ESTIMATION OF MAINTENANCE DREDGING FOR NAVIGATION CHANNELS

G H Lean BSc, ARCS, DIC

Hydraulics Research Station
Wallingford
Oxon OX10 8BA
Telephone 0491 35381

PREFACE

This report deals with methods of predicting maintenance dredging in those parts of navigation channels that are situated in the outer areas of estuaries and offshore. It discusses the behaviour of channels dredged across flow lines, and examines the influence of various factors on channel shape.

It deals in turn with methods of predicting infill due to the action of currents, gravity and wave activity, and the special problems that arise when the bed is composed of a cohesive material such as mud or silt. It discusses the application of tracers to estimate infill, and also trial dredging.

The report was prepared in response to a demand for a critical assessment of existing methods of predicting maintenance dredging in access channels, and is based on a review of available literature backed by experience gained in investigating practical problems at the Hydraulics Research Station.

CONTENTS

Page

Page

APPENDICES

LIST OF SYMBOLS

a = semi-width of top of channel

A = initial cross-section of channel

b = semi-width of bottom of channel

B = width between adjacent stream lines

c = kinematic wave velocity

c_s = velocity of kinematic shock

c_w = wave velocity

C = concentration of suspended particles in terms of the volume of compacted bed sediment per unit volume of water

C_* = concentration by volume of sediment particles in unit volume of the bed

C_f = friction factor = $\tau/\rho U^2$

C_u = undrained shear strength of sediment

d = thickness of movement of sediment

d_m = size of sediment particle

D = effective diffusion coefficient describing spread of channel in tidal flow

D_g = effective diffusion coefficient describing gravity infill

E = rate of erosion of sediment from bed, volume of compacted bed sediment per unit area per unit time

h = depth

H = wave height

I = infill rate per tide, in terms of volume of sediment per unit width across flow lines

k = wave number

k_s = constant

K = mean coefficient of vertical sediment diffusion due to turbulent fluctuations over depth

ℓ_1, ℓ_2 = average distance moved by sediment on flood and ebb

L = average distance that particles eroded from bed move before making subsequent contact with bed

m = velocity exponent

p = pressure

P = rate of deposition of sediment on bed, volume of bed sediment deposited per unit area per unit time

q = water discharge per unit width

s = specific gravity of bed particles

S = sediment flux in terms of volume of compacted bed sediment per unit width per unit time

S_w = sediment flux under waves in terms of volume of compacted bed sediment per unit width of wave crest per unit time

t = time

T = wave period

u,v,w = components of velocity in x,y,z directions

u_w = amplitude of orbit velocity at bed due to waves

U = mean velocity over the depth

U_* = friction velocity = $(\tau/\rho)^{1/2}$

U_d = drift near the bed under waves at a point

V_s = fall velocity of sediment particles

W = erosion velocity, ie upward velocity with which sediment particles are entrained from bed

W_d = particle drift near the bed under waves

x = longitudinal co-ordinate taken as positive downstream perpendicular to the channel

X = width of channel

y = horizontal transverse co-ordinate

z = vertical co-ordinate

Z = bed elevation above a reference level

$\overline{Z}$ = distance of centroid of suspended particles in a vertical above the bed

Z_f = depth of sliding

α_s = angle of sides of channel to horizontal

β = parameter characterising the velocity distribution over the depth

β_f = slope of sides for failure for coherent sediments

β_s = ratio of mass (sediment particle) to momentum diffusion in vertical direction

γ = $2(Dt)^{1/2}/a$

γ' = submerged weight per unit volume

δ = boundary layer thickness

η = change in water surface elevation over the channel

θ = $U_*^2/g(s-1)d_m$

θ_o = angle of channel to flow ($\theta_o = 90^o$ corresponds to a channel normal to the flow)

κ = Karman constant

λ_d = angle of dynamic friction of particles with bed

λ_i = angle of internal friction of bed sediment

λ_1 = $W_1 C_*/V_S\overline{C}_1$

ν = coefficient of kinematic viscosity

ξ = $V_S x/Uh$

ρ = water density

ρ_t = density of saturated bed sand

σ = density of sediment particles

σ_v^2 = variance of distribution of tracer after a single tide

τ = bed shear

$\phi(z)$ = function describing velocity distribution over vertical, ie $u = U\varphi(z)$

ψ = angle of particle paths on side slope to centre line of channel

$\psi(z)$ = function describing vertical diffusion distribution over vertical, ie $K_z = K\psi(z)$

ω = angular velocity of the wave

Γ = $-$ kx

Λ = hydraulic roughness of the bed

Suffixes

0,1 refer to quantities upstream and over the channel (respectively) at a certain phase of the tide

x,y,z refer to components of quantities in x,y,z directions

b refers to quantities measured close to the bed

1 INTRODUCTION

With the increasing size and draught of ships, port authorities are concerned with the cost of increasing the depths and widths in their access channels and dock areas, which includes the initial dredging cost and the continuing charges for maintaining the new dimensions. In the present report, attention is confined to the methods which can be used to predict maintenance dredging and is further restricted to those parts of the access channels which are situated in the outer parts of estuaries and offshore. Similar problems are encountered in the more sheltered areas inshore but here other factors such as the effect of littoral drift along the coast, the current pattern caused by the local geometry of the bed and shoreline and current drifts due to salinity and their inter-relationship with the enlarged channel make it impossible to give any general statement about channel infill; such cases must be treated individually.

Some idea of the size of the problem may be gathered from Table 1 which gives estimated quantities of sediment dredged from British ports during 1972 compiled from figures supplied to the National Ports Council.

Table 1
Quantities of sediment dredged from British Ports in 1972

Type of material	*Quantity dredged (tons, dry weight)*	*Cost (1972)*
Sand	5 077 000	
Silty sand	6 693 000	£13 500 000
Silt	8 588 000	
Fine silt	739 000	

The estimated cost includes capital charges based on the cost of a dredger assuming a 15 year life expectancy. Not all of the dredgings are taken from the approach channels, probably much of the 'fine silt' and 'silt' is deposited in the inshore areas. Generally, of course, deposition in a channel is brought about by the reduction of velocity consequent on the increased depth and is proportional to the amount of sediment carried in the water or the sediment flux. Often the sediment flux is extremely large (eg in the Outer Thames estuary the gross flux of fine silt and sand is about 4×10^6 m^3/annum/km width) so that even if a small fraction is deposited in the channel the maintenance toll will be quite high.

In the past, harbour engineers have tended to rely on the extrapolation of past and existing dredging rates to forecast future requirements. However, such methods become uncertain when large depth increases and associated channel extensions become necessary. In some cases entirely new ports need to be developed or alternative access routes are required where previous experience is lacking. As an example for a new port in the Outer Thames it was estimated that the eventual maintenance requirement to keep the access channel clear for 200 000 DWT tankers would be about 10×10^6 m^3/a, ie about half the dredging toll for the whole UK.

Some information can sometimes be obtained from an examination of dredging rates of existing channels at ports subject to similar hydraulic conditions. However, the type of bed sediment, wave and current climate, channel dimensions, are seldom sufficiently similar for more than crude estimates to be made in this way but, as experience accumulates, this method could prove most useful. Work is undoubtedly required which seeks to correlate dredging rates at existing ports with the factors mentioned.

Two methods are in current use to estimate the dredging required to maintain a new channel. In the first method trial trenches are dredged at one or more sites along the line of the channel and the infill is estimated from successive surveys. This method is direct but expensive since the trenches need to be large to avoid effects associated with the edges of the trench; this method is discussed further in Section 8. The second method is based on estimates of the sediment flux approaching the channel and its trapping efficiency, which in turn depends on the modification of the current pattern produced by the channel. Although simple in principle, the method suffers from lack of knowledge concerning the relationship between the sediment flux and the hydraulic parameters and the way in which the current pattern and flux is modified by the channel. Much of the report (Sections 2 and 3) is concerned with these questions.

For channels in line with the flow or inclined at very small angles to it, infill occurs mainly by gravity. Particles moving in contact with the sides of the channel move down the slope and the infill on this account has been quantified (Fredsoe, 1976). In some places where the currents are weak fine silts or muds accumulating on the surrounding bed could flow under gravity as a suspension into the channel. Both these effects are considered in Section 4. Generally speaking, fine particles of clay or silt in suspension in rivers flocculate when they come in contact with sea water and may deposit on the bed where they consolidate and offer considerable resistance to subsequent erosion. In some estuaries the areas of deposition are fairly localised in a region where the landward drifts of water at the bed caused by salinity gradients or wave (wind) effects balance the river flow. In other estuaries, when the tidal currents are smaller and the rivers larger, these areas of deposition may be more widespread. Navigation channels dredged in such environments infill largely due to the disturbance of the balance of depositional and erosional phases of the flow brought about by the reduced velocity over the channel (Section 5).

When waves are present, the oscillatory flow at the bed which they cause can increase the sediment flux in tidal currents and hence can increase the rate of infill into dredged channels. Even in the absence of tidal currents, waves can induce a set movement of sediment by setting up drift currents in the main body of the water or more directly by asymmetry of the oscillation, which is reduced in the increased depth over a channel thus causing infill. Methods of evaluating infill when waves are present using relationships connecting sediment flux with wave characteristics and by more direct field methods using tracer particles, are discussed in Sections 6 and 7.

Generally dredged channels in offshore areas are not sufficiently wide or deep to significantly affect the overall flow, so we are concerned only with local changes in the flow pattern due to its presence. In more restricted waters the presence of the channel may change the incident flow distribution and this change is usually evaluated by recourse to hydraulic or mathematical models. Such models are briefly referred to in Section 9. However, in some harbours, entrance channels intercept the coastal drift of sediment in the near beach area inshore of the breaker zone and the channel is subject to infill from this cause. The rate of infill may then depend on the redistribution of littoral sediment drift and subsequent moulding of the coastline by the presence of breakwaters, groynes etc, designed to protect the channel and beach areas against wave action; the problem of estimating infill is then very specific to the particular area involved. Such problems are not considered in the present report.

In considering the local flow over a channel we make the usual assumption that since the flow accelerations in tidal flow in deep water are small, the flow over the channel at a certain phase of the tide is the same as if the flow were steady. Thus the effects of the tidal flow are considered to be the same as the summation of a series of steady flows at the appropriate velocities, directions and depths. It may be noted here that while this may be valid for the flow it is less so for the sediment flux. The quantity of fine sediment in suspension, because of its low fall velocity, takes time to adjust to a reduction in velocity; thus there is a time lag between changes in velocity and changes in concentration. In addition, entrainment of sediment from the bed depends on the bed features (ripples or dunes) and their form is dependent on the flow, thus the sediment flux will be different in accelerated and steady flow on this account also. Strictly, therefore, sediment flux relationships derived for steady conditions cannot be applied in a tidal environment.

We start our discussion by considering the simplest case, viz the infill of a channel crossing the flow at right angles. This case is of practical importance and brings out several features which are of interest in more complex situations.

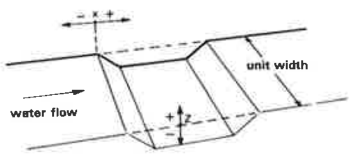
2 INFILL DUE TO ACTION OF CURRENTS

2.1 Channel at right angles to flow

Since the velocity over the channel will vary in inverse proportion to the depth (discharge constant), and the capacity of the flow to transport sediment varies with the velocity, there will be deposition over the upstream of the channel and erosion over the downstream slope. Thus in steady flow the channel would migrate downstream with no change in cross-sectional area. The latter statement may easily be verified by considering the equation of continuity for the sediment, viz

$$\frac{\partial S}{\partial x} + \frac{\partial Z}{\partial t} = 0 \qquad ....(2.1.1)$$

in which



S is the transport of sediment measured in terms of the volume of compacted bed sediment passing unit width across the flow per unit time;

Z is the bed elevation above the bed level of the approach flow;

x is the longitudinal co-ordinate taken as positive downstream;

t is the time.

Since S is the same, upstream and downstream of the channel, integrating (2.1.1) with respect to x shows that

$$\int_{-\infty}^{\infty} Z dx = \text{constant} \qquad ....(2.1.2)$$

Thus the cross-sectional area of the channel remains constant and equal to the initially dredged area.

However, the shape of the channel will change, eventually becoming wider and shallower, due to the fact that the sediment flux is not a linear function of velocity. If, for example, we write S as a function of the velocity U (U denotes the mean velocity over the depth) thus

$$S = S(U) \qquad ....(2.1.3)$$

for the constant discharge (q) per unit width across the channel; it will be evident that S is a function of Z only and multiplying or dividing (2.1.1) by

$$c = \left(\frac{\partial S}{\partial Z}\right)_{q \text{ constant}} \qquad ....(2.1.4)$$

we obtain

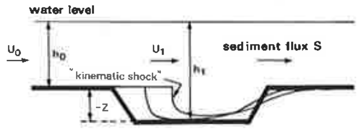
$$c\frac{\partial S}{\partial x} + \frac{\partial S}{\partial t} = 0 \qquad ....(2.1.5)$$

with a similar relation for Z, viz

$$c \frac{\partial Z}{\partial x} + \frac{\partial Z}{\partial t} = 0 \qquad \text{....(2.1.6)}$$

Thus Z and S are constant along waves travelling with velocity c. The behaviour of these waves, the so-called "kinematic waves" – since they arise solely from the equation of continuity – has been studied by Lighthill and Whitham (1955) in connection with flood waves and traffic flow problems. We note that if c were constant (c_o) and the dredged section given by $Z = F(x)$ initially, the solution of (2.1.6) is $Z = F(x-c_ot)$ and the channel will simply travel downstream at velocity c_o without change of form. However, since the velocity of the kinematic waves is a function of Z, the channel changes shape as it advances leading to the formation of discontinuities in the shape of the bed (and of sediment flux) due to the overtaking of slower waves by faster ones – the so-called "kinematic shocks". These travel with a velocity

$$c_s = \frac{S_U - S_D}{Z_U - Z_D} \qquad \text{....(2.1.7)}$$



Channel becomes shallower but cross-section area remains constant

in which S_U, Z_U and S_D, Z_D are the sediment flux and bed elevations in front and behind the shock.

For a "weak" shock, ie one in which (Z_U-Z_D) is small

$$c_s = \tfrac{1}{2}(c_U + c_D) \text{ approximately.} \qquad \text{....(2.1.8)}$$

To illustrate the way in which shocks are formed let us approximate S by a power law

$$S = k_s U^m \qquad \text{....(2.1.9)}$$

in which k_s is a constant *(see Note)*; m is usually a high power depending on the proximity of U to the threshold velocity at which sediment just moves.

Note:

k is a function of the size of specific gravity of the particles composing the bed sediments and the size of the bed features (ripples, dunes or sand waves) formed by the action of the current on the bed. It is also weakly dependent on the depth; partly because:

i) the shear velocity U_* ($= \tau/\rho$, τ = bed shear, ρ = density of sea-water), which governs the rate of erosion of particles from the bed, is not quite proportional to the velocity but is also dependent on the depth. [If Manning's law of resistance applied, the ratio U/U_* would vary as $(depth)^{1/6}$.]

ii) the size of the bed features themselves depends upon U_*.

The transport of fine sediments which travel largely in suspension is more dependent on depth and we shall see in Section 2.2 that this leads to an important contributory factor in channel infilling. For the moment, however, k is assumed independent of depth.

Then,

$$c = (dS/dU)dU/dZ = mk_sU^{m-1}(dU/dZ) \qquad \text{....(2.1.10)}$$

Since $q = U_oh_o = U(h_o-Z)$ = constant, U_o,h_o being the approach velocity and depth, (Z is negative for a channel), the kinematic wave velocity

$$c = mk_sU^{m+1}/q = c_o[h_o/(h_o-Z)]^{m+1} \qquad \text{....(2.1.11)}$$

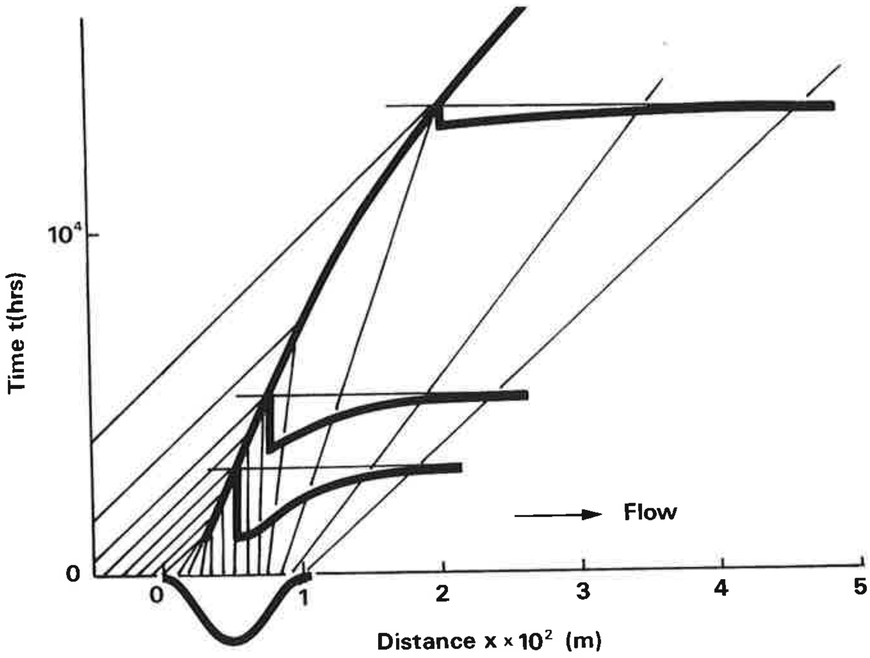
c_o being the wave velocity appropriate to the approach flow, ie the velocity with which a small bed disturbance would move, is given by

$$c_o = k_s\, m\, U_o{}^m/h_o = m(S_o/h_o) \qquad \text{....(2.1.12)}$$

Hence for Z negative, corresponding to a dredged channel, the waves travel more slowly as the local depth increases, their velocity reaching a minimum at the point of maximum depth. Thus the front of the depression steepens as the waves from the front overtake those over the upstream slope, Fig 1. The waves would ultimately cross but such an evantuality is physically unacceptable since it would give two values of Z at the same point. Evidently, continuity can only be satisfied by fitting a shock wave which proceeds with the velocity given by equation (2.1.7) or approximately the mean wave velocity on its upstream and downstream side (equation 2.1.8). Thus the shock gradually overtakes the point of maximum depression (its strength, measured by the change in Z across it, grows) passing this point in a time given by

$$X_o/2(c_o-c_M) \text{ (approximately)} \qquad \text{....(2.1.13)}$$

Figure 1
Evolution of channel section in unidirectional flow



after the channel has been dredged (X_o is the original width of the channel and c_M is the wave velocity at the point of maximum depression).

During this time the width of the channel remains nearly constant the wave velocity at the rear of the channel being approximately equal to the mean velocity of the shock, but its shape has altered considerably. Later, the shock strength gradually decays as more sediment from upstream is fed into it ($c_o > c_s$) and the shock velocity increases and slowly tends towards the velocity of waves travelling from the rear of the depression. In the latter phase of the motion, the width of the channel increases and is given by (Lighthill and Whitham, 1955)

$$X^2 = X_o(c_o - c_M)t \text{ (approximately)} \qquad \text{....(2.1.14)}$$

from which it appears that the maximum depth (Z_M) at time (t) is related to the initial maximum depth $(Z_M)_o$ in the channel by

$$\frac{Z_M}{(Z_M)_o} = \left(\frac{X_o}{(c_o - c_M)t}\right)^{½} \text{ (approximately)} \qquad \text{....(2.1.15)}$$

The above analysis relates to a channel which, once dredged, is allowed to progress without further attention. In fact navigation channels cannot be allowed to change their position in this way and would need to be continuously dredged to remove the sediment deposited from upstream. For a wide trapezoidal channel with a level bed the rate of dredging required to maintain its position on the upstream side in unidirectional flow is

$$(S_o - S_1) = (c_o + c_1)(Z_o - Z_1)/2 \text{ (approximately)} \qquad \text{....(2.1.16)}$$

in which

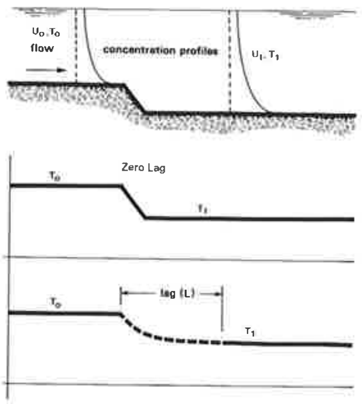
S_o = sediment flux approaching the channel,
S_1 = sediment flux over the bed of the channel.

To maintain the width of the channel, this material could simply be dumped at the foot of the downstream slope. If the sediment flux were a function of the local velocity only it is clear that in a tidal situation the channel would tend to migrate in the direction of the dominant drift of sediment. To maintain the position of the channel on the updrift side the required dredging over a tide would be

$$\int_E (S_0 - S_1)dt - \int_F (S_0 - S_1)dt \qquad ....(2.1.17)$$

where the first integral refers to the excess of the transport from the dominant direction (taken as the ebb direction) over that in the channel and the second integral refers to similar quantities in the opposite direction. The first integral is taken over the part of the tidal cycle occupied by flow in the dominant drift direction and refers to accretion of the updrift slope; the second integral is taken over the part of the tidal cycle when the flow is reversed and refers to the erosion of this slope. Thus, in a perfectly balanced tidal situation, the accretion on one half tidal cycle and erosion on the next would result in no net change of bed level and the channel would be self-maintaining. Experience shows that this desirable outcome does not prevail in practice and the channel slowly fills in. It appears that the initial premise (equation 2.1.3) that the sediment flux is dependent only on the local velocity is inadequate; clearly some time (or distance) lag must elapse before the sediment transport appropriate to the local flow state can be established. We shall see in the next section that this leads to effects of a diffusive character which tend to smooth out the discontinuities of bed slope and which for balanced tidal flow becomes the main factor for reduction of channel depth.

2.2 Lag effects



Note:

We note that W does not correspond to an upward flow of water since there can be no net movement of water at any level, although there can be a net upward flux of vertical momentum near the bed.

The way in which lag effects enter into the question of infill is most readily explained by considering an extreme example. We revert to the case of a steady uniform flow (U_o,h_o) approaching a channel (U_1,h_1) of large width and constant depth (h_1), the sediment being carried in suspension. According to the usual theory of suspension the sediment is distributed over the depth by the turbulence in the flow which opposes the tendency for the particles to fall to the bed at the rate V_S C per unit area, where V_S is the fall velocity of the sediment and C is their concentration (which we here measure in terms of the volume of compacted sediment per unit volume of water). At the bed the sediment is assumed to be maintained in suspension by re-entrainment at the rate

$$WC_* \qquad ....(2.2.1)$$

per unit area of the bed, in which

C_* is the concentration by volume of sediment particles in the bed ($C_* \gg C$) (for fine sand $C_* = 0.61$ approximately),

W is an erosion velocity *(see Note)*.

In a "diffusive" description of suspension, we may equate WC_* to

$$-\ (K_z\ \partial C/\partial z)_{bed}\ - \qquad ....(2.2.2)$$

in which K_z is the vertical diffusion coefficient for the sediment at the level of the bed forms and ($\partial C/\partial z$) is the concentration gradient at the bed.

Here, we consider only short term changes, ie we suppose the time to be sufficiently short after the channel has been dredged for changes in bed level to be negligible. For simplicity we also assume that the sediment is so fine (or the turbulence intensity so large) that it is distributed nearly uniformly over the depth. Since the approach flow is assumed steady and uniform the rates of erosion and deposition are assumed to be in balance, so

$$W_oC_* = V_SC_o \qquad ....(2.2.3)$$

and since the velocity is assumed uniform over the depth, the flux of sediment is given by

$$S_o = U_o h_o (W_o C_* / V_S) \qquad ....(2.2.4)$$

(Suffix o and suffix 1 refer to the approach and channel flow respectively.)

If the channel is sufficiently wide, at a great distance downstream from the upstream edge (x = 0), the flow will again be steady and uniform with a concentration C_1 determined by the new rate of erosion W_1 appropriate to the velocity U_1, thus

$$W_1 C_* = V_S C_1, \qquad ....(2.2.5)$$

and

$$S_1 = U_1 h_1 (W_1 C_* / V_S)$$

$$= U_o h_o (W_1 C_* / V_S) \qquad ....(2.2.6)$$

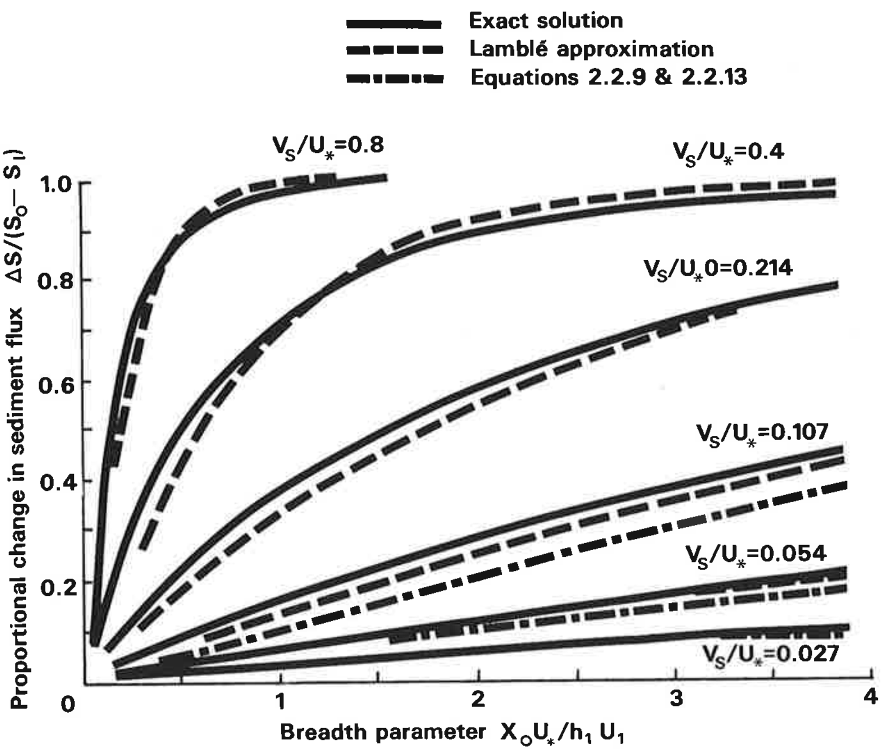
However the flux over the upstream edge cannot immediately decrease to the new entrainment rate, since the particles can only fall out at their settling velocity. The change in suspended concentration over the channel is evidently given from continuity viz:

$$U_1 h_1 \, dC/dx = W_1 C_* - V_S C \qquad ....(2.2.7)$$

This equation is easily solved for C to give the change in sediment flux over the channel

$$\Delta S = (S_o - S) \qquad ....(2.2.8)$$

Figure 2
Infill rate (ΔS) for channel, breadth X_o (finite depth)



thus

$$\Delta S = (S_o - S_1)(1 - e^{-x/L}) \qquad ....(2.2.9)$$

with

$$L = U_1 h_1 / V_S \qquad ....(2.2.10)$$

$$= 2(\overline{Z}/V_S) U_o \qquad ....(2.2.11)$$

in which $\bar{Z}$ is the centroid of the vertical distribution of concentration at entry to the channel (equal to half the water depth in this case).

It is not difficult to show that L can be identified with the average step length of the particles, defined as the distance particles move from the point at which they are eroded from the bed to subsequent contact with the bed (Lean, 1971). If the particles are less fine, they will be distributed over the vertical in accordance with the usual theory of suspension by turbulent motion (Rouse, 1949). One might expect that in this case the change of sediment flux over the channel would still be given approximately by equations (2.2.9) and (2.2.11) but with $\bar{Z}$ modified to take into account the non-uniformity of the sediment concentration in depth. Thus for a constant eddy diffusivity (K) over the depth

$$\bar{Z} = (K/V_S)[1-\{V_S h_o/K[\exp(V_S h_o/K-1]\}] \quad \text{....(2.2.12)}$$

For large water depths this becomes

$$\bar{Z} = K/V_S \quad \text{....(2.2.13)}$$

Note:

Taking the usual expression for the mean vertical diffusion coefficient of the sediment over a vertical viz:

$$K = \beta_s \kappa h_o U_*/6 \quad \text{....(2.2.16)}$$

in which

β_s = ratio of sediment to momentum diffusion coefficient

κ = Von Karman constant (0.4)

U_* = shear velocity

Thus taking $\beta = 1$, $U_*/U = .05$ for a rippled bed $h_o = 15$ m, then for $U = 0.3$ m/s, $L = 90$ m and 720 m for fine sand (0.12 mm diameter) and silt (0.06 mm diameter respectively. The width of the channel (X_o) may be of the order of 100–200 m.

which, when substituted in (2.2.11) and (2.2.9) gives flux changes in fair agreement with a more accurate treatment, (Lean, 1970).

For smaller depths, $V_S h/K$ or V_S/U_* small *(see Note)*, the agreement with the analytical solution (Appendix 1) is also good (Fig 2).

From equation (2.2.9) we see that for a channel of limited width (X_o) the reduction of flux rate at the downstream end of the channel is given by

$$\Delta S = (S_o - S_1)(1 - e^{-X_o/L}) \quad \text{....(2.2.14)}$$

This must evidently be equal to the infill rate.

Downstream of the channel, the bank will erode and the flux (S) will increase due to the higher velocity over the bank according to

$$(S_o - S) = (S_1 - S_o)\,[1 - e^{-X_o/L}]\, e^{-X_1/L} \quad \text{....(2.2.15)}$$

in which X_1 is the distance from the downstream edge of the channel.

In a typical tidal situation with a fine sand or silt bed L is often comparable with X_o (see note). Thus the infill is considerably less than ($S_1 - S_o$), the infill rate which would occur if S were a function of the local velocity only.

2.2.1 The interpretation of lag as a diffusive effect

It is evident from equation (2.2.9) that in the decelerated flow over the upstream edge of the channel, the flux lags that which would occur if the transport were a function of the local velocity only by an average distance L. Similarly in the accelerated flow over the downstream edge, the flux increase lags that appropriate to the local velocity by a distance L. In fact, over a gently sloping bed, it would seem reasonable to approximate the flux at x to that which would occur at the local velocity at (x–L), viz:

$$S(x-L) = S(x) - L\frac{\partial S}{\partial x} \quad \text{....(2.2.17)}$$

and

$$\frac{\partial S(x-L)}{\partial x} = \frac{\partial S}{\partial x} - L\frac{\partial^2 S}{\partial x^2} \quad \text{....(2.2.18)}$$

in which S(x) is a function of the local velocity or local bed elevation (Z) only. Thus in a spatially accelerated or retarded flow we must substitute ($\partial S/\partial x - L\partial^2 S/\partial x^2$) for $\partial S/\partial x$. The second term $-L\partial^2 S/\partial x^2$ may be regarded as a correction term to express the fact that S depends not only on the velocity but also on the rate of change of velocity.

With this substitution equation (2.1.5) becomes

$$\frac{\partial S}{\partial t} + c\frac{\partial S}{\partial x} - Lc\frac{\partial^2 S}{\partial x^2} = 0 \qquad ....(2.2.19)$$

with a similar relation for Z viz:

$$\frac{\partial Z}{\partial t} + c\frac{\partial Z}{\partial x} - Lc\frac{\partial^2 Z}{\partial x^2} = 0 \qquad ....(2.2.20)$$

(The latter equation can readily be deduced by differentiating (2.2.19) with respect to x and substituting for $\partial S/\partial x$ from the continuity equation (2.1.1).)

Equations (2.2.19), (2.2.20) are typical of equations representing the evolution of quantities (S,Z) diffusing about an origin moving with velocity c *(see Note)* with a diffusion coefficient given by

$$D = cL \qquad ....(2.2.21)$$

Note:

This is easily seen, if c were constant, by referring the quantities (Z, for example) to an origin moving at velocity c. In this case the new length co-ordinate $x_1 = (x-ct)$ and $\partial Z/\partial t$ must be replaced by $\partial Z/\partial t - c\,\partial Z/\partial x_1$.

Thus the equation (2.2.20) becomes

$$\partial Z/\partial t - D\partial^2 Z/\partial x_1^2 = 0 \qquad ....(2.2.22)$$

which is the well known form of the diffusion equation.

Generally c is not constant but a general treatment which allows the variation of Z and S along kinematic waves when diffusion is present is given by Lighthill and Whitham (1955). However, for channels of small incised depths c will be approximately constant and equal to c_o (equation (2.1.11)). A typical solution of (2.2.20) is

$$Z = \frac{A}{2\sqrt{\pi Dt}} \exp\left[-(x-c_o t)^2/4Dt\right] \qquad ....(2.2.23)$$

which represents the profile after a long time of a channel with an initial incised area A. Thus the maximum depth Z_m at $x = c_o t$ is given by

$$Z_m = A/2\sqrt{\pi Dt} \qquad ....(2.2.24)$$

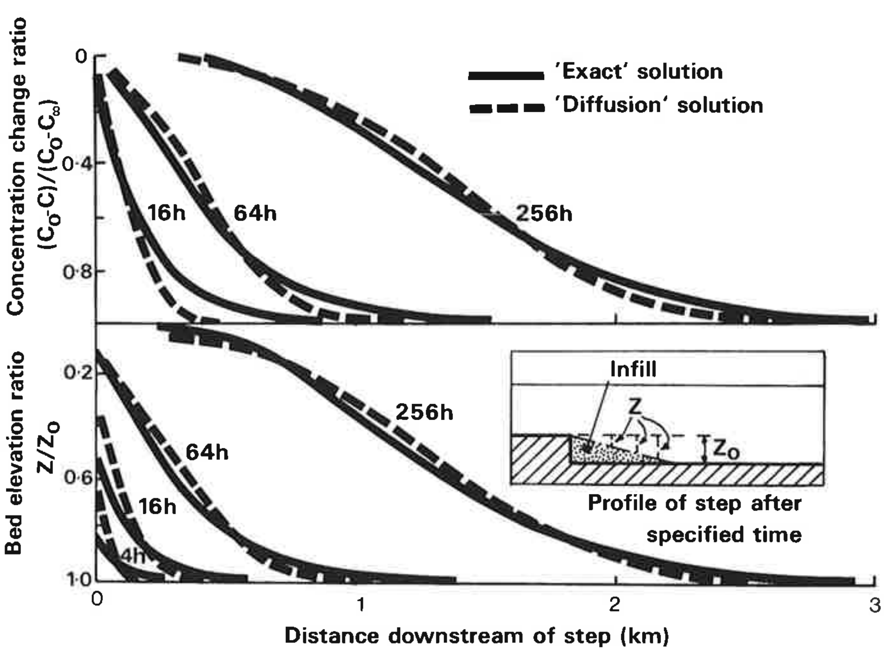
It will be clear from the derivation of (2.2.19), (2.2.20) that these equations cannot be expected to apply when there are discontinuities in S or $\partial S/\partial x$, as will occur, for example in the initial stages of the flow over a channel with abrupt side slopes. The equations then give infill rates tending to infinity as t approaches zero, which is physically unrealistic and contrary to a more accurate equation (2.2.26) which is applicable when t is small. However, they probably give reasonable estimates of infill rates when t is large.

This is borne out in the case of steady flow of a "well-mixed" suspension over an initial small step in the bed, when the bed is allowed to respond to changes in the sediment flux. For this comparatively simple case an analytical solution is available (Appendix 2) and the evolution of the step and flux changes can be traced, Fig 3. Initially the change in concentration and sediment flux is given by equation (2.2.9) but after a time, due to differential deposition (negative step) or erosion (positive step), the step is gradually smoothed out and the flux gradients become more gradual. The changes of flux and bed levels are then approximated by the diffusion equations (2.2.19) and (2.2.20) with a diffusion coefficient given by (2.2.21) (Appendix 2). The "exact" analytical solution and the solution from the diffusion equations are compared in Fig 3 for a particular example. It will be seen that after a long time the bed profiles lag the concentration of flux changes by the distance L (L = 0.125 km in the example). In the absence of lag changes in concentration would parallel those of the bed profile – as the bed changed the concentration at the section would respond immediately.

2.2.2 Diffusion in tidal flow

We now return to the case of a balanced tidal flow. It is clear that for times which are long compared with the tidal period, the convective term in equations (2.2.19), (2.2.20) will disappear and D may be considered as a long-term average over a flood or ebb cycle. With the convective term omitted, (2.2.19), (2.2.20) correspond to the classic diffusion equation and their solution for an initial step or "ramp" variation of Z is well known (Carslaw and Jaeger, 1959). From the latter

Figure 3
Concentration and bed profiles - "well stirred" case



equation, the changes of bed level (Z) at a distance x from the centre line of an initial trapezoidal shape channel, with a level bottom of width 2b, top width 2a, incised depth Z_O and side slopes $Z_O/(a-b)$ are given by

$$Z/Z_O = [a\,F(X,\gamma) - b\,F(X',\gamma')]/(a-b) \qquad ....(2.2.25)$$

in which

$$F(X,\gamma) = (1-X) + \frac{\gamma}{2}\left\{\text{ierfc}\left(\frac{1-X}{\gamma}\right) + \text{ierfc}\left(\frac{1+X}{\gamma}\right) - 2\text{ierfc}\,(X/\gamma)\right\}, \qquad ....(2.2.26)$$

$$X = x/a,\ X' = x/b, \qquad ....(2.2.27)$$

$$\gamma = 2(Dt)^{1/2}/a\ ,\ \gamma' = 2(Dt)^{1/2}/b \qquad ....(2.2.28)$$

and

$$\text{ierfc}(\zeta) = \int_{\zeta}^{\infty} \text{erfc}(\xi)\ .\ d\xi \qquad ....(2.2.29)$$

The latter function is tabulated by Carslaw and Jaeger (1959).

The bed profiles for a channel in which a = 2b, for various values of γ are given in Fig 4 (one side of channel only represented).

From equation (2.2.25), the average rate of deposition over the bed of the channel $(-b \leqslant x \leqslant b)$ for short times is given by

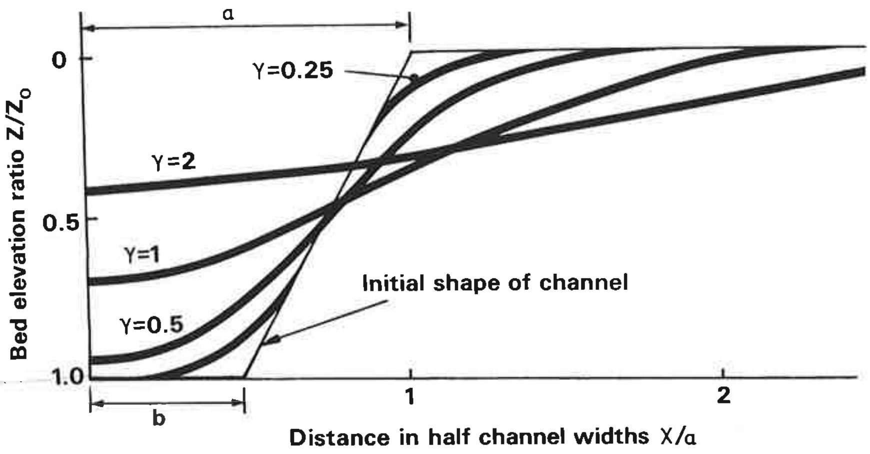
$$2b\ dZ/dt = D\tan\alpha_S \qquad ....(2.2.30)$$

in which $\tan\alpha_S$ = side slopes of channel.

This equation is probably unreliable for α_S large, since, as already mentioned, the diffusion equation is not an accurate description when gradients of γ or τ are large.

When the tidal flux on the ebb and flood are not balanced, the channel will still diffuse about its centre (change its shape) in the manner shown in Fig 4 but its centre will travel in the direction of the dominant drift with a velocity equal to the average value of c over the spring-neap cycle (= 0 in this case).

Figure 4
Infill of trapezoidal channel (a=2b)



As an example we may cite the case of a trial channel dredged at right angles to the coast near the port of Dunkirk (Lepetit, 1975). The channel, 300 m long was dredged in fine sand (D_{50} size 0.2 mm) below a mean water depth of about 6 m. The dominant sediment drift was in the flood direction (maximum current of the order of 1 m/s) parallel to the coast. The channel migrated in the flood direction at the rate of about 20 metres in 11 months (c = 0.0025 m/hour) and in the same period the maximum incised depth decreased by about half (3½ m to 1¾ m) corresponding to a value of $2(Dt)^{1/2}/a = 1.6$ (Fig 4). Since the half width of the channel (a) was 40 m, the effective value of D = cL = 0.127 m^2/hour corresponding to L = 50 metres (approximately). If we assume that the sand in movement was mainly composed of the finer fraction of the bed sand (0.12 mm), fall velocity V_S = 0.009 m/s, this value of L would correspond to that which would occur in 6 m water depth at a velocity of 0.32 m/s. This is not an unreasonable value bearing in mind that this current must be interpreted as the current which would give the same lag distance in steady flow as that for the alternating flood and ebb during the 11 month period.

2.3 Two phase description of flow

So far our discussion has centred on the continuity equation (2.1.1) and we have made certain simplifying assumptions with regard to the variation of S or Z with x in order to explain the infill mechanism. These have enabled us to show how, when the sediment is carried in suspension, lag effects can arise and how, in the long-term these effects in balanced or nearly balanced tidal situations are the main agent for determining infill. However, it is useful, especially when we need to consider short-term effects, to write the continuity equation in a form in which the boundary conditions at the bed are given explicitly. Since these conditions constitute one of the basic difficulties of the problem, they are set down here for reference. Broadly the equations given follow those of O'Connor (1975).

$$K_z \frac{\partial C}{\partial z} + V_S C = 0, \qquad z = h \qquad \text{....(2.3.1)}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + w\frac{\partial C}{\partial z} = \frac{\partial}{\partial z}(K_z \frac{\partial C}{\partial z} + V_S C), \qquad Z \leqslant z \leqslant h \qquad \text{....(2.3.2)}$$

$$E = -(K_z \frac{\partial C}{\partial z})_{bed} \qquad z = Z \qquad \text{....(2.3.3)}$$

$$C_* \frac{\partial Z}{\partial t} = P - E \qquad z = Z \qquad \text{....(2.3.4)}$$

in which

u, w are the horizontal and vertical components of velocity at (x,z,t)
K_Z is the vertical diffusion coefficient
P is the volume of compacted sediment deposited on the bed per unit area per unit time
E is the volume of compacted sediment eroded from the bed per unit area per unit time.

O'Connor includes a term $\partial S_b/\partial x$ on the left hand side of (2.3.2), S_b being the sediment transported along the bed by sliding or rolling, which to be consistent must here be measured in terms of the volume of compacted sand transported per unit width per unit time. Here S_b has been omitted since C is presumed to include all the material in motion independently of the way it moves. How far these equations can be used to describe the physical processes involved in bed movement is a matter of conjecture but it seems not unreasonable to combine suspended and bed transport together especially as the main use of S_b in practice is to define a rate of erosion (W C_*). Thus for cohesionless sediments it is usual to write

$$E = W C_* = V_S C_b^e \qquad \text{....(2.3.5)}$$

where C_b^e is the concentration of sediment in the water at the bed under uniform flow conditions corresponding to the local velocity and depth and

$$C_b^e = S_b/U_b\, d_b \qquad \text{....(2.3.6)}$$

in which U_b is the average velocity with which the bed load is moving and d_b is a certain thickness over which the particles engaged in bed movement are assumed to be spread (Appendix 3).

The bed load is assumed to respond very quickly to changes in the shear at the bed, or what amounts to the same thing, the erosion rate E is assumed to be a function of the local bed shear only. The formulation of E is probably the most difficult which besets the infill problem. No direct measurements of it in the field or laboratory exist and there are comparatively few field measurements of C_b^e from which it could be indirectly determined, (equation (2.3.5)).

Usually the erosion rate is inferred from total flux measurements assuming an equilibrium vertical distribution of concentration derived from the theory of suspended flow (see Section 2.4) or from equation (2.3.6) assuming a bed load formula which relates S_b to the bed shear (Appendix 3). Many formulae of this type exist, most of which are based on an analysis of measurements of the transport of coarse sediments in laboratory flumes, which move mainly in contact with the bed. A survey of such formulae is given by Yalin, 1970. However, it may be noted that the extrapolation of laboratory measurements to determine the flux or E is probably unreliable since these quantities are likely to be dependent on the scale of the bed features. The way in which these features affect erosion rates is not fully understood.

The deposition rate, P, for cohesionless sediments is simply

$$P = V_S C_b \qquad ....(2.3.7)$$

in which C_b is the local concentration of sediment close to the bed. C_b is determined from the solution of the equations (2.3.1) – (2.3.4). To determine Z(x,t) it is necessary to solve equation (2.3.2) with the initial values of Z(x,0) and boundary conditions (2.3.1), (2.3.2), (2.3.3) and (2.3.5). The new value of Z for a given time step is then obtained from (2.3.4) and the process repeated. Such a process is best done numerically (O'Connor, 1975).

We note that the continuity equation (2.1.1) can easily be recovered by integrating equation (2.3.2) over the depth with the conditions at the surface (2.3.1) and the bed (2.3.4) given by

$$\frac{\partial S}{\partial x} = \frac{\partial}{\partial x} \int_Z^h u\, C\, dz = E - P \qquad ....(2.3.8)$$

provided the term representing the sediment stored in suspension $\frac{\partial}{\partial t} \int_Z^h C\, dz$ is negligible. This can be shown to be the case since the rate at which the bed elevation changes (kinematic wave velocity c) is very small compared with the current velocity u (Engelund, 1972).

With this assumption we can recover the results of the last section for steady flow and in so doing obtain a rather more generalised form for the diffusion coefficient (D), equation (2.2.21). We adopt the method of Aris (1959) but make the additional simplifying assumptions that the incised depth of the channel is small, so that changes in velocity over the channel are significant only insofar as they affect the entrainment rate. The velocity (u) and vertical diffusion coefficient (K_Z) are then independent of x in equation (2.3.2) (and w = 0), although we may if we wish include a variation of these quantities over a section viz, $U\phi(z)$ and $K\psi(z)$ in which U and K represent mean values of u and K_Z over the section.

We shall assume initially (t = 0) that the channel is represented by a very localised depression in the bed (δ function) such that

$$\int_{-\infty}^{\infty} Z\, dx = A \qquad ....(2.3.9)$$

in which A is its cross-sectional area.

It is found (Appendix 4), as already anticipated, that the channel migrates downstream with velocity c given by equation (2.1.4) maintaining its cross-sectional area (A), and spreads out about its centre with an effective diffusion coefficient given by

$$D_e = c\, L_e \qquad ....(2.3.10)$$

with

$$L_e = 2 \int_0^h \left\{ f(z) \int_0^z U\phi(z')dz' \right\} dz \Big/ V_S \int_0^h f(z)dz \qquad ....(2.3.11)$$

where

$$f(z) = U\phi(z) \exp\left[- \int_0^z (V_S/K\psi(z)] \, dz\right. \qquad ....(2.3.12)$$

If U and K are independent of z, ie $\phi(z)$, $\psi(z) = 1$, L_e becomes equal to the lag distance L given by equations (2.2.11) and (2.2.12).

With a flat-bottomed channel, the initial rate of erosion (equation 2.3.5)) is equal to $V_S C_{b_1}$, where C_{b_1} is the bed concentration in uniform flow at the specified velocity and depth over the channel. Thus equation (2.3.8) then becomes

$$\partial S/\partial x = V_S(C_{b_1} - C_b) \qquad \text{....(2.3.13)}$$

and the initial infill rate given by the change of flux across the channel with Z constant, once C_{b_1} is known, depends only on the concentration at the bed C_b, which must be determined from the solution of equations (2.3.1) – (2.3.3). The equation (2.2.9) may be regarded as an approximate solution of these equations for this case with the initial condition $S = S_o$ at $x = 0$.

2.4 Maintenance dredging

In general we are less concerned with the way in which a channel would change if left to itself than the dredging required to maintain a given shape and position. Strictly, as maintenance dredging proceeds the bed levels outside the channel will fall and the dredging required to maintain a given width and depth will fall also. However in estimating dredging rates it is usual to ignore this long-term effect and treat the channel and surrounding bed as though it were static. Thus for a very wide channel during an ebb tide the infill rate will be

$$\oint_E (S_o - S_1)dt$$

and there will be erosion of the banks downstream at an equal rate, which is ignored. Similarly during a flood tide, the infill rate will be

$$\oint_F (S_o' - S_1')dt$$

Dredging requirements to regain position of original channel

and there will again be an equal rate of erosion downstream, which is ignored. Thus the tidal infill rate is

$$I = \oint_E (S_o - S_1)dt + \oint_F (S_o' - S_1')dt \qquad \text{....(2.4.1)}$$

In this expression

S_o, S_o' denote the sediment flux approaching the channel during ebb and flood tides respectively, and

S_1, S_1' denote the sediment flux appropriate to the reduced velocity over the channel during ebb and flood tides.

To obtain the annual rate of infill this expression must be summed over a year of tidal spring-neap cycles.

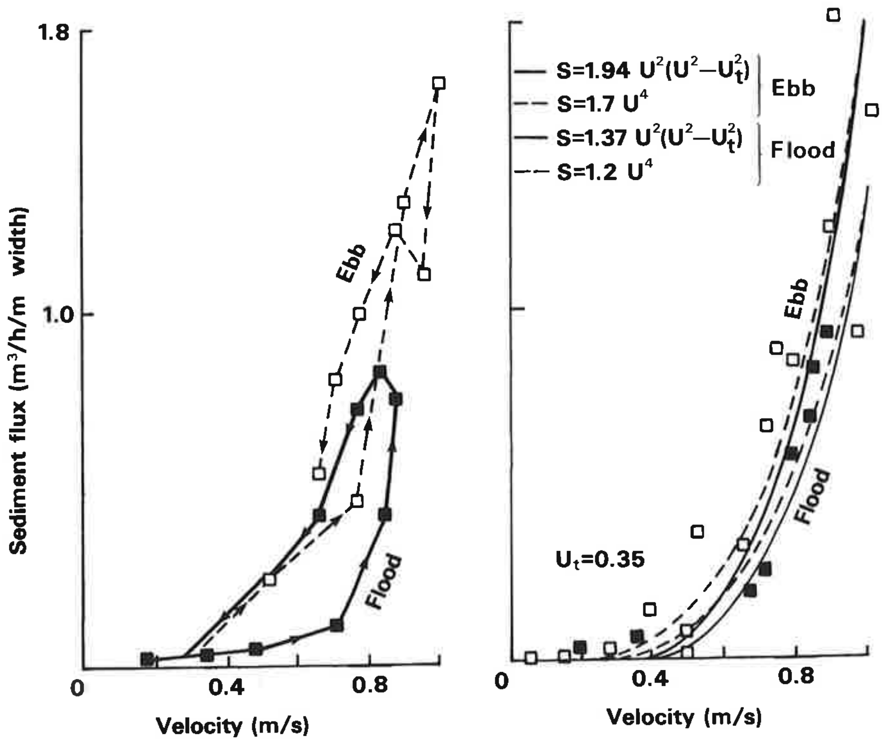
For channels of lesser width (X_o) the transport at the downstream edge will be greater than S_1 so that the infill will be reduced by the factor $(1-\exp(-X_o/L))$ which will vary with the fall velocity of the suspended sediment. For very fine sediments the lag distance (L) will be so large that the infill contributed by them will be small.

Two methods are currently used to estimate the values of sediment flux at different stages of the tide. The first involves an extrapolation of laboratory flume data to full scale depths and velocities. These are described by formulae which are usually based on physical arguments concerning the way in which the flux depends on certain non-dimensional parameters describing the flow and properties of the sediment. Many formulae of this type have been devised (US Task Committee for Preparation of Sediment Manual, 1971) but none has been widely accepted. The most recent (White, 1975), unlike most of the others, is based on a wide range of laboratory work. It gave fair agreement with observed transport rates of fine sand at sites in the Thames estuary.

In the second method, the sediment flux is measured directly at several stations along the proposed route of the channel over short time intervals through both neap and spring tides. Each measurement involves the integration of the product of the velocity and the concentration of the suspended solids over the depth for a range of size fractions, (Thorn, 1975). This enables both S_0 and S_1 (the flux appropriate to the velocity over the channel) to be estimated and hence the infill at all tidal stages.

However, in this estimation it is important to reduce the flux measurements to a series of steady flow conditions since to determine infill we are concerned with the change in erosion rate (E) over the channel due to the reduction of velocity (equation (2.3.13)). In general, in tidal flow, the flux lags the velocity, ie at a given velocity the flux is less when the current is increasing than when it is decreasing; thus the flux variation with velocity over an ebb or flood tide takes the form of an hysteresis loop. An example is shown in Fig 5(a) which relates to flux measurements at a station in the Thames estuary taken at consecutive half-hourly intervals starting at low water in a depth of about 17 metres.

Figure 5
Hysteresis of sediment flux in tidal flow



The same argument holds for the lag in time in the flux over the tidal cycle. The lag is due to the time taken for the sediment to fall out of suspension when the velocity decreases, the average time lag in flux consequent on a small change of velocity being (L/U) (Appendix 5). The flux measurements, reduced to a series of steady flow conditions by the method given in the Appendix, are shown in Fig 5(b). It will be noted that at this station for a given velocity the flux on the ebb is greater than on the flood, although at other stations the reverse situation applied. In Fig 5(b) two types of empirical formulae for representing the flux versus velocity variation are contrasted, namely the simple power law (equation 2.1.9)) and a formula of the type

$$S = k_1 U^2 (U^2 - U_t^2) \qquad ....(2.4.2)$$

in which U_t denotes the threshold velocity below which movement of sediment is presumed no longer to occur. Although empirical, the latter formula has some physical justification. It assumes that, for a given bed sediment, the rate of erosion from the bed (WC_*) is proportional to the excess of the bed shear above the threshold shear and the size of the bed forms is assumed not to change significantly with bed shear. The derivation of this equation is given in Appendix 6. It will be evident that for practical purposes either form of equation can be used.

An example in which this second method was used to determine infill rates occurred in connection with the proposed sea port at Maplin. Initially it was proposed that the access channel should follow a natural channel to a point opposite the port and would then cut across the estuary at right angles to the main tidal flow. The results showed that the sediment in motion consisted mainly of the finer fractions of the bed material (fine sand, mean size 0.15 mm). The finest size fractions (less than 0.06 mm) which made up the bulk of the sediment flux (but which was present in insignificant proportions in the bed) contributed a negligible amount to the infill. Typically the infill rates were for channels of incised depths 6–10 m and 500 m width and were estimated to be about 1.6–2.4 x 10^6 m^3/annum/km length of channel.

2.5 Alternative procedures for calculating infill

2.5.1 Method of Lamblé

Lamblé (1958) has presented a method for predicting the efficiency of sand traps in sewer mains which Vinje (1960) has applied to the estimation of the infill of dredged cuts in field situations without, however, giving details of the procedure. Essentially, Lamblé starts with the equations (2.3.1)–(2.3.3) for the steady flow of a suspension ($\partial C/\partial t = 0$) and determines approximate solutions applicable to cases in which the initial concentration at entry is an arbitrary function of z and the traps are such that sediment re-entrained from the bottom is negligible ("perfect settling").

We may apply Lamblé's approximation to the settlement of a suspension over a wide dredged channel by noting that if $C_o(z)$ is the initial distribution approaching the channel

$$C_o(z) = (W_o C_*/V_s) \exp(-V_s z/K_z) \qquad ....(2.5.1)$$

which satisfies equations (2.3.1) and (2.3.2) with $\partial C/\partial t = \partial C/\partial x = 0$ and the boundary condition for constant erosion rate ($W_c C_*$) at the bed, viz

$$K_z \, \partial C/\partial z = W_o C_* \qquad z = 0 \qquad ....(2.5.2)$$

Thus denoting the departure of the concentration over the channel from the initial concentration by $\underline{C}$ ie

$$\underline{C} = C - C_o \qquad ....(2.5.3)$$

$\underline{C}$ satisfies equations (2.3.1), (2.3.2). At the bed, if the depth over the channel is constant (h_1), the erosion rate will be constant and equal to a lower value from that at entry ($W_1 C_*$) so that equation (2.3.3) becomes

$$W_1 C_* = -(K_z \, \partial C/\partial z) \qquad z = 0 \qquad ....(2.5.4)$$

and substituting from (2.5.3), (2.5.2) we have

$$-K_z \, \partial \underline{C}/\partial z = (W_1 - W_o) C_* = \text{constant} \qquad z = 0 \qquad ....(2.5.5)$$

Thus with the substitution (2.5.3), the solution of the equations may be written

$$\underline{C} = [(W_1 - W_o)C_*/V_S]\, f(x,z) \qquad ....(2.5.6)$$

in which f(x,z) satisfies (2.3.1), (2.3.2) with

$$-(K_z/V_S)\partial f/\partial z = 1 \qquad z = 0 \qquad ....(2.5.7)$$

and $\underline{C}$ simply scales with $(W_1 - W_o)C_*/V_S$ ....(2.5.8)

Taking the mean value of $\underline{C}$ over the depth and writing

$$f_o(x) = \int_o^h f(x,z)dz/h \qquad ....(2.5.9)$$

we find from (2.5.6)

$$\overline{\underline{C}} = [(W_1 - W_o)C_*/V_S]\, f_o(x) \qquad ....(2.5.10)$$

For perfect settling, $W_1 = 0$ and the mean concentration (departure) is then given by

$$\overline{\underline{C}}_p = [-\, W_o C_*/V_S]\, f_o(x) \qquad ....(2.5.11)$$

For perfect settling with uniform vertical distribution of concentration at entry, Lamblé gives the following approximate expression for the ratio of the mean concentration $\overline{C}_p$ to that at entry $\overline{C}_o$

$$\overline{C}_p/\overline{C}_o = L(x) \qquad ....(2.5.12)$$

with

$$L(x) = \log^{-1}\{-\lambda_1\xi + (\lambda_1 - 1)^2 \log[(\xi + \lambda_1 - 1)/(\lambda_1 - 1)]\} \qquad ....(2.5.13)$$

in which

$$\xi = V_S x/\overline{U}h = V_S x/q \qquad ....(2.5.14)$$

and

$$\lambda_1 = W_1 C_*/V_S \overline{C}_1 \qquad ....(2.5.15)$$

with $\overline{C}_1$ denoting the mean concentration over the vertical at $x = \infty$.

Thus

$$\overline{\underline{C}}_p = \overline{C}_p - \overline{C}_o = [L(x) - 1]\,\overline{C}_o \qquad ....(2.5.16)$$

which gives with (2.5.11) and (2.5.10)

$$\overline{\underline{C}} = (W_1 - W_o)\,[1 - L(x)]\,\overline{C}_o/W_o \qquad ....(2.5.17)$$

We note that when $x = \infty$, $\overline{\underline{C}} = C_1 - \overline{C}_o$ and $L(x) = 0$, so

$$\overline{\underline{C}}/\overline{\underline{C}}_1 = [1 - L(x)] \qquad ....(2.5.18)$$

If the velocity is uniform over the depth, this is equivalent to

$$(S_o - S)/(S_o - S_1) = [1 - L(x)] \qquad ....(2.5.19)$$

and for constant eddy diffusivity ($K_z = K$)

$$\lambda_1 = (V_S h_1/K)\,[1 - \exp(-V_S h_1/K)]^{-1} \qquad ....(2.5.20)$$

Equation (2.5.19) with $V_S h_1/K = 15\, V_S/U_*$, is compared with the analytical solution for this case in Fig 2. The agreement is good for V_S/U_* small but less so for larger values of V_S/U_*. This is expected since the approximation (2.5.13) assumes uniform concentration over the depth in the approach flow, which is clearly invalid when V_S/U_* is large.

Although Lamblé gives a procedure for perfect settling which enables the rate of settlement to be calculated for arbitrary vertical distributions of concentration at entry, this procedure does not appear to be applicable to dredged channels where $\lambda_1 = \lambda_0$ (approximately).

2.5.2 *Method of Gole et al*

Gole et al (1971) suggest that the rate of infill per unit width transverse to the flow is given by the equation

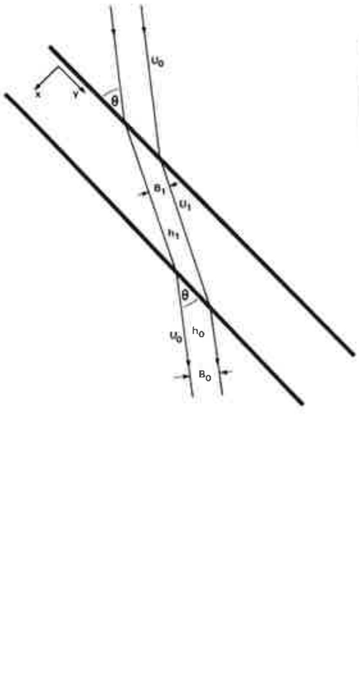
$$\Delta S = k_G C_o V_S X [1-(h_o/h_1)^2]/h_o \qquad ....(2.5.21)$$

in which k_G is a fitting constant with a value of about 0.3 for Indian ports.

The factor $C_o V_S X/h_o$ is evidently assumed to equal the rate of infill which would occur if the velocity over the channel were small enough to allow the sediment to settle completely ($h_1 \gg h_o$). The depth factor in equation (2.5.21) assumes that the carrying capacity of the current is proportional to (mean velocity)^2. Greater exponents would be expected for fine sands.

3 INCLINED CHANNELS

3.1 General remarks



It is often necessary for channels to be dredged at an angle to the dominant tidal flow directions. For ease of navigation, channels are usually straight for considerable distances and in many cases the bed levels may be fairly constant along such stretches. The channel may be subject to similar flow conditions from section to section, ie the velocity and flux of sediment will be independent of the distance (y) along the channel. Small scale experiments indicate that when steady flow occurs over a straight inclined channel the flow lines are refracted down the channel. Thus the width between adjacent stream lines decreases and this can lead to an increase or decrease in velocity over the channel depending on whether or not this width reduction is greater or less than the increase in depth. Generally if the angle between the channel and the flow direction is greater than about 30^o the velocity decreases, but at lower angles provided the ratio of the depth over the channel to that over the sides is sufficiently large, the velocity increases. Hence if S_x is the component of sediment flux normal to the channel (x direction) ie $S_x = U_x h\bar{C}$ (approximately) where U_x = velocity component normal to the channel, averaged over the depth h, the equation of continuity becomes

$$\partial S_x/\partial x + \partial Z/\partial t = 0 \qquad ....(3.1.1)$$

Assuming S_x will be the same at a great distance on each side of the channel, integration of (3.1.1) with respect to x shows, as for a channel normal to the flow, that the cross-sectional area of the channel and adjacent banks will remain constant and equal to that dredged initially, but the shape will change. To estimate the sediment flux it is necessary to determine the velocity changes over the channel.

It will be evident that to satisfy flow continuity over the channel

$$U_x h = q_x = \text{constant} \qquad ....(3.1.2)$$

and, by analogy with equation (2.1.4), the velocity c_x of kinematic waves becomes

$$c_x = (\partial S_x/\partial Z)_{q_x} \text{ constant} \qquad ....(3.1.3)$$

If the flow were inviscid and irrotational, the component of velocity in line with the channel (U_y) would be unchanged, ie

$$U_y = \text{constant} \qquad ....(3.1.4)$$

Thus the flow would be refracted along the channel, the velocity being reduced and the contraction between adjacent stream lines being less than the increase in depth.

If we consider a very wide channel (wide in the direction of the flow) of constant incised depth, and again use subscripts $_0$ and $_1$ to denote quantities upstream of and over the channel, then in steady flow the reduction of velocity over the upstream slope is given by

$$U_0 \{1 - [\cos^2\theta_0 + (\sin^2\theta_0/H)]^{½}\} \qquad ....(3.1.5)$$

where

θ_0 = angle of channel to flow ($\theta_0 = 90^0$ corresponds to a channel normal to the flow) and

$H = h_1/h_0$.

Thus if $h_1 - h_0$ is small the reduction in velocity becomes

$$U_0 \frac{h_1-h_0}{h_0} \sin^2\theta_0 \qquad ....(3.1.6)$$

with a similar increase in velocity over the downstream slope.

The channel will infill over the upstream slope and erode over the downstream slope at the rate

$$B_0 [S_0 - (\frac{B_1}{B_0}) S_1] \qquad ....(3.1.7)$$

in which B_0 is the width between streamlines approaching the channel and B_1 is the width between these streamlines over the channel. Thus from (3.1.1) and (3.1.2)

$$(B_1/B_0) = (\sin^2\theta_0 + H^2\cos^2\theta_0)^{-½} \qquad ....(3.1.8)$$

Comparison of the infill for a wide oblique channel with one at right angles to the flow, with S given by an equation of the form (2.1.9), indicates that the infill is markedly reduced as the angle to the flow becomes smaller. For such channels maintenance could be reduced at the expense of an increase in capital dredging costs.

As previously discussed, when the bulk of the sediment moves in suspension rather than as bed load, the infill will occur over an average distance L and erosion will occur downstream of the channel. Thus for a channel of lesser width the infill rate immediately after dredging will be given by

$$B_0 [S_0 - \frac{B_1}{B_0} S_1] [1 - \exp(-s/L)] \qquad ....(3.1.9)$$

where s is the length of the refracted flow lines over the channel. For a channel of small incised depth, where refraction is small, s = (channel width)/$\sin\theta_0$ with L given by (2.2.11).

This expression must be summed over the year for all tidal stages on neap and spring cycles to obtain the annual rate of infill.

In a balanced tidal situation, after a long time the sides will gradually flatten and the channel become shallower due to the lag effects already discussed. The kinematic wave velocity

Note:

Again, in a balanced tidal situation $C_x = 0$.

$$c_x(\theta_0) = \frac{\partial S_x}{\partial Z} = \frac{U_0 \sin\theta_0}{h_0} [(\frac{dS}{dU})_{U=U_0} \sin^2\theta_0 + (\frac{S}{U_0})\cos^2\theta_0] \qquad ....(3.1.10)$$

and the effective diffusion coefficient due to these effects is approximately

$$D_x(\theta_0) = c(\theta_0) \, . \, L \sin\theta_0 \qquad ....(3.1.11)$$

3.2 Distribution of velocity in depth

So far we have assumed that the velocity is uniform over the depth but generally the velocity is greatest at the surface, decreases slowly over most of the depth and falls sharply to zero near the bed. For example, over a rippled bed with a hydraulic roughness equivalent to 0.05 m in a depth of 15 m, the velocity falls from 40% of its surface value to zero in the bottom 10% of the depth. Engelund (1970) has found that the velocity outside a thin bottom layer in steady parallel flow conforms to that which would be expected if a fictitious "slip velocity" (U_{bo}) is introduced at the bed and the eddy viscosity (K_z) is assumed constant throughout the depth. According to Fredsoe (1974) a good approximation to the velocity at height z above the bed is

$$U_z = U_{bo} \cos[\beta(1 - z/h_o)] / \cos\beta \qquad ....(3.2.1)$$

in which β is given by

$$\beta^2 = 14\, U_{*o} \cos\beta / U_{bo} \qquad ....(3.2.2)$$

and the eddy viscosity

$$K_z = U_{*o} h_o / 13 \qquad ....(3.2.3)$$

Evidently from (3.2.1) the surface velocity is given by $U_{bo}/\cos\beta$ and $\beta = 0$ corresponds to uniform velocity over the depth.

Fredsoe (1974) using an inviscid form of the flow equations with vorticity present has calculated the perturbed flow, including wave disturbances, over an inclined channel. The deviation of the bed stream lines is greater than those at the surface because the pressure distribution in depth is hydrostatic so that changes in water surface elevation must be balanced at all levels by equal centrifugal forces U^2/r, where $1/r$ represents the stream line curvature. Hence r will decrease and the flow be deflected to a greater extent near the bed. For a shallow channel of cosine form inclined at large angles to the flow ($\theta_o = 45–60^o$), the lateral deviations of the bed and surface stream lines agreed with those calculated by Fredsoe. However, the calculations are cumbrous, although they can produce an approximate solution for a shallow channel with a flat bed at low Froude numbers (where surface disturbances are minimal) which correspond to the case of a navigation channel in deep water. In this case, neglecting vertical velocity components (w) the rise in water surface (η) over the channel is constant and given by

$$\eta = U^2_{bo} \left(\frac{h_1 - h_o}{h_o}\right) \sin^2\theta_o\, M(\beta) \quad \text{(approximately)} \qquad ....(3.2.4)$$

with

$$M(\beta) = 2\beta / [\cos\beta \,.\, \log\{(1 + \sin\beta)/(1 - \sin\beta)\}] \qquad ....(3.2.5)$$

and the velocity over the channel is independent of x,y. The fall of velocity over the channel at the bed and surface is given by

$$\Delta U_b = U_{bo} \left(\frac{h_1 - h_o}{h_o}\right) \sin^2\theta_o\, M(\beta) \qquad ....(3.2.6)$$

$$\Delta U_s = U_{so} \left(\frac{h_1 - h_o}{h_o}\right) \sin^2\theta_o\, M(\beta) \,.\, \cos^2\beta \qquad ....(3.2.7)$$

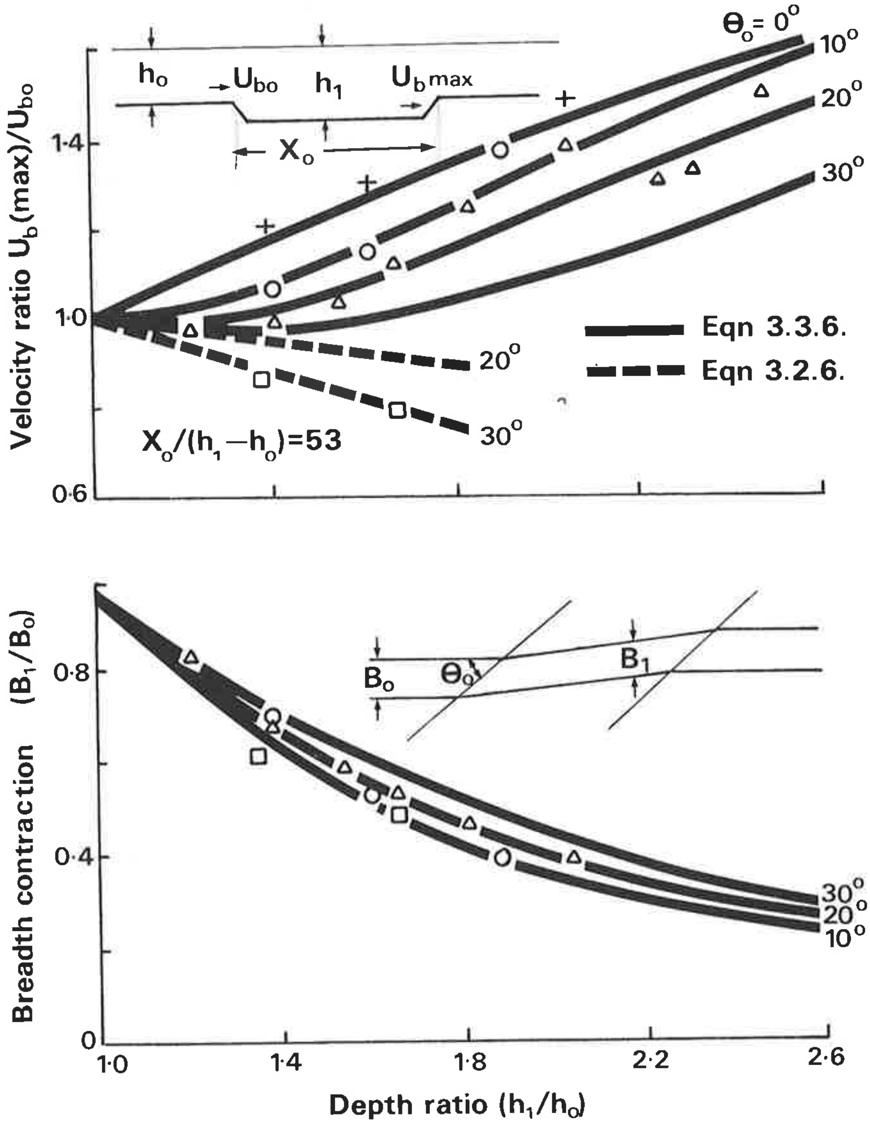
The change in bed velocity ($\Delta U_b/U_{bo}$) for a channel inclined at 30^o to the flow ($\theta_o = 30^o$) from (3.2.6) is in good agreement with experimental values obtained in a laboratory channel (Whillock, 1973) Fig 6. In these experiments the friction velocity $U_* = .05\,\bar{U}$ ($\bar{U}$ = mean velocity over vertical) which, from equations (3.2.1) and (3.2.2), gives a value of $\beta = 0.8$. At lower values of θ_o there are considerable divergences; these are discussed in the next section.

So far, in order to determine infill rates (equation (3.1.7)) it has been assumed that the sediment flux (S) is related to the mean velocity over the section obtained either from extrapolation of laboratory results or from direct field measurements. However, since erosion rates are likely to be more sensitive to the velocity near the bed (U_b) than the mean velocity it would seem more reasonable, in the present application, to relate S to U_b. For a hydraulically rough bed the slip velocity U_b has been defined by Fredsoe as

$$U_b = U_*[8.3 + 2.45 \ln(h/13k_s)] \quad ....(3.2.8)$$

where k_s is the equivalent hydraulic roughness of the bed and h is the depth. Thus assuming the usual logarithmic velocity distribution to apply, U_b is the velocity at h/13. The reduced flux over the channel may then be inferred from (3.2.6), or from the more accurate analysis given by Fredsoe, and hence the infill determined.

Figure 6
Maximum bed velocity ratios and breadth contractions over an inclined channel



3.3 Channels at small inclinations

3.3.1 *Preliminary remarks*

As the angle of the channel to the flow decreases, the flow lines over the channel lengthen and, for wide channels, the reduction of resistance consequent on the increased depth becomes a major factor in the determination of the velocity changes which occur. The essentially inviscid treatments described in sections 3.1 and 3.2 no longer apply without modification, in particular the assumption that the tangential momentum is constant over the channel (equation (3.1.2)) is no longer valid.

If, for the moment, we ignore momentum changes, these being small in comparison with friction changes over the bed, then we can say that the water surface slope due to friction along the channel will be equal to that over the sides. Adopting, for simplicity, a Chezy friction law coefficient (C_f) relating slope and mean velocity, equation (3.1.2) would then be replaced by

$$\frac{U_o^2}{C_{fo} h_o} \cos\theta_o = \frac{U_1^2}{C_{f_1} h_1} \cos\theta_1 \qquad \text{....(3.3.1)}$$

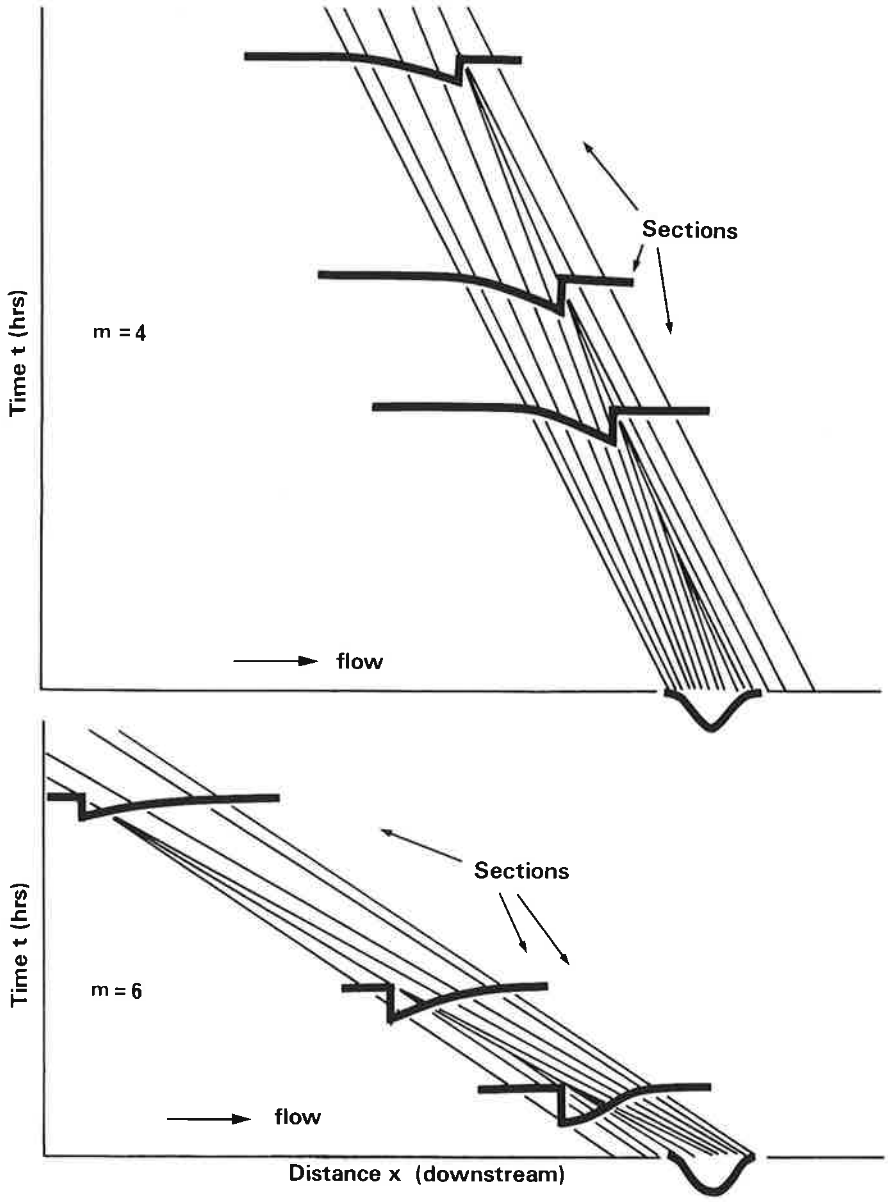
U_o, U_1 being mean velocities over the depth and θ_o, θ_1, the angles of the flow lines to the channel in the approach and cross-channel flow respectively.

Combining (3.3.1) with the continuity equation (3.1.2) it is easy to show that for $C_{fo} = C_{f_1}$ when $\theta_1 < 45^o$, the velocity over the channel is greater than the approach velocity for all values of (h_1/h_o). It will also be evident from (3.3.1) that for small inclinations,

$$U_1 = U_o (h_1/h_o)^{1/2} \qquad \text{....(3.3.2)}$$

so the breadth contraction of the flow lines is given by $B_1 = B_o(U_o/U_1)^3$. Thus if the sediment flux is related to the velocity by a simple power law (2.1.9), the flux between adjacent stream lines over the channel, ($B_1 S_1 = B_o S_o (U_1/U_o)^{m-3}$) will be greater than the incident flux ($B_o S_o$) when $m > 3$.

Figure 7
Upstream movement of slightly inclined channel



Over beds consisting of fine sand, values of m considerably greater than 3 are common, so we would expect there to be an increase in sediment transported over the upstream edge of the channel and a corresponding decrease over the downstream edge. Thus the channel would migrate upstream, the upstream edge eroding and the downstream edge accreting.

We note that the kinematic wave velocity given by equation (3.1.3) can be written, with $h = h_o - Z$,

$$c_x = -\ \partial(SU_x/U)/\partial h \qquad \text{....(3.3.3)}$$

and substituting for S from (2.1.9), with (3.1.2),

$$c_x = -\ k_s q_x \partial(U^{(m-1)}/h)/\partial h \qquad \text{....(3.3.4)}$$

If the variation of U with depth across the channel is given by (3.3.2), viz $U = U_o(h/h_o)^{1/2}$ it is not difficult to show that

$$c_x = -\ (S_{ox}/h_o)[(m-3)/2](h/h_o)^{(m-5)/2} \qquad \text{....(3.3.5)}$$

confirming that when $m > 3$, the channel moves upstream. It will also be noted that when $m = 5$, the channel moves upstream without change of form but for other values of m a discontinuity develops (kinematic shock) on the upstream or downstream side of the channel depending on whether m is greater or less than 5, ie whether c_x increases or decreases with h. The formation of the shock is sketched in Fig 7.

3.3.2 *Measured velocity changes in steady flow*

Usually the velocity changes over channels with relatively small incised depths are too small to be measured reliably in the field and recourse must be made to laboratory studies. Experiments have been made with steady flow over a trapezoidal channel with side slopes 1:12, bottom width about 20 times the depth covering the ranges $1.2 < H < 2$ and inclinations $\theta_o = 0^o, 10^o, 20^o$ and 30^o (Whillock, 1973). At the largest inclination and at $\theta_o = 20^o$ for (h_1/h_o) less than about 1.35, the bed velocities were fairly constant across the channel and lower than those in the approach flow; in this range, as already mentioned, the fall in bed velocity was in reasonable agreement with equation (3.2.6). The refraction of the streamlines across the channel is shown in Photo 1.

At $\theta_o = 20^o$ with higher values of (h_1/h_o) an initial fall in velocity over the upstream slope again occurred but was followed by an acceleration across the channel, a peak value of the bed velocity being attained at or slightly downstream of the downstream edge of the channel (Fig 8). Above a certain minimum value, $h_1/h_o = 1.55$, the average bed velocity in the accelerated part was greater than in the approach flow; this would evidently correspond to an initial eroding condition. Beyond the downstream edge the velocity fell gradually usually reaching the upstream value in a distance equal to about twice the channel width. Similar behaviour of the flow was exhibited at $\theta_o = 10^o$; here, however, the minimum depth ratio for accelerated flow was less $(h_1/h_o = 1.39)$ as might be expected. The maximum bed velocity ratios which occurred at the downstream edge of the channel are shown in Fig 6.

It is of course of great importance to be able to predict velocities. Fortunately, O'Connor and Lean (1977) have shown that the velocity changes across the channel and downstream can be fairly well predicted from the equation for the mean tangential momentum:

$$U_x\, d\, U_y/dx = \left(\frac{U_o^{\,2}}{C_{fo}^2 h_o} \cos\theta_o - \frac{U^2}{C_{f_1}^2 h_1} \cos\theta\right) \qquad \text{....(3.3.6)}$$

with

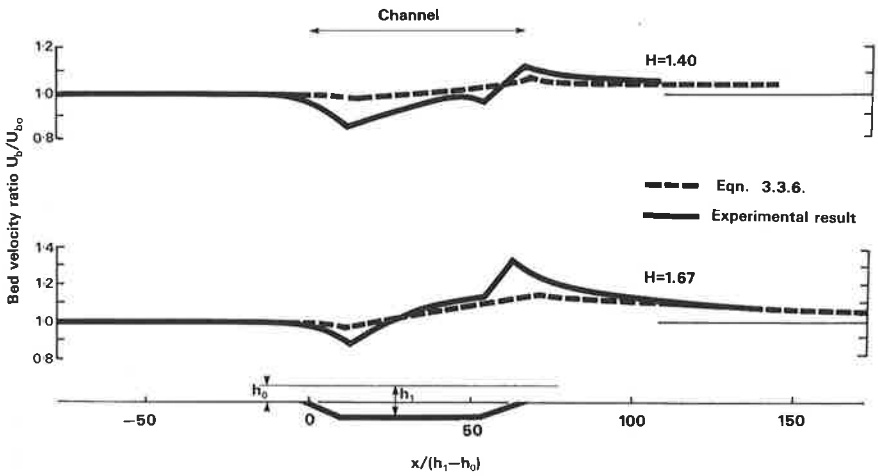
$$U^2 = U_x^{\,2} + U_y^{\,2} \qquad \text{....(3.3.7)}$$

and

U_x given by continuity equation (3.1.2), (ie $U_0 h_0 \sin\theta_0 / h_1$ over channel and $U_0 \sin\theta_0$ upstream and downstream).

This assumes that no change of tangential momentum occurs over the upstream and downstream slopes, ie equation (3.1.2) holds for flow over the slopes. From (3.3.6) it would seem that across the channel U increases and would eventually reach U_1 (given by equation (3.3.1)) for a very wide channel ($x \to \infty$).

Figure 8
Bed velocity distributions, channel at 20° to flow



Equation (3.3.6) assumes that the velocity is uniform in depth. However, in the experiments, as predicted by Fredsoe, the flow at the bed was refracted more than the surface flow so that the local bed shear was at an angle to the mean flow. So far an analysis which takes account of this factor is not available.

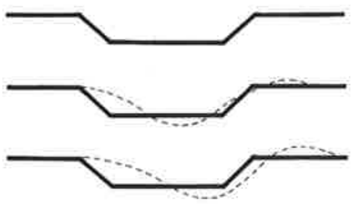
In Fredsoe's experiments, the behaviour of particles suspended in the flow was observed. The size and density of the particles was adjusted so that the ratio of their fall velocity to the flow velocity was approximately the same as for fine sand in typical tidal currents. However, since the bed of the channel and banks were moulded in smooth concrete, only the pattern of deposition and not erosion of the particles could be observed. Usually the quantity of sediment in circulation was adjusted to prevent continuous deposition on the bed upstream of the channel.

Deposition always occurred on the upstream slope of the channel, where velocities lower than those upstream had been shown to occur. The deposition patterns generally reflected the velocity changes quite closely. When $\theta_0 = 20^\circ$ and $h_1/h_0 = 1.2$, deposition was fairly general over the bed of the channel. At $h_1/h_0 = 1.6$ (when the average velocity over the channel was slightly greater than that upstream) deposition took place mainly on the upstream slope and only spread into the channel as this deposit grew. This also happened when $\theta_0 = 10^\circ$ and $h_1/h_0 = 1.4$. In all cases the downstream slope was swept clear by the flow and it seems likely that this slope would have eroded had it been composed of mobile material. This is perhaps most graphically illustrated in Photo 2. This was taken at the end of an experiment with the 10° channel in which the approach velocity had been reduced to allow the sediment to form incipient ripples over the bed upstream. The sparseness of the deposits beyond the centre line of the channel compared with that in the approach flow and the lack of deposits over the downstream slope and for a short distance downstream illustrates the likelihood of erosion in these areas; it is probable that if the bed had been erodible the channel would initially have migrated downstream.

3.3.3 *Sediment flux changes over the channel*

It will be evident from the preceding section that, in unidirectional flow, channels at small angles to the flow will migrate downstream, infilling on the upstream side and eroding over the downstream side. For h_1/h_0 less that the critical value the average velocity across the channel will be less than the approach velocity so that to maintain the channel in a given position sediment must be removed at rates given approximately by equation (3.1.7) with $B_1 S_1$ appropriate to the breadth of adjacent stream lines ($B_1 = B_0 U_0 h_0 / U_1 h_1$) having an average velocity (U_1) over the channel. In a tidal flow, this infill rate at each phase of the tide must be integrated over a year to give the annual infill.

For h_1/h_0 greater than the critical value, the average velocity over the channel is greater than the approach velocity but over its upstream portion it is less than the approach value. In unidirectional flow, to maintain the channel stationary the sediment deposited on this side must be removed. A somewhat crude measure of the infill may still be given by equation (3.1.7) but with the distance (s) now referring to the distance over which the velocity is less than the approach velocity and B_1, S_1 corresponding to the average breadth of the stream lines and sediment flux over this distance. However, over the downstream part of the channel ($x > s$) where the velocity is greater than the approach velocity, erosion would occur and the depth increase. Thus it would appear that initially the channel would be unstable, tending to become deeper on the downstream side as it migrates downstream. However, the sediment eroded from the channel will deposit in the decelerating flow downstream, so raising the bank level and decrease the velocity (equation (3.3.1)). Further, the rise in bank level and increased thickness of sediment will tend to slow down the rate at which the downstream slope moves downstream (and may even reverse it, Fig 7) causing the channel to narrow, so reducing the velocity increment over the channel and the tendency for further erosion. Also, it is likely that in the long term (unless the channel is successively trimmed back) the upstream slope would steepen due to the formation of a kinematic shock, causing flow separation, reduced bed velocities and increased deposition over the upstream part of the channel.



For h_1/h_0 greater than the minimum, similar effects might be expected in an unbalanced tidal flow with a pronounced flux bias in one direction. A small infill would again be expected on the dominant drift side since the deposition on this side – assuming the flux is simply velocity dependent – will not be entirely removed by the weaker scour by flow in the reverse direction. However, on the downdrift side, the channel is likely to erode initially and continue to do so until the bank level rises significantly. In a balanced or nearly balanced tidal situation it seems likely that the channel will erode on each side, the eroded sediment being deposited on each bank. In time the rise in bank levels will tend to refract the flow towards the normal, reducing velocity over the channel and hence erosion.

Since the critical value of h_1/h_0 decreases with angle (θ_0) even channels with small incised depths at very small angles to balanced tidal flows will be subject to erosion provided they are sufficiently wide to allow the velocity increase, due to the reduced resistances, to become effective. However, other factors, viz gravity and wave action, may then become predominant in causing infill. These factors are discussed in sections 4 and 5. In addition, it is well known that natural channels in line with the flow develop alternate shoals and deeps. Instabilities of this type may also be present along dredged channels. Besides creating additional friction losses and lower velocities over the channel, these instabilities may well cause the channel to meander, giving rise to changes in the flow directions over the channel and banks.

3.3.4 *Tidal flow in aligned channels*

So far we have assumed that the flow over a channel at a particular tidal phase is the same as that in the steady state. However, in a tidal flow only part of the head drop is expended in overcoming the resistance of the channel, the rest being used to accelerate the flow. In order to obtain an approximate estimate of the velocity changes on this account let us consider the simple case of a straight channel dredged in line with the flow along an estuary. We assume that (i) the effect of the channel on the flow external to it is negligible, ie the tidal elevations and currents outside the channel are the same as before the channel was dredged, and (ii) the water surface elevation over the channel is the same as over the banks. (Strictly to satisfy this condition and flow continuity it is necessary to include traverse currents into and out of the channel but these currents are assumed small enough to be ignored).

The linearised equations of motion for the currents over the bank and channel are respectively

$$-g\,\partial\eta/\partial x = \partial U_o/\partial t + f_o U_o \qquad \text{....(3.3.8)}$$

$$-g\,\partial\eta/\partial x = \partial U_1/\partial t + f_1 U_1 \qquad \text{....(3.3.9)}$$

where η is the surface elevation above mean tide level
and the quadratic friction term is approximately linearised by putting

$$f_o = 8\,C_{fo}|U_o|/3\pi\,h_o \qquad \text{....(3.3.10)}$$

$$f_1 = 8\,C_{f_1}|U_1|/3\pi\,h_1 \qquad \text{....(3.3.11)}$$

in which C_{fo}, C_{f_1} denote the friction factors
and $|U_o|$, $|U_1|$ are the amplitudes of the tidal currents over the banks and channel.

In most estuaries the phase difference between the surface slope ($\partial\eta/\partial x$) and currents differ only slightly from a quarter of the tidal period, depending on the shape and overall resistance of the estuary (Hunt, 1964).

Thus we may write,

$$-g\,\partial\eta/\partial x = g|\eta_x|\cos\sigma t \qquad \text{....(3.3.12)}$$

$$U_o = |U_o|\sin(\sigma t+\epsilon_o) \qquad \text{....(3.3.13)}$$

$$U_1 = |U_1|\sin(\sigma t+\epsilon_1) \qquad \text{....(3.3.14)}$$

in which

$$\sigma = 2\pi/T,\ T \text{ denotes tidal period.} \qquad \text{....(3.3.15)}$$

Substituting in (3.3.8), (3.3.9), we find

$$g|\eta_x| = |U_o|(\sigma^2+f_o^{\,2})^{1/2} = |U_1|(\sigma^2+f_1^{\,2})^{1/2} \qquad \text{....(3.3.16)}$$

in which f_o, f_1 are given by (3.3.10), (3.3.11). We note that when $\sigma = 0$ (steady flow) and $C_{fo} = C_{f_1}$, $U_1/U_o = (h_1/h_o)^{1/2}$, (equation 3.3.2).

As an example suppose $|U_o| = 1$ m/s, $h_o = 10$ m, $C_f = 0.0025$ and $h_1 = 20$ m; for $\sigma = 1.45 \times 10^{-4}\ s^{-1}$, we find from (3.3.16) that $|U_1| = 1.29$ m/s, which is appreciably less than for steady flow (1.41 m/s). For slightly inclined channels of large width where bed friction effects are important, currents in steady flow experiments will exceed those in tidal flows and the critical values of the ratio h_1/h_o for an initially eroding condition will be greater.

4 EFFECTS OF GRAVITY

4.1 Direct gravity infill

In some situations navigation channels follow the course of natural channels and are, therefore, nearly in line with the dominant flow direction. Clear water experiments (Whillock 1973) have shown that in wide channels of constant incised depth in line with the current, the velocity is greater than over the sides, approximately in proportion to the square root of the depth (equation (3.3.2)). Secondary circulations over the banks were too weak to be detected, the flow over the channel being directed parallel to the channel; dye plumes adjusted in position to be along the channel edges showed no pronounced tendency to diffuse more strongly into the channel than over the banks (Photo 3).

However, due to gravity, sediment particles resting on the banks will be subject to a force tending to cause them to move down the slope resulting in infill. It is easy to show that the resultant force acting on particles moving in contact with the bed will be inclined at an angle ψ to the channel given by

$$\tan\psi = \frac{\tan\alpha_s}{\tan\lambda_d} = -\frac{1}{\tan\lambda_d}\frac{\partial Z}{\partial x} \qquad \text{....(4.1.1)}$$

in which α_s is the inclination of the bank slope, $\tan\lambda_d$ is the coefficient of dynamic friction (x is again the co-ordinate perpendicular to the channel direction and Z is bed elevation above a horizontal datum).

Since the particles will be transported in the direction of the resultant force, it seems reasonable to assume that the transverse component of the bed sediment transport in the x direction is given by

$$S_{bx} = S_{bo}\tan\psi = -\frac{S_{bo}}{\tan\lambda_d}\frac{\partial Z}{\partial x} \qquad \text{....(4.1.2)}$$

in which S_{bo} is the transport per unit width of sediment in contact with the bed in the longitudinal direction and the negative sign is introduced to take account of the fact that S_{bx} is negative when

$\partial Z/\partial x > 0$.

Since there is no change of transport in the y direction (along the channel), the sediment continuity equation becomes

$$\frac{\partial S_{bx}}{\partial x} + \frac{\partial Z}{\partial t} = 0, \qquad \text{....(4.1.3)}$$

S_{bx} being measured in terms of the volume of compacted bed sediment in movement for unit length along the channel per second. Substituting for S_{bx} we find

$$\frac{\partial Z}{\partial t} = \frac{1}{\tan\lambda_d}\frac{\partial}{\partial x}\left(S_{bo}\frac{\partial Z}{\partial x}\right) \qquad \text{....(4.1.4)}$$

Provided the variation of Z is small compared with the water depth, S_{bo} can be taken as approximately constant and equation (4.1.4) then reduces to the standard diffusion equation

$$\frac{\partial Z}{\partial t} = D_g\frac{\partial^2 Z}{\partial x^2} \qquad \text{....(4.1.5)}$$

with the diffusion coefficient D_g given by

$$D_g = S_{bo}/\tan\lambda_d \qquad \text{....(4.1.6)}$$

With this value of the diffusion coefficient the degradation of the channel section will be similar to that shown in Fig 4. Fredsoe, 1976, to whom the previous analysis is due, has shown that for a channel with initial side slopes α_s and incised depth (h_1-h_o), the volume rate of infill per unit length of channel at time t after dredging is given by

$$(h_1-h_o)\frac{\sqrt{D_g}}{\sqrt{\pi}}\frac{1}{(t+t_o)^{1/2}} \qquad ....(4.1.7)$$

with

$$t_o = \frac{\pi}{64}\frac{(h_1-h_o)^2}{D_g(\tan\alpha_s)^2} \qquad ....(4.1.8)$$

Fair agreement was found between measured and theoretical sections in laboratory experiments on the evolution of an in line channel of initially trapezoidal form (Fredsoe 1976). These experiments were performed with coarse sand (1.2 mm mean diameter) which travelled entirely along the bed. The bed transport could thus be identified with the volume transport associated with migration of sand dunes over the bed. However, some difficulty would be experienced in estimating S_{bo} with more commonly occurring fine sands and silts which travel mainly in suspension. In suspension, the action of gravity will have no transverse component and the bed dunes move partly by differential erosion, and deposition of particles directly from suspension, as well as by rolling and sliding along the surface of the dunes. For such sediments it would seem reasonable to calculate S_{bo} by extrapolation to the finer grain size of one of the existing formulae describing the transport of coarser particles which travel solely in this way, eg Meyer–Peter and Muller, 1948.

Fredsoe suggests that in a tidal situation the above analysis is still approximately valid if S_{bo} is taken to be equal to the mean value of S_{bo} over the tidal period. Further, if the channel is very slightly inclined to the flow direction, the effect of gravity will be to enhance the transverse component of transport on the upstream side and diminish it by an equal amount on the downstream side so that the total infill will be the same as for a channel aligned with the flow. This argument takes no account of the changes in S_{bo} due to change in bed current over the channel discussed in section 3.3.3. However, it is clear that the effect of gravity (equation (4.1.2)) is important, tending to increase infill and reduce erosion.

4.2 Indirect gravity effects

The previous analysis refers to particles which are moving in close proximity to the bed such that their weight is directly taken by the bed. Fine particles (sand and silt) travel mainly in suspension and their weight is balanced by an increase in the hydrostatic pressure gradient (in the same way as, for example, a liquid of heterogeneous density). In a channel in line with the flow, due to the higher concentration of particles close to the bed, there will generally be a greater pressure at a particular bed level on the banks than at the same elevation over the channel. On the other hand, near the surface, the concentration and pressure will be greater over the channel than over the banks. These horizontal pressure gradients will tend to set up currents towards the channel at the bed and away from the channel at the surface. From continuity, the net inflow will be zero but since the water will be more heavily laden with solids near the bed there will be a greater flux of suspended solids into the channel than out of it. Some of this sediment will deposit leading to infill. So far such effects have not been quantified but in order to estimate the order of magnitude of the inflow of suspended sediment let us assume that the suspension can be idealised as a layer of uniform thickness, (d_o) of uniform concentration on each side of the channel. The slopes of the banks are generally steep enough to allow free flow under gravity of suspension laterally from the sides into the channel. The flow would be similar to that which would occur over a broad crested weir, the critical depth being given by

$$d_c = \frac{2}{3} d_o \qquad ....(4.2.1)$$

and the critical velocity

$$v_c = \sqrt{g' d_c} \qquad ....(4.2.2)$$

where

$$g' = g(\rho' - \rho)/\rho \qquad ....(4.2.3)$$

is the effective gravity acting on the suspension,

ρ' = density of the suspension

and

ρ = density of sea water ....(4.2.4)

Thus the volume rate of particles convected into the channel under gravity per unit length of channel

$$v_c d_c (\rho' - \rho)/\sigma \qquad ....(4.2.5)$$

is equivalent to a volume rate of compacted sand of

$$\frac{1}{C_*} \frac{\rho}{\sigma} \left(\frac{8g}{27}\right)^{1/2} \left[d_o \frac{(\rho' - \rho)}{\rho}\right]^{3/2} \qquad ....(4.2.6)$$

If $\bar{C}$ is the volume of particles per unit volume averaged over the depth, the weight of particles per unit area over the bed is

$$d_o(\rho' - \rho)g = h\bar{C}(\sigma - \rho) \qquad ....(4.2.7)$$

so that the rate at which particles are convected into unit length of the channel in terms of volume of compacted sand is

$$\frac{\bar{C}^{3/2}}{C_*} \left(\frac{8g}{27}\right)^{1/2} h^{3/2} \left(\frac{\sigma - \rho}{\rho}\right)^{3/2} \frac{\rho}{\sigma} \qquad ....(4.2.8)$$

As an example, along a channel in the middle of the Thames estuary where a navigation channel was proposed, the mean concentration of particles greater than 40μ diameter on a spring tide was 32 ppm by weight ($\bar{C}$ = 12 x 10^{-6}) and the depth at mean tide level was 15 m. This concentration should be reduced to 7.5 ppm. The calculated volume rate from (4.2.8) with C_* = 0.61 was 10^5 m^3/km/annum or double this quantity for both sides of the channel. If the channel were slightly inclined to the flow, the gravity current would be reduced on one side but reinforced on the other so that the gravity inflow will be the same.

There is little doubt that this volume rate is extreme since it has been assumed that the incised depth is large enough for critical flow to develop and that all the inflow is deposited in the channel. It was for this reason and because of its very small settling velocity that the size fraction less than 40μ was ignored in the case discussed. If particles in the size range 40–60μ were ignored for the same reason, the inflow would have been reduced by a factor of about 50%. Nevertheless the magnitudes involved suggest that infill of suspended solids due to induced gravity currents could be important.

It should also be noted that transfer of suspended particles into a channel can also occur by lateral diffusion in the absence of a gravity current. The rate is given by

$$K_x \frac{\partial C}{\partial x}$$

where K_x is the coefficient of diffusion in the lateral direction. The coefficient K_x will vary throughout the depth and the concentration gradient will change sign, the lateral transfer being directed away from the channel at the surface and towards the channel near the bed.

Where the currents in the upper layers are negligible Bagnold (1962) has argued from energy consideration that a suspension can maintain itself indefinitely in deep water over a gently sloping bed (angle α_S). The suspension will flow in a layer of uniform thickness d_B with mean velocity $\bar{u}_B$ given by

$$[(\sigma-\rho)g\bar{C}\, d_B/\bar{u}_B{}^2]\, \log_{10}{}^2(13.2\, d_B/k_r) \sin [\alpha_S - (V_S/\bar{u}_B)] = 0.045 \quad ....(4.2.9)$$

in which k_r is the hydraulic roughness of the boundary, provided the flow in the layer is turbulent.

Gravity currents caused by density differences resulting from the presence of suspended particles could evidently be an important contributory factor to the siltation of channels in areas where currents are very weak.

5 CHANNEL INFILL WITH BEDS OF COHESIVE SEDIMENTS

5.1 Erosion and depositional properties of cohesive sediments

In some areas, the tidal velocities are not sufficient to maintain the coarser fractions of sediment in motion and the bed may consist of fine particles which have been deposited from suspension. Such beds often contain silt (of order of 0.06 mm diameter) with a varying admixture of clay (less than 0.006 mm diameter) and differ from beds of fine sands in possessing distinctive cohesive properties. The primary clay particles are so fine that in a completely dispersed suspension they would be carried by even the weakest currents and their settling rate would be extremely slow, but when the concentration is large enough for the probability of collision to become significant, the particles can cohere to form clusters or flocs which have a larger rate of settlement than the individual particles. The coherence depends on the size grading, mineralogical composition and organic content of the particles as well as the salinity and temperatures of the sea water. For given particles, the size of the flocs (and their rates of settlement) besides depending on their concentration will also depend on the turbulence in the water. Relatively mild turbulence will increase flocculation by increasing the probability of particles colliding. Severe turbulence, on the other hand, will reduce flocculation because although the probability of collision is increased, the higher internal fluid shear rates will cause the floc bonds to be broken. Since these factors cannot be adequately reproduced in the laboratory, rates of settlement are best determined in the field, using instruments which allow the settling velocity of silt flocs to be measured in their natural state (Owen, 1971).

Fine sediments with cohesive properties also behave differently from incohesive sediments with regard to their erosional and depositional properties. Whereas for cohesionless particles the threshold shear (or velocity) required to move the bed sediment is the same for both erosion and deposition, with cohesive sediments the shear at which the bed starts to erode (τ_e) is greater than that required to allow deposition (τ_d).

For bed shears (τ) above τ_e, Partheniades (1962) has suggested that a linear relation exists between the rate of erosion and the shear stress, viz:

$$E = dm/dt = M(\tau-\tau_e)/\tau_e \; , \quad \tau > \tau_e \; , \; M = \text{constant} \quad ....(5.1.1)$$

Here dm/dt denotes the mass rate at which sediment is eroded per unit area of the bed material. (The bed shear is related to the friction velocity V_* by $V_* = (\tau/\rho)^{1/2}$, where ρ is the bulk density of the suspension.)

When silt is deposited on the bed it immediately begins to consolidate, gradually increasing in density at all levels over a long period of time unless the material is subsequently removed by erosion. Migniot (1968) and Owen (1970) have shown that the consolidation of deposited silt proceeds more rapidly for thin than for thick layers giving higher densities both during deposition and for several hours thereafter. The critical shear stress (τ_e) needed for erosion, which is partly a function of the density of the deposit, also increases rapidly as consolidation proceeds and appears to be also a function of the Bingham shear strength. This offers the possibility of determining τ_e from shear tests on "undisturbed" silt samples from the bed.

Silt eroded from the bed appears to be transported entirely in suspension, its distribution in depth being determined as for non-cohesive sediments with the settling velocity appropriate to the size of silt flocs as governed by the concentration and shear distribution. Concentration can be determined by the Owen tube method (Owen 1971) and the shear distribution from the velocity distribution over a section.

When the shear falls below τ_d, deposition from suspension occurs at a rate given by Krone (1962) viz

$$P = dm/dt = C_b V_S (\tau_d - \tau)/\tau_d \quad , \quad \tau < \tau_d \qquad ....(5.1.2)$$

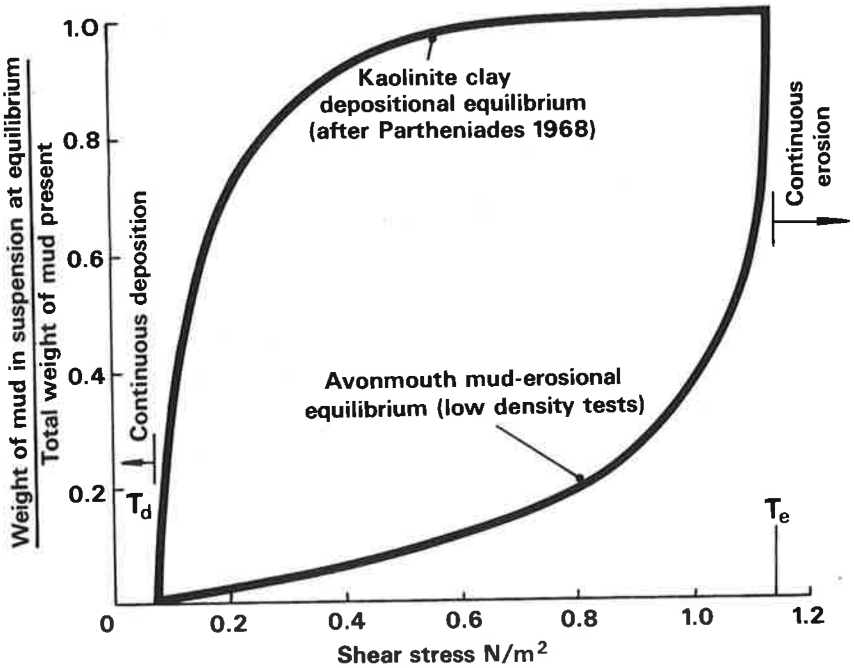
where $C_b V_S$ are the concentration and settling velocity of the silt near the bed.

The fall velocity of cohesive materials is a function of the floc size and therefore depends on the concentration, eg field tests on Thames mud showed that during spring tides with $50 < C < 3000$ ppm the fall velocity was given by

$$V_S = 2 \times 10^{-3} \text{ C mm/s (approximately)} \qquad ....(5.1.3)$$

At shears between τ_e and τ_d it appears that neither erosion nor deposition occurs and the bed plays no active part in the sediment movement.

Figure 9
Comparison of erosional and depositional equilibrium conditions



Almost any concentration of suspended sediment is stable between these limits. According to Mehta and Partheniades, 1974 in the "depositional" phase (τ decreasing, Fig 9) the concentration depends only on the amount of sediment initially present in suspension in the form of flocs which are too weakly bonded to attach themselves to the bed. Similarly according to recent work by Owen, 1975, in the "eroding" phase (τ increasing) the concentration of suspended silt present depends only on the proportion of silt that will ultimately be eroded. These remarks apply to steady state (equilibrium) conditions, which, in the depositional phase for example, is attained after τ has been decreased from a higher value and then held constant indefinitely, ie a step decrease in τ. The delay constant which is a measure of the rate at which the concentration assumes the new value (and the bed accretes) ranges from several minutes to several hours depending on the nature of the sediment. Methods have not yet been developed for quantifying the changes which occur in time dependent situations when $\tau_d < \tau < \tau_e$ and in tidal flow situations it is usual to assume that the net erosion and deposition is zero over this range.

5.2 Deposition in tidal channels

In the outer parts of estuaries where the concentration of fine silt particles is small the fall velocity of the sediment flocs can be so low (usually less than about 2 mm/s) that the suspended sediment concentration is nearly uniform throughout the depth. The change in concentration throughout a tidal period is also small and varies only over longer times, from spring to neap tides for example. Thus, contrary to the case of coarser sediment fractions, changes in sediment flux due to changes in concentration over the depth (equation (2.3.2)) can be ignored and changes in bed level (equation (2.3.4)) are given by the direct imbalance of erosion and deposition, with C taken as constant or changing only slowly with time. The amount of material eroded when $\tau > \tau_e$ and deposited when $\tau > \tau_d$ can then be determined by integrating (5.1.1) and (5.1.2) over the appropriate stages of the tidal flow. Over a dredged channel, the velocities are reduced (at a large angle to the flow) thus allowing a longer period during the tide for deposition and shorter times for erosion than outside the channel. Thus the rate at which siltation occurs in the channel is given by the difference between the net deposition outside and that over the channel.

Before equations (5.1.1), (5.1.2) can be used for estimating siltation, values of C, V_S and $\tau = \rho U_*^2$ need to be obtained from field measurements. Ideally the threshold values of τ_e, τ_d, and the erosion constant M at the field density of the bed deposits should be determined by tests in an experimental flume, but often such information is lacking and one must have recourse to comparison with estuarial muds in other locations for which these parameters are known. This was necessary for example in the case of nine channels in the estuary of the River Plate, which were subject to siltation from the suspended silt originating in the tributary rivers (Harrison and Owen, 1971). In this case the dredging rates to maintain the depths in the channels were known and this allowed a comparison between the rate given by equations (5.1.1) and (5.1.2) and that observed. Unfortunately, sufficient information was not available to allow the value of C . V_S to be determined. Instead a value was chosen to give agreement with the two most heavily silting channels and comparison made between the calculated and known dredging rates in the other channels using the same value. The agreement was close enough to justify the form, if not the actual values, of the theoretical predictions. Extrapolation allowed the increased siltation rates due to channel deepening to be estimated.

5.3 Fluid mud layers

It may be noted that in some estuaries, high concentrations of mud in suspension, with accompanying high settling rates and thick deposits of mud, exist along reaches which consequently may need to be dredged to maintain navigation depths. In the Thames, for example, accumulations of mud occur in the adjacent Halfway, Barking and Woolwich "mud" reaches although both upstream and downstream the bed is relatively hard. These accumulations appear to be due to the deposition of mud flocs convected into the region from both upstream and downstream as a result of drifts near the bed. These drifts can be set up by density or gravity horizontal gradients induced by differences in salinity between the fresh water flow and sea water. In estuaries in which the tidal currents are weak, there is a well marked interface between the fresh water and underlying salt water (the so-called "salt water wedge"), the shear at the interface causing a seaward flow of salt water at the interface and a landward drift near the bed. At the apex of the wedge, this landward drift is just balanced by the seaward drift of the river flow.

When the tidal currents are stronger, the density is more nearly homogeneous through the depth due to mixing but the drifts at the bed remain. In order to calculate the changes in suspended concentration and the pattern of siltation in such cases, Odd and Owen (1972) have used a two layer model to schematize the flow in the Thames estuary. They assumed that bed drift was confined to a relatively thin layer near the bed and that the bed had constant thickness (d_L). The flow in this layer is assumed to be induced by the interfacial shear (τ_i) between the layers, the mean longitudinal density distribution $(\partial\bar{\rho}/\partial x)/\bar{\rho}$ and mean depth variation $(\partial h/\partial x)$ along the estuary. It is not difficult to show that in such a model if $d_L \ll h$ (h denotes the depth), the position of zero drift in the lower layer occurs at a position where the tidal current amplitude in the upper layer $(|U_U^{(1)}|)$ satisfies

$$g(\partial\bar{\rho}/\partial x)/2\bar{\rho} = (4/\pi)\, k_i\, |U_U^{(1)}|\, Q_R/b\, h^2\, d_L \qquad ....(5.3.1)$$

in which

Q_R	= river discharge
b,h	= breadth and depth of estuary at null drift section
τ_i	= $k_i\, \rho(U_U - U_L)|U_U - U_L|$ ....(5.3.2)
U_U, U_L	= current velocity in upper and lower layers respectively.
k_i	= constant of proportionality.

It will be noted that the null position is very dependent on the assumed values of d_L and k_i. In the calculation the value of $(\partial\bar{\rho}/\partial x)$ was taken from the observed distribution of salinity (proportional to density) during a period when Q_R was approximately constant and d_L was chosen to give agreement with the observed tidal drifts near the bed; however, the use of a constant thickness d_L is unrealistic.

To determine the longitudinal distribution of mud concentration equations (5.1.1) and (5.1.2) were used to specify erosion and deposition respectively. Since the active part of the bed consists of freshly deposited flocs loosely compacted (consolidation period < 6 hours on a spring tide) its bulk density is low ($\gamma \approx 1.05 \times 10^3$ kg/m^3). Such "fluid" mud beds are easy to erode ($2 < \tau_e < 5$ dynes/cm^2) compared with more consolidated muds. It appears that when erosion takes place ($\tau > \tau_e$) it does so by the entrainment of particles directly into suspension rather than by surface disturbances and breaking as would occur with a low density liquid.

In the model, it was assumed that the exchange of suspended particles between the layers was due to the vertical movement of water. Accumulations in the mud reaches and the variation of concentration of suspended sediment during a spring tide in both layers were well represented. The model was used to predict the effect of a tidal barrier on the distribution of mud in the estuary.

6 INFILL DUE TO WAVE ACTION

6.1 Preliminary remarks (Tidal currents weak)

Channels in deep water are generally exposed to waves as well as currents. When the currents are weak channels may infill due to the action of waves alone. Even moderate wave activity can generate oscillatory currents at the bed which are large enough to move fine sands and silts in quite large depths (Fig 10a & b) (Komar and Miller, 1974). Measurements of the amplitude of the oscillatory currents required to move sediments with flat beds have been made, usually in the laboratory, but it is likely that over ripple beds previously moulded by wave motion, the threshold velocity would be less. This is because larger bed currents can occur over the ripple crests.

Figure 10 (a)
The near bottom orbital velocity for sediment movement under waves

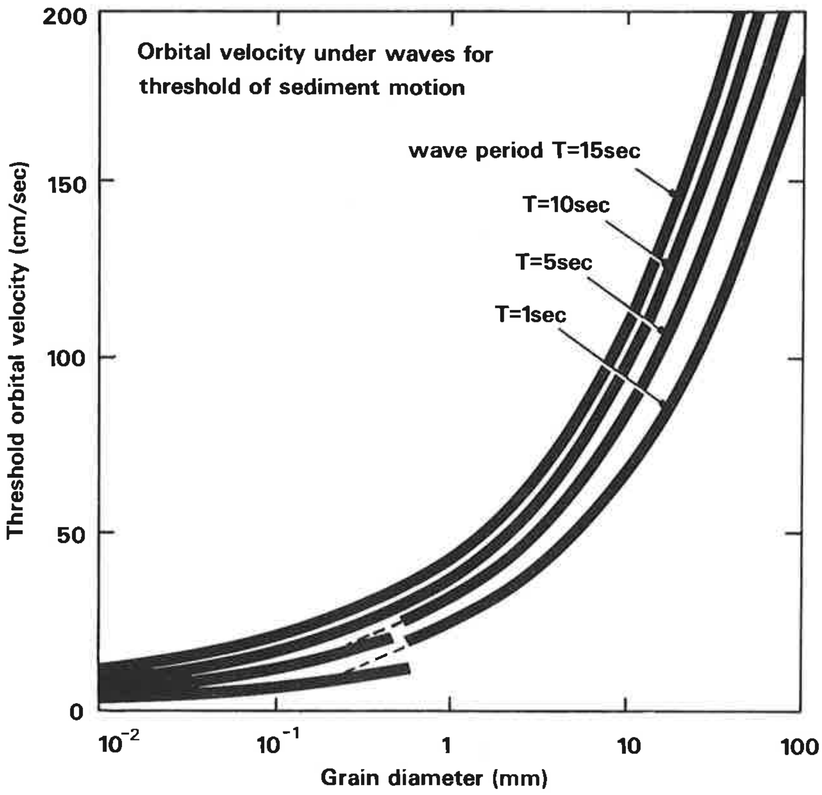
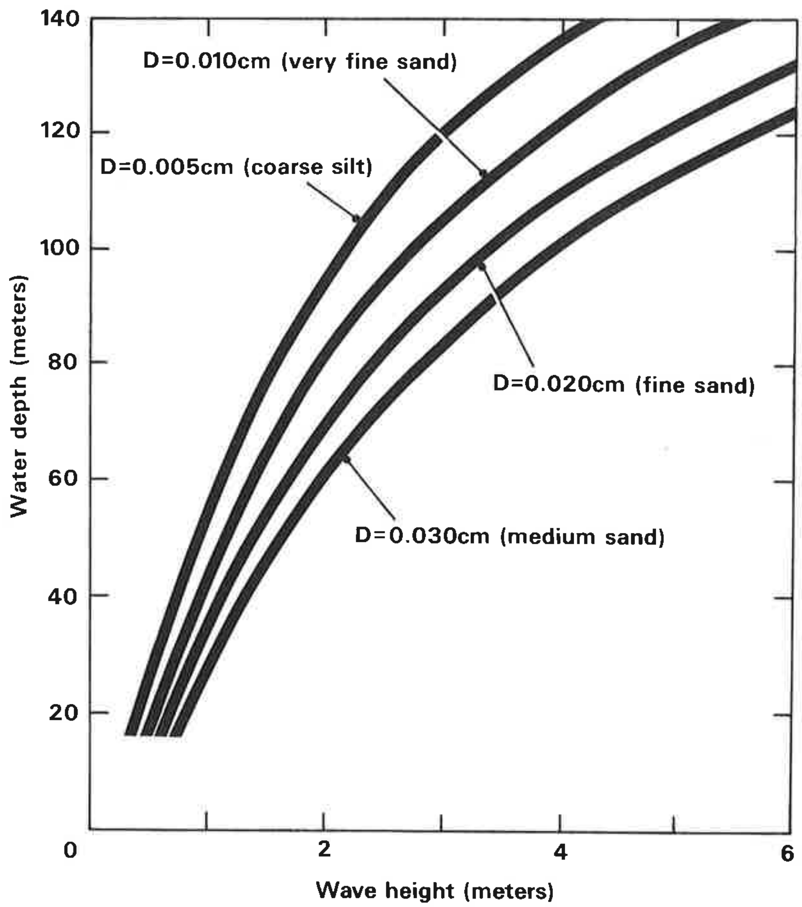


Figure 10 (b)
Water depth for threshold of sediment movement



There is a dearth of information on the sediment flux to be expected when these oscillatory currents are exceeded. Basically there are two mechanisms which cause sediments to be transported under waves. The first arises because the oscillatory velocity is not symmetric in time, the forward velocity (in direction of the waves) being greater when the crest passes than the backward velocity when the trough passes. Thus if the instantaneous transport is proportional to a high power of the velocity there will be a net movement of sediment in the direction of the waves even in the absence of a net drift of water at the bed.

The second mechanism arises from the drift current at the bed induced by the waves, which can transport sediment in the wave direction. This drift current is a consequence of the rapid attenuation of the oscillatory current by viscosity in a thin boundary layer close to the bed (thickness $\delta = 0(\nu^{1/2}/\omega^{1/2})$, ν = kinematic viscosity, ω = angular velocity of the waves, δ about 1 mm for a 10 s wave) and occurs whenever there is a spatial variation in the amplitude or phase of the oscillation above the boundary layer. The drift velocity outside the layer, Batchelor (1967), is given by

$$U_d = -\frac{3}{8\omega}\left[\frac{du_w^2}{dx} + 2u_w^2 \frac{d\Gamma}{dx}\right] \qquad \text{....(6.1.1)}$$

in which the orbit velocity at the bed due to the waves is given by

$$u_w \cos(\omega t - kx) \qquad \text{....(6.1.2)}$$

with

$$u_w = \pi H/(T \sinh kh) \qquad \text{....(6.1.3)}$$

H = wave height

ω = angular velocity of the wave = $2\pi/T$

k = wave number = 2π/(wave-length)

T = wave period

c_w = wave velocity = ω/k

Γ = $-kx$ ....(6.1.4)

Thus for waves in deep water with u_w = constant

$$U_d = \tfrac{3}{4}\frac{u_w^2}{c_w} = \tfrac{3}{4}u_w^2\,(k/\omega) \qquad \text{....(6.1.5)}$$

It is remarkable that U_d is independent of viscosity which is essentially a kinematic effect due to the variation in phase of the orbital velocity along the wave. (U_d must be distinguished from the drift velocity of a "water" particle under the waves which can occur in the absence of viscous effects.) Near the bed this drift is given by

$$\tfrac{1}{2}(u_w^2/c_w) \qquad \text{....(6.1.6)}$$

which happens to be of the same form as U_d. Thus the particle drift velocity (W_d) in the presence of viscous drift is given by

$$W_d = U_d + \tfrac{1}{2}\,u_w^2/c_w \qquad \text{....(6.1.7)}$$

The drift velocity given by (6.1.5) is however very small. For example, a wave 2 m high of 10 s period in 10 m depth of water gives U_d = 0.058 m/s, compared with an oscillatory current at the bed u_w = 0.85 m/s.

The variation of drift velocity in depth has been calculated by Longuet-Higgins, 1953, taking account of viscous boundary layer effects at the surface. Good agreement has been found with experimental results

(Russell and Osorio, 1958, over the range $0.7 < kh < 1.5$) despite the fact that the experiments were carried out with waves which generally produced turbulent flow at the bed. In all cases there was a down-wave drift at the bed and surface with a return flow in the opposite direction at intermediate levels. Similar profiles have been found in experiments with waves propagating up gently sloping bottoms (slopes 1:10–1:40) in deep water (Bijker et al, 1974) although the drift velocities were smaller than those given by equation (6.1.7).

6.2 Channel infill when tidal currents weak

Published equations relating to sediment transport due to wave motion in deep water are empirical or semi-empirical and have generally been derived from small scale experiments. An exception is that obtained by Rance, 1968, from an analysis of experiments in a deep flume (depth 5 m), in which the movement of 0.2 mm sand was derived from movement of the centroid of fluorescent particles subjected to a range of wave conditions. The transport equation was originally expressed in dimensionless parameters but for fine sand it may be written

$$S_w = 0.034 \frac{T}{h^2} u_w^6 \qquad \text{....(6.2.1)}$$

in which S_w = sediment transport rate in cubic metres per second of compacted sand per kilometre of wave crest and T, h, u_w are expressed in m/s units. It will be noted that the transport decreases strongly with increased depth due to the reduction of the orbit velocity (U_{max}) near the bed so that a channel subject to waves will infill at a rate given by the difference in the transport rates appropriate to the depths inside and outside the channel. More recent data from small scale experiments (Kamphuis, 1973) is in reasonable agreement with this formula.

Of course the transport formula (6.2.1) applies to a horizontal bed. In applications to infill calculations for channels on a sloping foreshore it is usual to resolve the flux S_w (which is normal to the wave crests) to components normal (S_{wn}) and tangential (S_{wt}) to the bed contours and to assume that the slope of the foreshore is in equilibrium with S_{wn} so that infill is due to S_{wt} only.

The equation (6.2.1) applies only to waves of a single frequency whereas seas in nature consist of a spectrum of waves of different frequency and height. Usually seas are defined in terms of a significant wave height (H_s) and wave period (T_s), the first quantity being equal to the mean of the highest one-third of all waves and the latter being equal to their mean period. Generally, the significant wave heights and periods are substituted in equation (6.2.1) to determine sediment transport rates.

In order to determine the annual siltation, it is necessary to consider the local wave climate over a long period (one year for example). For this purpose long-term, deep water wave-height statistics are required to give the annual distribution of significant wave heights, periods and directions for the site in question. These waves are then routed into the shallower water near the channel using standard wave refraction procedures (Abernethy and Gilbert, 1975) to determine the significant wave height and direction at different points along the channel.

In some situations, channels may be cut through shoals which are of sufficient height to cause the larger waves to break. As the waves move into shallower water the larger ones will continuously spill energy into turbulence thereby reducing their height to a value dictated mainly by the local depth, bed slope and wave length. In shallow water, waves are generally sufficiently long for their velocity to be dependent only on the depth (non-dispersive) so that single waves will tend to move as solitary waves without change of form. Thus those waves below the

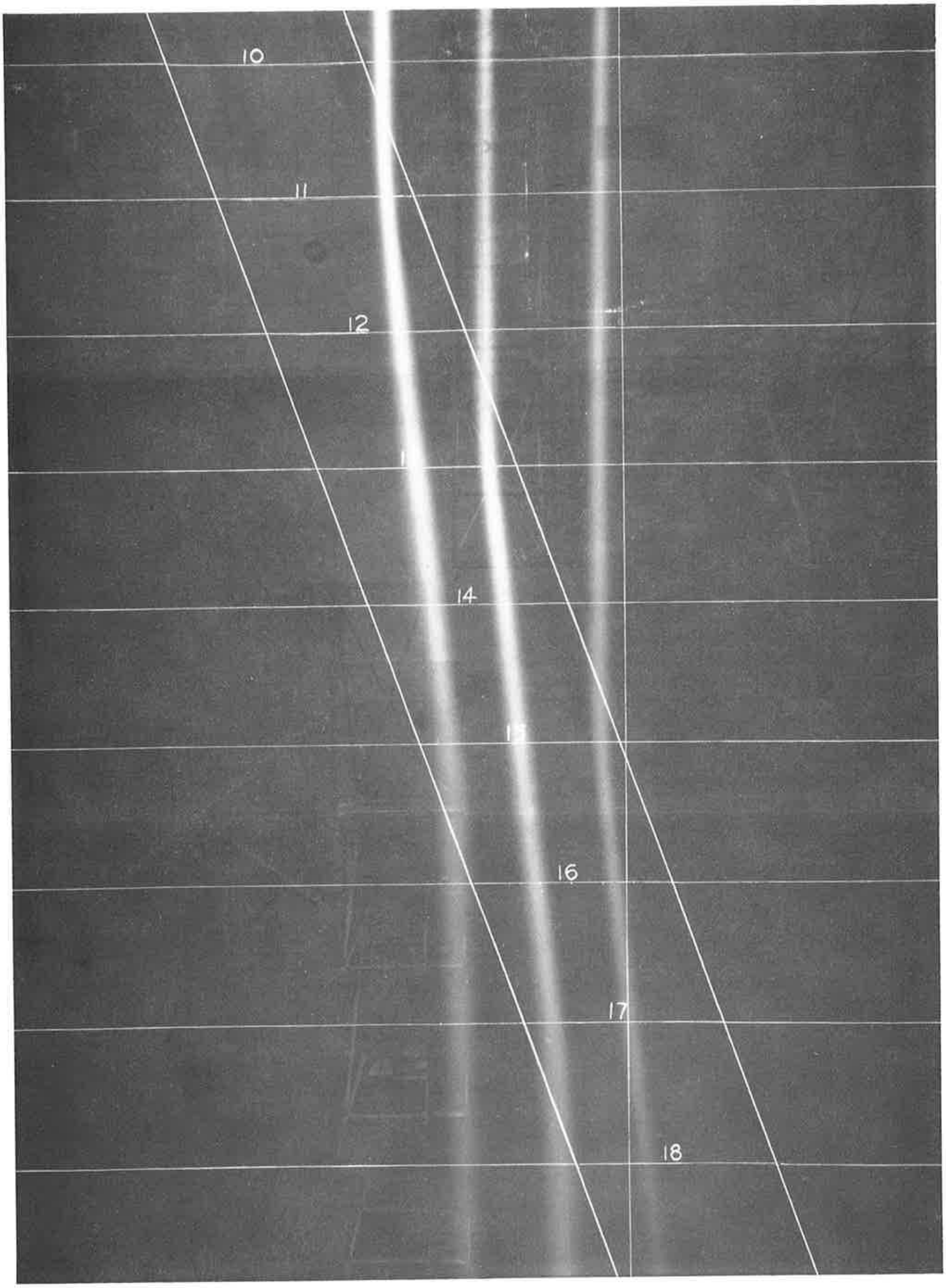


Plate 1 Refraction of dye plumes over channel (H = 1.4)

Channel inclination 20^{0}

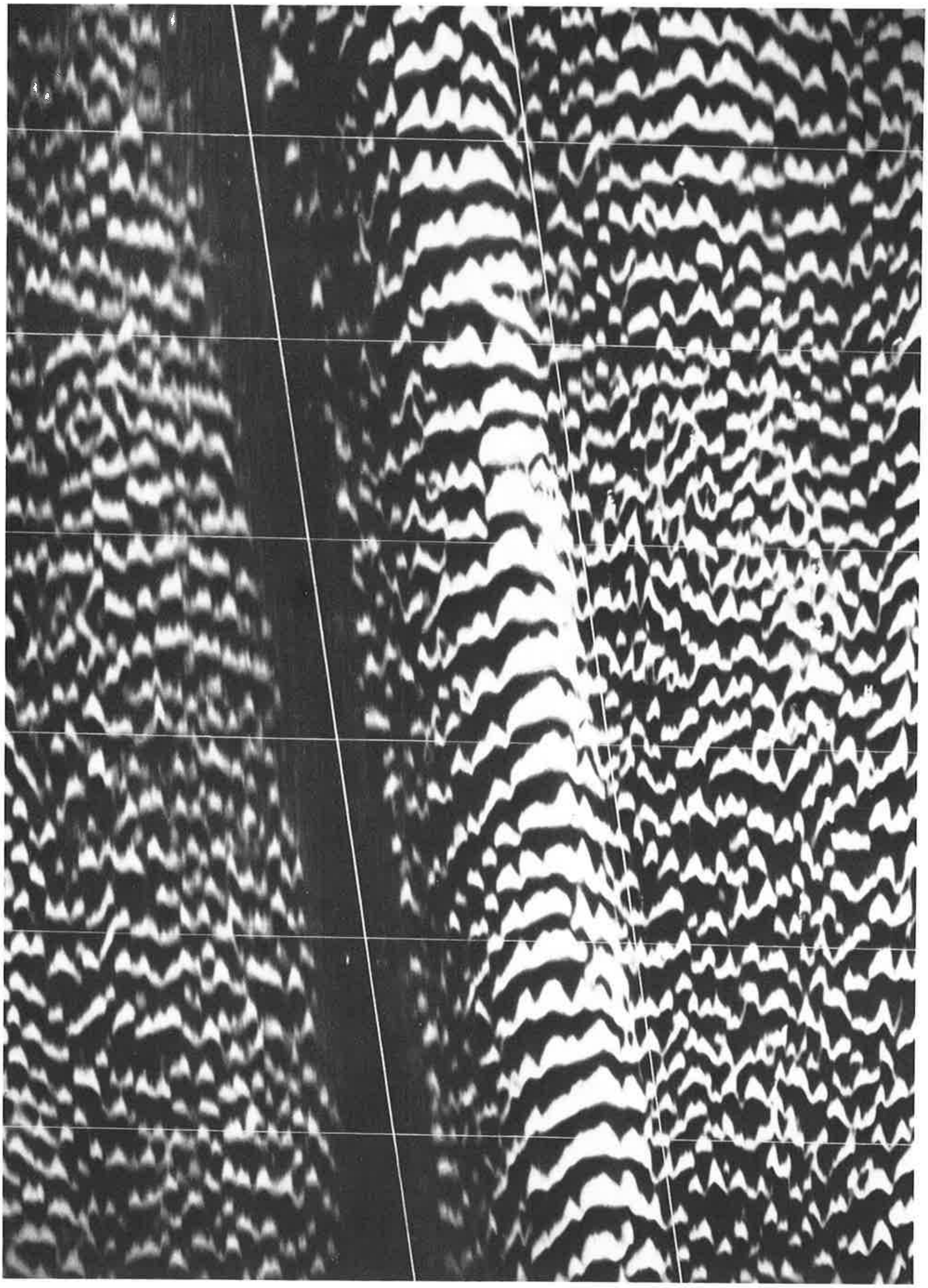


Plate 2 Incipient rippling over channel (H = 1.4)

Channel inclination 10^{o}

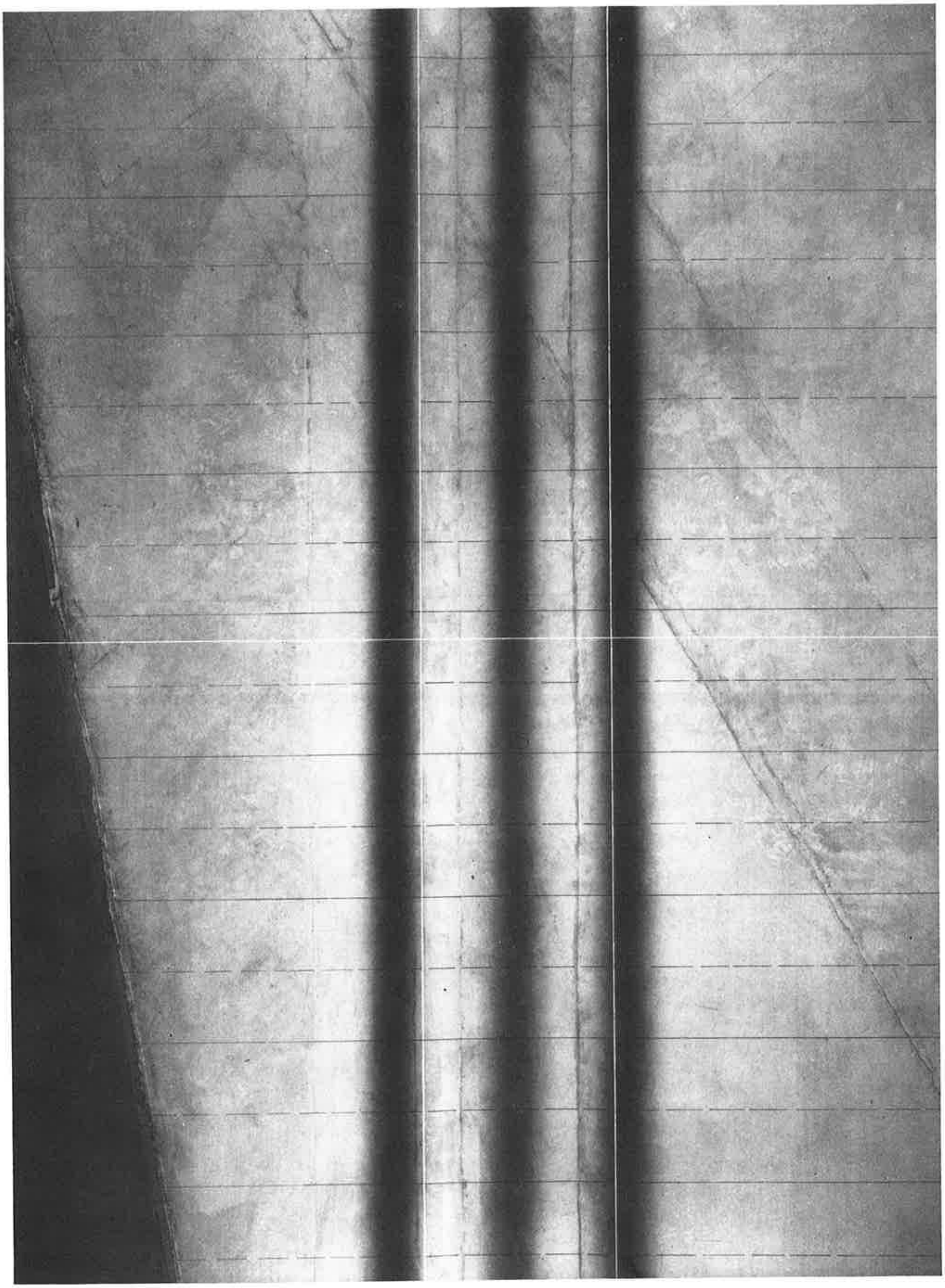


Plate 3   Diffusion of dye plumes along channel at 0^o (H = 1.6)

breaker height will be unaffected while the larger waves which originally exceeded this height are reduced to the breaker height.

Abernethy, 1971, using an empirical formula for the breaker height, viz:

$$H_b = 0.1\ \lambda_w \tanh(kh + 3\pi \tan\alpha_b) \qquad \text{....(6.2.2)}$$

$$\lambda_w = \frac{gT^2}{2\pi} \tanh(kh + 3\pi \tan\alpha_b) \qquad \text{....(6.2.3)}$$

where $\tan\alpha_b$ = bed slope, has calculated the effective wave height which must be substituted in (6.2.1) to derive the sediment transport rate for such situations.

Changes in water depth due to the tidal rise and fall have a direct effect on the transport as described by equation (6.2.1) and also an indirect effect by allowing larger waves to reach the channel at high tide without breaking. Both these effects must be taken into account when determining the siltation rate.

A procedure, taking account of the factors mentioned, has been used to estimate the infill of a channel dredged through an offshore bar at the entrance to Phitti Creek, which serves as a gateway to a new port (Qasim) to the east of Karachi (Abernethy, 1975). The channel will be subject to high wave activity during monsoon months and knowledge of likely infill was important in order to decide on the overdredging necessary in non-monsoon months to maintain a minimum given depth. The predicted annual infill was about 40% of that subsequently estimated from an analysis of the measured infill into a trial pit. However, the first prediction was based on insufficient wave data and the wave refraction techniques used to predict wave heights approaching the channel have since been improved.

6.3 Gravity effects in the presence of waves

Equation (6.2.1) describes the transport of sand under waves over a level bed. There will however be an increase in transport down the side slopes of the channel due to gravity. If, as seems likely, the transport under waves is linearly dependent on bed slope (positive downwards) for small slopes, one would expect a "diffusive" type widening and shallowing of a channel due to gravity (as discussed in section 4.1) as well as a lateral movement of the channel due to the convected flux changes governed by equation (6.2.1). Such effects would be additional to the diffusive type spread of the channel due to lag effects which occur if an appreciable fraction of the bed sediment moves in suspension in wave induced currents. However, the latter effects may be small when waves predominate.

6.4 Infill with waves and currents present

Little is known about the way in which sediment transport in currents is affected by wave action, although field observations suggest that quite small waves in moderate depths can significantly increase the concentration of suspended sediment over beds of fine sand. Bijker (1971) in discussing the transport along a foreshore by waves and currents suggested that the distribution of the concentration of suspended sediment is unchanged throughout most of the depth when waves are present, but that the concentration near the bed is increased due to the additional stirring action of the waves.

To quantify the transport of sediment with waves present Bijker used equation (2.3.6) with the bed transport (S_b) increased due to the waves. S_b may generally be written

$$S_b = U_* d_m f(\theta, \Lambda/d_m) \quad \text{....(6.4.1)}$$

in which the term $U_* d_m$ represents the convection of the particles engaged in bed movement and the function $f(\theta, \Lambda/d_m)$ represents a "stirring" parameter which governs the concentration near the bed.

In equation (6.4.1)

$$\theta = U_*^2/g(s-1)d_m \quad \text{....(6.4.2)}$$

Λ = hydraulic roughness of the bed forms

S_b = volume flux of sediment engaged in bed movement per second per unit width normal to the flow.

U_* = friction velocity $(\tau/\rho)^{1/2}$, τ = total shear stress at the bed and ρ is the water density.

d_m = size of sediment particles

s = specific gravity of sediment.

It may be noted that over self-formed beds the bed roughness (Λ) is mainly caused by ripples or dunes and may itself be a function of θ and the Reynolds number of the particles, Re = $U_* d_m/\nu$. Generally the depths are sufficiently great for the surface boundary to have no significant effect on the flow at the bed.

Byker applies Frijlink's empirical formula for the transport of sediment at the bed in currents alone, to quantify the bed concentration due to stirring, (Appendix 3). Omitting the ripple factor in the convective factor this formula gives

$$f(\theta, \Lambda/D) = 5 \exp[-0.27/\mu_r \theta] \quad \text{....(6.4.3)}$$

and Byker identifies this function with the stirring factor whose value will be changed when waves are present. Thus if θ_w denotes the value of θ with waves present and θ_c denotes the value of θ in currents alone he suggests that (6.4.3) should be replaced by

$$f_w(\theta_w, \Lambda_w/d_m) = 5 \exp[-0.27/\mu_w \theta_w] \quad \text{....(6.4.4)}$$

with

$$\theta_w = \theta_c [1 + \tfrac{1}{2}\zeta^2 (u_w^2/\bar{u}_c^2)] \quad \text{....(6.4.5)}$$

in which

u_w = amplitude of orbital velocity at the bed

$\bar{u}_c$ = mean current velocity

$\zeta = 0.18\ \bar{u}_c/U_*$ and ....(6.4.6)

$\mu_w = \mu_r$

Using these relations the transport (S_{bw}) of sediment engaged in bed movement under waves and currents, viz:

$$S_{bw} = U_* d_m f_w \quad \text{....(6.4.7)}$$

can be determined and hence the near bed concentration, C_{bw}, and the flux of sediment in suspension (Appendix 3).

To determine the initial infill rate from equation (3.1.9), the flux has to be calculated for the current and wave conditions in the approach flow (S_{ow}) and for the changed current and wave conditions over the channel (S_{1w}). For the latter it is important to take into account the change of

Note:

For waves which are long compared with the depth, at normal incidence the ratio of the wave height over the channel to the incident wave height is

$$2c_{w_0}/(c_{w_0} + c_{w_1}) \quad \text{....(6.4.8)}$$

in which c_{w_0} and c_{w_1} are the wave velocities over the banks and over the channel respectively (Lamb, Hydrodynamics, 6th Edition, p 263).

When waves are propagating towards the channel at a small angle of incidence, since they travel more slowly over the banks than over the channel they will be totally reflected when the angle of incidence is equal to the critical angle given by $\sin^{-1}(c_{w_0}/c_{w_1})$.

However, at larger incidences (near grazing angles) the increased wave activity along the channel edge is likely to act as a source of energy radiating waves across the channel. With waves of sufficient steepness along the channel edge, shorter waves at harmonics of the main frequency may occur over the channel.

wave height over the channel due to wave reflection as well as the reduction of orbit velocity due to the depth increase *(see Note).*

In a tidal situation, a channel under storm conditions is likely to change its shape in the manner discussed in sections 2 and 3, with an effective diffusive coefficient given by $c'L$ (equations (3.1.10) and (3.1.11) in which $c' = \partial S_{xw}/\partial Z$, S_{xw} being the component flux normal to the channel under the combined action of waves and currents). For a channel in line with the flow one might expect a gravity infill at the rate given by (4.1.2) but with S_{bo} modified by the presence of the waves S_w, equation (6.4.7).

6.5 Stability of bank slopes under waves

6.5.1 *Sand beds*

When breaking or near breaking waves pass over a sandy bed it is known that just prior to the passage of the crest the bottom of the bed can "explode" (Madsen, 1974). It appears that bed failure is due to the large horizontal pressure gradients associated with the high water surface slopes of the breaking waves rather than the vertical flow within the bed induced by these gradients. It is easily shown, by considering the horizontal forces on a thin slice of sediment at the bed-water interface, that incipient failure can occur at a critical horizontal pressure gradient given by

$$-\frac{1}{\rho g}\left(\frac{\partial p_b^+}{\partial x}\right)_c = \frac{\rho_t - \rho}{\rho}\tan\lambda_i \qquad \text{....(6.5.1)}$$

in which

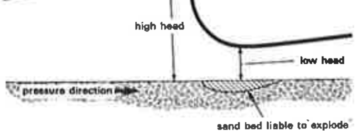
ρ_t = density of the saturated bed sand

ρ = water density

$\left(\frac{\partial p_b^+}{\partial x}\right)_c$ = critical excess pressure gradient at the bed due to the wave

and λ_i = angle of internal friction

For a horizontal bed of relatively loose sand in sea water the value of the right hand side is about 0.5; thus the waves must be quite steep for failure to occur. (The water surface slope of the wave will be comparable with the left hand side of equation (6.5.1) in shallow water.) The predicted failure should be interpreted as "momentary" in that it will take place for the short period of time during which the critical gradient is exceeded. Due to inertia the failure may not cause an appreciable parcel of bed sand to be forced out of the bed. Nevertheless the failure causes a slight motion in which individual grains slide past each other and exchange positions. Madsen's experiments suggest that grains could be disturbed in this way over a thickness $\delta = 0.11\ H_B$, where H_B is the height of the spilling wave.



On the side slopes of a channel, if the waves approach the channel normally, the horizontal gradients for failure would be expected to be less in the ratio

$[1-(\tan\alpha_s/\tan\lambda_i)]$

where α_s is the slope of the sides.

For $\tan\alpha_s = 1{:}12$, the right hand side of equation (6.5.1) is then reduced to about 0.42 which is only a little less than the maximum surface steepness for a deep water wave ($\tan 30^o = 0.58$). Thus it is unlikely that the sides of channels can fail due to this cause except perhaps in areas where channels are dredged through high shoals exposed to breaking waves.

6.5.2 *Cohesive beds*

Under storm conditions, pressure pulses due to waves have been thought to induce catastrophic instabilities in soft gently sloping cohesive sediments (Mitchell and Hall, 1974). Such instabilities could cause failure along the edges of steeply dredged channels. Two mechanisms have been put forward to explain such failures; one by Henkel, 1972, and a second by Wright and Dunham, 1972. In both cases, failure is assumed to occur when the horizontal pressure gradients under the waves near the bed ($\partial p^+/\partial x$) together with gravitational forces exceed the resisting forces derived from the strength of the undisturbed sediment. Thus the failure mechanism is akin to that for fine sand, but with soft sediments the angle of internal friction is considerably less. Henkel suggested that failure occurred by a sequence of circular arc slips whereas Wright et al attributed failure to excessive shearing stresses being developed at some depth in the soil.

Consideration of static forces shows that sediment on a slope will fail when the slope (β_f) is equal to a value given by

$$\sin 2\beta_f = 2C_u/\bar{\gamma}' Z_f \qquad ....(6.5.2)$$

in which

C_u = undrained shear strength of the sediment at depth Z_f

$\bar{\gamma}'$ = submerged weight per unit volume averaged over the depth Z_f

Z_f = depth of sliding.

For simply sedimented soil (that which is allowed to settle in a quiet state) it appears that the undrained shear strength at depth Z below the surface of the bed may be approximated by

$$C_u = A_s Z \bar{\gamma}' \qquad ....(6.5.3)$$

where A_s is a constant dependent on the degree of consolidation and nature of the soil. Thus at any degree of consolidation the failure angle depends simply on the value of A_s.

Mitchell et al, 1972 and 1974, have studied the stability of bed slopes (β_o) under wave action in the laboratory and compared the results with static tilting tests as well as vane shear strengths made before and after wave action. When $\beta_o \leqslant \beta_f/2$ no well defined slope failure occurred but under relatively high sub-surface pressures the bed gradually flattened by down-slope movement of sediment. At higher slopes, failure occurred by a mass movement of sediment down to considerable depths within the bed when the variation of pressure on the bed due to waves (calculated from linear theory) exceeded δp_c^+, given by

$$\delta p_c^+ \approx 2.5\,[\bar{\gamma}' Z_f]\,[\sin 2\beta_f - \sin 2\beta_o]/2 \qquad ....(6.5.4)$$

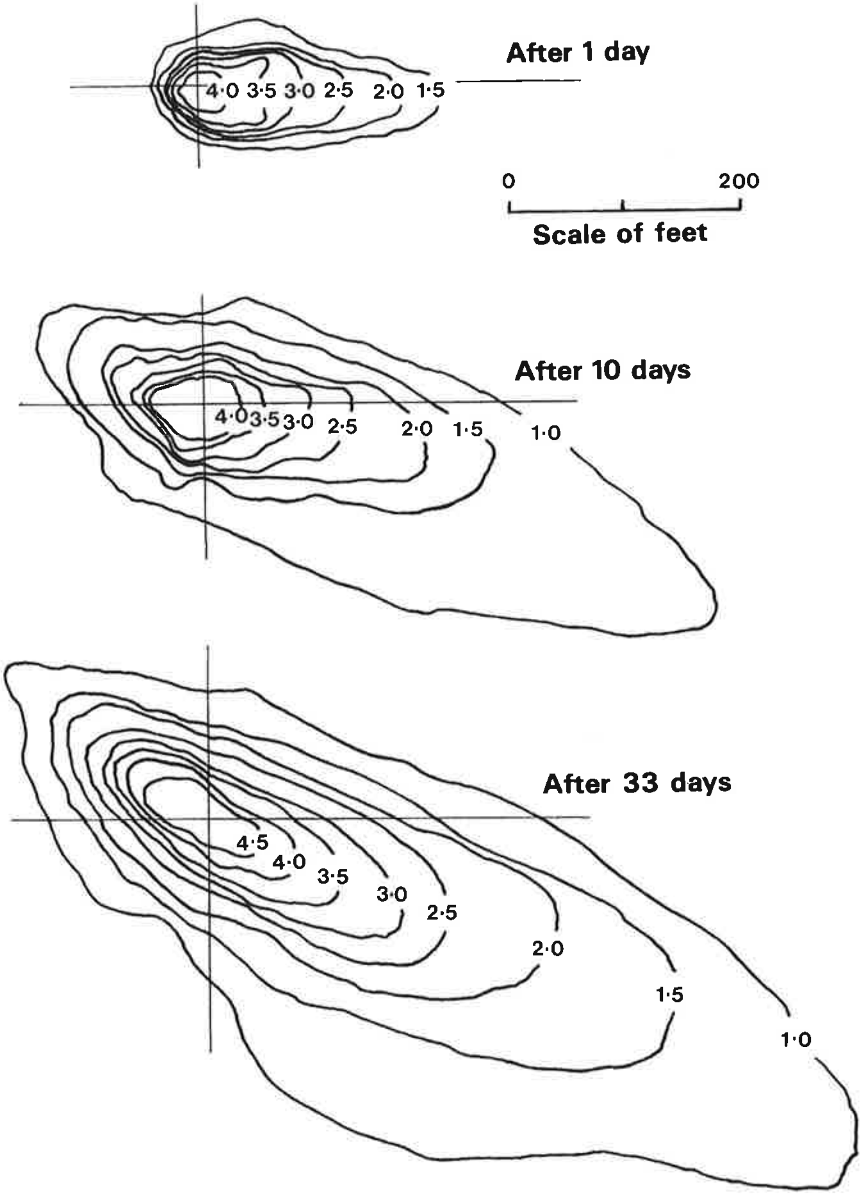
This failure was attributed to a loss of strength by "wave remoulding" as revealed by the tilting and vane shear tests. When the ratio of depth to wave length was less than 0.4, failure was marked by cyclic movements and fissure patterns resembling circular arcs. In the experiments the depth of failure was considerable (Z_f/λ = 0.2–0.6) and increased with sub-surface pressures, in general accordance with the relation proposed by Henkel.

To assess the likelihood of such failures Mitchell suggested that in situ field measurements of strength profiles should be carried out both before and immediately after storms.

7 APPLICATION OF TRACERS TO ESTIMATE INFILL

As explained in section 2.4, infill estimates are usually based on standard type field measurements of sediment flux taken at frequent intervals (half-hour) over several tides including springs and neaps. This method has the advantage that the flux can be related directly to the current, which allows both the flux approaching the channel and that appropriate to the changed current pattern over the projected dredged channel to be estimated, the difference giving the infill rate. Its main disadvantage is that it cannot be used in periods of high wave activity when conditions are too rough for survey vessels to operate. Thus its application is of limited utility in situations where the tidal currents are weak and infill occurs mainly when waves are present. In these situations alternative methods supplementing or replacing direct flux measurements are necessary.

Figure 11
Spacial tracer distributions



Spacial tracer distribution; lines of equal radioactivity denoted by the log of the count rate

For many years radioactive tracers have been used to indicate the direction and magnitude of sediment drift in estuaries and offshore regions. Usually the tracer has a fairly long half life so that it can be tracked for several weeks or months. In fact, the tracking period is more dependent on the dispersal of the tracer particles by the tidal currents than on its half life, so that from the point of view of embracing storm periods it is probably best applied in situations where the currents are weak.

To determine the sediment drift, a small quantity of sediment particles

is made highly radioactive and is released at the bed when the currents are low. The subsequent movements are mapped at different times over a period of several days or weeks. Contours of equal activity are then drawn and subsequently interpreted in terms of the proportions of the original weight of injected sediment, Fig 11. Core samples are taken, usually near the point of injection, to determine the different depths to which the particles become buried in the bed. The latter are used to estimate the thickness of the bed layer involved in movement.
In practice, injections are made at several points over the area of interest and mappings are carried out concurrently. Generally from each injection point the centroid of the activity distribution moves in the direction of the net tidal drift at the bed, the sediment transport being seldom balanced in the ebb and flood directions. The contours are spread about this point in approximately elliptical shape, the tracer particles being dispersed more in the direction of the currents (flood and ebb directions) than normal to it.

The net drift at each point is equal to the velocity of the centroid multiplied by the thickness of the bed layer. However, as already explained in section 2, for sediment travelling mainly in suspension, infill into a dredged channel is dependent on the sum of the settlements in the channel during the ebb and flood tides, rather than on the net drift. If, for example, sediment down to a certain thickness d' in the bed is assumed to move through average distances ℓ_1 and ℓ_2 during the flood and ebb tides, a channel normal to the flow will intercept quantities $\ell_1 d'$ and $\ell_2 d'$ respectively. Thus, if the channel has a trapping efficiency η ($\eta = 1$, for the channel behaving as a perfect trap) the rate of infill per unit width perpendicular to the flow will be

$$I = \eta(\ell_1 + \ell_2)d' \text{ per tide by volume} \qquad \text{....(7.1)}$$

ie, the sum of the average transport rates on ebb and flood multiplied by the trapping efficiency.

In these situations in which the sediment motion on the ebb and flood is nearly balanced, $I = 2\eta\ell_1 d'$ (approximately) and the problem is then reduced to determining ℓ_1 and d' and the trapping efficiency. The quantity (d') may be identified with the average depth below the mean bed level at which tracer particles are found from core samples. The length ℓ_1 must be inferred from the distribution of tracer particles after a number of tides, since the time required to make a survey does not permit its measurement after a single half tide.

It is usual to assume that ℓ_1 represents the average movement on a typical ebb or flood tide. The distribution of tracer after several tides is calculated by integration assuming that all tides are similar. Comparison of the calculated distribution with that measured then enables the value of ℓ_1 to be obtained. However, the result depends rather critically on the assumed sediment distribution after a single half tide.

To illustrate, suppose that after one half cycle the fraction of the sediment deposited on a strip of length dx originally in a strip of unit length normal to the flow is

$$f(x)dx \qquad \text{....(7.2)}$$

so that

$$\int_0^\infty f(x)dx = 1 \qquad \text{....(7.3)}$$

and

$$\ell_1 = \int_0^\infty dx \int_x^\infty f(x^1)dx^1 \qquad \text{....(7.4)}$$

Now suppose the sediment in the original strip is labelled with tracer of volume Vo. After one half cycle its distribution is given by

$$Vo\ f(x)dx \qquad ....(7.5)$$

and after one tide (two half cycles) its distribution $F_1(y)$ will be symmetric ($F_1(-y) = F_1(y)$) and given by

$$F_1(y) = Vo \int_{-\infty}^{0} f(-x)\ f(-x + y)dx \qquad ....(7.6)$$

The variance of this distribution (σ_V^2) will be given by

$$\sigma_V^2 = \int_{-\infty}^{\infty} F_1(y)\ .\ y^2\ .\ dy \qquad ....(7.7)$$

$$= \int_{0}^{\infty} y^2 dy \int_{-\infty}^{0} f(-x)f(-x + y)dx \qquad ....(7.8)$$

After n similar tides, the tracer distribution $F_n(y)$ may be obtained from the recurrence relation

$$F_n(y) = \int_{-\infty}^{\infty} F_{(n-1)}\ (X)\ F_1(X + y)dX \qquad ....(7.9)$$

and its variance is

$$n\sigma_V^2 \qquad ....(7.10)$$

whatever the form of f(x). Thus the measurement of the variance does not enable the value of ℓ_1 to be obtained from equation (7.4) unless some assumption is made about the form of f(x).

In theory it should be possible to obtain more information about the form of f(x) by taking fourth and higher moments of the tracer distribution but in practice the spread of the tracer is so great that these become impossible to measure with precision. In order to proceed one is therefore obliged to assume a form for f(x) which may be justified either by physical arguments or by comparison of predicted and measured distributions.

Hubbell and Sayre, 1964, produced a simplified model of the sediment motion in uniform, uni-directional flow. The particles moved in a series of steps such that at each step the fraction of sediment leaving a transverse strip was exponentially distributed. If a similar distribution, viz;

$$f(x) = e^{-x/\ell'} \qquad ....(7.11)$$

is also assumed to apply to the distribution of sediment from a strip of the bed after one half tide, it is easy to show that

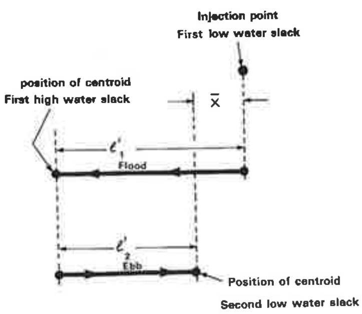
$$\ell_1 = \ell' \qquad ....(7.12)$$

and the variance of distribution of tracer from an initial point injection after n tides will be given by

$$\sigma_V^2 = 2\ell_1^2 n \qquad ....(7.13)$$

or

$$\ell_1 = \sqrt{\sigma_V}/n\sqrt{2} \qquad ....(7.14)$$



In an unbalanced tidal situation, having different length scales ℓ'_1, ℓ'_2 on the flood and ebb tides, the centroid of the distribution of tracer injected at a point at slack water will be displaced a distance given by

$$\bar{X} = (\ell'_1 - \ell'_2) \text{ per tide} \qquad ....(7.15)$$

and tracer will disperse about this point with a variance given by

$$(\ell'^2_1 + \ell'^2_2) \text{ per tide.} \qquad ....(7.16)$$

Thus the tracer distribution allows both $\ell_1 = \ell'_1$ and $\ell_2 = \ell'_2$ to be obtained and hence the infill rate.

The factor 2 appearing in equation (7.13) and the results given in (7.15) and (7.16) depend on the form assumed for f(x). In a particular case it is preferable to assume a form which gives a good fit to the tracer distributions obtained after a number of tides. [A procedure of this type was followed to predict the rate of maintenance dredging for a navigation channel in Botany Bay (Crickmore, 1967)]. It is interesting to note that whatever the form of f(x) it would be expected that after n similar tides the tracer would be approximately distributed about the centroid as though it were subject to a constant diffusion coefficient, ie the concentration of tracer at distance $x/\sqrt{n}$ from the centroid will be proportional to $n^{-\frac{1}{2}}$ or

$$F_1(x-\bar{x}) = \sqrt{n}\, F_n\left(\frac{x-\bar{x}_n}{\sqrt{n}}\right) \text{ (approximately)} \qquad \text{....(7.17)}$$

Such a relation allows $F_1(x)$ and hence f(x) to be approximately determined for a "typical" tide.

However, there are two main complications in this procedure. The first is associated with the irregularity in size of the bed forms (ripples and dunes). As these forms progress, over a period of several tides, the tracer particles become buried to deeper levels in the bed as the larger forms pass over them. Experiments in unidirectional flow in which the flux of sediment is constant have shown that the velocity of the centroid of the tracer distribution decreases with time as the depth of penetration increases. The product of this velocity and the depth of penetration is equal to the sediment flux (Crickmore and Lean, 1962). Similarly in tidal situations, it is found that the depth to which tracer particles become buried gradually decreases. This is reflected in a gradual reduction both in centroid velocity (in unbalanced tidal situations) and in the variance change per tide, ie the length scale of the distribution. Thus, an analysis seeking to determine the function $F_1(y)$ for a single tide (and hence f(x)) from the distribution after a number of tides must take account of this feature. One could speculate that it might be sufficient to scale down the measured radioactivity at a point and increase its distance from the injection point in proportion to the ratio of the depths of penetration at the time of survey to that after one tide.

The second complication arises from the effects of storms. High wave activity can lead to the tracer becoming buried deeper in the bed, due either to the creation of larger bed forms or to a general dilatation of the bed brought about by the oscillatory pressure gradients at the bed. Where tidal currents are strong as compared with drift currents generated by the waves, they will still be the main agency for conveying sediment. To obtain representative values of ℓ_1, ℓ_2 the derived values should be increased in proportion to the increased depth of penetration during the storm period.

So far tracer methods to determine infill have only been applied to situations where the proposed channel forms a highly effective trap ($\eta = 1$). In other situations the efficiency must be estimated.

If the effect of waves on the trapping efficiency of the channel is ignored, the trapping efficiency may be obtained from standard type measurements of sediment flux and currents taken during calm conditions together with a formula such as (3.1.9) which gives

$$\eta = \left[1 - \left(\frac{B_2T_2}{B_1T_1}\right)\right]\left[1 - e^{-s/\ell}\right], \quad B_2T_2 < B_1T_1 \qquad \text{....(7.18)}$$

To take account of the reduced wave action on the bed of the channel, as compared with that outside, T_2 and T_1 could be modified according to equations (6.4.1) and (6.4.4).

8 TRIAL DREDGES

Trial dredges are the most obvious method for determining the infill rates of proposed navigation channels. Usually one or more areas are dredged on the line of the channel to a depth similar to that proposed. The rates of infill are observed over an extended period by comparing successive surveys. Accurate methods are required both for position fixing and for echo-sounding to obtain reliable estimates. Usually sections are surveyed well beyond the limits of the dredged area to check the stability of the surrounding bed and to aid in positioning the dredge. Also samples of the bed sediment inside and outside the area are taken during infilling.

Ideally the depth, width and side slopes of the trial dredge should be the same as those of the proposed channel and its length sufficient to suppress end effects.

End effects constitute one of the main uncertainties in the application of the method since neither the flow nor the flux changes over a depression of limited length will be the same as over a long channel. The trial dredge will attract flow towards itself over its ends leading to higher velocities than would occur over a channel normal to the flow and lower celocities than for channel in line with the flow. In order to estimate the effect of the restricted dimensions, let us consider the case of a dredged hole elliptic in plan and of constant depth (h_1) in a uniform flow (velocity U_o, depth h_o) parallel to the major axis of the ellipse. We assume that the changes of velocity head are small enough to be ignored so that the flow is entirely governed by bed friction and that this friction can be linearised. It may then be shown (Appendix 7) that the velocity (U_E) over the elliptic depression is uniform and given by

$$\frac{U_E}{U_o} = [(\frac{b}{2a}H^2)^2 + H(1 + \frac{b}{a})]^{1/2} - \frac{bH^2}{2a} \qquad ....(8.1)$$

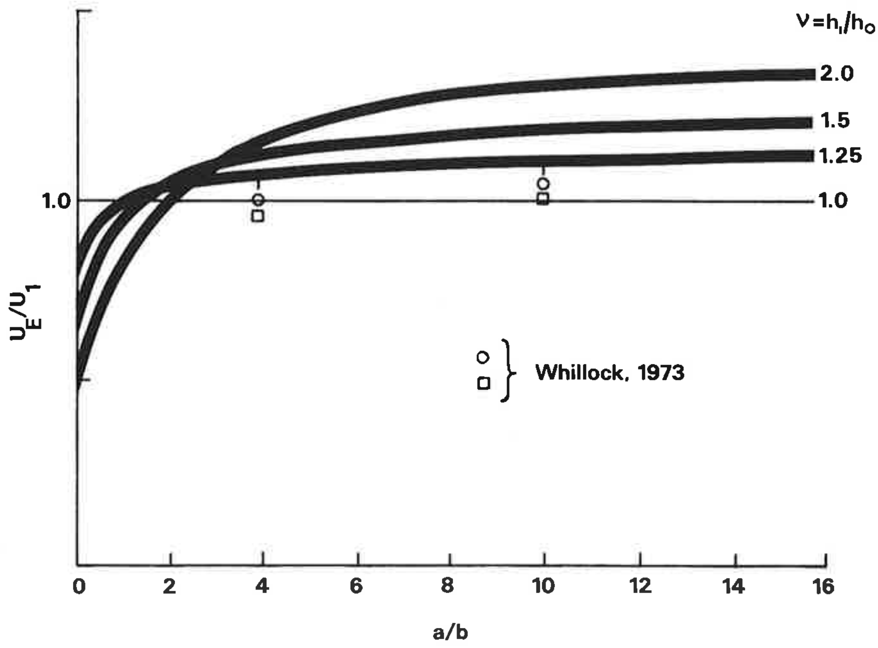
in which a, b are the major and minor areas of the ellipse.

We note that when $a \gg b$, corresponding to an in-line channel, $U_E/U_o = \sqrt{H}$ which is the result already obtained and when $a \ll b$, corresponding to a normal channel, $U_E/U_o = 1/H$. Values of U_E/U_o for other values of a/b are shown in Fig 12 together with model results (Whillock, 1973) for velocity ratios over the centre of rectangular holes of length/width ratios 9.8 and 3.8. We see that for moderate values of H (1–1.5) the length/width ratio must exceed 8 for the velocity ratio to approximate (within 5%) the value over a long channel. Similarly for a perpendicular channel the width/length ratio must exceed 8 for the velocity over the ellipse to fall to within 7% of that over a perpendicular channel.

With rectangular holes in unidirectional flow, the velocity at the bed showed a pronounced drop at entry which recovered along the length but again showed a small drop at the downstream end followed by an acceleration over the downstream edge. As might be expected, maximum deposition occurred at the upstream end, increasing in extent during the course of time. A similar result was obtained for trial dredges in an estuary with a bed of fine sand in which the flow on the ebb and flood was approximately balanced and parallel to the longer side of the trench. It filled in at each end but showed little change in the middle during a period of 7 months.

Generally the velocity changes over a trial hole are best determined by small scale laboratory experiments. This information together with field

Figure 12
Velocity ratio over an elliptic depression



measurements of sediment flux allows an estimate to be made of the infill of the hole (using the method outlined in Sections 2 and 3)which can be compared with that inferred from successive surveys. Such a comparison lends confidence to the extrapolation of the measured hole infill to that for the dredged channel. To the author's knowledge this procedure has been carried out in only one case, viz that of a trial hole in the Mouse Cut in the Thames estuary when the flow was inclined at 20^o to the proposed channel. In this case the estimated infill was about 25% less than that measured. However, the discrepancy may have been due to progressive infilling from the ends, which encroached on the central part of the hole giving a larger infill than would have occurred in a longer channel.

9 INFILL OF CHANNELS IN RESTRICTED AREAS (APPLICATION OF MODELS)

Channels dredged in offshore locations usually have little effect on the flow approaching them but in more limited areas, eg in inner estuaries, the dredged depths may cause a significant redistribution of the currents and in even more confined areas alter the overall currents in the estuary. In such situations it is usual to employ hydraulic or mathematical models to estimate the changed flow pattern. Both types of model are essentially two-dimensional since although in hydraulic models the bed friction can be adjusted to give tidal levels and average velocities similar to those occurring in nature, the large depth exaggerations (necessary to ensure turbulent flow in the model) distort the velocity profiles in depth. In cross-flow over a channel, for example, the exaggerated bed slopes can lead to flow separations (Section 10) where none exist in nature. However, flow reversals behind sharp lateral constrictions at the sides of estuaries and jetties, which can occur at the outlets of some estuaries, are probably better accounted for in hydraulic models than in conventional mathematical models. The latter basically solve the two-dimensional equations with bed friction included but without the addition of vorticity source terms which accompany sudden changes of geometry.

Early hydraulic models often had beds of sand or lightweight particles to determine the general shoaling and scour patterns consequent on large scale changes in estuaries and useful qualitative predictions have sometimes been made by such means. Similar predictions have been made by means of mathematical models using the equation of continuity for the sediment (equation (2.1.1)) and an empirical equation relating the sediment flux to mean velocity. The prediction of infill over a dredged channel depends on more detailed reproduction of the flow pattern in depth and the response of the bed silt to the flow. In hydraulic models, apart from the error in velocity profiles just mentioned and the effective foreshortening of the channel in terms of depth, the scouring properties (or erosion velocity) depend upon the size of the bed forms which in turn determine the turbulence and shear at the bed. However, bed forms on mobile beds in models do not develop to scale and they are often similar in size to the incised depth of the channel. This will alter the flow geometry, and the scour, which is probably a function of the energy loss over the channel, will be greater and hence the deposition less over the channel than would occur over the natural channel. While mathematical models do not suffer from this limitation, they demand a knowledge of the factors governing the flow structure, its inter-relation with the bed and scour rates, which we do not at the moment possess. Such knowledge can only be obtained by further experiment.

Hydraulic models with fixed beds of smaller areas are sometimes used to investigate flow patterns in tidal situations. These have the advantage that provided they are constructed to a large enough scale the exaggeration can be minimal and the velocity profiles in depth more representative. Such models are particularly useful where tidal currents are weak since the convected drifts generated mainly by wave momentum effects in inshore areas can be reproduced. However, the problem of interpreting the currents in terms of sediment flux still remains since the scaling laws which describe shoaling in channels when waves are present are not yet identified. Thus we are obliged to apply equations of type (6.4.1) and (6.4.3) pending more accurate data describing the sediment flux under tidal and wave conditions.

10 DISCUSSION

The methods for determining infill of channel exposed to tidal currents are discussed in sections 2 and 3, where it is assumed that the distribution of velocity and the turbulence properties in depth at a point in the channel are the same as that which would exist in uniform flow at the same depth and mean velocity. However, energy losses initiated in the expanded flow over the upstream face will generate additional turbulence, and over the bed of the channel the velocity distribution and reduced mean bed shear will only slowly revert to that for uniform flow. The effect of the increased turbulence is to increase the fluctuations of bed shear at a point and hence the erosion rate. This causes lower deposition in the channel than would otherwise occur (O'Connor, 1975). In an unbalanced tidal flow, the upstream slope (referred to the dominant drift direction) will get steeper as in Fig 1, and when the slope exceeds about 1:10 the main flow will separate from it and re-attach itself at a distance of about 7 (h_1-h_0) from the upstream edge, leaving a slow moving recirculating flow region between the slope and the main flow. This region is likely to behave as a trap for sediment falling from suspension and moving in contact with the bed, thus increasing the deposition.

Beyond the re-attachment point the turbulence is greater but the bed shear less than that appropriate to steady parallel flow. These quantities only slowly recover – in a distance of the order of 20 (h_1-h_0) (Tani 1968, Bradshaw and Wong, 1972). There is no evidence to show how the erosion rate is affected in such situations, although White, 1940, demonstrated that turbulence is an important factor. He conducted an experiment with venturi flow over a flat bed of sand and observed that bed movement first occurred in the more turbulent flow in the expansion downstream of the throat. This lack of information concerning the relation between the erosion rate, bed shear and turbulence is probably the chief impediment to progress in predicting infill rates. A field method of measuring such a relation seems to be outside the range of present techniques but useful information might be obtained in the laboratory with expanded flows over naturally formed beds. For cohesive sediments, where the beds are usually flat and sediment is taken directly into suspension without re-deposition, tracer methods would seem directly applicable provided the difficulty of incorporating the tracer sediment with the bed could be overcome.

It is desirable to have certain additional information to quantify infill. As described in Section 4, gravity can be an important factor but there is little published information on the way bed slope affects erosion and bed forms where fine sands are taken up into suspension. To study such effects laboratory experiments with sandy beds inclined transverse to the flow are clearly desirable.

When both waves and currents are present, the position is also unsatisfactory since field measurements of erosion rates or sediment flux have not been made even for level beds and we are obliged to rely on scaling factors obtained from model experiments to enable calm weather measurements to be extended to storm conditions. Since the bed forms in model experiments seldom develop to scale such extrapolations are dubious. Simultaneous field measurements of bed forms and concentrations of suspended sediment close to the bed with a self-recording meter to quantify the stirring function $f(\theta\ ,\ \Lambda/d_m)$ (equation (6.4.1)) would be valuable.

Finally, although the methods outlined in Sections 2 and 3 have been used to estimate the infill of channels there is no recorded case where such predictions have been compared with the subsequent maintenance dredging. Occasionally more limited comparisons have been made between predicted infill and that measured in a trial trench with fairly encouraging results but it would be valuable to compare the infill predicted from conventional flux measurements, supplemented by tracer experiments, with the dredging rates for an existing navigation channel.

11 CONCLUSIONS

1 To estimate the maintenance dredging required to prevent infill under the action of tidal currents, for channels at large angles to the flow ($90^o - 30^o$ approximately) it is probably sufficiently accurate to assume that for the base flow over the channel the velocity component tangential to the channel is constant; thus over the channel, the flow will be refracted and the velocity between adjacent stream lines will diminish. The infill rate can then be determined approximately by the application of equation (3.1.9), once the sediment flux at each phase of the tide is known. The latter is best obtained from measurements in the field.

A more accurate treatment, which has not so far been attempted, would require first, an integration of the flow equations over the channel, taking account of the velocity variation in depth and secondly an integration of equation (2.2.3) with the boundary conditions at the bed defining the way in which the rate of erosion (E) depends on the bed shear and turbulence.

2 Since the reductions in velocity and sediment flux are smaller when the channel is less inclined to the flow it is evidently an advantage to reduce the angle of the channel as much as possible, ie maintenance dredging can be reduced at the expense of greater capital dredging. Although the oblique channel is longer the reduction in sediment infill more than compensates for this increase in length.

3 For broad channels at small angles to the flow with gentle side slopes the reduced resistance over the channel due to the increased depth becomes important. Experiments indicate that for a given width, when the depth ratio exceeds a certain value the bed velocity averaged over the channel can be greater than the approach bed velocity. This value decreases with angle and width and channels with greater depth ratios in balanced or

nearly balanced tidal situations would, in the short term, be self-maintaining or eroding. In the long term, in such cases, it seems likely that the sediment eroded from the channel will build up the banks causing the local flow to refract towards the channel so increasing the effective angle of incidence and arresting erosion.

4 Channels dredged along the flow lines will infill mainly due to gravity. Sediment moving in close contact with the bed along the edges of the channel is subject to a gravity component causing it to be deflected towards the channel; the rate of infill due to this cause can be estimated approximately from equation (4.1.2) together with a "bed transport formula" relating the transport of sediment along the bed to the flow variables (velocity and depth). A similar gravity infill will occur in channels at a small angle to the flow.

Pressure changes due to the presence of suspended particles will tend to produce gravity currents towards the channel near the bed with a return flow at the surface and hence a net inflow of sediment. This could be an important factor causing infill in situations where the currents are weak and the suspended silt is mainly confined to a thin layer close to the bed.

5 In principle, the estimation of the rate of infill of channels dredged through beds of fine (cohesive) sediments is simpler than for cohesionless sediments in that erosion and deposition occur in distinct phases in the tidal cycle. When the bed shear exceeds a certain threshold value (τ_e), unlike cohesionless sediments, the net rate at which the bed erodes is independent of the quantity of sediment in suspension. When the bed shear is less than a lower threshold ($\tau_d < \tau_e$) deposition occurs at a rate which is proportional to the concentration of sediment in suspension near the bed. For shears intermediate between τ_e and τ_d it is usually assumed that neither deposition nor erosion takes place. In some cases the fall velocity of the sediment flocs is so small that the concentration of suspended sediment changes fairly slowly during a tide so that deposition in the channel can be obtained by differencing the quantities of sediment exchanged with the bed during the deposition and erosion phases over the tide.

The main problem in carrying out this procedure is that of quantifying the erosion and deposition rates. This involves the evaluation of τ_e and τ_d and the factor of proportionality M (equation (6.1.1)) by laboratory measurements and has in fact been carried out for comparatively few sediments. The other factors, namely the concentration and settling velocity of the sediment flocs, may be obtained directly from field measurements. The shear over the bed of the channel is usually inferred from directly measured velocity profiles over the undisturbed bed.

6 Formulae are available which relate the flux of fine sand to the wave induced oscillatory currents near the bed both without and with tidal currents present. The former is thought to be sufficiently firmly based to allow predictions of infill by extrapolation of measured infill rates from field trial dredges. When currents are present the additional bed shear caused by waves increases the rate at which sediment is eroded from the bed and convected by the current. Over a channel the added bed shear, which is dependent on the oscillatory current at the bed induced by the waves, will diminish due to the increased depth and the reduction of wave height caused by wave reflection so that the sediment flux will diminish and a larger infill occur than when waves are absent. The formula available relating the sediment flux to the current and the oscillatory bed velocities under waves is based on model experiments and there is an urgent need for field measurements.

7 Estimation of infill into dredged channels due to tidal currents is usually based on field measurements of currents and sediment flux along the proposed route. In some situations where currents are low and sediment flux is greatly increased during periods of high wave activity, tracer methods have then been employed to estimate infill. The main difficulty of such methods is that of inferring the average movement of the sediment on typical ebb and flood phases of the tide from the long term tracer distributions, since this can embrace periods of high wave activity when the thickness of the bed in movement can increase markedly. It is difficult to apply these methods in situations where the tidal currents are large and the tracer is rapidly dispersed.

8 Trial dredges are often employed to predict the likely infill into navigation channels and have the obvious advantage that they are subject to the same environmental conditions as the channel although for a limited time. Ideally the depth and width of the trial dredge should be the same as that of the navigation channel and its length should be great enough to avoid end effects. However, the method is expensive and is only practicable where dredgers are readily available. Occasionally the costs can be reduced by using smaller trial dredges (smaller depth and width than the projected channel) to "calibrate" infill determinations by conventional methods. Simple considerations suggest that for the flow and infill over the central part of the dredge to be similar to that over a long channel its length/width ratio should exceed about 8.

Where such methods have been employed agreement between predicted infill and that measured in the trial trench is within about 20%. Taken in conjunction with sediment flux, current and wave observations, they offer the best available means for the prediction of long term maintenance dredging. However such methods are costly, especially in areas where dredgers are not readily available. There is a need for comparison of these methods of estimating infill rates with the known dredging rates.

12 REFERENCES

1 Abernethy C L, Estimate of siltation in approach channels, Phitti Creek. Hydraulics Research Station, Wallingford, Report EX 575, 1971.

2 Abernethy C L, Trial dredging of Phitti Creek entrance. Hydraulics Research Station, Wallingford, Report EX 698, 1975.

3 Abernethy C L and Gilbert G, Refraction of wave spectra. Hydraulics Research Station, Wallingford, Report INT 117, 1975.

4 Antonia R A and Luxton R E, The response of a turbulent boundary layer to a step change in surface roughness, Part I, Smooth to rough. J Fluid Mech **48**, part 4, 721-761, 1971.

5 Aris R, Dispersion of a solute by diffusion, convection and exchange between phases. Proc Roy Soc 27, **252**, 538-550, 1959.

6 Bagnold R A, Autosuspension of transported sediment; turbidity currents. Proc Roy Soc A, **265**, 315-319, 1962.

7 Batchelor G K, An introduction to fluid dynamics. Camb Univ Press, 359-366, 1967.

8 Byker E W, Longshore transport computations. Proc ASCE Journ. Waterways, Harbour and Coastal Eng Div WWr 8546, 687-701, November 1971.

9 Byker E W, Kalwijk J P and Pieters T, Mass transport in gravity waves on a sloping bottom. Proc 14th Conf Coastal Eng, Copenhagen, **1**, Chap 25, 447-465, June 1974.

10 Bradshaw P and Wong F Y F, The reattachment and relaxation of a turbulent shear layer. J Fl Mech **52**, part 1, 113-135, 1972.

11 Carslaw H S and Jaeger J C, Conduction of heat in solids. Oxford Clarendon Press, 2nd Edition, 1959.

12 Crickmore M J and Lean G H, The measurement of sand transport by means of radioactive tracers. Proc Roy Soc A **266**, 402-421, 1962.

13 Crickmore M J, The use of tracers to determine infill rates in projected dredged channels, Proc Inst Civ Eng Symposium, London, 1967, Paper no 7, 55-60, 1967.

14 Engelund, F, Instability of erodible beds. J Fluid Mech, **42**, 225-244, 1970.

15 Engelund, F, Sediment dispersion inflow with moving boundaries. Discussion of a paper by Cheng-Lung Chen, Proc ASCE Journal Hyd Div, **98**, HY4, 726-728, 1972.

16 Fredsoe J, Rotational channel flow over small three-dimensional bottom irregularities. J Fluid Mech, **66**, part 1, 49-66, 1974.

17 Fresdoe, J, Sediment of river navigation channels. Submitted to ASCE for publication, June 1976.

18 Frijlink H C, Discussion des formules de debit solide de Kalinske, Einstein et Meyer-Peter et Mueller compte tenue des mesures recentes de transport dans les rivieres Neerlandaises. 2nd Journal Hydraulics Soc Hyd de France, Grenoble, France, 98-103, 1952.

19 Gole E V, Tarapore Z S and Brahms S B, Prediction of siltation in harbour basins and channels. Proc 14th IAHR Paper No 5, August 1971.

20 Harrison A J M and Owen M W, Siltation of fine sediments in estuaries. Proc 24th IAHR Conf, Paris, Paper D1, 1971.

21 Henkel D J, The role of waves in causing submarine land slides, Geotechnique 20, **1**, 75-80, 1972.

22 Hubbell D W and Sayre W W, Sand transport studies with radioactive tracers. Proc ASCE Jour Hyd Div **90**, HY3, Paper No 3900, 39-68, May 1964.

23 Hunt J N, Tidal oscillations in estuaries. Geophysical Jour of Royal Astronomical Society, **8**, No 4, 1964.

24 Kamphuis J W, Sediment transport by waves over a flat bed. 1st Australian Conf in Coastal Eng, 228-233, May 1973.

25 Komar P D and Miller M C, Sediment threshold under oscillatory waves. Proc 14th Conf Coastal Engineering, Copenhagen, Chap 44, 756-777, June 1974.

26 Krone R B, Flume studies of the transport of sediment in estuarial shoaling processes. Univ of California, Hyd Eng Lab, June 1962.

27 Lamble J, Methode de calcul approchee des ouvrages de decantation. La Houille Blanche Special B, 744-756, 1958.

28 Lean G H, Settling velocity of particles in channel flow. Stochastic hydraulics, Univ of Pittsburgh, Part 4, 339-352, May 1971.

29 Lepetit, J P, Stability of the access channel to the new harbour of Dunkirk. Proc 16th IAHR, Report No A28, 224-230, 1975.

30 Lighthill M J and Whitham G B, On kinematic waves, Part I and II. Proc Roy Soc A, **229**, 281-345, May 1955.

31 Longuet-Higgins M S, Mass transport in water waves. Phil Trans Roy Soc A **243**, 903,245,535-581, 1953.

32 Longuet-Higgins M S, The mechanism of the boundary layer near the bottom in a progressive wave. Proc 6th Conf Coastal Eng, Chap 10, 171-193, 1958.

33 Madsen O S, Stability of a sand bed under breaking waves. Proc 14th Conf Coastal Eng, Copenhagen, **II**, 776-794, June 1974.

34 Mehta A J and Partheniades E, On the depositioned properties of estuaries' sediments. Proc 14th Conf Coastal Eng, Copenhagen, **II**, Chap 72, 1232-1251, June 1974.

35 Meyre-Peter E and Muller R, Formulae for bed load transport. Proc 2nd Congress, IAHR, Stockholm, June 1948.

36 Migniot C, Etude des proprietes physique de different sediments tres fins et de leur comportement sans des actions hydrodynamiques. La Houille Blanche, Nov 7, 591-620, 1968.

37 Mitchell R J, Tsui K K and Sangrey D A, Failure of submarine slopes under wave action. Proc 13th Coastal Eng Conf, Vancouver, **2**, Chap 84, 1515-1541, 1972.

38 Mitchell R J and Hall J A, Stability and bearing capacity of bottom sediments. Proc 14th Coastal Eng Conf, Copenhagen, **2**, Chap 73, 1252-1273, 1974.

39 O'Connor B A, Siltation in dredged channels. First Int Symposium on Dredging Technology, E2, September 1975.

40 O'Connor B A and Lean G H, Estimation of siltation in dredged channels in open situations. PIANC Leningrad, Section II, Subject 2, September 1977.

41 Odd N V M and Owen M W, A two layer model of mud transport in the Thames estuary. Proc Inst Civil Eng, Paper No 75175 and discussion 1972.

42 Owen M W, Properties of a consolidating mud. Hyd Res Station, Wallingford, Report No INT 83, December 1970.

43 Owen M W, The effect of turbulence on the settling velocities of silt flows. Proc 14th IAHR Conf, Paris, Paper D4, 1971.

44 Owen M W, Erosion of Avonmouth mud. Hyd Res Station, Wallingford, Report No INT 150, Second Impression, Nov 1977.

45 Partheniades E, A study of erosion and deposition of cohesive soils in salt water. PhD Thesis, Univ of California, 1962.

46 Partheniades E, Erosion and deposition of cohesive soils. Proc ASCE, Hyd Div, **91**, HY1, 105-139, 1965.

47 Partheniades E, Cross R H and Ayora A, Further results on the deposition of cohesive sediments, Chap 47. Proc 11th Conf Coastal Eng, London, 1968.

48 Rance P J, Sand transport due to wave action. Proc 10th Conf Coastal Eng, Tokyo, Extra 6, 1-5, September 1968.

49 Rouse H, Engineering Hydraulics, John Wiley & Sons Inc. Proc 4th Hyd Conf, Iowa, Inst, Chap XII, 799-803, June 1949.

50 Russell R C H and Osorio J D C, An experimental investigation of drift

profiles in a closed channel. Proc 6th Conf Coastal Eng, Chap 10, 171-193, 1958.

51 Tani I, Review of some experimental results on the response of a turbulent boundary layer to sudden perturbation. Computations of Turbulent Layers Conference, **1**, 483-494, 1968.

52 Thorn M F C, Deep tidal flow over a fine sand bed. Proc 16th IAHR Congress, A128

53 US Task Committee for Preparation of Sediment Manual: H sediment discharge formulae. Jour Hyd Div Proc ASCE **96**, HY4, Paper No 8076 523-567, April 1971.

54 Vinje J J, Siltation in dredged trenches. Delft Hyd Lab Pub No 59, October, 1968.

55 Whillock A F, Laboratory studies of flow across dredged channels. Hydraulics Research Station, Wallingford, Report No EX 618, June 1973.

56 White C M, The equilibrium of sand grains on the bed of a stream. Proc Roy Soc A **174**, 322, 1940.

57 White W R, Mille H and Crabbe A D, Sediment transport theories, a review. Proc Inst Civ Eng. Part 2, **59**, 265-292, June 1975.

58 Yalin M S, Mechanics of sediment transport. Pergamon Press, 2nd Edition, 1977.

APPENDICES

APPENDIX 1

Initial infill rates into a channel in steady flow

We consider the case of steady flow (uniform velocity U_o, depth h_o) approaching a channel at right angles, represented by a step change in depth at x = 0, in which the concentration (C_o) of suspended solids over the depth in the incident flow is in equilibrium and the concentration (C_{ob}) at the bed from equation (2.2.3) is given by

$$V_S C_{ob} = W_o C_* \qquad \text{....(A1.1)}$$

The vertical diffusion coefficient K_Z is assumed constant (K) and equal to its mean value over a section. For a logarithmic distribution

$$K = \kappa U_* h/6 \qquad \text{....(A1.2)}$$

in which κ is the Von Karman constant and U_* is the friction velocity. For a given bed roughness U_*/U is approximately constant, hence K is the same over the channel as in the incident flow.

The rate at which sediment is dispersed upwards from the bed is

$$- K \, \partial C/\partial z, \qquad z = 0 \qquad \text{....(A1.3)}$$

so that in the incident flow

$$- K \, \partial C_o/\partial z = W_o C_* \qquad z = 0 \qquad \text{....(A1.4)}$$

At the upstream edge of the channel (x = 0) it is assumed that the velocity falls to U_1 giving rise to an immediate reduction in the erosion velocity to W_1, which remains constant over the channel. Thus the bed concentration over the channel is given by

$$- K \, \partial C/\partial z = W_1 C_* \qquad z = 0 \qquad \text{....(A1.5)}$$

In the steady flow of the suspension, the diffusive transport upward is balanced by the rate of fallout and convection of particles through a section, ie

$$K \, \partial^2 C/\partial z^2 + V_S \, \partial C/\partial z - U_1 \, \partial C/\partial x = 0 \qquad \text{....(A1.6)}$$

and to obtain the way in which the sediment flux changes over the channel, this equation must be solved with the bed condition (A1.4) and the condition of zero net transport at the surface, ie

$$K \, \partial C/\partial z + V_S C = 0 \qquad z = h_1 \qquad \text{....(A1.7)}$$

Integrating (A1.6) over the depth with the boundary conditions (A1.5) and (A1.7) gives

$$\partial S/\partial x = W_1 C_* - V_S C_b(x) \qquad \text{....(A1.8)}$$

which enables the change of sediment flux (S) and hence the infill rate to be obtained once the bed concentration has been determined.

We note that at a great distance downstream from the edge of the channel $\partial C/\partial x = 0$ and integrating (A1.6) over the depth then gives

$$C = C_b(\infty) \exp(-V_S z/K) \qquad \text{....(A1.9)}$$

and the transport, writing $S(\infty) = S_1$, becomes

$$S_1 = C_b(\infty)[U_1 K/V_S] \, [1 - \exp(-V_S h/K)] \qquad \text{....(A1.10)}$$

in which

$$C_b(\infty) = W_1 C_*/V_S \qquad \text{....(A1.11)}$$

Similar relationships apply to the vertical concentration distribution in the approach flow with the bed concentration given by (A1.1) and

$h = h_o$.

To simplify the problem we assume that the change in depth or velocity over the channel is significant only in so far as it effects the entrainment rate. Thus we assume that the initial concentration distribution at entry to the step is given by

$$C_o = (W_o C_*/V_S) \exp[-V_S z/K] \qquad x = 0 \qquad \text{....(A1.12)}$$

and

$$S_o = (W_o C_*/V_S)[U_1 K/V_S][1-\exp(-V_S h_1/K)] \qquad x = 0 \qquad \text{....(A1.13)}$$

In order to solve equation (A1.6) it is convenient to consider changes in the concentration rather than the concentration itself; thus we define

$$\underline{C}(x,z) = C(x,z) - C_o(z) \qquad \text{....(A1.14)}$$

which satisfies (A1.6) and the change in the sediment flux from the initial rate satisfies (A1.8) which may be written

$$\partial \underline{S}/\partial x = (W_1 - W_o)C_* - V_S \underline{C}_b(x) \qquad \text{....(A1.15)}$$

In addition, we remove the second term in (A1.6) by substituting

$$C_1(x,z) = \underline{C}(x,z) \exp(-V_S z/2K) \qquad \text{....(A1.16)}$$

so that this equation becomes

$$K\, \partial^2 C_1/\partial z^2 - V_S{}^2 C_1/4K - U_1\, \partial C_1/\partial x = 0 \qquad \text{....(A1.17)}$$

and the boundary conditions (A1.5), (A1.7) become

$$(V_S C_1/2) - K\, \partial C_1/\partial z = A \qquad z = 0 \qquad \text{....(A1.18)}$$

$$(V_S C_1/2) + K\, \partial C_1/\partial z = 0 \qquad z = h_1 \qquad \text{....(A1.19)}$$

and $C_1 = 0$ at $x = 0$ ....(A1.20)

Equation (A1.5) also becomes

$$\partial \underline{S}/\partial x = A - V_S C_{1b}(x) \qquad \text{....(A1.21)}$$

in which we have written

$$A = (W_1 - W_o)C_* \qquad \text{....(A1.22)}$$

To solve these equations we use the method of Laplace transforms defined by

$$\bar{C}(z) = \int_o^\infty C_1(x,z) e^{-px}\, dx \qquad \text{....(A1.23)}$$

Thus multiplying (A1.7) by e^{-px} and integrating with respect to x with the condition (A1.20), the equation is reduced to an ordinary linear differential equation

$$K\, d^2\bar{C}/dz^2 - (U_1 p + V_S{}^2/4K)\bar{C} = 0 \qquad \text{....(A1.24)}$$

which has the solution

$$\bar{C} = B_1 \exp(\lambda z) + B_2 \exp(-\lambda z) \qquad \text{....(A1.25)}$$

with

$$\lambda = (p + V_S{}^2/4KU_1)^{1/2}(U_1/K)^{1/2} \qquad \text{....(A1.26)}$$

in which the constants B_1, B_2 can be determined from the transformed boundary conditions (A1.18), (A1.19) viz:

$$(V_S\bar{C}/2 - K\, d\bar{C}/dz) = A/p \qquad z = 0 \qquad \text{....(A1.27)}$$

$$(V_S\bar{C}/2 + K\, d\bar{C}/dz) = 0 \qquad z = h_1 \qquad \text{....(A1.28)}$$

The transform of equation (A1.21) becomes

$$\bar{S} = A/p^2 - V_S\, \bar{C}_b/p \qquad \text{....(A1.29)}$$

in which from (A1.25)

$$\bar{C}_b = B_1 + B_2 \qquad \text{....(A1.30)}$$

which, substituting (A1.27) and (A1.28) in (A1.25), becomes after a little manipulation

$$\bar{S} = A[1-V_S\{\frac{\sinh\lambda h/2}{V_S\sinh\lambda h/2+2\lambda K\cosh\lambda h/2} + \frac{\cosh\lambda h/2}{V_S\cosh\lambda h/2+2\lambda K\sinh\lambda h/2}\}]/p^2 \qquad \text{....(A1.31)}$$

Transforming by standard methods we find

$$\bar{S} = AU_1K\{\frac{2\sinh 2\epsilon}{\cosh 2\epsilon+\sinh 2\epsilon} - 8\epsilon^3[S(\alpha_n) + S(\beta_n)]\}/V_S^2 \qquad \text{....(A1.32)}$$

where

$$\epsilon = V_S h_1/4K, \qquad \text{....(A1.33)}$$

α_n, β_n are the positive roots of

$$\alpha\tan\alpha = \epsilon, \qquad \text{....(A1.34)}$$

$$\beta\cot\beta + \epsilon = 0 \qquad \text{....(A1.35)}$$

and the sums $S(\alpha_n)$, $S(\beta)$ are given by

$$S(\theta_n) = \sum_{n=1}^{n=\infty} \theta_n^2 \exp[-(\theta_n^2+\epsilon^2)V_S x/\epsilon h_1 U_1]/(\theta_n^2+\epsilon^2)^2(\theta_n^2+\epsilon+\epsilon^2) \qquad \text{....(A1.36)}$$

We may express A in terms of the sediment flux change between that at entry S_o and that at a great distance downstream S_1 from equations (A1.13) and (A1.10). Thus

$$S_1 - S_o = (AU_1 K/V_S^2)[1-\exp(-4\epsilon)] \qquad \text{....(A1.37)}$$

and (A1.32) becomes

$$(S-S_o)/(S_1-S_o) = \{1 - 8\epsilon^3[S(\alpha_n) + S(\beta_n)]/[1-\exp(-4\epsilon)]\} \qquad \text{....(A1.38)}$$

When ϵ is very small, only the root (α_1) is important, which by (A1.34) becomes

$$\alpha_1 = \epsilon^{1/2} \qquad \text{....(A1.39)}$$

and (A1.38) becomes

$$(S-S_o)/(S_1-S_o) = 1-\exp(-V_S x/h_1 U_1) \qquad \text{....(A1.40)}$$

which agrees with equation (2.2.9).

In most practical cases, ϵ is small, and the first few terms of $S(\alpha_n)$ and $S(\beta_n)$ are required to achieve an adequate approximation.

The variation of the infill ratio given by (A1.38) has been calculated for different values of V_S/U_* using the relation (A1.2) which gives

$$\epsilon = 3\,V_S / 2\,\kappa\, U_* \qquad \text{....(A1.41)}$$

and taking $\kappa = 0.4$, which is applicable to suspensions in which the concentrations are weak.

APPENDIX 2

Bed and sediment flux changes downstream of a step in steady flow; "well-stirred case"

In order to clarify the way in which the finite settling velocity of the sediment gives rise to horizontal diffusive effects it is of interest to consider a simple case in which an analytical solution is available and to compare this solution with that assuming an effective diffusion coefficient (D) given by equation (2.2.19). We suppose that the sediment is sufficiently fine to be uniformly distributed over the depth at a certain concentration C and that to maintain a quantity in suspension particles falling to the bed are picked up again at the rate WC_*, in which C_* is the concentration in the bed and W is an erosion velocity.

Upstream of the channel the suspension and bed are in equilibrium at concentration C_o. Also Z = 0, so that if V_S is the settling velocity of the particles

$$W_oC_* = V_SC_o \qquad \text{....A2.1)}$$

and the sediment flux

$$S_o = qC_o = qW_oC_*/V_S \qquad \text{....(A2.2)}$$

At x = 0, t = 0 we suppose the flux encounters a small step in the bed of height Z_o (Z_o negative for a dredged channel) equivalent to a very wide channel and we shall examine the subsequent changes in C and Z.

Downstream of the step the erosion velocity changes to W ($W - W_o < 0$ for Z_o negative), which we assume is a function of the local velocity only. If the flux of sediment is represented by a power law with exponent m (equation (2.1.9)), and the discharge (of water) is constant, for small changes of bed elevation we may write

$$WC_* = W_oC_*(1 + mZ/h_o) \qquad \text{....(A2.3)}$$

And at a large distance downstream of the step ($x = \infty$), $Z = Z_o$ and the sediment flux (S_1) is given by

$$qC_\infty = qW_oC_*[1 + (m\,Z_o/h_o)]/V_S \qquad \text{....(A2.4)}$$

Thus the ultimate change of flux

$$(S_1 - S_o) = qW_oC_*m\,Z_o/h_o\,V_S = q\,m\,C_o\,Z_o/h_o \qquad \text{....(A2.5)}$$

(a) Analytical solution

If U, h are the velocity and depth over the step, from continuity of the sediment flux, we have

$$h\frac{\partial C}{\partial t} + Uh\frac{\partial C}{\partial x} = WC_* - V_SC \qquad \text{....(A2.6)}$$

$$-\frac{\partial Z}{\partial t} = WC_* - V_SC \qquad \text{....(A2.7)}$$

Substituting for WC_* from (A2.3), for Z small compared with h, these equations may be approximated by

$$\frac{\partial \underline{C}}{\partial t} + U_o\frac{\partial \underline{C}}{\partial x} = \frac{1}{h_o}[m\,W_oC_*\frac{Z}{h_o} - V_S\underline{C}] \qquad \text{....(A2.8)}$$

$$-\frac{\partial Z}{\partial t} = m\,W_oC_*\frac{Z}{h_o} - V_S\underline{C} \qquad \text{....(A2.9)}$$

in which $\underline{C}$ is the departure of the concentration from the initial concentration C_o, ie

$$\underline{C} = C - C_o \qquad \text{....(A2.10)}$$

Equations of this type are well known in the theory of heat conduction (Carslaw and Jaeger, p 393).

Putting

$b \;=\; m\,W_o C_*/h_o \;=\; m\,V_S\,C_o/h_o$ ....(A2.11)

$b^1 \;=\; V_S/h_o$ ....(A2.12)

$a \;=\; bb^1/U_o$ ....(A2.13)

their solution for the case of an initial step change Z_o and $t > x/U_o$ may be written

$$\underline{C} \;=\; \frac{b^1}{b+b^1}\,\frac{m\,C_o Z_o}{h_o}\,[N_1(x,t) + N_2(x,t)] \qquad \text{....(A2.14)}$$

with

$$N_1(x,t) = \frac{b}{U_o}\,e^{-bt}\int_o^x e^{(-b^1 x/U_o\,+\,b\,x^1/U_o)}\,I_o\,[4ax^1(t-\frac{x}{U_o})]^{\frac{1}{2}}\,dx^1, \qquad \text{....(A2.15)}$$

$$N_2(x,t) = \frac{b^1}{U_o}\,e^{-bt}\int_o^x e^{(bx/U_o\,-\,b^1 x^1/U_o)}\,I_o\,[4ax^1(t-\frac{x}{U_o})]^{\frac{1}{2}}\,dx^1 \qquad \text{....(A2.16)}$$

I_o is the modified Bessel function of the first kind and of zero order.

The bed elevation (Z) can be derived directly from equations (A2.9) and (A2.14).

Generally since the suspended concentrations are small and

$$\frac{b}{b^1} \;=\; \frac{m\,W_o C_*}{V_S} \;=\; m\,C_o \qquad \text{....(A2.17)}$$

we may ignore the first term in (A2.14) and thus we have

$$\underline{C} \;=\; \frac{m\,C_o Z_o}{h_o}\,N_2(x,t) \qquad \text{....(A2.18)}$$

$$Z \;=\; Z_o\left\{N_2(x,t) + e^{-bt}\,e^{-b^1 x/U_o}\,I_o\,[4ax(t-\frac{x}{U_o})]^{\frac{1}{2}}\right\} \qquad \text{....(A2.19)}$$

We note that when t is small and hence x is small, I_o = 1 (approximately) and

$$\underline{C} \;=\; \frac{m\,C_o Z_o}{h_o}\,e^{-bt}\,[1 - \exp(-b^1 x/U_o)] \qquad \text{....(A2.20)}$$

$$Z/Z_o = e^{-bt}[1 + bt(1 - \exp(-b^1 x/U_o))] \qquad \text{....(A2.21)}$$

Thus for b = 0 (corresponding to no change of bed elevation downstream of the step from equation (A2.6) with (A2.8)), the change of sediment flux downstream of the step, $q\underline{C}$, agrees with equations (2.2.9), (2.2.10) since $(q\,m\,C_o Z_o/h_o)$ is equal to $(S_o - S_1)$.

An example of the concentration and bed profiles $(\underline{C}/(m\,C_o Z_o/h_o), Z/Z_o)$ at different times calculated from equations (A2.18) and (A2.19) are shown in Fig 3. In this example, the approach velocity (U_o) and depth (h_o) have been taken as 1200 m/hour and 10 m respectively. The sediment is assumed to have a size of 0.2 mm (V_S = 96 m/hour) so that b^1 = 9.6 $(\text{hour})^{-1}$. At a great distance downstream of the step (Z_o = 1 m) the reduction of velocity is assumed to reduce the sediment transport, $(S_o - S_1)$, by 5.0 $m^3/h/m$ width of compacted sediment.

So from (A1.5) and (A1.17)

$b/b^1 = 1/240$

and the kinematic velocity

$$c = (\frac{dS}{dZ}) = \frac{S_o - S_1}{Z_o} \text{ (approximately)} = 5 \text{ m/hour.}$$

It will be noted that initially, the deposit downstream of the step thickens until it attains the upstream bed level and thereafter it slowly spreads downstream, the centroid of the bed profile moving downstream at the kinematic wave velocity. As expected, the bed profiles lag the concentration profiles by the distance L given by equation (2.2.10) or U_o/b^1, which in the present example is 125 metres.

(b) "Diffusion" solution

The flux of sediment (S) at a section is given by

$$S = qC \qquad \text{....(A2.22)}$$

in which q is the discharge of water per unit width ($q = U_o h_o$). The concentration (C) is given by

$$C = \frac{WC_*}{V_S} = \frac{W_o C_*}{V_S}(1 + mZ/h_o) \qquad \text{....(A2.23)}$$

from equation (A2.3). Thus the kinematic wave velocity (c) defined by equation (2.1.4) becomes

$$c = \frac{\partial S}{\partial Z} = m U_o W_o C_*/V_S = U_o b/b^1 \qquad \text{....(A2.24)}$$

from equation (A2.17).

Also the diffusion coefficient (D) for the present case is given by (equations (2.2.10) and (2.2.21)

$$D = cL = c U_o h_o/V_S = c U_o/b^1 \qquad \text{....(A2.25)}$$

Inserting $S = qC$ in equation (2.2.20) we find for the departure of the concentration from that in the approach flow

$$\frac{\partial \underline{C}}{\partial t} + c\frac{\partial \underline{C}}{\partial x} - D\frac{\partial^2 \underline{C}}{\partial x^2} = 0 \qquad \text{....(A2.26)}$$

The solution of this equation for an initial step change in C at x = 0, ie $\underline{C} = 0$ for $x = 0$ and $\underline{C}_\infty = C_o$ for $x > 0$ (C_∞ is given by equation (A2.4) and C_o by equation (A2.1)) is given by

$$\underline{C}/\underline{C}_\infty = 1 - ½ [\text{erfc}\frac{x-ct}{2\sqrt{Dt}} + e^{cx/D}\,\text{erfc}\frac{x+ct}{2\sqrt{Dt}} \qquad \text{....(A2.27)}$$

and this equation together with the equation of continuity (2.1.2) gives

$$Z/Z_o = 1 - ½ [\frac{ct}{\sqrt{Dt}}\frac{2}{\sqrt{\pi}}e^{-(x-ct)^2/4Dt} + \text{erfc}\frac{x-ct}{2\sqrt{Dt}}$$

$$- e^{cx/D}\{1 + \frac{c}{D}(x+ct)\}\,\text{erfc}\frac{x+ct}{2\sqrt{Dt}}] \qquad \text{....(A2.28)}$$

These expressions are compared with the analytical solution for the example previously given in Fig 3. It will be noted that after a long time, the agreement is fairly close and that in this case also, as might be expected, the bed profiles lag the concentration profiles by the distance L = 125 metres.

APPENDIX 3

Concentration of sediment in steady flow

Fine sands and silts which usually compose bed sediments in estuaries are mainly transported in suspension. The suspended load transport is given by

$$S_s = \int_b^h C(z)\, U(z)\, dz \qquad \text{....(A3.1)}$$

in which

$$C(z) = C_b \left(\frac{h-z}{z}\, \frac{b}{h-b}\right)^{Z*}, \qquad \text{....(A3.2)}$$

$$u(z) = (U_*/K) \ln (33z/k_s) \qquad \text{....(A3.3)}$$

$$Z_* = V_S/\kappa U_* \qquad \text{....(A3.4)}$$

κ = Von Karman constant usually taken as 0.4

k_s = equivalent hydraulic roughness of the bed forms

and

C_b = concentration at some reference level, usually taken at a small height b above the mean bed level.

The determination of C_b is central to the problem of relating the sediment flux or erosion rate ($WC_* = V_S C_b$) to the flow parameters. Usually it is assumed to be given by equation (2.3.6) viz:

$$C_b = S_b/U_b\, d_b \qquad \text{....(A3.5)}$$

in which S_b denotes the volume flux of sediment engaged in bed transport per unit width transverse to the flow and U_b, d_b are the average velocity and thickness of the layer over which this flux is assumed to be distributed. None of these quantities can be defined precisely and may, indeed, have little physical significance. Thus S_b is usually taken as referring to the fraction of the total particles in movement whose weight is directly taken by the bed, ie by contact with other bed particles. (In contrast the weight of the suspended particles is taken by the increased hydrostatic pressure.) However, in the case of fine sands, individual particles can sometimes move in suspension and sometimes in close contact with the upstream faces of the bed forms. This may account, to some extent, for the wide variety of formulae which have been proposed relating S_b to the hydraulic parameters *(see Note)*.

Note:

In view of these difficulties some investigators have abandoned the notion of separating the sediment flux into suspended and bed movement phases and deal only with the total flux, a quantity which is more readily measured in the laboratory. Unfortunately this approach, by itself, gives no information on the rate of erosion of sediments from the bed, which, as we have seen, is of crucial importance in channel infill.

It is helpful to realize that sediment load is derived from an equation containing C_b, which is derived from S_b which is bed transport.

In the case of coarse sands which at low transport rates move almost entirely in the bed phase, S_b is less equivocal and can be measured either by trapping or by measuring the average volume of the bed forms and the number passing a given point in unit time.

In order to prescribe S_b for finer sands, formulae derived from coarse sand experiments have been extrapolated to the fine sand range. Bijker, 1971, takes the formula suggested by Frijlink, 1952, which is based on measurements in the laboratory and in rivers and which takes account explicitly of the influence of bed forms, viz;

$$S_b = 5D\mu^{1/2}\, U_*\, \exp(-0.27/\mu_r\, \theta) \qquad \text{....(A3.6)}$$

in which

$$\theta = U_*^2/(s-1)\, gd_m \qquad \text{....(A3.7)}$$

In these formulae

S_b is the volume rate in terms of the volume of compacted sand per metre width/second, D is the particle diameter in metres,

s is the specific gravity of grains,

$U_* = \sqrt{\tau/\rho}$ in metres/second where τ = bed shear and ρ = density, and

μ_r is a ripple coefficient representing that part of the total bed shear (τ) that is available for transporting sediment. Generally, μ_r is taken equal to $(V_*(d_{90})/V_*)^{3/2}$

in which

$V_*(d_{90})$ = shear velocity which would crest at the given mean velocity if the bed were flat and composed of uniform grains of size exceeded by 10% of the bed material (see note).

Note:

If the high turbulence generated by the bed forms is assumed to reduce the boundary layer thickness over the surface below the size of the bed grains then

$$\mu = [\ln(33\frac{h}{k_s})/\ln(33\frac{h}{d_{90}})]^{3/2}$$

(approximately ....(A3.8)

To determine C_b, Bijker, 1971, suggests that the particles engaged in bed transport should be assumed to be uniformly distributed over a layer of thickness equal to the average elevation of the bed ripples, ie d_b = r where r = half the average ripple height so that the particles are travelling at the mean flow velocity in this layer. On the basis of some unpublished work by van Breugel, Bijker assumes that for a rippled bed z in equation (A3.2) should be measured from the average bed level, ie distance r below the ripple crests and that the effective roughness k_s is also equal to r. He then deduces that the average velocity in the layer is $6.35U_*$ so that the discharge in the layer is $6.35\ U_*r$ and

$$C_b = S_b/6.35\ U_*r \qquad \text{....(A3.9)}$$

The discharge of water in the rippled layer may be obtained by the following argument. The shear at the level of the ripple crests (ρU_*^2) is almost wholly resisted by the form drag of the ripples, and falls to zero at the level of the ripple troughs. It would seem reasonable to assume a linear variation of shear (τ) at levels above the troughs (z = 0) so that

$$\tau/\rho = U_*^2\ z/2r \qquad 0 < z < 2r \qquad \text{....(A3.10)}$$

If we assume that Prandtl's expression for the turbulent shear

$$\tau/\rho = l^2 \frac{dU}{dz} \left| \frac{dU}{dz} \right| \qquad \text{....(A3.11)}$$

can be applied in this region and that here also the mixing length (l) is given by

$$l = \kappa z \qquad \text{....(A3.12)}$$

in which k is the Karman constant equal to 0.4, we find

$$\frac{dU}{dz} = U_*/\kappa\,(2rz)^{1/2} \qquad \text{....(A3.13)}$$

so the velocity in the layer is given by

$$U = 2U_* z^{1/2}/\kappa\,(2r)^{1/2} \qquad \text{....(A3.14)}$$

Hence the discharge of water through the layer

$$\int_0^{2r} U dz = 6.7\ U_*r \qquad \text{....(A3.15)}$$

which is in fairly close agreement with that given by Bijker.

It may be noted that recent measurements (Antonia and Luxton, 1971) also support the half power velocity profile (equation (A3.14)) close to a rough bed although the arguments advanced for its existence are different from those given here.

The total transport in suspension (A3.1) is given by

$$S_s = 1.83\ S_b\ I_1\ \ln 33\ h/r + I_2 \qquad \text{....(A3.16)}$$

with

$$I_1 = 0.216\ (\frac{b}{h})^{(Z_*-1)}\ (1-\frac{b}{h})^{-Z_*} \int_{b/h}^{1} (1-\frac{z}{h})^{Z_*} (\frac{z}{h})^{-Z_*} d(\frac{z}{h}) \qquad \text{....(A3.17)}$$

and

$$I_2 = 0.216\ (\frac{b}{h})^{(Z_*-1)}\ (1-\frac{b}{h})^{-Z_*} \int_{b/h}^{1} (1-\frac{z}{h})^{Z_*} (\frac{z}{h})^{-Z_*} \ln(\frac{z}{h})\ d(\frac{z}{h})$$

....(A3.18)

APPENDIX 4

Application of Aris method to determine lag distance

(a) Derivation of equations

We assume that the rate of erosion (WC_*) is proportional to the flux of solids which would be carried by a stream flowing at the local depth and velocity, ie

WC_* is proportional to U^m

conforming with equation (2.1.9) or

WC_* is proportional to $[q/(h_o-Z]^m$

in which q is the water discharge per unit width and h_o is the depth upstream of the channel. Thus if (W_oC_*) is the rate of erosion at the bed upstream, for small changes of bed elevation (Z)

$WC_*-W_oC_* = (W_oC_*)\ mZ/h_o$

Thus equation (2.3.4) and (2.3.5) become

$$\frac{\partial Z}{ot} = V_S C_b - W_o C_*(1+mZ/h_o) \qquad \text{....(A4.1)}$$

$$-(K_z \frac{\partial C}{\partial z})_b = W_o C_*(1+mZ/h_o) \qquad \text{....(A4.2)}$$

in which subscript b refers to the bed.

It is convenient to consider departures of the concentration of suspended solids from that existing upstream, so we write

$$\underline{C} = C - C_o \qquad \text{....(A4.3)}$$

in which C_o is the concentration upstream given by

$$C_o = C_{ob} \exp [\int_o^z - \frac{V_S}{K\psi(z)}\ dz]\ , \qquad \text{....(A4.4)}$$

with

$$C_{ob} = W_o C_*/V_S \qquad \text{....(A4.5)}$$

and the equations (2.3.1) – (2.3.4) become

$$\left.\begin{array}{lll} K\psi(z)\frac{\partial \underline{C}}{\partial z} + V_S \underline{C} = 0 & z = h_o & \text{(a)} \\ \frac{\partial \underline{C}}{\partial t} + U\phi(z)\frac{\partial \underline{C}}{\partial x} = \frac{\partial}{\partial Z}(K\psi(z)\frac{\partial \underline{C}}{\partial z} + V_S \underline{C}) & 0 < z < h_o & \text{(b)} \\ \frac{\partial Z}{\partial t} = V_S\, \underline{C}_b - gZ & z = 0 & \text{(c)} \\ -K\psi(o)\frac{\partial \underline{C}}{\partial z} = gZ & z = 0 & \text{(d)} \end{array}\right\} \text{....(A4.6)}$$

in which

$$g = W_o C_*\, m/h_o$$
$$= V_S C_{ob}\, m/h_o \qquad \text{....(A4.7)}$$

(b) Derivation of moment equations

Following Aris, we cease to concern ourselves with the details of the variation of the concentration $\underline{C}$ (x,z,t) and bed elevation Z(x,t) over the channel but rather consider the moments of these quantities in the x direction referred to an origin which is moving with the channel.

Let U_c be the velocity of the origin, which for the moment is unspecified and let x, be the new co-ordinate in the x direction referred to this origin, ie

$$x_1 = x - U_c t \qquad \text{....(A4.8)}$$

We define moments of concentration and bed elevation thus

$$C^{(p)} = \int_{-\infty}^{\infty} x_1{}^p \, \underline{C}(x,z,t)dx_1 \qquad \text{....(A4.9)}$$

$$Z^{(p)} = \int_{-\infty}^{\infty} x_1{}^p \, Z(x_1,t)dx_1 \qquad \text{....(A4.10)}$$

and to obtain equations for these quantities we multiply equations (A4.6) by $x_1{}^p$ and integrate, noting that $\underline{C}$, Z and their derivatives are zero at the limits ($x_1 = \pm \infty$). Thus we find

$$\left.\begin{aligned}
&K\psi(z) \frac{\partial C^{(p)}}{\partial z} + V_S C^{(p)} = 0 && z = h_o && \text{(a)}\\
&\frac{\partial C^{(p)}}{\partial t} - (U\phi(z) - U_c)\, p\, C^{(p-1)} = \frac{\partial}{\partial z}(K\psi \frac{\partial C^{(p)}}{\partial z} + V_S C^{(p)}) && && \text{(b)}\\
& && 0 < z < h_o && \\
&\frac{\partial Z^{(p)}}{\partial t} + U_c\, p\, Z^{(p-1)} = V_S C_b{}^{(p)} - g\, Z^{(p)} && z = 0 && \text{(c)}\\
&- K\psi(0) (\frac{\partial C^{(p)}}{\partial z})_b = g\, Z^{(p)} && z = 0 && \text{(d)}
\end{aligned}\right\} \text{....(A4.11)}$$

Integrating (A4.11(b)) with respect to z over the depth (0,h) using the surface condition (A4.11(a)) and bed condition for

$C_b{}^{(p)}$ and $(\frac{\partial C^{(p)}}{\partial z})_b$ gives

$$\frac{d}{dt}[Z^{(p)} + \int_o^{ho} C^{(p)}\, dz] = - U_c\, p\, Z^{(p-1)} + \int_o^{ho} (U\phi - U_c)\, p\, C^{(p-1)}\, dz \qquad \text{....(A4.12)}$$

(c) Initial conditions

We assume that initially (t = 0) the channel can be represented by a delta function situated at x = 0 such that its area is given by

$$Z^{(o)} = Z_o X \qquad \text{....(A4.13)}$$

with Z = 0 except at $0 < x < X$ where X is small and

$$Z^{(1)},\ Z^{(2)} \text{ etc} = 0 \text{ at } t = 0 \qquad \text{....(A4.14)}$$

To find the pth moment from the equations (A4.11), which are essentially recurrence formulae, we need to solve for the (p−1)th moment. To simplify the calculations we shall ignore the time variation of concentration in equation (A4.11(b)), ie we assume that changes in bed elevation are entirely brought about by convection of the suspended solids by the flow and exchange with the bed. Comparison with the results of Aris in an analgous situation (where the time dependence was retained), shows that this amounts to examining the asymptotic behaviour of the moments after a long time.

(d) Zero moments $C^{(o)}$, $Z^{(o)}$

Substituting p = o in equations (A2.12) and (A2.11(c)) gives

$$Z^{(o)} = \text{constant} = Z_o X \qquad \text{....(A4.15)}$$

and

$$C_b^{(o)} = g Z_o X/V_S \qquad \text{....(A4.16)}$$

and solving (A2.11(b)) gives

$$C^{(o)} = C_b^{(o)} \exp\left[\int_o^z - \frac{V_S}{K\psi(z)}\, dz\right] \qquad \text{....(A4.17)}$$

which also evidently satisfies (A4.11(a)).

(e) First moments $C^{(1)}$, $Z^{(1)}$

Let us choose U_c so that $dZ^{(1)}/dt = 0$; since $Z^{(1)} = 0$ at $t = 0$ it follows that

$$Z^{(1)} = 0 \qquad \text{....(A4.18)}$$

always. Thus this choice of U_c is equivalent to adjusting the position of the origin of x to coincide with the centroid of the channel section.

To satisfy this condition we have from (A2.12)

$$U_c Z_o X = \int_o^{h_o} (U\phi - U_c) C^{(o)}\, dz \qquad \text{....(A4.19)}$$

or

$$U_c = \int_o^{h_o} U\phi\, C^{(o)}\, dz / (Z_o X + \int_o^{h} C^{(o)}\, dz) \qquad \text{....(A4.20)}$$

Generally the term $\int_o^{h_o} C^{(o)} dz$ which denotes the total quantity of material in suspension at any time is very small by comparison with the volume of the channel and hence can be neglected. Substituting for $C^{(o)}$ from (A4.17) in (A4.20) gives

$$U_c = \frac{g}{V_S} \int_o^{h_o} U\phi \exp\left[\int_o^z -(V_S/K\psi) dz\right] \text{ (approximately)} \qquad \text{....(A4.21)}$$

It is not difficult to show that

$$U_c = c \text{ (approximately)} \qquad \text{....(A4.22)}$$

in which c denotes the kinematic wave velocity defined by equation (2.1.4); thus since Z is small compared with h_o, the sediment flux is given by

$$S = \int_o^{h_o} U\phi\, C(z)\, dz \qquad \text{....(A4.23)}$$

$$= \int_o^{h_o} U\phi (Co + \underline{C})\, dz \qquad \text{....(A4.24)}$$

Thus

$$c = \frac{dS}{dZ} = \int_o^{h_o} U\phi \frac{\partial \underline{C}}{\partial Z}\, dz \qquad \text{....(A4.25)}$$

in which

$$\underline{C} = (W_o C_* m Z / V_S h_o) \exp\left[-\int_o^z (V_S/K\psi)\, dz\right] \qquad \text{....(A4.26)}$$

from which with (A4.7), equation (A4.22) follows.

To obtain $C^{(1)}$ we need to solve (A4.11(b)) with $p = 1$.

Neglecting the term $\partial C^{(1)}/\partial t$ and integrating gives

$$K\frac{\partial C^{(1)}}{\partial z} + V_S C^{(1)} = \int_z^{h_o} (U\phi(z) - U_c)C^{(o)}\,dz \qquad \text{....(A4.27)}$$

which evidently satisfies the surface condition (A4.11(a)).

We write

$$F(z) = \int_z^{h_o} (U(z') - U_c)\exp\left[\int_o^{z'} -\frac{V_S}{K\psi(z'')}\,dz''\right] dz' \qquad \text{....(A4.28)}$$

so that substituting for $C^{(o)}$ from (A4.17) and (A4.18), the equation (A4.27) gives

$$K\psi\frac{\partial C^{(1)}}{\partial z} + V_S C^{(1)} = \frac{g\,Z_o X}{V_S}F(z) \qquad \text{....(A4.29)}$$

which integrated gives

$$C^{(1)} = \frac{g\,Z_o X}{V_S}\left\{-\int_z^{h_o}\frac{F(z')}{K\psi(z')}\exp\left[\int_o^{z'}\frac{V_S}{K\psi(z'')}\,dz''\right]dz' + A\right\}\exp\left[-\int_o^{z}\frac{V_S}{K\psi(z)}\,dz\right]$$
$$\text{....(A4.30)}$$

in which the constant A is to be determined from the bed condition (A4.11(d)) viz:

$$-K\psi(0)\left(\frac{\partial C^{(1)}}{\partial Z}\right) = g\,Z^{(1)} = 0 \qquad \text{....(A4.31)}$$

(f) Second moment $Z^{(2)}$

With $p = 2$, equation (A4.12) becomes (since $Z^{(1)} = 0$)

$$\frac{dZ^{(2)}}{dt} = 2\int_o^{h_o} (U\phi - U_c)C^{(1)}\,dz \qquad \text{....(A4.32)}$$

$$\text{Thus } Z^{(2)} = 2(Z_o X)Dt \qquad \text{....(A4.33)}$$

in which D is an effective diffusion coefficient given by

$$D = \int_o^{h_o} (U\phi - U_c)C^{(1)}\,dz/Z_o X \qquad \text{....(A4.34)}$$

Substituting for $C^{(1)}$ from (A4.30) with (A4.31) is may be shown after a somewhat tedious calculation that

$$D = 2g\int_o^{h_o} (U\phi - U_c)F(z)\,dz/V_S^2 \qquad \text{....(A4.35)}$$

Generally $U_c = c$ (approximately) is small compared with $U\phi(z)$ so that (A4.35) becomes

$$D = 2g\int_o^{h_o} U\phi\,dz\int_z^{h_o} U\phi(z')\exp\left[\int_o^{z'} -\frac{V_S}{K\psi(z'')}\,dz''\right]dz'/V_s^2 \qquad \text{....(A4.36)}$$

(approximately)

which integrating by parts becomes

$$D = 2g\int_o^{h_o}\left\{U\phi\exp\left[\int_o^{z} -\frac{V_S}{K\psi(z')}\,dz'\right]\cdot\int_o^{z} U\phi(z')dz'\right\}dz/V_S^2 \qquad \text{....(A4.37)}$$

and writing

$$D_e = cL_e \qquad \text{....(A4.38)}$$

we find with (A4.21)

$$L_e = 2\left[\int_o^{h_o}\left\{f(z)\int_o^z U\phi(z')\,dz'\right\}dz/V_S\int_o^{h_o} f(z)dz\right] \qquad \text{....(A4.39)}$$

in which

$$f(z) = U\phi(z)\exp\left[-\int_o^z \frac{V_S}{K\psi(z')}\,dz'\right] \qquad \text{....(A4.40)}$$

If U, K are constant, viz U_o and K_o

$$f(z) = U_o \exp\left(-\frac{V_S}{K_o} z\right) \qquad \text{....(A4.41)}$$

and

$$L_e = L = 2U_o\int_o^{h_o} z\exp\left(-\frac{V_S z}{K}\right) / V_S\int_o^{h_o}\exp\left(-\frac{V_S z}{K}\right)dz \qquad \text{....(A4.42)}$$

$$= 2U_o\bar{z}/V_S \qquad \text{....(A4.43)}$$

in which $\bar{z}$ is given by equations (2.2.11) and (2.2.12).

APPENDIX 5

Derivation of flux versus velocity relation for steady flow from tidal flow measurements

Let us consider the oscillatory (tidal) motion of a "well-stirred' suspension in which the concentration and velocity at any time is uniform in depth. Further, we assume that the velocity at any phase of the motion is independent of x, ie U = U(t). Then considering the changes of concentration in a block of fluid of unit length we have

$$hDC/Dt = WC_* - V_S C \qquad \text{....(A5.1)}$$

in which, since the erosion velocity W is a function of U only, C is independent of x.

If the flow were steady at the velocity U corresponding to a particular phase of the tide, the concentration (C_e) would evidently be given by

$$WC_* = V_S C_e \qquad \text{....(A5.2)}$$

So (A5.1) may be written

$$hDC/Dt = V_S(C_e - C) \qquad \text{....(A5.3)}$$

which in terms of the sediment flux

$$S = UhC \qquad \text{....(A5.4)}$$

$$S_e = UhC_e \qquad \text{....(A5.5)}$$

becomes

$$hUDC/Dt = (S_e - S)V_S/h \qquad \text{....(A5.6)}$$

or neglecting variation of h with t

$$(\partial S/\partial t) - hC(\partial U/\partial t) = (S_e - S)V_S/h \qquad \text{....(A5.7)}$$

However, the second term on the left hand side of (A5.7) is solely due to the change of flux induced directly by changes in velocity. It would be present if the particles had a zero settling rate (a passive contaminant, for example); thus it cannot contribute to a lag or hysteresis effect and has therefore been neglected in the present context.

Thus the steady or equilibrium flux is related to that observed at a tidal phase by

$$S_e = S + (\partial S/\partial t)(h/V_S) \qquad \text{....(A5.8)}$$

S_e is greater than S if S (or U) is increasing with time and less if it is decreasing with time.

We note that for the "well-stirred" case, from equation (2.2.10)

$$h/V_S = L/U.$$

Thus it appears that (A5.8) may be generalized for cases in which the concentration is not uniformly distributed in depth by

$$S_e = S + (\partial S/\partial t)(L/U) \qquad \text{....(A5.9)}$$

For U and K_z uniform over the depth, L is given by equations (2.2.11) and (2.2.12). Knowing L, equation (A5.9) can be used to correct the values of S observed during a tidal cycle to give S_e.

APPENDIX 6

Derivation of flux equation (2.4.2) for steady flow

It may be shown from equations (2.3.1) – (2.3.3) that in steady flow, if the vertical diffusivity is constant ($K_z = K$), the vertical distribution of concentration is given by

$$C = C_b \exp(-V_S z/K) \quad ....(A6.1)$$

$$= (W_o C_*/V_S) \exp(-V_S z/K) \quad ....(A6.2)$$

Thus, if the velocity is approximately constant (U) over the depth (h) the flux is approximately given by

$$S = W_o C_* (UK/V_S^2)[1 - \exp(-V_S h/K)] \quad ....(A6.3)$$

and substituting for K its mean value over the depth, viz;

$$K = \kappa U_* h/6 \quad ....(A6.4)$$

we find

$$S = W_o C_* h \frac{\kappa}{6} \frac{U^2}{V_S^2} (\frac{U_*}{U}) [1 - \exp\{-\frac{6}{\kappa} (\frac{V_S}{U}) (\frac{U}{U_*})\}] \quad ....(A6.5)$$

For fine sand, V_S = 0.01 m/s, U/U_* = 20, κ = 0.4 and the exponential term is fairly small compared with unity (U = 1 m/s) so that for a given sand size and bed roughness.

$$S = c_T W_o C_* U^2 \text{ (approximately)}, \; c_T = \text{constant} \quad ....(A6.6)$$

If the rate of erosion is proportional to the excess shear above the threshold shear, ie

$$W_o C_* \text{ proportional to } (U_*^2 - U_{*t}^2)/V_S \quad ....(A6.7)$$

and

$$U_*/U = \text{constant} \quad ....(A6.8)$$

we have

$$S = c_T U^2 (U^2 - U_t^2), \; c_T = \text{constant} \quad ....(A6.9)$$

This equation may be compared with the simple power law form, viz;

$$S = k U^4 \quad ....(A6.10)$$

The values of c_T and k with S in units of m^3/hour/m width of compacted sand and U in m/s, taken from flux observations throughout a spring tide at a number of stations along a natural channel (Black Deep) in the Thames estuary are given in Table A6.1.

TABLE A6.1

Flux of sediment in tidal flow in size range 60–150 microns

The values of c_T and k show considerable variation, which may reflect the variable nature of the bed which was composed of fine sand with a small admixture of shell. However, the size of the sediment in suspension was remarkably similar at all stations.

Station	*Depth*	*Size of bed sand (microns)*			V_t	c_T	k	*Current direction*
	m	D_{10}	D_m	D_{90}	*m/s*			*E = Ebb* *F = Flood*
1	16	80	150	210	0.35	2.0	1.75	E & F
2	14	80	150	250	0.35	1.25	1.1	E & F
3	9	80	150	210	0.35	2.4	2.1	E & F
4	15	80	125	210	0.35	1.35	1.1	E
5	16	90	200	400	0.35	2.15	1.9	E & F
6	17				0.35	1.35	1.2	E
					0.35	0.50	0.45	F
7	17	90	150	250	0.35	1.95	1.7	E
					0.35	1.35	1.2	F
8	19	120	225	300	0.35	1.15	1.0	E & F
9	21	100	225	500	0.35	3.65	3.2	E & F
				Mean	0.35	1.93	1.65	

APPENDIX 7

Flow over an elliptic depression

Let us consider the flow over an elliptic depression (semi-axes a,b) in which the depth (h_1) is constant in an otherwise flat bed, depth h_0, the incident velocity having components U_0, V_0 parallel to the major and minor axes respectively. We assume that outside the ellipse, velocity head changes are small by comparison with losses due to friction and further that these losses are represented by a Chezy type formula, ie

$$\frac{\partial \zeta}{\partial s} = -f\frac{V^2}{h} \qquad \text{....(A7.1)}$$

in which

V = mean velocity over the vertical at point x,y

h = local depth

f = friction coefficient (the inverse of the Chezy coefficient) and

$\frac{\partial \zeta}{\partial s}$ = slope of the water surface in the direction of the velocity.

Thus, if u,v are the components of V in the x,y directions we have

$$\frac{\partial \zeta}{\partial x} = -fV\,u/h\ , \qquad \frac{\partial \zeta}{\partial y} = -fV\,v/h \qquad \text{....(A7.2)}$$

Since changes of V are assumed small outside the ellipse these equations reduce to

$$\frac{\partial \zeta}{\partial x} = -\left(\frac{f_0 W_0}{h_0}\right)u\ , \qquad \frac{\partial \zeta}{\partial x} = -\left(\frac{f_0 W_0}{h_0}\right)v \qquad \text{....(A7.3)}$$

in which

$$W_0 = (U_0{}^2 + V_0{}^2)^{1/2} \qquad \text{....(A7.4)}$$

These are similar to the equations for inviscid flow with a potential ϕ_0, which may be identified with ζ, thus

$$\phi_0 = (h_1/f_1 W_0)\zeta$$

To find solutions which satisfy the equation of continuity outside the ellipse with a uniform velocity W_E (components U_E, V_E inside the ellipse). Thus we write

$$\phi_0 = U_0\,x + V_0 y + Ae^{-\xi}\cos\eta + Be^{-\xi}\sin\eta,\ \text{outside ellipse} \qquad \text{....(A7.5)}$$

$$\phi_1 = U_E\,x + V_E\,y \qquad \text{inside ellipse} \qquad \text{....(A7.6)}$$

where the elliptic co-ordinates ξ, η are connected with x, y by the relations

$$x = \alpha\cosh\xi\,\cos\eta \qquad \text{....(A7.7)}$$

$$y = \alpha\sinh\xi\,\sin\eta \qquad \text{....(A7.8)}$$

and

$$\phi_1 = (h_1/f_1\,W_E)\zeta$$

We seek to determine A, B, U_E, V_E, which, at the edge of the ellipse ($\xi = \xi_o$), satisfy continuity of flux and of surface slope along the boundary ie

$$h_0\frac{\partial \phi_1}{\partial \xi} = h_1\frac{\partial \phi_2}{\partial \xi} \qquad \xi = \xi_o \qquad \text{....(A7.9)}$$

$$\frac{f_0 W_0}{h_0}\frac{\partial \phi_0}{\partial \eta} = \frac{f_1 W_E}{h_1}\frac{\partial \phi_1}{\partial \eta} \qquad \xi = \xi_o \qquad \text{....(A7.10)}$$

η, ξ are co-ordinates tangential and normal to the boundary and $\xi = \xi_o$.

Substituting for ϕ_0, ϕ_1 from (A7.5), (A7.6) in (A7.9), (A7.10), we obtain two equations for A, B, U_E, V_E, but since these equations must be satisfied at all points of the boundary the coefficients of $\sin\eta$ $\cos\eta$ must vanish in both equations, which gives the following four equations

$$[U_0 b - Ae^{-\xi_o}]h_0 = U_E b h_1 \qquad ....(A7.11)$$

$$[V_0 a - Be^{-\xi_o}]h_0 = V_E a h_1 \qquad ....(A7.12)$$

$$[-U_0 a - Ae^{-\xi_o}](f_0 W_0/h_0) = -U_E a (f_1 W_E/h_1) \qquad ....(A7.13)$$

$$[V_0 b + Be^{-\xi_o}](f_0 W_0/h_0) = V_E b (f_1 W_E/h_1) \qquad ....(A7.14)$$

since $a = \alpha \cosh \xi_o$, $b = \alpha \sinh\xi_o$. ....(A7.15)

Eliminating A, B from (A7.11), (A7.13) and (A7.12), (A7.14) we find the following equations for U_1, V_1,

$$(f_1 W_E/f_0 W_0)(U_E/U_0) + (H^2 b/a)(U_E/U_0) - H(a + b)/a = 0 \qquad ....(A7.16)$$

$$(f_1 W_E/f_0 W_1)(V_E/V_0) + (H^2 a/b)(V_E/V_0) - H(a + b)/b = 0 \qquad ....(A7.17)$$

in which we have written the depth ratio

$$H = h_1/h_0 \qquad ....(A7.18)$$

These equations allow the velocity $W_E = (U_E{}^2 + V_E{}^2)^{1/2}$ to be calculated for a given velocity W, at an angle of incidence θ to the major axis of the ellipse $U_0 = W_0 \cos\theta$, $V_0 = W_0 \sin\theta$.

We note that if the incident flow is parallel to the major axis of the ellipse, $V_0 = V_E = 0$ and the velocity over the ellipse (U_E) is given by equation (A4.16), thus ($f_0 = f_1$)

$$(U_E/U_0) = [(bH^2/2a)^2 + (H(a + b)/a)]^{1/2} - (bH^2/2a) \qquad ....(A7.19)$$

(162) Dd8013369 500 2/80 H P Ltd So'ton G3927