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Statistical physics of language dynamics

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Abstract. Language dynamics is a rapidly growing field that focuses on all processes related to the emergence, evolution, change and extinction of languages. Recently, the study of self-organization and evolution of language and meaning has led to the idea that a community of language users can be seen as a complex dynamical system, which collectively solves the problem of developing a shared communication framework through the back-and-forth signaling between individuals.

We shall review some of the progress made in the last few years and highlight potential future directions of research in this area. In particular, the emergence of a common lexicon and of a shared set of linguistic categories will be discussed, as examples corresponding to the early stages of a language. The extent to which synthetic modeling is nowadays contributing to the ongoing debate in cognitive science will be pointed out. In addition, the burst of growth of the web is providing new experimental frameworks. It makes available a huge amount of resources both as novel tools and data to be analized, allowing quantitative and large-scale analysis of the processes underlying the emergence of a collective information and language dynamics.

1. Introduction

Understanding the origins and evolution of language and meaning is currently one of the most promising areas of research in cognitive science.

Unprecedented results in information and communications technologies are enabling, for the first time, the possibility of mapping the interactions precisely, whether embodied and/or symbolic, of large numbers of actors, as well as the dynamics and transmission of information along social ties. At the same time, new theoretical and computational tools as well as synthetic modeling approaches have now reached sufficient maturity to contribute significantly to the long lasting debate in cognitive science. The combination of these two elements is opening terrific new avenues for studying the emergence and evolution of languages, new communication and semiotic systems. As was the case with biology, new tools and methods can trigger a significant boost in the ongoing transition of linguistics into an experimental discipline, where multiple evolutionary paths, timescales and dependence on the initial conditions can be effectively controlled and modeled.

Language as a social dynamical system Semiotic dynamics studies how populations of humans or agents can establish and share semiotic systems, typically driven by their use in communication. From this perspective, language is seen as an evolving [1] and self-organizing system, whose components are thus constantly being (re)shaped by language users in order to maximize communicative success and expressive power while at the same time minimizing articulatory effort. New words and grammatical constructions may be invented or acquired, new meanings may arise, the relation between language and meaning may shift (e.g., if a word adopts a new meaning), as well as the relation between meanings and the world may shift (e.g. if new perceptually grounded categories are introduced). All these changes happen at the level of the individual as well as at the group level. Here we focus on the interactions among the individuals, communicating both in a vertical (teacher-pupil) and in an horizontal (peer to peer) fashion. Communication acts are particular cases of language games, which, as already pointed out in [2], can be used to describe linguistic behavior, even though they can also include non linguistic behavior, such as pointing. Clark [3] argues that language and communication are social activities - joint activities - that require people to coordinate with each other as they speak and listen. Language use is more than the sum of a speaker speaking and a listener listening. It is the joint action that emerges when speakers and listeners [4] perform their individual actions in coordination, as ensembles. Again language is not seen as an individual process, but rather as a social process where a continuous alignment of mental representations [5] is taking place.

The landscape describing the large set of approaches to the study of language emergence and dynamics is extremely diversified, due to the flagrant complexity of a problem that can be addressed from many respects, with different methodologies, guided by often incompatible conceptual frameworks, and with different goals in mind.

A useful way to gain insights into such a variegated world is, therefore, that of focusing on few dimensions that allow for a coarse categorization of the ongoing research [6]. It is in general possible to identify broad paradigms that frame the problem in a particular way, focusing on specific aspects and addressing precise fundamental questions through concrete models and experiments [7]. Within each framework, then, the investigation can proceed through computational models, experiments with embodied agents, psychological experiments with human subjects and finally exploiting data made available either by in-house laboratory experiments as well as by large information systems like the Web.

Mathematical modeling of social phenomena Statistical physics has proven to be a very effective framework to describe phenomena outside the realm of traditional physics [8]. The last years have witnessed the attempt by physicists to study collective phenomena emerging from the interactions of individuals as elementary units in social structures [9]. This is the paradigm of the complex systems: an assembly of many interacting (and simple) units whose collective (i.e., large scale) behavior is not trivially deducible from the knowledge of the rules that govern their mutual interactions. This scenario is also true for problems related to the emergence of language.

From this new perspective, complex systems science turns out to be a natural ally in the quest for general mechanisms driving the collective dynamics whereby conventions can spread in a population, to understand how conceptual and linguistic coherence may arise through self-organization or evolution, and how concept formation and expression may interact to co-ordinate semiotic systems of individuals. One of the key methodological aspect of the modeling activity in the domains of complex systems is the tendency to seek simplified models to clearly pin down the assumptions and, in many cases, to make the models tractable from a mathematical point of view.

A crucial step in the modeling activity is represented by the comparison with empirical data in order to check whether the trends seen in real data are already compatible with plausible microscopic modeling of the individuals, or the latter requires additional ingredients. From this point of view, the Web may be a major source of help, both as a platform to perform controlled online social experiments, and as a repository of empirical data on large-scale phenomena. It is in this way that a virtuous cycle involving data collection, data analysis, modeling and predictions could be triggered, giving rise to an ever more rigorous and focused research approach to language dynamics.

It is worth stressing that the way the contributions are extended by the physicists, mathematicians and computer scientists should not be considered as alternatives to more traditional approaches. We rather posit that it would be crucial to foster the interactions across the different disciplines by promoting scientific activities with concrete mutual exchanges among all the interested scientists. This would help both in identifying the problems and sharpening the focus, as well as in devising the most suitable theoretical concepts and tools to approach the research.

Simple models of language dynamics Mathematical and computational modeling schemes play an essential role in all domains of sciences and they can clearly be helpful in studies related to the origins and evolution of language. Modeling can help us to understand what kind of mechanisms are necessary and sufficient for the origins and evolution of language. This approach makes it possible to examine through mathematical investigations and computational simulations whether certain basic assumptions of a theory are viable or not.

Most of the modeling efforts developed in the statistical physics of complex systems [9] are relatively new to more humanities oriented communities. One of the key methodological aspect is that of identifying and defining the simplest (minimal) models (i.e., algorithmic procedures) which could lead to efficient communication systems. It is important to stress the need in this field of shared and general models to create a common framework where different disciplines could compare their approaches and discuss the results. Moreover, the simplicity of the modeling schemes may allow for discovering underlying universalities, i.e., realizing that, behind the details of each single model, there could be a level where the mathematical structure is similar. This implies, on its turn, the possibility to perform mapping with other known models and exploit the background of the already acquired knowledge for those models. In this respect, statistical physics brings an important added value.

With this concept of *universality* in mind, an important open question concerns the quest for the best modeling schemes as well as the essential ingredients they should contain for a quantitative approach to the emergence and evolution of language structures. From this point of view, a first distinction concerns multi-agent models, in which one needs to define both the individuals' architectures and the social interactions, and macroscopic models in which populations are treated as a whole and one is interested in the evolution of aggregate quantities. Another dimension allows to discriminate between different approaches in the realm of multi-agent models according to the importance they give to cultural transmission (e.g., the Iterated Learning Model [10]), cognition and communication (Language Games [2, 11, 12, 13]) and biology (genetic evolution models [14, 15, 16, 17]). Also economic considerations, finally, have been pointed out [18, 19].

Some of the relevant general open questions include: What are the fundamental interaction mechanisms that allow for the emergence of consensus on an issue, a shared culture, a common language? What favors the homogenization process? What hinders it? Do spontaneous fluctuations slow down or even stop the ordering process? Does diversity of agents' properties strongly affect the model behavior? An additional relevant question concerns the effect of the topology of the social interaction network on the dynamical features of linguistic phenomena [9].

Language Games are particularly interesting since they provide a clue to describe and understand how shared conventions may emerge in a social group that constantly negotiate and reshape them. At present, Language games are investigated both through experiments involving embodied artificial agents (i.e., robots) and through multi-agent models. In particular, in the last few years, the methods and tools developed in statistical physics and complex systems science have turn out to be extremely powerful in providing more quantitative insights into the problem. While experiments have been tackling problems as complex as investigating the emergence of a shared grammar in a population, complex systems modeling has so far dealt with the most elementary, yet absolutely not trivial, problems of the emergence of a shared set of names (Naming Game) and categories (Category Game). The Category Game, in particular, is presently allowing for comparisons with data retrieved by psychological/anthropological experiments (e.g the World Color Survey).

The outline of the paper is as follows. We shall discuss problems of increasing complexity. We shall start with the so-called Naming Game that possibly represents the simplest example of the complex processes leading progressively to the establishment of complex human-like languages. Further we shall describe the so-called Category Game, which simulates the emergence of a shared set of linguistic categories, and we'll point out how the synthetic results obtained in this way agree quantitatively with the experimental ones. We shall conclude by highlighting a few open research challenges.

2. Naming Game

The Naming Game was expressively conceived to explore the role of self-organization in the evolution of language [11, 12] and it has acquired, since then, a paradigmatic role in the entire field of Semiotic Dynamics. The original paper [11] mainly focused on the formation of vocabularies, i.e., a set of mappings between words and meanings (for instance physical objects). In this context, each agent develops its own vocabulary in a random and private fashion. Nevertheless, agents are forced to align their vocabularies, through successive conversation, in order to obtain the benefit of cooperating through communication. Thus, a globally shared vocabulary emerges, or should emerge, as a result of local adjustments of individual word-meaning associations. The communication evolves through successive conversations, i.e., events that involve a certain number of agents (two, in practical implementations) and meanings. It is worth remarking that conversations are here particular cases of language games, which, as already pointed out by Wittgenstein [20, 2], are used to describe linguistic behavior but, if needed, can also include non-linguistic behavior, such as pointing.

This original seminal idea triggered a series of contributions along the same lines and many variants have been proposed along the years. It is worthwhile to mention here the work proposed in [21], who focuses on an imitation model which simulates how a common vocabulary is formed by agents imitating each other either using a mere random strategy or a strategy in which imitation follows the majority (which implies non-local information for the agents). A further contribution of the mentioned paper is the introduction of an interaction model which uses a probabilistic representation of the vocabulary. The probabilistic scheme is formally similar to the framework of evolutionary game theory [17, 22], since a *production* matrix and a *comprehension* matrix is associated to each agent. Unlike the approach of Evolutionary Language Games, the matrices are here dynamically transformed according to the social learning process and the cultural transmission rule. A similar approach has been proposed in [23].

Here we discuss in details a *minimal* version of the Naming Game which results in a drastic simplification of the model definition, while keeping the same overall phenomenology. This version of the Naming Game is suitable for massive numerical simulations and analytical approaches. Moreover its extreme simplicity allows for a direct comparison with other models introduced in other frameworks of statistical physics as well as in other disciplines.

2.1. The Minimal Naming Game

The simplest version of the Naming Game [13] is played by a population of N agents trying to bootstrap a common vocabulary for a certain number M of objects present in their environment. The objects can be people, physical objects, relations, web sites, pictures, music files, or any other kind of entity for which a population aims at reaching a consensus as far as their naming is concerned. Each player is characterized by an inventory of word-object associations he/she knows. All the inventories are initially empty ($t = 0$). At each time step ($t = 1, 2, ..$) two players are picked at random and one of them plays as speaker and the other as hearer. Their interaction obeys the following rules (see Figure 1):

- The speaker selects an object from the current context;
- The speaker retrieves a word from its inventory associated with the chosen object, or, if its inventory is empty, invents a new word;
- The speaker transmits the selected word to the hearer;
- If the hearer has the word named by the speaker in its inventory and that word is associated to the object chosen by the speaker, the interaction is a success and both players maintain in their inventories only the winning word, deleting **all** the others;
- If the hearer does not have the word named by the speaker in its inventory, or the word is associated to a different object, the interaction is a failure and the hearer updates its inventory by adding an association between the new word and the object.

The game is played on a fully connected network, i.e., each player can, in principle, play with all the other players, and makes two basic assumptions. One assumes that the number of possible words is so huge that the probability of a word to be re-invented is practically negligible (this means that homonymy is not taken into account here, though the extension is trivially possible). As a consequence, one can reduce, without loss of generality, the environment as consisting of only one single object ($M = 1$).

It is interesting to note that the authors in [24], have formally proven, adopting an evolutionary game theoretic approach, that languages with homonymy are evolutionarily

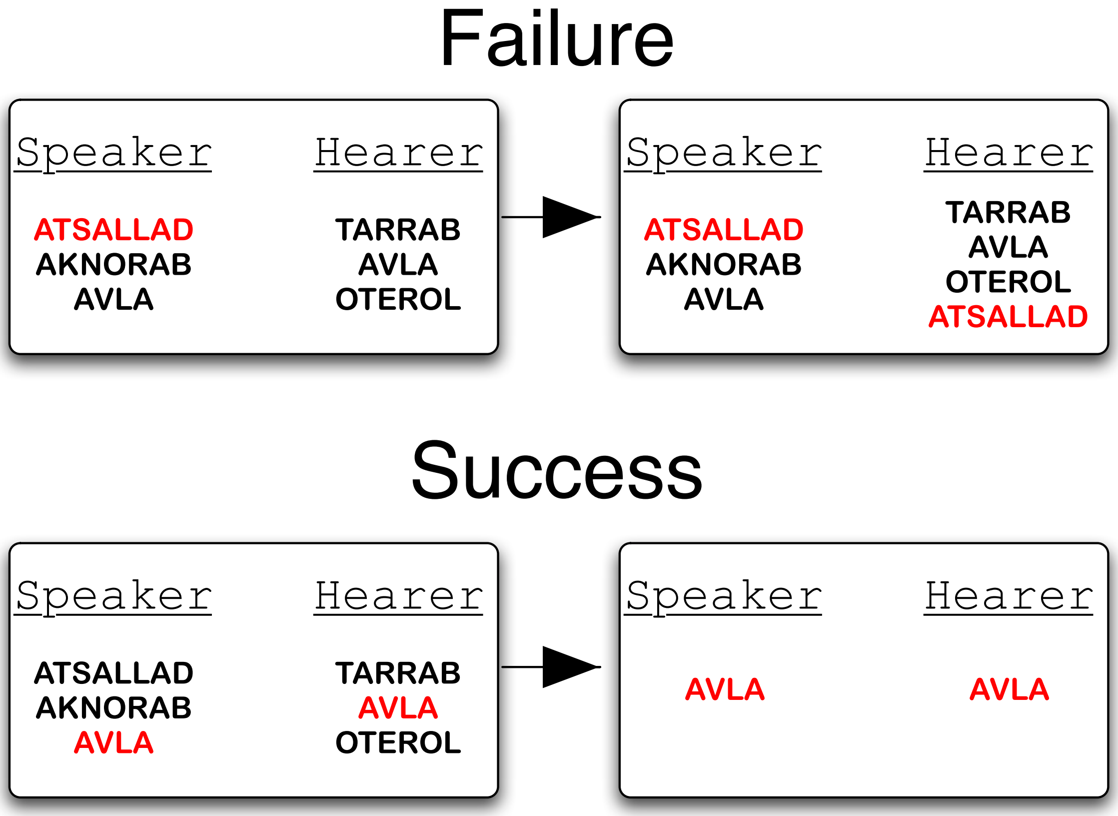

Figure 1. Naming Game. Examples of the dynamics of the inventories in a failed (top) and a successful (bottom) game. The speaker selects the word highlighted. If the hearer does not possess that word he includes it in his inventory (top). Otherwise both agents erase their inventories only keeping the winning word (bottom).

unstable. On the other hand, it is commonly observed that human languages contain several homonyms, while true synonyms are extremely rare. In [24] this apparent paradox is resolved noting that if we think of "words in a context", homonymy does indeed disappear from human languages, while synonymy becomes much more relevant. In the framework of the Naming game homonymy is not always an unstable feature (see next section about the Cateogry Game for an example [25]) and its survival depends in general on the size of the meaning and signal spaces [26].

A third assumption of the Naming Game consists ins assuming that the speaker and the hearer are able to establish whether a game was successful by subsequent actions performed in a common environment. For example, the speaker may refer to an object in the environment he wants to obtain and the hearer then hands the right object. If the game is a failure, the speaker may point (non-verbal communication) or get the object himself so that it is clear to the hearer which object was intended.

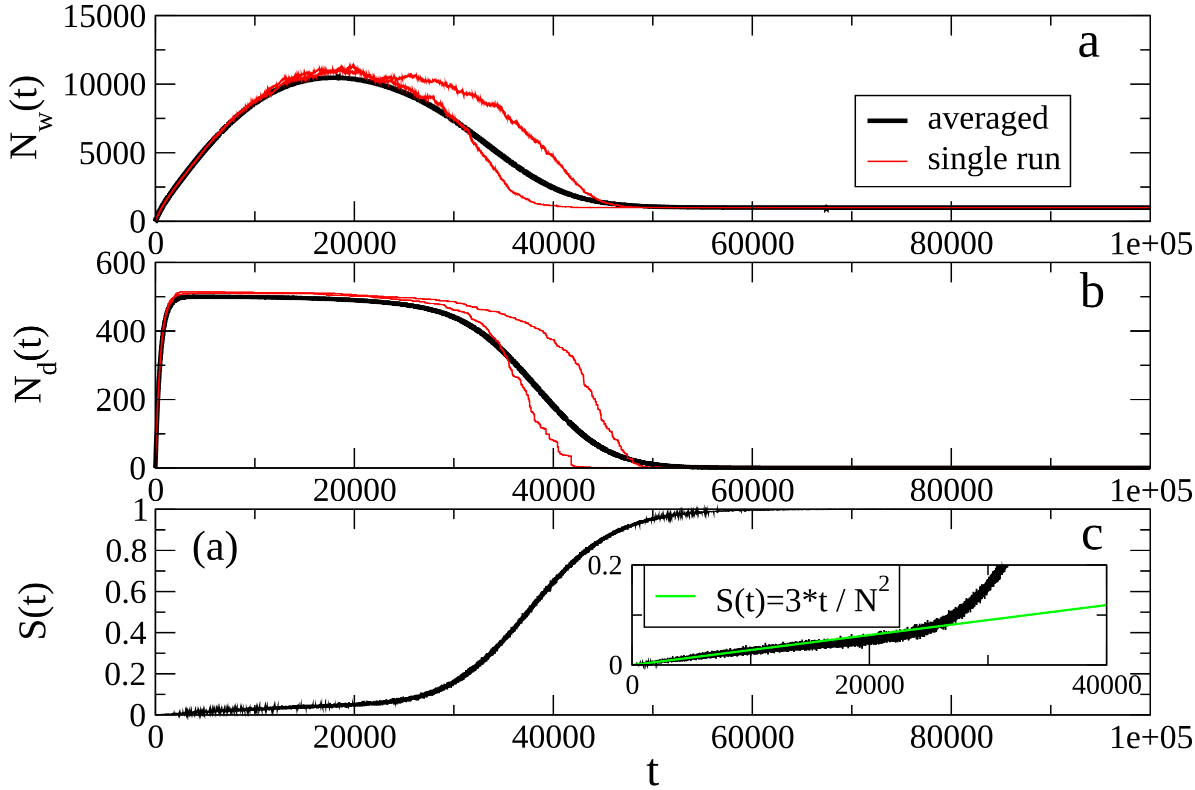

Figure 2. Naming Game. a) Total number of words present in the system, $N_w(t)$; b) Number of different words, $N_d(t)$; c) Success rate $S(t)$, i.e., probability of observing a successful interaction at time t. The inset shows the linear behavior of $S(t)$ at small times. The system reaches the final absorbing state, described by $N_w(t) = N$, $N_d(t) = 1$ and $S(t) = 1$, in which a global agreement has been reached.

2.2. Macroscopic analysis

Three main quantities allow to describe the dynamics of the model: The total number of words, $N_w(t)$, corresponding to the total memory required to the agents (i.e. to the sum of the sizes of their inventories); The number of different words, $N_d(t)$, telling us how many synonyms are present in the system at a given time; And the success rate $S(t)$, measuring the probability of observing a successful interaction at a given time. Figure 2 reports the evolution of these observables for the case in which one assumes that only two agents interact at each time step, but the model is perfectly applicable to the case where any number of agents interact simultaneously.

We can distinguish three phases in the behavior of the system. Very early, pairs of agents play almost uncorrelated games and the number of words hence increases over time as $N_w(t) = 2t$, while the number of different words increases as $N_d(t) = t$. In the second phase the success probability is still very small and agents' inventories start correlating, $N_w(t)$ curve presenting a well identified peak. The process evolves with an abrupt increase in the number of successes and a further reduction in the numbers of both total and different words. Finally, the dynamics ends when all agents have the same

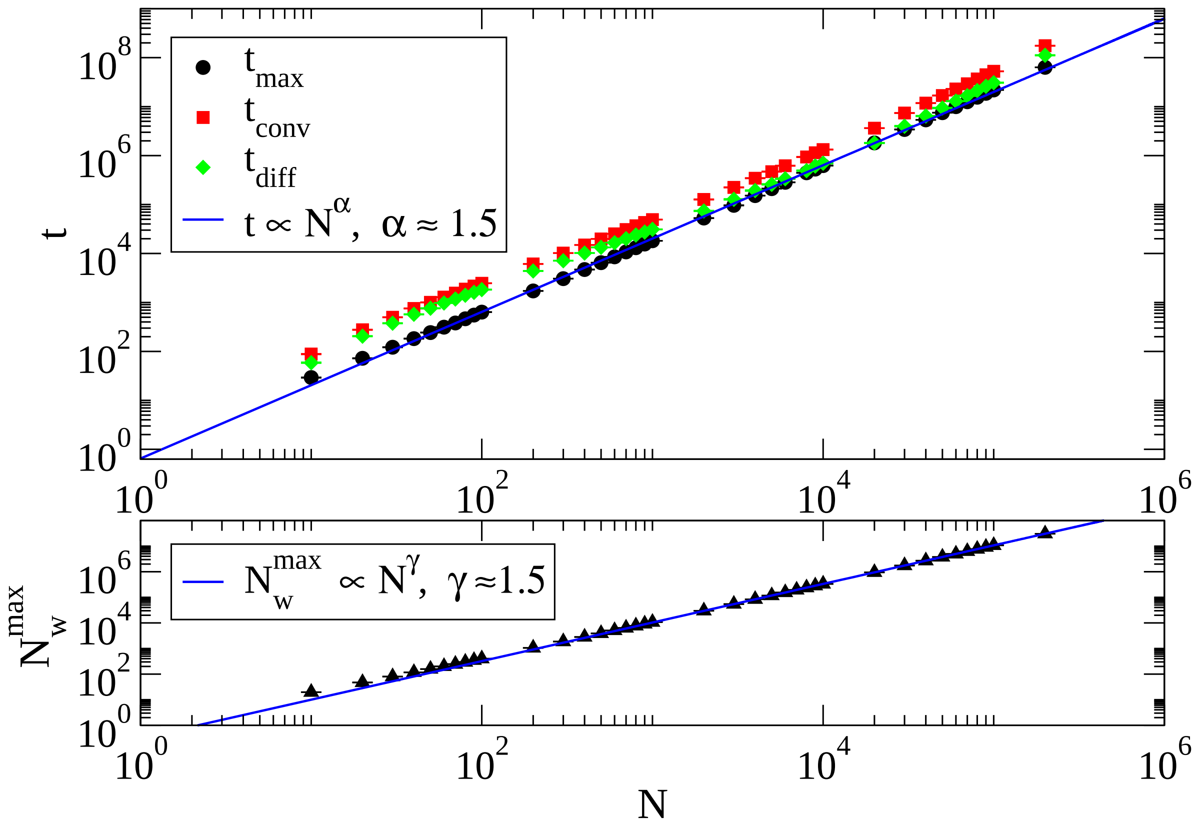

Figure 3. Naming Game. (Top) scaling of the peak and convergence time, t_{max} and t_{conv} along with their difference, t_{diff}. All curves scale with the power law $N^{1.5}$. (Bottom) the maximum number of words obeys the same power law scaling.

unique word and the system is in the attractive convergence state. It is worth noting that the developed communication system is not only *effective* (each agent understands all the others), but also *efficient* (no memory is wasted in the final state).

The system undergoes spontaneously a disorder/order transition to an asymptotic state where global coherence emerges, i.e., every agent has the same word for the same object. It is remarkable that this happens starting from completely empty inventories for each agent. The asymptotic state is one where a word invented during the time evolution took over with respect to the other competing words and imposed itself as the leading word. In this sense the system spontaneously selects one of the many possible coherent asymptotic states and the transition can thus be seen as a symmetry breaking transition.

Figure 3 shows the scaling behavior of the convergence time t_{conv}, and the time and height of the peak of $N_w(t)$, namely t_{max} and $N_w^{max} = N_w(t_{max})$. It turns out that all these quantities follow power law behaviors: $t_{max} \sim N^{\alpha}$, $t_{conv} \sim N^{\beta}$, $N_{max} \sim N^{\gamma}$ and $t_{diff} = (t_{conv} - t_{max}) \sim N^{\delta}$, with exponents $\alpha = \beta = \gamma = \delta \simeq 1.5$. A further timescale, namely $N^{5/4}$, rules the behavior of the success rate curve, whose abrupt jump appears therefore to be steeper and steeper as the population size grows, even on the convergence timescale. We do not enter here into more details on this point, but we refer the interested reader to [13], where in addition the values of all of these exponents are derived through simple scaling arguments.

2.3. Symmetry breaking: a controlled case

We concentrate now on a simpler case in which there are only two words at the beginning of the process, say A and B, so that the population can be divided into three classes: the fraction of agents with only A, n_A, the fraction of those with only the word B, n_B, and finally the fraction of agents with both words, n_{AB}. Describing the time evolution of the three species is straightforward:

$$\begin{aligned}
\dot{n}_A &= -\, n_A n_B + n_{AB}^2 + n_A n_{AB} \\
\dot{n}_B &= -\, n_A n_B + n_{AB}^2 + n_B n_{AB} \\
\dot{n}_{AB} &= +\, 2 n_A n_B - 2 n_{AB}^2 - (n_A + n_B) n_{AB}
\end{aligned} \tag{1}$$

The system of differential equations (1) is deterministic. It presents three fixed points in which the system can collapse depending on initial conditions. If $n_A(t=0) > n_B(t=0)$ $[n_B(t=0) > n_A(t=0)]$ then at the end of the evolution we will have the stable fixed point $n_A = 1$ $[n_B = 1]$ and, obviously, $n_B = n_{AB} = 0$ $[n_A = n_{AB} = 0]$. If, on the other hand, we start from $n_A(t=0) = n_B(t=0)$, then the equations lead to $n_A = n_B = 2n_{AB} = 0.4$. The latter situation is clearly unstable, since any external perturbation would make the system fall in one of the two stable fixed points. Indeed, it is never observed in simulations due to stochastic fluctuations that in all cases determine a symmetry breaking forcing a single word to prevail.

Eq.s 1 however, are not only a useful example to clarify the nature of the symmetry breaking process. In fact, they also describe the interaction among two different populations that converged separately on two distinct conventions. In this perspective, eq.s 1 predict that the population whose size is larger will impose its conventions. In the absence of fluctuations, this is true even if the difference is very small: B will dominate if $n_B(t=0) = 0.5 + \epsilon$ and $n_A(t=0) = 0.5 - \epsilon$, for any $0 < \epsilon \leq 0.5$ and $n_{AB}(t=0) = 0$. Data from simulations shows that the probability of success of the convention of the minority group n_A, decreases as the system size increases, going to zero in the thermodynamic limit ($N \to \infty$). A similar approach has been proposed to model the competition between two languages in the seminal paper [27]. It is worth remarking the formal similarities between modeling the competition between synonyms in a Naming Game framework and the competition between languages: in both cases a synonym or a language are represented by a single feature, e.g., the characters A or B, for instance, in equations (1). The similarity has been made more evident by the subsequent variants of the model introduced in [27] to include explicitly the possibility of bilingual individuals. In particular in [28, 29] deterministic models for the competition of two languages have been proposed which include bilingual individuals. In [30, 31] a modified version of the Voter model including bilinguals individuals has been proposed, the so-called AB-model. In a fully connected network and in the limit of infinite population size, the AB-model can be described by coupled differential equations for the fractions of individuals speaking language A, B or AB are, up to a constant normalization factor in the time-scale, identical to Eq.s 1.

In [32] it has been shown that the Naming Game and the AB-model are equivalent in the mean field approximation, though the differences at the microscopic level have non-trivial consequences. In particular the consensus-polarization phase transition taking place in the Naming Game (see section 2.5) is not observed in the AB-model. As for the interface motion in regular lattices, qualitatively, both models show the same behavior: a diffusive interface motion in a one-dimensional lattice, and a curvature driven dynamics with diffusing stripe-like metastable states in a two-dimensional one. However, in comparison to the Naming Game, the AB-model dynamics is shown to slow down the diffusion of such configurations. In general, the close connection of the AB model with the Naming Game suggests that the the latter can be fruitfully seen also as a framework to model language contact or, more speculatively, such issues as the emergence of new languages.

2.4. The role of the interaction topology

Social networks play an important role in determining the dynamics and outcome of language change [33, 34]. The first investigation of the role of topology was proposed, to the best of our knowledge, in 2004, at the 5th Conference on Language evolution, Leipzig [35]. Since then many approaches focused on adapting known models on topologies of increasing complexity: regular lattices, random graphs, scale-free graphs, etc.

The Naming Game model, as described above, is not well-defined on general networks. When the degree distribution is heterogeneous, it does matter if the first randomly chosen agent is selected as a speaker and one of its the neighbor as the hearer or viceversa: high-degree nodes are in fact more easily chosen as neighbors than low-degree vertices. Several variants of the Naming Game on generic networks can be defined. In the *direct Naming Game* (*reverse Naming Game*) a randomly chosen speaker (hearer) selects (again randomly) a hearer (speaker) among its neighbors. In a *neutral* strategy one selects an edge and assigns the role of speaker and hearer with equal probability to one of the two nodes [36].

Low-dimensional lattice On low-dimensional each agent can rapidly interact two or more times with its neighbors, favoring the establishment of a local consensus with a high success rate (Fig. 4, red squares for $1D$ and blue triangles for $2D$), i.e. of small sets of neighboring agents sharing a common unique word. Later on these "clusters" of neighboring agents with a common unique word undergo a coarsening phenomenon [37] with a competition among them driven by the fluctuations of the

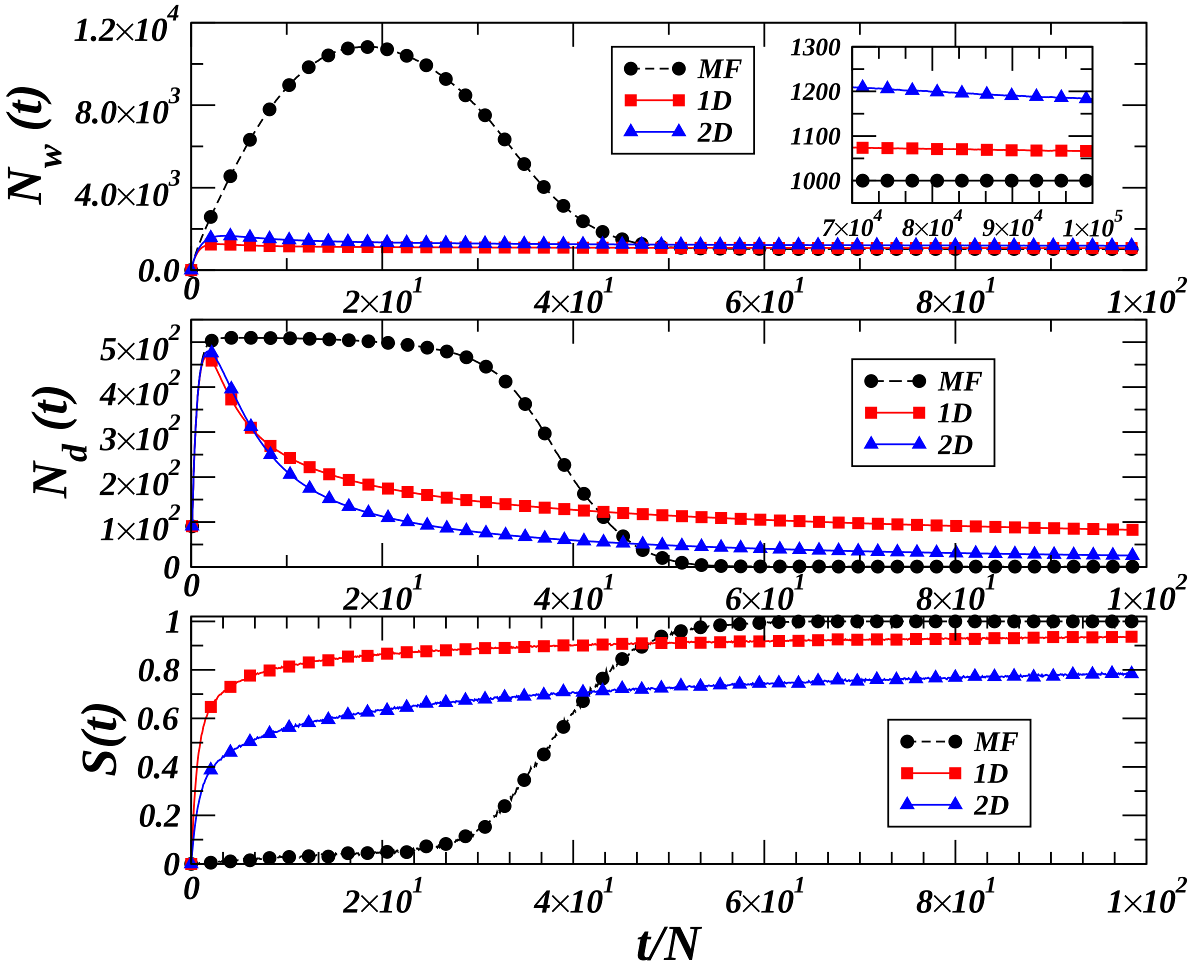

Figure 4. Evolution of the total number of words N_w (top), of the number of different words N_d (middle), and of the average success rate $S(t)$ (bottom), for a fully connected graph (mean-field, MF) (black circles) and low dimensional lattices ($1D$, red squares and $2D$, blue triangles) with $N = 1024$ agents, averaged over 10^3 realizations. The inset in the top graph shows the very slow convergence for low-dimensional systems.

interfaces [38]. The coarsening picture can be extended to higher dimensions and the scaling of the convergence time has been conjectured as being $\mathcal{O}(N^{1+1/d})$, where $d \leq 4$ is the dimensionality of the space. This prediction has been checked numerically. On the other hand the maximum total number of words in the system (maximal memory capacity) scales linearly with the system size, i.e., each agent uses only a finite capacity. In summary, low-dimensional lattice systems require more time to reach the consensus compared to mean-field, but a lower use of memory. A detailed analysis of the behaviour of the AB-model (whose mean-field deterministic version is equivalent, as we have seen above, to the deterministic Naming Game with only two possible words (Eqs 1)) on low-dimensional lattices has been carried out in [30]. Here the issue of memory is not important since the total number of words (or languages) is kept equal to two.

Small-world networks The effect of a small-world topology has been investigated in [39] in the framework of the Naming Game [13] and in [30] for the AB-model. Two different regimes are observed. For times shorter than a cross-over time, $t_{cross} = \mathcal{O}(N/p^2)$, one observes the usual coarsening phenomena as long as the clusters are typically one-dimensional, i.e., as long as the typical cluster size is smaller than $1/p$. For times much larger than t_{cross}, the dynamics is dominated by the existence of short-cuts and enters a mean-field like behavior. The convergence time is thus expected to scale as $N^{3/2}$ and not as N^3 (as in $d = 1$). Small-world topology allows thus to combine advantages from both finite-dimensional lattices and mean-field networks: on the one hand, only a finite memory per node is needed, in opposition to the $\mathcal{O}(N^{1/2})$ in mean-field; on the other hand the convergence time is expected to be much shorter than in finite dimensions. In [30] it has been studied the dynamics of the AB-model on a two-dimensional small world network. Also in this case a dynamical stage of coarsening is observed followed by a fast decay to the A or B absorbing states caused by a finite size fluctuation.

Complex networks The Naming Game has been studied also on complex networks. Here we only report about the global behaviour of the system and we refer to [36, 40] for an extensive discussion. Fig. 5 shows that the convergence time t_{conv} scales as N^{β} with $\beta \simeq 1.4 \pm 0.1$, for both Erdös-Renyi (ER) [41, 42] and Barabasi-Albert (BA) [43] networks. The scaling laws observed for the convergence time is a general robust feature that is not affected by further topological details, such as the average degree, the clustering or the particular form of the degree distribution. The value of the exponent β has been checked for various $\langle k \rangle$, clustering, and exponents γ of the degree distribution $P(k) \sim k^{-\gamma}$ for scale-free networks constructed with the uncorrelated configuration model (UCM) [44, 45, 46]. All these parameters have instead an effect on the other quantities such as the time and the value of the maximum of memory (see [36] for details). Finally, the presence of a strong community structure can in principle alter dramatically the overall dynamics, and we refer the interested reader to [36] (and to [47] for considerations on general ordering dynamics in this kind of networks).

2.5. Beyond consensus

A variant of the Naming Game has been introduced with the aim of mimicking the mechanisms leading to opinion and convention formation in a population of individuals [48]. In particular a new parameter, β ($\beta = 1$ corresponding to the Naming Game), has been added mimicking an *irresolute attitude* of the agents in making decisions. β is simply the probability that in a successful interaction both the speaker and the hearer update their memories erasing all opinions except the one involved in the interaction (see Figure 1). This negotiation process, as opposed to *herding-like* or *bounded confidence* driven processes, displays a non-equilibrium phase transition from an absorbing state in which all agents reach a consensus to an active (not-frozen as in the Axelrod model [49]) stationary state characterized either by polarization or

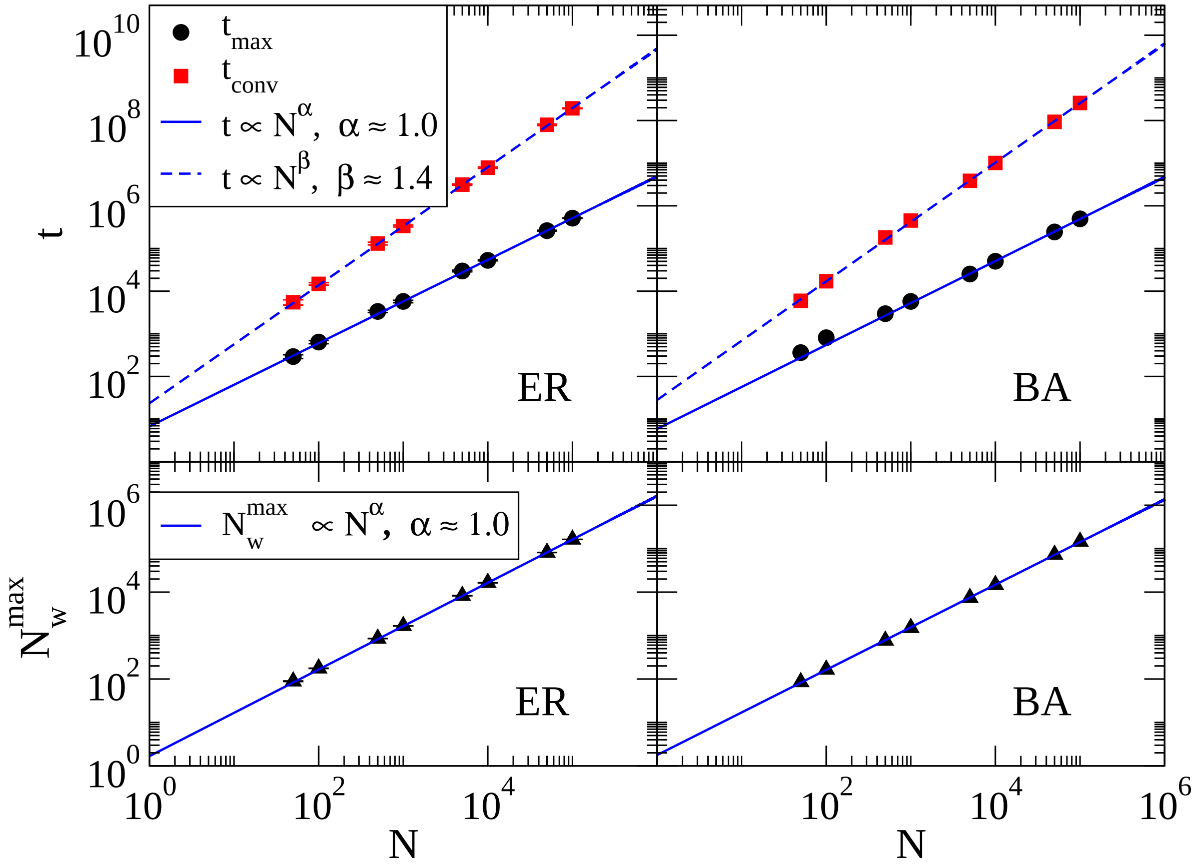

Figure 5. Top: scaling behavior with the system size N for the time of the memory peak (t_{max}) and the convergence time (t_{conv}) for ER random graphs (left) and BA scale-free networks (right) with average degree $\langle k \rangle = 4$. In both cases, the maximal memory is needed after a time proportional to the system size, while the time needed for convergence grows as N^{β} with $\beta \simeq 1.4$. Bottom: In both networks the necessary memory capacity (i.e. the maximal value N_w^{max} reached by N_w) scales linearly with the size of the network.

fragmentation in clusters of agents with different opinions. Figure 6 moreover shows that the transition at β_c is only the first of a series of transitions: when decreasing $\beta < \beta_c$, a system starting from empty initial conditions self-organizes into a fragmented state with an increasing number of opinions. At least two different universality classes exist, one for the case with two possible opinions and one for the case with an unlimited number of opinions. Very interestingly, the model displays the non-equilibrium phase transition also on heterogeneous networks, in contrast with other opinion-dynamics models, like for instance the Axelrod model [50], for which the transition disappears for heterogeneous networks in the thermodynamic limit.

3. Category Game

Categories are fundamental to recognize, differentiate and understand the environment. From Aristotle onwards, the issue of categorization has been subject to strong

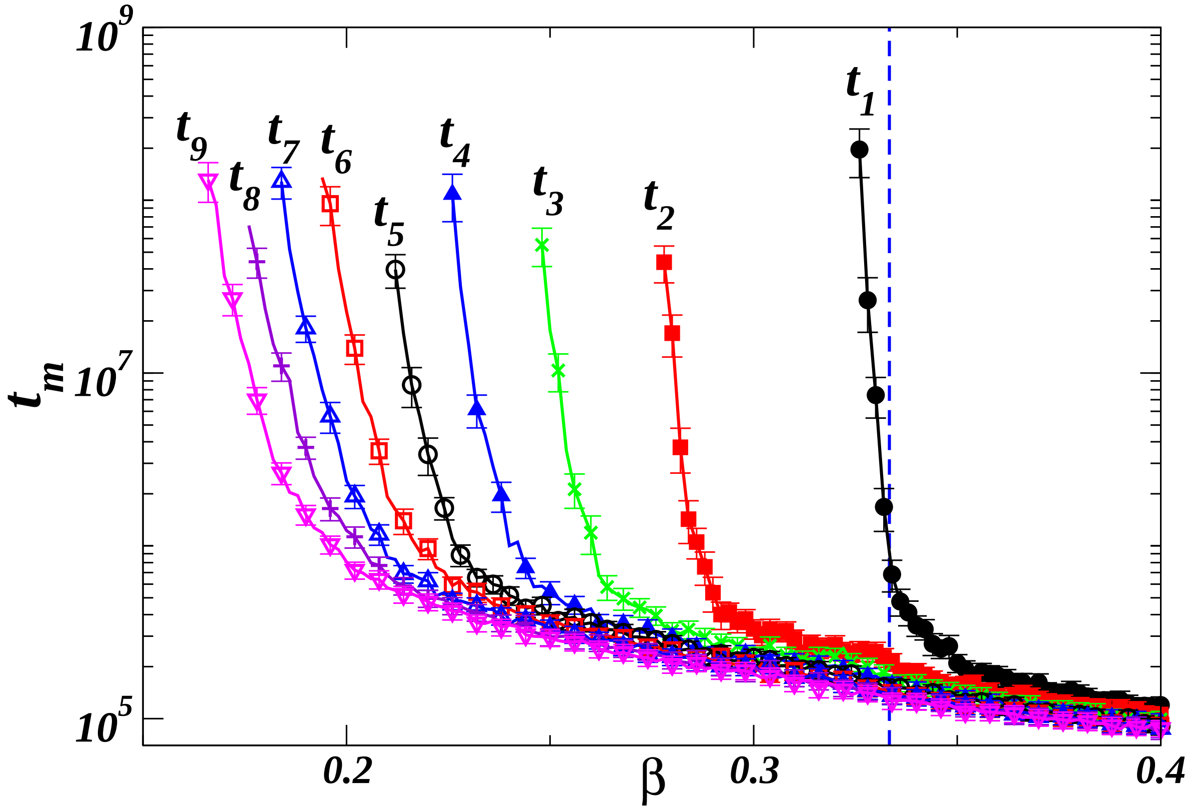

Figure 6. Time t_x required to a population on a fully-connected graph to reach a (fragmented) active stationary state with x different opinions. For every $m > 2$, the time t_m diverges at some critical value $\beta_c(m) < \beta_c$.

controversy in which purely cultural negotization mechanisms [2, 51] competed with physiological and cognitive features of the categorizing subjects [52]. A recent wave in cognitive science has induced a shift in viewpoint from the object of categorization to the categorizing subjects: categories are culture-dependent conventions shared by a given group. From this perspective, a crucial question is how they come to be accepted at a global level without any central coordination. Here we present the so-called Category Game, a scheme where an assembly of individuals with basic communication rules and without any external supervision may evolve an initially empty set of categories, achieving a non-trivial communication system.

The Category Game is a minimal model for linguistic categorization [53, 54, 55, 56, 57, 25, 58, 59, 60, 61], which is a more complex activity than naming a single object. In the spirit of reducing the rich spectrum of linguistic phenomena to essential aspects, prone to mathematical or numerical modeling, here we consider linguistic categorization as the elaboration of a map between a large set of perceptions or concepts and a small set of linguistic labels, typically nouns or attributes [62]. The paradigmatic case is offered by color naming: the potentially very large set of perceivable colors is mapped into a list of $5-10$ "basic color terms". The aim of the Category Game is not only reproducing in a realistic fashion the static (i.e., final) *categorization pattern* [63, 64], which is composed of a partition of the perceptual space and the dictionary connecting each category to a label, but to conjecture a plausible *dynamics* which brings to the light this final pattern in a large population of interacting individuals, all starting from an empty linguistic knowledge. A few simple rules for the interaction between pairs of individuals and samples of the external world amazingly generate, from scratch, a highly complex linguistic landscape, shared almost perfectly by all individuals, where the large set of perceptions is catalogued into a small set of linguistic categories [25].

The Category Game, originally conceived in [53], through a complex set of rules and detailed mechanisms, with the purpose of demonstrating the ability of numerical models to reproduce categorization patterns, posed from its birth a non-trivial problem: if the aim is the emergence of a pattern from scratch in a population, a discrimination activity where categories are refined with the purpose of separating different stimuli must be included in the rules of the game; this discrimination activity will continue until very close stimuli appear, requiring the introduction of a minimal distance between stimuli to set an endpoint for discrimination. This minimal distance is a quite natural parameter of any perceiving mechanism (being human or artificial), equivalent to a maximum resolution, often called "just noticeable difference" (JND) in the theories of perception. Such a parameter, anyway, trivially constrains the typical extension of categories, so that for very small JND one will end with a very large number of very small categories in the final categorization pattern. This problem was overcome in [25], where a minimal version of the Category Game was proposed, containing the essential ingredients to achieve the purpose: in particular, the solution to the problem consists in letting the model coagulate adjacent (small) perceptual categories through a *linguistic contagion* phenomenon: many neighboring categories with the same label will be considered as a unique linguistic category. The number of these large linguistic categories, quite surprisingly, remains much smaller than the number of tiny perceptual categories.

The other important step in demonstrating the relevance of simplified agent models for linguistic categorization was to make contact with experimental data. The perfect case study is offered by color categorization, where scientists in the past decades have collected a rich catalogue of data from tenths of different languages, building a very useful statistics of categorization patterns. The collection of these data is known as the World Colour Survey [65], which is freely available, and allowed some of us to test the similitude of patterns produced by the Category Game model with those observed in the human population, obtaining a remarkable agreement, as explained in details in the following [61].

3.1. Simple rules for the Category Game

Here we sketch the simplest rules for the Category Game, introduced in [25], using as an explanatory instance the case of color categorization. The Game involves a population of N artificial agents. Starting from scratch and without pre-defined color categories, the model dynamically generates, through a sequence of "games", a "categorization pattern" highly shared in the whole population of linguistic categories for the visible light spectrum. The model has the advantage of involving an extremely low number of parameters, basically the number of agents N and the JND curve $d_{min}(x)$, compared with its rich and realistic output.

For the sake of simplicity and not loosing the generality, color perception is reduced to a single analogical continuous perceptual channel, each light stimulus being a real number in the interval $[0, 1)$, which represents its normalized, rescaled wavelength. A categorization pattern is identified with a partition of the interval $[0, 1)$ in sub-intervals, or perceptual categories. Individuals have dynamical inventories of form-meaning associations linking perceptual categories with their linguistic counterparts, basic color terms, and these inventories evolve through elementary language games [2]. At each time step, two players (a speaker and a hearer) are randomly selected from the population and a scene of $M \geq 2$ stimuli is presented. Two stimuli cannot appear at a distance smaller than $d_{min}(x)$ where x is the value of one of the two. In this way, the JND is implemented in the model. On the basis of the presented stimuli, the speaker discriminates the scene, if necessary refining its perceptual categorization, and utters the color term associated to one of the stimuli. The hearer tries to guess the named stimulus, and based on their success or failure, both individuals rearrange their form-meaning inventories. New color terms are invented every time a new category is created for the purpose of discrimination, and are spread through the population in successive games.

To be more specific, we give a slightly more detailed insight into the rules for evolution of the agents. One of the objects, known only to the speaker, is the topic. The speaker checks if the topic is the unique stimulus in one of its perceptual categories. If both stimuli lie in one perceptual category, that category is divided into new categories, which inherit the words associated to the original category and are assigned a new word each; this process is called "discrimination" [53]. As a following step, the speaker utters the most relevant name of the category containing the topic (the most relevant name is the last name used in a winning game or the new name if the category has just been created). If the hearer does not have a category with that name, the game is a failure. If the hearer recognizes the name and there are many categories associated with the name, the hearer picks randomly one of these candidates (in the stable phase of the simulation and when M is not large, the hearer typically has a single candidate). Similarly, if the hearer recognizes the name and there are two or more objects in the corresponding category, it selects randomly one of them. If the picked candidate is the topic, the game is a success; otherwise, it is a failure. In case of failure, the hearer learns the name used by the speaker for the topic's category. In case of success, that name becomes the most relevant for that category and all other competing names are removed from both players' inventories.

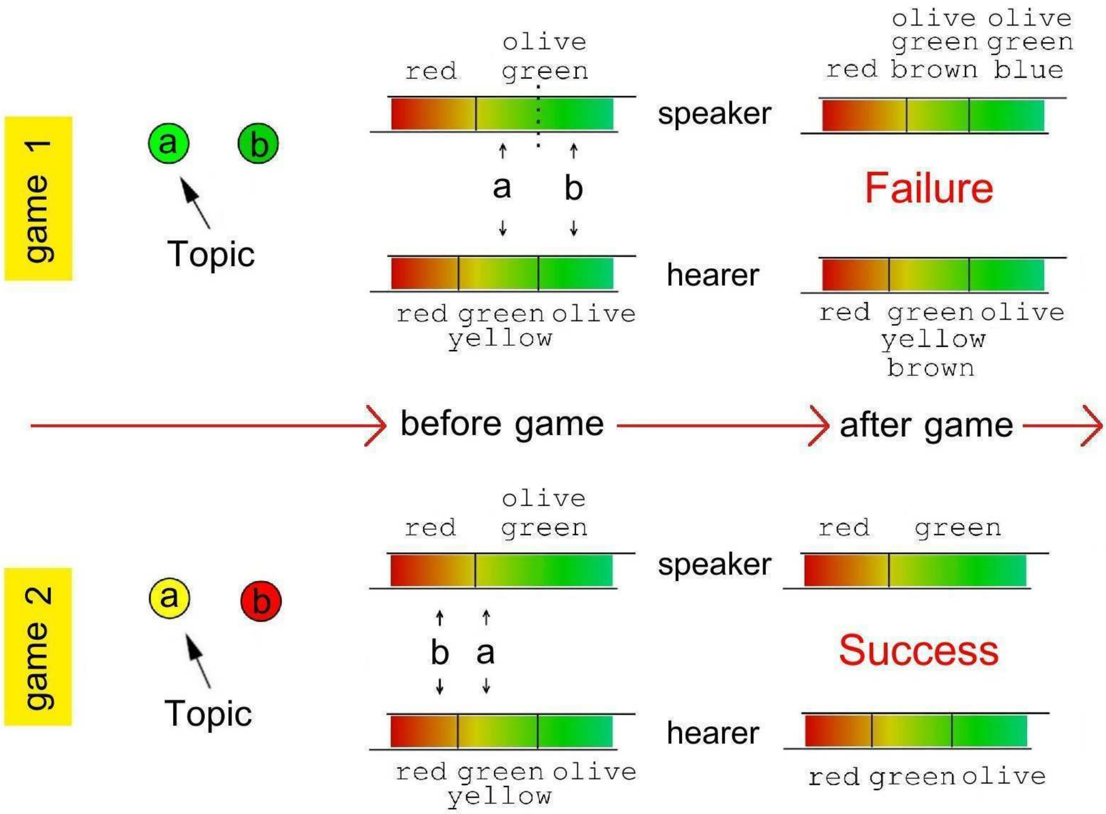

Figure 7. Rules of the category game. A pair of examples representing a failure (game 1) and a success (game 2), respectively. In a game, two players are randomly selected from the population. Two objects are presented to both players. The speaker selects the topic. In game 1 the speaker has to discriminate the chosen topic ("a" in this case) by creating a new boundary in his rightmost perceptual category at the position $(a+b)/2$. The two new categories inherit the words-inventory of the parent perceptual category (here the words "green" and "olive") along with a different brand new word each ("brown" and "blue"). Then the speaker browses the list of words associated to the perceptual category containing the topic. There are two possibilities: if a previous successful communication has occurred with this category, the last winning word is chosen; otherwise the last created word is selected. In the present example the speaker chooses the word "brown", and transmits it to the hearer. The outcome of the game is a failure since the hearer does not have the word "brown" in his inventory. The speaker unveils the topic, in a non-linguistic way (e.g. pointing at it), and the hearer adds the new word to the word inventory of the corresponding category. In game 2 the speaker chooses the topic "a", finds the topic already discriminated and verbalizes it using the word "green" (which, for example, may be the winning word in the last successful communication concerning that category). The hearer knows this word and therefore points correctly to the topic. This is a successful game: both the speaker and the hearer eliminate all competing words for the perceptual category containing the topic, leaving "green" only. In general when ambiguities are present (e.g. the hearer finds the verbalized word associated to more than one category containing an object), these are solved making an unbiased random choice.

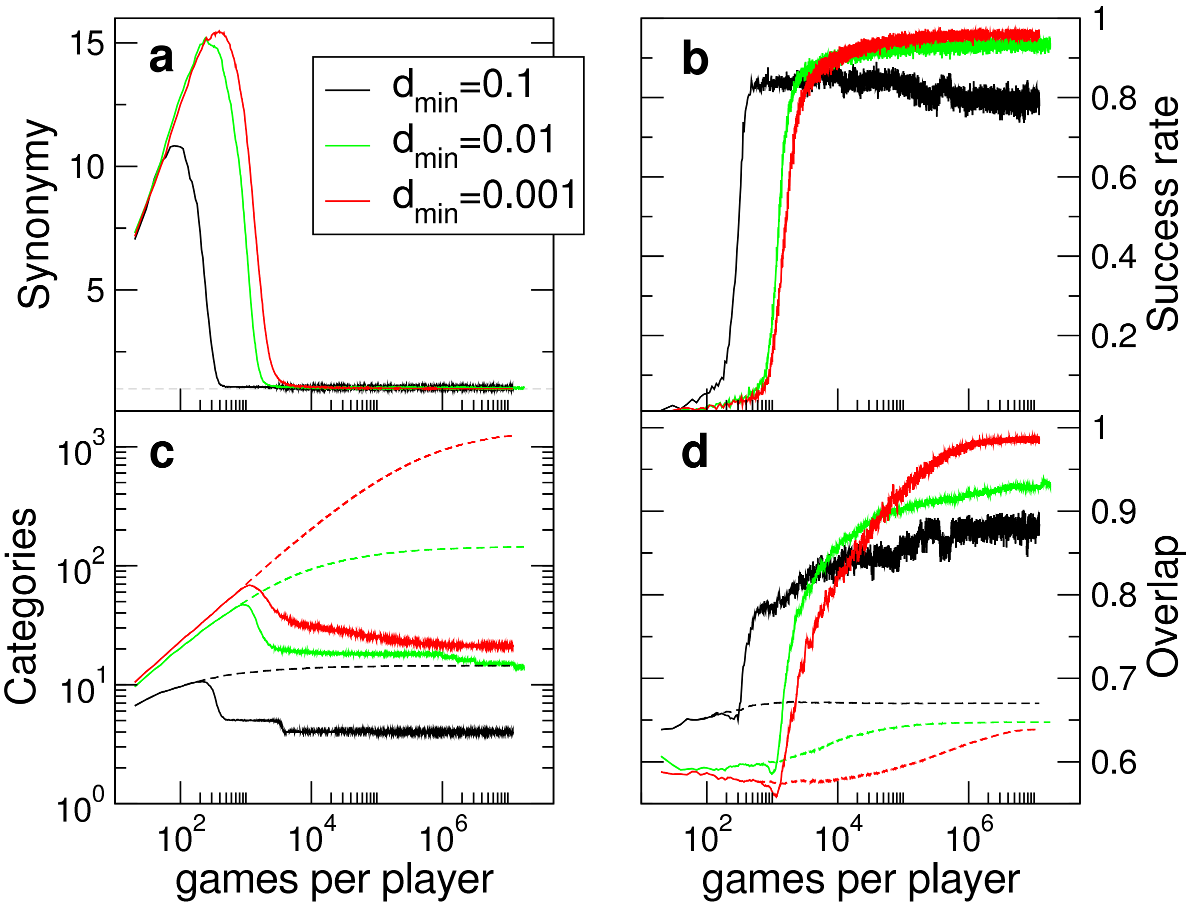

Figure 8. Results of simulations of the Category Game model with $N = 100$ and a flat (constant) $d_{min}(x) \equiv d_{min}$ curve with different values of d_{min}: a) Synonymy, i.e., average number of words per category; b) Success rate measured as the fraction of successful games in a sliding time windows games long; c) Average number of perceptual (dashed lines) and linguistic (solid lines) categories per individual; d) Averaged overlap, i.e., alignment among players, for perceptual (dashed curves) and linguistic (solid curves) categories.

3.2. From confusion to consensus

At the beginning all individuals have only the perceptual category $[0, 1)$ with no associated name. During a first phase of the evolution, the pressure of discrimination makes the number of perceptual categories increase, see dashed lines in Fig. 8c: at the same time, many different words are used by different agents for some similar categories. This kind of synonymy reaches a peak and then dries out (as displayed in Fig. 8a), in a similar way as in the Naming Game described before: when on average only one word is recognized by the whole population for each perceptual category, a second phase of the evolution intervenes. During this phase, words expand their dominion across adjacent perceptual categories, joining these categories to form new "linguistic categories". This is revealed by counting the number of these linguistic categories (solid lines in Fig. 8c), which decreases after some time. The coarsening of these categories becomes slower and slower, with a dynamical arrest analogous to the physical process in which supercooled liquids approach the glass transition [66]. In this long-lived almost stable phase, usually after 10^4 games per player, the linguistic categorization pattern has a degree of sharing between 90% and 100%; success is measured by counting in a small time window the rate of successful games (Fig. 8b), while the degree of sharing of categories is measured by an overlap function, which measure the alignment of category boundaries (both for perceptual or linguistic ones), displayed in Fig. 8d: for a mathematical definition of this function see [25]. The success rate and the overlap both remain stable for $10^5 \sim 10^6$ games per player [25]: we consider this pattern as the "final categorization pattern" generated by the model, which is most relevant for comparison with human color categories (see below). If one waits for a much longer time, the number of linguistic categories is observed to drop down: this non-realistic effect is due to the slow diffusion of category boundaries. Note that, at the level of the Category Game, categories can be equivalently described in terms of boundaries or prototypes, without any difference [25]. Slow diffusion of boundaries ultimately takes place due to small size effects. Recent investigations have demonstrated that this phase can occur on very long time-scale, with autocorrelation properties typical of an aging material, such as a glass.

The shared pattern in the long stable phase between 10^4 and 10^6 games per player is the main subject of the experiment described in the following section. It is remarkable, as already observed in [25] that the number of linguistic color categories achieved in this phase is of the order of 20 ± 10, even if the number of possible perceptual categories ranges between 100 and 10^4 and the number of agents ranges between 10 and 1000. For this reason it is plausible that the mechanism of spontaneous emergence of linguistic categories portrayed by this model is relevant for the problem of linguistic categorization in continuous spaces (such as color space) where no objective boundaries are present.

3.3. The role of parameters and the external world

As discussed above, the only parameters of the model are the size of the population N, the JND curve $d_{min}(x)$ and, eventually, the distribution function of the stimuli presented to the individuals. For the numerical results shown in the previous discussion we have considered a flat distribution where all stimuli between 0 and 1 were equally likely. In principle, one can model the role of environmental pressure through shaping this distribution function. It is interesting to discover that, while the general features of the dynamics are preserved, the final categorization pattern has a slight but observable sensitiveness to the distribution of stimuli. An example is offered by Figure 9, where stimuli distributions are sampled from different still pictures and where the final categorization pattern is portrayed for a few randomly selected individuals from a large population.

The role of N, as already discussed, is important in the stabilization of the plateau where the categorization pattern remains constant: this plateau, in time, is larger and larger as N increases [25]. On the other side, the role of $d_{min}(x)$ is crucial to obtain a

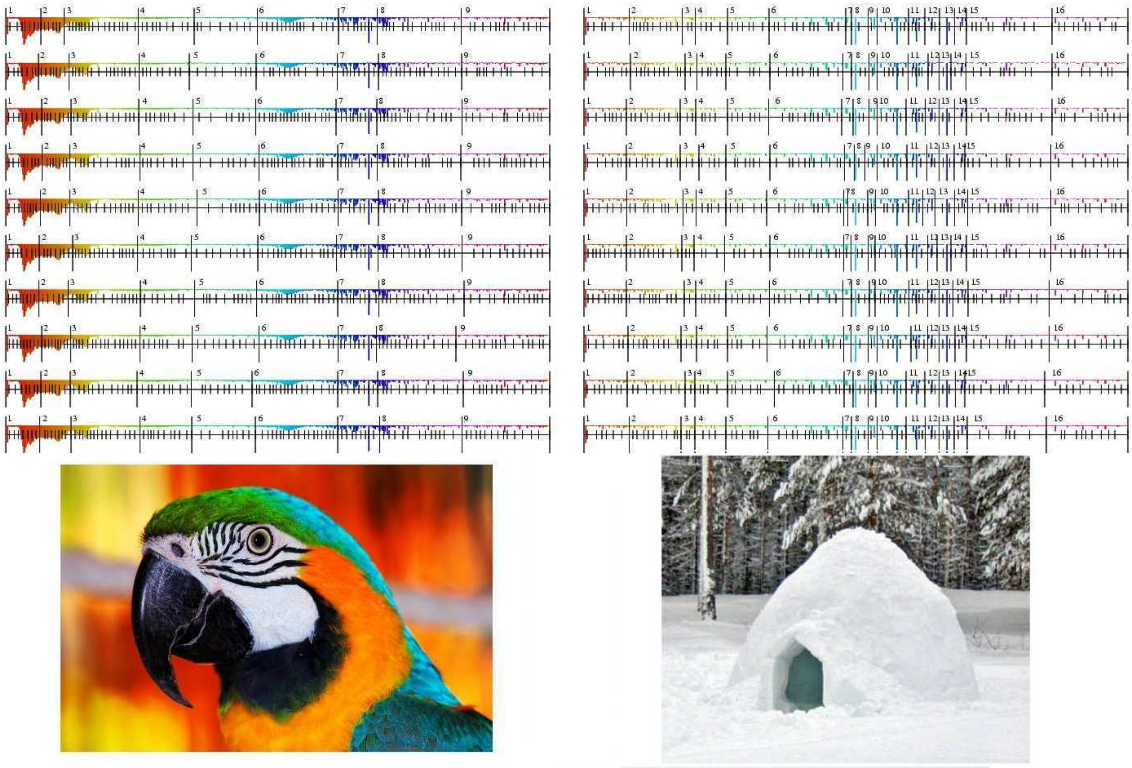

Figure 9. Categories and the pressure of environment. Inventories of 10 individuals randomly picked up in a population of $N = 100$ players, with $d_{min} = 0.01$, after 10^7 games. For each player the configuration of perceptual (small vertical lines) and linguistic (long vertical lines) category boundaries is superimposed to a colored histogram indicating the relative frequency of stimuli. The labels indicate the unique word associated to all perceptual categories forming each linguistic category. Two cases are presented with stimuli randomly extracted from the hue distribution of natural pictures. One can appreciate the perfect agreement of category names, as well as the good alignment of linguistic category boundaries. Moreover, linguistic categories tend to be more refined in regions where stimuli are more frequent: an example of how the environment may influence the categorization process.

close comparison with real data, as detailed in the next section.

4. Comparison with real-world data

A large amount of data on color categorization was gathered in the World Color Survey [67, 68], in which individuals belonging to different cultures had to name a set of colors. The results of the analysis of the categorization patterns obtained in this way have had a huge impact not only on such areas as Cognitive Science and Linguistics, but also Psychology, Philosophy and Anthropology (see for example, [62, 69, 70]). The main finding is that color systems across language are not random, but rather exhibit certain statistical regularities, thus implying that the classical theory of categorization, dating back to the work of Aristotle and claiming the arbitrariness of categorization, had to be reconsidered [69]. In this section, we describe how the Category Game model described above can be used to run a *Numerical* World Color Survey and point out that, remarkably, the synthetic results obtained in this way agree quantitatively with the experimental ones [61].

4.0.1. The World Color Survey P. Kay and B. Berlin [67] ran a first survey on 20 languages in 1969. From 1976 to 1980, the enlarged World Color Survey was conducted by the same researchers along with W. Merrifield and the data are public since 2003 on the website `http://www.icsi.berkeley.edu/wcs`. These data concern the basic color categories in 110 languages without written forms and spoken in small-scale, non-industrialized societies. On average, 24 native speakers of each language were interviewed. Each informant had to name each of 330 color chips produced by the Munsell Color Company that represent 40 gradations of hue and maximal saturation, plus 10 neutral color chips (black-gray-white) at 10 levels of value. The chips were presented in a predefined, fixed random order, to the informant who had to tag each of them with a "basic color term" is her language (in English, basic color terms would correspond to these would be "yellow","green", "red", etc. for more details see [67]).

After two decades of intense debate on this unique repository of data [69], Kay and Regier [68] performed a quantitative statistical analysis proving that the color naming systems obtained in different cultures and language are in fact not random. Through a suitable transformation they identified the most representative chip for each color name in each language and projected it into a suitable metric color space (namely, the CIEL*a*b color space). To investigate whether these points are more clustered across languages than would be expected by chance, they defined a dispersion measure on this set of languages S_0

$$D_{S_0} = \sum_{l,l^* \in S_0} \sum_{c \in l} \min_{c^* \in l^*} \text{distance}(c, c^*), \quad (2)$$

where l and l^* are two different languages, c and c^* are two basic color terms respectively from these two languages, and distance(c, c^*) is the distance between the points in color space in which the colors are represented. To give a meaning to the measured dispersion D_{S_0}, Kay and Regier created "new" datasets S_i ($i = 1, 2, .., 1000$) by random rotation of the original set S_0, and measured the dispersion of each new set D_{S_i}.

The human dispersion appears to be distinct from the histogram of the "random" dispersions with a probability larger than 99.9%. As shown in Figure 3a of [68], the average dispersion of the random datasets, $D_{neutral}$, is 1.14 times larger than the dispersion of human languages. Thus, human languages are more clustered, i.e., less dispersed, than their random counterparts and universality does exist [68].

4.0.2. The Numerical World Color Survey The key aspect of the statistical analysis described above is the comparison of the clustering properties of a set of *true* human languages against the ones exhibited by a certain number of randomized sets. In replicating the experiment it is therefore necessary to obtain two sets of synthetic data, one of which must have some human ingredient in its generation. The idea put forth in [61] is to act on the d_{min} parameter of the Category Game, describing, as discussed in the previous section, the discrimination power of the individuals to stimuli of a given wave-length. In fact, it turns out that human beings are endowed with a d_{min}, the "Just Noticeable difference" or JND, that is not continuous, but rather is a function of the frequency of the incident light (see the inset in Fig. 10) ‡. Technically, psychophysiologists define the JND as a function of wavelength to describe the minimum distance at which two stimuli from the same scene can be discriminated [71, 72]. The equivalence with the d_{min} parameter is therefore clear and different artificial sets can be created:

- *"Human"* categorization patterns are obtained from populations whose individuals are endowed with the rescaled human JND (i.e., d_{min});
- *Neutral* categorization patterns are obtained from populations in which the individuals have constant JND $d_{min} = 0.0143$, which is the average value of the human JND (as it is projected on the $[0, 1)$ interval, Fig. 10 (inset)).

In analogy to the WCS experiment, the randomness hypothesis in the NWCS for the neutral test-cases is supported by symmetry arguments: in neutral simulations there is no breakdown of translational symmetry, which is the main bias in the"human" simulations.

Thus, the difference between "human" and neutral data originates from the perceptive architecture of the individuals of the corresponding populations. A collection of "human" individuals form a "human" population, and will produce a corresponding "human" categorization pattern. In a hierarchical fashion, finally, a collection of populations is called a *world*, which in [61] is formed either by all "human" or by all non-"human" populations. To each world it corresponds a value of the dispersion D defined in Eq. (2), measuring the amount of dispersion of the languages (or categorization patterns) belonging to it. In the actual WCS there is of course only one human World (i.e., the collection of 110 experimental languages), while in [61] several worlds have been generated to gather statistics both for the "human" and non-"human" cases.

The main results of the NWCS are presented in Figure 10. Since the dispersion D defined in Eq. (2) [68] depends on the number of languages, the number of colors, and the space units used, every measure of D in the NWCS is normalized by the average value obtained in the "human" simulations, and every measure of D from the WCS experiment is divided by the value obtained in the original (non-randomized) WCS analysis (as in [68]). Thus, both the average of the "human worlds" and the value based on the WCS data are represented by 1 in Figure 3. In the same plot, the probability density of observing a value of D in the "neutral world" simulations is also shown by the red

‡ The attention is here on the human Just Noticeable Difference for the hue, see [61].

histogram bars. Finally, the Figure contains also the data reported in the histogram of the randomized datasets in Figure 3a of [68], whose abscissa is normalized by the value of the non-randomized dataset and frequencies are rescaled by the width of the bins.

Figure 10 illustrates the main results. The Category Game Model informed with the human $d_{min}(x)$ (JND) curve produces a class of "worlds" that has a dispersion lower than and well distinct from that of the class of "worlds" endowed with a non-human, uniform $d_{min}(x)$. Strikingly, moreover, the ratio observed in the NWCS between the average dispersion of the "neutral worlds" and the average dispersion of the "human worlds" is $D_{neutral}/D_{human} \sim 1.14$, very similar to the one observed between the randomized datasets and the original experimental dataset in the WCS. In the Supplementary information of [61], finally, it is shown that these findings are robust against changes in such parameters as the population size N, the distribution of the stimuli, the number of object is a scene M, the time of measurement (as long as a measure is taken in the temporal region in which a categorization pattern exists) etc.

These findings are important for a series of reasons. First of all, it is the first case in which the outcome of a numerical experiments in this field is comparable at any level with true experimental data. Second, as discussed above, the results of the NWCS are not only in qualitative, but also in quantitative agreement with the results of the WCS. Third, the very design of the model suggests a possible mechanisms lying at the roots of the observed universality. Human beings share certain perceptual bias that, even though are not strong enough to deterministically influence the outcome of a categorization, are on the other hand capable of influencing category patterns in a way that becomes evident only through a statistical analysis performed over a large number of languages. This explanation for the observed universality had already been put forth based on theoretical analysis (see, for instance [70, 73]), but the NWCS represents the first numerical evidence supporting it.

5. Conclusions and open problems

All the efforts outlined in the previous sections indicate that a complex cognitive phenomena as human language can be understood through a purely cultural route. In particular, human language is related to a community of individuals that interact with each other by means of a set of simple rules. Two important problems, Naming and Categorization, already provide us with enough evidence on how languages can evolve and change over time within different linguistic societies resulting, without any centralized control, into emergent regularized patterns. Most strikingly, the numerical findings of particular models show excellent quantitative agreement with real data.

Of course, these results are far from setting an endpoint in the research in cognitive science. Quite the reverse, this area is rich with many more and equally (or in fact more) challenging problems. In this spirit, we conclude by listing a few directions where the research in language dynamics is already moving or could possibly head to.

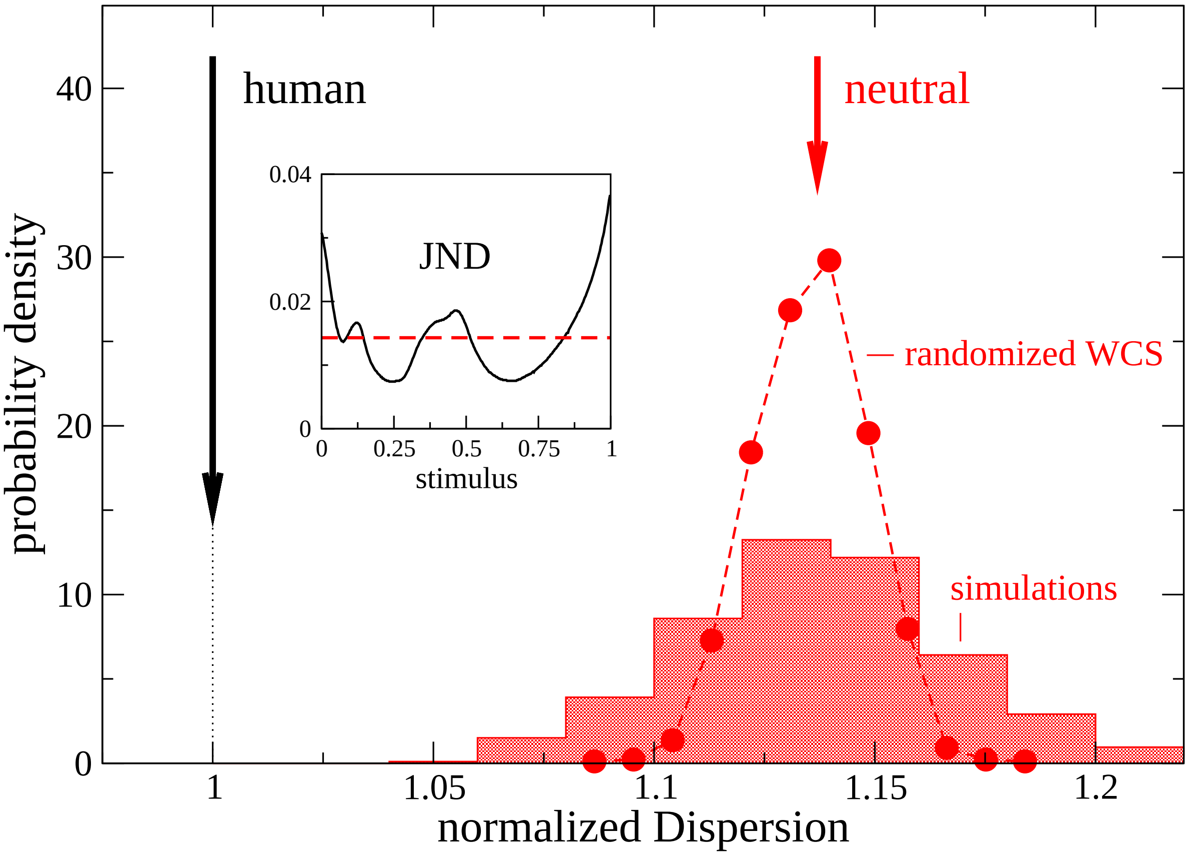

Figure 10. "Neutral worlds", $D_{neutral}$, (histogram) are significantly more dispersed than "human worlds", D_{human}, (black arrow), as also observed in the WCS data (the filled circles extracted from [68] and the black arrow). The abscissa is rescaled so that the human D (WCS) and the average "human worlds" D both equal 1. The histogram has been generated from 1500 neutral worlds, each made of 50 populations of 50 individuals, and $M = 2$ objects per scene. Categorization patterns have been considered after the population had evolved for a time of10^6 games per agents. The inset figure is the human JND function (adapted from [72]). On the vertical axis: the probability density $\rho(x_i)$ equals the percentage $f(x_i)$ of the observed measure in a given range $[x_i - \Delta/2, x_i + \Delta/2]$ centering around x_i, divided by the width of the bin Δ, i.e., $\rho(x_i) = f(x_i)/\Delta$. This procedure allows for a comparison between the histogram coming from the NWCS [61] and that obtained in the study on the WCS [68], where the bins have a different width.

5.1. Category formation

Again sticking on colors, the categorization problem is not a closed challenge, despite highly significative steps have been done in this direction. For instance, the emergence in a population of complex color terms has still to be explained: how fine-grained color terms like “crimson”, “magenta” etc. do emerge and coexist with basic color terms like “red”, “blue”, etc.? Are these the outcome of a special society of individuals for whom the set of basic terms is not sufficient for explaining the whole spectrum (e.g., painters) or there is a hierarchy of category structures to which people resort depending of the difficulty of their specific linguistic task? The two answers are not mutually esclusive, since a finer categorization could be driven by an uneven distribution of the stimuli.

In this area many questions remain open: how the number of emerging categories depend on factors like the population size, the dimension and structure of the semantic space, the network of acquaintances, the environment where the population live, genetic-driven perceptual endowments, specific cognitive abilities, etc. It would be important to investigate each of these elements to make the general modeling scheme closer to a larger set of realistic situations where categories emerge in a non-trivial way, so that specific predictions can be compared with real data.

For instance, a fundamental open question about the emergence of linguistic categories, and more generally of shared linguistic structures, concerns the role of timescales. How to reconcile the apparent static character of most of the linguistic structures we learned with the evidences of a fluid character of modern communication systems? Very preliminary studies suggest that well established linguistic structures can undergo *aging* [74, 75]: at relatively early stages changes are very frequent but they become progressively more rare as the system ages; a phenomenon whose intensity increases with the population size. From this point of view, shared linguistic conventions would not emerge as attractors of a language dynamics, but rather as metastable states.

Categorization is of course a far larger problem than partitioning a possibly continuous space of perceptions. It concerns the formation of a common lexicon and the emergence of labels and tags as well as the bootstrapping of syntactic/semantic categories for grammar. Yet, little is known about the collective dimensions of categorization. Understanding and capturing the interactive aspects of categorization process is a central challenge both for basic research and for future technologies. Furthermore, communication about complex information requires sophisticated conceptualizations, i.e., ways to encode knowledge at a conceptual level (for instance the notion of perspective reversal as *right of you*). Despite many studies concerning the topology of the space to be categorized and its impact on the categorization process [76], a satisfactory mathematical and computational scheme is still lacking.

5.2. Emergence of complex linguistic structures

Languages are extraordinarily complex because they are multi-layered distributed systems (sound, words, morphology, syntax, grammar) and large parts are not visible to direct observation. Despite many interesting attempts (e.g., generative grammar, unification based grammar, fluid construction grammar, etc.), we are still far from having a full picture and a flexible theoretical and computational framework for the emergence and the evolution of grammar systems. For instance, a very interesting direction concerns the emergence of *compositionality*: which are the mechanisms that bring us to associate different "features" to an object instead of using a finer categorization of just one preferred feature? In other words, how terms like "red square" or "big blue circle" emerge in a linguistic society? This will be a founding stone in explaining how human beings acquired the remarkable capacity of compositional semantics.

The experience of complex systems brings us to face this set of problems with a step-by-step approach, by starting with relatively simple cases while progressively aiming at more complex situations. In this perspective, one of the first natural question concerns the notion of complexity for a linguistic system. Here the word complexity is intended, in the spirit of the Algorithmic Complexity and Information Theory [77], as the minimal amount of information needed to specify a body of knowledge. Is it possible to introduce a suitable definition of complexity for a linguistic system? Is this notion of complexity related to the intuitive functional efficiency of the system? Can this complexity be interpreted as a sort of fitness function driving the evolution of linguistic structures? A natural starting point for studies in this direction is represented by the numeral systems [78, 79].

From a general perspective it is tempting to face the problem of simple grammars by exploiting their potential mapping to complex graphs and applying notions and tools of data and graph compression [80]. An interesting line of research concerns how much the hierarchy of patterns and motifs found by a data compression approach are related to specific grammatical or syntactic rules.

It is worth mentioning how the association between entropic properties and language structures has a long tradition. In evolutionary language games [17] the notion of *linguistic error limit* [22, 81] is introduced as the number of distinguishable signals in a protolanguage and therefore the number of objects that can be accurately described by this language. Increasing the number of signals would not increase the capacity of information transfer. An interesting parallel has been drawn between the formalism of evolutionary language game with that of information theory [82]. A possible way out is that of combining signals into words [83], opening the way to a potentially unlimited number of objects to refer to. More recently it has been conjectured that compression could aid in generalization as well as to make languages evolve towards smooth string spaces and that more complex language evolve more rapidly [84]. Recent approaches have exploited the notion of algorithmic complexity for the reconstruction of language trees [85] and that of Shannon entropy to investigate the presence of linguistic structures in Indus script [86] and Pictish symbols [87].

5.3. New tools for experimental semiotics

While the research field of semiotics may traditionally be considered a conceptual discipline, the cognitive turn has recently brought central semiotic questions and insights into the laboratories and a new discipline, dubbed experimental semiotics [88], is about to be born. A few important examples have already shown the viability of this approach: from coordination game with interconnected computers [89, 90] to experimental tests for Iterated Learning Models [91].

Though only a few years old, the growth of the World Wide Web and its effect on the society have been astonishing, spreading from the research in high-energy physics into other scientific disciplines, academe in general, commerce, entertainment, politics and almost anywhere where communication serves a purpose. Innovation has widened the possibilities for communication. Social media like blogs, wikis and social bookmarking tools allow the immediacy of conversation, with unprecedented levels of communication speed and community size. Millions of users now participate in managing their personal collection of online resources by enriching them with semantically meaningful information in the form of freely chosen tags and by coordinating the categories they imply. Wikipedia, Yahoo Answers and the ESP Game [92] are systems where users volunteer their human computation because they value helping others, participating in a community, or playing a game. These new types of communities are showing a very vital new form of semiotic dynamics. From a scientific point of view, these developments are very exciting because they can be tracked in real time and the tools of complex systems science and cognitive science can be used to study them.

From this perspective the web is acquiring the status of a platform for *social computing*, able to coordinate and exploit the cognitive abilities of the users for a given task and it is likely that the new social platforms appearing on the web, could rapidly become a very interesting laboratory for social sciences in general [93], and for studies on language emergence and evolution in particular. These recent advances are enabling for the first time the possibility of precisely mapping the interactions of large numbers of people at the same time as observing their behavior and in a reproducible way. In particular the dynamics and transmission of information along social ties can nowadays be the object of a quantitative investigation of the processes underlying the emergence of a collective information and language dynamics.

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