Uncertainty Model For Quantitative Precipitation Estimation Using Weather Radars

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***Abstract* — This paper introduces an uncertainty model for the quantitatively estimate precipitation using weather radars. The model considers various key aspects associated to radar calibration, attenuation, and the tradeoff between accuracy and radar coverage. An S-band-radar case study is presented to illustrate particular fractional-uncertainty calculations obtained to adjust various typical radar-calibration elements such as antenna, transmitter, receiver, and some other general elements included in the radar equation. This paper is based in "Guide to the expression of Uncertainty in measurement" [1] and the results show that the fractional uncertainty calculated by the model was 40 % for the reflectivity and 30% for the precipitation using the Marshall Palmer Z-R relationship.**

***Keywords* — Quantitative Precipitation Estimation, Radar Measurements, Uncertainty, Weather Radar.**

I. Introduction

Most methods to quantitatively estimate precipitation often fail to provide information about the type of uncertainty associated to the corresponding measurement technique; therefore it seems compelling to quantify the technique-associated uncertainty together with the actual estimation results [2].

The general purpose of measuring is to determine the actual value of a particular magnitude (i.e. the variable), which in our case corresponds to precipitation. According to [3], measurement uncertainty is a way to express the idea whereby for a magnitude and its given measurement there is no unique value but an infinite number of values scattered in the vicinity of the suggested result; additionally, these values are consistent with all observations, data and knowledge gathered from the physical world, and so can be attributed to the measured magnitudes with different degrees of reliability.

Just like other measurement instruments, weather radars rely on indirect measurements of actual precipitation; that is, radar receivers use antennas to perceive information associated to hydrometeors-echo power. This information is subsequently transformed using the radar equation (for radar reflectivity) and then, a whole data interpretation process begins in order to finally yield a quantitative precipitation estimation by means of a given algorithm (or method). From this point onwards, there are a considerable number of aspects that affect radar measurements and so influence data; therefore such aspects have a great impact on the final result (i.e. precipitation levels). These aspects include the radar equation itself, calibration issues associated to the observation system (antenna, receiver and transmitter), space-time variability of the measurements and precipitation's own micro-physics, which cause quantitative precipitation estimations to have considerable uncertainty associated to its measuring process.

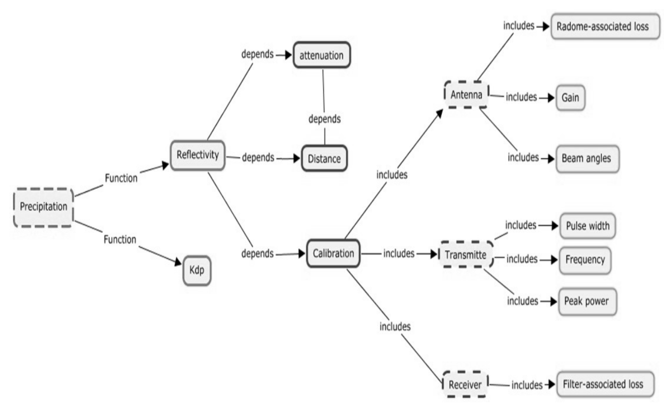
This paper provides a comprehensive uncertainty analysis intended for radar measurement procedures. A particular case study is analyzed in order to show quantitative results on the amount of uncertainty in the measurement system (the extent of such uncertainty). Section 2 of the paper illustrates the methodology applied to obtain the uncertainty model and section 3 shows specific case-study results. Conclusions are drawn in the final part of the paper.

II. Methodology

The methodology applied in this work involves various stages. There is an initial specification of the variable to be measured (i.e. precipitation), which is a function of other variables that are previously defined in the radar equation. Subsequently, the uncertainty sources associated to this process are identified e.g. anomalous propagation, attenuation, ground echo, beam partial filling, resolution, beam height, calibration, type of rain, DSD variety, evaporation and condensation [4].

A. Conceptual Model

Once the uncertainty sources were identified, a sub-set of uncertainty sources was selected, namely those related to radar calibration (i.e. antenna, transmitter and receiver), attenuation and the distance-sensitive radar resolution loss. Fig. 1 shows the conceptual map associated to the model:



Fig, 1. Conceptual Model

B. Mathematical Model

The mathematical uncertainty-measuring model was taken from "Guide to the expression of Uncertainty in measurement". The following is a description of the model [1]:

In most cases the value of variable Y (precipitation) is not measured directly; instead, such a value results from measuring other N magnitudes, namely $X_1, X_2, X_3, \ldots..X_N$, using a functional relationship as follows:

$$Y = f(X_1, X_2, X_3, \ldots..X_N) \tag{1}$$

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In some cases, the best estimation "y" of variable "Y" can be obtained from gathering "n" observations, namely:

$$y = \bar{Y} = \frac{1}{n}\sum_{k=1}^{n} Y_k = \frac{1}{n}\sum_{k=1}^{n} f(X_{1,k}, X_{2,k}, X_{3,k}, \ldots\ldots X_{N,k}) \tag{2}$$

The type of uncertainty associated to output estimation "y" is referred to as combined typical uncertainty and it is denoted by u_c .This particular uncertainty can be determined from the estimated typical deviation associated to each input estimate x_i, referred to as typical uncertainty and denoted by $\mathrm{u}(x_i)$.

Each of the input estimations x_i as well as their corresponding uncertainty $\mathrm{u}(x_i)$ is obtained from a distribution of possible values of each input magnitude x_i. This probability distribution might be based on a series of "n" observations $\mathrm{x}_{i,k}$ taken from the set of observations x_i (type A), or else, the distribution may be assumed (type B).

The type-A uncertainty assessment is obtained from various observations and it is calculated using the following expression:

$$u(x) = \sqrt{\frac{\sum_{j=1}^{n}(x_j - \bar{x})^2}{n(n-1)}} \tag{3}$$

Where:

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$$

For a single estimation $\mathrm{x_i}$ associated to an input magnitude $\mathrm{x_i}$ that was not obtained from repeated observations, the corresponding associated estimated variance, namely $\mathrm{u}^2(\mathrm{x_i})$, or the typical uncertainty $\mathrm{u}(\mathrm{x_i})$, can be established through scientific decision that is based on all the available information about the possible variability of X_i. The sort of information gathered may include:

- Results from previous measurements.
- Experience or general knowledge about the behavior and the properties of the specific materials and instruments.
- Manufacturing specifications (from suppliers).
- Data provided by certified calibrations or other types of certified processes
- Uncertainty previously assigned to reference values, which may be taken from books and manuals.

The values of $u^2(x_i)$ and $u(x_i)$ that are assessed as stated are called Type-B variance and Type-B typical uncertainty, respectively.

The uncertainty of x_i is not always expressed as a multiple of a typical deviation. Instead, it is possible to define a specific interval that corresponds to a particular reliability level (e.g. 90%, 95% or 99%). Unless stated otherwise, it can be assumed that a normal distribution has been used to calculate uncertainty; this yields the typical uncertainty of x_i simply by dividing the given uncertainty value by the corresponding factor in the normal distribution. Such a factor for the three aforementioned reliability levels is 1.64, 1.96 and 2.58, respectively.

In order to obtain the combined typical uncertainty estimation, namely $\mathrm{u_c}(\mathrm{y})$, in the case where all input magnitudes are independent, we take the positive square root of combined variance $\mathrm{u}_c^2(\mathrm{y})$, given by:

$$u_c^2(y) = \sum_{i=1}^{N} \left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i) \tag{4}$$

Where f is the function that defines the variable itself as a function of variables X_i.

Each $\mathrm{u}(x_i)$ represents a single typical uncertainty that is assessed as previously described (either Type-A assessment or Type-B assessment). The combined typical uncertainty $\mathrm{u_c}(\mathrm{y})$ is a typical deviation that is estimated and characterized according to the dispersion of the potential reasonable values that can be attributed to variable "Y".

Partial derivatives (i.e. ∂f/∂xi), also known as sensitivity coefficients, describe the variation of output estimation "y" as a function of the variations in the input estimation values $x_1, x_2, x_3, \ldots x_N$.

It is worth mentioning that the previous equation holds only if the input magnitudes x_i are independent or uncorrelated. In case some of the values of x_i are highly correlated, it is essential to consider such correlations as follows:

$$u_c^2(y) = \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j)$$
$$= \sum_{i=1}^{N} \left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i)$$
$$+ 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j) \tag{5}$$

Where x_i and x_j correspond to the estimations of $\mathrm{X_i}$ and $\mathrm{X_j}$; and $\mathrm{u}(x_i, x_j) = \mathrm{u}(x_j, x_i)$ represents the estimated covariance associated to x_i and x_j. The extent of the correlation between x_i and x_j is determined by the correlation coefficient as follows:

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i)\,\mathrm{u}(x_j)} \tag{6}$$

Where $r(x_i, x_j) = r(x_j, x_i)$ and $-1 \leq r(x_i, x_j) \leq +1$. If estimations x_i and x_j are independent, $r(x_i, x_j) = 0$, and so a variation in one of the estimation values does not imply a variation in the other value.

Fractional uncertainty is defined as the ratio of typical uncertainty to the value of the best magnitude estimate, namely:

$$Fractional\ Uncertainty = \frac{u(x_i)}{x_i} \tag{7}$$

Although $u_c(\mathrm{y})$ can be generally employed as an expression of uncertainty for a particular measurement result, it is often necessary (in certain commercial, industrial or regulatory applications as well as in the fields of health and security) to provide uncertainty measurements that clearly define an interval in which most of the distribution (reasonably attributed values) of the variable are expected to fall.

The new expression for uncertainty, which satisfies the interval-definition requirement, is referred to as expanded uncertainty and is denoted by U. Expanded uncertainty (U) is obtained by multiplying the combined typical uncertainty $u_c(\mathrm{y})$ by a given coverage factor (k):

$$U = ku_c(y) \tag{8}$$

In general, k takes values between 2 and 3. Experience and broad knowledge about the way measurement results should be handled may facilitate choosing a convenient value for k.

C. Uncertainty due to calibrations involving radar, distance and attenuation

Taking the radar equation defined by Probert Jones [5] as a starting point, a standard uncertainty calculation can be stated as follows:

$$P_r = \frac{\pi^3 P_{tx} G^2 \theta\phi h |K|^2 Z}{1024 \ln(2)\, \lambda^2\, r^2} \tag{9}$$

Where : P_r: received power at the radar

P_{tx}: radiated power from the transmitter

λ : wavelength (c/f)

Z : Radar-reflectivity factor

K : refraction complex index

h : spatial pulse length (c τ)

G: antenna gain

r: distance between the target and the radar's antenna

θ, ϕ : beam angles of the radar

By solving for reflectivity, and also considering that h=c τ, $\theta = \phi$ and λ =c/f, the following expression is obtained:

$$Z = \frac{2^{10} \ln(2)\, c}{\pi^3 |K|^2 G^2 \theta^2 P_{tx} \tau\, f^2} P_r r^2 \tag{10}$$

The addition of some terms that were not included by Jones, such as propagation loss (L_p), receiving filter loss (L_{MF}) [6][7] and a radome-associated loss (L_{RD}), yields the following expression:

$$Z = \frac{2^{10} \ln(2)\, c \;\; L_{RD}^2 \;\;\; L_{MF}}{\pi^3 |K|^2 \;\; G_A^2 \theta^2 \; P_{tx} \tau f^2} L_p^2 P_r r^2 \tag{11}$$

For radars that are in normal operation conditions, it is always possible to simplify the radar equation since most of the terms are constants; thus the following expression can be used:

$$Z = C_R L_p^2 P_r\, r^2 \tag{12}$$

Where C_R is the radar constant, defined by:

$$C_R = \frac{2^{10} \ln(2) c \;\; L_{RD}^2 \;\;\; L_{MF}}{\pi^3 |K|^2 \;\; G_A^2 \theta^2 \; P_{tx} \tau f^2} \tag{13}$$

By further elaborating on the previous expression, we obtain:

$$CTE = \frac{2^{10} \ln(2) c}{\pi^3 |K|^2} \tag{14}$$

Where:

$$CTE = \frac{2^{10} \ln(2) c}{\pi^3 |K|^2} \text{ y } C_{ANT} = G_A^2 \theta^2$$

Aspects such as antenna gain, beam angles, frequency accuracy, pulse length, and loss estimations (in radome and in the receiving filters) constitute some of the many factors that should be considered in a radar calibration process [8]. The following is a list of the most relevant factors to radar calibration:

TABLE I
FACTORS TO BE CONSIDERED IN UNCERTAINTY MODELS [8].

Elements	Factors to be considered in calibration	Standard uncertainty
Radar Calibration Antenna	• Gain (G_A). • -3dB beam-width angles (θ, ϕ). • Antenna constant $C_{ANT} = G_A^2 \theta^2$. • Radome-associated loss (L_{RD}).	• $U(G_A)$ • $U(\theta, \phi)$ • $U(C_{ANT})$ • $U(L_{RD})$
Radar Calibration Transmitter	• Pulse length (τ). • Frequency (f) and PRF. • Peak power (P_{tx}).	• $U(\tau)$ • $U(f)$ • $U(P_{tx})$
Radar Calibration Receiver:	• Gain (G). • Filter-associated loss (L_{MF}).	• $U(G)$ • $U(L_{MF})$
Attenuation	• Propagation loss (L_p).	• $U(L_p)$
Resolution Space	• Coverage range (r).	• $U(r)$
Physical constants:	• Wave speed c=2.9978*10^8 m/s. • Dielectric constant $\|K_W\|^2$=0.93.	• $U(c)$ • $U(K_W)$
Measurements	• Reflectivity (Z). • Received power (P_r).	• $U(Z)$ • $U(P_r)$

Regarding the various uncertainty sources that appear in the table above as independent (except for the existing correlation between the transmitting antenna gain and its -3 dB beam angles, which are handled together within the antenna constant), the following general expression is thus obtained in order to determine the combined uncertainty that lies in the corresponding reflectivity (Z) measurement:

$$u_c^2(Z) = [\tfrac{\partial Z}{\partial C_R}]^2 u^2(C_R) + [\tfrac{\partial Z}{\partial P_r}]^2 u^2(P_r) + [\tfrac{\partial Z}{\partial L_p}]^2 u^2(L_p) + [\tfrac{\partial Z}{\partial r}]^2 u^2(r) \tag{15}$$

Similarly, for C_R:

$$u_c^2(C_R) = [\tfrac{\partial C_R}{\partial C_{ANT}}]^2 u^2(C_{ANT}) + [\tfrac{\partial C_R}{\partial L_{RD}}]^2 u^2(L_{RD}) + [\tfrac{\partial C_R}{\partial \tau}]^2 u^2(\tau) + [\tfrac{\partial C_R}{\partial f}]^2 u^2(f) + [\tfrac{\partial C_R}{\partial P_{tx}}]^2 u^2(P_{tx}) + [\tfrac{\partial C_R}{\partial L_{MF}}]^2 u^2(L_{MF}) \tag{16}$$

The following table shows the results in terms of partial derivatives:

TABLE II
PARTIAL DERIVATIVES FOR THE UNCERTAINTY MODEL

		$Z = C_R L_p^2 P_r\, r^2$	
C_1	$\frac{\partial Z}{\partial C_R}$	$L_p^2 P_r\, r^2$	$\frac{Z}{C_R}$
C_2	$\frac{\partial Z}{\partial L_p}$	$2 C_R L_p P_r\, r^2$	$2\frac{Z}{L_p}$
C_3	$\frac{\partial Z}{\partial P_r}$	$C_R L_p^2\, r^2$	$\frac{Z}{P_r}$
C_4	$\frac{\partial Z}{\partial r}$	$2 C_R L_p^2 P_r$ r	$2\frac{Z}{r}$
		$C_R = CTE \frac{L_{RD}^2 L_{MF}}{C_{ANT} P_{tx} \tau f^2}$	
C_A	$\frac{\partial C_R}{\partial C_{ANT}}$	$-2CTE \frac{L_{RD}^2 L_{MF}}{C_{ANT}^2 P_{tx} \tau f^2}$	$-2\frac{C_R}{C_{ANT}}$
C_B	$\frac{\partial C_R}{\partial L_{RD}}$	$2 * CTE \frac{L_{RD}\, L_{MF}}{C_{ANT}\, P_{tx} \tau f^2}$	$2\frac{C_R}{L_{RD}}$
C_C	$\frac{\partial C_R}{\partial \tau}$	$-CTE \frac{L_{RD}^2\, L_{MF}}{C_{ANT}\, P_{tx} \tau^2 f^2}$	$-\frac{C_R}{\tau}$
C_D	$\frac{\partial C_R}{\partial f}$	$-2 * CTE \frac{L_{RD}^2\, L_{MF}}{C_{ANT}\, P_{tx} \tau f^3}$	$-2\frac{C_R}{f}$
C_E	$\frac{\partial C_R}{\partial P_{tx}}$	$-CTE \frac{L_{RD}^2\, L_{MF}}{C_{ANT}\, P_{tx}^2 \tau f^2}$	$-\frac{C_R}{P_{tx}}$
C_F	$\frac{\partial C_R}{\partial L_{MF}}$	$CTE \frac{L_{RD}^2\, L_p^2}{C_{ANT}\, P_{tx} \tau f^2}$	$\frac{C_R}{L_{MF}}$

By using the partial derivatives, the combined uncertainty that occurs when measuring reflectivity Z can be expressed as follows:

$$u_c^2(Z) = [\frac{Z}{C_R}]^2\, u^2(C_R) + [2\frac{Z}{L_p}]^2\, u^2(L_p) + [\frac{Z}{P_r}]^2\, u^2(P_r) + [2\frac{Z}{r}]^2\, u^2(\mathrm{r}) \tag{17}$$

The previous expression can be rewritten as follows:

$$[\frac{u_c(z)}{Z}]^2 = [\frac{u(C_R)}{C_R}]^2 + [2\frac{u(L_p)}{L_p}]^2 + [\frac{u(P_r)}{P_r}]^2 + [2\frac{u(r)}{r}]^2 \tag{18}$$

This expression serves to compute the fractional uncertainty associated to reflectivity measurements by assuming independence of all the variables involved. However, given the direct relation between attenuation and the radar's coverage range, and also assuming a corresponding correlation coefficient of 1, the following is obtained:

$$[\frac{u_c(z)}{Z}]^2 = [\frac{u(C_R)}{C_R}]^2 + [2\frac{u(L_p)}{L_p} + 2\frac{u(r)}{r}]^2 + [\frac{u(P_r)}{P_r}]^2 \tag{19}$$

Similarly, for the radar constant, the following is obtained:

$$u_c^2(C_R) = [-2\frac{C_R}{C_{ANT}}]^2\, u^2(C_{ANT}) + [-2\frac{C_R}{L_{RD}}]^2\, u^2(L_{RD}) + [-\frac{C_R}{\tau}]^2\, u^2(\tau) + [-2\frac{C_R}{f}]^2\, u^2(\mathrm{f}) + [-\frac{C_R}{P_{tx}}]^2\, u^2(\mathrm{P_{tx}}) + [\frac{C_R}{L_{MF}}]^2\, u^2(\mathrm{L_{MF}}) \tag{20}$$

$$[\frac{u_c(C_R)}{C_R}]^2 = [\frac{2u(C_{ANT})}{C_{ANT}}]^2 + [\frac{2u(L_{RD})}{L_{RD}}]^2 + [\frac{u(\tau)}{\tau}]^2 + [\frac{2u(\mathrm{f})}{f}]^2 + [\frac{u(\mathrm{P_{tx}})}{P_{tx}}]^2 + [\frac{u(\mathrm{L_{MF}})}{L_{MF}}]^2 \tag{21}$$

The uncertainty associated to the antenna constant, which depends on the gain and also on the -3db angles, is assumed as a correlated uncertainty with a correlation coefficient equal to 1, thus the corresponding expression y computed as follows:

$$C_{ANT} = G_A^2 \theta^2 \tag{22}$$

$$[\frac{u_c(C_{ANT})}{C_{ANT}}]^2 = [\frac{u(G_A)}{G_A} + \frac{u(G_A)}{G_A}]^2 \tag{23}$$

Considering that the actual variable of interest is precipitation (R), instead of reflectivity (Z), and also involving the Marshall & Palmer Z-R relation together with an algorithm that uses both reflectivity and the specific phase differential (for precipitation estimation), the following uncertainty expressions are obtained:

$$R = f(Z, K_{dp}) \qquad o \quad R = f(Z) \tag{24}$$

$$u_c^2(R) = [\frac{\partial R}{\partial Z}]^2\, u^2(Z) + [\frac{\partial R}{\partial K_{dp}}]^2\, u^2(K_{dp})$$
$$u_c^2(R) = [\frac{\partial R}{\partial Z}]^2\, u^2(Z) \tag{25}$$

III. Case Study

For this particular case study, technical information of radar, located in Brisbane-Australia, was considered (see Table II).

TABLE II
CHARACTERISTICS OF RADAR BRISBANE-AUSTRALIA

Frequency	**Band**	**PPI**	**Latitude**	**Longitude**	**Altitude**	**PRF**	**Range**
2,753 GHz	S	10	-27,669	152,862	168	999	150 Km

Fractional uncertainty for each of the variables involved in the radar equation was assessed as Type-B uncertainty, also considering the information provided by suppliers (manufacturers).

In the cases where suppliers provide the maximum error for a particular variable, the following expression serves to compute the fractional uncertainty of a triangular distribution:

$$\frac{u(x)}{x} = \frac{\Delta \mathrm{X}}{\sqrt{6}} \tag{26}$$

Similarly, for a normal distribution we can use:

$$\frac{u(x)}{x} = \frac{\Delta \mathrm{X}}{3} \tag{27}$$

On the other hand, if suppliers provide relative uncertainty in dB, it becomes necessary to calculate the corresponding fractional uncertainty as follows [9]:

$$\frac{u(x)}{x} = 10^{\frac{u(x)dB}{10}} - 1 \tag{28}$$

A. Antenna

TABLE III
ANTENNA UNCERTAINTY

Parameter	Value	Error ±ΔX	Relative uncertainty u(x)	Linear scale Error ±ΔX	Fractional uncertainty Triangular $\frac{u(x)}{x}$	Fractional uncertainty Normal $\frac{u(x)}{x}$
Gain	45 dB	-----	0.5 dB	----	0.122	0.122
Angle	1 °	0.03 °		0.03	0.012	0.01

B. Radome Associated Loss

Regarding the information provided by antenna suppliers, the standard uncertainty due to radome loss is about 0.02 dB.

TABLE IV
RADOME LOSS

Parameter	Value	Relative Uncertainty u(x)	Linear scale	Fractional uncertainty $\frac{u(x)}{x}$
Radome Loss	0.5 dB	0.02 dB	0.0046	0.0046

C. Pulse length τ

TABLE V
PULSE-LENGTH UNCERTAINTY T.

Parameter	Value	Error ±ΔX	Fractional uncertainty Triangular $\frac{u(x)}{x}$	Fractional uncertainty –Normal $\frac{u(x)}{x}$
Pulse length τ	0,0002185979 seg	0.000020 seg	0.037	0.03049

D. Frequency

TABLE VI
FREQUENCY UNCERTAINTY

Parameter	Value	Error ±ΔX	Linear scale	Fractional uncertainty Triangular $\frac{u(x)}{x}$	Fractional uncertainty Normal $\frac{u(x)}{x}$
Frequency	2,753 GHz	0.004GHz	0.004	0.0016	0.0013

E. Transmitted power

TABLE VII
TRANSMITTED-POWER UNCERTAINTY

Parameter	Value	Relative uncertainty $u(x)$	Fractional uncertainty $\frac{u(x)}{x}$
Power	250 KW	25KW	0.10

F. Receiving-filter loss

TABLE VIII
RECEIVING-FILTER LOSS (UNCERTAINTY)

Parameter	Value	Relative uncertainty $u(x)$	Linear scale	Fractional uncertainty $\frac{u(x)}{x}$
Receiving-filter loss		0.02 dB	0.0471	0.0471

Finally, by using equation 21, it is possible to calculate the fractional uncertainty associated to that radar constant, as shown in Table IX:

TABLE IX
RADAR-CONSTANT UNCERTAINTY

$$\left[\frac{u_c(C_R)}{C_R}\right]^2 = \left[\frac{2u(C_{ANT})}{C_{ANT}}\right]^2 + \left[\frac{2u(L_{RD})}{L_{RD}}\right]^2 + \left[\frac{u(\tau)}{\tau}\right]^2 + \left[\frac{2u(\mathrm{f})}{f}\right]^2 + \left[\frac{u(\mathrm{P_{tx}})}{P_{tx}}\right]^2 + \left[\frac{u(\mathrm{L_{MF}})}{L_{MF}}\right]^2$$

Fractional Uncertainty	Triangular value	Normal value
$\frac{u(C_{ANT})}{C_{ANT}}$	0.134	0.132
$\frac{u(L_{RD})}{L_{RD}}$	0.0046	0.0046
$\frac{u(\tau)}{\tau}$	0.037	0.030
$\frac{2u(\mathrm{f})}{f}$	0.0016	0.0013
$\frac{u(\mathrm{P_{tx}})}{P_{tx}}$	0.10	0.10
$\frac{u(\mathrm{L_{MF}})}{L_{MF}}$	0.0471	0.0471
$\frac{u_c(C_R)}{C_R}$	0.2924	0.2879

Using one example of the received power, namely -85 dBm at a distance of 10 kilometers, the following uncertainty calculations were obtained in terms of reflectivity:

G. Received power

TABLE X
RECEIVED-POWER UNCERTAINTY

Parameter	Value	Relative uncertainty $u(x)$	Fractional uncertainty $\frac{u(x)}{x}$
Potencia	-85 dBm	0.7dB	0.17

H. Distance to target

TABLE XI
RADAR-DISTANCE UNCERTAINTY

Parameter	Value	Relative uncertainty $u(x)$	Fractional uncertainty $\frac{u(x)}{x}$
Distance	10 Km	0.1 km	0.10

I. Attenuation loss

The reciprocal value of L_p ranges from 0 to 1, and, when having ideal conditions, this factor is assumed to be equal to 1; L_p is defined as follows:

$$L_p(r_o) = e^{\int_0^{ro}(k_g+k_n+k_p)dr} \tag{29}$$

Where k_g $(\frac{\mathrm{dB}}{\mathrm{Km}})$ is the attenuation coefficient of gases, $\mathrm{k_n}$ (dB/Km) corresponds to the absorption coefficient of clouds, and $\mathrm{k_p}$ $(\frac{\mathrm{dB}}{\mathrm{Km}})$ is the attenuation coefficient of rain. All these coefficients take approximate values intended for an S-band radar [10]:

Specifically, for an S-band radar, attenuation of gases is negligible, thus, $k_g = 0\ dB/Km$, k_n = 0.004 dB/Km, and k_p = 0.004 dB/Km, for precipitations of about 12 mm/h [11] .According to these values, we obtain $\mathrm{L_p} = 1.072$ together with its reciprocal ${L_p}^{-1} = 0.9323$.

TABLE XII
UNCERTAINTY DUE TO ATTENUATION

Parameter	Value ${L_p}^{-1}$	Relative Uncertainty $u(x)$	Fractional Uncertainty $\frac{u(x)}{x}$
Attenuation loss	0.9323	0.01	0.010

In order to calculate the fractional uncertainty of reflectivity, we use equation (19). These results can be observed in Table XIII.

TABLE XIII
REFLECTIVITY UNCERTAINTY

$$\left[\frac{u_c(z)}{Z}\right]^2 = \left[\frac{u(C_R)}{C_R}\right]^2 + \left[2\frac{u(L_p)}{L_p} + 2\frac{u(r)}{r}\right]^2 + \left[\frac{u(P_r)}{P_r}\right]^2$$

Fractional Uncertainty	Triangular Value	Normal Value
$\frac{u(C_R)}{C_R}$	0.2924	0.2879
$\frac{u(L_p)}{L_p}$	0.010	0.010
$\frac{u(r)}{r}$	0.10	0.10
$\frac{u(P_r)}{P_r}$	0.17	0.17
$\frac{u_c(z)}{Z}$	0.4034	0.4002

By using these data and also considering a sample reflectivity measurement of 40 dB (10000 mm^6/m^3), the following values are obtained:

Linear scale:

$$z = 10000 \pm 4002 \ \frac{mm^6}{m^3}$$

Logarithmic scale:

$$Z = 40dB \pm 1.5 \ dB$$

Provided there is considerable reflectivity uncertainty, it is worth noting that all this uncertainty propagates over the final precipitation estimation.

For the computation of precipitation uncertainty, the most common Z-R empirical relation is adopted [12]

$$Z = aR^b \tag{29}$$

Solving for R, we obtain:

$$R = (\frac{Z}{a})^{\frac{1}{b}} \tag{30}$$

Teni Using equation (25), the following expression is obtained to compute the precipitation uncertainty:

$$u_c^2(R) = [\frac{\partial R}{\partial Z}]^2 \ u^2(Z) + [\frac{\partial R}{\partial a}]^2 \ u^2(a) + [\frac{\partial R}{\partial b}]^2 \ u^2(b) \tag{31}$$

The following table shows the results obtained from the partial derivatives:

TABLE XIV
PARTIAL DERIVATIVES FOR THE PRECIPITATION UNCERTAINTY MODEL

$R = (\frac{Z}{a})^{\frac{1}{b}} = \sqrt[b]{\frac{Z}{a}}$			
C_1	$\frac{\partial R}{\partial Z}$	$\frac{1}{bZ}\sqrt[b]{\frac{Z}{a}}$	$\frac{R}{bZ}$
C_2	$\frac{\partial R}{\partial a}$	$-\frac{1}{ba}\sqrt[b]{\frac{Z}{a}}$	$-\frac{R}{ba}$
C_3	$\frac{\partial R}{\partial b}$	$-\frac{1}{b^2}\sqrt[b]{\frac{Z}{a}} ln\,[\frac{Z}{a}]$	$-\frac{R}{b^2}\ ln\,[\frac{Z}{a}]$

By substituting the partial derivatives we obtain:

$$u_c^2(R) = [\frac{R}{bZ}]^2 \ u^2(Z) + [\frac{R}{ba}]^2 \ u^2(a) + [\frac{R}{b^2} \ ln\,[\frac{Z}{a}]]^2 \ u^2(b)$$

$$[\frac{u_c(R)}{R}]^2 = [\frac{u^2(Z)}{bZ}]^2 + [\frac{u^2(a)}{ba}]^2 + [\frac{u^2(b)}{b^2} \ ln\,[\frac{Z}{a}]]^2 \tag{32}$$

Finally, the following expression can be obtained to find the fractional uncertainty of precipitation using the Z-R relation:

$$\frac{u_c(R)}{R} = \frac{1}{b}\sqrt{[\frac{u^2(Z)}{Z}]^2 + [\frac{u^2(a)}{a}]^2 + [ln\,[\frac{Z}{a}]]^2 \ [\frac{u^2(b)}{b}]^2} \tag{33}$$

Based on the previous expression, it is possible to calculate the Type-A fractional uncertainty for constants "a" and "b" from the data provided in the literature regarding various Z-R relations.

TABLE XV
A SUMMARY OF DIFFERENT VALUES FOR "A" AND "B" REGARDING Z-R RELATIONS [13], [14].

a	b	a	b	a	b
200	1,6	300	1,4	124	1,64
300	1,35	21	1,71	667	1,33
176,5	1,29	486	1,37	500	1,5
215,9	1,35	300	1,5	450	1,46
171,9	1,19	250	1,2	200	1,6
172,8	1,33	130	2	800	1,6
371	1,24	75	2	348	1,81
162	1,48	140	1,5	134	1,55
167,8	1,26	250	1,2	162	1,48
65,5	1,69	436	1,43	371	1,24

To calculate the Type-A standard uncertainty of "a" and "b" equation (3) was used, yielding the following values:

TABLE XVI
PRECIPITATION UNCERTAINTY

Fractional Uncertainty	Triangular value	Normal value
$\frac{u_c(z)}{Z}$	0,4034	0,4002
$\frac{u^2(a)}{a}$	0,1197	0,1197
$\frac{u^2(b)}{b}$	0,02624	0,02624
$\frac{u_c(R)}{R}$	0.2920	0.290

Specifically, when reflectivity is equal to 10000 $\frac{mm^6}{m^3}$, a=271.58, and b = 1.476, the following uncertainty values can be calculated for precipitation (R):

$$R = (\frac{10000}{271.58})^{\frac{1}{1.476}}$$

$$R = 11.49 \ mm \pm 29\%$$

$$R = 11.49 \ \pm 3.33 \ mm$$

IV. Conclusions

The antenna represents the element that most contributes to introducing uncertainty in the radar's constant; therefore, antennas play a crucial role in calibration processes. Additionally, a lot of effort should go into estimating the actual antenna gain and beam-width angles due to the nature of these variables as well as to the external uncertainty sources involved.

Although the fractional uncertainty associated to the radar's constant is 40%, when propagating towards the Marshall & Palmer Z-R relation used herein (for precipitation estimation), its value decreases down to 29%. The previous result follows from the very nature of the resulting expression for estimating precipitation in the Z-R relation.

The present work does not deal with all the possible uncertainty sources that affect quantitative precipitation estimation using radars. Specifically, aspects such as the space-time variation of the drop size distribution (dsd) would undoubtedly contribute to having greater uncertainty in the final results of similar studies. Furthermore, other uncertainty sources were also omitted, e.g. the uncertainty related to the non-linearity of transmitters and receivers, the partial beam filling, evaporation and condensation. These other uncertainty sources represent an interesting challenge for future research on precipitation estimation uncertainty using weather radars.

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