2021

An Analysis of a Hybrid Steel Bridge

Andrew Iglehart

AN ANALYSIS OF A HYBRID STEEL BRIDGE

A Capstone Experience/Thesis Project Presented in Partial Fulfillment

of the Requirements for the Degree Bachelor of Science

with Mahurin Honors College Graduate Distinction

at Western Kentucky University

By

Andrew D. Iglehart

May 2021

*****

CE/T Committee:

Dr. Shane Palmquist, Ph.D., P.E.

Tyler Baker, M.S., E.I.T

Joel Lenoir, P.E.

ABSTRACT

The American Institute of Steel Construction hosts a competition for graduating college seniors each year. The competition is designing and fabricating a scaled steel bridge within certain parameters. Each year the parameters change to allow different seniors to face similar challenges without copying the previous year's work. Before the outbreak of COVID-19, a bridge was fabricated as per the rules in the 2020, and the same bridge was used for the 2021 competition. With a bridge already fabricated and being used for the competition, a question arose about analyzing the bridge.

This thesis encompasses the entire analysis of the steel bridge completed by the honors student. The challenge of this project is due to the nature of the bridge being both a truss and a beam. This style of bridge does not have cookie cutter formulas to analyze the bridge, and approaches were made to analyze the bridge in all forms available. Multiple forms of analysis were used to analyze the bridge including hand calculations and a computer model. Different methods of hand calculations were used to verify the results of the computer model.

I dedicate this thesis to my parents, David and Gina Iglehart, who raised me into the man I am today. I also dedicate this thesis to my loving wife, Sarah Iglehart, who has stood by my side. Lastly, I dedicate this to God who has blessed me in every part of my life.

ACKNOWLEDGEMENTS

I would like to acknowledge Dr. Shane Palmquist who has been my faculty advisor, mentor, and friend through this process. I would also like to acknowledge Tanner Allen and Chase Schnell who also learned the computer software for the analysis. I would also like to acknowledge the WKU Civil Engineering Department for guiding each student through their program.

VITA

EDUCATION

Western Kentucky University, Bowling Green, KY B.A. in Civil Engineering – Mahurin Honors College Graduate Honors CE/T: *An Analysis of a Hybrid Steel Bridge*	May 2021
Woodford County High School, Versailles, KY	May 2017

PROFESSIONAL EXPERIENCE

The Engineering Center for Innovation, WKU Research Assistant	Jan. 2019- Present
Scotty's Internship	May 2020- Dec. 2020
Morley Internship	May 2019- Aug. 2019
Mindel Scott Internship	May 2018- Aug 2018

AWARDS & HONORS

Summa Cum Laude, WKU, May 2021

PROFESSIONAL MEMBERSHIPS

American Society of Civil Engineers (ASCE)
American Institute of Steel Construction (AISC)

CONTENTS

Abstract ii

Acknowledgements iv

Vita iv

List of Figures vii

List of Tables viii

Introduction 1

Computer Analysis 3

Hand Calculations 5

Results 9

Conclusions 11

References 12

Appendix A: Examples of Trusses and Beams 13

Appendix B: Loading Specifications 16

Appendix C: Visual Analysis 18

Appendix D: Shear and Moment Diagrams 19

Appendix E: Moment of Inertia Calculation 21

Appendix F: Superposition Deflection 23

Appendix G: Castigliano's Theorem 29

Appendix H: Internal Forces of the Supports 32

Appendix I: Buckling Forces 33

Appendix J: Final Deflections 34

LIST OF FIGURES

Figure A-1: Example of truss bridge 13

Figure A-2: Example of truss bridge 13

Figure A-3: Example of a beam bridge.. 14

Figure A-4: Example of a beam bridge.. 14

Figure A-5: Picture of the final completed bridge 15

Figure B-1: Lateral load test specifications 16

Figure B-2: Vertical load test specifications... 17

Figure C-1: 3D model of the final bridge design 18

Figure D-1: Free body diagram of the steel bridge.. 19

Figure D-2: Shear and moment diagram of the steel bridge 20

Figure E-1: Cross section of the components affecting the moment of inertia 21

Figure G-1: Diagram of the analyzed truss 29

LIST OF TABLES

Table B-1: Table showing the various loading cases outlined by AISC 17

Table C-1: Table of deflections for various load cases.. 18

Table E-1: Given values for calculating the moment of inertia....................................... 21

Table E-2: Calculations for the moment of inertia 22

Table E-3: Worst case scenario with a single point load of 2500 pounds located in the center of the bridge 22

Table F-1: Series of tables displaying constants needed to solve the differential equations to calculate the deflection of the beam for Load Case 1 26

Table F-2: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 1 27

Table F-3: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 2 27

Table F-4: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 3 27

Table F-5: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 4 28

Table F-6: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 5 28

Table F-7: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 6 28

Table G-1: Constants and calculations used to calculate the deflection of the steel bridge using Castigliano’s Theorem 30

Table G-2: Castigliano's Table to find the deflection at point D 31

Table H-1: Given values of the bridge... 32

Table H-2: Reactions for the South side of the bridge... 32

Table H-3: Reactions for the North side of the bridge... 32

Table I-1: Table giving the constants to solve Equation H2, the resulting Euler Load, and the Force across the main compression members of the truss. 33

Table J-1: Gauge deflections for varying weights being placed in the center of the bridge. 34

Table J-2: Table summarizing all the maximum deflections for each load case. 34

INTRODUCTION

The American Institute of Steel Construction (AISC) is dedicated to improving the steel industry and helping structural engineers grow and make connections. The organization offers a competition every year for seniors in college completing a degree in civil engineering. This competition is outlined through a problem statement, rules, etc. to design, fabricate, and construct a scaled down steel bridge. This bridge must meet various specifications outlined by AISC, and the rules change each year to ensure that the upcoming seniors face a different challenge.

In the spring of 2020, COVID-19 broke out across the world, and the competition was cancelled for that year. The rules outlined in 2021 allowed for bridges designed in 2020 to compete in that year's competition. An issue was brought up concerning analyzing the bridge. This was not completed in the previous year and needed to be completed this year for the competition.

The bridge was designed to be a combination of two different kinds of bridges: a beam and a truss. A beam can be described as a bridge with a continuous deck or driving surface with no pieces or members outside the pieces that stretch the entire length of the bridge. Trusses are bridges where the members form triangles and connect to make joints. Examples of real beams and trusses can be seen in Appendix A, along with a picture of the final bridge construction.

This combination of both a beam and a truss proved to be an interesting challenge since there is no outlined equation or method to analyze a hybrid bridge like this. The competition also outlined for the legs to be offset from each other, and that is not a trivial task. The analysis was complete using two primary forms of analysis: computer modeling and hand calculations. The computer modeling considers the offset nature of the legs and the fact of the bridge being both a combination of a beam and a truss. The hand calculations analyzed the bridge as a beam and a truss and analyzed other aspects of the bridge including the internal forces of some members and the buckling of other members. The results of the analysis were deflections. Deflections can be defined simply as how much something moved. In the case of the steel bridge, different parts of the bridge deflect more than others, so only certain deflections are necessary for competition and general engineering practice.

The competition outlined load cases where a 1000-pound weight is placed at a location on the bridge and a 1500-pound weight is placed at another location on the bridge. The deflection is measured in the middle of these two weights and is used for final awards for the end of the competition. Another test, called the lateral load test, was outlined where a 50-pound weight pulled the bridge horizontally. This test was designed to test the sway of the bridge which is how much the bridge will deflect in the horizontal direction. Diagrams of how the bridge will be tested can be seen in Appendix B. For the analysis, deflections were measured in the middle of the weights and the maximum deflection was calculated (Student, 2020).

COMPUTER ANALYSIS

Numerous attempts were made to create a working computer model of the bridge. Programs like SOLIDWORKS were used in the beginning stages of the analysis, but the results were inconclusive due to the complexity of the program. Another program was sought after that was more versatile that would help with the analysis. A program called Visual Analysis was used to complete the final analysis due to the ease and simplicity of the program.

Visual Analysis was useful for taking each member of the bridge and reducing it down to a near stick figure. Each member was given a certain cross-sectional area and moment of inertia based off the shape of each member. The ends of each member were then designated to have a simple connection or rigid connection. A simple connection, or a pinned connection, is a connection where members are connected at a point, but the members can still rotate around said point. A rigid connection is a connection where the members meet at a point, and the members are welded into place to keep the members from moving. Making these distinctions is what made the computer model accurate.

Some aspects of the bridge could not be perfectly placed into Visual Analysis. This can easily be seen with the stringers of the bridge. The stringers are the members run along the under part of the bridge. It can be seen in the photo of the bridge that two tubes of steel are welded together to compose the stringer, and Visual Analysis only has one piece where the stringer is. This was resolved by finding a shape in the Visual Analysis that had a similar cross-sectional area and moment of inertia to the original stringer.

Visual Analysis was able to put the loading cases outlined by AISC and put them on the bridge. Due to the various loading locations, a setting had to be made for each of the six load cases specified plus an extra setting for the lateral load test. These deflections and a diagram of the completed Visual Analysis Model can be seen in Appendix C, but these results are only useful if they can be verified.

HAND CALCULATIONS

The necessary hand calculations required more assumptions than the Visual Analysis. As stated previously, the bridge is a combination of both a truss and a beam, and there is no simple equation to analyze the bridge. The analysis was complete by analyzing the bridge with two different methods. The first portion of the analysis involved assuming the bridge was a beam and solving the respective differential equations using the principle of superposition. The second portion of the analysis was done by assuming the bridge was a truss, and Castigliano's Method was used to complete that portion of the analysis. Along with the two methods of analysis, the reactions were also calculated for each of the legs. The reactions are simply the amount of force needed to support the weight which is provided by the legs of the bridge. Along with calculating the reactions, the top pieces of the bridge, also known as the compression members, were analyzed for buckling forces. These were all the forms of analysis taken to fully complete the analysis.

Treating the bridge as a beam allows the analyzer to create some helpful visual aids and give insightful information on the status of the bridge. One of the visual aids is a free body diagram which shows the length of the bridge and the locations of the loads. The free body diagram is also the foundation of creating a shear and moment diagram. The shear and moment diagram show the internal forces of the bridge. This diagram is also the beginning point of completing the needed differential equations to analyze the bridge, and both can be seen in Appendix D.

One of the main problems with assuming the bridge to be a beam is the inconsistent moment of inertia from one side of the bridge to the other. The moment of inertia can be approximated by doing a calculation as if the compression member were extended from one side of the bridge to the other. This, then, allows a constant moment of inertia to be applied to the entire bridge, and the solving of the deflection differential equations can be done. A table showing the values and equations can be seen in Appendix E, along with an extra table that shows the maximum deflection with a concentrated load in the center of the bridge.

The final step with analyzing the bridge as a beam is solving the differential equations that give the deflection of the bridge at every point along the beam. This is done by integrating the shear equation four times to give the final deflection equation. An example of the differential equations can be seen in Appendix F. To accomplish this, the principle of superposition was invoked to give the correct deflections. The principle of deflection simply states that the total deflection of a beam with multiple loads is the sum of deflections of the beam analyzed with one load at a time. This means that a deflection can be calculated with only the first load on the beam, a second deflection can be calculated for the second load, and the total deflection is the sum of the two deflections. This, then, means that the system of differential equations must be solved twice to account for each load. Solving the differential equations is a simple matter when the equation is continuous and differential, but the system has a discontinuity where each load is concentrated. This can be easily seen in the drops and sharp turns displayed in the shear and moment diagrams in Diagram D2. This forces the differential equations to be solved from one edge of the bridge to the load, and then from the load to the other side of the bridge. This can be seen in the series of tables in Appendix F displaying the constants of the differential equations. This method was chosen because an equation exists (listed in the FE handbook and the AISC Steel Construction Manual page 3-210) that gives the deflection of the beam, but the equation is only applicable to one side of the beam. To complete this analysis, the deflection had to be known for every point across the bridge, but the equation is useful for checking the final deflection. Once the differential equations are solved, the process can be repeated for all the load cases specified by AISC.

The next step in the analysis was to analyze the bridge as a truss using Castigliano's Theorem (Hibbeler, 2006). This method required two major assumptions. The first assumption required the two tension members in the middle of the bridge to meet at a single point on the bridge. The reason for this is because a truss cannot be analyzed unless it is statically determinate. This is based off the number of joints and members of the truss, and the only way to make it statically determinate was to combine the two tension members. The second assumption required both loads to be combined into a single point and placed at the joint where the two tension members met. This is also since a truss can only be analyzed with loads located at the joints of the truss. The deflection calculation can only be done for a single point on the bridge, and the point where the load was located was chosen to find the deflection. This is because the maximum deflection and the measured deflection for the competition would be closest to this point. These assumptions create a more conservative approach to the analysis because this forces the bridge to face a worst-case scenario. The results for Castigliano's Theorem can be seen in Appendix G.

Another aspect of verifying if Visual Analysis completed a reasonable analysis is verifying if some of the internal forces in the members are similar through hand calculations. This is done through calculating the reactions of each of the four legs supporting the bridge. Due to the offset nature of the legs, each side of the bridge had to be analyzed individually. An assumption that is made is each side of the bridge assumes half of each load. This is reasonable because if each side did not assume half of the load, then the bridge would become unstable and possibly dynamic. Calculating the reactions in this way allows the offset legs to be considered, and the forces generated by the legs can be easily shown. The results can then be compared with the internal forces calculated by Visual Analysis. The results of this portion of the analysis can be seen in Appendix H.

The final portion of the hand calculations required the buckling of some of the members to be checked. When considering which members to check, the legs were considered, but were dismissed due to how short the members were. This is because for calculating the buckling force, the shorter the member, the higher the force must be to cause the member to buckle (Zill, 2016). This left the top compression members as prime members to be checked. This is due to how long the members were and how critical the members were to the stability of the bridge. To calculate the force needed to cause the member to buckle, a differential equation had to be solved to calculate the Euler buckling force, which is the lowest buckling force. The solution to the differential equation gives an expression that is dependent on the length of the member, the moment of inertia of the member, and the modulus of elasticity of the material. Then, the Euler buckling force can be calculated for the desired members, and the buckling force can be compared with the internal forces calculated by Visual Analysis. The results can be seen in Appendix I.

RESULTS

The results of each of the analyses is what verifies each of the methods used. When comparing the deflections of Visual Analysis with the beam deflection and Castigliano's Theorem, the deflections are within reason of each other. Since the hand calculations required assumptions to be made, it is reasonable to understand why the deflections are less conservative than the deflections estimated by Visual Analysis. As far as the hand calculations were concerned, Castigliano's Theorem was the most conservative, and was closest to the deflection of Load Case 2. This is reasonable since Load Case two is the load case where the loads are closest to the center of the bridge. For the calculation of the internal forces of the legs, the values were very close with eachother. This is another proof of how Visual Analysis is trustworthy and aligns with much of common knowledge and practice in engineering. The calculation of the Euler buckling force produced a number that was in the thousands of pounds which is to be expected. The calculated internal forces from Visual Analysis for the members yielded values that did not break one thousand pounds. This is because the members are not meant to reach buckling, and if the members did reach buckling, then the members would need to be strengthened, replaced, or redesigned.

Another comparison to how correct the analysis is, is by comparing the results of Visual Analysis and the hand calculations with results from load testing the physical bridge itself. Due to the bridge being load tested in competition for only one Load Case, the actual deflection of the bridge is only given for Load Case 2. Other data was gathered by placing varying weights in the center of the bridge and averaging dial gauges to obtain deflections. These deflections can be seen in Appendix J along with a table outling the maximum deflections calculated and measured for each load case. The deflection for Load Case 2 was measured to be 0.86 inches during the competition, and the maximum deflection for Load Case 2 for the analysis was 0.15 inches. This discrepancy is due to the “give” in the connections across the bridge. “Give” is simply the small gaps between each connection to ensure the bridge can be built effectively and quickly. This is also practiced in engineering, and it is readily taught that a bolt with a half inch diameter will not fit into a half inch diameter hole. Even a millimeter per connection has vast effects for the total deflection of the bridge. With all these considerations considered, it can be determined that the completed analysis is a correct and reasonable analysis.

CONCLUSIONS

The completed analysis yielded results that were within a magnitude of the actual deflections of the bridge which is fantastic considering the small gaps between each connection in the bridge. The completed analysis consisted of a computer model on Visual Analysis and hand calculations analyzing the bridge as a truss and as a beam. The analysis also considered internal forces for the supports and buckling forces for the top compression members. All of which produced answers that show excellent analysis methods. The use of combined methods, new programs, and advanced analysis show the complexity of this problem and the need for it to be taught more.

It would be an interesting project to perform an analysis with a more exact software and one that did true finite element analysis. Using that software would yield more accurate results and give more certainty to the processes already learned. It would also be prudent to educate the upcoming students on this software or software's like Visual Analysis to give students a better understanding of analysis for their future careers.

REFERENCES

American Institute of Steel Construction. (2017). *STEEL CONSTRUCTION MANUAL* (15th edition).

Hibbeler, R. C. (2006). *Structural analysis*. Upper Saddle River, N.J: Pearson/Prentice Hall.

Student Steel Bridge Supplemental Competition. (2020) Retrieved from https://www.aisc.org/education/university-programs/student-steel-bridge-competition/ssbc-rules-and-clarifications/

Zill, D. G. (2016). *Advanced Engineering Mathematics, 6th Edition*. Place of publication not identified: Jones & Bartlett Learning.

A) APPENDIX A: EXAMPLES OF TRUSSES AND BEAMS

Figure A-1: Example of truss bridge

Figure A-2: Example of truss bridge

Figure A-3: Example of a beam bridge

Figure A-4: Example of a beam bridge

Figure A-5: Picture of the final completed bridge

B) APPENDIX B: LOADING SPECIFICATIONS

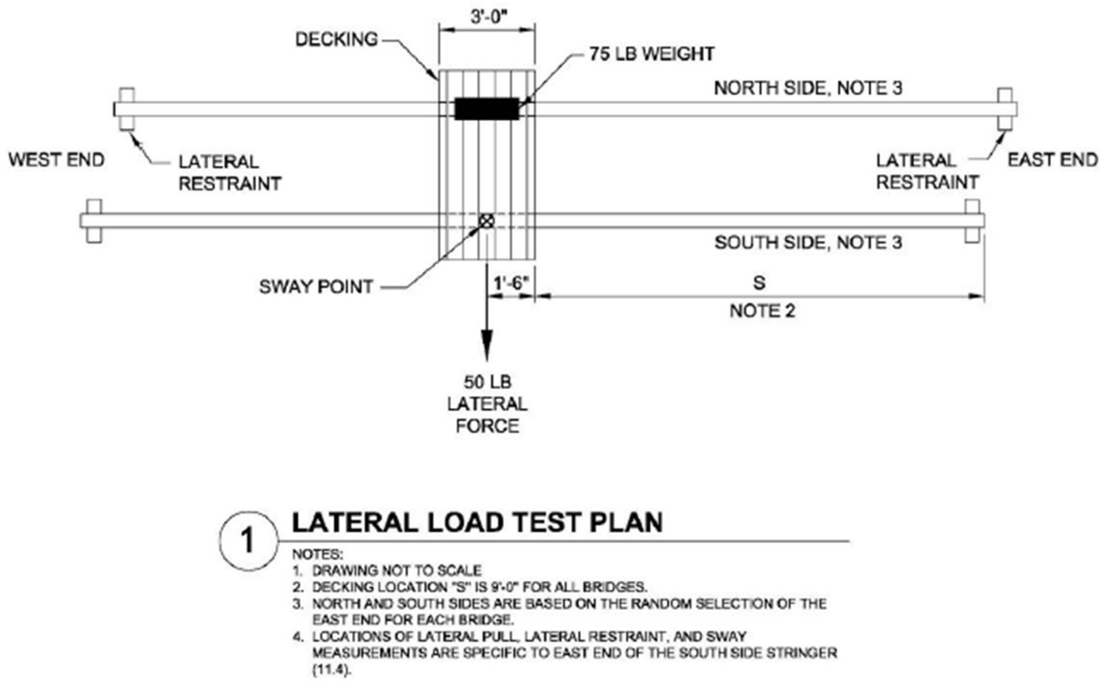

Figure B-1: Lateral load test specifications

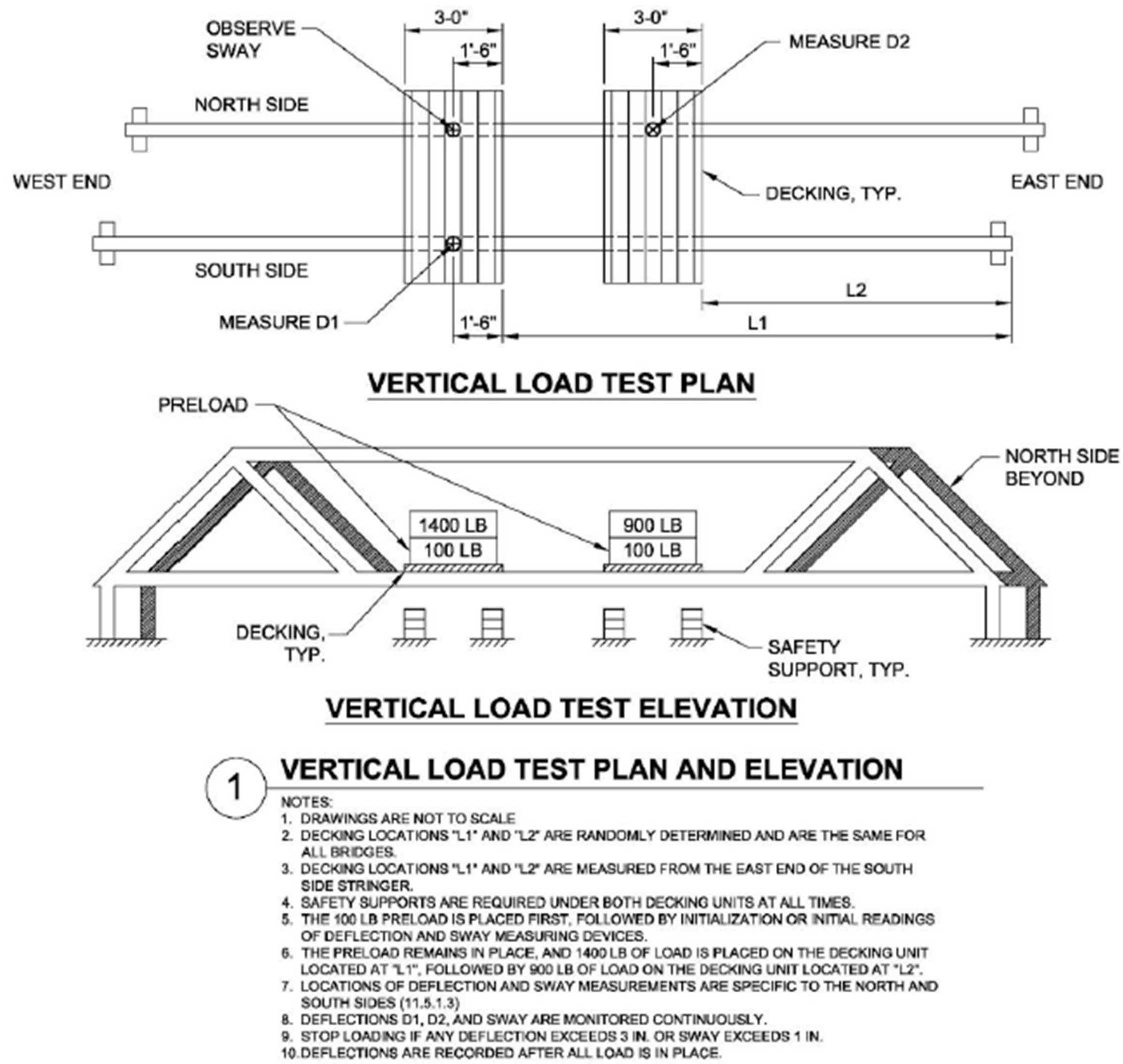

Figure B-2: Vertical load test specifications

	L1	**L2**	**S**
Load Case 1	8'-0"	3'-0"	9'-0"
Load Case 2	10'-0"	4'-0"	9'-0"
Load Case 3	11'-0"	7'-0"	9'-0"
Load Case 4	12'-0"	3'-6"	9'-0"
Load Case 5	12'-6"	6'-0"	9'-0"
Load Case 6	13'-0"	8'-5"	9'-0"

Table B-1: Table showing the various loading cases outlined by AISC

C) APPENDIX C: VISUAL ANALYSIS

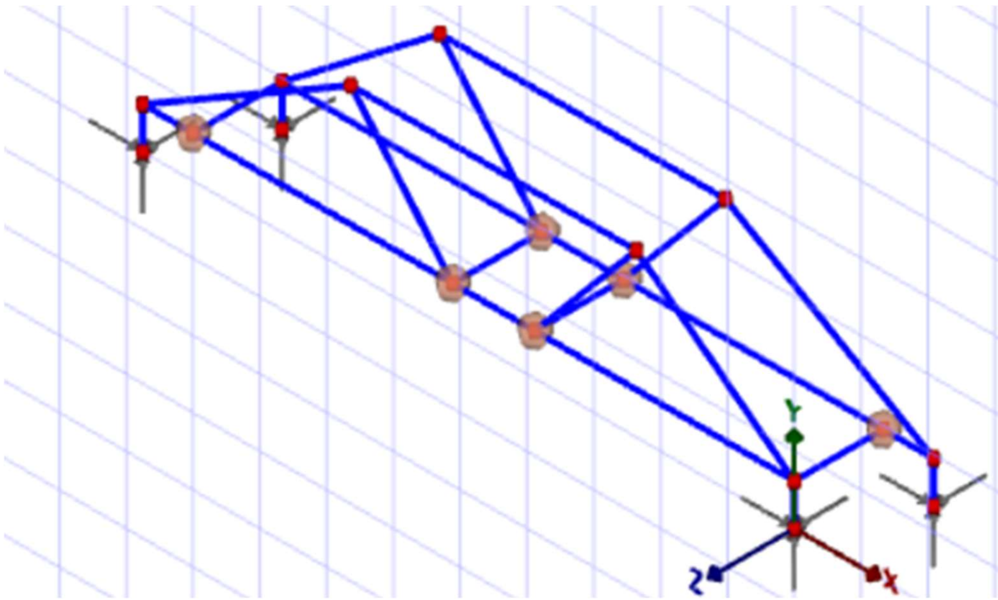

Figure C-1: 3D model of the final bridge design

	Max Vertical Deflection	**Max Sway in x Direction**	**Max Sway in z Direction**
Lateral Load Test	0.012 in	0.006 in	0.121 in
Load Case 1	0.247 in	0.079 in	0.081 in
Load Case 2	0.154 in	0.026 in	0.022 in
Load Case 3	0.336 in	0.118 in	0.116 in
Load Case 4	0.307 in	0.063 in	0.068 in
Load Case 5	0.370 in	0.110 in	0.120 in
Load Case 6	0.506 in	0.181 in	0.187 in

Table C-1: Table of deflections for various load cases

D) APPENDIX D: SHEAR AND MOMENT DIAGRAMS

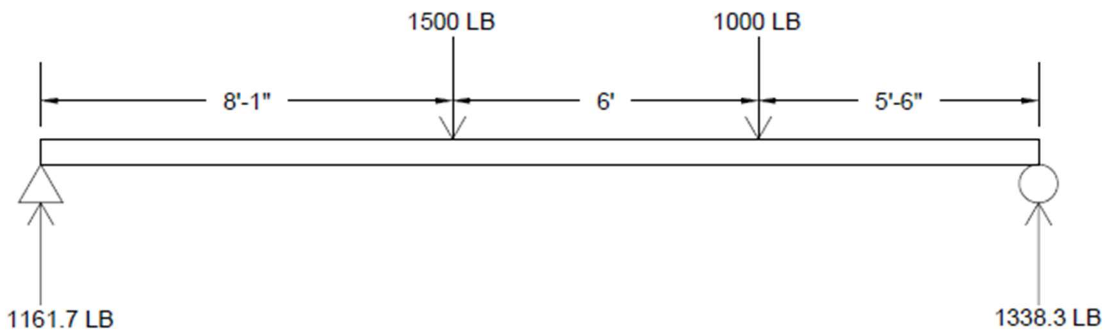

Figure D-1: Free body diagram of the steel bridge

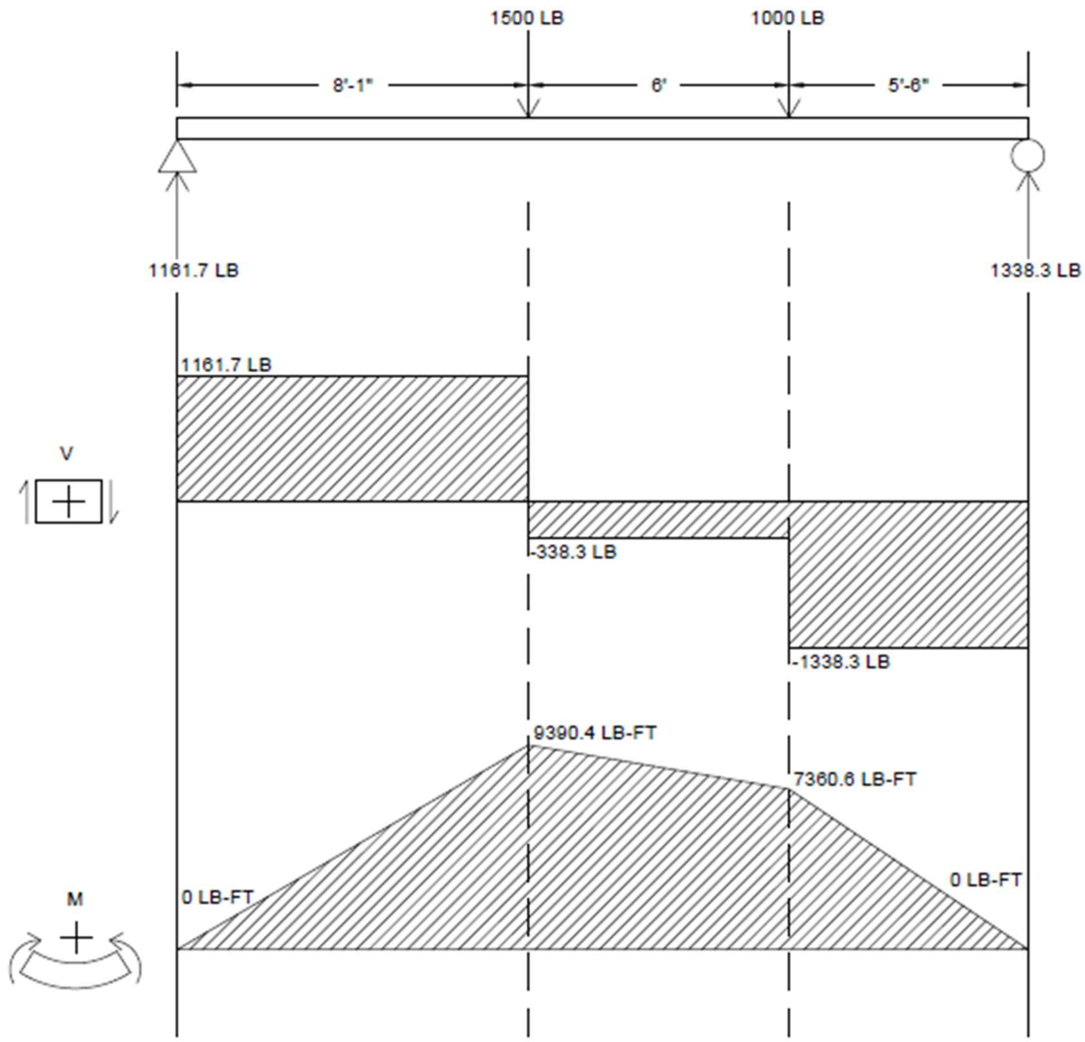

Figure D-2: Shear and moment diagram of the steel bridge

E) APPENDIX E: MOMENT OF INERTIA CALCULATION

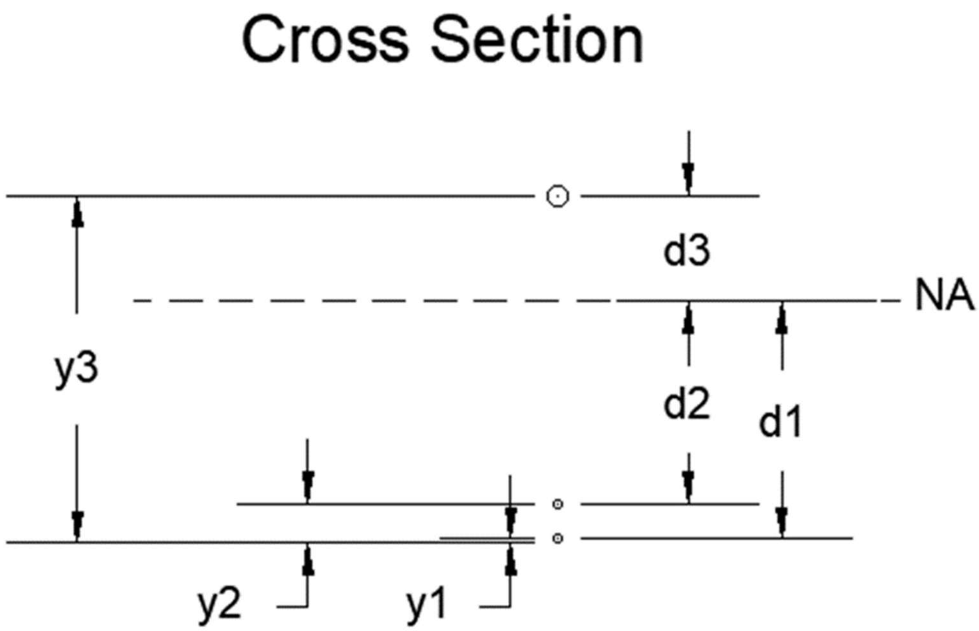

Figure E-1: Cross section of the components affecting the moment of inertia

Moment of Inertia Givens:

y_1=	0.5	in	(distance from bottom of truss to center of member)
y_2=	5	in	(distance from bottom of truss to center of member)
y_3=	44	in	(distance from bottom of truss to center of member)
D_1=	1	in	(diameter)
D_2=	1	in	(diameter)
D_3=	2.5	in	(diameter)
t_1=	0.058	in	(thickness of tubing)
t_2=	0.058	in	(thickness of tubing)
t_3=	0.095	in	(thickness of tubing)
ID_1=	0.884	in	(inner diameter)
ID_2=	0.884	in	(inner diameter)
ID_3=	2.31	in	(inner diameter)
L=	235	in	(length of bridge)
P=	2500	lb	(load)
E=	29000	ksi	(modulus of elasticity)

Table E-1: Given values for calculating the moment of inertia

Moment of Inertia Calculations:

A_1=	((PI*D_1^2)/4)-((PI*ID_1^2)/4)	A_1=	0.1716	in^2
A_2=	((PI*D_2^2)/4)-((PI*ID_2^2)/4)	A_2=	0.1716	in^2
A_3=	((PI*D_3^2)/4)-((PI*ID_3^2)/4)	A_3=	0.7178	in^2
y_{bar}/NA=	(A_1*y_1+A_2*y_2+A_3*y_3)/(A_1+A_2+A_3)	y_{bar}=	30.6543	in
d_1=	y_{bar}-y_1	d_1=	30.1543	in
d_2=	y_{bar}-y_2	d_2=	25.6543	in
d_3=	y_3-y_{bar}	d_3=	13.3457	in
I_1=	((PI*D_1^4)/64)-((PI*ID_1^4)/64)	I_1=	0.0191	in^4
I_2=	((PI*D_2^4)/64)-((PI*ID_2^4)/64)	I_2=	0.0191	in^4
I_3=	((PI*D_3^4)/64)-((PI*ID_3^4)/64)	I_3=	0.5198	in^4
I_{tot}=	I_1+I_2+I_3+A_1*d_1^2+A_2*d_2^2+A_3*d_3^2	I_{tot}=	397.4385	in^4

Table E-2: Calculations for the moment of inertia

Deflection=	**(P*L^3)/(48*E*I)**	**Deflection=**	**0.05865**	**in**

Table E-3: Worst case scenario with a single point load of 2500 pounds located in the center of the bridge

F) APPENDIX F: SUPERPOSITION DEFLECTION

The following equations display the series of differential equations to solve for the final deflection of any point along the bridge. The variable "x" is the distance from the left side of the bridge to the desired point of measurement. The constants, c_n, denote constants that are calculated based on certain characteristics of the bridge.

$$Shear: V(x) = c_1$$
$$Moment: M(x) = \int V(x) = c_1 x + c_2$$
$$Angle: \theta(x) = \int \left(\frac{1}{EI}\right) M(x) = \left(\frac{1}{EI}\right)\left(\frac{c_1}{2}x^2 + c_2 x + c_3\right)$$
$$Deflection: y(x) = \int \left(\frac{1}{EI}\right) \theta(x) = \left(\frac{1}{EI}\right)\left(\frac{c_1}{6}x^3 + \frac{c_2}{2}x^2 + c_3 x + c_4\right)$$

Equation F-1: Series of differential equations to solve for the deflection of a beam.

The series of tables show the constants to solve the differential equations in Equation F1. The sections that show check make sure that the deflection match the equation listed in the FE handbook and that the deflection at the location of the load matches from one side of the beam to the other.

Superposition Deflection Calculations

Deflection	-0.0503	in
x=	10.083	ft

Information

Total L	235	in
L1	8	ft
L2	3	ft
F1	1500	lb
F2	1000	lb
E	29000	ksi
I	397	in^4

Distance to center of Loads		1.5 ft
Total L	19.583 ft	
0-L1	10.083 ft	
L1-L2	5 ft	
L2-Tl	4.5 ft	
0-L2	15.083 ft	
L1-Tl	9.5 ft	

Load 1

Reactions

Ay	727.7
By	772.3

Shear

0<x<L1	Const
V(x)=	727.7

L1<x<TL	Const
V(x)=	-772.3

Moment

0<x<L1	x	Const
M(x)=	727.7	0

L1<x<TL	x	Const
M(x)=	-772.3	15125

Angle

0<x<L1	x^2	x	Const
θ(x)=	363.8	0	-35565

L1<x<TL	x^2	x	Const
θ(x)=	-386.2	15125	-111820

Deflection

0<x<L1	x^3	x^2	x	Const
y(x)=	121.3	0	-35565	0

L1<x<TL	x^3	x^2	x	Const
y(x)=	-128.7	7562.5	-111820	256302

Check

at x=L1;	y1=y2	y1=	-234282	Good
		y2=	-234282	

0<x<L1;	y1=FE	y1=	-0.03512	Good
		FE=	-0.03512	

Load 1 Deflection

y(Load 1)=	-0.03512	in

<u>Load 2</u>

Reactions

Ay	229.8
By	770.2

Shear

0<x<L1	Const
V(x)=	229.8

L1<x<TL	Const
V(x)=	-770.2

Moment

0<x<L1	x	Const
M(x)=	229.8	0

L1<x<TL	x	Const
M(x)=	-770.2	15083

Angle

0<x<L1	x^2	x	Const
θ(x)=	114.9	0	-13912

L1<x<TL	x^2	x	Const
θ(x)=	-385.1	15083	-127665

Deflection

0<x<L1	x^3	x^2	x	Const
y(x)=	38.30	0	-13912	0

L1<x<TL	x^3	x^2	X	Const
y(x)=	-128.4	7541	-127665	571927

Check

at x=L2;	y1=y2	y1=	-101016	Good
		y2=	-80182.4	

0<x<L2;	y1=FE	y1=	-0.01514	Good
		FE=	-0.01514	

Load 2 Deflection

y(Load 2)=	-0.01514	in

Table F-1: Series of tables displaying constants needed to solve the differential equations to calculate the deflection of the beam for Load Case 1

Load Case 1			
L1	8	ft	
L2	3	ft	
Load 1	1500	lb	
Load 2	1000	lb	
Deflection at x=L1	x=10.083 ft	-0.0503	in
Deflection at x=L2	x=15.083 ft	-0.0345	in
Max Deflection	x= 10.167 ft	-0.0503	in

Table F-2: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 1

Load Case 2			
L1	10	ft	
L2	4	ft	
Load 1	1500	lb	
Load 2	1000	lb	
Deflection at x=L1	x=8.083 ft	-0.0494	in
Deflection at x=L2	x=14.083 ft	-0.0398	in
Max Deflection	x=9.783 ft	-0.0514	in

Table F-3: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 2

Load Case 3			
L1	11	ft	
L2	7	ft	
Load 1	1500	lb	
Load 2	1000	lb	
Deflection at x=L1	x= 7.083 ft	-0.0501	in
Deflection at x=L2	x= 11.083ft	-0.0527	in
Max Deflection	x= 9.539 ft	-0.0544	in

Table F-4: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 3

Load Case 4			
L1	12	ft	
L2	3.5	ft	
Load 1	1500	lb	
Load 2	1000	lb	
Deflection at x=L1	x= 6.083 ft	-0.0383	in
Deflection at x=L2	x= 14.583 ft	-0.0323	in
Max Deflection	x= 9.572 ft	-0.0450	in

Table F-5: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 4

Load Case 5			
L1	12.5	ft	
L2	6	ft	
Load 1	1500	lb	
Load 2	1000	lb	
Deflection at x=L1	x= 5.583 ft	-0.0392	in
Deflection at x=L2	x= 12.083 ft	-0.0447	in
Max Deflection	x= 9.530 ft	-0.0486	in

Table F-6: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 5

Load Case 6			
L1	13	ft	
L2	8.417	ft	
Load 1	1500	lb	
Load 2	1000	lb	
Deflection at x=L1	x= 5.083 ft	-0.0375	in
Deflection at x=L2	x= 9.667 ft	-0.0485	in
Max Deflection	x= 9.248 ft	-0.0486	in

Table F-7: Deflection at the measured locations of L1 and L2 and the maximum deflection for Load Case 6

G) APPENDIX G: CASTIGLIANO'S THEOREM

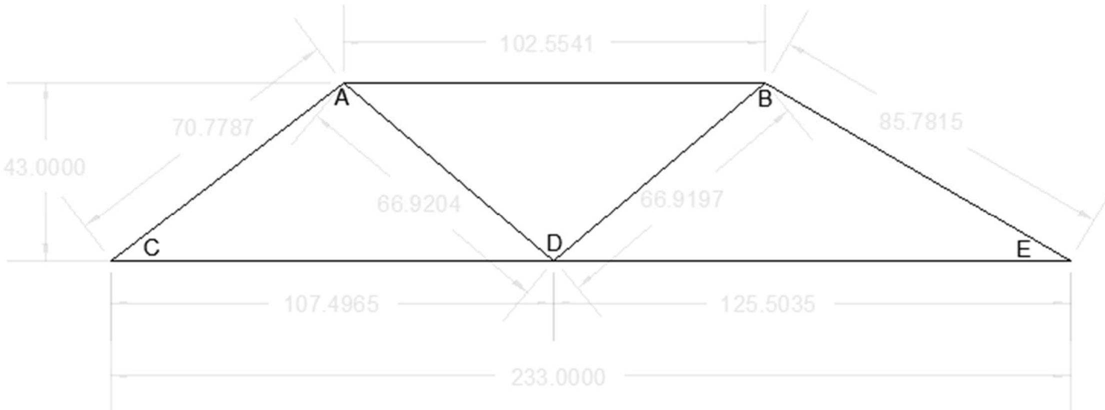

Figure G-1: Diagram of the analyzed truss

The following series of tables outlines the constants for competing Castigliano's Theorem for analyzing trusses. Each constant is calculated by analyzing each joint of the bridge to find the internal forces acting on each of the members. The load is applied at point D on the diagram of the truss, and the measure deflection is also at point D.

Castigliano Deflection

Deflection	**0.1182**	**in**
Load	**2500**	**lb**

Dimensions

Member	Length (in)	X component	Y component	Out dia (in)	In Dia (in)	A (in^2)	E (ksi)
AB	103	1	0	2.5	2.31	0.718	29000
AC	70.8	0.794	0.608	2.5	2.31	0.718	29000
AD	66.9	0.766	0.643	1	0.9	0.149	29000
BD	66.9	0.766	0.643	1	0.9	0.149	29000
BE	85.8	0.865	0.501	2.5	2.31	0.718	29000
CD	107	1	0	1	0.884	0.343	29000
DE	126	1	0	1	0.884	0.343	29000
height	43						
length	233						

Reactions

	Load	P
Ey	1153	0.461
Cy	1347	0.539

Joint C

		Load	P
Fy	Fac	-2217	-0.887
Fx	Fcd	1761	0.704

Joint A

		Load	P
Fy	Fad	2096	0.838
Fx	Fab	-3366	-1.35

Joint E

		Load	P
Fy	Fbe	-2301	-0.920
Fx	Fde	-1991	-0.796

Joint B

		Load	P
Fy	Fbd	1795	0.718

Member	Force norm	P
AB	-3366	-1.347
AC	-2217	-0.887
AD	2096	0.838
BD	1795	0.718
BE	-2301	-0.920
CD	1761	0.704
DE	-1991	-0.796

Table G-1: Constants and calculations used to calculate the deflection of the steel bridge using Castigliano's Theorem

Castigliano's Table

Member	δN/δP (-)	N p=0 (lb)	L (ft)	A (in^2)	E (ksi)	Combined	
AB	-1.347	-3366	8.55	0.718	29000	1.861	
AC	-0.887	-2217	5.90	0.718	29000	0.557	
AD	0.838	2096	5.58	0.149	29000	2.264	
BD	0.718	1795	5.58	0.149	29000	1.661	
BE	-0.920	-2301	7.15	0.718	29000	0.727	
CD	0.704	1761	8.96	0.343	29000	1.116	
DE	-0.796	-1991	10.46	0.343	29000	1.666	
					Sum=	**0.00985**	**ft**

Table G-2: Castigliano's Table to find the deflection at point D

H) APPENDIX H: INTERNAL FORCES OF THE SUPPORTS

Assuming a point load located at L1 and L2 (plus 1.5'), half of the load will be distributed to one side of the bridge and half to the other. This will give each leg their own reactions. L1 and L2 will be shifted 1.5' closer to the West side on the North Side. The Reactions for Load Case 2 are seen in Table H2 and Table H3.

Total L	235	in	19.58 ft
F1=	1500	lb	
L1=	10	ft	
F2=	1000	lb	
L2=	4	ft	

Table H-1: Given values of the bridge

	South Side		**Visual Analysis**	
SWy	580.8511	lb	571.94	lb
SEy	669.1489	lb	678.07	lb

Table H-2: Reactions for the South side of the bridge

	North Side		**Visual Analysis**	
NWy	676.5957	lb	690.88	lb
Ney	573.4043	lb	559.11	lb

Table H-3: Reactions for the North side of the bridge

I) APPENDIX I: BUCKLING FORCES

$$\boldsymbol{EI\frac{d^2y}{d^2x} = -Py}$$

Equation I-1: Differential Equation describing the buckling force for a member.

$$P = \frac{\pi^2 EI}{L^2}$$

Equation I-2: Solution to the differential equation describing the lowest buckling force that is also called the Euler Load.

	E (ksi)	**I (in^4)**	**L (in)**	**Buckling Force (lb)**	**Visual Analysis Force (lb)**
SW Compression	29000	0.5198	86.3	20,000	997
SE Compression	29000	0.5198	71.3	29,200	991
Top Truss	29000	0.5198	102.6	14,100	1290

Table I-1: Table giving the constants to solve Equation H2, the resulting Euler Load, and the Force across the main compression members of the truss.

J) APPENDIX J: FINAL DEFLECTIONS

	Gauge (Deflections: inches)		
	1000 lbs	750 lbs	500 lbs
gauge 1 (in)	0.38	0.311	0.234
gauge 2 (in)	0.338	0.299	0.201
Digital gauge (in)	0.604	0.28	0.208
Average (in)	**0.441**	**0.297**	**0.214**

Table J-1: Gauge deflections for varying weights being placed in the center of the bridge.

	Actual Deflection	**Visual Analysis**	**Superposition**
Load Case 1	N/A	0.247 in	0.0503 in
Load Case 2	0.86 in	0.154 in	0.0514 in
Load Case 3	N/A	0.336 in	0.0544 in
Load Case 4	N/A	0.307 in	0.0450 in
Load Case 5	N/A	0.370 in	0.0486 in
Load Case 6	N/A	0.506 in	0.0486 in

Table J-2: Table summarizing all the maximum deflections for each load case.