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DEPENDENCE AND THE ASYMPTOTIC BEHAVIOR OF LARGE CLAIMS REINSURANCE

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Abstract

We consider an extension of the classical compound Poisson risk model, where the waiting time between two consecutive claims and the forthcoming claim are no longer independent. Asymptotic tail probabilities of the reinsurance amount under ECOMOR and LCR treaties are obtained. Simulation results are provided in order to illustrate this.

Keywords: Dependence, ECOMOR and LCR reinsurance, Long-tailed distribution, Tail probability

1 Introduction

Insurance companies often seek reinsurance to protect themselves against catastrophic losses. Such reinsurance comes in many forms. Excess of loss and stop loss coverages are

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common, and the risks associated with these coverages have been thoroughly studied in the literature. Two lesser-known reinsurances are ECOMOR (excédent du coût moyen relatif) and LCR (largest claims reinsurance). This may be due to their mathematical complexity. Under ECOMOR, the reinsurer pays the sum of the exceedances of the l largest claims over the $l+1$st largest claim. Under LCR, the reinsurer pays the sum of the l largest claims. These forms of reinsurance were introduced to actuaries by Thépaut (1950) and Ammeter (1964), respectively.

The purpose of this paper is to establish the asymptotic tail probabilities of the reinsurance amount under ECOMOR and LCR. This problem is considered by Ladoucette and Teugels (2006a and b) under the assumption that the claim amounts are iid and independent of the claim arrival process. Kremer (1998) provides an upper bound for the reinsurance premium when the claim amounts are not necessarily independent. In this paper, we consider a different dependence assumption. That is, we assume that the interarrival time and the forthcoming claim size are dependent. In the context of ruin theory, similar risk models are discussed by Albrecher and Boxma (2004), Albrecher and Teugels (2006) and Boudreault *et al.* (2006).

We consider a risk process for which the claim sizes $X_i, i = 1, 2, \ldots$ are assumed to be positive iid rvs with common distribution function F. Moreover, the claim arrival process $\{N(u), u \geq 0\}$ is assumed to be a homogeneous Poisson process with intensity $\lambda > 0$. Let $X_{N(t),1} \geq X_{N(t),2}, \ldots$ be the order statistics corresponding to the claim sizes occurring on the time horizon of interest, $[0, t]$. Then the reinsurance amounts under ECOMOR and LCR are given by

$$E_l(t) = \sum_{i=1}^{l} (X_{N(t),i} - X_{N(t),l+1}) I_{\{N(t)>l\}}, \tag{1}$$

and

$$L_l(t) = \sum_{i=1}^{l} X_{N(t),i} I_{\{N(t)\geq l\}}. \tag{2}$$

As stated above, our primary objective is to obtain asymptotic tail probabilities for the reinsurance amount under ECOMOR and LCR reinsurance treaties. These results can be used in analyzing risk measures associated with these contracts.

2 Preliminaries

2.1 Definitions

The dependence structure associated with the distribution of a random vector can be characterized in terms of a *copula*. A two-dimensional copula is a bivariate distribution function defined on $[0,1]^2$ with uniformly distributed marginals. Due to Sklar's Theorem (see Sklar, 1959), if F is a joint distribution function with continuous marginals F_1 and F_2 respectively, then there exists a unique copula, C, given by

$$C(u,v) = F(F_1^{\leftarrow}(u), F_2^{\leftarrow}(v)),$$

where $h^{\leftarrow}(u) = \inf\{x : h(x) \geq u\}$. Similarly, the *survival copula* is defined as the copula relative to the joint survival function and is given by

$$\widehat{C}(u,v) = u + v - 1 + C(1-u, 1-v).$$

A more formal definition and properties of copulas are given in Nelsen (1999).

There are many characterizations of heavy-tailed distributions, but one of the largest families is the class $\mathcal{L}$ of long-tailed distributions. By definition, a df $F = 1 - \bar{F}$ belongs to $\mathcal{L}$ if

$$\lim_{t\to\infty} \frac{\bar{F}(t+x)}{\bar{F}(t)} = 1, \text{ for all } x \in \Re.$$

Note that, the convergence is uniform on compact subsets of $\Re$. One of the most important subclasses is the class $\mathcal{S}$ of sub-exponential distributions. By definition, a df F with positive support belongs to $\mathcal{S}$ if

$$\lim_{x\to\infty} \frac{\Pr(X_1 + X_2 > x)}{\Pr(X > x)} = 2,$$

where X_1 and X_2 are independent copies of X. For more details on heavy-tailed distributions, we refer the reader to Bingham *et al.* (1987) and Embrechts *et al.* (1997).

2.2 Assumptions and Examples

Let W_i be the time between the $(i-1)^{st}$ and i^{th} claims. This model relaxes the usual assumption of independence between W_i and X_i. The following assumptions for the underlying dependence structure are sufficient to establish our main results.

Assumption 1 *The random vectors* (X_i, W_i), $i = 1, \ldots, N(t)$, *are mutually independent and identically distributed, and the generic pair* (X_1, W_1) *has absolutely continuous copula* C *with corresponding survival copula* $\widehat{C}$.

Assumption 2 *There exists a* $v_0 \in (0,1)$ *and a function* g *such that*

$$\lim_{u \downarrow 0} \frac{\widehat{c}_2(u,v)}{u} = g(v), \text{ for all } v \in [v_0, 1],$$

where $\widehat{c}_2(u,v) := \partial_v \widehat{C}(u,v)$.

Below are some examples of copulas given in Nelsen (1999) which satisfy Assumptions 1 and 2.

Example 1 *Independence*

$$C(u,v) = uv,$$

with $g(v) = 1$.

Example 2 *Ali-Mikhail-Haq*

$$C(u,v) = \frac{uv}{1 - \theta(1-u)(1-v)}, \ \theta \in [-1, 1],$$

with $g(v) = 1 + \theta(1 - 2v)$.

Example 3 *Clayton*

$$C(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}, \ \theta \in (0, \infty),$$

with $g(v) = (1+\theta)(1-v)^{\theta}$.

Example 4 *Farlie-Gumbel-Morgenstern*

$$C(u,v) = uv + \theta uv(1-u)(1-v), \ \theta \in [-1, 1],$$

with $g(v) = 1 + \theta(1 - 2v)$.

Example 5 *Frank*

$$C(u,v) = -\frac{1}{\theta} \ln \left(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1} \right), \ \theta \in \Re \setminus \{0\},$$

with $g(v) = \theta e^{\theta(1-v)}/(e^{\theta} - 1)$.

Example 6 *Plackett*

$$C(u,v) = \frac{1 + (\theta-1)(u+v) - \sqrt{(1+(\theta-1)(u+v))^2 - 4uv\theta(\theta-1)}}{2(\theta-1)}, \ \theta \in \Re_+ \setminus \{1\},$$

with $g(v) = \theta/(1+(\theta-1)v)^2$.

Note that, while all six of the above examples involve a symmetric copula, this is not necessary. In particular, Assumptions 1 and 2 are satisfied by the asymmetric copula,

$$C_{k,l}(u,v) = u^{1-k}v^{1-l}C(u^k, v^l),$$

for many of the well-known absolutely continuous symmetric copulas C given in Nelsen (1999) and $0 < k, l < 1$. This construction of an asymmetric copula was proposed by Khoudraji (1995).

We also note that Assumptions 1 and 2 imply the existence of the limit

$$\lim_{x\to\infty} \Pr(W_1 \le w \mid_{X_1 > x}).$$

This is a special case of the characterization of random vectors with one extreme component given by Heffernan and Resnick (2007).

3 Main results

3.1 Order statistics

In the first part of this section, we derive the asymptotic behavior of the lth largest order statistic $X_{N(t),l}$. Recall that the joint pdf of the interarrival times conditioned on the number of claims by time t is

$$f_{W_1,\ldots,W_n|_{N(t)=n}}(w_1,\ldots,w_n) = \frac{n!}{t^n}, \text{ on } D_n = \left\{(w_1,\ldots,w_n) : 0 < \sum_{j=1}^{n} w_j < t, i = 1,\ldots,n\right\}$$

(see, for example, Embrechts *et al.*, 1997, p. 187), and the marginals are identically distributed with common density

$$f_{W|_{N(t)=n}}(w) = \frac{n(t-w)^{n-1}}{t^n}, \ 0 < w < t.$$

Proposition 1 *If Assumptions 1 and 2 are satisfied with $v_0 = e^{-\lambda t}$, then for any integer $l \geq 1$ we have*

$$\Pr(X_{N(t),l} > s) \sim [\Pr(X_1 > s)]^l \, K(l) \text{ as } s \to \infty,$$

where

$$K(l) = \int_0^t \int_0^{t-\omega_1} \cdots \int_0^{t-\sum_{i=1}^{l-1}\omega_i} h(t - \sum_{i=1}^{l} \omega_i, l) \prod_{i=1}^{l} g(e^{-\lambda \omega_i}) \, d\boldsymbol{w}$$

and

$$h(x, l) = e^{-\lambda t} \lambda^l \sum_{n=0}^{\infty} \frac{(\lambda x)^n}{n!} \binom{n+l}{l}.$$

Proof. For simplicity, we focus on the case in which $l = 1$. Extensions to $l > 1$ follow the same logic. We have

$$\begin{aligned} \Pr(X_{N(t),1} > s) &= \sum_{n=1}^{\infty} e^{-\lambda t} \frac{(\lambda t)^n}{n!} \Pr\left(X_{N(t),1} > s \,|_{N(t)=n}\right) \\ &= \sum_{n=1}^{\infty} e^{-\lambda t} \lambda^n \int_{D_n} \Pr\left(X_{N(t),1} > s \,|_{\boldsymbol{W}=\boldsymbol{w},\, N(t)=n}\right) \, d\boldsymbol{w} \\ &= \sum_{n=1}^{\infty} e^{-\lambda t} \lambda^n \int_{D_n} \left\{ 1 - \prod_{i=1}^{n} [1 - \Pr(X_1 > s \,|_{W_1=w_i})] \right\} \, d\boldsymbol{w}. \end{aligned} \tag{3}$$

Now,

$$\begin{aligned} &\sum_{n=1}^{\infty} e^{-\lambda t} \lambda^n \int_{D_n} \left\{ \sum_{i=1}^{n} \frac{\Pr\left(X_1 > s \,|_{W_1=w_i}\right)}{\Pr(X_1 > s)} \right\} \, d\boldsymbol{w} \\ &= \sum_{n=1}^{\infty} e^{-\lambda t} \frac{(\lambda t)^n}{n!} n^2 \int_0^t \frac{\Pr\left(X_1 > s \,|_{W_1=w}\right)}{\Pr(X_1 > s)} \times \frac{(t-w)^{n-1}}{t^n} \, dw. \end{aligned} \tag{4}$$

And since the inequality

$$n \int_0^t \frac{\Pr\left(X_1 > s \,|_{W_1=w}\right)}{\Pr(X_1 > s)} \times \frac{(t-w)^{n-1}}{t^n} \, dw < e^{\lambda t} \frac{n}{\lambda} \int_0^t \frac{(t-w)^{n-1}}{t^n} \, dw = e^{\lambda t}/\lambda$$

holds for any $s > 0$, we can apply the Dominated Convergence Theorem to show that (4) is asymptotically equivalent to

$$\begin{aligned} &\sum_{n=1}^{\infty} e^{-\lambda t} \frac{\lambda^n}{(n-1)!} n \int_0^t g(e^{-\lambda w})(t-w)^{n-1} \, dw \\ &= e^{-\lambda t} \lambda \int_0^t g(e^{-\lambda w}) \sum_{n=0}^{\infty} \frac{(n+1)}{n!} [\lambda(t-w)]^n \, dw. \end{aligned}$$

Note that we used the fact that $\Pr\left(X_1 > s \mid_{W_1=w}\right) \sim \Pr(X_1 > s)g(e^{-\lambda w})$, which is a straightforward implication of Assumption 2. The interchange of the summation and integral is due to Pratt's Lemma (see Pratt, 1960). In a similar manner, the remaining terms of (3) can be shown to be $o(\Pr(X_1 > s))$. The proof for the case $l = 1$ is now complete. ■

Some examples with a simple closed form for the asymptotic constant $K(1)$ from Proposition 1 are now given. In Example 1, the explicit form of the asymptotic constant is $K(l) = (\lambda t)^l / l!$, which is the l^{th} factorial moment of the counting process. That is,

$$K(l) = \mathbb{E}\left\{\frac{N(t)(N(t)-1)\ldots(N(t)-l+1)}{l!}\right\}.$$

Examples 2 and 4 imply that $K(1) = \lambda t - (1 - e^{-2\lambda t})\theta/2$. In the case of Example 6, we have

$$K(1) = 1 - \frac{\theta}{\theta - 1 + e^{\lambda t}} - \frac{\lambda t + \theta \ln(\theta) - \theta \ln(\theta - 1 + e^{\lambda t})}{\theta - 1}.$$

For other cases, including $l > 1$, if a closed form is obtainable it is long and complicated.

3.2 ECOMOR and LCR reinsurance

This section contains the main results of this paper. More specifically, the asymptotic tail probability results for the ECOMOR and LCR reinsurances on finite horizon $[0, t]$ are obtained. Recall that we allow dependence between claim amount and interarrival time and the number of claims process is assumed to be homogeneous Poisson. These results are motivated by the work of Ladoucette and Teugels (2006a) which assumes that the claim and number of claims processes are independent; the counting process is assumed to be a mixed Poisson process. They provide explicit results for the ECOMOR reinsurance when the horizon is finite. Specifically,

$$\Pr(E_l(t) > s) \sim \Pr(X_{N(t),1} > s) \text{ as } s \to \infty,$$

for any $l \geq 1$, provided that $X_1 \in \mathcal{L}$. We conclude that the same results follow under our assumptions for both reinsurances and sub-exponential claim size.

Theorem 1 *If Assumptions 1 and 2 are satisfied with $v_0 = e^{-\lambda t}$, and $F \in \mathcal{S}$, then for any integer $l \geq 1$ we have*

$$\Pr(E_l(t) > s) \sim \Pr(L_l(t) > s) \sim \Pr(X_{N(t),1} > s) \text{ as } s \to \infty.$$

Proof. We first prove the LCR case. Clearly,

$$\Pr(X_{N(t),1} > s) \leq \Pr(L_l(t) > s) = \Pr(X_{N(t),1} > s) + \Pr(L_l(t) > s, X_{N(t),1} \leq s). \quad (5)$$

Now, by following the steps as in the proof of Proposition 1, one may obtain that

$$\begin{aligned} &\Pr(L_l(t) > s, X_{N(t),1} \leq s) && (6)\\ &\leq \sum_{n=l}^{\infty} e^{-\lambda t}\lambda^n \int_{D_n} \sum_{i_1 \neq i_2 \neq \ldots \neq i_l} \Pr\left(\sum_{j=1,\ldots,l} X_{i_j} > s, \max_{j=1,\ldots,l} X_{i_j} \leq s \,|_{\boldsymbol{W}=\boldsymbol{w},\, N(t)=n}\right) d\boldsymbol{w}. \end{aligned}$$

Recall that due to our assumptions the random variables $X_i \,|_{W_i=w_i}$ are independent and $\Pr(X_i > s \,|_{W_i=w_i}) \sim \bar{F}(s)g(e^{-\lambda w_i})$. These and the fact that $F \in \mathcal{S}$ allow us to apply Theorem 1 from Cline (1986) which gives that

$$\Pr\left(\sum_{j=1,\ldots,l} X_{i_j} > s, \max_{j=1,\ldots,l} X_{i_j} \leq s \,|_{\boldsymbol{W}=\boldsymbol{w}}\right) = o(\bar{F}(s)),$$

holds for all distinct integers $1 \leq i_1, \ldots, i_l \leq n$. The latter together with equations (5) and (6) complete the proof for the LCR case, provided that the Dominated Convergence Theorem can be applied to equation (6). From equation (6)

$$\begin{aligned} &\Pr(L_l(t) > s, X_{N(t),1} \leq s)/\bar{F}(s)\\ &\leq \sum_{n=l}^{\infty} e^{-\lambda t}\lambda^n n^l \int_{D_n} \Pr\left(\sum_{i=1}^{n} X_i > s \,|_{\boldsymbol{W}=\boldsymbol{w},\, N(t)=n}\right) d\boldsymbol{w}/\bar{F}(s)\\ &\leq \sum_{n=l}^{\infty} e^{-\lambda t}\lambda^n n^l \int_{D_n} \Pr\left(\sum_{i=1}^{n} Y_i > s\right) d\boldsymbol{w}/\bar{F}(s), \qquad (7) \end{aligned}$$

where $Y_1, Y_2, \ldots$ are iid random variables with df $G(s) = \max\left\{0, 1 - \frac{e^{\lambda t}}{\lambda}\bar{F}(s)\right\}$. Note that the last inequality follows due to

$$\Pr(X_i > s \,|_{W_i=w_i}) \leq \frac{e^{\lambda t}}{\lambda}\bar{F}(s).$$

Since $F \in \mathcal{S}$ and $\Pr(Y_1 > s) \sim \frac{e^{\lambda t}}{\lambda}\bar{F}(s)$, Theorem 1 from Cline (1986) gives that $G \in \mathcal{S}$. The latter and Lemma 1.3.5 from Embrechts *et al.* (1997) implies that there exists a finite constant C such that for all s

$$\frac{\Pr\left(\sum_{i=1}^{n} Y_i > s\right)}{\bar{F}(s)} \leq C2^n \min\left\{1, \frac{e^{\lambda t}}{\lambda}\right\},$$

which together with equation (7), allow us to conclude that the Dominated Convergence Theorem can be applied to equation (6). The proof is now complete for the LCR case.

It is easy to get

$$\Pr(X_{N(t),1} > s) - \Pr(E_l(t) \leq s, X_{N(t),1} > s) \leq \Pr(E_l(t) > s) \leq \Pr(L_l(t) > s),$$

which implies that

$$\Pr(E_l(t) \leq s, X_{N(t),1} > s) = o(\bar{F}(s)) \tag{8}$$

is sufficient to prove in order to conclude the ECOMOR case.

Again, as in the proof of Proposition 1 the following holds

$$\Pr(E_l(t) \leq s, X_{N(t),1} > s) \leq \sum_{n=l+1}^{\infty} e^{-\lambda t}\lambda^n \int_{D_n} \sum_{i_1 \neq i_2 \neq \ldots \neq i_{l+1}} \tag{9}$$

$$\Pr\left(X_{i_1} > s, \sum_{j=1}^{l}(X_{i_j} - X_{i_{l+1}}) \leq s, X_{i_1} \geq X_{i_2} \geq \ldots \geq X_{i_{l+1}} |_{\boldsymbol{W}=\boldsymbol{w},\, N(t)=n}\right) d\boldsymbol{w}.$$

We first prove that each summation term is $o(\bar{F}(s))$. Let $Z_i = X_i |_{W_i = w_i}$. Now,

$$\begin{aligned}
&\Pr\left(Z_1 > s, \sum_{j=1}^{l}(Z_i - Z_{l+1}) \leq s, Z_1 \geq Z_2 \geq \ldots \geq Z_{l+1}\right) \\
&= \Pr\left(Z_1 > s \geq Z_2 \geq \ldots \geq Z_{l+1}, \sum_{j=1}^{l}(Z_i - Z_{l+1}) \leq s\right) + o(\bar{F}(s)). \qquad (10)
\end{aligned}$$

The remaining term from the above equation is bounded by

$$\int \cdots \int_{\{ly_{l+1} - \sum_{i=2}^{l} y_i \geq 0\}} \Pr\left(s < Z_1 \leq s + ly_{l+1} - \sum_{i=2}^{l} y_i, (Z_2, \ldots, Z_{l+1}) \in d\boldsymbol{z}\right) = o(\bar{F}(s)), \tag{11}$$

where the last step is a consequence of the Dominated Convergence Theorem and the fact that the df of rvs Z_i belong to the class $\mathcal{L}$.

From equation (9), for any s we have

$$\begin{aligned}
&\frac{\Pr(E_l(t) \leq s, X_{N(t),1} > s)}{\bar{F}(s)} \\
&\leq \sum_{n=l+1}^{\infty} e^{-\lambda t}\lambda^n \int_{D_n} \sum_{i_1 \neq i_2 \neq \ldots \neq i_{l+1}} \frac{\Pr\left(X_{i_1} > s \mid_{\boldsymbol{W}=\boldsymbol{w},\, N(t)=n}\right)}{\bar{F}(s)}\, d\boldsymbol{w} \\
&\leq \sum_{n=l+1}^{\infty} \lambda^{n-1} n^{l+1} \frac{t^n}{n!}.
\end{aligned}$$

This allows us to apply the Dominated Convergence Theorem in equation (9), which together with equations (10) and (11) we get (8). This completes the proof for the ECOMOR case. ■

3.3 Another Dependence Model

Boudreault *et al.* (2006) consider a risk process for which each claim amount is dependent on the waiting time until the claim as follows:

$$\Pr(X_1 > x \mid_{W_1=w}) = e^{-\beta w}\bar{F}_1(x) + (1 - e^{-\beta w})\bar{F}_2(x),$$

where $F_1 = 1 - \bar{F}_1$ and $F_2 = 1 - \bar{F}_2$ are distribution functions of positive random variables such that F_2 has a heavier tail than F_1. It follows that

$$\frac{\Pr(X_1 > x \mid_{W_1=w})}{\Pr(X_1 > x)} \sim \frac{\lambda + \beta}{\beta}(1 - e^{-\beta w}),\ \ x \to \infty. \tag{12}$$

Therefore, Proposition 1 holds with $g(e^{-\lambda w})$ replaced by the right hand side of (12), and Theorem 1 holds. This illustrates that, even when we cannot explicitly characterize the dependence structure of W_1 and X_1 via the copula, we can still obtain the asymptotic results as long as the limit of $\Pr(X_1 > x \mid_{W_1=w})/\Pr(X_1 > x)$ exists.

4 Simulation Study

To explore the results given in Proposition 1 and Theorem 1, a simulation study was performed. It was assumed that claim amounts have a Pareto distribution with distribution function

$$F_{X_1}(x) = 1 - (1 + x)^{-\alpha},\ \ x \geq 0$$

with α equal to 1 and 2. The dependence of the claim amount and the waiting time until the claim is given by the Ali-Mikhail-Haq copula given in Example 2 with values of θ equal to -0.9, 0.1 and 0.9.

Each analysis consists of 1,000,000 simulations of the risk process with $\lambda = 1$ and time horizon $t = 50$. For each simulation, the values of $X_{N(50),1}$, $L_2(50)$ and $E_1(50)$ were calculated. Probabilities associated with these three random variables were then estimated from the empirical distributions arising from the simulated samples of size 1,000,000. Probabilities associated with the random variable X_1, were estimated from the empirical distribution of all simulated claim amounts. These estimates were used to obtain the ratios presented in Tables 1, 2, 3 and 4.

For the ratios in Tables 1 and 2, the speed of convergence increases with θ, the strength of dependence. For $\alpha = 2$ the ratios converge quickly to 1.

Table 1: Estimated probability ratios, $\Pr(X_{N(50),1} > s)/K(1)\Pr(X_1 > s)$, when $\alpha = 1$ and $\theta \in \{-0.9, 0.1, 0.9\}$.

s	-0.9	0.1	0.9
500	0.9413	0.9533	0.9623
1000	0.9654	0.9772	0.9852
2000	0.9782	0.9884	0.9978
2500	0.9806	0.9908	0.9997
4000	0.9843	0.9942	1

Table 2: Estimated probability ratios, $\Pr(X_{N(50),1} > s)/K(1)\Pr(X_1 > s)$, when $\alpha = 2$ and $\theta \in \{-0.9, 0.1, 0.9\}$.

s	-0.9	0.1	0.9
50	0.9815	0.9924	0.9999
150	0.9907	0.9997	1.0074
250	0.9912	0.9998	1.0088
500	0.9912	1.0010	1.0088
1000	0.9912	1.0010	1.0088

The probabilities involving $L_2(50)$ and $E_1(50)$ are compared with those involving $X_{N(50),1}$ in Tables 3 and 4 for $\theta \in \{-0.9, 0.1, 0.9\}$ and $\alpha = 1$, $\alpha = 2$, respectively. For both cases, there does not appear to be an effect from θ, indicating that unlike the maximum, LCR and ECOMOR are not affected by the strength of dependence. In addition, when $\alpha = 2$, the rate of convergence is faster than when $\alpha = 1$.

Table 3: Estimated probability ratios, $\Pr(L_2(50) > s)/\Pr(X_{N(50),1} > s)$ and $\Pr(E_1(50) > s)/\Pr(X_{N(50),1} > s)$, when $\alpha = 1$ and $\theta \in \{-0.9, 0.1, 0.9\}$.

	LCR			ECOMOR		
$s\backslash\theta$	-0.9	0.1	0.9	-0.9	0.1	0.9
500	1.2169	1.2155	1.2165	0.8394	0.8401	0.8395
1000	1.1456	1.1443	1.1422	0.8931	0.8928	0.8944
2000	1.0906	1.0853	1.0876	0.9334	0.9590	0.9332
2500	1.0740	1.0750	1.0755	0.9389	0.9402	0.9461
4000	1.0535	1.0509	1.0533	0.9563	0.9613	0.9594

Table 4: Estimated probability ratios, $\Pr(L_2(50) > s)/\Pr(X_{N(50),1} > s)$ and $\Pr(E_1(50) > s)/\Pr(X_{N(50),1} > s)$, when $\alpha = 2$ and $\theta \in \{-0.9, 0.1, 0.9\}$.

	LCR			ECOMOR		
$s\backslash\theta$	-0.9	0.1	0.9	-0.9	0.1	0.9
50	1.5466	1.5630	1.5508	0.7270	0.7268	0.7269
150	1.1964	1.1744	1.1797	0.8787	0.8873	0.8820
250	1.1023	1.0966	1.1006	0.9223	0.9136	0.9301
500	1.0506	1.0598	1.0276	0.9545	0.9620	0.9585
1000	1	1.0189	1.0204	0.9821	0.9783	1

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