HANDOFF ISSUES IN A TRANSMIT DIVERSITY SYSTEM

A Thesis

by

KAVITA JASWAL

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2003

Major Subject: Electrical Engineering

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ABSTRACT

Handoff Issues in a Transmit Diversity System. (December 2003)

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This thesis addresses handoff issues in an WCDMA system with space-time block coded transmit antenna diversity. Soft handoff has traditionally been used in CDMA systems because of its ability to provide an improved link performance due to the inherent macro diversity. Next generation systems will incorporate transmit diversity schemes employing several transmit antennas at the base station. These schemes have been shown to improve downlink transmission performance especially capacity and quality. This research investigates the possibility that the diversity obtained through soft handoff can be compensated for by the diversity obtained in a transmit diversity system with hard handoff. We analyze the system for two performance measures, namely, the probability of bit error and the outage probability, in order to determine whether the improvement in link performance, as a result of transmit diversity in a system with hard handoffs obviates the need for soft handoffs.

To my parents and brother, for making this possible.

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CHAPTER I

INTRODUCTION

The next generation wireless systems are required to support services such as high speed internet access, cellular video conferencing and video streaming. The aim is towards achieving unparalleled wireless access as wasn't possible earlier. Achieving all this needs support for high data rates and capacity within the confines of the wireless channel. The main constraints are the presence of multipath fading, inter-symbol interference (caused by the multipath scenario), and the presence of noise. These need to be countered to get a relatively error-free communication link. The power transmitter limitations of the mobiles owing to weight and government safety regulations makes this task difficult at the mobile end, shifting the emphasis onto the base stations.

On the reverse link, the detrimental effects of fading and interference can be countered by the employment of receive diversity, multiuser detection and interference cancellation. On the forward link, however, techniques such as transmit diversity acquire significance in such a scenario where the need is to improve the capacity without expanding the bandwidth. This is especially significant in low-mobility environments where frequency diversity or time diversity is not available. Such next generation systems will incorporate transmit diversity schemes employing several transmit antennas at the base station. These schemes have been shown to improve downlink transmission performance especially capacity and quality.

Handoff is a very important feature of cellular code-division multiple-access (CDMA) systems. Traditionally, soft handoffs have been used in CDMA systems be-

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cause of their ability to provide an improved reverse link performance and extended cell coverage because of the inherent macro diversity. Also, there is no capacity loss as no extra channels are needed to perform soft handoff. On the forward link however, there is capacity loss due to two traffic channels assigned to a user in the handoff. Moreover there is additional co-channel interference because of the same additional traffic channels. In comparison to this, the absence of capacity loss as well as the interference increase for hard handoff presents a debatable contrast, especially for next generation systems employing transmit diversity.

Acting on this motivation, this thesis investigates the possibility that the diversity obtained through soft handoff can be compensated for by the diversity obtained in a transmit diversity system with hard handoff. The major contribution of this work is the analysis of a CDMA system for two performance measures, namely, the probability of bit error and the outage probability, for soft as well as hard handoff, in the presence or absence of transmit diversity. The aim is to demonstrate whether the improvement in link performance as a result of transmit diversity in a system with hard handoffs obviates the need for soft handoffs.

In this chapter, we first present an introduction to the cellular concept in section A followed by an introduction to the concept of handoffs in section B. We then talk about transmit diversity along with an emphasis on its importance with respect to the next generation system requirements in section C. In Section D, we cover the issue of handoffs in a transmit diversity system. A review of literature relevant to the thesis is also presented in this section. Finally, a brief description of the organization of the thesis is given in section E.

A. Cellular Concept

The cellular concept is the idea that called for replacing large, single high power transmitter cells with several small, low power transmitter cells. Each of these cells would provide coverage to a small portion of the coverage area. This effectively solved the problem of limited user capacity and spectral congestion, by offering high capacity in a limited spectral allocation without any major system overhauls [16]. A base station transmitter at the center of the cell is assumed to service all the mobile stations within its cell area. This can be seen in figure 1.

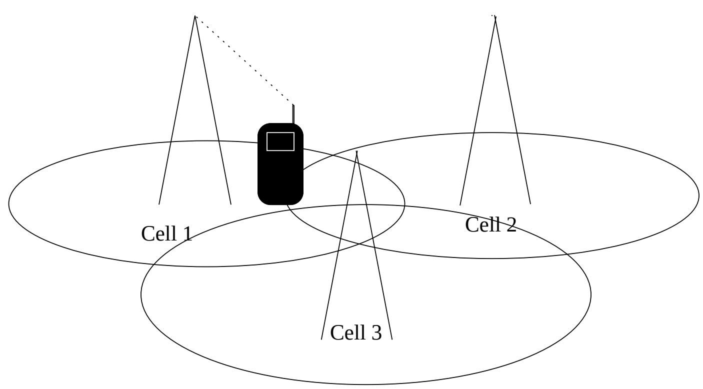

Fig. 1. Cellular Concept

B. The Handoff Procedure

The 3GPP vocabulary [2] defines a handoff (handover) as 'The transfer of a user's connection from one radio channel to another (can be the same or different cell)'. As a mobile moves towards the boundary of its serving cell, the movement causes dynamic changes in the interference levels and the link quality. This may cause the mobile to transfer communication to or to migrate to a different base station. This change of serving base station is called a handoff. A handoff may be an inter cell handoff (between different cells and hence needing a change in network connections), an intra-cell handoff (between different sectors of a cell) or an inter-system handoff (between networks using different radio systems, for example UMTS and GSM).

Hard Handoff, also known as a 'break-before-make' handoff, is the category of handoff procedures in which the mobile switches to a new radio link after breaking connection with the old radio link. The switch-over takes place when the pilot strength from the new base station exceeds the pilot strength from the serving base station by an amount called *hysteresis*. At any time, the active set (set of base stations with which the user is in communication) will have only one base station. This can be seen in figure 2. In the figure, base station one is referred to as BS1 and base station two as BS2.

Soft Handoff, on the other hand, is the handoff procedure in which a mobile has connection with more than one radio link simultaneously during handoff. Once the signal from a single radio link is considerably stronger than the others, a decision will be made to communicate with that one only. Hence in comparison to hard handoff, the active set may contain more than one base station. The addition and removal of base stations into and from the active set is dependent on parameters such as the *add threshold*, *drop threshold* and *drop timer*. A base station is added to the active set when its pilot signal strength exceeds the add threshold. A base station is removed from the active set when its pilot signal strength drops below the drop threshold and stays below it for the time specified by the drop timer. The process is illustrated in figure 3.

Each handoff procedure has its advantages and disadvantages. Soft handoff results in reduced ping-pong effect (The handing-off back and forth of a mobile several

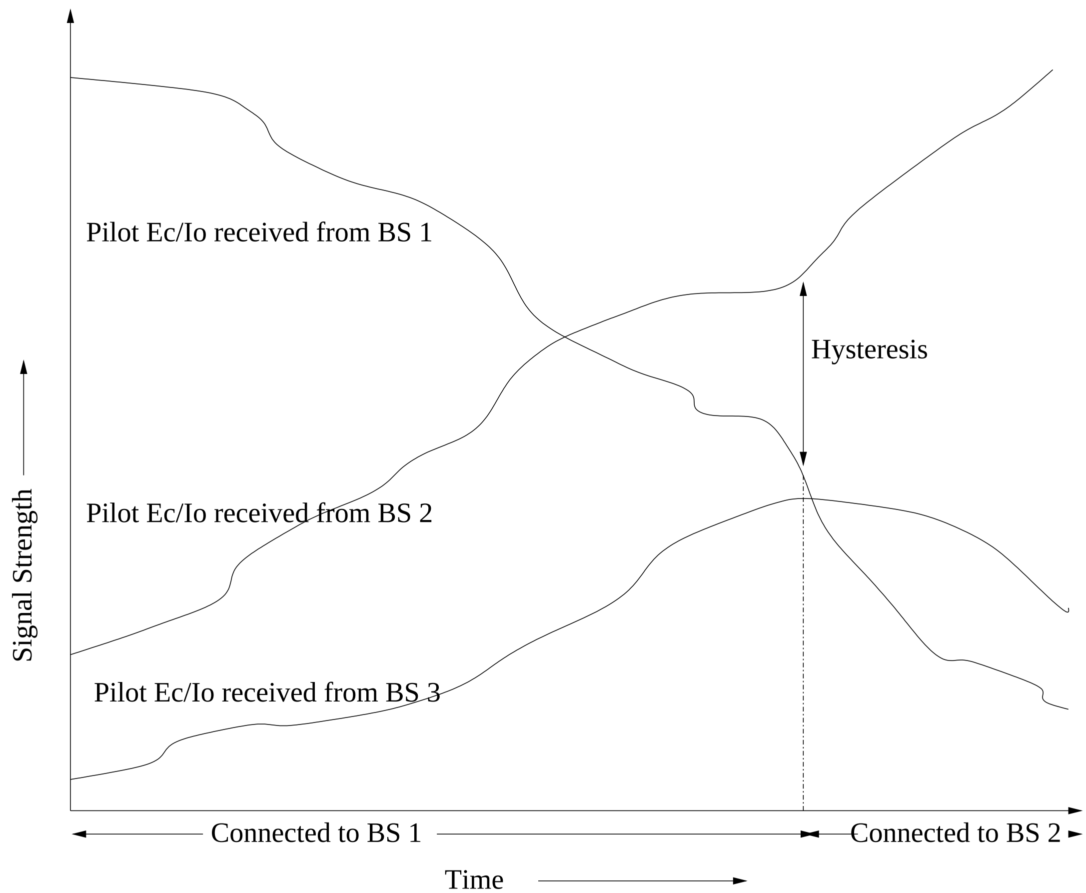

Fig. 2. Hard Handoff Procedure

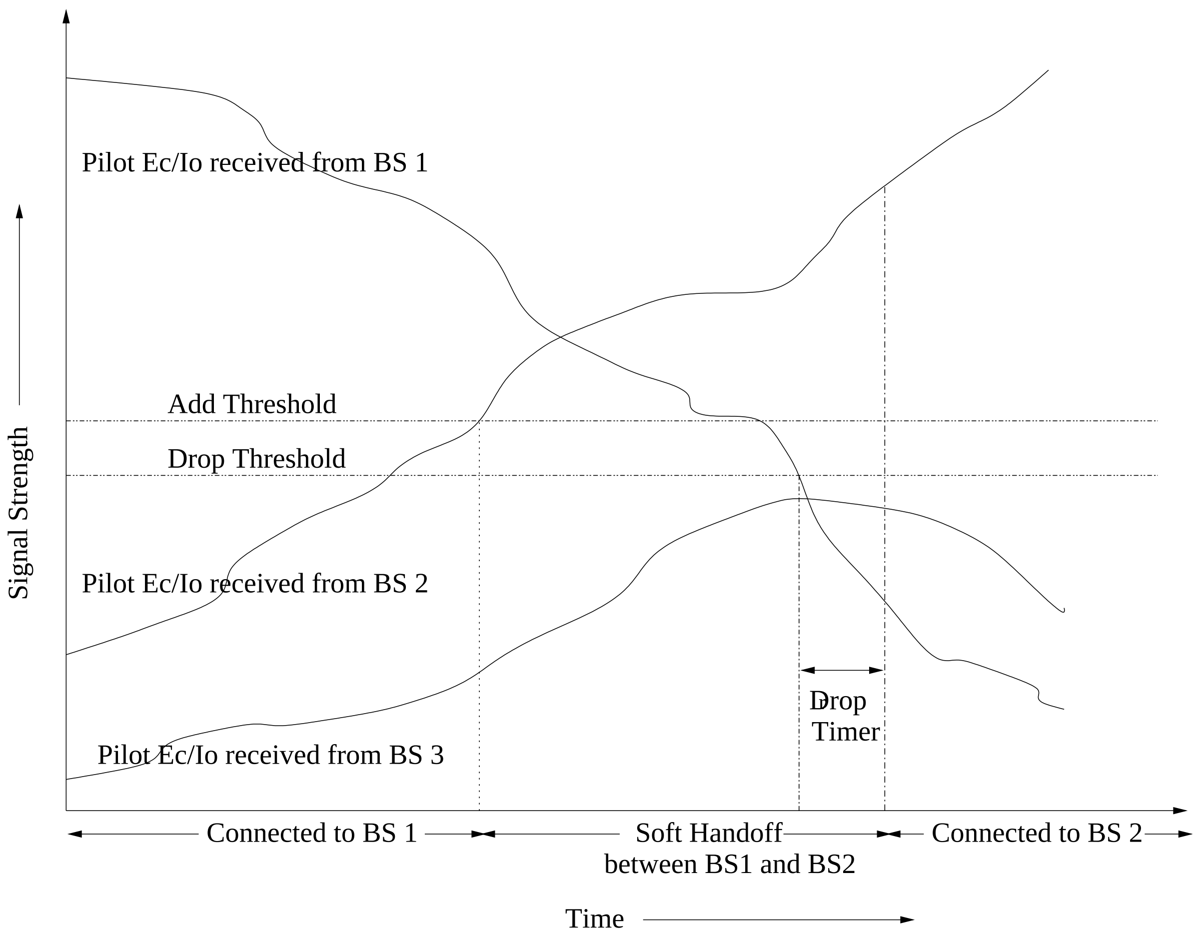

Fig. 3. Soft Handoff Procedure

times between two (or more) base stations in a relatively short period of time is known as the "ping-pong" effect.)[1], reduced fade margins (for fixed outage probability and base station separation), higher uplink capacity and fewer time constraints on the network [20]. The trade-off is an increase in downlink interference, the requirement of additional network resources and a complex implementation [25] .

C. Transmit Diversity

In a transmit diversity system, the transmitter has multiple antennas(Figure 4). Bits from a serial bit stream are distributed to parallel substreams and mapped to waveforms which are transmitted from the respective antenna. The mapping is according to the space-time block code used. The channel introduces distortion in the signal through the fading and the added noise. In addition to this, the signals experience interference also. At the receiver, simple signal processing is carried out with the received distorted and superimposed waveforms to get estimates of the sent data.

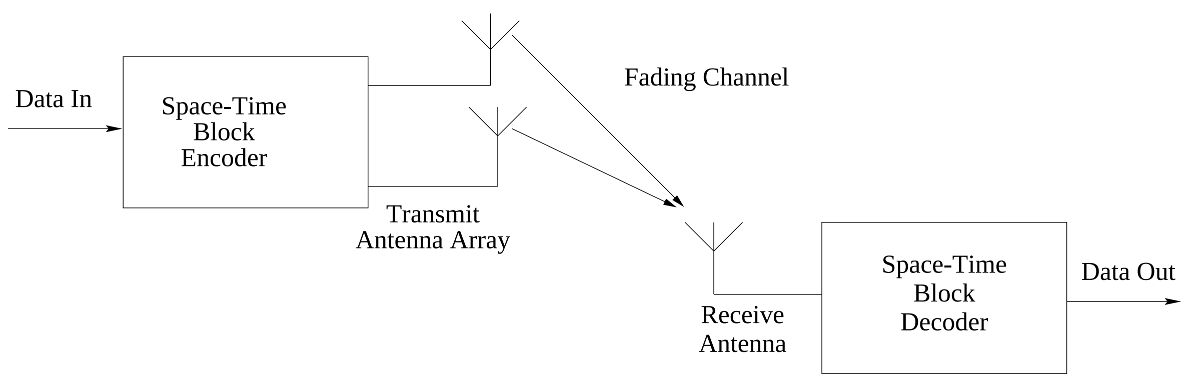

Fig. 4. Communication System Utilizing Space Time Block Coded Transmit Diversity

[1]The analogy is to the movement of a ping-pong ball in a ping-pong game.

Transmit diversity improves performance as a result of the redundancy in the form of the added channel(s) from the transmitter to the receiver. In case one of the channels goes into a fade the other can be used to recover the data. The probability of all channels failing together is small as compared to the failure of a single channel. This means a lower probability of bit error as well as a lower outage probability for the system. Figure 5 [17] compares the probability of bit error for a non diversity system with a transmit diversity system, the system having 6 multipaths for a vehicular channel. Figure 6 ([16] page 329) compares the probability of outage for a single antenna system vs a 2 antenna MRC system. Improved performance due to transmit diversity can help us attain higher data rates, capacity and spectral efficiency. For the scope of our thesis, we will deal with the two transmit antenna, one receive antenna transmit diversity scheme called space-time block coded transmit diversity as shown in fig 4.

D. Handoffs in a Transmit Diversity System

Soft Handoff has traditionally been considered as the handoff of choice for CDMA systems because of its ability to provide an improved link performance due to the inherent macro diversity. Considerable research has demonstrated the advantages of soft handoff over hard handoff. The work done in [20] shows that soft handoff results in increased capacity as well as coverage on the reverse link. Simulations of GSM hard handoffs and CDMA soft handoffs have been compared in [7]. The results show a 2.6-3.6 dB higher fade margin for a GSM hard handoff as compared to a CDMA soft handoff. Further work was carried out in [6] where a propagation model considering path loss, lognormal shadowing and Rayleigh fading was considered. Here, coverage areas for reverse and forward links for hard as well as soft handoff were computed

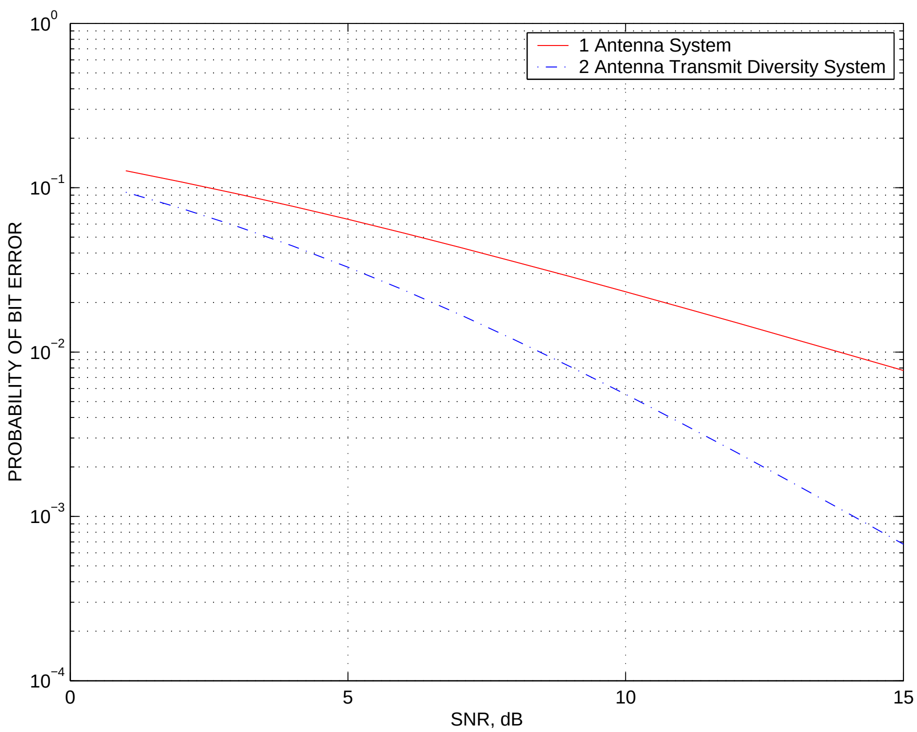

Fig. 5. Probability of Bit Error

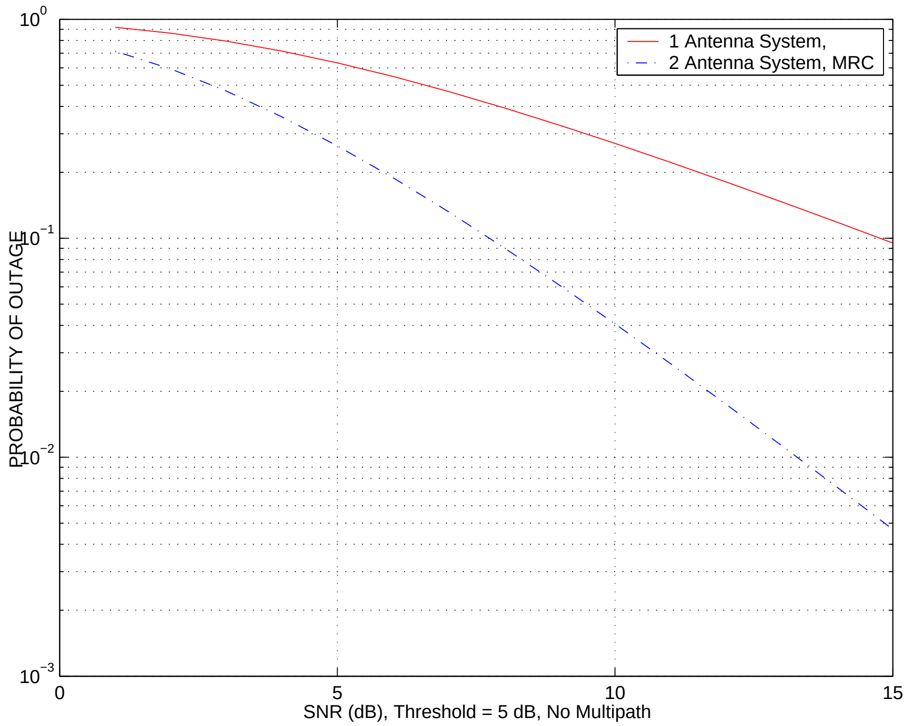

Fig. 6. Probability of Outage

analytically. Soft Handoff coverage on both the forward as well as the reverse link was shown to be around twice that of the hard handoff coverage and increasing with an increase in the outage threshold. However, for the forward link computations the paper doesn't consider the increased interference to the system because of soft handoff. The increased interference is because the new base station now transmits an additional signal for the mobile station [19, 11]. Recent research in this direction has shown that soft handoff on the forward links actually hurts system capacity [27].

As stated in the previous sections antenna diversity has been proven to be a practical and effective technique to mitigate the effects of fading [22]. In fact transmit diversity has been employed as a means of improving the downlink capacity in the standardization of the third generation wireless systems [9]. Dabak et. al. [8], presents the space time block coded transmit antenna diversity scheme for WCDMA and performance results for link level simulations under various doppler rates, channel rates and environments as well as for a system undergoing soft handoff . The results show a performance gain of 1.3 dB for transmit diversity with soft handoff over the non diversity case with soft handoff. However no attempt was made to consider hard handoffs in a system with transmit diversity.

On the analysis front, the analysis for uncoded bit error probabilities for various open loop (does not require any knowledge of the channel at the transmitter) and closed loop (requires knowledge of channel at the transmitter) transmit diversity schemes was carried out in a paper by Sandell [17]. Further work was carried out by Bjerke et. al. [5] which builds on Sandell's results to derive the uncoded bit error probability for various receive antenna diversity schemes in addition to transmit diversity. Our research takes off from this point to analyze the two performance measures, namely the probability of outage and the probability of bit error for a system with hard or soft handoff, with and without transmit diversity.

E. Organization of the Thesis

This thesis is organized into five chapters following this introduction. Chapter II reviews in brief the background of the WCDMA downlink transmit diversity concept. Next in chapter III, we discuss soft as well as hard handoffs in non-diversity and transmit diversity systems and analyze their performance with respect to the probability of outage. Chapter IV contains the analysis for the probability of bit error. In chapter V, we compare the analytical results for the above for different channel models, number of users, percentage of users in handoff , outage thresholds and SNRs. Finally, chapter VI presents the concluding remarks and suggestions for future work.

CHAPTER II

WCDMA TRANSMIT DIVERSITY CONCEPT

In this chapter, we review the Direct Sequence Code-Division Multiple-Access (DS-CDMA) concept followed by a subsection on the receivers used for CDMA systems in a frequency selective environment, namely RAKE receivers. We also talk about the transmit diversity concept in brief and follow it by giving an overview of open loop space-time block coded transmit diversity based on Alamouti's scheme [4].

A. Background

1. DS-CDMA Concept

Direct Sequence- Code Division Multiple Access (DS-CDMA) is a multiple access technique based on spread spectrum techniques, in which all users share the same transmission bandwidth. The user's unique spreading code is used to modulate the users data. The rate of the spreading code (Rc) is usually much higher than that of the user data (Rs). The ratio Rc/Rs is called the spreading gain of the system. This gives a measure of the bandwidth expansion caused due to the multiplication by the spreading sequence. The capacity of a conventional DS-CDMA system depends on the spreading gain of the system.

The spread signal is m(t)= d(t)×s(t), where d(t) is the user data and s(t) is the spreading sequence. The spread signal can be despread back to the original signal by multiplying the received sequence with the same sequence used for spreading.

As seen in figure 7(a), the original signal with a small bandwidth($2/T_s$) is interfered by the interference and may be distorted such that it is irrecoverable. After spreading, the signal is now spread to $2/T_c$ ($\gg 2/T_s$) as in 7(b). After despreading at the receiver, the signal is despread to the original bandwidth, while the interference is spread to the larger bandwidth. However, the interference is weakened and may not be strong enough to corrupt the signal extensively 7(c). This shows the interference rejection as well as jamming robustness properties of DS-CDMA.

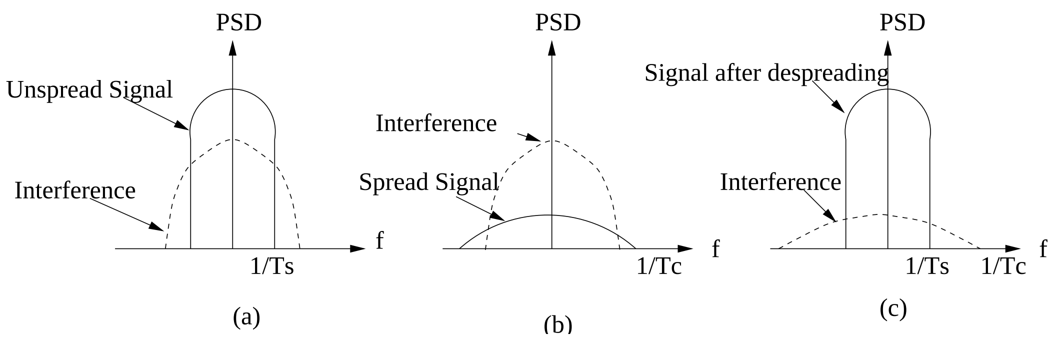

Fig. 7. Spreading and Despreading of a CDMA Signal

2. RAKE Receiver

In a wireless communication system, the transmitted signal can reflect off natural obstacles like buildings, trees before reaching the receiver. These reflections may arrive at the receiver with different attenuation, phase shift and time delay. Some of the in-phase paths may combine constructively leading to a strengthening of the signal whereas some out of phase paths may combine destructively resulting in a weaker signal. This phenomenon is called fading and is usually modelled as a complex random variable with its magnitude as a rayleigh random variable and its phase as a uniform random variable between $[0, 2\pi)$. If the time delay of an received signal path τ is more than the symbol interval T_s, then the delayed signal might interfere with the next symbols leading to a kind of signal distortion known as inter symbol interference(ISI).

A frequency selective channel is a type of ISI channel such that the time delays are approximately equal to or larger than the symbol interval T_s. A frequency selective channel with L paths, for a bandpass signal with a bandwidth W, can be modelled as a filter with a time domain impulse response given by

$$h(t) = \sum_{k=0}^{L-1} h_k e^{j\phi_k} \delta(t - \tau_k) = \sum_{k=0}^{L-1} h_k(t)$$

where h_k is the channel gain of the k^{th} path and is rayleigh distributed. ϕ_k is the phase shift of the k^{th} path and is uniformly distributed between $[0, 2\pi)$. Also, τ_k is the time delay of the k^{th} path. The transfer function of h(t) is given by

$$H(f) = \sum_{k=0}^{L-1} h_k e^{(j\phi_k + 2\pi f \delta \tau_k)}$$

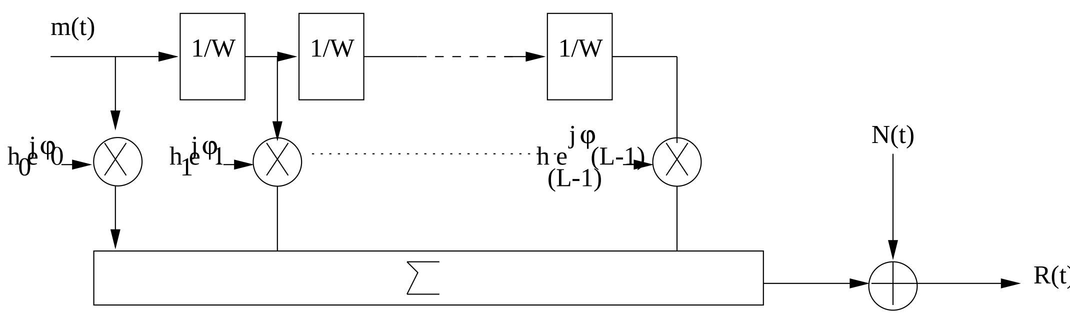

Fig. 8. Tapped Delay Line Model of a Frequency Selective Channel

It can be interpreted from the transfer function that the channel response H(f), changes its magnitude when the frequency f is varied, hence earning it the name of a frequency selective channel. A frequency channel offers frequency diversity as all frequency components may not undergo fading at the same time. We can represent such a channel by the tapped delay line model as shown in figure 8. Frequency selective channels can give upto L^{th} order diversity if the received signal is processed by the receiver in an optimum manner. This is done by a RAKE receiver.

A RAKE receiver exploits the fact that, for a spread spectrum signal, each path of the transmitted signal does not interfere with each other after despreading as a consequence of its having a low auto correlation. The structure of a rake receiver is as shown in figure 9. The received signal is passed through a filter whose tap coefficients are derived from the (estimates of) the channels gains. The output of the filter is then despread and integrated. This can be written as

$$
\begin{aligned}
Z &= Re\left[\sum_{k=0}^{L-1}\int_0^T h_k^* R(t-(L-1-k)/W)s^*(t-(L-1)/W)\,dt\right] \\
&= Re\left[\sum_{k=0}^{L-1} h_k^* \sum_{\hat{k}=0}^{L-1} h_{\hat{k}} \int_0^T s(t-(L-1-k+\hat{k})/W)s^*(t-(L-1)/W)\,dt\right]
\end{aligned}
$$

If the autocorrelation of the code sequences is negligible,

$$
\int_0^T s(t-(\hat{k}-k)/W)s^*(t)\,dt \approx 0, \qquad for \qquad \hat{k} \neq k
$$

then the output of the rake receiver is given by,

$$
Z = \quad Re\left[\sum_{k=0}^{L-1} |h_k|^2 \int_0^T |s(t)|^2\,dt\right]
$$

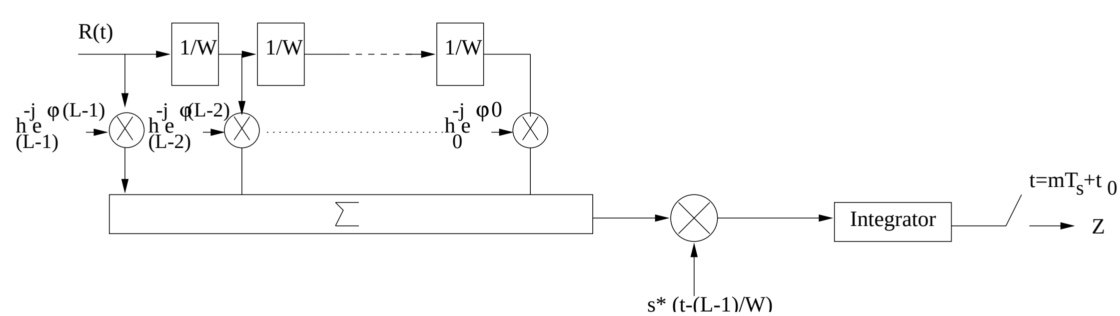

Fig. 9. Rake Receiver

Hence, the output of the rake receiver is similar to a L^{th} order diversity combiner since its output is proportional to $\sum_{k=0}^{L-1} |h_k|^2$.

3. Diversity- Concept and Background

If multiple copies of the same signal are transmitted over independently fading channels, the possibility that all of them are corrupted is quite improbable. This is the fundamental principle of diversity. Of the various forms of diversity available for communications systems, the most popular ones are temporal diversity, spatial diversity and frequency diversity.

Temporal diversity is obtained by transmitting the same signal separated in time, with the time separation being more than the coherence time (time duration over which the channel impulse response is essentially invariant) of the channel. For frequency diversity, the signals are sent over different frequency bands, where the separation between the different frequency bands is greater than the coherence bandwidth (range of frequencies over which the channel can be considered 'flat') of the channel. When the same data is sent over different antennas, separated in space, it results in spatial diversity. It has been noticed that, if the separation between the antennas is more than half a wavelength, then the channels are uncorrelated.

The choice of a diversity scheme depends on the characteristics of the channel such as the coherence time or the coherence bandwidth. It has been observed, however, that regardless of the channel characteristics, spatial diversity is always effective, as long as the antenna separation is sufficient to ensure uncorrelated channels.

Delay Diversity and other schemes [23, 24, 22, 18] were the initial schemes proposed to use transmit diversity. Delay diversity is a simple scheme in which signals are transmitted on two different antennas with a delay between them. Another simple transmit diversity scheme was proposed in [4] which used two antennas at the base station and simple signal processing at the receiver to achieve diversity. This elegant and simple scheme went on to become a paradigm in the field of transmit diversity. In fact, this scheme has been employed as a means of improving the downlink capacity in the standardization of the third generation wireless systems. The next section explains this scheme in detail.

B. Space Time Transmit Diversity

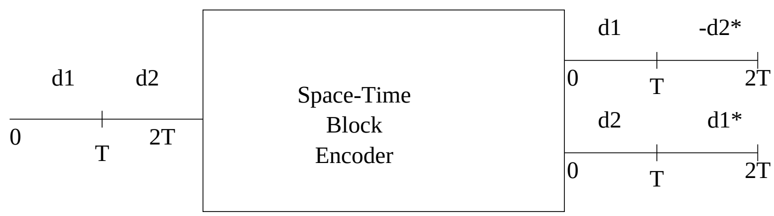

Fig. 10. STTD Encoder

The transmitter has two transmit antennas and one receive antenna. QPSK symbols d_1 and d_2 are transmitted simultaneously from the two antennas in the first symbol period. In the next symbol period symbol $-d_2^*$ is transmitted from antenna one and d_1^* is transmitted from antenna two (complex conjugate operation is denoted by '*'.). The transmission sequence is as shown in figure 10. Assuming the fading to be constant over two symbol intervals, the signal received in two consecutive intervals can be shown to be [17]

$$r_1(t) = \sum_{k=0}^{L-1}[d_1 h_{1,k}(t) + d_2 h_{2,k}(t)]s(t-\tau_k) + n_1(t), \tag{2.1}$$

$$r_2(t) = \sum_{k=0}^{L-1}[-d_2^* h_{1,k}(t) + d_1^* h_{2,k}(t)]s(t-\tau_k) + n_2(t), \quad (2.2)$$

where $h_{i,k}(t)$ is the complex-valued channel coefficient for the k^{th} multipath of the i_{th} diversity antenna. s(t) is the spreading sequence and n_i(t) is the additive white gaussian noise for the i^{th} diversity antenna. The channel has L multipaths with τ_k representing the delay of the $k^t h$ path. The output of the k^{th} RAKE finger is given by

$$\begin{aligned} r_{1,k} &= \int_0^T r_1(t)s^*(t-\tau_k)dt \\ &= d_1 h_{1,k} + d_2 h_{2,k} + n_{1,k} \quad (2.3) \\ r_{2,k} &= \int_0^T r_2(t)s^*(t-\tau_k)dt \\ &= -d_2^* h_{1,k} + d_1^* h_{2,k} + n_{2,k} \quad (2.4) \end{aligned}$$

$n_{i,k}$ being noise samples at the i^{th} diversity antenna. This includes the noise as well as the multiple access interference due to the multipaths. An analysis of this interference is detailed in appendix A. The mobile does the following linear processing to generate soft outputs for the transmitted signals. Hence, we get estimates of the transmitted data as

$$\begin{aligned} \hat{d}_1 &= \sum_{k=0}^{L-1}[h_{1,k}^* r_{1,k} + h_{2,k} r_{2,k}^*] \\ &= \sum_{k=0}^{L-1}[|h_{1,k}|^2 + |h_{2,k}|^2]d_1 + h_{1,k}^* n_{1,k} + h_{2,k} n_{2,k}^* \quad (2.5) \end{aligned}$$

and,

$$\hat{d}_2 = \sum_{k=0}^{L-1}[h_{2,k}^* r_{1,k} - h_{1,k} r_{2,k}^*]$$

$$= \sum_{k=0}^{L-1}[|h_{1,k}|^2 + |h_{2,k}|^2]d_2 - h_{1,k}n_{2,k}^* + h_{2,k}^*n_{1,k} \qquad (2.6)$$

The resulting diversity order is equal to that obtained from maximal ratio receiver combining (MRRC) with two receive antennas, being $2L$. This is twice the diversity obtained from non-diversity systems. Having explained the space-time block coded transmit diversity system used, we now deal with the analysis of the system in the next two chapters.

CHAPTER III

PERFORMANCE MEASURES - PROBABILITY OF OUTAGE

In this chapter, we derive the probability of outage (An outage occurs when, due to a limitation on the maximum transmitter power, the measured signal to noise ratio of a connection is lower than the target signal to noise ratio.) for the forward link of a wireless system. We take into consideration four cases. We consider the case of a non-diversity system undergoing soft handoff or hard handoff. Also a system incorporating transmit diversity is considered while undergoing soft handoff or hard handoff.

A. Background

The analysis for uncoded bit error probabilities for various open loop and closed loop transmit diversity schemes was carried out in a paper by Sandell [17]. Further work was carried out by Bjerke et. al. [5] which builds on Sandell's results to derive the uncoded bit error probability for various receive antenna diversity combining schemes, in addition to transmit diversity. We take inspiration from both these results to derive our performance measures.

We consider a mobile unit moving in a straight line between the two base stations with the distance between the base stations being R. The signal from the base stations is subject to Rayleigh fading. The average path loss is assumed to be inversely proportional to the n^{th} power of the distance.

Fading multipath channel is modeled as a tapped delay line with L time-varying, complex-valued gaussian distributed tap coefficients. Fading is assumed to be constant over at least 2 consecutive symbols. The paths between each transmit antenna and the receive antenna are assumed to be independent with an identical exponential delay profile. The instantaneous signal strength of the paths is given by

$$\begin{aligned}\Omega_l &= \Omega_0 e^{-l\delta} \qquad (3.1)\\ \Omega_0 &= \text{instantaneous signal strength of the first incoming path.}\\ \delta &= \text{rate of power decay.If } \delta \text{ is } 0 \text{ , constant propagation profile.}\end{aligned}$$

We also assume perfect channel vector estimation and maximal ratio combining at the RAKE receiver leading to a time diversity of the order L. Hence, the SINR at the RAKE output in a system with a single antenna is given by

$$\Upsilon = \sum_{k=0}^{L-1} \Upsilon_k \qquad (3.2)$$

where Υ_k is the instantaneous SINR on the k^{th} RAKE finger. In case of open loop transmit diversity, the SINR at the RAKE will be [17]

$$\Upsilon = \frac{1}{2} \sum_{k=0}^{L-1} [\Upsilon_{1,k} + \Upsilon_{2,k}] \qquad (3.3)$$

where the factor 1/2 is because of the sharing of transmit power between the antennas. $\Upsilon_{i,k}$ is the SINR from the kth path due to antenna i. Having explained the above, we proceed with the derivations.

B. Outage Probability with Soft Handoff, No Diversity

For soft handoff, the RAKE receiver performs maximal ratio combining of the signal received from the two base stations. The instantaneous SINR at the output of the RAKE receiver at a distance r from base station one (hence referred to as BS1) and a distance $(R-r)$ from base station two (BS2) is given by

$$\Upsilon = \sum_{k=0}^{L-1} \Upsilon_{1,k} + \sum_{k=0}^{L-1} \Upsilon_{2,k} \tag{3.4}$$

where,

$$\Upsilon_{1,k} = \frac{E_b \Omega_l}{N_e r^n} \tag{3.5}$$

and

$$\Upsilon_{2,k} = \frac{E_b \Omega_l}{N_e (R-r)^n} \tag{3.6}$$

Here, $\Upsilon_{i,k}$ is the instantaneous SINR on the k^{th} rake finger for signal from BSi at a distance r. E_b is the energy per transmitted bit and N_e is the equivalent power spectral density including AWGN and total interference (approximated as WGN). The analysis for the interference is dealt with in Appendix A. Hence, the characteristic function of Υ is

$$\begin{aligned} \psi_{\Upsilon}(j\upsilon) &= \prod_{k=0}^{L-1} \frac{1}{(1-j\upsilon\overline{\Upsilon}_{1,k})(1-j\upsilon\overline{\Upsilon}_{2,k})} \\ &= \sum_{k=0}^{L-1} \left[\frac{C_{1,k}}{(1-j\upsilon\overline{\Upsilon}_{1,k})} + \frac{C_{2,k}}{(1-j\upsilon\overline{\Upsilon}_{2,k})} \right] \end{aligned} \tag{3.7}$$

with the coefficients of the partial fraction expansion as,

$$C_{1,k} = \prod_{l=0,\, l\neq k}^{L-1} \frac{\overline{\Upsilon}_{1,k}}{\overline{\Upsilon}_{1,k} - \overline{\Upsilon}_{1,l}} \times \prod_{l=0}^{L-1} \frac{\overline{\Upsilon}_{1,k}}{\overline{\Upsilon}_{1,k} - \overline{\Upsilon}_{2,l}} \tag{3.8}$$

and

$$C_{2,k} = \prod_{l=0,\, l\neq k}^{L-1} \frac{\overline{\Upsilon}_{2,k}}{\overline{\Upsilon}_{2,k} - \overline{\Upsilon}_{2,l}} \times \prod_{l=0}^{L-1} \frac{\overline{\Upsilon}_{2,k}}{\overline{\Upsilon}_{2,k} - \overline{\Upsilon}_{1,l}} \tag{3.9}$$

$\overline{\Upsilon}_{i,k}$ is the average SINR on path k from BSi, assumed to be different from average

SINR's on the other paths from BSi. The characteristic function is inverse-fourier transformed to get the probability density function.

$$p_\Gamma(\Upsilon) \;=\; \sum_{k=0}^{L-1}\left[\frac{C_{1,k}}{\overline{\Upsilon}_{1,k}}e^{-\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{C_{2,k}}{\overline{\Upsilon}_{2,k}}e^{-\Upsilon/\overline{\Upsilon}_{2,k}}\right] \qquad ,\Upsilon \geq 0 \tag{3.10}$$

The outage probability, that is, the probability that Υ is less than some SINR threshold Υ_T, for a system under soft handoff without transmit diversity is given by

$$Pr(\Upsilon \leq \Upsilon_T) \;=\; \int_0^{\Upsilon_T} p_\Gamma(\Upsilon)d\Upsilon \tag{3.11}$$

$$= \int_0^{\Upsilon_T}\sum_{k=0}^{L-1}\left[\frac{C_{1,k}}{\overline{\Upsilon}_{1,k}}e^{-\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{C_{2,k}}{\overline{\Upsilon}_{2,k}}e^{-\Upsilon/\overline{\Upsilon}_{2,k}}\right]d\Upsilon$$

$$= \sum_{k=0}^{L-1}\left[C_{1,k}(1-e^{-\Upsilon_T/\overline{\Upsilon}_{1,k}}) + C_{2,k}(1-e^{-\Upsilon_T/\overline{\Upsilon}_{2,k}})\right] \tag{3.12}$$

C. Outage Probability with Hard Handoff, No Diversity

For the case of hard handoff, to make our analysis tractable, we make the following assumption. Allowing for the ping-pong effect, we assume the hysteresis to be zero such that the mobile is allowed to select the larger of the signals from the two base stations at any time. Hence, the receiver is assumed to perform selection diversity combining. In that case, the SINR at the output of the RAKE is given by

$$\Upsilon = \max(\Upsilon_1, \Upsilon_2) \tag{3.13}$$

Where, Υ_i is given by

$$\Upsilon_i = \sum_{k=0}^{L-1}\Upsilon_{i,k} \qquad i = \{1,2\} \tag{3.14}$$

The corresponding characteristic function is

$$\psi_{\Upsilon_i}(jv) = \prod_{k=0}^{L-1} \frac{1}{(1 - jv\overline{\Upsilon}_{i,k})} \qquad i = \{1, 2\}$$

$$= \sum_{k=0}^{L-1} \frac{A_{i,k}}{(1 - jv\overline{\Upsilon}_{i,k})} \qquad i = \{1, 2\} \tag{3.15}$$

Where, the coefficient $A_{i,k}$ is given by,

$$A_{i,k} = \prod_{l=0,\, l\neq k}^{L-1} \frac{\overline{\Upsilon}_{i,k}}{\overline{\Upsilon}_{i,k} - \overline{\Upsilon}_{i,l}} \qquad i = \{1, 2\} \tag{3.16}$$

The pdf of Υ can be determined from the following [14]

$$F_{\Gamma}(\Upsilon) = F_{\Gamma_1}(\Upsilon)F_{\Gamma_2}(\Upsilon) \tag{3.17}$$

$$p_{\Gamma}(\Upsilon) = p_{\Gamma_1}(\Upsilon)F_{\Gamma_2}(\Upsilon) + p_{\Gamma_2}(\Upsilon)F_{\Gamma_1}(\Upsilon) \tag{3.18}$$

Where, the pdf and cdf of Υ_i are given as

$$p_{\Gamma_i}(\Upsilon) = \sum_{k=0}^{L-1} \frac{A_{i,k}}{\overline{\Upsilon}_{i,k}} e^{-\Upsilon/\overline{\Upsilon}_{i,k}} \qquad i = \{1, 2\} \tag{3.19}$$

$$F_{\Gamma_i}(\Upsilon) = \sum_{k=0}^{L-1} A_{i,k}(1 - e^{-\Upsilon/\overline{\Upsilon}_{i,k}}) \qquad i = \{1, 2\} \tag{3.20}$$

Hence, the pdf of Υ is

$$p_{\Gamma}(\Upsilon) = \sum_{k=0}^{L-1} \frac{A_{1,k}}{\overline{\Upsilon}_{1,k}} e^{-\Upsilon/\overline{\Upsilon}_{1,k}} \times \sum_{k'=0}^{L-1} A_{2,k'}(1 - e^{-\Upsilon/\overline{\Upsilon}_{2,k'}}) +$$

$$\sum_{k=0}^{L-1} \frac{A_{2,k}}{\overline{\Upsilon}_{2,k}} e^{-\Upsilon/\overline{\Upsilon}_{2,k}} \sum_{k'=0}^{L-1} A_{1,k'}(1 - e^{-\Upsilon/\overline{\Upsilon}_{1,k'}}) \tag{3.21}$$

$$= \sum_{k=0}^{L-1} \sum_{k'=0}^{L-1} [\frac{A_{1,k}A_{2,k'}}{\overline{\Upsilon}_{1,k}} (e^{-\Upsilon/\overline{\Upsilon}_{1,k}} - e^{-\Upsilon(\frac{1}{\overline{\Upsilon}_{1,k}} + \frac{1}{\overline{\Upsilon}_{2,k'}})})$$

$$+\frac{A_{2,k}A_{1,k'}}{\overline{\Upsilon}_{2,k}}(e^{-\Upsilon/\overline{\Upsilon}_{2,k}} - e^{-\Upsilon(\frac{1}{\overline{\Upsilon}_{2,k}}+\frac{1}{\overline{\Upsilon}_{1,k'}})})] \qquad ,\Upsilon \geq 0 \qquad (3.22)$$

Hence, the outage probability for hard handoff without transmit diversity is given by

$$\begin{aligned} P_o &= \int_0^{\Upsilon_T} p_\Gamma(\Upsilon)\, d\Upsilon \\ &= \int_0^{\Upsilon_T} \sum_{k=0}^{L-1}\sum_{k'=0}^{L-1}[\frac{A_{1,k}A_{2,k'}}{\overline{\Upsilon}_{1,k}}(e^{-\Upsilon/\overline{\Upsilon}_{1,k}} - e^{-\Upsilon(\frac{1}{\overline{\Upsilon}_{1,k}}+\frac{1}{\overline{\Upsilon}_{2,k'}})}) \quad + \\ &\quad \frac{A_{2,k}A_{1,k'}}{\overline{\Upsilon}_{2,k}}(e^{-\Upsilon/\overline{\Upsilon}_{2,k}} - e^{-\Upsilon(\frac{1}{\overline{\Upsilon}_{2,k}}+\frac{1}{\overline{\Upsilon}_{1,k'}})})]\, d\Upsilon \\ &= \sum_{k=0}^{L-1}\sum_{k'=0}^{L-1}[A_{1,k}A_{2,k'}(1 - e^{-\Upsilon_T/\overline{\Upsilon}_{1,k}} - \overline{\Upsilon}_{2,k'}\alpha_1(1 - e^{-\Upsilon_T/\Delta_1})) + \\ &\quad A_{2,k}A_{1,k'}(1 - e^{-\Upsilon_T/\overline{\Upsilon}_{2,k}} - \overline{\Upsilon}_{1,k'}\alpha_2(1 - e^{-\Upsilon_T/\Delta_2}))] \qquad (3.23) \end{aligned}$$

With, α_1, α_2, Δ_1 and Δ_2 being defined as,

$$\alpha_1 = \frac{1}{(\overline{\Upsilon}_{1,k} + \overline{\Upsilon}_{2,k'})}, \quad \alpha_2 = \frac{1}{(\overline{\Upsilon}_{1,k'} + \overline{\Upsilon}_{2,k})}, \qquad (3.24)$$

$$\Delta_1 = \overline{\Upsilon}_{1,k}\overline{\Upsilon}_{2,k'}\alpha_1, \qquad \Delta_2 = \overline{\Upsilon}_{1,k'}\overline{\Upsilon}_{2,k}\alpha_2 \qquad (3.25)$$

D. Outage Probability with Soft Handoff, Transmit Diversity

For a system having transmit diversity and undergoing soft handoff, the instantaneous SINR at the output of the RAKE receiver is given by

$$\Upsilon = \frac{1}{2}\sum_{k=0}^{L-1}[\Upsilon_{11,k} + \Upsilon_{12,k}] + \frac{1}{2}\sum_{k=0}^{L-1}[\Upsilon_{21,k} + \Upsilon_{22,k}] \tag{3.26}$$

Where, $\Upsilon_{ij,k}$ is the received instantaneous SINR from path k of the signal from the jth diversity antenna of BSi. Assuming that the k th paths (due to multipath) of both diversity channels are identical, we get

$$\Upsilon_{11,k} = \Upsilon_{12,k} = \Upsilon_{1,k} \tag{3.27}$$

and

$$\Upsilon_{21,k} = \Upsilon_{22,k} = \Upsilon_{2,k} \tag{3.28}$$

Hence, the characteristic function of Υ is given by

$$\begin{aligned}\psi_{\Upsilon}(jv) &= \prod_{k=0}^{L-1}\frac{1}{(1-\frac{jv\overline{\Upsilon}_{1,k}}{2})^2(1-\frac{jv\overline{\Upsilon}_{2,k}}{2})^2} \\ &= \sum_{k=0}^{L-1}\left[\frac{D_{11,k}}{(1-\frac{jv\overline{\Upsilon}_{1,k}}{2})} + \frac{D_{12,k}}{(1-\frac{jv\overline{\Upsilon}_{1,k}}{2})^2} + \frac{D_{21,k}}{(1-\frac{jv\overline{\Upsilon}_{2,k}}{2})} + \frac{D_{22,k}}{(1-\frac{jv\overline{\Upsilon}_{2,k}}{2})^2}\right]\end{aligned} \tag{3.29}$$

with,

$$D_{11,k} = 2\,{C_{1,k}}^2\left[\sum_{l=0,\,l\neq k}^{L-1}\frac{\overline{\Upsilon}_{1,l}}{(\overline{\Upsilon}_{1,l}-\overline{\Upsilon}_{1,k})} + \sum_{l=0}^{L-1}\frac{\overline{\Upsilon}_{2,l}}{(\overline{\Upsilon}_{2,l}-\overline{\Upsilon}_{1,k})}\right] \tag{3.30}$$

$$D_{12,k} = {C_{1,k}}^2 \tag{3.31}$$

and

$$D_{21,k} = 2\,{C_{2,k}}^2 \left[\sum_{l=0,\, l\neq k}^{L-1} \frac{\overline{\Upsilon}_{2,l}}{(\overline{\Upsilon}_{2,l} - \overline{\Upsilon}_{2,k})} + \sum_{l=0}^{L-1} \frac{\overline{\Upsilon}_{1,l}}{(\overline{\Upsilon}_{1,l} - \overline{\Upsilon}_{2,k})} \right] \tag{3.32}$$

$$D_{22,k} = {C_{2,k}}^2 \tag{3.33}$$

Where, the coefficients $C_{1,k}$ and $C_{2,k}$ are as given by equations 3.8 and 3.9 .From equation 3.29, the pdf of Υ is given by

$$p_\Gamma(\Upsilon) = \sum_{k=0}^{L-1} [\frac{2D_{11,k}}{\overline{\Upsilon}_{1,k}} e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{4D_{12,k}\Upsilon}{{\overline{\Upsilon}_{1,k}}^2} e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{2D_{21,k}}{\overline{\Upsilon}_{2,k}} e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} + \frac{4D_{22,k}\Upsilon}{{\overline{\Upsilon}_{2,k}}^2} e^{-2\Upsilon/\overline{\Upsilon}_{2,k}}] \quad , \Upsilon \geq 0 \tag{3.34}$$

Hence, the outage probability for soft handoff with transmit diversity is given by

$$\begin{aligned} P_o &= \int_0^{\Upsilon_T} p_\Gamma(\Upsilon) d\Upsilon \\ &= \int_0^{\Upsilon_T} \sum_{k=0}^{L-1} [\frac{2D_{11,k}}{\overline{\Upsilon}_{1,k}} e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{4D_{12,k}\Upsilon}{{\overline{\Upsilon}_{1,k}}^2} e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{2D_{21,k}}{\overline{\Upsilon}_{2,k}} e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} \\ &\quad + \frac{4D_{22,k}\Upsilon}{{\overline{\Upsilon}_{2,k}}^2} e^{-2\Upsilon/\overline{\Upsilon}_{2,k}}] d\Upsilon \\ &= \sum_{k=0}^{L-1} [D_{11,k}(1 - e^{-2\Upsilon_T/\overline{\Upsilon}_{1,k}}) + D_{12,k}(1 - e^{-2\Upsilon_T/\overline{\Upsilon}_{1,k}} - \frac{2\Upsilon_T}{\overline{\Upsilon}_{1,k}} e^{-2\Upsilon_T/\overline{\Upsilon}_{1,k}}) \\ &\quad + D_{21,k}(1 - e^{-2\Upsilon_T/\overline{\Upsilon}_{2,k}}) + D_{22,k}(1 - e^{-2\Upsilon_T/\overline{\Upsilon}_{2,k}} - \frac{2\Upsilon_T}{\overline{\Upsilon}_{2,k}} e^{-2\Upsilon_T/\overline{\Upsilon}_{2,k}})] \end{aligned} \tag{3.35}$$

E. Outage Probability with Hard Handoff, Transmit Diversity

For hard handoff with transmit diversity, the receiver first performs maximal ratio combining of the signal received from the diversity antennas followed by selection diversity combining of the two signals from the two base stations. The SINR at the output of the RAKE is

$$\Upsilon = \max(\Upsilon_1, \Upsilon_2) \tag{3.36}$$

Υ_1 and Υ_2 are given by

$$\Upsilon_i = \frac{1}{2}\sum_{k=0}^{L-1}\left[\Upsilon_{i1,k} + \Upsilon_{i2,k}\right], \qquad i = \{1,2\} \tag{3.37}$$

Assuming the the kth paths of both diversity channels to be identical, as in equations 3.27 and 3.28, we get the corresponding characteristic function as

$$\begin{aligned}\psi_{\Upsilon_i}(jv) &= \prod_{k=0}^{L-1}\frac{1}{(1-\frac{jv\overline{\Upsilon}_{i,k}}{2})^2}, \qquad i=\{1,2\} \\ &= \sum_{k=0}^{L-1}\frac{B_{i1,k}}{(1-\frac{jv\overline{\Upsilon}_{1,k}}{2})} + \frac{B_{i2,k}}{(1-\frac{jv\overline{\Upsilon}_{1,k}}{2})^2} \end{aligned}\tag{3.38}$$

with,

$$B_{i1,k} = 2\,{A_{i,k}}^2 \sum_{l=0,\, l\neq k}^{L-1}\frac{\overline{\Upsilon}_{i,l}}{(\overline{\Upsilon}_{i,l}-\overline{\Upsilon}_{i,k})} \tag{3.39}$$

$$B_{i2,k} = {A_{i,k}}^2 \tag{3.40}$$

In the above equations, $A_{i,k}$ is as defined in 3.16. As determined in equations 3.17-3.20, we get the pdf and cdf of Υ_i as

$$p_{\Gamma_i}(\Upsilon) = \sum_{k=0}^{L-1}\left[\frac{2B_{i1,k}}{\overline{\Upsilon}_{i,k}}e^{-2\Upsilon/\overline{\Upsilon}_{i,k}} + \frac{4B_{i2,k}\Upsilon}{{\overline{\Upsilon}_{i,k}}^2}e^{-2\Upsilon/\overline{\Upsilon}_{i,k}}\right], \qquad i=\{1,2\} \tag{3.41}$$

$$F_{\Gamma_i}(\Upsilon) = \sum_{k=0}^{L-1}\left[(B_{i1,k}+B_{i2,k})(1-e^{-2\Upsilon/\overline{\Upsilon}_{i,k}}) - \frac{2B_{i2,k}}{\overline{\Upsilon}_{i,k}}e^{-2\Upsilon/\overline{\Upsilon}_{i,k}}\right],$$

$$i = \{1,2\} \tag{3.42}$$

Hence, the pdf of Υ is

$$\begin{aligned}
p_{\Gamma}(\Upsilon) &= \sum_{k=0}^{L-1}[\frac{2B_{11,k}}{\overline{\Upsilon}_{1,k}}e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{4B_{12,k}\Upsilon}{\overline{\Upsilon}_{1,k}^{\ 2}}e^{-2\Upsilon/\overline{\Upsilon}_{1,k}}] \times \\
& \sum_{k'=0}^{L-1}[(B_{21,k'}+B_{22,k'})(1-e^{-2\Upsilon/\overline{\Upsilon}_{2,k'}}) - \frac{2B_{22,k'}\Upsilon}{\overline{\Upsilon}_{2,k'}}e^{-2\Upsilon/\overline{\Upsilon}_{2,k'}}] + \\
& \sum_{k=0}^{L-1}[\frac{2B_{21,k}}{\overline{\Upsilon}_{2,k}}e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} + \frac{4B_{22,k}\Upsilon}{\overline{\Upsilon}_{2,k}^{\ 2}}e^{-2\Upsilon/\overline{\Upsilon}_{2,k}}] \times \\
& \sum_{k'=0}^{L-1}[(B_{11,k'}+B_{12,k'})(1-e^{-2\Upsilon/\overline{\Upsilon}_{1,k'}}) - \frac{2B_{12,k'}\Upsilon}{\overline{\Upsilon}_{1,k'}}e^{-2\Upsilon/\overline{\Upsilon}_{1,k'}}] \\
&= \sum_{k=0}^{L-1}\sum_{k'=0}^{L-1}[\frac{2B_{11,k}(B_{21,k'}+B_{22,k'})}{\overline{\Upsilon}_{1,k}}(e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} - e^{-2\Upsilon/\Delta_1}) \\
& \quad + \frac{2B_{21,k}(B_{11,k'}+B_{12,k'})}{\overline{\Upsilon}_{2,k}}(e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} - e^{-2\Upsilon/\Delta_2}) \\
& \quad + \frac{4B_{12,k}(B_{21,k'}+B_{22,k'})\Upsilon}{\overline{\Upsilon}_{1,k}^{\ 2}}(e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} - e^{-2\Upsilon/\Delta_1}) \\
& \quad + \frac{4B_{22,k}(B_{11,k'}+B_{12,k'})}{\overline{\Upsilon}_{2,k}^{\ 2}}(\Upsilon e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} - \Upsilon e^{-2\Upsilon/\Delta_2}) \\
& \quad - \frac{4B_{11,k}B_{22,k'}}{\overline{\Upsilon}_{1,k}\overline{\Upsilon}_{2,k'}}\Upsilon e^{-2\Upsilon/\Delta_1} - \frac{4B_{21,k}B_{12,k'}}{\overline{\Upsilon}_{1,k'}\overline{\Upsilon}_{2,k}}\Upsilon e^{-2\Upsilon/\Delta_2}
\end{aligned}$$

$$-\frac{8B_{12,k}B_{22,k'}}{\overline{\Upsilon}_{1,k'}{}^2\overline{\Upsilon}_{2,k}}\Upsilon^2 e^{-2\Upsilon/\Delta_1} - \frac{8B_{22,k}B_{12,k'}}{\overline{\Upsilon}_{1,k'}\overline{\Upsilon}_{2,k}{}^2}\Upsilon^2 e^{-2\Upsilon/\Delta_2}] \qquad (3.43)$$

With, α_1, α_2, Δ_1 and Δ_2 as defined in equations 3.24 and 3.25 respectively. Hence, the outage probability is given by

$$\begin{aligned}
P_o &= \int_0^{\Upsilon_T} p_\Gamma(\Upsilon)d\Upsilon \\
&= \sum_{k=0}^{L-1}\sum_{k'=0}^{L-1}[B_{11,k}(B_{21,k'}+B_{22,k'})(1-e^{-2\Upsilon_T/\overline{\Upsilon}_{1,k}} - \overline{\Upsilon}_{2,k'}\alpha_1(1-e^{-2\Upsilon_T/\Delta_1})) \\
&\quad + B_{21,k}(B_{11,k'}+B_{12,k'})(1-e^{-2\Upsilon_T/\overline{\Upsilon}_{2,k}} - \overline{\Upsilon}_{1,k'}\alpha_2(1-e^{-2\Upsilon_T/\Delta_2})) \\
&\quad + B_{12,k}(B_{21,k'}+B_{22,k'})(1-e^{-2\Upsilon_T/\overline{\Upsilon}_{1,k}} - \frac{2\Upsilon_T}{\overline{\Upsilon}_{1,k}}e^{-2\Upsilon_T/\overline{\Upsilon}_{1,k}} + \frac{2\Upsilon_T\alpha_1\overline{\Upsilon}_{2,k'}}{\overline{\Upsilon}_{1,k}} \\
&\quad \times e^{-2\Upsilon_T/\Delta_1} - \overline{\Upsilon}_{2,k'}{}^2\alpha_1{}^2(1-e^{-2\Upsilon_T/\Delta_1})) + B_{22,k}(B_{11,k'}+B_{12,k'}) \\
&\quad (1-e^{-2\Upsilon_T/\overline{\Upsilon}_{2,k}} - \frac{2\Upsilon_T}{\overline{\Upsilon}_{2,k}}e^{-2\Upsilon_T/\overline{\Upsilon}_{2,k}} + \frac{2\Upsilon_T\alpha_2\overline{\Upsilon}_{1,k'}}{\overline{\Upsilon}_{2,k}}e^{-2\Upsilon_T/\Delta_2} - \overline{\Upsilon}_{1,k'}{}^2\alpha_2{}^2 \\
&\quad (1-e^{-2\Upsilon_T/\Delta_2})) - B_{11,k}B_{22,k'}(\overline{\Upsilon}_{1,k'}\overline{\Upsilon}_{2,k}\alpha_1{}^2(1-e^{-2\Upsilon_T/\Delta_1}) - 2\Upsilon_T\alpha_1 e^{-2\Upsilon_T/\Delta_1}) \\
&\quad - B_{21,k}B_{12,k'}(\overline{\Upsilon}_{2,k'}\overline{\Upsilon}_{1,k}\alpha_2{}^2(1-e^{-2\Upsilon_T/\Delta_2}) - 2\Upsilon_T\alpha_2 e^{-2\Upsilon_T/\Delta_2}) - B_{12,k}B_{22,k'} \\
&\quad \times(2\overline{\Upsilon}_{2,k'}{}^2\overline{\Upsilon}_{1,k}\alpha_1{}^3(1-e^{-2\Upsilon_T/\Delta_1}) - 4\Upsilon_T\alpha_1 e^{-2\Upsilon_T/\Delta_1}(\frac{\Upsilon_T}{\overline{\Upsilon}_{1,k}} + \overline{\Upsilon}_{2,k'}\alpha_1)) - B_{22,k}
\end{aligned}$$

$$\times B_{12,k'}(2\overline{\Upsilon}_{1,k'}{}^2\overline{\Upsilon}_{2,k}\alpha_2{}^3(1-e^{-2\Upsilon_T/\Delta_2})-4\Upsilon_T\alpha_2 e^{-2\Upsilon_T/\Delta_2}(\frac{\Upsilon_T}{\overline{\Upsilon}_{2,k}}+\overline{\Upsilon}_{1,k'}\alpha_2))] \tag{3.44}$$

CHAPTER IV

PERFORMANCE MEASURES - PROBABILITY OF BIT ERROR

In the last chapter, we analyzed the probability of outage for a transmit diversity system as well as for a non-diversity system, undergoing soft handoff and hard handoff. We will proceed similarly in this chapter, but our performance measure to be analyzed will be the probability of bit error. We will consider a non-diversity system undergoing soft handoff or hard handoff. Also, the same will be analyzed for a system incorporating transmit diversity. The system as well as the assumptions for the system are the same as in the preceding chapter.

A. Probability of Bit Error with Soft Handoff, No Diversity

The probability of bit error is obtained by integrating the conditional error probability ($Q(\sqrt{2\Upsilon})$) over the pdf of Υ (3.10). The pdf and hence, the probability of bit error is

$$p_\Gamma(\Upsilon) = \sum_{k=0}^{L-1}\left[\frac{C_{1,k}}{\overline{\Upsilon}_{1,k}}e^{-\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{C_{2,k}}{\overline{\Upsilon}_{2,k}}e^{-\Upsilon/\overline{\Upsilon}_{2,k}}\right] \qquad ,\Upsilon \geq 0$$

$$Pb = \int_0^\infty Q(\sqrt{2\Upsilon})p_\Gamma(\Upsilon)d\Upsilon \qquad (4.1)$$

$$= \int_0^\infty Q(\sqrt{2\Upsilon})\sum_{k=0}^{L-1}[\frac{C_{1,k}}{\overline{\Upsilon}_{1,k}}e^{-\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{C_{2,k}}{\overline{\Upsilon}_{2,k}}e^{-\Upsilon/\overline{\Upsilon}_{2,k}}]d\Upsilon$$

$$= \sum_{k=0}^{L-1}\frac{1}{2}\left[C_{1,k}\left(1-\sqrt{\frac{\overline{\Upsilon}_{1,k}}{\left(1+\overline{\Upsilon}_{1,k}\right)}}\right) + C_{2,k}\left(1-\sqrt{\frac{\overline{\Upsilon}_{2,k}}{\left(1+\overline{\Upsilon}_{2,k}\right)}}\right)\right] \qquad (4.2)$$

B. Probability of Bit Error with Hard Handoff, No Diversity

In the case of hard handoff without diversity, the pdf of Υ (3.22) and hence, the probability of bit error is given by

$$
\begin{aligned}
p_\Gamma(\Upsilon) &= \sum_{k=0}^{L-1} \frac{A_{1,k}}{\overline{\Upsilon}_{1,k}} e^{-\Upsilon/\overline{\Upsilon}_{1,k}} \times \sum_{k'=0}^{L-1} A_{2,k'}(1 - e^{-\Upsilon/\overline{\Upsilon}_{2,k'}}) + \\
& \sum_{k=0}^{L-1} \frac{A_{2,k}}{\overline{\Upsilon}_{2,k}} e^{-\Upsilon/\overline{\Upsilon}_{2,k}} \sum_{k'=0}^{L-1} A_{1,k'}(1 - e^{-\Upsilon/\overline{\Upsilon}_{1,k'}}) \\
&= \sum_{k=0}^{L-1} \sum_{k'=0}^{L-1} [\frac{A_{1,k}A_{2,k'}}{\overline{\Upsilon}_{1,k}} (e^{-\Upsilon/\overline{\Upsilon}_{1,k}} - e^{-\Upsilon(\frac{1}{\overline{\Upsilon}_{1,k}} + \frac{1}{\overline{\Upsilon}_{2,k'}})}) \\
& \quad + \frac{A_{2,k}A_{1,k'}}{\overline{\Upsilon}_{2,k}} (e^{-\Upsilon/\overline{\Upsilon}_{2,k}} - e^{-\Upsilon(\frac{1}{\overline{\Upsilon}_{2,k}} + \frac{1}{\overline{\Upsilon}_{1,k'}})})] \qquad , \Upsilon \geq 0 \\
Pb &= \int_0^\infty Q(\sqrt{2\Upsilon}) \sum_{k=0}^{L-1} \sum_{k'=0}^{L-1} [\frac{A_{1,k}A_{2,k'}}{\overline{\Upsilon}_{1,k}} (e^{-\Upsilon/\overline{\Upsilon}_{1,k}} - e^{-\Upsilon(\frac{1}{\overline{\Upsilon}_{1,k}} + \frac{1}{\overline{\Upsilon}_{2,k'}})}) \\
& \quad + \frac{A_{2,k}A_{1,k'}}{\overline{\Upsilon}_{2,k}} (e^{-\Upsilon/\overline{\Upsilon}_{2,k}} - e^{-\Upsilon(\frac{1}{\overline{\Upsilon}_{2,k}} + \frac{1}{\overline{\Upsilon}_{1,k'}})})] \, d\Upsilon \\
&= \int_0^\infty Q(\sqrt{2\Upsilon}) \sum_{k=0}^{L-1} \sum_{k'=0}^{L-1} [\frac{A_{1,k}A_{2,k'}}{\overline{\Upsilon}_{1,k}} (e^{-\Upsilon/\overline{\Upsilon}_{1,k}} - e^{-\Upsilon/\Delta_1}) \\
& \quad + \frac{A_{2,k}A_{1,k'}}{\overline{\Upsilon}_{2,k}} (e^{-\Upsilon/\overline{\Upsilon}_{2,k}} - e^{-\Upsilon/\Delta_2})] \, d\Upsilon \\
&= \sum_{k=0}^{L-1} \sum_{k'=0}^{L-1} [\frac{A_{1,k}A_{2,k'}}{2} ((1 - \sqrt{\frac{\overline{\Upsilon}_{1,k}}{1 + \overline{\Upsilon}_{1,k}}}) - \overline{\Upsilon}_{2,k'}\alpha_1(1 - \sqrt{\frac{\Delta_1}{1 + \Delta_1}})) \\
& \quad + \frac{A_{2,k}A_{1,k'}}{2} ((1 - \sqrt{\frac{\overline{\Upsilon}_{2,k}}{1 + \overline{\Upsilon}_{2,k}}}) - \overline{\Upsilon}_{1,k'}\alpha_2(1 - \sqrt{\frac{\Delta_2}{1 + \Delta_2}}))] \qquad (4.3)
\end{aligned}
$$

Δ_1 and Δ_2 are as defined in equation 3.25.

C. Probability of Bit Error with Soft Handoff, Transmit Diversity

Taking the pdf of Υ as in equation 3.34 and solving as in [17], the probability of bit error is

$$\begin{aligned} p_\Gamma(\Upsilon) &= \sum_{k=0}^{L-1}[\frac{2D_{11,k}}{\overline{\Upsilon}_{1,k}}e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{4D_{12,k}\Upsilon}{\overline{\Upsilon}_{1,k}^{\;2}}e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{2D_{21,k}}{\overline{\Upsilon}_{2,k}}e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} \\ &\quad + \frac{4D_{22,k}\Upsilon}{\overline{\Upsilon}_{2,k}^{\;2}}e^{-2\Upsilon/\overline{\Upsilon}_{2,k}}] \qquad , \Upsilon \geq 0 \end{aligned}$$

$$\begin{aligned} Pb &= \int_0^\infty Q(\sqrt{2\Upsilon})\sum_{k=0}^{L-1}[\frac{2D_{11,k}}{\overline{\Upsilon}_{1,k}}e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{4D_{12,k}\Upsilon}{\overline{\Upsilon}_{1,k}^{\;2}}e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} \\ &\quad + \frac{2D_{21,k}}{\overline{\Upsilon}_{2,k}}e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} + \frac{4D_{22,k}\Upsilon}{\overline{\Upsilon}_{2,k}^{\;2}}e^{-2\Upsilon/\overline{\Upsilon}_{2,k}}]\; d\Upsilon \\ &= \sum_{k=0}^{L-1}[\frac{D_{11,k}}{2}(1-\mu_1) + \frac{D_{12,k}}{4}(2 - 3\mu_1 + \mu_1^{\;3}) \\ &\quad + \frac{D_{21,k}}{2}(1-\mu_2) + \frac{D_{22,k}}{4}(2 - 3\mu_2 + \mu_2^{\;3})] \end{aligned} \tag{4.4}$$

With, μ_1 and μ_2 as

$$\mu_1 = \sqrt{\frac{\overline{\Upsilon}_{1,k}}{2+\overline{\Upsilon}_{1,k}}}, \qquad \mu_2 = \sqrt{\frac{\overline{\Upsilon}_{2,k}}{2+\overline{\Upsilon}_{2,k}}} \tag{4.5}$$

D. Probability of Bit Error with Hard Handoff, Transmit Diversity

For hard handoff with transmit diversity, the pdf of Υ (3.43) and the probability of bit error will be

$$
\begin{aligned}
p_\Gamma(\Upsilon) \;=\; & \sum_{k=0}^{L-1}[\frac{2B_{11,k}}{\overline{\Upsilon}_{1,k}}e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} + \frac{4B_{12,k}\Upsilon}{\overline{\Upsilon}_{1,k}^{\;2}}e^{-2\Upsilon/\overline{\Upsilon}_{1,k}}] \times \\
& \sum_{k'=0}^{L-1}[(B_{21,k'} + B_{22,k'})(1 - e^{-2\Upsilon/\overline{\Upsilon}_{2,k'}}) - \frac{2B_{22,k'}\Upsilon}{\overline{\Upsilon}_{2,k'}}e^{-2\Upsilon/\overline{\Upsilon}_{2,k'}}] + \\
& \sum_{k=0}^{L-1}[\frac{2B_{21,k}}{\overline{\Upsilon}_{2,k}}e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} + \frac{4B_{22,k}\Upsilon}{\overline{\Upsilon}_{2,k}^{\;2}}e^{-2\Upsilon/\overline{\Upsilon}_{2,k}}] \times \\
& \sum_{k'=0}^{L-1}[(B_{11,k'} + B_{12,k'})(1 - e^{-2\Upsilon/\overline{\Upsilon}_{1,k'}}) - \frac{2B_{12,k'}\Upsilon}{\overline{\Upsilon}_{1,k'}}e^{-2\Upsilon/\overline{\Upsilon}_{1,k'}}] \\
= \; & \sum_{k=0}^{L-1}\sum_{k'=0}^{L-1}[\frac{2B_{11,k}(B_{21,k'} + B_{22,k'})}{\overline{\Upsilon}_{1,k}}(e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} - e^{-2\Upsilon/\Delta_1}) \\
& + \frac{2B_{21,k}(B_{11,k'} + B_{12,k'})}{\overline{\Upsilon}_{2,k}}(e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} - e^{-2\Upsilon/\Delta_2}) \\
& + \frac{4B_{12,k}(B_{21,k'} + B_{22,k'})\Upsilon}{\overline{\Upsilon}_{1,k}^{\;2}}(e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} - e^{-2\Upsilon/\Delta_1}) \\
& + \frac{4B_{22,k}(B_{11,k'} + B_{12,k'})}{\overline{\Upsilon}_{2,k}^{\;2}}(\Upsilon e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} - \Upsilon e^{-2\Upsilon/\Delta_2}) \\
& - \frac{4B_{11,k}B_{22,k'}}{\overline{\Upsilon}_{1,k}\overline{\Upsilon}_{2,k'}}\Upsilon e^{-2\Upsilon/\Delta_1} - \frac{4B_{21,k}B_{12,k'}}{\overline{\Upsilon}_{1,k'}\overline{\Upsilon}_{2,k}}\Upsilon e^{-2\Upsilon/\Delta_2} \\
& - \frac{8B_{12,k}B_{22,k'}}{\overline{\Upsilon}_{1,k'}^{\;2}\overline{\Upsilon}_{2,k}}\Upsilon^2 e^{-2\Upsilon/\Delta_1} - \frac{8B_{22,k}B_{12,k'}}{\overline{\Upsilon}_{1,k'}\overline{\Upsilon}_{2,k}^{\;2}}\Upsilon^2 e^{-2\Upsilon/\Delta_2}]
\end{aligned}
$$

$$
\begin{aligned}
Pb &= \int_0^\infty Q(\sqrt{2\Upsilon}) \sum_{k=0}^{L-1} \sum_{k'=0}^{L-1} [\frac{2B_{11,k}(B_{21,k'}+B_{22,k'})}{\overline{\Upsilon}_{1,k}}(e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} - e^{-2\Upsilon/\Delta_1}) \\
&+\frac{2B_{21,k}(B_{11,k'}+B_{12,k'})}{\overline{\Upsilon}_{2,k}}(e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} - e^{-2\Upsilon/\Delta_2}) \\
&+\frac{4B_{12,k}(B_{21,k'}+B_{22,k'})}{\overline{\Upsilon}_{1,k}{}^2}(\Upsilon e^{-2\Upsilon/\overline{\Upsilon}_{1,k}} - \Upsilon e^{-2\Upsilon/\Delta_1}) \\
&+\frac{4B_{22,k}(B_{11,k'}+B_{12,k'})}{\overline{\Upsilon}_{2,k}{}^2}(\Upsilon e^{-2\Upsilon/\overline{\Upsilon}_{2,k}} - \Upsilon e^{-2\Upsilon/\Delta_2}) \\
&-\frac{4B_{11,k}B_{22,k'}}{\overline{\Upsilon}_{1,k}\overline{\Upsilon}_{2,k'}}\Upsilon e^{-2\Upsilon/\Delta_1} - \frac{4B_{21,k}B_{12,k'}}{\overline{\Upsilon}_{1,k'}\overline{\Upsilon}_{2,k}}\Upsilon e^{-2\Upsilon/\Delta_2} \\
&-\frac{8B_{12,k}B_{22,k'}}{\overline{\Upsilon}_{1,k'}{}^2\overline{\Upsilon}_{2,k}}\Upsilon^2 e^{-2\Upsilon/\Delta_1} - \frac{8B_{22,k}B_{12,k'}}{\overline{\Upsilon}_{1,k'}\overline{\Upsilon}_{2,k}{}^2}\Upsilon^2 e^{-2\Upsilon/\Delta_2}]\, d\Upsilon \\
&= \frac{1}{2}\sum_{k=0}^{L-1}\sum_{k'=0}^{L-1}[B_{11,k}(B_{21,k'}+B_{22,k'})(1-\mu_1-\overline{\Upsilon}_{2,k'}\alpha_1(1-\beta_1)) \\
&+B_{21,k}(B_{11,k'}+B12,k')(1-\mu_1-\overline{\Upsilon}_{1,k'}\alpha_2(1-\beta_2)) \\
&-\frac{B_{11,k}B_{22,k'}\overline{\Upsilon}_{1,k}\overline{\Upsilon}_{2,k'}\alpha_1{}^2}{2}(2-3\beta_1+\beta_1{}^3) \\
&-\frac{B_{21,k}B_{21,k'}\overline{\Upsilon}_{2,k}\overline{\Upsilon}_{1,k'}\alpha_2{}^2}{2}(2-3\beta_2+\beta_2{}^3) \\
&+\frac{B_{12,k}(B_{21,k'}+B_{22,k'})}{2}(2-3\mu_1+\mu_1{}^2-\overline{\Upsilon}_{2,k'}{}^2\overline{\Upsilon}_{1,k}(2-3\beta_1+\beta_1{}^3)
\end{aligned}
$$

$$+\frac{B_{22,k}(B_{11,k'}+B_{12,k'})}{2}(2-3\mu_2+{\mu_2}^2-{\overline{\Upsilon}_{1,k'}}^2\overline{\Upsilon}_{2,k}(2-3\beta_2+{\beta_2}^3)$$

$$-4B_{12,k}B_{22,k'}\overline{\Upsilon}_{1,k}{\overline{\Upsilon}_{2,k'}}^2{\alpha_1}^3(\frac{1}{2}-\frac{15\beta_1}{16}-\frac{5{\beta_1}^3}{8}+\frac{3{\beta_1}^5}{16})$$

$$-4B_{22,k}B_{12,k'}\overline{\Upsilon}_{2,k}{\overline{\Upsilon}_{1,k'}}^2{\alpha_2}^3(\frac{1}{2}-\frac{15\beta_2}{16}-\frac{5{\beta_2}^3}{8}+\frac{3{\beta_2}^5}{16})] \quad (4.6)$$

Where, μ_1 and μ_2 are as defined in equation 4.5, α_1 and α_2 are as defined in equation 3.24. Also, β_1 and β_2 are

$$\beta_1=\sqrt{\frac{\overline{\Upsilon}_{1,k}\overline{\Upsilon}_{2,k'}}{2(\overline{\Upsilon}_{1,k}+\overline{\Upsilon}_{2,k'})+\overline{\Upsilon}_{1,k}\overline{\Upsilon}_{2,k'}}} \quad \beta_2=\sqrt{\frac{\overline{\Upsilon}_{2,k}\overline{\Upsilon}_{1,k'}}{2(\overline{\Upsilon}_{2,k}+\overline{\Upsilon}_{1,k'})+\overline{\Upsilon}_{2,k}\overline{\Upsilon}_{1,k'}}} \quad (4.7)$$

In this chapter and the preceding chapter, we analyzed the system for performance in terms of outage and bit error rate. In the next chapter, we present the results of this analysis.

CHAPTER V

RESULTS

In this chapter, we compare the analytical results for the performance of an uncoded non-diversity and an uncoded two antenna transmit diversity system, in terms of the probability of outage and the probability of bit error, for different channel models, number of users, percentage of users in handoff, outage threshold's and SNR's.

The propagation loss was modeled by a macro cell propagation model as suggested in [3] and detailed in Appendix B. Interference was modeled by the gaussian approximation taking into consideration the two cells concerned only. This is as detailed in Appendix A. The spreading factor was assumed to be 256, the spectral efficiency of the unspread system (for BPSK) was $\frac{1}{2}$ and the number of users (unless specified other wise) was 21. Some of the other system parameters used are as listed in table I.

Table I. System Parameters

Cell Radius	1 Km
Number of Soft Handoff legs	2
Orthogonality Factor	0.4
Maximum Base Station Transmission Power	20 Watts
Transmitter Antenna Gain	10 dB
Receiver Antenna Gain	0 dB
Thermal Noise Density	-95 dBm/Hz
Overhead to Pilot and other common control channels	15 %
Percent of Users in Handoff	15 %

A. Probability of Outage

1. Various Channel Models

We compared the four cases for the three channel models considered in [17] namely Office A, Pedestrian A and Vehicular A, with the characteristics as given in table II. The system characteristics were as detailed in table I. Probability of outage was determined for the movement of the mobile in the handoff region (from two-thirds of the distance equal to the radius of the cell to one-thirds of the radius of the cell beyond the cell boundary). Outage probability is plotted as a function of the distance for all three channel models. Outage threshold is assumed to be 5 dB.

Table II. WCDMA Channel Models

Channel	Mean powers(dB)	Delays(μs)
Office A	$0, -10, -30$	$0, 0.24, 0.485$
Pedestrian A	$0, -12.5, -25$	$0, 0.24, 0.485$
Vehicular A	$0, -1, -9, -10, -15, -20$	$0, 0.31, 0.71, 1.09, 1.73, 2.51$

On the whole, the system exhibited the best performance for the Vehicular A channel, for all the four cases, giving the lowest probability of outage. The results for the Vehicular A channel were the best followed by the Office A channel and then the Pedestrian A channel, for all four cases. This is because of the availability of two dominant paths.

Hard handoff with diversity fared better than soft handoff in a non-diversity system for all three channel models. Comparatively, hard handoff with diversity fared the best in the Pedestrian A channel followed by the Office A channel and the

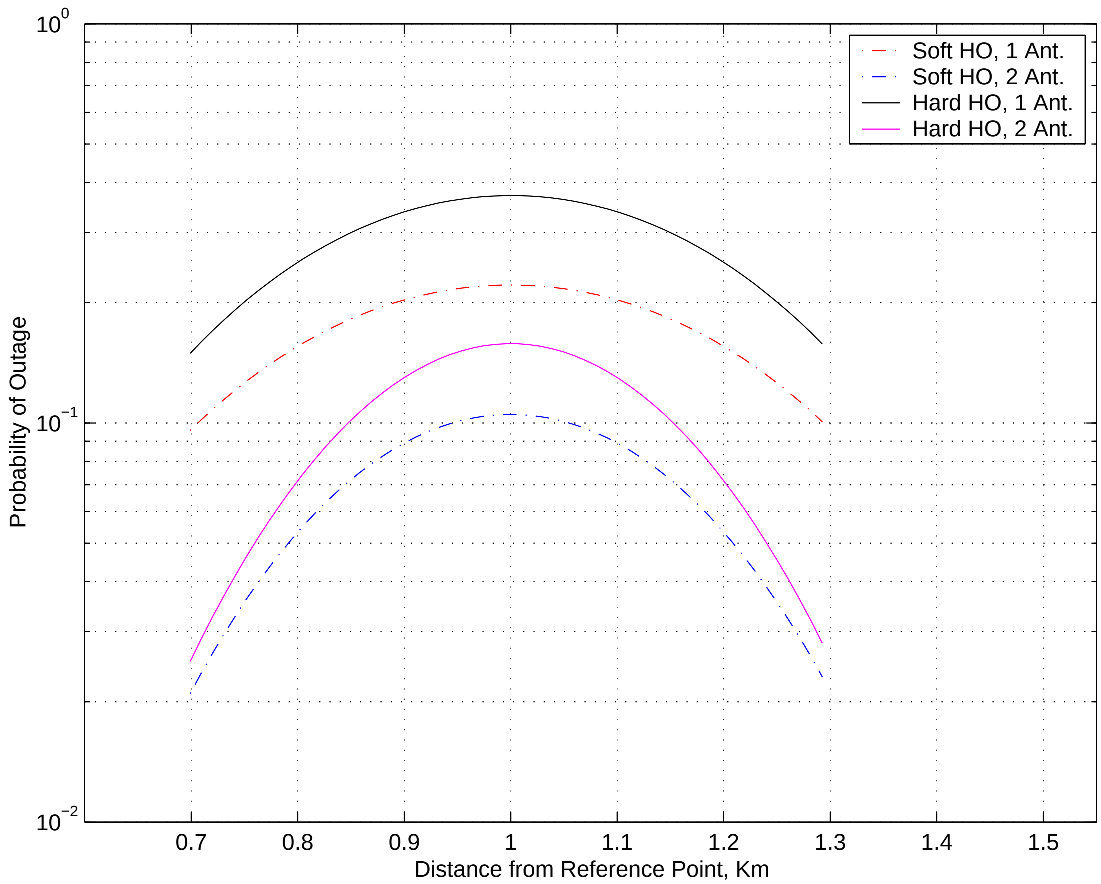

Fig. 11. Probability of Outage in the Office A Channel, 15 Percent of Users in Handoff

Vehicular A channel respectively, when compared to soft handoff without diversity. This is because a single dominant path and a steep decay profile doesn't give MRC a lot of advantage over SDC. As expected, soft handoff with transmit diversity shows the best performance. The results are a shown in figures 11, 12 and 13.

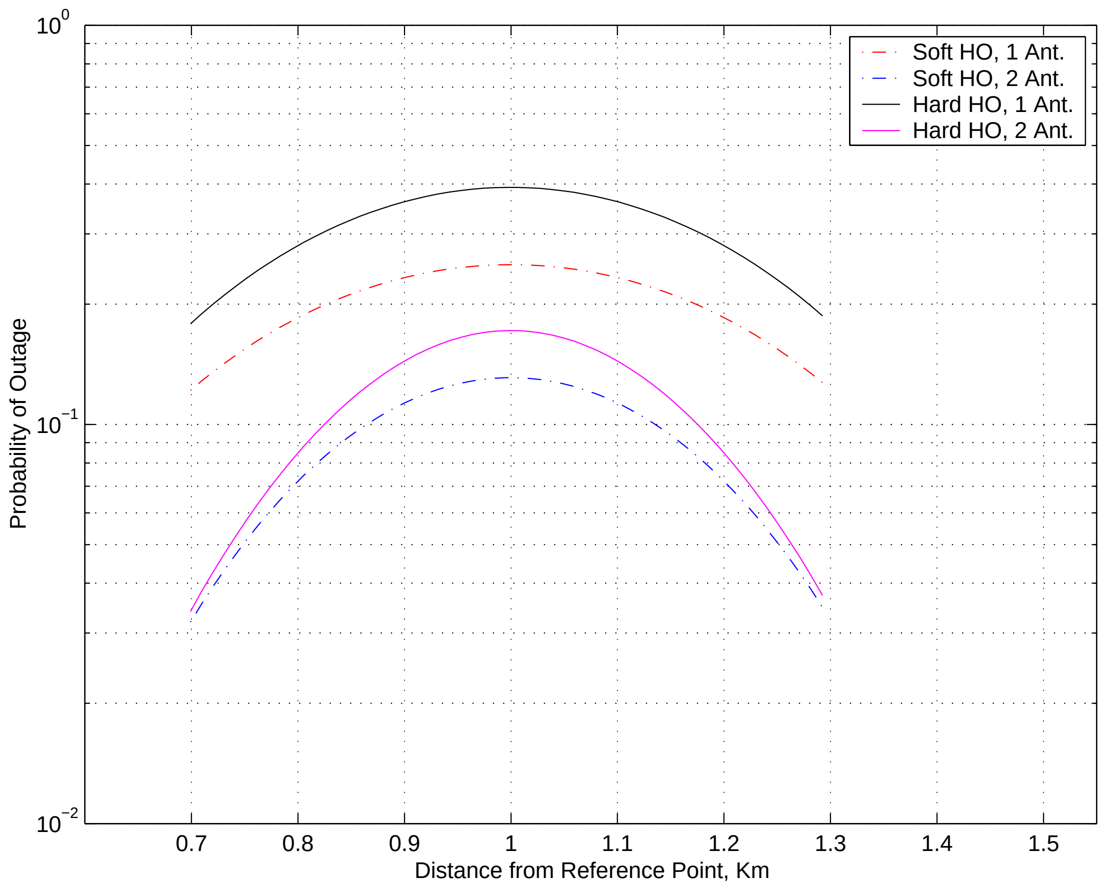

Fig. 12. Probability of Outage in the Pedestrian A Channel, 15 Percent of Users in Handoff

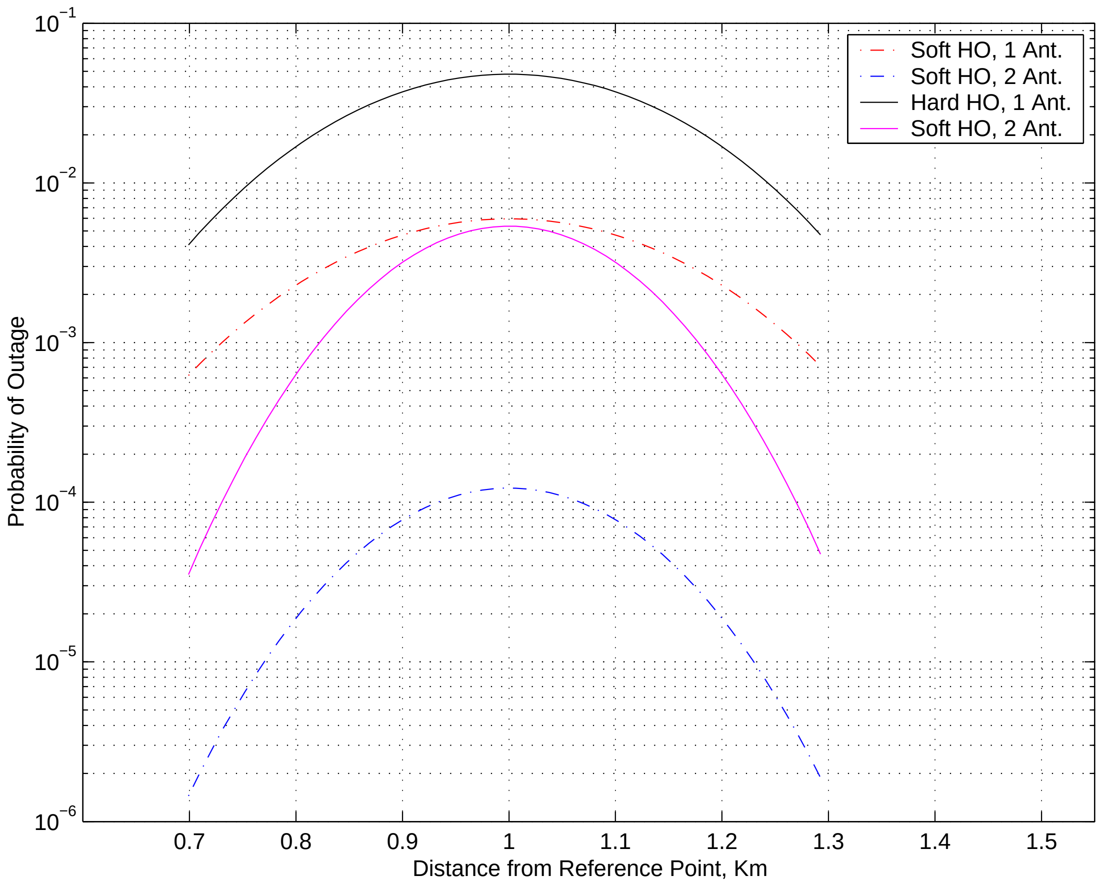

Fig. 13. Probability of Outage in the Vehicular A Channel, 15 Percent of Users in Handoff

2. Varying the Transmit SNRs

We compared the system for different transmit signal to noise ratios. This was done by varying the transmitter power and determining the probability of outage at the boundary of the cell. The channel model considered was the Vehicular A channel. The probability of outage was plotted against the signal to noise ratio (excluding interference) at the cell boundary. The results are as shown in figure 14.

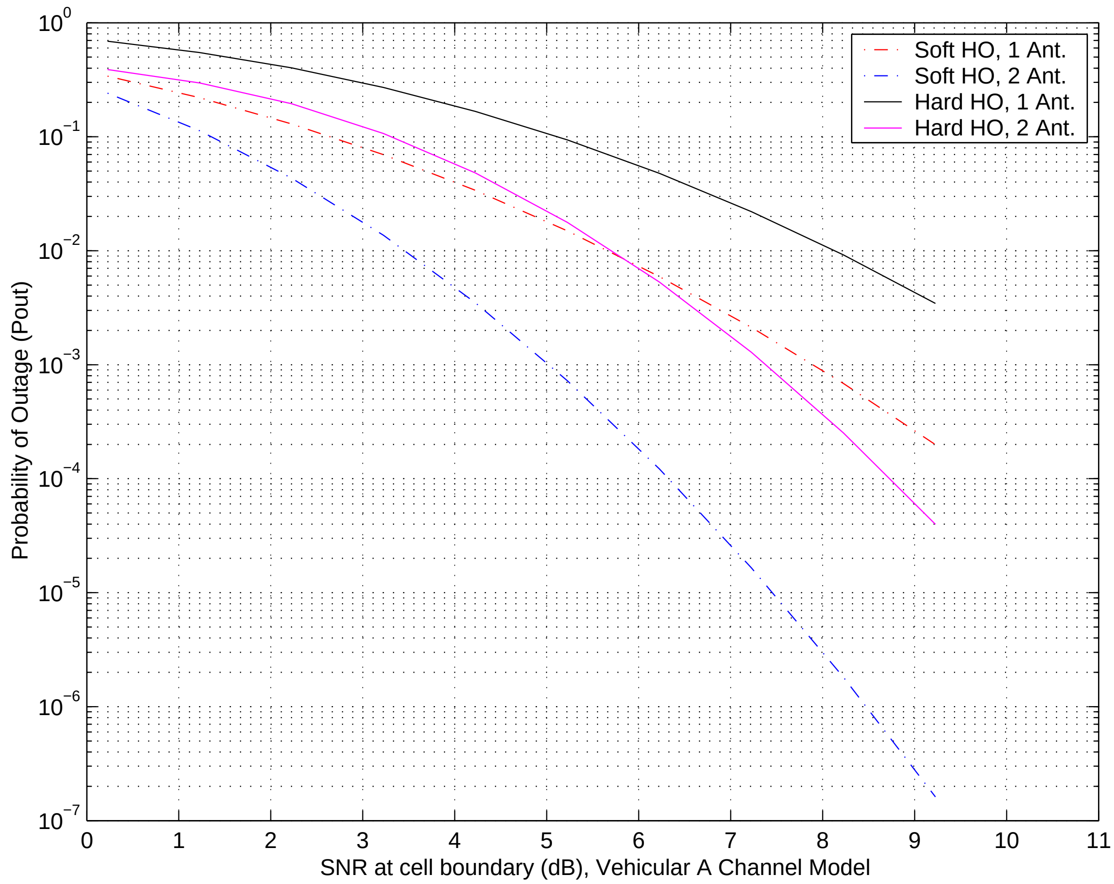

Fig. 14. Probability of Outage at the Cell Boundary for Different SNR's, in the Vehicular A Channel, 15 Percent of Users in Handoff

Soft handoff with diversity gave the best performance and hard handoff without diversity fared the worst. For low values of signal to noise ratios, soft handoff without diversity performed better than hard handoff with diversity. However, hard handoff with diversity showed an improvement over soft handoff with diversity over higher values of SNR's. For an outage probability of 10^{-3}, soft handoff with diversity has an advantage of 2.5 dB over hard handoff with diversity. For the same probability of outage, hard handoff with diversity showed a 0.5 dB gain over soft handoff without diversity.

3. Varying the Number of Users

We compared the system for different number of users. The probability of outage at the cell boundary for different number of users was determined and plotted as in figure 15. We see a gradual rise in the probability of outage which saturates for large number of users. Soft handoff with transmit diversity fares best with hard handoff without diversity giving the worst peformance. Hard handoff with diversity performs better than soft handoff without diversity for lower number of users but loses out for a large nmber of users. This is because more users contribute to an increased interference as well as a decreased transmit power allocated to each user for a fixed total transmitter power. This results in a decrease in the SNR and hence the results as detailed above.

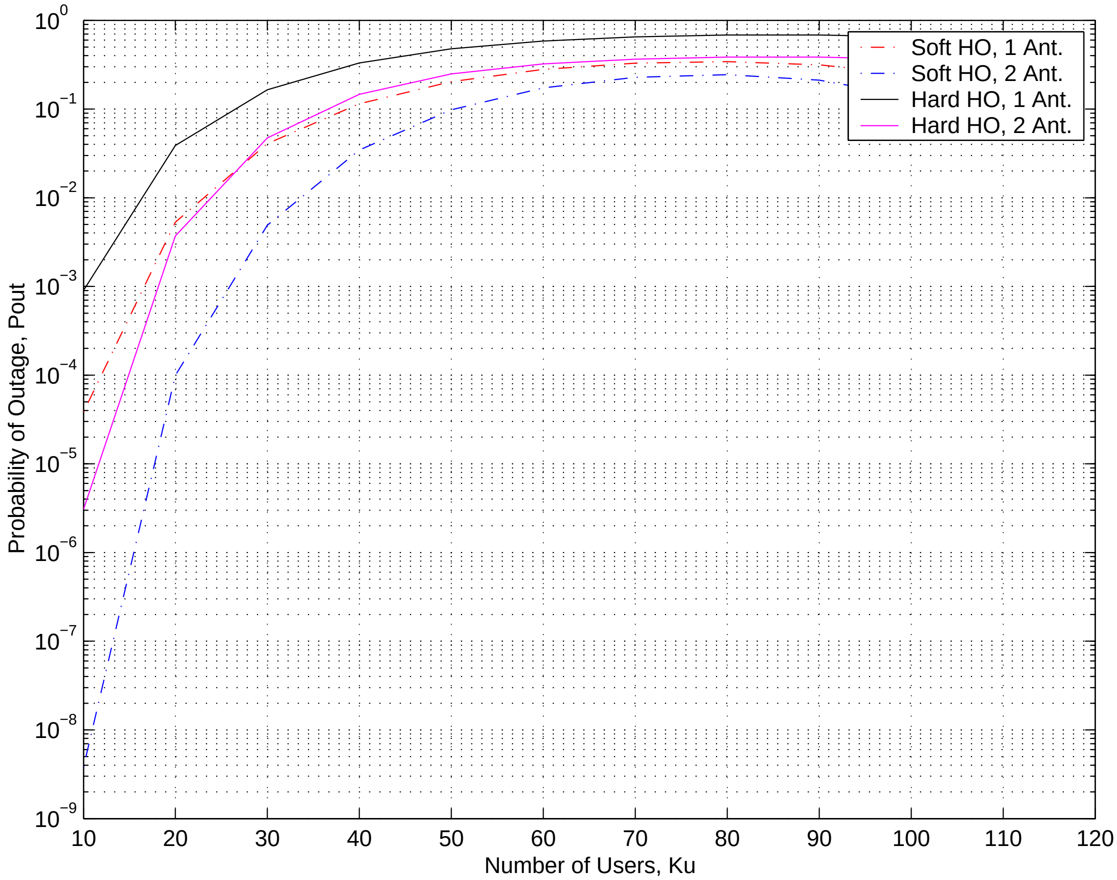

Fig. 15. Probability of Outage at the Cell Boundary for Different Number of Users, in the Vehicular A Channel, Percentage of Users in Handoff = 30, Transmit Power = 20 W

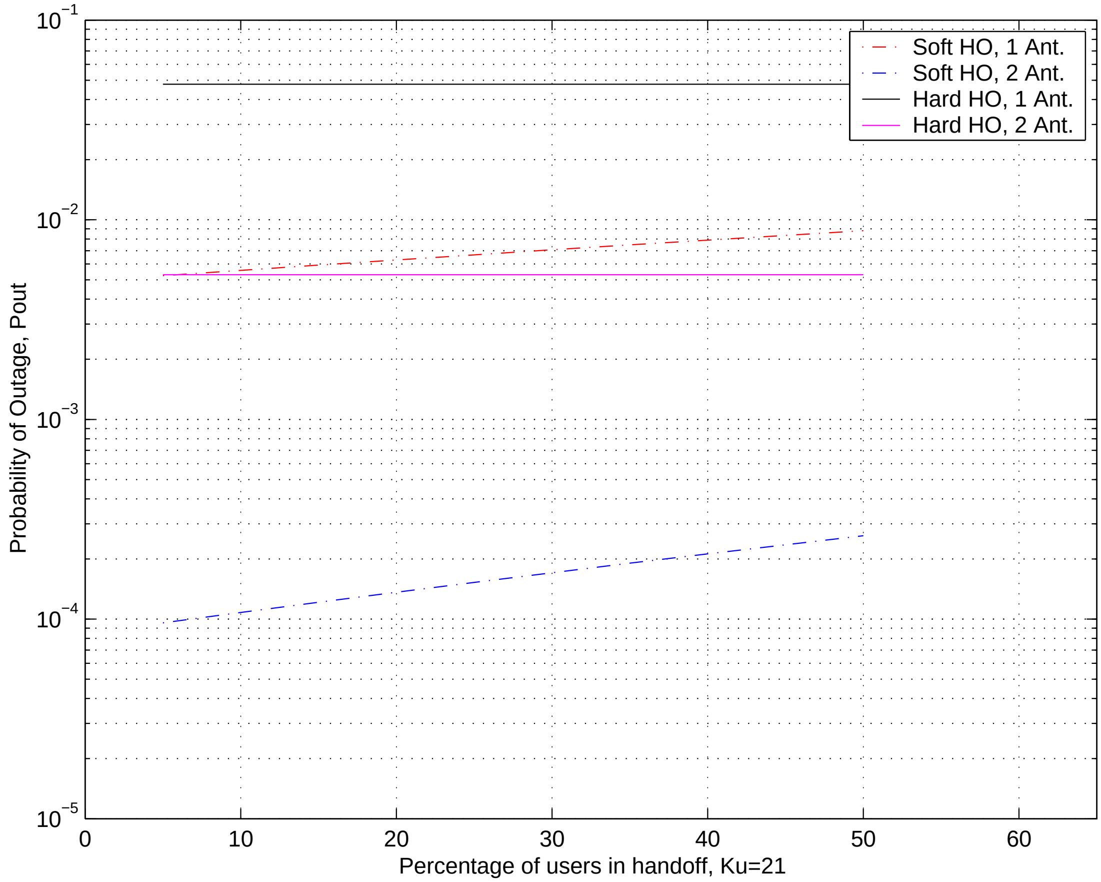

Fig. 16. Probability of Outage at the Cell Boundary for Different Percentage of Users Under Handoff, in the Vehicular A Channel, Number of Users Ku=21, Transmit Power = 20 W

4. Varying the Percentage of Users in Handoff

Here, we compared the outage probability of the system at the cell boundary while varying the percentage of users in handoff. Varying the pecentage of users in handoff varies the loading on the cell and hence the interference associated with the system. For hard handoff, there isn't any change in the loading as is evinced by the straight lines seen in figure 16, both with and without transmit diversity. For soft handoff, as the loading increases, we see an increase in the outage probability.

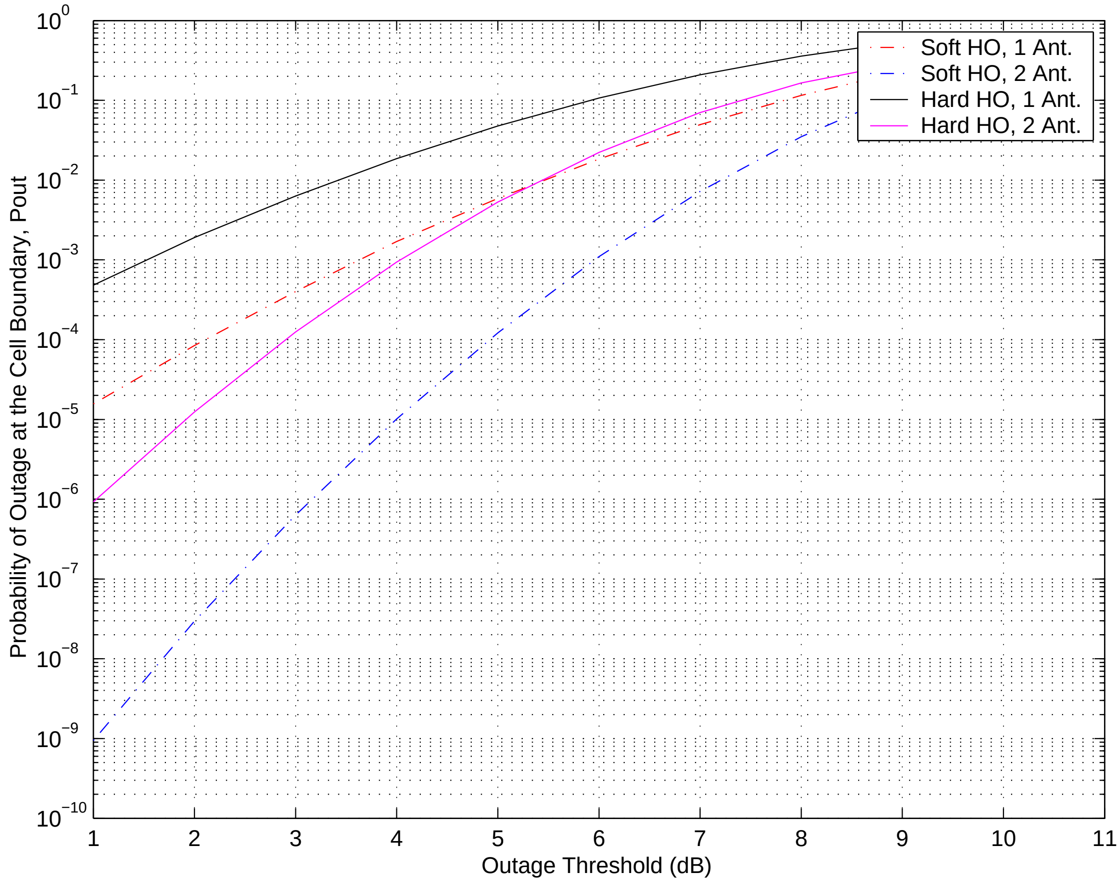

Fig. 17. Probability of Outage at the Cell Boundary for Different Outage Thresholds, in the Vehicular A Channel, Percentage of Users in Handoff = 15, Transmit Power = 20 W

5. Varying the Outage Thresholds

Here, we varied the outage threshold of the system while keeping all other factors constant. As seen in figure 17, we plotted the probability of outage at the boundary of the cell against the outage threshold. Soft handoff with diversity gives the best performance, hard handoff without diversity the worst. Hard handoff with diversity fares better than soft handoff without diversity at lower outage thresholds but loses out at higher values. At the threshold value of 4 dB(to ensure a probability of outage of 10^{-3}), hard handoff with diversity shows a slightly better performance than soft handoff without diversity.

B. Probability of Bit Error

1. Various Channel Models

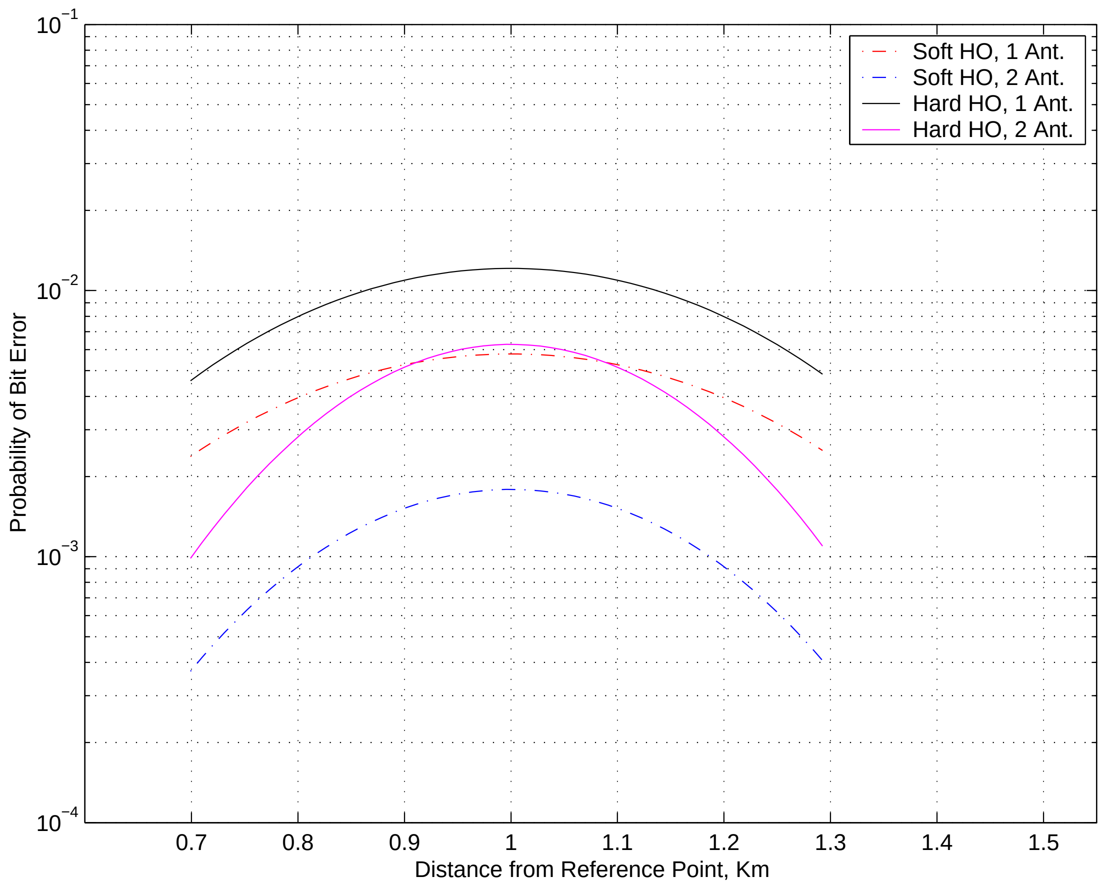

Fig. 18. Probability of Bit Error in the Office A Channel, 15 Percent of Users in Handoff

As with the probability of outage plots, we plotted the probability of bit error curves for different channel models against the distance from the center of the cell.

This is as seen in figures 18, 19 and 20. Of all the three channel models, the systems exhibit the best performance in the Vehicular A channel because of the presence of the two dominant paths.

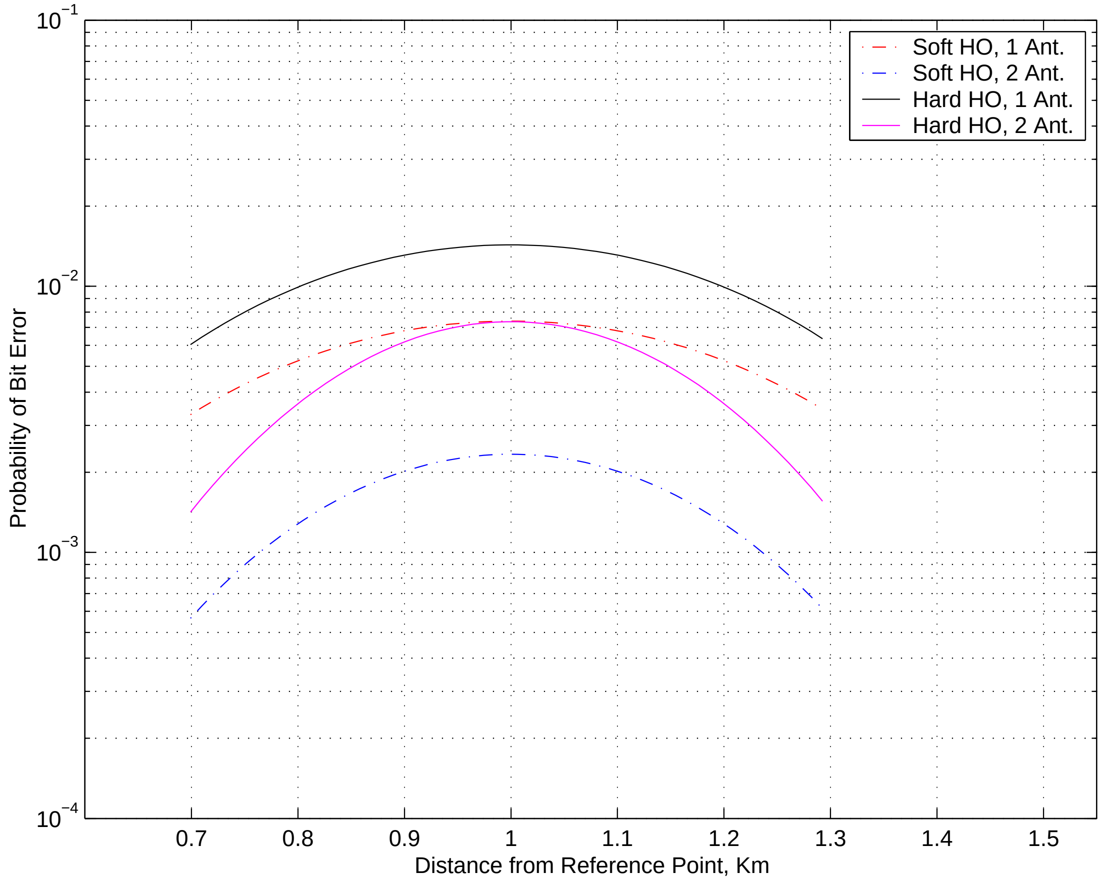

Fig. 19. Probability of Bit Error in the Pedestrian A Channel, 15 Percent of Users in Handoff

However, as far as an comparison between hard handoff with diversity and soft handoff without diversity is concerned, the best results are obtained for the Pedestrian A channel, with hard handoff in a transmit diversity system achieving comparable bit error probability statistics as a non-diversity system with soft handoff. In the Vehicular A channel, the performance is much worse for the handoff with transmit diversity case in comparison to soft handoff without any diversity. Also, soft handoff with transmit diversity shows the best performance.

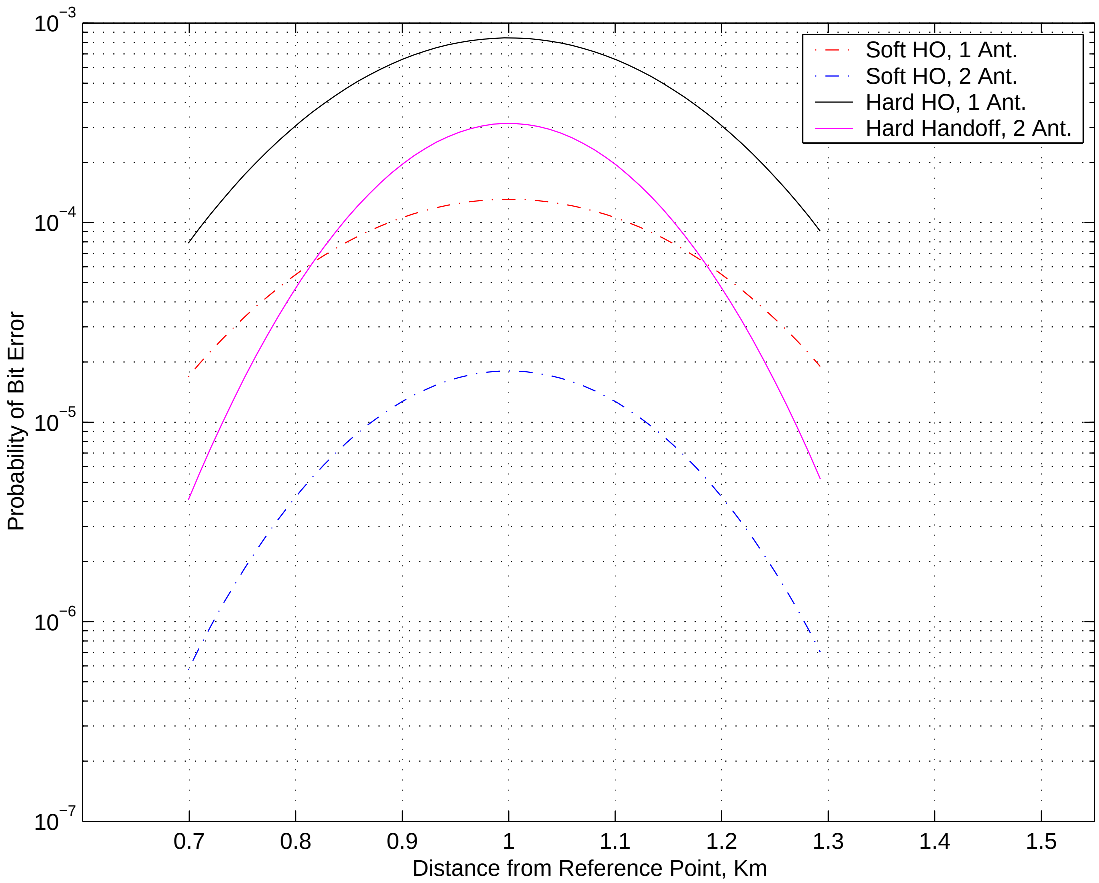

Fig. 20. Probability of Bit Error in the Vehicular A Channel, 15 Percent of Users in Handoff

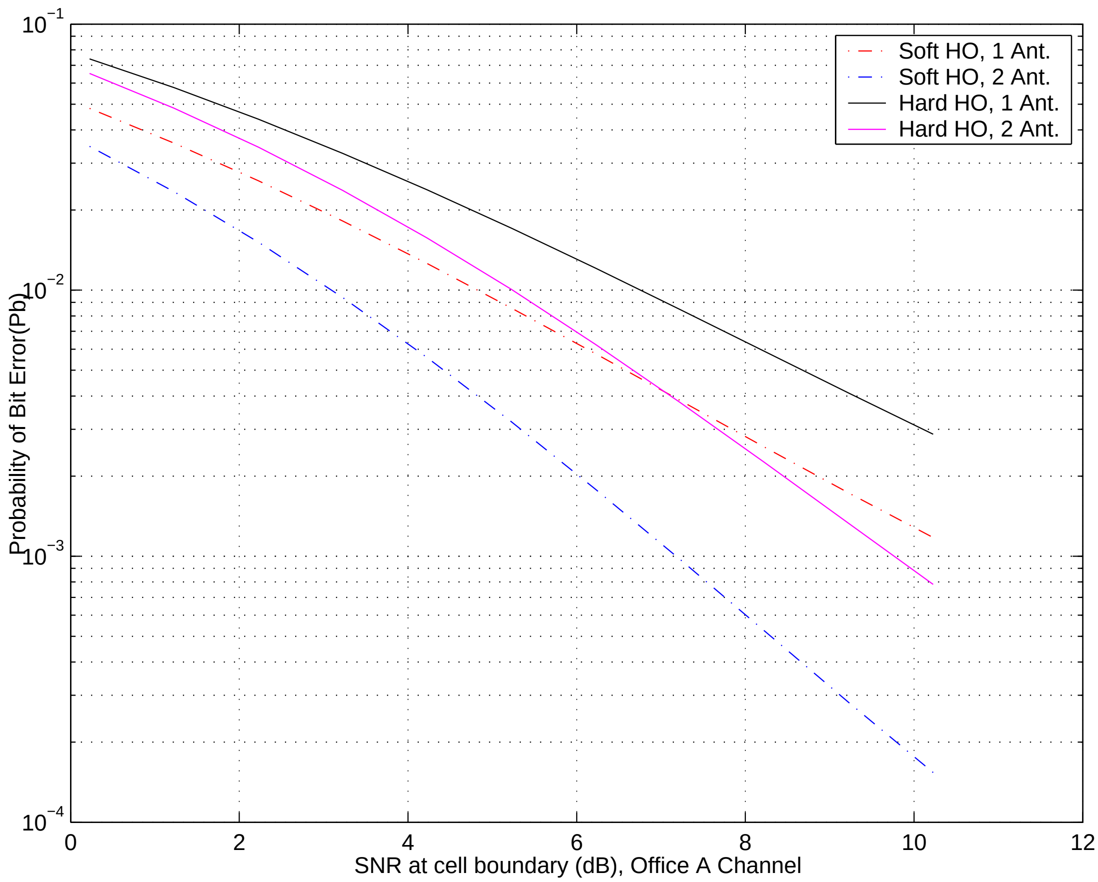

Fig. 21. Probability of Bit Error at the Cell Boundary for Different SNR's, in the Vehicular A Channel, 15 Percent of Users in Handoff

2. Varying the Transmit SNRs

We determined the probability of bit error at the cell boundary for different transmitter powers and hence for different signal to noise ratios. The channel model considered was the Office A channel. The rest of the parameters were as detailed in the table I. Not surprisingly, soft handoff with transmit diversity showed the best performance with hard handoff with no diversity showing the worst. Hard handoff with transmit diversity showed better performance than soft handoff without diversity at higher signal to noise ratios. For bit error probability of 10^{-3}, soft handoff with diversity showed a gain between $2-3$ dB over hard handoff with transmit diversity, while hard handoff with transmit diversity showed a gain of around 0.5 dB over soft handoff without diversity. These results are as illustrated in figure 21.

3. Varying the Number of Users

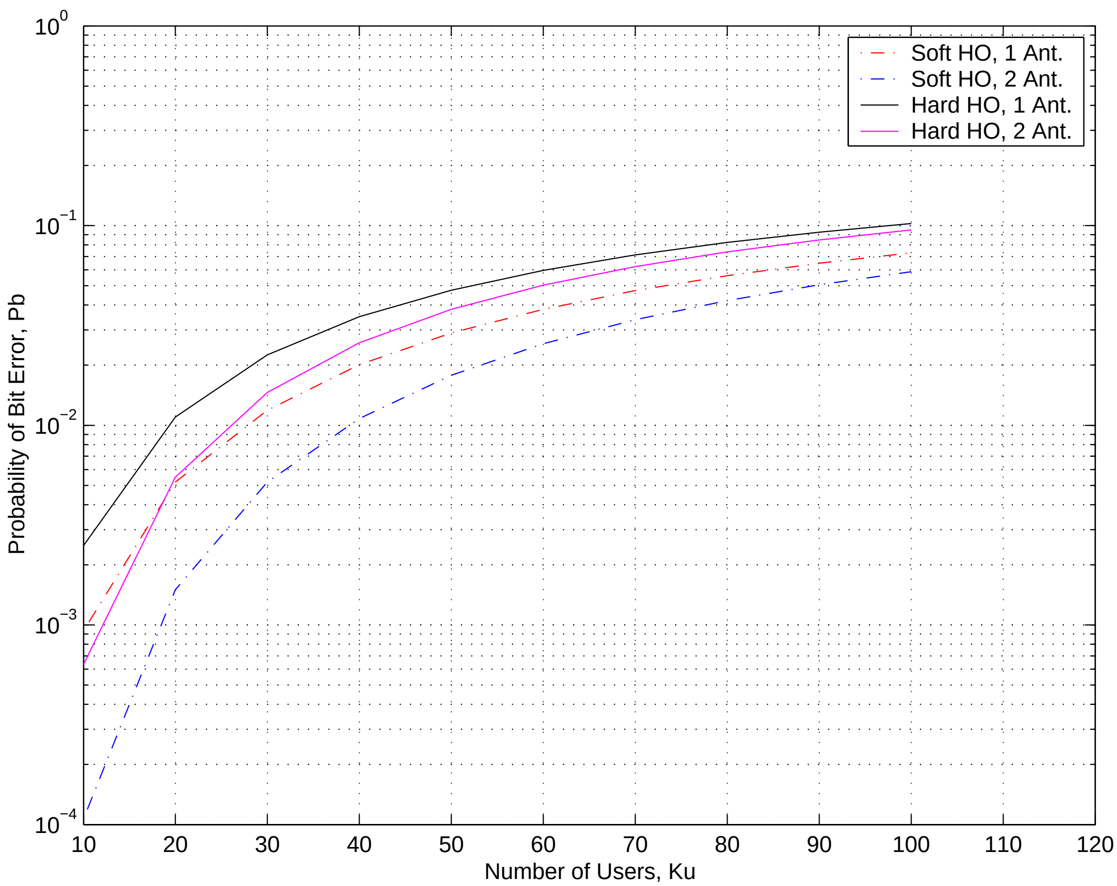

Fig. 22. Probability of Bit Error at the Cell Boundary for Different Number of Users, in the Office A Channel, Percentage of Users in Handoff = 30, Transmit Power = 20 W

Probability of bit error at the cell boundary was determined by varying the number of users and the results were plotted. This is plotted in figure 22. As expected, the probability of bit error rises with the increase in the number of users. Note that soft handoff with transmit diversity gives the best performance and hard handoff without diversity gives the worst performance. Hard handoff with transmit diversity performs better than soft handoff with no diversity for small number of users, but its performance degrades with an increase in inteference due to the added users.

4. Varying the Percentage of Users in Handoff

As the case for probability of outage, we plot the probability of bit error at the cell boundary against the percentage of users in handoff in figure 23. We observe that the curves for hard handoff stay steady. However, the curves for soft handoff see a fall in performance with an increase in the percentage of users under handoff. The channel model being considered is the Office A channel. Soft handoff with diversity gives the best performance. This is followed by soft handoff without diversity, but only till more than one-third the number of users are under handoff.

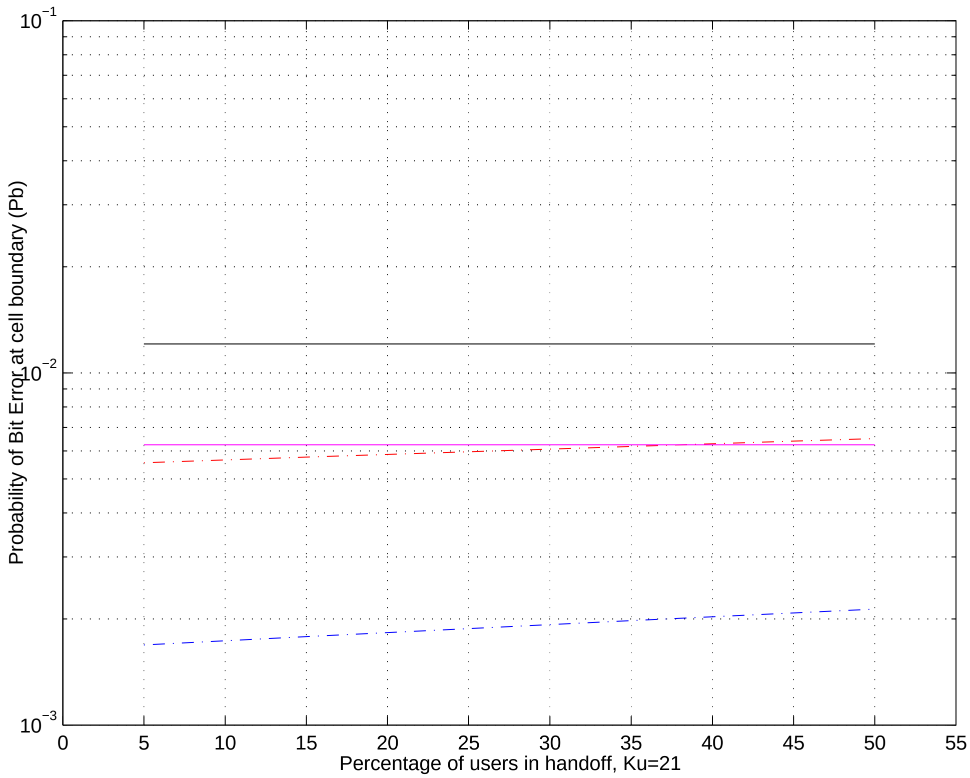

Fig. 23. Probability of Bit Error at the Cell Boundary for Different Percentage of Users Under Handoff, in the Office A Channel, Number of Users Ku=21, Transmit Power = 20 W

CHAPTER VI

CONCLUSIONS AND FUTURE RESEARCH

In this thesis, we have focused on handoff issues in a WCDMA system with transmit diversity at the transmitting base station.The major contribution of this work is the analysis of a CDMA system for two performance measures, namely, the probability of bit error and the outage probability, for soft as well as hard handoff, in the presence or absence of transmit diversity. We also investigated the possibility that the diversity obtained through soft handoff can be compensated for by the diversity obtained in a transmit diversity system with hard handoff. We also compared the analytical results for the above system for different channel models, number of users, percentage of users in handoff , outage thresholds and SNRs.

From the probability of outage results, we found that for the vehicular A channel, hard handoff with diversity performs as well as soft handoff without diversity. For the other multipath models, it gives a superior performance as compared to soft handoff without diversity. From the probability of bit error results, we found that for the pedestrian A channel, hard handoff with diversity is able to meet the performance of soft handoff without diversity. But this is not the case for a vehicular A and office A channel where it exhibits a worser performance. We also observed that hard handoff with transmit diversity out performs soft handoff without diversity, but only at high signal to noise ratios and with a light loading (less number of users), for both our performance measures. We also noted the slightly detrimental effect of the increase in the percentage of users undergoing soft handoff, as against no effect on a system with hard handoff.

However, the interference assumption that we made was that of an additive white gaussian noise model while considering the two concerned base stations only. More work could be done to model the interference in a more accurate manner, perhaps along the lines of the modeling done in [27]. Also, simulations of the actual system could be carried out. A more complex analysis, modelling the handoffs according to the parameters like hysteresis, and the add and the drop thresholds could also be attempted.

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APPENDIX A

MACRO PROPAGATION MODEL

We used the macro cell propagation model [3] which is applicable for the test scenarios in urban and suburban areas, outside the high rise core where the buildings are of nearly uniform height. The path loss for this model was given by,

$$L = 40(1 - 4 \times 10 - 3D_{hb})Log10(R) - 18Log10(D_{hb}) + 21Log10(f) + 80\ dB \quad (A.1)$$

Where R is the base station to the mobile station separation in kilometers, f is the carrier frequency of 2000 MHz and D_{hb} is the base station antenna height, in meters, measured from the average rooftop level. The base station antenna height is fixed at 15 meters above the average rooftop (D_{hb} = 15 m). Considering a carrier frequency of 2000 MHz and a base station antenna height of 15 meters, the formula becomes,

$$L = 128.1 + 37.6Log10(R) \quad (A.2)$$

Hence, the received power in the forward link for the macro cell environment can be expressed as,

$$RX_{PWR} = TX_{PWR} - Max((L - G_{Tx} - G_{RX}), MCL) \quad (A.3)$$

where RX_{PWR} is the received signal power, TX_{PWR} is the transmitted signal power, G_{Tx} is the transmitter antenna gain and G_{RX} is the receiver antenna gain.

Another important parameter used in the above formula is the minimum coupling loss (MCL), the minimum loss in signal due to fact that the base stations are always placed much higher than the mobile stations(s). Minimum Coupling Loss (MCL) is defined as the minimum distance loss including antenna gain measured between antenna connectors. MCL is assumed to be 70 dB for the macro-cellular environment.

APPENDIX B

ACCOUNTING FOR THE INTERFERENCE IN SOFT HANDOFF AND HARD HANDOFF

A CDMA system is an interference limited system. The interference analysis for the forward link is complicated as compared to the reverse link. In the forward link a single base station transmits (synchronously) to many mobiles in the cell area. Therefore, the interference received by the mobiles is received from concentrated large sources (neighbouring base stations). At each base station, transmission power is shared by the users. For ease of analysis, we assume an equal allocation of power to all mobile stations. We also make a gaussian approximation to get a simpler analysis, though a much more complicated analysis has been attempted in [27].

Assuming user i to be controlled by Base station 1 and to be the mobile station under consideration. We assume a system with two cells, the serving cell and the target cell. Hence, the interference received by mobile station i will be the other cell interference from base station 2. Some intra-cell interference will also be present because of the loss of orthogonality as a result of multipath. This effect is modelled by the incorporation of a factor called the orthogonality factor, represented as f_{orth}.

Since we assume an equal allocation of power to all the mobiles under a base station, each mobile in a cell contributes $P_{j,k}/2W_{ss}$ to the total power spectral density of the signal received by the mobile i , where j is the base station servicing the k^{th} mobile. $P_{j,k}$ is the power transmitted by base station j and intended for mobile k which adds to the interference seen by i. Since the number of users in any cell , K_u, is $\gg 1$, we approximate the interference (sum of all base station signals and hence, sum of signals intended for all users) as a gaussian random process via the central limit theorem [14]. W_{ss} is the spread bandwidth of the system. The power spectral density of the noise and interference, received by mobile station i is given by,

$$PSD_{I+N} = \frac{N_o}{2} + (1 - f_{orth}) \sum_{k=1,k\neq i}^{K_u} \frac{P_{1,k}}{2W_{ss}} + \sum_{k=1}^{K_u} \frac{P_{2,k}}{2W_{ss}} \quad \text{(B.1)}$$

$$= \frac{N_o}{2}\left[1 + (1 - f_{orth})\frac{R_b}{W_{us}}\frac{1}{SF}\sum_{k=1,k\neq i}^{K_u}\frac{Eb_{1,k}}{N_o} + \frac{R_b}{W_{us}}\frac{1}{SF}\sum_{k=1}^{K_u}\frac{Eb_{2,k}}{N_o}\right] \quad \text{(B.2)}$$

Where, R_b/W_{us} is the spectral efficiency of the unspread system. $Eb_{j,k}/N_o$ is the signal to noise ratio of the k^{th} mobile under the base station j and SF is the spreading factor. Now, in the handoff region the signal to noise ratios from both the base stations to the mobile can be assumed to be approximately equal (justifying the handoff taking place). Hence assuming, $\frac{Eb_{1,k}}{N_o} \approx \frac{Eb_{2,k}}{N_o}$. Also since we assume equal power allocation to all mobiles, the psd of noise and interference becomes,

$$PSD_{I+N} = \frac{N_o}{2}\left[1 + (1 - f_{orth})\frac{R_b}{W_{us}}\frac{1}{SF}\sum_{k=1,k\neq i}^{K_u}\frac{Eb}{N_o} + \frac{R_b}{W_{us}}\frac{1}{SF}\sum_{k=1}^{K_u}\frac{Eb}{N_o}\right]$$

$$= \frac{N_o}{2}\left[1 + (1 - f_{orth})\frac{R_b}{W_{us}}\frac{1}{SF}(K_u - 1)\frac{Eb}{N_o} + \frac{R_b}{W_{us}}\frac{1}{SF}K_u\frac{Eb}{N_o}\right] \quad \text{(B.3)}$$

Hence, the effective signal to noise ratio for the i^{th} mobile from j^{th} base station and from the l^{th} multipath, assuming a BPSK system under hard handoff, will be given by,

$$\left(\frac{Eb}{N_o}\right)_{eff\_HHO,i} = \frac{\frac{Eb_{j,l}}{N_o}}{1 + \frac{R_b}{W_{us}}\frac{1}{SF}\frac{Eb}{N_o}\left[(1 - f_{orth})(K_u - 1) + K_u\right]}$$

(B.4)

During soft handoff, the target base station also has to transmit the same signal to the mobile as the serving base station. Hence, the cell loading increases. We assume a fraction $g > 1$ of all users to be in soft handoff and only two cells to be involved in any handoff. Going by the equal power allocation assumption, both the base stations allocate equal power to the mobile. This is effectively the same as increasing the number of mobiles in the cell. Hence, the cell loading in the cell increases by a factor of g and becomes $(1+g)K_u$ from K_u.Then the effective signal to noise ratio for the i^{th} mobile from j^{th} base station and from the l^{th} multipath, assuming a BPSK system under soft handoff will be given by,

$$\begin{aligned}\left(\frac{Eb}{N_o}\right)_{eff\_SHO,i} &= \frac{\frac{Eb_{j,l}}{N_o}}{1+\frac{R_b}{W_{us}}\frac{1}{SF}\frac{Eb}{N_o}(1+g)\left[(1-f_{orth})(K_u-1)+(K_u-1)\right]} \\ &= \frac{\frac{Eb_{j,l}}{N_o}}{1+\frac{R_b}{W_{us}}\frac{1}{SF}\frac{Eb}{N_o}(1+g)\left[(2-f_{orth})(K_u-1)\right]}\end{aligned} \quad \text{(B.5)}$$

In the above two equations, $\frac{Eb_{j,l}}{N_o}$ is the signal to noise ratio for the l^{th} multipath from base station j (This is after taking into consideration the path loss).

VITA

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