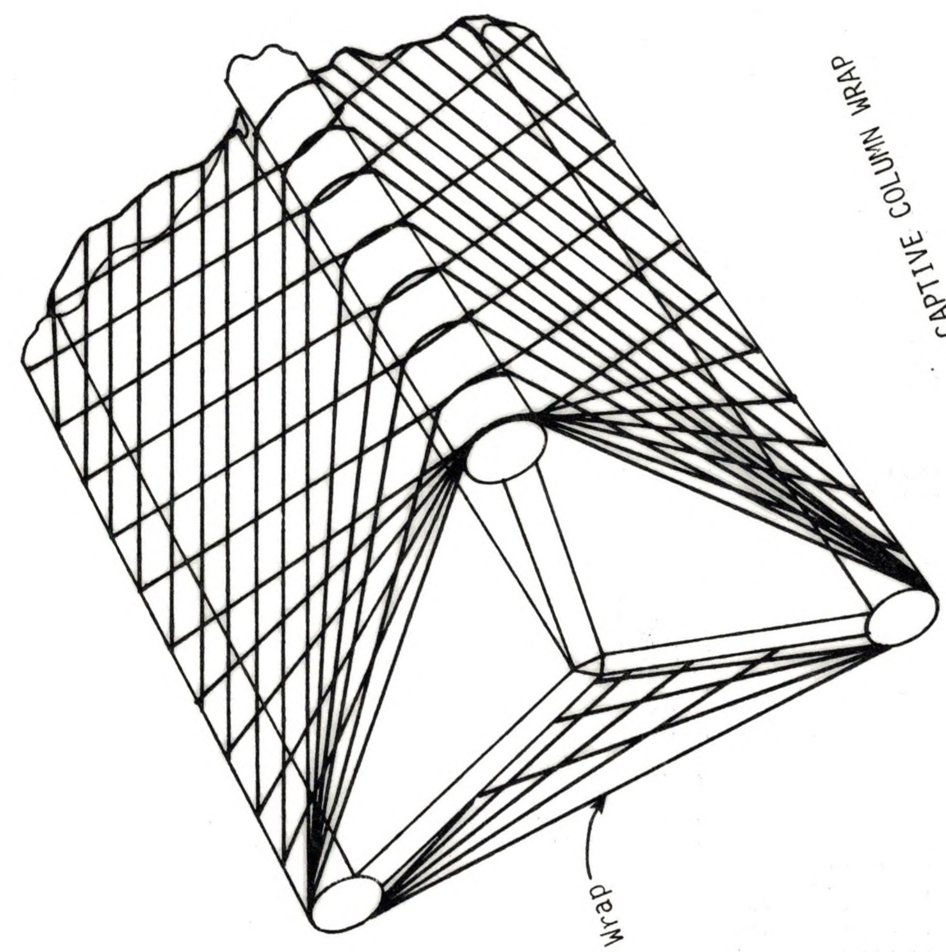


FIGURE 3 - CAPTIVE COLUMN WRAP

lightweight materials, with specific applicable properties, for the core and wrap.

Further, the inventor of the captive column concept, Mr. Lawrence Bosch, believes that the core experiences only compressive forces, no moments, and that the rigidity of the structure is determined by the compressive strength of the core, as well as the tensile strength of the filament windings. Also, shear forces and torsional forces which may act upon a column are resolved into tensile and compressive forces in the structure [4].

Obviously, to utilize the maximum strength capabilities of each component in such a concept, while also maintaining structural rigidity at a low overall weight, it is necessary to identify, and design, for the loads placed upon each component. These loads change for each specific geometry, loading condition, and material combination, thereby providing the impetus for this research effort.

Core

The core members provide continuous support for the cap elements, thereby preventing inward buckling. At the outer edge of each rib, the caps are secured by the appropriate adhesive. The ribs of the core are also joined by an adhesive where they meet, typically at the centrodial axis of the column. Balsa wood has been the primary core material. Since the core is to restrict inward deflection of the caps, the grain of the balsa wood is oriented perpendicular to these caps, utilizing the largest modulus of elasticity of anisotropic balsa wood. Douglas fir, fiberglass, and other plastics are possible alternative core materials (see Figure 1).

Caps

Each cap, thought to be the primary load carrying element, extends without interruption through the entire length of the column. They are prevented from buckling by the wrap and core elements. Normally, they are high ultimate strength unidirectional fiberous rods. Any geometry or material for the cap is possible with 1/8" to 1/2" circular rods being the most common shapes to date (see Figure 2).

Wrap

The third basic captive column element is the tension wrap or filament winding. This filament is oriented in a spiral fashion with one-half of the filament spiraling in one direction along the column and the other half spiraling in the opposite direction along the structure. Each filament of the wrap is joined, where it passes over a cap, by an appropriate adhesive. Various degrees of pitch may be employed on the wrap, with 30^{0} to 60^{0} being the most common. The helical wrap may be formed from a variety of high tension materials such as Dupont Kevlar, other synthetic fibers, or various metallic wires (see Figure 3).

The wrap is placed on the column with some tension, called pre-tension, typically in the one to three pound range. When the column is not under a load all wrap fibers are in tension. However, since compressive axial forces cannot be transmitted by the wrap, numerous fibers do relax when the column is loaded.

Columns Constructed for Testing

The ten captive columns twenty-eight inches long built for experimental testing incorporated either the fiberous epoxy resin fiberglass rods (E of 6×10^{6}psi) or the carbon steel rods (E of 30×10^{6}psi). The two different cap materials were never combined. That is, all the caps on any one column were either fiberglass or steel. Thus, five columns had fiberglass caps while the other five had steel caps. Cap diameters were 1/8 and 1/4 inches. The caps were joined to the core, along their entire length, by Minnesota Mining and Manufacturing (3M) structural adhesive #1838 B/A.

Eight of the columns had 3/16 inch thick balsa wood cores while the other two columns had 3/16 inch acrylic cores. The material properties of the cores are presented in Table 4. The balsa wood ribs were joined to a pine centerpiece, at the neutral axis, by Elmer's wood glue. The grain of the balsa wood extended radially outward from the core center, so that it was perpendicular to the caps. The ribs of the acrylic core were also joined at the neutral axis by the adhesive K-Lux Solvent Cement. However, because of the isotropic properties of acrylic plastic, a different centerpiece was not required. Dupont Kevlar was the only wrap material used. It was 0.0078 inches in diameter and had a modulus of elasticity of 18×10^6 psi. The wrap angle was 45^o in all cases and the wrap density was 20.

Hence, two different captive column cross sections, each utilizing fiberglass and steel caps, were built, tested, and modelled. Note that the two different cross sections are sized so that the moment of inertia of the caps, about the centrodial axis of the column, are equal. That is, the moment of inertia of three 1/8 inch caps on the triangular cross section is equal to the moment of inertia of four 1/8 inch caps on the square cross section. The same is true, approximately, less than two percent error, for the columns constructed with 1/4 inch caps. Observe that the core and wrap are completely neglected in this calculation. Also, the two acrylic core columns were fabricated only in the square cross section.

Shown in Table 2 are the sizes and materials of the caps used on the respective cores. The cross-sectional dimensions of the columns tested in are shown in Figure 6.

Construction

The captive column is capable of being constructed in a vast array of configurations. For clarity, a number of geometries and variations are shown in Figure 4.

Presently, with the exception of a wrapping machine, the captive columns are constructed manually. Four to five inch balsa wood core sections, with the grain running radially outward, are glued together until the desired column length is reached. A 1/16 inch groove is machined on the end of each rib to facilitate gluing of the cap to the core. Precise construction is important. After the core has been constructed and the caps are properly attached the column is wrapped. The wrapping machine traverses the entire column applying the wrap at a specific angle. Trivial cross sectional distrotions, due to construction oversight, torsion along the length of the column, and unsatisfactory glues comprise some of the more important construction faults. Additionally, the core center piece should be a material which has a compressive strength, in all directions, equal to, or greater than, the compressive strength of the material used for the core ribs.

Experience indicates construction technique is as important to the structural integrity of the captive column concept as material and geometrical considerations.

TABLE 2

COMPONENTS AND CONFIGURATIONS OF THE TEN CAPTIVE COLUMNS BUILT FOR TESTING

Core Material	Cap Material (Dia)		Wrap
3/16" Balsa Wood	Fiberglass	Steel	
Square Cross Section	1/8" and 1/4"	1/8" and 1/4"	Kevlar
Triangle Cross Section	1/8" and 1/4"	1/8" and 1/4"	Kevlar
3/16" Acrylic			
Square Cross Section	1/8"	1/8"	Kevlar

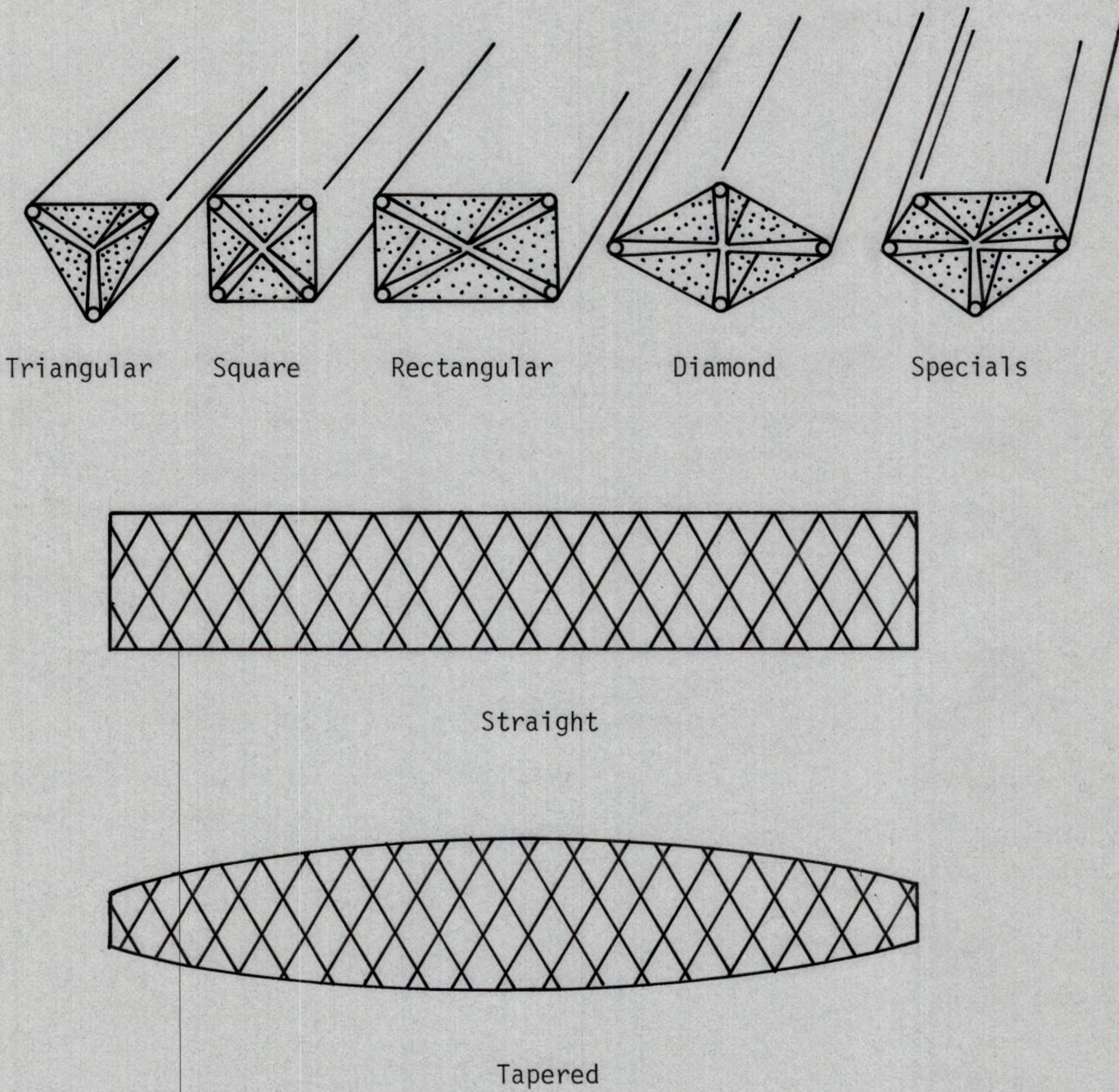


FIGURE 4 - CAPTIVE COLUMN GEOMETRIES

CHAPTER 3

THEORETICAL GOVERNING EQUATIONS

This chapter delineates the equations employed for the theoretical calculations of captive column midspan deflections and captive column core stresses. As previously stated, these deflections and stresses are calculated so that comparisons can be made with computer and laboratory results. These calculated deflections and stresses will provide another data base around which verification and/or improvements in the computer model can be made. These comparisons are presented in Chapter 6.

For the most part, the calculations draw upon classical strength of material methods and can be explored, in much more detail, in any introductory text on the subject [5][6]. Two variances, however, do arise. One is due to the composite nature of captive columns; in essence, a column (or beam) of two materials. The other variance is due to the anisotropic material properties of balsa wood cores; the most common core material and the material used in eight of the ten columns in this analysis. Both of these aberrations are discussed under the following subtitle.

Deflection

Classical beam theory states that a simply supported lineraly elastic beam under a concentrated midspan load will deflect according to the following formula:

$$\delta = \frac{FL^3}{48\ EI} \qquad (1)$$

where: δ = deflection

F = applied load

L = length of beam

E = modulus of elasticity

I = moment of inertia

Equation (1) is ideal and therefore implies many assumptions. Two of these assumptions must be reviewed in this discussion. First, the beam is assumed to be constructed of one homogenous material, and therefore, a single modulus of elasticity applies to the entire cross section and, subsequently, to the entire moment of inertia. Second, it is assumed that the modulus of elasticity used in this equation applies in the direction in which the beam will experience tension and compression during bending. For a beam made with an isotropic material, the given value of E naturally applies. However, for a beam material with two or three different values of E (anisotropic), the appropriate value must be defined. In the case of the captive column, at least three different materials are used, complicating the EI calculation of Equation (1) and introducing the first variance. Thus, it is necessary to derive an equivalent EI combination, $(EI)_{eq}$, for the composite captive column. Additionally, selecting the correct value of E to be used in the calculations for orthotropic balsa wood precipitates the second variance from the elementary beam deflection calculation.

The equivalent EI (flexural rigidity) developed for a captive column is shown below:

$$(EI)_{eq} = E_{core}\,I_{core} + E_{cap}\,I_{cap} \tag{2}$$

This substituted into Equation (1) yields:

$$\delta = \frac{FL^3}{48(EI)_{eq}} = \frac{FL^3}{48(E_{core}\,I_{core} + E_{cap}\,I_{cap})} \tag{3}$$

Notice that the wrap is neglected in the calculation. The wrap's moment of inertia, in comparison to the cap's and core's moment of inertia, is so small that it has a negligible effect on the computed deflection. This is not to say, however, that the wrap does not influence the captive column rigidity; it performs the important task of maintaining the cross sectional geometry during deflection.

Shown in A and B of Figure 5 are the two captive column cross sections used in the captive columns which were modelled and tested. In order to determine different moments of inertia for the caps or core when different cap diameters or core thicknesses are used, the following formulas were developed.

Square Cross Section:

$$I_{cap} = 4 \cdot I_c + 2 \cdot A_c L_3 \quad (4)$$

$$I_{core} = I_x' + 2 \cdot I_x + 2.A_{cr} L_4 \quad (5)$$

where: $I_c = \left(\frac{1}{4}\right) \pi r^4$ (6)

$A_c = \pi r^2$ (7)

$L_3 = (h + r + .5W)^2$ (8)

$I_x = \left(\frac{1}{12}\right) Wh^3$ (9)

$I_y = \left(\frac{1}{12}\right) hW^3$ (10)

$I_x' = \left(\frac{1}{12}\right) (2h + W)(W)^3$ (11)

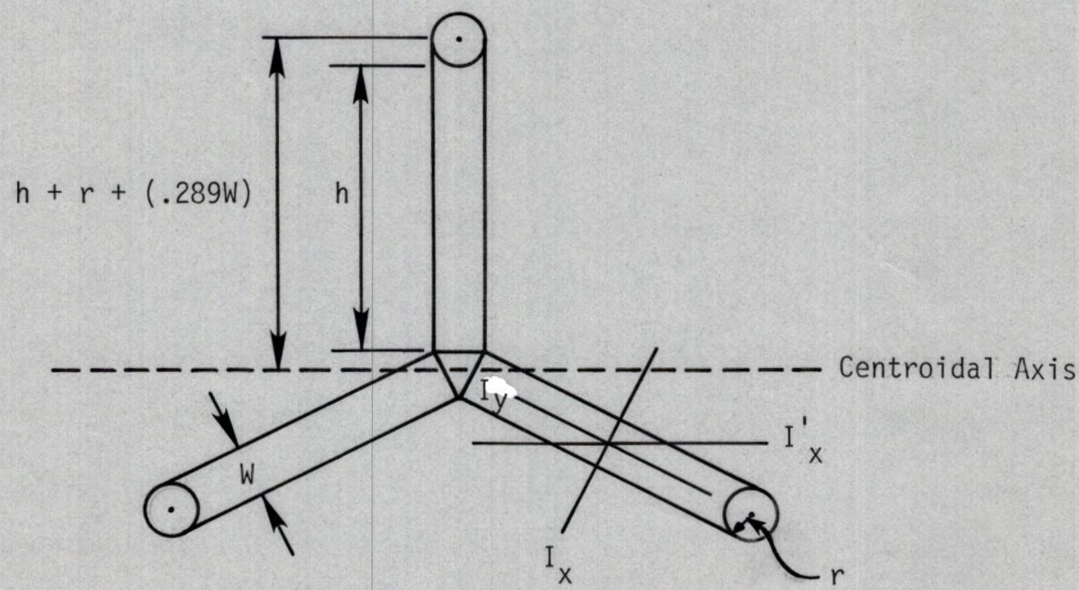
$L_4 = [\left(\frac{1}{12}\right) (W + h)]^2$ (12)

$A_{cr} = Wh$ (13)

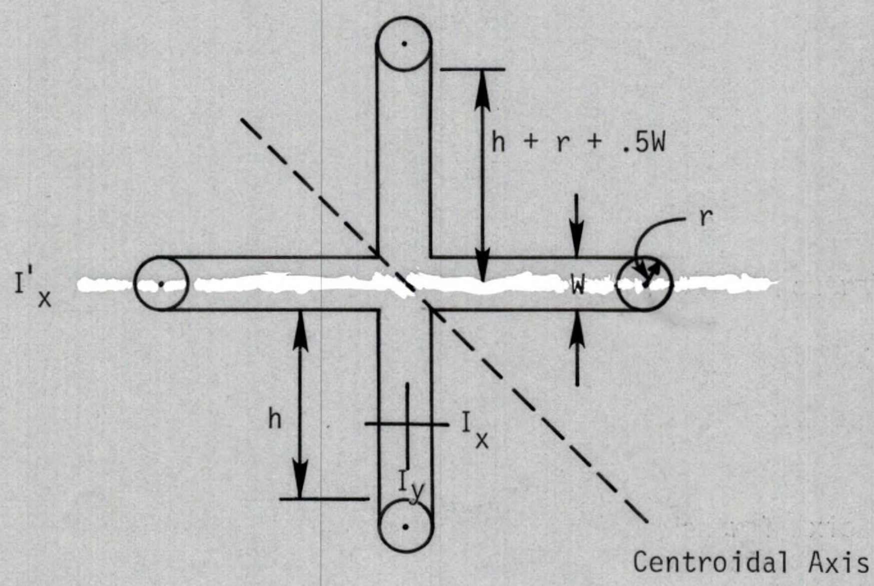
Triangular Cross Section:

$$I_{cap} = 3I_c + 1.5A_c L_1 \quad (14)$$

$$I_{core} = 1.5I_x + 1.5I_y + 1.5A_{cr} L_2 \quad (15)$$



A. TRIANGULAR CROSS SECTION



B. SQUARE CROSS SECTION

FIGURE 5 - MOMENT OF INERTIA NOMENCLATURE

where: $$I_c = \left(\frac{1}{4}\right)\pi r^4 \tag{16}$$

$$A_c = \pi r^2 \tag{17}$$

$$L_1 = (h + r + (.289W))^2 \tag{18}$$

$$I_x = \left(\frac{1}{12}\right) Wh^3 \tag{19}$$

$$I_y = \left(\frac{1}{12}\right) hW^3 \tag{20}$$

$$A_{cr} = W \cdot h \tag{21}$$

$$L_2 = (h/2 + (.289 \cdot W))^2 \tag{22}$$

The equations are derived from standard moment of inertia calculations and are presented here only for clarity and completeness. These computed moments of inertia are multiplied by the appropriate values of E to determine the equivalent flexural rigidity as shown in Equation (2).

The modulus of elasticity used for a balsa wood core in the deflection calculations is the value for the direction parallel to the caps; that is, the smaller E of 13,400 psi. Typically, the columns are built with the largest E (400,000 psi), in the direction perpendicular to the caps, thereby providing the greatest restraint against inward cap movement.

Table 3 lists the respective sizes and moments of inertia for the captive columns built and modelled.

Stress

Strain is measured via rectangular strain rossettes, mounted on the ribs of 3/16" plexiglass cores. The strain gauges are mounted in the center of the rib, 3.5 inches from the middle of the column (see Figure 18). These experimentally determined strains are substituted into the following equations to derive the principal stresses. These stresses are compared

TABLE 3

SIZES AND MOMENTS OF INERTIA OF
THE CAPTIVE COLUMNS BUILT FOR TESTING

Cross Section Geometry and Size	Moments of Inertia[a] (In^4)	
Triangular Cross Section (1.875 inches on a side)[b]	Caps	Core
1/8" caps	0.0229	.1106
1/4" caps	0.1023	.1106
Square Cross Section (1.325 inches on a side)[b]		
1/8" caps	0.0231	0.0937
1/4" caps	0.1025	0.0937

(a) With respect to the column's centrodial axis

(b) Distance given is from the cap center to the adjacent cap center to the stresses obtained from the computer program via the plane stress element core.

$$\sigma_p = E\left(\frac{\varepsilon_1 + \varepsilon_3}{2(1 - \nu)} + \frac{1}{2(1 + \nu)} \sqrt{(\varepsilon_1 - \varepsilon_3)^2 + (2\varepsilon_2 - \varepsilon_1 - \varepsilon_3)^2}\right) \qquad (23)$$

$$\sigma_q = E\left(\frac{\varepsilon_1 + \varepsilon_3}{2(1 - \nu)} - \frac{1}{2(1 + \nu)} \sqrt{(\varepsilon_1 - \varepsilon_3)^2 + (2\varepsilon_2 - \varepsilon_1 - \varepsilon_3)^2}\right) \qquad (24)$$

$$\text{Tan } 2\emptyset = \frac{2\varepsilon_2 - \varepsilon_1 - \varepsilon_3}{\varepsilon_1 - \varepsilon_3} \qquad (25)$$

where: E = modulus of elasticity

ε_1 = strain from strain gauge 1

ε_2 = strain from strain gauge 2

ε_3 = strain from strain gauge 3

ν = Poisson's ratio

σ_p = maximum principal stress

σ_q = minimum principal stress

$\emptyset$ = orientation of the axis of the maximum principal stress, measured from strain gauge one in the direction of strain gauge three

See Appendix F for the computer program which determines the principal stresses and principal direction, given the three strains ε_1, ε_2, ε_3.

CHAPTER 4

COMPUTER MODEL

As previously stated, the objective of this research effort is to develop a finite element computer model that can be used in the investigation of the captive column design. Specifically, the purpose of such a model is to study the effects of design modification--geometry, material, and loading patterns--before costly prototypes are built, while also analyzing the interrelationships of these variables in the captive column concept. This chapter describes the computer model evolved through this endeavor.

Background

The finite element method has only recently become a useful tool for such an analysis, primarily because of the availability of fast computers with large storage space. It is still, however, as much an art as it is a very precise and exacting science. The user must be experienced in choosing elements, placing loads and constraints, numbering elements in the proper sequence, sizing elements, and other basic modelling tasks. For these reasons, the final computer program(s) used to analyze any structure is best arrived at by an interactive process of varying most or all of the above variables. Because there are so many such opportunities for even an experienced analyst to make errors, models should be verified whenever possible with test data. Typically, this is done by comparing computer model deflections and stresses with laboratory deflections and stresses. This report concentrates on this phase of model verification. That is, comparing classical beam theory deflections, experimental deflections and stresses, and deflections and stresses from the computer model. This is the important first step in the development of a reliable computer model, which can then be employed in the design and optimization of captive column structures.

Two different types of captive columns were modelled. They are triangular and square cross-sectioned, simply-supported beams. The detailed properties and methods of fabrication are described in Chapter 2. Each captive column is 28 inches long. The equalateral triangular cross section is 1.875 inches on a side, between cap centers, while the square cross section is 1.325 inches on a side from cap center to cap center. Both columns when tested and modelled are loaded with a transverse force at the top in the middle of the column (see Figure 6).

Structural Analysis Program

The computer model uses the Finite Element Structural Analysis Program (SAP IV) [7], available through the University of North Dakota Computer Center. This finite element program has a number of elements that can be used independently or in conjunction with one another to mathematically model a structure which, in this case, is the captive column. The accuracy of the model depends upon the correct combination, orientation, and physical size of these elements.

Input into the SAP program, besides defining nodal positions and element types, includes the following information:

Elements:
1) shape
2) cross-sectional area
3) moments of inertia
4) moduli of elasticity, moduli of rigidity
5) temperatures
6) poisson ratios

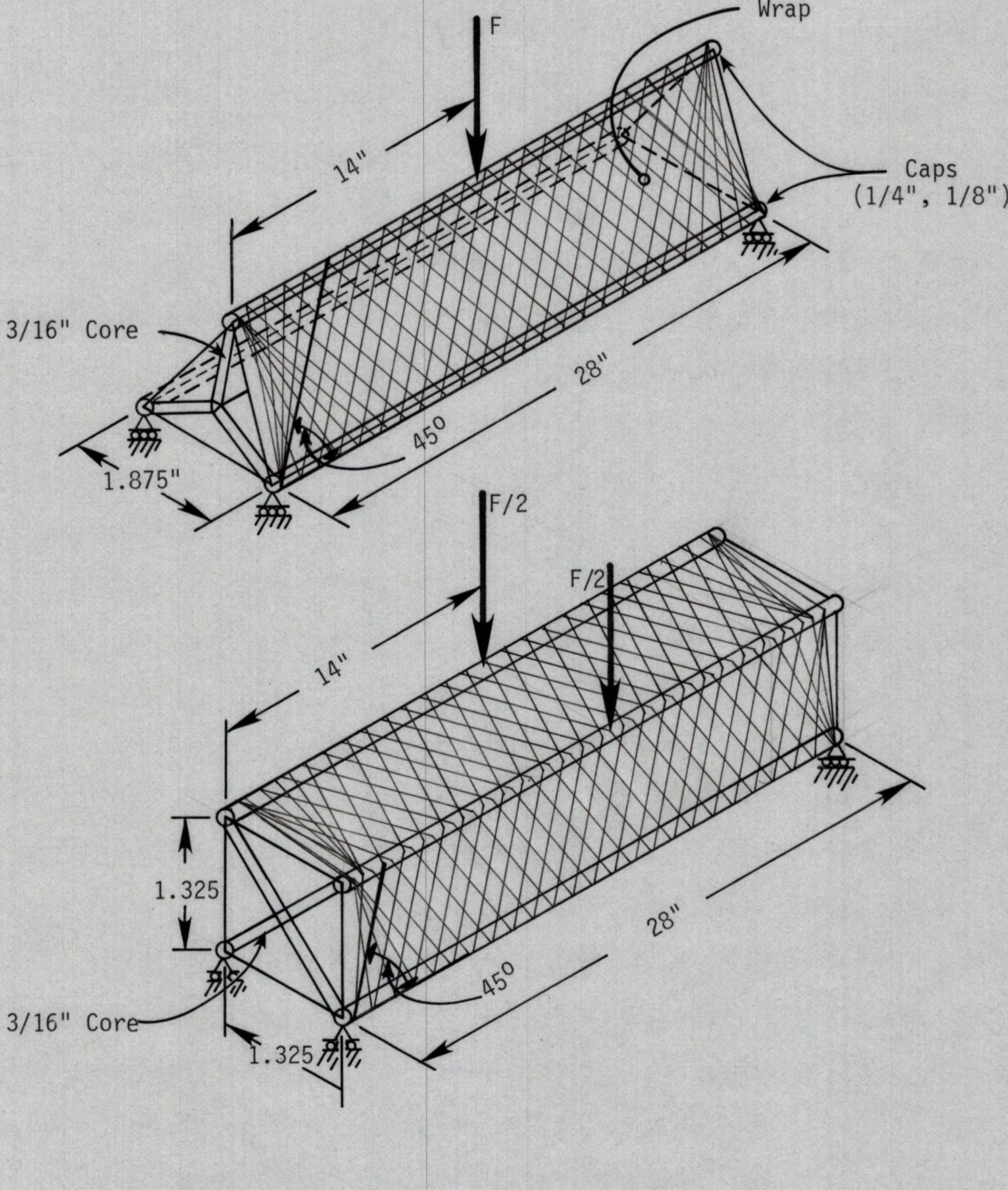


FIGURE 6 - CAPTIVE COLUMN TEST SPECIMENS

7) coefficients of thermal expansion

8) orthotropic directions

<u>Nodes</u>: 1) temperatures

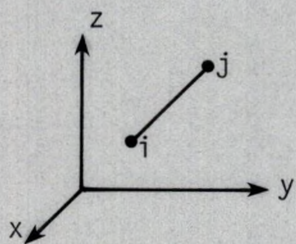
2) degrees of freedom

3) applied forces and moments

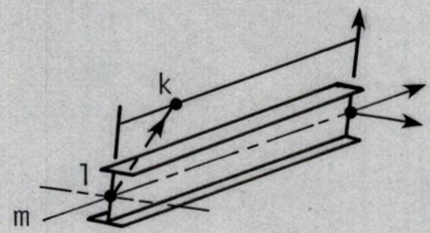
Shown in Figure 7 are seven of the element types available in the SAP program. Figure 8 pictorially shows simply one, two, and three dimensional finite element structures. Shown in Figures 9 and 10 are the finite element nodal positions and numbering for the above mentioned captive columns. These nodal points define the physical size and geometry of the model and are used to connect elements to one another.

The nodal numbering, which is purely arbitrary, has an influence on the bandwidth. Band width can best be described as the length of the longest column of elements, in the stiffness matrix, from the diagonal to the last nonzero entry. The larger this bandwidth, the longer the computer solution time required. For a given structure, all numbering schemes lead to the same size stiffness matrix and the same number of nonzero terms; however, different numbering schemes lead to different arrangements of nonzero terms, which affect the bandwidth. Thus, to minimize bandwidth, a simple procedure is to number across the small dimension at one extremity of the structure and then number in succeeding adjacent rows until the whole structure has been covered [8,9]. This scheme has been used in both models.

In the future, especially with larger computer models, strict attention should be paid to this matter. For example, the ratio of computer solution time for the triangular column numbered along its longest dimension and the present system, numbered across the smallest dimension, is approximately five to one.

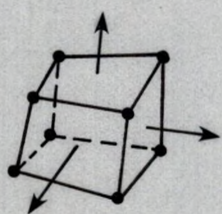


A. TRUSS ELEMENT

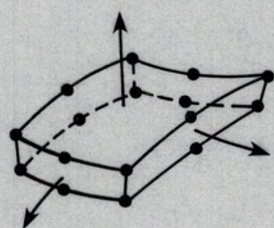


B. THREE-DIMENSIONAL BEAM ELEMENT

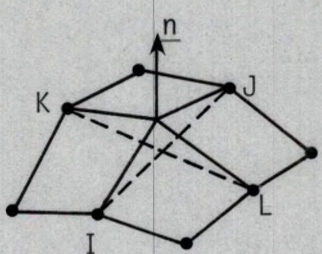
C. PLANE STRESS, PLANE STRAIN, AND AXISYMMETRIC ELEMENTS



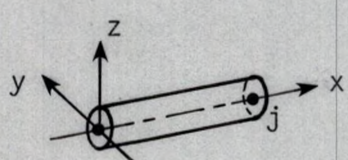
D. THREE-DIMENSIONAL SOLID



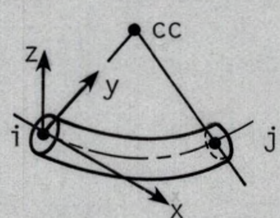
E. VARIABLE-NUMBER-NODES THICK SHELL AND THREE-DIMENSIONAL ELEMENT



F. THIN SHELL AND BOUNDARY ELEMENT



TANGENT



BEND

G. PIPE ELEMENT

FIGURE 7 - ELEMENT LIBRARY OF THE STRUCTURAL ANALYSIS PROGRAM

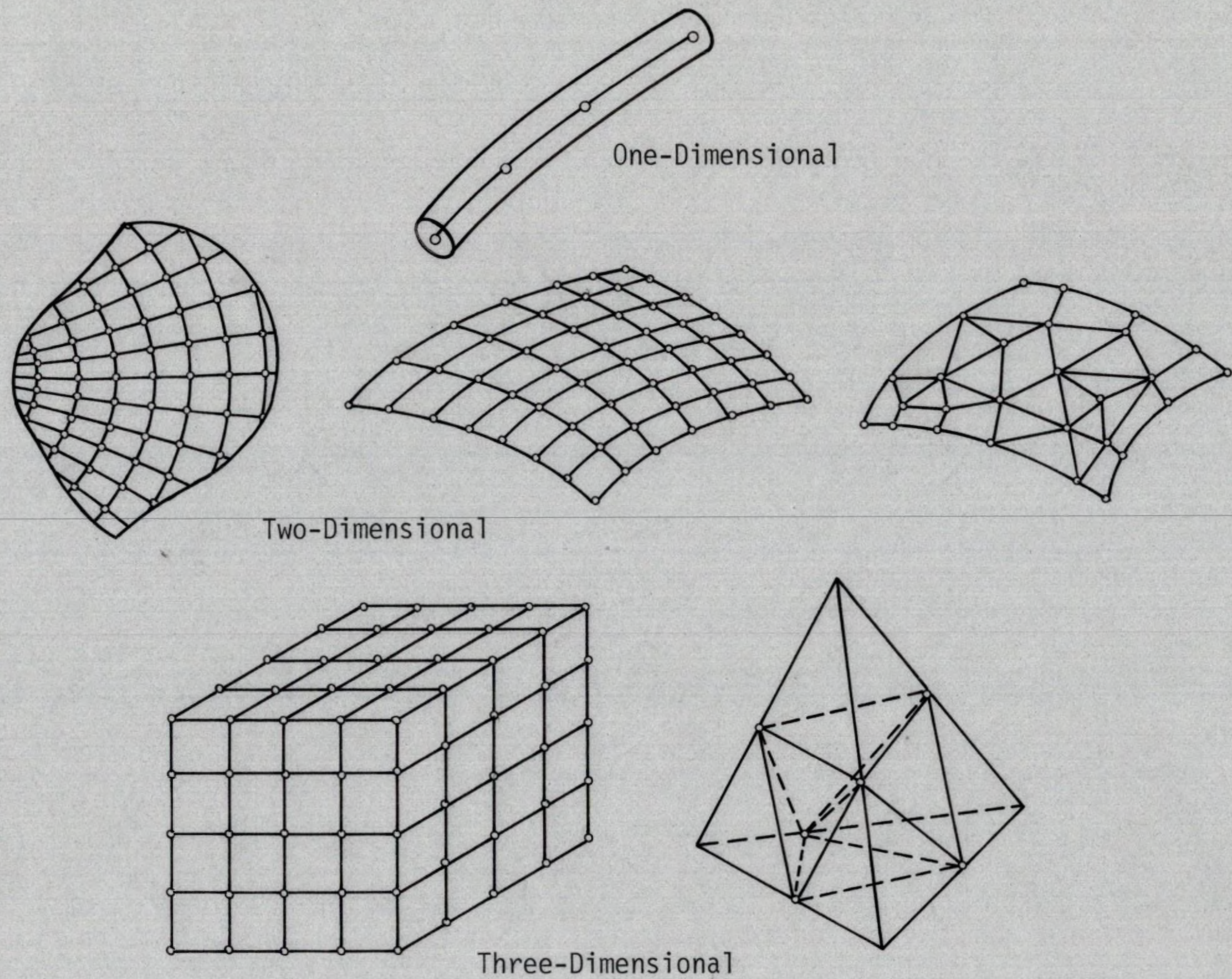
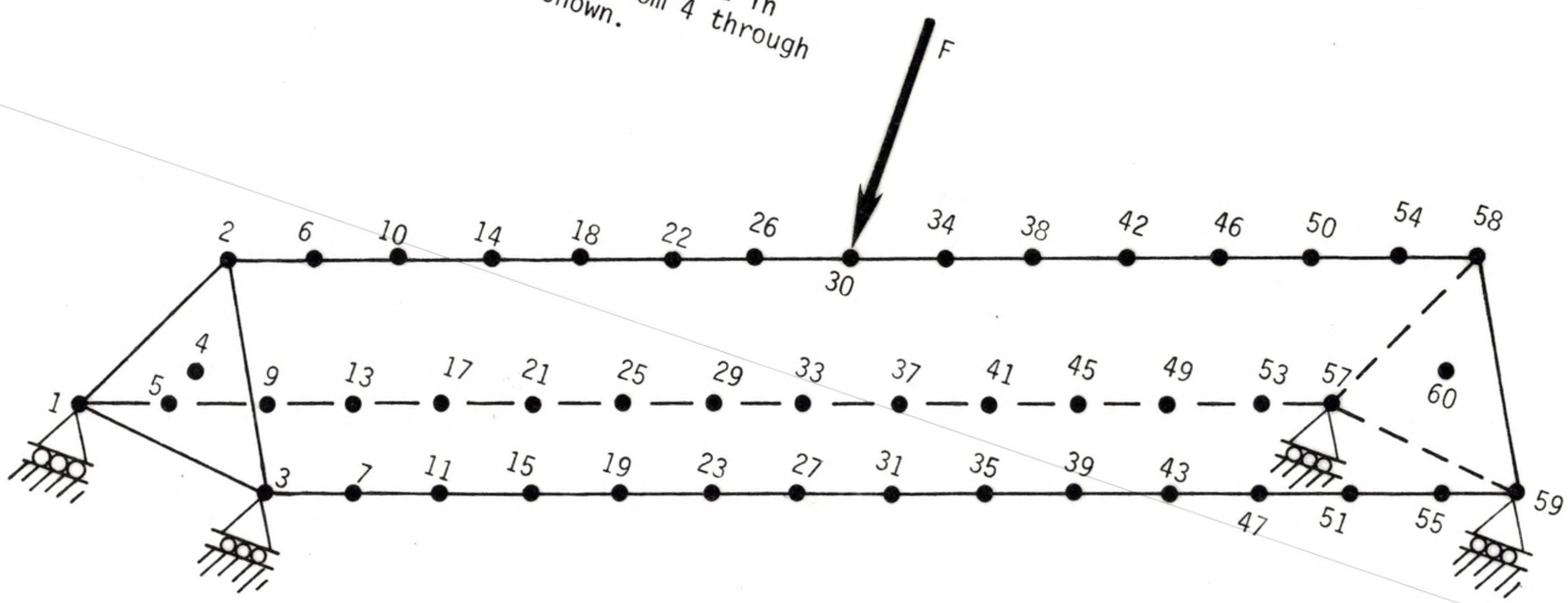


FIGURE 8 - FINITE ELEMENT STRUCTURES

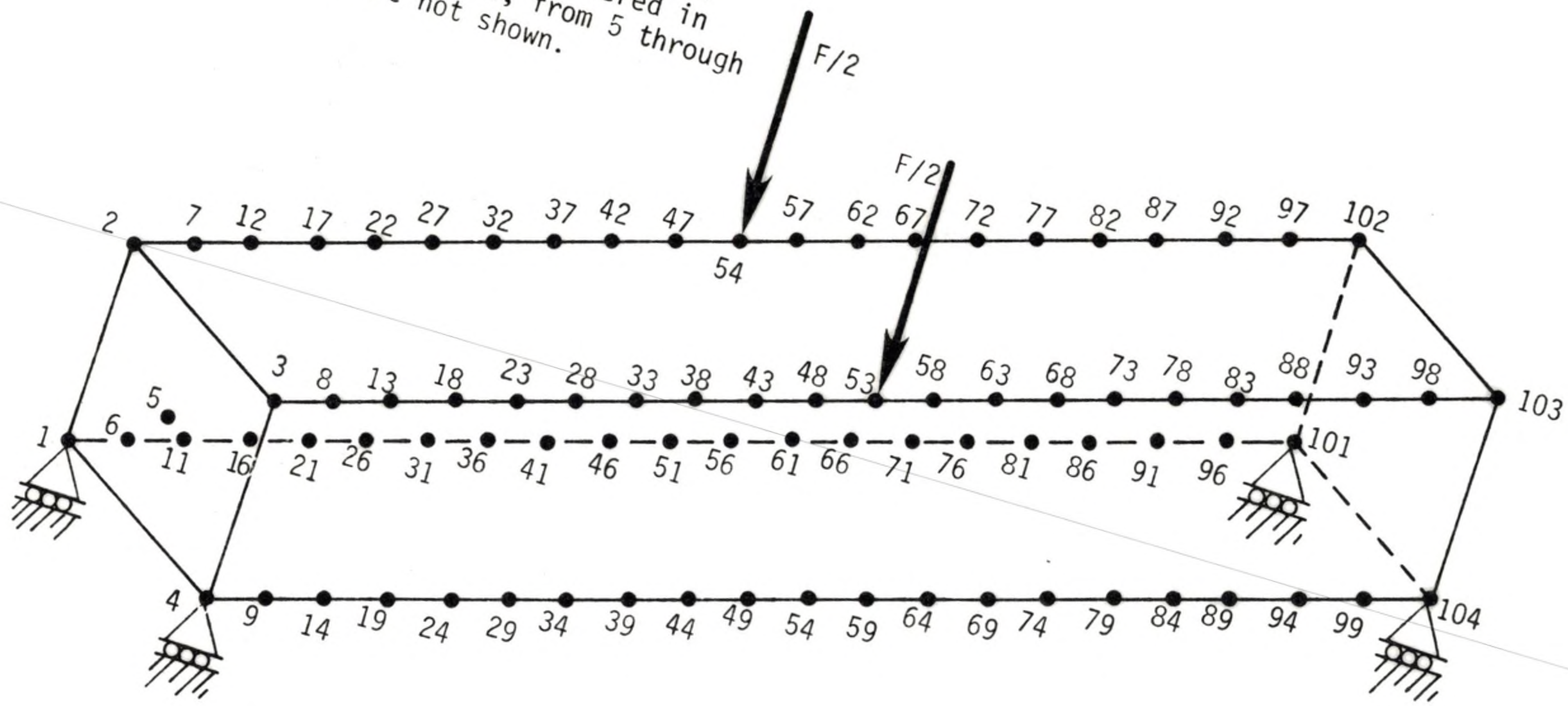
Note: Nodal points on the neutral axis, which are numbered in multiples of 4, from 4 through 60, are not shown.



Number of Nodes = 60
Number of Elements = 213

FIGURE 9 - NODAL NUMBERING FOR THE TRIANGULAR CROSS SECTION CAPTIVE COLUMN

Note: Nodal points on the neutral axis, which are numbered in multiples of 5, from 5 through 105, are not shown.



Number of Nodes = 105
Number of Elements = 404

FIGURE 10 - NODAL NUMBERING FOR THE SQUARE CROSS SECTION CAPTIVE COLUMN

Three element types are used in the computer model. They are described in Table 4.

Shown in Figures 11 through 13 are the physical representations of the captive column components in the finite element computer model. Naturally, many variations of the model are possible. This particular combination, as with all finite element models, is the culmination of intuition, experience, and trial and error. The material properties, the most definitive aspect of the model, were selected from the best available sources, usually the material manufacturer.

Even though the final element and material selections can only be justified by the validity of the model, certain element type applications are mandated. These will be discussed in detail for each captive column component--cap, core, and wrap--along with some of the other element types and configurations considered. Additionally, the two physical discrepencies occurring between the computer model and the real column are discussed. They are: 1) the difference in the number of wrap elements used in the model and those on the actual column, and 2) using both beam and plane stress elements, which theoretically occupy a portion of the same physical space in the core. Both of these discrepencies are considered in detail under the following subtitles.

Output from the finite element structural analysis program is as follows:

1. Translations along and rotations about the x, y, and z axes for each nodal point.
2. Axial and shear forces, plus torsion and bending moments, at both ends (nodal points) of each beam element.
3. Normal, shear, and principal stresses and corresponding directions for each plane stress element.
4. Axial stress and force in each truss element.

TABLE 4

ELEMENT TYPES AND PROPERTIES USED IN THE COMPUTER MODEL

	Caps	Core		Wrap
ELEMENT TYPE	BEAM	PLANE STRESS	BEAM	TRUSS
Size	1/8 in. and 1/4 in. dia. 2 in. long (T) 1.4 in. long (S)	3/16 in. thick 2 in. x 1 in. (T) 1.4 in. x .94 in. (S)	3/16 in. x 3/16 in. 1 in. long (T) .94 in. long (S)	0.0474 in. dia (T) 0.0395 in. dia (S) 2.739 in. long (T) 1.928 in. long (S)
Modulus of Elasticity	6×10^6 psi (FG) 30×10^6 psi (ST)	E_N = 13,400 psi (B) E_S = 400,000 psi (B) E_T = 13,400 psi (B) E = 450,000 psi (A)	1.0 psi	18×10^6 psi
Shear Modulus	NA	18,000 psi (B) 173,076 psi (A)	NA	NA
Poissons Ratio	.3	γ_{NS}=.3 (B) γ_{NT}=.3 (B) γ_{ST}=.04 (B) γ =.3 (A)	.3	.3

where: (T) = Triangular cross section
(S) = Square cross section
(B) = Balsa wood
(A) = Acrylic
(FG) = Fiberglass
(ST) = Steel

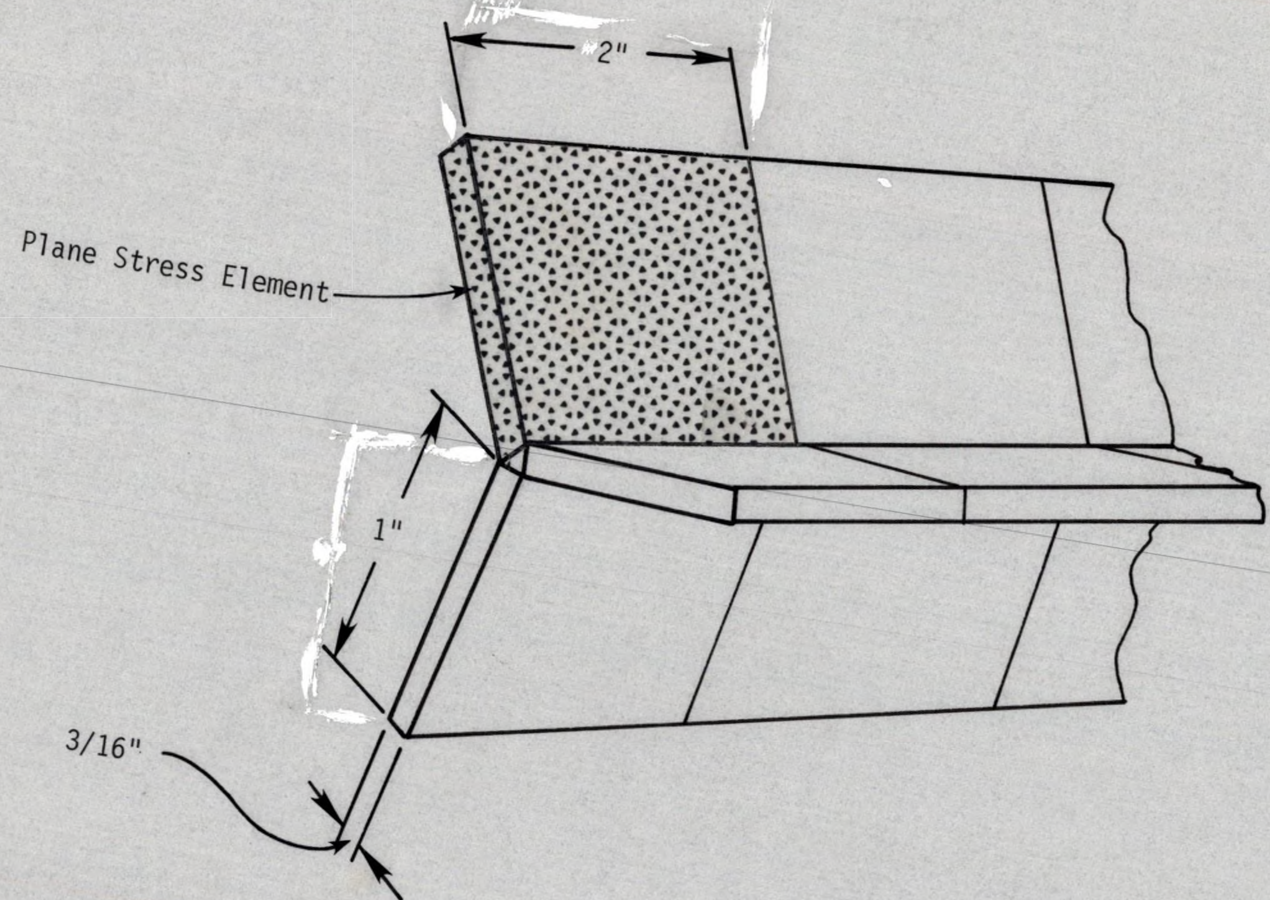


FIGURE 11 - FINITE ELEMENT CAPTIVE COLUMN CORE

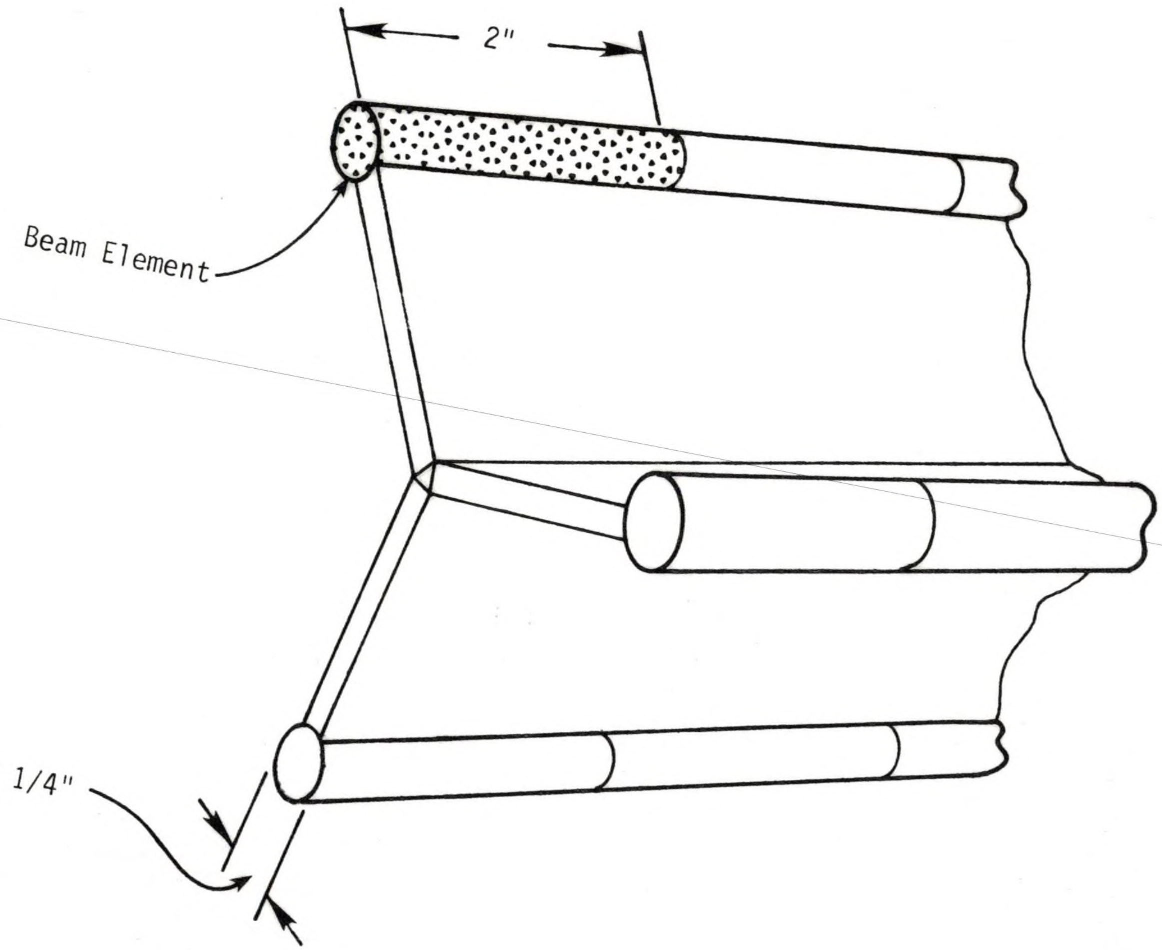


FIGURE 12 - FINITE ELEMENT CAPTIVE COLUMN CAP

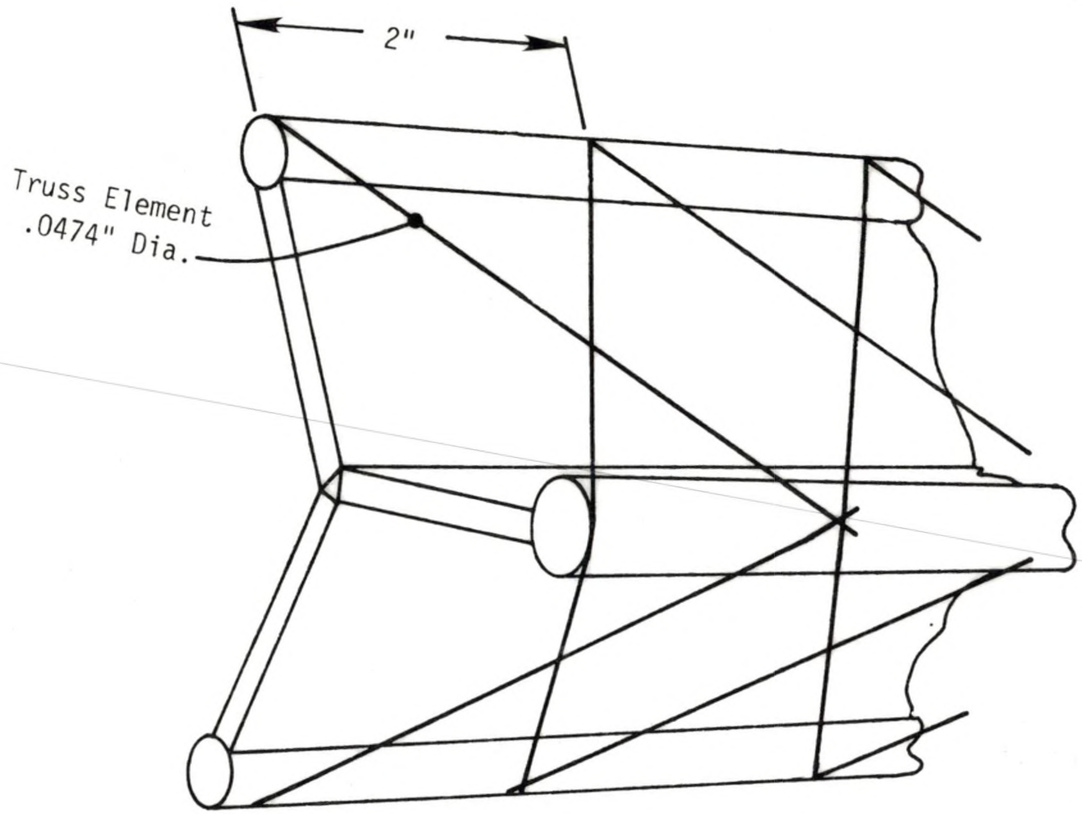


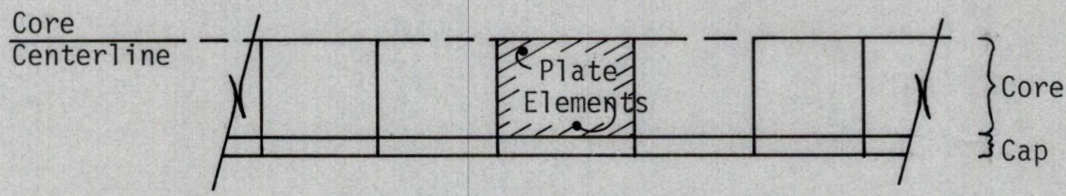
FIGURE 13 - FINITE ELEMENT CAPTIVE COLUMN WRAP

Core

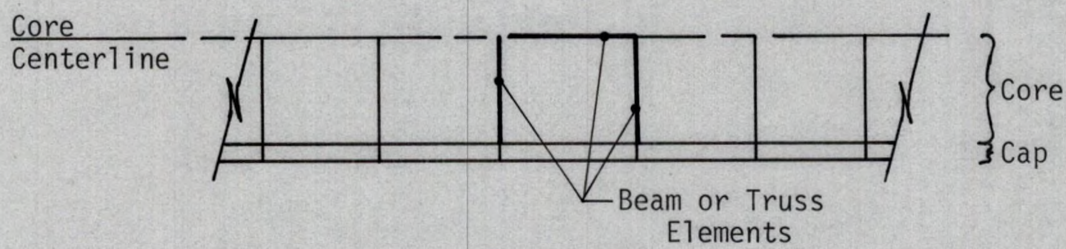
By far, the core was the most difficult part of the captive column to model. Shown in Figure 14 are some of the element combinations attempted in modelling individual ribs of the core. Shown in Figure 15 is the final combination of elements selected. They are 3/16 inch thick plane stress elements (same thickness as the actual balsa wood or plexiglass core) and 3/16 inch by 3/16 inch beam elements. Notice that the beam elements theoretically occupy a portion of the same physical space as the plane stress elements, an apparent disparity. The beam elements are included only to insure stiffness perpendicular to the plane stress elements. That is, plane stress elements withstand loads only in the two dimensional plane of the element. Loads perpendicular to the plane stress elements generate zeroes on the diagonal of the stiffness matrix, rendering the matrix, and ultimately the program, insolvable. Thus, to overcome this problem, beam elements are sandwiched in the core between the plane stress elements. However, to minimize their duplicative effect, the beam element modulus of elasticity is set at one psi or 2/10,000 of one percent of the plane stress elements' modulus of elasticity. This, consequently, nullifies any effect the core beam elements have on the computer model. Therefore, for all practical purposes, the core is modelled only by plane stress elements. The beam elements are included only to guarantee equation compatability in the mathematical solution.

Caps

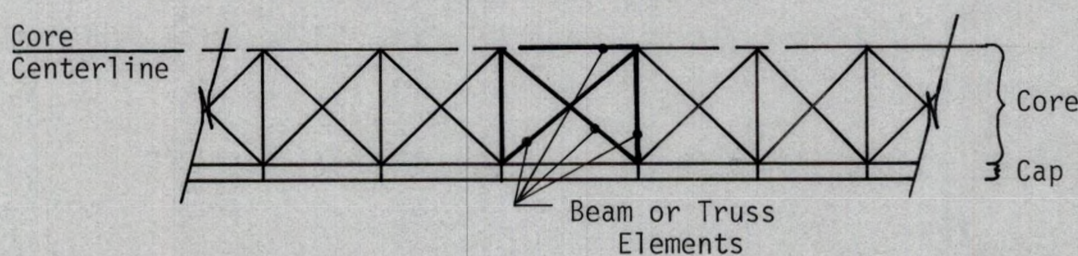
This element was the easiest component of the column to model. The caps carry not only axial loads, but very small moments as well. Therefore, circular SAP beam elements, equal in diameter and of the same material



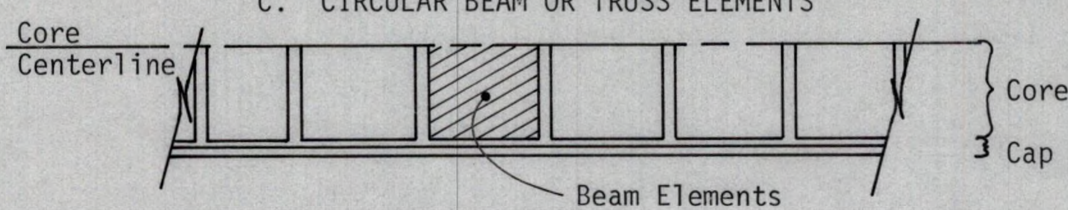
A. PLATE ELEMENTS



B. CIRCULAR BEAM OR TRUSS ELEMENTS



C. CIRCULAR BEAM OR TRUSS ELEMENTS



D. BEAM ELEMENTS SHAPED LIKE PLATES

FIGURE 14 - ELEMENT TYPES AND PATTERNS EXAMINED IN MODELING THE CORE

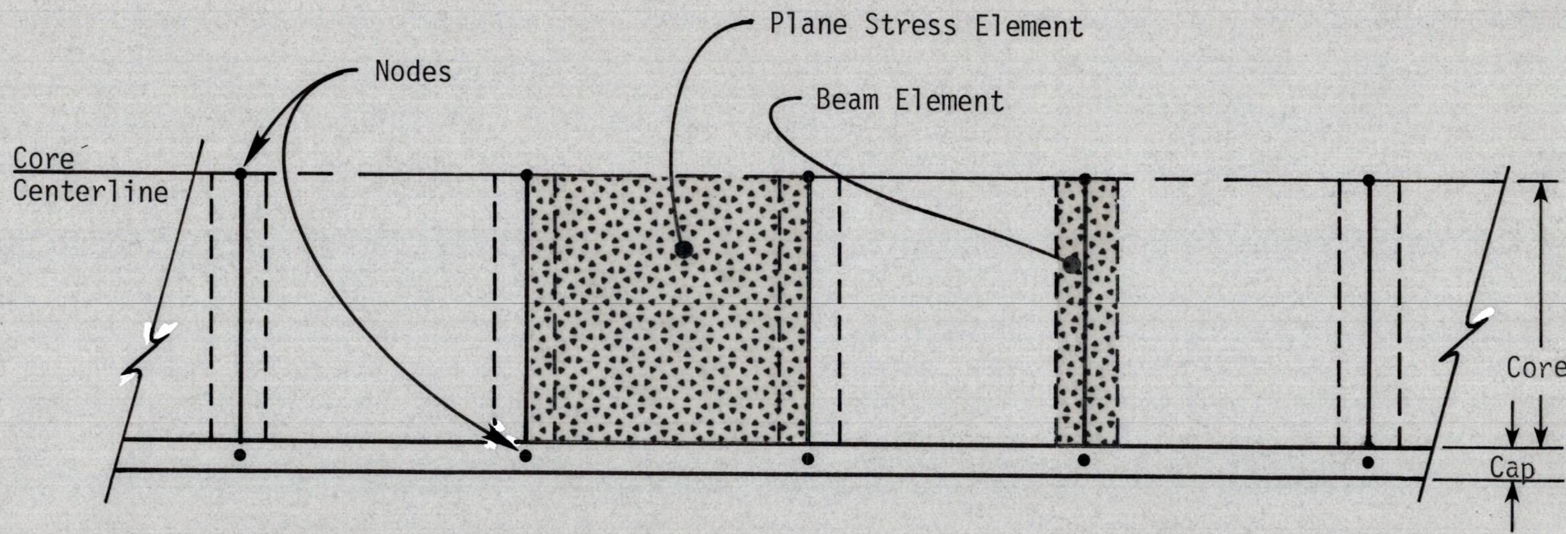


FIGURE 15 - ELEMENT CONFIGURATION OF ONE RIB IN THE FINITE ELEMENT CORE

properties as the actual caps, are specified. The only other possible SAP element type that could be used is the truss element, which should perhaps be considered in future modelling of larger systems because the number of degrees of freedom is reduced. Truss members do not carry a bending moment, but the moments in the actual cap are small when the diameter is relatively small.

Wrap

The triangular cross section computer model has one .0474 inch diameter truss element, every two inches, on each side of the three sided column, for each of the two directions of wrap. Thus, six truss elements, two on each side of the column, every two inches, represent the wrap along the longitudinal axis of the column. A similar situation exists for the square cross section model. However, in this model the nodes, and therefore, the wrap elements, are 1.4 inches apart instead of two inches. This was done purposely so that the truss elements representing the wrap would be at 45 degrees, in both models, just as they are in the physical column. Thus, there are 84 wrap elements in the triangular column and 160 wrap elements in the square column.

The actual column has approximately twenty .0078 inch diameter Kevlar strands uniformly distributed per inch along the column. The truss element area in the computer model was set equal to the area of the Kevlar strands which it displaces. For example, one truss element in the triangular model displaces forty Kevlar strands, requiring a .0474 inch diameter truss element.

$$\left(40\left(\left(\frac{.0075}{2}\right)^2 \cdot \pi\right)\right) = \left(\left(\frac{.0474}{2}\right)^2 \cdot \pi\right)$$

The computer model wrap is an ostensible simplification of the actual column wrap. Ideally, each individual wrap filament would be modelled in the program. However, to define a truss element in the program, as mentioned before, a nodal point is required at each end of the element. This would require a total of 2,240 nodes for the triangular model or 2,800 nodes for the square model. This is obviously beyond the storage and computational capacity of the computer. Thus, one truss element represents forty Kevlar wraps in the triangular column and 28 Kevlar wraps in the square column. This is believed to be a reasonable approximation.

The wraps are modelled as trusses because the actual Kevlar wrap can transmit only axial forces and no moments. In fact, filamentous Kevlar transmits only axial tension forces and no compressive forces. Kevlar simply relaxes when in compression and does not carry a load. SAP truss elements will act in tension or compression according to the modulus of elasticity that is specified. Two different moduli cannot be specified, one for tension and one for compression, nor can the truss elements be directed to act only in tension or compression. This presented a significant problem in modelling the wraps because certain wrap elements do relax on the column when a load is applied. Therefore, to accurately model this phenomenon, the program must account for zero compressive forces in the truss elements that relax. This is done by identifying those truss elements that act in compression and assigning to them a modulus of elasticity of one psi. This compares to a modulus of $18 \times (10)^6$ psi for those wrap elements that act in tension. This task of identifying the tension and compression members for each loading pattern and material combination necessitates at least two runs of the computer model. In the first run, all the truss elements are assigned the higher value of E (18×10^6psi), and the column is loaded. Those members that act in compression during this run are identified and assigned the lower value of E (1 psi) for the next run.

This procedure is done until all of the remaining wrap elements act in tension. This final program then represents the captive column for a specific loading pattern and material combination. Typically, for the bending loads considered, about one half of the wraps are removed (see Figures 30 and 31).

CHAPTER 5

EXPERIMENTAL APPARATUS AND PROCEDURES

This chapter describes the apparatus used and procedures followed in experimentally determining simply supported captive column midspan deflection and strain in the core. Four triangular and six square cross section captive columns were tested; all are of the size, shape, and construction as described in Chapter 2.

Deflection Measurement

The deflection of the top cap or caps was measured directly under the load with a Soil Test Inc. dial gauge. The gauge reads in .001 inch increments. The load was continuously applied, .15 inches/minute, by a motorized Dillon Universal machine. The load was read from a 500 pound, 2 pound increment, scale. A 3/4 inch wide composite hardwood block, notched to fit the upper cap(s), transmited the load to the column. This block was positioned at the center of the column, 14 inches from either simply supported renetion (see Figures 16 and 17).

Each column was tested three times with the column rotated clockwise between tests so that a different cap(s) was the top, or load bearing, cap for each test. All of the columns were loaded past the 100 pound point, but not to destruction. All load-deflection test data is presented in Appendix .

Strain Measurement

Experimental core strain was determined by strain gauging two captive column cores.

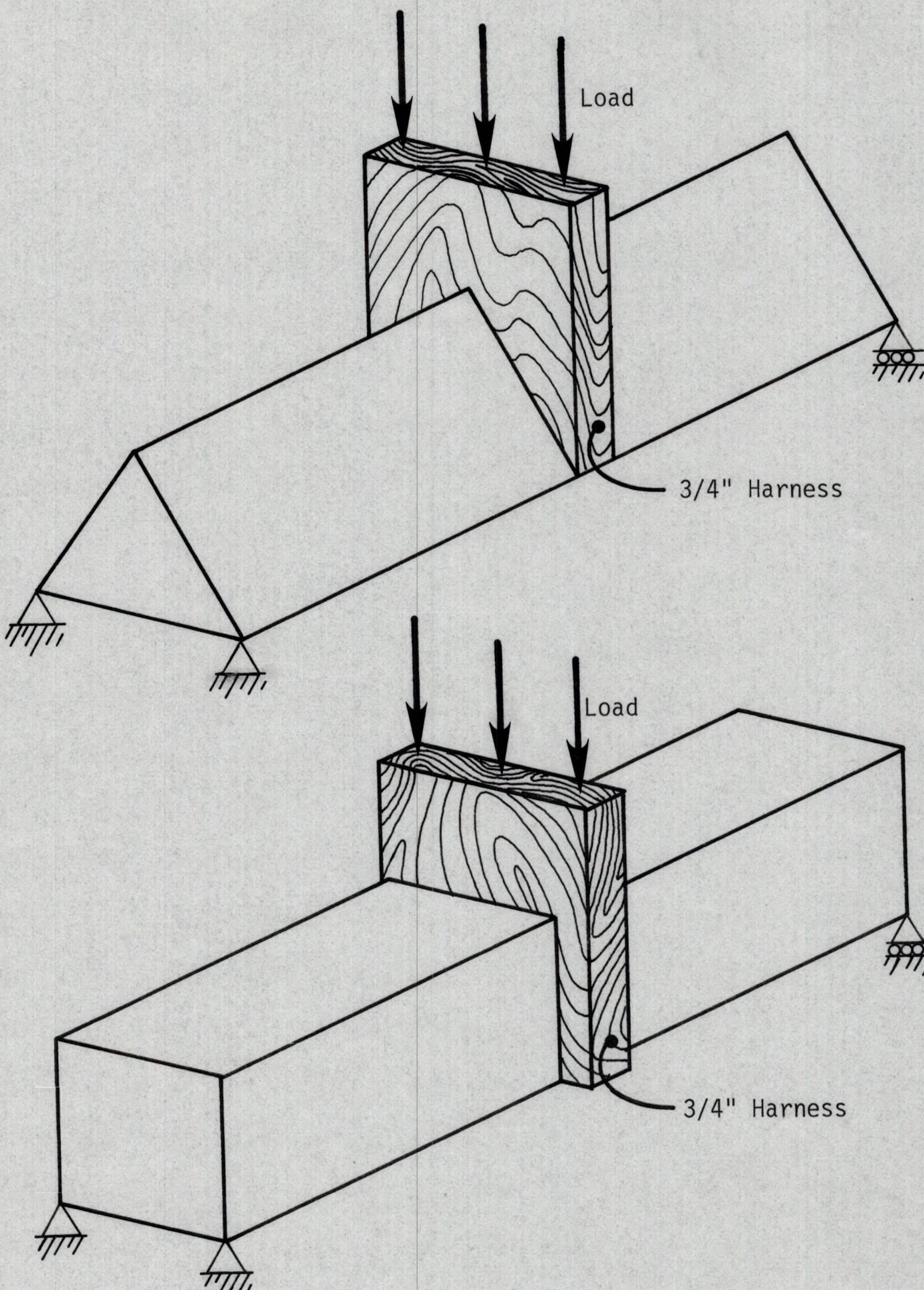


FIGURE 16 - MIDSPAN LOAD APPLICATION

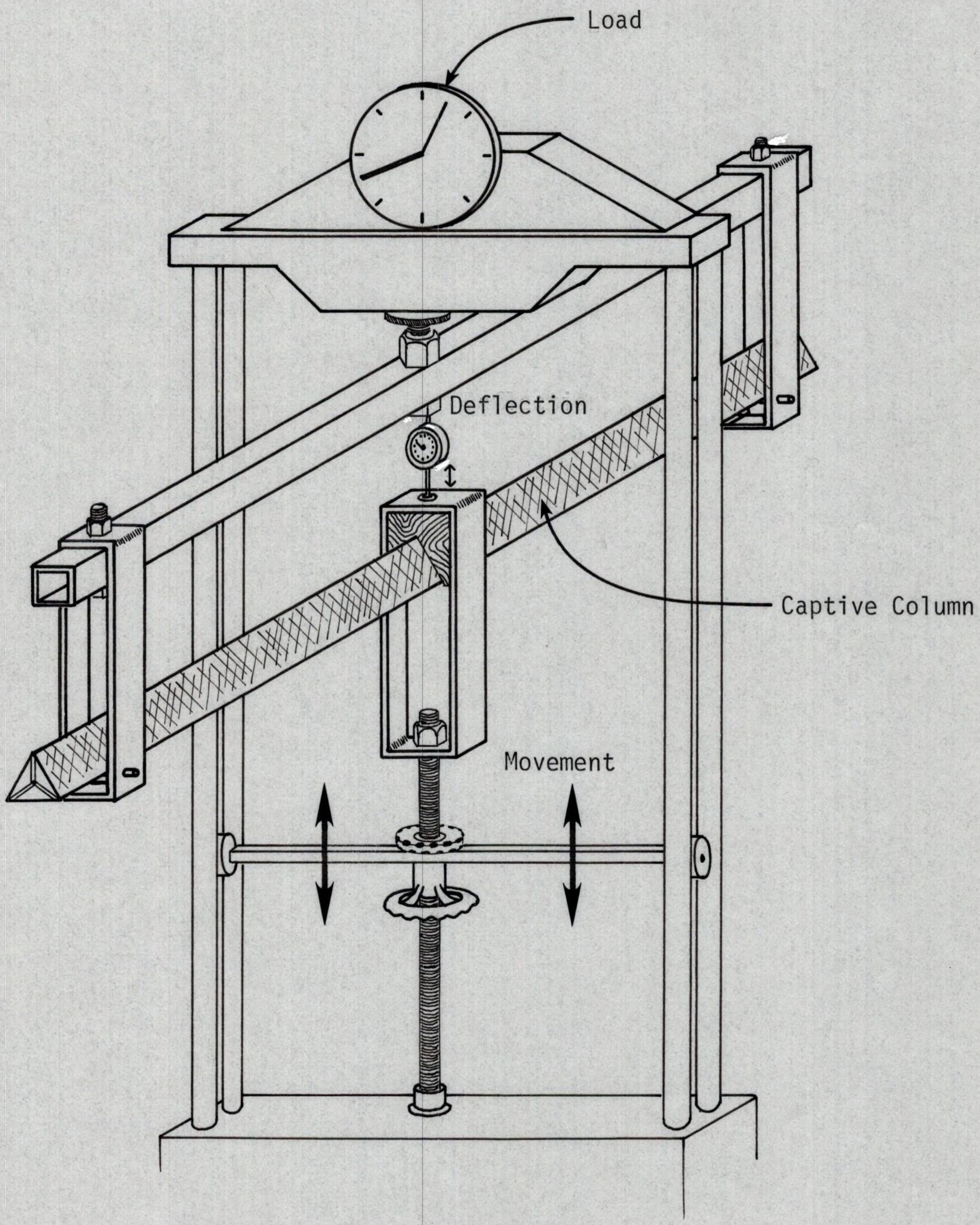


FIGURE 17 - TEST CONFIGURATION FOR LOAD-DEFLECTION AND LOAD-STRAIN MEASUREMENTS

One rectangular strain rosette was epoxied to each of the two, acrylic core, square cross section, captive columns (each of the above mentioned columns were also tested for load deflection data). Each column had one strain rosette in the center of rib A, 3.5 inches from the middle of the column (see Figure 18). The three strain outputs from the rosette -- ε_1, ε_2, ε_3 -- were input into the formulas given in Chapter 3 to calculate the principal core stresses.

Both of these strain gauged captive columns were unique in that they had 3/16 inch acrylic (Plexiglass) cores. Strain gauges were not placed on the other eight captive columns constructed with balsa wood cores because the epoxy used to attach the rosette penetrated into the wood and unpredictably altered its material properties. An acrylic core was chosen because it could readily be strain gauged and, also, it had a modulus of elasticity, similar to balsa wood, in the direction perpendicular to the caps.

Each column was tested for strain eight times. Each time the strain rosette was located in a different position, relative to the load. This was done by first rotating the column clockwise four times and then swinging the column end for end -- positioning the strain rosette on the other side of the load -- before rotating four times again. For clarity the eight different locations are shown in Figure 19. Note that the strain rosette is never removed from the rib to which it was originally attached, it is simply rotated into the eight different positions.

The same strain gauge pattern, and output, could have been achieved by placing eight strain gauges on the column and loading just once. However, strain rosette frugality dictated the use of one rosette and

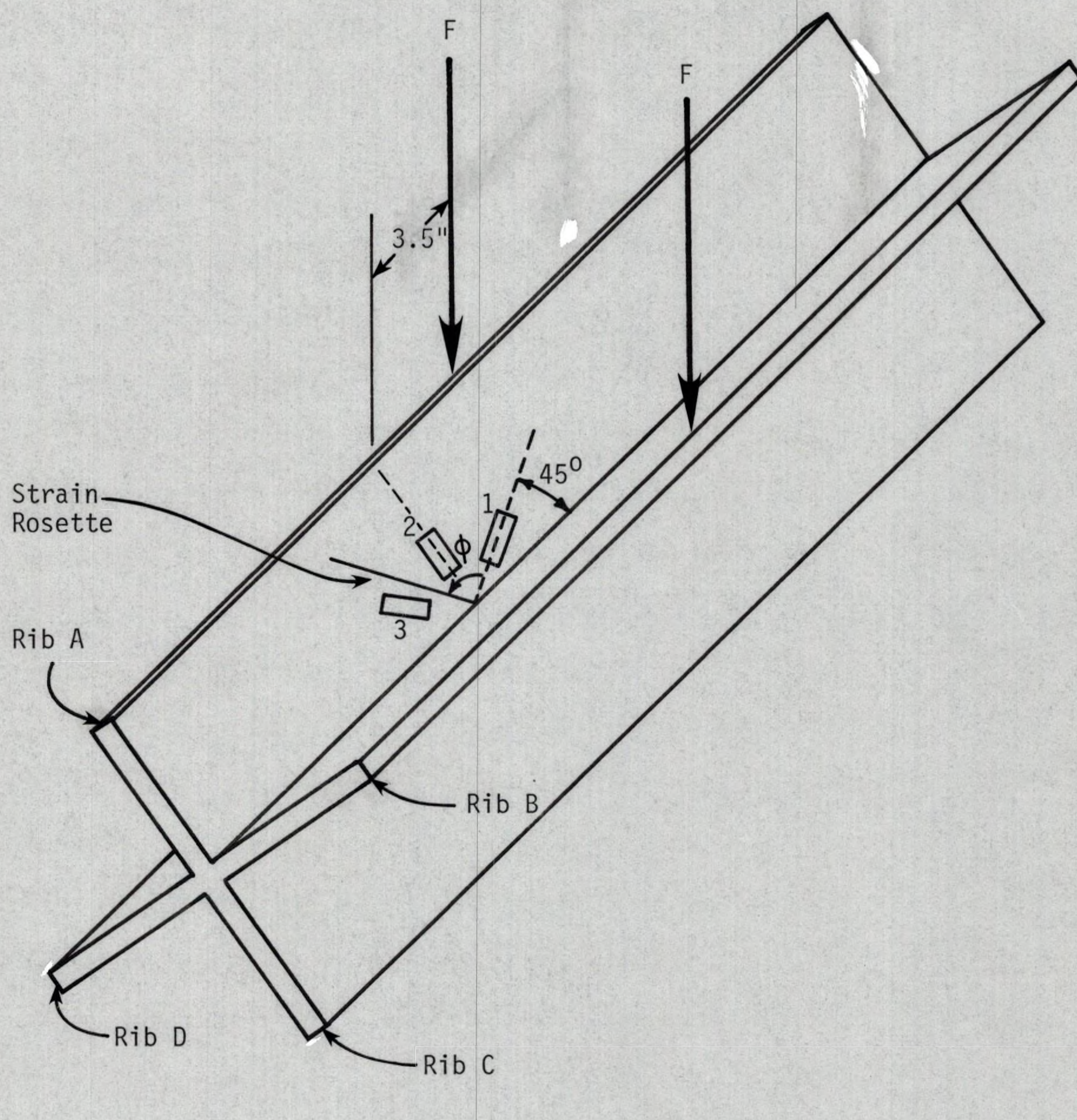
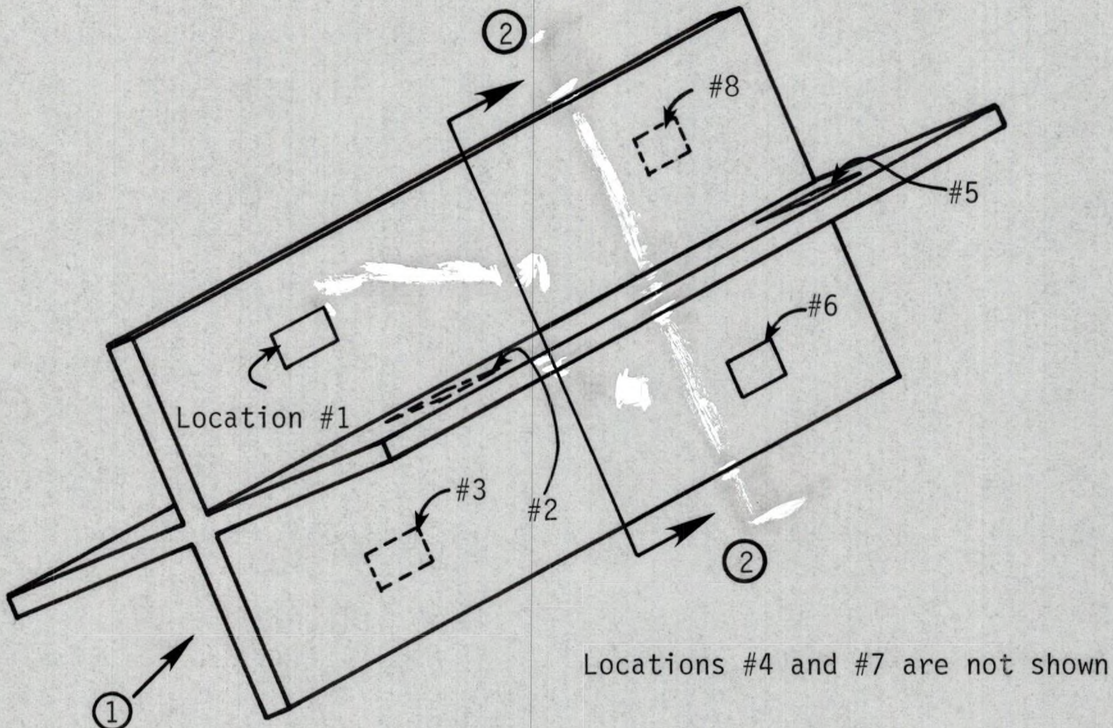


FIGURE 18 - STRAIN ROSETTE ORIENTATION ON THE SQUARE CROSS SECTION CAPTIVE COLUMN



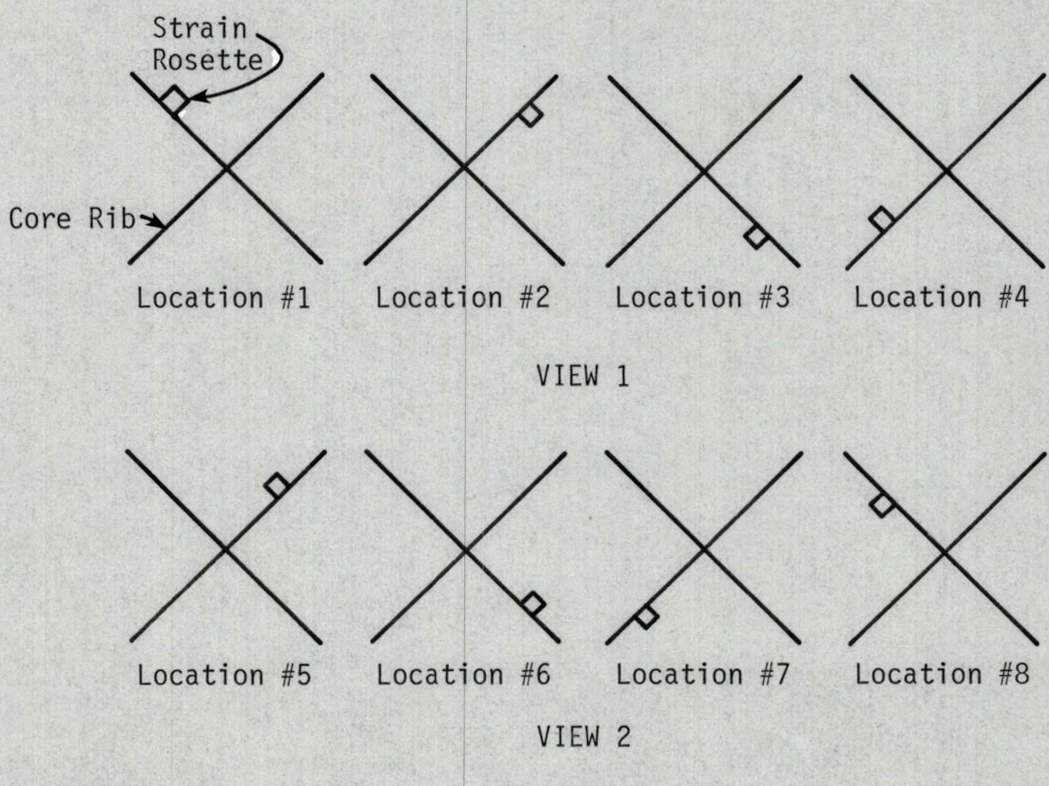


FIGURE 19 - THE EIGHT STRAIN ROSETTE LOCATIONS ON THE SQUARE CROSS SECTION CAPTIVE COLUMN

eight loadings. (For the sagacious reader; four rosettes and two loadings or two rosettes and four loadings would also confer the same amount of strain information.)

The strain rosette outputs from the eight locations -- with three strain outputs per location and loading -- were recorded for two reasons. First, to average the strains from similar locations, thereby improving the experimental data, and second, to "average to zero" the plate bending stresses.

It should be apparent that the strains, and therefore the stresses, are symmetric about the midspan load. In fact, two planes of symmetry exist. One vertical plane passes through the point of load application, while the other vertical plane of symmetry extends along the longitudinal centrodial axis, through the center of the column (see Figure 20). Thus, the strain readings from locations 1 and 5, 2 and 8, 3 and 7, and 4 and 6 (refer to Figure 19) should be equal due to the two planes of symmetry, and are therefore averaged to minimize any deviation due to experimental error. These four averaged strain locations -- I, II, III, and IV -- with each location still having the three strains -- ε_1, ε_2, ε_3 -- are shown in Figure 21. Keep in mind that this is the orientation from view one of Figure 19. Therefore, the locations correspond to locations 1, 2, 3, and 4 of that figure. Observe that since strains are symmetric about the load, locations 5, 6, 7, and 8 could have been chosen without affecting the data, or the eventual comparison to the computer stresses. The three averaged strains at each of the four locations of Figure 21 are now used to compute the two principal stresses and one principal direction at each location. However, a problem exists.

The strain gauges, unlike the plane stress computer elements, account for both possible types of bending out of the plane of

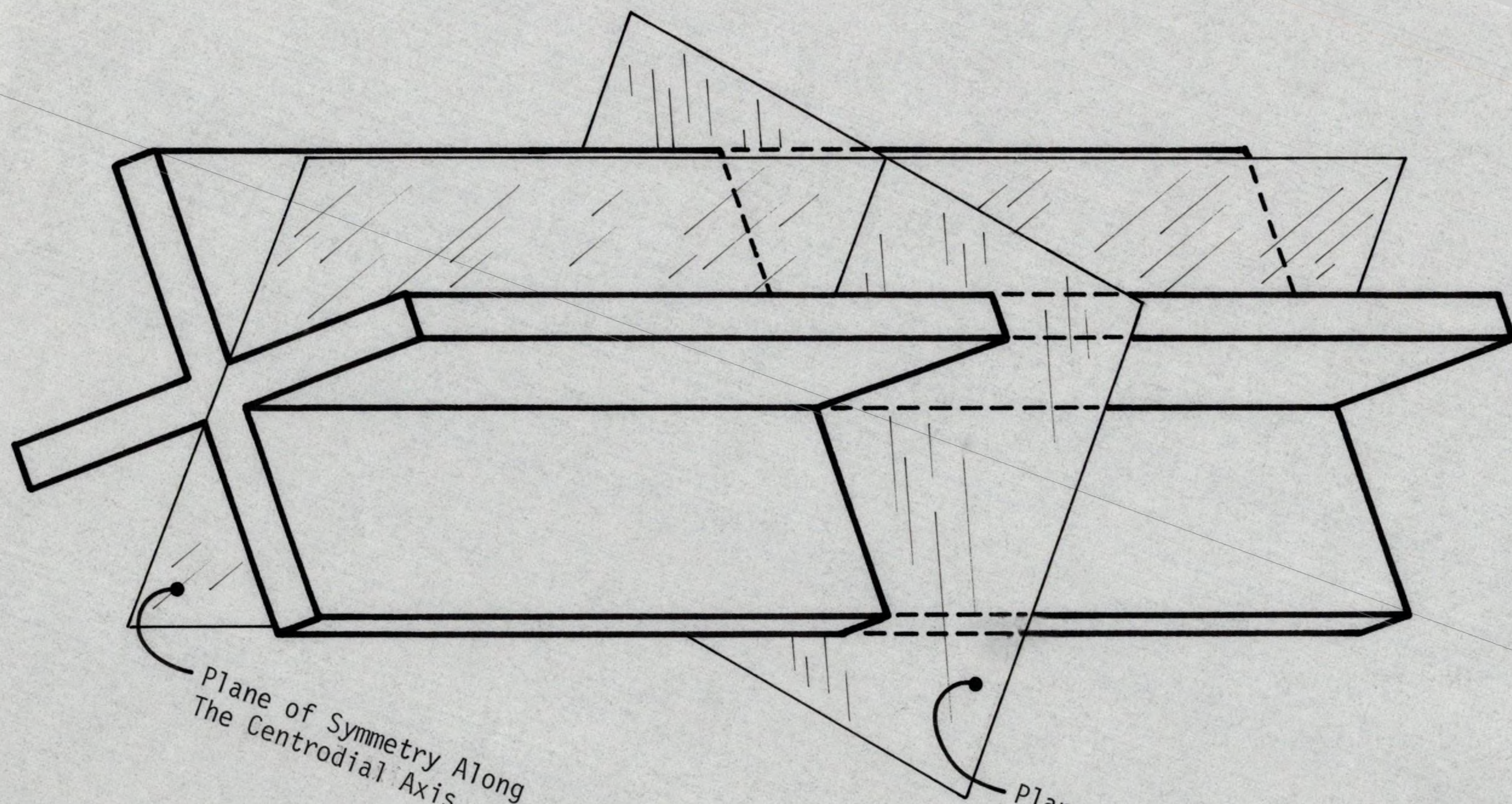
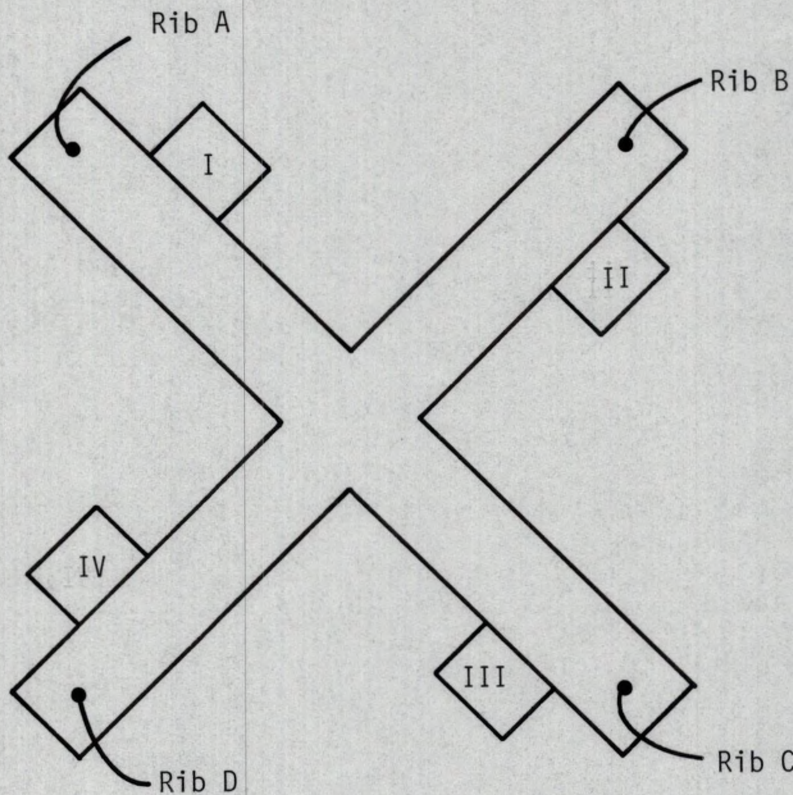


FIGURE 20 - THE TWO PLANES OF STRAIN SYMMETRY FOR THE SQUARE CROSS SECTION CAPTIVE COLUMN



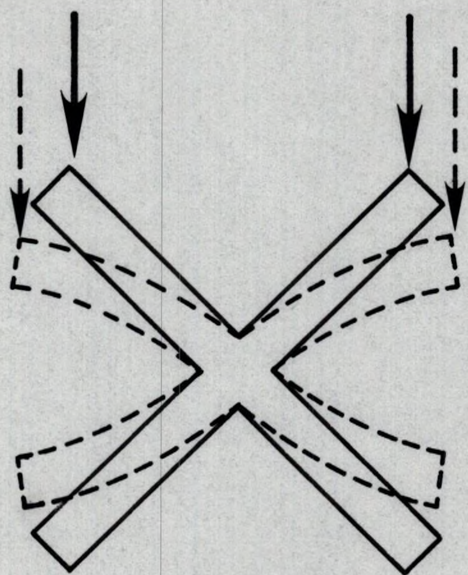
VIEW 1

FIGURE 21 - THE FOUR AVERAGED STRAIN LOCATIONS

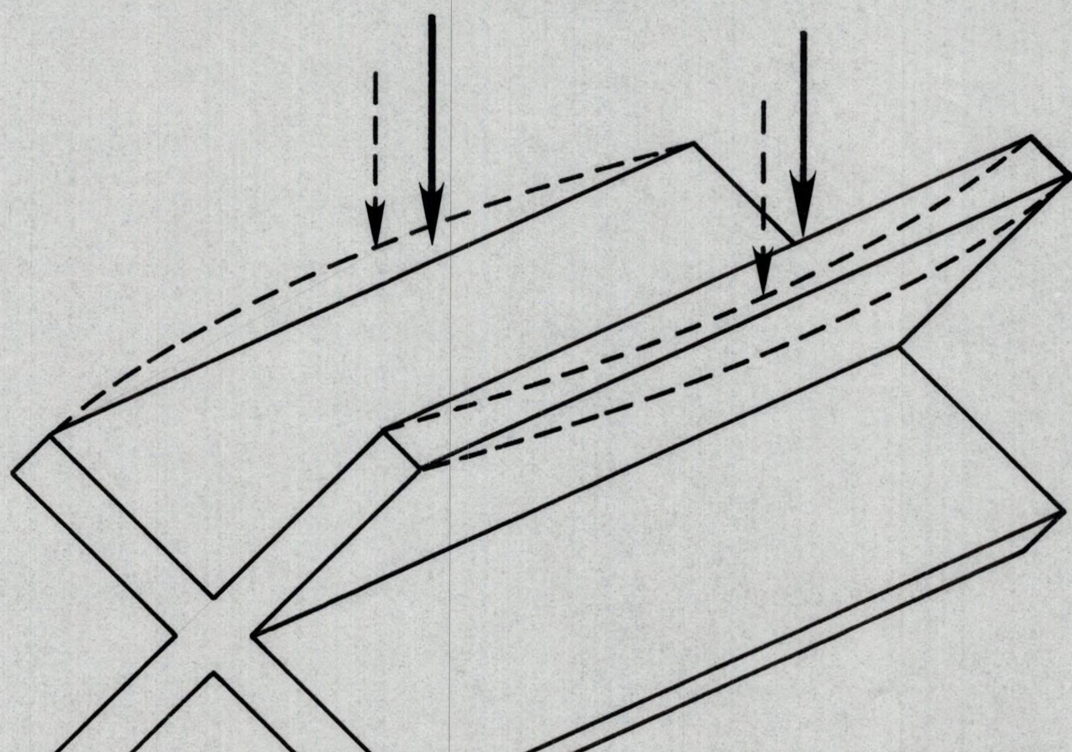
the core rib; that is, plate bending (see Figure 22). In order to compare the experimental results with the computer results, this plate bending, detected by the strain gauges, must be "subtracted" or averaged out from the principal stresses to leave only the in-plane stresses. The finite element plane stress core, as modelled in the computer simulation, can compute only in-plane stresses (see Chapter 4 for further discussion). The strains induced by plate bending are removed from the experimental data by averaging the strains of locations I and II, and locations III and IV of Figure 23A.

Because, as shown in Figure 20, a vertical plane, through and along, the column's centrodial axis is a plane of strains symmetry, Figure 23A can be shown as Figure 23B. Observe that experimentally determined strains are now available for each side of ribs A and D and/or ribs B and C, whichever is preferable. Averaging the strains of location I and II cancels the tensile bending strain of location I with the compressive bending strain of location II or vice versa, yielding values for only the in-plane strain. This averaging technique also applied for locations III and IV. Principal stress and directions are then determined from these strains via the computer program in Appendix F. These stresses are now directly comparable to the computer plane stress element output.

To summarize, three strains at eight different locations are reduced to two principal stresses and one principal angle at two locations. One location, above the neutral axis, provides the in-plane principal stresses and direction in rib A and/or rib B 3.5 inches on either side of the load. While the other location, below the neutral axis, provides the in-plane principal stresses and direction in rib C and/or rib D 3.5 inches on either side of the load. This condensation of raw strain data from eight positions



A. Transverse Bending



B. Longitudinal Bending

FIGURE 22 - TWO TYPES OF BENDING OUT OF THE PLANE OF THE CORE RIBS

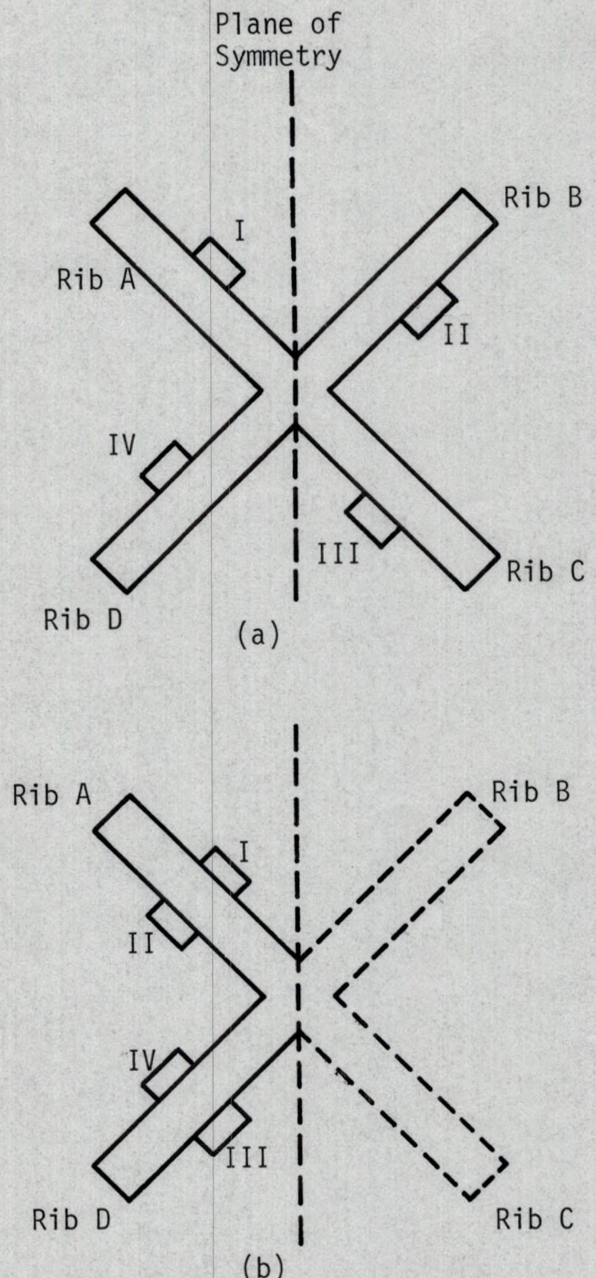


FIGURE 23 - REARRANGEMENT OF LOCATION II AND III BECAUSE OF SYMMETRY

down to principal stresses and directions at two positions is possible because of the two planes of symmetry, with respect to strain, through the square cross section captive column.

CHAPTER 6

RESULTS

The results of the finite element computer model in predicting captive column midspan deflections and captive column core stresses are discussed in this chapter. Also discussed are the loads experienced by the captive column caps and wraps, as analytically computed and computer simulated; and how these elements, with the information from this research effort, can now be designed given the captive column load condition.

Additionally, the load deflection curves for four captive columns, that were tested to failure, are presented. Finally, discussions concerning the ability of the finite element computer model to simulate captive column pretension and column type loading are addressed.

Deflection Comparisons

Table 5 compares the slopes of the deflection versus load curves (lines) for the ten captive columns that were experimentally tested, computer modelled, and analytically calculated. Note that Table 5 does not list specific deflections for any given load. Rather, each number represents the slope of a linear deflection versus load line which passes through the origin. A specific deflection, in inches, is calculated by multiplying the load in question times the applicable number from Table 5. Recall that this is the deflection of the top cap(s), directly under the load, at the midspan of a captive column. For example, from Table 5, a 100 pound midspan load deflects the top 1/8 inch diameter steel cap of a triangular cross section balsa wood core column -- .157 inches experimentally, .133 inches in the computer model, and .066 inches theoretically.

TABLE 5

THE SLOPES OF THE DEFLECTION VERSUS LOAD CURVES FOR THE TOP CAP(S) OF A 28 INCH LONG CAPTIVE COLUMN, LOADED AT THE MIDSPAN

Columns	Triangular Cross Section			Square Cross Section		
	Experimental (10^{-3} in/lb)	Computer (10^{-3} in/lb)	Theoretical (10^{-3} in/lb)	Experimental (10^{-3} in/lb)	Computer (10^{-3} in/lb)	Theoretical (10^{-3} in/lb)
Balsa Wood Core:						
1/8 inch dia. steel caps	1.57	1.33	0.66	1.71	1.38	0.66
1/8 inch dia. fiberglass caps	4.67	4.18	3.27	4.44	4.07	3.27
1/4 inch dia. steel caps	0.81	0.73	0.148	0.76	0.75	0.148
1/4 inch dia. fiberglass caps	1.67	1.49	0.79	1.74	1.52	0.74
Acrylic Core:						
1/8 inch dia. steel caps	NT	NT	NT	1.04	0.92	0.62
1/8 inch dia. fiberglass caps	NT	NT	NT	2.93	2.6	2.5

NT = Not Tested

The data is presented in slope format, rather than deflections for a specified load, because each of the three cases -- experimental, computer, and theoretical -- generate linear deflection versus load curves (lines) passing through the origin. This is predictable for both the linear computer model and the theoretical calculations, but not intuitively obvious for the experimental case.

The experimental data that was recorded during the ten captive column tests plot into a definite linear relationship. Recall that the columns were not loaded to failure (Appendix B tabulates all of the deflection versus load data for the ten columns). This linear experimental relationship was quantified by a least squares analysis. The analysis generated the slope of the best fit line through the recorded data points. This computed slope is presented in Table 5 along with the computer derived slopes and the theoretical slopes.

A number of observations concerning the deflections comparisons can be made by examining Table 5.

1. The computer derived slope for each column (except for one case) is closer to the actual experimental slope than to the theoretical slope.

Table 6 shows the percent difference between the slopes of the three cases. Observe that the computer derived slope averages 11.6 percent and 10.4 percent less than the actual experimental slope (this compares to 55.5 percent and 56.6 percent difference between the experimental and theoretical slopes). This says that at loads below the yield point of the column the computer derived deflections are approximately 10 to 12 percent less than the actual deflections. This is considered good agreement and lends significant credibility to the finite element computer model for predicting captive column deflections.

TABLE 6

PERCENT DIFFERENCES BETWEEN THE THREE SLOPE CASES OF TABLE 5

Columns	Triangular Cross Section			Square Cross Section		
	Experimental vs. Computer	Experimental vs. Theoretical	Computer vs. Theoretical	Experimental vs. Computer	Experimental vs. Theoretical	Computer vs. Theoretical
Balsa Wood Core:						
1/8 inch dia. steel caps	15.3	58.0	50.3	19.3	61.4	52.2
1/8 inch dia. fiberglass caps	10.5	30.0	21.8	8.3	26.4	19.7
1/4 inch dia. steel caps	9.8	81.5	79.5	1.3	80.4	80.5
1/4 inch dia. fiberglass caps	10.7	52.6	47.0	12.6	58.1	52.0
AVERAGE:	11.6	55.5	49.7	10.4	56.6	51.1
Acrylic Core:						
1/8 inch dia. steel caps	NT	NT	NT	11.5	40.4	32.6
1/8 inch dia. fiberglass caps	NT	NT	NT	11.3	14.7	3.8
AVERAGE:				11.4	27.6	18.2

NT = Not Tested

2. The order of slope magnitude for each column is experimental, computer, and theoretical. Experimental always having the largest slope (i.e. largest deflection for a given load).

Ideally, experimental, computer, and theoretical deflections would have been equal for each column tested. However, as seen in Table 5, this is not true. There relationship is, however, explainable in light of the assumptions inherent in the determinations of each of the slope. Primarily, the assumption of the computer analysis more closely approximate the real captive column behavior than the assumption inherent in the theoretical analysis.

The theoretical calculations assume ideal conditions. That is, no slippage occurs between the glued core and caps; the caps remain equidistant from each other at all times; the column retains its original geometry during loading; there is zero local deformation under the point of load application; and it assumes an ideal cross section, one where the strain surface remains plane. In short, this theoretical calculation, as applicable to the captive column, is probably not reasonable, ever attainable, but still useful. It serves as a bench mark by showing the least possible deflection, for a given loading pattern, of a given captive column.

The computer model is a step, and a significant step, toward modelling the real column. By defining specific properties for each incremental volume, via the finite element, the cap, core, and wrap deform, translate, and rotate according to the loads placed upon them while, simultaneously, satisfying the given material properties of the captive column element it models. Moreover, the cross section of the column is not constrained to remain symmetric.

3. The captive columns constructed with 1/4 inch diameter caps have smaller slopes -- deflect less for a given load -- than comparable captive columns with 1/8 inch diameter caps. Also, captive columns constructed with steel caps have smaller slopes than comparable columns with fiberglass caps.

This comparison verifies what is already known via beam theory. An increase in the moment of inertia, or the modulus of elasticity, of the load carrying area increases the modulus of rigidity of the beam.

However, a five fold increase in the modulus of elasticity of the caps -- 6×10^6 for fiberglass to 30×10^6 for steel -- does not, as theory predicts, decrease the experimental slope by five times. Instead, the steel capped columns deflect only one-half to one-third the amount that comparable fiberglass capped columns do at a given load. Additionally, increasing the moment of inertia of the caps 4.5 times -- 0.02298 in^4 to 0.1023 in^4 -- via 1/8 inch diameter caps to 1/4 inch diameter caps, does not, as theory predicts, decrease the experimental slope by a factor of 4.5. Rather, the slope decreases by a factor of 1.7 to 2.7. Note that these comparisons are done on the captive columns with balsa wood cores. It is assumed, as suggested in Table 5, that the acrylic core assists the caps in carrying part of the applied load. Therefore, the acrylic core comparisons introduce another variable and are, for that reason, neglected here.

4. The columns built with 3/16 inch acrylic cores have smaller slopes than comparable columns built with 3/16 inch balsa wood cores.

The acrylic core captive column was built for two reasons. First, it could be strain gauged (see Chapter 5), and second, because it had a larger modulus of elasticity than balsa wood in the direction parallel to the caps; 450,000 psi compared to 13,400 psi. This longitudinal modulus of elasticity of the core was thought to have an impact on the columns resistance to bending. By testing columns whose only difference is this longitudinal modulus of elasticity, the modulus of elasticity effect can be seen in the comparison of Table 5.

Note that the acrylic core columns deflect approximately one-third the amount of comparable balsa wood core columns. Also keep in mind that fiberglass and steel capped acrylic core columns weigh, respectively, 2.2 and 3.4 times their comparable balsa wood core columns.

5. The experimental slopes, in Table 5, for the triangular and square captive columns agree to within eight percent of each other. Likewise for the computer and theoretical slopes. For example, a triangular captive column with 1/4 inch diameter steel caps and a balsa wood core has a slope of $.81 \times 10^{-3}$ inch/lb. While its square cross section counterpart has a slope of $.76 \times 10^{-3}$ inch/lb; yielding a 6.1 percent difference in the deflection versus load slope. A similar comparison can be made between the computer and theoretical slopes for these columns. This slope agreement between triangular and square cross section captive columns that have identical caps is no coincidence. The triangular and square cross section columns were designed so that the caps' moments of inertia, about the neutral axis, are equal for both geometries.

This comparison demonstrates two points. First, the EI product, modulus of elasticity times moment of inertia, of the caps -- and only the caps -- determines the slope of the midspan deflection versus load curve for beams with balsa wood cores. The geometry of the columns cross section is of secondary importance. The geometry-square or trianuglar, etc. is important primarily in the design of the most efficient I for the caps; That is, in the design of a column with the largest ratio of cap moment of inertia to a column weight per lineal foot. Second, as shown in the two computer columns of Table 5, the computer program accurately models this phenomenon.

Notice that the theoretical slopes also show this relationship. However, this is to be expected from examining the theoretical calculation of the $(EI)_{eq}$ for a column:

$$(EI)_{eq} = E_{core}I_{core} + E_{cap}I_{cap}$$

This $(EI)_{eq}$ is used in the following formula to determine deflections and, ultimately, slopes (see Chapter 3 for further details).

$$\delta = \frac{FL^3}{48(EI)_{eq}}$$

where: E_{core} = modulus of elasticity of the core

E_{cap} = modulus of elasticity of the caps

$(EI)_{eq}$ = calculated EI equivalent of the core and cap

F = load applied to the captive column

I_{cap} = moment of inertia of all the caps about the columns neutral axis

I_{core} = moment of inertia of the core about the columns neutral axis

L = length of the captive column

δ = midspan deflection

The calculated $E_{cap}I_{cap}$ value for balsawood core columns is typically 100 times the $E_{core}I_{core}$ value for columns with fiberglass caps, and 450 times the $E_{core}I_{core}$ value when steel caps are used. Thus, it is easy to see why, in the theoretical calculations, that the core ($E_{core}I_{core}$) has little impact upon the determination of $(EI)_{eq}$, and therefore, little impact upon the deflection (or slope) calculation.

Therefore, in summary, when comparing square and triangular captive columns that have identical caps (E_{cap}), and identical cap moments of inertia (I_{cap}), it's expected -- due to the insignificance of the balsa wood $E_{core}E_{core}$ term -- that the theoretical, experimental, and computer slopes of the two columns be similar.

Core Stress Comparisons

The results of the computer model derived core stresses and the experimentally determined core stresses are compared in Figures 24 through 27. Recall that these plane stress elements diagram the stresses at a point 3.5 inches to the left of a 100 pound midspan load. One of the plane stress elements is centered in rib A, one of the two ribs above the neutral axis, while the other plane stress element is centered in rib D, one of the two ribs below the neutral axis (see Chapter 5). Three points should be made when reviewing these figures.

First, good correlation exists between the computer model stresses and actual stresses. Not only does the order of magnitude of the principal stresses agree but the orientation of the two dimensional stress elements agrees. Indeed, for the case of the acrylic core with steel caps, Figure 24, computer and experimental principal stresses differ by no more than twenty percent, while principal directions both agree to within eleven percent. In the case of the acrylic core with fiberglass caps, Figure 25, the computer and experimental stresses differ by 95 percent for the tensile stress in rib A and 74 percent for the compressive stress in rib D. However, the other principal stresses on each element agree very well, with less than eight percent difference. Additionally, the principal directions differ by 2 percent for rib A and 28 percent for rib D.

The second point to be made regards the magnitude and direction of the normal and shear stresses. Shown in Figures 26 and 27 are the same two

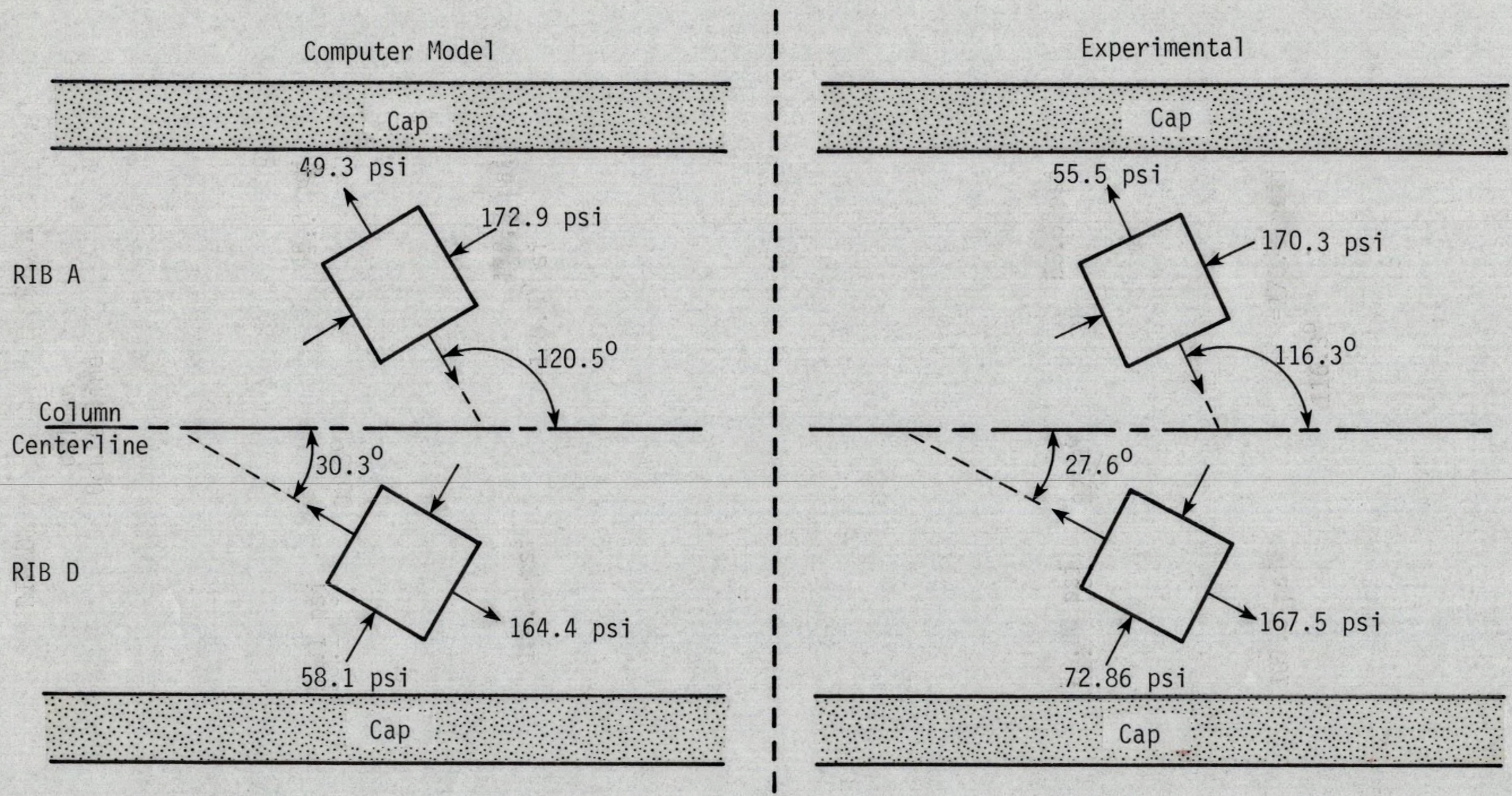


FIGURE 24 - PRINCIPAL CORE STRESSES FOR A 3/16 INCH ACRYLIC CORE CAPTIVE COLUMN WITH 1/8 INCH DIAMETER STEEL CAPS

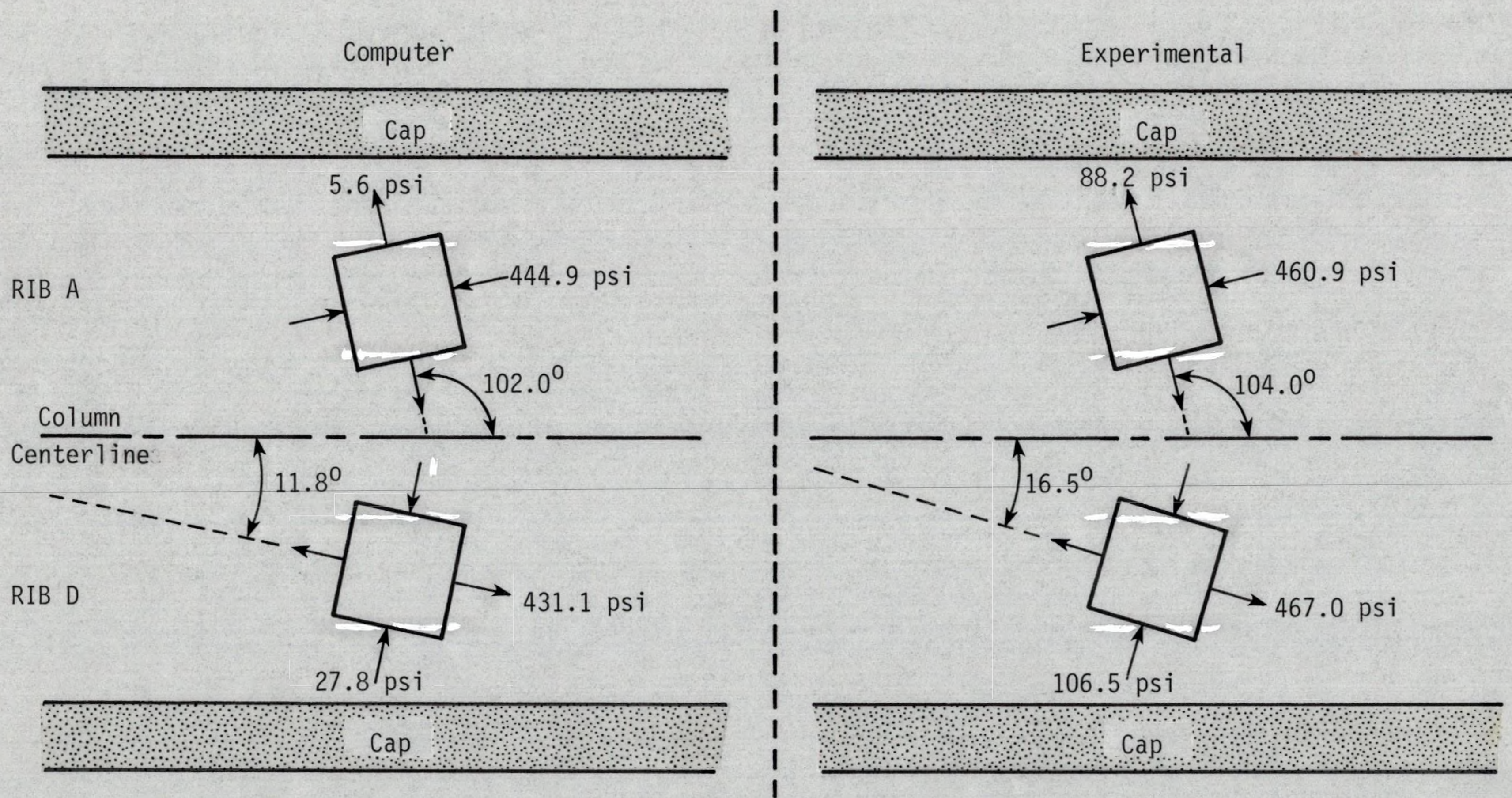


FIGURE 25 - PRINCIPAL CORE STRESSES FOR A 3/16 INCH ACRYLIC CORE CAPTIVE COLUMN WITH 1/8 INCH DIAMETER FIBERGLASS CAPS

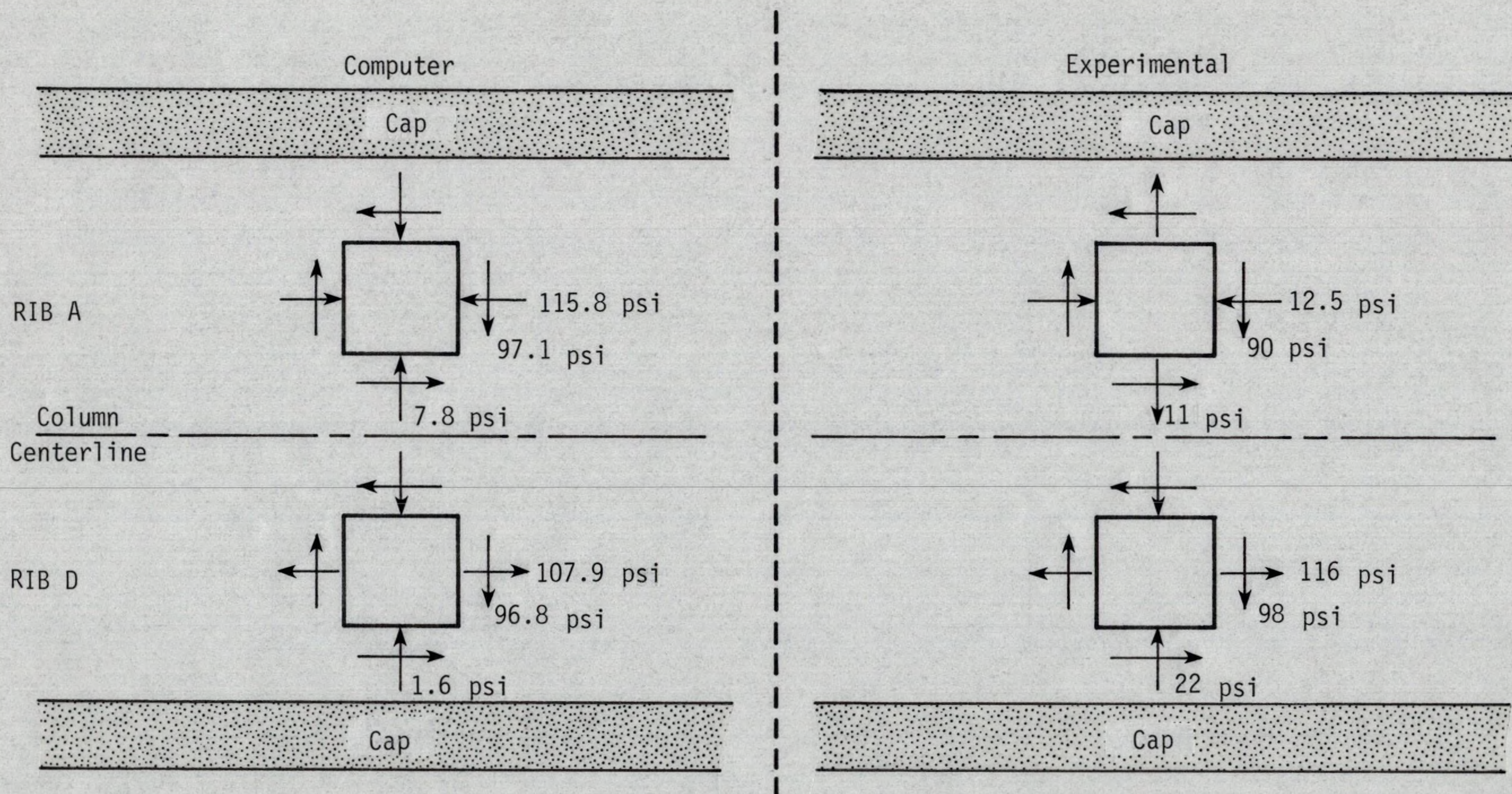


FIGURE 26 - NORMAL AND SHEAR STRESSES FOR A 3/16 INCH ACRYLIC CORE CAPTIVE COLUMN WITH 1/8 INCH DIAMETER STEEL CAPS

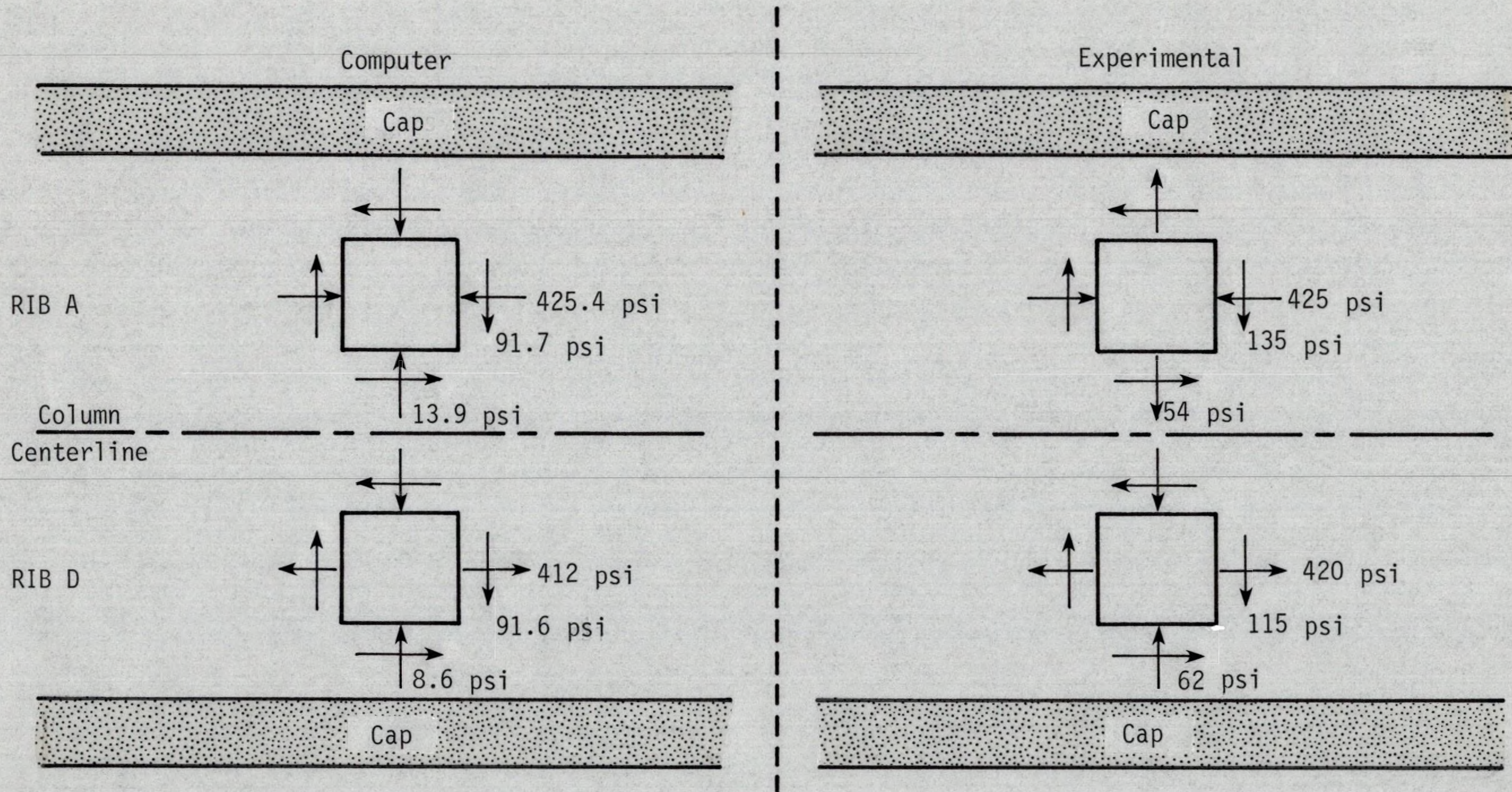


FIGURE 27 - NORMAL AND SHEAR STRESSES FOR A 3/16 INCH ACRYLIC CORE CAPTIVE COLUMN WITH 1/8 INCH DIAMETER FIBERGLASS CAPS

dimensional plane stress elements of Figures 24 and 25. However, the elements have been rotated by the use of Mohr's circle, so they are aligned, or square, with the core. Now instead of principal stresses, normal stresses and shear stresses are diagramed.

Observe that the normal stress perpendicular to the caps is in all cases relatively small (1.6 to 62 psi). Also observe that the computer model indicates that this stress is, in all four cases, compressive while the experimental results indicate a compressive stress in two out of the four cases. This discrepancy in the direction and magnitude of experimental stresses can possibly be attributed to the averaging technique, used to "subtract" or average out the two possible modes of plate bending stress (see Chapter 5), or errors in the computer model as well. However, for design purpose the determination of this tensile or compressive stress is possibly insignificant. Since, as mentioned before, the stress magnitude is small, relative to the other normal stresses at 100 lb. and relative to the tensile and compressive strength of balsa wood, for both the computer and experimental results. Also, observe that the magnitude and direction of normal stresses parallel to the caps agree to within 7.9 percent, while shear stresses compare to within 7.3 percent for the steel capped column and 47.2 percent for the fiberglass capped column.

Implied by the good stress comparisons of Figures 24 through 27 is the potential usefulness of the finite element model in future captive column core designs. Stresses, and therefore forces, in any direction, at any location, in the core can be predicted. Specifically, the stresses in the core due to inward cap buckling, beam bending, and shear for any given loading pattern can be analyzed and ultimately designed for.

The third, and decisive, point to be discussed is what the plane stress diagrams say concerning the purpose and function of the core in the captive column concept. These observations are now outlined.

Computer and experimental stresses perpendicular to the caps are observed to be insignificant. Initially, it was hypothesized that inward cap buckling placed the largest loads on the core. Promoting this line of reasoning was the current construction practice of orientating the balsa wood grain perpendicular to the caps. Balsa wood, having the largest modulus of elasticity in the direction of the grain -- 400,000 psi compared to 13,400 psi against the grain -- is, naturally, the strongest or most rigid in this direction. Now, in light of the core stress analysis, this reasoning, but not the construction practice, appears in jeopardy. Captive column construction experience proves, without doubt, that the grain of balsa wood cores must run perpendicular to the caps. Any other orientation of the balsa wood yields a captive column core that cannot even withstand the stress imposed upon it during wrapping. That is, the column torques, bends, and deforms beyond use during the wrapping process. Therefore, the following theory of balsa wood orientation is presented for discussion and future analysis.

The grain of the balsa wood is orientated perpendicular to the caps primarily to withstand the stress induced in the core due to wrap pretension. Recall that neither the computer model stresses nor the experimental stresses account for the possibility of prestress in the core due to the wrap pretension. Both sets of data only model or record the core stresses induced because of the load applied to the column. Therefore, stresses in the core because of wrap pretension were neither computer modelled nor experimentally measured. Modelling the wrap pretension in the computer program was attempted but, as explained under subtitle -- wrap pretension -- of this chapter, was not successful.

In the case of the experimentally determined stress, the strain gauges are zeroed when the core is in the prestressed condition. Understandably, then, the gauges measure only the strain in the core due to the applied loading. That is, if the initial condition of the core is one of a nonzero stress, the strain gauges will not detect this stress because they are forced to assume zero strain, and therefore zero stress, at the no load condition.

However, at the conclusion of the experimental testing, all of the Kevlar wraps on the square, acrylic core, 1/8 inch diameter steel capped captive column were cut. The strain gauges were set at zero strain, and therefore zero stress, in this uncut, no load, mode. Therefore, cutting the wraps relieved the initial prestress enabling the zeroed strain gauges to measure, in a negative direction, this wrap induced core prestrain (prestress). The results show a two dimensional principal core stress element, for rib A, orientated at 78.4 degrees with the horizontal and having normal tensile stresses of 408 psi and 632 psi (actually wrap induced compressive stresses). These numbers were compared with those of rib A, Figure 24, which are for a 100 pound midspan load. Notice that the magnitude of the normal wrap induced core prestress, for this, the only core tested, is 3.7 and 7.4 times greater than the normal core stresses resulting from a 100 pound midspan load.

Perhaps, then, the construction and material properties of the core are dependent upon the wrapping pretension rather than the load induced normal stresses.

<u>Axial Cap Forces</u>

Table 7 compares the computer program axial cap forces, at a point directly under a 100 pound applied load, to theoretically calculated axial

TABLE 7

MAXIMUM AXIAL CAP FORCES FOR A 100 POUND MIDSPAN LOAD

Column	Triangular Cross Section (Top Cap)		Square Cross Section (Top Caps)	
	Calculated (lb)	Computer (lb)	Calculated (lb)	Computer (lb)
Balsa Wood Core:				
1/8 inch dia. steel caps	431	412	264	252
1/8 inch dia. fiberglass caps	431	405	264	251
1/4 inch dia. steel caps	431	386	264	234
1/4 inch dia. fiberglass caps	431	404	264	246
AVERAGE:	431	401	264	245
Acrylic Core:				
1/8 inch dia. steel caps	NT	NT	264	227
1/8 inch dia. fiberglass caps	NT	NT	264	171
AVERAGE:			264	199

NT = Not Tested

cap forces. This calculation assumes that the caps carry the entire applied load. That is, the core and wrap are insignificant as load bearing members.

The calculation to determine this cap force (F_C) is computed as follows:

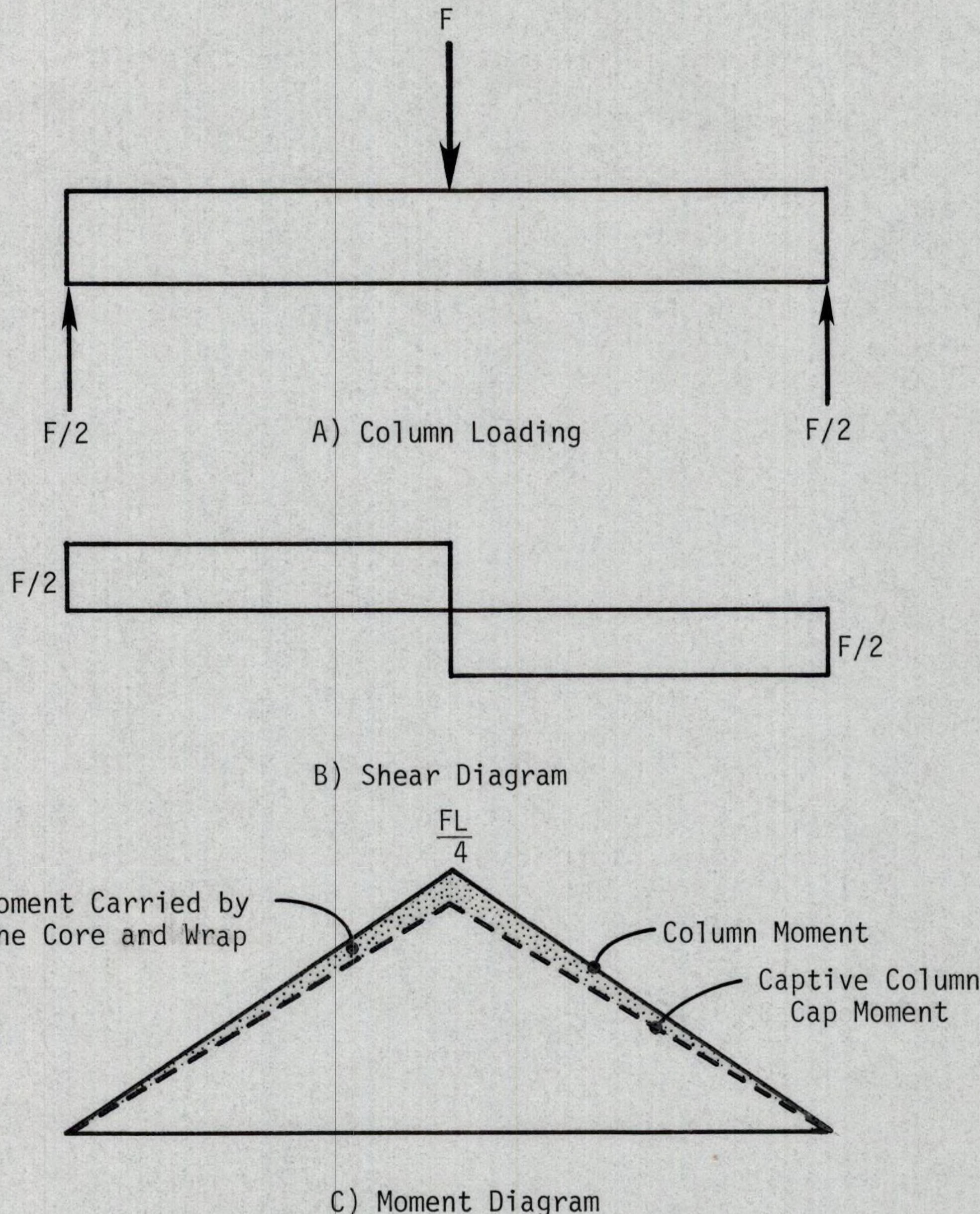
$$\left(\frac{F}{2}\right)\left(\frac{L}{2}\right) = M = N \cdot F_C \cdot D \qquad (1)$$

where:
- F = applied load
- L = length of the captive column
- M = moment
- N = number of caps above the neutral axis
- F_C = axial force in the caps
- D = distance between caps (i.e. distance between the lines of action of the two forces; a couple)

In effect, this calculation assumes that the caps form a couple of magnitude M which develops the internal resisting moment.

The comparisons of Table 7 indicate three points.

1) The computer model derived axial cap loads agree to within 10.4 percent of the calculated axial cap loads. This excellent agreement proves, as hypothesized, that the caps form a couple of magnitude $M = N \cdot F_C \cdot D$ forming the internal resisting moment. Further, an extension of this line of reasoning says that the balsa wood core contributes very little to the load carrying capacity of the column. This can be shown more clearly by two simple diagrams. First, shown in Figure 28 are typical shear and moment diagrams for a beam (captive column) carrying a mid-span load. Superimposed on the beam moment diagram, Figure 28C, is the computer calculated moment diagram for just the captive column caps. The small shaded area on the moment diagram represents that minute moment which is



<u>FIGURE 28</u> - BEAM LOADING PLUS THE CORRESPONDING SHEAR AND MOMENT DIAGRAMS

not carried by the caps, and therefore, must be carried by other components of the captive column.

Second, Figure 29A diagrams the bending stress distribution through a captive column cross section. Note the large stress concentration at the caps. This stress pattern is quite different from the linear distribution shown in Figure 29B for beams constructed of one material.

2) From point 1 above, it is apparent that a method now exists to calculate the axial force in the caps for a given loading pattern. This makes it possible to design the caps of a captive column so that the cap stress is below the yield stress of the cap material.

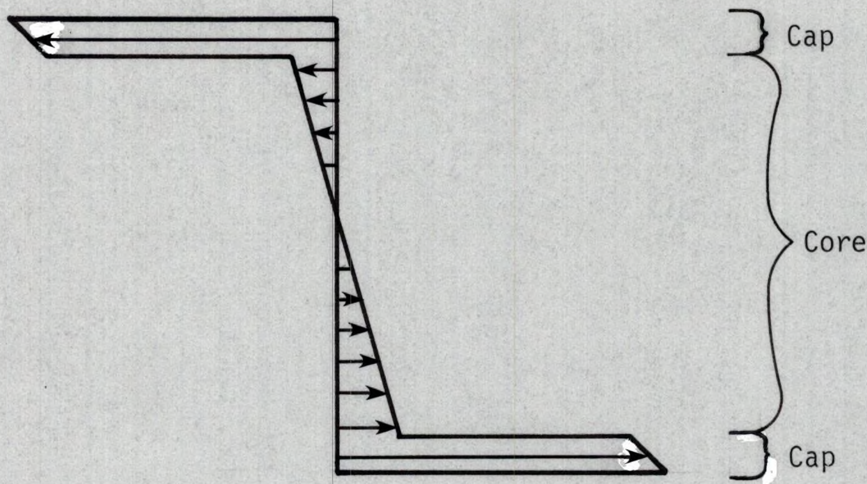
For instance, from Table 7, the caps of the captive column with 1/8 inch diameter steel caps, balsa wood core, and a square cross section will experience an axial force of approximately 264 pounds per cap for an applied midspan load of 100 pounds. This creates a normal stress of:

$$\frac{F_C}{A} = \frac{264 \text{ lbs}}{\frac{\pi(1/8)2}{4}} = 21{,}512 \text{ psi} \qquad (2)$$

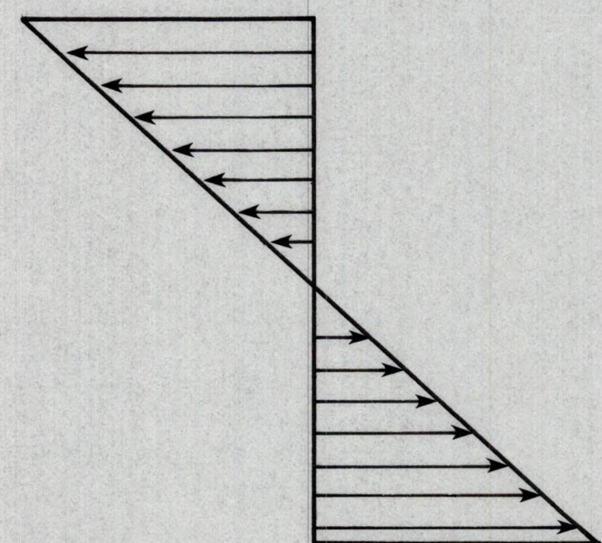
which is well below the 60,000 psi tensile or compressive yield stress of steel. Therefore, if the 100 pound midspan load is the largest load expected on this column the diameter of the caps could be reduced or a different, perhaps lighter, material, with a smaller yield stress, could be used for the caps. Also, this design approach is conservative.

3) The third point to be discussed from Table 7 is the large difference, 24.6 percent, between the computed and computer derived axial cap forces for the columns built with acrylic cores.

Recall that the calculated forces of Table 7 assume, via formula 1, that the entire applied load is carried by the caps. In the computer program this constraint is not made. Therefore, the difference between the



A) Bending stress distribution of
a captive column cross section



B) Cross section showing a typical
bending stress distribution for
a beam constituted of one material

FIGURE 29 - BENDING STRESS DISTRIBUTION

calculated and computer derived axial forces is the amount which is carried by the core and/or the wrap. Observe that this difference is much greater (24.6 percent versus 7.2 percent) for columns with acrylic cores than for columns with balsa wood cores. Since the only difference between the balsa wood and the acrylic core columns is the modulus of elasticity parallel to the caps, the following conclusions can be made. Increasing the modulus of elasticity of the core in the direction parallel to the caps increases the load carrying capacity of the core, while decreasing the forces on the caps. Moreover, as this modulus of elasticity is increased, until it equals the modulus of elasticity of the caps, the bending stress distribution approaches the diagram of Figure 29B.

Wrap Elements

Figures 30 and 31 show the computer wrap elements remaining in tension for triangular and square cross section captive columns which experience a 100 pound midspan load. Recall from Chapter 4 the laborious process of identifying and redefining the modulus of elasticity for compressive wrap members. Although these two figures show representative wrap elements that remain in tension, the other eight columns do differ slightly in the number and location of tensile wraps. However, the wraps in tension on the sides of Figure 30 and 31 are the same for all triangular and square cross section columns. The difference, then, in the number and location of the tension wraps occurs only on the bottom of the triangular cross section column; and on the bottom and top of the square cross section column. Table 8 gives the number of wraps remaining in tension for each column under a 100 lb. midspan load.

The force experienced by the computer model wrap elements ranges from 10 to 59 lbs. for the triangular cross section column, and 3 to 33 lbs. for

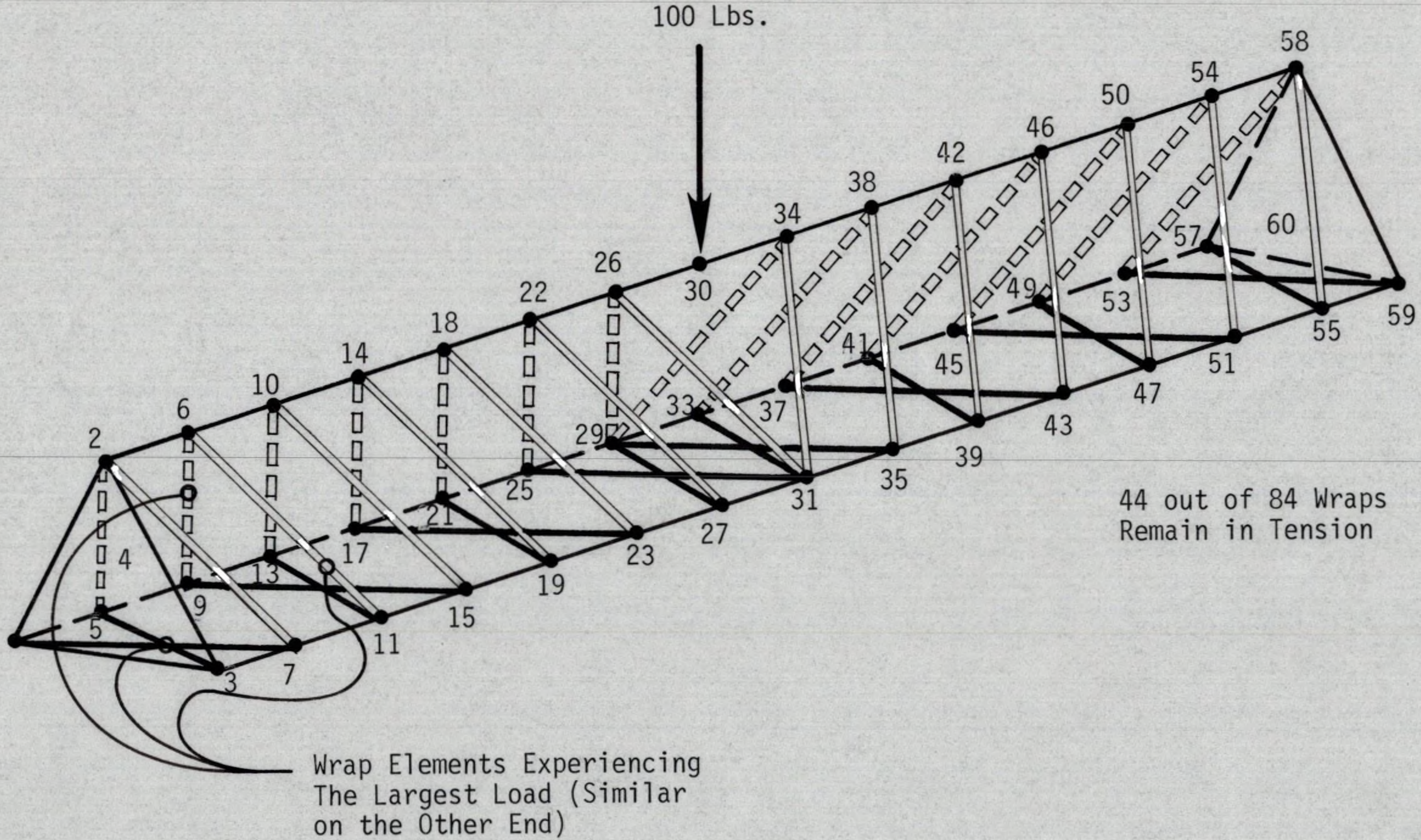


FIGURE 30 - COMPUTER WRAP ELEMENTS REMAINING IN TENSION FOR A 1/8 INCH DIAMETER FIBERGLASS CAPPED CAPTIVE COLUMN, WITH A 3/16 BALSA WOOD CORE

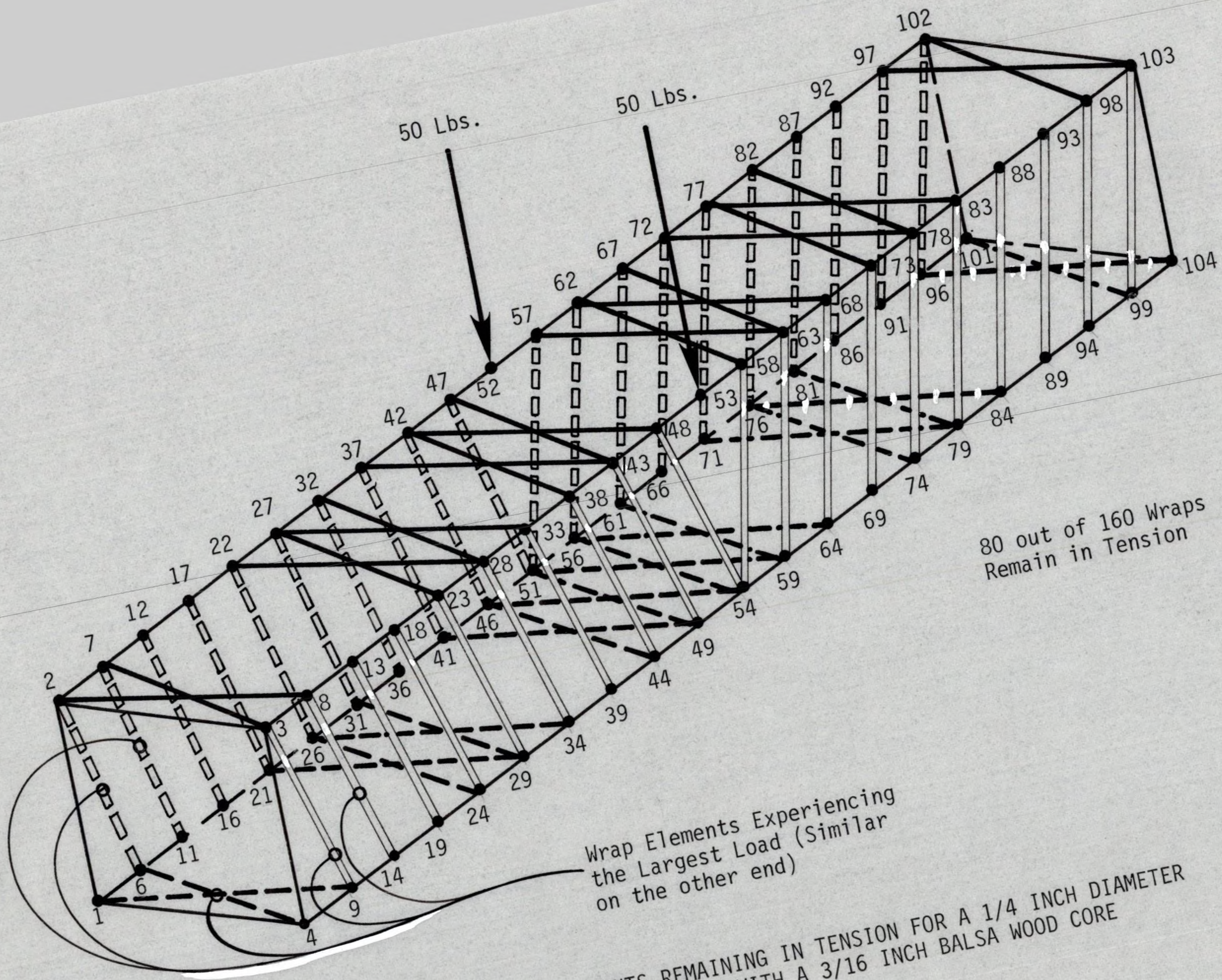


FIGURE 31 - COMPUTER WRAP ELEMENTS REMAINING IN TENSION FOR A 1/4 INCH DIAMETER STEEL CAPPED CAPTIVE COLUMN, WITH A 3/16 INCH BALSA WOOD CORE

TABLE 8

THE NUMBER OF COMPUTER WRAP ELEMENTS REMAINING IN TENSION FOR A 100 POUND MIDSPAN LOAD

Column	Triangular Cross Section (84 possible)	Square Cross Section (160 possible)
Balsa Wood Core:		
1/8 inch dia. steel caps	44	80
1/8 inch dia. fiberglass caps	44	84
1/4 inch dia. steel caps	40	80
1/4 inch dia. fiberglass caps	44	76
Acrylic Core:		
1/8 inch dia. steel caps	NT	64
1/8 inch fiberglass caps	NT	88

NT = Not Tested

the square cross section column. Since each computer model wrap represents 40 Kevlar strands in the triangular column and 28 Kevlar strands in the square column (see Chapter 4), the maximum wrap force, and wrap stress, can be calculated for each column. For the triangular case 59 pounds/40 strands equals 1.475 pound per strand or 30,900 psi. For the square column 33 pounds/28 strands equals 1.178 pounds per strand or 24,700 psi. Both of these stresses are well below the 400,000 psi tensile strength of the .0078 inch diameter Kevlar that was used to wrap the tested columns.

Observe that this type of computer analysis could be employed in the design of captive column wrap, identifying those areas of high wrap loading while also specifying adequate wrap material and the proper wrap density (wrap density is the term coined to describe the number of wraps per lineal inch along the column).

Two additional points should, however, be mentioned. First, the method of determining wrap forces is based upon the assumption of a rigid cap to wrap connection. If this epoxied connection is not as strong as the wrap itself, the epoxy, not the wrap, becomes the limiting design feature. Second, the wrap forces derived by the computer model do not include the initial wrap pretension (see the following subtitle -- Wrap Pretension) which ranges from two to five pounds per strand. This pretension must be measured or selected during the wrapping process and then added to the computer derived wrap forces in order to adequately design the wrap for a given loading pattern. For instance, adding a two pound wrap pretension to the computer derived force of 1.475 pounds, computed above, yields a total force of 3.475 pounds or 72,700 psi tensile stress. Likewise for the square cross section column, 1.178 pounds plus a 2 pound wrap pretension yields a total wrap force of 3.178 pounds or 66,500 psi tensile stress per strand.

Wrap Pretension

The wrapping machine applies, during the final phase of captive column construction, a filament wrap at 2 to 5 pounds of tension (See Chapter 2 for further details). The original finite denent computer models attempted to model this pretension by making use of the induced thermal load capabilities of the finite element SAP IV program. However, for reasons discussed below, this wrap pretension simulation was dropped from the computer analysis.

Nodal and element temperatures along with an element coefficient of linear thermal expansion can be input into the program. The computer program averages the two nodal temperatures for each wrap element and subtracts this temperature from the specified wrap element temperature. The difference is multiplied by the coefficient of expansion which induces expansion or contraction (element contraction in this case) of the element. The contraction or expansion of the wrap elements induce a pretension, or precompression, for each wrap element. It was thought that this thermally induced pretension would accurately model the construction pretension.

It was discovered, however, that this temperature induced pretension does not alter the fundamental computer stiffness matrix. Rather, it shifts the load-deflection curve to the right or left, depending upon whether contraction or expansion is induced. In this case, it shifted the curve to the right, since thermal contraction was induced.

Looking at Figure 32 two observations become apparent. First, the curves are parallel, and second, because of the induced pretension the top cap(s) of the column is theoretically deflected in the unloaded condition. Realistically the top cap(s) would deflect slightly toward the center of the column when wrapped. However, for computer model verification purposes

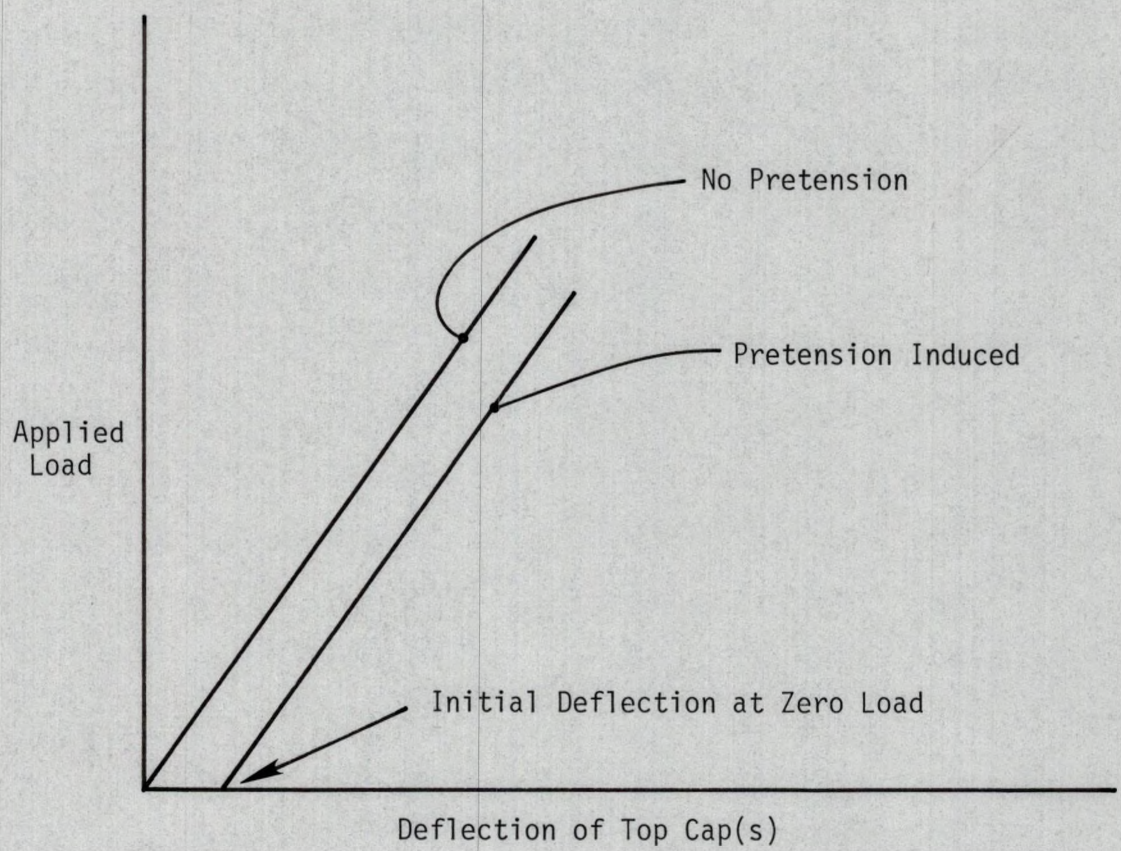


FIGURE 32 - LOAD-DEFLECTION COMPARISON FOR A COMPUTER MODEL WITH, AND WITHOUT, WRAP PRETENSION

the primary concern is differential deflection between the loaded and unloaded condition. Since both curves are parallel, identical slopes, it is desirable to use the curve passing through the origin for load-deflection comparisons. Thus computer pretension was not used in any of the ten programs which model their respective captive columns.

Captive Columns Tested To Failure

The ten captive columns that were tested for deflection and core stress data were not loaded to failure. They were saved and will be used for further computer model verification. However, four similar captive columns were tested to failure, or more specifically, loaded past their ultimate strength. Their load-deflection curves are given in Figures 33 through 36 to provide the reader with an idea of the relative strength and behavior of the captive column.

A comparison of the ultimate strengths for the different columns, or an investigation into the significant factors influencing the ultimate strength, were not undertaken in this research effort. It can be mentioned, however, that the observed mode of failure in most cases for these columns, and other columns loaded past their ultimate strength, by a midspan point load, is a localized horizontal side translation, directly under the load, of the top loadbearing cap(s). This translation is diagramed in Figure 37 for a triangular cross section column. A similar situation exists for the square cross section column.

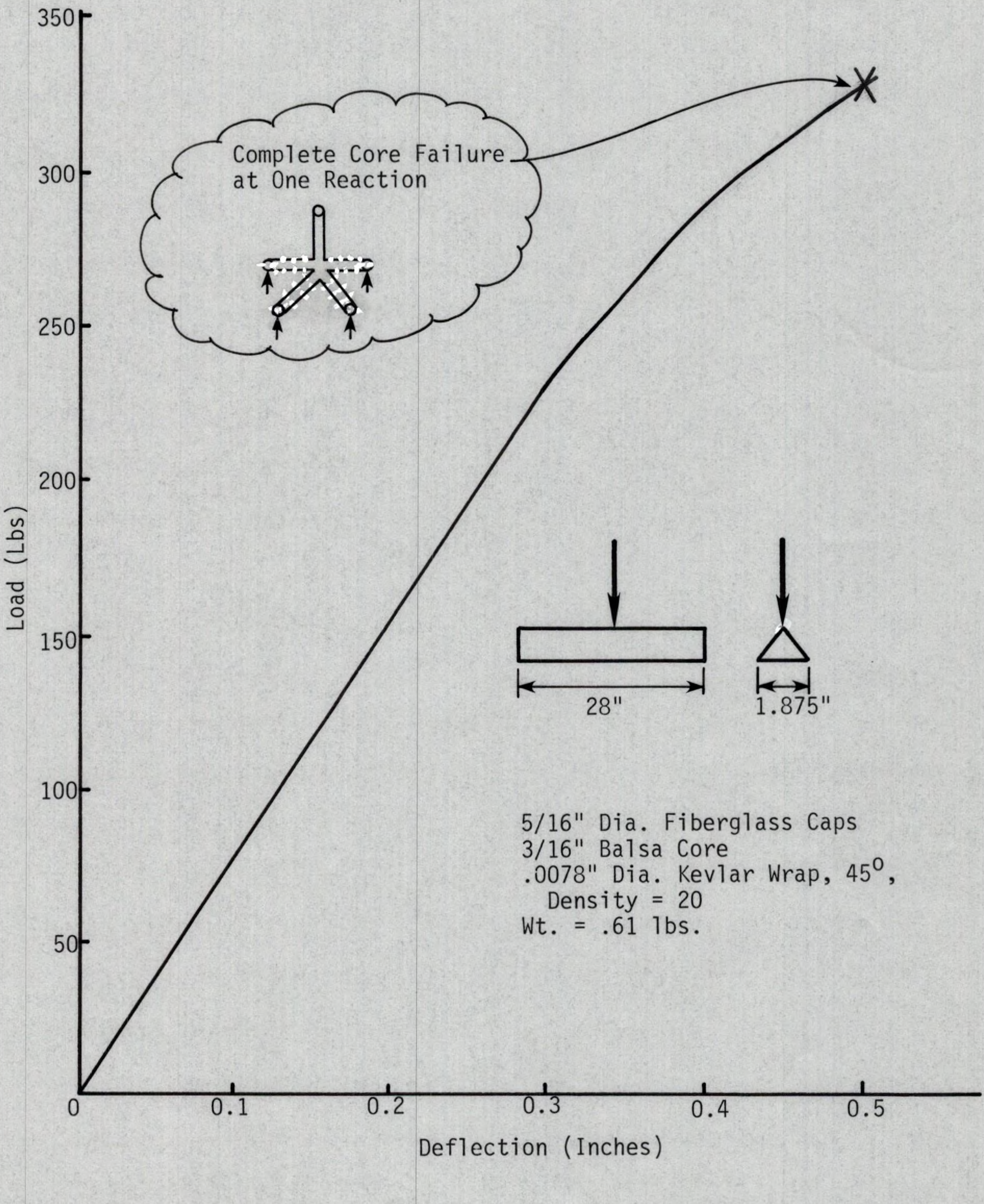


FIGURE 33 - FLEXURE TEST TO DESTRUCTION OF A TRIANGULAR COLUMN WITH 5/16 INCH DIAMETER FIBERGLASS CAPS

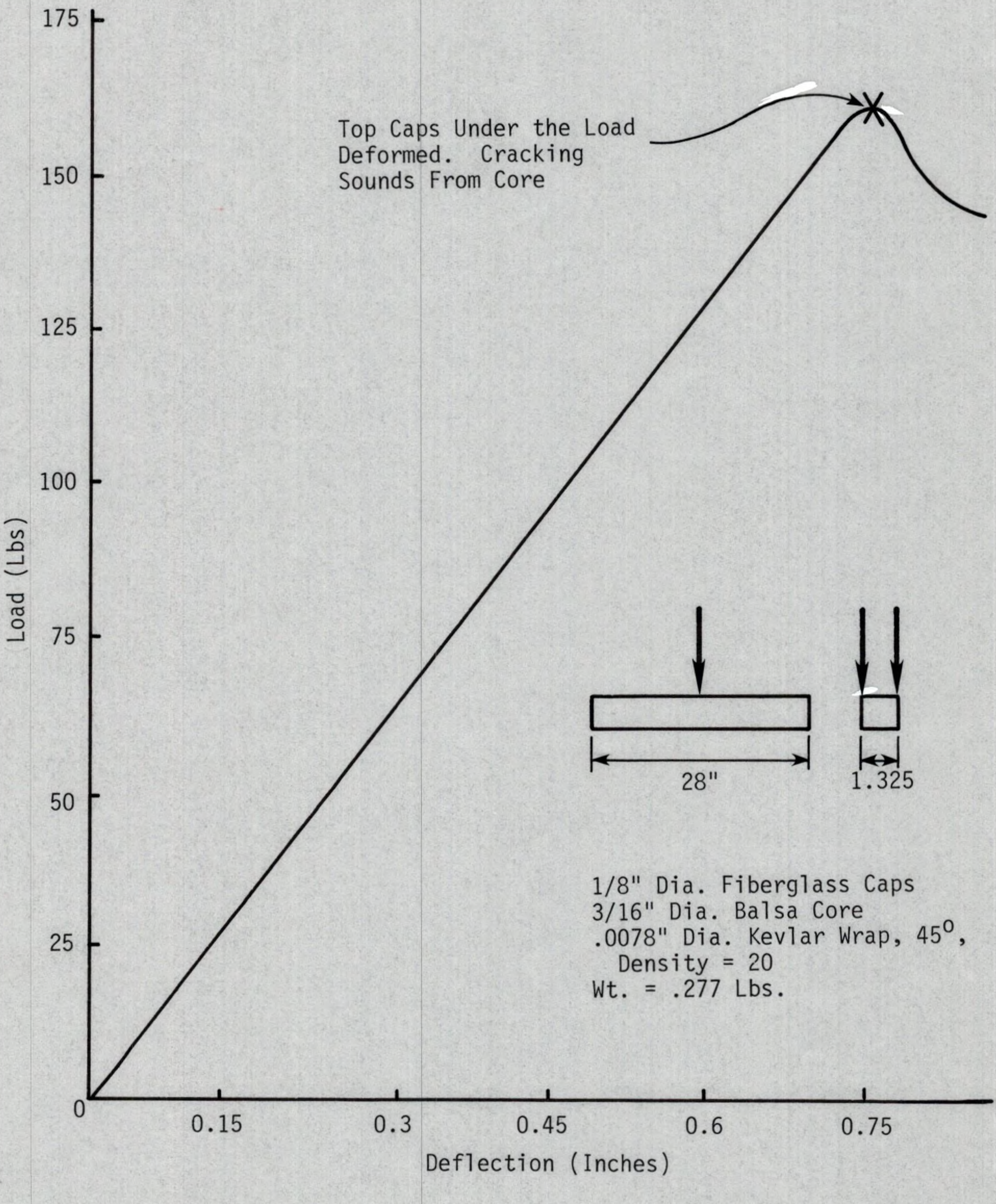


FIGURE 34 - FLEXURE TEST TO DESTRUCTION OF A SQUARE COLUMN WITH 1/8 INCH DIA. FIBERGLASS CAPS

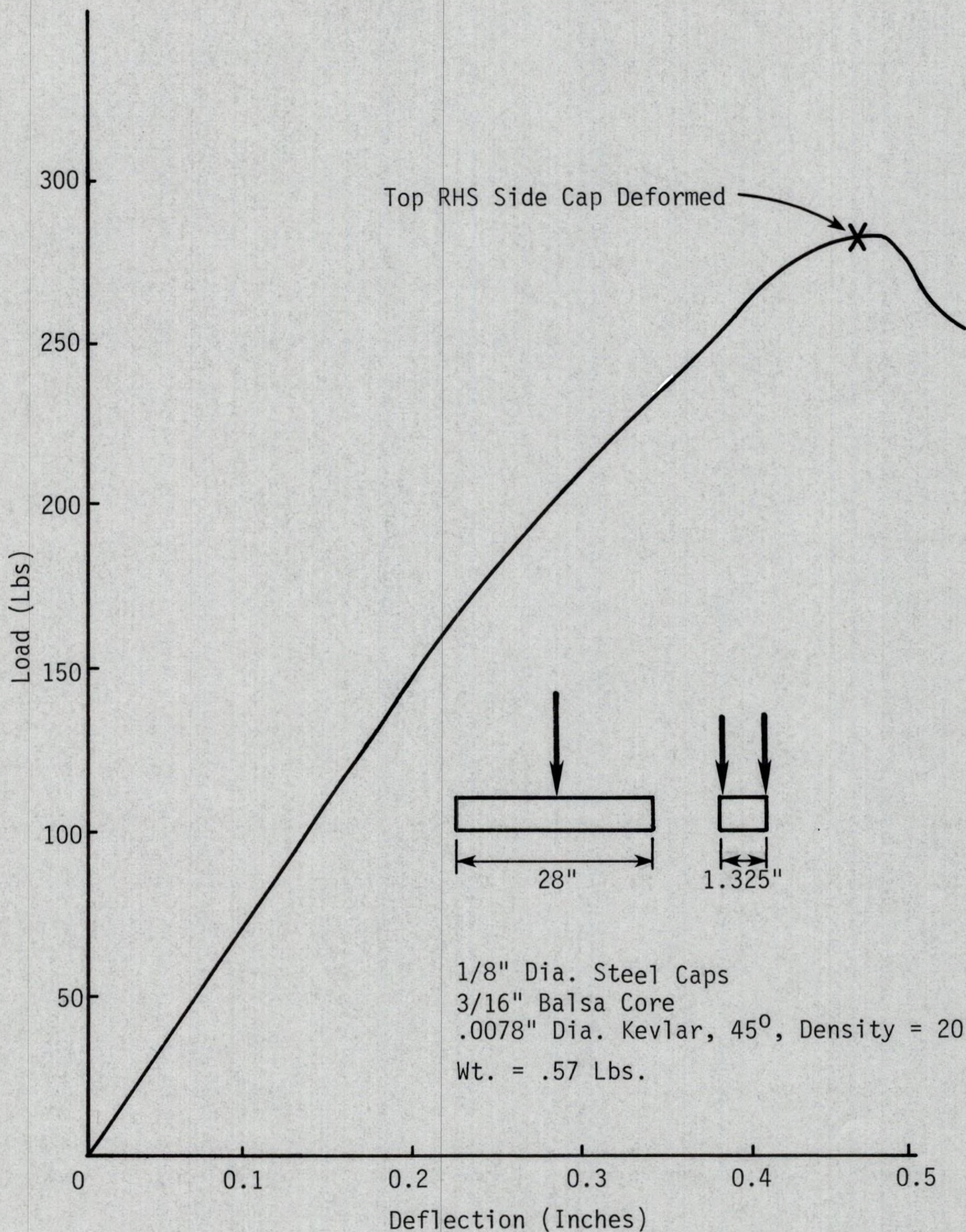


FIGURE 35 - FLEXURE TEST TO DESTRUCTION OF A SQUARE COLUMN WITH 1/8 INCH DIA. STEEL CAPS

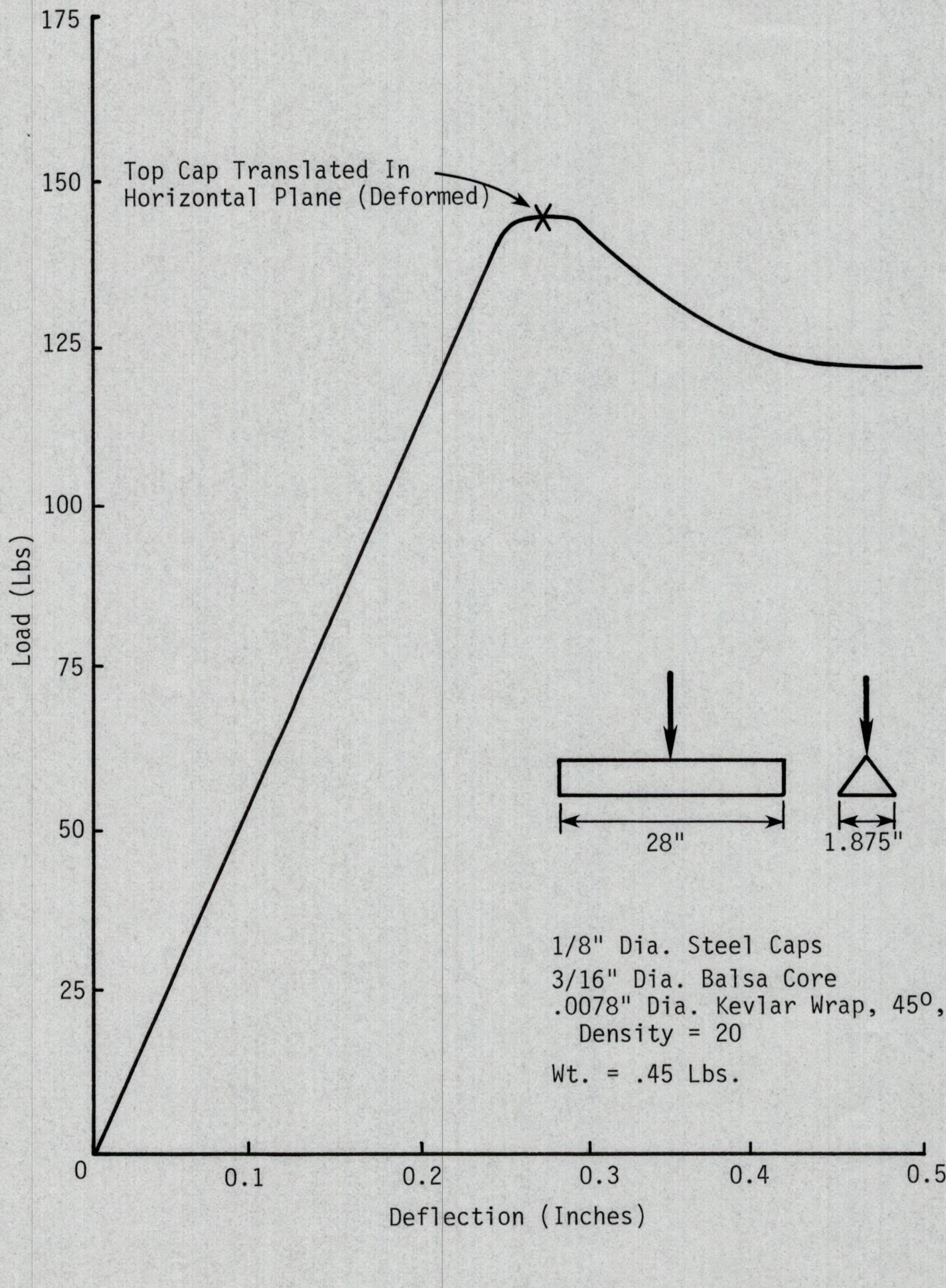


FIGURE 36 - FLEXURE TEST TO DESTRUCTION OF A TRI-ANGULAR COLUMN WITH 1/8" DIA. STEEL CAPS

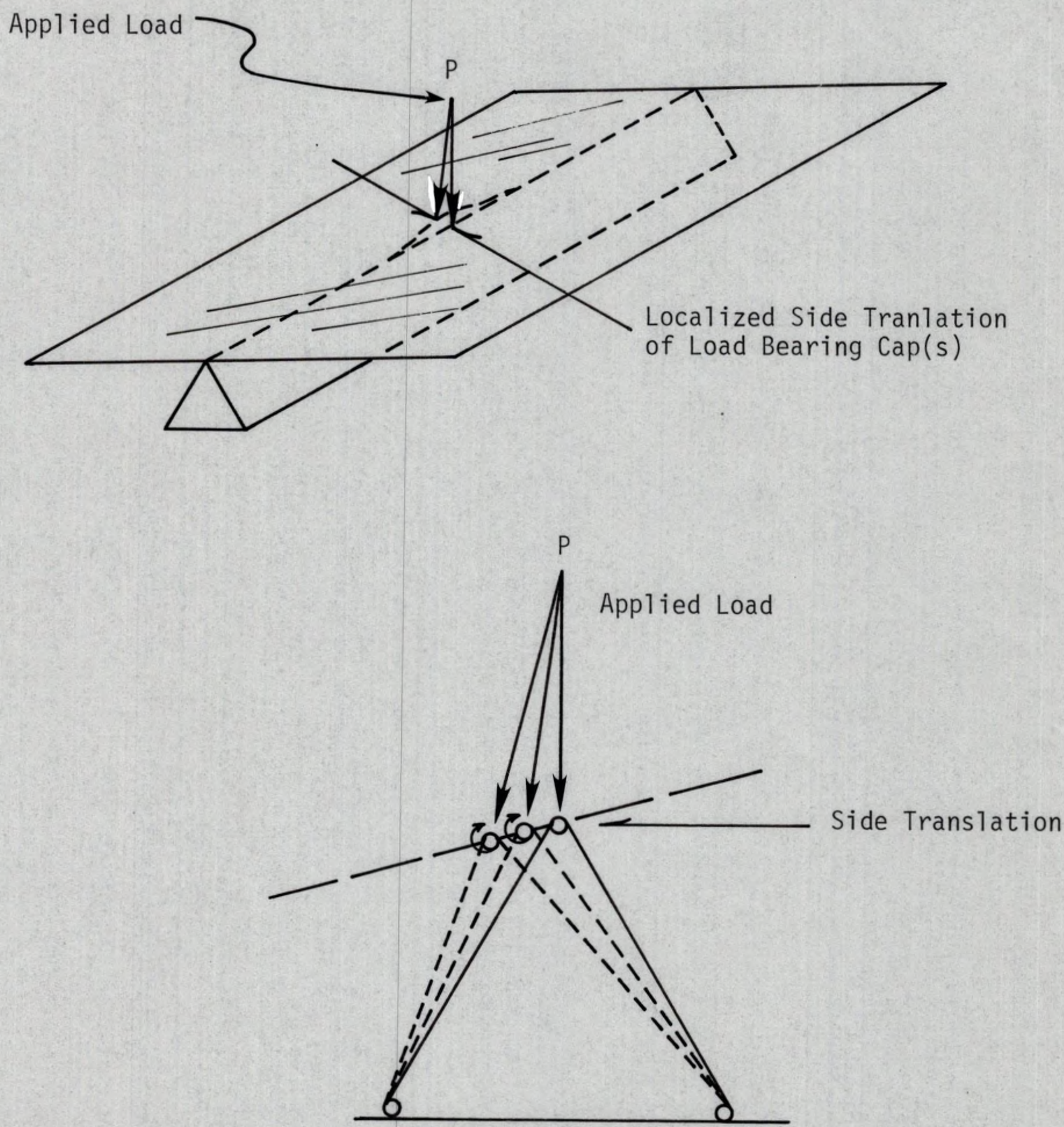


FIGURE 37 - OBSERVED MODE OF FAILURE FOR CAPTIVE COLUMNS LOADED PAST THEIR ULTIMATE STRENGTH

CHAPTER 7

CONCLUSION

A general method of modelling the captive column using finite element techniques has been established. Specifically, two finite element computer programs were developed . One models a triangular cross section captive column 1.875 inches on a side and 28 inches long. The other models a square cross section captive column 1.325 inches on a side and 28 inches long. A total of ten captive columns, with these dimensions, were also constructed and statically tested under a midspan load. The validity of the computer models were corroborated by comparing the computer model midspan deflections and the internal core stresses with the actual experimental test data. The results of these comparisons are as follows.

Computer model deflections at the midspan of the column, under a concentrated load, were 10 to 12 percent less than the actual experimentally measured deflections. Furthermore, for the captive columns with steel caps, the computer model core stresses, at a point 3.5 inches on either side of the midspan load, differ by no more than 20 percent from the experimentally measured core stresses. For the captive columns with fiberglass caps, the computer model core stresses differ by 95 percent and 74 percent for the algebraically smallest principal stress and less than 8 percent for the other, larger, principal stress. Principal directions of the two dimensional stress element differed by no more than 11 percent for the steel capped captive column and from 2 to 28 percent for the fiberglass capped captive column.

Besides confirming the validity of finite element techniques in modelling captive columns, the computer models can also be used in the design and specification of the three captive column components. Specifically, a method was developed for designing captive column caps -- given the loading pattern, column geometry, and maximum applied load -- so that the axial load in the caps would not exceed the yield stress of the cap material. Also, a method was developed for designing the captive column wraps. This method considers both the initial wrap pretension and the individual wrap forces experienced due to the applied load. Furthermore, with the aid of the computer model, variable wrap densities along the length of the column can be specified. Finally, in Chapter 6, the significance of the anisotropic modulus of elasticity of the balsa wood core was discussed. It was suggested that the largest modulus of elasticity of the core should be in the direction perpendicular to the caps primarily to restrict inward cap deflection during the construction wrapping process. Also, increasing the modulus of elasticity of the core material in the direction parallel to the caps increases the flexural load carrying capacity of the core, while decreasing the axial forces in the caps.

Discussion

Understanding, with the intent of designing, the three captive column components will require more than computer and theoretical verification of the experimentally observed phenomena. It will require an understanding of the relationships between the caps, the core, and the wraps. That is, discerning how these conjunctive captive column components act and react; how a design variation in one element impacts the other two elements; and most importantly how the properties of the captive column relate back to established and indisputable beam and column theory. These questions are of course the intent of this research effort and have not, at this time, been completely answered. However, at this, the summation of one phase of the research effort, it is imperative to regress from the geometrical progression of investigating smaller and smaller units of the problem and stop to integrate those discrete bits of information into an abstract concept.

As mentioned in Chapter 2, beam theory establishes that an increase in either, or both, the moment of inertia or modulus of elasticity of a beam increases it's flexural rigidity and therefore it's load carrying capacity at a given deflection. Ideally then, a beam should be constructed with a material having a large modulus of elasticity (and ultimate strength), while incorporating as much of this material as far from the neutral axis as possible. This, of course, is the rational behind the structural I-beam. However, two problems exist. First, total beam weight, and second, physical size. Both of these are important design and economic considerations.

The captive column addresses both of these problems. Generally, in common structural members, the material used to support the flanges (in the case of the captive column, the caps) away from the neutral axis is the same material as that of the flanges (caps). This significantly increases the weight of the beam without increasing it's load carrying capability. However, if these flanges (caps) could be rigidly supported by a lightweight web (in the case of the captive column, the core), the total weight of the beam could be reduced without sacrificing the beams load carrying capacity. Furthermore, the larger the distance from the flanges (caps) to the neutral axis and the larger the modulus of elasticity of the flanges (caps), the smaller the cross sectional area of the flanges (caps) needs to be in order to maintain the same flexural rigitity. Again, this leads to a weight reduction.

Conversely, a smaller cross section captive column can be achieved, for a given flexural rigidity, by increasing the caps modulus of elasticity and correspondingly decreasing the moment of inertia by reducing the distances of the caps from the neutral axis.

The key requirement, and the reason for wrapping the captive column, is to rigidly support the caps in their original position. Known lightweight core materials, when used alone to support the caps away from the neutral axis, do not have the structural integrity to maintain the caps in their original position, relative to each other, during load application. The core, along with the attached caps, twist, bend, and deform rendering the entire beam useless. However, the application of a lightweight, high strength wrap material assists the core in supporting and captivating the load bearing caps in their original geometry, while also uniting the three components into an integral unit.

Thus, the captive column can be viewed as a refinement of well-known structural design techniques. By selecting lightweight, high-strength, and high modulus of elasticity material such as balsa wood, glass reinforced polyester, and Dupont Kevlar, a lightweight structural composite with a high strength to weight ratio can be assembled.

This, of course, hinges on the important, and as yet not completely identified, captive column design criteria; the determination of the loads experienced by the cap, core, and wrap. Once the design variables are understood, a lightweight core and wrap can be specified which will withstand the same maximum applied load as the load bearing caps, thereby creating a structural composite where the three components will fail simultaneously under the maximum applied load.

APPENDICES

APPENDIX A

COMPUTER CONTROL CARD DESIGNATING SYSTEM

Because of the large number of different computer models tried it was necessary to design a control card designating system to inventory the program decks. This system is presented below for those who continue my work and may need to use these programs.

Column	Symbol	Explanation	Designating The
3	F	Flexure	Loading Pattern
	C	Axial Compression	
4	T	Truss Element	Cap Elements
	B	Beam Element	
	P	Plate Element	
	0	No Element	
5	T	Truss Element	Core Elements
	B	Beam Element	
	P	Plate Elements	
	S	Plane Stress Elements	
	0	No Elements	
6	T	Truss Elements	Wrap Elements
	B	Beam Elements	
	P	Plane Elements	
	0	No Elements	
7	A	24 Node 28" Long Triangular Column	Computer Model Size and Geometry
	B	180 Node 28" Long Triangular Column	
	C	60 Node 28" Long Triangular Column (Final Model)	
	D	106 Node 28" Long Square Column (Final Model)	
8	A	Circular Truss and Beam Elements Extending Radially Outward	Elements Used For The Core
	B	Members of A Plus Circular Truss and Beam Elements Crisscrossing in the Plane of the Rib	
	C	Beam Elements With Core Dimensions	
	D	Both Beam and Plane Stress Elements	
	E	Both Beam and Plate Elements	
9	0	Single Load Applied at the Midspan	Loading
	D	Dual Load Applied at the Midspan	
	E	Single Load Applied 8" From Support	
	F	Dual Load Applied 8" From Support	
	T	Trial Run Designation	

APPENDIX B

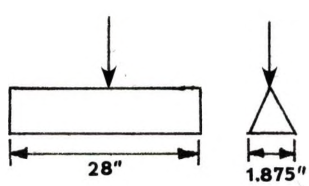
EXPERIMENTAL DATA

TABLE 9

LOAD-DEFLECTION DATA FOR A TRIANGULAR CROSS SECTION CAPTIVE COLUMN WITH 1/8 INCH DIAMETER STEEL CAPS; 3/16 INCH BALSA WOOD CORE; 45^{o}, .0078 INCH DIAMETER, 20 DENSITY, KEVLAR WRAP, AND A PINE CENTERPIECE

Dial Gage Reading 10^{-3}inch	1st Defl 10^{-3}inch	1st Load Lbs	2nd Defl 10^{-3}inch	2nd Load Lbs	3rd Defl 10^{-3}inch	3rd Load Lbs
775	25	10	25	12	25	10
750	50	25	50	27	50	26
725	75	36	75	43	75	42
700	100	54	100	58	100	57
675	125	72	125	75	125	76
650	150	89	150	92	150	94
625	175	103	175	108	175	108
600	200	116	200	120	200	120
	Side Wraps Very Loose, Can Probably Take a Larger Load		Same as (1)		Same as (1)	

Notes:
1) Load applied with triangular harness
2) Dillon machine with 500 lb scale
3) 1/8" steel caps, 3/16" balsa core with glued sections and a pine center, 45^{o} Kevlar wrap
4) Column built by Dave
5) No twist in the column



Wt = 205.7 gm = 0.45 lbs

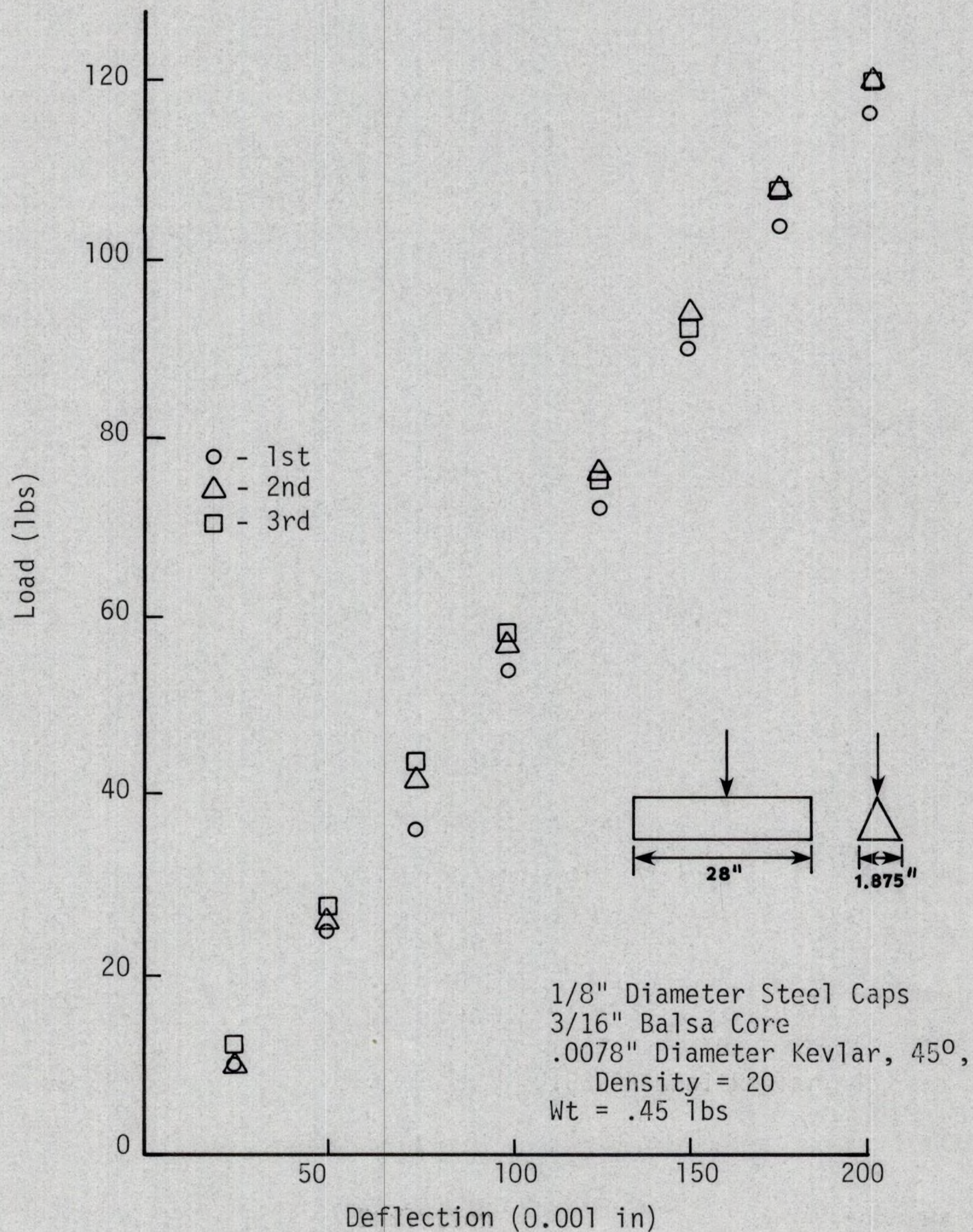


FIGURE 38 - LOAD VERSUS DEFLECTION GRAPH OF THE TABLE 9 DATA

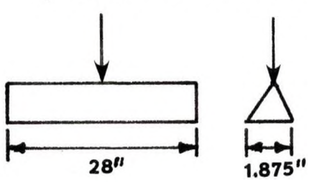
TABLE 10

LOAD-DEFLECTION DATA FOR A TRIANGULAR CROSS SECTION CAPTIVE COLUMN WITH 1/8 INCH DIAMETER FIBERGLASS CAPS; 3/16 INCH BALSA WOOD CORE; 45^{o}, .0078 INCH DIAMETER, 20 DENSITY, KEVLAR WRAP, AND A PINE CENTERPIECE

Dial Gage Reading 10^{-3}inch	1st Defl 10^{-3}inch	1st Load Lbs	2nd Defl 10^{-3}inch	2nd Load Lbs	3rd Defl 10^{-3}inch	3rd Load Lbs
800	25	8	0	0	0	0
775	50	12	25	8	25	7
750	75	16	50	13	50	14
725	-	-	75	21	75	21
700	125	25	100	29	100	31
650	175	32	150	36	150	41
600	225	45	200	50	200	46
550	275	54	250	58	250	54
500	325	66	300	68	300	64
450	375	77	350	80	350	77
400	425	89	400	93	400	90
350	475	103	-	-	-	-

Side Wrap Very Loose | Wrap not quite as loose as (1) | Same as (2)

Notes:
1) Load applied with triangular harness
2) Dillon machine with 500 lb scale
3) 1/8" fiberglass caps, 3/16" balsa core with glued sections and a pine center, 45^{o} Kevlar wrap
4) Column made by Dave
5) 3/16" twist over 28" length, slight bow to the column



Wt = 108.8 gm = 0.24 lbs

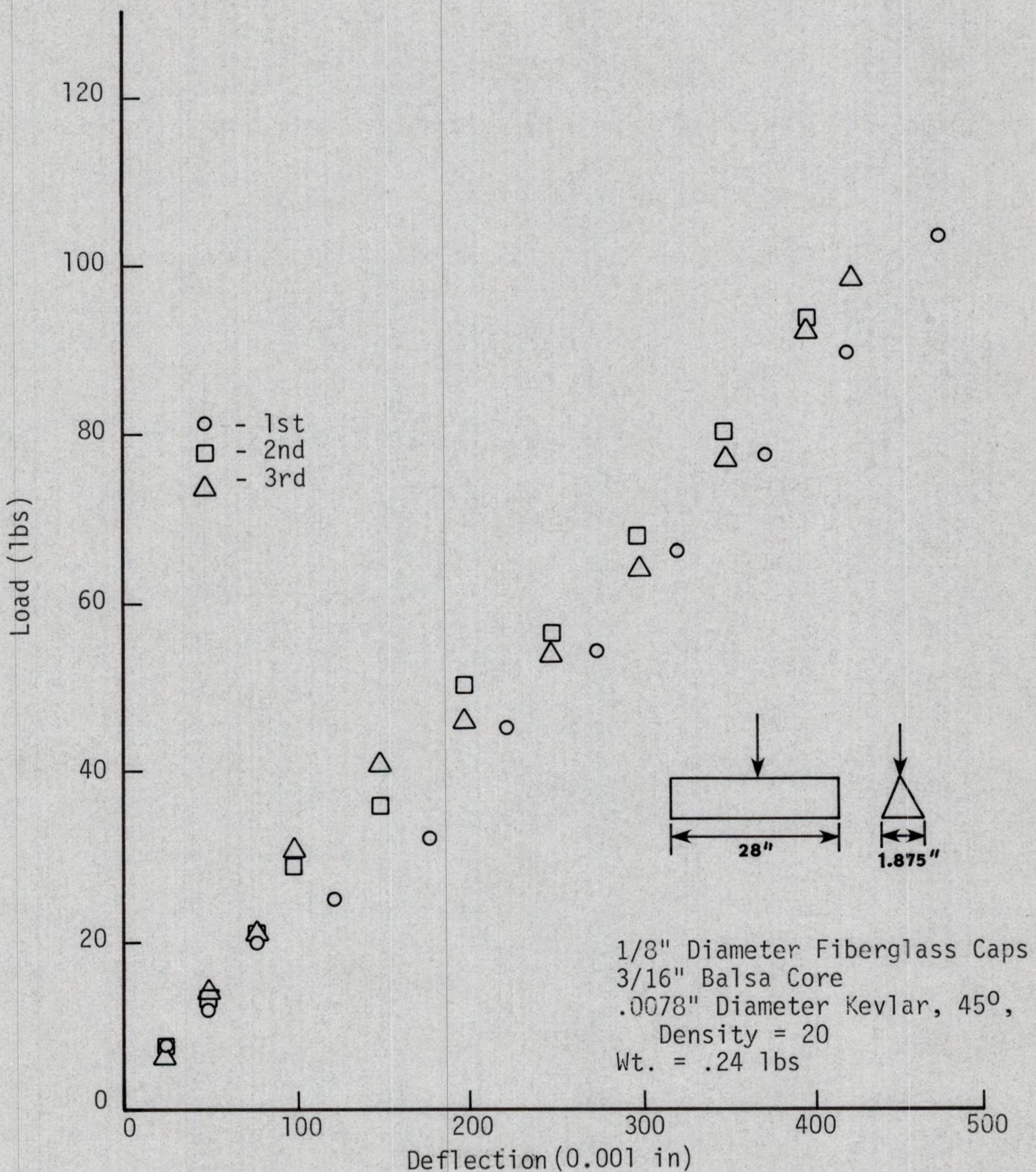


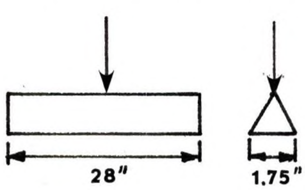
FIGURE 39 - LOAD VERSUS DEFLECTION GRAPH OF THE TABLE 10 DATA

TABLE 11

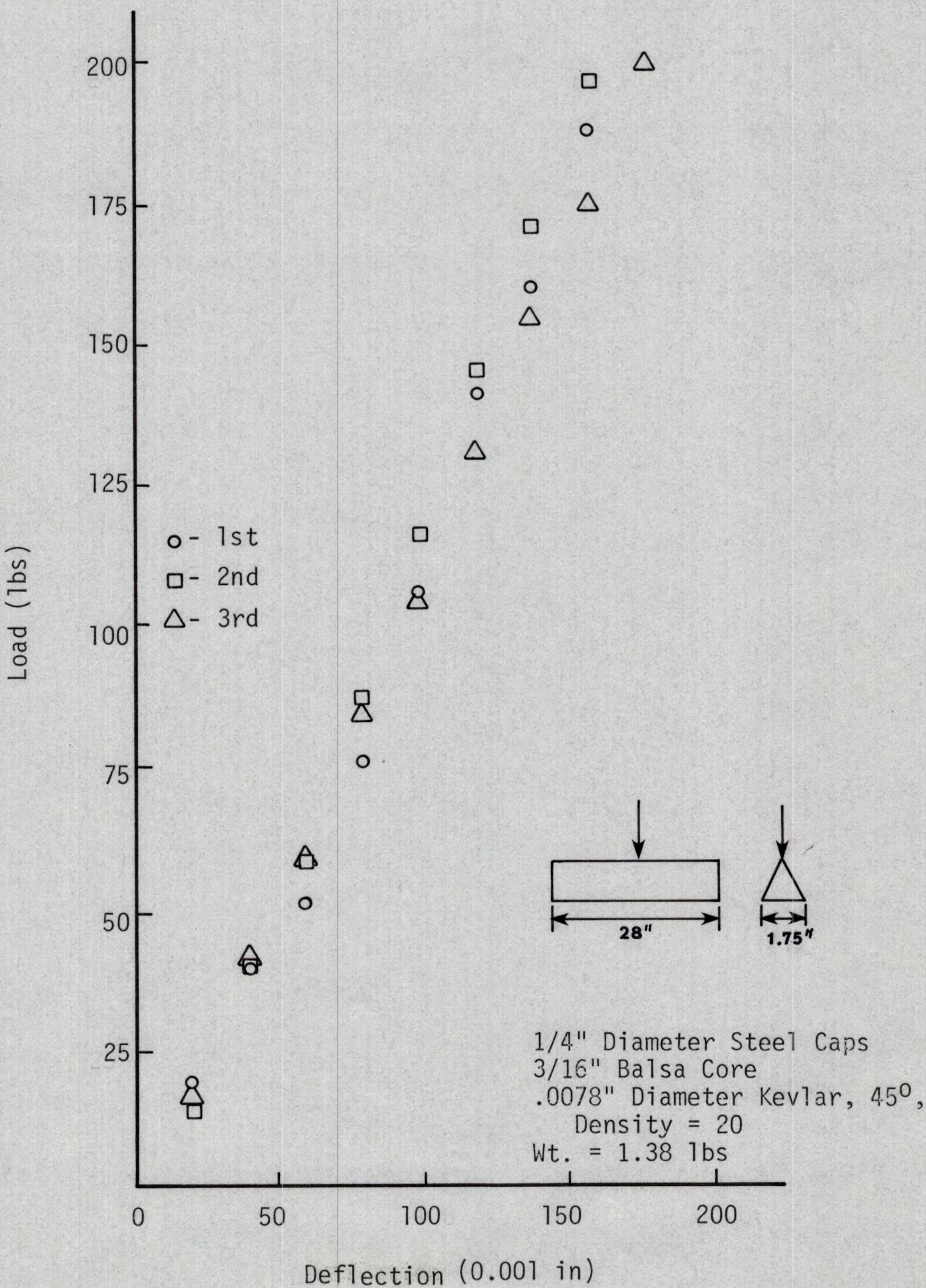
LOAD-DEFLECTION DATA FOR A TRIANGULAR CROSS SECTION CAPTIVE COLUMN WITH 1/4 INCH DIAMETER STEEL CAPS; 3/16 INCH BALSA WOOD CORE; 45^o, .0078 INCH DIAMETER, 20 DENSITY, KEVLAR WRAP AND A PINE CENTERPIECE

Dial Gage Reading 10^{-3}inch	1st Defl 10^{-3}inch	1st Load Lbs	2nd Defl 10^{-3}inch	2nd Load Lbs	3rd Defl 10^{-3}inch	3rd Load Lbs
840	20	18	0	0	20	16
820	40	33	20	14	40	36
800	60	50	40	34	60	58
780	80	75	60	58	80	79
760	100	105	80	86	100	104
740	120	135	100	115	120	130
720	140	159	120	144	140	153
700	160	186	140	169	160	174
680	-	-	160	195 (cracking noise)	180	198
	Side wraps loose, could probably take more load if core wouldn't fail		Same as (1)		Same as (1)	

Notes:
1) Load applied with triangular harness
2) Dillon machine with 500 lb scale
3) 1/4" steel caps, 3/16" balsa core with sections glued and a pine center, 45^o Kevlar wrap
4) Column made by Dave
5) No twist in column



Wt = 628.0 gm = 1.38 lbs



<u>FIGURE 40</u> - LOAD VERSUS DEFLECTION GRAPH OF THE TABLE 11 DATA

TABLE 12

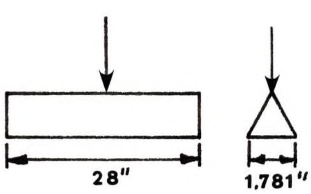
LOAD-DEFLECTION DATA FOR A TRIANGULAR CROSS SECTION CAPTIVE COLUMN WITH 1/4 INCH DIAMETER FIBERGLASS CAPS; 3/16 INCH BALSA WOOD CORE; 45^{o}, .0078 INCH DIAMETER, 20 DENSITY, KEVLAR WRAP AND A PINE CENTERPIECE

Dial Gage Reading 10^{-3}inch	1st Defl 10^{-3}inch	1st Load Lbs	2nd Defl 10^{-3}inch	2nd Load Lbs	3rd Defl 10^{-3}inch	3rd Load Lbs
900	25	7	-	-	25	8
875	50	20	0	0	50	21
850	75	35	25	9	75	28
825	100	50	50	17	100	39
800	125	66	75	26	125	52
775	150	80	100	38	150	69
750	175	97	125	52	175	86
725	200	114	150	68	200	102
700	225	132	175	84	225	118
675	250	146	200	99	250	135
650	-	-	225	116	275	150
625	-	-	250	134	-	-

Side wraps loose (not quite as loose as square with 1/4 fiberglass)

Same as (1)

Notes:
1) Load applied with triangular harness
2) Dillon machine with 500 lb scale
3) 1/4" fiberglass caps, 3/16" balsa core with glued sections and a pine center, 45^{o} Kevlar wrap
4) Column made by Dave
5) 1/16" twist over 28" length



Wt = 211.9 gm = 0.47 lbs

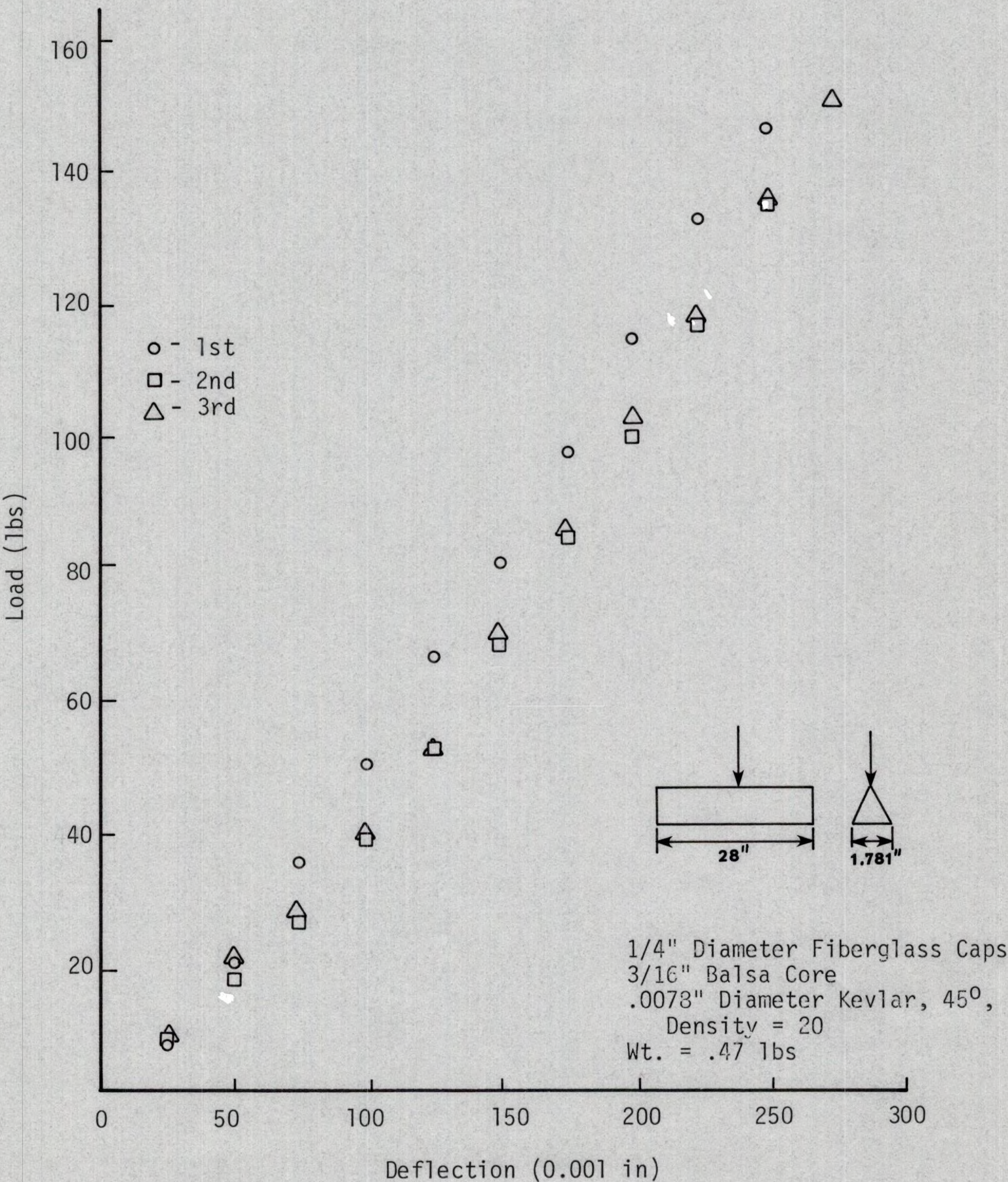
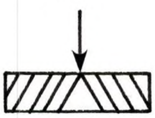


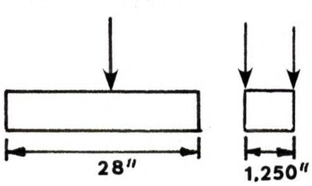
FIGURE 41 - LOAD VERSUS DEFLECTION GRAPH OF THE TABLE 12 DATA

TABLE 13

LOAD-DEFLECTION DATA FOR A SQUARE CROSS SECTION CAPTIVE COLUMN WITH 1/8 INCH DIAMETER STEEL CAPS; 3/16 INCH BALSA WOOD CORE; 45^{o}, .0078 INCH DIAMETER, 20 DENSITY, KEVLAR WRAP AND A PINE CENTERPIECE

Dial Gage Reading	1st		2nd		3rd		4th	
	Defl	Load	Defl	Load	Defl	Load	Defl	Load
10^{-3}inch	10^{-3}in	Lbs	10^{-3}in	Lbs	10^{-3}in	Lbs	10^{-3}in	Lbs
775	25	11	-	-	0	0	-	-
750	50	25	-	-	25	11	0	0
725	75	41	0	0	50	29	25	10
700	100	56	25	10	75	45	50	22
675	125	70	50	28	100	59	75	35
650	150	84	75	43	125	74	100	47
625	175	101	100	57	150	89	125	63
600	200	116	125	72	175	105	150	78
575	-	-	150	87	200	118	175	93
550	-	-	175	100	-	-	200	105
525	-	-	200	114	-	-	225	118
	Side Wraps Loose		Same as (1)		Same as (1)		Side Wraps looser than before - cracking noise in core on last two tests	

Notes:
1) Load applied with square harness
2) Dillon machine with 500 lb scale
3) 1/8" steel caps, 3/16" balsa core with glued sections and a pine center, 45^{o} Kevlar wrap
4) Column made by Dave
5) No apparent twist in column



Wt = 256.2 gm = 0.56 lbs

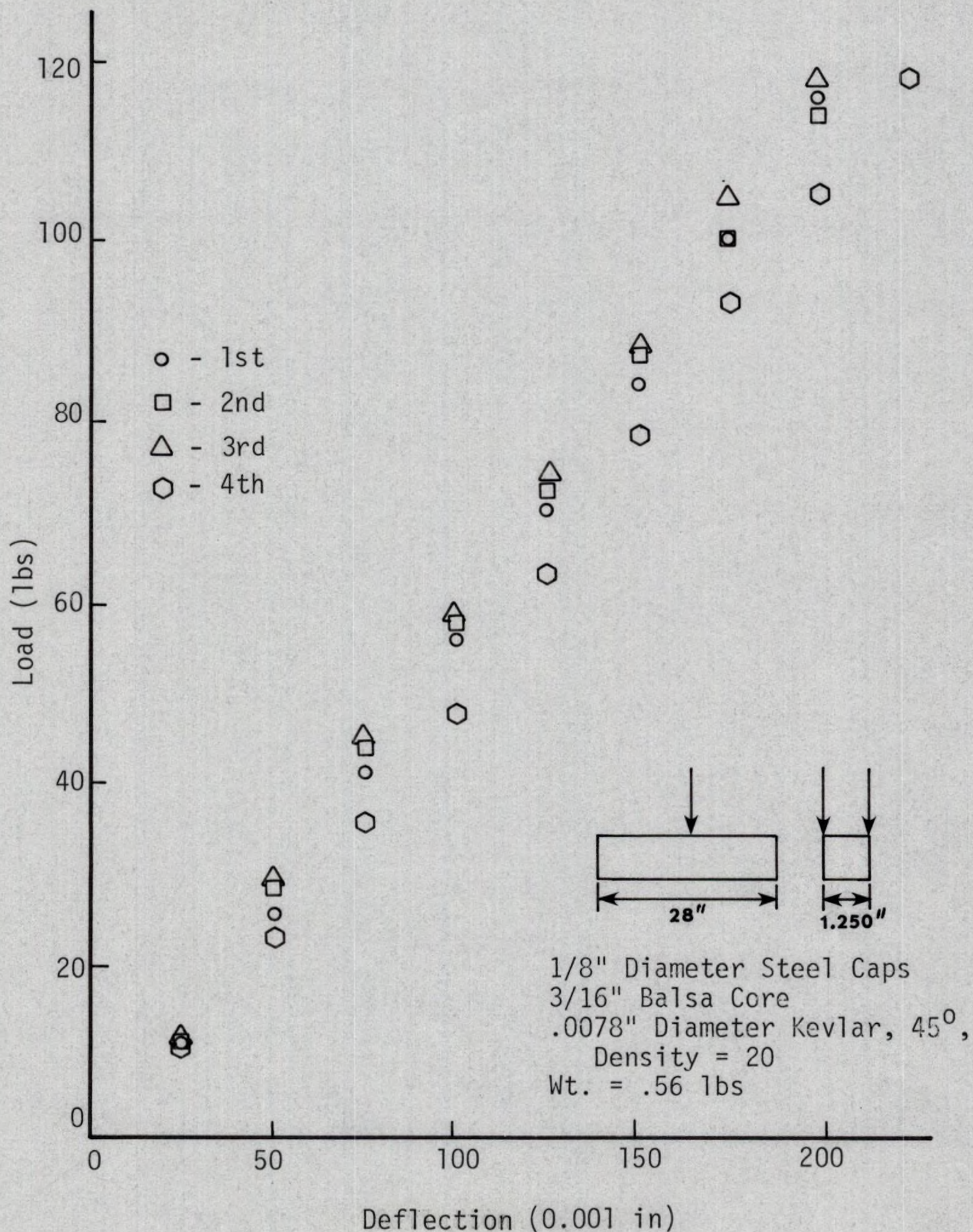
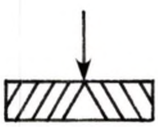


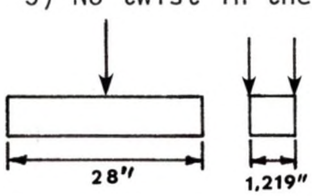
FIGURE 42 - LOAD VERSUS DEFLECTION GRAPH OF THE TABLE 13 DATA

TABLE 14

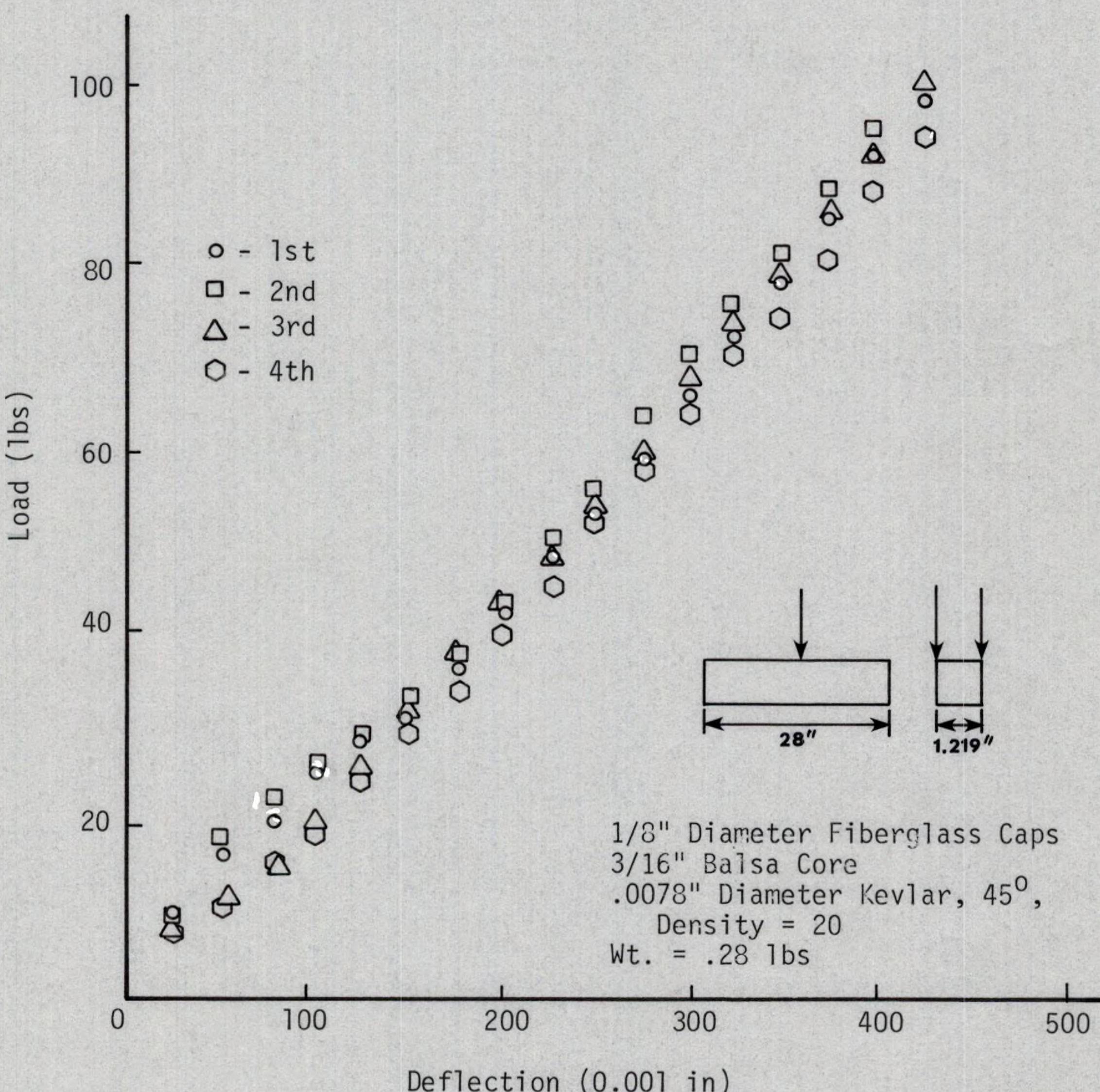
LOAD-DEFLECTION DATA FOR A SQUARE CROSS SECTION CAPTIVE COLUMN WITH 1/8 INCH DIAMETER FIBERGLASS CAPS; 3/16 INCH BALSA WOOD CORE; 45^o, .0078 INCH DIAMETER, 20 DENSITY, KEVLAR WRAP AND A PINE CENTERPIECE

Dial Gage Reading 10^{-3}inch	1st Defl 10^{-3}in	1st Load Lbs	2nd Defl 10^{-3}in	2nd Load Lbs	3rd Defl 10^{-3}in	3rd Load Lbs	4th Defl 10^{-3}in	4th Load Lbs
650	25	11	0	0	25	8	25	8
625	50	16	25	10	50	12	50	11
600	75	20	50	18	75	15	75	15
575	100	25	75	22	100	20	100	19
550	125	28	100	26	125	26	125	24
525	150	31	125	29	150	32	150	29
500	175	36	150	33	175	38	175	34
475	200	42	175	38	200	43	200	40
450	225	48	200	43	225	48	225	45
425	250	53	225	50	250	54	250	52
400	275	59	250	56	275	60	275	58
375	300	66	275	63	300	68	300	64
350	325	72	300	70	325	74	325	70
325	350	78	325	75	350	79	350	76
300	375	85	350	81	375	86	375	81
275	400	92	375	88	400	92	400	88
250	425	98	400	94	425	100	425	94
	Side Wrap Very Loose		Same as (1)		Same as (1)		Same as (1)	

Notes:
1) Load applied with square harness
2) Dillon machine with 500 lb scale
3) 1/8" fiberglass caps, 3/16" balsa core with glued sections and a pine center, 45^o Kevlar wrap
4) Column made by Dave
5) No twist in the column



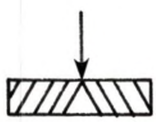
Wt = 126.1 gm = 0.28 lbs



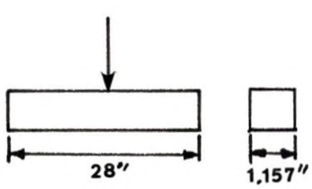
<u>FIGURE 43</u> - LOAD VERSUS DEFLECTION GRAPH OF THE TABLE 14 DATA

TABLE 15

LOAD-DEFLECTION DATA FOR A SQUARE CROSS SECTION CAPTIVE COLUMN WITH 1/4 INCH DIAMETER STEEL CAPS; 3/16 INCH BALSA WOOD CORE; 45^{o}, .0078 INCH DIAMETER, 20 DENSITY, KEVLAR WRAP AND A PINE CENTERPIECE

Dial Gage Reading	1st		2nd		3rd		4th	
	Defl	Load	Defl	Load	Defl	Load	Defl	Load
10^{-3}inch	10^{-3}in	Lbs	10^{-3}in	Lbs	10^{-3}in	Lbs	10^{-3}in	Lbs
800	0	0	0	0	0	0	0	0
780	20	13	20	18	20	8	20	20
760	40	38	40	42	40	26	40	43
740	60	68	60	68	60	46	60	70
720	80	97	80	98	80	70	80	97
700	100	126	100	127	100	95	100	120
680	120	153	120	156	120	122	120	142
660	140	178	140	181	140	148	140	168
640	160	204	160	206	160	174	-	-
620	-	-	-	-	180	200	-	-
	Side Wraps Loose, Can Take a Greater Load (≈300#)		Same as (1)		Same as (1)			

Notes:
1) Load applied with square harness
2) Dillon machine with 500 lb scale
3) 1/4" steel caps, 3/16" balsa core with glued sections and a pine center, 45^{o} Kevlar wrap
4) Column made by Dave
5) 1/16" twist over 28" length



Wt = 823.4 gm = 1.81 lbs

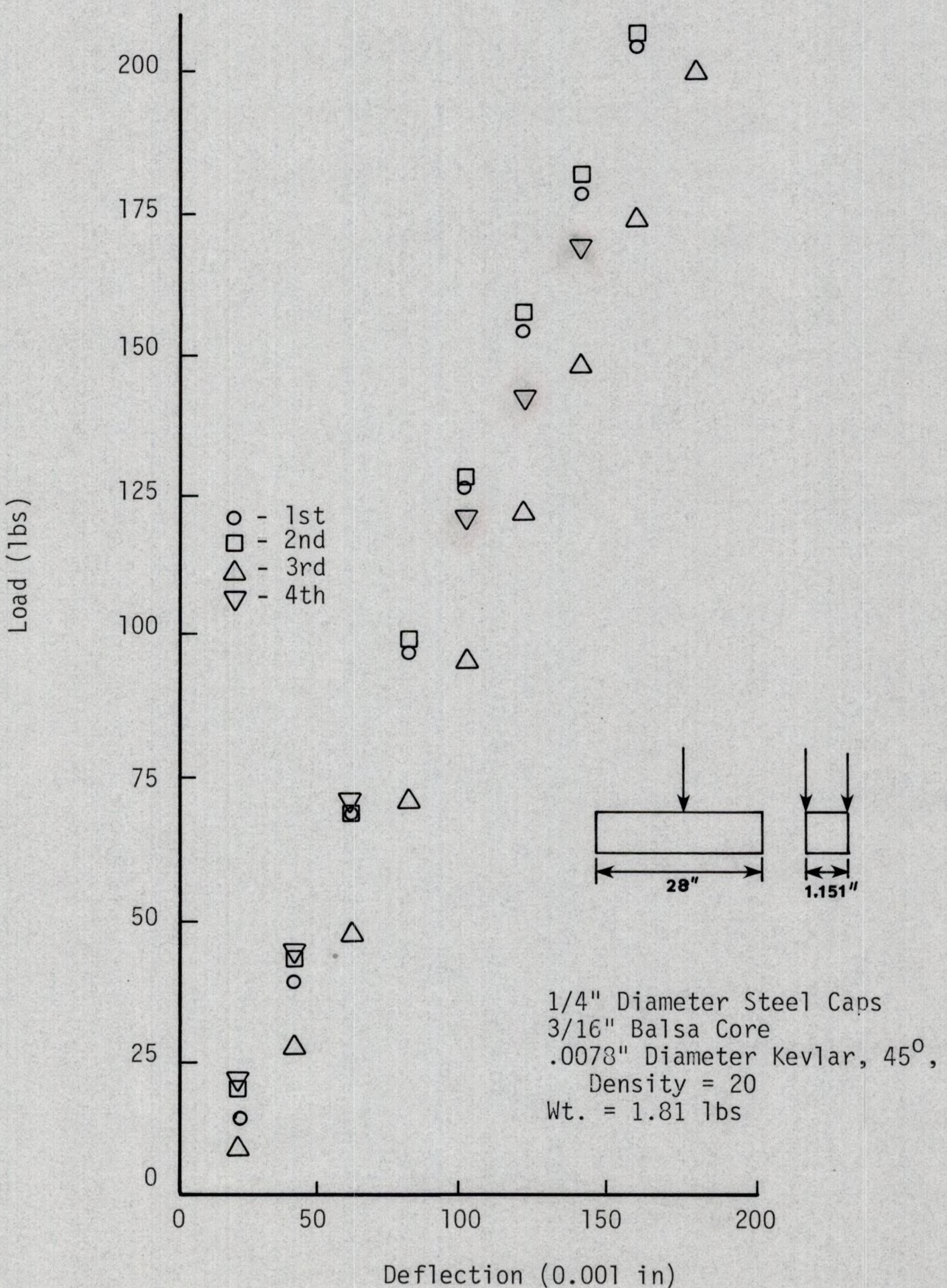
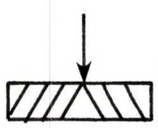


FIGURE 44 - LOAD VERSUS DEFLECTION GRAPH OF THE TABLE 15 DATA

TABLE 16

LOAD-DEFLECTION DATA FOR A SQUARE CROSS SECTION CAPTIVE COLUMN WITH 1/4 INCH DIAMETER FIBERGLASS CAPS; 3/16 INCH BALSA WOOD CORE; 45^o, .0078 INCH DIAMETER, 20 DENSITY, KEVLAR WRAP AND A PINE CENTERPIECE

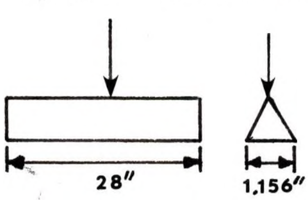
Dial Gage	1st		2nd		3rd		4th	
Reading	Defl	Load	Defl	Load	Defl	Load	Defl	Load
10^{-3}inch	10^{-3}in	Lbs	10^{-3}in	Lbs	10^{-3}in	Lbs	10^{-3}in	Lbs
800	25	9	0	0	25	10	0	0
775	50	21	25	8	50	25	25	6
750	75	35	50	21	75	40	50	19
725	100	51	75	36	100	52	75	35
700	125	65	100	50	125	66	100	48
675	150	77	125	65	150	79	125	61
650	175	91	150	79	175	93	150	75
625	200	106	175	95	200	106	175	89
600	225	122	200	108	225	120	200	105
575	250	138	225	124	250	135	225	121
550	-	-	250	138	275	150	250	136
525	-	-	275	154	-	-	-	-



Side Wrap Very Loose, Can Probably Take More Load

Side Wrap is Looser, Not Much More Load

Notes:
1) Load applied with square harness
2) Dillon machine with 500 lb scale
3) 1/4" fiberglass caps, 3/16" balsa core with sections glued and a pine center, 45^o Kevlar wrap
4) Column made by Dave
5) 1/8" twist over 28" length



Wt = 273.2 gm = 0.60 lbs

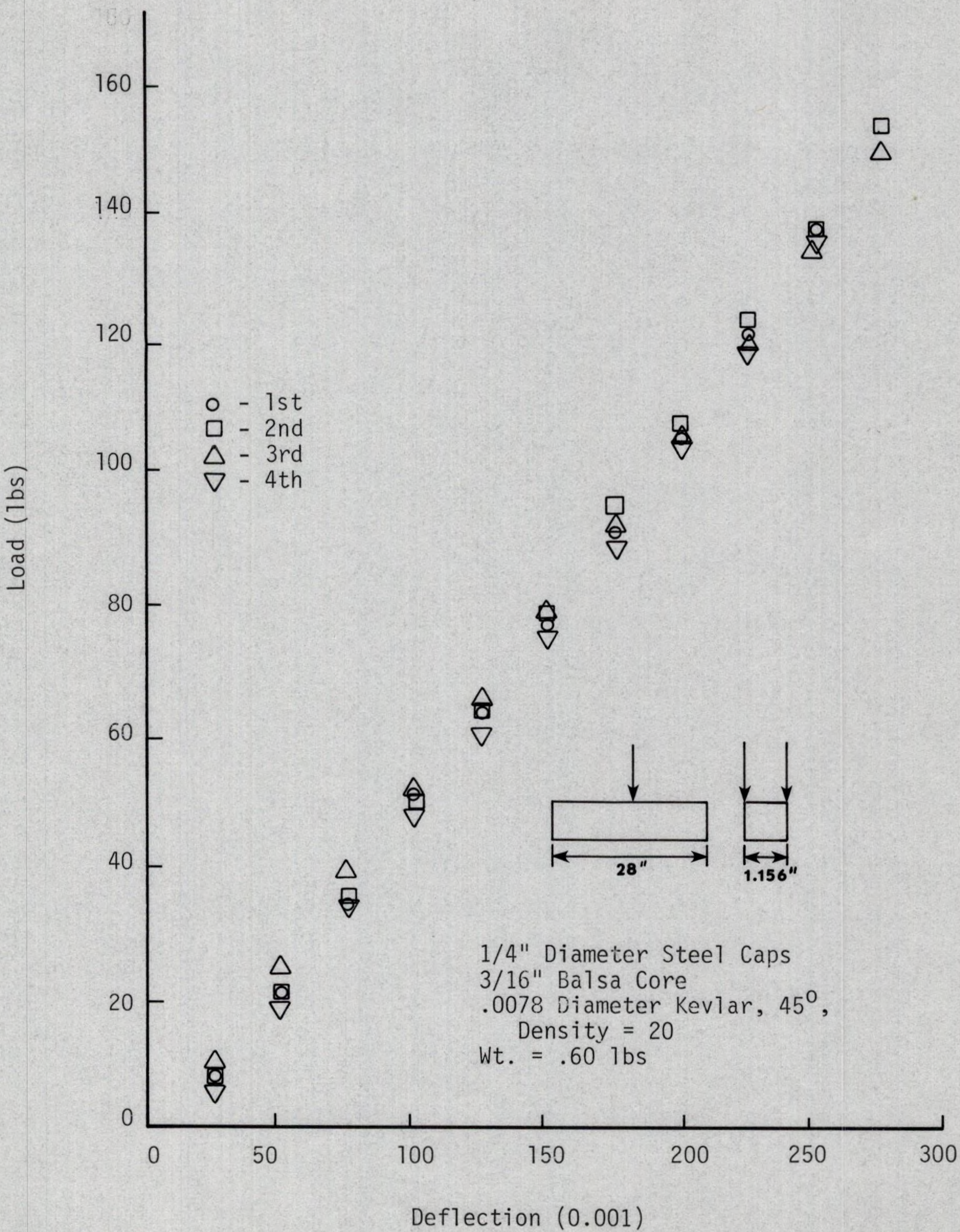


FIGURE 45 - LOAD VERSUS DEFLECTION GRAPH OF THE TABLE 16 DATA

TABLE 17

LOAD-STRAIN DATA FOR A SQUARE CROSS SECTION CAPTIVE COLUMN WITH 1/8 INCH DIAMETER STEEL CAPS; 3/16 INCH ACRYLIC CORE; AND 45^{o}, .0078 INCH DIAMETER, 20 DENSITY KEVLAR WRAP

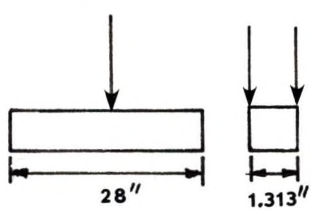
Case	Deflection (10^{-3} inch)	Load (Lbs)	ε (10^{-6} in/in)	ε_2 (10^{-6} in/in)	ε_3 (10^{-6} in/in)
1	25	10	-66	35	30
	50	30	-145	83	78
	75	51	-221	127	125
	100	76	-309	174	176
	125	104	-405	235	241
2	25	20	-52	-2	10
	50	43	-116	-1	26
	75	64	-188	1	49
	100	88	-264	2	80
	125	113	-356	6	113
3	25	17	33	-34	-18
	50	27	73	-73	-46
	75	46	141	-125	-95
	100	72	216	-172	-141
	125	94	299	-226	-201
4	25	15	48	-8	-21
	50	30	157	-14	-83
	75	50	262	-19	-142
	100	74	361	-28	-191
	125	102	468	-38	-246
5	25	15	-43	30	22
	50	35	-129	89	77
	75	58	-210	138	127
	100	87	-299	190	183
	125	115	-394	243	243
6	25	22	79	-7	-42
	50	45	184	-16	-96
	75	72	278	-26	-140
	100	96	375	-33	-192
7	25	22	31	-28	-19
	50	46	105	-84	-72
	75	69	172	-126	-118
	100	94	239	-171	-164

TABLE 17, Cont.

Case	Deflection (10^{-3}inch)	Load (Lbs)	ε (10^{-6}in/in)	ε_2 (10^{-6}in/in)	ε_3 (10^{-6}in/in)
8	25	14	-45	7	10
	50	31	-116	11	29
	75	52	-175	14	47
	100	78	-246	17	74
	125	108	-338	21	113

$\delta = 1.0446F + 12.1687$ (least squares fit of data)

Notes: 1) Load applied with square harness
2) Dillon machine with 500 lb scale
3) 1/8" steel caps, 3/16" acrylite core with solvent cement, 3M-1838 B/A for caps force, 45° Kevlar wrap
4) Core made by Mike, caps and wrap by Steve and Dave
5) Slight twist present (1/16" over 31" length)



Wt = 564.8

TABLE 18

LOAD-STRAIN DATA FOR A SQUARE CROSS SECTION CAPTIVE COLUMN WITH 1/8 INCH DIAMETER FIBERGLASS CAPS; 3/16 INCH ACRYLIC CORE; AND 45^o, .0078 INCH DIAMETER, 20 DENSITY KEVLAR WRAP

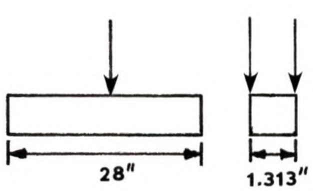
Case	Deflection (10^{-3}inch)	Load (Lbs)	ε (10^{-6}in/in)	ε_2 (10^{-6}in/in)	ε_3 (10^{-6}in/in)
1	50	16	-88	116	8
	100	30	-174	238	37
	150	48	-268	377	66
	200	66	-367	507	91
	250	86	-461	636	126
	275	93	-508	701	145
2	50	12	-89	0	-24
	100	22	-185	1	-37
	150	40	-298	6	-43
	200	56	-412	9	-51
	250	80	-533	16	-43
	300	95	-661	26	-45
3	50	9	97	-100	-33
	100	26	197	-211	-80
	150	44	288	-321	-108
	200	62	377	-438	-131
	250	82	467	-547	-155
	300	99	552	-663	-182
4	50	13	87	-13	1
	100	24	203	-40	-11
	150	38	333	-62	-27
	200	54	453	-84	-37
	250	76	574	-98	-45
	300	93	695	-121	-60
5	50	12	-71	109	5
	100	24	-154	251	24
	150	39	-241	376	49
	200	60	-346	514	79
	250	78	-449	648	107
	300	96	-551	786	142

TABLE 18, Cont.

Case	Deflection (10^{-3}inch)	Load (Lbs)	ε (10^{-6}in/in)	ε_2 (10^{-6}in/in)	ε_3 (10^{-6}in/in)
6	50	14	128	-18	-2
	100	28	259	-19	-19
	150	44	399	-41	-35
	200	58	533	-51	-52
	250	74	670	-76	-73
	300	91	810	-102	-83
7	50	12	98	-117	-40
	100	23	208	-241	-83
	150	42	313	-357	-121
	200	54	406	-475	-169
	250	74	499	-593	-189
	300	92	588	-711	-219
8	50	15	-68	2	-28
	100	26	-173	16	-54
	150	42	-280	30	-70
	200	59	-395	41	-77
	250	80	-510	57	-78
	300	96	-634	72	-87

$\delta = 2.9258F + 22.2762$ (least squares fit of data)

Notes:
1) Load applied with square harness
2) Dillon machine with 500 lb scale
3) 1/8" fiberglass caps, 3/16" acrylite core with solvent cement, 3M-1838 B/A for caps to core, 45^{o} Kevlar wrap
4) Core made by Mike, caps and wrap by Steve and Dave
5) No twist in the column



Wt = 437.1 gms

APPENDIX C

TRIANGULAR CROSS SECTION FINITE ELEMENT COMPUTER PROGRAM

TRIANGULAR CAPTIVE COLUMN-FBSTCDT- 1/4IN. FIBERGLASS CAPS-BALSA CORE-KEV

C O N T R O L   I N F O R M A T I O N

```
     NUMBER OF NODAL POINTS     =    61
     NUMBER OF ELEMENT TYPES    =     4
     NUMBER OF LOAD CASES       =     1
     NUMBER OF FREQUENCIES      =     0
     ANALYSIS CODE (NDYN)       =     0
       EQ.0,  STATIC
       EQ.1,  MODAL EXTRACTION
       EQ.2,  FORCED RESPONSE
       EQ.3,  RESPONSE SPECTRUM
       EQ.4,  DIRECT INTEGRATION
     SOLUTION MODE (MODEX)      =     0
       EQ.0,  EXECUTION
       EQ.1,  DATA CHECK
     NUMBER OF SUBSPACE
     ITERATION VECTORS (NAD)    =     0
     EQUATIONS PER BLOCK        =     0
     TAPE10 SAVE FLAG (N10SV) =       0
```

NODAL POINT INPUT DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	NODAL POINT COORDINATES X	Y	Z		T
1	0	1	0	0	0	0	0.0	0.0	0.0	0	0.0
5	0	0	0	0	0	0	0.0	0.0	2.000	0	0.0
29	0	0	1	0	0	0	0.0	0.0	14.000	4	0.0
33	0	0	0	0	0	0	0.0	0.0	16.000	0	0.0
57	0	1	0	0	0	0	0.0	0.0	28.000	4	0.0
2	0	0	0	0	0	0	0.938	1.624	0.0	0	0.0
30	1	0	1	0	0	0	0.938	1.624	14.000	4	0.0
34	0	0	0	0	0	0	0.938	1.624	16.000	0	0.0
58	0	0	0	0	0	0	0.938	1.624	28.000	4	0.0
3	0	1	0	0	0	0	1.875	0.0	0.0	0	0.0
7	0	0	0	0	0	0	1.875	0.0	2.000	0	0.0
31	0	0	1	0	0	0	1.875	0.0	14.000	4	0.0
35	0	0	0	0	0	0	1.875	0.0	16.000	0	0.0
59	0	1	0	0	0	0	1.875	0.0	28.000	4	0.0
4	0	0	0	0	0	0	0.938	0.541	0.0	0	0.0
32	1	0	1	0	0	0	0.938	0.541	14.000	4	0.0
36	0	0	0	0	0	0	0.938	0.541	16.000	0	0.0
60	0	0	0	0	0	0	0.938	0.541	28.000	4	0.0
61	1	1	1	1	1	1	0.938	0.541	30.000	0	0.0

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	NODAL POINT COORDINATES X	Y	Z	T
1	0	1	0	0	0	0	0.0	0.0	0.0	0.0
2	0	0	0	0	0	0	0.938	1.624	0.0	0.0
3	0	1	0	0	0	0	1.875	0.0	0.0	0.0
4	0	0	0	0	0	0	0.938	0.541	0.0	0.0
5	0	0	0	0	0	0	0.0	0.0	2.000	0.0
6	0	0	0	0	0	0	0.938	1.624	2.000	0.0
7	0	0	0	0	0	0	1.875	0.0	2.000	0.0
8	0	0	0	0	0	0	0.938	0.541	2.000	0.0
9	0	0	0	0	0	0	0.0	0.0	4.000	0.0
10	0	0	0	0	0	0	0.938	1.624	4.000	0.0
11	0	0	0	0	0	0	1.875	0.0	4.000	0.0
12	0	0	0	0	0	0	0.938	0.541	4.000	0.0
13	0	0	0	0	0	0	0.0	0.0	6.000	0.0
14	0	0	0	0	0	0	0.938	1.624	6.000	0.0
15	0	0	0	0	0	0	1.875	0.0	6.000	0.0
16	0	0	0	0	0	0	0.938	0.541	6.000	0.0
17	0	0	0	0	0	0	0.0	0.0	8.000	0.0
18	0	0	0	0	0	0	0.938	1.624	8.000	0.0
19	0	0	0	0	0	0	1.875	0.0	8.000	0.0
20	0	0	0	0	0	0	0.938	0.541	8.000	0.0
21	0	0	0	0	0	0	0.0	0.0	10.000	0.0
22	0	0	0	0	0	0	0.938	1.624	10.000	0.0
23	0	0	0	0	0	0	1.875	0.0	10.000	0.0
24	0	0	0	0	0	0	0.938	0.541	10.000	0.0
25	0	0	0	0	0	0	0.0	0.0	12.000	0.0
26	0	0	0	0	0	0	0.938	1.624	12.000	0.0
27	0	0	0	0	0	0	1.875	0.0	12.000	0.0
28	0	0	0	0	0	0	0.938	0.541	12.000	0.0
29	0	0	1	0	0	0	0.0	0.0	14.000	0.0
30	1	0	1	0	0	0	0.938	1.624	14.000	0.0
31	0	0	1	0	0	0	1.875	0.0	14.000	0.0
32	1	0	1	0	0	0	0.938	0.541	14.000	0.0
33	0	0	0	0	0	0	0.0	0.0	16.000	0.0
34	0	0	0	0	0	0	0.938	1.624	16.000	0.0
35	0	0	0	0	0	0	1.875	0.0	16.000	0.0
36	0	0	0	0	0	0	0.938	0.541	16.000	0.0
37	0	0	0	0	0	0	0.0	0.0	18.000	0.0
38	0	0	0	0	0	0	0.938	1.624	18.000	0.0
39	0	0	0	0	0	0	1.875	0.0	18.000	0.0
40	0	0	0	0	0	0	0.938	0.541	18.000	0.0

41	0	0	0	0	0	0	0.0	0.0	20.000	0.0
42	0	0	0	0	0	0	0.938	1.624	20.000	0.0
43	0	0	0	0	0	0	1.875	0.0	20.000	0.0
44	0	0	0	0	0	0	0.938	0.541	20.000	0.0
45	0	0	0	0	0	0	0.0	0.0	22.000	0.0
46	0	0	0	0	0	0	0.938	1.624	22.000	0.0
47	0	0	0	0	0	0	1.875	0.0	22.000	0.0
48	0	0	0	0	0	0	0.938	0.541	22.000	0.0
49	0	0	0	0	0	0	0.0	0.0	24.000	0.0
50	0	0	0	0	0	0	0.938	1.624	24.000	0.0
51	0	0	0	0	0	0	1.875	0.0	24.000	0.0
52	0	0	0	0	0	0	0.938	0.541	24.000	0.0
53	0	0	0	0	0	0	0.0	0.0	26.000	0.0
54	0	0	0	0	0	0	0.938	1.624	26.000	0.0
55	0	0	0	0	0	0	1.875	0.0	26.000	0.0
56	0	0	0	0	0	0	0.938	0.541	26.000	0.0
57	0	1	0	0	0	0	0.0	0.0	28.000	0.0
58	0	0	0	0	0	0	0.938	1.624	28.000	0.0
59	0	1	0	0	0	0	1.875	0.0	28.000	0.0
60	0	0	0	0	0	0	0.938	0.541	28.000	0.0
61	1	1	1	1	1	1	0.938	0.541	30.000	0.0

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	1	0	2	3	4	5
2	6	7	8	9	10	11
3	12	0	13	14	15	16
4	17	18	19	20	21	22
5	23	24	25	26	27	28
6	29	30	31	32	33	34
7	35	36	37	38	39	40
8	41	42	43	44	45	46
9	47	48	49	50	51	52
10	53	54	55	56	57	58
11	59	60	61	62	63	64
12	65	66	67	68	69	70
13	71	72	73	74	75	76
14	77	78	79	80	81	82
15	83	84	85	86	87	88
16	89	90	91	92	93	94
17	95	96	97	98	99	100
18	101	102	103	104	105	106
19	107	108	109	110	111	112
20	113	114	115	116	117	118
21	119	120	121	122	123	124
22	125	126	127	128	129	130
23	131	132	133	134	135	136
24	137	138	139	140	141	142
25	143	144	145	146	147	148
26	149	150	151	152	153	154
27	155	156	157	158	159	160
28	161	162	163	164	165	166
29	167	168	0	169	170	171
30	0	172	0	173	174	175
31	176	177	0	178	179	180
32	0	181	0	182	183	184
33	185	186	187	188	189	190
34	191	192	193	194	195	196
35	197	198	199	200	201	202
36	203	204	205	206	207	208
37	209	210	211	212	213	214
38	215	216	217	218	219	220
39	221	222	223	224	225	226
40	227	228	229	230	231	232

41	233	234	235	236	237	238
42	239	240	241	242	243	244
43	245	246	247	248	249	250
44	251	252	253	254	255	256
45	257	258	259	260	261	262
46	263	264	265	266	267	268
47	269	270	271	272	273	274
48	275	276	277	278	279	280
49	281	282	283	284	285	286
50	287	288	289	290	291	292
51	293	294	295	296	297	298
52	299	300	301	302	303	304
53	305	306	307	308	309	310
54	311	312	313	314	315	316
55	317	318	319	320	321	322
56	323	324	325	326	327	328
57	329	0	330	331	332	333
58	334	335	336	337	338	339
59	340	0	341	342	343	344
60	345	346	347	348	349	350
61	0	0	0	0	0	0

3 / D B E A M   E L E M E N T S

NUMBER OF BEAMS	=	42
NUMBER OF GEOMETRIC PROPERTY SETS	=	1
NUMBER OF FIXED END FORCE SETS	=	0
NUMBER OF MATERIALS	=	1

MATERIAL PROPERTIES

MATERIAL NUMBER	YOUNG*S MODULUS	POISSON*S RATIO	MASS DENSITY	WEIGHT DENSITY
1	0.6000D 07	0.3000	0.0	0.0

BEAM GEOMETRIC PROPERTIES

SECTION NUMBER	AXIAL AREA A(1)	SHEAR AREA A(2)	SHEAR AREA A(3)	TORSION J(1)	INERTIA I(2)	INERTIA I(3)
1	0.4900D-01	0.0	0.0	0.3830D-03	0.1917D-03	0.1917D-03

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0	0.0	0.0	0.0
Y-DIR	0.0	0.0	0.0	0.0
Z-DIR	0.0	0.0	0.0	0.0

3/D BEAM ELEMENT DATA

BEAM NUMBER	NODE -I	NODE -J	NODE -K	MATERIAL NUMBER	SECTION NUMBER	ELEMENT END LOADS A	B	C	D	END CODES -I	-J
1	1	5	61	1	1	0	0	0	0	0	0
2	5	9	61	1	1	0	0	0	0	0	0
3	9	13	61	1	1	0	0	0	0	0	0
4	13	17	61	1	1	0	0	0	0	0	0
5	17	21	61	1	1	0	0	0	0	0	0
6	21	25	61	1	1	0	0	0	0	0	0
7	25	29	61	1	1	0	0	0	0	0	0
8	29	33	61	1	1	0	0	0	0	0	0
9	33	37	61	1	1	0	0	0	0	0	0
10	37	41	61	1	1	0	0	0	0	0	0
11	41	45	61	1	1	0	0	0	0	0	0
12	45	49	61	1	1	0	0	0	0	0	0
13	49	53	61	1	1	0	0	0	0	0	0
14	53	57	61	1	1	0	0	0	0	0	0
15	2	6	61	1	1	0	0	0	0	0	0
16	6	10	61	1	1	0	0	0	0	0	0
17	10	14	61	1	1	0	0	0	0	0	0
18	14	18	61	1	1	0	0	0	0	0	0
19	18	22	61	1	1	0	0	0	0	0	0
20	22	26	61	1	1	0	0	0	0	0	0
21	26	30	61	1	1	0	0	0	0	0	0
22	30	34	61	1	1	0	0	0	0	0	0
23	34	38	61	1	1	0	0	0	0	0	0
24	38	42	61	1	1	0	0	0	0	0	0
25	42	46	61	1	1	0	0	0	0	0	0
26	46	50	61	1	1	0	0	0	0	0	0
27	50	54	61	1	1	0	0	0	0	0	0
28	54	58	61	1	1	0	0	0	0	0	0
29	3	7	61	1	1	0	0	0	0	0	0
30	7	11	61	1	1	0	0	0	0	0	0
31	11	15	61	1	1	0	0	0	0	0	0
32	15	19	61	1	1	0	0	0	0	0	0
33	19	23	61	1	1	0	0	0	0	0	0
34	23	27	61	1	1	0	0	0	0	0	0
35	27	31	61	1	1	0	0	0	0	0	0
36	31	35	61	1	1	0	0	0	0	0	0
37	35	39	61	1	1	0	0	0	0	0	0
38	39	43	61	1	1	0	0	0	0	0	0
39	43	47	61	1	1	0	0	0	0	0	0
40	47	51	61	1	1	0	0	0	0	0	0
41	51	55	61	1	1	0	0	0	0	0	0
42	55	59	61	1	1	0	0	0	0	0	0

```
PLANE STRESS ANALYSIS
MEMBRANE ELEMENTS
INCOMPATIBLE MODES SUPPRESSED


NUMBER OF ELEMENTS     =     42
NUMBER OF MATERIALS    =      1
MAXIMUM TEMPERATURES
PER MATERIAL           =      1
ANALYSIS CODE          =      2
CODE FOR INCLUSION
OF BENDING MODES       =      1
   EQ.0, INCLUDE
   GT.0, SUPPRESS




MATERIAL I.D. NUMBER     =      1
NUMBER OF TEMPERATURES =      1
WEIGHT DENSITY           =     0.0
MASS   DENSITY           =     0.0
BETA ANGLE               =     0.0

TEMPERATURE          E(N)           E(S)           E(T)      NU(NS)     NU(NT)     NU(ST)
        0.0   0.1340D 05    0.4000D 06    0.1340D 05    0.3000     0.3000     0.0400
```

G(NS)	ALPHA(N)	ALPHA(S)	ALPHA(T)
0.1800D 05	0.1000D-03	0.1000D-03	0.1000D-03

ELEMENT LOAD MULTIPLIERS

LOAD CASE	TEMPERATURE	PRESSURE	X-GRAVITY	Y-GRAVITY	Z-GRAVITY
A	0.0	0.0	0.0	0.0	0.0
B	0.0	0.0	0.0	0.0	0.0
C	0.0	0.0	0.0	0.0	0.0
D	0.0	0.0	0.0	0.0	0.0

ELEMENT NUMBER	I	J	K	L	MATL TYPE	REFERENCE TEMPERATURE	I-J FACE PRESSURE	STRESS OPTION	KG	THICKNESS
1	4	8	5	1	1	0.0	0.0	4	1	0.1875
2	8	12	9	5	1	0.0	0.0	4	4	0.1875
3	12	16	13	9	1	0.0	0.0	4	4	0.1875
4	16	20	17	13	1	0.0	0.0	4	4	0.1875
5	20	24	21	17	1	0.0	0.0	4	4	0.1875
6	24	28	25	21	1	0.0	0.0	4	4	0.1875
7	28	32	29	25	1	0.0	0.0	4	4	0.1875
8	32	36	33	29	1	0.0	0.0	4	4	0.1875
9	36	40	37	33	1	0.0	0.0	4	4	0.1875
10	40	44	41	37	1	0.0	0.0	4	4	0.1875
11	44	48	45	41	1	0.0	0.0	4	4	0.1875
12	48	52	49	45	1	0.0	0.0	4	4	0.1875
13	52	56	53	49	1	0.0	0.0	4	4	0.1875
14	56	60	57	53	1	0.0	0.0	4	4	0.1875
15	4	8	6	2	1	0.0	0.0	4	1	0.1875
16	8	12	10	6	1	0.0	0.0	4	4	0.1875
17	12	16	14	10	1	0.0	0.0	4	4	0.1875
18	16	20	18	14	1	0.0	0.0	4	4	0.1875
19	20	24	22	18	1	0.0	0.0	4	4	0.1875
20	24	28	26	22	1	0.0	0.0	4	4	0.1875
21	28	32	30	26	1	0.0	0.0	4	4	0.1875
22	32	36	34	30	1	0.0	0.0	4	4	0.1875
23	36	40	38	34	1	0.0	0.0	4	4	0.1875
24	40	44	42	38	1	0.0	0.0	4	4	0.1875
25	44	48	46	42	1	0.0	0.0	4	4	0.1875
26	48	52	50	46	1	0.0	0.0	4	4	0.1875
27	52	56	54	50	1	0.0	0.0	4	4	0.1875
28	56	60	58	54	1	0.0	0.0	4	4	0.1875
29	4	8	7	3	1	0.0	0.0	4	1	0.1875
30	8	12	11	7	1	0.0	0.0	4	4	0.1875
31	12	16	15	11	1	0.0	0.0	4	4	0.1875
32	16	20	19	15	1	0.0	0.0	4	4	0.1875
33	20	24	23	19	1	0.0	0.0	4	4	0.1875
34	24	28	27	23	1	0.0	0.0	4	4	0.1875
35	28	32	31	27	1	0.0	0.0	4	4	0.1875
36	32	36	35	31	1	0.0	0.0	4	4	0.1875
37	36	40	39	35	1	0.0	0.0	4	4	0.1875
38	40	44	43	39	1	0.0	0.0	4	4	0.1875
39	44	48	47	43	1	0.0	0.0	4	4	0.1875
40	48	52	51	47	1	0.0	0.0	4	4	0.1875
41	52	56	55	51	1	0.0	0.0	4	4	0.1875
42	56	60	59	55	1	0.0	0.0	4	4	0.1875

3 / D B E A M E L E M E N T S

NUMBER OF BEAMS = 45
NUMBER OF GEOMETRIC PROPERTY SETS= 1
NUMBER OF FIXED END FORCE SETS = 0
NUMBER OF MATERIALS = 1

MATERIAL PROPERTIES

MATERIAL NUMBER	YOUNG*S MODULUS	POISSON*S RATIO	MASS DENSITY	WEIGHT DENSITY
1	0.1000D 01	0.3000	0.0	0.0

BEAM GEOMETRIC PROPERTIES

SECTION NUMBER	AXIAL AREA A(1)	SHEAR AREA A(2)	SHEAR AREA A(3)	TORSION J(1)	INERTIA I(2)	INERTIA I(3)
1	0.3520D-01	0.0	0.0	0.2060D-03	0.1030D-03	0.1030D-03

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0	0.0	0.0	0.0
Y-DIR	0.0	0.0	0.0	0.0
Z-DIR	0.0	0.0	0.0	0.0

3/D BEAM ELEMENT DATA

BEAM NUMBER	NODE -I	NODE -J	NODE -K	MATERIAL NUMBER	SECTION NUMBER	ELEMENT END LOADS A	B	C	D	END CODES -I	-J
1	4	1	61	1	1	0	0	0	0	0	0
2	8	5	61	1	1	0	0	0	0	0	0
3	12	9	61	1	1	0	0	0	0	0	0
4	16	13	61	1	1	0	0	0	0	0	0
5	20	17	61	1	1	0	0	0	0	0	0
6	24	21	61	1	1	0	0	0	0	0	0
7	28	25	61	1	1	0	0	0	0	0	0
8	32	29	61	1	1	0	0	0	0	0	0
9	36	33	61	1	1	0	0	0	0	0	0
10	40	37	61	1	1	0	0	0	0	0	0
11	44	41	61	1	1	0	0	0	0	0	0
12	48	45	61	1	1	0	0	0	0	0	0
13	52	49	61	1	1	0	0	0	0	0	0
14	56	53	61	1	1	0	0	0	0	0	0
15	60	57	61	1	1	0	0	0	0	0	0
16	4	2	61	1	1	0	0	0	0	0	0
17	8	6	61	1	1	0	0	0	0	0	0
18	12	10	61	1	1	0	0	0	0	0	0
19	16	14	61	1	1	0	0	0	0	0	0
20	20	18	61	1	1	0	0	0	0	0	0
21	24	22	61	1	1	0	0	0	0	0	0
22	28	26	61	1	1	0	0	0	0	0	0
23	32	30	61	1	1	0	0	0	0	0	0
24	36	34	61	1	1	0	0	0	0	0	0
25	40	38	61	1	1	0	0	0	0	0	0
26	44	42	61	1	1	0	0	0	0	0	0
27	48	46	61	1	1	0	0	0	0	0	0
28	52	50	61	1	1	0	0	0	0	0	0
29	56	54	61	1	1	0	0	0	0	0	0
30	60	58	61	1	1	0	0	0	0	0	0
31	4	3	61	1	1	0	0	0	0	0	0
32	8	7	61	1	1	0	0	0	0	0	0
33	12	11	61	1	1	0	0	0	0	0	0
34	16	15	61	1	1	0	0	0	0	0	0
35	20	19	61	1	1	0	0	0	0	0	0
36	24	23	61	1	1	0	0	0	0	0	0
37	28	27	61	1	1	0	0	0	0	0	0
38	32	31	61	1	1	0	0	0	0	0	0
39	36	35	61	1	1	0	0	0	0	0	0
40	40	39	61	1	1	0	0	0	0	0	0
41	44	43	61	1	1	0	0	0	0	0	0
42	48	47	61	1	1	0	0	0	0	0	0
43	52	51	61	1	1	0	0	0	0	0	0
44	56	55	61	1	1	0	0	0	0	0	0
45	60	59	61	1	1	0	0	0	0	0	0

NUMBER OF TRUSS MEMBERS= 84
NUMBER OF DIFF. MEMBERS= 2

TYPE	E	ALPHA	DEN	AREA	WT
1	0.1800000D 08	0.2000000D-04	0.0	0.1767000D-02	0.0
2	0.1000000D 01	0.2000000D-04	0.0	0.1767000D-02	0.0

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0	0.0	0.0	0.0
Y-DIR	0.0	0.0	0.0	0.0
Z-DIR	0.0	0.0	0.0	0.0
TEMP	0.0	0.0	0.0	0.0

N	I	J	TYPE	TEMP	BAND
1	1	6	2	0.0	31
2	5	10	2	0.0	33
3	9	14	2	0.0	33
4	13	18	2	0.0	33
5	17	22	2	0.0	33
6	21	26	2	0.0	33
7	25	30	2	0.0	30
8	29	34	1	0.0	27
9	33	38	1	0.0	33
10	37	42	1	0.0	33
11	41	46	1	0.0	33
12	45	50	1	0.0	33
13	49	54	1	0.0	33
14	53	58	1	0.0	32
15	2	5	1	0.0	20
16	6	9	1	0.0	21
17	10	13	1	0.0	21
18	14	17	1	0.0	21
19	18	21	1	0.0	21
20	22	25	1	0.0	21
21	26	29	1	0.0	20
22	30	33	2	0.0	16
23	34	37	2	0.0	21
24	38	41	2	0.0	21
25	42	45	2	0.0	21
26	46	49	2	0.0	21
27	50	53	2	0.0	21
28	54	57	2	0.0	20
29	3	6	2	0.0	20
30	7	10	2	0.0	21
31	11	14	2	0.0	21
32	15	18	2	0.0	21
33	19	22	2	0.0	21
34	23	26	2	0.0	21
35	27	30	2	0.0	18
36	31	34	1	0.0	18
37	35	38	1	0.0	21
38	39	42	1	0.0	21
39	43	46	1	0.0	21
40	47	50	1	0.0	21
41	51	54	1	0.0	21
42	55	58	1	0.0	20
43	2	7	1	0.0	32
44	6	11	1	0.0	33
45	10	15	1	0.0	33

46	14	19	1	0.0	33
47	18	23	1	0.0	33
48	22	27	1	0.0	33
49	26	31	1	0.0	29
50	30	35	2	0.0	28
51	34	39	2	0.0	33
52	38	43	2	0.0	33
53	42	47	2	0.0	33
54	46	51	2	0.0	33
55	50	55	2	0.0	33
56	54	59	2	0.0	31
57	1	7	1	0.0	37
58	5	11	2	0.0	39
59	9	15	1	0.0	39
60	13	19	1	0.0	39
61	17	23	2	0.0	39
62	21	27	2	0.0	39
63	25	31	1	0.0	35
64	29	35	1	0.0	33
65	33	39	2	0.0	39
66	37	43	2	0.0	39
67	41	47	1	0.0	39
68	45	51	1	0.0	39
69	49	55	2	0.0	39
70	53	59	1	0.0	37
71	3	5	1	0.0	14
72	7	9	2	0.0	15
73	11	13	1	0.0	15
74	15	17	1	0.0	15
75	19	21	2	0.0	15
76	23	25	2	0.0	15
77	27	29	1	0.0	14
78	31	33	1	0.0	12
79	35	37	2	0.0	15
80	39	41	2	0.0	15
81	43	45	1	0.0	15
82	47	49	1	0.0	15
83	51	53	2	0.0	15
84	55	57	1	0.0	14

E Q U A T I O N   P A R A M E T E R S

```
TOTAL NUMBER OF EQUATIONS          =   350
BANDWIDTH                          =    45
NUMBER OF EQUATIONS IN A BLOCK     =    74
NUMBER OF BLOCKS                   =     5
```

N O D A L   L O A D S   (S T A T I C)   O R   M A S S E S   (D Y N A M I C)

NODE NUMBER	LOAD CASE	X-AXIS FORCE	Y-AXIS FORCE	Z-AXIS FORCE	X-AXIS MOMENT
30	1	0.0	-0.10000D 03	0.0	0.0

Y-AXIS MOMENT	Z-AXIS MOMENT
0.0	0.0

STRUCTURE LOAD CASE	ELEMENT LOAD MULTIPLIERS A	B	C	D
1	0.0	0.0	0.0	0.0

APPENDIX D

SQUARE CROSS SECTION FINITE ELEMENT COMPUTER PROGRAM

SQUARE CAPTIVE COLUMN -FESTDDD- 1/8IN. STEEL CAPS - PLEXIGLASS CORE KEVL

C O N T R O L   I N F O R M A T I O N

```
    NUMBER OF NODAL POINTS     =   106
    NUMBER OF ELEMENT TYPES    =     4
    NUMBER OF LOAD CASES       =     1
    NUMBER OF FREQUENCIES      =     0
    ANALYSIS CODE (NDYN)       =     0
      EQ.0,  STATIC
      EQ.1,  MODAL EXTRACTION
      EQ.2,  FORCED RESPONSE
      EQ.3,  RESPONSE SPECTRUM
      EQ.4,  DIRECT INTEGRATION
    SOLUTION MODE (MODEX)      =     0
      EQ.0,  EXECUTION
      EQ.1,  DATA CHECK
    NUMBER OF SUBSPACE
    ITERATION VECTORS (NAD)    =     0
    EQUATIONS PER BLOCK        =     0
    TAPE10 SAVE FLAG (N10SV)   =     0
```

NODAL POINT INPUT DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	NODAL POINT COORDINATES X	Y	Z		T
1	0	1	0	0	0	0	0.0	0.0	0.0	0	0.0
6	0	0	0	0	0	0	0.0	0.0	1.400	0	0.0
51	0	0	1	0	0	0	0.0	0.0	14.000	5	0.0
56	0	0	0	0	0	0	0.0	0.0	15.400	0	0.0
101	0	1	0	0	0	0	0.0	0.0	28.000	5	0.0
2	0	0	0	0	0	0	0.0	1.325	0.0	0	0.0
52	1	0	1	0	0	0	0.0	1.325	14.000	5	0.0
57	0	0	0	0	0	0	0.0	1.325	15.400	0	0.0
102	0	0	0	0	0	0	0.0	1.325	28.000	5	0.0
3	0	0	0	0	0	0	1.325	1.325	0.0	0	0.0
53	1	0	1	0	0	0	1.325	1.325	14.000	5	0.0
58	0	0	0	0	0	0	1.325	1.325	15.400	0	0.0
103	0	0	0	0	0	0	1.325	1.325	28.000	5	0.0
4	0	1	0	0	0	0	1.325	0.0	0.0	0	0.0
9	0	0	0	0	0	0	1.325	0.0	1.400	0	0.0
54	0	0	1	0	0	0	1.325	0.0	14.000	5	0.0
59	0	0	0	0	0	0	1.325	0.0	15.400	0	0.0
104	0	1	0	0	0	0	1.325	0.0	28.000	5	0.0
5	0	0	0	0	0	0	0.663	0.663	0.0	0	0.0
55	0	0	1	0	0	0	0.663	0.663	14.000	5	0.0
60	0	0	0	0	0	0	0.663	0.663	15.400	0	0.0
105	0	0	0	0	0	0	0.663	0.663	28.000	5	0.0
106	1	1	1	1	1	1	0.663	0.663	30.000	0	0.0

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	NODAL POINT COORDINATES X	Y	Z	T
1	0	1	0	0	0	0	0.0	0.0	0.0	0.0
2	0	0	0	0	0	0	0.0	1.325	0.0	0.0
3	0	0	0	0	0	0	1.325	1.325	0.0	0.0
4	0	1	0	0	0	0	1.325	0.0	0.0	0.0
5	0	0	0	0	0	0	0.663	0.663	0.0	0.0
6	0	0	0	0	0	0	0.0	0.0	1.400	0.0
7	0	0	0	0	0	0	0.0	1.325	1.400	0.0
8	0	0	0	0	0	0	1.325	1.325	1.400	0.0
9	0	0	0	0	0	0	1.325	0.0	1.400	0.0
10	0	0	0	0	0	0	0.663	0.663	1.400	0.0
11	0	0	0	0	0	0	0.0	0.0	2.800	0.0
12	0	0	0	0	0	0	0.0	1.325	2.800	0.0
13	0	0	0	0	0	0	1.325	1.325	2.800	0.0
14	0	0	0	0	0	0	1.325	0.0	2.800	0.0
15	0	0	0	0	0	0	0.663	0.663	2.800	0.0
16	0	0	0	0	0	0	0.0	0.0	4.200	0.0
17	0	0	0	0	0	0	0.0	1.325	4.200	0.0
18	0	0	0	0	0	0	1.325	1.325	4.200	0.0
19	0	0	0	0	0	0	1.325	0.0	4.200	0.0
20	0	0	0	0	0	0	0.663	0.663	4.200	0.0
21	0	0	0	0	0	0	0.0	0.0	5.600	0.0
22	0	0	0	0	0	0	0.0	1.325	5.600	0.0
23	0	0	0	0	0	0	1.325	1.325	5.600	0.0
24	0	0	0	0	0	0	1.325	0.0	5.600	0.0
25	0	0	0	0	0	0	0.663	0.663	5.600	0.0
26	0	0	0	0	0	0	0.0	0.0	7.000	0.0
27	0	0	0	0	0	0	0.0	1.325	7.000	0.0
28	0	0	0	0	0	0	1.325	1.325	7.000	0.0
29	0	0	0	0	0	0	1.325	0.0	7.000	0.0
30	0	0	0	0	0	0	0.663	0.663	7.000	0.0
31	0	0	0	0	0	0	0.0	0.0	8.400	0.0
32	0	0	0	0	0	0	0.0	1.325	8.400	0.0
33	0	0	0	0	0	0	1.325	1.325	8.400	0.0
34	0	0	0	0	0	0	1.325	0.0	8.400	0.0
35	0	0	0	0	0	0	0.663	0.663	8.400	0.0
36	0	0	0	0	0	0	0.0	0.0	9.800	0.0
37	0	0	0	0	0	0	0.0	1.325	9.800	0.0
38	0	0	0	0	0	0	1.325	1.325	9.800	0.0
39	0	0	0	0	0	0	1.325	0.0	9.800	0.0
40	0	0	0	0	0	0	0.663	0.663	9.800	0.0

41	0	0	0	0	0	0	0.0	0.0	11.200	0.0
42	0	0	0	0	0	0	0.0	1.325	11.200	0.0
43	0	0	0	0	0	0	1.325	1.325	11.200	0.0
44	0	0	0	0	0	0	1.325	0.0	11.200	0.0
45	0	0	0	0	0	0	0.663	0.663	11.200	0.0
46	0	0	0	0	0	0	0.0	0.0	12.600	0.0
47	0	0	0	0	0	0	0.0	1.325	12.600	0.0
48	0	0	0	0	0	0	1.325	1.325	12.600	0.0
49	0	0	0	0	0	0	1.325	0.0	12.600	0.0
50	0	0	0	0	0	0	0.663	0.663	12.600	0.0
51	0	0	1	0	0	0	0.0	0.0	14.000	0.0
52	1	0	1	0	0	0	0.0	1.325	14.000	0.0
53	1	0	1	0	0	0	1.325	1.325	14.000	0.0
54	0	0	1	0	0	0	1.325	0.0	14.000	0.0
55	0	0	1	0	0	0	0.663	0.663	14.000	0.0
56	0	0	0	0	0	0	0.0	0.0	15.400	0.0
57	0	0	0	0	0	0	0.0	1.325	15.400	0.0
58	0	0	0	0	0	0	1.325	1.325	15.400	0.0
59	0	0	0	0	0	0	1.325	0.0	15.400	0.0
60	0	0	0	0	0	0	0.663	0.663	15.400	0.0
61	0	0	0	0	0	0	0.0	0.0	16.800	0.0
62	0	0	0	0	0	0	0.0	1.325	16.800	0.0
63	0	0	0	0	0	0	1.325	1.325	16.800	0.0
64	0	0	0	0	0	0	1.325	0.0	16.800	0.0
65	0	0	0	0	0	0	0.663	0.663	16.800	0.0
66	0	0	0	0	0	0	0.0	0.0	18.200	0.0
67	0	0	0	0	0	0	0.0	1.325	18.200	0.0
68	0	0	0	0	0	0	1.325	1.325	18.200	0.0
69	0	0	0	0	0	0	1.325	0.0	18.200	0.0
70	0	0	0	0	0	0	0.663	0.663	18.200	0.0
71	0	0	0	0	0	0	0.0	0.0	19.600	0.0
72	0	0	0	0	0	0	0.0	1.325	19.600	0.0
73	0	0	0	0	0	0	1.325	1.325	19.600	0.0
74	0	0	0	0	0	0	1.325	0.0	19.600	0.0
75	0	0	0	0	0	0	0.663	0.663	19.600	0.0
76	0	0	0	0	0	0	0.0	0.0	21.000	0.0
77	0	0	0	0	0	0	0.0	1.325	21.000	0.0
78	0	0	0	0	0	0	1.325	1.325	21.000	0.0
79	0	0	0	0	0	0	1.325	0.0	21.000	0.0
80	0	0	0	0	0	0	0.663	0.663	21.000	0.0
81	0	0	0	0	0	0	0.0	0.0	22.400	0.0
82	0	0	0	0	0	0	0.0	1.325	22.400	0.0

83	0	0	0	0	0	0	1.325	1.325	22.400	0.0
84	0	0	0	0	0	0	1.325	0.0	22.400	0.0
85	0	0	0	0	0	0	0.663	0.663	22.400	0.0
86	0	0	0	0	0	0	0.0	0.0	23.800	0.0
87	0	0	0	0	0	0	0.0	1.325	23.800	0.0
88	0	0	0	0	0	0	1.325	1.325	23.800	0.0
89	0	0	0	0	0	0	1.325	0.0	23.800	0.0
90	0	0	0	0	0	0	0.663	0.663	23.800	0.0
91	0	0	0	0	0	0	0.0	0.0	25.200	0.0
92	0	0	0	0	0	0	0.0	1.325	25.200	0.0
93	0	0	0	0	0	0	1.325	1.325	25.200	0.0
94	0	0	0	0	0	0	1.325	0.0	25.200	0.0
95	0	0	0	0	0	0	0.663	0.663	25.200	0.0
96	0	0	0	0	0	0	0.0	0.0	26.600	0.0
97	0	0	0	0	0	0	0.0	1.325	26.600	0.0
98	0	0	0	0	0	0	1.325	1.325	26.600	0.0
99	0	0	0	0	0	0	1.325	0.0	26.600	0.0
100	0	0	0	0	0	0	0.663	0.663	26.600	0.0
101	0	1	0	0	0	0	0.0	0.0	28.000	0.0
102	0	0	0	0	0	0	0.0	1.325	28.000	0.0
103	0	0	0	0	0	0	1.325	1.325	28.000	0.0
104	0	1	0	0	0	0	1.325	0.0	28.000	0.0
105	0	0	0	0	0	0	0.663	0.663	28.000	0.0
106	1	1	1	1	1	1	0.663	0.663	30.000	0.0

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	1	0	2	3	4	5
2	6	7	8	9	10	11
3	12	13	14	15	16	17
4	18	0	19	20	21	22
5	23	24	25	26	27	28
6	29	30	31	32	33	34
7	35	36	37	38	39	40
8	41	42	43	44	45	46
9	47	48	49	50	51	52
10	53	54	55	56	57	58
11	59	60	61	62	63	64
12	65	66	67	68	69	70
13	71	72	73	74	75	76
14	77	78	79	80	81	82
15	83	84	85	86	87	88
16	89	90	91	92	93	94
17	95	96	97	98	99	100
18	101	102	103	104	105	106
19	107	108	109	110	111	112
20	113	114	115	116	117	118
21	119	120	121	122	123	124
22	125	126	127	128	129	130
23	131	132	133	134	135	136
24	137	138	139	140	141	142
25	143	144	145	146	147	148
26	149	150	151	152	153	154
27	155	156	157	158	159	160
28	161	162	163	164	165	166
29	167	168	169	170	171	172
30	173	174	175	176	177	178
31	179	180	181	182	183	184
32	185	186	187	188	189	190
33	191	192	193	194	195	196
34	197	198	199	200	201	202
35	203	204	205	206	207	208
36	209	210	211	212	213	214
37	215	216	217	218	219	220
38	221	222	223	224	225	226
39	227	228	229	230	231	232
40	233	234	235	236	237	238

41	239	240	241	242	243	244
42	245	246	247	248	249	250
43	251	252	253	254	255	256
44	257	258	259	260	261	262
45	263	264	265	266	267	268
46	269	270	271	272	273	274
47	275	276	277	278	279	280
48	281	282	283	284	285	286
49	287	288	289	290	291	292
50	293	294	295	296	297	298
51	299	300	0	301	302	303
52	0	304	0	305	306	307
53	0	308	0	309	310	311
54	312	313	0	314	315	316
55	317	318	0	319	320	321
56	322	323	324	325	326	327
57	328	329	330	331	332	333
58	334	335	336	337	338	339
59	340	341	342	343	344	345
60	346	347	348	349	350	351
61	352	353	354	355	356	357
62	358	359	360	361	362	363
63	364	365	366	367	368	369
64	370	371	372	373	374	375
65	376	377	378	379	380	381
66	382	383	384	385	386	387
67	388	389	390	391	392	393
68	394	395	396	397	398	399
69	400	401	402	403	404	405
70	406	407	408	409	410	411
71	412	413	414	415	416	417
72	418	419	420	421	422	423
73	424	425	426	427	428	429
74	430	431	432	433	434	435
75	436	437	438	439	440	441
76	442	443	444	445	446	447
77	448	449	450	451	452	453
78	454	455	456	457	458	459
79	460	461	462	463	464	465
80	466	467	468	469	470	471
81	472	473	474	475	476	477
82	478	479	480	481	482	483
83	484	485	486	487	488	489
84	490	491	492	493	494	495
85	496	497	498	499	500	501

86	502	503	504	505	506	507
87	508	509	510	511	512	513
88	514	515	516	517	518	519
89	520	521	522	523	524	525
90	526	527	528	529	530	531
91	532	533	534	535	536	537
92	538	539	540	541	542	543
93	544	545	546	547	548	549
94	550	551	552	553	554	555
95	556	557	558	559	560	561
96	562	563	564	565	566	567
97	568	569	570	571	572	573
98	574	575	576	577	578	579
99	580	581	582	583	584	585
100	586	587	588	589	590	591
101	592	0	593	594	595	596
102	597	598	599	600	601	602
103	603	604	605	606	607	608
104	609	0	610	611	612	613
105	614	615	616	617	618	619
106	0	0	0	0	0	0

3 / D   B E A M   E L E M E N T S

NUMBER OF BEAMS = 80
NUMBER OF GEOMETRIC PROPERTY SETS= 1
NUMBER OF FIXED END FORCE SETS = 0
NUMBER OF MATERIALS = 1

MATERIAL PROPERTIES

MATERIAL NUMBER	YOUNG*S MODULUS	POISSON*S RATIO	MASS DENSITY	WEIGHT DENSITY
1	0.3000D 08	0.3000	0.0	0.0

BEAM GEOMETRIC PROPERTIES

SECTION NUMBER	AXIAL AREA A(1)	SHEAR AREA A(2)	SHEAR AREA A(3)	TORSION J(1)	INERTIA I(2)	INERTIA I(3)
1	0.1230D-01	0.0	0.0	0.2400D-04	0.1200D-04	0.1200D-04

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0	0.0	0.0	0.0
Y-DIR	0.0	0.0	0.0	0.0
Z-DIR	0.0	0.0	0.0	0.0

3/D BEAM ELEMENT DATA

BEAM NUMBER	NODE -I	NODE -J	NODE -K	MATERIAL NUMBER	SECTION NUMBER	ELEMENT END LOADS A	B	C	D	END CODES -I	-J
1	1	6	106	1	1	0	0	0	0	0	0
2	6	11	106	1	1	0	0	0	0	0	0
3	11	16	106	1	1	0	0	0	0	0	0
4	16	21	106	1	1	0	0	0	0	0	0
5	21	26	106	1	1	0	0	0	0	0	0
6	26	31	106	1	1	0	0	0	0	0	0
7	31	36	106	1	1	0	0	0	0	0	0
8	36	41	106	1	1	0	0	0	0	0	0
9	41	46	106	1	1	0	0	0	0	0	0
10	46	51	106	1	1	0	0	0	0	0	0
11	51	56	106	1	1	0	0	0	0	0	0
12	56	61	106	1	1	0	0	0	0	0	0
13	61	66	106	1	1	0	0	0	0	0	0
14	66	71	106	1	1	0	0	0	0	0	0
15	71	76	106	1	1	0	0	0	0	0	0
16	76	81	106	1	1	0	0	0	0	0	0
17	81	86	106	1	1	0	0	0	0	0	0
18	86	91	106	1	1	0	0	0	0	0	0
19	91	96	106	1	1	0	0	0	0	0	0
20	96	101	106	1	1	0	0	0	0	0	0
21	2	7	106	1	1	0	0	0	0	0	0
22	7	12	106	1	1	0	0	0	0	0	0
23	12	17	106	1	1	0	0	0	0	0	0
24	17	22	106	1	1	0	0	0	0	0	0
25	22	27	106	1	1	0	0	0	0	0	0
26	27	32	106	1	1	0	0	0	0	0	0
27	32	37	106	1	1	0	0	0	0	0	0
28	37	42	106	1	1	0	0	0	0	0	0
29	42	47	106	1	1	0	0	0	0	0	0
30	47	52	106	1	1	0	0	0	0	0	0
31	52	57	106	1	1	0	0	0	0	0	0
32	57	62	106	1	1	0	0	0	0	0	0
33	62	67	106	1	1	0	0	0	0	0	0
34	67	72	106	1	1	0	0	0	0	0	0
35	72	77	106	1	1	0	0	0	0	0	0
36	77	82	106	1	1	0	0	0	0	0	0
37	82	87	106	1	1	0	0	0	0	0	0
38	87	92	106	1	1	0	0	0	0	0	0
39	92	97	106	1	1	0	0	0	0	0	0
40	97	102	106	1	1	0	0	0	0	0	0

41	3	8	106	1	1	0	0	0	0	0	0
42	8	13	106	1	1	0	0	0	0	0	0
43	13	18	106	1	1	0	0	0	0	0	0
44	18	23	106	1	1	0	0	0	0	0	0
45	23	28	106	1	1	0	0	0	0	0	0
46	28	33	106	1	1	0	0	0	0	0	0
47	33	38	106	1	1	0	0	0	0	0	0
48	38	43	106	1	1	0	0	0	0	0	0
49	43	48	106	1	1	0	0	0	0	0	0
50	48	53	106	1	1	0	0	0	0	0	0
51	53	58	106	1	1	0	0	0	0	0	0
52	58	63	106	1	1	0	0	0	0	0	0
53	63	68	106	1	1	0	0	0	0	0	0
54	68	73	106	1	1	0	0	0	0	0	0
55	73	78	106	1	1	0	0	0	0	0	0
56	78	83	106	1	1	0	0	0	0	0	0
57	83	88	106	1	1	0	0	0	0	0	0
58	88	93	106	1	1	0	0	0	0	0	0
59	93	98	106	1	1	0	0	0	0	0	0
60	98	103	106	1	1	0	0	0	0	0	0
61	4	9	106	1	1	0	0	0	0	0	0
62	9	14	106	1	1	0	0	0	0	0	0
63	14	19	106	1	1	0	0	0	0	0	0
64	19	24	106	1	1	0	0	0	0	0	0
65	24	29	106	1	1	0	0	0	0	0	0
66	29	34	106	1	1	0	0	0	0	0	0
67	34	39	106	1	1	0	0	0	0	0	0
68	39	44	106	1	1	0	0	0	0	0	0
69	44	49	106	1	1	0	0	0	0	0	0
70	49	54	106	1	1	0	0	0	0	0	0
71	54	59	106	1	1	0	0	0	0	0	0
72	59	64	106	1	1	0	0	0	0	0	0
73	64	69	106	1	1	0	0	0	0	0	0
74	69	74	106	1	1	0	0	0	0	0	0
75	74	79	106	1	1	0	0	0	0	0	0
76	79	84	106	1	1	0	0	0	0	0	0
77	84	89	106	1	1	0	0	0	0	0	0
78	89	94	106	1	1	0	0	0	0	0	0
79	94	99	106	1	1	0	0	0	0	0	0
80	95	104	106	1	1	0	0	0	0	0	0

3 / D BEAM ELEMENTS

NUMBER OF BEAMS	=	84
NUMBER OF GEOMETRIC PROPERTY SETS	=	1
NUMBER OF FIXED END FORCE SETS	=	0
NUMBER OF MATERIALS	=	1

MATERIAL PROPERTIES

MATERIAL NUMBER	YOUNG*S MODULUS	POISSON*S RATIO	MASS DENSITY	WEIGHT DENSITY
1	0.1000D 01	0.3000	0.0	0.0

BEAM GEOMETRIC PROPERTIES

SECTION NUMBER	AXIAL AREA A(1)	SHEAR AREA A(2)	SHEAR AREA A(3)	TORSION J(1)	INERTIA I(2)	INERTIA I(3)
1	0.3520D-01	0.0	0.0	0.2060D-03	0.1030D-03	0.1030D-03

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0	0.0	0.0	0.0
Y-DIR	0.0	0.0	0.0	0.0
Z-DIR	0.0	0.0	0.0	0.0

3/D BEAM ELEMENT DATA

BEAM NUMBER	NODE -I	NODE -J	NODE -K	MATERIAL NUMBER	SECTION NUMBER	ELEMENT END LOADS A	B	C	D	END CODES -I	-J
1	5	1	106	1	1	0	0	0	0	0	0
2	10	6	106	1	1	0	0	0	0	0	0
3	15	11	106	1	1	0	0	0	0	0	0
4	20	16	106	1	1	0	0	0	0	0	0
5	25	21	106	1	1	0	0	0	0	0	0
6	30	26	106	1	1	0	0	0	0	0	0
7	35	31	106	1	1	0	0	0	0	0	0
8	40	36	106	1	1	0	0	0	0	0	0
9	45	41	106	1	1	0	0	0	0	0	0
10	50	46	106	1	1	0	0	0	0	0	0
11	55	51	106	1	1	0	0	0	0	0	0
12	60	56	106	1	1	0	0	0	0	0	0
13	65	61	106	1	1	0	0	0	0	0	0
14	70	66	106	1	1	0	0	0	0	0	0
15	75	71	106	1	1	0	0	0	0	0	0
16	80	76	106	1	1	0	0	0	0	0	0
17	85	81	106	1	1	0	0	0	0	0	0
18	90	86	106	1	1	0	0	0	0	0	0
19	95	91	106	1	1	0	0	0	0	0	0
20	100	96	106	1	1	0	0	0	0	0	0
21	105	101	106	1	1	0	0	0	0	0	0
22	5	2	106	1	1	0	0	0	0	0	0
23	10	7	106	1	1	0	0	0	0	0	0
24	15	12	106	1	1	0	0	0	0	0	0
25	20	17	106	1	1	0	0	0	0	0	0
26	25	22	106	1	1	0	0	0	0	0	0
27	30	27	106	1	1	0	0	0	0	0	0
28	35	32	106	1	1	0	0	0	0	0	0
29	40	37	106	1	1	0	0	0	0	0	0
30	45	42	106	1	1	0	0	0	0	0	0
31	50	47	106	1	1	0	0	0	0	0	0
32	55	52	106	1	1	0	0	0	0	0	0
33	60	57	106	1	1	0	0	0	0	0	0
34	65	62	106	1	1	0	0	0	0	0	0
35	70	67	106	1	1	0	0	0	0	0	0
36	75	72	106	1	1	0	0	0	0	0	0
37	80	77	106	1	1	0	0	0	0	0	0
38	85	82	106	1	1	0	0	0	0	0	0
39	90	87	106	1	1	0	0	0	0	0	0
40	95	92	106	1	1	0	0	0	0	0	0

41	100	97	106	1	1	0	0	0	0	0	0
42	105	102	106	1	1	0	0	0	0	0	0
43	5	3	106	1	1	0	0	0	0	0	0
44	10	8	106	1	1	0	0	0	0	0	0
45	15	13	106	1	1	0	0	0	0	0	0
46	20	18	106	1	1	0	0	0	0	0	0
47	25	23	106	1	1	0	0	0	0	0	0
48	30	28	106	1	1	0	0	0	0	0	0
49	35	33	106	1	1	0	0	0	0	0	0
50	40	38	106	1	1	0	0	0	0	0	0
51	45	43	106	1	1	0	0	0	0	0	0
52	50	48	106	1	1	0	0	0	0	0	0
53	55	53	106	1	1	0	0	0	0	0	0
54	60	58	106	1	1	0	0	0	0	0	0
55	65	63	106	1	1	0	0	0	0	0	0
56	70	68	106	1	1	0	0	0	0	0	0
57	75	73	106	1	1	0	0	0	0	0	0
58	80	78	106	1	1	0	0	0	0	0	0
59	85	83	106	1	1	0	0	0	0	0	0
60	90	88	106	1	1	0	0	0	0	0	0
61	95	93	106	1	1	0	0	0	0	0	0
62	100	98	106	1	1	0	0	0	0	0	0
63	105	103	106	1	1	0	0	0	0	0	0
64	5	4	106	1	1	0	0	0	0	0	0
65	10	9	106	1	1	0	0	0	0	0	0
66	15	14	106	1	1	0	0	0	0	0	0
67	20	19	106	1	1	0	0	0	0	0	0
68	25	24	106	1	1	0	0	0	0	0	0
69	30	29	106	1	1	0	0	0	0	0	0
70	35	34	106	1	1	0	0	0	0	0	0
71	40	39	106	1	1	0	0	0	0	0	0
72	45	44	106	1	1	0	0	0	0	0	0
73	50	49	106	1	1	0	0	0	0	0	0
74	55	54	106	1	1	0	0	0	0	0	0
75	60	59	106	1	1	0	0	0	0	0	0
76	65	64	106	1	1	0	0	0	0	0	0
77	70	69	106	1	1	0	0	0	0	0	0
78	75	74	106	1	1	0	0	0	0	0	0
79	80	79	106	1	1	0	0	0	0	0	0
80	85	84	106	1	1	0	0	0	0	0	0
81	90	89	106	1	1	0	0	0	0	0	0
82	95	94	106	1	1	0	0	0	0	0	0
83	100	99	106	1	1	0	0	0	0	0	0
84	105	104	106	1	1	0	0	0	0	0	0

PLANE STRESS ANALYSIS
MEMBRANE ELEMENTS
INCOMPATIBLE MODES SUPPRESSED

```
NUMBER OF ELEMENTS    =     80
NUMBER OF MATERIALS   =      1
MAXIMUM TEMPERATURES
PER MATERIAL          =      1
ANALYSIS CODE         =      2
CODE FOR INCLUSION
OF BENDING MODES      =      1
  EQ.0, INCLUDE
  GT.0, SUPPRESS
```

```
MATERIAL I.D. NUMBER    =     1
NUMBER OF TEMPERATURES  =     1
WEIGHT DENSITY          =     0.0
MASS   DENSITY          =     0.0
BETA ANGLE              =     0.0
```

TEMPERATURE	E(N)	E(S)	E(T)	NU(NS)	NU(NT)	NU(ST)
0.0	0.1340D 05	0.4000D 06	0.1340D 05	0.3000	0.3000	0.0400

G(NS)	ALPHA(N)	ALPHA(S)	ALPHA(T)
0.1800D 05	0.1000D-03	0.1000D-03	0.1000D-03

ELEMENT LOAD MULTIPLIERS

LOAD CASE	TEMPERATURE	PRESSURE	X-GRAVITY	Y-GRAVITY	Z-GRAVITY
A	0.0	0.0	0.0	0.0	0.0
B	0.0	0.0	0.0	0.0	0.0
C	0.0	0.0	0.0	0.0	0.0
D	0.0	0.0	0.0	0.0	0.0

ELEMENT NUMBER	I	J	K	L	MATL TYPE	REFERENCE TEMPERATURE	I-J FACE PRESSURE	STRESS OPTION	KG	THICKNESS
1	5	10	6	1	1	0.0	0.0	4	1	0.1875
2	10	15	11	6	1	0.0	0.0	4	5	0.1875
3	15	20	16	11	1	0.0	0.0	4	5	0.1875
4	20	25	21	16	1	0.0	0.0	4	5	0.1875
5	25	30	26	21	1	0.0	0.0	4	5	0.1875
6	30	35	31	26	1	0.0	0.0	4	5	0.1875
7	35	40	36	31	1	0.0	0.0	4	5	0.1875
8	40	45	41	36	1	0.0	0.0	4	5	0.1875
9	45	50	46	41	1	0.0	0.0	4	5	0.1875
10	50	55	51	46	1	0.0	0.0	4	5	0.1875
11	55	60	56	51	1	0.0	0.0	4	5	0.1875
12	60	65	61	56	1	0.0	0.0	4	5	0.1875
13	65	70	66	61	1	0.0	0.0	4	5	0.1875
14	70	75	71	66	1	0.0	0.0	4	5	0.1875
15	75	80	76	71	1	0.0	0.0	4	5	0.1875
16	80	85	81	76	1	0.0	0.0	4	5	0.1875
17	85	90	86	81	1	0.0	0.0	4	5	0.1875
18	90	95	91	86	1	0.0	0.0	4	5	0.1875
19	95	100	96	91	1	0.0	0.0	4	5	0.1875
20	100	105	101	96	1	0.0	0.0	4	5	0.1875
21	5	10	7	2	1	0.0	0.0	4	1	0.1875
22	10	15	12	7	1	0.0	0.0	4	5	0.1875
23	15	20	17	12	1	0.0	0.0	4	5	0.1875
24	20	25	22	17	1	0.0	0.0	4	5	0.1875
25	25	30	27	22	1	0.0	0.0	4	5	0.1875
26	30	35	32	27	1	0.0	0.0	4	5	0.1875
27	35	40	37	32	1	0.0	0.0	4	5	0.1875
28	40	45	42	37	1	0.0	0.0	4	5	0.1875
29	45	50	47	42	1	0.0	0.0	4	5	0.1875
30	50	55	52	47	1	0.0	0.0	4	5	0.1875
31	55	60	57	52	1	0.0	0.0	4	5	0.1875
32	60	65	62	57	1	0.0	0.0	4	5	0.1875
33	65	70	67	62	1	0.0	0.0	4	5	0.1875
34	70	75	72	67	1	0.0	0.0	4	5	0.1875
35	75	80	77	72	1	0.0	0.0	4	5	0.1875
36	80	85	82	77	1	0.0	0.0	4	5	0.1875
37	85	90	87	82	1	0.0	0.0	4	5	0.1875
38	90	95	92	87	1	0.0	0.0	4	5	0.1875
39	95	100	97	92	1	0.0	0.0	4	5	0.1875
40	100	105	102	97	1	0.0	0.0	4	5	0.1875
41	5	10	8	3	1	0.0	0.0	4	1	0.1875
42	10	15	13	8	1	0.0	0.0	4	5	0.1875
43	15	20	18	13	1	0.0	0.0	4	5	0.1875
44	20	25	23	18	1	0.0	0.0	4	5	0.1875
45	25	30	28	23	1	0.0	0.0	4	5	0.1875

46	30	35	33	28	1	0.0	0.0	4	5	0.1875
47	35	40	38	33	1	0.0	0.0	4	5	0.1875
48	40	45	43	38	1	0.0	0.0	4	5	0.1875
49	45	50	48	43	1	0.0	0.0	4	5	0.1875
50	50	55	53	48	1	0.0	0.0	4	5	0.1875
51	55	60	58	53	1	0.0	0.0	4	5	0.1875
52	60	65	63	58	1	0.0	0.0	4	5	0.1875
53	65	70	68	63	1	0.0	0.0	4	5	0.1875
54	70	75	73	68	1	0.0	0.0	4	5	0.1875
55	75	80	78	73	1	0.0	0.0	4	5	0.1875
56	80	85	83	78	1	0.0	0.0	4	5	0.1875
57	85	90	88	83	1	0.0	0.0	4	5	0.1875
58	90	95	93	88	1	0.0	0.0	4	5	0.1875
59	95	100	98	93	1	0.0	0.0	4	5	0.1875
60	100	105	103	98	1	0.0	0.0	4	5	0.1875
61	5	10	9	4	1	0.0	0.0	4	1	0.1875
62	10	15	14	9	1	0.0	0.0	4	5	0.1875
63	15	20	19	14	1	0.0	0.0	4	5	0.1875
64	20	25	24	19	1	0.0	0.0	4	5	0.1875
65	25	30	29	24	1	0.0	0.0	4	5	0.1875
66	30	35	34	29	1	0.0	0.0	4	5	0.1875
67	35	40	39	34	1	0.0	0.0	4	5	0.1875
68	40	45	44	39	1	0.0	0.0	4	5	0.1875
69	45	50	49	44	1	0.0	0.0	4	5	0.1875
70	50	55	54	49	1	0.0	0.0	4	5	0.1875
71	55	60	59	54	1	0.0	0.0	4	5	0.1875
72	60	65	64	59	1	0.0	0.0	4	5	0.1875
73	65	70	69	64	1	0.0	0.0	4	5	0.1875
74	70	75	74	69	1	0.0	0.0	4	5	0.1875
75	75	80	79	74	1	0.0	0.0	4	5	0.1875
76	80	85	84	79	1	0.0	0.0	4	5	0.1875
77	85	90	89	84	1	0.0	0.0	4	5	0.1875
78	90	95	94	89	1	0.0	0.0	4	5	0.1875
79	95	100	99	94	1	0.0	0.0	4	5	0.1875
80	100	105	104	99	1	0.0	0.0	4	5	0.1875

NUMBER OF TRUSS MEMBERS= 160
NUMBER OF DIFF. MEMBERS= 2

TYPE	E	ALPHA	DEN	AREA	WT
1	0.1800000D 08	0.0	0.0	0.1230000D-02	0.0
2	0.1000000D 01	0.0	0.0	0.1230000D-02	0.0

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0	0.0	0.0	0.0
Y-DIR	0.0	0.0	0.0	0.0
Z-DIR	0.0	0.0	0.0	0.0
TEMP	0.0	0.0	0.0	0.0

N	I	J	TYPE	TEMP	BAND
1	1	7	2	0.0	37
2	6	12	2	0.0	39
3	11	17	2	0.0	39
4	16	22	2	0.0	39
5	21	27	2	0.0	39
6	26	32	2	0.0	39
7	31	37	2	0.0	39
8	36	42	2	0.0	39
9	41	47	2	0.0	39
10	46	52	2	0.0	36
11	51	57	1	0.0	32
12	56	62	1	0.0	39
13	61	67	1	0.0	39
14	66	72	1	0.0	39
15	71	77	1	0.0	39
16	76	82	1	0.0	39
17	81	87	1	0.0	39
18	86	92	1	0.0	39
19	91	97	1	0.0	39
20	96	102	1	0.0	38
21	2	6	1	0.0	26
22	7	11	1	0.0	27
23	12	16	1	0.0	27
24	17	21	1	0.0	27
25	22	26	1	0.0	27
26	27	31	1	0.0	27
27	32	36	1	0.0	27
28	37	41	1	0.0	27
29	42	46	1	0.0	27
30	47	51	1	0.0	26
31	52	56	2	0.0	21
32	57	61	2	0.0	27
33	62	66	2	0.0	27
34	67	71	2	0.0	27
35	72	76	2	0.0	27
36	77	81	2	0.0	27
37	82	86	2	0.0	27
38	87	91	2	0.0	27
39	92	96	2	0.0	27
40	97	101	2	0.0	26
41	2	8	1	0.0	38
42	7	13	2	0.0	39
43	12	18	1	0.0	39
44	17	23	2	0.0	39
45	22	28	2	0.0	39

46	27	33	1	0.0	39
47	32	38	2	0.0	39
48	37	43	1	0.0	39
49	42	48	1	0.0	39
50	47	53	2	0.0	34
51	52	58	2	0.0	33
52	57	63	1	0.0	39
53	62	68	1	0.0	39
54	67	73	2	0.0	39
55	72	78	1	0.0	39
56	77	83	2	0.0	39
57	82	88	2	0.0	39
58	87	93	1	0.0	39
59	92	98	2	0.0	39
60	97	103	1	0.0	38
61	3	7	1	0.0	26
62	8	12	2	0.0	27
63	13	17	1	0.0	27
64	18	22	2	0.0	27
65	23	27	2	0.0	27
66	28	32	1	0.0	27
67	33	37	2	0.0	27
68	38	42	1	0.0	27
69	43	47	1	0.0	27
70	48	52	2	0.0	24
71	53	57	2	0.0	23
72	58	62	1	0.0	27
73	63	67	1	0.0	27
74	68	72	2	0.0	27
75	73	77	1	0.0	27
76	78	82	2	0.0	27
77	83	87	2	0.0	27
78	88	92	1	0.0	27
79	93	97	2	0.0	27
80	98	102	1	0.0	26
81	4	8	2	0.0	26
82	9	13	2	0.0	27
83	14	18	2	0.0	27
84	19	23	2	0.0	27
85	24	28	2	0.0	27
86	29	33	2	0.0	27
87	34	38	2	0.0	27
88	39	43	2	0.0	27
89	44	48	2	0.0	27
90	49	53	2	0.0	22
91	54	58	1	0.0	25
92	59	63	1	0.0	27

93	64	68	1	0.0	27
94	69	73	1	0.0	27
95	74	78	1	0.0	27
96	79	83	1	0.0	27
97	84	88	1	0.0	27
98	89	93	1	0.0	27
99	94	98	1	0.0	27
100	99	103	1	0.0	26
101	3	9	1	0.0	38
102	8	14	1	0.0	39
103	13	19	1	0.0	39
104	18	24	1	0.0	39
105	23	29	1	0.0	39
106	28	34	1	0.0	39
107	33	39	1	0.0	39
108	38	44	1	0.0	39
109	43	49	1	0.0	39
110	48	54	1	0.0	33
111	53	59	2	0.0	35
112	58	64	2	0.0	39
113	63	69	2	0.0	39
114	68	74	2	0.0	39
115	73	79	2	0.0	39
116	78	84	2	0.0	39
117	83	89	2	0.0	39
118	88	94	2	0.0	39
119	93	99	2	0.0	39
120	98	104	2	0.0	37
121	4	6	1	0.0	14
122	9	11	2	0.0	15
123	14	16	1	0.0	15
124	19	21	2	0.0	15
125	24	26	2	0.0	15
126	29	31	2	0.0	15
127	34	36	1	0.0	15
128	39	41	1	0.0	15
129	44	46	2	0.0	15
130	49	51	1	0.0	14
131	54	56	1	0.0	13
132	59	61	2	0.0	15
133	64	66	1	0.0	15
134	69	71	1	0.0	15
135	74	76	2	0.0	15
136	79	81	2	0.0	15
137	84	86	2	0.0	15
138	89	91	1	0.0	15
139	94	96	2	0.0	15

140	99	101	1	0.0	14
141	1	9	1	0.0	49
142	6	14	2	0.0	51
143	11	19	1	0.0	51
144	16	24	2	0.0	51
145	21	29	2	0.0	51
146	26	34	2	0.0	51
147	31	39	1	0.0	51
148	36	44	1	0.0	51
149	41	49	2	0.0	51
150	46	54	1	0.0	45
151	51	59	1	0.0	44
152	56	64	2	0.0	51
153	61	69	1	0.0	51
154	66	74	1	0.0	51
155	71	79	2	0.0	51
156	76	84	2	0.0	51
157	81	89	2	0.0	51
158	86	94	1	0.0	51
159	91	99	2	0.0	51
160	96	104	1	0.0	49

E Q U A T I O N   P A R A M E T E R S

```
 TOTAL NUMBER OF EQUATIONS           =  619
 BANDWIDTH                           =   57
 NUMBER OF EQUATIONS IN A BLOCK =   59
 NUMBER OF BLOCKS                    =   11
```

N O D A L   L O A D S   (S T A T I C)   O R   M A S S E S   (D Y N A M I C)

NODE NUMBER	LOAD CASE	X-AXIS FORCE	Y-AXIS FORCE	Z-AXIS FORCE	X-AXIS MOMENT	Y-AXIS MOMENT	Z-AXIS MOMENT
52	1	0.0	-0.50000D 02	0.0	0.0	0.0	0.0
53	1	0.0	-0.50000D 02	0.0	0.0	0.0	0.0

ELEMENT LOAD MULTIPLIERS

STRUCTURE LOAD CASE	A	B	C	D
1	0.0	0.0	0.0	0.0

APPENDIX E

SAMPLE OUTPUT FROM FINITE ELEMENT
COMPUTER PROGRAM

T W O - D I M E N S I O N A L   F I N I T E   E L E M E N T S

1. CENTROID STRESSES REFERENCED TO LOCAL Y-Z COORDINATES.
2. MID-SIDE STRESSES ARE NORMAL AND PARALLEL TO ELEMENT EDGES.

ELEMENT (1)

LOAD	LOC	S11	S22	S33	S12
1	CEN	-0.24654D 01	-0.18617D 03	0.0	0.31696D 02

ELEMENT (2)

LOAD	LOC	S11	S22	S33	S12
1	CEN	-0.91740D 00	-0.63346D 02	0.0	0.40710D 02

ELEMENT (3)

LOAD	LOC	S11	S22	S33	S12
1	CEN	0.22534D 00	-0.65997D 02	0.0	0.65623D 02

ELEMENT (4)

LOAD	LOC	S11	S22	S33	S12
1	CEN	0.11804D 01	-0.25778D 02	0.0	0.88734D 02

S-MAX	S-MIN	ANGLE
0.28496D 01	-0.19149D 03	9.52

S-MAX	S-MIN	ANGLE
0.19168D 02	-0.83431D 02	26.26

S-MAX	S-MIN	ANGLE
0.40618D 02	-0.10639D 03	31.61

S-MAX	S-MIN	ANGLE
0.77453D 02	-0.10205D 03	40.68

.....BEAM FORCES AND MOMENTS

BEAM NO.	LOAD NO.	AXIAL R1	SHEAR R2	SHEAR R3	TORSION M1	BENDING M2	BENDING M3
1	1	1.880D 01	3.591D-02	2.159D 00	-3.338D-06	-9.326D-07	9.791D-07
		-1.880D 01	-3.591D-02	-2.159D 00	3.338D-06	-3.022D 00	5.028D-02
2	1	-3.043D 01	-3.082D-01	-2.087D 00	1.134D-06	3.022D 00	-5.028D-02
		3.043D 01	3.082D-01	2.087D 00	-1.134D-06	-1.004D-01	-3.812D-01
3	1	-4.812D 01	4.332D-02	4.422D-01	5.041D-06	1.004D-01	3.812D-01
		4.812D 01	-4.332D-02	-4.422D-01	-5.041D-06	-7.195D-01	-3.205D-01
4	1	-9.127D 01	2.916D-01	-1.886D 00	4.648D-06	7.195D-01	3.205D-01
		9.127D 01	-2.916D-01	1.886D 00	-4.648D-06	1.921D 00	8.771D-02
5	1	-1.177D 02	5.969D-01	3.708D-01	-1.643D-06	-1.921D 00	-8.771D-02
		1.177D 02	-5.969D-01	-3.708D-01	1.643D-06	1.402D 00	9.233D-01
6	1	-1.381D 02	-4.904D-01	1.878D 00	-8.025D-06	-1.402D 00	-9.233D-01
		1.381D 02	4.904D-01	-1.878D 00	8.025D-06	-1.227D 00	2.367D-01
7	1	-1.490D 02	-4.029D-01	8.263D-01	-7.953D-06	1.227D 00	-2.367D-01
		1.490D 02	4.029D-01	-8.263D-01	7.953D-06	-2.383D 00	-3.273D-01
8	1	-1.838D 02	8.010D-01	-2.485D 00	-4.144D-06	2.383D 00	3.273D-01
		1.838D 02	-8.010D-01	2.485D 00	4.144D-06	1.095D 00	7.940D-01

TRUSS MEMBER ACTIONS

MEMBER	LOAD	STRESS	FORCE
1	1	-0.00453	-0.000
2	1	-0.00187	-0.000
3	1	-0.00225	-0.000
4	1	-0.00464	-0.000
5	1	-0.00695	-0.000
6	1	-0.00638	-0.000
7	1	-0.00371	-0.000
8	1	-0.00235	-0.000
9	1	-0.00294	-0.000
10	1	-0.00251	-0.000
11	1	11911.55987	14.651
12	1	16923.97245	20.816
13	1	13491.75850	16.595

N O D E   D I S P L A C E M E N T S / R O T A T I O N S

NODE NUMBER	LOAD CASE	X-TRANSLATION	Y-TRANSLATION	Z-TRANSLATION	X-ROTATION	Y-ROTATION	Z-ROTATION
106	1	0.0	0.0	0.0	0.0	0.0	0.0
105	1	-0.41309D-13	-0.68156D-02	-0.52665D-03	-0.81517D-02	0.13893D-15	0.13067D-13
104	1	0.57617D-02	0.0	0.44968D-02	-0.17098D-01	0.66507D-02	0.66479D-02
103	1	0.84539D-03	-0.84990D-02	-0.53964D-02	-0.10342D-01	-0.12678D-02	-0.29302D-02
102	1	-0.84539D-03	-0.84990D-02	-0.53964D-02	-0.10342D-01	0.12678D-02	0.29302D-02
101	1	-0.57617D-02	0.0	0.44968D-02	-0.17098D-01	-0.66507D-02	-0.66479D-02
100	1	-0.40671D-13	-0.20497D-01	-0.47380D-03	-0.72328D-02	0.51466D-16	0.78337D-14
99	1	-0.16423D-02	-0.21965D-01	0.45681D-02	-0.12873D-01	0.25644D-02	0.66479D-02
98	1	0.22885D-02	-0.22993D-01	-0.52543D-02	-0.10373D-01	-0.55676D-03	-0.29302D-02
97	1	-0.22885D-02	-0.22993D-01	-0.52543D-02	-0.10373D-01	0.55676D-03	0.29302D-02
96	1	0.16423D-02	-0.21965D-01	0.45681D-02	-0.12873D-01	-0.25644D-02	-0.66479D-02
95	1	-0.40492D-13	-0.35358D-01	-0.29969D-03	-0.55677D-02	-0.27175D-15	0.94505D-15
94	1	-0.11097D-02	-0.36225D-01	0.44527D-02	-0.90345D-02	-0.21840D-02	0.66479D-02

```
      COMMON STNP(4,4), STNQ(4,4),STSP(4,4),STSQ(4,4), PHI
(4,4), ASTN(4,4,3)
      DIMENSION SH(3), SL(3), STN(8,4,3), ASTN2(2,4,3)
      DO 10 I=1,8
      WRITE (6,1) I
    1 FORMAT (/,' CASE ',I1)
      DO 10 J=1,4
      LOAD=25*J
      WRITE (6,3) LOAD
    3 FORMAT (' ENTER LOW AND HIGH DATA FOR ',I3,' POUND F
ORCE')
      READ (5,*) FL, SL(1), SL(2), SL(3), FH, SH(1), SH(2)
, SH(3)
      DO 10 K=1,3
   10 STN(I,J,K)=(J*25-FL)*(SH(K)-SL(K))/(FH-FL)+SL(K)
      WRITE (6,22)
   22 FORMAT (/,/,' ENTER MODULUS OF ELASTICITY AND POISON
S RATIO')
      READ (5,*) ELAS, POI
      WRITE (6,4)
    4 FORMAT (/,/,/,/,' INTERPOLATED STRAIN VALUES')
      DO 11 I=1,8
      WRITE (6,5) I
    5 FORMAT (/,' CASE ',I1,'   LOAD  STRAIN-1  STRAIN-2
STRAIN-3')
      DO 11 J=1,4
      LOAD=25*J
   11 WRITE (6,6) LOAD, STN(I,J,1), STN(I,J,2), STN(I,J,3)
    6 FORMAT (10X,I3,3X,F8.2,2X,F8.2,2X,F8.2)
      WRITE (6,7)
    7 FORMAT (/,/,/,' AVERAGE STRAINS WITH PRINCIPAL STRAI
NS AND STRESSES')
      DO 12 I=1,4
      WRITE (6,5) I
```

```
      WRITE (6,23)
      DO 17 I=1,2
      WRITE (6,8) I
      DO 17 J=1,4
      LOAD=25*J
      DO 18 K=1,3
   18 ASTN(I,J,K)=ASTN2(I,J,K)
      CALL STRESS(I,J,ELAS,POI)
   17 WRITE (6,9) LOAD, STNP(I,J), STNQ(I,J), STSP(I,J), S
TSQ(I,J), PHI(I,J)
      STOP
      END
      SUBROUTINE STRESS(I,J,ELAS,POI)
      COMMON STNP(4,4),STNQ(4,4), STSP(4,4), STSQ(4,4), PH
I(4,4), ASTN(4,4,3)
      A=2*ASTN(I,J,2)-ASTN(I,J,1)-ASTN(I,J,3)
      B=ASTN(I,J,1)-ASTN(I,J,3)
      AB=SQRT(A**2+B**2)*0.5
      C=(ASTN(I,J,1)+ASTN(I,J,3))*0.5
      STNP(I,J)=C+AB
      STNQ(I,J)=C-AB
      ANG=ATAN2(A,B)*0.5
      PHI(I,J)=ANG*57.29558+45.0
      G=C/(1-POI)
      H=AB/(1+POI)
      STSP(I,J)=ELAS*(G+H)
      STSQ(I,J)=ELAS*(G-H)
      RETURN
      END
```

```
      I2=I*2
      I1=I2-1
      DO 12 J=1,4
      LOAD=25*J
      DO 13 K=1,3
   13 ASTN(I,J,K)=(STN(I1,J,K)+STN(I2,J,K))/2
   12 WRITE (6,6) LOAD, ASTN(I,J,1), ASTN(I,J,2), ASTN(I,J
     ,3)
      WRITE (6,23)
   23 FORMAT ('  ')
      DO 14 I=1,4
      WRITE (6,8) I
    8 FORMAT (/,' CASE ',I1,'   LOAD  STRAIN-P  STRAIN-Q
     STRESS-P  STRESS-Q  ANGLE')
      DO 14 J=1,4
      CALL STRESS(I,J,ELAS,POI)
      LOAD=25*J
   14 WRITE (6,9) LOAD, STNP(I,J), STNQ(I,J), STSP(I,J), S
     TSQ(I,J), PHI(I,J)
    9 FORMAT (10X,I3,3X,F8.2,2X,F8.2,2X,F8.2,2X,F8.2,2X,F6
     .1)
      WRITE (6,21)
   21 FORMAT (/,/,/,' SECOND AVERAGING: STRAINS WITH PRINC
     IPAL STRAINS AND STRESSES')
      DO 15 I=1,2
      WRITE (6,5) I
      I2=I*2
      I1=I2-1
      DO 15 J=1,4
      LOAD=25*J
      DO 16 K=1,3
   16 ASTN2(I,J,K)=(ASTN(I2,J,K)+ASTN(I1,J,K))/2
   15 WRITE (6,6) LOAD, ASTN2(I,J,1), ASTN2(I,J,2), ASTN2(
     I,J,3)
```

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