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THE PARABOLIC METHOD FOR NUMERICAL
MODELLING OF WATER WAVES

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ABSTRACT

This report describes a recently developed numerical method, known as the parabolic method, for computing wave transformations in coastal waters. This method has potential advantages over traditional ray tracing methods and has undergone rapid development since the late nineteen-seventies. A review of technical literature during this time is contained in the report, and the present stage of development and future requirements are assessed. Some results are presented from a computational model based on the parabolic method which is being developed at Bristol University and Hydraulics Research Limited.

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1 INTRODUCTION

The prediction of wave conditions in coastal regions forms an essential part of investigations in all types of coastal engineering problems. An important step in wave prediction exercises is the determination of how waves transform from deep to shallow water. The sea areas that have to be considered in these exercises are often too big (of the order of several kilometres square) to be represented in scaled physical models. Numerical modelling is therefore the most common method used.

Until quite recently, ray tracing models were the most popular type of numerical method used by research organisations and coastal engineers. These models do have some important drawbacks and can give inaccurate predictions in coastal regions with irregular depth contours. Within the last few years a new type of numerical method suitable for coastal areas has been developed which potentially can overcome many of the drawbacks with ray models. This new approach is known as the parabolic method.

During this short time there has been a great deal of interest shown in the parabolic method and many publications have appeared in research journals. Numerical models based on the parabolic method are now beginning to be used as tools in practical wave prediction problems. However, because of the short time in which these developments have taken place, many coastal engineers are probably unaware of the existence of this type of numerical model and the possibilities that it can offer.

The purpose of this report is to assess the stage that has been reached in the development of the parabolic method and to suggest further work necessary for the method to become a standard numerical modelling tool. In Chapter 2 the deficiencies of ray tracing methods and the possibility of overcoming them with a parabolic method are explained. In the next chapter a brief description of the theoretical basis of the parabolic method is given and an indication of how it can be represented in a computer model. In Chapter 4 there is a review of published work on the subject, and in the following chapter some results from a parabolic model being developed jointly at Bristol University and Hydraulics Research Limited are presented. Finally, in Chapters 6 and 7 the present stage of development is summarised and suggestions for further work are made.

2 BACKGROUND TO THE PARABOLIC METHOD

Until the late nineteen-seventies most numerical models of wave transformation in shallow water used

ray tracing or finite difference methods. These are collectively known as "refraction methods" and models based on them as "refraction models". In these methods the equations to be solved represent the wave processes of refraction and shoaling while wave diffraction, which also occurs in quite general coastal situations, is ignored. The need to incorporate diffraction is the main motive behind the development of the parabolic method. Before considering this method it is important to understand why refraction models can in some cases be inaccurate.

Refraction models have been most successfully applied to coastal regions where there is a fairly regular and gentle decrease in depth from deep water to the coastline. In fact, for a seabed with perfectly straight and parallel depth contours and with a gentle slope, no diffraction occurs and the refraction models give accurate results. Many coastal regions do approximate quite well to this idealised depth profile, and refraction models have proved useful in such situations. However, as the bathymetry becomes more irregular, diffraction effects generally become more important. In coastal areas with highly irregular depth profiles, refraction models often give poor results.

The effect of diffraction is generally to smooth out (in a spatial sense) the extremes of wave height and steepness that are quite frequently predicted by the refraction models. Various improvements to these models have been made (Refs 1, 5, 27) by introducing numerical smoothing processes to mimic the effect of diffraction, or by using a spectral representation of the wave field (this reduces the importance of not incorporating diffraction of individual components of the spectrum). However, these improvements are essentially a numerical redistribution of wave energy and are not a genuine modelling of the diffraction process. In some situations, for instance where waves propagate over a system of shoals and channels, diffraction effects can be so dominant that they cannot be adequately represented by numerical smoothing. In these circumstances it is desirable to have a numerical wave model which incorporates diffraction behaviour in the basic governing equations.

It is possible, in principle at least, to return to the full wave equation incorporating diffraction as well as refraction and shoaling. However, a model based on this equation would have two important disadvantages. Firstly, the model would be set up as a boundary value problem requiring specification of wave conditions on all boundaries of the grid. Except

in the simplest open-sea situations this would be impossible to do with any accuracy. A second disadvantage is that a minimum number of grid elements per wavelength is needed. If N is the number of elements along one side of the grid, the computational effort is proportional to N^4 . The computational effort therefore increases rapidly with the total grid size per wavelength, and grid sizes in coastal wave problems are typically of the order of hundreds of wavelengths square. This type of model is therefore often too costly and inflexible to be readily used.

It is for these reasons that in recent years a lot of research effort has been devoted to the parabolic approximation to the full wave equation. In this approximation there are certain restrictions to the types of waves that can be modelled, but diffraction terms are retained, the problem with boundary conditions is overcome, and much less computing time is required compared with models using the full wave equation (for parabolic models the computing effort is proportional to N^2).

The principal restriction required by the parabolic approximation is that waves everywhere in the study area should travel within a relatively narrow directional range either side of a specified "main propagation" direction. In practice this range can often be quite large, typically up to 45° either side of the main direction. It can be seen therefore that the parabolic approximation is well suited to many coastal applications in which most of the wave energy travels towards the coast in a relatively narrow directional band with little backscatter or deviation from this band. The method would not be applicable where there is significant reflected wave energy, for instance in front of structures or within harbours. The method may also break down where there are strong refraction effects causing waves to bend too far from the main propagation direction.

3 DESCRIPTION OF THE PARABOLIC METHOD

3.1 The governing equation

The full linear wave equation with time-dependence removed is:

$$\nabla \cdot (c c_g \nabla \eta) + \frac{\omega^2 c_g \eta}{c} = 0 \quad (1)$$

In this equation c is the wave celerity, c_g the group velocity, η the complex wave amplitude and ∇ the two-dimensional horizontal gradient operator. This

equation describes the propagation of water waves over regions of arbitrarily varying depth provided the bed slopes are not too great. Eq 1 is usually known as the "mild-slope equation" and derivations are given in Berkhoff (1976) and Smith and Sprinks (1975). The mild-slope equation is of the elliptic type, and its method of solution requires that boundary conditions are specified along the entire boundary of the study area.

To derive the parabolic approximation to Eq 1, the complex wave amplitude, η , is split into two components representing transmitted (forward-scattered) waves and reflected (back-scattered) waves. In the derivation, the reflected wave field is neglected as being small in comparison with the transmitted wave field, and the following equation is obtained.

$$\frac{\partial \eta}{\partial x} = \left[ik - \frac{1}{2kcc_g} \frac{\partial}{\partial x} (kcc_g) \right] \eta + \frac{i}{2kcc_g} \frac{\partial}{\partial y} (cc_g \frac{\partial \eta}{\partial y}) \quad (2)$$

In this equation the x-coordinate is in the main wave propagation direction and the y-coordinate is in the transverse direction. This equation was first derived in the late nineteen-seventies independently by Radder (1979) and Lozano and Liu (1980). The reader is referred to these papers for derivations. Eq 2 is of the parabolic type, and this has important consequences for its method of solution.

Before describing the computational model based on Eq 2, it is worthwhile to review briefly the physical processes incorporated in Eq 1 and Eq 2. Refraction and shoaling are incorporated in both equations. Diffraction is additionally incorporated in Eq 1, while in Eq 2 diffraction in the transverse direction is included but not in the longitudinal direction (the direction of wave travel). Diffraction effects in the transverse direction, however, are usually more important. Waves from all directions are modelled by Eq 1, but Eq 2 is restricted to waves from a relatively narrow directional band centred on the main propagation direction. Reflected or back-scattered waves are therefore not modelled. The effects of currents on wave refraction are not included in either equation, nor are any dissipative or generative effects such as bottom friction, wave breaking and wind growth. Both equations assume linear waves, and therefore no non-linear effects are modelled. A consequence of the linear treatment is that a random sea (represented as a period and directional spectrum) can be modelled by a simple superposition of the period and directional components.

3.2 Numerical representation

A numerical solution to a parabolic equation such as Eq 2 can be achieved by a "marching" finite-difference scheme. This method involves superimposing a rectangular grid over a plan of the coastal area of interest, and taking depth values at each grid intersection point (Fig 1). The grid should be aligned with one axis (conventionally the y-axis) roughly parallel to the shoreline. The marching solution method requires that boundary conditions are specified on the offshore boundary (in the form of an incident wave condition) and along the side boundaries. The solution process is to start at the offshore boundary (the first row) and to calculate wave conditions at all points along the second row using the incident wave values on the first row and a finite difference formulation of Eq 2. The calculated wave parameters on the second row are then used to determine wave values on the third row. This process is repeated until the last row, furthest inshore, is reached.

Various schemes exist for a finite-difference formulation of Eq 2, but the standard Crank-Nicholson scheme is by far the most common choice in published work. Examples of this scheme are given in Radder (1979) and Kirby and Dalrymple (1983).

3.3 Boundary conditions

(a) Offshore boundary.

This takes the form of an incident wave condition specified at each point on the offshore row. A single wave energy, period and direction should be specified at each point. It is possible to supply different values of these parameters at different points along the offshore row (to simulate a spatial variation in the incident wave) but most practical applications would involve the use of identical values along the whole offshore row. If all wave processes are treated linearly, a wave spectrum can be represented by repeated running of the model with different initial period and direction values although this may involve considerable computational effort.

(b) Inshore boundary

No boundary condition is required if the inshore row has a finite water depth. Land boundaries can be represented by an appropriate boundary condition, but it is often simpler to represent land areas as sea areas with a small constant depth. This "thin-film" technique is also often

the best method of representing internal land areas such as small islands.

(c) Side boundaries

Boundary conditions need to be specified along the side boundaries. Generally, however, in open sea situations it is not possible to specify these boundary conditions correctly because the incoming wave along the side boundaries is not known. The usual method to overcome this problem is to assume a simple boundary condition (such as a linear wave absorption condition for waves from a given direction), and then to extend the grid laterally so that the errors which arise from the boundary condition do not "propagate in" to the region of interest. Fig 1 shows the areas adjacent to each side of the grid that are affected in this way. This method of overcoming the problem does involve some extra computational effort (but not an excessive amount) because of the larger grid size needed.

4 REVIEW OF RECENT DEVELOPMENTS

4.1 Incorporation of additional wave processes

Since the late nineteen-seventies much interest has been shown in applying parabolic methods to water-wave propagation, and much research effort has been devoted to extending the basic equation, Eq 2, to include other wave phenomena.

Booij (1981) has introduced a term describing wave energy dissipation at the seabed, and this work has been followed up by Dalrymple, Kirby and Hwang (1984) and Liu and Tsay (1985). Additional terms to describe the effects of currents on the refraction of waves have been incorporated into Eq 2 by Booij (1981), Liu (1983) and Kirby (1984 and 1986). An iterative scheme to include weak reflections has been described by Liu and Tsay (1983a). Most recently, the parabolic method has been extended to incorporate non-linear deep-water waves (Kirby and Dalrymple (1983 and 1984), Liu and Tsay (1984)) and shallow-water waves (Liu, Yoon and Kirby (1985)).

Wave breaking is a particularly complex phenomenon, and an accurate numerical representation is not yet available. Present methods for including breaking in parabolic models attempt only to determine the amount of wave energy dissipated and are not concerned with the other details of the breaking process. Booij (1981) and Dalrymple, Kirby and Hwang (1984) use a

general dissipation term which can include energy loss from breaking. Calculation of this breaking term is commonly based on the limiting wave height allowed by the breaking process at a given depth. Once a wave height has been calculated, it is compared with the wave height at which breaking starts to occur at that depth. If it exceeds this breaker height it is reduced to that value. Recently more sophisticated methods have been advanced. Dally, Dean and Dalrymple (1985) describe a method similar to wave energy decay in a hydraulic jump which has been incorporated in a parabolic model by Kirby and Dalrymple (1986). Dingemans et al (1984) use a similar approach in their parabolic model based on the analogy with the energy dissipated in a tidal bore (Battjes and Janssen (1978), Battjes and Stive (1985)). The inclusion of wave breaking also allows a convenient computation of waves around islands by means of the "thin film" procedure. This involves representing the land area on the island as a shoal with a constant, very shallow water depth. The breaking process will ensure that negligible energy will travel over these land areas. The advantage of this method is that there is no need to specify special boundary conditions for internal land areas (section 3.3).

The problem of widening the allowed directional band centred on the main propagation direction has also been investigated. Booij (1981) has proposed additional terms to Eq 2 which improve general accuracy and extend the width of the allowed band by about 10° . Kirby (1986) provides a more rigorous derivation of Booij's correction terms and discusses other possible approaches to more accurate schemes, including a "mini-max" procedure to extend the width of the allowed direction band further still. With this method wave angles out to about 70° from the main direction can be computed accurately.

4.2 Comparisons with physical models and different types of numerical model

Parabolic models have been tested quite extensively against other types of numerical model and laboratory experiments. These comparisons have usually involved simplified bathymetries representing test cases in which refraction theory breaks down and diffraction effects become important. The following are the most common test cases encountered in the recent technical literature.

(a) Single breakwater on a flat seabed.

Dalrymple, Kirby and Hwang (1984) have investigated the problem of wave diffraction around a single, semi-infinite, perfectly reflecting breakwater on a flat seabed. Comparison was made between a parabolic model and the analytical solution to this problem (Sommerfeld (1896), Penney and Price (1952)) for the wave field in the shelter of the breakwater. Two angles of incidence were investigated, 90° and 60° (see Fig 5 for definition of incident angle). Good agreement for the 90° case was obtained, but the 60° incident angle gave poor agreement in the shelter, with the parabolic model underpredicting by up to 50%. The single breakwater problem is investigated further in Section 5.2 of this report.

(b) Circular shoal on a flat seabed.

This is another classical test case which has been investigated by Radder (1979) and Kirby and Dalrymple (1983). It represents a particularly severe test for the parabolic method. The refraction method predicts a cusped caustic and therefore breaks down with strong diffraction effects occurring. No analytical solution exists for this problem, but it has been investigated extensively with other types of model based on a full wave equation. Radder has compared his parabolic model predictions with published results from these alternative models. There is qualitative agreement over and behind the shoal, but there are areas where significant differences in wave height occur. Kirby and Dalrymple investigated the effect of including their deep-water non-linear term. Generally they found that this term had the effect of reducing wave heights still further at the focus, the effect becoming more important with higher incident waves. Although Kirby and Dalrymple did not compare their results with other numerical or laboratory models, other researches (see below) have found that better predictions are made with the inclusion of this type of non-linearity.

(c) Elliptic shoal on a sloping seabed.

This is a similar but somewhat more complex test case to the previous one. As in that case, a cusped caustic is obtained by refraction theory, but the regular refraction behaviour caused by the sloping seabed is superimposed on the caustic. This problem was first investigated by Berkhoff, Booij and Radder (1982) who compared predictions

from their parabolic model with those from a ray tracing model and a full wave equation model. Laboratory tests were additionally carried out. The parabolic model compared well with both the full wave equation model and the laboratory measurements, and was somewhat better than the ray tracing model. However, there were significant deviations from the measured wave heights at certain locations behind the shoal. The problem was subsequently investigated by Liu and Tsay (1983b) and Kirby and Dalrymple (1984). Liu and Tsay obtained similar results, while Kirby and Dalrymple obtained improved results using their deep-water non-linear parabolic model. Liu and Tsay extended the problem by introducing a single breakwater in different areas, but did not compare their results with other numerical models or laboratory measurements.

(d) Circular shelf.

As in the previous two cases, this type of depth profile causes a focussing of wave rays and therefore creates strong diffraction effects. Whalin (1971) carried out a series of laboratory experiments on wave propagation with this bathymetry. Lozano and Liu (1980) compared their linear parabolic model with Whalin's data with reasonably good agreement. Liu and Tsay (1984) compared their deep-water non-linear parabolic model with some of Whalin's measurements (at shorter periods, corresponding to deep water conditions). Better wave height predictions were obtained, particularly near the focus, compared with the linear model. Energy exchange between the first and second harmonics was also predicted. Liu, Yoon and Kirby (1985) used their shallow water non-linear parabolic model for a similar comparison with Whalin's measurements at longer periods. Up to five harmonic components were considered in this numerical model. Wave height predictions for the first three harmonics (the only harmonics for which measured data were available) were in reasonable agreement with measurements.

(e) Submerged island with uniform slopes.

Tsay and Liu (1982) and Liu and Tsay (1983a) have investigated the problem of waves travelling over a submerged island with a profile consisting of uniformly sloping sides and a flat or pointed top. The purpose was to test the ability of the parabolic model to include weak reflections. Comparisons were made with wave heights from a full wave equation model with good agreement,

although the reflected wave energy was found to be small.

4.3 Comparisons with field data

Published comparisons of parabolic models with laboratory experiments and alternative numerical models are quite extensive. In contrast, there exists little published material comparing the predictions of parabolic models with measured wave data.

Booij (1981) has compared his linear parabolic model with wave data from the Oosterschelde Estuary in Holland. He concluded that the model is in good agreement with the field data, and the reader is referred to Vrijling and Bruinsma (1980) for the detailed comparison. Booij also investigated the effects of tidal currents in his model and discovered that they created little difference in the overall spatial distribution of wave heights, but that at some locations the differences could be significant. The currents tested were, however, quite small, and more severe effects could well occur in areas with stronger tidal currents.

Liu and Tsay (1985) have compared their linear parabolic model with two sets of wave measurements from the research pier of the American Coastal Engineering Research Centre at Duck, North Carolina. The bathymetry of the area provides an interesting test. The depth contours are generally reasonably straight and parallel, but at the tip of the pier there is a depression which causes waves to refract away to either side leaving reduced wave heights along the length of the pier. Wave measurements were made at spatial intervals along the pier, and Liu and Tsay achieved mixed success with their model. The first set of measurements were well predicted with the inclusion of an appropriate friction factor in their model. The second set of measurements were, however, significantly underpredicted. In both cases the model performed much better than a pure refraction model. Unfortunately, in these comparisons, no measured offshore data were available and therefore the input conditions to the model were somewhat unreliable.

A systematic comparison of results from the parabolic model developed by the University of Delft, Holland with field data from the Dutch coastline near the Haringvliet sluice has been carried out (Dingemans et al (1984)). This area has a complex, shallow bathymetry in which strong diffraction effects can be expected. Six inshore waverider buoys were deployed, with one buoy capable of recording wave directions deployed further offshore to provide the incident boundary conditions for the parabolic model. It was

found that the parabolic model gave good results (with a 15% standard deviation) for those cases where the offshore waves approximated reasonably well to a single frequency and direction. Results were less accurate after waves had broken.

In view of the sparsity of field data/parabolic model comparisons, it is worth mentioning two other sets of wave measurements which have been compared with pure refraction models and which could be reanalysed using a parabolic model. These studies are reported in Wang and Yang (1981) who took measurements at Sylt on the Dutch coast, and Tucker, Carr and Pitt (1983) who investigated wave transmission over Dunwich Bank on the East Anglian coast.

5 SOME RESULTS AND COMPARISONS WITH A PARABOLIC MODEL

In this section, some results from a parabolic model being developed at Bristol University and Hydraulics Research Limited are presented. Two classical test cases, namely the circular shoal and single breakwater, are investigated.

5.1 Circular shoal

The size of the modelled area, and the location and dimensions of the shoal are shown in Fig 2. The depth profile over the shoal is defined by the equation:

$$h = h_o + \frac{(h_o - h_m)r^2}{R^2} \quad (3)$$

in which h = depth at a general point on the shoal, h_o is the constant depth outside the shoal, h_m is the minimum depth at the centre of the shoal, R is the radius of the shoal and r is the distance from the general point to the centre of the shoal. In these tests, values of $R = 5\text{m}$, $h_o = 0.9375\text{m}$, $h_m = 0.3125\text{m}$ and a wavelength, L_o , of 2.5m on the flat area were used. Monochromatic waves were incident at one side of the grid in the direction of the x -axis (the main propagation direction). An element size of 0.3125m in the x -direction by 0.15625m in the y -direction was used in these tests. The values of R , h_o , h_m , L_o and element size correspond to those used by Radder (1979) who investigated the same problem. A boundary condition $\partial\eta/\partial y = 0$ was used on both side boundaries.

Results are presented in Fig 3 as contours of wave height coefficient over the whole modelled area. A very similar pattern to Radder's results is obtained.

This test was repeated with extra correction terms introduced in the governing equation as suggested by Kirby (1986). The governing equation with these terms included reads:

$$\frac{\partial \eta'}{\partial x} = \frac{3}{4k} \left[i + \frac{1}{2k} \frac{\partial k}{\partial x} \right] \frac{\partial^2 \eta'}{\partial y^2} + \left[ik - \frac{1}{2k} \frac{\partial k}{\partial x} \right] \eta' - \frac{1}{4k^2} \frac{\partial^3 \eta'}{\partial x \partial y^2} \quad (4)$$

where $\eta' = \eta (cc_g)^{\frac{1}{2}}$ (5)

Fig 4 shows wave height coefficients using Eq 4 in the circular shoal problem. The effect of the correction terms is to slightly enlarge the area of big wave heights at the focus and to shift this area upwave from the shoal. This effect is, however, very small, and even the original equation gave good results. An important reason for the small effect of the correction terms is that only small deviations in the direction of wave travel occur in this problem. In the single breakwater test case described below, the effect of larger angles between the x-direction and the wave direction is investigated.

5.2 Single breakwater

The position of the breakwater was chosen to lie along the right-hand side of the grid in the x-direction as shown in Fig 5. The tip was at the upwave end of the grid, and the breakwater was assumed to continue indefinitely in the downwave direction. A flat seabed was assumed throughout, and the problem was non-dimensionalised by measuring distances in terms of wavelengths. An element size of 0.1 wavelength in the x-direction by 0.125 wavelength in the y-direction was used. An area of 20 wavelengths (x-direction) by 10 wavelengths (y-direction) was modelled, giving a total of 200 by 80 elements. Separate tests were carried out for incident angles of 10°, 20°, 30°, 40° and 50° (the incident angle is defined in Fig 5). Wave height coefficients have been measured along a section normal to the breakwater (ie in the y-direction) at a distance of five wavelengths downwave from the tip of the breakwater. Results from the parabolic model are compared with the analytical solution to the problem. These results are presented in Figs 6 to 9.

The comparison for the 'non-corrected' parabolic model (Figs 6 and 7) indicates reasonable agreement for 10° and 20° incident angles, but becomes progressively worse at larger angles. The differences are worst in the transition region between the unsheltered area to the left of the figures and the shelter on the right. When the tests were repeated with the 'corrected' parabolic model the same trend towards greater error

at large incident angles was apparent (Figs 8 and 9). However, results are generally much improved in comparison with the non-corrected version, particularly in the transition region.

6 SUMMARY AND CONCLUSIONS

- (a) Much research work has been carried out since the late nineteen-seventies on the application of parabolic methods to the propagation of waves in coastal regions. Traditionally, methods which incorporate the refraction and shoaling of waves, but not diffraction, have been used for these types of problems. These methods, known as refraction methods, have proved successful in areas where the seabed bathymetry is reasonably regular. However, in areas of irregular bathymetry, such as where shoals and channels occur, these methods break down completely, and a type of method that incorporates diffraction is required. The parabolic method is one such type. It is particularly well-suited to open-sea coastal modelling because known boundary conditions are required only on the offshore boundary of the grid, and because the numerical solution process involves considerably less computing effort than alternative models which incorporate diffraction.
- (b) A lot of the research effort has been devoted to extending the basic parabolic equation (Eq 2) to include other wave phenomena such as bottom friction, wave breaking, current refraction, and various types of non-linearity. Most of the published work has considered monochromatic and monodirectional input wave conditions. Very little consideration has been given to problems concerned with random seas.
- (c) Most of the published comparisons of parabolic models with alternative numerical models and laboratory experiments have involved idealised depth profiles which create important diffraction effects. In these comparisons, parabolic models invariably give better wave height predictions than pure refraction models. The agreement with laboratory measurements appears to be generally adequate for most engineering purposes, although commonly there are fairly large areas, downwave of the diffracting feature, where significant differences occur. There have been few published comparisons involving irregular bathymetries as would typically occur in practical problems.
- (d) Little comparison work with field data has been carried out, and that which has been done is inconclusive.

- (e) There has been little published work on the accuracy of parabolic models with respect to element size per wavelength, and on numerical techniques to improve accuracy and increase the allowed element size. The information that has been published usually forms a small part of a paper whose main theme is the comparison with other types of model. However, it is probable that unpublished work of this sort has been carried out and incorporated into practical models in use at research institutions.

7 FUTURE REQUIREMENTS

- (a) The computing effort in a parabolic model is strongly dependent on the maximum allowable element size. The accuracy of predicted wave heights with respect to element sizes should be systematically investigated and if possible a criterion for a maximum element size per wavelength should be established.
- (b) Further development of parabolic models is necessary to incorporate incident wave spectra, and the additional computational effort compared with monochromatic and monodirectional incident waves should be assessed.
- (c) Systematic comparisons of parabolic models (and indeed all types of numerical wave model) with field data are an urgent requirement. Different types of bathymetry, seabed composition and the effects of strong tidal currents should be investigated.
- (d) The usefulness of parabolic models to coastal engineers will ultimately depend on their accuracy, cost and flexibility when used in a commercial environment. This is likely only to become established by experience when the models are used on a regular and fairly routine basis. At the present time parabolic models are still something of a novelty, and therefore experience gained in their use in real or realistically simulated problems will be invaluable.

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Figures

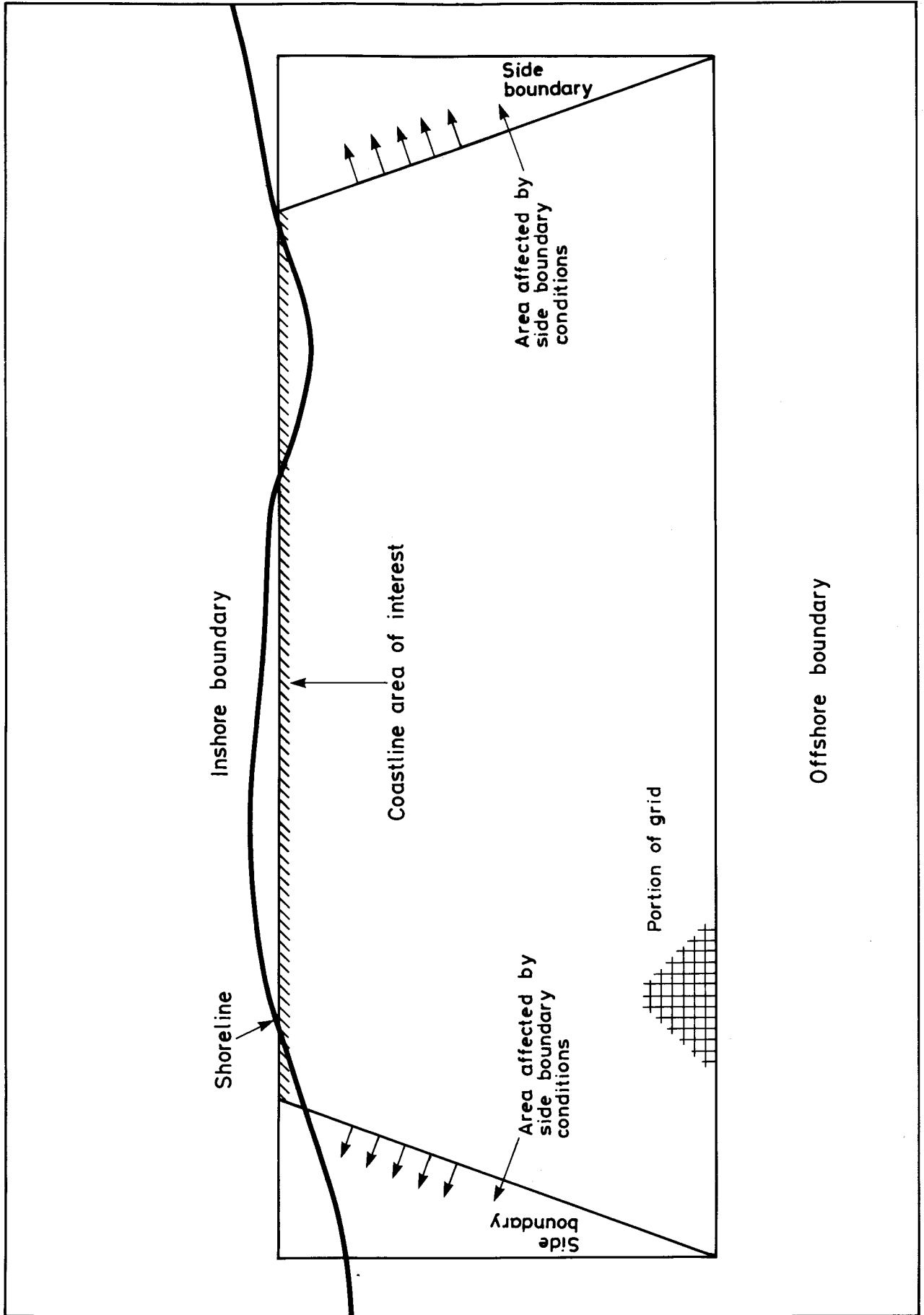


Fig 1 Parabolic model grid system for a general coastal area

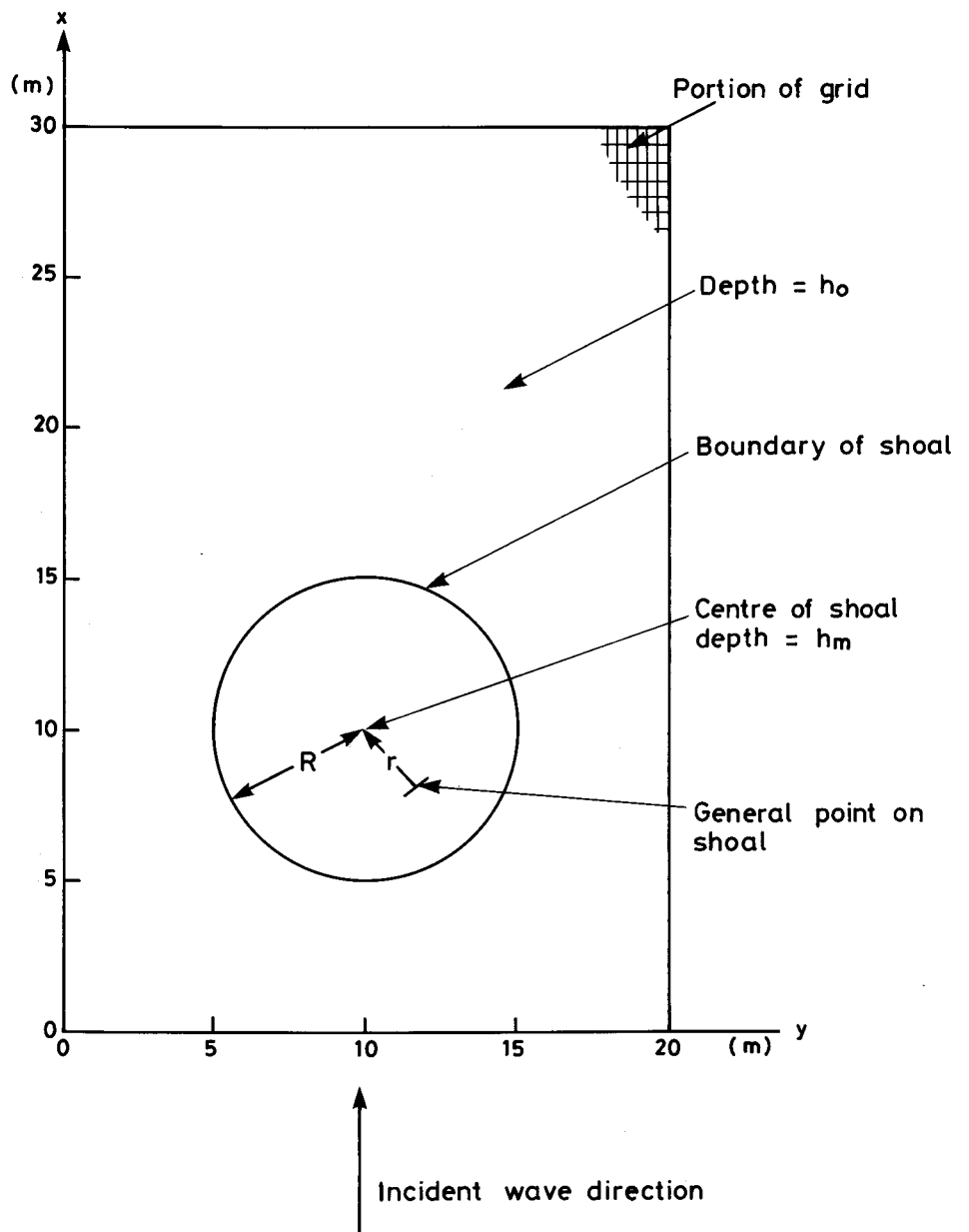


Fig 2 Model layout and grid for the circular shoal problem

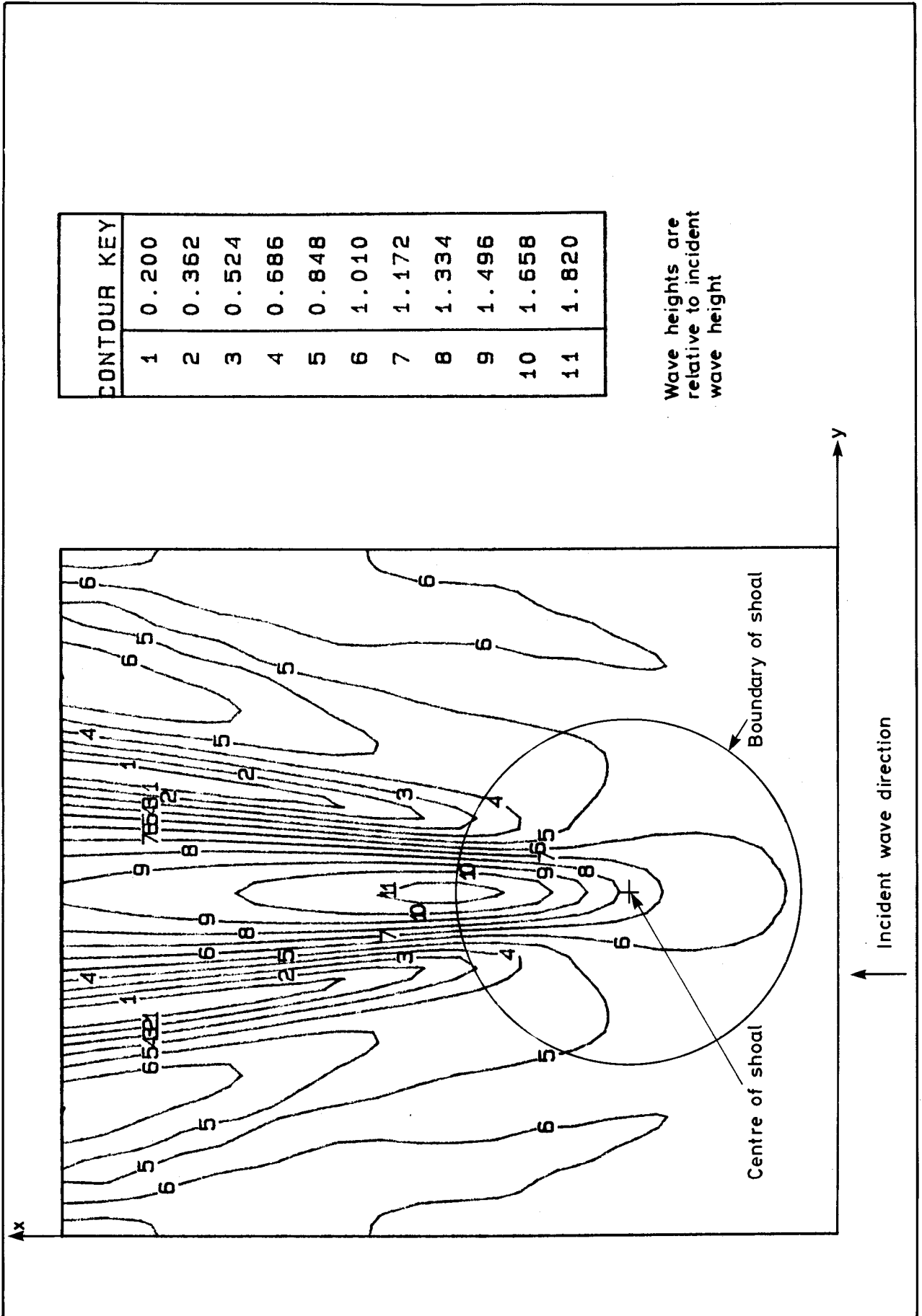


Fig 3 Wave height contours. Circular shoal problem using parabolic model without correction terms

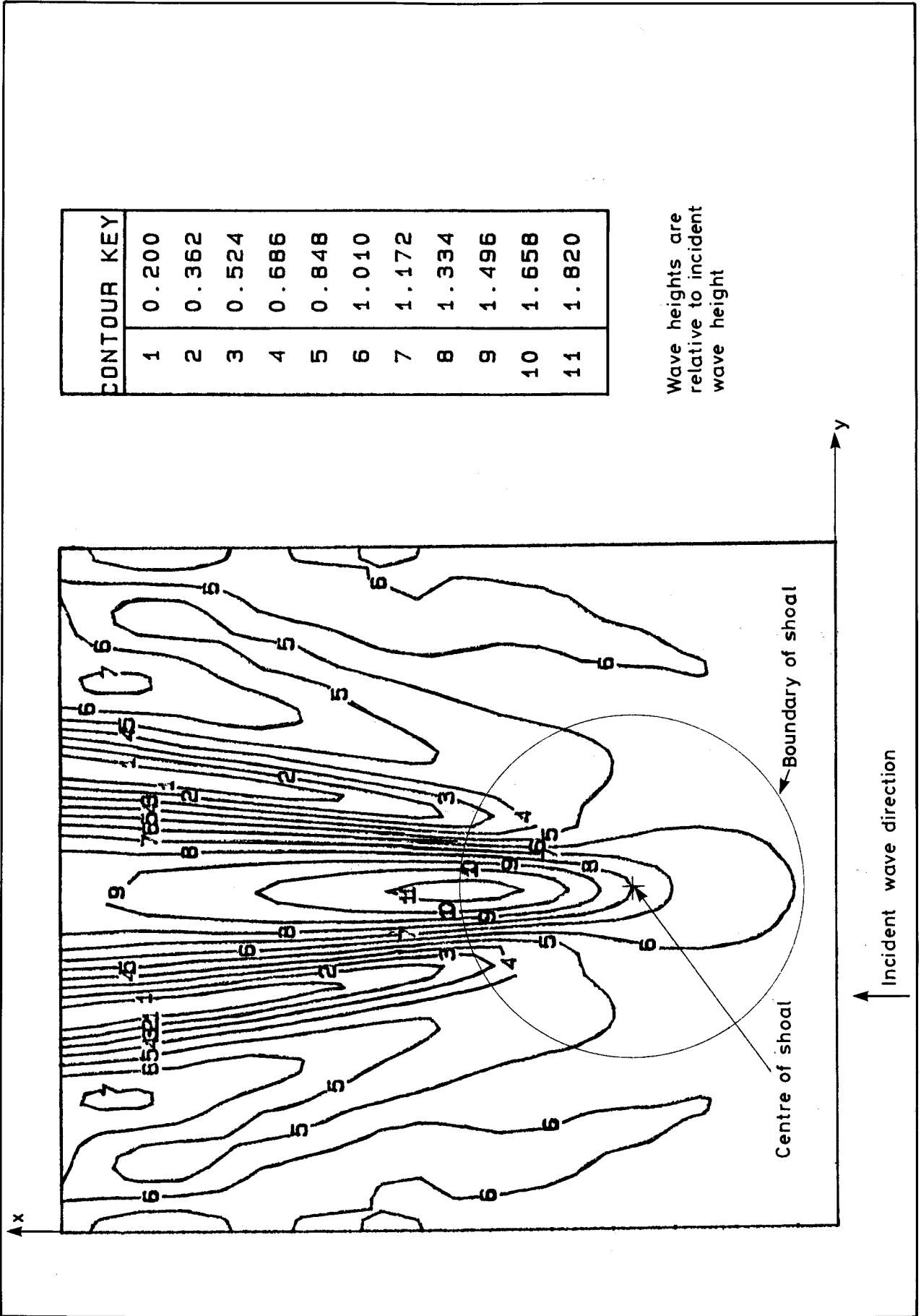


Fig 4 Wave height contours. Circular shoal problem using parabolic model with correction terms

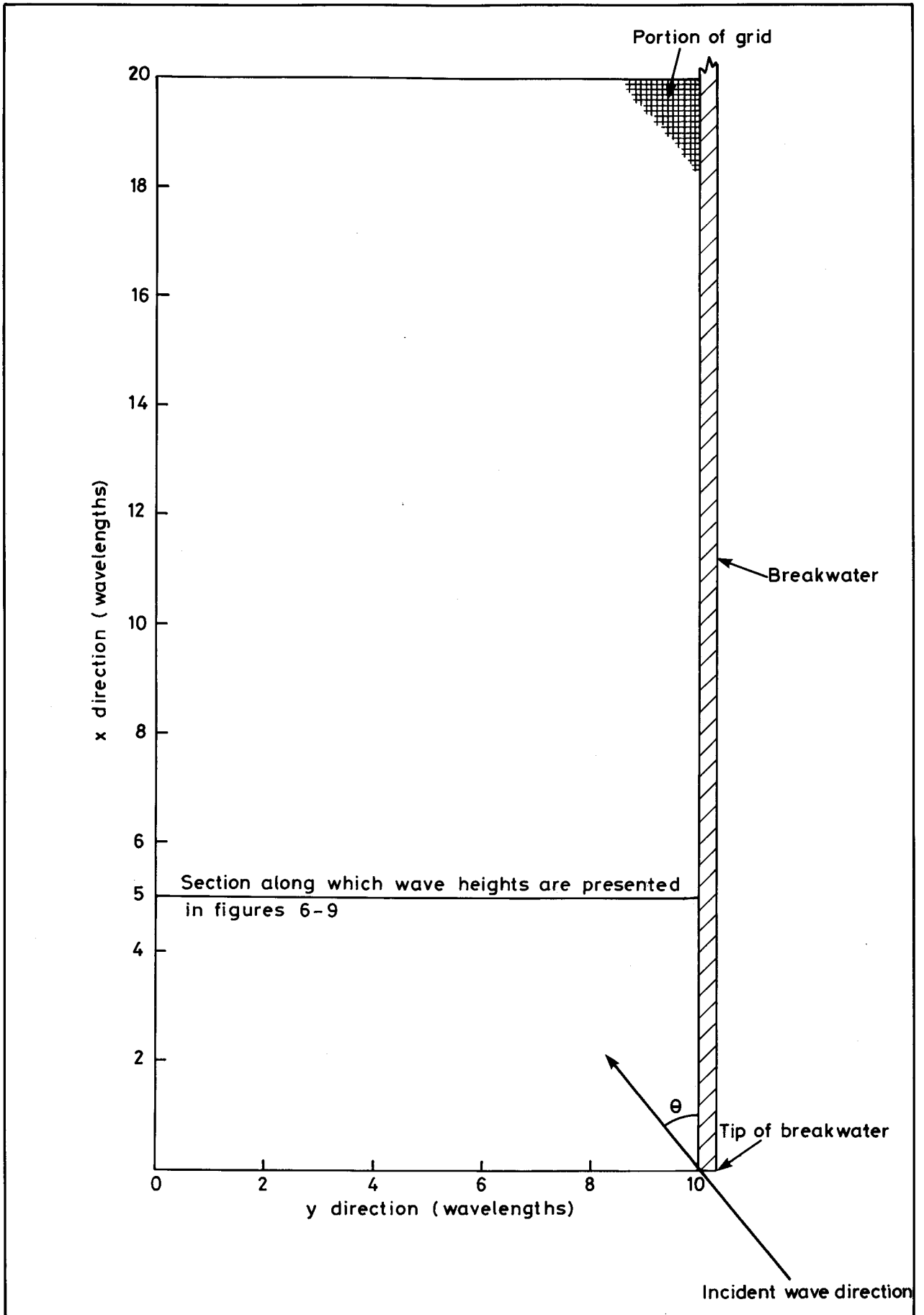


Fig 5 Model layout and grid for the single breakwater problem.
 θ is the incident wave angle

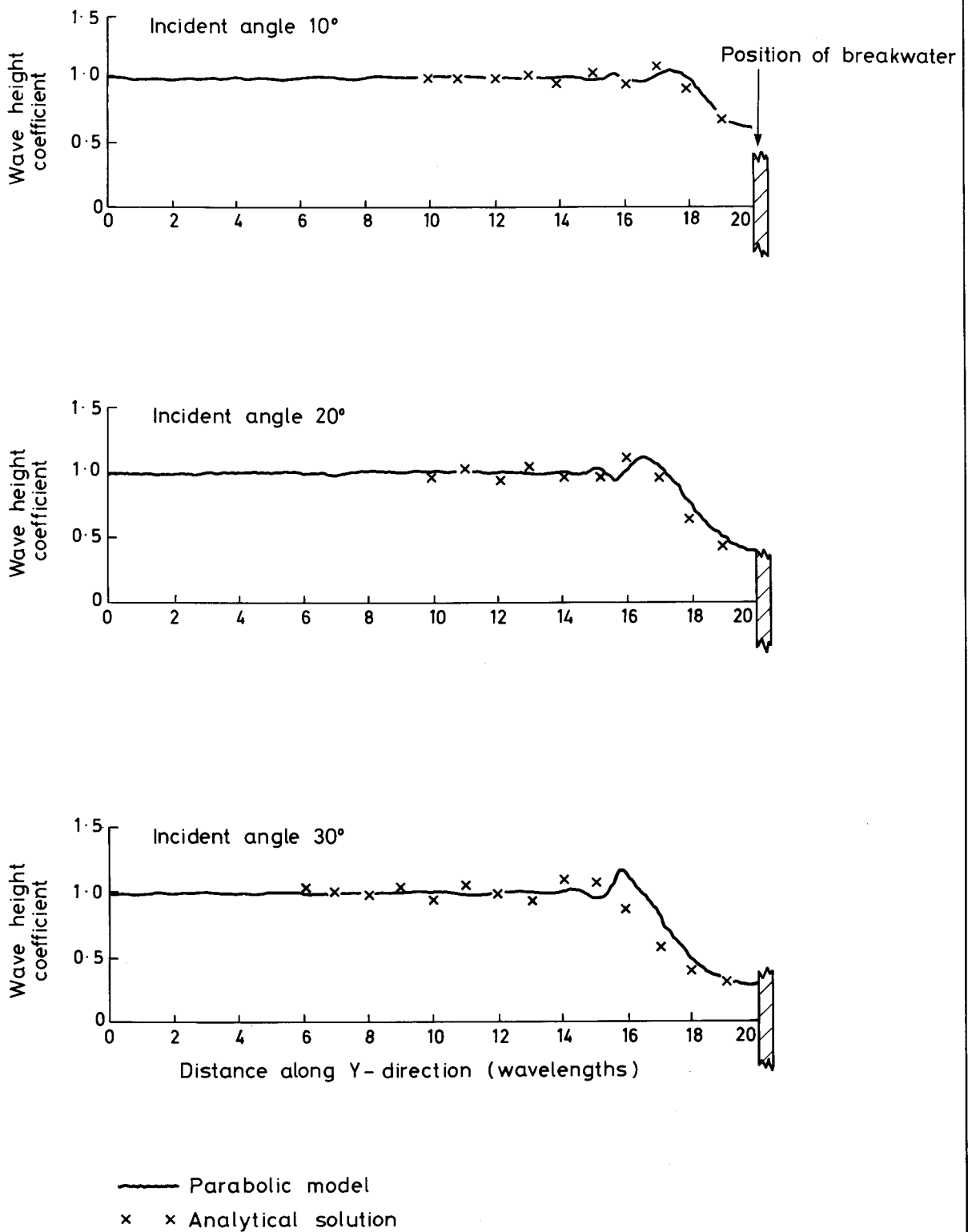
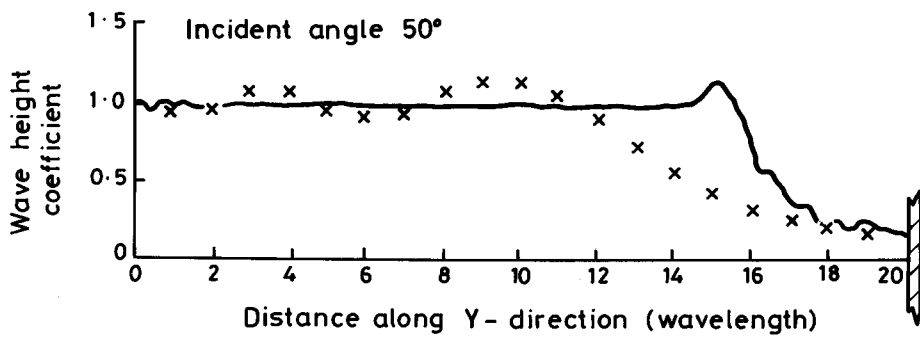
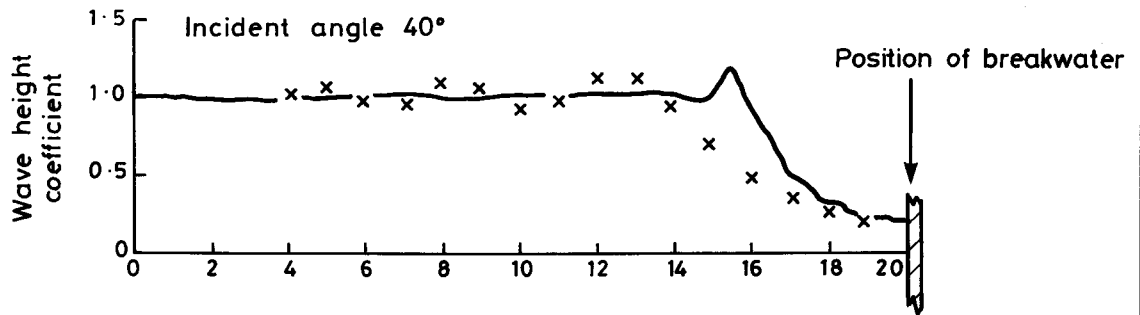


Fig 6 Comparison of parabolic model (without correction term) with analytical solution. Cross-sections at 5 wavelengths from breakwater tip



— Parabolic model
 x x Analytical solution

Fig 7 Comparison of parabolic model (without correction term) with analytical solution. Cross-sections at 5 wavelengths from breakwater tip

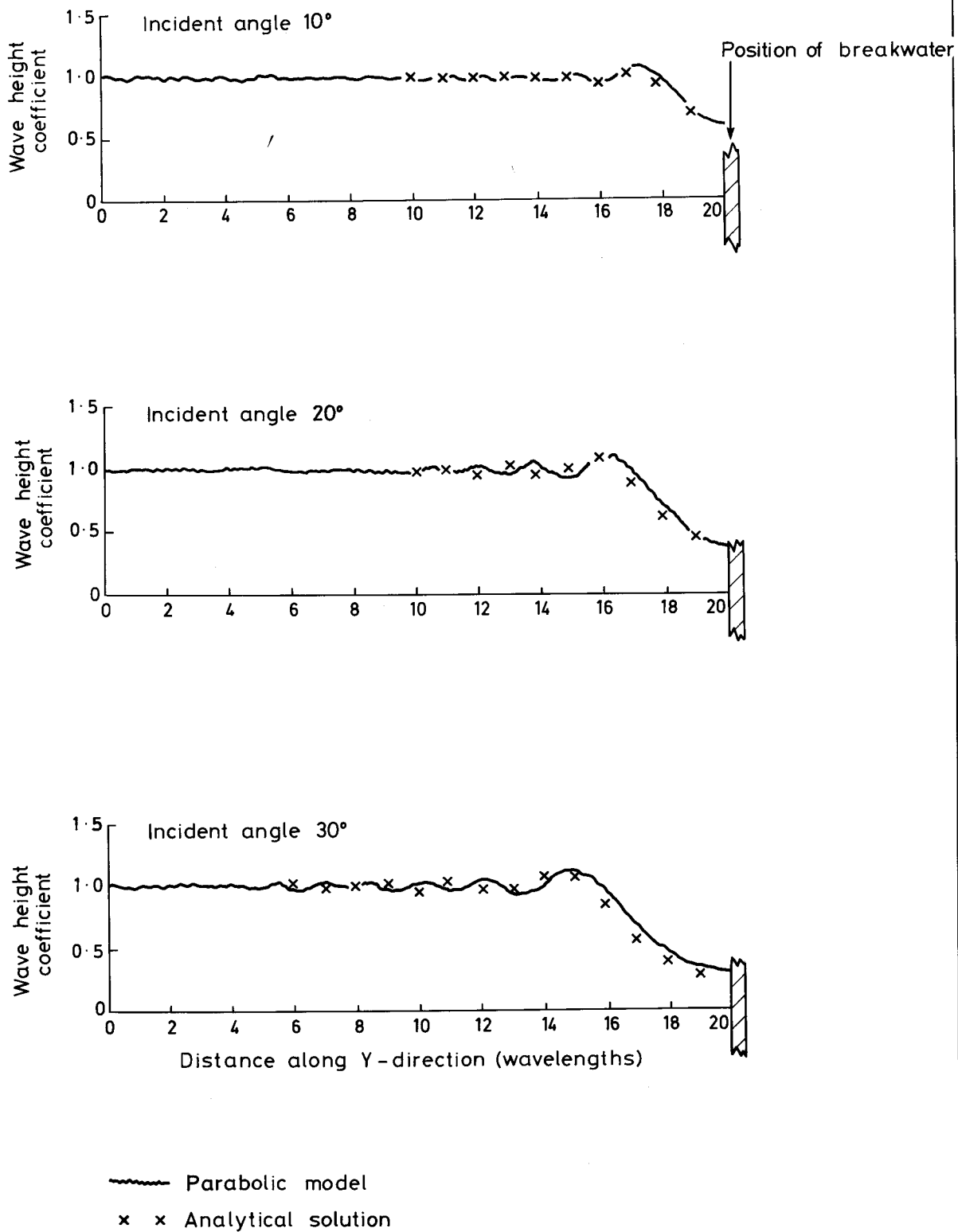
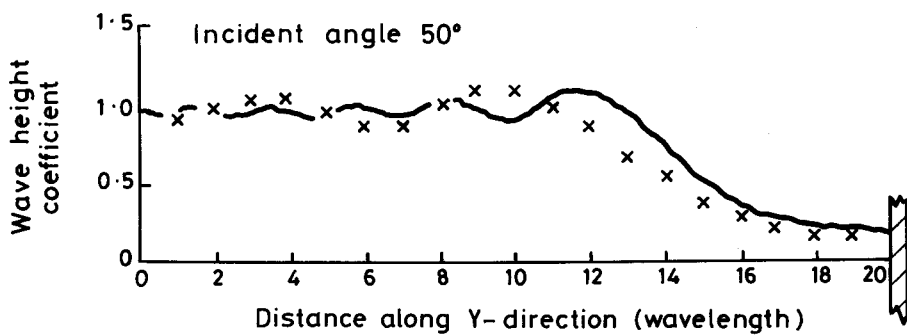
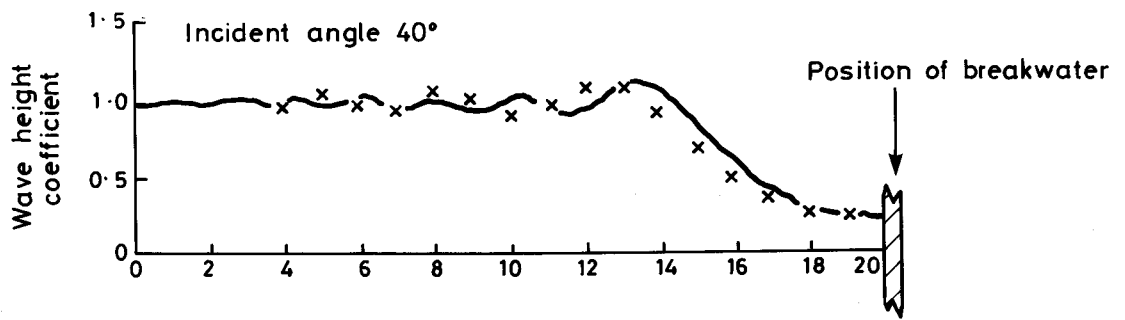


Fig 8 Comparison of parabolic model (with correction term) with analytical solution. Cross-sections at 5 wavelengths from breakwater tip



— Parabolic model
 x x Analytical solution

Fig 9 Comparison of parabolic model (with correction term) with analytical solution. Cross-sections at 5 wavelengths from breakwater tip

