ABSTRACT

This thesis addresses the question “How can mathematics provide knowledge of physical objects?” which was provoked by Thomas Aquinas’s inclusion of “intermediate” (i.e., physico-mathematical) sciences in his *Division and Methods of the Sciences.* In examining Aquinas’s process of division, I paid special attention to the way he distinguishes the sciences according to the formal *ratio* of their objects, an important development upon Boethius’s framework. This led me to discuss the modes of abstraction proper to each science and, in turn, how their distinct epistemic foundations seem to prevent one science from being meaningfully applied to the study of another. However, in the case of physics, the accident quantity is implicitly included in the definition of its objects, suggesting that mathematics can, in some way, inform their study (even though mathematical propositions themselves are neither true nor false from the standpoint of extramental reality). I concluded that the knowledge obtained through physico-mathematical sciences is conditional in an ontological sense, for the mathematical systems that these sciences employ cannot be more than hypothetical depictions of observed phenomena. Nevertheless, insofar as the conclusions of a given mathematical model are corroborated by physical data, the hypothesis of the model is validated. In fact, mathematics’ indifference to the material world is of remarkable value to the physicist. As an ordered system of the imagination, mathematics enables the physicist to reinterpret the material world according to its quantitative aspects in an idealized setting. In this way, mathematics can become an indispensable tool in the physicist’s quest to locate and abstract the universal natures of physical bodies.

THE BRANIFF GRADUATE SCHOOL

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UNIVERSITY OF DALLAS

THE RELATION BETWEEN PHYSICS AND MATHEMATICS IN THOMAS AQUINAS'S *DIVISION AND METHODS OF THE SCIENCES*

by

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B.A., THOMAS AQUINAS COLLEGE, MAY 17, 2014

A thesis submitted to the Graduate Faculty of the University of Dallas in partial fulfillment of requirements for the degree of Master of Arts in the Department of Humanities.

APRIL 23, 2021

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To Gabriela

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PREFACE

In this thesis, I will examine the relation between physics and mathematics, as set forth by Thomas Aquinas in his *Division and Methods of the Sciences* (i.e., Questions Five and Six of the *Expositio super librum Boethii De Trinitate,* his commentary on Boethius's *De Trinitate*).[1] In this text, Aquinas divides speculative science into three branches—physics (i.e., natural science),[2] mathematics, and metaphysics—and defines the objects and methods proper to each. Although it is one of his earlier works (likely written between 1255 and 1259), it is also his most detailed on the subject. What is interesting about his treatment of the sciences here is that it is something of a tangent in a primarily theological work. Aquinas's chief objective is to discuss the science of Sacred Doctrine, but in order to do this, he must first show Sacred Doctrine's place among the speculative sciences. Thus, a systematic outline is needed.[3]

We should note right away that the term *science* has broader signification for Aquinas than it does in our present day. For him, the terms *science* and *philosophy* are interchangeable, generally signifying "knowledge of things through their causes."[4] Armand Maurer explains:

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1. Thomas Aquinas, *The Division and Methods of the Sciences: Questions V and VI of His Commentary on the "De Trinitate" of Boethius,* 4th ed., trans. Armand Maurer (Toronto: Pontifical Institute of Mediaeval Studies, 1986), hereafter *SBT*.

2. Following Aquinas's terminology, I will use the term *physics* interchangeably with *natural science* ("*physica sive scientia naturalis*," *SBT*, q. 5, a. 1, c.) to generally denote the science of physical objects. Subsequent connotations of the term (e.g., "Newtonian physics") should not be inferred, as it is crucial for physics to be understood in distinction from mathematics in Aquinas's framework. In spite of the danger of this confusion, I prefer using the term *physics* to denote Aquinas's natural science, as it more clearly reflects this science's place among the others. Modern types of physics—such as Newtonian physics—may more aptly be termed *physico-mathematical sciences,* given their distinctive use of applied mathematics. Since this distinction pertains directly to the main question of this thesis, I will thus only use the term *physics* to denote the non-mathematical study of physical objects, unless I specifically state otherwise.

3. Armand Maurer, "Introduction," in Aquinas, *SBT*, vii.

4. Ibid., vii–ix.

> As Aristotle said before him, it is knowledge not only of fact, but of reasoned fact. It reaches its ideal, not simply when it records observable connections in nature and calculates them in mathematical terms, but rather when it accounts for observable phenomena and the properties of things by bringing to light their intelligible relations to their causes. Metaphysics reaches this goal when, for example, it explains the contingent universe through God, mathematics when it explains the properties of a triangle through its definition, natural philosophy when it accounts for change through efficient and final causes and the intrinsic principles of bodies, matter and form.[5]

In short, ontological knowledge is the hallmark of science from the classical point of view. This, of course, stands in contrast with the modern emphasis on empiriological knowledge within the sciences.[6] While it is not my purpose here to compare the merits of these two approaches, my path of inquiry will end up touching upon both. This is because, in considering the relation between physics and mathematics, I will focus on the question, “How can mathematics provide knowledge of physical objects?”

I will begin by discussing Aquinas’s process of division, whereby he classifies objects both according to their modes of existence (i.e., with regard to separation from matter and motion) and according to their modes of knowability. This second criterion is an important way in which Aquinas builds upon Boethius’s work, for it enables him to explain how different sciences can consider the same real objects. The way this development impacts the distinction between physics and mathematics will bear directly upon the central question of this thesis. This will become evident right away, for I will next discuss the modes of abstraction that are distinctive of each of the three sciences. In this context, I will examine Aquinas’s solutions to various difficulties that arise, such as questions on the knowability of matter and the ability of quantity to be considered apart from sensible substance. This will then lead me to discuss the

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5. Maurer, “Introduction,” in Aquinas, *SBT*, vii–ix.

6. Ibid., x.

methods unique to each science, where we will find Aquinas setting forth what is perhaps the most important doctrine of all—namely, that the truth of each science is verified through a different human faculty (i.e., the senses in physics, the imagination in mathematics, and the intellect in metaphysics). Why this doctrine is so significant, as I will discuss, is because it implies that the objects of each science can only be known through that science's own proper method.

This is what sets the stage for a place of real tension in Aquinas's division of the sciences. Given how sharply he distinguishes the epistemic foundation of physics from that of mathematics, it should strike us as odd that there could be such things as *intermediate sciences* (i.e., sciences possessing characteristics of both physics and mathematics). Nevertheless, Aquinas himself recognizes such physico-mathematical sciences, and he names several (astronomy most notably). This, of course, is what provokes the central question of this thesis, "How can mathematics provide knowledge of physical objects?" My challenge will be to reconcile Aquinas's sharp differentiation between physical and mathematical knowledge with his simultaneous recognition of sciences that somehow combine the two. In order to resolve this difficulty, I will begin by investigating how quantity, as an accident, is present in physical bodies. In doing so, I will look for some way in which mathematics, the science of quantity, is able to inform the science of physical bodies. Of course, the distinct epistemic foundations of each science will render this investigation all the more challenging, for it appears that whatever mathematics has to offer in the study of physical bodies, it is not in the realm of ontological knowledge.

Through this discussion, I am confident that Aquinas's threefold division—far from being disproven or made obsolete by later developments in the physico-mathematical sciences—

will display a real strength in its ability to explain mathematics' role in the study of the physical world. Although we will find that mathematics, of itself, cannot provide ontological knowledge of physical bodies, it can nevertheless help us intuit physical truth in a new way, by providing a unique framework for reinterpreting our observations of the physical world. In other words, what I aim to show is how mathematics lets us reimagine, in an idealized manner, the interactions of real physical beings, and how, consequently, it places us in a better position to abstract their essences.

1 DIVISION OF THE SCIENCES ACCORDING TO MATTER AND MOTION

To begin, let us examine Aquinas's criteria for his division of the sciences, which he initiates with the question, "Is Speculative Science Appropriately Divided into these Three Parts: Natural, Mathematical, and Divine?"[1] Here, it will be necessary to gain a firm grasp of what sets apart a speculative science generally, as well as what distinguishes the speculative sciences from each other specifically. In particular, we must understand how Aquinas classifies the objects of these sciences both according to their modes of existence and also according to our modes of knowing them. The way that Aquinas develops upon his Boethian source material in this context—i.e., his choice to distinguish the objects of physics and mathematics according to the way they are known by us—is what plants the seed for the ultimate source of tension addressed by this thesis.

Aquinas determines the general character of a speculative science by distinguishing it from a practical science. This will enable him to identify essential differences within the speculative sciences, according to which he can then establish his division. Explaining Aquinas's initial process, Ralph McInerny states, "In order for there to be kinds or types of *speculabile* and thereby kinds or types of theoretical knowing or science, we need to know what belongs to the speculable as such and, having found this out, to ask what variations in its essential or characteristic notes are possible."[2] The speculative and practical sciences correspond respectively to the speculative intellect (ordered toward truth for the sake of contemplation) and the practical intellect (ordered toward truth for the sake of action). Accordingly, a speculative

1. Aquinas, *SBT*, q. 5, a. 1, p. 9.

2. Ralph McInerny, *St. Thomas Aquinas* (Notre Dame, IN: University of Notre Dame Press, 1977), 82.

science aims at knowledge for its own sake, as opposed to a practical science which aims at knowledge for the sake of action.[3] Thus, since a speculative science studies things simply as they exist, Aquinas determines that his division will be made according to differences in the things themselves.[4]

Having defined its general character, Aquinas next establishes the specific differences within speculative science. In fact, he has already laid the groundwork for this division in his literal commentary on the second chapter of *De Trinitate.* In this preamble to the fifth question, he discusses how both the thing known and the way in which it is known need to be considered when classifying the sciences. He notes how Boethius exhorts his readers to "examine each subject as far as it can be grasped and understood," and he explains the significance of Boethius's choice of words.[5] *Understood* implies that the methods of the sciences "conform to things," while *grasped* implies that the subjects also "conform to us" insofar as they are knowable by the intellect.[6] It is based on these two criteria that Aquinas divides speculative science.

The second of these two criteria, the way in which the subject is knowable by the intellect, is a crucial way in which Aquinas builds upon the Boethian foundation of his work. It

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3. Ralph McInerny, *Boethius and Aquinas* (Washington, D.C.: Catholic University of America Press, 1990), 132.

4. John F. Wippel, *The Metaphysical Thought of Thomas Aquinas: From Finite Being to Uncreated Being* (Washington, D.C.: Catholic University of America Press, 2000), 5.

5. Aquinas, *SBT,* literal commentary on ch. 2, p. 4.

6. Ibid., p. 5.

leads him to categorize the subject of a science in terms of its formal *ratio,* rather than simply in terms of its state in reality.[7] Maurer notes:

> In this Article St. Thomas shows the essential role played by the operations of the intellect in the determination of the subjects of the sciences. The sciences are no longer considered as differentiated according to a distinction of forms ready-made in the world, but according to distinctions the mind itself makes in the course of its investigation of reality. Thus he changes the very notion of the object of a science.[8]

In short, the "conform to us" criterion enables Aquinas to allow for different sciences considering the same real objects, as long as they do so under different formal perspectives. This will prove especially important for understanding the distinction between physics and mathematics.[9]

Aquinas sets out his division of speculative science according to three kinds of subject matter: what is in matter and in motion (physics), what is in matter without motion (mathematics), and what is without matter and without motion (metaphysics).[10] We will see how this division according to matter and motion both conforms to things and conforms to us. Since the objects are speculative, Aquinas identifies their essential characteristics from the standpoint of the intellect, which requires him to consider the objects both as they are and as they are known. He proposes two essential characteristics: immateriality and necessity, which we could consider to be the genus and differentia of speculative objects (in the sense that immateriality is a characteristic shared with the objects of practical science, while necessity is not).

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7. Maurer, "Introduction," in Aquinas, *SBT*, xvi.

8. Ibid.

9. Wippel, *Metaphysical Thought*, 7.

10. The fourth potential combination, *without matter but in motion,* is a logical contradiction (to be discussed).

As McInerny explains, "Thomas holds that there are two things which pertain to the object of speculative or theoretical knowing as such, one of them deriving from the faculty or capacity in play, namely, the intellect, the other deriving from that which qualifies or perfects the intellect in this activity, namely, science."[11] *Immateriality* corresponds with the capacity of the intellect: for a thing to be an object of the intellect, the thing must be immaterial, since the intellect itself is immaterial (in accordance with the principle that, as John F. Wippel states it, "whatever is received in something is received in accord with the mode of being or the capacity of that which receives it").[12] *Necessity* corresponds with the science that perfects the intellect: for a thing to be an object of speculative science, predications concerning it must be necessary.[13] Although Aquinas takes the immateriality of the intellect and the necessity of scientific knowledge as premises in this context, we should note that he does so only due to the limits of his scope, and not because he believes they require no justification, as McInerny explains:

> Obviously this account relies on the cogency of the proof of the immateriality of the intellect and on the account of science, both Aristotelian doctrines [that] Thomas accepts and argues for elsewhere. To do so here would have taken him so far afield he might never have returned to the question at hand. But we should not imagine that he thinks the immateriality of the intellect self-evident or that the requirements for *scientia* do not have to be painstakingly established.[14]

Having named immateriality and necessity as the two distinguishing attributes of a speculative object, Aquinas goes one step further. He seeks a definition of a speculative object that names the primary attributes of that object. Even though *immateriality* fits this criterion, *necessity* does not seem to name an attribute of the object *per se;* rather, it seems to name a

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11. McInerny, *St. Thomas Aquinas,* 82.

12. Wippel, *Metaphysical Thought*, 6.

13. Aquinas, *SBT,* q. 5, a. 1, c., p. 13.

14. McInerny, *Boethius and Aquinas,* 133.

property resulting from a prior attribute. This attribute, he concludes, is immobility, because necessity can only be said of things that are not subject to change. Wippel explains, "But every necessary thing insofar as it is necessary must be immobile. What is moved is capable of being and of not being, either in the absolute sense, or at least in a qualified sense."[15]

Thus, Aquinas establishes the two criteria which he will use to identify the essential differences between speculative objects: *immateriality* and *immobility.* Applying these criteria, he first considers immaterial objects that depend on matter for their being, i.e., for their existence outside the mind. He subdivides these objects into two categories: those which depend upon matter for their being understood, and those which do not. Corporeal objects, studied in physics, must include the notion of sensible matter in their definitions. The example he provides is the definition of a human being, which must include the notion *flesh and bones.*[16]

On the other hand, quantities, studied in mathematics, do not include the notion of sensible matter in their definitions, even though they only exist outside the mind in sensible matter. Objects that do not depend on matter for their being (e.g., God and the angels, also substance, quality, being, potency, act, one and many, etc.) are studied in metaphysics.[17]

We can see how Aquinas's use of the "conform to us" criterion has enabled him to distinguish mathematics from physics. Since the objects of both sciences are in matter and subject to motion outside the mind, we cannot distinguish between them according to this criterion alone. Nevertheless, we can distinguish physics and mathematics according to their formal perspectives, since their objects are grasped by us in different ways. This difference is an

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15. Wippel, *Metaphysical Thought*, 6–7.

16. Ibid., 7.

17. Aquinas, *SBT,* q. 5, a. 1, c., p. 14.

essential one—and therefore a basis for dividing one science from another—for it pertains to the very nature of the objects as speculable. What it means, however, for an object to be grasped under different formal perspectives will be made clearer once I discuss the different modes of abstraction. Suffice it to say, at this point, that each of the three speculative sciences has its own unique mode.[18]

Aquinas concludes his survey of possible speculative objects by noting the fourth combination of the criteria *dependent/not dependent on matter for their existence* and *dependent/not dependent on matter for their being understood.* To review: physics studies objects that are dependent on matter for their existence and dependent on matter for their being understood; mathematics studies objects that are dependent on matter for their existence and not dependent on matter for their being understood; metaphysics studies objects that are not dependent on matter for their existence and not dependent on matter for their being understood. This leaves the fourth combination, *not dependent on matter for their existence* and *dependent on matter for their being understood.*

Aquinas, however, states that there is no fourth science corresponding to this combination, because such an object simply cannot exist. The reason for this is the immateriality of the intellect. This justification may seem strange, given that Aquinas has already allowed that physical objects depend upon matter for their being understood (in fact, this phenomenon itself presents a difficulty that he will address in the next article). Nevertheless, he is not inconsistent in ruling out a fourth science based on this justification, as Wippel explains:

> An immaterial intellect can hardly impose dependency upon matter in order for objects to be understood if those same objects do not depend on matter in the order of being. This is an important point, since it indicates that in Thomas's eyes the threefold division of

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18. Yves R. Simon, "Maritain's Philosophy of the Sciences," *Thomist* 5, no. 1 (1943): 87.

> theoretical science is exhaustive. There is no fourth distinctive kind of object of theoretical science which might lead one to postulate a fourth theoretical science.[19]

If an object does not depend on matter for its existence, then it could only be said to depend upon matter for its being understood based upon some consideration of the intellect itself. But since the intellect is immaterial, it could not be on account of the intellect that an object requires matter for its being understood. Therefore, there can be no object that falls into this fourth potential category, meaning that there is no such category of speculative science.[20]

At this point, we have studied Aquinas's basic criteria for distinguishing the speculative sciences, i.e., the extent to which their objects depend upon matter and motion, both for their being and for their being understood. In the context of this thesis, what is most important to note is that the only difference between physical and mathematical objects, as Aquinas understands them, is that physical objects depend upon matter for their being understood, while mathematical objects do not. This, of course, will eventually lead to the central question of this thesis (regarding the physico-mathematical sciences), but it also creates some immediate difficulties that need to be addressed.

Thus, as I continue to generally follow Aquinas's path of inquiry in the upcoming chapters, I will focus particularly on problems that arise from his accounts of both physics and mathematics. In the next two chapters, I will focus on physics and the question of whether matter can be known (Aquinas's definition of a physical object implies that it can). In the following two chapters, I will then shift my focus to mathematics and the question of whether quantity can be

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19. Wippel, *Metaphysical Thought*, 9.

20. Nevertheless, this provokes a question as to how the material objects of physics can be known by the intellect at all, since matter, as such, is not a suitable object of the intellect. When examining the second article, I will discuss how Aquinas resolves this difficulty.

known apart from sensible matter (likewise, Aquinas's definition of a mathematical object implies that it can). After this, I will be able to devote the remaining chapters to the main question of this thesis, "How can mathematics provide knowledge of physical objects?" first by comparing the epistemic foundations of physics and mathematics, then by examining the relation between these distinct modes of knowledge in the intermediate sciences.

2 WHAT IS MATTER?

As we examine Aquinas's use of *matter* as a criterion for his division of the sciences, it will be helpful to briefly review his understanding of the concept. My primary objective here is to outline basic notions and distinctions necessary for understanding the relation between physics and mathematics. After doing so, however, I will also briefly discuss the way that we, as twenty-first century readers, ought to understand Aquinas's notion of matter in the context of modern physics. I will do this to prevent us from misattributing certain contemporary notions of matter to his framework.

Aquinas understands matter both relatively and ultimately.[1] Both concepts of matter are inherently tied to the potential for change.[2] In *De principiis naturae*, Aquinas explains:

> Moreover, for each existence there is something in potency. Something is in potency to be man, as sperm or the ovum, and something is in potency to be white, as man. Both that which is in potency to substantial existence and that which is in potency to accidental existence can be called matter: for example sperm is the matter of man and man is the matter of whiteness.[3]

Matter is inferred to be that which underlies change. Change occurs through the interaction of contraries, whereby one form overpowers its contrary form or privation in the underlying matter. In accidental change, substance serves as the underlying matter. For example, when a bronze statue is sculpted, the matter of the statue (as statue) is the bronze itself. It underlies the change from the shapeless bronze to the bronze statue. Without the bronze underlying this change, there

1. The terms *ultimate matter* and *prime matter* are interchangeable. It is called "ultimate" according to our order of knowing (i.e., from effect to cause) and "prime" according to the order of being.

2. Patrick Suppes, "Aristotle's Concept of Matter and Its Relation to Modern Concepts of Matter," *Synthese* 28, no. 1 (1974): 28.

3. Thomas Aquinas, *The Principles of Nature,* in *An Introduction to the Philosophy of Nature,* ed. R. A. Kocourek (St. Paul: North Central Publishing, 1948), hereafter *PN,* par. 2.

would be no connection between the two (i.e., the shapeless bronze could not be said to have become the bronze statue). Further on, Aquinas continues:

> In order that there be generation three things are required: *being in potency* which is matter, *non-existence in act* which is privation, and *that through which something comes to be in act* which is form. For example when a statue is made from bronze, the bronze which is in potency to the form of the statue is the *matter;* the shapeless or undisposed something is the *privation;* and the shape because of which it is called a statue is the *form.* But it is not a substantial form because the bronze, before it receives the shape, has existence in act and its existence does not depend upon that shape; rather it is an accidental form, because all artificial forms are accidental. Art operates only on that which is already constituted in existence by nature.[4]

In this example, however, the bronze in the statue is only considered matter in relation to the statue as such (i.e., the bronze underlies its artificial form). In itself, the bronze possesses its own substantial form, which remains unchanged. Thus, the change from the shapeless bronze to the bronze statue is but an accidental change. Accordingly, the continuant of accidental change—substance—is not matter in the ultimate sense, but only in the relative sense.

When it comes to substantial change, there is a "problem of the continuant."[5] Put succinctly, this phenomenon provokes the question, "What underlies substantial change?" In other words, what is the connection between the generated and the corrupted substance? There is no problem of the continuant with accidental change, as the continuant is clearly substance. But the problem quickly manifests itself once we draw the analogy between accidental and substantial change. What is it that acts as the bronze did in the context of one substance becoming another? Whatever underlies this change would have to be matter in the most primal sense. Since it could not be a substance, it would therefore be entirely devoid of form of itself.

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4. Aquinas, *PN*, par. 8.

5. John D. Kronen, Sandra Menssen, and Thomas D. Sullivan, "The Problem of the Continuant: Aquinas and Suárez on Prime Matter and Substantial Generation," *Review of Metaphysics* 53, no. 4. (2000): 867.

Lacking form, and therefore act, it follows that this matter can best be understood as pure potency. This, more or less, is the line of reasoning that leads to the notion of *prime matter*.[6] Wippel concludes:

> In sum, it is clear that from the beginning to the end of his career Thomas defends the view that prime matter includes no form or actuality within its nature, since it is the first or ultimate subject both for form and for privation. Frequently enough he refers to prime matter as potency (or as "in potency") only. This is really equivalent to his other way of describing it—as pure potency. In both cases he wishes to stress the point that it neither is nor contains any actuality in and of itself, and yet that it is not sheer nothingness. Nor is it reducible to privation. On the contrary, it is a real intrinsic principle which must be present in every corporeal being both to account for the fact that such a being is capable of undergoing substantial change and to allow for the possibility that a given kind of being can be multiplied in numerically distinct individuals which belong to the same species.[7]

Prime matter can never exist on its own. In fact, such a claim would be self-contradictory. It only actually exists when brought into act by a substantial form—otherwise, it can only be said to exist in potency.[8] In this way, therefore, it can be said to occupy some strange territory between pure nothingness and being in act.[9] Since, as Wippel puts it, prime matter "includes no determination or actuality in itself," it is impossible to define, except "by analogy or proportion, or by means of the form that actualizes it."[10] For example, we can say it underlies substantial form as substance underlies accidental form (by analogy), or that it is the material principle of a given composite substance (by the form).[11] As Wippel notes:

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6. Kronen, Menssen, and Sullivan, "Problem of the Continuant," 870–71.

7. Wippel, *Metaphysical Thought*, 317.

8. Aquinas, *PN,* par. 17.

9. Kronen, Menssen, and Sullivan, "Problem of the Continuant," 873.

10. Wippel, *Metaphysical Thought*, 313.

11. Aquinas, *PN,* par. 14.

> Since these two approaches are not mutually exclusive, we conclude that according to Thomas we can know prime matter in both ways—by analogy or proportion, and by reason of the form which actualizes it. Because of its purely potential character, we cannot know it directly in and of itself. In and of itself it possesses no form or actuality which would render it intelligible to us.[12]

Thus, any attempt to fully conceptualize prime matter will be unsatisfying. Since prime matter cannot be an object of the imagination, it almost seems better to imagine it as "nothing" rather than "something." Short of trying to imagine it, we might think of it as providing the necessary conditions for three-dimensional extension, allowing substantial forms to interact and (through their contrary influences) bring about the phenomena of substantial change.[13] This seems to be along the lines of what Aquinas describes in the following passage:

> Again, notice that prime matter is said to be numerically one in all things. But to be numerically one can be said in two ways: that which has a determined numerically one form, as Socrates; prime matter is not said to be numerically one in this way, since it does not have in itself a form. Also, something is said to be numerically one because it is without the dispositions which would cause it to differ numerically; prime matter is said to be numerically one in this way, because it is understood without all the dispositions which would cause it to differ numerically.[14]

At this point, I will note that we must be careful not to confuse the Thomistic notion of matter (especially prime matter) with notions encountered in modern physics. These two approaches simply have different goals. As discussed, the idea of prime matter was derived as a solution to the metaphysical problem of the continuant. Understood in this way, matter is not knowable in itself, for it is devoid of act. John D. Kronen, Sandra Menssen, and Thomas D. Sullivan provide helpful context for the doctrine of prime matter by noting that, "Aristotle, Aquinas, and Suárez (among others) believed that in order to explain substantial generation it is

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12. Wippel, *Metaphysical Thought*, 327.

13. Suppes, "Aristotle's Concept of Matter," 48-49.

14. Aquinas, *PN,* par. 16.

necessary to postulate the existence of prime matter. The notion of prime matter is like any other theoretical notion in modern physics: it is postulated when necessary for explanation."[15]

The experimental physicist, on the other hand, studies the properties of matter, as it is encountered in the physical world. Understood in this way, matter is knowable, since, as Aquinas would put it, it possesses act through its substantial form. Thus, we can infer that there is something of an equivocation between the two uses of the term *matter.* In brief, it seems that what the experimental physicist would call "matter" is what Aquinas would call "relative matter." Patrick Suppes details the reasoning for this claim below:

> It is wrong headed from his [Aristotle's] standpoint to ask of a substance what is its form and what is its ultimate matter and then to ask for properties of the matter. This view of matter seems contrary to that of contemporary physics with its talk about the quantity of matter or mass as an invariant property of matter. It must be realized that in talking about matter in this way physicists are not talking about matter in the way that Aristotle does. In abstract classical dynamics, for example, the only property of matter that is admitted is its mass, but even this admission is not consistent with Aristotle's doctrine of matter as pure potentiality. The evident contradiction between these two ways of talking about matter does not mean that one is wrong and the other is correct – it means that the word matter, or its translation in various natural languages, is being used in more than one sense.[16]

Given the limitations here, I will conclude by simply reiterating that the Thomistic concept of matter is not a concept of matter to be encountered in modern physics. Without attempting to compare the merits of these different concepts, we must note that the term *matter* has different referents in the Thomistic and modern contexts. As a result, at least for our purposes here, we can rest assured that neither concept necessarily poses a challenge to the other.

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15. Kronen, Menssen, and Sullivan, "Problem of the Continuant," 869.

16. Suppes, "Aristotle's Concept of Matter," 29–30.

3 DESIGNATED AND UNDESIGNATED MATTER

We are now in a position to confront head-on a significant difficulty in Aquinas's account of physics. Once again, this difficulty results from his claim that physical objects depend upon matter for their being understood. Thus, we must examine how physical objects are known by the intellect by turning to the second article of question five. Here, Aquinas asks, "Does Natural Philosophy Treat of What Exists in Motion and Matter?"[1] In this article, there are actually two related questions he must answer, but both follow from the fact that matter is included in a physical object's definition. The first is, given that material objects are subject to motion (i.e., change), how can there be speculative knowledge of them? The second is, given that the intellect itself is immaterial, how does it even have the capacity for knowing material objects?

To address these problems, Aquinas begins by referencing Plato's belief that there could not be speculative science of objects in motion, given the change that occurs in the sensible world. He then examines Plato's response to this problem, the theory of Ideas, and summarizes it in the following way: "So he claimed that there were substances separated from the sense world, which might serve as the objects of science and of definitions."[2] As Jacques Maritain notes, "Plato is very near Heraclitus … from the point of view of the knowledge of nature. He too sought being and found sensible flux and therefore he too, discouraged by this flux, declared that the world of sensible nature can only be the object of opinion, δόχα, not of science."[3]

1. Aquinas, *SBT,* q. 5, a. 2, p. 25.

2. Ibid., q. 5, a. 1, c., p. 27.

3. Jacques Maritain, *Philosophy of Nature,* trans. Imelda C. Byrne (New York: Philosophical Library, 1951), 5.

Not surprisingly, Aquinas is dissatisfied with Plato's account, and he undertakes to show how Plato erred in denying that a science of the material world could be had. He states that Plato "made this mistake because he failed to distinguish what is essential from what is accidental" (and amusingly notes that "it happens that by accident even the wise often fall into error").[4] He then presents an alternative solution to the problem facing Plato, a solution that affirms the science of physics. Citing Aristotle's *Metaphysics,* he distinguishes between the whole sensible object (i.e., the composite) and its nature or form. These natures and forms are not subject to motion *per se*, but rather, they are only subject to motion insofar as their composites are. Since, he argues, "anything can be thought of without all the items that are not essentially related to it," these natures and forms "can be the objects of sciences and of definitions."[5] And it is through our scientific knowledge of these forms that the very composites of which they are principles, i.e., mobile bodies, become objects of scientific knowledge themselves. In other words, the abstraction of a universal from a particular makes the science of physics possible. In Aquinas's words:

> Natures of this sort, thus abstracted, can be considered in two ways. First, in themselves; and then they are thought of without motion and designated matter. This happens to them only by reason of the being they have in the intellect. Second, they can be viewed in relation to the things of which they are the natures; and these things exist with matter and motion. Thus they are principles by which we know these things, for everything is known through its form.[6]

Aristotle and Aquinas do not disagree with Plato's premise that scientific knowledge can only be had of essences directly. They recognize, however, that there is no need (and therefore

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4. Aquinas, *SBT,* q. 5, a. 1, c., p. 27.

5. Ibid., p. 28.

6. Aquinas, *SBT,* q. 5, a. 2, c., p. 29. Maurer translates *materia signata* and *materia non signata* as "determinate matter" and "indeterminate matter," respectively. For the sake of consistency, I have inserted "designated" and "undesignated" into the translation wherever Maurer has "determinate" and "indeterminate."

no basis) for positing self-subsistent essences of sensible objects if the mind is able to abstract essences from the individuals themselves. The characteristics of Plato's Ideas that made them so appealing—namely, positive unity and separation from things—are characteristics that likewise belong to the Aristotelian universal.[7] What makes this Aristotelian notion a breakthrough, and not simply an alternate theory, is that it allows for our ability to have scientific knowledge of individual material bodies—something unthinkable in the Platonic model. Each particular is known through its form, which, in turn, is known by the intellect as a universal essence. Nevertheless, the fact that everything is known through its form enables us to truly say that we have scientific knowledge of bodies in matter and in motion.[8]

Aquinas now proceeds to address the second problem. Once again, the difficulty here is to reconcile the immateriality of the intellect with the materiality of physical objects, if we are to consider them speculative objects. This leads him to distinguish between *designated matter* (*materia signata*) and *undesignated matter* (*materia non signata,* also called *common matter*). He states, "Consequently, natures of this kind, which make possible sciences of things subject to motion, must be thought of without designated matter and everything following upon such matter; but not without undesignated matter, because on its notion depends the notion of form that determines matter to itself."[9]

Aquinas appeals to the distinction between a particular and a universal. To use his example, we might use the phrase *flesh and bones* when referring to a particular instance of

7. Maritain, *Philosophy of Nature*, 5–6.

8. Ariberto Acerbi, "Aquinas's Commentary on Boethius's *De Trinitate*," *Review of Metaphysics* 66, no. 2 (2012): 328.

9. Aquinas, *SBT,* q. 5, a. 1, c., pp. 28–29.

matter, such as *Socrates's flesh and bones.* In this case, we would be speaking of designated matter. In itself, designated matter can only be known through sensation, for it belongs to non-abstracted material objects. On the other hand, we might use the phrase *flesh and bones* when referring to matter more generally, i.e., to matter considered with the basic characteristics common to a species, and not as belonging to a particular individual. In this case, we would be speaking of undesignated matter.[10]

It is the notion of undesignated matter that remains when the intellect abstracts a universal from a particular. Once again, the key distinction to be made here is that the intellect does not apprehend the matter of a composite directly, but rather, through the mediation of its form. Although the characteristics of matter common to a species are truly material characteristics, they exist through the act of the form, and therefore they are included in the definition. Thus, through its apprehension of the universal, the intellect can have scientific knowledge of material beings. As a matter of fact, earlier in his commentary (q. 4, a. 2), Aquinas gives a detailed explanation of the relation between an individual and its form. I will present Wippel's summary of the argument here:

> As regards individuals within the same species, Thomas develops this explanation. If the parts of a genus and a species are matter and form, the parts of an individual are *this* matter and *this* form. Hence it is this matter and this form that account for numerical diversity within species. But no form insofar as it is a form is this form (or individual) of itself. (He explains that he has added "insofar as it is a form" because of the rational soul. In a way it is this individual something [*hoc aliquid*] of itself, but not insofar as it is a form.) He also notes that our intellect can attribute to many things any form which can be received in something as in matter or a subject. But this ability to be predicated of many is contrary to the nature of an individual. Therefore, form becomes this form or individual by being received in matter. But matter is indistinct in and of itself. Hence it cannot individuate the form it receives unless it is rendered divisible. This follows, Thomas explains, because a form is not individuated simply because it is received in matter. It is individuated only insofar as it is received in *this* matter which is distinct and determined to the here and now. And matter is rendered divisible only through quantity. In other

10. Wippel, *Metaphysical Thought*, 358.

> words, it is only by reason of quantity that matter can be divided into parts and rendered subject to the here and now. Therefore, Thomas continues, matter is rendered *this* and designated only insofar as it is subject to dimensions.[11]

In summary, the intellect apprehends the forms of composites in themselves (i.e., in abstraction from motion and matter), but it can also view them in relation to the things of which they are the natures.[12] In doing so, the intellect is thereby able to have scientific knowledge of physical bodies themselves. Thus, in spite of the apparent challenges posed by their materiality and mobility, physical bodies, apprehended through the abstraction of the universal from the particular, are truly speculative objects. In its ability to explain this phenomenon, the Aristotelian understanding of nature has a remarkable advantage over the Platonic one. As Aquinas puts it, "in natural science we know mutable and material things existing outside the soul through natures of this kind; that is to say, natures that are immobile and considered without particular matter."[13]

Taking a step back, I should point out that we have just made important progress toward addressing the main question of this thesis by ironing out the doctrine of undesignated matter. Of course, this doctrine has had the immediate effect of resolving a potential problem in Aquinas's definition of physics, but it will also prove critical for understanding the relation between physics and mathematics in the long run. However, I will avoid getting too far ahead of myself right now and instead proceed to my discussion of quantity as a mathematical object. As I examine this topic over the next two chapters, I will especially focus on the issue of quantity being knowable

11. Wippel, *Metaphysical Thought*, 360–61.

12. Aquinas, *SBT,* q. 5, a. 2, c., p. 29.

13. Ibid.

apart from sensible matter. Right now, I will simply note that we will eventually return to the concept of undesignated matter, surprisingly, in the context of mathematics.

4 MODES OF ABSTRACTION

Once again, a key feature of Aquinas's division of the sciences is that a science is distinguished not only according the way its objects exist outside the mind, but also according to the way its objects are apprehended by the intellect. The full impact of this second criterion is felt in the third article of question five, where Aquinas considers the question, "Does Mathematics Treat, Without Motion and Matter, of What Exists in Matter?"[1] Although this article is meant to address the subject of mathematics, it becomes important for another reason altogether. To distinguish mathematics from the other sciences (particularly physics), Aquinas reveals that each science is characterized by a unique mode of abstraction. As a result, this article will prove critical for understanding the different epistemic foundations of physics and mathematics, and thus it will play a key role in addressing the central question of this thesis.

Abstractio vs. Separatio

To begin, Aquinas names the two operations of the intellect: the understanding of natures and predication (i.e., apprehension and judgment). Distinguishing between these two operations, and between the modes of abstraction proper to each, will be the first step towards distinguishing the sciences according to their modes of apprehension. Thus, the first distinction that must be made is between *abstractio* and *separatio.*[2]

The first operation of the intellect, simple apprehension of intelligible objects, is the operation by which we know the essence of a thing. Thus, we are able to express its nature in a

1. Aquinas, *SBT,* q. 5, a. 3, p. 32.

2. Douglas C. Hall, *The Trinity: An Analysis of St. Thomas Aquinas' "Expositio" of the "De Trinitate" of Boethius* (Leiden: Brill, 1992), 89–90; McInerny, *Boethius and Aquinas,* 137.

definition and know its place in the hierarchy of beings, i.e., whether it is a substance or an accident, a whole or a part. The second operation of the intellect, judgment, is the operation by which we join and divide intelligible objects. Having grasped these objects through the first operation of the intellect, we can make affirmative or negative predications with them through the second. Thus, through the second operation of the intellect, we consider the existence of a thing, i.e., whether it exists through the joining of the principles of a composite, or separately, as a simple substance. For example, by the first operation, we could understand the notions of *grass* and *green* individually; by the second, we could join these notions in the affirmation, "Grass is green." Likewise, we could divide *grass* and *red* by the second operation in the negation, "Grass is not red." Again, it is necessary to distinguish these two operations here, because each operation has its own proper mode of abstraction.[3]

Abstraction according to simple apprehension is, not surprisingly, the act of considering a nature absolutely in itself, that is, without reference to any other form that may accompany it in reality. For example, one might consider *greenness* without considering *grass,* or vice versa. In this mode of abstraction, which is termed *abstraction* (*abstractio*) in the strictest sense (since forms accompanying each other in reality are being drawn away from each other in the mind),[4] there is no consideration of the form's state of existence in reality. In other words, when considering *greenness* or *grass* in this manner, it is of no concern whether or not greenness exists in grass.[5]

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3. Hall, *The Trinity,* 89–90; McInerny, *St. Thomas Aquinas,* 82–83; McInerny, *Boethius and Aquinas,* 137–39; Maurer, "Introduction," in Aquinas, *SBT*, xvii.

4. "Like the Greek term *aphaeresis,* the Latin *abstractio,* from *ab* and *trahere,* meant the process of drawing one thing away from another, as to pull an apple from a tree, or hew stone from stone." Charles De Koninck, "Abstraction from Matter (II)," *Laval Théologique et Philosophique* 16, no. 1 (1960): 53.

5. McInerny, *St. Thomas Aquinas,* 82–83; Maurer, "Introduction," in Aquinas, *SBT*, xvii.

On the other hand, abstraction according to judgment, the second operation of the intellect, requires knowledge of the intellectual object's state of existence in reality. For example, in the negative predication, "Grass is not red," the form *grass* is being considered separately from the form *red,* which is only possible because they already exist separately in reality. Whereas in simple apprehension we posit nothing about the existence of the abstract form, in judgment we do. Thus, when we judge that object A does or does not exist with object B, our judgment can only be true if it conforms with reality. For this reason, abstraction according to judgment (i.e., negative judgment) is more fittingly termed *separation* (*separatio*), since the intellect simply recognizes an already-existent separation between forms in reality.[6]

To be clear, however, it is not through *separatio* that we know an object exists, simply speaking. Since *separatio* is an act of negative judgment (i.e., nothing is positively predicated of the subject), it presupposes that an object exists when stating what it is not. For example, the statement, "Grass is not red," considered by itself, does not necessitate that grass exists any more than the statement "Unicorns are not red," necessitates that unicorns exist. However, when we already know that an object exists, *separatio* reveals something about that object's mode of existence by setting it apart from other objects. Thus, it is not by the statement, "Grass is not red" that we know grass exists; however, given its existence (which we would ascertain through positive judgment), *separatio* tells us something about the way it exists (i.e., apart from redness).

Having distinguished between the two operations of the intellect and the modes of abstraction proper to each, we can now distinguish the sciences according to these modes of abstraction. Right away, we can identify *separatio* as the mode of abstraction proper to

6. Maurer, "Introduction," in Aquinas, *SBT*, xviii; McInerny, *St. Thomas Aquinas,* 82–83; McInerny, *Boethius and Aquinas,* 137–39.

metaphysics. This is due to the fact that the knowledge obtained through *separatio* is not knowledge of an object's essence, but rather, only of its existence. In other words, simply perceiving that the subject exists apart from some other form is the extent to which we can apprehend a subject through *separatio.* This stands in stark contrast to *abstractio,* through which we can apprehend the form (i.e., the essence itself) of the subject.[7] As to why metaphysics is the science characterized by *separatio,* Douglas C. Hall explains:

> This type of judgement is not an "abstraction," simply as such, but only in the more general sense of a "removal," though specifically by way of negation. Its application to divine science is that by means of *separatio* the intellect acknowledges and asserts that "being," "esse," need not be identified with material, sensible, changing "being," or even "being" of a particular or determined kind. Accordingly, the notion of "God," as well as any other notional "objects" of divine science, cannot be abstracted, as such.[8]

That metaphysics is uniquely characterized by this mode of abstraction should not come as a surprise, since metaphysics is the study of exclusively immaterial objects. What is true for metaphysics, namely, that the intellect has no need to abstract (*abstrahere*) its objects from matter, is not true for either physics or mathematics. This, in fact, is why Aquinas takes the time to critique the Platonic notion of Ideas. We might say that it fails to distinguish between essence and existence—or, at a minimum, between the distinct operations through which they are grasped, apprehension and judgment. Thus, without making this distinction, there would no grounds for distinguishing between *abstractio* and *separatio.*[9] As Maurer puts it:

> Because an object of thought is intelligible in itself, Plato thought that it must exist in itself. In fact, what is one in existence can be conceived in multiple fashion in simple

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7. This can begin to explain why a science characterized by *separatio* (i.e., metaphysics) is inherently less knowable to us than sciences characterized by *abstractio*. Even though *abstractio* is not in itself concerned with the existence of the abstract object, the fact that the object can exist is already a given (granting that its actual state of existence remains undetermined). Through *separatio,* on the other hand, the intellect is unable to apprehend anything of an object's essence.

8. Hall, *The Trinity,* 89–90.

9. Aquinas, *SBT,* q. 5, a. 3, c., p. 41.

> apprehension. Only in negative judgment do we grasp the separation of one thing from another in existence. So the fact that we can consider a nature without considering the individuals in which it exists is no indication of the separate existence of that nature.[10]

Since, however, my overall purpose is to examine the relation between physics and mathematics, I will now set aside *separatio* and examine *abstractio* more closely. With *abstractio* coming into play in both physics and mathematics, the task ahead will be to distinguish how it is uniquely performed in each.

Abstractio Subdivided

Although *abstractio,* abstraction in the narrow sense, belongs to both physics and mathematics, we will now see how the distinction between the proper objects of each corresponds to a distinction within *abstractio* itself. What allows us to abstract physical object A from designated matter B is necessarily different from what allows us to abstract mathematical object A from sensible matter B. For instance, given *this stone,* it is evident that the operation enabling us to consider *stone* universally is different from the operation enabling us to consider *sphere of such-and-such volume* (i.e., the quantity of *this stone*). Thus, in identifying this distinction, Aquinas names two different modes of *abstractio: abstractio totalis* (i.e., abstraction of the whole from parts) and *abstractio formalis* (i.e., abstraction of the form from sensible matter). Each of these modes corresponds to a different science.[11]

Before distinguishing these modes of abstraction, Aquinas first presents a principle which restricts the ways in which *abstractio* can be validly performed. This restriction can be described

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10. Maurer, "Introduction," in Aquinas, *SBT*, xix.

11. McInerny, *St. Thomas Aquinas,* 88–91.

as the "principle of dependent intelligibility."[12] He states, "Therefore, when the nature itself is related to, and depends on something else, with regard to that which forms the definition (*ratio*) of the nature, and through which the nature itself is understood, clearly we cannot know the nature without that other thing."[13]

Aquinas sets out categories of objects which depend upon another for their intelligibility: 1) a part that is defined in reference to a whole (e.g., a foot is not knowable in itself, but it is only knowable insofar as it is a part of a whole animal); 2) the form of a composite, which must include matter in its definition (which is especially relevant in physics and the reason why undesignated matter must be considered); 3) a part that is defined through its relationship to another part (he does not provide an example, but we might think of a thumb only being understood in relation to the other fingers); 4) an accident that is defined in reference to a particular subject, (e.g., *snub,* as said of a nose); 5) a thing that is defined through its relation to another thing, even though they exist separately in reality (e.g., *son* and *father*). In each one of these pairs, the antecedent cannot be abstracted from the consequent, for each consequent is included in the definition of its antecedent.[14]

There are two ways, however, in which an object does not depend upon another for its intelligibility, even though the objects exist together in reality.[15] In such cases, *abstractio* can be validly performed. One of these scenarios occurs when an object exists with another but is in

12. Thomas C. Anderson, "Intelligible Matter and the Objects of Mathematics in Aquinas," *New Scholasticism* 43, no. 4 (1969): 560–61.

13. Aquinas, *SBT,* q. 5, a. 3, c., p. 36.

14. McInerny, *St. Thomas Aquinas,* 88–91.

15. The fact that such objects exist together in reality is the reason why *abstractio,* rather than *separatio,* is necessary.

itself a whole. What distinguishes this scenario from the prohibited part/whole relationship mentioned above is the ability to define an object in this relationship without reference to the other. The example Aquinas provides is, "as *letter* can be understood without *syllable*, but not vice versa, and *animal* without *foot*, but not conversely."[16] An abstraction in this case is termed an *abstraction of a whole* (or *abstraction of the universal from the particular*), "in which we consider a nature absolutely, according to its essential character, in independence of all parts that do not belong to the species but are accidental parts."[17] As previously discussed, this mode of abstraction corresponds to physics. The other scenario occurs when an object exists with another as a form in matter or as an accident in a subject. The example Aquinas provides is, "*whiteness* can be understood without *man* and vice versa."[18] An abstraction in this case is termed an *abstraction of a form from sensible matter,* and it corresponds to mathematics. In mathematics, the form being abstracted from sensible matter is, of course, the accident quantity. Aquinas assigns a hierarchy to these two modes of *abstractio*, placing *abstractio totalis* (abstraction in physics) in the first order and *abstractio formalis* (abstraction in mathematics) in the second order.[19]

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16. Aquinas, *SBT,* q. 5, a. 3, c., p. 36. Aquinas's *letter/syllable* example may seem odd, for one is probably inclined to think of the letter as the part and the syllable as the whole in this relationship. Actually, it is precisely in the awkwardness of this example that its value lies. The reason why a letter can be understood apart from a syllable is because the letter is, in itself, a whole. It is accidental to the letter as such whether it be found within a syllable or not. In a certain way, this example is the better of the two, for it is a case of something, that is itself a whole, being found *within* another (as the form of the species is found within an individual).

17. Ibid., p. 40.

18. Ibid., p. 37.

19. McInerny, *St. Thomas Aquinas,* 88–91; McInerny, *Boethius and Aquinas,* 139–41; Maritain, *Philosophy of Nature*, 15–22.

In *abstractio totalis,* Maurer notes, "the individuals are, as it were, 'parts' from which the nature as a 'whole' is abstracted."[20] Although this mode of abstraction is characteristic of physics, it is not restricted to this science. Insofar as all sciences examine only what is essential and ignore what is accidental to their objects, they all use *abstractio totalis.*[21] Accordingly, the examples themselves that Aquinas provides are not limited to physics. While we cannot think of a syllable without letters, a triangle without lines, or a compound without elements (*parts of the species and of the form,* i.e., parts included in the definition), we can, for example, think of a circle without semicircles (*parts of matter,* i.e., parts not included in the definition), since these are non-essential parts.[22] This latter example, evidently, is taken from mathematics. Nevertheless, *abstractio totalis* is most distinctive of physics, insofar as the physicist studies the natures of material things. Although form and matter are both included in the proper objects of physics, the physicist may abstract from designated matter, since the particular matter of a given individual is accidental to its nature. Thus, *flesh and bones* are rightly understood as parts of the species, whereas *this flesh and these bones* are understood as parts of matter.[23] Charles De Koninck explains:

> For instance, I can consider man as an animal, abstracting from the fact that he is an animal of a very special kind; and I can consider man without considering *this* one who is Socrates. But I cannot conceive man without conceiving animal, nor this man without conceiving man. Animal is prior to man inasmuch as an animal is not necessarily a man, even as a man is not necessarily Socrates. Both examples convey abstraction of universal from particular. In the first case we abstract a universal, animal, from a less universal, man; in the second, the particular is a singular. It is likewise called abstraction of the whole from the subjects or 'subjective parts' of which it can be said. (This term 'part' is

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20. Maurer, "Introduction," in Aquinas, *SBT*, xx.

21. Aquinas, *SBT,* q. 5, a. 3, c., p. 41.

22. Ibid., p. 39.

23. McInerny, *Boethius and Aquinas,* 140–41; Maurer, "Introduction," in Aquinas, *SBT*, xxi; McInerny, *St. Thomas Aquinas,* 90–91.

an analogical term, for Socrates is not part of man in the sense that his head is part of the whole that is Socrates; nor is horse a part of animal in this early sense of part.)[24]

In *abstractio formalis,* the principle of dependent intelligibility likewise applies, for abstraction is only possible "if the essential nature of the form does not depend on that particular kind of matter."[25] This, as it turns out, is no small qualification, since an accident cannot be understood apart from substance (for "it is the nature of every accident to depend upon substance").[26] How, then, can a formal abstraction ever be possible? The key distinction to make is that while an accident cannot be understood apart from some substance (i.e., the definition of an accident includes the notion of *inhering in a substance*), an accident can nevertheless be understood apart from any particular substance. For example, when we observe green grass, we can understand *greenness* in abstraction from the particular substance of grass, easily conceiving of the same form of greenness in a different substance instead. Thus, while it is possible to consider an accident apart from a particular substance (e.g., *this grass*), it is not possible to consider an accident apart from substance generally. Furthermore, it is possible to consider certain accidents without considering other accidents, based on their order of being. This is why *abstractio formalis* is mathematics' characteristic mode of abstraction. Because quantity, as Aquinas states, is first in being in the order of accidents, it "can be thought of in substance before the sensible qualities" and therefore "does not depend upon sensible matter but only upon intelligible matter" (a somewhat paradoxical notion that I will examine shortly).[27] Although quantity, like the other accidents, cannot be abstracted from substance generally, it can, for this

24. De Koninck, "Abstraction from Matter," 53.

25. Aquinas, *SBT,* q. 5, a. 3, c., p. 37.

26. Ibid.

27. Ibid., p. 38.

reason, be considered in abstraction from the sensible accidents. Thus, it is possible to consider quantity apart from sensible substance.[28]

Abstraction as an Analogous Term

Aquinas concludes the article by showing how the three modes of abstraction relate to his threefold division of the speculative sciences. *Separatio,* done according to the intellect's operation of composing and dividing, is characteristic of metaphysics. *Abstractio,* done according to the intellect's operation of apprehending natures, is characteristic of mathematics when a form is abstracted from sensible matter, but it is characteristic of physics when a universal is abstracted from a particular. Nevertheless, as I discussed above, Aquinas notes that the abstraction of a universal from a particular is, in a certain way, characteristic of "all the sciences in general, because science disregards accidental features and treats of necessary matters."[29] An example McInerny provides, which will be relevant in the next chapter, is that a mathematician will need to "consider circle apart from this circle," whenever considering a circle's essential properties.[30]

In this article, it is clear that Aquinas has accomplished more than simply describing the nature of mathematical objects. More importantly, he has revealed the ontological framework of his division of the speculative sciences, showing how the intellect's three modes of abstraction correspond to three degrees of objective immateriality. In effect, he has synthesized his two initial criteria for dividing the sciences (i.e., the ways in which they conform to things and

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28. McInerny, *St. Thomas Aquinas,* 90; McInerny, *Boethius and Aquinas,* 140–41; Anderson, "Intelligible Matter," 560–61.

29. Aquinas, *SBT,* q. 5, a. 3, c., p. 41.

30. McInerny, *St. Thomas Aquinas,* 91.

conform to us).[31] In this context, however, we should be careful not to regard these three modes of abstraction as differing simply along the lines of "more and less." Rather, the heterogeneity among these modes of abstraction (and, by extension, among their respective sciences) is something that Aquinas himself maintains.[32] Maritain elaborates on this point:

> That is why St. Thomas teaches in his *Commentary on the Trinity of Boethius,* that the aim or term of knowledge (which pertains to judgement, for it is in the judgement that cognition is perfected) *is not always of the same kind* in the different types of speculative knowledge. Physical knowledge terminates in the sensible; mathematical knowledge terminates in the imaginable; metaphysical knowledge in the pure intelligible.[33]

This doctrine regarding the unique termination of knowledge in each of the sciences will be useful in addressing the main question of this thesis, so I will return to it later. At this moment, I will simply note that *abstraction,* as applied to the three modes discussed here, is best characterized as an analogous term. In each of the three modes, one concept is drawn away from another (and herein lies the analogy)—but these modes of abstraction are distinguished according to the different modes of being of the abstract objects themselves. In other words, with *separatio* belonging to substance, *abstractio formalis* to quantity, and *abstractio totalis* to quality, we find ourselves using a single term—*abstraction*—to describe operations proper to three distinct categories, thus rendering it analogous.[34]

At this point, however, there is more to discuss with regard to mathematical objects. I have so far discussed the way Aquinas distinguishes them from physical objects, which is based on how they are known by us (i.e., through different modes of abstraction). However, the

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31. Maritain, *Philosophy of Nature*, 13–14.

32. Aquinas, *SBT,* q. 6, a. 2, c., p. 77.

33. Maritain, *Philosophy of Nature*, 24.

34. De Koninck, "Abstraction from Matter," 53.

question of how quantity can be considered apart from sensible quality has not been entirely answered. Even though we have noted Aquinas's explanation that quantity is prior in being to sensible quality, the fact remains that quantity depends upon matter in some way. Thus, we must examine what is meant by the term *intelligible matter,* and how, in particular, any such non-sensible matter can be included in a mathematical object.

5 INTELLIGIBLE MATTER

In the previous chapter, I introduced Aquinas's use of the term *intelligible matter* in the context of mathematical abstraction, i.e., *abstractio formalis.* This term denotes the fact that quantity depends upon substance for its being and for its being understood, but not upon sensible matter. Unlike physical abstraction *(abstractio totalis),* in which the notion of undesignated matter is retained, mathematical abstraction strips away an object's sensible qualities, leaving it devoid of any sensible matter, designated or undesignated. On the one hand, this situation does not violate the principle of dependent intelligibility, for quantity does not depend upon quality for its being or for its being understood. Nevertheless, there does seem to be a problem on a related front: namely, that quantity—even in the abstract—depends upon matter. This we can infer from the following passage in the *Summa:*

> In order to resolve this point, we may observe that all plurality is a consequence of division. Now division is twofold; one is material, and is division of the continuous; from this results number, which is a species of quantity. Number in this sense is found only in material things which have quantity. The other kind of division is called formal, and is effected by opposite or diverse forms; and this kind of division results in a multitude, which does not belong to a genus, but is transcendental in the sense in which being is divided by one and by many. This kind of multitude is found only in immaterial things.[1]

Here, Aquinas sets out two ways in which plurality can occur: either through the division of the continuous (material) or through formal distinction (immaterial). That quantity belongs to the first category (as he states) is made evident by our ability to distinguish individual mathematical objects having the same form (e.g., two congruent triangles). Thus, while quantity can be considered apart from sensible matter, it cannot be considered apart from matter simply, due to the need for a principle of individuation. It was precisely this problem that led Aristotle to coin

1. Thomas Aquinas, *The Summa Theologiæ of Saint Thomas Aquinas,* 2nd ed., trans. Fathers of the English Dominican Province (London: Burns Oates & Washbourne, 1920), I, q. 30, a. 3, c.

the term *intelligible matter.* Of course, no accident inheres in matter immediately. Accidents inhere in substance, in a way analogous to that in which substantial form inheres in matter. Thus, in the case of mathematical quantity, we can describe the individuating principle (i.e., intelligible matter) as "non-sensible material substance,"[2] or, as Aquinas defines it, "substance as subject to quantity."[3] Maurer explains the reasoning for this notion:

> The presence of substance in the object of mathematics is also required as the intelligible matter of quantity. As Aristotle maintains, there must be some matter wherever there is form instantiated in several individuals. Since there are many mathematical circles, triangles, and numbers, there must be a material principle of individuation in mathematics, not perceptible to the senses but known by the intellect. This principle, according to Aquinas, is substance.[4]

Of course, substance as a material principle is understood relatively.[5] Nevertheless, relative or otherwise, this notion of a nonsensible material principle is still noteworthy, if not altogether odd. Regardless of what has been said thus far, the very term *intelligible matter* seems self-contradictory, as things are only intelligible insofar as they are immaterial.[6] So, what exactly is this "immaterial" material?

Aristotle only references intelligible matter three times throughout his work (*Metaphysics* Z.10, 11; H.6). In *Metaphysics* Z.10, he distinguishes between sensible circles (e.g., made of

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2. Anderson, "Intelligible Matter," 563.

3. Aquinas, *ST*, I, q. 85, a. 1, ad 2.

4. Armand Maurer, "Thomists and Thomas Aquinas on the Foundation of Mathematics," *The Review of Metaphysics* 47, no. 1 (1993): 49.

5. See the distinction between relative and ultimate matter in chapter 2.

6. John Thorp, "Intelligible Matter in Aristotle," *Society for Ancient Greek Philosophy Newsletter* (April 2010), https://orb.binghamton.edu/cgi/viewcontent.cgi?article=1384&context=sagp: 1.

bronze or wood) and intelligible circles (i.e., mathematical objects). Both of these types of circles he characterizes as individual circles.[7]

> But when we come to the concrete thing, e.g. this circle, i.e. one of the individual circles, whether sensible or intelligible (I mean by intelligible circles the mathematical, and by sensible circles those of bronze and wood.… But matter is unknowable in itself. And some matter is sensible and some intelligible, sensible matter being for instance bronze and wood and all matter that is changeable, and intelligible matter being that which is present in sensible things not *qua* sensible, i.e. in the objects of mathematics.[8]

It is in *Metaphysics* Z.11 that Aristotle presents intelligible matter as a solution to the problem of individuation among mathematical objects. Here, he observes "that even some things which are not perceptible must have matter; for there is some matter in everything which is not an essence and a bare form but a 'this'….and while one kind of matter is perceptible, there is another which is intelligible."[9]

It is Aristotle's concept of abstraction that allows him to approach the problem in this way. Quantities exist, first and foremost, in sensible objects. They are, so to speak, "laid bare" and made suitable objects of mathematics when the posterior accidents (sensible qualities, etc.) are set aside. He explains this in *Metaphysics* K.3:

> [T]he mathematician investigates abstractions: before beginning his investigation he strips off all the sensible qualities, e.g. weight and lightness, hardness and its contrary, and also heat and cold and the other sensible contrarieties, and leaves only the quantitative and continuous, sometimes in one, sometimes in two, sometimes in three dimensions, and the attributes of these *qua* quantitative and continuous, and does not consider them in any other respect, and examines the relative positions of some and the attributes of these, and the commensurabilities and incommensurabilities of others, and the ratios of others.[10]

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7. Thorp, "Intelligible Matter," 1.

8. Aristotle, *Metaphysics,* in *Complete Works of Aristotle: The Revised Oxford Translation,* ed. Jonathan Barnes (Princeton, NJ: Princeton University Press, 1984), 2:1036a2–12.

9. Ibid., 1036b36–1037a5.

10. Ibid., 1061a29–b1.

Looking at these different texts together, we should recognize that a mathematical quantity, having been abstracted from sensible matter, is no less of an individual than it was before the abstraction occurred. It is still individuated by matter—the same matter, in fact, by which it had been individuated previously. All that has changed is that extraneous accidents are no longer considered. This is how we ought to understand Aristotle's claim that intelligible matter is present in sensible things, but not *qua* sensible.[11]

Aristotle's third reference to intelligible matter is found in *Metaphysics* H.6. In this context, he is considering the unity of a definition, and how the genus and differentia correspond to matter and form, respectively.

> Of matter some is intelligible, some perceptible, and in a formula there is always an element of matter as well as one of actuality; e.g. the circle is 'a plane figure'.[12]

Aristotle's language here seems to imply that intelligible matter ought to be understood as the principle of potency in an essence—that, in other words, it is represented by the genus of a definition. This would seem to make sense, since an essence is intelligible, and a genus can be considered, analogously, as its material principle.[13] Nevertheless, as there is disagreement about how to read this H.6 text in conjunction with the Z.10 and Z.11 texts, we ought to examine Aquinas's interpretation. Doing so will likely reveal the way in which these texts informed his own work in his commentary on *De Trinitate.* Additionally, his approach has the feature of being able to reconcile the meanings of the Z.10, Z.11, and H.6 texts, suggesting that he does, in fact, provide the best possible interpretation.

11. Thorp, "Intelligible Matter," 2.

12. Aristotle, *Metaphysics* 1045a33–6.

13. Thorp, "Intelligible Matter," 2.

Looking, therefore, to Aquinas's reading of this text, we find that he does not understand intelligible matter as simply the principle of potency in an essence. Rather, he understands intelligible matter as the principle of potency in mathematical objects specifically, thereby placing greater significance in Aristotle's mathematical example than other readers might.[14] To understand Aquinas's reasoning, we must first distinguish between the essences of individual sensible objects and the essences of individual intelligible objects. An essence, of course, is itself a non-sensible, intelligible thing. Universally predicable, the only notion of matter that an essence can include is undesignated (common) matter. Nevertheless, even individual (i.e., non-universally predicable) objects themselves can vary in their degrees of materiality.

I will give a concrete example. Just as there is a single essence common to each individual elephant, so too is there a single essence common to each individual mathematical circle. Now, it is evident that these two situations are not entirely alike. The individual in the first case, *this elephant,* is a non-abstract, material object, while the individual in the second case, *this mathematical circle,* is an abstract, immaterial object. Consequently, when considering the essences corresponding to each, we find that while the essence *elephant* only requires one abstractive operation (*abstractio totalis*), the essence *mathematical circle* requires two (*abstractio formalis* and *abstractio totalis*).[15] As John Thorp explains, "within the class of essences, all of which are intelligible objects, there is one group that makes reference to sensible things, things with sensible matter, and another that makes reference to intelligible things, things with intelligible matter."[16]

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14. Thorp, "Intelligible Matter," 4.

15. McInerny, *St. Thomas Aquinas,* 91.

16. Thorp, "Intelligible Matter," 5.

How does this distinction shed light on Aquinas's interpretation of the H.6 text? In brief, it allows us to understand the use of *intelligible matter* in a way that is consistent with the use of the term in the Z.10 and Z.11 texts. In the H.6 text, we can read Aristotle as simply making the argument that an essential definition makes reference to both form and matter. He takes for granted that no one would dispute this claim when the definition is of an object in sensible matter. For this reason, he focuses on the less obvious scenario, namely, when the definition is of an intelligible object (i.e., a mathematical essence). In other words, he is stating that, among the definable essences, some essences are of intelligible objects (i.e., mathematical objects), but that even these objects have a material principle referenced in their definitions (i.e., intelligible matter). In all three texts, therefore, Aristotle, as Aquinas understands him, depicts intelligible matter as the non-sensible material substance underlying quantities as such.[17]

Although it seems unlikely that we will ever have a perfectly clear picture of what intelligible matter is (in fact, a "picture" of a non-sensible is actually quite impossible), nevertheless, in light of what we have considered above, we can have a good sense of what intelligible matter is by way of analogy. This is the approach that Aquinas takes in the *Summa,* presenting intelligible matter as analogous to the more knowable sensible matter: "sensible matter is corporeal matter as subject to sensible qualities, such as being cold or hot, hard or soft, and the like: while intelligible matter is substance as subject to quantity."[18]

In a similar vein, De Koninck presents a satisfying way of understanding the paradoxical term *intelligible matter,* also by way of analogy to sensible matter:

> Now the ineffable individuals of mathematics, like those of physics, must require something extrinsic to 'what' they are to distinguish them from one another, some subject

17. Thorp, "Intelligible Matter," 5; Anderson, "Intelligible Matter," 565.

18. Aquinas, *ST,* I, q. 85, a. 1, ad 2; Anderson, "Intelligible Matter," 565–66.

> analogous to the designatable matter of the bowling pins. Yet there is a profound difference. In the first case it is *this* individual sensible matter. The latter too is a *this,* and in the nature of matter, but not sensible, for we neither can, nor need verify it in the sense experience. The mind nevertheless does reach it, inasmuch as we are quite clear about 'two or more circles of the same radius,' even though we could never designate them to external sense. So we call the matter of these mathematical individuals 'intelligible', in the sense that it can be reached only by mind, and is not the individual matter of the external sense experience.[19]

This compels me to make one final observation about intelligible matter. While this point may have been understood as implicit in my discussion already, I believe it ought to be stated as clearly as possible. It brief, it should be noted that intelligible matter, like sensible matter, admits of the distinction between *designated* and *undesignated.* I alluded to this fact previously by mentioning that mathematical essences require both *abstractio formalis* and *abstractio totalis.* In other words, a mathematical individual, brought forth through *abstractio formalis,* is distinct from other mathematical individuals of the same form through designated intelligible matter (e.g., multiple congruent triangles). To consider, however, what is common to each of these individuals, the additional operation of *abstractio totalis* is required. This operation, in turn, yields a single mathematical essence, the intelligible matter of which is undesignated or common. We can find Aquinas himself making this distinction in the following passage from *De Veritate:*

> There are two kinds of matter from which abstraction is made: intelligible matter and sensible matter. … Each, however, can be taken in two ways: as designated and as not designated. I call matter *designated* if it is considered together with the determination of its dimensions, that is, with these or those dimensions. I call it *not designated,* however, if it is considered without the determination of its dimensions. In this connection, it must be noted that designated matter is the principle of individuation, from which every intellect abstracts inasmuch as it is said to abstract from the here and now. The intellect of the natural philosopher however, does not abstract from non-designated sensible matter; for it considers man, flesh and bone, in whose definitions non-designated sensible matter is

19. De Koninck, "Abstraction from Matter," 64.

> included. The intellect of the mathematician, however, abstracts entirely from sensible matter, though not from non-designated, intelligible matter.[20]

To summarize: through mathematics' characteristic mode of abstraction, *abstractio formalis,* the mathematician will always abstract from sensible matter, both designated and undesignated. The mathematician will also abstract from designated intelligible matter (through *abstractio totalis*) when considering a mathematical essence rather than an individual. The mathematician, however, can never abstract from undesignated intelligible matter, which would be an attempt to consider quantity, an accident, without an underlying substance (a violation of the principle of dependent intelligibility). For this reason, undesignated intelligible matter necessarily belongs to the universal object of mathematics.[21]

Before proceeding to the next chapter, it might be beneficial to review the various concepts of matter I have discussed so far, especially with regard to their relation to physics and mathematics. First, there is the distinction between *prime matter* and *relative matter,* i.e., between "pure potency underlying substantial change" and "material substance underlying accidental change." Second, there is the distinction between *designated matter* and *undesignated matter,* i.e., between "particular matter" and "universal matter." The first distinction is made with regard to matter's role as an underlier of change, while the second distinction is made with regard to its knowability (i.e., whether it is knowable by the senses or by the intellect). Finally, we now have the distinction between *sensible matter* and *intelligible matter,* i.e., between "matter belonging to physical objects" and "matter belonging to mathematical objects."

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20. Thomas Aquinas, *The Disputed Questions on Truth,* Vol. I: Questions I–IX, trans. Robert W. Mulligan (Chicago: Henry Regnery, 1952), q. 2, a. 6, ad 1.

21. Anderson, "Intelligible Matter," 570.

When considering all of these notions of matter in the context of physics and mathematics, we should note the following: 1) although not studied directly, prime matter plays a role in physics insofar as it underlies substantial change; 2) since mathematical objects do not undergo substantial change, prime matter is not particularly relevant to mathematics; 3) both physics and mathematics are concerned with relative matter, i.e., material substance underlying accidents; 4) although both physics and mathematics examine individuals (and are thus concerned with designated matter), it is the ability to abstract undesignated matter from these individuals—i.e., to consider their essences in the abstract—that makes both sciences possible; 5) as discussed, physical objects include sensible matter, while mathematical objects only include intelligible matter, although both kinds of matter admit of the distinction between *designated* and *undesignated.*

Having thus examined our ability to know mathematical objects apart from sensible matter, I can now begin to address the central question of this thesis more directly. To do so, I will turn to question six, where Aquinas discusses the methods proper to each science. In this context, we will need to rely heavily upon our understanding of each science's formal *ratio* and distinct mode of abstraction, as discussed over these past several chapters. We will begin to see how our different modes of knowing play a crucial role in our ability to ascertain truth in each of the sciences, which, in turn, will lend greater weight to our ultimate investigation into the nature of intermediate sciences.

6 SCIENTIFIC METHODS

Up to this point, we have followed Aquinas in his examination of the subjects of each science—"what they are about," so to speak. His discussion has addressed how each science's proper objects exist in reality, as well as how they exist in the mind, resulting in a discussion on the three modes of abstraction. Now, Aquinas will examine each science's characteristic methods of procedure—"how they are done"—and maintain that each has its own unique method proper to its subject matter. This position bears directly on the main question of this thesis. In effect, by holding that each science has a method proper to it, Aquinas dissuades us from attempting to overextend the method of one science into the domain of another. Why is this significant? As we shall see in the discussion below, there is one science—mathematics—which produces greater certainty than the other two. While I have yet to discuss the reasons for this, we should anticipate that this will produce further questions about the relation between physics and mathematics. In particular, if the methods of mathematics and physics are unequal in the certitude they produce, how are the intermediate (i.e., physico-mathematical) sciences impacted?[1] Let us turn to the first article of question six, where Aquinas considers the question, "Must we Proceed according to the Mode of Reason in Natural Science, according to the Mode of Learning in Mathematics, and according to the Mode of Intellect in Divine Science?"[2]

The Method of Physics

Aquinas proposes that the method of physics ought to be considered *according to the mode of reason.* Before we continue, it is worth noting the similarity between the term *reason*

1. Maurer, "Introduction," in Aquinas, *SBT*, xxxi–xxxii.

2. Aquinas, *SBT,* q. 6, a. 1, p. 59.

and the terms descriptive of the other two methods, especially *intellect* in the case of metaphysics. His uses of these terms should be distinguished. Maurer clarifies:

> For St. Thomas, reason and intellect are not really distinct powers of man. They are one and the same intellectual power by which we know in different ways. Through reason we move from the known to the unknown, advancing from one thing to another in our conquest of truth. Through intellect we grasp an intelligible truth simply and intuitively, without any movement or discourse of the mind.[3]

Accordingly, Aquinas sets out three ways in which a scientific method could be considered rational. The first is by proceeding according to its principles (e.g., "beginning with mental beings, like genus, species, opposite," etc.), which is to follow the "propositions taught in logic."[4] This method, however, is not distinctive of physics. Rather, it is necessary to follow this method in any science, for it is only through proceeding logically from the principles of a science that we are guaranteed certitude. Nevertheless, this method is particularly characteristic of logic itself (which he describes as "having a teaching function in the other sciences"), and also of metaphysics, "because both are universal sciences and in a sense treat of the same subject."[5] This refers to his earlier discussion on the subject of metaphysics, in which he shows that metaphysics treats of the universal principles of all beings. In this way, metaphysics and logic are particularly alike, in that they are both concerned with mental beings (genus, species, opposite, etc.) as universal principles.

The second way in which a method can be considered rational is according to "the end that terminates the thinking process."[6] Aquinas explains, "For the ultimate end that rational

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3. Maurer, "Introduction," in Aquinas, *SBT*, xxxiii.

4. Aquinas, *SBT,* q. 6, a. 1a, c., p. 63.

5. Ibid., pp. 63–64.

6. Ibid., p. 64.

inquiry ought to reach is the understanding of principles, in which we resolve our judgments. And when this takes place, it is not called a rational procedure or proof but a demonstration."[7] Demonstration, therefore, is the ultimate goal of any rational process. Because of this, any scientific method that is directed toward demonstrative knowledge can be considered rational according to this second way. This second way, however, is subdivided into two categories. In brief, this subdivision is made because in any method ordered toward demonstrative knowledge, demonstrative knowledge will either be achieved, or it will not. But in either case, the method can be considered rational insofar as it is ordered to demonstrative knowledge as an end. In the cases when demonstrative knowledge is not achieved, the rational method is still useful in crafting probable arguments, which use logic as an instrument. Probable arguments, while not demonstrative in themselves, help pave the way for demonstrative proofs, since they follow the principles of logic and are ultimately ordered toward demonstrative knowledge.[8]

Both of the methods described above employ the principles of logic to achieve demonstrative knowledge. Neither, however, are unique to physics. This brings us to the third way in which a scientific method can be considered rational. In this way, a method is rational, not on account of the logical principles used, but rather, on account of the rational power of the soul. It is so called because "we follow the manner proper to the rational soul in knowing."[9] It is in this way that we say the rational method is proper to physics.

Examining this third way more closely, Aquinas identifies two senses in which physics follows the method of the rational soul. First, physics "proceeds from what is better known to us

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7. Aquinas, *SBT,* q. 6, a. 1a, c., p. 64.

8. Ibid., pp. 64–65.

9. Ibid., p. 65.

and less knowable in its own nature."[10] This follows the learning of the rational soul, since we first learn through the senses. The process of acquiring knowledge begins with the building of individual sensible experiences, which, in turn, allow us to arrive at universal knowledge through analysis. Since the objects we study in physics are sensible objects (and we seek universal knowledge of these objects), the science itself is structured in such a way that is directly imitative of the human reasoning process. In other words, in physics, we begin by looking at sensible effects, and then we move from those sensible effects to the non-sensible causes of those effects. Every time we abstract the nature of a sensible object, this is what we do. Accordingly, knowledge of causes through their effects is characteristic of rational thought, for it is a kind of movement from "sign to the thing signified."[11]

Second, physics follows the manner of the rational soul by moving from knowledge of one thing to knowledge of another. Now, this is not exclusively true of physics, in the sense that this movement from one thing to another can be between things distinct in the mind but not in reality (e.g., in a demonstration through a proper definition or formal cause). Nevertheless, physics is unique in that its demonstrations can move between things distinct in reality. Such a movement occurs when one moves from knowledge of an effect to knowledge of its extrinsic cause (i.e., either an efficient or a final cause).[12] In other words, the demonstrations in physics are uniquely characteristic of reason, for they move from one thing to another in the most

10. Aquinas, *SBT,* q. 6, a. 1a, c., p. 65.

11. Maurer, "Introduction," in Aquinas, *SBT*, xxxiv.

12. Aquinas, *PN,* par. 20.

complete sense. Thus "moving from one thing to another" is most appropriately said of physics, causing its method to be aptly considered *according to reason.*[13]

The Method of Mathematics

Aquinas next investigates the method of mathematics, proposing that it ought to be characterized as *according to the mode of learning.*

> I reply that mathematical science is said to proceed according to the mode of learning, not because it alone does so, but because this is especially characteristic of it. For, since learning is nothing else than the taking of knowledge from another, we are said to proceed according to the mode of learning when our procedure leads to certain knowledge, which is called science. Now this occurs particularly in the mathematical sciences. Because mathematics is situated between natural and divine science, it is more certain than either.[14]

According to the mode of learning may seem to be a strange way of distinguishing mathematics from the other sciences. Does not the taking of knowledge from another occur in all the sciences? Aquinas does not deny this. Rather, he simply holds that learning is most characteristic of mathematics because the knowledge taken in mathematics is more certain than the knowledge taken in the other sciences. He proceeds to show how mathematics is the most certain science, and, therefore, most aptly characterized by the mode of learning.

Reason's use in mathematics is similar to its use in physics, with the difference being, as noted, that mathematics does not demonstrate through efficient or final causes. Rather, mathematics only demonstrates through formal causes. Such demonstrations begin from an object's definition or principles, and they end by arriving at a previously unknown property of that object. For example, from the definition of a triangle, we can prove that the sum of its

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13. Maurer, "Introduction," in Aquinas, *SBT*, xxxiv.

14. Aquinas, *SBT,* q. 6, a. 1b, c., p. 67.

interior angles is equal to two right angles. Since, however, mathematical demonstrations move from one thing to another in a more limited way than physical demonstrations, we do not consider reason to be particularly distinctive of its mode, but rather learning. We ascribe this term to mathematics simply because it is the easiest of the three sciences to learn, insofar as it is the clearest and the most certain to us.[15] While we might initially balk at this claim (rightly objecting that mathematics can be very difficult to learn), to understand Aquinas's point, we need look no farther than the modern predilection for quantifying knowledge. Today, quantification is typically considered the gold standard for objective certitude, even in subjects that are not strictly mathematical. As this article suggests, such an outlook is not entirely a good one, but it does reveal just how much trust we place in the certitude of mathematical knowledge.

With regard to certainty, Aquinas understands mathematics as something of a "mean" between the less certain "extremes" of physics and metaphysics. According to their subjects' inherent knowability, the three sciences can be viewed in an ascending scale (i.e., from less to more knowable), beginning with physics and ending with metaphysics. However, according to their subjects' knowability to the human intellect, the scale is inverted, such that the order of less-to-more knowable begins with metaphysics and ends with physics. Viewed from either direction, mathematics is the only science which can never be considered "least knowable." In this way, we can begin to understand why Aquinas considers it the most certain of the three.

Mathematics is more certain than physics because it abstracts from motion and sensible matter, meaning that it is further removed from change and contingency:

> It is more certain than natural science because its investigation is not bound up with motion and matter, while the investigation of natural science centers upon matter and motion. Now from the very fact that natural science deals with matter, its knowledge depends upon many factors: upon the consideration of matter itself, of form, and of the

15. Maurer, "Introduction," in Aquinas, *SBT*, xxxv.

> material dispositions and properties accompanying form in matter. And whenever there are many factors to be considered in order to know something, knowledge is more difficult.[16]

Elaborating, Aquinas points to motion as a cause of diminished certitude in physics. Demonstrations in physics, he says, "are often valid only in the majority of cases" because the objects "are mobile and lack uniformity."[17] Further, particularity (which results from the composition of form and matter) presents more factors that need to be considered in a demonstration. Where there are more factors to be considered, there is a greater chance that something will be omitted, resulting in an error (e.g., medicine, alchemy, and ethics). For the same reason, the variability of particulars also increases the likelihood of error in physics.[18]

As noted, in physics we reason from visible effects to unseen causes. Because of this, there is a greater possibility for us to err in our reasoning. Aquinas points to Ptolemaic astronomy as an example.[19] Although it appears to describe the motions of the heavenly bodies, Aquinas notes that the Ptolemaic account is not definitive. He explains that it could simply be "saving the appearances," i.e., that the apparent motions of the heavenly bodies could be accounted for in a different way.[20] This example touches on the main question of this thesis, for it directly concerns the epistemology of astronomy, an intermediate science. It indicates that not even a mathematical approach (the most certain of the scientific methods) can ensure mathematical certitude in the study of physical objects. Although this shortcoming results from

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16. Aquinas, *SBT,* q. 6, a. 1b, c., pp. 67–68.

17. Ibid., p. 68.

18. Ibid.

19. Aquinas, *ST,* I, q. 32, a. 1, ad 2.

20. Maurer, "Introduction," in Aquinas, *SBT*, xxxvi.

the materiality of the objects, the question, nonetheless, seems to be, "What good is mathematics in the physical arena, if it is so limited by the uncertainty of sensible objects? How could mathematics contribute to our understanding of these things?" Of course, there is still a good deal more to examine before proposing an answer.

Mathematics is more certain than metaphysics, simply because the objects of metaphysics are above human reason. We are not able to fully comprehend them, so our knowledge of them is less certain to us (i.e., we do not know how well we are understanding these objects).[21] As Aquinas explains, "The method of mathematics is also more certain than the method of divine science, because the objects of divine science are further removed from sensible things, from which our knowledge takes its origin."[22] The objects of metaphysics, whether separate substances (i.e., immaterially subsisting) or common principles (i.e., not immaterially subsisting—substance, act, potency, etc.), are not sensible in themselves because of their immateriality and, for the same reason, are not able to be imagined. Rather, both separate substances and common principles are knowable to us through their sensible effects. The objects of mathematics, on the other hand, are able to be imagined, since quantity is sensible.[23] For this reason, therefore, "human intellect, which takes its knowledge from images, knows these things with greater ease and certainty" than the objects of metaphysics.[24]

Thus, Aquinas concludes that "mathematical inquiry is easier and more certain than physical and theological, and much more so than that of the other sciences that are practical; and

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21. Maurer, "Introduction," in Aquinas, *SBT*, xxxvii.

22. Aquinas, *SBT,* q. 6, a. 1b, c., p. 68.

23. See the following chapter, where I discuss the role of imagination in mathematics.

24. Ibid., pp. 68–69.

for this reason it is said especially to proceed according to the mode of learning."[25] Having thus discussed the methods of both physics and mathematics, I will next give a brief overview of the method of metaphysics, even though it does not directly factor into the main question of this thesis. Since I will subsequently address judgment in the sciences (i.e., how the truth of each is verified), presenting all three scientific methods here will provide the best context for understanding the critical doctrine we are about to examine.

The Method of Metaphysics

Aquinas concludes by investigating the method of metaphysics, proposing that it ought to be characterized as *according to the method of the intellect.* He begins by distinguishing reason from the intellect:

> Now reason differs from intellect as multitude does from unity. Thus Boethius says that reasoning is related to understanding as time to eternity and as a circle to its center. For it is distinctive of reason to disperse itself in the consideration of many things, and then to gather one simple truth from them.… Conversely, intellect first contemplates a truth one and undivided and in that truth comprehends a whole multitude, as God, by knowing his essence, knows all things.[26]

Here, Aquinas explains that intellectual thinking is both the beginning and end of rational thinking. Intellectual thinking is the end of rational thinking insofar as it is the ultimate goal of rational thinking, terminating the process. This occurs through analysis, "in which reason gathers one simple truth from many things."[27] Intellectual thinking is the beginning of rational thinking insofar as it allows the intellect to "comprehend a multiplicity in unity."[28] This occurs through

25. Aquinas, *SBT,* q. 6, a. 1b, c., p. 69.

26. Ibid., q. 6, a. 1c, c., pp. 70–71.

27. Ibid., p. 71.

28. Ibid.

synthesis. Both of these statements are true because, according to the human mode of knowing, analysis precedes synthesis. In other words, knowledge of causes (and, implicitly, of their multitude of effects) comes about by universalizing from particulars—the process of analysis. The perfection of this process, however, is when knowledge of these particulars corresponds to the order of being; this is the process of synthesis, in which the intellect apprehends the act of a principle so thoroughly that it can anticipate the effects that it brings into being.

Metaphysics deals with entirely non-sensible and non-imaginable objects. They are purely intelligible, which cannot be said of the objects of the other sciences. This seems to be the reason first and foremost why *method of the intellect* is appropriately said of metaphysics. Of course, it still employs a rational process because it is a human science. But unlike in physics, where extensive processes of reason are employed, in metaphysics, the reasoning is so brief and simple that it closely approximates intellectual intuition. And unlike in physics, where analysis predominates due to our need for drawing a universal from a multitude of objects, in metaphysics, synthesis predominates due to our need for drawing out a multitude of truths from one.[29]

Metaphysics also imitates the intellect in that it is the last science that ought to be studied, being of things furthest removed from the senses. This is in itself imitative of the fact that intellectual understanding of non-sensibles is the terminus of reasoning, which begins with sensibles. This is true whether we are reasoning to God as the First Cause, or to being and its properties as the most universal concepts.[30] Thus, Aquinas states that “all rational thinking in all

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29. Maurer, “Introduction,” in Aquinas, *SBT*, xxxvii.

30. Ibid.

the sciences, following the way of analysis, terminates in the knowledge of divine science."[31] The objects of all other sciences are none other than the diverse effects of the first principles of being; therefore, if analyses of these many particulars are taken to their ultimate terminus, the intellect arrives at their first principles, the proper objects of metaphysics (i.e., separate substances). Thus, divine science is called "first philosophy" insofar as intellectual thought is the beginning of rational thought (i.e., by giving principles to the other sciences), but it is called "metaphysics" according to the order of learning.[32]

Termination of Judgment

In the second article of question six, Aquinas addresses whether imagination is used in the study of metaphysics. To answer this question, he discusses the epistemic foundations of each science, distinguishing how physics, mathematics, and metaphysics are verified through the senses, imagination, and intellect, respectively.[33] As I have previously indicated, this discussion will prove to be of vital importance for understanding the relation between physics and mathematics, particularly with regard to their distinct epistemic foundations.

First, Aquinas notes that since all human knowledge begins in the senses, all the sciences must begin there as well. Likewise, our knowledge ends in the intellect; however, the intellect forms its judgments differently for the different kinds of objects it apprehends. Thus, judgments are made differently in each of the different sciences, and they are accordingly said to *terminate* in different faculties of knowledge (either in the senses, imagination, or intellect). In other words,

31. Aquinas, *SBT,* q. 6, a. 1c, c., p. 72.

32. Ibid., p. 73.

33. Maurer, "Introduction," in Aquinas, *SBT*, xxxix.

each faculty is the basis of verifying the truth of judgments in one of the three respective sciences.

In physics, judgment terminates in the senses, meaning that our scientific knowledge is based entirely upon what we have perceived with our senses. Although demonstrations certainly do occur in physics, they are nonetheless based on premises taken from sense experience. As such, it is all the more evident why physics ought to be considered the least certain of the sciences. Analogously, in mathematics, judgment terminates in the imagination. This means that it is the imagination that provides evidence for mathematical judgment, although at this point it may not be entirely clear as to why. I will investigate this notion further in the upcoming chapter. Finally, in metaphysics, judgment terminates in the intellect. This must be the case, for neither the senses nor the imagination can grasp purely immaterial objects. Now, this does not mean that the metaphysician cannot begin with knowledge obtained through the senses and the imagination—this is altogether necessary. Nevertheless, the judgments he makes are based entirely upon his intellectual apprehension of such objects.[34]

To conclude this chapter, I should highlight the significance of Aquinas's position that there are distinct methods proper to each science. The implication seems to be that although mathematics is the most certain of the sciences, attempting to take a mathematical approach to the other sciences would be altogether misguided, no matter how tempting.[35] This claim, however, seems to preclude the very possibility of intermediate sciences, sciences that Aquinas himself acknowledges. Thus, it is Aquinas's own system that creates an apparent contradiction,

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34. Maurer, "Introduction," in Aquinas, *SBT*, xxxix.; Acerbi, "Aquinas's Commentary," 322–25.

35. Maurer, "Introduction," in Aquinas, *SBT*, xxxix.

provoking the main question of this thesis: “How can mathematics provide knowledge of physical objects?”

To lay my final groundwork before addressing this question, I will once again look closely at mathematical objects, this time to investigate their basis in reality. In particular, I will confront the fact that mathematical objects reside in the imagination, which makes it appear uncertain whether mathematical truths can apply to physical objects. This last inquiry will be critical for understanding the way mathematics relates to the physical world, and it will allow us to ascertain its value there in the following chapter.

7 MATHEMATICS' BASIS IN REALITY

There is a frequently asked question that might be worth addressing at this point. The question is whether mathematical objects would exist if there were no mathematicians to conceive of them. Of course, one might reply, "Yes, because they would still exist in the divine mind." However, this answer is unsatisfying in that it sidesteps the question really being asked: "To what proximate efficient cause do mathematical objects owe their existence?"

It is likely—if not altogether certain—that certain "perfect" mathematical forms are never actually encountered in the physical world. Now, I say "perfect" colloquially, as one would describe, for example, a "perfect circle." Nevertheless, this use of the word can lead to some confusion. Saying "perfect" of certain mathematical forms seems to imply that other forms are somehow "imperfect" as mathematical objects (e.g., the "slightly misshapen circle"). In fact, both forms (considered in the abstract) are equal in their status as mathematical objects. The properties of a figure that closely approximates a circle can be known mathematically just as those of a true circle, albeit with greater difficulty.

To speak precisely, the notions of such mathematical entities are actually composed of various elements. In the example of a perfect circle, the notion includes both: 1) a line, and 2) equidistance from a center. Our reason allows us to compose these two ideas, having recognized that they are not opposed to each other. Thus, when investigating the properties of a perfect circle, we can arrive at valid and necessary conclusions, without having to depend on the actual existence of such a circle in the physical world. By the same reasoning, the imperfectly circular figure is just a more complex figure (i.e., composed of more elements) than the perfect circle. The fact that we may only encounter imperfectly circular figures in the physical world presents no problem for our ability to know a perfect circle. Incidentally, it may be the case that our ability to know figures without actually seeing them (e.g., the perfect circle) is what led geometers at one time to regard their objects as Platonic Ideas, rather than real quantities abstracted from nature. In other words, they failed to recognize the constructive role of our reason in bringing forth these "perfect" figures from the elements of quantity.[1]

To return, therefore, to the initial question, we must distinguish between composed mathematical objects (e.g., circles, numbers, etc.) and mathematical elements and principles (e.g., lines, surfaces, unity, equality, etc.). From what has been said above, it seems fair to say that composed mathematical objects are, in a manner of speaking, immaterial human artifacts, for they come to be through the composition of elements according to human ingenuity. Thus, to wonder whether these mathematical "artifacts" would exist if there were no mathematicians to conceive of them seems no different from wondering whether artifacts of any kind (e.g., houses, automobiles, paintings, etc.) would exist if there were no artificers to conceive of them. The answer is simply, "No." Of course, God would still know the infinitude of potential artifacts that could be derived from the principles of nature, but to point this out is not especially helpful. Without an artificer, artifacts can only exist in potency.

But how should we approach this question with regard to mathematical elements? Would points, lines, and surfaces exist if there were no mathematician to conceive of them? If these mathematical elements were inherent in natural bodies, we would certainly hold that they exist without the need for a mathematician. But the question remains, "How can mathematical elements exist in natural bodies?" If we consider a natural body at the atomic or subatomic level, it seems impossible to say that Euclidean points, lines, and surfaces belong to the body in

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1. Jacques Maritain, *The Degrees of Knowledge,* trans. Gerald B. Phelan (Notre Dame, IN: University of Notre Dame Press, 1995), 153.

actuality, even though there is the potential for designating their positions. Such a designation, however, would be physically arbitrary and thus require the work of a mathematician. In other words, since being abstracted from matter is essential to the nature of a mathematical object, we must hold that no mathematical object, simple or composed, would exist without an abstractive act of the mind. Maritain explains:

> Although mathematical entities can exist only in matter—to the extent that they can exist outside the mind—nevertheless they do not exist in matter as mathematical entities, or in a mathematical state. "Straight line," "circle," "whole numbers" are all realized in sensible things, but only by lacking the conditions of ideal purity that the mode of existing mathematically imposes upon them.[2]

We should note that this does not imply that quantity itself is somehow a construct of the mind. The claim that mathematical objects are produced by an activity of the mind may appear to suggest this, but Aquinas would certainly reject this idea. The distinction we must make is between real extramental quantities and abstract mathematical objects. Our first task is to understand how the two are related.[3] Following this, we will need to consider how the basis in reality of mathematics relates to that of physics. In other words, we must yet again compare the *abstractio formalis* of mathematics with the *abstractio totalis* of physics, this time anticipating what the epistemic implications will be for the intermediate sciences. This will lay the final groundwork for addressing the question of this thesis, "How can mathematics provide knowledge of physical objects?"

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2. Maritain, *Degrees of Knowledge,* 57–58.

3. Thomas C. Anderson, "Aristotle and Aquinas on the Freedom of the Mathematician," *Thomist* 36, no. 2 (April 1, 1972): 243–44.

Concepts with a Remote Basis in Reality

Maurer notes that Aquinas's fullest treatment of mathematics is found in his commentary on *De Trinitate,* which, of course, is the very text we have been examining.[4] Perhaps as a consequence of Aquinas's limited writing on the subject, there is not a full consensus regarding his mathematical epistemology.[5] Nevertheless, Maurer recommends a relatively obscure *quaestio disputata* to complement Aquinas's treatment of mathematics in this work. In this *quaestio,* Aquinas distinguishes three ways in which a concept can relate to extramental reality, and I will present Maurer's own summary here:[6]

> (1) A concept may be a likeness of a reality outside the mind, for example 'man.' A concept of this sort has an immediate foundation in extramental reality: the reality causes the truth of the concept through the conformity of mind and reality, and the term signifying the concept is properly predicated of the reality.
>
> (2) A concept may not be a likeness of an extramental reality, but the mind may devise *(adinvenit)* it as a consequence of our way of knowing extramental reality. A concept of this sort has only a remote foundation in reality; its immediate basis is an activity of the mind itself. An example is the concept of genus. There is nothing outside the mind corresponding to this concept, but from the fact that we know there are many species of animals we attribute to animal the notion of genus. The proximate foundation of a concept of this sort is a constructive act of the mind; but the concept has a remote basis in the extramental world, so the mind is not mistaken in forming it.
>
> Another example of this type of concept suggested by Aquinas is the abstraction of the mathematicians, or the abstraction of mathematics (*abstractio mathematicorum*). He is not referring to the mathematician's act of abstracting but to the mathematical concept or *intentio* he forms by means of this act. . . .
>
> (3) A concept may have neither a proximate nor a remote foundation in reality, like the concept of a chimera. This is not a likeness of anything in the world, nor do we

4. Maurer, "Foundation of Mathematics," 59–60.

5. Ibid., 47.

6. "Composed at Rome between 1265 and 1267, Aquinas regarded the *quaestio* as so important that he inserted it into his commentary on the *Sentences,* which he had written a decade earlier (1252–1256). Thus the *quaestio* dates from his mature years, when he was a master in theology, unlike the rest of the commentary, which he wrote as a bachelor of the *Sentences.* It cannot be dismissed, therefore, as an early expression of Aquinas's teaching on mathematics" (ibid., 52).

> form it as a consequence of our way of knowing the world. Hence Aquinas calls it a false concept.[7]

Of course, what is noteworthy here is that Aquinas places mathematical entities in the same category as logical notions. As a matter of fact, this is a significant way in which he develops the Aristotelian understanding of mathematics. He is in agreement with Aristotle by maintaining that mathematical objects exist only in the mind, but he also suggests something further. Rather than speaking of mathematical objects as merely the things "remaining" after sensible qualities have been disregarded, he regards them as similar to logical notions, insofar as they are "intentions which our intellect devises (*adinvenit*) because of its knowledge of extramental things."[8] In other words, he attributes a more constructive role to the intellect in producing mathematical objects. To offer an analogy: whereas Aristotle's mathematician is like a sculptor, chiseling a statue from rock, Aquinas's mathematician is more like an architect, building an entirely new structure.

Now the extent to which Aquinas intends to compare mathematical with logical notions may be open to interpretation. For instance, Maurer and De Koninck each interpret Aquinas's comparison slightly differently. While Maurer states that "it seems inevitable to conclude that both are *entia rationis* and 'second intentions,' though not of the same kind,"[9] De Koninck states plainly that mathematical objects "differ from second intentions."[10] Nevertheless, although they may not be in complete agreement as to whether mathematicals are rightly considered second intentions, Maurer and De Koninck seem to agree on the following points: 1) mathematical

7. Maurer, "Foundation of Mathematics," 53.

8. Anderson, "Freedom of the Mathematician," 243–44.

9. Maurer, "Foundation of Mathematics," 60–61.

10. De Koninck, "Abstraction from Matter," 56.

notions and logical notions differ in that mathematical notions are known for their own sake (applied mathematics notwithstanding), whereas logical notions are simply tools, known only for the sake of knowing other subjects (not to be confused with mathematical logic, a branch of mathematics studied for its own sake); 2) nevertheless, neither mathematical nor logical notions have a direct referent in extramental reality; 3) consequently, both mathematical and logical notions owe their existence to constructive acts of the mind.[11]

Incidentally, these second two points are supported by the phenomenon of quantities seeming to gain certain properties in the abstract. The clearest example of this is the mathematical line, defined by Euclid as a "breadthless length."[12] Such a line cannot be perceived in itself—but, rather, only inferred—in the material world (e.g., "there is a distance between object A and object B that can be understood in terms of only one dimension"). Strangely, however, it seems that the Euclidean line does not simply lack properties from the material world. That it also appears to receive properties from the mind becomes evident when considering the single point of "contact" between the circle and its tangent. In the material world, it is impossible for two bodies to touch at just one point, if a point is defined as "that which has no part."[13] Yet, this is precisely what occurs when the boundaries of these bodies—surfaces and lines—are considered in the abstract.[14] Maurer notes:

> This is true not only in geometry but also in arithmetic. Aquinas distinguishes between a multitude that is numbered or numerable and the number by which we number it. In a sense things numbered or numerable can be called a number, as we speak of ten men or horses. Number itself, however, is that by which they are numbered. This numeration or

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11. Maurer, "Foundation of Mathematics," 60–61; De Koninck, "Abstraction from Matter," 56.

12. Euclid, *Elements,* trans. Thomas L. Heath (Santa Fe: Green Lion Press, 2017), I, definition 2.

13. Ibid., I, definition 1.

14. Maurer, "Foundation of Mathematics," 50–51.

> counting is an act of the human mind. The existence of the multitude is due to the divine mind; its numeration is owing to ours.[15]

To thus return to the initial question of this chapter, we must affirm that all mathematical objects—whether elements or composed quantities—owe their existence to the human intellect as their proximate efficient cause. In other words, to the question of whether mathematical objects would exist if there were no mathematicians to conceive of them, we must reply, "No." Real quantities, i.e., accidents of material substances, are not themselves the proper objects of mathematics.

For greater clarity on this point, we should examine the distinction between the first two categories of concepts distinguished by Maurer, which hinges on predicability. Concepts of the first category (e.g., *man,* considered as an individual) can be validly predicated of things outside the mind, as is the case when we say, "Socrates is a man." On the other hand, concepts of the second category (e.g., *genus, species,* etc.) cannot be predicated of things outside the mind. To illustrate this, we may consider that for the statement, "S is a species," there is no extramental object that can validly stand in the place of subject S. This is likewise the situation with mathematical entities, which can be illustrated with a similar example.

Let us consider a square tabletop. Now, first of all, it is of no consequence whether the tabletop is perfectly square down to the molecular level or not. As I discussed previously, it is not the shape of an object that inhibits its ideal purity as a mathematical object, but rather, it is the fact that the object itself is material. In other words, it is from the very fact that we are speaking of this "square" as being bounded by molecules rather than by breadthless lengths that

15. Maurer, "Foundation of Mathematics," 51–52.

we have our problem. So, for purposes of discussion, let us assume that the tabletop is perfectly square. Given this situation then, we might say, "This tabletop is square." Now, if we meant that, "The quantity inhering in the matter of this tabletop is square," then our statement would be true. If, however, we meant that "This tabletop is a mathematical square," our statement would be as absurd as if we had said, "Socrates is the species man."

But what about the former interpretation? Does not the truth of the statement "The quantity inhering in the matter of this tabletop is square," indicate that there can indeed be an extramental referent for mathematical concepts? Actually, it does not. Notice what happens if I try to modify my predicate to *mathematical square:* "The quantity inhering in the matter of this tabletop is a mathematical square." This is a self-contradictory statement. A mathematical quantity, as defined, is an abstracted quantity. Thus, this modified statement effectively states, "This quantity inhering in matter is an abstracted quantity," which, of course, is absurd. The quantity of which we are speaking is either inhering in matter or it is abstracted from matter—it cannot be both. Through this example, we can see why Aquinas includes mathematical entities in the second category of concepts rather than in the first. Thomas C. Anderson explains:

> Once the nature of this abstraction is grasped it becomes clear how Thomistic commentators, by emphasizing different features of abstracted quantity, can attribute positions to St. Thomas which seem contradictory. … Each is simply stressing a different side of Aquinas' (and Aristotle's) teaching, *viz.,* that mathematics does consider real quantities but not *qua* real. In other words, the mathematician considers real quantities as abstracted.
>
> To say this is to say that mathematical quantities with their peculiar characteristics exist only because of the act of abstraction. And this is to say that mathematical quantities actually exist as such only in the mind of the mathematician for it is only in thought that real quantities are nonsensible, immobile, nowhere, uni, bi, or even no dimensional, and freed from sensible matter.[16]

16. Anderson, "Intelligible Matter," 557–58.

Truth in Mathematics

Having thus considered Aquinas's understanding of the relation between extramental quantities and mathematical objects, we must now consider the epistemic implications. In the following passage from Maurer, we should note that a question of truth immediately arises as a result of Aquinas's account:

> There are important consequences of Aquinas's placing the notions of mathematics in the second order of his *quaestio disputata* instead of the first. Unlike concepts on the first level, those on the second do not properly speaking exist outside the mind. Their proper subject of existence is the mind itself. They are not signs of anything in the external world. Hence mathematical terms cannot properly be predicated of anything real: there is no referent in the external world for a mathematical line, circle, or number. Finally, mathematical notions are not false; but neither are they said to be true, in that they conform to anything outside the mind. Aquinas does not suggest that they might be true in some other sense.[17]

If we hold that mathematical objects are produced by the human intellect, in what way is mathematics still meaningful as a science? Is mathematics merely a system that, while consistent in itself, has no bearing on reality?[18] And with regard to the question of this thesis, how can we understand mathematics in context of the intermediate sciences? Maurer's claims above warrant further examination, for they call into question the possibility that mathematics can be fruitfully applied to the study of physical objects. If—from the standpoint of extramental reality—mathematical notions are neither true nor false, how can physical truths be known through applied mathematics? To answer this question, we must investigate the process by which the mind produces mathematical objects.

We should thus return to the notion of mathematical judgment terminating in the imagination, which was initially addressed in the previous chapter. As discussed, Aquinas holds

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17. Maurer, "Foundation of Mathematics," 56.

18. De Koninck, "Abstraction from Matter," 53–54.

that there is a different termination of knowledge in each of the three speculative sciences, due to our different modes of knowing their respective objects. So, what does it mean for judgment in mathematics to terminate in the imagination, and how does this bear upon the question of truth in mathematics?[19] In what remains of this chapter, I will first discuss how mathematical objects exist in the imagination, and then I will discuss how the intellect refers to the imagination to answer the two questions that, as Maritain holds, every science must address: "first, the question, AN EST—whether the thing exists; and then the question, QUID EST—what is its nature."[20]

The role of the imagination is crucial to Aquinas's understanding of the way in which mathematical objects are known. In fact, that mathematical entities are imaginable (and yet free from sensible matter) is the reason he considers mathematics to be the most certain science. This does, however, present a difficulty, for mathematical objects have already been said to lack all sensible qualities. How can the imagination apprehend something lacking sensible qualities? There is clearly no way to account for Aquinas's claim without admitting that he must, in some way, consider mathematical objects to be sensible.[21]

Fortunately, we can resolve this dilemma by recognizing that quantity itself is a sensible—a common sensible, that is. As Aristotle understands it, a common sensible is something sensed directly, but not as a proper object of one of the five senses. Thus, common sensibles are never sensed apart from one of the proper sensibles. Nevertheless, as something sensed, a common sensible can be apprehended by the imagination. Of course, this is in accord with experience—we can, after all, imagine things as having dimension. In different respects,

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19. Anderson, "Freedom of the Mathematician," 249–50.

20. Maritain, *Degrees of Knowledge,* 57.

21. Anderson, "Freedom of the Mathematician," 246.

therefore, we can think of mathematical objects as both nonsensible and sensible. Insofar as they lack the sensible qualities, which would otherwise render them the proper objects of one of the five senses, they are rightly regarded as nonsensible. To return to the example of the square tabletop, this is why we would be wrong to consider the quantity of the tabletop (as belonging to the tabletop) to be a mathematical object (i.e., since the quantity of *this tabletop* is imbued with sensible qualities). Nevertheless, mathematical objects are rightly regarded as sensible insofar as quantity is a common sensible. Their ability to be objects of science, however, is only realized due to the imagination's apprehension of quantity in the abstract.[22]

As Maritain notes, it is precisely through the fact that quantity precedes sensible qualities in the order of accidents—and yet is itself a common sensible—that we can understand the role of the imagination in mathematics. Although the imagination follows sense experience in a certain way, it is altogether independent from it in another way. Rather than clinging to the extramental conditions that accompany our sense perception of quantity, the imagination does away with these conditions through its abstraction of quantity from its posterior accidents.[23] As Maritain puts it:

> And so it is that the intuitive schemes of the imagination—which are not at all the object itself of mathematics but only the sensible symbol or illustration of that object—manifest to us in a sensible way, though independent of every experimental condition, essences and properties which of themselves precede the sensible order and are independent of it.[24]

Although, of course, mathematics employs logic in its demonstrations (as discussed in the previous chapter on method), nevertheless, mathematical objects are not purely intelligible. For

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22. Anderson, "Freedom of the Mathematician," 246–48; Anderson, "Intelligible Matter," 568.

23. Maritain, *Degrees of Knowledge,* 153.

24. Ibid., 153.

this reason, the inherent possibility of each object's existence—*an est*—can ultimately only be demonstrated through its constructability. This is done through the aid of the imagination, which allows us to verify, sensibly, whether a mathematical object can exist in the ideal realm. In other words, it is through the imagination that the mathematician verifies that an object belongs to Aquinas's second category of concepts (i.e., those having a remote foundation in reality), rather than to the third (i.e., false concepts).[25]

Aquinas describes this process as an *operational demonstration,* which he details in the following passage. Note that once the question *an est* has been answered about the object through this kind of demonstration, the question *quid est* can subsequently be answered, i.e., through demonstrations about the object's properties. Anderson notes:

> There is supposed in these [mathematical] sciences those things which are first in the genus of quantity such as unity and line and surface and other such. These being presupposed, certain other things are sought by demonstration.... These demonstrations are said to be, so to speak, operational, as is: On a given straight line to construct an equilateral triangle. This having been proved, certain further passions are proved, as that its angles are equal or some other such thing....[26]

From this, we can understand more fully what it means for mathematical judgment to terminate in the imagination and, consequently, address the question of truth in mathematics. Operational demonstrations, i.e., demonstrations by way of construction, of course only produce individual mathematical objects. However, by constructing a mathematical individual, operational demonstrations confirm the possible existence of that individual's essence. In this way, the imagination presents evidence allowing the intellect to verify that the concept it apprehends is not a false concept, but rather a true mathematical essence (*an est*). Likewise, the imagination

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25. Maritain, *Degrees of Knowledge,* 153–154; Anderson, "Freedom of the Mathematician," 250–51.

26. Anderson, "Freedom of the Mathematician," 251–52.

also provides the basis for subsequent demonstrations about the properties of this essence (*quid est*). This is what it means for mathematical judgment to terminate in the imagination: judgments that the intellect makes about a mathematical object are either true or false depending upon the object's status in the imagination.[27]

Thus, when Maurer says that "mathematical notions are not false; but neither are they said to be true, in that they conform to anything outside the mind,"[28] he is speaking only about "the conformity of intellect and thing"[29] outside the mind. Of course, mathematical notions cannot be true (or false) in this sense, since there are no extramental referents for mathematical objects. However, a mathematical notion can be true insofar as a conformity of intellect and thing occurs when the intellect makes a correct judgment about a mathematical object as it exists in the imagination. In other words, our intellect's notion of a mathematical essence could be regarded as true if a corresponding individual were shown to exist in the imagination (and, conversely, this mathematical individual could be regarded as a *true thing* if it accurately represented a mathematical essence known by the intellect).[30] In short, because mathematical

27. Anderson, "Freedom of the Mathematician," 244–45, 250–51; De Koninck, "Abstraction from Matter," 66–67. Maritain notes that the intellect's conclusions may be verified in the imagination "either directly or analogically," taking into account areas of mathematics that tend toward pure idealism (e.g., non-Euclidean geometry); he considers these areas to be systems "stemming" from intuitive notions (e.g., Euclidean geometry), "and in which they may find an analogical interpretation" (*Degrees of Knowledge,* 58). Such pure idealism in mathematics, he argues, is valid due to the fact that "mathematical beings … abstract not only from existence but even from any order to existence," resulting in mathematics' ultimate indifference toward extramental reality (154). While further discussion here would take me beyond my scope of applied mathematics, it is worth noting that mathematics' basis in the imagination is likely even more liberating than the ancients had realized.

28. Maurer, "Foundation of Mathematics," 56.

29. Aquinas, *ST,* I, q. 16, a. 2.

30. Ibid., aa. 1–2.

objects are inherently abstract, mathematics as a science attains truth when its propositions conform to objects in the imagination.[31] De Koninck provides an example:

> When we demonstrate that there is a triangle whose sides are equal, we never imply that there is such a figure in reality, either with or without sensible matter. We merely show that there *is* such a definable subject in the sense of truth, and that whatever is demonstrated of it is true of it qua abstracted from all sensible matter.[32]

To conclude, I should return the initial question posed in this chapter: “To what proximate efficient cause do mathematical objects owe their existence?” Of course, my answer is, “The imagination”—but here I will add the qualifier, “directed by the intellect.” As discussed previously, *abstractio formalis* (by which a mathematical object is generated) must occur in the imagination, since what remains—quantity—is a common sensible and can only be apprehended by a sense power. Nevertheless, it belongs to the intellect to define mathematical essences and to thereby guide the imagination in its generation of individuals. We should also note the intellect’s reciprocal dependence upon the imagination, insofar as the intellect relies on the imagination’s ability to construct individuals in order to verify the truth of its own definitions. In short, we must hold that there is an interdependence between the imagination and the intellect in the generation of mathematical objects.[33]

What remains now is to address the main question of this thesis: “How can mathematics provide knowledge of physical objects?” Having just discussed the way in which mathematics is able to yield truth, we can finally confront this question head-on. I have maintained that mathematics, considered in itself, does indeed yield truth in an unequivocal sense. Nevertheless,

31. Anderson, “Freedom of the Mathematician,” 249–50.

32. De Koninck, “Abstraction from Matter,” 63.

33. Ibid., 66–67.

given that the objects of mathematics reside in the imagination, the truth of the science cannot extend to extramental objects as such. From what I have discussed so far, it seems that, at best, mathematics is simply indifferent to the extramental world.[34] How, in spite of this, mathematics can still be applied to the study of physical objects will be discussed in the upcoming chapter.

34. Maritain, *Degrees of Knowledge,* 154.

8 THE INTERMEDIATE "DIVISION"

By Maurer's estimation, Aquinas "gives us the best analysis of physico-mathematical science written in the Middle Ages" in his commentary on *De Trinitate;* nevertheless, these sciences "play a very minor role in St. Thomas' scheme," likely due to the fact that they were still undeveloped and poorly understood at the time.[1] Still, even though the medievals did not often distinguish the various types of natural science (with the result that the term *physics* could refer indifferently to either experimental science or natural philosophy), they, like the ancients before them, noticed something unusual about astronomy, harmony, and optics. Unlike other sciences of physical objects, these sciences make overt use of mathematics in their interpretation of observations. Thus, ancient and medieval Aristotelians regarded these sciences as *scientia media,* intermediate sciences.[2]

As Aquinas understands it, an intermediate science is materially physical and formally mathematical. As Maritain explains, it is formally mathematical "because its rule of interpretation, its rule of analysis and deduction is mathematical," but "it is materially physical because what it assembles and interprets by the help of mathematical intelligibility … is physical reality, physical data."[3] Aquinas's own account of the respective objects and methods of physics and mathematics has provoked the main question of this thesis. How can mathematics be applied to the study of physical bodies in such a way that we achieve an improved understanding of these bodies? This problem is especially difficult because of the limited number of times Aquinas

1. Maurer, "Introduction," in Aquinas, *SBT*, xi.

2. Simon, 96–97; Maritain, *Philosophy of Nature*, 37.

3. Maritain, *Philosophy of Nature*, 37–38.

discusses physico-mathematical sciences in his *Division and Methods.* I will briefly summarize them here.

In the first article of question five, Aquinas describes physico-mathematical sciences as *subalternated* to mathematics (i.e., taking mathematical conclusions as their own premises). In his reply to the fifth objection, he notes that "[t]his is how music is contained under arithmetic."[4] In his reply to the ninth objection, he notes that astronomy "presupposes the whole of mathematics."[5]

Aquinas, however, gives his most significant treatment of physico-mathematical sciences in the third article of question five, discussing them in his replies to objections five through eight. In his reply to the fifth objection, he makes an important point about mathematics' measuring function, which I will discuss further:

> By its very nature motion is not in the category of quantity, but it partakes somewhat of the nature of quantity from another source, namely, according as the division of motion derives from either the division of space or the division of the thing subject to motion. So it does not belong to the mathematician to treat of motion, although mathematical principles can be applied to motion. Therefore, inasmuch as the principles of quantity are applied to motion, the natural scientist treats of the division and continuity of motion, as is clear in the *Physics*. And the measurements of motions are studied in the intermediate sciences between mathematics and natural science: for instance, in the science of the moved sphere and in astronomy.[6]

In his reply to the sixth objection, Aquinas notes that the intermediate sciences are cases of applied mathematics, rather than parts of mathematics. He explains that the principles of a more abstract science can be applied to other sciences (but not vice versa), as is the case with mathematics and physics. Significantly, he argues that physico-mathematical sciences "have a

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4. Aquinas, *SBT,* q. 5, a. 1, ad 5, p. 21.

5. Ibid., ad 9, p. 23.

6. Ibid., q. 5, a. 3, ad 5, p. 43.

closer affinity to mathematics, because in their thinking that which is physical is, as it were, material, whereas that which is mathematical is, as it were, formal."[7] Thus, although the physico-mathematical sciences make demonstrations about physical objects, potentially even arriving at the same conclusions as pure physics, they do so in different ways, a point he further develops in his reply to the seventh objection (with the example, "the natural scientist proves that the earth is round from the movement of heavy bodies, while the astronomer proves it by considering eclipses of the moon").[8]

Finally, in his reply to the eighth objection, Aquinas notes that the heavenly bodies' uniformity of motion is what allows them to be effectively measured by mathematics. Describing the heavenly bodies as "mobile and incorruptible," he distinguishes them from "mobile and corruptible" beings, which cannot be studied mathematically.[9] Disregarding the archaic distinction between corruptible and "incorruptible" physical objects, it seems that the general principle is nevertheless valid and can be applied to the other physico-mathematical sciences as well. In other words, physical sciences which have relatively few material factors to consider (e.g., harmony and optics, in addition to astronomy) are the ones suited for a mathematical approach. This corresponds with the explanation he gives in article six as to why mathematics is a more certain science than physics.[10]

Maritain, however, highlights another time when Aquinas discusses the intermediate sciences—not in his commentary on *De Trinitate*—that would be helpful to include here. As

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7. Aquinas, *SBT,* q. 5, a. 3, ad 6, p. 45.

8. Ibid., ad 7, p. 45.

9. Ibid., ad 8, p. 46.

10. "And whenever there are many factors to be considered in order to know something, knowledge is more difficult." Ibid., q. 6, a. 1b, c., pp. 67–68.

Maritain notes, "In his Commentary on the second book of Aristotle's *Physics,* St. Thomas draws attention to the fact that, while these sciences are formally mathematical they are nevertheless more physical because, says he, their term,—the terminus in which judgement is completed and verified,—is sensible nature."[11] Given his claim in the *De Trinitate* commentary that intermediate sciences "have a closer affinity to mathematics,"[12] it is certainly surprising that Aquinas would claim elsewhere that the intermediate sciences are more physical. In fact, it appears that a misinterpretation of Aristotle is actually what led him to characterize the intermediate sciences in this way.[13] In the passage in question, Aristotle discusses the branches of mathematics and refers to what modern translators understand as "the more physical of the branches of mathematics."[14] Maritain notes that "St. Thomas, on the contrary, in his 3rd lesson on the second book of the *Physics,* understands this expression to mean not the more physical branches of mathematics but sciences that are more physical than mathematical, *magis naturales quam mathematicae.*"[15]

Regardless, however, of what Aristotle had meant to say, Aquinas justifies his own interpretation of Aristotle's text by pointing to the termination of scientific judgment. Maritain explains:

> That enables him to state a very important point of doctrine, namely that the while they remain formally mathematical, these sciences are more physical: *quia harum scientiarum consideratio TERMINATUR ad materiam naturalem, licet per principia mathematica*

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11. Maritain, *Philosophy of Nature*, 38.

12. Aquinas, *SBT,* q. 5, a. 3, ad 6, p. 45.

13. "In Chapter 2, 194a, 7, lib. II of the *Physics,* Aristotle is speaking of mathematical knowledge and he speaks of the branches of mathematics which are more physical than others" (Maritain, *Philosophy of Nature,* 38).

14. Ibid.

15. Ibid., 38–39.

> *procedat.* Their weight as science draws them toward physical existence although their rule of interpretation and deduction is mathematical.[16]

In short, we are faced with two seemingly opposed positions held by Aquinas: on the one hand, the intermediate sciences are more physical than mathematical in that their truth must be verified by the senses; on the other hand, the intermediate sciences are more mathematical than physical in their manner of interpretation and explanation.[17] I would maintain, however, that this does not reflect any inconsistency on Aquinas's part. Rather, this apparent opposition is actually the key to understanding the relation between physics and mathematics in the intermediate sciences.

Since the physico-mathematical sciences are subalternate to their purely mathematical counterparts, we should review the concept of *subalternation,* stated here by Maritain:

> A science is said to be subalternated to another when it derives its *principles* from this other science, which is called the subalternant. The subalternate science does not by itself resolve its conclusions into the first principles of reason, into self-evident principles, but the subalternant science resolves its own conclusions into first principles and these conclusions of the subalternant serve as principles for the subalternate science.[18]

The objects of a subalternate science are the same as those of its subalternant, with the exception that the subalternate science adds an accidental difference (from the standpoint of the subalternant) to the *ratio* of its objects. For example, optics, as subalternate to geometry, studies lines *as visual*; acoustics, as subalternate to arithmetic, studies number *as sounding;* astronomy, as subalternate to "the whole of mathematics,"[19] studies quantity *as mobile.* Again, this is why Aquinas understands these sciences to be formally mathematical. From the standpoint of a

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16. Maritain, *Philosophy of Nature,* 39.

17. Ibid.

18. Ibid., 103.

19. Aquinas, *SBT,* q. 5, a. 1, ad 9, p. 23.

physico-mathematical science's methodology, its object's physicality is but an accidental attribute.[20]

We should keep in mind that the way quantity is approached in physics is different from the way it is approached in mathematics. In physics, quantity is considered as real, as the first accident of material substances. Maritain notes:

> It is precisely under the conditions and modalities of this real quantity, or, to put it in another way, it is as quantitatively measured and regulated, that the interacting causes in nature develop their qualitative activities. *In mensura, pondere et numero.* Physical reality abounds with entitative riches that are irreducible to quantity. But, by reason of its materiality, and from the fact that it emanates from corporeal substance through the intermediary of quantity, this world of qualities is subjected intrinsically to quantitative determinations (and that is why it is accessible to our extrinsic and artificial mensurations).[21]

In this way, it could be said that physics only considers quantity as a part in relation to a whole or to other parts—i.e., in relation to the whole substance to which it belongs or in relation to the qualities that it determines. Physics does not consider quantity simply in itself. Rather, it is mathematics that does this, and it must therefore consider quantity as imaginary—i.e., apart from sensible qualities, through *abstractio formalis*. Although no longer considered as real, quantity can in this manner be understood "from the point of view of the very relations of order and measurement which the objects of thought discernible in it, as forms or essences proper to it, maintain among themselves."[22]

We should also keep in mind the different ways each of these sciences answer the questions *an est* and *quid est* for their respective objects. In physics, these two questions are

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20. Maritain, *Philosophy of Nature*, 103.

21. Maritain, *Degrees of Knowledge,* 151–52.

22. Ibid., 152.

answered by the intellect based on evidence presented by the senses (i.e., judgment terminates in the senses). When physical natures remain obscure (as is often the case in experimental physical sciences), probable arguments are made in place of demonstrations.[23] These arguments, which are built from sense experiences (as opposed to essential definitions), yield scientific laws—laws whose veracity are provisionally accepted, based upon their ability to account for all the data available. When physical natures can be known through abstraction (as is often the case in natural philosophy, e.g., the vegetative soul), true demonstrations regarding their properties can be made. Nevertheless, because these demonstrations are of things sensed, they are only true insofar as their principles are based on true abstractions. In either case, therefore, it is necessary that knowledge of physical objects has its basis in the senses.[24]

In mathematics, the questions *an est* and *quid est* are answered by the intellect based on evidence presented by the imagination (i.e., judgment terminates in the imagination). Accordingly, mathematical demonstrations are concerned with the possible or ideal existence of quantities (*an est*) and, subsequently, with their properties (*quid est*). Since, however, the proper objects of this science are confined to the ideal realm, demonstrations are made through connections in the intellect alone. In other words, after the initial act of *abstractio formalis,* the mathematician operates with indifference toward extramental reality, both with regards to the epistemic basis of his demonstrations, as well as to the viability of his conclusions.[25]

What, then, are the implications for the intermediate sciences, given that they take a mathematical approach to the study of physical objects? Although the objects studied are real

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23. Aquinas, *SBT,* q. 6, a. 1a, c., p. 64.

24. Maritain, *Degrees of Knowledge,* 58–59.

25. Ibid., 57–58.

physical objects, their quantitative attributes cannot be approached as real quantities; rather, they can only be approached as mathematical quantities. In other words, there is evidently a disconnect between the actual objects of a physico-mathematical science (i.e., real physical objects) and the abstract quantities that represent them within the mathematical systems built for their measurement. Thus, perhaps disappointingly, we must conclude that mathematics actually cannot enable us to know physical objects in an ontological sense. Mathematics can, however, enable us to measure a physical object—which, as Aquinas himself suggests, is where the real value of a physico-mathematical science lies.[26]

In fact, this distinction between ontological knowledge and measurement accounts for the apparent contradiction in Aquinas's description of the intermediate sciences (i.e., that they are formally mathematical but also that they are more physical than mathematical). A physico-mathematical science is formally mathematical in that the knowledge obtained is mathematical. Again, the physicality of the objects studied in a subalternate science is but accidental from the standpoint of the subalternant science (i.e., mathematics). Since mathematical judgment terminates in the imagination, its certitude cannot extend into the physical realm. Nevertheless, mathematics can be applied to physical bodies as an extrinsic means of measurement. It is in this sense that the intermediate sciences are rightly regarded as more physical than mathematical: they exist for the sake of elucidating the physical, and consequently, they are only regarded as true insofar as they are corroborated by observation (i.e., judgment terminates in the senses).[27]

Mathematical entities, however, have no inherent connection with the physical bodies they measure. Consequently, a mathematical approach to the study of physical bodies is

26. Aquinas, *SBT,* q. 5, a. 3, ad 5, p. 43.

27. Maritain, *Degrees of Knowledge,* 62–63; Simon, 98–99.

incapable of providing us with ontological knowledge of them. Unlike the natural scientist—who, through chains of efficient and formal causality, traces observable phenomena back to their principles—the physico-mathematical scientist represents (and predicts) observable phenomena through mathematical systems. Since, however, these mathematical representations can only be made by hypothesis, what is demonstrated through them is thus only hypothetically true. Again, this is simply due to the differing epistemic foundations of physics and mathematics.[28]

Nevertheless, although this precludes a mathematical ontology of physical objects, it still allows for mathematical representation and measurement. We must not forget that mathematics has a basis—albeit a remote one—in the physical real. Because of this, in a sense, mathematical entities reenter the physical realm as intelligible signs of their material, quantitative counterparts. To put it another way, by applying mathematical ideals to physical objects, we simply return to that from which we derived our mathematical ideals in the first place—the physical world. Armed now with an *epistēmē* of quantity itself (which, although giving determination to the physical world, had been previously unknowable to us in its naturally material state), we are able to recognize in the real what we have analyzed in the ideal. We can thus idealize the physical realm by imposing a mathematical framework upon it; nevertheless, this framework will only bridge the gap between matter and mind to the extent that it imitates observable phenomena. Mathematics will never allow us to fully comprehend the nature of physical objects, for since quantity is posterior to substance, we will always be required to look at a physical object's nature "from the outside in." What all of this means, in short, is that mathematical measurement of the real is possible, but nonetheless it is also limited, contingent upon the observational data

28. Maritain, *Degrees of Knowledge,* 62–63; Simon, 101.

available at a given time, and therefore open to revision whenever new data challenges the current model.[29] As Maritain puts it:

> In physico-mathematical sciences, deductive theory and the system of notions they elaborate hark back to experimental results to verify whether that theory is apt accurately to express those experimental results in an appropriate technical vocabulary. Here the substitute for the ontological *quid est* is not an inductively established law, but a mathematics *quid est,* an algorithm of the physical real.[30]

As evidence that Aquinas himself holds this view, we can look to the following passage from the *Summa,* where he distinguishes demonstrative proofs from quasi-proofs, which are still useful in their ability to explain and expound upon observable phenomena. Although, once again, the context is a discussion on the Trinity, the example he provides is particularly relevant here, for it compares a proof from pure physics with one from astronomy, an intermediate science.

> Reason may be employed in two ways to establish a point: firstly, for the purpose of furnishing sufficient proof of some principle, as in natural science, where sufficient proof can be brought to show that the movement of the heavens is always of uniform velocity. Reason is employed in another way, not as furnishing a sufficient proof of a principle, but as confirming an already established principle, by showing the congruity of its results, as in astrology the theory of eccentrics and epicycles is considered as established, because thereby the sensible appearances of the heavenly movements can be explained; not, however, as if this proof were sufficient, forasmuch as some other theory might explain them.[31]

As I indicated previously, this example from Ptolemaic astronomy illustrates the kind of certitude attainable through the physico-mathematical sciences in general. In fact, it can help us understand why Aquinas would regard them as intermediate sciences, rather than belonging their own separate division: the certitude encountered in the physico-mathematical sciences is purely mathematical. Mathematical systems can lead to necessary conclusions within themselves, but

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29. Maritain, *Degrees of Knowledge,* 62–63, 155–56.

30. Ibid., 59.

31. Aquinas, *ST,* I, q. 32, a. 1, ad 2.

there is no necessity that such conclusions are true to physical reality, since the application of the mathematical system was based on hypothesis in the first place. Thus, mathematics can lend credence to a physical hypothesis; nevertheless, the ultimate verification of a physical hypothesis belongs to the senses. Considered in this way, it is evident that there is no unique mode of speculative knowledge provided by the physico-mathematical sciences. Hence, there is no "intermediate division"—at least not in the proper sense.

However, in maintaining that a mathematical model cannot demonstratively prove a physical hypothesis, I do not imply that such models have no value. Quite the contrary, mathematical models can often be the best ways of depicting physical phenomena—and I do not just mean this from the standpoint of their predictive ability either. Rather, it is often the case that our only way of gleaning any sort of insight into the nature of a physical object is by measuring its behaviors mathematically. To take perhaps the most dramatic example: when Newton showed that the heavenly bodies follow the same inverse square law as projectiles on earth, he did more than simply provide us with a reliable means of studying their motion. Rather, he also revealed that the heavenly bodies, as mobile objects, are of the same nature as terrestrial bodies. While this revelation did not come by means of an ontological proof, the fact that both heavenly and terrestrial motion can be depicted in the same way mathematically is such overwhelming evidence in support of this claim that the burden of proof clearly rests on those who would maintain otherwise.

Of course, there is always the possibility of an explanatory model being proven inferior to another explanatory model, as Ptolemy's was to Newton's. (I am avoiding using the phrase "proven false" since mathematical systems are neither true nor false from the standpoint of extramental reality). What confidence ought we to have, then, in the usefulness of any given model? Maritain proposes the following rule of thumb: “The more the mathematical is reduced to the role of enabling one by measurement and calculation to get a surer grasp of the undiluted physical and of those causes and conditions whose character as *entia realia* the philosopher has no reason to question, the more does the result deserve to be considered a fact.”[32] In other words, the more a physico-mathematical science can be grounded in sense experience and integrated with our philosophical understanding of it, the more reliable it will be.

In light of the foregoing discussion, we would do well by letting this viewpoint stand as our key takeaway. Even though mathematics is indifferent to extramental reality, when it is used to represent and measure observable phenomena, it can direct our imagination in such a way that is uniquely beneficial for our understanding of physical objects. Since mathematics is firmly grounded in the imagination, it gives us the advantage of being able to depict the interactions of real physical beings—insofar as they are characterized by extension, locomotion, and time—in an idealized manner. Furthermore, as we continue to develop and refine these idealized depictions, we continue to hone our own ability to see through the irregularities of the material world and thus identify essences and universal laws with greater precision. Finally, since judgment of physical objects terminates in the senses, the mathematician’s inclination to meander into the ideal realm will be continually held in check, as long as observational reasoning remains the final arbiter for questions of truth.[33]

32. Maritain, *Degrees of Knowledge,* 63.

33. Ibid., 155–56.

CONCLUSION

In this thesis, I have examined the relation between physics and mathematics in Thomas Aquinas's *Division and Methods of the Sciences,* considering the question, "How can mathematics provide knowledge of physical objects?" Given how sharply he distinguishes between not only the objects, but also the methods proper to physics and mathematics respectively, his admission of intermediate (i.e., physico-mathematical) sciences presents a challenge to the integrity of his framework.

We began by studying Aquinas's process of dividing the speculative sciences, which is based on two criteria: the way the speculative object exists and the way it is known. Since immateriality and immobility are essential to speculative objects as such, he thus categorizes them insofar as they depend on matter for their existence and for their knowability. With the latter criterion, he builds upon Boethius's framework by distinguishing the sciences according to the way their objects are studied. As a result, different sciences can consider the same real objects, which bears directly upon the distinction between physics and mathematics. In physics, we consider an object insofar as it is in motion and matter. Because sensible matter belongs to the formal *ratio* of physical objects, the study of motion likewise belongs to this science. In mathematics, however, we consider an aspect of a physical object that is not dependent on sensible matter for its being understood (i.e., quantity), and thus motion is not considered.

We then examined the three modes of abstraction from matter, which correspond to three degrees of objective immateriality and are each characteristic of a different science. To apprehend physical objects, *abstractio totalis* (abstraction of the universal from the particular) is necessary. Although designated matter cannot be directly known by the intellect, undesignated matter is included in the definitions of all physical objects. To apprehend quantity by itself, (in other words, to apprehend mathematical objects), *abstractio formalis* (abstraction of a form from sensible matter) is necessary. It is possible to consider quantity apart from sensible qualities only because quantity is first among the accidents in the order of being. Thus, although the definitions of mathematical objects do not include sensible matter, they do include intelligible matter (which is postulated due to quantity's need for a principle of individuation).

Next, we followed as Aquinas set out the unique methods of each science. Of special importance was his doctrine that judgment is verified through the senses in physics, through the imagination in mathematics, and through the intellect in metaphysics. This is deeply connected to the fact that abstract quantities, as opposed to real quantities (i.e., accidents of material substances), are the proper objects of mathematics. Maintaining that mathematical entities have no direct referent in extramental reality, Aquinas regards them as similar to logical notions in that they are produced through a constructive act of the mind. Because quantity is a common sensible, the imagination is able to apprehend it, but in a way that is not as limited as the other sense powers. In generating mathematical objects, the imagination does away with the extramental conditions (i.e., the posterior accidents) that accompany our sense perception of quantity. In short, mathematical judgments can be true, but only insofar as their referents are objects existing in the imagination. This, of course, is quite unlike physical judgments, whose referents are extramental objects. Thus, it is evident that Aquinas understands physics and mathematics as having distinct—if not wholly incompatible—epistemic foundations.

Thus, we arrived at the problem resulting from Aquinas's inclusion of the intermediate sciences. Is it possible to differentiate physical from mathematical knowledge the way he does, while simultaneously admitting of sciences that somehow combine the two? As I discussed in the previous chapter, the solution to this problem begins with the fact that quantity, as an accident, is implied in the definition of physical bodies. Thus, the science of quantity—mathematics—must, in some way, be able to inform the science of physical bodies. But the question is, "How?" Not to the extent of providing ontological knowledge of physical bodies—at least, not through itself. Again, mathematical judgment terminates in the imagination, meaning that mathematical propositions can be neither true nor false from the standpoint of extramental reality.

As I discussed, Aquinas attributes a measuring role to mathematics when it is applied to physical bodies, and herein lies the solution to the question of this thesis. Being formally mathematical, the intermediate sciences provide a conditional knowledge of the physical objects they study. Although mathematics is certainly capable of achieving *epistēmē* for its own (abstract) objects, a breakdown in certitude occurs with the hypothesis that the mathematical entities employed are, indeed, true representatives of the physical phenomena observed. This, of course, is evidenced by the fact that mathematical models of physical phenomena are continually open to review with the presence of new data.

Nevertheless, insofar as such models are corroborated by physical data, the hypotheses are validated. Actually, the fact that mathematics is indifferent to extramental reality is, in a certain way, an advantage when it is applied to physics. It is precisely because mathematics is indifferent to the material world that it offers a fresh perspective to the physicist. It allows him to reimagine and reinterpret observational data and to recognize when preconceived notions of the natural order may have warped his perspective (e.g., the inverse square law challenging presuppositions about the nature of heavenly bodies). Thus, to the central question of this thesis, "How can mathematics provide knowledge of physical objects?" I offer the following answer: Being itself an ordered system of the imagination, mathematics allows the physicist to idealize the material world according to its quantitative aspects, thus assisting him in his task of locating and abstracting the immaterial side of the material world—i.e., the universal natures of physical bodies.

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