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Nicolas Bouché

*Max Planck Institute for Extraterrestrial Physics*

Jeffrey P. Gardner

*Pittsburgh Supercomputing Center*

Neal Katz

*University of Massachusetts Amherst*

David H. Weinberg

*The Ohio State University*

Romeel Davé

*The University of Arizona*

*See next page for additional authors*

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**Authors**

Nicolas Bouché, Jeffrey P. Gardner, Neal Katz, David H. Weinberg, Romeel Davé, and James D. Lowenthal

## MEASURING THE HALO MASS OF $Z \sim 3$ DAMPED LY-ALPHA ABSORBERS FROM THE ABSORBER-GALAXY CROSS-CORRELATION

NICOLAS BOUCHÉ

Max Planck Institut für extraterrestrische Physik, Giessenbachstrasse, Garching, D-85748, Germany; nbouche@mpe.mpg.de

JEFFREY P. GARDNER

Pittsburgh Supercomputing Center, 4400 5th Ave Pittsburgh, PA 15213; gardnerj@psc.edu

NEAL KATZ

Department of Astronomy, University of Massachusetts-Amherst, Amherst, MA 01003; nsk@kaka.astro.umass.edu

DAVID H. WEINBERG

Department of Astronomy, The Ohio State University, 140 W. 18th Ave, Columbus, OH 43210-1173; dhw@astronomy.ohio-state.edu

ROMEEL DAVÉ

Astronomy Department, University of Arizona, 933 N. Cherry Ave., Tucson, AZ 85721; rad@as.arizona.edu

JAMES D. LOWENTHAL

Five College Astronomy Department, Smith College, McConnell Hall, Northampton, MA 01063; james@ast.smith.edu

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### ABSTRACT

We test the reliability of a method to measure the mean halo mass of absorption line systems such as Damped Ly $\alpha$  absorbers (DLAs). The method is based on measuring the ratio of the cross-correlation between DLAs and galaxies to the auto-correlation of the galaxies themselves, which is (in linear theory) the ratio of their bias factor  $\bar{b}$ . We show that the ratio of the projected cross- and auto-correlation functions ( $w_{\text{dg}}(r_\theta)/w_{\text{gg}}(r_\theta)$ ) is also the ratio of their bias factor irrespective of the galaxy distribution, provided that one uses the same galaxies for  $w_{\text{dg}}(r_\theta)$  and  $w_{\text{gg}}(r_\theta)$ . Thus, the method requires only multi-band imaging of DLA fields, and is applicable to all redshifts. Here, we focus on  $z = 3$  DLAs. We demonstrate that the cross-correlation method robustly constrains the mean DLA halo mass using smoothed particle hydrodynamics (SPH) cosmological simulations that resolve DLAs and galaxies in halos of mass  $M_h \gtrsim 5.2 \times 10^{10} M_\odot$ . If we use the bias formalism of Mo & White (2002) with the DLA and galaxy mass distributions of these simulations, we predict a ratio  $w_{\text{dg}}/w_{\text{gg}}$  of 0.771. Direct measurement of these correlation functions from the simulations yields  $w_{\text{dg}}/w_{\text{gg}} = \bar{b}_{\text{DLA}}/\bar{b}_{\text{gal}} = 0.73 \pm 0.08$ , in excellent agreement with that prediction. Equivalently, inverting the measured correlation ratio to infer a mean DLA halo mass yields (logarithmic averaging, in solar units)  $\langle \log M_{\text{DLA}} \rangle = 11.13^{+0.13}_{-0.13}$ , in excellent agreement with the true value in the simulations:  $\langle \log M_{\text{DLA}} \rangle = 11.16$  is the probability weighted mean mass of the DLA host halos in the simulations. The cross-correlation method thus appear to yield a robust estimate of the average host halo mass even though the DLAs and the galaxies occupy a broad mass spectrum of halos, and massive halos contain multiple galaxies with DLAs. If we consider subsets of the simulated galaxies with high star formation rates (representing Lyman break galaxies [LBGs]), then both correlations are higher, but their ratio still implies the same DLA host mass, irrespective of the galaxy sub-samples, i.e. the cross-correlation technique is also reliable. The inferred mean DLA halo mass,  $\langle \log M_{\text{DLA}} \rangle = 11.13^{+0.13}_{-0.13}$ , is an upper limit since the simulations do not resolve halos less massive than  $\sim 10^{10.5} M_\odot$ . Thus, our results imply that the correlation length between DLAs and LBGs is predicted to be, at most,  $\sim 2.85 h^{-1}$  Mpc given that  $z = 3$  LBGs have a correlation length of  $r_0 \simeq 4 h^{-1}$  Mpc. While the small size of current observational samples does not allow strong conclusions, future measurements of this cross-correlation can definitively distinguish models in which many DLAs reside in low mass halos from those in which DLAs are massive disks occupying only high mass halos.

*Subject headings:* cosmology: theory — galaxies: evolution — galaxies: high-redshift — quasars: absorption lines

### 1. INTRODUCTION

Damped Ly $\alpha$  absorbers (DLAs), which cause the strongest absorption lines found in quasar spectra, have neutral hydrogen (HI) column densities greater

than  $2 \times 10^{20} \text{ cm}^{-2}$ . Their integrated column density distribution implies that DLAs contain the largest reservoir of neutral hydrogen (HI) at high redshifts (e.g. Lanzetta et al. 1991, 1995; Ellison et al. 2001; Péroux et al. 2003). They contain more neutral HI than all the absorption line clouds in the Ly-alpha forest combined; and in an  $\Omega_M = 1$  universe, they contain as much hydrogen as the co-moving mass density of stars in disk galaxies today. This led Wolfe et al. (1986) to put forward the hypothesis that DLAs are large thick gaseous disk galaxies. This hypothesis has been debated since. On the one hand, absorption line velocity profiles of low-ionization species of DLAs seem to be consistent with those expected from lines of sight intercepting rotating thick gaseous disks (Wolfe et al. 1995; Prochaska & Wolfe 1997b; Ledoux et al. 1998). Prochaska & Wolfe (1997b) argue that the most likely rotation velocity is  $\sim 225 \text{ km s}^{-1}$ , i.e. that DLAs are typically as massive ( $10^{12} M_\odot$ ) as  $L^*(z=0)$  galaxies. On the other hand, McDonald & Miralda-Escudé (1999) and Haehnelt et al. (1998, 2000) have shown that a large range of structures and morphologies, rather than a single uniform type of galaxy, can account for the observed DLA kinematics. At least at low redshifts ( $z < 1$ ), this is supported by observations (Le Brun et al. 1997; Kulkarni et al. 2000; Rao & Turnshek 2000).

Early predictions of DLA properties were made using cosmological simulations (Katz et al. 1996b) and semi-analytical simulations of galaxy formation (Kauffmann 1996). Then, Gardner et al. (1997a) extended the results of Katz et al. (1996b) to predict the DLA statistics (e.g.  $\frac{dN}{dz}$ ) accounting for the limited resolution of those simulations. They developed a semi-analytical method to correct the numerical predictions for the contribution of unresolved low-mass halos, and found that roughly half of these systems reside in halos with circular velocities  $V_c \geq 100 \text{ km s}^{-1}$ , and half in halos with  $35 \text{ km s}^{-1} \leq V_c \leq 100 \text{ km s}^{-1}$ . Interestingly, Gardner et al. (1997b) found that ‘a CDM model with  $\Omega_0 = 0.4$ ,  $\Omega_\Lambda = 0.6$  gives an acceptable fit to the observed absorption statistics’, whereas other models did not match the observations so well. More recently, Gardner et al. (2001) found that there was an anti-correlation between the absorber cross-section and the projected distance to the nearest galaxy, and that DLAs arise out to 10–15 kpc. Indeed, they found that the mean cross section for DLA absorption is much larger than what one would estimate based on the collapse of the baryons into a centrifugally supported disk. To match the observed DLA abundances, they required an extrapolation of the mass function to small halos down to a cut-off of  $V_c = 50\text{--}80 \text{ km s}^{-1}$ .

Other work, such as that of Mo et al. (1999), Haehnelt et al. (1998, 2000), Nagamine et al. (2004), and Okoshi et al. (2004), indicates that DLAs are mostly faint (sub- $L_{z=0}^*$ ) galaxies in small dark matter halos with  $V_c \ll 100 \text{ km s}^{-1}$ . However, the exact fraction of DLAs in such halos is a strong function of resolution, as shown by Nagamine et al. (2004). Fynbo et al. (1999) and Schaye (2001) used cross-section arguments and reached similar conclusions. For instance, Schaye (2001) argued that the observed Lyman break galaxy (LBG) number density alone ( $n = 0.016 h^3 \text{ Mpc}^{-3}$  down to  $0.1 L^*$ ) can account for all DLA absorptions at  $z \sim 3$  if the cross-section for

DLA absorption is  $\pi r^2$  with  $r = 19 h^{-1} \text{ kpc}$ , much larger than the luminous parts of most LBGs (Lowenthal et al. 1997). However, Schaye (2001) pointed out that the cross-section can be much smaller than this, if a fraction of DLA systems arise in outflows or if  $n$  is much larger (i.e. there are many LBGs or other galaxies not yet detected). In the semi-analytical models of Maller et al. (2000), DLAs arise from the combined effects of massive central galaxies and a number of smaller satellites within  $100 h^{-1} \text{ kpc}$  in virialized halos. From all these studies, it appears that the low-mass hypothesis is favored against the thick gaseous disk model of Wolfe et al. (1986, 1995) and Prochaska & Wolfe (1997a). A strong constraint on the nature of DLA will come from a measure of the typical DLA halo mass.

In order to constrain the mass of  $z \simeq 3$  DLAs, several groups are using Lyman break galaxies (LBGs) as large scale structure tracers (Gawiser et al. 2001; Adelberger et al. 2003 [hereafter ASSP03]; Bouché & Lowenthal 2003; Bouché 2003; Bouché & Lowenthal 2004 [hereafter BL04]; and Cooke et al., 2004, private communication) to measure the DLA-LBG cross-correlation, given that in hierarchical galaxy formation models, different DLA masses will lead to different clustering properties with the galaxies around them. Specifically, the DLA-galaxy cross-correlation yields a measurement of the dark matter halo mass associated with DLAs relative to that of the galaxies. In particular, if the galaxies are less (more) correlated with the DLAs than with themselves, this will imply that the halos of DLAs are less (more) massive than those of the galaxies.

The purpose of this paper is to use cosmological simulations in order to demonstrate that cross-correlation techniques will uniquely constrain the mean DLA halo mass, and to compare the results with observations. The advantage of using cosmological simulations is that one can check the reliability of the clustering results given that the mean halo mass of any population is a known quantity in the simulations. As we will show, we find that the DLA-galaxy cross-correlation implies a mean DLA halo mass of  $\langle \log M_{\text{DLA}} \rangle \simeq 11.13_{-0.13}^{+0.13}$  close to the  $\log M_{\text{DLA}} = 11.16$  expected from the DLA halo mass distribution. The method is generally applicable to any redshifts, but we focus here on  $z = 3$ .

Section 2 presents the numerical simulations used in this paper. Section 3 lays the foundations of our clustering analysis. Our results are presented in section 4 along with a comparison to current observational results. A discussion of the implications of our results is presented in section 5.

## 2. SIMULATIONS

We use the Tree-SPH simulations of Katz et al. (1996a) parallelized by Davé et al. (1997), which combine smoothed particle hydrodynamics (SPH, Lucy 1977) with the tree algorithm for computation of the gravitational force (Hernquist 1987). This formulation is completely Lagrangian, i.e. it follows each particle in space and time. The simulations include dark matter, gas, and stars. Dark matter particles are collision-less and influenced only by gravity, while gas particles are influenced by pressure gradients and shocks in addition to gravity, and can cool radiatively. Gas particles are

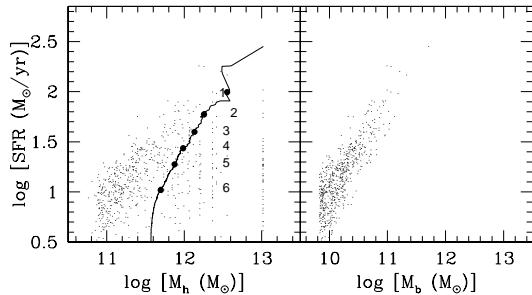


FIG. 1.— The star formation rate (SFR) as a function of halo mass (DM+baryons)  $M_h$  is shown on the left panel. The streak of points at  $10^{13} M_\odot$  corresponds to several resolved galaxies. The line shows the running mean (in  $\log M$ ) with a decreasing SFR threshold. The filled circles show the SFR threshold versus the mean mass ( $\langle \log M \rangle$ ) of the six sub-samples. The six sub-samples are the 7, 25, 50, 100, 200, 400 most star-forming galaxies, labeled 1 to 6. The right panel shows the SFR as a function of the baryonic mass  $M_b$  (right).

transformed into collision-less stars when the following conditions are met: the local density reaches a certain threshold ( $n_H \geq 0.1 \text{ cm}^{-3}$ ), and the particles are colder than a threshold temperature ( $T \leq 30,000 \text{ K}$ ) and are part of a Jeans unstable convergent flow (see Katz et al. 1996a, for details). A Miller & Scalo (1979) initial mass function of stars is assumed. Stars of mass greater than  $8 M_\odot$  become supernovae and inject  $10^{51} \text{ erg s}^{-1}$  of pure thermal energy into neighboring gas particles. Thus, the star formation rate (SFR) is known for each galaxy. Photo-ionization by a spatially uniform UV background (Haardt & Madau 1996) is included.

The simulation was run from redshift  $z = 49$  to redshift  $z = 0$  with the following cosmological parameters:  $\Omega_M = 0.4$ ,  $\Omega_\Lambda = 0.6$ ,  $h \equiv H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1}) = 0.65$ ,  $\Omega_b = 0.02 h^{-2}$ , a primordial power spectrum index  $n = 0.93$ , and  $\sigma_8 = 0.8$  for the amplitude of mass fluctuations. In this paper, we use the  $z = 3$  output. The simulation has  $128^3$  dark matter particles and the same number of gas particles in a periodic box of  $22.222 h^{-1} \text{ Mpc}$  (co-moving) on a side with a gravitational softening length of  $3.5 h^{-1} \text{ kpc}$  (Plummer equivalent). The mass of a dark matter particle is  $8.2 \times 10^8 M_\odot$ , and the mass of a baryonic particle is  $1.09 \times 10^8 M_\odot$ . We identify dark matter halos by using a ‘friends-of-friends’ algorithm (Davis et al. 1985) with a linking length of 0.173 times the mean interparticle separation. There are 1770 resolved dark matter halos with a minimum of 64 dark matter particles ( $5.2 \times 10^{10} M_\odot$ ).

We use the group finding algorithm ‘spline kernel interpolative denmax’ (SKID) (Katz et al. 1996a) to find galaxies in the simulations. We refer the reader to Kereš et al. (2004) for a detailed discussion of the SKID algorithm. There are 651 galaxies resolved with a minimum of 64 SPH particles (or  $6.9 \times 10^9 M_\odot$ ). Fig. 1 shows the SFR as a function of total halo mass (dark matter + baryons; left) and baryonic mass (right) for the 651 SKID-identified galaxies. The line shows the running mean (in  $\log M$ ) with a decreasing SFR threshold.

The rest-UV spectra and colors of observed LBGs are dominated by the light from massive stars (Lowenthal et al. 1997; Pettini et al. 2001). To simulate various ‘flux-simulated’ LBG samples in the simulations,

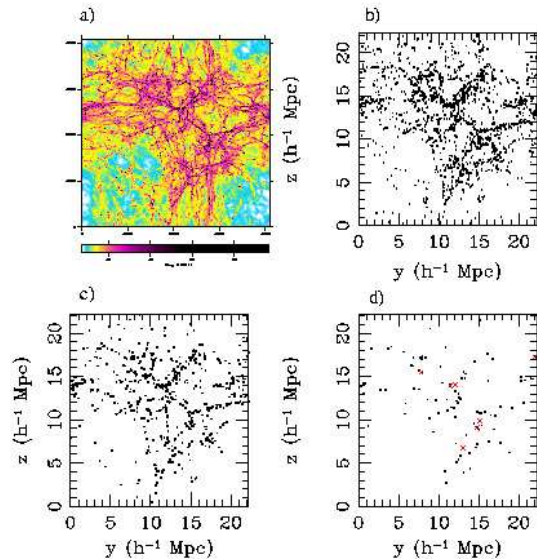


FIG. 2.— (a): Column density map of HI in the  $22.222 h^{-3} \text{ Mpc}^3$  volume projected along the  $x$  axis on a  $4096^2$  pixel grid. Potential DLAs with  $N_{\text{HI}}$  larger than  $> 10^{20.3} \text{ cm}^{-2}$  appear black. (b): Position of potential DLAs projected along the  $x$  axis. (c): Position of the 651 galaxies that have a baryonic mass  $M_b$  larger than the resolution  $6.8 \times 10^9 M_\odot$ . (d): Position of the 100 most star-forming galaxies. The red crosses show the positions of the seven most star-forming galaxies.

we selected 6 sub-samples of galaxies according to their SFR, consisting of the 7, 25, 50, 100, 200, 400 most star-forming galaxies. The corresponding SFR thresholds and mean masses  $\langle \log M \rangle$  for each of the sub-samples are marked with the filled circles in Fig. 1 (left) labeled 1 to 6. Naturally, real LBGs are color-selected, so this SFR selection can only be an approximation. Davé et al. (1999) discuss the properties of LBGs in numerical simulations similar to this one.

We select DLAs from the simulations as follows. We compute the HI column density ( $N_{\text{HI}}$ ) from the gas density projected onto a uniform grid with  $4096^2$  pixels, each  $5.43 \text{ kpc}$  co-moving (or  $2 \text{ kpc}$  physical) in size, corresponding to the smoothing length. Each gas particle is projected onto the grid in correct proportions to the pixel(s) it subtends given its smoothing length. Since DLAs occur in dense regions, however, the smoothing lengths are typically equal or smaller than the pixel size. We first assume the gas is optically thin, and then correct the column densities for the ionization background using a self-shielding correction as in Katz et al. (1996b). The HI column density projected along the  $x$  axis is shown in Fig. 2(a). A pixel is selected as a DLA from the column density map if  $N_{\text{HI}}$  is greater than  $10^{20.3} \text{ cm}^{-2}$ . There are approximately 115,000 pixels that meet this criterion, shown in Fig. 2(b). We assume that each such pixel is a potential DLA. Fig. 2(c) shows the positions of the 651 galaxies that have a baryonic mass  $M_b$  larger than the resolution  $6.8 \times 10^9 M_\odot$ . Fig. 2(d) shows the positions of the 100 galaxies with the highest star formation rate, and the positions of the simulated LBGs as red crosses. From Fig. 2, one can already see that the galaxies and the DLAs are correlated.

The left panel of Fig. 3 shows the mass probability

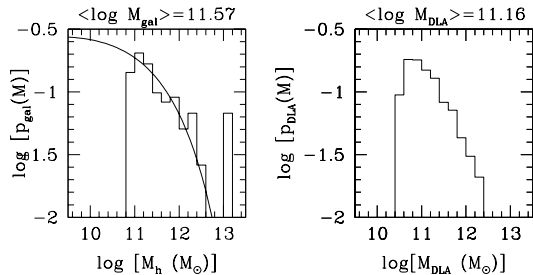


FIG. 3.— Left: The halo mass (DM+baryons) probability function  $p(M)$  of all the SKID-identified galaxies with baryonic masses larger than the resolution  $6.8 \times 10^9 M_\odot$ , corresponding to 64 SPH particles. For comparison, the curve is the dark matter mass function from the extended Press-Schechter formalism of Mo & White (2002) in this cosmology, scaled arbitrarily (i.e. not a fit). Right: The DLA dark matter mass distribution,  $p_{\text{DLA}}(M) (\propto \frac{dN}{dz d \log M})$ . It was found by matching the 2-D DLA positions with the nearest resolved halo. The shape of the distribution will not be constrained by the DLA-galaxy cross-correlation, but its first moment ( $\langle \log M \rangle$ ) will be.

distribution of all the resolved galaxies. The line shows the halo mass distribution obtained from Press-Schechter (PS) formalism (Mo & White 2002). The mean mass (logarithmic average) for all the 651 galaxies is shown ( $\langle \log M_h(M_\odot) \rangle = 11.57$ ). The right panel of Fig. 3 shows the DLA halo mass distribution. The halo mass of a given DLA was obtained by matching the projected DLA positions (2-D) with those of the resolved halos. The projected distance distribution (between halos and DLAs) peaks at 8 kpc, with a tail to  $\sim 20$  kpc (physical units) (see also Gardner et al. 2001), and there is very little ambiguity in identifying the halo of a DLA. Practically all the DLAs reside in halos with more than 64 dark matter particles. Note that, at  $z = 0$ , the DLA distribution appears to be broadly peaked at around  $V_{\text{rot}} = 200 \text{ km s}^{-1}$  (Zwaan et al. [2005], in preparation) and is even broader with respect to luminosity (Rosenberg & Schneider 2003).

As mentioned, the purpose of this paper is to show that cross-correlation techniques will uniquely constrain the mean of this distribution, but it will not constrain its shape. We refer the reader to Gardner et al. (2001) and Nagamine et al. (2004) for a detailed discussion of the DLA halo mass distribution in numerical simulations. Typically, in order to match the observed DLA statistics, they require an extrapolation of the DLA mass function below the mass resolution. Here we make no attempt to include halos smaller than our resolution since it would require putting in the appropriate cross-correlation signal by hand for halos smaller than our resolution.

### 3. CORRELATION FUNCTIONS IN HIERARCHICAL MODELS

In this section, we describe the fundamental clustering relations necessary to understand how one can determine the halo mass of DLAs.

A widely used statistic to measure the clustering of galaxies is the galaxy auto-correlation function  $\xi_{\text{gg}}(r)$ . Similarly, one can define the cross-correlation  $\xi_{\text{dg}}$  between DLAs and galaxies from the conditional probability of finding a galaxy in a volume  $dV$  at a distance

$r = |\mathbf{r}_1 - \mathbf{r}_o|$ , given that there is a DLA at  $\mathbf{r}_o$ :

$$P(\text{LBG}|\text{DLA}) = \bar{n}_u(1 + \xi_{\text{dg}}(r))dV, \quad (1)$$

where  $n_u$  is the unconditional background galaxy density, i.e. the density when  $\xi = 0$ .

At a given redshift, the auto-correlation and cross-correlation functions are related to the dark matter correlation function  $\xi_{\text{DM}}$  through the mean bias  $\bar{b}(M)$ :

$$\xi_{\text{gg}}(r) = \bar{b}^2(M_{\text{gal}}) \xi_{\text{DM}}(r), \quad (2)$$

$$\xi_{\text{dg}}(r) = \bar{b}(M_{\text{DLA}}) \bar{b}(M_{\text{gal}}) \xi_{\text{DM}}(r), \quad (3)$$

where  $M_{\text{gal}}$  is the mean galaxy halo mass, and  $M_{\text{DLA}}$  is the mean DLA halo mass, and  $\bar{b}(M)$  is given by:

$$\int_M^\infty p(M') b(M') dM', \quad (4)$$

where  $p(M)$  is the halo mass probability distribution and  $b(M)$  is the bias function, which can be computed using the extended PS formalism (e.g. Mo et al. 1993; Mo & White 2002). Thus, from Eq. 2–3, if both  $\xi_{\text{dg}}$  and  $\xi_{\text{gg}}$  are power-laws ( $\xi \propto r^\gamma$ ) with the same slope  $\gamma$ , the amplitude ratio of the cross- to auto-correlation is a measurement of the bias ratio  $\bar{b}(M_{\text{DLA}})/\bar{b}(M_{\text{gal}})$  from which one can infer the halo masses  $M_{\text{DLA}}/M_{\text{gal}}$ . The details will be presented in section 4.2. Briefly, given that  $b(M)$  is a monotonic increasing function of  $M$ , if  $\xi_{\text{dg}}$  is greater (smaller) than  $\xi_{\text{gg}}$ , then the halos of DLAs are more (less) massive than those of the galaxies.

In the remainder of this work, we will use only projected correlation functions,  $w(r_\theta)$ , where  $r_\theta = D_A(1+z)\theta$  in co-moving Mpc with  $D_A$  the angular diameter distance. This is necessary since (1) the gas column density distribution is a 2-D quantity, and (2) this corresponds to the situation when one relies on photometric redshifts (e.g. Bouché & Lowenthal 2003, BL04).  $w(r_\theta)$  is directly related to spatial correlation functions  $\xi(r)$  if the selection function is known. Following Phillipps et al. (1978) and Budavári et al. (2003), the projected auto-correlation function,  $w_{\text{gg}}$ , of galaxies with a redshift distribution  $\frac{dN}{dz}$  is:

$$\begin{aligned} w_{\text{gg}}(r_\theta) &= \int_0^\infty dz \left( \frac{dN}{dz} \right)^2 g(z)^{-1} \times (f(z)\theta)^{1-\gamma} r_0^\gamma H_\gamma \\ &= (r_\theta)^{1-\gamma} r_{0,\text{gg}}^\gamma H_\gamma \int_0^\infty dl \left( \frac{dN}{dl} \right)^2, \end{aligned} \quad (5)$$

where  $\frac{dN}{dl}$  is the galaxy redshift distribution in physical units,  $f(z) = D_A(1+z)$  is the co-moving line-of-sight distance,  $g(z) = dr/dz = c/H(z)$ , and  $H_\gamma = \frac{\Gamma(\frac{1}{2})\Gamma(\frac{\gamma-1}{2})}{\Gamma(\frac{\gamma}{2})}$  (see Appendix A). The projected cross-correlation  $w_{\text{dg}}$  between a given absorber at a given redshift and the galaxies (with a distribution  $\frac{dN}{dz}$ ) is:

$$w_{\text{dg}}(r_\theta) = \int \frac{dN}{dl} \xi(\sqrt{r_\theta^2 + l^2}) dl, \quad (6)$$

For galaxies distributed in a top-hat redshift distribution  $\frac{dN}{dl}$  of width  $W_z$  (normalized such that  $\int \frac{dN}{dl} dl = 1$ ), as in the case here, Equations 5 and 6 imply that the amplitudes of both  $w_{\text{dg}}(r_\theta)$  and  $w_{\text{gg}}(r_\theta)$  are inversely proportional to  $W_z$  (see Appendix A for the derivations):

$$w_{\text{gg}}(r_\theta) \simeq (r_\theta)^{1-\gamma} r_{0,\text{gg}}^\gamma H_\gamma \times \left( \frac{1}{W_z} \right)^2 W_z, \quad (7)$$

$$w_{\text{dg}}(r_\theta) \simeq (r_\theta)^{1-\gamma} r_{0,\text{dg}}^\gamma H_\gamma \times \frac{1}{W_z}. \quad (8)$$

Therefore, the ratio of the amplitudes of the two projected correlation functions  $w_{\text{dg}}$  to  $w_{\text{gg}}$  is simply  $(r_{0,\text{dg}}/r_{0,\text{gg}})^\gamma$ , or the bias ratio  $\bar{b}(M_{\text{DLA}})/\bar{b}(M_{\text{gal}})$  from which we infer the mean DLA halo mass, regardless of the redshift distribution. This is an important result for surveys that rely on photometric redshifts: the ratio of the *projected* correlation is a true measure of the bias ratio, regardless of contamination or uncertainty in the actual redshift distribution, provided that the same galaxies are used for  $w_{\text{dg}}$  to  $w_{\text{gg}}$ .

#### 4. RESULTS

In section 4.1, we quantify the amplitude of the DLA-galaxy cross-correlation relative to the galaxy-galaxy auto-correlation in the SPH simulations. We show how to invert the cross-correlation results into a mass constraint in section 4.2. We show that this method is independent of the galaxy sample that one uses (§ 4.3). Finally, we compare these results to observational results in section 4.4.

##### 4.1. DLA-galaxy cross-correlation

The filled circles in Figure 4 show the DLA-galaxy cross-correlation  $w_{\text{dg}}$  using the entire sample of 115,000 DLAs and the 651 resolved galaxies. We computed  $w_{\text{dg}}(r_\theta)$  with the following estimator:

$$1 + w_{\text{dg}}(r_\theta) = \left\langle \frac{N_{\text{obs}}(r_\theta)}{N_{\text{exp}}(r_\theta)} \right\rangle, \quad (9)$$

where  $N_{\text{obs}}(r_\theta)$  is the observed number of galaxies between  $r_\theta - dr/2$  and  $r_\theta + dr/2$  from a DLA and  $N_{\text{exp}}(r_\theta)$  is the expected number of galaxies if they were uniformly distributed, i.e.  $N_{\text{exp}}(r_\theta) = 2\pi r_\theta \Sigma_g dr$  where  $\Sigma_g$  is the galaxy surface density.  $\langle \rangle$  denotes the average over the number of selected DLAs ( $N_{\text{DLA}}$ ). In counting the pairs, we took into account the periodic boundary conditions of the simulations.

There are several reasons not to use other estimators such as the Landy & Szalay (1993) (LS) estimator. First, we want to duplicate as closely as possible the method (and estimator) used in the observations of Bouché & Lowenthal (2003) and BL04. But, more importantly, the LS estimator is symmetric under the exchange galaxy-absorber, whereas here and for the observations of BL04, the symmetry is broken. This is due to the absorber redshift being well known, while galaxies have photometric redshifts with larger uncertainties and, therefore, are distributed along the line of sight. This broken symmetry is also fundamental in the derivation of Eqs. 5–6. Had we used spectroscopic redshifts and  $\xi(r)$  instead of  $w(r_\theta)$ , the LS estimator would be superior.

Given that we use the galaxy surface density  $\Sigma_g$  to estimate the unconditional galaxy density (see Eq. 1), the integral of  $\Sigma_g(1 + w_{\text{dg}})$  over the survey area  $A$  will be equal to the total number of galaxies, i.e.  $\int_A \Sigma_g(1 + w_{\text{dg}})dA = N_g$ . As a consequence,  $\int \Sigma_g w_{\text{dg}}dA = 0$  and the correlation will be negative on the largest scales, i.e. biased low. This is the known ‘integral constraint’. In the case of our  $22.222h^{-2}$  Mpc<sup>2</sup> survey geometry, we estimated the integral constraint to be  $C = 0.04$ , or 2% of the cross-correlation strength at  $1h^{-1}$  Mpc. We added  $C$  to  $w_{\text{dg}}$  estimated from Eq. 9.

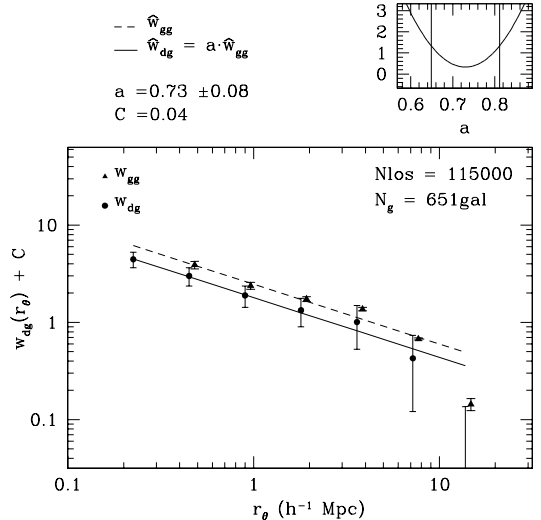


FIG. 4.— The filled circles show the projected DLA-galaxy cross-correlation  $w_{\text{dg}}(r_\theta)$  at  $z = 3$  in this  $22.222h^{-3}$  Mpc<sup>3</sup> simulation. The solid triangles show the projected auto-correlation  $w_{\text{gg}}(r_\theta)$  (offsetted by 0.03dex in the  $x$ -axis for clarity). The full sample of 115,000 DLAs and 651 resolved galaxies were used. The amplitude ratio  $a = 0.73 \pm 0.08$ , which is  $w_{\text{dg}}/w_{\text{gg}}$ , is found by fitting  $w_{\text{dg}}$  to the model  $\hat{w}_{\text{dg}} = a \times \hat{w}_{\text{gg}}$ , where  $\hat{w}_{\text{gg}}$  is the fit to the galaxy-galaxy auto-correlation (shown by the dashed line). The small panel shows the  $1\sigma$  contour of the  $\chi^2$  distribution. An integral constraint of  $C = 0.04$  was used.

The uncertainty to  $w_{\text{dg}}$ ,  $\sigma_w$ , has two terms, the Poisson noise and the clustering variance (see Eisenstein 2003, references therein, and Appendix B). In Appendix B, we show that  $\sigma_w$  will be proportional to  $1/\sqrt{N_{\text{DLA}}}$  (Eq. B11).

There are several ways to compute  $\sigma_w$  in practice. The proper way to compute  $\sigma_w$  would be to resample the DLAs, since this would include the uncertainty due to the finite number of lines of sight. However, this is valid for independent lines of sight, as in the case of an observational sample (provided that  $N_{\text{DLA}}$  is large, say greater than 10) and will not be correct here given that we have only one simulation, and that we have to use the same galaxies for each simulated line of sight. The uncertainty  $\sigma_w$  must then reflect that we used only one realization of the large scale structure. For this reason, we elected to use the jackknife estimator (Efron 1982), i.e. by dividing the  $22.222h^{-2}$  Mpc<sup>2</sup> area into 9 equal parts and each time leaving one part out. This will accurately reflect the uncertainty in  $w_{\text{dg}}$  due to the one large scale structure used, but the S/N ratio ( $\text{SNR} \equiv w_{\text{dg}}/\sigma_w$ ) will not increase with  $\sqrt{N_{\text{DLA}}}$  as expected (Appendix B); it will saturate after a certain value of  $N_{\text{DLA}}$ . We find that indeed the SNR saturates at  $N_{\text{DLA}} \simeq 40$  (not shown). This is a major difference from observational samples, where each field is independent. In that case, Eq. B11 applies and the SNR is proportional to  $\sqrt{N_{\text{DLA}}}$ .

We computed the full covariance matrix from the  $N_{\text{jack}} = 9$  realizations as follows:

$$\text{COV}_{ij} = \frac{N_{\text{jack}} - 1}{N_{\text{jack}}} \sum_{k=1}^{N_{\text{jack}}} [w_k(r_{\theta_i}) - \bar{w}(r_{\theta_i})] \cdot [w_k(r_{\theta_j}) - \bar{w}(r_{\theta_j})] \quad (10)$$

where  $\omega_k$  is the  $k$ th measurement of the cross-correlation and  $\bar{\omega}$  is the average of the  $N_{\text{jack}}$  measurements of the cross-correlation. The error bars in Fig. 4 show the diagonal elements of the covariance matrix, i.e.  $\sigma_w \equiv \sqrt{\text{COV}_{ii}}$ .

We computed the projected auto-correlation  $w_{\text{gg}}(r_\theta)$  of the same simulated galaxies used for  $w_{\text{dg}}(r_\theta)$  in a similar manner. We used the estimator shown in Eq. 9 to compute  $w_{\text{gg}}(r_\theta)$ , where  $N_{\text{obs}}(r)$  is now the number of galaxies between  $r - dr/2$  and  $r + dr/2$  from another galaxy. The open triangles in Fig. 4 show the projected auto-correlation  $w_{\text{gg}}(r_\theta)$  of the 651 galaxies.

We fitted the galaxy auto-correlation with a power law model ( $\hat{w}_{\text{gg}} = A_{\text{gg}} r_\theta^\beta$ ) by minimizing  $\chi^2 \propto [\mathbf{w} - \hat{\mathbf{w}}]^T \text{COV}^{-1} [\mathbf{w} - \hat{\mathbf{w}}]$ , where  $\mathbf{w}$  and  $\hat{\mathbf{w}}$  are the vector data and model, respectively, and  $\text{COV}^{-1}$  is the inverse of the covariance matrix. We used Single Value Decomposition (SVD) techniques to invert the covariance matrix,  $\text{COV}$ , since it is singular and the inversion is unstable (see discussion in Bernstein 1994).

We then use that fit as a template to constrain the amplitude of  $w_{\text{dg}}$ , i.e.

$$\hat{w}_{\text{dg}} = a \times \hat{w}_{\text{gg}}, \quad (11)$$

where  $a$  is the amplitude ratio  $A_{\text{dg}}/A_{\text{gg}}$  of the correlation functions. This assumes that the two correlation functions have the same slope (see section 3). This method also closely matches the method used by BL04 (see section 4.4 below), and will make comparison to those observations straightforward.

The solid line in Fig. 4 shows the fit to  $w_{\text{dg}}$  using Eq. 11, where the best amplitude  $a$  is

$$a = 0.73 \pm 0.08. \quad (12)$$

The top panel shows the  $\chi^2(a)$  distribution with the  $1\sigma$  range. In other words, the bias ratio  $\bar{b}(M_{\text{DLA}})/\bar{b}(M_{\text{gal}})$  is  $0.73 \pm 0.08$ . This can be converted into a correlation length for  $w_{\text{dg}}$  of  $a^{1/1.8} \simeq 84\%$  times that of the galaxy auto-correlation, i.e.  $r_{0,\text{dg}} \simeq 0.84 \times r_{0,\text{gg}}$ .

Several authors (e.g. Berlind & Weinberg 2002; Berlind et al. 2003, and reference therein) have shown that the small scales ( $r < 1$  Mpc) of the correlation function are the scales sensitive to variations in the halo occupation number. At those scales,  $\xi(r)$  is very susceptible to galaxy pairs that are in the same halo. Therefore, when we repeated our analysis with the six sub-samples, we restricted ourselves to  $r_\theta > 1h^{-1}$  Mpc. In this case, for the full sample, we find the amplitude ratio to be  $a = 0.70 \pm 0.18$ , in good agreement with Eq. 12.

The reader should not use these results (e.g. Eq. 12), obtained with 651 galaxies and 115,000 DLAs, to scale the errors to smaller samples because we use the same large scale structure for all the 115,000 simulated DLAs. As mentioned earlier, the large scale structure dominates the uncertainty at large  $N_{\text{DLA}}$ , and this is seen in the fact that the SNR saturates after  $N_{\text{DLA}} \simeq 40$ . We come back to this point at the end of section 4.4.

#### 4.2. The mass of DLA halos from the amplitude of $w_{\text{dg}}$

Eq. 12, i.e. the bias ratio  $\bar{b}(M_{\text{DLA}})/\bar{b}(M_{\text{gal}})$ , can be converted into a mean halo mass for DLAs if one knows the functional form of  $b(M)$  and  $M_{\text{gal}}$ . One can use

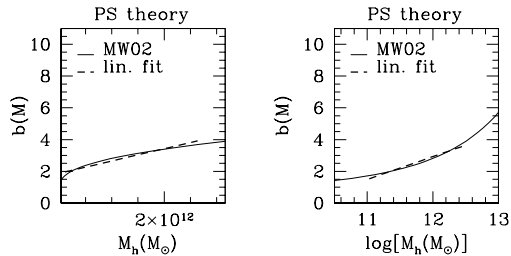


FIG. 5.— Left: The  $z = 3$  bias  $b(M)$  as a function of halos of mass  $M$  from the extended Press-Schechter theory as in Mo & White (2002). Right: Same in  $\log M$  space. In both panels, the dashed line is a linear fit to the curve over the mass range  $\log M \sim 11.5$ –12.5.

the PS formalism (e.g. Mo & White 2002) or the auto-correlation of several galaxy sub-samples to constrain the shape of  $b(M)$ . We will refer to these as the ‘theoretical method’ and as the ‘empirical method’, respectively.

##### 4.2.1. Theoretical biases $b(M)$

One can compute the theoretical biases for any population (Eq. 4) and predict the bias ratio a priori if the mass probability distribution  $p(M)$  is known. Naturally,  $p(M)$  is known in our simulation (Fig. 3). We will show that the predicted bias ratio is well within the  $1\sigma$  range of our results (Eq. 12), demonstrating the reliability of the method.

Given that galaxies and the DLAs actually lie in halos of different masses, the theoretical biases are found from Eq. 4, i.e.:

$$\bar{b}_{\text{DLA}}(> M) = \int_M^\infty p_{\text{DLA}}(M') b(M') d \log M', \quad (13)$$

$$\bar{b}_{\text{gal}}(> M) = \int_M^\infty p_{\text{gal}}(M') b(M') d \log M', \quad (14)$$

where  $p(M)$  is the appropriate mass distribution ( $p(M) \equiv \frac{dn}{d \log M}$ ) (normalized such that  $\int p(M) d \log M = 1$ ) and  $b(M)$  is the bias of halos of a given mass  $M$ . The bias function  $b(M)$  is also a function of redshift  $z$ , i.e.  $b(M, z)$ , and can be computed at a given  $z$  from the extended PS formalism (e.g. Mo & White 2002). It is shown in Fig. 5 for  $z = 3$  on a linear-linear (left) and log-linear (right) plot.

The mass distributions  $p_{\text{DLA}}$  and  $p_{\text{gal}}$  were shown in Fig. 3. Because  $p(M)$  is bounded at some low mass limit  $M_{\text{min}}$  (due to limited resolution or to observational selection), the mean bias  $\bar{b}$  of a given galaxy sample is defined by  $\bar{b}_{\text{gal}} = \bar{b}(> M_{\text{min}})$ .

The predicted biases  $\bar{b}$  are shown in the left panel of Fig. 6. The predicted biases  $\bar{b}$  for the sub-samples, the 651 galaxies, and the DLAs are represented by open squares, the filled square and the filled circle, respectively. From  $\bar{b}$  for the 651 galaxies (filled square) and for DLAs (filled circle), the theoretical bias ratio  $\bar{b}_{\text{DLA}}/\bar{b}_{\text{gal}}$  is found to be 0.771, very close to the bias ratio measured from the clustering of galaxies around the DLAs (Eq. 12).

When the distributions  $p(M)$  are not known, we can infer a mass ratio from the bias ratio using the approx-



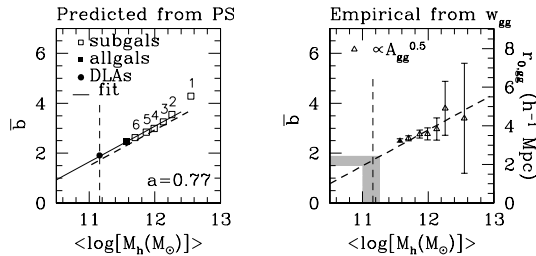


FIG. 6.— Left: The symbols show the mean bias  $\bar{b}$  as a function of the mean halo mass  $\langle \log M_h \rangle$  for the DLAs (circle), full galaxy sample (filled square) and the sub-samples (open squares), labeled 1 to 6. The bias is calculated using Eqs. 13–14 and the distributions  $p_{\text{DLA}}$  and  $p_{\text{gal}}$  (shown in Fig. 1). From the filled symbols, the predicted bias ratio is found to be 0.771. The solid line is a linear fit to the points and is 5% away from the linear approximation (Eq. 5) shown by the thick dashed line. Right: Same as left, but using the empirical method (see text) instead of the mass distributions. The triangles show the mean bias for the different galaxy samples using  $\bar{b} \propto \sqrt{A_{\text{gg}}}$  (Eq. 2) where the normalization was adjusted to match the amplitude of the dashed line. The bias for the 651 galaxies is represented by the filled triangle. The open triangles show  $\bar{b}$  for the sub-samples. The right axis scale shows the corresponding correlation length  $r_0$ . For the DLAs, the shaded areas show the measured bias ratio  $a = 0.73 \pm 0.08$  and the corresponding halo mass range  $\langle \log M_{\text{DLA}}(M_\odot) \rangle = 11.13^{+0.13}_{-0.13}$  using the mean mass of the 651 resolved galaxies:  $\langle \log M_{\text{gal}} \rangle = 11.57$ . In both panels, the vertical dashed line shows the ‘true’ mean  $\langle \log M_{\text{DLA}} \rangle = 11.16$  obtained from the DLA mass distribution  $p_{\text{DLA}}(M)$  shown in Fig. 3. Both panels are for  $z = 3$ .

imation  $b(M) = b_0 + b_1 \log M$ <sup>1</sup>, over a restricted mass range. In each panel in Fig. 5, the dashed line shows such a linear fit over the mass range  $\log M \sim 11$ –12.5. Using this approximation, the mean bias  $\bar{b}$  is given by

$$\begin{aligned} \bar{b}(> M_{\text{min}}) &= \int_{M_{\text{min}}}^{\infty} p(M') b(M') d \log M' \\ &= b_0 + b_1 \int_{M_{\text{min}}}^{\infty} p(M') \log M' d \log M' \\ &= b_0 + b_1 \langle \log M \rangle, \end{aligned} \quad (15)$$

where  $\langle \log M \rangle$  is the first moment of the distribution  $p(M)$ . Thus, the mean bias for the galaxies and the DLAs are  $\bar{b}_{\text{gal}} = b(\langle \log M_{\text{gal}} \rangle)$ , and  $\bar{b}_{\text{DLA}} = b(\langle \log M_{\text{DLA}} \rangle)$ , respectively, where  $\langle \rangle$  is the first moment of the appropriate mass distribution.

In Figure 6 (left panel) the solid line is a linear fit to the theoretical biases  $\bar{b}(\langle \log M \rangle)$ , and is 5% away (in amplitude) from the linear approximation (Eq. 15) shown by the thick dashed line. The vertical dashed line indicates the mean DLA halo mass  $\langle \log M_{\text{DLA}} \rangle$  that is found from the first moment of the mass distribution in Fig. 3. This shows that using a linear approximation of  $b(M)$  is equivalent to using the bias function  $b(M)$  from Mo & White (2002), provided that the DLA-galaxy mass ratio is not larger than a decade. Indeed, the 5% difference in amplitude cancels out when taking the bias ratio.

#### 4.2.2. Empirical method for $b(M)$

<sup>1</sup> One can use  $b(M) = b'_0 + b'_1 M$  instead, and replace  $\langle \log M \rangle$  by  $\langle M \rangle$  in the remaining of the discussion.

To infer  $\langle \log M_{\text{DLA}} \rangle$  from Eq. 12 or from observations, one needs to find the coefficients  $b_0$ , and  $b_1$ . To do so, one can either use the PS formalism (Mo & White 2002) or use the fact that  $\bar{b}$  is proportional to  $\sqrt{A_{\text{gg}}}$  (Eq. 2), where  $A_{\text{gg}}$  is measured for each of the galaxy sub-samples covering the mass range  $\log M \sim 11.5$ –12.5. Figure 6 (right panel) illustrates this point. The thick dashed line is again the linear approximation shown in Fig. 6. The open (filled) triangles show the mean biases  $\bar{b}$  of the sub-samples (full sample) assuming that  $\bar{b} \propto \sqrt{A_{\text{gg}}}$  (Eq. 2). The normalization is set to match the dashed line, and is not relevant since we measure a ratio of two biases. This shows that one can use either the PS formalism (Mo & White 2002) or use  $\sqrt{A_{\text{gg}}}$  to find the coefficients  $b_0$ , and  $b_1$ .

In the case where the auto-correlation length  $r_{0,\text{gg}}$  has been determined, one can use the right  $y$ -axis scale of Fig. 6 to infer the DLA halo mass from the measured bias ratio.

#### 4.2.3. The mean DLA halo mass

To actually determine  $\langle \log M_{\text{DLA}} \rangle$  from our cross-correlation result (Eq. 12), we used (i) the linear approximation to the PS bias (thick dashed line in Fig. 6), and (ii)  $\langle \log M_{\text{gal}} \rangle = 11.57$  for the 651 galaxies. We infer a mean DLA halo mass of  $\langle \log M_{\text{DLA}} \rangle = 11.13^{+0.13}_{-0.13}$ , shown by the vertical shaded area on the right panel of Fig. 6. Our cross-correlation result (Eq. 12) is shown by the horizontal shaded area. The ‘true’ DLA mass derived from  $p_{\text{DLA}}$  (Fig. 3) and Eq. 13 is shown by the vertical dashed line at  $\log M_{\text{DLA}} = 11.16$ . Similarly, using fits to  $b(M)$  in linear space (left panel of Fig. 5), we infer  $\langle M_{\text{DLA}} \rangle = 2.12^{+2.96}_{-2.0} \times 10^{11}$ , close to ‘true’ mean  $1/N_{\text{DLA}} \sum_i M_{\text{DLA},i} = 3.94 \times 10^{11} M_\odot$ .

In summary, the amplitude of  $w_{\text{dg}}$  relative to  $w_{\text{gg}}$ ,  $a = 0.73 \pm 0.08$  (Eq. 12), measured in this simulation implies that DLAs have halos of (logarithmically) averaged mass:

$$\langle \log M_{\text{DLA}}(M_\odot) \rangle = 11.13^{+0.13}_{-0.13}, \quad (16)$$

close to the true 11.16. This shows that the cross-correlation technique uniquely constrain the mean of the halo mass distribution, despite the fact that DLAs occupy a range of halo masses and some halos contain multiple galaxies and multiple DLA systems. In the next section, we will show that the technique is reliable in the sense that it will lead to the same answer regardless of the galaxy sample used.

From the right panel of Fig. 6, we can now predict the cross-correlation strength for real  $z = 3$  LBGs, which have a correlation length of  $r_{0,\text{gg}} \simeq 4$  Mpc (e.g. ASSP03; Adelberger et al. 2004), corresponding to halos mass of  $M_h \simeq 10^{12} M_\odot$ . From the figure, one expects that the correlation ratio or the bias ratio is  $\sim 1.75/3 = 0.58$ , and thus the DLA-LBG cross-correlation would have a correlation length  $r_{0,\text{dg}} = 4 \times (0.58)^{1/1.6} \simeq 2.85$  Mpc.

Potential systematics include the few massive halos ( $M_h > 10^{13} M_\odot$ ) that are missed due to the limited volume ( $22.222h^{-3} \text{ Mpc}^3$ ) of our simulation. However, since DLAs are cross-section selected these few massive halos are too scarce to change the mean  $\langle \log M_{\text{DLA}} \rangle$  of the DLA mass probability distribution (Fig. 3, right). Naturally, if there were such massive halos in our simulations, the amplitude of the cross-correlation would be different.

We address this point in a general way in the next section (4.3) and show that the derived  $\langle \log M_{\text{DLA}} \rangle$  is independent of the galaxy sample used.

Our treatment of feedback is limited to energy injection of supernovae, and thus does not treat phenomenon like winds. Nagamine et al. (2004) included winds in similar simulations and showed that the DLA abundance decreases with increasing wind strength, but the mean DLA halo mass will be shifted towards higher mass in the presence of winds. Nagamine et al. (2004) also showed that the DLA abundance (extrapolated to  $10^8 M_{\odot}$ , i.e. below the resolution limit, using the PS formalism) also decreases with increasing resolution, but again, the mean DLA halo mass will be shifted towards higher mass in higher resolution runs.

Given that (i) a fraction of DLAs are expected to arise in halos below our mass resolution of  $M_h \gtrsim 5.2 \times 10^{10} M_{\odot}$  and (ii) our total DLA abundance extrapolated to  $10^8 M_{\odot}$ , as in Nagamine et al. (2004), over-predicts the observed DLA abundance, Eq. 16 is an upper limit. Furthermore, given the results of Nagamine et al. (2004) showed that both winds and better resolution increase the mean DLA halo mass, we conclude that a simulation with SNe winds and with a better mass resolution would lower our mean DLA mass. Reading from Fig. 5 in Nagamine et al. (2004), we estimate that  $\langle \log M_{\text{DLA}} \rangle$  is  $\sim 10.6$  for their high resolution run with strong winds, or a factor of  $\sim 5$  smaller than here. Thus, Eq. 16 is an upper limit.

#### 4.3. The cross-correlation is independent of the galaxy sample

From Eqs. 2 and 3, we expect the relative amplitude  $a$  to vary as a function of the halo mass of the galaxy sample  $M_h$ . We therefore performed the same cross-correlation calculations for each of the six sub-samples presented in section 2 (see also Fig. 1), and ask the question: is the inferred  $\langle M_{\text{DLA}} \rangle$  the same in each case? We restricted ourselves to scales  $r_{\theta} > 1$  Mpc (from the discussion in § 4.1).

Fig. 7 shows the measured amplitude or bias ratio  $a$  for each of the sub-samples. The amplitude ratio  $a$  for the sub-samples (full sample) is represented by the open squares (filled circle) with solid error bars. The filled circle with dotted error bars represents the full sample shown in Fig. 4 from which we inferred  $a = 0.73 \pm 0.08$  and  $\log M_{\text{DLA}} = 11.13^{+0.13}_{-0.13}$ . As expected,  $a$  increases with larger sub-samples, or with decreasing galaxy halo mass  $M_h$ .

For the method to be self-consistent, the derived DLA halo mass  $\langle \log M_{\text{DLA}} \rangle$  should be the same for all the sub-samples. Given Eq. 15,  $\langle \log M_{\text{DLA}} \rangle = 11.13$  determined in the previous section, and a mean galaxy halo mass  $\langle \log M_h \rangle$ , we can predict the bias ratio  $a$ . The solid line in Fig. 7 represents this prediction. One sees that the measured bias ratio for the sub-samples (open squares) follow the expected bias ratio (solid line). For comparison, the dashed lines show the expected amplitude ratios  $a$  if DLAs were in halos of mean mass  $\langle \log M_{\text{DLA}} \rangle = 10.5, 11.5$  and  $12$  (from bottom to top) instead of the inferred  $\langle \log M_{\text{DLA}} \rangle = 11.13$ .

We conclude that the method is self-consistent: the mass  $\langle \log M_{\text{DLA}} \rangle$  is independent of the galaxy sample used and that the clustering statistics of DLAs with

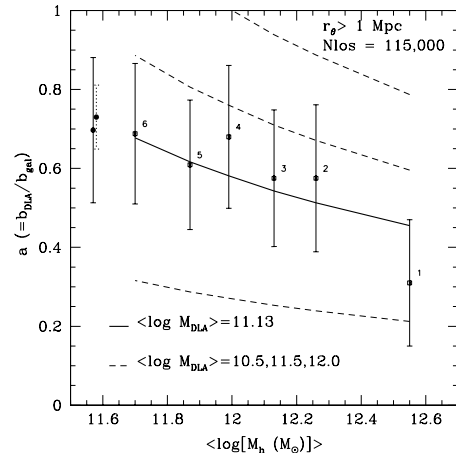


FIG. 7.— The DLA-LBG cross- to auto-correlation amplitude ratio,  $a$  (defined in Eq. 11), as a function of the mean galaxy halo mass  $\langle \log M_h \rangle$  using the full DLA sample of  $\sim 115,000$  lines of sight. Note that  $a$  is also  $b(M_{\text{DLA}})/b(M_{\text{gal}})$ . Each of the sub-samples (see Fig. 1) is labeled 1 to 6. The filled circle with solid error bars shows  $a$  for the entire sample of 651 galaxies and 115,000 DLAs. The filled circle with dotted error bars (offsetted along the  $x$ -axis) shows  $a$  fitted over all scales for the entire sample of 651 galaxies and 115,000 DLAs from which we infer a mean DLA halo mass of  $\langle \log M_{\text{DLA}} \rangle = 11.13^{+0.13}_{-0.13}$ . For this mass, the expected amplitude ratio  $a$  for the six sub-samples is shown by the solid line. The expected  $a$  follows closely the values found for the sub-samples, showing that one will get the same DLA halo mass for any galaxy sub-sample, as long as the same galaxies are used for  $w_{\text{gg}}$  and  $w_{\text{dg}}$ , i.e. that the method is self-consistent. For comparison, the dashed lines show the expected  $a$  if the mean DLA halo mass  $\langle \log M_{\text{DLA}} \rangle$  were 10.5, 11.5 and 12 (from bottom to top) instead.

galaxies can be used to infer their mass, and that large observational samples will shed new light on their nature. A direct observational measure of the relative amplitude  $a$  (Cooke 2005, private communication), will show whether or not DLAs are massive disks ( $10^{12} M_{\odot}$ ) as proposed by Wolfe et al. (1986, 1995); Prochaska & Wolfe (1997b).

#### 4.4. Comparison to observations

In this section, we first briefly review past and recent observations of clustering between galaxies and DLAs (§ 4.4.1). We then (§ 4.4.2) focus on comparing the simulated DLA-LBG cross-correlation to the observational results of BL04, in a meaningful way, i.e. with a sample of similar size.

##### 4.4.1. Observations of the DLA-galaxy cross-correlation at $z = 3$

Early attempts to detect diffuse Ly $\alpha$  emission from DLAs at  $z > 2$  using deep narrow band imaging (Lowenthal et al. 1995) did not reveal the absorber but unveiled a few companion Ly $\alpha$  emitters (Lowenthal et al. 1991), hinting at the clustering of galaxies around DLAs. This prompted Wolfe (1993) to calculate the two-point correlation function at  $\langle z \rangle = 2.6$  and to conclude that, indeed, Ly $\alpha$  emitters are clustered near DLAs at the 99% or greater confidence level. Some recent Ly $\alpha$  searches have succeeded in unveiling the absorber (e.g. Fynbo et al. 1999).

Francis & Hewett (1993) reported the discovery of super-clustering of sub-DLAs at  $z \sim 2.4$  and  $z \sim 2.9$ :

a total of four HI clouds are seen in a QSO pair separated by  $8'$ , each being at the same velocity. Recent results from narrow-band imaging of the Francis & Hewett field shows that spectroscopically confirmed Ly $\alpha$  emitters are clustered at the redshift of the strongest HI cloud at  $z = 2.9$  ( $\log N_{\text{HI}} = 20.9$ ) towards Q2138-4427 (Fynbo et al. 2003). Roche et al. (2000) identified eight Lyman-alpha emitting galaxies near the DLA at  $z = 2.3$  towards PHL 957 in addition to the previously discovered Coup Fourré galaxy (Lowenthal et al. 1991), implying the presence of a group, filament, or proto-cluster associated with the DLA. Other evidence of clustering include the work of Ellison et al. (2001), who found that the DLA at  $z_{\text{abs}} = 3.37$  towards Q0201+1120 is part of a concentration of matter that includes at least four galaxies (including the DLA) over transverse scales greater than  $5h^{-1}$  Mpc; and D'Odorico et al. (2002) who showed that out of ten DLAs in QSO pairs, five are matching systems within  $1000\text{km s}^{-1}$ . They concluded that this result indicates a highly significant over-density of strong absorption systems over separation lengths from  $\sim 1$  to  $8 h^{-1}$  Mpc.

Gawiser et al. (2001) studied the cross-correlation of LBGs around one  $z \sim 4$  DLA. Probably due to the high redshift of their DLA, Gawiser et al. (2001) found that  $w_{\text{dg}}(r_\theta)$  is consistent with 0, i.e. they found that the distribution of the eight galaxies in that field (with spectroscopic redshifts) is indistinguishable from a random distribution. Their data did not allow them to put limits on the amplitude of  $w_{\text{dg}}$ .

ASSP03 found a lack of galaxies near four DLAs and concluded that the DLA-LBG cross-correlation is significantly weaker than the LBG-LBG auto-correlation at the 90% confidence level. They found two LBGs within  $r_\theta = 5.7h^{-1}$  Mpc and within  $W_z < 0.0125$  ( $\sim 8h^{-1}$  Mpc) whereas  $\sim 6$  were expected if the cross-correlation has the same amplitude as the galaxy auto-correlation. Because of the field of view available, both of these studies were not sensitive to scales larger than  $r_\theta \sim 5h^{-1}$  Mpc, which is important since the relevant scales to measure the DLA-LBG cross-correlation extend up to  $r_\theta \sim 10h^{-1}$  Mpc.

However, the results of ASSP03 can be used to put an upper limit on  $w_{\text{dg}}/w_{\text{gg}}$  through the following steps: First, note that the two galaxies (in  $N_{\text{DLAs}} = 4$  fields) observed by Adelberger et al. give

$$\langle N_{\text{obs}} \rangle = \langle N_{\text{exp}} \rangle (1 + \bar{\xi}_{\text{dg}}) = 0.5(2/4), \quad (17)$$

and the six galaxies expected if  $\bar{\xi}_{\text{dg}} = \bar{\xi}_{\text{gg}}$  give  $\langle N_{\text{exp}} \rangle (1 + \bar{\xi}_{\text{gg}}) = 6/4$ , where  $\bar{\xi}$  is the volume average of the correlation function. Second, we find  $\bar{\xi}_{\text{gg}} \simeq 1.1$  for the LBG auto-correlation published in ASSP03 and assuming that the cell used is a sphere centered on the DLAs with an effective radius of  $\sim 6h^{-1}$  Mpc, i.e. with the same volume as the cylindrical cell used by ASSP03. Thus, the expected number of galaxies per DLA field is  $\langle N_{\text{exp}} \rangle = 0.68$  if  $\bar{\xi}_{\text{dg}} = 0$ , and the total number of galaxies is  $4 \times \langle N_{\text{exp}} \rangle = 2.85$ . Clearly their measurement of 2 galaxies is consistent with no cross-correlation. From Eq. 17, we can infer that  $1 + \xi_{\text{dg}} = 0.7$  using  $\langle N_{\text{exp}} \rangle = 0.68$ . Third, the uncertainty to  $\bar{\xi}_{\text{dg}}$ ,  $\sigma_\xi$ , can be estimated using the results shown in Appendix B. The variance  $V(\xi) = \sigma_\xi^2$  is made of two

terms, the shot noise variance  $V_{sn}$  and the clustering variance  $V_{cl}$ . The shot noise variance to  $\langle N_{\text{obs}} \rangle$  is  $V(\langle N_{\text{obs}} \rangle)_{sn} = \langle N_{\text{obs}} \rangle = 0.5$  (Eq. B2). The 2pt clustering variance (Eq. B6) is simply  $\bar{N}^2(A\bar{\xi}_{\text{gg}}) = \bar{N}^2(2.50)$  where  $A = J_2/K_1 = 2.28$ . The 3pt clustering variance (Eq. B7) is  $\sim 0$  since  $\xi_{\text{dg}} \simeq 0$ . Finally, from Eq. B11,  $\sigma_\xi = \frac{1}{\sqrt{4}} \frac{1}{\sqrt{2.85/4}} \sqrt{(1 + \frac{2.5}{0.7})} \sqrt{0.7} \simeq 1.06$ , and a 1-sigma (2-sigma) upper limit to  $\bar{\xi}_{\text{dg}}$  is  $\bar{\xi}_{\text{dg}} + 1(2)\sigma_\xi = -0.3 + 1.06(2.12) = 0.76(1.82)$ , respectively. Since  $\bar{\xi}_{\text{gg}} = 1.1$ , the 1-sigma (2-sigma) upper limit to the amplitude ratio is  $\bar{\xi}_{\text{dg}}/\bar{\xi}_{\text{gg}} \lesssim 0.70(1.65)$ , respectively. This rough calculation is quite consistent with Adelberger's results where it was found that  $\xi_{\text{dg}} < \xi_{\text{gg}}$  at the 90% confidence level using Monte Carlo simulations.

Given that the relevant scales to measure the DLA-LBG cross-correlation extend up to  $r_\theta \sim 10h^{-1}$  Mpc, Bouché & Lowenthal (2003) were able to first detect and measure a DLA-LBG cross-correlation signal (BL04) using the wide-field ( $0.35 \text{ deg}^2$  or  $\sim 40^2 \text{ Mpc}^2$  co-moving at redshift  $z = 3$ ) imager MOSAIC on the Kitt Peak 4m telescope. Bouché & Lowenthal (2003) showed that there was an over-density of LBGs by a factor of  $\sim 3$  (with 95% confidence) around the  $z_{\text{abs}} \simeq 3$  DLA towards the quasar APM 08279+5255 ( $z_{\text{em}} = 3.91$ ) on scales  $2.5 < r_\theta < 5h^{-1}$  Mpc. Extending the results of Bouché & Lowenthal (2003) to three  $z \sim 3$  DLA fields, BL04 probed the DLA-LBG cross-correlation on scales  $r_\theta \sim 5\text{--}20h^{-1}$  Mpc and found (i) a DLA-LBG cross-correlation with a relative amplitude  $w_{\text{dg}} = (1.62 \pm 1.32) \times w_{\text{gg}}$  that is greater than zero at the  $\sim 95\%$  confidence level, and (ii)  $w_{\text{dg}}$  is most significant on scales  $5\text{--}10h^{-1}$  Mpc. In other words, DLAs are clustered with LBGs, but unfortunately the sample size did not allow BL04 to test whether  $a$  is greater or smaller than 1. Soon, the ongoing survey of  $z \simeq 3$  DLAs of Cooke et al. (2004) will double the sample of BL04.

In a slightly different context Bouché et al. (2004) applied successfully the technique presented here to 212  $z \simeq 0.5$  Mg II systems (of which 50% are expected to be DLAs) using luminous red galaxies (LRGs) in the Sloan Digital Sky Survey Data Release 1. They found that the Mg II-LRG cross-correlation has an amplitude  $0.67 \pm 0.09$  times that of the LRG-LRG auto-correlation, over co-moving scales up to  $r_\theta = 13 h^{-1}$  Mpc. Since LRGs have halo-masses greater than  $3.5 \times 10^{12} M_\odot$  for  $M_R \lesssim -21$ , this relative amplitude implies that the Mg II host-galaxies have halo-masses greater than  $\sim 2\text{--}8 \times 10^{11} M_\odot$ . These results show how powerful the cross-correlation technique is.

To summarize the current observational situation on the  $z = 3$  DLA-LBG cross-correlation, ASSP03 finds that the amplitude ratio is  $\bar{\xi}_{\text{dg}}/\bar{\xi}_{\text{gg}} \lesssim 0.70$ , and BL04 finds that  $\bar{\xi}_{\text{dg}}/\bar{\xi}_{\text{gg}} \gtrsim 0.30$ , both at the 1-sigma level. Using Monte-Carlo simulations, ASSP03 finds  $\bar{\xi}_{\text{dg}}/\bar{\xi}_{\text{gg}} < 1.0$ , at the 90% confidence level, and BL04 finds  $\bar{\xi}_{\text{dg}}/\bar{\xi}_{\text{gg}} > 0.0$ , at the 95% confidence level. The DLA halo mass range allowed by these observations is still large: it covers  $\log M_{\text{DLA}} \sim 10\text{--}12 M_\odot$ <sup>2</sup>.

<sup>2</sup> After this paper submission, we learned that Monaco et

#### 4.4.2. Simulation of present observations: $w_{\text{dg}}$ with small samples

There are many significant differences between the observational sample of BL04 and the present simulated one. First, the shape of the volume is very different: The survey volume of BL04 is  $40 \times 40 \times 100h^{-3} \text{ Mpc}^3$  (co-moving) deep, while these simulations are  $22.222h^{-1} \text{ Mpc}$  (co-moving) on a side. Given that the survey of BL04 contains about 80–120 LBGs per field, their observed LBG number density corresponds to about seven galaxies per  $22.222h^{-3} \text{ Mpc}^3$ . Naturally, seven galaxies are not a fair sample of the LBG luminosity function. This is an inherent problem due to the size of the simulation, rendering the comparison between the observed and the simulated cross-correlation difficult. Second, as mentioned in § 2, the simulated LBGs are selected according to their SFR, while the observed LBGs are color selected. Third, the same galaxies are used for every simulated line of sight. These differences limit our ability to perform a direct comparison to observations.

With these caveats in mind, we can repeat our analysis of section 4.2 in the limit of small  $N_{\text{DLA}}$  and with similar galaxy number densities. Because, to first order,  $\sigma_w \propto (\sqrt{N_{\text{DLA}}N_{\text{gal}}})^{-1}$  (Eq. B11), a sample made of 10 DLAs and 25 galaxies per  $22.222^2h^{-2} \text{ Mpc}^2$  ‘field’ is expected to have similar errors to the sample of BL04 made of 3 DLAs and 100 galaxies per  $40^2h^{-2} \text{ Mpc}^2$  field. As for the full sample, we restricted ourselves to scales  $r_\theta > 1h^{-1} \text{ Mpc}$ , which also corresponds to the most relevant scales 5– $10h^{-1} \text{ Mpc}$  of the observations of BL04. We find that the relative amplitude of the cross-correlation with 10 lines of sight and 25 galaxies is  $a = 0.77 \pm 0.53$ , whereas BL04 found  $a = 1.62 \pm 1.32$ , i.e. both with the same signal-to-noise ratio.

This confirms the results of BL04. But, more importantly, one can now use the result for this sample made of 10 DLAs and 25 galaxies (with a surface density  $\Sigma_g \sim 0.05 \text{ Mpc}^{-2}$ ) as a benchmark to predict the SNR for the larger samples of future observations, given that the SNR will be proportional to  $\sqrt{N_{\text{gal}}N_{\text{DLA}}}$  (Eq. B11).

## 5. CONCLUSIONS

Motivated by the fact that (i) the amplitude of the cross-correlation is a measurement of the mean DLA halo mass and (ii) observational constraints (Gawiser et al. 2001; ASSP03; BL04 and Cooke et al. 2005, private communication) are reaching a turning point and the DLA halo masses are starting to be constrained, we tested the cross-correlation technique using Tree-SPH cosmological simulations. The method uses the ratio of the cross-correlation between DLAs and high-redshift galaxies to the auto-correlation of the galaxies themselves, which is (in linear theory) the ratio of their bias factor, to infer the mean DLA halo mass.

In a Tree-SPH simulation (Katz et al. 1996a) parallelized by Davé et al. (1997) with  $128^3$  particles in a volume  $22.222^3 h^{-3} \text{ Mpc}^3$  (co-moving), we find that:

1. scales  $r_\theta > 1\text{--}15 h^{-1} \text{ Mpc}$  are the most relevant

al.(2005, private communication) constrained the halo mass of a few individual DLAs to be around  $5 \times 10^{11} M_\odot$ . Their mass estimates come from the emission-absorption redshift difference as a proxy for a rotation curve.

scales to constrain the mean DLA halo mass using the projected cross-correlation  $w_{\text{dg}}(r_\theta)$ ;

2. the DLA-galaxy cross-correlation has an amplitude  $w_{\text{dg}} = (0.73 \pm 0.08) \times w_{\text{gg}}$ , close to the predicted value of 0.771 using the Mo & White (2002) formalism;
3. the inferred mean DLA halo mass is

$$\langle \log M_{\text{DLA}}(M_\odot) \rangle = 11.13_{-0.13}^{+0.13}, \quad (18)$$

in excellent agreement with the true values of the simulations, i.e.  $\log M_{\text{DLA}} = 11.16$ . We can thus conclude that the cross-correlation technique yields the first moment of the DLA halo mass distribution, even when DLAs and galaxies occupy a broad range of halos with massive halos containing multiple galaxies with DLAs;

4. if we consider subsets of the simulated galaxies with higher star-formation rates (representing LBGs), the cross-correlation technique is self-consistent, i.e. the DLA mass inferred from the ratio of the correlation functions does not depend on the galaxy sample used. This demonstrates the reliability of the method;
5. for real  $z = 3$  LBGs with a correlation length  $r_{0,\text{gg}} \simeq 4h^{-1} \text{ Mpc}$  (Adelberger et al. 2003, 2004), our results imply that the DLA-LBG cross-correlation is expected to have a correlation length  $r_{0,\text{dg}} \simeq 2.85h^{-1} \text{ Mpc}$ ;
6. with small samples (with 10 lines of sight and 25 galaxies) matching the statistics of BL04; the relative amplitude of the cross-correlation is  $a = 0.77 \pm 0.53$ , i.e. with a signal-to-noise ratio (SNR  $\sim 1.3\text{--}1.5$ ) comparable to BL04, where they found  $a = 1.62 \pm 1.32$ .

In short, the cross-correlation between galaxies and DLAs is a powerful and self-consistent technique to constrain the mean mass of DLAs, and we have demonstrated its reliability. Given the resolution limits of the simulation used here ( $M_h \gtrsim 5.2 \times 10^{10} M_\odot$ ), our values are strictly upper limits. These simulation results suggest that DLAs are expected to be less massive than  $z = 3$  LBGs by a factor of at least  $\sim 4.8$ .

Recently, Cassata et al. (2004) studied the morphology of K-selected galaxies at redshifts up to  $z = 2.5$  and found that the late type fraction drops beyond  $z > 2$ . Erb et al. (2004) show that the kinematics of 13  $z > 2$  morphologically elongated galaxies are not consistent with those of an inclined disk. Furthermore, the virial mass of these galaxies is in the range of a few  $10^{10} M_\odot$  up to  $5 \times 10^{10} M_\odot$ . These results and the ones presented here disfavor the presence of large massive  $10^{12} M_\odot$  disk at  $z > 2$  and therefore the massive disk hypothesis for DLAs.

Current observational samples are just starting to put constraints on  $w_{\text{dg}}/w_{\text{gg}}$  for  $z = 3$  DLAs. BL04 found  $w_{\text{dg}}/w_{\text{gg}} > 0$  at the 95% confidence level, and ASSP03 found  $< 1$  at the 90% confidence level, allowing the mass range  $\langle \log M_{\text{DLA}} \rangle \sim 10\text{--}12 M_\odot$ . Future observations will be able to distinguish between models in which DLAs reside in low mass halos from those in which DLAs are

massive disks occupying only high mass halos thanks to planned wide-field imagers.

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#### APPENDIX

##### CROSS-CORRELATION AND AUTO-CORRELATION FUNCTIONS

For a given absorber with galaxies distributed with  $\frac{dN}{dz}$ , one may think that the projected auto-correlation  $w_{p,gg}(r_\theta)$  is proportional to  $\int \left(\frac{dN}{dz}\right)^2 dz$  while the cross-correlation  $w_{dg}(r_\theta)$  is proportional to  $\int \left(\frac{dN}{dz}\right) dz$ . Thus, at first glance, their ratio is not very useful. Below we show the situation to be not so trivial. In this appendix, we merely connect results previously published to show that the amplitude of both  $w_{gg}(r_\theta)$  and  $w_{dg}(r_\theta)$  are proportional to  $1/W_z$ , where  $W_z$  is the redshift width of the galaxy distribution (determined by the box size or by observational selections such as photometric techniques).

First, we list some definitions and three results (Eq. A1–A3) that will be useful later. For a 3D correlation function  $\xi(r) = (r/r_0)^{-\gamma}$ , the projected correlation function  $w_p(r_p)$  is (Davis & Peebles 1983):

$$\begin{aligned} w_p(r_p) &= \int_{-\infty}^{\infty} dy \xi(r_p, y) = \int_{-\infty}^{\infty} dy \xi(\sqrt{r_p^2 + y^2}) \\ &= (r_p)^{1-\gamma} r_0^\gamma H_\gamma \end{aligned} \quad (\text{A1})$$

where  $\xi(r_p, y)$  is the 3D correlation function decomposed along the line of sight  $y$  and on the plane of the sky  $r_p$ , i.e.  $r^2 = y^2 + r_p^2$ .  $H_\gamma$  is in fact the Beta function  $B(a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt$  evaluated with  $a = 1/2$  and  $b = (\gamma - 1)/2$ , i.e.  $H_\gamma = B(\frac{1}{2}, \frac{\gamma-1}{2}) = \frac{\Gamma(\frac{1}{2})\Gamma(\frac{\gamma-1}{2})}{\Gamma(\frac{\gamma}{2})}$ .

In appendix C of ASSP03, one finds the expected number of neighbors between  $r_\theta - dr/2$  and  $r_\theta + dr/2$  within a redshift distance  $|\Delta_z| < r_z$ :

$$\begin{aligned} w_p(r_\theta; < r_z) &= \frac{1}{r_z} \int_0^{r_z} dl \xi(\sqrt{r_\theta^2 + l^2}) \\ &= \frac{1}{2r_z} (r_\theta)^{1-\gamma} r_0^\gamma H_\gamma I_x\left(\frac{1}{2}, \frac{\gamma-1}{2}\right) \end{aligned} \quad (\text{A2})$$

where  $x = r_z^2/(r_z^2 + r_\theta^2)$  and  $I_x$  is the incomplete Beta function  $B_x(a, b) = \int_0^x t^{a-1} (1-t)^{b-1} dt$  normalized by  $B(a, b)$ :  $I_x(a, b) \equiv B_x(a, b)/B(a, b)$ .

Many papers (Phillipps et al. 1978; Peebles 1993; Budavári et al. 2003) have shown that the angular correlation function is:

$$w(\theta) = (\theta)^{1-\gamma} r_0^\gamma H_\gamma \times \int_0^\infty dz \left(\frac{dN}{dz}\right)^2 g(z)^{-1} f(z)^{1-\gamma} \quad (\text{A3})$$

where  $g(z) = dr/dz = c/H(z)$  and  $f(z) = D_c(z)$  is the co-moving line-of-sight distance to redshift  $z$ , i.e.  $D_c(z) = \int_0^z dt \frac{c}{H(t)}$ .

Equation A3 can be derived from the definitions of the angular and 3D correlation functions,  $w(\theta)$  and  $\xi(r)$  (e.g. Phillipps et al. 1978). We reproduce the derivation here and extend it to projected auto- and cross-correlation functions. The probabilities of finding a galaxy in a volume  $dV_1$  and another in a volume  $dV_2$  at a distance  $r = |\mathbf{r}_2 - \mathbf{r}_1|$ , along two lines of sight separated by  $\theta$  are:

$$dP(\theta) = \mathcal{N}^2 d\Omega_1 d\Omega_2 [1 + w(\theta)] \quad (\text{A4})$$

$$dP(r) = n(z)^2 dV_1 dV_2 [1 + \xi(r)] \quad (\text{A5})$$

where  $\mathcal{N}$  is the number of galaxies per solid angle, i.e.  $dN/d\Omega$ , and  $n(z)$  is the number density of galaxies, which can be a function of redshift. Given that  $\mathcal{N} = \frac{1}{d\Omega} \int n(z) dV(z)$  and that  $dV = f^2(z)g(z) d\Omega dz$ ,  $\mathcal{N} \equiv \int dz \frac{dN}{dz} = \int dz n(z) f^2(z) g(z)$ .

To relate  $w(\theta)$  and  $\xi(r)$ , one needs to integrate Eq. A5 over all possible lines-of-sight separated by  $\theta$  (i.e. along  $z_1$  and  $z_2$ ) and equate it with Eq. A4:

$$\begin{aligned} \mathcal{N}^2 [1 + w(\theta)] &= \int_0^\infty dz_1 f(z_1)^2 g(z_1) n(z_1) \cdot \\ &\quad \int_0^\infty dz_2 f(z_2)^2 g(z_2) n(z_2) [1 + \xi(r_{12})] . \end{aligned} \quad (\text{A6})$$

In the regime of small angles, the distance  $r_{12}$  (in co-moving Mpc) can be approximated by:

$$\begin{aligned}
r_{12}^2 &= r_1^2 + r_2^2 - 2r_1r_2 \cos \theta \\
&\simeq (r_1 - r_2)^2 + r^2\theta^2 \quad \text{with } r = \frac{r_1 + r_2}{2} \\
&\simeq (g(z)(z_1 - z_2))^2 + f(z)^2\theta^2 \quad \text{with } z = \frac{z_1 + z_2}{2} \\
&\simeq g(z)^2y^2 + f(z)^2\theta^2 \quad \text{with } y = z_1 - z_2.
\end{aligned} \tag{A7}$$

Changing variables in Eq. A6 from  $(z_1, z_2)$  to  $(z, y)$ , assuming the the major contribution is from  $z_1 \simeq z_2$  and using Eq. A7, the angular correlation function is

$$w(\theta) = \frac{\int_0^\infty dz f(z)^4 g(z)^2 n(z)^2 \int_{-\infty}^\infty dy \xi(\sqrt{f(z)^2\theta^2 + g(z)^2y^2})}{[\int_0^\infty dz f^2(z)g(z)n(z)]^2}. \tag{A8}$$

Changing variables to  $l = g(z)y$ , using Eq. A1 and using a normalized redshift distribution, i.e.  $\int dz \frac{dN}{dz} = 1$ , Eq. A8 becomes

$$w(\theta) = \int_0^\infty dz \left(\frac{dN}{dz}\right)^2 g(z)^{-1} \times (f(z)\theta)^{1-\gamma} r_0^\gamma H_\gamma \tag{A9}$$

which leads to Eq. A3 (Eq. 9 in Budavári et al. 2003) and is one version of Limber's equations.

In this paper, we measured the projected auto-correlation of the LRGs,  $w_{\text{gg}}(r_\theta)$ , where  $r_\theta = f(z)\theta$ <sup>3</sup>. Following the same steps as above with  $r_\theta$  instead of  $\theta$ , and  $dV = (dr_\theta)^2 g(z) dz$ ,  $w_{\text{gg}}(r_\theta)$  is:

$$w_{\text{gg}}(r_\theta) = r_\theta^{1-\gamma} r_{0,\text{gg}}^\gamma H_\gamma \int_0^\infty dz \left(\frac{dN}{dz}\right)^2 g(z)^{-1} \tag{A10}$$

In the case of the projected cross-correlation,  $w_{\text{dg}}(r_\theta)$ , the conditional probability of finding a galaxy in the volume  $dV_2$  given that there is an absorber at a known position  $\mathbf{r}_1$  is, by definition (e.g. Eisenstein 2003),

$$dP(2|1)(r_\theta) = \mathcal{N}_g d\Omega_2 [1 + w_{\text{dg}}(r_\theta)] \tag{A11}$$

$$dP(2|1)(r) = n_g(z) dV_2 [1 + \xi_{\text{dg}}(r)]. \tag{A12}$$

Using the same approximations (Eq. A7) and one integral along the line of sight  $z_2$  (keeping the absorber at  $z_1$ ), one finds that the projected cross-correlation is:

$$\begin{aligned}
w_{\text{dg}}(r_\theta) &= \int_0^\infty dz_2 f(z_2)^2 g(z_2) n(z_2) \xi_{\text{dg}}(r_{12}) \\
&= \int_0^\infty dz \left(\frac{dN}{dz}\right) \xi_{\text{dg}}(\sqrt{r_\theta^2 + g(z)^2(z_1 - z_2)^2}) \\
&= \int_0^\infty dy g(z) \left(\frac{dN}{dz}\right) g(z)^{-1} \xi_{\text{dg}}(\sqrt{r_\theta^2 + g(z)^2y^2}) \\
&= \int_0^\infty dl \frac{dN}{dl} \xi_{\text{dg}}(\sqrt{r_\theta^2 + l^2})
\end{aligned} \tag{A13}$$

$$\simeq \frac{1}{W_z} \times (r_\theta)^{1-\gamma} r_{0,\text{dg}}^\gamma H_\gamma, \tag{A14}$$

where we approximated  $\frac{dN}{dz}$  with a normalized top-hat of width  $W_z = 2r_z$ , used Eq. A2, and the fact that  $I_x \simeq 1$  since  $x = r_z^2/(r_z^2 + r_\theta^2) \simeq 1$  for a redshift width  $W_z$  of  $20h^{-1}$  Mpc and  $r_\theta = 1h^{-1}$  Mpc<sup>4</sup>. Thus, as one would have expected, the cross-correlation is inversely proportional to the width of the galaxy distribution.

Naturally, in Eq. A11 and A12, the redshift of galaxy 1 (i.e. the absorber) is assumed to be known with good precision. If the absorber population had poorly known redshifts, one would need to add an integral to Eq. A13, washing out the cross-correlation signal further.

For the projected auto-correlation (Eq. A10), if one approximates  $\frac{dN}{dz}$  by a top-hat function of width  $W_z$ , then

$$\begin{aligned}
w_{\text{gg}}(r_\theta) &= (r_\theta)^{1-\gamma} r_{0,\text{gg}}^\gamma H_\gamma \times \int_0^\infty dz g(z) \left(\frac{dN}{dz}\right)^2 g(z)^{-2} \\
&= (r_\theta)^{1-\gamma} r_{0,\text{gg}}^\gamma H_\gamma \times \int_0^\infty dl \left(\frac{dN}{dl}\right)^2 \\
&\simeq \left(\frac{1}{W_z}\right)^2 W_z \times (r_\theta)^{1-\gamma} r_{0,\text{gg}}^\gamma H_\gamma,
\end{aligned} \tag{A15}$$

<sup>3</sup> In general this should be  $D_A(1+z)\theta$  where  $D_A$  is the angular distance. For a flat universe,  $D_A(1+z) = D_M = D_C = f(z)$  where  $D_M$  is the co-moving transverse distance, using D. Hogg's notations (Hogg 1999).

<sup>4</sup>  $I_x = 0.94$  for  $W_z = 22.222 h^{-1}$  Mpc and  $r_\theta = 1 h^{-1}$  Mpc.

which shows that the auto-correlation depends on the redshift distribution of the galaxies in the same way as the cross-correlation, i.e.  $\propto 1/W_z$ . The reason for this is that the redshift distribution  $\frac{dN}{dz}$  has a very different role with respect to the correlation functions, which can be seen by comparing Eqs. A10 and A13. It is this very different role that leads to the same  $1/W_z$  dependence.

In the case of a Gaussian redshift distribution  $\frac{dN}{dz}$ , the ratio of cross- and auto-correlations may not be exactly  $r_{0,dg}/r_{0,gg}$  if the approximation leading to A14 breaks down. Using mock galaxy samples (from the GIF2 collaboration, Gao et al. 2004) selected in a redshift slice of width,  $W_z$ , equal to their artificial Gaussian redshift errors  $\sigma_z$ , we find that the cross-correlation is overestimated by  $25 \pm 10$  per cent. This correction factor is independent of the width of the redshift distribution as long as  $\sigma_z \simeq W_z$  or as long as it is Gaussian. This implies that the ratio of the correlation functions ( $w_{dg}/w_{gg}$ ) will be insensitive to errors in photometric redshifts.

## THE ERRORS TO CORRELATION FUNCTIONS

In this appendix, we list the basic properties of the errors to correlation functions.

From the definition of the cross-correlation  $\xi_{dg}$  shown in Eq. 1, the expected number of galaxies in a cell of volume  $\Delta V$  centered on a DLA is given by the counts of neighbor galaxies:

$$\langle N_{\text{obs}} \rangle = \bar{N}(1 + \bar{\xi}_{dg}(r)), \quad (\text{B1})$$

where  $\bar{N} = n_u \Delta V$ .

Various text books (e.g. Peebles 1980, section 36) have shown that the variance of the number of neighbor galaxies  $N_{\text{obs}}$  near a DLA is the sum of the shot noise

$$V(N_{\text{obs}})_{sn} = N_{\text{obs}} \quad (\text{B2})$$

and the clustering variance  $V(N_{\text{obs}})_{cl}$ . The clustering variance is itself the sum of the two terms,  $V_{2pt}$  and  $V_{3pt}$ :

$$V(N_{\text{obs}})_{2pt} = \bar{N}^2 \frac{1}{(\Delta V)^2} \int_{\Delta V} \int_{\Delta V} \xi_{gg}(|r_2 - r_1|) \quad (\text{B3})$$

$$V(N_{\text{obs}})_{3pt} = \bar{N}^2 \frac{1}{(\Delta V)^2} \int_{\Delta V} \int_{\Delta V} [\zeta_{dgg}(r_1, r_2) - \xi_{dg}(r_1)\xi_{dg}(r_2)] dV_1 dV_2, \quad (\text{B4})$$

where  $\bar{N} = n_u \Delta V = N_{\text{exp}}$ ,  $\xi_{gg}$  is the galaxy-galaxy auto-correlation, and  $\zeta_{dgg}$  is the three-point correlation function.  $\zeta$  can be written as a product of two-point correlations (Peebles 1980):

$$\zeta_{dgg}(\mathbf{r}_1, \mathbf{r}_2) = Q [\xi_{dg}(r_1)\xi_{dg}(r_2) + \xi_{dg}(r_1)\xi_{gg}(|\mathbf{r}_1 - \mathbf{r}_2|) + \xi_{dg}(r_2)\xi_{gg}(|\mathbf{r}_1 - \mathbf{r}_2|)], \quad (\text{B5})$$

where  $r_1 = |\mathbf{r}_o + \mathbf{r}|$ ,  $r_2 = |\mathbf{r}_o + \mathbf{r}_2|$  and  $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$ .

For a spherical volume  $\Delta V$ , the integrals B3–B4 can be written as (using the results in Peebles 1980, section 59):

$$V(N_{\text{obs}})_{2pt} = \bar{N}^2 \left(\frac{r_{gg,0}}{r}\right)^\gamma J_2 = \bar{N}^2 \left(\frac{J_2}{K_1}\right) \bar{\xi}_{gg} \quad (\text{B6})$$

$$\begin{aligned} V(N_{\text{obs}})_{3pt} &= \bar{N}^2 \left[ Q \left( K_1^2 \left(\frac{r_{dg,0}}{r}\right)^{2\gamma} + 2K_2 \left(\frac{r_{dg,0}}{r}\right)^\gamma \left(\frac{r_{gg,0}}{r}\right)^\gamma \right) - K_1^2 \left(\frac{r_{dg,0}}{r}\right)^{2\gamma} \right], \\ &= \bar{N}^2 \left[ Q \left( \bar{\xi}_{dg}^2 + 2\frac{K_2}{K_1} \bar{\xi}_{gg} \bar{\xi}_{dg} \right) - \bar{\xi}_{dg}^2 \right] \end{aligned} \quad (\text{B7})$$

where  $\bar{\xi} = 1/\Delta V \int_0^r \xi dV = (r_0/r)^\gamma K_1$ ,  $J_2 = 72/[(3-\gamma)(4-\gamma)(6-\gamma)2^\gamma]$ ,  $K_1 = 3/(3-\gamma)$  and  $K_2$  can only be computed numerically. For  $\gamma = 1.6$ ,  $J_2 = 4.87$ ,  $K_1 = 2.14$ , and  $K_2 \simeq 4$ .

The variance  $V(\xi)$  of the estimator of  $\xi$  can be computed analytically. From Landy & Szalay (1993), it is:

$$\begin{aligned} V(\xi) &= V \left( \left\langle \frac{N_{\text{obs}}}{N_{\text{rand}}} \right\rangle \right) \\ &\simeq \frac{V(\langle N_{\text{obs}} \rangle)}{\langle N_{\text{rand}} \rangle^2} + \frac{V(\langle N_{\text{rand}} \rangle) \langle N_{\text{obs}} \rangle^2}{\langle N_{\text{rand}} \rangle^4} \\ &\simeq \left[ \frac{V(\langle N_{\text{obs}} \rangle)}{\langle N_{\text{obs}} \rangle^2} + \frac{V(\langle N_{\text{rand}} \rangle)}{\langle N_{\text{rand}} \rangle^2} \right] (1 + \bar{\xi}_{dg})^2. \end{aligned} \quad (\text{B8})$$

The shot noise of the random sample in Eq. B8,  $V(\langle N_{\text{rand}} \rangle)/\langle N_{\text{rand}} \rangle^2$ , can be neglected because the random sample of galaxies is intentionally much larger than the sample of observed galaxies. Thus, the rms ( $1\sigma$ ) of  $\xi_{dg}$  is

$$\sigma_\xi \simeq \frac{\sigma_{\langle N_{\text{obs}} \rangle}}{\langle N_{\text{obs}} \rangle} [1 + \bar{\xi}_{dg}] \quad (\text{B9})$$

where  $\sigma_{\langle N_{\text{obs}} \rangle} \equiv \frac{1}{N_{\text{DLA}}} \sqrt{V(N_{\text{obs}})}$ , and  $V(N_{\text{obs}})$  is given by the sum of Eqs. B2–B4. If we approximate the clustering variance of  $N_{\text{obs}}$  (Eqs. B3–B4) by  $V_{cl} = \bar{N}^2 (A\bar{\xi}_{gg} + B\bar{\xi}_{dg}^2 + C\bar{\xi}_{dg}\bar{\xi}_{gg})$ , where  $A = J_2/K_1$ ,  $B = Q - 1$  and  $C = 2QK_2/K_1^2$

are constants (Eq. B6–B7), then  $\sigma_{\langle N_{\text{obs}} \rangle}$  becomes

$$\begin{aligned}\sigma_{\langle N_{\text{obs}} \rangle} &= \frac{1}{\sqrt{N_{\text{DLA}}}} \sqrt{V(N_{\text{obs}})_{sn} + V(N_{\text{obs}})_{cl}} \\ &= \frac{1}{\sqrt{N_{\text{DLA}}}} \sqrt{\langle N_{\text{obs}} \rangle + \bar{N}^2 (A\bar{\xi}_{gg} + B\bar{\xi}_{dg}^2 + C\bar{\xi}_{dg}\bar{\xi}_{gg})},\end{aligned}\quad (\text{B10})$$

Therefore, the expected rms of the cross-correlation function  $\sigma_w$ , Eq. B9, becomes:

$$\sigma_{\xi} = \frac{1}{\sqrt{N_{\text{DLA}}}} \frac{1}{\sqrt{\langle N_{\text{obs}} \rangle}} \cdot \sqrt{1 + \bar{N}^2 \frac{A\bar{\xi}_{gg} + B\bar{\xi}_{dg}^2 + C\bar{\xi}_{dg}\bar{\xi}_{gg}}{\langle N_{\text{obs}} \rangle}} \cdot (1 + \bar{\xi}_{dg}),$$

or

$$\sigma_{\xi} = \frac{1}{\sqrt{N_{\text{DLA}}}} \frac{1}{\sqrt{\bar{N}}} \cdot \sqrt{1 + \frac{A\bar{\xi}_{gg} + B\bar{\xi}_{dg}^2 + C\bar{\xi}_{dg}\bar{\xi}_{gg}}{(1 + \bar{\xi}_{dg})}} \cdot \sqrt{(1 + \bar{\xi}_{dg})},\quad (\text{B11})$$

using Eq. B1. This expression is proportional to  $\frac{1}{\sqrt{N_{\text{DLA}}}} \frac{1}{\sqrt{\bar{N}}}$  as one might have expected. Thus, the noise in  $\langle \xi \rangle$  goes as the inverse of the square root of the number of DLAs,  $N_{\text{DLA}}$ , and as the inverse of the square root of the number of galaxies  $N$  in the cell of volume  $\Delta V$ .

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