Theoretical and Empirical Contributions to Monetary Policy Analysis

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Declaration

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Abstract

This thesis collects three different contributions to monetary macroeconomics, covering both theoretical and empirical aspects.

First chapter builds on the DSGE models of New Keynesian tradition, and studies monetary policy around a non efficient steady state. Using a two-stage approach developed by Levine, McAdam, and Pearlman (2007), I show that in the presence of backward looking firms, the central planner improves social welfare when it allows for a steady state rate of inflation marginally above zero.

In the second chapter, I estimate a simple two-country DSGE model to study the behaviour of the Eastern European central banks, obtaining some innovative important results. First, a simple monetary policy rule mimicking an optimal rule together with the assumption about the existence of non-zero steady state rate of inflation deliver a significantly better fit to the data. Furthermore, the empirical hypothesis that central banks systematically target CPI inflation rather than PPI inflation is rejected for all the investigated Eastern European countries (EEC).

In the third chapter, I use a Bayesian VAR with economically interpretable structural restrictions and zero restrictions on lags, to analyse the transmission channels of external shocks to an extended set of EEC. I study to what extent monetary policy shocks originating from the US and from Germany can explain fluctuations on Eastern European markets. To carry out the Bayesian inference, I use a Gibbs sampling approach. I find that the US monetary policy influences the EEC macroeconomic variables at least as much as its German counterpart.

Preliminary Definitions

Actual level of a variable: The nominal value that the generic variable D_t takes when both stochasticity and (real and nominal) market imperfections are considered. Written in relative/real terms, the variable is denoted by $\tilde{D}_t = D_t/P_t$.

Stationary level of a variable: The value that the variable takes when only (real and nominal) market imperfections are considered. It is denoted by D if in nominal value and by $\bar{D}$ if in real/relative terms.

Natural level of a variable: The value that the variable takes when only real market imperfections are considered. It is denoted by D^n.

Deviation of the actual level of a variable from its stationary level: The ratio between the actual and the stationary level of a variable. It is denoted by $\hat{D}_t \equiv D_t/D$.

Log-transformation of a variable: The value that obtains by applying logarithms to the value of the variable. It is denoted by $d_t \equiv \log D_t$. Likewise, $\tilde{d}_t \equiv \log \tilde{D}_t$, $d^n \equiv \log D^n$, $\hat{d}_t \equiv \log \hat{D}_t$.

Introduction

My research interest focuses on the analysis of monetary policy issues, and covers both theoretical aspects, which I study through Dynamic Stochastic General Equilibrium (DSGE) modelling, and empirical aspects, which I study using Bayesian estimation techniques. My theoretical work, developed in Chapter 1, aims at investigating whether real and nominal frictions may represent a source of trend inflation. My empirical work consists of two exercises. The first one, illustrated in Chapter 2, aims at bringing to the data the theoretical predictions delivered by my theoretical analysis. In particular, I investigate whether a DSGE model with non-zero inflation performs empirically better than an analogous model abstracting from trend inflation. This exercise consists a set of three estimations performed in a context with a large economy (represented by Germany) and a small open economy (represented, in turn, by the Czech Republic, Hungary and Poland). The second exercise, described in Chapter 3, extends that performed in Chapter 2 and aims at assessing whether the transmission channel studied there is the most relevant when looking at Eastern European countries (to the three listed above, here I also consider Slovakia), or international transmission of monetary shocks from the US, either directly or indirectly through its effect of the German economy, may also play a significant role.

In order to achieve my theoretical goal, I build on the literature of New Keynesian DSGE models containing real and nominal frictions. The debate regarding economic rigidities has a long history. Phillips (1958) can be considered as a cornerstone for this debate. In the monetary economics literature, many arguments suggesting that monetary authorities should take into account the trade-off between inflation and output were built upon his empirical findings. Monetarist economists criticised this line of reasoning, considering them somehow too "static". In their view, the crux of the matter was the assumption that individuals are only able to form inflationary expectations based on past inflation. This controversy led to the traditional acceleration Phillips curve, characterised by backward looking components. After the famous critique by Robert E. Lucas in 1976, macroeconomics was micro-founded, and the resulting theory had individuals making their choices on fully rational decisions. In fact, it was in response to Lucas' critique of ad-hoc modelling that Kydland and Prescott (1982) formulated a theory of micro-founded business cycle models, known as the Real Business Cycle (RBC) theory. This is a general equilibrium models in which the economy responds efficiently to shocks to the total factor productivity that generate fluctuations. It is because of the efficiency of these responses that monetary policy has no real effects on the economy. Their model became an important building block for the New Keynesian models (NKM), which represent the class of models that central banks most commonly rely upon at present.

NKMs postulate the existence of a Phillips curve, essentially generated by market frictions, also in the presence of fully rational firms that maximize the discounted value of profits in the form of Calvo's contracts. Roberts (2001), Eichenbaum and Fisher (2003) and Dupuis (2004), show that the standard New Keynesian

Phillips curve (NKPC) with a simple forward looking component does not correspond well with empirical results, as it is unable to capture inflation persistence. Having recognised the importance of the inflation inertia, economists have been looking for a way to create a Phillips curve that could simultaneously have both forward and backward looking elements. One of the first attempts to develop a "two-sided Phillips curve" was Fuhrer and Moore (1995), which used two-period Taylor contracts.

Another approach proposes an indexation of inflation. This is possible by using either the method of static indexation, as suggested by *e.g.*, Yun (1996), or the dynamic one as in Eichenbaum and Fisher (2003) and Christiano, Eichenbaum and Evans (2005). Yun (1996) uses the idea that firms that cannot re-optimise in a given period, may increase prices in the attempt to anticipate potential inflationary phenomena, which they take statically into account at a fixed rate. Static inflation can be seen as the long term average rate of inflation and, therefore, long term inflation inertia is introduced into the model. Similarly, in the model of dynamic indexation from Christiano *et al.* (2005), firms that are not allowed to re-optimise may raise prices by adding a fraction of the last observed rate of inflation. Both approaches are nevertheless criticised in the literature because they assume that firms adjust prices every period, which contradicts the evidence on price stickiness. Galí and Gertler (1999) introduce a variation to this approach by assuming that only a fraction of firms change their prices, and provide also empirical support to their approach. In their model, one part of the firms allowed to adjust prices behave rationally, while the complementary part indexes their prices according to a rule of thumb.

Nonetheless, the empirical evidence about the importance of the backward looking component in the NKPC delivers ambiguous results. Most of the estimates suggest a hybrid NKPC with a backward looking component. A value of lagged inflation should be around 0.3 to 0.5, in line with other empirical findings such as Galí and Gertler (1999) and Galí, Gertler, and Lopez-Salido (2001). Benigno and Lopez-Salido (2002) test the Galí-Gertler model on five countries in the Euro Area, showing that German firms behave rationally and, therefore, inflation is strongly forward looking. By contrast, in France, Spain and Italy, the opposite is true. In these countries, firms are characterized by backward looking price setting behaviour, strongly linking their prices to past conditions. These asymmetries are important as they complicate the determination of unique monetary policy within the Euro Area. Using Bayesian estimation for euro area data and model with NKPC based on Christiano *et al.* (2005) approach, Smets and Wouters (2003) show, that the forward looking component clearly dominates, however, the backward looking one is also important. Values for both parameters are in line with the findings above. Conversely, some authors, *e.g.*, Levine, Pearlman, Perendia, and Yang (2012), show that including habit formation improves the performance of the model more than a backward looking component of the Phillips curve. For comparison, the estimates for habit formation are usually large: the literature refers to a value around 0.8.

In my theoretical analysis, developed in Chapter 1, I build on the study by Levine *et al.* (2012) and study optimal monetary policy around a non efficient steady state, using a two-stage approach developed by Levine, McAdam, and Pearlman (2007). The goal is to show that in the presence of backward looking firms (as in Galí and Gertler, 1999; and Steinsson, 2003), the central planner can improve social welfare by allowing a steady state rate of inflation marginally above zero. In such an environment, there are two forces in the steady state that tend to balance out. On the one hand, given a positive rate of inflation, a non-zero price dispersion arises in steady state, decreasing welfare. On the other hand, positive inflation causes leisure to decline, and thereby includes a rise in output, bringing it closer to its efficient level. This means that the steady state output gap is lower than it would be in the absence of inflation and welfare rises. There is no obvious analytical solution to the central planner problem, so I solve the model numerically using the Levine-Pearlman Dynare-based system, known as ACES (Analysis and Control of Economic Systems).

My empirical work, developed in Chapters 2 and 3, is based on the Bayesian estimation approach, which I apply to two questions. I estimate a simple two-country DSGE model, similar to Lubik and Schorfheide (2007), with a non-zero steady state inflation, to study the behaviour of the Eastern European central banks. The Bayesian analysis, which I carry out using a Metropolis-Hastings algorithm, suggests that my model performs better than the benchmark in several aspects. First, I can show that assuming the existence of trend inflation, *i.e.*, modelling the log-linearised Phillips curve similarly to Ascari and Ropele (2007), delivers a significantly better fit to the data. Second, although a number of theoretical contributions, *e.g.*, Galí and Monacelli (2005), argue that PPI inflation targeting performs better than CPI inflation targeting in terms of welfare loss, the empirical literature mainly concentrates on simple rules with CPI inflation targeting. Using a posterior odds test, I show that the empirical hypothesis that central banks systematically target CPI inflation rather than PPI inflation is rejected for all the investigated Eastern European countries (EEC), *i.e.*, Czech Republic, Hungary, and Poland.

Second, I use a Bayesian VAR with economically interpretable structural restrictions and zero restrictions on lags to analyse the transmission channels of external shocks to an extended set of EEC, which also includes Slovakia (along with Czech Republic, Hungary, and Poland). Following Mackowiak (2006), I study to what extent monetary policy shocks originating from the US and from Germany can explain fluctuations on Eastern European markets. To carry out the Bayesian inference, I use a Gibbs sampling approach. I find that the US monetary policy influences the EEC macroeconomic variables at least as much as its German (later ECB's) counterpart.

Chapter 1

Welfare Analysis in a Model with Inflation Persistence and Non-Zero Steady State Inflation

1.1 Introduction

In the New Keynesian literature, the long run equilibrium is typically defined as a steady state with zero inflation. Casual observations however suggest that inflation exhibits a non-zero trend, and several recent empirical contributions point out that trend inflation plays an important role in shaping many macroeconomic features. In particular, a significant deal of this literature seems to focus on the effects of the existence of trend inflation on monetary policy conduct. This chapter attempts to complement these studies, and investigates whether monetary policy may be a possible cause of trend inflation. Specifically, I seek to find an answer to the question: "Might non-zero steady state inflation be welfare improving?"

In order to explore this idea, I develop a New Keynesian model where a social planner seeking to maximise welfare optimally chooses the level of steady state inflation. My model features two key aspects. First, I consider a hybrid Phillips curve, containing both a forward-looking and a backward-looking component, as in Galí and Gertler (1999) and Steinsson (2003). Second, I depart from studying the equilibrium around the efficient steady state of the model and, following Levine, McAdam, and Pearlman (2007), I use the Hamiltonian approach rather than the traditional linear-quadratic approximation to derive the welfare loss function. As I show and discuss in the next sections of this chapters, both aspects are key in that overlooking either of them would result in the social planner trivially choosing zero steady state inflation. That is, the traditional New Keynesian prediction about trend inflation results as a particular case of the more general model that I develop in this chapter.

My analysis shows that, around a non efficient steady state, the magnitude of the share of backward looking firms influences the optimal level of the steady state inflation. In particular, the social planner chooses zero inflation if just the forward-looking component of the Phillips curve exists. More generally, the presence of a backward looking component leads instead to an equilibrium with positive steady state inflation, which also generates higher steady state output. The reason is that, although non-zero steady state inflation generates greater price dispersion and thereby a fall in aggregate output, positive inflation also leads to an increase in real marginal labour cost. This has a positive effect on output via the substitution effect set in motion by the change in real wage. For sufficiently low levels of inflation, the latter effect dominates, the net impact of trend inflation on output is positive.

A natural question that arises in this context is how the welfare loss resulting from the Hamiltonian approach relates to the one obtained by the traditional linear-quadratic approximation. To this aim, I investigate the effect on welfare of the intervention of a central bank (acting as a social planner), introduced using a number of simple and optimal policy rules. In particular, I analyse the individual performance of these rules and their impact on the social welfare in the presence of a TFP shock. I find that the presence backward looking component is paramount in determining the difference in the welfare losses computed using the traditional and the Hamiltonian approach. Such a difference is best observed when simple rules are adopted. In line with the literature, I also find that, using either approach and under a common calibration, the pure inflation targeting rule performs best, as it lowers significantly more the volatility of inflation relative to the other simple rules.

1.2 Literature Review

Despite being a relatively recent feature in monetary economics, trend inflation has already been studied in conjunction with a number of issues in this literature. For instance, Ascari (2004) log-linearises the baseline New Keynesian Model (NKM) around an exogenously determined positive steady state inflation to argue that this significantly changes the dynamics of the model relative to the framework with zero inflation; Ireland (2007) studies the relationship between trend inflation and the Taylor rule against US data, and find that the Federal Reserve's inflation target changed many times during the last four decades of the last century. My contribution departs from these by endogenising trend inflation and investigating its effects on a variety of different monetary policy rules.

The material discussed in this chapter relates to a number of articles studying social welfare in the presence of trend inflation. For example, Ascari and Ropele (2007) study the optimal monetary policy in a NKM with non-zero steady state inflation to point out that the level of trend inflation has a strong impact on social welfare. Schmitt-Grohé and Uribe (2007) show that, when one considers positive steady state inflation, exogenously given and calibrated to match US data, price dispersion is a first order source of social welfare inefficiency. My work differs from these contributions as the effects on social welfare are investigated with references to a level of inflation optimally chosen by a benevolent.

Another branch of the literature that is linked to the analysis developed in this chapter studies trend inflation in relation to specific aspects on the behaviour of inflation itself. In particular, building on the literature modelling trend inflation as a random walk without drift (*e.g.*, Cogley and Sargent, 2005Cogley and Sargent (2005); Stock and Watson, 2007), Cogley and Sbordone (2008) find that a purely forward looking New Keynesian Phillips Curve (NKPC) with a trend inflation is empirically capable to explain a great deal of the observed inflation persistence in the US data. As such, trend inflation contributes a highly persistent component to actual inflation, from a source that is quite different from any intrinsic persistence implied by the dynamics of price adjustment. My research complements this contribution by investigating whether inflation persistence may be an endogenous source, as well as a consequence, of non-zero steady state inflation.

Regarding the impact of different monetary policy rules on social welfare, most contributions in the literature compute the welfare loss function using a traditional linear-quadratic approach. In the benchmark model (see, *e.g.*, Clarida, Galí and Gertler, 1999), welfare loss is a function of inflation and output gap. Although, as Steinsson (2003) shows, this traditional method can be applied to a model with built-in inflation persistence (with the welfare loss function simply featuring an additional term), the standard linear-quadratic approximation can only be used if small deviations from the efficient steady state are considered. In the efficient steady state, nominal and real frictions are neutralised, hence output is at its efficient level and, more importantly, inflation displays a zero trend. As a result, the traditional approach is unsuitable to deal with my analysis. For this reason, I depart from this literature and use the approach developed by Levine et al. (2007). These authors develop a linear-quadratic approximation around a non efficient steady state, known as the Hamiltonian approach, which I adapt to the analysis of monetary policy with endogenous trend inflation and inflation persistence. The advantages of this alternative method are that nominal and real frictions need not be eliminated in the steady state (in particular, inflation need not display a zero trend), and that the welfare loss function can be derived also for bigger deviations from the steady state.

1.3 The Model: Demand Side

The demand side of my model economy is represented by the traditional aggregate demand curve, in the New Keynesian version of the DSGE models, known as the expectational IS curve. It serves as an intertemporal tool to characterise the households optimal choice. Often, it is represented by a log-linearized approximation of the forward looking Euler equation, which is the solution of the representative consumer's dynamic optimisation problem.

1.3.1 The Household Optimisation Problem

The economy consists of a continuum of infinitely lived households, which all have identical preferences. The representative household maximises the discounted stream of instantaneous utility functions over current and future periods

$$U = E_t \sum_{t=0}^{\infty} \beta^t \left[U \left(C_t, N_t \right) \right], \tag{1.1}$$

by optimally choosing, at each date $t \in [0, \infty)$, C_t consumption units and N_t working hours. The preference parameter $\beta \in (0, 1]$ is the subjective household discount factor. I assume that $U_C > 0$, $U_N < 0$, $U_{CC} < 0$, $U_{NN} > 0$ and $U_{C,N} = 0$. In solving this problem, the household is subject to the budget constraint

$$B_t + W_t N_t + T_t + D_t \geq C_t P_t + E_t \left[Q_{t,t+1} B_{t+1} \right], \tag{1.2}$$

where $E_t \left[Q_{t,t+1} \right]$ represents the price of the riskless bond held in B_{t+1} units, $C_t P_t$ are consumption expenditures, W_t is the nominal wage, T_t is a lump sum transferand D_t is the net profit that the firms pay to the shareholders (households), which does not affect any other variables in the model.

The Lagrangian that corresponds to the representative agent's optimisation takes the following form

$$L = E_t \sum_{t=0}^{\infty} \beta^t \left[U \left(C_t, N_t \right) \right] + \lambda_t \left(B_t + W_t N_t + T_t + D_t - C_t P_t - Q_{t,t+1} B_{t+1} \right).$$

Solving the maximization problem, I obtain the first order conditions

$$\frac{\partial U}{\partial C_t} = U_{C,t} - \lambda_t P_t = 0, \tag{1.3}$$

$$\frac{\partial U}{\partial N_t} = U_{N,t} + \lambda_t W_t = 0, \tag{1.4}$$

$$\frac{\partial U}{\partial B_{t+1}} = -\beta\lambda_{t+1} + \lambda_t Q_{t,t+1} = 0, \tag{1.5}$$

$$\frac{\partial U}{\partial \lambda_t} = B_t + W_t N_t + T_t + D_t - C_t P_t - Q_{t,t+1} B_{t+1} = 0, \tag{1.6}$$

where λ_t is the Lagrangian multiplier and can be interpreted as the shadow price of consumption.

Euler Equation

The gross return of a one year riskless bond R_t equals the reciprocal of its price

$$R_t = \frac{1}{E_t\left[Q_{t,t+1}\right]},$$

where $E_t\left[Q_{t,t+1}\right]$ is also known as the dynamic stochastic discount factor. Note that $R_t = (1 + i_t)$, where i_t is the nominal interest rate. From the condition (1.3) I obtain the conventional stochastic Euler equation

$$\beta E_t\left[\frac{U_{C,t+1}}{U_{C,t}}\frac{P_t}{P_{t+1}}\right] = E_t\left[Q_{t,t+1}\right], \tag{1.7}$$

which represents the intertemporal optimality conditions linking current and future consumption choices.

Labour Supply

From condition (1.4), I obtain the equation that determines household's labour supply

$$\tilde{W}_t = -\frac{U_{N,t}}{U_{C,t}}, \tag{1.8}$$

according to which the marginal rate of substitution between labour and consumption equals the real wage $\tilde{W}_t = W_t/P_t$.

1.3.2 Consumption Index and Consumer Price index

I assume there exists a unit continuum of differentiated goods available in the goods market. I abstract from government spending so that all goods are consumed by domestic households. C_t is a composite consumption good, which takes the familiar Dixit-Stiglitz form

$$C_t = \left(\int_0^1 [C_t(i)]^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1.9}$$

where the parameter $\varepsilon > 1$ represents the (constant) consumption elasticity of substitution, and $C_t(i)$ represents the consumption level of good i. As I show in Appendix 1.A, I obtain the demand for an individual consumption good

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t, \tag{1.10}$$

as a function of its price $P_t(i)$ relative to the price index P_t, of the composite consumption level C_t, and of the elasticity of substitution parameter ε_t.

Using the market clearing conditions $C_t(i) = Y_t(i)$, for all i and t, and $C_t = Y_t$, for all t, I can rewrite (1.10) to measure production of firm i

$$Y_t(i) = \left(\frac{P_t\,(i)}{P_t}\right)^{-\varepsilon} Y_t, \tag{1.11}$$

which represents the relationship linking individual output to aggregate output and the relative price. Price dispersion in (1.11) is an important source of potential welfare losses in a closed economy. Using equation (1.10), I also obtain an expression for the consumer price index in a closed economy as a function of each good's price, formally

$$P_t = \left[\int_0^1 (P_t(i))^{1-\varepsilon}\, di\right]^{\frac{1}{1-\varepsilon}}. \tag{1.12}$$

It is worth mentioning that, in this closed economy model, the consumer price index and the producer price index are identical.

1.4 The Model: Supply Side

In order to obtain the Phillips curve, it is convenient to look at the behaviour of each firm in the goods markets. The reason is that I assume that only some firms will set the optimal price as they would under perfect price flexibility, while the others will not. Moreover, firms supply differentiated goods to the markets, and this allow them to have a certain degree of market power. As a result, firms are not price-takers, but rather incorporate the demand function into their decision regarding how much they should produce.

I closely follow the price setting mechanism in Calvo (1983) and Galí and Gertler (1999). There are two types of firms. As illustrated in Figure 1.1, a first

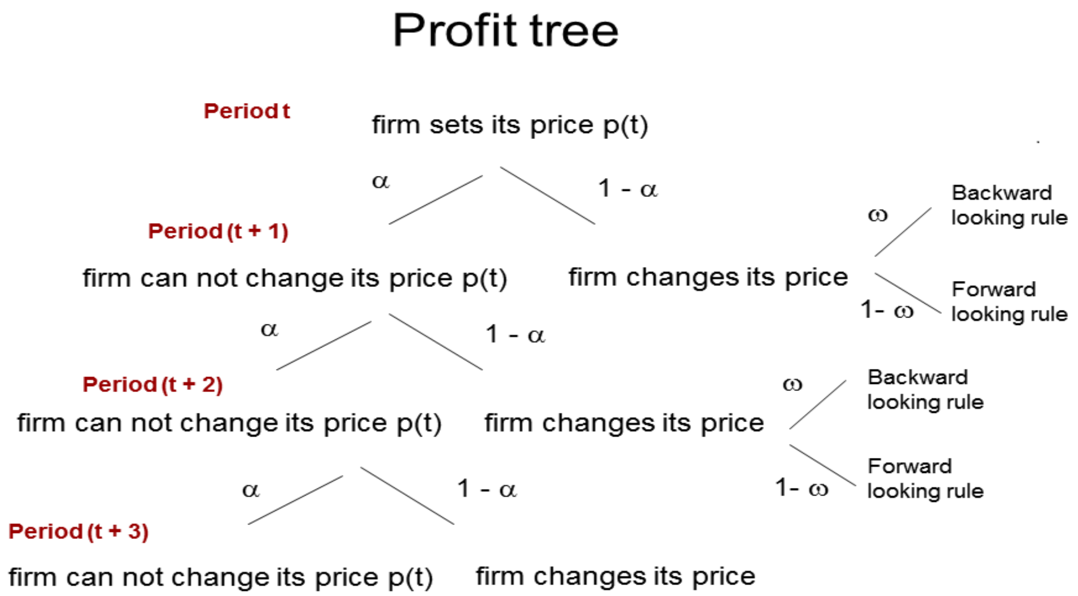

Figure 1.1: Profit tree: A share of $1-\alpha$ firms are allowed to change their prices, from which ω firms choose a price according rule of thumb.

group of firms, with measure $1-\omega$, set the price optimally. The complementary measure ω set the price according to a rule of thumb. Firms may face two different situations: i) they are allowed to set their price (with probability $1-\alpha$); ii) they are not allowed to do so (with probability α). Hence, at each time t, a measure $(1-\omega)(1-\alpha)$ set the price optimally, and are labelled f; a measure $\omega(1-\alpha)$ set the price according to a rule of thumb, and are labelled b; finally, a measure α leave the price unchanged, and are labelled s.

Optimal Prices

Consider first one of the $(1-\omega)(1-\alpha)$ firms, labelled f, that at time t are allowed to change the price and do so optimally. Each firm in this group sets price

$P_t^f(i)$ to maximise its expected stream of profits $G_t(i)$ subject to the technological constraint (represented by its production function)

$$\begin{aligned} &\max_{P_t^f(i)} \quad \sum_{j=0}^{\infty} \alpha^j E_t \left[Q_{t,t+j} \left(P_t^f(i) Y_{t+j}(i) - W_{t+j} N_{t+j}(i) \right) \right], \\ &\text{subject to:} \quad Y_t(i) = A_t N_t(i), \text{ and } Y_t(i) = \left(\tfrac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t, \end{aligned} \tag{1.13}$$

where A_t is total factor productivity, which is common to all firms. As already mentioned above, assume that the stock of capital is fixed, so labour is the only variable input in the production function, and its demand function given by equation (1.11). Hence, the first order condition for this problem is

$$\sum_{j=0}^{\infty} \alpha^j E_t \left[Q_{t,t+j} \left((1-\varepsilon) Y_{t+j} \left(\frac{P_t^f(i)}{P_{t+j}} \right)^{-\varepsilon} + \varepsilon \frac{W_{t+j}}{P_t^f(i)} \frac{Y_{t+j}}{A_{t+j}} \left(\frac{P_t^f(i)}{P_{t+j}} \right)^{-\varepsilon} \right) \right] = 0.$$

Defining the nominal marginal cost as

$$MC_t(i) = W_t / A_t = MC_t, \tag{1.14}$$

and using (1.7) together with the fact that, at time t, $j = 0$ and so $\alpha^j = 1$ and $Q_{1,1} = 1$, I can rewrite this expression to obtain

$$\begin{aligned} P_t^f(i) E_t &\left[\sum_{j=0}^{\infty} \alpha^j \beta^j \frac{U_{C,t+j}}{U_{C,t}} \frac{P_t}{P_{t+j}} \left(\frac{1}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \right] \\ &= \frac{\varepsilon}{\varepsilon - 1} E_t \left[\sum_{j=0}^{\infty} \alpha^j \beta^j \frac{U_{C,t+j}}{U_{C,t}} Y_{t+j} MC_{t+j} \frac{P_t}{P_{t+j}} \left(\frac{1}{P_{t+j}} \right)^{-\varepsilon} \right]. \end{aligned}$$

Multiplying both sides by $(1/P_t)^{\varepsilon}$ and $U_{C,t}$ yields

$$\begin{aligned}\tilde{P}_t^f(i)\, E_t\left[\sum_{j=0}^{\infty}\alpha^j\beta^j U_{C,t+j}Y_{t+j}\left(\frac{P_{t+j}}{P_t}\right)^{\varepsilon-1}\right]& \\ =\frac{\varepsilon}{\varepsilon-1}E_t\left[\sum_{j=0}^{\infty}\alpha^j\beta^j U_{C,t+j}Y_{t+j}\widetilde{MC}_{t+j}\left(\frac{P_{t+j}}{P_t}\right)^{\varepsilon}\right]&,\end{aligned} \tag{1.15}$$

where $\tilde{P}_t^f(i) = P_t^f(i)/P_t$ is the relative forward looking price and $\widetilde{MC}_t = MC_t/P_t$ is the real marginal cost. This equation can be written in terms of forward looking price and inflation, using $P_{t+j}/P_t = \Pi_{t+j}\cdot\Pi_{t+j-1}\cdot\ldots\cdot\Pi_{t+1} = \prod_{k=1}^{j}\Pi_{t+k}$, where $\Pi_t = P_t/P_{t-1}$ is inflation in period t. It therefore holds that

$$\tilde{P}_t^f(i) = \frac{J_t}{H_t}. \tag{1.16}$$

The variables J_t and H_t are respectively given by

$$J_t = \mu U_{C,t}Y_t\widetilde{MC}_t + \alpha\beta E_t\left[(\Pi_{t+1})^{\varepsilon} J_{t+1}\right] \tag{1.17}$$

and

$$H_t = U_{C,t}Y_t + \alpha\beta E_t\left[(\Pi_{t+1})^{\varepsilon-1} H_{t+1}\right]. \tag{1.18}$$

The expression $\mu = \varepsilon/(\varepsilon-1)$ is the constant markup that the firm charges by exploiting its monopolistic power. Recall that if the firm faced perfect competition, this markup would disappear and firms would set their (relative) prices equal to their real marginal costs, which by symmetry would be identical to all firms.

Rule of Thumb and Staggered Prices

Consider now the $\omega(1-\alpha)$ firms, labelled b, that set their price at time t. These firms do so according to a rule of thumb, *i.e.*, by indexing the price to the last observed rate of inflation, Π_{t-1}, yielding

$$P_t^b(i) = \Pi_{t-1} X_{t-1} = P_t^b, \tag{1.19}$$

where X_{t-1} denotes the index of the prices set at date $t-1$. The rule of thumb in relative terms, using the definition of relative prices $\tilde{P}_t^b \equiv P_t^b / P_t$ and $\tilde{X}_{t-1} \equiv X_{t-1}/P_{t-1}$, is given by

$$\tilde{P}_t^b = \frac{\Pi_{t-1}}{\Pi_t} \tilde{X}_{t-1}. \tag{1.20}$$

The new average price set in period t is then defined by

$$X_t \equiv \left[(1-\omega) P_t^{f(1-\varepsilon)} + \omega P_t^{b(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}. \tag{1.21}$$

and equivalently in relative terms by

$$\tilde{X}_t \equiv \left[(1-\omega) \tilde{P}_t^{f(1-\varepsilon)} + \omega \tilde{P}_t^{b(1-\varepsilon)} \right]^{\frac{1}{1-\varepsilon}}. \tag{1.22}$$

1.4.1 Derivation of the Phillips Curve

Aside from the firms that set their prices optimally and are forward looking, and those that set their prices according to the inflation index and are backward looking, there is a third group of firms at each period t of measure α that do not change their prices at all so that for them

$$P_t^s(i) = P_{t-1}, \tag{1.23}$$

which in relative terms is

$$\tilde{P}_t^s(i) = \frac{1}{\Pi_t}. \tag{1.24}$$

I obtain the Phillips curve representing the supply side of the model by aggregating the prices chosen by the different types of firms. Hence, from equation (1.12), it follows that

$$1 = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon} di$$
$$= \left[\int_0^{(1-\alpha)(1-\omega)} \left[\tilde{P}_t^f(i)\right]^{1-\varepsilon} di + \int_{(1-\alpha)(1-\omega)}^{1-\alpha} \left[\tilde{P}_t^b(i)\right]^{1-\varepsilon} di + \int_{1-\alpha}^{1} \left[\tilde{P}_t^s(i)\right]^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}.$$

To simplify, I assume that all firms face the same shock and share the same technology. Therefore, all firms of the same type will set the same price when allowed to change it. Therefore $\tilde{P}_t^f(i) = \tilde{P}_t^f$ and $\tilde{P}_t^b(i) = \tilde{P}_t^b$ and the Phillips curve is

$$1 = (1-\alpha)(1-\omega)\left(\tilde{P}_t^f\right)^{1-\varepsilon} + (1-\alpha)\,\omega\left(\tilde{P}_t^b\right)^{1-\varepsilon} + \alpha\left(\Pi_t\right)^{\varepsilon-1}. \tag{1.25}$$

The Phillips curve is therefore function of both the forward looking price index and the backward looking price index. A more tractable version of (1.25) is obtained by log-linearising the model 1.B.

1.4.2 Price Dispersion

The dispersion between the firm's price and the aggregate price level is an important source of social inefficiencies. Given that some of the individual prices are staggered and that the aggregate price level changes, the relative prices of individual goods change. Dispersion in relative prices Z_t leads to output dispersion that in turn leads to an inefficient level of aggregate production.

Aggregating the individual output across the firms by using individual production function (1.13) yields

$$\int_0^1 Y_t(i)\, di = A_t \int_0^1 N_t(i)\, di.$$

From the definition of aggregate labour: $N_t = \int_0^1 N_t(i)\, di$, this means that

$$N_t = \int_0^1 \frac{Y_t(i)}{A_t} di.$$

Multiplying and dividing by Y_t, and using equation (1.11)

$$N_t = \frac{Y_t}{A_t} Z_t, \tag{1.26}$$

where Z_t is a measure of relative price dispersion, given by

$$Z_t = \int_0^1 \frac{Y_t(i)}{Y_t} di = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di.$$

Together with equation (1.25), the measure of price dispersion can be rewritten as

$$Z_t = (1-\alpha) \left(\tilde{X}_t \right)^{-\varepsilon} + \alpha Z_{t-1} \Pi_t^{\varepsilon}. \tag{1.27}$$

Note that $Y_t \neq \int_0^1 Y_t(i)\, di$; using a composite consumption together with the market cleaning condition for closed economy, I get the composed expression for output

$$Y_t = \left(\int_0^1 [Y_t(i)]^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

As Schmitt-Grohé and Uribe (2007) demonstrate, relative price dispersion is bounded below by $Z_t \geq 1$. Additionally, price dispersion can be disregarded in the first order approximation of the model. Woodford (2003) and Schmitt-Grohé and Uribe (2007) also show that, given a steady state with zero inflation ($\Pi = 1$), price dispersion is deterministic and follows a AR(1) process, and as such does not have any real effect.

1.5 Non Policy Equilibrium and Steady state

After a brief discussion about a non-policy equilibrium of this model, in this section I introduce a deterministic environment around a steady state with non-zero inflation, and I compare it to a steady state with zero inflation, illustrating the similarities and pointing out the differences.

1.5.1 Non Policy Equilibrium

Assume that the instantaneous utility function is time-separable and takes the form

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}, \tag{1.28}$$

where σ and η are positive parameters, interpretable as constant relative risk aversion (CRRA) coefficients, which determine the elasticities of substitution of consumption and labour supply. The marginal utility of consumption and marginal disutility of labour are given respectively by

$$U_{C,t} = C_t^{-\sigma}$$

and

$$U_{N,t} = -N_t^{\eta}.$$

For a closed economy, the equilibrium is specified by $C_t = Y_t$ and $B_t = 0$, so that the Euler equation takes the form

$$\beta E_t \left[\left(\frac{Y_{t+1}}{Y_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] = E_t \left[Q_{t,t+1} \right]. \tag{1.29}$$

1.5.2 The Steady State

In the literature, it is common to assume that inflation in steady state equals zero (*i.e.,* the gross rate of inflation is $\Pi = 1$). However, as I demonstrate in this section, a moderate long run inflation, that is, a steady state non-zero inflation, can improve social welfare. This fact is exploited by the central planner in the case that $\omega > 0$, *i.e.*, in a model with (some) backward looking agents, as described in more details in Section 1.6.1. Prior to this, it is necessary to define the non-stochastic steady state of the model, in which exogenous disturbances are absent and it holds that $A = 1$. Real variables and variables written in relative terms are constant and only nominal variables increase, following the steady state inflation process Π.

In short, I can summarise the overall effect of a non-zero steady state inflation as follows. On the one hand, positive inflation generates greater price dispersion Z, so the aggregate output tends to decrease. On the other hand, as I show below, positive (small) inflation leads to an increase in real marginal labour cost. Since wages are flexible, marginal cost equals real wage, thus positive (small) steady state inflation leads to a higher real wage. Higher real wage has both income and substitution effects. The income effect that follows from an increase in real wages will cause an increase in both leisure and consumption. Therefore, labour supply decreases. However, because of the higher wage, workers also substitute their leisure with labour. A rise in the labour input induces an increase in aggregate output. Hence, as I will demonstrate below the effect of inflation on output is determined by two countervailing forces, a positive effect generated by inducing higher real wages and a negative effect generated by inducing greater price dispersion.

From the Euler equation in the steady state, using equation (1.29), it follows that the market discount factor equals its subjective counterpart

$$\frac{\beta}{\Pi} = Q, \tag{1.30}$$

where Π is steady state inflation. Therefore, it holds that if inflation in steady state is non-zero, the discount factor Q is lower than the subjective household's discount factor β.

Using equation (1.8), the steady state labour supply can be determined by equating the marginal rate of substitution between labour and consumption to the steady state real wage

$$\bar{W} = -\frac{U_N}{U_C}. \tag{1.31}$$

From the supply side, using equations (1.20), (1.22) and (1.25), all firms allowed to change the prices set them equally, as a function of the level of steady state inflation, formally

$$\bar{P}^b = \bar{X}$$

and

$$\bar{P}^f = \bar{X},$$

where:

$$\bar{X} = \left(\frac{1 - \alpha\,(\Pi)^{\varepsilon - 1}}{1 - \alpha} \right)^{\frac{1}{1-\varepsilon}}. \tag{1.32}$$

Note that, if $\Pi = 1$, then also the new and old prices equal the CPI, and therefore the model delivers unit relative prices. In the case that the steady state inflation is positive, the nominal prices increase steadily. It follows from the definition of the price index that this aggregator averages new and unchanged prices, so it holds that $P < X = P^f = P^b$. Thus, the new nominal price grows more than the price index, which is pulled down by the lower prices chosen in the last periods, making the new relative price larger than one, formally

$$\bar{P}^f = \frac{P^f}{P} > 1.$$

The fact that zero inflation implies that all relative and nominal prices equal one also implies unit price dispersion, *i.e.*, $Z = 1$. It can be shown that, when $\Pi > 1$,

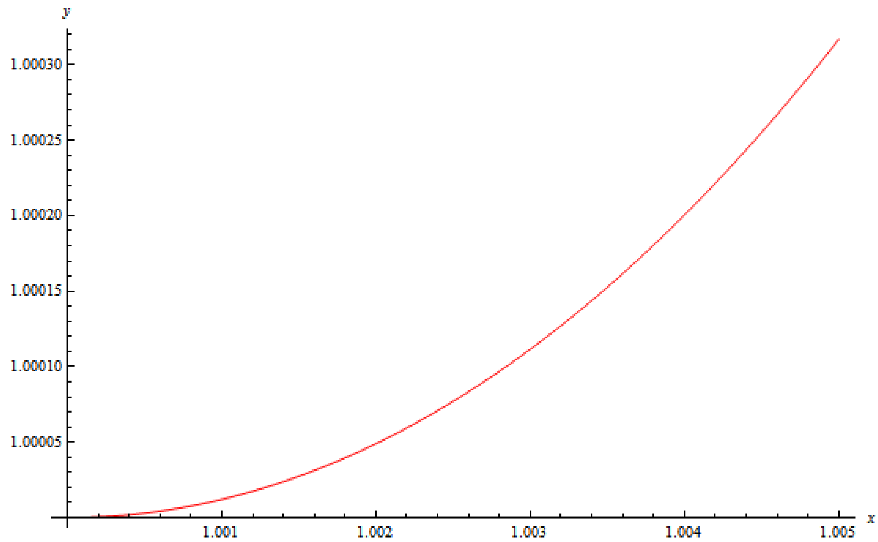

Figure 1.2: The relationship between steady state inflation (horizontal axis) and price dispersion (vertical axis).

the price dispersion is above one. Using equation (1.27) in steady state, price dispersion is a function of the new price and inflation, formally

$$Z = \frac{(1-\alpha)\,\bar{X}^{-\varepsilon}}{1-\alpha\Pi^{\varepsilon}}. \tag{1.33}$$

Price dispersion is directly related to inflation as long as the gross rate of inflation is close to one. Figure 1.2 shows that a higher inflation generates larger steady state price dispersion. Solving (1.16) for marginal costs in terms of the rate of inflation yields

$$\overline{MC} = \frac{1}{\mu}\left(\frac{1-\alpha\beta\Pi^{\varepsilon}}{1-\alpha\beta\Pi^{\varepsilon-1}}\right)\bar{P}^{f}. \tag{1.34}$$

If $\Pi = 1$, marginal costs are the reciprocal of the forward looking price over markup, otherwise the real marginal costs also depend on the level of the steady state inflation. On one hand, if Π is close enough to one, the marginal costs are directly related to the rate of inflation. On the other hand, with higher steady state inflation, real marginal costs decrease.A small increase in the steady state inflation leads to higher real marginal costs. Given that the real marginal costs are proportional to the real wage, gross inflation slightly higher than 1 leads to an increase in the real wage.

Inserting the equality $\overline{MC} = \bar{W}$ into equation (1.34) together with (1.32) and rearranging, I get

$$\bar{W} = \frac{1}{\mu}\left(\frac{1-\alpha\beta\Pi^{\varepsilon}}{1-\alpha\beta\Pi^{\varepsilon-1}}\right)\bar{X}. \tag{1.35}$$

At the firm-level, for the production function and for the aggregate production function in steady state

$$Y(i) = N(i)$$

and

$$Y = \frac{N}{Z}.$$

The wedge between aggregate output and labour is the reciprocal of the price dispersion.

Finally, the level of output in steady state can be expressed as a function of steady state inflation

$$Y = \left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\eta}}\left[\left(\frac{1-\alpha\Pi^{\varepsilon-1}}{1-\alpha}\right)^{\frac{\varepsilon(\eta+1)}{\varepsilon-1}} \cdot \frac{(1-\alpha)^{1+\eta}\left(1-\alpha\beta\Pi^{\varepsilon-1}\right)}{\left(1-\alpha\beta\Pi^{\varepsilon}\right)\left(1-\alpha\Pi^{\varepsilon-1}\right)\left(1-\alpha\Pi^{\varepsilon}\right)^{\eta}}\right]^{-\frac{1}{\sigma+\eta}}. \tag{1.36}$$

The steady state level of output (1.36) differs from the natural level of output Y^n. The natural level of output arises when all prices are flexible (*i.e.*, $\alpha = 0$), so all firms set their prices equal. For this reason, at the natural level of output all prices are equal to the consumer price index, *i.e.*, $P_t(i) = P_t$, and it follows that $\bar{X} = \bar{P}^f = \bar{P}^b = 1$. The natural level of output itself can be expressed as

$$Y^n = \mu^{-\frac{1}{\sigma+\eta}}.$$

Note that, in the case of zero inflation, steady state output equals its natural level, because any effects from the presence of nominal distortions are neutralised by the equality of nominal and real prices for all firms. Therefore, the output gap, defined as the ratio between actual and natural level of output $\tilde{Y} = \bar{Y}/\bar{Y}^n$, equals one.

In the case of non-zero inflation, the output gap is instead given by

$$\tilde{Y} = \left[\left(\frac{1 - \alpha (\Pi)^{\varepsilon-1}}{1-\alpha} \right)^{\frac{\varepsilon}{\varepsilon-1}(\eta+1)} \frac{(1-\alpha)^{1+\eta} \left(1 - \alpha\beta\Pi^{\varepsilon-1}\right)}{\left(1-\alpha\beta\Pi^{\varepsilon}\right)\left(1-\alpha\Pi^{\varepsilon-1}\right)\left(1-\alpha\Pi^{\varepsilon}\right)^{\eta}} \right]^{-\frac{1}{\sigma+\eta}}.$$

Figure 1.3 shows that a small level of inflation can increase steady state output over its natural level. However, higher inflation is distortionary and leads to a negative output gap, *i.e.*, $\tilde{Y} < 1$.

Additionally, it can be shown that the aggregate output produced in the steady state differs from the efficient level of output, defined as the output in an economy without any nominal and real distortions, such that the marginal product of labour

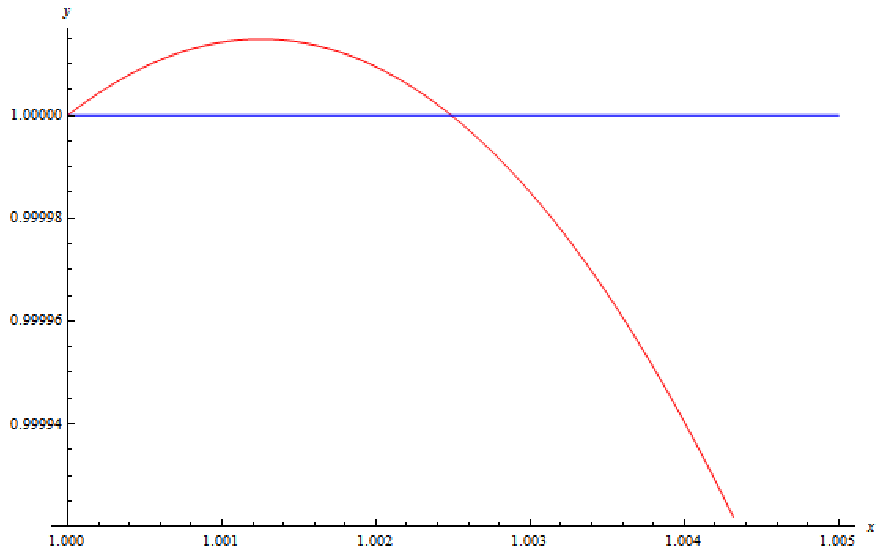

Figure 1.3: The relationship between steady state inflation (horizontal axis) and steady state output gap (vertical axis).

MPN equals the marginal rate of substitution $MRS_{C,N}$.

The wedge between steady state output and its efficient level therefore reflects the wedge between the marginal product of labour $MPN = Y/N$ and the marginal rate of substitution $MRS_{C,N} = -U_N/U_C$, which can be illustrated using (1.35) in conjunction with (1.36)

$$\frac{Y}{N} = -\mu \frac{U_N}{U_C} \frac{\left(1 - \alpha\beta\Pi^{\varepsilon-1}\right)\left(1 - \alpha\Pi^{\varepsilon}\right)}{\left(1 - \alpha\beta\Pi^{\varepsilon}\right)\left(1 - \alpha\Pi^{\varepsilon-1}\right)}. \tag{1.37}$$

Note that without any nominal distortions, *i.e.*, in the case that $\alpha = 0$, or in the standard case of zero steady state inflation, *i.e.*, $\Pi = 1$, the fraction disappears and the wedge between output and labour is generated only by real distortions,

σ	1.5	inverse intertemporal elasticity of substitution
η	1	inverse elasticity of labour supply
β	0.99	subjective discount factor
α	2/3	parameter of price stickiness
ε	4	elasticity of substitution between goods

Table 1.1: Calibration of the model parameters

captured by the mark-up μ. Note that assuming perfect competition, $\varepsilon \to \infty$ leads to a zero mark-up, where $\mu = 1$.

Rotemberg and Woodford (1998) emphasise that the aim of the policy maker is to stabilise the economy around the efficient level of output Y_t^e. To this aim, they approximate the welfare of the representative household around the efficient level of output. As mentioned above, nominal distortions in steady state disappear if inflation equals one. Real distortions generated by monopolistic competition can be offset by introducing employment subsidies that increase the output up to the desired efficient level. Using equation (1.37), it follows that it is sufficient to introduce an output subsidy with rate τ such that $(1-\tau)\,\mu = 1$. In other words, the subsidies can be expressed as $\tau = \varepsilon^{-1} > 0$, and the higher the elasticity of substitution, *i.e.*, the lower the markup, the lower the subsidies required to obtain the efficient level of output.

The introduction of employment subsidies lowers the MPN, directly leading to increases in employment and output to their efficient levels. Therefore the natural level and the efficient level of output are identical, *i.e.*, $Y^n = Y^e$, and the output gap equals the welfare relevant output gap.

1.5.3 Innovations and Calibration of the model

In the last section, I described my model in its deterministic, perfect foresight, steady state. In what follows I study the behaviour of the model in a stochastic environment where the source of uncertainty is represented by shocks to total factor productivity (TFP). The TFP innovation A_t measures the aggregate technology process, which (log-linearised) follows the stochastic AR(1) process

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \xi_{a,t},$$

where $\hat{a}_t \equiv (\log A_t - \log A)$ denotes the deviation, in log terms, of TFP from its steady state value, $\rho_a \in [0, 1)$ is the autocorrelation parameter, and $\xi_a \sim i.i.d.\,(0, \sigma_a^2)$ is a white noise shock.

In assigning numerical values to the structural parameters, I closely follow the literature. Table 1.1 reports the calibrated values for the parameters of the model. The parameter σ is the inverse of the intertemporal elasticity of substitution (IES), and the elasticity is usually assumed to take values between 0.5 and 1. I choose $\sigma = 1.5$ so that the IES equals 0.67. The labour disutility parameter η is set equal to one, in accordance with the estimates from Smets and Wouters (2003).

Each time period in this model corresponds to a quarter of a year, hence $\beta = 0.99$ corresponds to just over an annual subjective discount rate of 4 per cent. The price stickiness parameter, $\alpha = 2/3$, means that on average a given firm changes its price every three periods, computed as $1/(1 - \alpha)$. This corresponds to the usual value estimated for the US market. Finally, setting $\varepsilon = 4$ implies that the average markup over the marginal costs is set to 33 percent.

1.6 Welfare Loss Function and Welfare Loss Measure

There are several linear-quadratic approaches in the literature to approximating the welfare loss function. Generally, a linear-quadratic approach uses the first order approximation of the structural equations to obtain a second order approximation of the utility function to represent the social welfare function. There is an extensive literature devoted to the analytical derivation of the welfare loss function. This paper provides a welfare based analysis to measure the effectiveness of different monetary policy rules comparing two different approaches: the traditional linear-quadratic approach and the Hamiltonian approach.

The first approach is based on Levine, McAdam, and Pearlman (2007) and their Hamiltonian two-stage linear-quadratic approach. I show that, for a Ramsey planner and in the presence of backward looking firms, it is desirable to impose a non-zero steady state rate of inflation.

In their work, Levine *et al.* (2007) develop a two-stage Ramsey policy, where in the first stage a steady state in line with a social planner allocation is found and corresponds to a best possible deterministic outcome. In the second stage, a monetary policy rule for a stochastic environment is determined, which leads to the lowest possible welfare loss (and therefore the closest to the steady state value derived in the first stage). In this case, however, it is not possible to derive an analytical solution for the welfare loss function, so the model is solved numerically using Dynare/ACES software.

The second approach, which has an analytical solution, is based on Steinsson (2003) and similar to Rotemberg and Woodford (1999), and enables me to compare my results with the numerical solution. Following Rotemberg and Woodford (1999), the second order approximation of the welfare loss function is derived together with the first order approximation of the equilibrium conditions. Two conditions are necessary to derive an accurate second order approximation of the welfare using this method. First, the real distortions following from monopolistic competition have to be eliminated by assuming labour subsidies, thus allowing only for small distortions. Therefore, in the steady state, the output gap is zero and the model is log-linearised around its efficient level. Second, the approximation has to be calculated in a closed neighborhood of zero steady state inflation, so that in the steady state the price distortions are eliminated.

1.6.1 Welfare Loss Function using the Hamiltonian Approach

In this section, I explain how the welfare loss is calculated using the two-stage approach by Levine *et al.* (2007). In the first stage, the social planner chooses a steady state inflation level, which maximises steady state welfare. In the second stage, welfare in a stochastic environment around the first-stage steady state is maximised.

Generally, the goal of the central bank's monetary policy is to maximise a social welfare function Ω, which consists of the discounted stream of the households' utility over the current and the future periods

$$\Omega = \sum_{t=0}^{\infty} \beta^t U_t, \tag{1.38}$$

where U_t is the instantaneous utility function of the representative household. Using (1.28), the nonlinear optimisation problem for the Ramsey planner is defined by

$$\Omega = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{j1-\sigma}}{1-\sigma} - \left(\int_0^1 \frac{N_t^j(i)}{1+\eta} di \right)^{1+\eta} \right],$$

where $C_t^j = C_t$ for all households j, given the assumption that all household have identical preferences and resources. An individual j supplies N_t^j labour in total, assuming that to each firm i it supplies equal amount of labour $N_t^j(i)$. Given (1.26), the Ramsey planner problem can be rewritten as

$$\Omega = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{j1-\sigma}}{1-\sigma} - \frac{1}{1+\eta} \left(\frac{Y_t}{A_t} Z_t \right)^{1+\eta} \right]$$

and is maximised subject to the structural conditions (1.25), (1.16), (1.17), (1.18), (1.20), (1.22) and (1.27). In this first stage, it is only necessary to find the first order conditions in a deterministic steady state. Thus, only the optimal steady state is identified.

Using a Dynare based MATLAB software package called ACES (Analysis and Control of Economic Systems), and assuming that inflation is the instrument controlled by the central bank, the steady state of this problem is identified as a non-zero steady state inflation when $\omega > 0$. As illustrated in Figure 1.3, a moderate steady state rate of inflation leads to output rising above its natural level (*e.g.*, $Y > Y^n$) and it can be shown that the social planner chooses a positive rate of steady state inflation whenever $\omega > 0$. Recall from Section 1.5.2 that, for $\Pi > 1$, price dispersion is higher than one in steady state but, at the same time, if inflation is sufficiently small, then the equilibrium in the labour market implies

ω	Π	$\bar{W}$	N	$C = Y$	$\bar{\Omega}$
0	1	0.75	0.891299	0.891299	-251.5670
0.3	1.00006	0.750002	0.8913	0.8913	-251.5657
0.5	1.00013	0.750006	0.891302	0.891302	-251.5656
0.8	1.00052	0.750021	0.891311	0.891308	-251.5657

Table 1.2: Hamiltonian steady state outcomes for different shares of backward looking firms.

that output may rise above its natural level. Therefore, there is a certain trade off between these two effects, where the positive effects outweigh the negative ones if inflation is positive but very small.

Given (1.38), social welfare in the steady state is measured as

$$\bar{\Omega} = \frac{1}{1-\beta} U\left(C, N\right). \tag{1.39}$$

As it is shown in Table 1.2, the social welfare rises as the value of the backward looking parameter ω is increased.[1] The higher the share of backward looking companies, the higher the trend inflation chosen by the social planner. Higher inflation leads to a higher real wage and an increase in labour because the substitution effect dominates the income effect. Although output increases with the level of steady state inflation, its rise is lower than that of labour, due to the additional price dispersion that higher inflation generates.

From the equations in Section 1.5.2, it is straightforward to notice that the

[1] For reference, note that the level of welfare generated in an efficient level of output is $\bar{\Omega} = -250$, with $\Pi = \bar{W} = N = C = Y^{eff} = 1$.

steady state outcomes are not directly influenced by the parameter ω. As a result, one might be tempted to conclude that the social planner may choose steady state inflation independent of this parameter. However, Table 1.2 clearly points towards the opposite direction: the optimal social planner decision is affected by the magnitude of the fraction of backward looking firms. Zero inflation is optimally chosen only when all firms are forward looking, and this also represents the steady state with the lowest level of output and welfare. As the fraction of backward looking firms increases, both inflation and welfare grow. This is suggestive of a positive ω being beneficial for the economy.

One possible rationale for this finding relies on the price dispersion being, under certain conditions and off steady state, a negative function of ω.[2] In the presence of backward looking firms, in fact, the social planner would exploit this negative link by letting inflation reach an optimal positive level in the steady state. In particular, the larger ω, the higher the optimal steady state inflation. As explained above, a positive inflation increases welfare through the marginal costs channel, and decreases it by raising price dispersion. For any given $\omega > 0$, it is then possible to find a level of positive inflation that would result in better outcomes than a zero inflation (both in transition and, eventually, in the steady state). Suppose that the social planner attempted to impose the same level of steady state inflation

[2] More precisely, I can rewrite (1.27) as

$$Z_t = (1-\alpha)\left[(1-\omega)\left(P_t^f/P_t\right)^{-\varepsilon} + \omega\left(P_t^b/P_t\right)^{-\varepsilon}\right] + \alpha Z_{t-1}\Pi_t^{\varepsilon},$$

from which it follows that $P_t^f > P_t^b$ entails $dZ_t/d\omega < 0$. This means that, whenever forward looking firms (f) set a higher price than their backward looking counterparts (b), price dispersion is inversely related with the fraction of b firms. The condition $P_t^f > P_t^b$ seems natural to assume in the case of an unanticipated inflation rise, given that while f firms are fully rational in their reaction to a rise in inflation, b firms reaction is lagged one period, and always considers only the last period inflation (which is in turn the 'average' outcome of the choices made by f and b).

with a lower ω. Then, the fraction of forward looking firms would possibly be sufficiently large to command a high price dispersion, whose negative effects may overcome the positive ones due to the rise in labour supply. Of course, the social planner could not possibly exploit this channel to improve welfare when $\omega = 0$, as the forward looking NKPC would lack any inflation inertia.

The aim of the second stage of the Hamiltonian approach is to derive an accurate linear-quadratic approximation in a stochastic environment around a steady state level identified in the first stage. Calculating the first order Taylor approximation (log-linearisation) of the first order conditions and the constraints, together with the second order Taylor approximation of the Hamiltonian with the Lagrangian problem, delivers a quadratic approximation of the utility expressed in terms of the state variables. In the presence of non-zero inflation, an analytical derivation of this function is not possible. Thus, at this point, I merely compare the welfare loss function obtained from analytical solution of simple linear-quadratic approach with the numerical results obtained directly using ACES.

1.6.2 Analytical Approximation of the Welfare Loss Function

Because the analytical derivation of the welfare function around a steady state with non-zero inflation is not possible, I compare my numerical result, obtained using the Hamiltonian approach described in previous section, with the result derived from the traditional approach, analogous to the one implemented by Steinsson (2003). This method is based on a linear-quadratic approach using the first order approximation of the structural equations and the second order approximation of the welfare function. The approximation of the second order of the welfare function is detailed in the appendix. The analytical welfare loss function also proves useful for a derivation of the optimal monetary policy under commitment.

I study the model in the neighborhood of its non-stochastic perfect-foresight steady state. There are two points worth mentioning. First, without any further assumptions, this steady state does not achieve the long run efficient level of output because of the presence of nominal and real distortions. In particular, non-zero inflation in the steady state generates a degree of price dispersion that, together with the firms' monopolistic power, implies lower output. This causes problems when calculating the second order approximation of the welfare, and therefore these distortions have to be neutralised, as discussed in Section 1.5.2.

Log-Linearisation

To derive analytically the loss function around the steady state with zero inflation, it is necessary to approximate the structural equations of the model in a close neighborhood of the model's steady state. The log-linearisation of a generic variable D_t around its steady state value D can be generally expressed as

$$D_t = De^{\log D_t/D} = De^{\log D_t - \log D} = De^{(d_t - d)} = De^{\hat{d}} \approx D\left(1 + \hat{d}_t\right).$$

However, some of the structural equations are linear in their logs so I can write directly $\log D_t = d_t$. This holds for the Euler equation (1.29), whose log-linearisation is

$$y_t = E_t\left[y_{t+1} + \frac{1}{\sigma}\left(\pi_{t+1} - i_t - \varrho\right)\right]$$

with $\varrho = -\log \beta$ and where $p_{t+1} - p_t \equiv \pi_{t+1}$ and $i_t = -\log (Q_{t,t+1})$. Assuming that the variables do not deviate much from their steady state values, the Euler equation can be written as a deviation from its steady state level as

$$\hat{y}_t = E_t \left[\hat{y}_{t+1} + \frac{1}{\sigma} (\pi_{t+1} - \hat{\imath}_t) \right], \tag{1.40}$$

where I employ the definitions $\hat{p}_{t+1} - \hat{p}_t \equiv \pi_{t+1}$ and $\hat{\imath}_t \equiv -[\log (Q_{t,t+1}) - \log (Q)]$.[3] It is clear from (1.40) that, in a closed economy, the real interest rate, $i_t - \pi_{t+1}$, is pinned down by the real output growth rate. Hence, if prices are fully flexible and firms are fully rational, central bank intervention modifies the nominal interest rate, i_t. This change is fully anticipated by firms that, by setting their prices accordingly, induce a proportional shift in the inflation rate π_{t+1}. If prices are not fully flexible and/or firms are not fully rational, then changes in the nominal interest rate induced by the central bank may have real effects, as prices do not respond proportionally to variations in the nominal interest rate. As a result, inflation only partially responds to the central bank intervention, and real output growth, $y_t - y_{t-1}$, also changes. The final result is that the Euler equation holds with a modified real rate of interest and a modified degree of output growth.

For this reason, it is important to introduce the concept of natural equilibrium, where all firms are fully rational and may adjust their prices every period. Denote the log-linearisation of the generic variable D_t in its natural level as $d_t^n \equiv \log D_t^n$. Following the definition of the natural level of a variable, I can also rewrite the log-linearised Euler equation in terms of the output gap (the deviation of the real variables from their natural levels), in turn defined as the log of the ratio between

[3]Note that since the steady state level of inflation is set to be zero, we can write $\hat{\pi}_t = \pi_t$.

actual output and natural output, and denoted by $\tilde{y}_t = y_t - y_t^n$. Using (1.40) yields

$$\tilde{y}_t = E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma} E_t\left[i_t - \pi_{t+1} - r_t^n\right], \tag{1.41}$$

where r_t^n is the Wicksellian natural rate of interest and it holds that $r_t^n = \varrho + \sigma E_t\left(\Delta y_{t+1}^n\right)$. For future reference, I also express (1.41) as a deviation from the steady state level, formally

$$\tilde{y}_t = E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma} E_t\left[\hat{\imath}_t - \pi_{t+1} - \hat{r}_t^n\right]. \tag{1.42}$$

In this case, $\hat{r}_t^n$ is the deviation of the Wicksellian natural rate of interest from its steady state level, and it holds that $\hat{r}_t^n = \sigma E_t\left[\Delta y_{t+1}^n\right]$.

Additionally, the log-linearisation of the labour supply delivers the relationship between the real wage, labour supply and output

$$\hat{w}_t = \eta \hat{n}_t + \sigma \hat{y}_t. \tag{1.43}$$

Furthermore, as I describe in Appendix 1.B in more detail, the supply curve shows how inflation reacts to variations in real marginal costs as well as future and past inflation. I rewrite this as

$$\pi_t = \chi^f E_t\left[\pi_{t+1}\right] + \chi^b \pi_{t-1} + \kappa_{mc} \widehat{mc}_t \tag{1.44}$$

with the parameters

$$\begin{aligned}\Psi &= \alpha+\omega\left(1-\alpha\left(1-\beta\right)\right)\\ \chi^{f} &= \alpha\beta/\Psi, \chi^{b}=\omega/\Psi\\ \kappa_{mc} &= \left(1-\omega\right)\left(1-\alpha\right)\left(1-\alpha\beta\right)/\Psi,\end{aligned}$$

where $0<\left\{\chi^{f},\chi^{b},\kappa_{mc}\right\}<1$ and $\Psi>1$. Notice that, when $\beta\rightarrow 1$, $\chi^{f}+\chi^{b}=1$. Hence, inflation is a function of the convex combination of future and past inflation.

The measure of price dispersion in (1.27), log-linearised around the zero inflation steady state, delivers the AR(1) process

$$z_{t}=\alpha z_{t-1},$$

which, as discussed in Schmitt-Grohé and Uribe (2007), is purely deterministic up to the first order, and does not have any real effect on the rest of the model.

Similar to the Euler equation, the Phillips curve can also be rewritten in terms of the output gap. As mentioned earlier, in the natural state of the economy firms can reset their prices in each period. Since I assume that all firms are identical, they will all choose the same price, which equals the aggregate price level. Therefore, all relative prices in the natural level of output equal one.

Formally, it is straightforward to see that the natural level of the relative price (in logarithms) equals the real marginal cost in the natural level mc_{t}^{n} plus the markup μ, where the latter can be disregarded when considering the deviation from its steady state level

$$\widehat{mc}_t^n = 0. \tag{1.45}$$

This is because the real marginal cost always equals the mark up, which does not deviate from its steady state level. Log-linearisation of equation (1.14), together with (1.8), yields

$$\widehat{mc}_t = \eta \hat{n}_t + \sigma \hat{y}_t - a_t. \tag{1.46}$$

Additionally, log-linearisation of (1.26), together with the fact that $\hat{z}_t$ can be disregarded, delivers

$$\hat{n}_t = \hat{y}_t - a_t.$$

Inserting this back into (1.46), yields the real marginal cost

$$\widehat{mc}_t = (\eta + \sigma)\,\hat{y}_t - (1 + \eta)\,a_t.$$

Combining the last expression with (1.45) delivers the natural level of output

$$\hat{y}_t^n = \frac{(1+\eta)}{(\eta+\sigma)} a_t, \tag{1.47}$$

and I can therefore express the the relationship between the marginal cost and output gap as

$$\widehat{mc}_t - \widehat{mc}_t^n = (\eta + \sigma)\,(\hat{y}_t - \hat{y}_t^n) - (1 + \eta)\,a_t + (1 + \eta)\,a_t.$$

Rearranging, and employing the definition of the output gap $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$, yields

$$\widehat{mc}_t = (\eta + \sigma)\,\tilde{y}_t. \tag{1.48}$$

Plugging this equation into (1.44) delivers a hybrid NKPC in terms of the output gap, that is log-linearised around a steady state equilibrium with zero inflation

$$\pi_t = \chi^f E_t [\pi_{t+1}] + \chi^b \pi_{t-1} + \kappa \tilde{y}_t. \tag{1.49}$$

The parameter $\kappa = \kappa_{mc} (\eta + \sigma) > 0$ is the slope of the Phillips curve expressed in terms of the output gap, and reflects the relationship between inflation and output.

Following Steinsson (2003), I consider four different values of the backward looking parameter ω. The values for the composed parameters under these values of ω are shown in Table 1.3. The first row corresponds to the standard NKPC with a purely forward looking component, as postulated by most contributions to the literature (*e.g.*, Woodford, 2003). The second row corresponds to the values in the Galí-Gertler estimation for a rule of thumb model.[4] As in Steinsson (2003), I refer to the third row as a *midway case*, where half of the firms are rational, and half of them follow a rule of thumb. The fourth row correspond to a model with a large share of backward looking firms and is a good proxy for the Fuhrer-Moore (1995) model. Even when omega is set this high, the Phillips curve is both forward and backward looking, with a slightly higher weight on the latter component. However, the curve becomes very inelastic to changes in the output gap.

Assuming that all firms are purely backward looking, *i.e.*, $\omega \to 1$, does not directly reduce the Phillips curve to an *acceleration Phillips curve*, since $\chi^f \to \alpha\beta/(1 + \alpha\beta)$. However, Steinsson (2003) shows that taking the limit $\omega \to 1$,

[4] Galí and Gertler (1999) test empirically this hybrid NKPC and argue that, depending on the conditions, the estimates for ω are between 0.26 and 0.49, which correspond to estimates for χ^b between 0.25 and 0.38 and for χ^f between 0.59 and 0.68.

ω	χ^f	χ^b	κ
0	0.99	0	0.43
0.3	0.68	0.31	0.21
0.5	0.57	0.43	0.12
0.8	0.45	0.55	0.04

Table 1.3: Structural parameters of the hybrid NKPC for different shares of backward looking firms.

the equation (1.49) has a unique bounded solution in which $\pi_t = \pi_{t-1}$. Hence, if all firms are backward looking, the price index in period t simply equals the price index in period $t-1$, and it holds that $\left\{\chi^f, \chi^b, \kappa\right\} \rightarrow \{0, 1, 0\}$. In this case, the Phillips curve is vertical, which implies the absence of a trade-off between output and inflation.

Welfare Loss Function for a Model with Zero Steady State Inflation

Using the technique described in Rotemberg and Woodford (1999) and Steinsson (2003), and assuming that output is on its efficient level in steady state, the analytical derivation of the second order approximation of the objective function is straightforward and is described in more technical details in Appendix 1.C.

Ignoring terms higher than the second order and those independent of policy, the welfare loss function can be approximated by

$$\Omega_0 = \Theta E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda_\Delta \Delta\pi_t^2 + \lambda_y \tilde{y}_t^2\right),$$

where Θ, λ_y and λ_Δ are functions of the structural parameters of the model. In contrast to the traditional loss function derived by Rotemberg and Woodford (1999) for a model with purely forward looking NKPC, this expression contains the relative weight on the inflation growth fluctuation term $\lambda_\Delta = \alpha^{-1}\omega/(1-\omega)$, which is positively related to the parameter ω. That is, the larger the fraction of backward looking firms, the higher the welfare loss caused by a fluctuation in the rate of inflation (relative to the level of inflation), since the share of prices based on past information is relatively larger. However, this effect is twofold, because a higher share of backward looking firms also leads to higher inflation persistence, and therefore to smaller changes in inflation growth.

The relative weight on output gap fluctuations is increasing in σ and η, and given by $\lambda_y = (1/\alpha - \beta)(1-\alpha)(\eta+\sigma)/\varepsilon$. These two parameters govern the wedge between marginal rate of substitution and marginal product of labour, which measures the level of inefficiency in the economy. The value of λ_y is also decreasing in α and ε, which in turn govern the weight of output gap fluctuations relative to inflation fluctuations. With the calibration given in Section 1.5.3, the term λ_Δ varies positively with the parameter ω. The relative weight on inflation growth strengthens as the value of ω is raised, since $\partial^2\lambda_\Delta/\partial\omega^2 > 0$, although it should be noted that increasing inflation persistence also implies a decreasing value of squared inflation growth deviation.

As long as β approaches 1, the average welfare loss can be written as the unconditional expected average welfare loss

$$\Omega_0 = \Theta\left[var\left\{\pi\right\} + \lambda_\Delta var\left\{\Delta\pi\right\} + \lambda_y var\left\{\tilde{y}\right\}\right]. \tag{1.50}$$

The traditional loss function can be found as a special case of (1.50) by imposing $\omega = 0$, which leads to $\lambda_\Delta = 0$. This loss function corresponds the one derived by Woodford (2003) or Galí (2008), formally

$$\Omega_0 = \Theta \left[var\left\{\pi\right\} + \lambda_y var\left\{\tilde{y}\right\}\right]. \tag{1.51}$$

In the literature, most contributions opt for this more conservative loss function, even when (1.50) would possibly be the most suitable one, *i.e.*, when the model actually presents backward looking components (see, *e.g.*, Clarida, Galí and Gertler, 1999).

In the next section, I compare the social welfare loss obtained using the Hamiltonian approach from Section 1.6.1 with that resulting from the generalised loss function (1.50).

1.6.3 Welfare Measure

The welfare loss can be rewritten in terms of a compensating differential, a measure of how much households should be compensated in order to neutralise the welfare loss. To calculate the compensating differential between two regimes, I use the definition of welfare given in (1.38). In this paper, the compensating differential is calculated as a difference between the welfare obtained using the analysed monetary policy rule and the steady state value.

Generally it holds that the analysed rule generates welfare

$$\Omega = \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right),$$

which is compared with the benchmark welfare

$$\Omega^* = \sum_{t=0}^{\infty} \beta^t U\left(C_t^*, N_t^*\right).$$

I am looking for a consumption compensating differential ζ that equates welfare Ω to its analogous Ω^*. Formally

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) = \sum_{t=0}^{\infty} \beta^t U\left(\left(1-\zeta\right) C_t^*, N_t^*\right).$$

Putting the specific utility function (1.28), welfare reads

$$\Omega = \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(\left(1-\zeta\right) C_t^*\right)^{1-\sigma}}{1-\sigma} - \frac{N_t^{*1+\eta}}{1+\eta} \right],$$

which can be expressed as

$$\Omega = \Omega^* + \left(\left(1-\zeta\right)^{1-\sigma} - 1\right) \sum_{t=0}^{\infty} \beta^t \frac{C_t^{*1-\sigma}}{1-\sigma}.$$

Solving for the compensating differential, and using the term $\Omega_C = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{*1-\sigma}}{1-\sigma}$ as the welfare following directly from consumption, I obtain

$$\zeta = 1 - \left[\frac{\Omega - \Omega^*}{\Omega_C} + 1 \right]^{\frac{1}{1-\sigma}}$$

When compared to the steady state value, welfare is given by

$$\bar{\Omega} = \frac{1}{1-\beta} U\left(\bar{C}, \bar{N}\right)$$

so that the compensating differential can be expressed similarly to the one obtained by Leith, Moldovan, and Rossi (2012)

$$\zeta = 1 - \left(\frac{\Omega - \bar{\Omega}}{\bar{\Omega}_C} + 1 \right)^{\frac{1}{1-\sigma}},$$

with $\bar{\Omega}_C = \frac{1}{1-\beta} \frac{\bar{C}^{1-\sigma}}{1-\sigma}$.

1.7 Analysis of Monetary Policy

The preceding sections outlined the demand side and the supply side of a model for a closed economy without any central bank intervention. In this section, I introduce four interest rate targeting rules in simple and optimal form: the Taylor rule and the forward looking Taylor rule, which targets both output and inflation in current and forward looking form, respectively. Furthermore, I analyse the pure inflation targeting rule, which neglects output and concentrates only on inflation, and finally the Taylor rule with an interest smoothing component. First, I assume the unique calibration for all rules given in Section 1.5.3, and focus only on different values for the parameter ω, since the goal of this exercise is to show the relevance of this parameter is for the model economy. Second, I examine the optimal form of all these rules. In the next section, I analyse the individual performance of these rules and their impact on social welfare in the presence of a TFP shock.

Taylor Rule

Generally, a short interest rate such as the federal funds rate in the US serves as the instrument in a Taylor rule (TR). According to this rule, the central bank adjusts the instrument in response to variations in inflation and output

$$\hat{\imath}_t = \varrho + \phi_\pi \pi_t + \phi_y \hat{y}_t. \tag{1.52}$$

The parameter ϱ is described by McCallum (2002) as the equilibrium interest rate or equivalently, by Galí (2008), as the households' discount rate. The coefficients $\phi_\pi$and ϕ_y are non-negative and determined by the central bank. They describe the strength of the interest rate $\hat{\imath}_t$ response to changes in inflation π_t and output $\hat{y}_t$, respectively. According to this rule, if inflation or output increase, the interest rate should increase as well, resulting in a contractionary monetary policy. For the calibration, I let $\phi_\pi = 1.5$ and $\phi_y = 0.125$, which correspond to the values given by Taylor (1993).

Assuming that $\varrho = \hat{r}_t^n$, the interest rate set by the central bank moves one to one with the natural rate of interest. The idea of having the natural rate of interest as part of the Taylor rule can be traced directly to Taylor (1999), who shows that the intercept in the model should equal the equilibrium real interest rate if targeted inflation is zero. This is desirable for all shocks which cause movements in natural rate of the economy, as in fact a TFP shock does. The TFP shock affects $\hat{r}_t^n$ directly, and it is desirable that the interest rate follows the same path. Since a positive TFP shock increases the natural level of output, it follows that, if the policy maker aims at stabilising prices, aggregate output must be allowed to fluctuate. Under this assumption, the interest rate must decrease in response to temporary increases in productivity.[5]

[5] See Woodford (2003, p. 263) for a discussion on monetary policy in the presence of demand shocks.

Pure Inflation Targeting Rule

Consider the special case where $\phi_y = 0$ in (1.52). This is the pure inflation targeting rule (PITR)

$$\hat{\imath}_t = \varrho + \phi_\pi \pi_t. \tag{1.53}$$

PITR differs from a strict inflation targeting rule insofar as inflation is kept constant at its steady state level over time. It is worth noting that a strict rule can also target output, and the difference between strict and flexible targeting is simply reflected in the weights placed on the endogenous variables. Pure inflation targeting should then be understood as a rule where inflation is the only variable that the central bank targets (see, *e.g.*, Svensson (2010)).

Rule with Interest Rate Smoothing Component

Another version of a Taylor rule often used in the literature is interest rate smoothing rule (ISR), which takes the form

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_\pi \pi_t + \phi_y \hat{y}_t. \tag{1.54}$$

The parameter ρ_i is the interest rate smoothing parameter, which prevents the interest rate from becoming a jump variable, and reflects the relative weight placed on interest rate smoothing. If $\rho_i > 1$, this rule is known in the literature as the *superinertial* rule, and Woodford (2003, ch. 4) shows that a unique equilibrium fulfilling the Blanchard-Kahn conditions simply requires that one of the remaining coefficients is positive. The calibrated value of this parameter, usually lies between zero and one in the literature. I calibrate the rule using $\rho_i = 0.5$.

Forward Looking Taylor Rule

The rule that most central banks follow, *e.g.*, the Bank of England or Bank of Sweden, is the forward looking Taylor rule (FLTR), which targets both the expected future inflation and the expected output gap

$$\hat{\imath}_t = \varrho + \phi_\pi E_t \left[\pi_{t+1}\right] + \phi_y E_t \left[\hat{y}_{t+1}\right]. \tag{1.55}$$

As shown in the Section 1.8, rule (1.55) delivers higher welfare losses in response to a TFP shock than the Taylor Rule.

Optimal Simple Targeting Rules

Rather than calibration, the parameters of the monetary policy rules can be chosen in such a way that the welfare loss function is minimised. In the context of this paper, a simple rule is considered to be optimal if it delivers a locally unique equilibrium. Overall, there are many values of the parameters available, which can be considered as locally optimal. Since I use two different software packages to analyse the two approaches (*i.e.*, I use Dynare for the traditional linear-quadratic approach, and ACES for the Hamiltonian approach), the results may differ depending on the starting values. Therefore, it is necessary to fix the starting points in a way that the local area for both programmes is identical, and the resulting optimised parameters are comparable with each other. Additionally, similar to Schmitt-Grohé and Uribe (2007), I restrict the monetary rule parameters for output and inflation to the interval $[-5, 5]$, and the interest rate smooth parameter to the interval $[0, 1]$.[6]

[6] In fact, I extend the interval from Schmitt-Grohe and Uribe, who restrict their analysed policy rules to the interval $[-3, 3]$.

1.8 Results

In this section, I study the differences resulting from the welfare loss analysis of the traditional linear-quadratic approach and the Hamiltonian approach. In particular, I compare the welfare loss and the volatility of the endogenous variables included in the welfare loss function (1.50). The actual welfare loss is then measured in the form of a compensating differential from the steady state welfare. As discussed in Section 1.5.2, steady state welfare is independent of monetary policy. As the traditional linear-quadratic approach disregards any real or nominal distortion when computing the steady state, welfare always equals its maximum level, regardless of the share of backward looking firms ω. Conversely, the Hamiltonian approach entails that welfare varies with the parameter ω, since distortions are taken into account when looking at the steady state.

As Levine *et al.* (2007) emphasise, the welfare loss function resulting from the Hamiltonian approach is comparable with the one resulting from the traditional approach if certain conditions are fulfilled. In particular, they argue that "it should be stressed that this is only welfare based under the extreme restrictions that NKM has no capital, no habit, no indexing, has only one shock A_t and uses a separable utility [function]" (p. 24).

The model considered in this chapter fulfills all the above conditions, except for the assumption regarding price indexing. It is therefore appropriate to assume that the difference between these two approaches results from the different values of ω. As a matter of fact, when $\omega = 0$, the two approaches deliver identical dynamics for all endogenous variables. Table 1.4 reports the variances of inflation, change in inflation and output gap, and summarises the welfare losses obtained

Taylor rule	For linearised model				For Hamiltonian			
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
var(π)	0.7859	0.7387	0.6919	0.5513	0.7859	0.7606	0.6934	0.5569
var(Δπ)	0.0786	0.0392	0.0233	0.0074	0.0786	0.0380	0.0232	0.0073
var(x)	0.0154	0.0278	0.0570	0.3553	0.0154	0.0269	0.0566	0.2481
Welfare	-4.7991%	-4.6686%	-4.4547%	-3.7674%	-5.1125%	-4.9099%	-4.5298%	-3.7188%
PITR								
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
var(π)	0.1175	0.1094	0.1016	0.0789	0.1175	0.1015	0.1015	0.0763
var(Δπ)	0.0118	0.0059	0.0036	0.0012	0.0118	0.0053	0.0036	0.0011
var(x)	0.0023	0.0044	0.0096	0.0468	0.0023	0.0042	0.0095	0.0433
Welfare	-0.6966%	-0.6722%	-0.6377%	-0.5386%	-1.5123%	-1.4540%	-1.4184%	-1.2808%
ISR								
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
var(π)	0.1561	0.1486	0.1422	0.1235	0.1561	0.1544	0.1424	0.1251
var(Δπ)	0.0224	0.0120	0.0077	0.0029	0.0224	0.0118	0.0077	0.0029
var(x)	0.0072	0.0179	0.0359	0.1477	0.0072	0.0176	0.0358	0.1458
Welfare	-0.9288%	-0.9371%	-0.9330%	-0.9282%	-1.7214%	-1.7352%	-1.6461%	-1.5643%
FLTR								
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
var(π)	0.9773	0.8859	0.7963	0.5888	0.9773	0.9072	0.7979	0.6061
var(Δπ)	0.0977	0.0380	0.0201	0.0063	0.0977	0.0366	0.0201	0.0062
var(x)	0.0192	0.0164	0.0276	0.1797	0.0192	0.0157	0.0271	0.1753
Welfare	-6.0197%	-5.5892%	-5.0632%	-3.9077%	-6.1787%	-5.6796%	-5.0502%	-3.9111%

Table 1.4: Variances and welfare loss under simple policy rules.

using the calibrated model with simple monetary policy rules. The table shows that, when $\omega = 0$, the volatilities are equal for both approaches across all analysed rules. Furthermore, the welfare loss delivered by each simple monetary policy rule is always higher using the Hamiltonian approach. The reason is that the approximation of the welfare loss function in the traditional approach disregards some terms, including the terms independent of the policy.[7] The Hamiltonian numerical approximation in this case is more precise. A closer look on the welfare loss also suggests that the difference between the traditional and the Hamiltonian welfare loss decreases with the value of ω.

[7] For a detailed discussion, see Appendix 1.C.

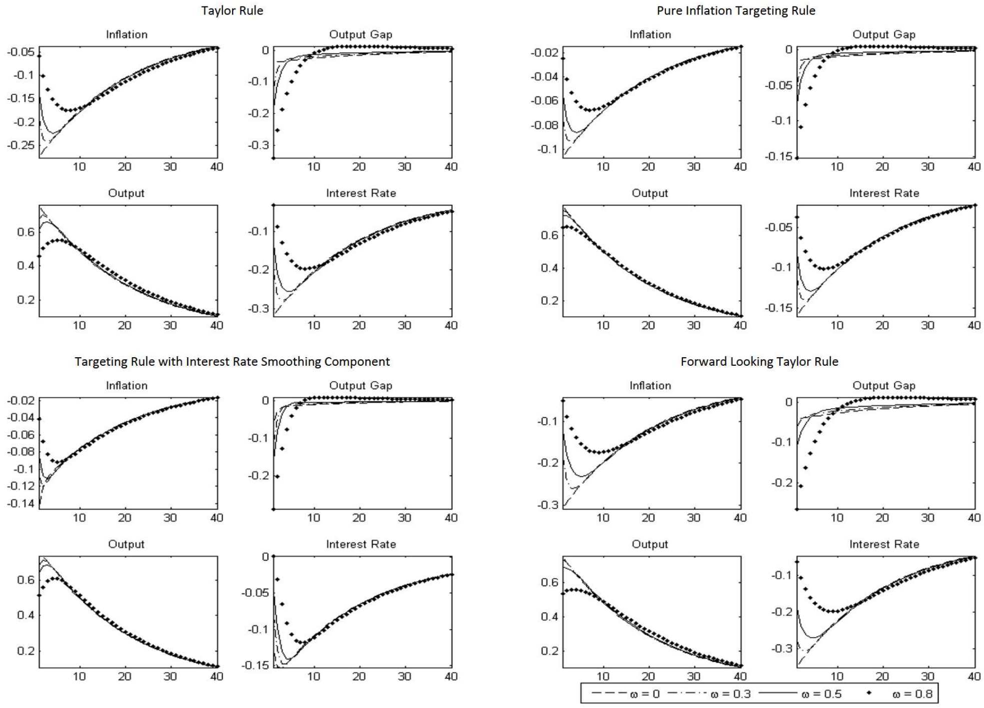

Figure 1.4: Impulse response functions under simple policy rules.

Using the Taylor rule, the welfare loss difference is more than 0.3 for $\omega = 0$. As the level of ω rise, the difference lowers, reaching a mere 0.05 when $\omega = 0.8$. The reason is that, although the inflation volatility increases marginally, the output gap variance is significantly lower using the Hamiltonian approach. Similar trends can be observed when using every simple rule.

Despite of the "horizontal" differences generated at different levels of the parameter ω, the welfare losses generated by the two approaches exhibit a clear pattern across the different rules. In the case of a TFP shock and under a common calibration, PITR performs best, closely followed by ISR. The Taylor rule and its forward looking equivalent perform alike, though producing a higher welfare loss. This corresponds to the results often discussed in the literature, according to which pure inflation targeting is the most appropriate simple rule.[8]

A visual representation of the impulse responses is given in Figure 1.4. Because of the similarities in the dynamic behavior generated by the two approaches, I only report the responses resulting from the traditional linear-quadratic approach.

The figure shows that a higher ω leads to a higher output gap volatility and lower inflation volatility, which require a weaker reaction from the central bank, in turn resulting in a lower volatility of the interest rate.

Using the optimal simple rules instead, the difference between the results generated by the two approaches is larger. Although variances are negligible and very close to zero for all the variables, and therefore the welfare loss generated by the traditional linear-quadratic approach converges zero, the Hamiltonian welfare loss is around 0.9 percent. Table 1.5 reports the variances and welfare losses for the optimal simple rules. For both approaches, welfare loss is very low and stable across the values of ω. The only exception is the optimised PITR that performs very badly in comparison to the other rules. This is due to the restricted value of parameter ϕ_π which is not high enough to achieve zero inflation volatility, similar to Schmitt-Grohé and Uribe (2007). In the case of no restrictions on ϕ_π, the optimal policy calls for setting the inflation coefficient ϕ_π to be very high. With a high enough value for this parameter, this rule performs similarly to the others resulting in negligible welfare losses.

Under the stated restrictions on ϕ_π, the optimal rules that target aggregate output perform better than PITR. For all these rules, the optimised parameter ϕ_y

[8] See, *e.g.*, Galí and Monacelli (2005).

Taylor rule	For linearised model				For Hamiltonian			
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
var(π)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
var(Δπ)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
var(x)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002
Welfare	0.0000%	0.0000%	0.0000%	0.0000%	-0.8979%	-0.8980%	-0.8993%	-0.9062%
PITR								
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
var(π)	0.0022	0.0021	0.0021	0.0018	0.0022	0.0020	0.0021	0.0018
var(Δπ)	0.0002	0.0001	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001
var(x)	0.0000	0.0002	0.0006	0.0039	0.0000	0.0003	0.0006	0.0039
Welfare	-0.0132%	-0.0133%	-0.0135%	-0.0153%	-0.9097%	-0.9103%	-0.9098%	-0.9144%
ISR								
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
var(π)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
var(Δπ)	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000
var(x)	0.0001	0.0002	0.0003	0.0006	0.0001	0.0005	0.0004	0.0007
Welfare	-0.0002%	-0.0002%	-0.0002%	-0.0003%	-0.8982%	-0.8982%	-0.8996%	-0.9064%
FLTR								
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
var(π)	0.0017	0.0000	0.0000	0.0000	0.0017	0.0001	0.0000	0.0000
var(Δπ)	0.0002	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000
var(x)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
Welfare	-0.0098%	0.0000%	0.0000%	0.0000%	-0.8968%	-0.8982%	-0.8993%	-0.9062%

Table 1.5: Variances and welfare loss under optimal policy rules.

is slightly negative. In the presence of a TFP shock, output increases one to one with its natural level so that the output gap is low and inflation decreases. When the central bank targets inflation, it decreases the interest rate in order to push inflation back to its equilibrium. A decrease in inflation causes a further increase in output. A positive and high coefficient on inflation ensures that inflation volatility is minimised. Therefore, should the inflation coefficient be allowed to take (unreasonably) high values, the PITR would perform better than any other rule.

It should be emphasised that targeting output with a positive coefficient ϕ_y pushes output back to its equilibrium and increases the output gap. As a result, output gap volatility in Table 1.4 and Figure 1.4 increases. Conversely, a negative coefficient on output implies that the output deviates less from its natural level, leading to a lower welfare loss. From the variance analysis, it follows that, as the share of backward looking firms increases, inflation variance decreases and output gap variance increases. Increasing the parameter ω causes higher inflation persistence, hence the central bank find it easier to control inflation volatility. However, this fact is offset by the higher volatility of output gap.

The optimised simple rule coefficients for both the linearised approach, using Dynare, and the Hamiltonian approach, using ACES, are shown in Table 1.6 of Appendix 1.D. The two software packages allow for different magnitude of deviations from the steady state (ACES allowing for the largest) and Dynare does not allow for parameter restriction (while ACES does). For these reasons, I chose for the linearised approach to use ACES results as the starting values. From Table 1.6, it follows that the two approaches deliver similar values for the optimal parameters. Because the TFP shock belongs to the group of efficient shocks, which can be offset by an appropriate policy of the social planner, the welfare loss values by optimal rules are practically zero if the coefficients are close to the reported values.[9]

1.9 Concluding Remarks

This chapter has investigated whether trend inflation might be the endogenous outcome of a deliberate choice by a social planner, who aims at minimising welfare losses around a non efficient steady state. In particular, I have founded my analysis

[9] Similarly, a preference shock can be considered an efficient shock. Conversely, the cost push shock belongs to the group of inefficient shocks and cannot be fully accommodated by the social planner. For further detail, see, *e.g.*,

on a model that considers backward looking as well as forward looking agents, and investigated the issue under a number of different monetary policy rules. I have first characterised the steady state of the economy in a deterministic environment, and then I have extended the model to consider a stochastic environment, where the source of uncertainty has been represented by a shock hitting firms' TFP. I have derived the welfare loss function around the steady state following two approaches: the traditional linear-quadratic approach and the Hamiltonian approach. The two approaches differ in that, while the former assumes an efficient steady state with zero inflation, the latter relaxes these assumptions allowing for nominal and real distortions in the steady state. Finally, I have compared the predictions of the resulting models, using a number of simple and optimal monetary policy rules.

Two innovative outcomes are worth noting. First, the welfare loss that a Hamiltonian social planner obtains in the steady state depends on the level of inflation. In particular, a positive (and sufficiently low) inflation may be beneficial for the economy, as the negative effect due to a rise in the price dispersion may be more than offset by the positive effect that nominal rigidities generate on labour supply. Second, the share of backward looking firms has an impact on the differential in welfare losses that obtains using the two approaches (with social welfare, by construction, always higher under the traditional approach). In particular, the higher this share, the narrower the difference between the predicted welfare levels. The reason is that the Hamiltonian planner is able to exploit the behaviour of the backward looking agents, and can thus increase inflation to improve welfare. This is an option that is not available under the traditional linear-quadratic approach.

The analysis has been conducted with reference to a stochastic environment associated with a single shock to TFP. This is chosen to allow for a direct comparison between the welfare losses generated by a generalised version of the traditional linear-quadratic approach and by the alternative Hamiltonian approach, which is not feasible when more sources of uncertainty are considered. An obvious extension of the model would consider a cost push shock as an alternative to the stochastic TFP, as suggested by a strand of the literature (see, *e.g.*, Clarida, Galí and Gertler, 1999; and Steinsson, 2003). Another interesting extension, particularly when considering the findings by Bhattarai, Eggertsson and Schoenle (2014), would consider a demand shock. A preliminary study of this case, however, leads to results very similar to those discussed in this chapter, with the exception that inflation inertia appears to have a relatively higher impact on the dynamics of the endogenous variables. More generally, I believe that investigating monetary policy under different sources of uncertainty in the presence of a non-zero steady state inflation in presence of inflation inertia might represent a promising topic for further research.

Finally, the study conducted in this chapter has only considered a closed economy. Being a closed economy model, my framework mainly lends itself to the analysis of any country that could be considered as a large economy (in isolation or with trading partners regarded as small economies) in some particular context. One example of this model's application is featured in Chapter 2, where I consider Germany as a large economy and a number of Eastern European countries as small economies. My contribution there focuses on the empirical performance of the different rules. I leave the theoretical study of these issues to future investigation.

1.A Consumer Price Index

This appendix describes the derivation of household demand for individual goods, and the derivation of the price index. Firstly, I assume that the index of consumption goods takes the familiar Dixit–Stiglitz form

$$C_t = \left(\int_0^1 \left[C_t(i) \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{1.56}$$

The representative household maximises the composite consumption C_t by choosing $C_t(i)$, subject to the budget constraint

$$\int_0^1 P_t(i)\, C_t(i) di = Z_t, \tag{1.57}$$

taking the available amount of resources Z_t, which equals $P_t C_t$ in (1.57), as given.

I can write the Lagrangian

$$L_C = \left(\int_0^1 \left[C_t(i) \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} - \Lambda_t \left(\int_0^1 P_t(i)\, C_t(i) di - Z_t \right). \tag{1.58}$$

The first-order conditions are

$$\frac{\partial L}{\partial C_t(i)} = \frac{\varepsilon}{\varepsilon-1} \left(\int_0^1 \left[C_t(i) \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}-1} \frac{\varepsilon-1}{\varepsilon} \left[C_t(i) \right]^{\frac{\varepsilon-1}{\varepsilon}-1} - \Lambda_t P_t(i) = 0 \tag{1.59}$$

$$\frac{\partial L}{\partial \Lambda_t} = \int_0^1 P_t(i)\, C_t(i) di - Z_t = 0. \tag{1.60}$$

I can eliminate the Lagrangian multiplier. From condition (1.59) I obtain

$$\Lambda_t P_t(i) = \left(\int_0^1 [C_t(i)]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}-1} [C_t(i)]^{\frac{\varepsilon-1}{\varepsilon}-1}. \tag{1.61}$$

To get the value of Λ_t, multiply both sides by $C_t(i)$ and integrate over goods

$$\Lambda_t P_t(i) C_t(i) = \left[\int_0^1 (C_t(i))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}-1} (C_t(i))^{\frac{\varepsilon-1}{\varepsilon}},$$

resulting in

$$\Lambda_t = \frac{\left[\int_0^1 (C_t(i))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}}{\int_0^1 P_t(i) C_t(i) di} = \frac{C_t}{Z_t} \equiv \frac{1}{P_t}.$$

Demand for Consumption Goods From (1.61), I also get the demand function for each good i

$$\frac{P_t(i)}{P_t} = \left[\int_0^1 (C_t(i))^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{1}{\varepsilon-1}} (C_t(i))^{-\frac{1}{\varepsilon}} = (C_t C_t(i))^{-\frac{1}{\varepsilon}},$$

which results to the demand for an individual consumption good (1.10)

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t.$$

Price Index By replacing $C_t(i)$ with the last expression in (1.56), I obtain an expression for the price index as a function of each good's price

$$C_t^{\frac{\varepsilon-1}{\varepsilon}} = C_t^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{1}{P_t}\right)^{1-\varepsilon} \int_0^1 P_t(i)^{1-\varepsilon} \, di,$$

which rearranging leads to a price index

$$P_t = \left[\int_0^1 (P_t(i))^{1-\varepsilon} \, di\right]^{\frac{1}{1-\varepsilon}}. \tag{1.62}$$

1.B Phillips Curve

The log-linearisation of the equations (1.16), (1.17) and (1.18) straightforwardly leads to

$$\hat{p}_t^f = \hat{\jmath}_t - \hat{h}_t$$

with

$$\hat{\jmath}_t = (1-\alpha\beta)\left((1-\sigma)\,\hat{y}_t + \widehat{mc}_t\right) + \alpha\beta\left(\varepsilon\pi_{t+1} + \hat{\jmath}_{t+1}\right)$$

and

$$\hat{h}_t = (1-\alpha\beta)\left((1-\sigma)\,\hat{y}_t\right) + \alpha\beta\left((\varepsilon-1)\,\pi_{t+1} + \hat{h}_{t+1}\right),$$

from which I can obtain the forward looking price in log-linear terms

$$\hat{p}_t^f = (1-\alpha\beta)\,\widehat{mc}_t + \alpha\beta\pi_{t+1} + \alpha\beta\hat{p}_{t+1}^f. \tag{1.63}$$

The log-linearisation of (1.20), (1.22) and (1.25) yields

$$\hat{p}_t^b = \hat{x}_{t-1} + \pi_{t-1} - \pi_t, \tag{1.64}$$

together with

$$\hat{x}_t = (1-\omega)\,\hat{p}_t^f + \omega \hat{p}_t^b, \tag{1.65}$$

and

$$\hat{x}_t = \frac{\alpha}{1-\alpha}\pi_t. \tag{1.66}$$

Using last equation with (1.65) together with (1.64) and (1.63) and simplifying delivers the hybrid NKPC

$$\pi_t = \frac{\alpha\beta}{\alpha + \omega\,(1-\alpha\,(1-\beta))}\pi_{t+1} + \frac{\omega}{\alpha + \omega\,(1-\alpha\,(1-\beta))}\pi_{t-1} + \frac{(1-\omega)\,(1-\alpha)\,(1-\alpha\beta)}{\alpha + \omega\,(1-\alpha\,(1-\beta))}\widehat{mc}_t.$$

1.C Central Bank Loss Function

In this appendix I derive the central bank's objective function. The welfare criterion is the stream of expected representative household's utility, which the central bank is willing to maximise

$$\Omega = E_0\left[\sum_{t=0}^{\infty}\beta^t U_t\right]$$

with

$$U_t = \frac{Y_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta},$$

where in the second expression I use the fact that $Y_t = C_t$. The second-order Taylor series approximation of utility function yields[10]

$$U\left(C_t, N_t\right) \approx \frac{\bar{Y}^{1-\sigma}}{1-\sigma} - \frac{\bar{N}^{1+\eta}}{1+\eta} + \bar{Y}^{1-\sigma}\frac{Y_t - \bar{Y}}{\bar{Y}} - \bar{N}^{1+\eta}\frac{N_t - \bar{N}}{\bar{N}} + \frac{1}{2}\left[-\sigma\bar{Y}^{1-\sigma}\left(\frac{Y_t - \bar{Y}}{\bar{Y}}\right)^2 - \eta\bar{N}^{1+\eta}\left(\frac{N_t - \bar{N}}{\bar{N}}\right)^2\right].$$

So that the previous expression takes the following form

$$U\left(C_t, N_t\right) \approx \frac{\bar{Y}^{1-\sigma}}{1-\sigma} - \frac{\bar{N}^{1+\eta}}{1+\eta} + \bar{Y}^{1-\sigma}\left(\tilde{y}_t + \frac{1}{2}\tilde{y}_t^2\right) - \bar{N}^{1+\eta}\left(\tilde{n}_t + \frac{1}{2}\tilde{n}_t^2\right) - \frac{\sigma}{2}\bar{Y}^{1-\sigma}\tilde{y}_t^2 - \frac{\eta}{2}\bar{N}^{1+\eta}\tilde{n}_t^2$$

and thus

$$U_t - \bar{U} \approx \bar{Y}^{1-\sigma}\left(\tilde{y}_t + \frac{1}{2}\tilde{y}_t^2\right) - \bar{N}^{1+\eta}\left(\tilde{n}_t + \frac{1}{2}\tilde{n}_t^2\right) + o\left(\left\|a^3\right\|\right),$$

where $\bar{U} = \frac{\bar{Y}^{1-\sigma}}{1-\sigma} - \frac{\bar{N}^{1+\eta}}{1+\eta}$ and $o\left(\|a^n\|\right)$ represents terms of order n or higher.

Recall from Section (1.4.2), equation (1.26) that

$$N_t = \frac{Y_t}{A_t}Z_t,$$

[10] The formula for second-order Taylor approximation is given by

$$f\left(x_t, y_t\right) \approx f\left(X, Y\right) + f_x\left(X, Y\right)\left(x_t - X\right) + f_y\left(X, Y\right)\left(y_t - Y\right) + + f_{xy}\left(X, Y\right)\left(x_t - X\right)\left(y_t - Y\right) + \frac{1}{2}\left[\begin{array}{c} f_{xx}\left(X, Y\right)\left(x_t - X\right)^2 + \\ + f_{yy}\left(X, Y\right)\left(y_t - Y\right)^2 \end{array}\right]$$

log-linearised as

$$\hat{n}_t = \hat{y}_t + z_t,$$

where $z_t \equiv \log \int_0^1 (P_t(i)/P_t)^{-\varepsilon} di$, $\tilde{a}_t = \mathring{z}_t = 0$. Hence, isolating the terms independent of policy

$$U_t - \bar{U} \approx \bar{Y}^{1-\sigma}\left(\tilde{y}_t + \frac{1-\sigma}{2}\tilde{y}_t^2\right) - \bar{N}^{1+\eta}\left(\tilde{y}_t + z_t + \frac{1+\eta}{2}\tilde{y}_t^2\right) + t.i.p. + o\left(\left\|a^3\right\|\right),$$

where $t.i.p.$ abbreviates "terms independent of policy". Using the facts that, in a fully efficient equilibrium, the marginal product of labour $MPN = \frac{Y}{N}$ equals the marginal rate of substitution between consumption and labour, *e.g.*, $MRS_{C,N} = \frac{U_N}{U_C} = \frac{N^\eta}{Y^{-\sigma}}$, I can write

$$\begin{aligned} U_t - \bar{U} &\approx \bar{Y}^{1-\sigma}\left(\frac{1-\sigma}{2}\hat{y}_t^2 - z_t - \frac{1+\eta}{2}\hat{y}_t^2\right) + t.i.p. + o\left(\left\|a^3\right\|\right) \\ &= -\bar{Y}^{1-\sigma}\left(z_t + \frac{\eta+\sigma}{2}\hat{y}_t^2\right) + t.i.p. + o\left(\left\|a^3\right\|\right). \end{aligned}$$

Recall that I have used the fact that z_t^2 is higher than second order, which is shown in what follows.

Let $\dot{p}_t(i) \equiv \log P_t(i)/P_t$, $\dot{p}(i) \equiv \log \bar{P}(i)/\bar{P} = 0$. Consider that

$$(P_t(i)/P_t)^{1-\varepsilon} = \exp\left[(1-\varepsilon)\dot{p}_t(i)\right]$$

than

$$\begin{aligned} (P_t(i)/P_t)^{1-\varepsilon} &= \exp\left[(1-\varepsilon)\dot{p}(i)\right] + (1-\varepsilon)\exp\left[(1-\varepsilon)\dot{p}(i)\right]\left[\dot{p}_t(i) - \dot{p}(i)\right] \\ &\quad + (1-\varepsilon)^2\exp\left[(1-\varepsilon)\dot{p}(i)\right]\left[\dot{p}_t(i) - \dot{p}(i)\right]^2/2 + o\left(\left\|a^3\right\|\right) \end{aligned}$$

$$\left(P_t\left(i\right)/P_t\right)^{1-\varepsilon} = 1 + \left(1-\varepsilon\right)\dot{p}_t\left(i\right) + \left(1-\varepsilon\right)^2\left[\dot{p}_t\left(i\right)\right]^2/2 + o\left(\left\|a^3\right\|\right).$$

Integrate over i delivers

$$\frac{1}{\left(P_t\right)^{1-\varepsilon}}\int_0^1 \left(P_t\left(i\right)\right)^{1-\varepsilon} di = 1 + \left(1-\varepsilon\right)\int_0^1 \dot{p}_t\left(i\right) di + \frac{\left(1-\varepsilon\right)^2}{2}\int_0^1 \left[\dot{p}_t\left(i\right)\right]^2 di.$$

Notice that, from the definition of price index, the LHS equals one, *e.g.*, $\frac{1}{\left(P_t\right)^{1-\varepsilon}}\int_0^1 \left(P_t\left(i\right)\right)^{1-\varepsilon} di = 1$. Thus it holds

$$\int_0^1 \dot{p}_t\left(i\right) di = \frac{\varepsilon-1}{2}\int_0^1 \left[\dot{p}_t\left(i\right)\right]^2 di.$$

Furthermore, notice that, as firms are uniformly distributed between zero and one, I can write

$$E_i\left[\dot{p}_t\left(i\right)\right] = \frac{\varepsilon-1}{2}E_i\left[\left(\dot{p}_t\left(i\right)\right)^2\right].$$

Given that

$$\begin{aligned} E_i\left[\left(\dot{p}_t\left(i\right)\right)^2\right] &= E_i\left[\left(p_t\left(i\right)-p_t\right)^2\right] \\ &= E_i\left[\left(p_t\left(i\right)-E_i\left(p_t\left(i\right)\right)\right)^2\right] \\ &= var_i\left\{p_t\left(i\right)\right\}. \qquad (1.67) \end{aligned}$$

I obtain

$$E_i\left[\left(\frac{P_t\left(i\right)}{\left(P_t\right)}\right)^{1-\varepsilon}\right] = 1,$$

which implies that

$$E_i\left[(p_t(i))^{1-\varepsilon}\right] = p_t^{1-\varepsilon}.$$

Note also that

$$(P_t(i)/P_t)^{-\varepsilon} = 1 - \varepsilon \dot{p}_t(i) + \varepsilon^2 [\dot{p}_t(i)]^2/2 + o\left(\|a^3\|\right).$$

Integrating over i and using the result of equation (1.67)

$$\begin{aligned}
\int_0^1 (P_t(i)/P_t)^{-\varepsilon} di &= 1 - \varepsilon \int_0^1 \dot{p}_t(i)\, di + \frac{\varepsilon^2}{2}\int_0^1 [\dot{p}_t(i)]^2 di + o\left(\|a^3\|\right) \\
&= 1 - \varepsilon E_i[\dot{p}_t(i)] + \frac{\varepsilon^2}{2} E_i\left\{[\dot{p}_t(i)]^2\right\} + o\left(\|a^3\|\right) \\
&= 1 - \varepsilon\frac{\varepsilon-1}{2} E_i\left\{[\dot{p}_t(i)]^2\right\} + \frac{\varepsilon^2}{2} E_i\left\{[\dot{p}_t(i)]^2\right\} + o\left(\|a^3\|\right) \\
&= 1 + \frac{\varepsilon}{2} E_i\left\{[\dot{p}_t(i)]^2\right\} + o\left(\|a^3\|\right) \\
&= 1 + \frac{\varepsilon}{2} var_i[p_t(i)] + o\left(\|a^3\|\right),
\end{aligned}$$

which finally results in

$$z_t = \log \int_0^1 (P_t(i)/P_t)^{-\varepsilon} di = \frac{\varepsilon}{2} var_i[p_t(i)] + o\left(\|a^3\|\right).$$

I have assumed that a fraction $1-\alpha$ of the households in the economy are able to change their prices in each period. Consequently, the distribution of prices, $\{p_t(i)\}$, at time t consists of α times the distribution of prices at time $t-1$; plus two atoms of size $(1-\alpha)(1-\omega)$ and $(1-\alpha)\omega$ at the two new prices, p^f and p^b, respectively. Define

$$p_t \equiv E_i[p_t(i)] \text{ and } \Delta_t \equiv var_i[p_t(i)],$$

than using the recursive characterisation of the distribution of prices, I can write

$$p_t = \alpha E_i \left[p_{t-1}(i)\right] + (1-\alpha)(1-\omega) E_i p_t^f + (1-\alpha)\omega E_i p_t^b.$$

Taking the one-lag time difference yields

$$\begin{aligned} p_t - p_{t-1} &= \alpha E_i \left[p_{t-1}(i) - p_{t-1}\right] + (1-\alpha)(1-\omega)\left(p_t^f - p_{t-1}\right) \\ &\quad + (1-\alpha)\omega\left(p_t^b - p_{t-1}\right) \\ &= (1-\alpha)(1-\omega)\left(p_t^f - p_{t-1}\right) + (1-\alpha)\omega\left(p_t^b - p_{t-1}\right) \\ &= (1-\alpha)(x_t - p_{t-1}) \end{aligned}$$

and

$$x_t - p_{t-1} = \frac{p_t - p_{t-1}}{1-\alpha} = \frac{\pi_t}{1-\alpha}.$$

Similarly

$$\begin{aligned} \Delta_t &\equiv var_i\left[p_t(i)\right] = var_i\left[p_t(i) - \mu_{t-1}\right] \\ &= E_i\left\{\left[p_t(i) - \mu_{t-1}\right]^2\right\} - \left[E_i p_t(i) - \mu_{t-1}\right]^2. \end{aligned}$$

Again, using the recursive characterisation of the distribution of prices I can write

$$\begin{aligned} E_i\left\{\left[p_t(i) - p_{t-1}\right]^2\right\} &= \alpha E_i\left\{\left[p_{t-1}(i) - p_{t-1}\right]^2\right\} \\ &\quad + (1-\alpha)(1-\omega)\left(p_t^f - p_{t-1}\right)^2 + (1-\alpha)\omega\left(p_t^b - p_{t-1}\right)^2. \end{aligned}$$

Noting that

$$x_t = (1-\omega) p_t^f + \omega p_t^b$$

and

$$p_t^b = x_{t-1} + \pi_{t-1},$$

I can further state that

$$\begin{aligned} p_t^b - p_{t-1} &= x_{t-1} + \pi_{t-1} - p_{t-1} \\ &= x_{t-1} + \pi_{t-1} - p_{t-2} - (p_{t-1} - p_{t-2}) \\ &= x_{t-1} + \pi_{t-1} - p_{t-2} - \pi_{t-1} \\ &= x_{t-1} - p_{t-2} \end{aligned}$$

and that

$$\begin{aligned} p_t^f - p_{t-1} &= \frac{x_t}{1-\omega} - \frac{\omega p_t^b}{1-\omega} - p_{t-1} \\ &= \frac{x_t - p_{t-1}}{1-\omega} - \frac{\omega\left(p_t^b - p_{t-1}\right)}{1-\omega} \\ &= \frac{x_t - p_{t-1}}{1-\omega} - \frac{\omega\left(x_{t-1} - p_{t-2}\right)}{1-\omega}. \end{aligned}$$

Hence

$$\Delta_t = \alpha E_i\left\{\left[p_{t-1}(i) - p_{t-1}\right]^2\right\} + (1-\alpha)(1-\omega)\left(\frac{x_t - p_{t-1}}{1-\omega} - \frac{\omega\left(x_{t-1} - p_{t-2}\right)}{1-\omega}\right)^2$$
$$+ (1-\alpha)\,\omega\left(x_{t-1} - p_{t-2}\right)^2 - \left[E_i p_t(i) - p_{t-1}\right]^2$$

$$\begin{aligned} \Delta_t \;=\; & \alpha\Delta_{t-1} + (1-\alpha)\left(\frac{\left(x_t - p_{t-1}\right)^2}{1-\omega} + \frac{\omega^2\left(x_{t-1} - p_{t-2}\right)^2}{1-\omega}\right. \\ & \left. - \frac{2\omega\left(x_t - p_{t-1}\right)\left(x_{t-1} - p_{t-2}\right)}{1-\omega}\right) + (1-\alpha)\,\omega\left(x_{t-1} - p_{t-2}\right)^2 - \left[p_t - p_{t-1}\right]^2 \end{aligned}$$

$$\Delta_t = \alpha\Delta_{t-1} + (1-\alpha)\left(\frac{\pi_t^2}{(1-\alpha)^2(1-\omega)} + \frac{\omega^2\pi_{t-1}^2}{(1-\alpha)^2(1-\omega)} - \frac{2\omega\pi_t\pi_{t-1}}{(1-\alpha)^2(1-\omega)}\right) + \frac{(1-\alpha)\,\omega\pi_{t-1}^2}{(1-\alpha)^2} - \pi_t^2$$

$$\Delta_t = \alpha\Delta_{t-1} + \frac{\pi_t^2}{(1-\alpha)(1-\omega)} - \frac{2\omega\pi_t\pi_{t-1}}{(1-\alpha)(1-\omega)} + \frac{\omega^2\pi_{t-1}^2}{(1-\alpha)(1-\omega)} + \frac{\omega\pi_{t-1}^2}{1-\alpha} - \pi_t^2$$

$$\Delta_t = \alpha\Delta_{t-1} + \frac{\pi_t^2}{(1-\alpha)(1-\omega)} - \frac{2\omega\pi_t\pi_{t-1}}{(1-\alpha)(1-\omega)} + \frac{\omega^2\pi_{t-1}^2 + (1-\omega)\,\omega\pi_{t-1}^2}{(1-\alpha)(1-\omega)} - \pi_t^2$$

Note that

$$\omega^2\pi_{t-1}^2 + (1-\omega)\,\omega\pi_{t-1}^2 = \omega^2\pi_{t-1}^2 + \omega\pi_{t-1}^2 - \omega^2\pi_{t-1}^2 = \omega\pi_{t-1}^2,$$

thus

$$\Delta_t = \alpha\Delta_{t-1} + \frac{\pi_t^2}{(1-\alpha)(1-\omega)} - \frac{2\omega\pi_t\pi_{t-1}}{(1-\alpha)(1-\omega)} + \frac{\omega\pi_{t-1}^2}{(1-\alpha)(1-\omega)} - \pi_t^2.$$

Notice that

$$\frac{\pi_t^2}{(1-\alpha)(1-\omega)} = \frac{\pi_t^2}{1-\alpha} + \frac{\omega\pi_t^2}{(1-\alpha)(1-\omega)}.$$

Hence

$$\Delta_t = \alpha\Delta_{t-1} + \frac{\pi_t^2}{1-\alpha} + \frac{\omega\left(\pi_t^2 - 2\pi_t\pi_{t-1} + \pi_{t-1}^2\right)}{(1-\alpha)(1-\omega)} - \pi_t^2$$

and

$$\Delta_t = \alpha\Delta_{t-1} + \frac{\alpha}{1-\alpha}\pi_t^2 + \frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_t^2.$$

Solving this expression forward

$$\Delta_{t-1} = \alpha\Delta_{t-2} + \frac{\alpha}{(1-\alpha)}\pi_{t-1}^2 + \frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_{t-1}^2$$
$$\Delta_{t-2} = \alpha\Delta_{t-3} + \frac{\alpha}{(1-\alpha)}\pi_{t-2}^2 + \frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_{t-2}^2$$

and replacing into the first expression delivers

$$\Delta_t = \alpha^2\left(\alpha\Delta_{t-3} + \frac{\alpha}{(1-\alpha)}\pi_{t-2}^2 + \frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_{t-2}^2\right) + \alpha\left(\frac{\alpha}{(1-\alpha)}\pi_{t-1}^2 + \frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_{t-1}^2\right) + \frac{\alpha}{(1-\alpha)}\pi_t^2 + \frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_t^2$$

and

$$\Delta_t = \alpha^3\Delta_{t-3} + \frac{\alpha}{(1-\alpha)}\left[\pi_t^2 + \alpha\pi_{t-1}^2 + \alpha^2\pi_{t-2}^2\right] + \frac{\omega}{(1-\alpha)(1-\omega)}\left[\Delta\pi_t^2 + \alpha\Delta\pi_{t-1}^2 + \alpha^2\Delta\pi_{t-2}^2\right].$$

Generalising to period $t = -1$

$$\Delta_t = \alpha^{t+1}\Delta_{-1} + \frac{\alpha}{(1-\alpha)}\sum_{s=0}^{t}\alpha^s\pi_{t-s}^2 + \frac{\omega}{(1-\alpha)(1-\omega)}\sum_{s=0}^{t}\alpha^s\Delta\pi_{t-s}^2.$$

Simplifying, and reverting the index

$$\Delta_t = \alpha^{t+1}\Delta_{-1} + \sum_{s=0}^{t} \alpha^{t-s} \left(\frac{\alpha}{(1-\alpha)} \pi_s^2 + \frac{\omega}{(1-\alpha)(1-\omega)} \Delta\pi_s^2 \right),$$

thus

$$\Delta_t = \alpha^t \left[\alpha\Delta_{-1} + \sum_{s=0}^{t} \alpha^{-s} \left(\frac{\alpha}{(1-\alpha)} \pi_s^2 + \frac{\omega}{(1-\alpha)(1-\omega)} \Delta\pi_s^2 \right) \right].$$

Therefore, I have

$$\begin{aligned} z_t &= \frac{\varepsilon}{2} var_i \left[p_t(i) \right] + o\left(\left\| a^3 \right\| \right) \\ &= \frac{\varepsilon}{2} \Delta_t + o\left(\left\| a^3 \right\| \right) \\ &= \frac{\varepsilon}{2} \alpha^t \left[\alpha\Delta_{-1} + \sum_{s=0}^{t} \alpha^{-s} \left(\frac{\alpha}{(1-\alpha)} \pi_s^2 + \frac{\omega}{(1-\alpha)(1-\omega)} \Delta\pi_s^2 \right) \right] + o\left(\left\| a^3 \right\| \right) \end{aligned}$$

and

$$\begin{aligned} U_t - \bar{U} = -\frac{\bar{Y}^{1-\sigma}}{2} \Bigg\{ \varepsilon\alpha^t \Bigg[\alpha\Delta_{-1} + \textstyle\sum_{s=0}^{t} \alpha^{-s} \left(\frac{\alpha}{(1-\alpha)} \pi_s^2 \right. \\ \left. + \frac{\omega}{(1-\alpha)(1-\omega)} \Delta\pi_s^2 \right) \Bigg] + (\eta+\sigma)\tilde{y}_t^2 \Bigg\} + t.i.p. + o\left(\left\| a^3 \right\| \right). \end{aligned}$$

Finally, I get

$$\begin{aligned} \Omega_0 = -\varepsilon \frac{\bar{Y}^{1-\sigma}}{2} E \sum_{t=0}^{\infty} (\alpha\beta)^t \left[\alpha\Delta_{-1} + \sum_{s=0}^{t} \alpha^{-s} \left(\frac{\alpha}{(1-\alpha)} \pi_s^2 + \frac{\omega}{(1-\alpha)(1-\omega)} \Delta\pi_s^2 \right) \right] \\ - \frac{\bar{Y}^{1-\sigma}}{2} E \sum_{t=0}^{\infty} \beta^t (\eta+\sigma)\tilde{y}_t^2 + t.i.p. + o\left(\left\| a^3 \right\| \right). \end{aligned}$$

The first present value on the RHS of this equation is

$$\sum_{t=0}^{\infty}\beta^t\Delta_t=\sum_{t=0}^{\infty}(\alpha\beta)^t\left[\alpha\Delta_{-1}+\sum_{s=0}^{t}\alpha^{-s}\left(\frac{\alpha}{(1-\alpha)}\pi_s^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_s^2\right)\right].$$

Following Steinsson, assume that Δ_{-1} is a term independent on policy

$$\sum_{t=0}^{\infty}\beta^t\Delta_t=\sum_{t=0}^{\infty}\left[(\alpha\beta)^t\sum_{s=0}^{t}\alpha^{-s}\left(\frac{\alpha}{(1-\alpha)}\pi_s^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_s^2\right)\right]+t.i.p.$$

The first the on the RHS of this equation is, for $t=0$

$$\frac{\alpha}{(1-\alpha)}\pi_0^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_0^2,$$

for $t=1$

$$\begin{aligned}(1+\alpha\beta)&\left(\frac{\alpha}{(1-\alpha)}\pi_0^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_0^2\right)\\&+\beta\left(\frac{\alpha}{(1-\alpha)}\pi_1^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_1^2\right),\end{aligned}$$

for $t=2$

$$\begin{aligned}\left(1+\alpha\beta+\alpha^2\beta^2\right)&\left(\frac{\alpha}{(1-\alpha)}\pi_0^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_0^2\right)\\&+\left(\beta+\alpha\beta^2\right)\left(\frac{\alpha}{(1-\alpha)}\pi_1^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_1^2\right)\\&+\beta^2\left(\frac{\alpha}{(1-\alpha)}\pi_2^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_2^2\right),\end{aligned}$$

for $t=3$

$$\begin{aligned}
&\left(1+\alpha\beta+\alpha^2\beta^2+\alpha^3\beta^3\right)\left(\frac{\alpha}{(1-\alpha)}\pi_0^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_0^2\right)\\
&\quad+\left(\beta+\alpha\beta^2+\alpha^2\beta^3\right)\left(\frac{\alpha}{(1-\alpha)}\pi_1^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_1^2\right)\\
&\quad\quad+\left(\beta^2+\alpha\beta^3\right)\left(\frac{\alpha}{(1-\alpha)}\pi_2^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_2^2\right)\\
&\quad\quad\quad+\beta^3\left(\frac{\alpha}{(1-\alpha)}\pi_3^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_3^2\right).
\end{aligned}$$

Generalising

$$\begin{aligned}
&\sum_{t=0}^{T}(\alpha\beta)^t\left(\frac{\alpha}{(1-\alpha)}\pi_0^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_0^2\right)\\
&\quad+\sum_{t=0}^{T-1}(\alpha\beta)^t\beta\left(\frac{\alpha}{(1-\alpha)}\pi_1^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_1^2\right)\\
&\quad+\sum_{t=0}^{T-2}(\alpha\beta)^t\beta^2\left(\frac{\alpha}{(1-\alpha)}\pi_2^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_2^2\right)\\
&\quad\quad+\sum_{t=0}^{T-3}(\alpha\beta)^t\beta^3\left(\frac{\alpha}{(1-\alpha)}\pi_3^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_3^2\right)+...
\end{aligned}$$

For $T\rightarrow\infty$ (and taking out from the sums the terms constant with respect to the index t)

$$\left(\frac{\alpha}{(1-\alpha)}\pi_0^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_0^2\right)\sum_{t=0}^{\infty}(\alpha\beta)^t$$
$$+\beta\left(\frac{\alpha}{(1-\alpha)}\pi_1^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_1^2\right)\sum_{t=0}^{\infty}(\alpha\beta)^t$$
$$+\beta^2\left(\frac{\alpha}{(1-\alpha)}\pi_2^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_2^2\right)\sum_{t=0}^{\infty}(\alpha\beta)^t$$
$$+\beta^3\left(\frac{\alpha}{(1-\alpha)}\pi_3^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_3^2\right)\sum_{t=0}^{\infty}(\alpha\beta)^t+...$$

Considering that $\sum_{t=0}^{\infty}(\alpha\beta)^t=(1-\alpha\beta)^{-1}$, and generalising

$$\sum_{t=0}^{\infty}\beta^t\Delta_t=\frac{1}{1-\alpha\beta}\sum_{t=0}^{\infty}\beta^t\left(\frac{\alpha}{(1-\alpha)}\pi_t^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_t^2\right)+t.i.p.$$

Plugging this back into the welfare loss function yields

$$\Omega_0=-\frac{\bar{Y}^{1-\sigma}}{2}\sum_{t=0}^{\infty}\beta^t\frac{\varepsilon}{1-\alpha\beta}\left(\frac{\alpha}{(1-\alpha)}\pi_t^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_t^2\right)$$
$$-\frac{\bar{Y}^{1-\sigma}}{2}\sum_{t=0}^{\infty}\beta^t(\eta+\sigma)\tilde{y}_t^2+t.i.p.+o\left(\left\|a^3\right\|\right)$$

which can be rewritten as

$$\Omega_0=-\frac{\bar{Y}^{1-\sigma}}{2}\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varepsilon}{1-\alpha\beta}\left(\frac{\alpha}{(1-\alpha)}\pi_t^2+\frac{\omega}{(1-\alpha)(1-\omega)}\Delta\pi_t^2\right)+(\eta+\sigma)\tilde{y}_t^2\right\}$$
$$+t.i.p.+o\left(\left\|a^3\right\|\right).$$

Assuming that all firms are forward looking only, *e.g.*, $\omega=0$, I get a policy maker's objective function without backward looking component, which is identical with those from Woodford (2003). More generally it holds that

$$\Omega_0 = -\Theta E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda_\Delta \Delta \pi_t^2 + \lambda_y \tilde{y}_t^2 \right) + t.i.p. + o\left(\left\| a^3 \right\| \right), \tag{1.68}$$

where $\Theta = \frac{\bar{Y}^{1-\sigma}}{2} \frac{\varepsilon}{1-\alpha\beta} \frac{\alpha}{(1-\alpha)}$, $\lambda_y = \frac{1-\alpha\beta}{\varepsilon} \frac{1-\alpha}{\alpha} (\eta + \sigma)$ and $\lambda_\Delta = \frac{\omega}{\alpha(1-\omega)}$ are functions of the structural parameters of the model.

1.D Optimal Rules Coefficients

This appendix compares the optimised simple rules coefficients resulting from the traditional linearised approach and Hamiltonian approach, given various monetary policy rules and levels of parameter omega. The estimated optimal coefficients differ for two reasons: different methodologies and different algorithms.

The results in Table 1.6 for the linearised approach are obtained using the ACES results as the starting values. The reason is that the two software packages allow for different magnitude of deviations from the steady state (ACES allowing for the largest) and Dynare does not allow for parameter restriction (while ACES does), hence using the ACES results are stating values provides parameters estimates for the linearised approach that are comparable to those obtained by the Hamiltonian approach. In other words, using Dynare for the linear approach leads to very small (nearly zero) welfare losses in many local minima, with the end values always looking very close to the starting ones.

As Table 1.6 illustrates, there is virtually no difference between the two approaches when one considers optimal rules, at least as far as TFP shocks are concerned. Even when differences seem to arise (*i.e.*, when $\omega = 0.8$, in the optimal

TR optimal	Linearised				Hamiltonian			
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
ϕπ	1.014	5.000	5.000	4.463	1.014	5.000	5.000	5.000
ϕy	-0.075	-0.075	-0.075	-0.0750002	-0.075	-0.075	-0.075	-0.063
PITR optimal								
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
ϕπ	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000
ISR optimal								
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
ϕπ	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000
ϕy	-0.037	-0.038	-0.039	-0.041	-0.021	0.002	-0.021	-0.034
ρi	0.517	0.516	0.515	0.511	0.699	0.870	0.680	0.400
FLTR optimal								
ω	0	0.3	0.5	0.8	0	0.3	0.5	0.8
ϕπ	5.000	5.000	5.000	5.000	5.000	5.000	5.000	2.976
ϕy	-0.015	-0.079	-0.079	-0.079	-0.005	-0.063	-0.075	-0.075

Table 1.6: Optimal rules coefficients for linearised and Hamiltonian approach.

Taylor rule and in the forward-looking Taylor rule), welfare that would obtain from one approach using the optimal coefficient of the other approach is only marginally different from the one reported in Table 1.6. Therefore, comparing the Hamiltonian approach with its traditional counterpart appears to make much more sense when simple rather than optimal rules are involved.

Chapter 2

Estimation of a Small New Keynesian Model with Trend Inflation for Eastern European Countries

2.1 Introduction

The Bayesian estimation of the DSGE models has recently attracted the attention of an increasing number of economists investigating whether the predictions of the DSGE models match the statistical properties of the empirical data, which are the transmission channels for the exogenous shocks, and what is the behaviour of the central bank. In this paper, I study the performance of a simple small open economy (SOE) model, and investigate the conduct of monetary policy in Eastern European countries.

There are two innovative aspects that I consider in this chapter. First, my analysis assesses to what extent the assumption of a zero steady state inflation influences the model's empirical fit, in order to sheds new light on the importance of introducing trend inflation into these type of models. Previous contributions mainly point out the importance of the trend inflation on theoretical grounds (*e.g.*, Ascari and Ropelle, 2007). Empirical estimations of models with trade inflation are rare in the literature. One of the few exceptions is by Cogley and Sbordone, 2008, who estimate the NKPC as a single equation with a time varying trend inflation. Unlike these authors, I focus on a more structured DSGE model in which the NKPC is embedded. I estimate the model using the Bayesian methodology. Second, in order to estimate the Phillips curve more accurately, I do not treat the marginal cost as a latent variable, as it is common in the Bayesian literature, but I use real unit labour cost data as a proxy for it. Using the resulting framework, I show that the backward looking component in the Phillips curve is important, and the model performs significantly better when accounting for non-zero steady state inflation.

Beside analysing how important the innovations on the supply side of the model are, my interest lies in estimation of a suitable monetary policy rule for both a large and a small economy. When considering SOE monetary policy, a number of theoretical contributions argue that PPI inflation targeting performs better than CPI inflation targeting in terms of welfare loss. Nonetheless, the empirical literature mainly concentrates on simple rules with CPI inflation targeting. Using a posterior odds test, I analyse whether the central banks of some selected Eastern European countries (EEC) systematically target CPI inflation rather than PPI inflation. My results suggest that this hypothesis is empirically rejected for all the investigated Eastern European countries.

Lastly, I analyse to what extent the central bank of a small Eastern European country, such as the Czech Republic, Hungary or Poland, responds to variations in the exchange rate. This question, originally posed by Lubik and Schorfheide (2007) with regard to Australia, Canada, New Zealand and the UK, is particularly interesting in the context of the emerging EEC, since these countries are potential candidates to join the Eurozone. The decision on whether to join the Eurozone may well be determined at least in part by the gains and losses these countries may face when abandoning a flexible exchange rate regime. I identify Germany as the large economy, since this country represents the largest trading partner of all of the selected EEC.[1] The results are mixed. I show that the Czech central bank is likely to target exchange rates, whereas its Hungarian and Polish counterparts are not.

The paper includes two large sections. Section 2.3 describes the theoretical model, which builds on the New Keynesian literature with non zero inflation trend such as Ascari and Ropele (2007). I generalise the model for a SOE and I introduce an incomplete pass-through and a home bias in the representation of consumer preferences. The reason is that, although it is still common in the New Keynesian literature to assume that the law of one price and the PPP hold, both assumptions strongly contradict the well-estiblished empirical evidence.

My results are discussed in Section 2.4, where I describe the Bayesian methodology adopted to estimate the structural parameters of the model, particularly

[1]Germany attracts between 25 and 30 percent of the total exports from each of the EEC. The fact that this trade partnership is not reciprocal (less than 4 percent) allows me to conclude that Germany behaves as a large economy relative to the EEC.

those specifying the Phillips curve. This approach also allows me to analyse the implications of modifying the Phillips curve to account for a non-zero steady state inflation. For robustness, my findings are derived using different monetary policy rules. Among them, however, I show that a simple monetary policy rule capturing the essential features of the optimal one, as derived by, *e.g.*, Steinsson (2003), significantly improves the fit to the data.

2.2 Literature Review

The structure of the model closely relates to Galí and Monacelli (2005), Tuesta and Rabanal (2006) and De Paoli (2009). Furthermore, I add some assumptions to the model that are motivated by the empirical evidence. I assume incomplete pass-through following Monacelli (2003).[2] I also assume home bias in consumption, which leads to deviation from power purchasing parity. The intratemporal elasticity of substitution between domestic and foreign goods differs from unity, allowing the SOE central bank to manipulate the terms of trade, which now relates to the relative domestic price. The reason for introducing these variations to the benchmark model is twofold. On the one hand, Devereux and Engel (2003) show that optimal monetary policy, in case of less than perfect (incomplete) pass-through of the exchange rate to the local currency prices, should involve some consideration of exchange rate volatility. On the other hand, although it is typically assumed in the literature that the elasticity of substitution between domestic and foreign goods is one (as in, *e.g.*, Corsetti and Pesenti (2001), Devereux and Engel (2003), and Ob-

[2] We refer to this article for a discussion about the different strategies regarding the modeling of incomplete pass-through, and more generally about the differences between producer currency pricing and local currency pricing.

stfeld and Rogoff (2002)), empirical estimations suggest larger elasticities. Using this result, Sutherland (2006) argues that the central bank should add targeting of the exchange rate to monetary policy.

The supply side is characterised by a hybrid New Keynesian Phillips Curve, which is derived using a rule of thumb following Galí and Gertler (1999). A similar Phillips Curve specification is also used by Benigno and Lopez-Salido (2002), who analyse the effect of asymmetric supply shocks across countries within a monetary union. Additionally, I follow Ascari and Ropele (2007) and log-linearise this Phillips curve around a non-zero steady state, and show that this assumption improves the fit of the model significantly. The monetary policy is specified as by using different Taylor type rules for both, the closed and the open economy. The aim is twofold. First, different rules serve the robustness check for my results. However, I can also identify the best suitable monetary policy rule.

There is a large literature using Bayesian techniques to estimate monetary policy rules in DSGE models. The first important work in this field is Smets and Wouters (2003), who estimate structural parameters of a closed economy model using Euro Area data. This work has since been extended for the SOE model. Lubik and Schorfheide (2005) create a two symmetric country model and estimate it using U.S. and Euro Area data. Using a similar dataset, Tuesta and Rabanal (2006) estimate and compare models with complete and incomplete financial markets. In their later paper, Lubik and Schorfheide (2007) estimate how central banks in Australia, Canada, New Zealand and UK respond to exchange rate changes, estimating composite structural parameters. Similarly, Adolfson, Laséen, Lindé, and Villani (2008) and Liu (2006) investigate similar questions while assuming incomplete pass-through, using data for Sweden and New Zealand, respectively. Justiniano and Preston (2010) identify the optimal policy rule within a generalized class of Taylor-type rule, which they estimate using data from Australia, Canada and New Zealand. They show that these rules do not respond to the nominal exchange rates. Negro and Schorfheide (2009) also study the effect of changes in the monetary policy rule, using data for Chile.

The use of Bayesian techniques to estimate the NKPC have so far only yielded mixed results. Schorfheide (2008) reviews the identification and the estimates for the Phillips curve coefficients, obtained using U.S. data. He demonstrates how estimates of marginal costs treated as latent variables or measured in terms of an output gap vary widely with observable marginal costs, measured by unit labour costs. He concludes that the estimated values are more robust if marginal costs are explicitly included. Galí and Gertler (1999) estimate a NKPC with unit labour costs as a proxy for marginal costs as well as output gap, using the general method of moments (GMM). They show that using unit labor costs delivers better estimates than using the output gap. Similarly, Sbordone (2002) argues that estimating the NKPC with the output gap is successful as long as the output gap is a good measure of marginal costs and Cogley and Sbordone (2008) use this proxy to estimate the NKPC with time varying trend inflation.

The rest of the paper is organised as follows. In Section 2.3, I specify the model assuming two countries that may or may not differ in size. After describing the demand and supply side of the model in details, I specify the monetary policy rules as nominal interest rate rules for each country, and log-linearise the model around its non-zero inflation steady state. In Section 2.4, I describe the estimation methodology, the dataset, and the choice of prior. I also present the estimation results and the model fit following from the Bayesian estimation, and analyse the impulse responses. Section 2.5 concludes.

2.3 The Model

I first specify the model for two generic countries. The framework encapsulates both the scenario where there are two symmetric countries or where there are two countries that differ in size and openness. I then specify the model for a SOE, which interacts with a large economy. Section 2.3.1 describes in detail the household preferences, its optimisation problem as well as total and aggregate demand for both domestic and foreign country. Section 2.3.2 describes the supply side of the model. The whole model is log-linearised around its steady state in Section 2.3.3, and monetary policy rules in simple form are described in more detail in Section 2.3.4.

2.3.1 Demand Side

I consider two countries; country H, also called home or domestic country, and F, the foreign country. A continuum of agents of unit mass populate the world economy, where the population in the segment $[0, n)$ belongs to country H and the population in the segment $(n, 1]$ belongs to country F. Consumption C is a Dixit-Stiglitz aggregator of home and foreign goods. Home consumers preferences are represented by

$$C_t = \left[\gamma^{\frac{1}{\theta}} \left(C_{H,t}\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} \left(C_{F,t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{2.1}$$

with the intratemporal elasticity of substitution between domestic and foreign goods, θ, not necessary equal to one. Sutherland (2006) surveys the empirical literature and concludes that the majority of the empirical evidence suggests $\theta \in (5, 10)$.

The parameter γ introduces a home bias in consumption, and its value is given by

$$1 - \gamma = (1 - n)\,\lambda \quad \rightarrow \quad \gamma = 1 - (1 - n)\,\lambda, \tag{2.2}$$

where $(1 - n)$ is the relative size of country F and $\lambda \in [0, 1)$ is the degree of openness of country H. If $\lambda = 0$, the domestic economy is autarkic and only domestic goods are consumed. Furthermore, as the size of the economy (n) increases, consumers buy relatively more domestic goods and imports become less relevant. Therefore, in a large economy, where $n \rightarrow 1$, people mainly consume domestically produced goods, whereas for a small open economy where $n \rightarrow 0$, international trade is more important. A small economy is also more strongly influenced by foreign innovations.

Similar preferences are specified for the foreign consumer:

$$C_t^* = \left[(\gamma^*)^{\frac{1}{\theta}} \left(C_{H,t}^* \right)^{\frac{\theta-1}{\theta}} + (1 - \gamma^*)^{\frac{1}{\theta}} \left(C_{F,t}^* \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{2.3}$$

where the parameter γ^* is determined by the size and openness of the foreign economy, that is

$$\gamma^* = n\lambda. \tag{2.4}$$

Note that the specifications of γ and γ^* imply that the power purchasing parity (PPP) does not hold in this model.

The sub-indices for domestic consumption of domestic (respectively, imported) goods $C_{H,t}$ (resp., $C_{F,t}$) are

$$C_{H,t} = \left(\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n \left[C_{H,t}(i) \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.5}$$

$$C_{F,t} = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 \left[C_{F,t}(i) \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.6}$$

Analogously, for foreign consumption $C^*_{H,t}$ (resp., $C^*_{F,t}$) it holds

$$C^*_{H,t} = \left(\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n \left[C^*_{H,t}(i) \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.7}$$

$$C^*_{F,t} = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 \left[C^*_{F,t}(i) \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.8}$$

where $C_{H,t}$ is the home consumption of domestically produced goods and $C_{F,t}$ is the home consumption of imported goods. Analogously, $C^*_{H,t}$ is foreign consumption of domestic exports (goods produced in the home country) and $C^*_{F,t}$ is foreign consumption of goods produced abroad. Finally, $C_t(i)$ is the total consumption of a generic good (i).[3] The parameter ε is the elasticity of substitution between the differentiated goods produced in one country and holds unchanged across countries.

I assume that the consumption choices of all households from one country are identical. From the consumption maximisation problem of the representative domestic household, I obtain the domestic demand function for a domestic good

[3]Generally, starred variables are expressed in foreign currency, unstarred in domestic currency. However, this rule does not apply to consumption, which is expressed in real terms: in this case, it is only used to distinguish between consumption at home and abroad.

$C_{H,t}(i)$ and foreign good $C_{F,t}(i)$ as follows

$$C_{H,t}(i) = \frac{1}{n} C_{H,t} \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon}, \quad C_{F,t}(i) = \frac{1}{1-n} C_{F,t} \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon}. \tag{2.9}$$

Analogously, it holds that the foreign demand for a domestic good $C^*_{H,t}(i)$ and for a foreign good $C^*_{F,t}(i)$ are respectively given by

$$C^*_{H,t}(i) = \frac{1}{n} C^*_{H,t} \left(\frac{P^*_{H,t}(i)}{P^*_{H,t}} \right)^{-\varepsilon}, \quad C^*_{F,t}(i) = \frac{1}{1-n} C^*_{F,t} \left(\frac{P^*_{F,t}(i)}{P^*_{F,t}} \right)^{-\varepsilon}. \tag{2.10}$$

The aggregate domestic demand for domestic good and for foreign goods (imports) can be written in terms of aggregate world consumption

$$C_{H,t} = \gamma C_t \left(\frac{P_{H,t}}{P_t} \right)^{-\theta}, \quad C_{F,t} = (1-\gamma) C_t \left(\frac{P_{F,t}}{P_t} \right)^{-\theta}, \tag{2.11}$$

and the aggregate foreign demand function for domestic goods (in other words, exports from the point of view of the home country) and for goods produced abroad can be written as

$$C^*_{H,t} = \gamma^* C^*_t \left(\frac{P^*_{H,t}}{P^*_t} \right)^{-\theta}, \quad C^*_{F,t} = (1-\gamma^*) C^*_t \left(\frac{P^*_{F,t}}{P^*_t} \right)^{-\theta}. \tag{2.12}$$

By manipulation of the demand functions, the consumption-based price indices for domestic and foreign country can be expressed respectively as

$$P_t = \left[\gamma (P_{H,t})^{1-\theta} + (1-\gamma)(P_{F,t})^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{2.13}$$

$$P^*_t = \left[\gamma^* \left(P^*_{H,t}\right)^{1-\theta} + (1-\gamma^*) \left(P^*_{F,t}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{2.14}$$

The price sub-index $P_{z,t}$ ($P^*_{z,t}$) for goods produced in country $z \in \{H, F\}$ can be expressed in the domestic (foreign) currency as

$$P_{H,t} = \left(\frac{1}{n}\int_0^n [P_{H,t}(i)]^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}}, \quad P_{F,t} = \left(\frac{1}{1-n}\int_n^1 [P_{F,t}(i)]^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}} \quad (2.15)$$

$$P^*_{H,t} = \left(\frac{1}{n}\int_0^n \left[P^*_{H,t}(i)\right]^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}, \quad P^*_{F,t} = \left(\frac{1}{1-n}\int_n^1 \left[P^*_{F,t}(i)\right]^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}} \quad (2.16)$$

with the producer price index of the domestically produced goods $P_{H,t}$ and the importer price index for the goods from foreign country $P_{F,t}$ both expressed in the domestic currency. Analogously, $P^*_{F,t}$ ($P^*_{H,t}$) is the producer price index in foreign country (price of the imported goods from the point of view of consumers abroad) in foreign currency.

The Law of one Price and the Real Exchange Rate

There is strong empirical evidence that the law of one price (LOP) does not hold, which could be because of different producer pricing or because importers face monopolistic competition similar to producers and therefore charge a mark-up over their price. Hence, it is very common in New Open Economy Macroeconomic Models (NOEM) to assume incomplete pass-through. In this paper I follow Monacelli (2003) and assume that the law of one price holds when the goods arrive "at the dock", but setting the price in domestic currency causes a deviation from the LOP. This is explained in more detail in Section 2.3.2, where it is shown that the domestic retailers set the price of the imported good in monopolistic competition.

The LOP gap is defined as

$$\Psi_t = S_t \frac{P^*_{F,t}}{P_{F,t}}, \tag{2.17}$$

where the nominal exchange rate S_t denotes the price of the foreign currency in terms of the domestic currency.[4] Additionally, given the different degrees of home bias in consumption between the two countries, *i.e.* $\gamma \neq \gamma^*$, it follows from equation (2.13) that the PPP does not hold, and the CPI in each country differs, formally

$$P_t \neq S_t P^*_t.$$

Hence, the real exchange rate differs from one, and I can express it as the price of foreign goods in term of domestic goods

$$RS_t = \frac{S_t P^*_t}{P_t}. \tag{2.18}$$

Note that a decrease in the nominal exchange rate S_t and analogously, *ceteris paribus*, in the real exchange RS_t implies an appreciation of the domestic currency.

The terms of trade, which is given by a ratio between importer and domestic producer prices is also expressed in terms of relative prices

$$TOT_t = \frac{P_{F,t}}{P_{H,t}} = \frac{\tilde{P}_{F,t}}{\tilde{P}_{H,t}}, \tag{2.19}$$

where $\tilde{P}_{F,t} = P_{F,t}/P_t$ and $\tilde{P}_{H,t} = P_{H,t}/P_t$. Combining the last equation with equation (2.13) shows that the relative domestic price can be easily expressed as

[4] Note however that for the domestic price, from the point of view of domestic producer, the law of one price holds, because he gets the price "at the dock".

a function of the terms of trade

$$\tilde{P}_{H,t} = \left[\gamma + (1-\gamma)(TOT_t)^{1-\theta}\right]^{-\frac{1}{1-\theta}}.$$

The relationship between domestic and CPI inflation is given by the relationship between domestic relative prices of the current and past period

$$\frac{\Pi_{H,t}}{\Pi_t} = \frac{\tilde{P}_{H,t}}{\tilde{P}_{H,t-1}}. \tag{2.20}$$

and the relationship between imported and CPI inflation can be expressed as the relationship between relative prices of the imports in domestic currency of the current and past period

$$\frac{\Pi_{F,t}}{\Pi_t} = \frac{\tilde{P}_{F,t}}{\tilde{P}_{F,t-1}}. \tag{2.21}$$

The Household Optimisation Problem

Both domestic and foreign economies consist of a continuum of identically infinite-lived agents. The preferences of the domestic representative agent is given by the instantaneous utility function of the same form as in Chapter 1

$$U_t(C,N) = \frac{\varepsilon_t C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}, \tag{2.22}$$

where the function (U) is separable in consumption (C) and working hours (N), so that $U_{C,N} = 0$, and where the preference shock ε_t affects the rate of intertemporal substitution in consumption for domestic households, similar to the one in Tuesta and Rabanal (2006). The utility function is also time-separable and the parameters σ and η are both positive CRRA (constant relative risk aversion) parameters determining the elasticity of substitution.

The representative agent maximises its discounted stream of instantaneous utility functions over current and future periods

$$U = E_t \sum_{t=0}^{\infty} \beta^t \left[U_t \left(C, N \right) \right], \tag{2.23}$$

where $\beta \in (0,1]$ is the subjective discount factor, by choosing $\{C_t, N_t\}_{t=0}^{\infty}$. She also holds international bonds B_t denominated in the national currency which yields a gross return of R_t at the end of the period.

Her budget constraint, is given by

$$B_t + W_t N_t + T_t + D_t \geq P_t C_t + E_t \left[Q_{t,t+1} B_{t+1} \right]. \tag{2.24}$$

Note that

$$P_t C_t = \int_0^n P_H(i) \, C_H(i) \, di + \int_n^1 P_F(i) \, C_F(i) \, di,$$

where $C_H(i)$ ($C_F(i)$) is consumption of domestic (foreign) good i, given its price $P_H(i)$ ($P_F(i)$), and P_t is the overall consumer price index. Notice also that the agent consumes all goods at any time t. The nominal bonds denominated in domestic currency at the end of the period t are denoted by B_t.[5] W_t is the nominal wage, and T_t and D_t are the lump sum transfer and the profits of the companies held by the household, respectively. $E_t[Q_{t,t+1}]$ is the dynamic stochastic discount

[5] It holds that $B_t + B_t^* = 0$, so the world-wide stock of international bonds equal zero for all periods.

factor between period t and $t+1$, for which it holds

$$E_t\left[Q_{t,t+1}\right]=\frac{1}{R_t},$$

where R_t is the gross return on a riskless one year nominal bond, with a yield assumed to be small. Furthermore, for sufficiently small values of i_t, it holds that $\log\left(R_t\right)\approx i_t$, where i_t is the riskless short term nominal interest rate.

From the first order condition of the maximisation problem of the domestic representative household, I obtain the Euler equation for the domestic economy

$$E_t\left(\left[\frac{C_t}{C_{t+1}}\right]^{-\sigma}\Pi_{t+1}\frac{\varepsilon_t}{\varepsilon_{t+1}}\right)=\beta R_t \tag{2.25}$$

with domestic CPI inflation given by $\Pi_{t+1}=P_{t+1}/P_t$. Following Steinbach *et al.* (2009), the expression $\varepsilon_{t+1}/\varepsilon_t$ can be also interpreted as a risk premium on asset holding, *i.e.*, the wedge between the interest rate set by central bank and the actual return on assets. The domestic households labour supply is given by

$$\tilde{W}_t=\frac{N_t^{\eta}}{C_t^{-\sigma}}, \tag{2.26}$$

where $\tilde{W}_t$ is the real domestic wage. I assume that labour is immobile across countries. Assuming that the foreign household faces the same maximisation problem, the Euler equation and the labour supply for a foreign economy are derived analogously.

The Asset Market Structure

In my model, I ignore the transaction costs and assume that financial markets are such that consumers from either country have access to both domestic and foreign bonds. The market price of a domestic riskless bond equals the expected nominal return of the bond, and is given by $1/R_t = E_t\left[Q_{t,t+1}\right]$. Similarly for a foreign bond expressed in domestic currency, it holds that $S_t/\left(R_t^*\right) = E_t\left[S_{t+1}Q_{t,t+1}\right]$. With no possibility of arbitrage, the expected returns of these two bonds must be equal, and the two equations can be combined. Therefore, the uncovered interested parity holds and is expressed as

$$E_t\left[Q_{t,t+1}\left\{R_t - \frac{R_t^* S_{t+1}}{S_t}\right\}\right] = 0,$$

where S_t is the nominal exchange rate, expressed as the price of foreign currency in terms of domestic currency. Using the last equation together with (2.18), the uncovered interest parity equation can be written as the expected change in the real exchange rate RS_t and the ratio between domestic and foreign real interest rate

$$\frac{R_t}{R_t^*}E_t\left[\frac{\Pi_{t+1}^*}{\Pi_{t+1}}\right] = E_t\left[\frac{RS_{t+1}}{RS_t}\right]. \tag{2.27}$$

Under the assumption of complete securities markets with no uncertainty, consumption risk is perfectly shared and the stochastic discount factor, expressed in the same currency, is equal across the countries. Using the Euler equation (2.25) and its equivalent for the foreign country, and recalling that the constant subjective discount factor β is shared by both countries, delivers

$$E_t\left[\frac{\varepsilon_t}{\varepsilon_{t+1}}\left(\frac{C_t}{C_{t+1}}\right)^{-\sigma}\frac{S_t}{S_{t+1}}\Pi_{t+1}\right] = E_t\left[\frac{\varepsilon_t^*}{\varepsilon_{t+1}^*}\left(\frac{C_t^*}{C_{t+1}^*}\right)^{-\sigma}\Pi_{t+1}^*\right]. \tag{2.28}$$

Using again equation (2.18), (2.28) can be rewritten as a function of the real exchange rate

$$E_t\left[\frac{RS_{t+1}}{RS_t}\right]=\frac{E_t\left[\left(\frac{C_{t+1}^*}{C_{t+1}}\right)^{-\sigma}\frac{\varepsilon_{t+1}^*}{\varepsilon_{t+1}}\right]}{\left(\frac{C_t^*}{C_t}\right)^{-\sigma}\frac{\varepsilon_t^*}{\varepsilon_t}}.$$

Given the fact that this equation holds in all periods t, including the steady state condition of zero net foreign assets and the ex-ante identical environment, I obtain the optimal risk sharing, under complete financial markets

$$RS_t=\kappa_c\left(\frac{C_t^*}{C_t}\right)^{-\sigma}\frac{\varepsilon_t^*}{\varepsilon_t}.$$

The constant κ_c is determined by the initial market equilibrium for state-contingent bonds, which reflects the initial wealth differences. Without loss of generality, following Galí and Monacelli (2005), I can assume that the initial distribution of wealth is such that $\kappa_c=1$ and the risk sharing equation can be written analogously to the one in Tuesta and Rabanal (2006)[6]

$$RS_t=\left(\frac{C_t^*}{C_t}\right)^{-\sigma}\frac{\varepsilon_t^*}{\varepsilon_t}. \tag{2.29}$$

This equation reflects the fact that if power purchasing parity (PPP) holds, *e.g.,* $RS_t=1$, the marginal utility of consumption, *i.e.* the consumption level is equal across the countries. However, deviations from PPP imply different consumption levels across the two countries caused by the changes in the real exchange rate. Hence, the ratio of marginal utilities across the two countries is equal to the ratio of aggregate prices denote here by the real exchange rate.

[6] This result holds in the case of symmetric perfect foresight steady state and symmetric initial relative net asset position. Further details in Section 1.5.2 .

Aggregate Demand for Domestic and Foreign Goods

The total demand for a domestically produced good i consists of the weighted average of n domestic and $(1-n)$ foreign demand

$$Y_t(i) = nC_{H,t}(i) + (1-n)\, C^*_{H,t}(i).$$

Using (2.9) and (2.11) together with (2.10) and (2.12), it is possible to express the demand for good i in terms of price dispersion and the real exchange rate, where the prices are expressed in domestic currency

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left(\tilde{P}_{H,t}\right)^{-\theta} \left[\gamma C_t + \frac{1-n}{n}\gamma^* C^*_t (RS_t)^{\theta}\right]. \qquad (2.30)$$

Thus, an appreciation of the currency leads to a decrease in output of domestic good i, $Y_t(i)$. Furthermore, the aggregate demand for domestic output can be written as a sum of the amounts produced domestically of good i

$$Y_t = \left(\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n [Y_t(i)]^{\frac{\varepsilon-1}{\varepsilon}}\, di\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Plugging this into equation (2.30) together with (2.15), the aggregate demand in the domestic country yields

$$Y_t = \left(\tilde{P}_{H,t}\right)^{-\theta} \left[\gamma C_t + \frac{1-n}{n}\gamma^* C^*_t (RS_t)^{\theta}\right], \qquad (2.31)$$

hence the demand for a home-produced good is inversely related to an appreciation of the exchange rate. The reason is that foreign consumption decreases in terms of the home currency. Therefore, the degree to which appreciation influences the domestic production of good i depends on size of the foreign economy and its (domestic) openness as much as of the price dispersion given by the elasticities of substitutions. The higher the elasticity of substitution between domestic and foreign goods, the more sensitive the output of the domestic economy to the changes in the currency.

Combining (2.30) and (2.31), the total demand for good i, written in terms of domestic aggregate output is

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t, \tag{2.32}$$

which depends directly on the aggregate domestic output, the price of good i relative to the overall domestic price level, as well as the elasticity of substitution between domestic goods.

Analogously, the total demand for a foreign produced good i is

$$Y_t^*(i) = nC_{F,t}(i) + (1-n)\,C_{F,t}^*(i)$$

and can be rewritten as

$$Y_t^*(i) = \left(\frac{P_{F,t}^*(i)}{P_{F,t}^*}\right)^{-\varepsilon} \left(\tilde{P}_{F,t}^*\right)^{-\theta} \left[\frac{n}{1-n}(1-\gamma)\,C_t\,(RS_t)^{-\theta} + (1-\gamma^*)\,C_t^*\right], \tag{2.33}$$

where $\tilde{P}_{F,t}^* = P_{F,t}^*/P_t^*$ is the relative foreign producer price index expressed in foreign currency.

The aggregate demand for foreign output can be written as a sum of the foreign production of all goods

$$Y_t^* = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 \left[Y_t^* (i) \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

Using (2.33) together with (2.16), the aggregate demand for foreign output is given by

$$Y_t^* = \left(\tilde{P}_{F,t}^* \right)^{-\theta} \left[\frac{n}{1-n} (1-\gamma) C_t (RS_t)^{-\theta} + (1-\gamma^*) C_t^* \right] . \qquad (2.34)$$

Thus, combining (2.33) and (2.34), I obtain total demand for the foreign good i in terms of foreign aggregate output

$$Y_t^* (i) = \left(\tilde{P}_{F,t}^* \right)^{-\varepsilon} Y_t^*, \qquad (2.35)$$

Large Economy versus Small Open Economy

I can rewrite the key equations by assuming that the size of the foreign economy (domestic) market is sufficiently large that it is hardly influenced by the SOE. In this sense, analogous to Galí and Monacelli (2005), the large economy behaves as if it is autarkic and its associated economic variables are exogenous from the point of view of the SOE. Using the definition (2.2) and (2.4), and assuming that the domestic economy is small, *i.e.*, $n \to 0$, the aggregate consumption of domestic and foreign goods given by (2.1) and (2.3) becomes

$$C_t = \left[(1-\lambda)^{\frac{1}{\theta}} (C_{H,t})^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}} (C_{F,t})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} .$$

For the foreign large economy, defined as the rest of the world, the quantity of imports from the SOE are so marginal that I can assume

$$C_t^* = C_{F,t}^*.$$

Given (2.13), the relative domestic price index equation yields

$$P_t = \left[(1-\lambda)(P_{H,t})^{1-\theta} + \lambda (P_{F,t})^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{2.36}$$

Note that for the foreign large economy there is no dispersion between producer and consumer price index, formally

$$P_t^* = P_{F,t}^*. \tag{2.37}$$

and it follows from equation (2.34) that the aggregate demand for goods produced in large foreign economy is given as $Y_t^* = C_t^*$.

Thus, the LOP gap (2.17) can be written in terms of real exchange rate and the terms of trade

$$\Psi_t = \frac{RS_t}{\tilde{P}_{F,t}}. \tag{2.38}$$

The total demand for a generic domestic good i given (2.30)

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left(\tilde{P}_{H,t}\right)^{-\theta} C_t \left[1-\lambda+\lambda RS_t^{\theta-\frac{1}{\sigma}}\right] \tag{2.39}$$

depends on the openness of the domestic economy λ, the price dispersion between domestic producer and consumer price indexes and the real exchange rate. The real depreciation of the exchange rate leads to an increase in production of good

i, the domestic good is cheaper and therefore the consumption of the good abroad increases. Analogous to the closed economy case, the higher the dispersion between the price of a particular good i and the domestic price index caused by the price stickiness, the lower the demand for good i. Additionally, for the SOE, there is a wedge between producer and consumer price indexes, which lowers domestic output.

The aggregate demand for domestic goods, assuming all the conditions associated with a SOE yields

$$Y_t = \left(\tilde{P}_{H,t}\right)^{-\theta} C_t \left[1 - \lambda + \lambda RS_t^{\theta - \frac{1}{\sigma}}\right]. \tag{2.40}$$

2.3.2 Firm Optimisation: The Phillips Curve

The supply side of the domestic economy consists of two parts. There are producers and import retailers, both setting prices in the manner described by Calvo (1983) and Galí and Gertler (1999). As described in Chapter 1, each producer (resp., retailer) belongs to one of two types of firms. A measure $1 - \omega$ (resp., $1 - \omega^F$) set the price optimally, and are labelled f. A measure ω (resp., ω^F) set the price according to a rule-of-thumb, and are labelled b. Firms may face two different situations: i) either they are allowed to set their price with probability $1 - \alpha$ (resp., $1 - \alpha^F$); ii) or they are not allowed to do so with probability α (resp., α^F). Hence, at each time t, a measure $(1 - \omega)(1 - \alpha)$ (resp., $[1 - \omega^F][1 - \alpha^F]$) sets the price optimally; a measure $\omega(1 - \alpha)$ (resp., $\omega^F[1 - \alpha^F]$) sets the price according to a rule-of-thumb; a measure α holds the price unchanged.

Price Setting Mechanism for Final Goods Producers

First consider one of the $(1-\omega)(1-\alpha)$ firms in country H that, at time t, are allowed to set their price optimally. Each producer in this group sets price $P_t^f(i)$ to maximise its expected stream of profits $G_t(i)$

$$\max_{P_t^f(i)} \quad \sum_{j=0}^{\infty} \alpha^j E_t \left[Q_{t,t+j} \left(P_t^f(i) Y_{t+j}(i) - W_{t+j} N_{t+j}(i) \right) \right], \tag{2.41}$$
$$\text{subject to:} \quad Y_{t+j}(i) = A_{t+j} N_{t+j}(i), \quad \text{and} \quad Y_{t+j}(i) = \left(\frac{P_t^f(i)}{P_{H,t+j}} \right)^{-\varepsilon} Y_{t+j},$$

where A_t is total factor productivity, and the constraints respectively represent the production technology, and the demand function (2.32). The first order condition for the SOE producers delivers the optimal choice of the forward looking price $P_t^f(i)$

$$\frac{P_t^f(i)}{P_t} \sum_{j=0}^{\infty} (\alpha\beta)^j E_t \left[(C_{t+j})^{-\sigma} Y_{t+j} \frac{P_t}{P_{t+j}} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\varepsilon} \right]$$
$$= \frac{\varepsilon}{\varepsilon - 1} \sum_{j=0}^{\infty} (\alpha\beta)^j E_t \left[\left(\frac{Y_{t+j}}{A_{t+j}} \right)^{\eta+1} \left(\frac{P_{H,t}}{P_{H,t+j}} \right)^{-\varepsilon} \right].$$

Denoting by $\tilde{P}_t^f(i) = P_t^f(i)/P_t$ the relative forward looking price of domestic firm i, the last equation can be rewritten in terms of difference equations

$$\tilde{P}_t^f(i) = \frac{J_t}{H_t} \tag{2.42}$$

with

$$J_t = \mu V_t \left(\frac{Y_t}{A_t} \right)^{\eta+1} + \alpha\beta E_t \left[(\Pi_{H,t+1})^{\varepsilon} J_{t+1} \right] \tag{2.43}$$

and

$$H_t = C_t^{-\sigma} Y_t + \alpha\beta E_t \left[(\Pi_{H,t+1})^{\varepsilon} (\Pi_{t+1})^{-1} H_{t+1} \right], \tag{2.44}$$

where $\mu = \varepsilon / (\varepsilon - 1)$ is the domestic mark-up. I also introduce the mark-up shock V_t

$$\log \frac{V_t}{\bar{V}} = \rho_v \log \frac{V_{t-1}}{\bar{V}} + \varepsilon_{v,t},$$

where $\bar{V}$ is the steady state value of the mark-up innovation and $\varepsilon_{v,t}$ is an i.i.d. shock. Given equilibrium on the labour market, the first expression in (2.43) can be written in terms of real marginal costs $\widetilde{MC}_t$ and the relative domestic price $\tilde{P}_{H,t}$

$$\left(\frac{Y_t}{A_t} \right)^{\eta+1} = C_t^{-\sigma} Y_t \widetilde{MC}_t \tilde{P}_{H,t},$$

where $\widetilde{MC}_t = MC_t / P_{H,t}$. The forward looking price therefore depends on domestic and CPI inflation, and the relative domestic price.

The remaining $\omega (1 - \alpha)$ domestic firms set prices at time t according to the rule of thumb, indexing it to the last observed price index. In terms of the rate of domestic producer inflation $\Pi_{H,t-1}$

$$P_t^b = \Pi_{H,t-1} X_{t-1}, \tag{2.45}$$

where X_{t-1} denotes an index of the prices set at date $t-1$, given by

$$X_t \equiv \left[(1 - \omega) P_t^{f^{(1-\varepsilon)}} + \omega P_t^{b^{(1-\varepsilon)}} \right]^{\frac{1}{1-\varepsilon}}. \tag{2.46}$$

The aggregate producer price level then follows the law of motion

$$P_{H,t} = \left[(1-\alpha) X_t^{1-\varepsilon} + \alpha \left(P_{H,t-1}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{2.47}$$

The set of equations (2.42)-(2.47) constitute the hybrid New Keynesian Phillips curve, which characterises the producer side of country H. The set of equations leading to the Phillips curve for country F is derived analogously. The hybrid NKPC for country F has the same form as in Chapter 1. However, the hybrid NKPC for country H, because of the dispersion between PPI and CPI, can be written as a function of the consumer price index and the terms of trade, as shown by, Benigno and Benigno (2003).

Price Setting Mechanism for Importing Retailers

Following Monacelli (2003), I assume that for retailers, who import differentiated goods into the domestic economy, the law of one price holds "at the dock". Similar to the domestic producers, domestic importing retailers also face a downward sloping demand curve. Under monopolistic competition, they set their prices, in terms of domestic currency, accordingly. The deviation between the prices of the imported good in domestic and foreign currency therefore generates a LOP gap.

Consider the $\alpha^F \omega^F$ share of local retailers importing good j at a cost $S_t P_{F,t}^{*}(i)$, and setting the price of the imported good in a domestic currency to maximise their profits

$$\begin{aligned} &\max_{P_t^f(i)} \sum_{j=0}^{\infty} \alpha_F^j E_t \left[Q_{t,t+j} \left(P_t^{F,f}(i) - S_t P_t^{F*}(i) \right) C_{F,t+j}(i) \right], \\ &\text{subject to: } C_{F,t}(i) = \tfrac{1}{1-n} \left(\tfrac{P_t^{F,f}(i)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t}. \end{aligned} \tag{2.48}$$

$P_t^{F,f}(i)$ is the price of the imported good in domestic currency set by a forward looking retailer, $P_t^{F*}(i)$ is the price of the same good in the currency of the producer and α^F is the probability that this price holds unchanged the next period. In generally, it is assumed that the parameter α^F can differ from those associated with producers, denoted by α. The problem is solved analogously to the one solved by the domestic producer. The first order condition delivers the optimal choice of the relative forward looking price, $\tilde{P}_t^{F,f}(i) = P_t^{F,f}(i)/P_t$

$$\tilde{P}_t^{F,f}(i) = E_t\left[\frac{\mu\sum_{j=0}^{\infty}(\alpha^F\beta)^j(C_{t+j})^{-\sigma}C_{F,t+j}\Psi_{t+j}V_{t+j}^F\left(\prod_{k=1}^{j}\Pi_{F,t+k}\right)^{\varepsilon}\tilde{P}_{F,t+j}}{\sum_{j=0}^{\infty}(\alpha^F\beta)^j(C_{t+j})^{-\sigma}C_{F,t+j}\left(\prod_{k=1}^{j}\Pi_{F.t+k}\right)^{\varepsilon}\left(\prod_{k=1}^{j}\Pi_{t+k}\right)^{-1}}\right],$$

where I also use equation (2.38). In terms of difference equation, the first order condition delivers

$$\tilde{P}_t^{F,f}(i) = \frac{J_t^F}{H_t^F} \tag{2.49}$$

with

$$J_t^F = \mu V_t^F C_t^{-\sigma}C_{F,t}\Psi_t\tilde{P}_{F,t} + (\alpha^F\beta)E_t\left[(\Pi_{F,t+1})^{\varepsilon}J_{t+1}^F\right], \tag{2.50}$$

where V_t^F is the importers mark up shock, with analogous characteristics as the producer's; and

$$H_t^F = C_t^{-\sigma}C_{F,t} + (\alpha^F\beta)E_t\left[(\Pi_{F,t+1})^{\varepsilon}(\Pi_{t+1})^{-1}H_{t+1}^F\right]. \tag{2.51}$$

The remaining $\omega^F(1-\alpha^F)$ importers set their prices at time t according to the rule of thumb by indexing them to the last observed rate of import inflation

$\Pi_{F,t-1}$

$$P_t^{F,b} = \Pi_{F,t-1} X_{F,t-1}, \tag{2.52}$$

where $X_{F,t-1}$ denotes an index of the prices of imported goods set at date $t-1$, given by

$$X_{F,t} \equiv \left[(1-\omega) P_t^{F,f^{(1-\varepsilon)}} + \omega \left(P_t^{F,b} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{2.53}$$

Assuming that all firms face the same shock, I can write the aggregate importer price level

$$P_{F,t} = \left[(1-\alpha) X_{F,t}^{1-\varepsilon} + \alpha \left(P_{F,t-1} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{2.54}$$

Equations (2.49) to (2.54) characterise the import price inflation hybrid NKPC.

2.3.3 Steady State and Log-linearised Form of the Model

Before the actual estimation, the equations characterising the non-policy part of the model should be log-linearised around the steady state, assuming that $n \to 0$. Monetary policy is described in more details in the next section. In this section, I assume a perfect-foresight steady state for both economies with zero income growth and stable technology. Furthermore, I normalise the steady state nominal exchange rate to unity, formally $S = 1$. One additional assumption about the steady state: prices of imports increase at the same rate as prices of domestically produced goods. Therefore, inflation is the same across both countries, so that the real exchange rate in steady state is stable. This restriction is reasonable because any equilibrium with an explosive exchange rate would not be sustainable.

Since in the steady state all prices change at the same rate, and the price of the imports increases at the same rate as the price of the domestically produced goods, I can normalise the price indices by imposing $\bar{P}_H = \bar{P}_F$.[7] Therefore, from equation (2.13), it follows that the consumer and producer price index are equal, formally, $\bar{P} = \bar{P}_H$. Inflation, as well as the relative prices, do not change and it holds that

$$\bar{\Pi}_H = \bar{\Pi}_F = \bar{\Pi} = \bar{\Pi}^*.$$

Furthermore, denoting growth factors by G, from the definition of real exchange rate it follows that

$$G_{RS} = 1,$$

which in conjunction with (2.27) leads to

$$R = R^*.$$

Together with (2.17), I can then write

$$G_{RS} = G_S = G_\Psi = 1.$$

Note that, since in steady state production per capita is equal across countries, and recalling that the nominal exchange rate equals one, then it must be that price indices are also equal across countries. Hence, considering also the definition of the real exchange rate, it follows that

$$RS = \Psi = S = 1.$$

[7]This assumption follows De Paoli (2009), the price indices in steady state are normalised such as $\bar{P}_H = \bar{P}_F$ and $\bar{P}_H^* = \bar{P}_F^*$, so that the producer prices are in the steady state the same for both countries.

Using (2.29) in steady state, consumption is equalised across countries, and given by $\bar{C} = \bar{C}^*$. Market clearing implies $\bar{Y} = \bar{C}$ and $\bar{Y}^* = \bar{C}^*$. Given the previous results, it holds that $\bar{Y} = \bar{Y}^*$, so the domestic and foreign country have the same per capita income. Therefore, as long as the production technology is the same for both countries, $\bar{N} = \bar{N}^*$.

The structural equations characterising the non-policy part of the model can be written in the (log-)linearised form around their steady state. Linearising equation (2.36) defines the relationship between producer and importer relative price

$$1 = (1 - \lambda)\, \tilde{p}_{H,t} + \lambda \tilde{p}_{F,t}. \tag{2.55}$$

The relationships between relative producer price and inflation and relative importer price and inflation are given respectively by

$$\tilde{p}_{H,t} - \tilde{p}_{H,t-1} = \hat{\pi}_{H,t} - \hat{\pi}_t \tag{2.56}$$

and

$$\tilde{p}_{F,t} - \tilde{p}_{F,t-1} = \hat{\pi}_{F,t} - \hat{\pi}_t. \tag{2.57}$$

The LOP gap (2.38) and the real exchange rate (2.18), written in first difference, are given respectively by

$$\hat{\Psi}_t = \widehat{rs}_t - \tilde{p}_{F,t} \tag{2.58}$$

and

$$\Delta \widehat{rs}_t = \Delta \hat{s}_t + \hat{\pi}^*_t - \hat{\pi}_t + \varepsilon_{rs,t}, \tag{2.59}$$

where I add $\varepsilon_{rs,t}$, an unobservable shock, to capture possible measurement error in the data and to relax the potentially tight cross-equation restrictions in the model.

The domestic Euler equation (2.25) can be rewritten in terms of deviations from the steady state as

$$\hat{c}_t = E_t\left[\hat{c}_{t+1}\right] - \frac{1}{\sigma}\left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right] + E_t\left[\Delta\epsilon_{t+1}\right]\right), \tag{2.60}$$

where I have used again the approximation $log\left(R_t\right) \approx \hat{\imath}_t$. The term $\Delta\epsilon_{t+1} = \log\varepsilon_{t+1} - \log\varepsilon_t$ is the first differences of the structural preference shock. The linearisation of the uncovered interest parity delivers (2.27)

$$\left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right) - \left(\hat{\imath}_t^* - E_t\left[\hat{\pi}_{t+1}^*\right]\right) = E_t\left[\widehat{rs}_{t+1}\right] - \widehat{rs}_t. \tag{2.61}$$

The UIP equation describes the relationship between real interest rate and real exchange rate.

The optimal risk sharing from equation (2.29) becomes

$$\widehat{rs}_t = \sigma\left(\hat{c}_t - \hat{c}_t^*\right) + \epsilon_t^* - \epsilon_t, \tag{2.62}$$

where the difference between the world and the domestic preference shock $\left(\epsilon_t^* - \epsilon_t\right)$ captures the deviations from optimal risk sharing. The risk sharing equation describes the link between real exchange rate and consumption. Assuming complete markets, both equations hold, making Euler equation for the domestic country redundant. The risk sharing equation ensures that the marginal utility is the same in both countries. Assuming everyone in the world shares the same preferences, the level of consumption is the same across the countries. Because the UIP holds, the domestic real interest rate moves along with the interest rate abroad. The

UIP ensures that there is no arbitrage between the foreign and domestic financial markets, thereby determining a relationship that renders the Euler equation redundant.

The good market clearing condition, represented by (2.40), yields

$$\hat{y}_t = -\theta \tilde{p}_{H,t} + \hat{c}_t + \lambda \left(\theta - \frac{1}{\sigma} \right) \widehat{rs}_t. \tag{2.63}$$

The log-linearisation of the supply side is given in more details in Appendix 2.B and leads to a hybrid NKPC with a non-zero steady state inflation

$$\hat{\pi}_{H,t} = \chi^f E_t \left[\hat{\pi}_{H,t+1} \right] + \chi^b \hat{\pi}_{H,t-1} + \kappa_{mc} \left(\widehat{mc}_t + v_t \right) + \chi^\pi \left(\hat{h}_t - \left(\hat{y}_t - \sigma \hat{c}_t \right) \right), \tag{2.64}$$

where the real marginal costs $\widehat{mc}_t = \widehat{mc}_t^{nom} - \hat{p}_{H,t}$ are expressed by

$$\widehat{mc}_t = \eta \hat{y}_t + \sigma \hat{c}_t - (\eta + 1) a_t - \tilde{p}_{H,t} \tag{2.65}$$

and

$$\hat{h}_t = \left(1 - \alpha \beta \bar{\Pi}^{\varepsilon - 1} \right) \left(\hat{y}_t - \sigma \hat{c}_t \right) + (\alpha \beta) \bar{\Pi}^{\varepsilon - 1} E_t \left[\varepsilon \hat{\pi}_{H,t+1} - \hat{\pi}_{t+1} + \hat{h}_{t+1} \right]. \tag{2.66}$$

Analogously, the NKPC for imported prices can be log-linearised to obtain

$$\hat{\pi}_{F,t} = \chi_F^f E_t \left[\hat{\pi}_{F,t+1} \right] + \chi_F^b \hat{\pi}_{F,t-1} + \kappa_F \left(\widehat{\Psi}_t + v_t^F \right) + \chi_F^\pi \left(\hat{h}_t^F - \left(\hat{c}_t^F - \sigma \hat{c}_t \right) \right) \tag{2.67}$$

with

$$\hat{h}_t^F = \left(1 - \alpha^F \beta \bar{\Pi}^{\varepsilon - 1} \right) \left(\hat{c}_t^F - \sigma \hat{c}_t \right) + \left(\alpha^F \beta \right) \bar{\Pi}^{\varepsilon - 1} E_t \left[\varepsilon \hat{\pi}_{F,t+1} - \hat{\pi}_{t+1} + \hat{h}_{t+1}^F \right] \tag{2.68}$$

and, from (2.11),

$$\hat{c}_{F,t} = \hat{c}_t - \theta \tilde{p}_{F,t}. \tag{2.69}$$

For country F, the producer's NKPC is log-linearised analogously to the producer's NKPC of the SOE. The large economy works as in autarky, (imports and exports of this country can be seen as negligible,) so that this NKPC is identical to the one derived in Chapter 1 (and are reported here for convenience). The market clearing condition is

$$\hat{y}_t^* = \hat{c}_t^*, \tag{2.70}$$

the Euler equation is

$$\hat{c}_t^* = E_t\left[\hat{c}_{t+1}^*\right] - \frac{1}{\sigma}\left(\hat{\imath}_t^* - E_t\left[\hat{\pi}_{t+1}^*\right] + E_t\left[\Delta\varepsilon_{t+1}^*\right]\right), \tag{2.71}$$

the Phillips curve with a backward looking and non-zero inflation component is

$$\hat{\pi}_t^* = \chi_f^* E_t\left[\hat{\pi}_{t+1}^*\right] + \chi_b^* \hat{\pi}_{t-1}^* + \kappa_{mc}^*\left(\widehat{mc}_t^* + v_t^*\right) + \chi_\pi^*\left[\hat{h}_t^* + (\sigma - 1)\,\hat{y}_t^*\right], \tag{2.72}$$

where

$$\hat{h}_t^* = \left(1 - \alpha\beta\bar{\Pi}^{\varepsilon-1}\right)\left(\hat{y}_t^* - \sigma\hat{c}_t^*\right) + (\alpha\beta)\,\bar{\Pi}^{\varepsilon-1} E_t\left[(\varepsilon - 1)\,\hat{\pi}_{t+1}^* + \hat{h}_{t+1}\right] \tag{2.73}$$

and the marginal costs are

$$\widehat{mc}_t^* = (\eta + \sigma)\,\hat{y}_t^* - (1 + \eta)\,a_t^*. \tag{2.74}$$

To estimate this model all that is needed now is a monetary policy rule. In this chapter, I use simple interest rate rules of a Taylor type with producer inflation targeting and consumer inflation targeting, and strict exchange rate targeting. The monetary policy rules are described in more details below.

2.3.4 Monetary Policy Rules

To close the model, I need to specify the policy chosen by the monetary authority. For estimation purposes, most of the recent papers, *e.g.,* Smets and Wouters (2003), use a generalised Taylor rule, where the central bank systematically responds to the changes in inflation, output and, in the case of a SOE, to the exchange rate. Analysing the effect of simple rules has some advantages relative to the optimal monetary policy, as they are more likely to be used in practice because they are more easily implemented. Additionally, their parameters are more robust to the model specification than the structural parameters of the optimal rule. The best known example of a simple nominal interest rate rule is the Taylor rule, which uses the interest rate as the instrument to implement the policy.

This paper compares a number of different simple targeting rules of the Taylor type for both economies. For the relatively large closed economy, three monetary policy rules are analysed. The first one is a common Taylor rule with an interest rate smoothing component, $\rho_i^* \hat{\imath}_{t-1}^*$, which is typically used in the literature to improve the fit of the empirical estimation as it incorporates observed interest rate persistence. The rule has the following form

$$\hat{\imath}_t^* = \rho_i^* \hat{\imath}_{t-1}^* + \phi_\pi^* \hat{\pi}_t^* + \phi_y^* \hat{y}_t^* + \varepsilon_{u,t}^*, \tag{2.75}$$

where $\varepsilon_{u,t}^*$ is an exogenous monetary policy shock. Alternatively, following Smets and Wouters (2003), the central bank also responds to the speed of inflation $\Delta\pi_t^*$

$$\hat{\imath}_t^* = \rho_i^* \hat{\imath}_{t-1}^* + \phi_\pi^* \hat{\pi}_t^* + \phi_y^* \hat{y}_t^* + \phi_{\Delta\pi}^* \Delta\hat{\pi}_t^* + \varepsilon_{u,t}^*. \tag{2.76}$$

The third analysed rule takes the form of the optimal monetary policy rule identified using a welfare loss function from Chapter 1, where I approximate the optimal behaviour of the central bank

$$\hat{\imath}_t^* = \rho_i^* \hat{\imath}_{t-1}^* + \phi_\pi^* \hat{\pi}_t^* + \phi_y^* \hat{y}_t^* + \phi_{\Delta 1}^* \Delta\hat{\pi}_t^* + \phi_{\Delta 2}^* \Delta\hat{\pi}_{t+1}^* + \phi_{\Delta y}^* \Delta\hat{y}_t^* + \varepsilon_{u,t}^*. \tag{2.77}$$

The aim of using three different rules is to find out whether the European central bank targets acting as the large economy in this model, conducts monetary policy using a simple Taylor rule (2.75) or incorporates any of the additional terms in (2.76) and (2.77). I choose the rule that best fits for each case, when modeling the economies of the Czech Republic, Hungary or Poland.

For these small economies I again specify the three main monetary policy rules, but modified for a SOE. The first one is similar to (2.76), where in addition to the traditional Taylor Rule, the central bank targets the change in inflation and in the exchange rate

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \phi_\Delta \Delta\hat{\pi}_t + \phi_S \Delta\hat{s}_t + \varepsilon_{u,t}. \tag{2.78}$$

The second rule is analogous to the rule of optimal type (2.77)

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \phi_{\Delta 1} \Delta\hat{\pi}_t + \phi_{\Delta 2} \Delta\hat{\pi}_{t+1} + \phi_{\Delta y} \Delta\hat{y}_t + \phi_S \Delta\hat{s}_t + \varepsilon_{u,t}. \tag{2.79}$$

Alternatively, I also assume that the central bank targets exchange rate strictly, following Adolfson, Laséen, Lindé, and Villani (2008)

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_S \Delta \hat{s}_t + \varepsilon_{u,t}. \tag{2.80}$$

I am interested in answering two main questions regarding the monetary policy rule for a SOE. First, there are many studies that modify the simple instrumental rule to match the needs of a small open economy. Although the theoretical work emphasises that a targeting PPI inflation performs better in terms of welfare loss, the empirical literature usually assumes a simple rule with consumer inflation targeting. In fact, by moving the interest rate, the central bank can either target producer domestic inflation or CPI inflation. However, Galí and Monacelli (2005) as well as Sutherland (2002) point out that if the economy's non-stochastic steady state is at its optimum and no (or only very small) cost push distortions are present, the optimal monetary policy is pure domestic inflation targeting (*e.g.*, $\hat{\pi}_{H,t} = 0$). Strict producer-price targeting has a smoother effect on domestic variables without any distortion to the foreign economy. However, Sutherland also argues that when cost push shocks have larger variance, CPI targeting may obtain better results.

To investigate whether the central bank targets domestic producer inflation instead of CPI inflation, I compare (2.78) and (2.79) with the corresponding rules in terms of PPI inflation, simply obtained by replacing $\hat{\pi}_t$ with $\hat{\pi}_{H,t}$, and reported here for convenience

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_\pi \hat{\pi}_{H,t} + \phi_y \hat{y}_t + \phi_\Delta \Delta \hat{\pi}_{H,t} + \phi_S \Delta \hat{s}_t + \varepsilon_{u,t} \tag{2.81}$$

and

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + \phi_\pi \hat{\pi}_{H,t} + \phi_y \hat{y}_t + \phi_\Delta \Delta\hat{\pi}_{H,t} + \phi_{\Delta 2} \Delta\hat{\pi}_{H,t+1} + \phi_{\Delta y} \Delta\hat{y}_t + \phi_S \Delta\hat{s}_t + \varepsilon_{u,t}. \quad (2.82)$$

As I show later, in both cases, the difference in the model fit is significant.

Second, following Lubik and Shorfheide (2007), I study to what extent the central banks of the EEC countries respond not only to the changes in inflation and output, but also to the changes in inflation and exchange rate, *e.g.*, whether the parameter ϕ_S plays an important rule. I compare the simple rules (2.81) and (2.82) with their equivalents by assuming that $\phi_S = 0$.

Summary of the model and exogenous disturbances

To summarise, the model consists of a non-policy part determined by equations (2.55) to (2.74), a monetary policy rule specified above and a set of exogenous shocks, which follow an autoregressive process given in a log-linearised form.

The country-specific TFP for domestic and foreign country are defined respectively by

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \varepsilon_{a,t}, \\ a_t^* &= \rho_{a^*} a_{t-1}^* + \varepsilon_{a,t}^*; \end{aligned}$$

the preference innovations are given for domestic and foreign consumers respectively by

$$\begin{aligned}\epsilon_t &= \rho_e \epsilon_{t-1} + \varepsilon_{e,t}, \\ \epsilon_t^* &= \rho_{e^*} \epsilon_{t-1}^* + \varepsilon_{e,t}^*.\end{aligned}$$

Finally, the cost push for domestic producers and for domestic retailers are expressed by

$$\begin{aligned}v_t &= \rho_v v_{t-1} + \varepsilon_{v,t}, \\ v_t^F &= \rho_{v^F} v_{t-1}^F + \varepsilon_{v^F,t},\end{aligned}$$

whereas for foreign producers they are

$$v_t^* = \rho_{v^*} v_{t-1}^* + \varepsilon_{v,t}^*.$$

The stochastic AR(1) processes are driven by exogenous shocks, of which seven are white noise, $\varepsilon_{a,t}$, $\varepsilon_{a,t}^*$, $\varepsilon_{e,t}$, $\varepsilon_{e,t}^*$, $\varepsilon_{v,t}$, $\varepsilon_{v^F,t}$, $\varepsilon_{v,t}^*$, plus two exogenous monetary policy shocks, $\varepsilon_{u,t}$ and $\varepsilon_{u,t}^*$, and one measurement error, $\varepsilon_{rs,t}$.

2.4 Model Estimation and Estimation Results

This section illustrates the estimation of the model, and is divided into three parts. First, I discuss the Bayesian methodology and estimation technique I use in detail. Then, after a brief look at the data, I describe my choice of priors in the context of the existing literature on this field. Finally, I present the estimation results, including the posterior distribution, impulse responses and variance decomposition.

The estimated model consists of a set of equilibrium equations. All equations are log-linearised, and the variables are expressed in terms of the deviation from their respective steady state levels, both for the small and the large economy, as described in a previous sections. The small open economy case is estimated on data from the EEC countries, such as the Czech Republic, Hungary and Poland. The large economy is represented by Germany.

2.4.1 Methodology

For the empirical analysis of my DSGE model, I adopt a Bayesian estimation approach, which has many advantages. First, the Bayesian approach allows me to incorporate priors based on theoretical considerations or other research. Second, the Bayesian approach is a full information method in contrast to a single equation method such as GMM and therefore it is more likely to produce better estimates.[8] Furthermore, using the estimated log data density of the model, facilitates comparisons of the goodness of fit of different models. In comparison to the Gibbs Sampling method, which I use in Chapter 3 to estimate a structural VAR model, the estimation of a DSGE model requires a more general algorithm. The reason is that the conditional posterior distributions are not available. Following most of the literature, I use a random walk Metropolis-Hasting algorithm to approximate the posterior distribution of the estimated parameters that I briefly describe below.[9]

Suppose that the aim is to draw a sample from a target density $\pi(\Phi)$. Note that Φ is a $(K \times 1)$ vector of parameters of interest. The target density is a

[8] See Linde (2005).

[9] For more details, see An and Schorfheide (2007) and Blake and Mumtaz (2012).

posterior distribution, which is too complex to allow a direct sample. Therefore an indirect method is needed. The steps describing a random walk Metropolis-Hastings algorithm are the following:

1. Set a prior distribution for each parameter $p(\Phi)$.

2. Find the mode of the posterior distribution $\pi(\Phi)$ via numerical maximisation. Denote the estimates of the parameters at the mode by $\Phi^{\max}$, and their covariance matrix, which is the inverse Hessian matrix, by $H^{\max}$.

3. To approximate $\pi(\Phi)$, the following algorithm is used:

   (a) Specify a candidate density $q\left(\Phi^{G+1}/\Phi^{G}\right)$, where G is an index of draws.

   (b) Set the initial estimates of the parameters Φ^{G} with $G = 0$.

   (c) Generate a candidate value Φ^{G+1} from the candidate density. I use a random walk version of this algorithm with the candidate density specified as a random walk, such as

   $$\Phi^{G+1} = \Phi^{G} + e,$$

   where e is a K-vector random walk with a normal distribution

   $$e \sim N(0, \Sigma).$$

   (d) Compute the acceptance probability. The candidate Φ^{G+1} is accepted with probability α, given by

   $$\alpha = \min\left(\frac{\pi\left(\Phi^{G+1}\right)/q\left(\Phi^{G+1}/\Phi^{G}\right)}{\pi\left(\Phi^{G}\right)/q\left(\Phi^{G}/\Phi^{G+1}\right)}; 1\right),$$

where the numerator is the target density evaluated at the new draw of the parameters $\pi\left(\Phi^{G+1}\right)$ relative to the candidate density evaluated at the new draw parameters $q\left(\Phi^{G+1}/\Phi^{G}\right)$, and the denominator is the same expression evaluated at the previous draw of the parameters. Again, using a random walk version together with the fact that the normal distribution is symmetric, the acceptance probability simplifies to

$$\alpha = \min\left(\frac{\pi\left(\Phi^{G+1}\right)}{\pi\left(\Phi^{G}\right)}; 1\right).$$

Step 3 is repeated M times. The first $(M-J)$ iterations are discarded. The last J draws are instead retained to estimate the posterior marginal distribution. For the results, I use four chains of $M = 200,000$ draws, each starting from a different value. From each chain, the last $J = 0.55 \times M$ draws are used to approximate the empirical distribution of the parameters.

Note that using Gibbs sampling, used in Chapter 3, the probability α is equal to 1 for every draw, because the target density is known and identical for all draws. However, using the Metropolis-Hastings algorithm, the acceptance rate depends on the variance Σ, which is set manually. It holds that the higher the variance, the more volatile the drawings. Therefore, a lower acceptance is to be expected in this case. On the other hand, if Σ is set too low, the volatility of the drawings is low as well. Therefore, the estimation of the parameters is likely to be close to the prior. Drawing a random number u from a uniform distribution $u \sim U(0,1)$, it holds that the candidate Φ^{G+1} is accepted if $\alpha > u$, otherwise it is rejected.

The acceptance rate, given as the ratio between the accepted draws and the total number of draws, should lie between 20% and 40%. Some researchers are more specific and suggest that, for multivariate estimations, the acceptance rate should optimally be set to approximately 23%. The convergence of the chains is checked according to a Brooks and Gelman (1998) convergence diagnostic. The visual comparison between chains variance of the main results for selected estimations are shown in Figures 2.1 and 2.2 and described in more details in Section 2.4.3.

I use posterior odds test to compare the performance across models. Assume the null hypothesis that a model M1 is preferred to a model M2. The marginal data density is given for M1 by $\pi_{0,T}$, and for M2 by $\pi_{1,T}$. The posterior odds test is computed as the ratio of the marginal data density of M1 to M2. Following Lubik and Schorfheide (2007), the posterior odds can be interpreted as follows:

- $\frac{\pi_{0,T}}{\pi_{1,T}} > 1$, the null hypothesis is supported;
- $1 > \frac{\pi_{0,T}}{\pi_{1,T}} > 10^{-1/2}$, there is only indecisive evidence against the null hypothesis;
- $10^{-1/2} > \frac{\pi_{0,T}}{\pi_{1,T}} > 10^{-1}$, there is substantial evidence against the null hypothesis;
- $10^{-1} > \frac{\pi_{0,T}}{\pi_{1,T}} > 10^{-3/2}$, there is strong evidence against the null hypothesis;
- $10^{-3/2} > \frac{\pi_{0,T}}{\pi_{1,T}} > 10^{-2}$, there is very strong evidence against the null hypothesis;
- $10^{-2} > \frac{\pi_{0,T}}{\pi_{1,T}}$, there is decisive evidence against the null hypothesis.

2.4.2 Choice of Prior

The model represented in the theoretical part of this chapter has 27 endogenous macro variables and for the empirical estimation I use 9 time series. It is based on a data sample over the period 1996 to 2012 for Germany and the Czech Republic, and 1998 to 2012 for Hungary and Poland. The sources of the raw data are Datastream and the Fred database and the details on each of the particular time series are given in Appendix 2.A. I use variables that are common in the literature, such as inflation, output growth, interest and exchange rate. Additionally, I follow Sbordone (2002) and Galí and Gertler (1999), who estimate the NKPC using unit labour costs as a proxy for real marginal costs. Most of the empirical papers take the marginal costs as a latent variable and, as Schorfheide (2008) describes, the estimation results on the NKPC parameters may vary significantly, however these authors show that unit labour costs are more appropriate measure for the NKPC than the output gap. Additionally, it is worth to mention, that the number of time series is lower than number of shocks to prevent problem of stochastic singularity.

The corresponding measurement equation is given as

$$\begin{aligned} Y_t &= \begin{pmatrix} \Pi_{H,t}^{OBS} & Y_t^{OBS} & MC_t^{OBS} & i_t^{OBS} & S_t^{OBS} & \Pi_t^{*OBS} & Y_t^{*OBS} & MC_t^{*OBS} & i_t^{*OBS} \end{pmatrix}^T \\ &= \begin{pmatrix} \bar{\Pi} & 0 & 0 & R & 0 & \bar{\Pi}^* & 0 & 0 & R^* \end{pmatrix}^T \\ &\quad + \begin{pmatrix} \pi_{H,t} & \hat{y}_t - \hat{y}_{t-1} & mc_t - mc_{t-1} & i_t & s_t - s_{t-1} & \pi_t^* & \hat{y}_t^* - \hat{y}_{t-1}^* & mc_t^* - mc_{t-1}^* & i_t^* \end{pmatrix}^T \end{aligned}$$

Note that for the A1 approach, the value of steady state domestic and foreign inflation $\bar{\Pi} = \bar{\Pi}^*$ is nil.

I choose Bayesian estimation over maximum likelihood estimation because it permits me to incorporate a prior distribution. Incorporating priors means introducing additional general information about subjective beliefs of the parameter distribution, or information coming from previous econometric and theoretical studies. In the case that just a small sample of data is available, a prior distribution is additional information that enables more stability in the optimisation algorithm. However, selecting an appropriate prior is one of the most difficult tasks associated with the use of the Bayesian approach.

I use German data to estimate the parameters for the large economy. The selection of the prior distribution follows closely Smets and Wouters (2003), and are represented in Table 2.1. For parameters that are restricted to the interval $(0, 1)$, I use a Beta distribution. Non-negative parameters are then Gamma distributed. As for the autoregressive parameters of the shocks, I use a Beta distribution with a mean of 0.8 and a standard deviation of 0.1. The variances of the shocks are inverse gamma, with prior distribution $\sigma^2 \sim \Gamma^{-1}(1, 10)$. The standard errors are set such that the domain covers a reasonable range of parameter values.

The priors for the interest rate rule coefficients have rather wide confidence intervals. They are distributed around a mean given by the Taylor rule, following Lubik and Schorfheide (2005). Additionally, the prior distribution for the parameter ϕ_π has a lower bound of one, to satisfy the Taylor principle. Priors for the rest of the parameters in the monetary policy rule are Gamma distributed, with mean and standard error as those chosen by Smets and Wouters (2003) and Lubik and Schorfheide (2005).

Parameter	Distribution	Mean	Standard error
$\sigma\left(\varepsilon_a^*\right)$	Inverse Gamma	1	10
$\sigma\left(\varepsilon_e^*\right)$	Inverse Gamma	1	10
$\sigma\left(\varepsilon_v^*\right)$	Inverse Gamma	1	10
$\sigma\left(\varepsilon_u^*\right)$	Inverse Gamma	1	10
ρ_a^*	Beta	0.8	0.1
ρ_e^*	Beta	0.8	0.1
ρ_v^*	Beta	0.8	0.1
ρ_i^*	Beta	0.5	0.2
ϕ_π^*	Gamma	1.5	0.1
ϕ_y^*	Gamma	0.125	0.05
$\phi_{\Delta 1}^*$	Gamma	0.3	0.1
$\phi_{\Delta 2}^*$	Gamma	0.3	0.1
$\phi_{\Delta y}^*$	Gamma	0.0625	0.05
χ_f^*	Beta	0.5	0.2
χ_b^*	Beta	0.5	0.2
κ_{mc}^*	Gamma	0.1	0.05
χ_π^*	Normal	0	0.05
Π	Gamma	1.005	0.003

Table 2.1: Prior Distribution for Large Economy

Parameter	Distribution	Mean	Standard error
$\sigma(\varepsilon_a)$	Inverse Gamma	1	10
$\sigma(\varepsilon_e)$	Inverse Gamma	1	10
$\sigma(\varepsilon_u)$	Inverse Gamma	1	10
$\sigma(\varepsilon_v)$	Inverse Gamma	1	10
$\sigma(\varepsilon_{v^F})$	Inverse Gamma	1	10
$\sigma(\varepsilon_{rs})$	Inverse Gamma	1	10
ρ_a	Beta	0.8	0.1
ρ_e	Beta	0.8	0.1
ρ_v	Beta	0.8	0.1
ρ_{v^F}	Beta	0.8	0.1
ρ_i	Beta	0.5	0.2
ϕ_π	Gamma	1.5	0.1
ϕ_y	Gamma	0.125	0.05
$\phi_{\Delta 1}$	Gamma	0.3	0.1
$\phi_{\Delta 2}$	Gamma	0.3	0.1
$\phi_{\Delta y}$	Gamma	0.0625	0.05
ϕ_S	Gamma	0.3	0.1
χ^f	Beta	0.5	0.2
χ^b	Beta	0.5	0.2
κ_{mc}	Gamma	0.1	0.05
χ^π	Normal	0	0.05
χ_F^f	Beta	0.5	0.2
χ_F^b	Beta	0.5	0.2
κ_F	Gamma	0.1	0.05
χ_F^π	Normal	0	0.05
Π	Gamma	1.005	0.003

Table 2.2: Prior Distribution for Small Open Economy

Following Lubik and Schorfheide (2007), I estimate the composite structural coefficients of the NKPC rather than the underlying primitives, to avoid identification issues. The values of the NKPC parameters χ^b, χ^f and κ_{mc} reported in the literature are controversial. Therefore, the priors chosen here are consistent with the middle case, with a standard deviation large enough to ensure that the estimate is mainly determined by the data. Consistent with Lubik and Schorfheide (2007), the parameters χ^b, χ^f are beta distributed, and the parameter κ_{mc} is gamma distributed. The minimum level of the prior is consistent with the findings by Galí and Gertler (1999). The parameter χ^π is normally distributed around a zero mean, since it might take both positive and negative values. The prior of the inflation trend Π is gamma distributed around the average of the trend value, given by the HP filter, and it is lower-bounded at one. For Germany, the average inflation of the estimated sample corresponds to $\Pi = 1.005$.

The parameters for the SOE have similar priors as those for the closed economy. The priors for the importer NKPC parameter are set analogously to the producer NKPC. The prior for ϕ_S is gamma distributed, with mean equal to 0.3. The steady state inflation Π is the trend inflation given the HP filter for the observed period. It is the same for the Czech Republic and Germany, and for Hungary and Poland, it corresponds to $\Pi = 1.0153$ and $\Pi = 1.0154$, respectively. The degree of openness λ is set to 0.6 for the Czech Republic, corresponding to the average Import/GDP ratio over the data sample. For Hungary and Poland, it is set to be 0.7 and 0.36, respectively.

Most of the parameters are not imposed to be the same for all countries, but it is merely assumed that they have identical priors. This also mirrors the fact that the countries have a similar economic history and have undergone similar structural changes since the end of the Cold War. Some parameters are identical for all countries. For example, the parameter β, which is fixed and not estimated. Instead I follow the convention and set it at 0.99.

2.4.3 Estimation Results

The composite structural parameters are estimated in two steps. The first step contains the estimation of the model for the closed economy, obtained using German data. The estimation for Germany can also be seen as the empirical estimation of the model from Chapter 1. In this part, I focus on three main issues. First, I generally estimate the NKPC for Germany, and show the importance of the backward looking component. Second, I am interested in whether the estimate for χ_π^* is significant. In other words, if the assumption of non-zero inflation in steady state improves the fit to the data. The third issue, which is important for further estimation and analysis of different simple rules, is to find the one rule that fits best the German data.

In the second part, the model for the SOE is estimated, using the data from EEC. I use the best fitting monetary policy rule for the closed economy, and estimate domestic and foreign parameters using EEC and German data together. Along with the estimates for the SOE Phillips curve, where I analyse the importance of the non-zero inflation part of the Phillips curve given by parameters χ_π and $\chi_{\pi^F t}$, I wish to identify what monetary policy fits the data best. Therefore, I first investigate whether the EEC central bank responds to a CPI inflation,

Monetary policy rule	Log Data Density		Posterior odds
	A1	A2	
Rule 1 (2.75)	−178.15	−173.8	0.013
Rule 2 (2.76)	−170.67	−161.55	0.000
Rule 3 (2.77)	−166.39	−156.05	0.000

Table 2.3: Posterior Odd Test

Note: the table reports posterior odds test for German data on the hypothesis H0: $\chi_\pi^* = 0$ against the alternative $\chi_\pi^* \neq 0$.

I then show that the data suggests that PPI inflation targeting performs better. I then concentrate on understanding how important are the exchange rate movements in the simple rules for the central bank, and whether the EEC central banks systematically respond to such changes.

Results for Germany

In this section, I use three different simple rules for the closed economy, specified in (2.75), (2.76) and (2.77). I estimate each of them applying two different approaches, to assess the importance of the estimation of the non-zero steady state inflation part in the NKPC. The first approach (A1) assumes that the steady state inflation is zero, as is common in the literature, which leads to a backward looking NKPC with $\chi_\pi^* = 0$. The second approach (A2) estimates the parameter χ_π^* as well as the steady state inflation Π. The log marginal data densities and the odds for these two specifications are portrayed in Table 2.3. The chains converge to the target distribution for all estimations. Figure 2.1 reports the convergence

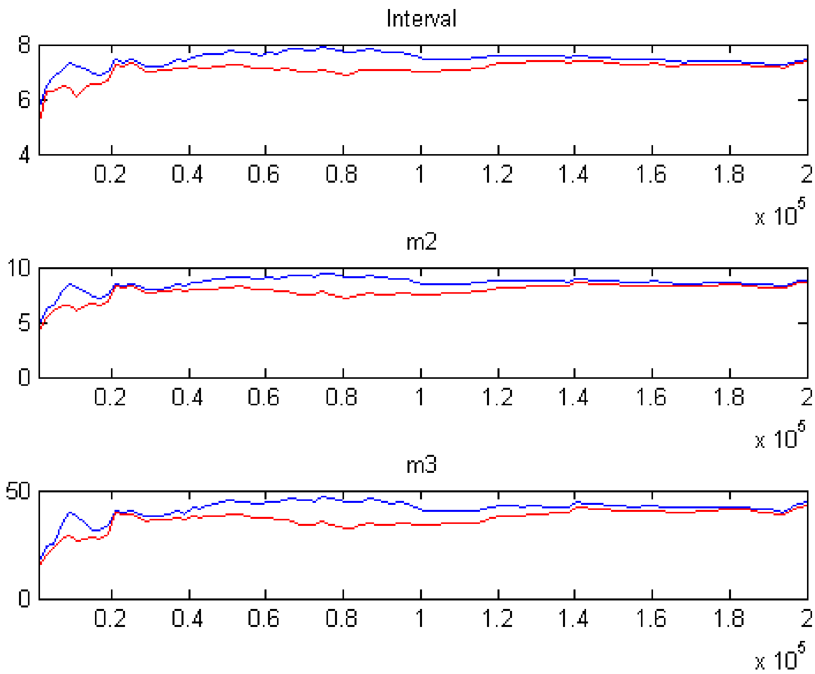

Figure 2.1: Multivariate Convergence Diagnostic for Germany

diagnostic for the estimation using the second approach and rule (2.77). There are three measures in each figure. Interval refers to an 80 % confidence interval around mean, m2 refers to the variance measure and m3 is based on the third moment of the aggregate measure. The convergence of the chains to the target distribution occurs if the between-chain measure (blue line) and the within-chain measure (red line) are relatively constant and converge.

Two results emerge from the analysis of the log marginal likelihood and posterior odds. First, the estimation of the model with the second approach improves the fit to the data relatively to imposing a steady state rate of inflation that is zero. The posterior odds show that the hypothesis H0 of the steady state zero inflation can be rejected. Thus, this approach is used also for the SOE estimation in step 2. Second, the monetary policy rule (2.77) is clearly the best fit for the data. It follows that the more complex the rule is, the better the performance of the model. The traditional Taylor rule from (2.75) performs worse, whereas the "optimal" simple rule fits the data best. This evidence suggests that the central bank takes into account all the elements following from the welfare maximisation of the loss function, as derived in presence of backward looking firms, as done in Chapter 1. Given the log density, it is shown that including inflation change targeting improves the fit significantly.

The Bayesian estimated posterior distribution, based on the second approach and the monetary rule (2.77), is reported in Table 2.4. The table displays the mode and standard error resulting from the posterior maximisation. It also details the estimation results obtained through the Metropolis-Hastings algorithm, such as the posterior mean and the 90% posterior probability interval for both the estimated parameters and the standard deviation of shocks.

For all values, the highest posterior density intervals suggest that the estimated parameters are not equal to zero. Focusing on the two parameters that show how important is the non-zero steady state inflation, Table 2.4 shows that my estimation proposes a value around 0.2 for parameter χ_{π}^{*}, which is higher than that assumed in the prior distribution; and a value around 1.005 for the estimated trend inflation Π, implying a steady state rate of inflation of 2% percent per year. The values are robust and lie in the confidence interval using both approaches. The estimates for the parameter χ_{π}^{*} are lower when assuming the simple Taylor rule (2.75) – around 0.13 for both approaches. For the remaining two other rules,

Parameter	**Mode**	**S.D.**	**10%**	**Mean**	**90%**
$\sigma\left(\varepsilon_a^*\right)$	1.0194	0.0790	0.9022	1.0321	1.1597
$\sigma\left(\varepsilon_e^*\right)$	5.5972	0.6983	3.0235	6.8599	11.2815
$\sigma\left(\varepsilon_v^*\right)$	0.3324	0.0515	0.28711	0.3526	0.4181
$\sigma\left(\varepsilon_u^*\right)$	0.5434	0.0417	0.3976	0.5278	0.6536
ρ_a^*	0.9944	0.0052	0.9866	0.9924	0.9985
ρ_e^*	0.9802	0.0064	0.9699	0.9802	0.9937
ρ_v^*	0.8417	0.0128	0.7074	0.8168	0.9313
ρ_i^*	0.9588	0.0256	0.8988	0.9418	0.9892
ϕ_π^*	1.4353	0.0316	1.3081	1.4642	1.6021
ϕ_y^*	0.0199	0.0055	0.0100	0.0232	0.0361
$\phi_{\Delta 1}^*$	0.5348	0.0195	0.2440	0.4504	0.6584
$\phi_{\Delta 2}^*$	0.3584	0.0328	0.1939	0.3845	0.5728
$\phi_{\Delta y}^*$	0.0716	0.0074	0.0084	0.0995	0.1724
χ_f^*	0.9452	0.0352	0.8041	0.8970	0.9889
χ_b^*	0.3026	0.0566	0.1288	0.2717	0.4158
κ_{mc}^*	0.4861	0.0110	0.4860	0.6213	0.7924
χ_π^*	0.2709	0.0244	0.1569	0.2248	0.2975
Π	1.0033	0.0005	1.0006	1.0045	1.0083

Table 2.4: Parameter Estimation Results for Germany

	Log Data Density		Czech Rep.	Hungary	Poland
A2	CPI targeting, $\phi_S > 0$	Rule 1 (2.78)	-659.26	-630.38	-740.57
		Rule 2 (2.79)	-659.94	-633.51	-723.96
	PPI targeting, $\phi_S > 0$	Rule 1 (2.81)	-637.50	-603.98	-714.12
		Rule 2 (2.82)	-640.11	-595.45	-705.25
	PPI targeting, $\phi_S = 0$	Rule 1	-641.27	-602.56	-711.64
		Rule 2	-648.80	-594.13	-705.05
	Pure exchange rate	Rule 3 (2.80)	-706.85	-630.78	-842.91
A1	PPI targeting, $\phi_S > 0$	Rule 2 (2.82)	-646.27	-604.06	-721.94

Table 2.5: Marginal Data Densities under Different Approaches and Monetary Policy Rules Regimes

the values are surprisingly stable, and lie between 0.22 and 0.26. The result for the steady state inflation Π is very similar for all three monetary policy rules in the second approach.

My estimate suggests a value of lagged inflation χ_b^* of around 0.3, in line with other empirical findings such as Galí and Gertler (1999) and Galí, Gertler, and Lopez-Salido (2001). With the exception of the cost push shock, all the autoregressive parameters for the shocks are estimated to be higher than the value of 0.8 assumed in the prior distribution. Surprisingly, the TFP shock is also very persistent, with the AR parameter around 0.99, a much higher value than the 0.83 estimated by Smets and Wouters (2003). Moreover, the monetary policy rules parameters are very robust and they all lie, independent of the estimation approach and rule, in the confidence interval given in Table 2.4. These parameters are all consistent with the values found in the literature.

Results for European Emerging Markets

In this section, I analyse the monetary policy rules for the SOE from Section 2.3.4. I use different assumptions to understand the behavior of the central banks in the EEC. The summary of the marginal data densities from the different estimations can be found in Table 2.5. The results of the estimations are explained below.

I report the results obtained using the second approach outlined in the previous subsection, *i.e.*, assuming that the steady state inflation differs from zero, which provides significantly better results than those delivered by the first approach.[10] It is straightforward to demonstrate that a pure exchange rate targeting policy can be rejected as the policy being implemented by at least two of the three countries, since this rule performs the worst for both Czech and Polish data. Adolfson *et al.* (2008) reach a similar conclusion investigating the Swedish economy.

Finally I test whether the central bank targets CPI or PPI inflation. The results of the posterior odds test, with a null hypothesis that the central bank is focusing on CPI inflation rather the PPI inflation, are displayed in Table 2.6. The null hypothesis can be rejected for both rules and all countries. I can thus conclude that there is a clear evidence in favor of PPI inflation targeting over CPI inflation targeting. This is in line with the theoretical literature, which shows that responding to the PPI inflation rather than the CPI delivers lower welfare losses.

[10] For comparison purposes, the best fit obtained using the first method is also reported.

	Rule 1			Rule 2		
	H0	H1	Post. Odds	H0	H1	Post. Odds
Czech Rep	−659.26	−637.50	0.000	−659.94	−640.11	0.000
Hungary	−630.38	−603.98	0.000	−633.51	−595.45	0.000
Poland	−740.57	−714.12	0.000	−723.96	−705.25	0.000

Table 2.6: Posterior Odd Test

Notes: hypothesis H0 that the central bank uses a CPI inflation targeting (2.78) and (2.79) vs hypothesis H1 that the central bank uses (2.81) and (2.82).

I then perform the posterior odds test to show how important it is to include the non-zero component into the Phillips curve. Similar to what I did for Germany, I estimate the model for both rules (2.81) and (2.82). On the one hand, I assume that $\chi^{\pi} = 0$. On other hand, my estimations are obtained when assuming that $\chi^{\pi} \neq 0$. The marginal data densities displayed in Table 2.7 suggest that including an estimation of χ^{π} improves the fit to the data. The posterior odds ratio is zero in all cases, rejecting the null hypothesis that χ^{π} equals zero for all three countries.

Furthermore, I am interested in whether the central bank responds to changes in the exchange rate. To answer this question, I follow Lubik and Schorfeide (2007) and first estimate both rules (2.81) and (2.82) assuming that $\phi_S > 0$. Second, I estimate the same rules, but assume that the central bank is not interested in exchange rate targeting, and set $\phi_S = 0$. The null hypothesis is that the central bank does not respond to the exchange rate changes. The results for both the TR2 and TR3 rule are given in Table 2.8. The null hypothesis can be rejected

	H0	H1	Posterior Odds
Czech Rep	-646.27	-640.11	0.002
Hungary	-604.06	-595.45	0.000
Poland	-721.94	-705.25	0.000

Table 2.7: Posterior Odd Test

Note: The table reports posterior odds test for EEC on the hypothesis H0: $\chi_\pi = 0$ and $\chi_{\pi^F} = 0$ against the alternative $\chi_\pi \neq 0$ and $\chi_{\pi^F} \neq 0$.

only for the Czech Republic. This suggests that the Czech National Bank targets the exchange rate, but the Central Banks of Hungary and Poland do not.

The estimated parameters are similar for all three countries. They can be found in Tables 2.9 - 2.11. For the estimation using the second approach, with reference to the Czech Republic, the convergence diagnostic is illustrated in Figure 2.2.[11]. As the convergence diagnostics suggest, all the chains converge to the target distribution. The backward looking component for producer inflation lies between 0.2 and 0.35 for all countries. Compared to Germany, the non-zero steady state inflation component is lower, but still positive and significantly different from zero. For the retailers' Phillips curve, the parameter χ_F^π is slightly negative for Czech Republic and Hungary, and all are significantly different from zero. The prior distribution, the posterior distribution and the posterior mode of these two parameters is visually illustrated in Figure 2.3 and 2.4.

[11]The remaining diagnostic illustrations are available from the author upon request.

	H0	H1	Posterior Odds
Czech Rep	−641.27	−637.50	0.023
Hungary	−602.56	−603.98	4.161
Poland	−711.64	−714.12	11.876

Table 2.8: Posterior Odd Test

Note: The table reports posterior odds test for EEC on the hypothesis H0: $\phi_S = 0$ against the alternative $\phi_S \neq 0$.

Finally, the monetary policy rule parameters are close to those reported in the literature. The central bank of all three countries respond much more actively to inflation (both to current and past changes) than to output (and its change). The estimates for exchange rate targeting in the monetary policy rule are higher, for all the three countries, than the prior values. ($\phi_S^{cz} = 0.14$, $\phi_S^{Hun} = 0.15$, $\phi_S^{Pol} = 0.11$).

Parameter	Mode	S.D.	10%	Mean	90%
$\sigma(\varepsilon_a)$	0.7227	0.1556	0.5380	0.7714	0.9883
$\sigma(\varepsilon_e)$	3.0276	0.4826	2.3178	3.1085	3.9074
$\sigma(\varepsilon_u)$	1.5490	0.1977	1.2970	1.6581	2.0258
$\sigma(\varepsilon_v)$	1.7389	0.2642	1.3114	1.5946	1.8639
$\sigma(\varepsilon_{v^F})$	10.7510	3.0950	0.2225	10.6040	20.3485
$\sigma(\varepsilon_{rs})$	4.9429	0.4970	4.2216	5.0141	5.7741
ρ_a	0.9247	0.0110	0.7732	0.8873	0.9888
ρ_e	0.8766	0.0176	0.8282	0.8763	0.9307
ρ_v	0.7025	0.0237	0.6174	0.7254	0.8318
ρ_{v^F}	0.8752	0.0225	0.7582	0.8508	0.9686
ρ_i	0.9256	0.0416	0.8079	0.8948	0.9843
ϕ_π	1.3994	0.0314	1.3291	1.4661	1.6052
ϕ_y	0.0539	0.0176	0.0229	0.0640	0.1049
$\phi_{\Delta 1}$	0.3508	0.0376	0.2291	0.3760	0.5248
$\phi_{\Delta 2}$	0.3387	0.0239	0.1735	0.3276	0.4908
$\phi_{\Delta y}$	0.1024	0.0118	0.0013	0.0476	0.0929
ϕ_S	0.1392	0.0210	0.0871	0.1440	0.2050
χ^f	0.9077	0.0364	0.6890	0.8258	0.9671
χ^b	0.3063	0.0318	0.1148	0.2788	0.4334
κ_{mc}	0.2922	0.0123	0.3357	0.4149	0.5098
χ^π	0.1036	0.0191	0.0130	0.0770	0.1472
χ^f_F	0.6355	0.1010	0.2573	0.5403	0.7991
χ^b_F	0.1928	0.0260	0.0628	0.2377	0.3873
κ_F	0.0586	0.0104	0.0202	0.0691	0.1214
χ^π_F	0.0076	0.0080	-0.0989	-0.0126	0.0709
Π	1.0041	0.0004	1.0007	1.0053	1.0097

Table 2.9: Parameter Estimation Results for the Czech Republic

Parameter	Mode	S.D.	10%	Mean	90%
$\sigma(\varepsilon_a)$	0.5770	0.1148	0.5589	0.7989	1.0704
$\sigma(\varepsilon_e)$	6.5825	0.5749	5.2662	6.6646	8.0768
$\sigma(\varepsilon_u)$	1.9111	0.2379	1.6213	2.0654	2.4917
$\sigma(\varepsilon_v)$	1.7238	0.2345	1.3308	1.6808	2.0113
$\sigma(\varepsilon_{v^F})$	10.5052	1.1955	0.2216	1.9184	5.2044
$\sigma(\varepsilon_{rs})$	7.0219	0.4958	5.8129	6.9522	8.0079
ρ_a	0.7869	0.0202	0.8477	0.9042	0.9626
ρ_e	0.9143	0.0089	0.8872	0.9088	0.9314
ρ_v	0.6560	0.0097	0.5739	0.6655	0.7501
ρ_{v^F}	0.8515	0.0160	0.7020	0.8312	0.9723
ρ_i	0.8868	0.0174	0.7804	0.8709	0.9634
ϕ_π	1.5100	0.0351	1.3622	1.5075	1.6407
ϕ_y	0.0404	0.0060	0.0185	0.0522	0.0854
$\phi_{\Delta 1}$	0.2777	0.0253	0.1725	0.2758	0.3749
$\phi_{\Delta 2}$	0.4318	0.0206	0.1201	0.3077	0.4223
$\phi_{\Delta y}$	0.0510	0.0095	0.0004	0.0372	0.0748
ϕ_S	0.1451	0.0107	0.0772	0.1521	0.2302
χ^f	0.8227	0.0357	0.6184	0.7971	0.9564
χ^b	0.3723	0.0468	0.1998	0.3399	0.4819
κ_{mc}	0.4320	0.0090	0.4160	0.4889	0.5653
χ^π	0.0741	0.0056	-0.0210	0.0565	0.1283
χ_F^f	0.4061	0.0358	0.0880	0.4329	0.7770
χ_F^b	0.2545	0.0164	0.0394	0.2188	0.3606
κ_F	0.0843	0.0076	0.0105	0.0493	0.0873
χ_F^π	-0.0715	0.0041	-0.1092	-0.0415	0.0235
Π	1.0097	0.0004	1.0017	1.0062	1.0102

Table 2.10: Parameter Estimation Results for Hungary

Parameter	Mode	S.D.	10%	Mean	90%
$\sigma(\varepsilon_a)$	1.1451	0.1256	0.8682	1.0746	1.2679
$\sigma(\varepsilon_e)$	7.0226	0.7007	5.1819	6.5509	7.8704
$\sigma(\varepsilon_u)$	1.5254	0.2327	1.3060	1.6315	1.9452
$\sigma(\varepsilon_v)$	1.3775	0.1799	1.0422	1.2815	1.5081
$\sigma(\varepsilon_{v^F})$	0.4572	0.4848	0.2284	0.8527	1.5687
$\sigma(\varepsilon_{rs})$	6.9759	0.5465	5.9946	7.0510	8.0882
ρ_a	0.9627	0.0101	0.9144	0.9471	0.9807
ρ_e	0.9128	0.0103	0.8989	0.9181	0.9394
ρ_v	0.7359	0.0133	0.4996	0.6548	0.7981
ρ_{v^F}	0.8895	0.0180	0.7818	0.8811	0.9770
ρ_i	0.8601	0.0267	0.5576	0.7165	0.8671
ϕ_π	1.5237	0.0138	1.3934	1.5019	1.6300
ϕ_y	0.0599	0.0082	0.0452	0.0866	0.1357
$\phi_{\Delta 1}$	0.3017	0.0165	0.1375	0.2259	0.3056
$\phi_{\Delta 2}$	0.3714	0.0260	0.1770	0.3229	0.4748
$\phi_{\Delta y}$	0.0939	0.0106	0.0055	0.1105	0.2116
ϕ_S	0.1120	0.0343	0.0702	0.1127	0.1551
χ^f	0.9221	0.0200	0.6402	0.7843	0.9490
χ^b	0.3732	0.0375	0.1794	0.3276	0.4820
κ_{mc}	0.4439	0.0137	0.4857	0.5697	0.6630
χ^π	0.0665	0.0109	0.0174	0.0800	0.1756
χ_F^f	0.3279	0.0369	0.3319	0.5075	0.7006
χ_F^b	0.3022	0.0486	0.4359	0.5423	0.6778
κ_F	0.0373	0.0053	0.0010	0.0106	0.0188
χ_F^π	0.0509	0.0107	-0.0115	0.0415	0.1096
Π	1.0025	0.0006	1.0006	1.0035	1.0062

Table 2.11: Parameter Estimation Results for Poland

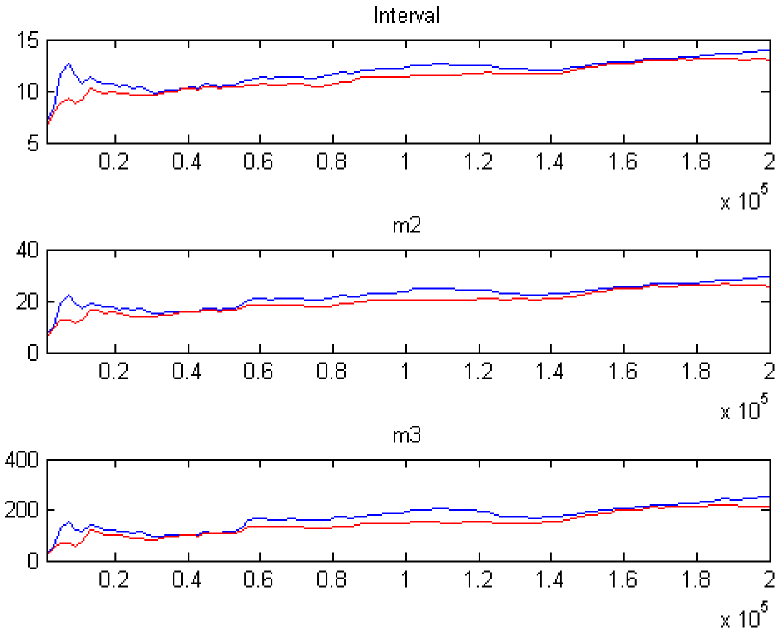

Figure 2.2: Multivariate Convergence Diagnostic for the Czech Republic

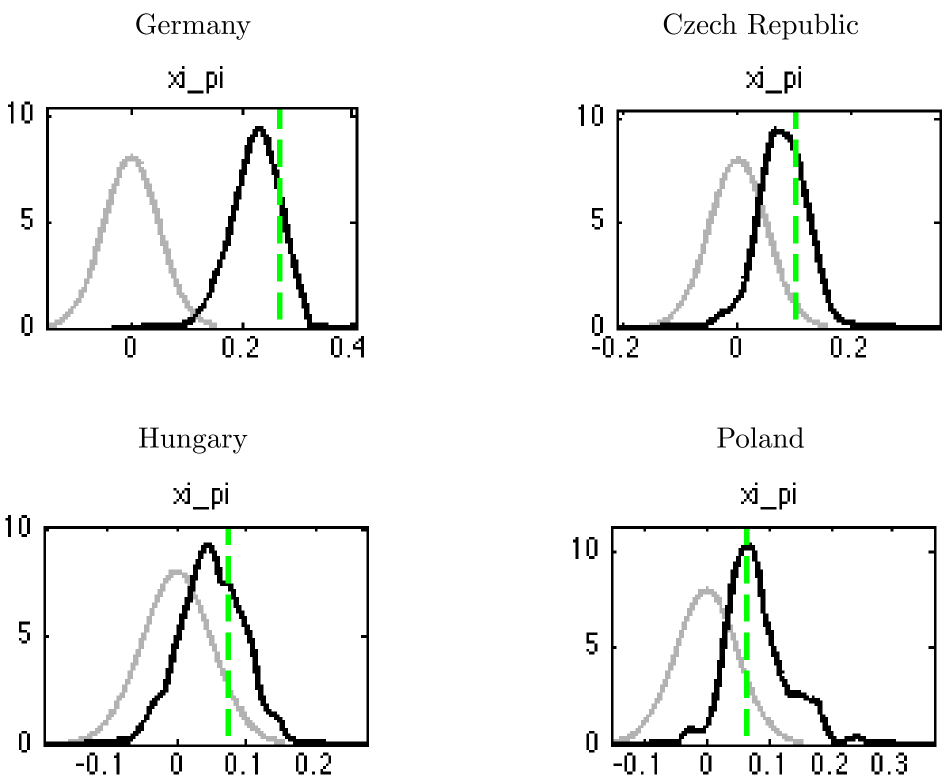

Figure 2.3: Prior and Posterior Distribution and Posterior Mode for the Parameter χ_π

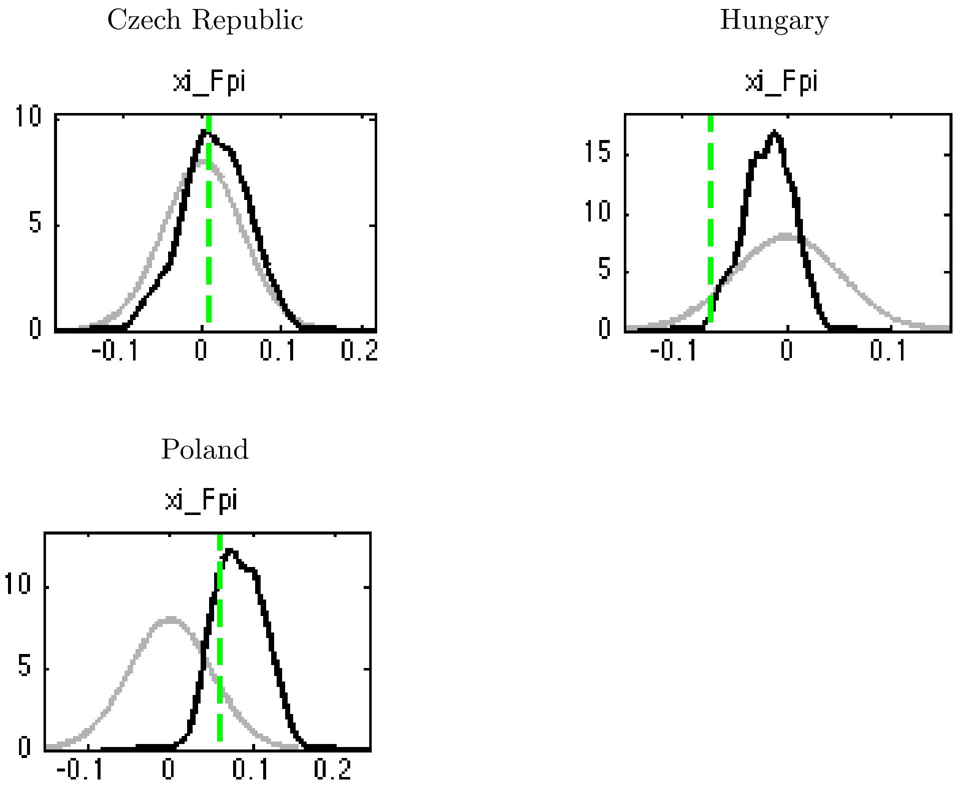

Figure 2.4: Prior and Posterior Distribution and Posterior Mode for the Parameter χ_F^π

Impulse Response Functions Analysis

In this section, I explain how the endogenous variables such as inflation, output, interest rate and real exchange rate respond to each structural shock over next 10 periods (*i.e.*, 2.5 years). The responses are illustrated in Figures 2.5-2.13 and, because of the similarities in the dynamic behavior of the three EEC, I only report the results of the estimates relative to the Czech Republic.[12] In what follows, I

[12]The quantitative difference between A1 and A2 approach, *i.e.*, model with zero steady state inflation and with trend inflation, can be found in Appendix 2.C.

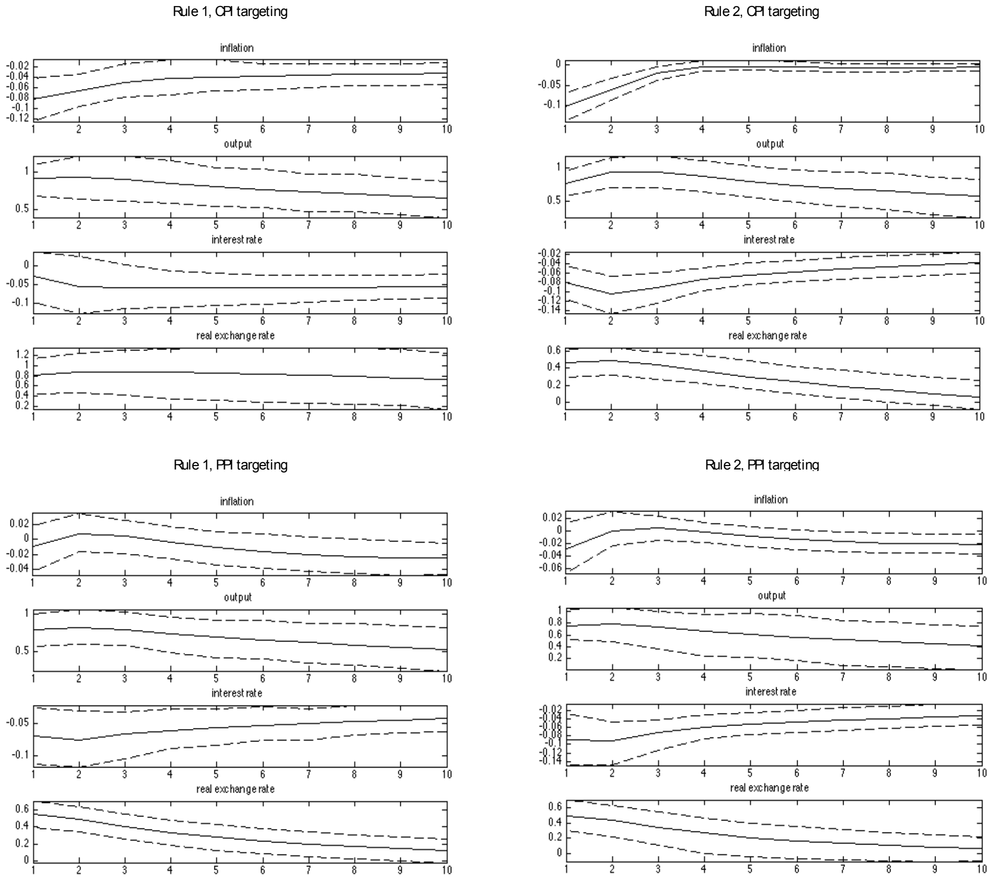

Figure 2.5: Impulse Responses to a Domestic TFP shock

compare the monetary policy rules in (2.78)-(2.79) and (2.81)-(2.82) to identify potential differences between CPI and PPI inflation targeting. The solid line is the median response, and the area within the dashed lines represents the 90% HPD interval.

Figure 2.5 displays the responses of the domestic variables to a positive domestic TFP shock. Independently of the monetary policy rule, output reacts positively to the TFP shock, and stronger than all other variables. This result can be interpreted as follows. An increase in productivity leads to lower marginal costs, hence to a decrease in producer prices and domestic PPI inflation. Therefore, the relative prices of imported good increase, and the aggregate demand shifts towards the cheaper domestic goods. This, in turn, implies a rise in domestic aggregate output. Foreign inflation relates positively to a change in LOP gap, which is a function of the real exchange rate and the foreign (relative) price. A rise in real exchange rate leads to an increase in LOP gap, whereas a higher $\tilde{p}_t^F$ implies a lower LOP gap. As a result, the LOP gap increases less than proportionally with the shock, leading to a modest rise in imported inflation. CPI inflation, given by the combination of domestic and foreign inflation, decreases overall since the drop in the producer prices variation entails stronger effects than the higher imported inflation.

In response to lower inflation, the central bank opts for an expansionary monetary policy, which implies a fall in the interest rate. Because the interest rate of the large economy remains constant, but the uncovered interest parity holds, the nominal and real exchange rate depreciates. These results are in line with the theoretical findings in Galí and Monacelli (2005).

It also follows from Figure 2.5 that the overall inflation decreases more in the case of CPI inflation targeting than with PPI inflation targeting. The reason is that when producer inflation is targeted, it fluctuates less and therefore the price for the domestic good is more stable. The decrease in PPI inflation is partly offset by the increase in imported inflation and thus, overall inflation is less volatile than

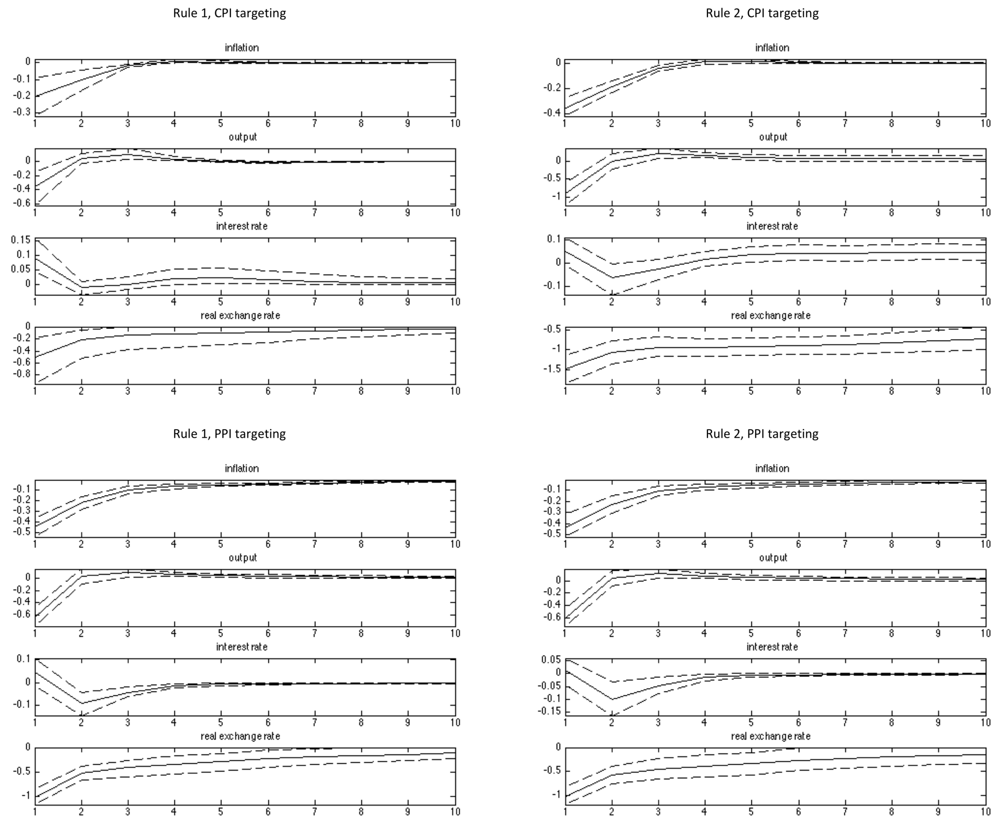

Figure 2.6: Impulse Responses to a Domestic Monetary Shock

in the case of CPI targeting. It may also be noted that output and real exchange rate vary only marginally, regardless the choice of the policy target.

The responses to a domestic monetary shock are presented in Figure 2.6. An unexpected increase in the interest rate leads to a lower aggregate output. First, a higher interest rate implies a higher return on domestic assets, and therefore makes the domestic currency more attractive. The nominal appreciation, making imports cheaper, leads to a drop in the demand for domestic goods. In turn, a downward shift in demand for domestic goods results in lower inflation and aggregate output.

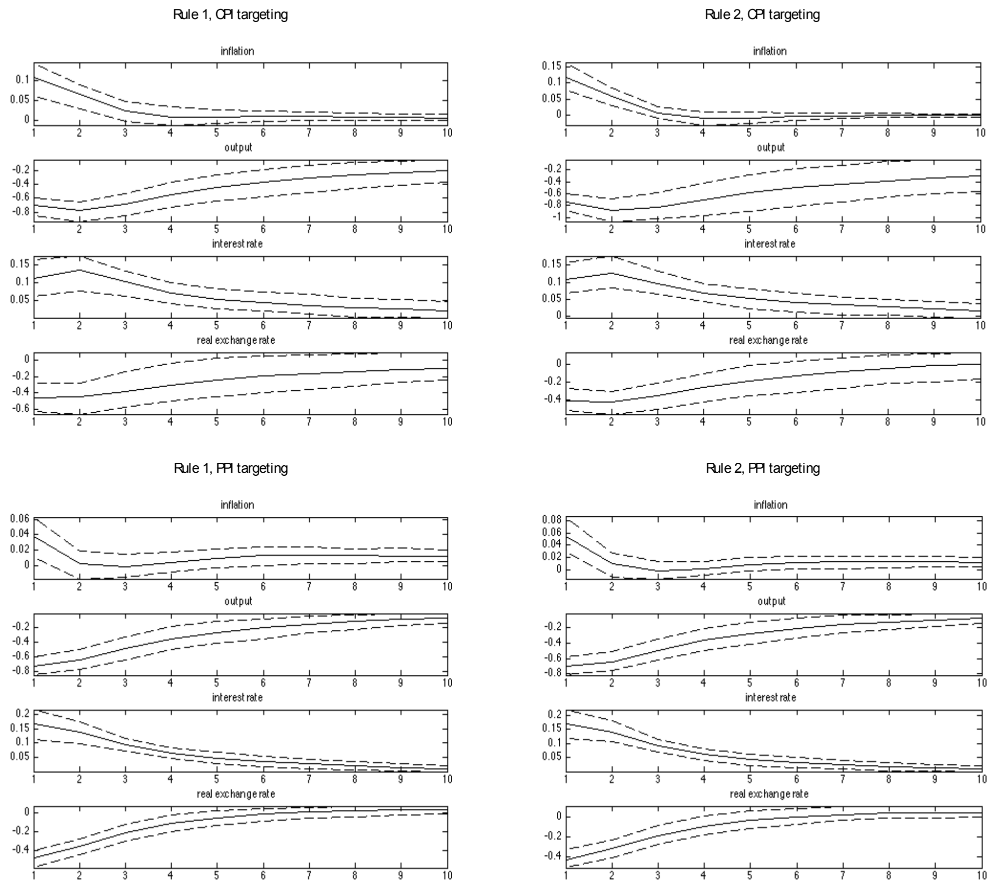

Figure 2.7: Impulse Responses to a Domestic Producer Cost Push Shock

Figure 2.7 plots the responses to a domestic cost push shock. This shock immediately increases producer inflation. The higher relative domestic price reduces the overall demand for domestic good, and therefore results in a drop in aggregate domestic output. Overall inflation also increases. Thus, the central bank reacts by raising the interest rate, which leads to an appreciation of the exchange rate and, furthermore, depresses the competitiveness of the domestic goods in the international markets. Also in the case of a cost push shock, overall inflation is less volatile

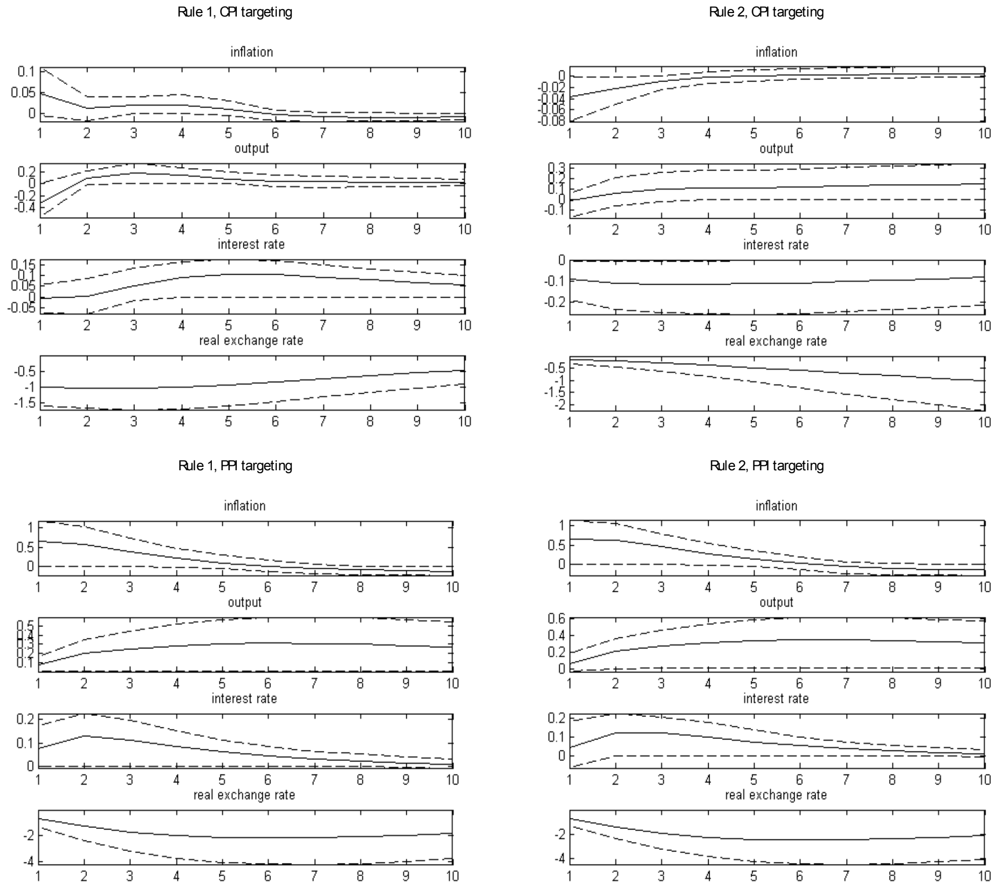

Figure 2.8: Impulse Responses to an Importer Cost Push Shock

if the central bank directly targets producer inflation. The initial response of the aggregate output, however, is seemingly independent of the policy rule adopted.

In the presence of an importer cost push shock, the difference between the PPI inflation and CPI inflation targeting is more obvious than in the previous cases. Depending on the rule, the very dynamics of the main economic variables change. The impulse responses are illustrated in Figure 2.8. An importer cost push shock

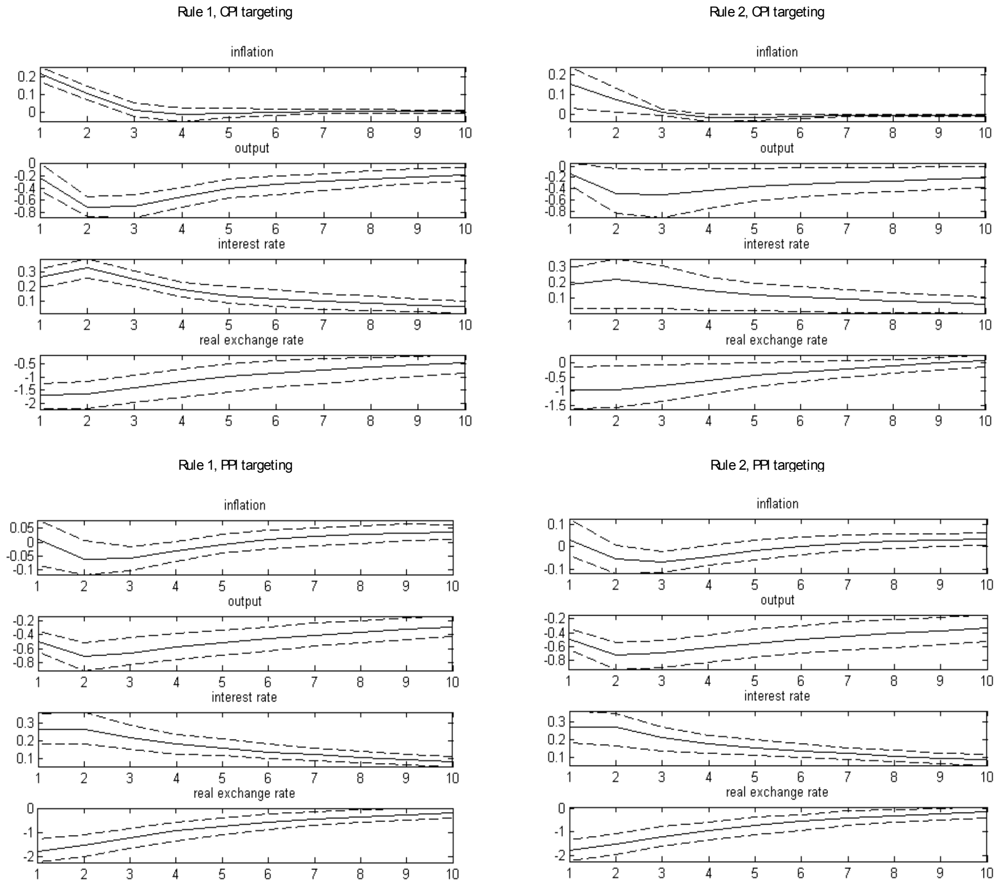

Figure 2.9: Impulse Responses to a Domestic Preference Shock

increases immediately the inflation of the imported goods. Thus, the price of these goods increases relative to the price of the domestically produced goods. Imports fall, and overall domestic consumption decreases, increasing the marginal utility of consumption. However, a rise in domestic production occurs, due to a higher domestic demand for domestic goods. Given the fact that the agents desire to diversify risk, the real exchange rate appreciates, which reduces competitive advantage on the international market. Therefore, the resulting effect on the domestic output is ambiguous. In the case of PPI targeting, the response of the

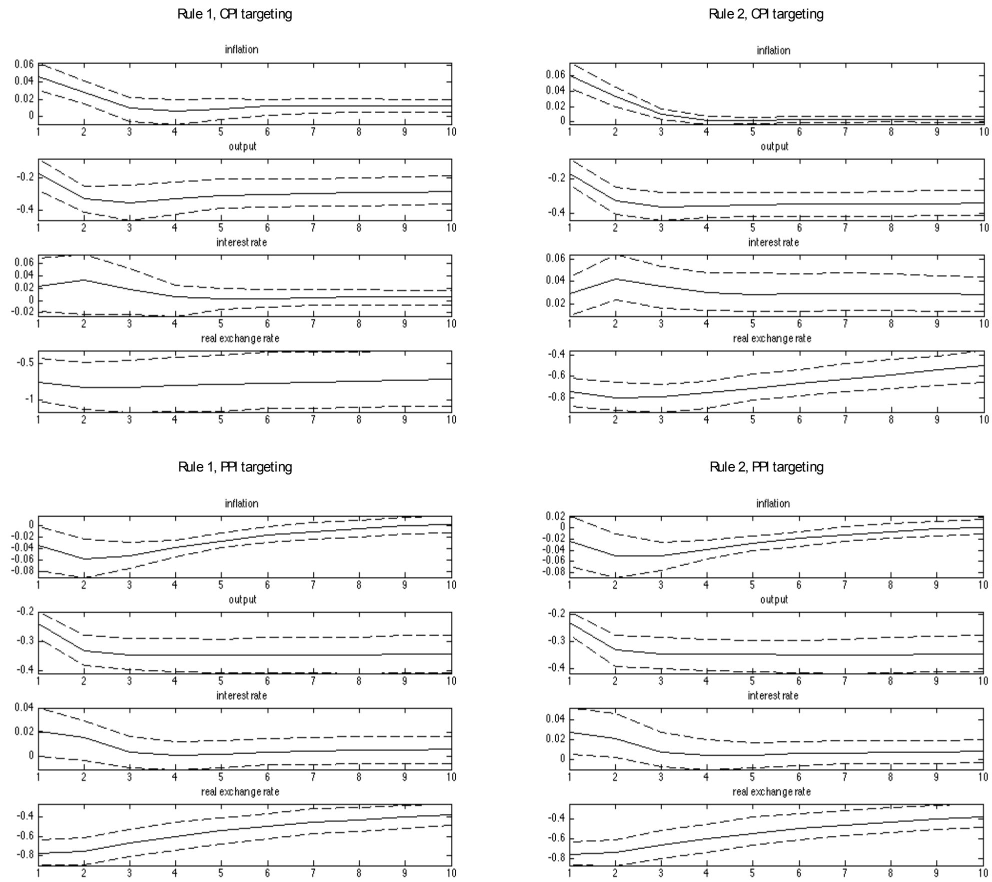

Figure 2.10: Impulse Responses to a Foreign TFP Shock

central bank to the rising inflation is milder. As a result, output slightly increases, but this also implies a substantial rise in inflation. Under CPI targeting, the central bank intervention is stronger: this entails lower output, but inflation growth is very small.

A domestic demand shock, illustrated in Figure 2.9, increases overall consumption. Since agents share risk internationally, the resulting decrease in the marginal utility of consumption implies an appreciation in the domestic currency, and therefore an increase in relative domestic price. As a consequence, the LOP gap decreases, and so does imported inflation. The demand for domestic goods decreases, whereas demand for foreign goods increases more than proportionally. With the increase in consumption, the marginal utility of consumption decreases and the wage increases. This is due to the fact that when agents optimise they equate the ratio of marginal disutility of labour to the marginal utility of consumption and also to the real wage. This would imply that whenever consumption increases, agents tend to lower their labour supply for a given wage. Since in equilibrium labour does not decrease sufficiently to keep the ratio constant, the real wage grows. This leads to an increase in marginal costs which is partly offset due to the increase in the relative domestic price. Finally, an increase in marginal costs leads to a rise in producer inflation. The overall rate of inflation increases. Thus, the central bank tightens its policy by increasing the interest rate.

If a TFP shock hits a foreign large economy, the rate of inflation in that country falls, domestic aggregate output increases, and the central bank lowers the interest rate. The impact on the domestic variables is shown in Figure 2.10. The domestic currency appreciates relative to the foreign currency. The relative price for foreign good decrease, hence demand shifts toward the foreign produced goods. Foreign inflation lowers (decrease in LOP gap) and domestic inflation rises (increase in real wage, real marginal costs). In the case of PPI targeting, the overall inflation may fall, however by CPI targeting, the CPI inflation increases initially. After the initial drop, the output decreases further as a consequence of the rise in the interest rate, having its trough in the second to third period, and afterwards returning back to its equilibrium very slowly.

A positive foreign monetary policy shock, illustrated in Figure 2.11, causes an

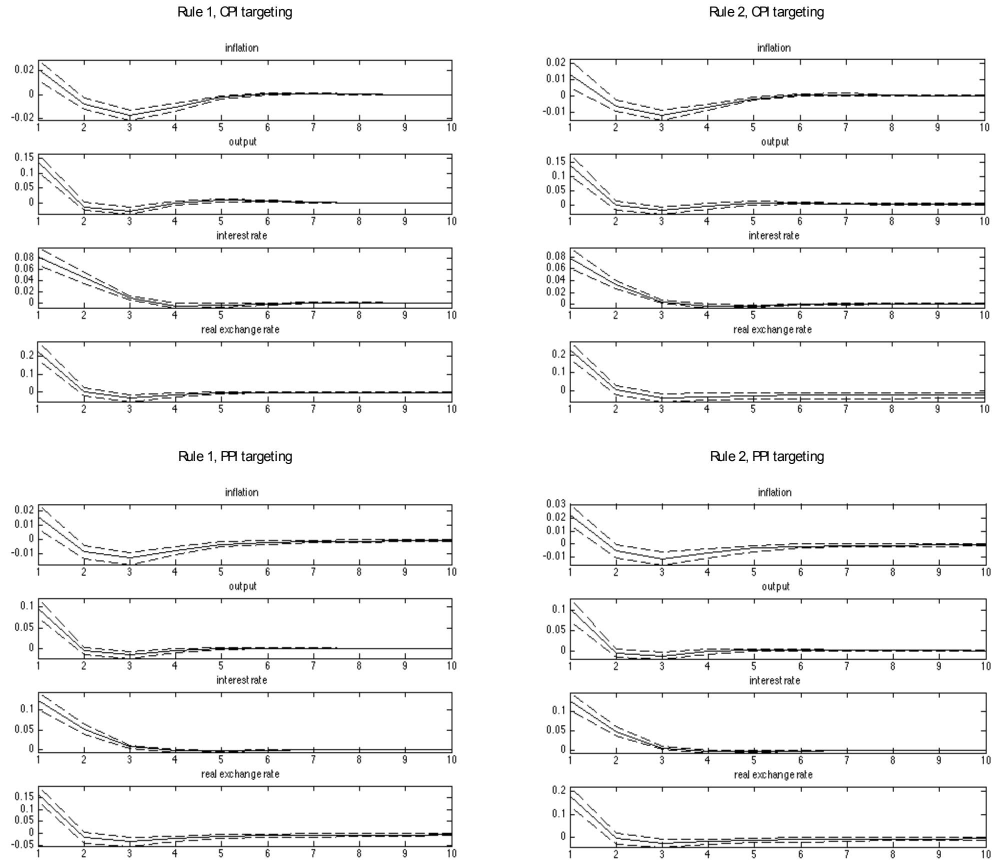

Figure 2.11: Impulse Responses to a Foreign Monetary Shock

immediate appreciation in the foreign currency. The domestic currency depreciates and, as a consequence, the domestic goods become cheaper relative to the foreign one. Thus, the demand for domestic good increases and so does aggregate domestic output. The overall inflation rises as well, as a consequence of an increase in domestic inflation. Therefore, the central bank opts for a contractionary monetary policy, which entails a return of the exchange rate quickly - after two periods - back to its equilibrium.

Figure 2.12 shows that a foreign cost push shock leads to a currency depreci-

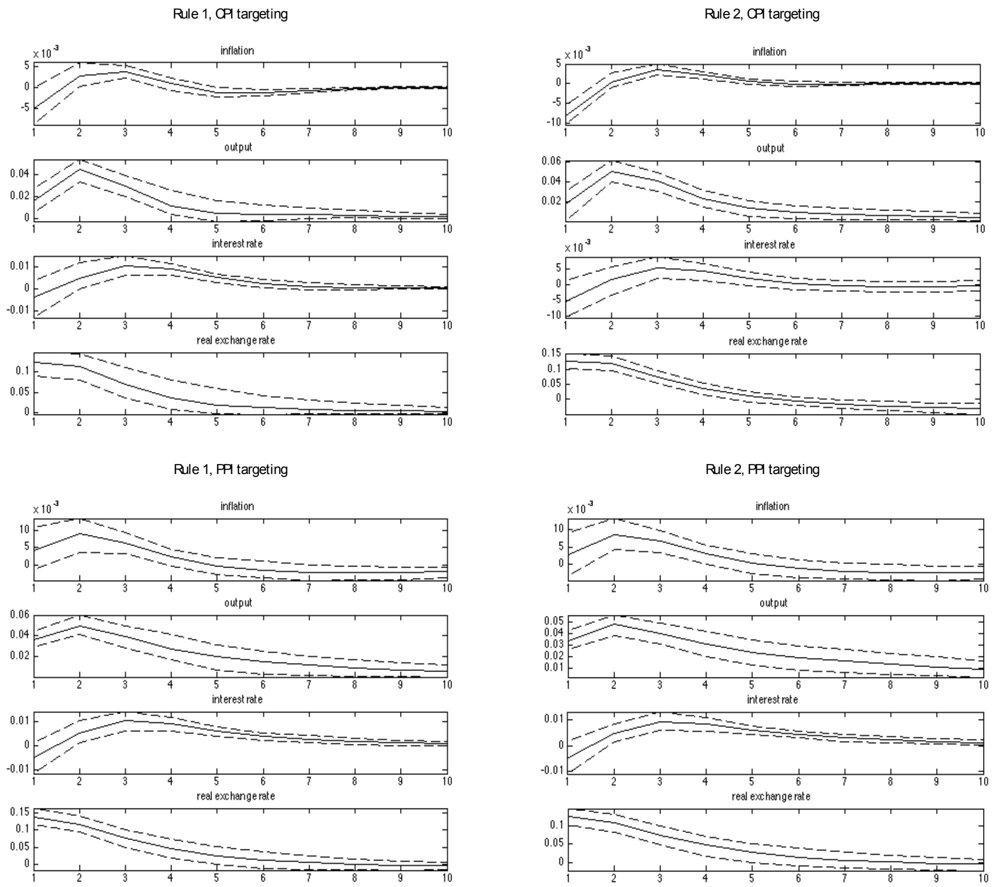

Figure 2.12: Impulse Responses to a Foreign Cost Push Shock

ation and a rise in domestic aggregate output. For the large foreign economy, the shock leads to an increase in inflation and a drop in consumption and output. The central bank increases the interest rate. As a consequence, the domestic currency depreciates and domestic goods gain a relative price advantage, which results in a demand shift toward domestic goods. Domestic aggregate output increases, but overall domestic consumption falls due to higher prices. This leads to a decrease in the real wage, and a drop in the real marginal costs. Thus, PPI inflation decreases. Overall inflation decreases if it is subject to the central bank's targeting.

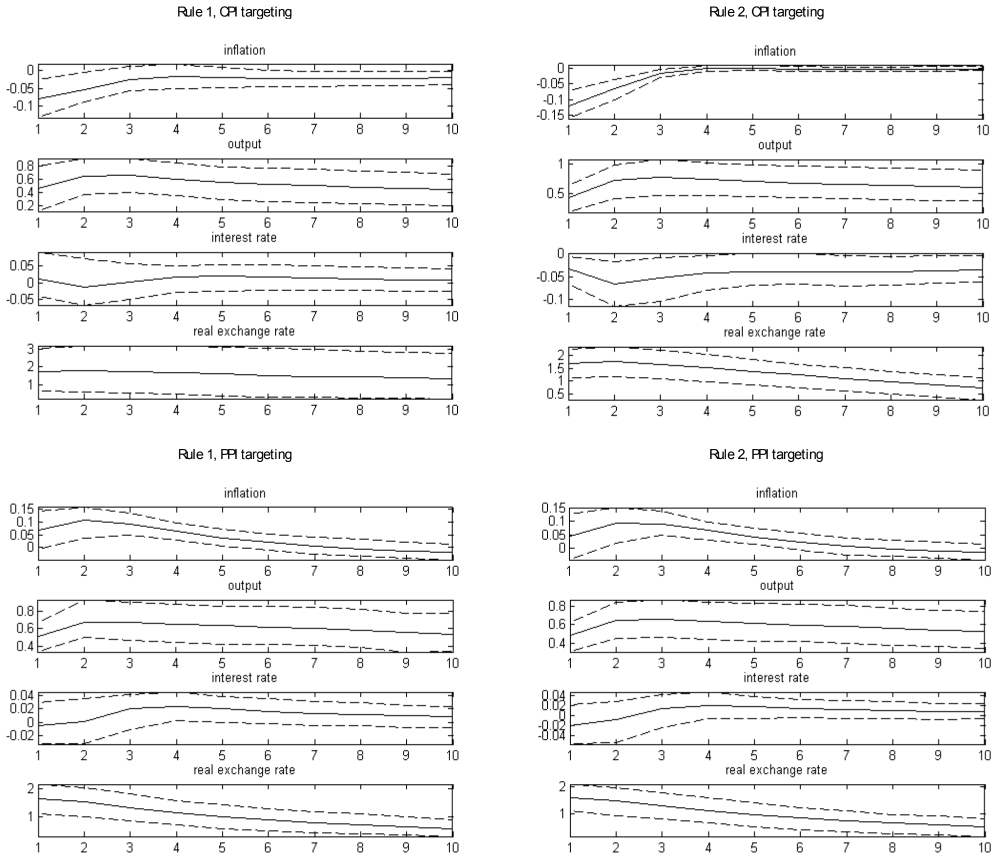

Figure 2.13: Impulse Responses to a Foreign Preference Shock

Nevertheless, if the central bank targets PPI inflation, the overall inflation may increase, since the rise in foreign inflation outweighs the effect of the decrease in PPI inflation. All in all, the central bank decreases the interest rate in response to a foreign cost push shock.

Similarly to a foreign cost push shock, the foreign demand shock increases domestic output. Foreign consumption initially increases, leading to a lower marginal utility of consumption in the large economy and the foreign central bank reacts with an increase in the interest rate, which has the effect of domestic currency to depreciates. The responses to the shock are presented in Figure 2.13. If the domestic central bank targets the PPI, overall inflation may increase. By contrast, targeting CPI leads to a drop in overall inflation. As a consequence, the latter results in an expansionary monetary policy.

To conclude, note that in most of the cases, targeting PPI leads to lower volatility in CPI inflation than with CPI targeting. The effect of different inflation targets on output is not that strong, hence it causes only limited changes to output. In line with the typical arguments in the theoretical literature, which maintain that PPI targeting leads to lower welfare losses, my impulse responses clearly show that such welfare gains are mainly due to the different effects on inflation generated by targeting the two alternative price indices.

2.5 Concluding Remarks

This chapter considered the performance and characteristics of simple monetary policy rules using a two-country model. First, I developed a small-scale two-country DSGE model similar to Lubik and Schorfheide (2007), with a micro-founded Phillips curve. I assumed imperfect pass-through and non-unit intratemporal elasticity of substitution between domestic and foreign goods and log-linearised the model around a steady state with non-zero inflation.

I carried out Bayesian inference, using a Metropolis Hastings sampling approach, to measure the performance of this model against German data. The novel feature has been the inclusion of real unit labour costs for a better measurement of the Phillips curve. Firstly, using only the part of the model related to the large economy, which is identical to the one built in Chapter 1, I have tested several simple nominal interest rate rules. I have shown that a simple monetary policy rule mimicking an optimal rule, similar to the one derived in Chapter 1, gives the best outcome. Additionally, I have shown that the estimation of the structural parameters of the model are robust to the choice of the monetary policy rule, and that the non-zero inflation part included in the Phillips curve improves the model fit significantly.

To study the model for the SOE, I have used the data of EEC, such as the Czech Republic, Hungary and Poland. Using a posterior odds test, I found evidence that the central banks of all these countries target a PPI inflation instead of CPI inflation, contrary to what is usually assumed in the empirical literature. I have shown that, also in the case of a SOE, the model with a non-zero steady state inflation performs substantially better. If I compare the non-zero steady state inflation component between the three EEC and Germany, I find that the magnitude is lower for the latter, though it remains positive and significantly different from zero. Further analysis about the monetary policy rules shows that a pure exchange rate target can be rejected for all three EEC, and that only the Czech Republic appears to respond to exchange rate movements.

2.A Data

All observations are quarterly, seasonally adjusted using the defaults settings of the X12 filter in Eviews 6. The empirical estimation is based on a data sample over the period 1996 to 2012 for Germany and the Czech Republic, and 1998 to 2012 for Hungary and Poland.

The FRED database was used as a source for following time series:

- Inflation, defined as the log difference of the consumer or producer price index multiplied by 100
  - Consumer Price Index: All Items in Germany (DEUCPIALLQINMEI)
  - Consumer Price Index: All Items for Czech Republic (CZECPIALLMINMEI)
  - Domestic Producer Prices Index: Manufacturing for Czech Republic (CZEPPDMQINMEI)
  - Consumer Price Index: All Items for Hungary (HUNCPIALLMINMEI)
  - Domestic Producer Prices Index: Manufacturing for Hungary (HUNPPDMQINMEI)
  - Consumer Price Index: All Items for Poland (POLCPIALLMINMEI)
  - Domestic Producer Prices Index: Manufacturing for Poland (POLPPDMQINMEI)
- Output growth, constructed as the log difference of real output that is defined as a nominal output divided by a deflator, multiplied by 100

- Current Price Gross Domestic Product in Germany (DEUGDPNQDSMEI)
- GDP Implicit Price Deflator in Germany (DEUGDPDEFQISMEI)
- Current Price Gross Domestic Product in Czech Republic(CZEGDPNQDSMEI)
- GDP Implicit Price Deflator in Czech Republic (CZEGDPDEFQIS-MEI)
- Current Price Gross Domestic Product in Hungary (HUNGDPNQDSMEI)
- GDP Implicit Price Deflator in Hungary (HUNGDPDEFQISMEI)
- Current Price Gross Domestic Product in Poland (POLGDPNQDSMEI)
- GDP Implicit Price Deflator in Poland (POLGDPDEFQISMEI)

- Unit labour cost is defined as the percentage change of the ratio between total labour costs and real GDP
  - Benchmarked Unit Labor Costs - Total for Germany (DEUULCTOTQP-NMEI)
  - Benchmarked Unit Labor Costs - Manufacturing for Czech Republic (CZEULCMANQPNMEI)
  - Benchmarked Unit Labor Costs - Manufacturing for Hungary (HUNUL-CMANQPNMEI)
  - Benchmarked Unit Labor Costs - Manufacturing for Poland (POLUL-CMANQPNMEI)

- To compute average import/GDP ratio, I use
  - Imports of Goods and Services in Czech Republic (CZEIMPORTADSMEI)

- Imports of Goods and Services in Hungary (HUNIMPORTADSMEI)
- Imports of Goods and Services in Poland (POLIMPORTADSMEI)

For the analysis, Datastream was a source for following data:

- The interest rate is an annualised quarter to quarter interest rate monthly average, divided by four so as to be expressed in quarterly terms
  - German Day to Day money market rate monthly average (BDSU0101R)
  - Czech Discount Rate (640015045)
  - Hungarian Central Bank Base Rate (870002307)
  - Polish Central Bank Rediscount Rate (POOIR037)
- The quarterly change in the exchange rate is computed as a log difference of the bilateral nominal exchange rate between Euro and EEC currency
  - German Mark to US $ (USWGMRK)
  - Czech Koruna to US $ (USCZECK)
  - Hungarian Forint to US $ (USHUNGF)
  - Polish Zloty to US $ (USPOLZL)

2.B Log-linearisation of the Phillips Curve

The log-linearisation of the equations (2.42), (2.43) and (2.44) straightforwardly leads respectively to

$$\hat{p}_t^f = \hat{\jmath}_t - \hat{h}_t,$$

$$\hat{\jmath}_t = (1 - \alpha\beta (\Pi_H)^{\varepsilon}) (\hat{y}_t - \sigma\hat{c}_t + \widehat{mc}_t + \tilde{p}_{H,t} + v_t) + \alpha\beta (\Pi_H)^{\varepsilon} (\varepsilon\hat{\pi}_{H,t+1} + \hat{\jmath}_{t+1}),$$

$$\hat{h}_t = \left(1 - \alpha\beta\bar{\Pi}^{\varepsilon-1}\right) (\hat{y}_t - \sigma\hat{c}_t) + (\alpha\beta)\,\bar{\Pi}^{\varepsilon-1} \left(\varepsilon\hat{\pi}_{H,t+1} - \hat{\pi}_{t+1} + \hat{h}_{t+1}\right),$$

from which I can obtain the forward looking price in log-linearised term as

$$\begin{aligned}\hat{p}_t^f = \alpha\beta \left[\bar{\Pi}^{\varepsilon-1} - \bar{\Pi}^{\varepsilon}\right] (\hat{y}_t - \sigma\hat{c}_t) + (1 - \alpha\beta (\Pi)^{\varepsilon}) (\widehat{mc}_t + \tilde{p}_{H,t} + v_t) \\ + \alpha\beta \left\{\Pi^{\varepsilon}\varepsilon\hat{\pi}_{H,t+1} - \bar{\Pi}^{\varepsilon-1}\varepsilon\hat{\pi}_{H,t+1}\right\} \\ + (\alpha\beta)\,\bar{\Pi}^{\varepsilon-1}\hat{\pi}_{t+1} + \alpha\beta \left[\bar{\Pi}^{\varepsilon}\hat{\jmath}_{t+1} - \bar{\Pi}^{\varepsilon-1}\hat{h}_{t+1}\right].\end{aligned}$$

The log-linearisation of equations (1.20),(1.22) deliver respectively

$$\tilde{p}_t^b = \tilde{x}_{t-1} + \hat{\pi}_{H,t-1} - \hat{\pi}_t,$$

$$\hat{x}_t = (1 - \omega)\,\hat{p}_t^f + \omega\hat{p}_t^b.$$

The domestic price dynamics in relative terms given in equation (2.47) is log-linearised as

$$\hat{\pi}_t = \frac{1 - \alpha\Pi^{\varepsilon-1}}{\alpha\Pi^{\varepsilon-1}}\tilde{x}_t + \frac{\alpha\Pi^{\varepsilon-1}\tilde{p}_{H,t-1} - \tilde{p}_{H,t}}{\alpha\Pi^{\varepsilon-1}}.$$

Combining all these equations together delivers a hybrid NKPC of a form

$$\left(\alpha\bar{\Pi}^{\varepsilon-1}+\omega\left(1-\alpha\bar{\Pi}^{\varepsilon}\left(\bar{\Pi}^{-1}-\beta\right)\right)\right)\left(\hat{\pi}_t+\Delta\tilde{p}_{H,t}\right)=\alpha\beta\bar{\Pi}^{\varepsilon}\left(\hat{\pi}_{t+1}+\Delta\tilde{p}_{H,t+1}\right)$$
$$+\,\omega\left(\hat{\pi}_{t-1}+\Delta\tilde{p}_{H,t-1}\right)+\left(1-\alpha\Pi^{\varepsilon-1}\right)(1-\omega)\left(1-\alpha\beta\left(\Pi\right)^{\varepsilon}\right)\left(\widehat{mc}_t+v_t\right)$$
$$+\left(1-\alpha\Pi^{\varepsilon-1}\right)(1-\omega)\,\alpha\beta\left(\bar{\Pi}^{\varepsilon}-\bar{\Pi}^{\varepsilon-1}\right)$$
$$\cdot\left[\hat{h}_{t+1}+(\varepsilon-1)\left(\hat{\pi}_{t+1}+\Delta\tilde{p}_{H,t+1}\right)+\left(\sigma\hat{c}_t-\hat{y}_t\right)+\Delta\tilde{p}_{H,t+1}\right]$$

Using (2.20), which log-linearised delivers

$$\hat{\pi}_t+\Delta\tilde{p}_{H,t}=\hat{\pi}_{H,t},$$

simplifies the hybrid NKPC. After collecting the terms together and using the definition of $\hat{h}_t$ above, the Phillips curve with backward looking firms linearising around a non-zero steady state inflation yields

$$\left(\alpha\bar{\Pi}^{\varepsilon-1}+\omega\left(1-\alpha\bar{\Pi}^{\varepsilon}\left(\bar{\Pi}^{-1}-\beta\right)\right)\right)\hat{\pi}_{H,t}=\alpha\beta\bar{\Pi}^{\varepsilon}\hat{\pi}_{H,t+1}+\omega\hat{\pi}_{H,t-1}$$
$$+\left(1-\alpha\Pi^{\varepsilon-1}\right)(1-\omega)\left(1-\alpha\beta\left(\Pi\right)^{\varepsilon}\right)\left(\widehat{mc}_t+v_t\right)$$
$$\cdot\left(1-\alpha\Pi^{\varepsilon-1}\right)(1-\omega)\left(\bar{\Pi}^{-1}-1\right)\alpha\beta\bar{\Pi}^{\varepsilon-1}\left[\hat{h}_{t+1}+\varepsilon\hat{\pi}_{H,t+1}-\hat{\pi}_{t+1}+\sigma\hat{c}_t-\hat{y}_t\right].$$

Written in terms of parameters

$$\hat{\pi}_{H,t}=\chi^{f}E_t\left[\hat{\pi}_{H,t+1}\right]+\chi^{b}\hat{\pi}_{H,t-1}+\kappa_{mc}\left(\widehat{mc}_t+v_t\right)+\chi^{\pi}\left(\hat{h}_t-\left(\hat{y}_t-\sigma\hat{c}_t\right)\right)$$

with parameters

$$
\begin{aligned}
\Psi &= \alpha\bar{\Pi}^{\varepsilon-1} + \omega\left(1 - \alpha\bar{\Pi}^{\varepsilon}\left(\bar{\Pi}^{-1} - \beta\right)\right) \\
\chi^{f} &= \alpha\beta\bar{\Pi}^{\varepsilon}/\Psi, \chi^{b} = \omega/\Psi, \\
\kappa_{mc} &= \left(1 - \alpha\Pi^{\varepsilon-1}\right)(1-\omega)\left(1 - \alpha\beta\Pi^{\varepsilon}\right)/\Psi \\
\chi^{\pi} &= \left(1 - \alpha\Pi^{\varepsilon-1}\right)(1-\omega)\left(\bar{\Pi}^{-1} - 1\right).
\end{aligned}
$$

2.C Quantitative Implications of Positive Trend Inflation

In this appendix I illustrate the quantitative implications of positive trend inflation on the IRFs, *e.g.*, the quantitative difference between the two approaches A1 and A2. For illustration, I use the estimation results for Czech Republic and PPI inflation targeting.

Domestic TFP Shock

Inflation			Interest Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
0.0012	-0.0068	0.0106	-0.0155	-0.0362	0.0018
0.0045	0.0067	0.0112	-0.0076	-0.0254	-0.0050
0.0044	0.0022	0.0020	-0.0021	-0.0121	-0.0089
0.0038	0.0039	-0.0017	0.0011	0.0025	-0.0008
0.0035	0.0059	-0.0018	0.0031	0.0026	-0.0001
0.0035	0.0097	-0.0001	0.0046	0.0038	0.0054
0.0037	0.0092	-0.0006	0.0057	0.0091	0.0110
0.0040	0.0101	-0.0003	0.0065	0.0109	0.0134
0.0043	0.0113	0.0008	0.0070	0.0130	0.0139
0.0047	0.0112	0.0009	0.0074	0.0105	0.0095
Output			**Real Exchange Rate**		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
0.0236	0.0031	0.0614	0.0180	-0.0142	0.0731
-0.0023	-0.0822	-0.0040	0.0013	-0.0726	0.0065
-0.0255	-0.0974	0.0290	-0.0107	-0.1082	0.0076
-0.0428	-0.1610	0.0030	-0.0172	-0.1309	0.0104
-0.0552	-0.1304	0.0336	-0.0200	-0.1281	0.0158
-0.0641	-0.1394	0.0165	-0.0203	-0.1113	0.0199
-0.0705	-0.1686	-0.0302	-0.0191	-0.1081	0.0111
-0.0749	-0.1451	-0.0197	-0.0171	-0.0975	0.0222
-0.0779	-0.1353	-0.0336	-0.0146	-0.0889	0.0190
-0.0796	-0.1159	-0.0383	-0.0119	-0.0692	0.0277

Domestic Monetary Policy Shock

Inflation			Interest Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
0.0019	0.0154	0.0181	-0.0127	-0.0044	-0.0312
-0.0007	-0.0136	0.0064	0.0000	-0.0049	0.0114
-0.0025	0.0004	-0.0034	0.0004	-0.0026	0.0047
-0.0016	0.0008	-0.0086	-0.0005	-0.0020	-0.0008
-0.0005	0.0005	0.0001	-0.0006	-0.0004	-0.0007
0.0000	-0.0019	0.0080	-0.0004	0.0000	-0.0009
0.0001	-0.0071	0.0021	-0.0002	-0.0002	-0.0003
0.0001	-0.0054	0.0029	-0.0001	-0.0002	0.0001
0.0001	-0.0037	0.0021	-0.0001	-0.0001	0.0004
0.0001	-0.0026	0.0008	-0.0001	0.0000	0.0005

Output			Real Exchange Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
0.0213	0.0404	-0.0018	0.0224	0.0434	0.0657
-0.0028	-0.0194	-0.0204	0.0078	-0.0346	-0.0175
-0.0034	-0.0144	-0.0158	0.0085	-0.0258	0.0145
-0.0001	0.0016	-0.0013	0.0115	-0.0069	0.0378
0.0011	-0.0011	0.0041	0.0125	-0.0025	0.0249
0.0010	-0.0050	0.0007	0.0125	0.0406	0.0519
0.0006	-0.0043	-0.0013	0.0122	0.0369	0.0345
0.0002	-0.0020	-0.0018	0.0118	0.0323	0.0160
0.0000	0.0010	0.0003	0.0115	0.0216	0.0029
-0.0003	0.0010	0.0002	0.0113	0.0138	-0.0037

Domestic Cost Push Shock

Inflation			Interest Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
-0.0090	-0.0025	-0.0057	-0.0060	-0.0069	-0.0093
-0.0055	-0.0049	-0.0088	-0.0078	-0.0009	-0.0121
-0.0027	0.0027	-0.0031	-0.0042	-0.0022	-0.0124
-0.0019	0.0025	-0.0044	-0.0015	0.0024	-0.0005
-0.0017	0.0028	-0.0030	-0.0002	0.0015	-0.0002
-0.0015	0.0006	-0.0041	0.0004	0.0013	0.0015
-0.0012	-0.0010	-0.0032	0.0006	0.0014	0.0021
-0.0009	-0.0004	-0.0010	0.0007	-0.0002	0.0009
-0.0006	0.0008	0.0005	0.0008	0.0009	0.0014
-0.0004	-0.0002	0.0005	0.0008	0.0004	0.0014

Output			Real Exchange Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
-0.0031	0.0107	0.0246	-0.0144	-0.0121	0.0139
0.0044	0.0112	0.0109	-0.0114	-0.0197	0.0012
-0.0009	0.0330	0.0275	-0.0137	-0.0277	-0.0105
-0.0064	0.0317	0.0186	-0.0152	-0.0330	-0.0049
-0.0091	-0.0258	-0.0216	-0.0147	-0.0214	0.0139
-0.0100	-0.0166	-0.0036	-0.0132	-0.0312	0.0092
-0.0099	-0.0123	0.0049	-0.0114	-0.0223	0.0052
-0.0093	-0.0197	0.0002	-0.0096	-0.0093	0.0154
-0.0086	-0.0239	-0.0032	-0.0079	-0.0197	0.0003
-0.0077	-0.0180	0.0005	-0.0065	-0.0042	0.0072

Domestic Importer Cost Push Shock

Inflation			Interest Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
0.0465	0.0032	-0.0448	0.0041	-0.0192	0.0155
0.0602	0.0048	-0.0345	0.0141	0.0004	0.0154
0.0503	0.0022	-0.0077	0.0149	-0.0013	0.0112
0.0341	0.0045	0.0179	0.0123	0.0006	0.0160
0.0188	-0.0066	0.0219	0.0091	0.0002	0.0104
0.0064	-0.0064	0.0062	0.0063	0.0003	0.0045
-0.0028	-0.0386	0.0007	0.0040	0.0001	0.0019
-0.0091	-0.0205	0.0054	0.0022	0.0002	-0.0004
-0.0132	-0.0290	0.0074	0.0009	-0.0009	-0.0015
-0.0155	-0.0227	0.0087	0.0000	-0.0024	-0.0032

Output			Real Exchange Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
-0.0214	-0.0309	-0.0002	-0.0710	0.0264	-0.0044
0.0108	0.0000	-0.0187	-0.1134	0.0637	-0.0081
0.0298	0.0008	-0.0045	-0.1595	0.1690	-0.0129
0.0417	0.0034	0.0055	-0.1950	0.1948	-0.0075
0.0486	0.0017	0.0051	-0.2168	0.1497	-0.0141
0.0517	0.0045	0.0146	-0.2265	0.0585	-0.0270
0.0523	0.0053	0.0163	-0.2266	0.0830	-0.0307
0.0511	0.0040	0.0047	-0.2198	0.1432	-0.0326
0.0488	0.0046	0.0034	-0.2085	0.2364	-0.0129
0.0457	0.0038	0.0065	-0.1944	0.1664	-0.0100

Domestic Preference Shock

Inflation			Interest Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
-0.0116	-0.0003	-0.0056	-0.0190	-0.0246	-0.0174
-0.0097	-0.0082	-0.0124	-0.0214	-0.0377	-0.0284
-0.0079	-0.0175	-0.0068	-0.0152	-0.0213	-0.0193
-0.0079	-0.0145	0.0049	-0.0099	-0.0141	-0.0098
-0.0080	-0.0204	-0.0042	-0.0063	-0.0053	-0.0045
-0.0077	-0.0155	-0.0051	-0.0037	-0.0010	0.0023
-0.0069	-0.0028	-0.0004	-0.0018	-0.0009	0.0021
-0.0060	-0.0018	-0.0038	-0.0002	0.0028	0.0038
-0.0049	0.0007	-0.0008	0.0011	0.0010	0.0034
-0.0038	0.0032	0.0006	0.0021	0.0018	0.0051

Output			Real Exchange Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
-0.0404	-0.0428	-0.0390	-0.0382	-0.1017	-0.0753
-0.0246	-0.0460	-0.0437	-0.0456	-0.0617	-0.0027
-0.0300	-0.0626	-0.0538	-0.0573	-0.1117	-0.0278
-0.0374	-0.0610	-0.0386	-0.0646	-0.1145	-0.0473
-0.0422	-0.0437	-0.0152	-0.0666	-0.1586	-0.0770
-0.0448	-0.0419	-0.0048	-0.0648	-0.1793	-0.0816
-0.0462	-0.0651	-0.0203	-0.0609	-0.0877	0.0307
-0.0468	-0.0811	-0.0230	-0.0557	-0.1295	0.0003
-0.0468	-0.0841	-0.0276	-0.0500	-0.1369	-0.0004
-0.0464	-0.0896	-0.0246	-0.0440	-0.1268	0.0074

Foreign TFP Shock

Inflation			Interest Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
-0.0048	-0.0078	-0.0006	-0.0037	-0.0040	-0.0011
-0.0043	-0.0087	-0.0041	-0.0030	-0.0062	0.0033
-0.0034	-0.0056	-0.0062	-0.0021	-0.0019	0.0042
-0.0029	-0.0019	-0.0025	-0.0016	-0.0034	0.0014
-0.0025	-0.0022	-0.0018	-0.0013	-0.0037	-0.0017
-0.0023	-0.0049	-0.0037	-0.0012	-0.0025	-0.0020
-0.0021	-0.0003	-0.0017	-0.0011	-0.0011	-0.0010
-0.0020	0.0008	-0.0007	-0.0010	-0.0019	-0.0018
-0.0019	0.0006	-0.0017	-0.0010	-0.0018	-0.0020
-0.0017	0.0008	-0.0021	-0.0010	-0.0011	-0.0010

Output			Real Exchange Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
-0.0116	-0.0191	-0.0182	-0.0215	-0.0120	-0.0013
-0.0090	-0.0249	-0.0073	-0.0163	-0.0230	-0.0159
-0.0113	-0.0160	-0.0020	-0.0147	-0.0142	-0.0101
-0.0131	-0.0239	-0.0153	-0.0137	-0.0045	-0.0022
-0.0141	-0.0208	-0.0126	-0.0128	0.0255	0.0228
-0.0147	-0.0327	-0.0230	-0.0122	0.0221	0.0069
-0.0152	-0.0284	-0.0165	-0.0116	0.0243	0.0102
-0.0156	-0.0258	-0.0111	-0.0111	0.0127	0.0086
-0.0160	-0.0233	-0.0065	-0.0107	0.0162	0.0185
-0.0163	-0.0244	-0.0080	-0.0104	0.0010	0.0105

Foreign Monetary Policy Shock

Inflation			Interest Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
-0.0035	0.0011	-0.0056	-0.0073	-0.0054	-0.0113
-0.0005	0.0000	-0.0019	-0.0002	0.0008	0.0000
-0.0006	-0.0008	-0.0001	0.0000	0.0002	0.0008
-0.0007	-0.0014	-0.0005	-0.0005	-0.0010	-0.0005
-0.0004	-0.0010	-0.0005	-0.0004	-0.0008	-0.0002
-0.0002	-0.0002	0.0000	-0.0002	-0.0003	0.0000
-0.0001	-0.0001	0.0001	0.0000	-0.0001	0.0000
-0.0001	-0.0002	0.0002	0.0000	0.0000	0.0000
-0.0001	-0.0004	0.0001	0.0000	0.0000	0.0000
-0.0001	-0.0003	0.0001	0.0000	0.0000	0.0000

Output			Real Exchange Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
0.0068	0.0000	0.0086	-0.0056	-0.0107	-0.0024
0.0065	0.0051	0.0076	-0.0062	-0.0135	-0.0025
0.0000	-0.0014	0.0022	-0.0063	-0.0065	-0.0014
-0.0008	-0.0025	-0.0006	-0.0038	-0.0038	-0.0013
-0.0001	-0.0003	0.0005	-0.0020	-0.0030	0.0003
0.0002	0.0000	0.0006	-0.0013	-0.0031	0.0008
0.0003	-0.0001	0.0005	-0.0010	-0.0026	0.0005
0.0002	-0.0001	0.0004	-0.0009	-0.0011	0.0012
0.0002	0.0000	0.0004	-0.0008	-0.0012	0.0004
0.0001	-0.0001	0.0002	-0.0007	-0.0008	0.0004

Foreign Cost Push Shock

Inflation			Interest Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
0.0010	0.0010	0.0012	0.0017	0.0019	0.0021
0.9111	0.7828	0.0001	-0.0007	0.0001	-0.0016
0.1635	0.4806	0.0005	-0.0010	0.0002	-0.0015
0.3048	-0.3643	0.0000	-0.0006	-0.0007	-0.0006
0.4432	-0.0753	-0.0002	-0.0003	-0.0002	-0.0004
0.3688	0.4028	0.0001	-0.0001	0.0004	-0.0002
0.2209	0.5370	-0.0002	0.0000	0.0001	-0.0005
0.0852	0.3754	-0.0004	0.0000	0.0001	-0.0005
-0.0199	-0.3155	-0.0006	0.0000	0.0002	-0.0003
-0.0963	-0.7545	0.0002	0.0000	0.0002	-0.0002

Output			Real Exchange Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
0.0021	0.0052	0.0018	0.0004	0.0003	-0.0009
0.0005	-0.0002	-0.0002	-0.0001	-0.0008	0.0016
0.0021	0.0031	0.0003	0.0012	0.0015	-0.0008
0.0026	0.0045	0.0006	0.0014	-0.0006	-0.0032
0.0024	0.0047	0.0017	0.0009	-0.0055	-0.0052
0.0021	0.0028	0.0007	0.0004	-0.0017	0.0028
0.0018	0.0029	0.0017	-0.0002	-0.0038	0.0046
0.0015	0.0028	0.0024	-0.0006	-0.0066	0.0043
0.0013	0.0025	0.0028	-0.0009	-0.0067	0.0046
0.0011	0.0013	0.0022	-0.0011	-0.0096	0.0019

Foreign Preference Shock

Inflation			Interest Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
0.0029	0.0093	-0.0088	0.0047	0.0054	0.0134
-0.0013	-0.0040	-0.0287	0.0056	0.0007	0.0069
-0.0023	0.0109	0.0005	0.0032	-0.0042	-0.0016
-0.0013	-0.0045	-0.0099	0.0019	0.0017	0.0035
0.0001	0.0035	0.0019	0.0015	0.0017	0.0005
0.0011	0.0012	-0.0007	0.0014	0.0049	0.0021
0.0020	0.0011	-0.0028	0.0014	0.0047	0.0019
0.0025	0.0044	-0.0017	0.0014	0.0065	0.0029
0.0029	0.0106	0.0019	0.0013	0.0057	0.0008
0.0031	0.0061	-0.0018	0.0013	0.0068	0.0017

Output			Real Exchange Rate		
Mean	HPDinferior	HPDsuperior	Mean	HPDinferior	HPDsuperior
-0.0159	-0.0355	-0.0465	-0.0927	-0.0336	-0.1075
-0.0351	-0.0231	-0.0402	-0.0920	-0.0914	-0.1739
-0.0334	-0.0108	-0.0124	-0.0812	-0.0600	-0.1391
-0.0305	-0.0149	-0.0139	-0.0713	-0.0989	-0.2120
-0.0290	-0.0136	-0.0071	-0.0637	-0.0438	-0.1453
-0.0283	-0.0076	-0.0033	-0.0577	-0.0501	-0.1602
-0.0276	-0.0041	-0.0027	-0.0529	-0.0065	-0.1005
-0.0270	-0.0064	-0.0083	-0.0490	-0.0660	-0.1382
-0.0264	-0.0055	-0.0134	-0.0458	-0.0068	-0.0422
-0.0259	-0.0056	-0.0144	-0.0431	-0.0613	-0.0759

Chapter 3

Impact of Foreign Monetary Policy on Eastern European Countries

3.1 Introduction

The goal of this paper is to show the impact of US monetary policy shocks on real output and price levels in several Eastern European countries (EEC). Particularly, I am interested in assessing how much of the movement is generated directly by the US monetary movements in the policy shock and how much indirectly through changes in German aggregate demand caused by this shock, under the assumption that the economic performances in Germany and the EEC are closely related. My starting point is the presumption that US monetary policy shocks might have a significant influence on these countries. Nonetheless, it can be argued that Germany is a major trading partner for all the EEC. It attracts between 25 to

30 percent of the total exports from each of these countries and the EEC are also substantial importers of goods produced in Germany. These relationships are not reciprocal, which suggest that the EEC can be characterised as SOE relative to Germany. It follows from the data that the openness of the EEC towards Germany should be significantly stronger than the one towards the US market. An analogous relationship can be found between Germany and the US. Since the US is an important export partner for Germany, covering a 7 percent export share, but not vice versa, Germany might be therefore regarded as a small open economy (SOE) relative to the US.

Because the data suggests that the EEC are more open towards Germany than the US, one may expect that the effect of the US shock is significantly weaker than the one generated by German Bundesbank/ECB. In this paper, I show that this is not the case: even if I control for the US impact through Germany (by including German variables), the strength of the effects of both shocks on EEC variables are comparable.

I choose the Czech Republic, Hungary, Poland and the Slovak Republic as representatives of the EEC, countries that share similar characteristics. They started their economic transition in the early 1990's and rapidly opened their economies to Western trade and investment. For these countries the early 90's were characterised by higher inflation especially in Hungary and Poland caused by price liberalisation. During this period the exchange rate was pegged to a basket of currencies but during the second half of 1990's, they all adopted flexible exchange rate regime. The main part of the transition was finished and the economy system of these countries stabilised.

In this chapter, I run three different estimations. First, I estimate the direct influence of the U.S. monetary policy shock on the crucial variables of the EEC. Second, I repeat this estimation using Germany monetary policy shock instead. Both estimations are in line with the mainstream theory. According to the Dornbush-Mundell-Fleming model, the unanticipated increase in the large economy interest rate can have two contradictory effects on the variables of a SOE; the expenditure switching effect and the income absorption effect.[1] The resulting impact can lead to different reactions of the domestic output when the interest rate in the large country changes. The third estimation contains, beside the EEC, both large countries, Germany and the US, where it is assumed that Germany is open towards the US and closed towards the EEC. The objective is to investigate how much of the monetary policy shock generated by the FED is absorbed by Germany and how much is instead directly transmitted to the EEC.

The analysis is performed using a Bayesian Vector Autoregressive Model (BVAR). In contrast to the maximum likelihood method, whose estimates are based purely on the information contained in the data, Bayesian analysis departs from this approach by allowing me to incorporate prior beliefs about the parameters into the estimation process. Therefore, I can identify the foreign monetary shock using sign contemporaneous restrictions and also threat all foreign variables as exogenous using zero restrictions in a long run. Following Banbura et al. (2008), I implement the natural conjugate prior via artificial observation. This involves generating artificial data from the model assumed under the prior and mixing this with the actual data. The weight placed on the artificial data regulates how tightly the prior is imposed and I explain the all procedure in more details later.

[1] This mechanism is described in more details in Section 3.4.

The dataset available for the EEC is not very rich. To allow for a cross country comparison, I am forced to opt for relatively short time series. As a result, I use data starting in 1994. Therefore, the number of observations is limited, hence I restrain the number of variables too, by focusing on the movement in key macroeconomic variables such as CPI inflation and GDP growth.

The structure of this paper is as follows. The next section discusses the relevant literature. Section 3.2 gives more details about the VAR model adopted for the estimations. Section 3.3 describes the structural analysis for each country, including the impulse response functions, forecast error variance decomposition (FEVD) and historical decomposition. Section 3.4 compares results of this chapter with the ones of Chapter 2. Finally, Section 3.5 concludes.

3.1.1 Literature Review

A vast recent literature analyses exogenous disturbances generated at home or abroad and their impact on other macroeconomic variables. Several studies, including Gordon and Leeper (1994), Uhlig (2005) and Canova and Gambetti (2003), investigate the US monetary policy shock and its impact on the US macroeconomic variables. Similarly, Kim (1999) studies the effects of domestic monetary policy shocks in individual G-7 countries and Kim (2001a) shows the effect of the (domestic) monetary policy shock on the trade balance in small European countries such as France, UK and Italy, using German and US interest rate as a proxy for a world-wide short term interest rate. Furthermore, various authors study the impact of foreign shocks on SOE. Kim (2001b) analyses the effect of US monetary policy on the exchange rate and foreign trade balances on other G-6 countries. He shows that an expansionary US monetary policy shock generates positive spillover effects. Canova (2005) studies the transmission of US shocks on Latin countries and finds that the foreign monetary policy shock produces more fluctuations than real demand and supply shocks generated abroad. Additionally, Mackowiak (2007) finds that US monetary shocks are an important source of macroeconomic fluctuations for small emerging markets in South East Asia and Latin America. These shocks explain more of the variation of real aggregate output and the price level in those countries than the domestic monetary shocks.

Some authors also investigate the effect of monetary policy shocks on EEC. For example, Anzuini and Levy (2007) examine the effects of an EEC domestic monetary policy shock in a given EEC on its own key macroeconomic variables. Mackowiak (2006) studies the effect of ECB monetary policy shocks on those variables. My work is closely related to these two papers; I am investigating a new channel of foreign monetary policy influence. Using a method similar to Kim (2001b) and Canova (2005), I am interested in the impact of US monetary policy shocks on macroeconomic variables on the EEC. I use Mackowiak's (2006) argument that these countries are open to exogenous disturbances and show that a monetary shock that originates in the US can explain at least the same amount of EEC macroeconomic fluctuations as a shock generated by the European Central Bank (and previously by the Deutsche Bundesbank).

There are several possibilities for how to define a monetary policy shock. In most papers, including Mackowiak (2007), authors use interest rates set by central banks, whereas in other contributions, including Canova (2005), economists use the slope of the term structure of nominal interest rate and real balances. In this work, I follow the mainstream and use the quarterly change in the Federal Reserve funds rate as a source of the monetary policy shock.

In this paper, the BVAR methodology adopted relates to the methodology used by Kim (2001 and 2001b), Canova (2005) and Mackowiak (2006, 2007). The long-run zero restrictions for SOE are based on different findings from Cushman and Zha (1995), Kim and Roubini (2000) and Kim (1999). The sign restrictions are generated in a similar fashion as in Canova (2005) and Scholl and Uhlig (2005), using an algorithm developed by Ramirez *et al.* (2010). Finally, I impose the prior in my model using artificial observations following the work from Banbura *et al.* (2008).

The field of VAR econometrics is wide and several alternative approaches can be found in the literature. A number of papers raise some concerns about small scale SVAR and develop alternative methods. For example, Factor-augmented VARs (FAVARs), developed by Bernanke *et al.* (2005) incorporate more information so that the monetary policy shock can be better identified. Mumtaz and Surico (2007) use this approach to analyse the effect of world wide monetary policy shocks on a SOE. They show that an expansionary monetary policy shock causes a domestic nominal exchange rate appreciation and an increase in prices and in GDP (a prosper thy-neighbor situation). The Global VAR (GVAR) approach, proposed by Pesaran *et al.* (2002) and di Mauro *et al.* (2007), employs a vector error correction model for individual countries and combines the result to generate an estimate for all the variables simultaneously. Yet another methodology can be found in Canova and Ciccarelli (2006). Using a multi-country panel VAR model with time varying coefficients and cross unit interdependencies, these authors study the transmission of different shocks on G7 countries focusing on GDP growth and CPI inflation, and emphasise that this model is suitable for the study of the transmission of monetary policy shocks across economic areas and sectors.

3.2 Methodology

Testing the impact of monetary and fiscal policy is not a new idea. For example, Anderson and Jordan (1964) investigate the impact of a change in the monetary base on real GNP using a simple autoregressive model. However, their approach was criticized by Sims (1980) because of the missing feedback between GNP and the monetary base. This author went on proposing a vector autoregressive (VAR) model to analyse the monetary policy shock and its impact on endogenous variables. Since then, VAR models have been frequently used to identify the channels of monetary policy transmission. During the following twenty years, VAR models became very popular as an econometric technique for analysing the relationships between different endogenous variables. Ever since, the VAR approach has mainly been used to analyse the effects of a shock on macroeconomic variables using tools such as impulse response functions, FEVD and historical decomposition.

The literature generally distinguishes between reduced-form and structural VAR models, depending on whether the shocks are correlated or orthogonal to each other. Furthermore, putting some restrictions into the model leads to a restricted VAR, otherwise the model is described as unrestricted VAR. Orthogonalising the shocks can either follow directly from a Cholesky decomposition of the error terms covariance matrix, or from using restrictions derived from an economic interpretation of the model. The first is also known in the literature as recursive VAR.[2] At the moment, it appears that no clear consensus has formed in the literature regarding whether restrictions following from the Cholesky decomposition should be based on theory. On the one hand, Stock and Watson (2001) p.18 emphasise, "It is tempting to develop economic "theories" that, conveniently, lead to a particular recursive ordering of the variables. Rarely does it add value to repackage a recursive VAR and sell it as structural." On the other hand, there are authors, *e.g.* Gottschalk (2001), stating that when using a Cholesky decomposition, the restrictions need to be supported by theoretical interpretation. Canova (2007) argues that a Cholesky decomposition without any economic interpretations may be misleading.

To explore my research question in this chapter, I follow Canova (2007) and use a structural VAR with sign restrictions. I aim to apply a model that is consistent with the economic theory, mainly to avoid a misspecification of the monetary policy shock that, if positive, should lead to a contemporaneous increase in interest rate and a contemporaneous fall in output and inflation and not a contemporaneous zero response of output and inflation as implied by Cholesky decomposition, for example.

3.2.1 The reduced-form VAR

The reduced-form VAR model is the most general formulation within the VAR family and can be described as follows. Consider T observations of m variables. Take a VAR(p) process, where p is the number of lags of the process with linear

[2] C.f. Stock and Watson (2005).

structure, to estimate the relationship among a set of endogenous variables as follows

$$Y_t = BX_t + \epsilon_t \tag{3.1}$$

with $X_t = (Y_{t-1}, ...Y_{t-p}, 1)'$ and $B = (B_1, ...B_p, C)$, where Y_t is a $m \times 1$ vector of endogenous variables in period t. The intercept term C is a $m \times 1$ vector, which allows for the possibility of a nonzero $E[Y_t]$, $B\,(j)$, for $j = 1, .., p$, is a $m \times m$ matrix of regressors. The residual ϵ_t is a Gaussian white noise with zero mean (*i.e.* $E[\epsilon_t] = 0$) and variance-covariance matrix Σ exhibiting the following characteristics

$$\begin{aligned} E[\epsilon_t \epsilon_s'] &= \Sigma \text{ if } t = s, \\ E[\epsilon_t \epsilon_s'] &= 0 \text{ if } t \neq s. \end{aligned} \tag{3.2}$$

It is possible to incorporate some restrictions that have an economic interpretation into this model. However, as long as the number of restriction is insufficient, there is a lack of structure in this model. That means the error terms are correlated with each other and the identification of the individual shocks is difficult.

3.2.2 The Structural VAR

Several techniques can be adopted to impose structure on the VAR model. All aim to obtain consistent estimators, which means that the shocks must be orthogonolised and identified uniquely. Shocks can be orthogonalized by using a Cholesky decomposition of the covariance matrix.[3] This recursive VAR approach can lead

[3] This technique is used by many authors, *e.g.*, Mackowiak (2006).

to restrictions that are not consistent with the theory.[4] For this reason, many authors prefer to use a structural VAR (SVAR) approach, developed by Sims (1986), which includes restrictions that are in line with economic theory. However, the high number of restrictions needed to properly specify the model complicates its implementation. Therefore, the version of SVAR most frequently used in the literature is a hybrid approach, which uses a Cholesky decomposition together with economically meaningful restrictions. The model presented here belongs to this hybrid approach. Along with a Cholesky decomposition, I restrict the model by imposing specific signs and zero values.

To obtain an orthogonolised error term from equation (3.1), I can use an $m \times m$ matrix A such as

$$Y_t = BX_t + Ae_t,$$

where e_t is an orthogonal white noise vector following from $\epsilon_t = Ae_t$ with an identity variance-covariance matrix given by

$$\begin{aligned} E[e_t e_s'] &= I_m \text{ if } t = s, \\ E[e_t e_s'] &= 0 \text{ if } t \neq s. \end{aligned} \tag{3.3}$$

It follows from equation (3.2) that

$$E[Ae_t e_t' A'] = AA' = \Sigma.$$

In the contemporaneous period, the sign restrictions are implemented in such a way that the impulse responses to a monetary policy shock are consistent with

[4] C.f. Krolzig (2003), Lütkepohl (2005).

the theory. In this respect, I follow Ramirez *et al.* (2010), who provide an efficient algorithm to find the structural impact matrix $\tilde{A}$, which is consistent with impulse responses of certain signs.

To compute the structural impact matrix $\tilde{A}$, I draw some matrix $J \sim N(0,1)$, and take the QR decomposition $J = QR$ to find an orthonormal matrix Q such that it holds $QQ' = I_m$ and $\tilde{A} = AQ$. Therefore, I can write

$$\begin{aligned} \Sigma &= AQQ'A' \qquad (3.4) \\ \Sigma &= \tilde{A}\tilde{A}'. \end{aligned}$$

It is important that the matrix $\tilde{A}$ satisfies the sign restrictions set out below and it still holds that

$$\epsilon_t = \tilde{A}e_t. \qquad (3.5)$$

If it does not, I draw another matrix J from the normal distribution and repeat the procedure.

Assume that the identification scheme takes the following form

$$\begin{pmatrix} \epsilon\{IR^*\} \\ \epsilon\{\Delta GDP^*\} \\ \epsilon\{\Delta CPI^*\} \\ \epsilon\{\Delta GDP^{EEC}\} \\ \epsilon\{\Delta CPI^{EEC}\} \\ \epsilon\{XR^{EEC}\} \end{pmatrix} = \begin{pmatrix} + & . & . & 0 & 0 & 0 \\ - & . & . & 0 & 0 & 0 \\ - & . & . & 0 & 0 & 0 \\ . & . & . & . & 0 & 0 \\ . & . & . & . & . & 0 \\ . & . & . & . & . & . \end{pmatrix} \begin{pmatrix} e\{IR^*\} \\ e\{\Delta GDP^*\} \\ e\{\Delta CPI^*\} \\ e\{\Delta GDP^{EEC}\} \\ e\{\Delta CPI^{EEC}\} \\ e\{XR^{EEC}\} \end{pmatrix}.$$

The vector $a_j \in R^m$ is called an impulse vector if there is some matrix $\tilde{A}$

such that $\tilde{A}\tilde{A}' = \Sigma$ holds and a_j is the jth column of $\tilde{A}$. The impulse vector yields the instantaneous impulse response of all variables to the structural shock associated with that vector and, in my specification needs to have the following signs: $a_{11} > 0$, $a_{21} < 0$, and $a_{31} < 0$. In other words, the sign restrictions on large economy variables ensure that positive shocks in the interest rate implies a fall in GDP growth and inflation in the US. The impulse responses for the rest of the variables remain unrestricted on sign.

The identification is completed by using zero restrictions on contemporaneous structural parameters so as to ensure that the SOE does not influence the large economy contemporaneously. The dots correspond to freely estimated parameters.[5]

Beside the contemporaneous restrictions, I impose restrictions on lags. Note that the model can be divided into two parts, a first part with a $m_1 \times 1$ vector of the foreign large economy variables (Y_t^*), and a complementary $(m - m_1) \times 1$ vector of domestic SOE variables (Y_t^{EEC})[6]

$$Y_t = \begin{pmatrix} Y_t^* \\ Y_t^{EEC} \end{pmatrix}, \epsilon_t = \begin{pmatrix} \epsilon_t^* \\ \epsilon_t^{EEC} \end{pmatrix}. \tag{3.6}$$

Therefore, the matrix $B\left(j\right)$, $j = 1, ..., p$, can be rewritten as

$$B\left(j\right) = \begin{pmatrix} B_{11}(j) & B_{12}(j) \\ B_{21}(j) & B_{22}(j) \end{pmatrix}, \tag{3.7}$$

[5]For a critical survey on contemporanous restrictions, see Fry and Pagan (2011).

[6]In this paper, I assume that Y^* includes interest rate, foreign output, price level, while Y^{EEC} includes domestic output, price level and exchange rate.

where $B_{11}(j)$ is the $m_1 \times m_1$matrix of coefficients between foreign variables. $B_{12}(j)$ is $(m - m_1) \times m_1$ matrix of coefficients which demonstrate the impact of domestic SOE variables on the variables in the large economy. Following Cushman and Zha (1995), I impose zero restrictions on the prior beliefs that domestic economies are small and cannot influence the US economy with their action at any time, so that $B_{12}(j) = 0$. Analogously, the $m_1 \times (m - m_1)$ matrix $B_{21}(j)$ includes coefficients that measure the impact of the foreign large economy on the domestic SOE variables. The matrix $B_{22}(j)$ is a matrix of coefficients showing the relationship between domestic variables.

Assume that the vector Y_t^* includes three variables of the large economy with the following ordering: the interest rate IR^*, the GDP growth ΔGDP^* and the CPI inflation ΔCPI^*, formally

$$Y_t^* = \begin{pmatrix} IR^* \\ \Delta GDP^* \\ \Delta CPI^* \end{pmatrix}. \tag{3.8}$$

The first line of the system (3.1) represents the monetary policy rule of the large economy with parameters in the first row of $B_{11}(j)$, having the form of a Taylor rule with an interest rate smoothing component.

In the whole system, I identify only the foreign monetary policy shock. Beside the impulse-response functions, the matrix $\tilde{A}$ is used to calculate the FEVD as well as the historical decomposition. As emphasised in the literature, it is not customary to report the estimated VAR coefficients. The reasons are that there is a large number of them, and more importantly, many are not significant, especially for further lags. Therefore it is usual to report the results of the estimations in a summarised way. The most powerful tools in this case are impulse response functions, FEVD and the historical decomposition.

Impulse Response Functions

To isolate the effect of a monetary policy shock, I consider the VAR(p) model in companion form[7]

$$Z_t = \tilde{B} Z_{t-1} + v_t$$

with $Z_t = X_{t+1} = (Y_t, ...Y_{t-p+1}, 1)'$ and $v_t = (\epsilon_t, 0, ..., 0)'$ are $((m \times p) + 1) \times 1$ vectors of endogenous variables and error terms respectively, where the first row of this is identical to equation (3.1) and the remain rows are trivial. $\tilde{B}$ is a $((m \times p) + 1) \times ((m \times p) + 1)$ matrix given as

$$\tilde{B} = \begin{pmatrix} B_1 & B_2 & ... & B_{p-1} & B_p & C \\ I_m & 0 & ... & & 0 & 0 \\ & \ddots & & & & \vdots \\ 0 & 0 & & I_m & 0 & 0 \\ 0 & 0 & ... & 0 & 0 & 1 \end{pmatrix}.$$

I then solve backwards to obtain a moving average (MA) representation, which by ignoring deterministic terms delivers

$$Z_t = \tilde{B}^t Z_0 + \sum_{j=0}^{t-1} \tilde{B}^j v_{t-j}. \tag{3.9}$$

[7] The companion form transforms a VAR(p) into a VAR(1) model, see *e.g.* Canova (2007), chapter 4.

The required impulse response functions are represented in the first m rows of matrix $\tilde{B}^j$. Therefore, I can ignore deterministic terms because they are not important for the impulse responses and rewrite the equation (3.9) in terms of the reduced-form VAR form. Writing the first $m \times m$ part of matrix $\tilde{B}^j$ given as $\tilde{B}_1^j = \Psi_j$, I obtain

$$Y_t = \sum_{j=0}^{t-1} \Psi_j \epsilon_{t-j}. \tag{3.10}$$

Together with (3.5), this equation can be rewritten in terms of orthogonolised shocks as

$$Y_t = \sum_{j=0}^{t-1} \Phi_j e_{t-j}, \tag{3.11}$$

where $\Phi_j = \Psi_j \tilde{A}$. The equation (3.11) says that as one shock hits the economy

(*e.g.,* interest rate increases by one standard deviation point in period zero), one can see the effect to the system for the next s periods when no further shocks hit the economy. The generic (k, l) element of matrix Φ_j given by $\theta_{kl,j}$ represents the reaction of variable k (in period $t + j$) to a unit shock experienced by variable l, in period t.

Historical Decomposition

Following Lütkepohl (2011), the historical decomposition of the time series describes the contribution of the structural shock to the observed series. Using equation 3.11 , the variable k in period t can be represented as a sum of the structural shocks $l = 1, ...L$

$$y_{k,t} = \sum_{j=0}^{t-1} \left(\theta_{k1,j} e_{1,t-j} + \theta_{k2,j} e_{2,t-j} + ...\theta_{kL,j} e_{L,t-j}\right).$$

Therefore, it can be shown that the variable $y_{k,t}$ can be decomposed into the sum of the structural shocks. Thus,

$$y_{k,t}^{l}=\sum_{j=0}^{t-1}\theta_{kl,j}e_{l,t-j}$$

is the contribution of a specific structural shock l to the time series of variable y_k.

Forecasting Error Variance Decomposition[8]

Using equation 3.10, I can identify the error in forecasting the VAR s periods into the future as

$$Y_{t+s}-y_{t+s|t}=\epsilon_{t+s}+\Psi_1\epsilon_{t+s-1}+\Psi_2\epsilon_{t+s-2}+...+\Psi_{s-1}\epsilon_{t+1}. \tag{3.12}$$

The FEVD is given as a mean square error (MSE) of this s-period ahead forecast, expressed as

$$\begin{aligned} MSE\left(y_{t+s|t}\right) &= E\left[\left(Y_{t+s}-y_{t+s|t}\right)\left(Y_{t+s}-y_{t+s|t}\right)'\right] \\ &= \Sigma+\Psi_1\Sigma\Psi_1'+\Psi_2\Sigma\Psi_2'+...+\Psi_{s-1}\Sigma\Psi_{s-1}', \end{aligned} \tag{3.13}$$

where $\Sigma = E\left[\epsilon_t\epsilon_t'\right]$. The FEVD shows how each of the orthogonolised disturbances $(e_{1t},...,e_{mt})$ contributes to this MSE. In terms of structural shocks, I can

[8]For more details, see Hamilton (1994) chapter 11 and Canova (2007) chapter 4.

equivalently write

$$\begin{aligned} MSE\left(y_{t+s|t}\right) &= \Sigma + \Psi_1 \Sigma \Psi_1' + \Psi_2 \Sigma \Psi_2' + ... + \Psi_{s-1} \Sigma \Psi_{s-1}' \\ &= \tilde{A}\tilde{A}' + \Psi_1 \tilde{A}\tilde{A}' \Psi_1' + \Psi_2 \tilde{A}\tilde{A}' \Psi_2' + ... + \Psi_{s-1} \tilde{A}\tilde{A}' \Psi_{s-1}' \\ &= \Phi_0 \Phi_0' + \Phi_1 \Phi_1' + \Phi_2 \Phi_2' + ... + \Phi_{s-1} \Phi_{s-1}' \end{aligned}$$

with $\Phi_0 = \tilde{A}$. The kth diagonal element of $\Phi_j \Phi_j'$ is the sum of the squares of the elements in the kth row of Φ_j. Moreover, the sum of the kth diagonal elements of $\Phi_0 \Phi_0', ..., \Phi_{s-1} \Phi_{s-1}'$ is the MSE of forecast error variance of the s-step forecast of variable Y_j. Hence, the fraction of the variance in $y_{k,t+s|t}$ due to $\epsilon_{l,t}$ is

$$VD_{k.l} = \frac{\theta_{kl,0}^2 + \theta_{kl,1}^2 + ... + \theta_{kl,s-1}^2}{MSE\left(y_{k,t+s|t}\right)},$$

where $\theta_{kl,j}^2$ is the (k,l) element of Φ_j.

3.2.3 BVAR with Gibbs Sampling Estimation

In the early stages of the VAR literature, econometric models were usually estimated by using ordinary least squares (OLS) methods. Recently, Bayesian methods have attracted increased attention, because they are generally more precise than standard estimation approaches. Compared to the standard methodology, Bayesian estimations incorporate subjective beliefs or theoretical restrictions about the state of the coefficients. Bayesian methods were introduced by Zellner (1971) and have become more popular in the last twenty years, when the computer software and hardware developed and techniques such as Gibbs sampler were introduced.[9] In this work, I follow the contemporary literature and apply Bayesian estimation methods using Gibbs sampling to estimate the parameters of the SVAR model presented above.

The Bayesian estimation combines a subjective prior together with sample information. It is based on the Bayes' theorem, which states that

$$\text{posterior distribution} \propto \text{likelihood} \times \text{prior distribution}.$$

The likelihood function is taken from the OLS estimation of the data sample.[10] Equivalently, it can be written as

$$R\left(vec\left(B\right)\backslash\Sigma, Y_t\right) \propto F\left(Y_t\backslash vec\left(B\right), \Sigma\right) \times P\left(vec\left(B\right), \Sigma\right),$$

the posterior distribution $R\left(vec\left(B\right)\backslash\Sigma, Y_t\right)$ is proportional to the product of the prior distribution $P\left(vec\left(B\right), \Sigma\right)$ and distribution of the sample as given by the likelihood function $F\left(Y_t\backslash vec\left(B\right), \Sigma\right)$. The vector $vec\left(B\right)$ is a matrix of regressors B in vector form and Σ the variance-covariance matrix. The prior density and the likelihood function are both very important for the correct estimation of the model and therefore it is necessary to specify them fully. There exist several approaches to set the prior. Many authors use the Minnesota prior, developed by Litterman (1986), because of its simplicity, but since I incorporate a prior belief with zero

[9] C.f. Greene (2003), chapter 18 for more details.

[10] The likelihood function for B and Σ given the data can be expressed as

$$F\left(Y\backslash vec\left(B\right), \Sigma\right) = \left(2\pi\right)^{-Tm/2}\left|\Sigma^{-1}\right|^{T/2}\exp\left[-\frac{1}{2}\left(y - vec\left(B\right)' X\right)' \Sigma^{-1}\left(y - vec\left(B\right)' X\right)\right].$$

For more details, see e.g. Hamilton (1994), chapter 11.

restrictions, as discussed in Section 3.2, I opt for an independent normal inverse Wishart prior. Technically, I impose this prior by following Banbura *et al.* (2008) and incorporate additional artificial data.

It can be assumed, given the nature of the data, that the matrix of coefficients B is normally distributed

$$P\left(vec\left(B\right)\right) \sim N\left(vec\left(B_0\right), H\right), \tag{3.14}$$

where $vec\left(B_0\right)$ is the $\left(\left(m \times \left(m \times p+1\right)\right) \times 1\right)$ vector of prior means for the elements of matrix B. The matrix H is a $\left(\left(m \times \left(m \times p+1\right)\right) \times \left(m \times \left(m \times p+1\right)\right)\right)$ diagonal matrix, whose elements are the prior variances for each corresponding coefficient from matrix B_0. As discussed earlier, I impose the strong prior belief that the elements of matrices $B_{12}(j)$, $B_{13}(j)$ and $B_{23}(j)$ equal zero. Therefore, the prior variances of the corresponding elements in matrix H are set to be very low.

Following Zellner (1971), the conjugate prior for a positive definite variance-covariance matrix Σ is an Inverse Wishart prior

$$P\left(\Sigma\right) \sim IW\left(\bar{S}, \alpha\right) \tag{3.15}$$

with the prior scale matrix $\bar{S}$ and prior degrees of freedom α.

Given the fact that conjugate prior on B is normal distributed, it can be shown that the posterior distribution of the coefficients conditional on the variance-covariance matrix Σ is given by

$$R\left(vec\left(B\right) \backslash \Sigma, Y_t\right) \sim N\left(M^*, V^*\right),$$

where M^* and V^* are the mean and the variance of this normal distribution, respectively. As shown in Kadiyala and Karlsson (1997), the mean and the variance of the conditional posterior distribution are respectively given by

$$\begin{aligned} M^* &= \left(H^{-1} + \Sigma^{-1} \otimes X_t' X_t\right)^{-1} \left(H^{-1} vec\left(B_0\right) + \Sigma^{-1} \otimes X_t' Y_t\right) \\ V^* &= \left(H^{-1} + \Sigma^{-1} \otimes X_t' X_t\right)^{-1}. \end{aligned} \tag{3.16}$$

Note that M^* is a weighted average of the prior mean $vec\left(B_0\right)$ and the OLS estimator, given by $X_t' Y_t$, weighted by the reciprocal of the corresponding variance-covariance matrices. The smaller the values of matrix H elements, the higher the weight on the prior relative to the conditional posterior estimates. In the case where there are no beliefs about the prior, *i.e.* the value of matrix H elements are very large, then the posterior estimates are identical to the maximum likelihood estimator.

Given the prior in equation (3.15), the posterior distribution for Σ conditional on B is Inverse Wishart

$$R\left(\Sigma \backslash vec\left(B\right), Y_t\right) \sim IW\left(\bar{\Sigma}, T + \alpha\right),$$

where $\bar{\Sigma} = \bar{S} + \left(Y_t - X_t B\right)' \left(Y_t - X_t B\right)$, with T observations and α degrees of freedom.

Returning to the issue of how to incorporate prior beliefs into the estimation of my VAR model, I follow Banbura *et al.* (2008) and implement prior information by adding artificial data to the system. The artificial data Y_D and X_D are formed by four independent blocks as follows

$$Y_D = \begin{pmatrix} \frac{diag(\chi_1\sigma_1 ... \chi_m\sigma_m)}{\lambda} \\ 0_{((m\times(p-1))\times m)} \\ -------------- \\ diag\,(\sigma_1 ... \sigma_m) \\ -------------- \\ 0_{(1\times m)} \\ -------------- \\ \frac{diag(\chi_1\mu_1 ... \chi_m\mu_m)}{\tau} \end{pmatrix}, \; X_D = \begin{pmatrix} \frac{J_p \otimes diag(\sigma_1 ... \sigma_m)}{\lambda} & 0_{(mp\times 1)} \\ ---------------- & ------ \\ 0_{(m\times mp)} & 0_{(m\times 1)} \\ ---------------- & ------ \\ 0_{(1\times mp)} & c \\ ---------------- & ------ \\ \frac{J_p \otimes diag(\chi_1\mu_1 ... \chi_m\mu_m)}{\tau} & 0_{(m\times 1)} \end{pmatrix}. \tag{3.17}$$

The first block in each matrix imposes the prior beliefs on the autoregressive coefficients. The second block implements the prior for the variance-covariance matrix and the third block reflects the uninformative prior for the intercept. By adding artificial data in the last row, I incorporate the prior that incorporates the belief that the sum of the coefficients on lags of the dependent variable in each equation sum to 1, *i.e.* that each variable has a unit root. The matrix J_p is given as $J_p = diag\,(1...p)$. As in Banbura *et al.* (2008), the variance of the prior distribution is defined by hyperparameters that regulate the variation around the prior. The hyper-parameter $\lambda > 0$ controls the overall tightness of the prior so that as $\lambda \to 0$, the prior is implement more tightly, whereas the larger the value of this parameter the more the posterior approaches an OLS estimation of the VAR model. The hyperparameter τ controls for the degree of shrinkage. If τ is large, the prior is imposed loosely. I set $\lambda = 10$ and $\tau = 10\lambda$, implying that the prior on these data is not very informative. The parameter χ_i measures the persistence of variable i, and follows from the OLS estimation of AR(1). Literally, it is a prior mean for the coefficient on the first lag of dependent variable i. The parameter

μ_i is a sample mean of the variable i, and σ_i is a sample standard deviation of error terms. They can both be calculated as sample averages of the time series y_i from the OLS estimation. The matrix Y_D is the $(m(p+2)+1) \times m$ matrix and X_D is a $(m(p+2)+1) \times (mp+1)$ matrix adding $(m(p+2)+1)$ dummies to each time series. These artificial data are mixing with the actual data and the hyperparameters placed on them determine how tightly the prior is imposed. This approach also helps to alleviate the curse of dimensionality in the VAR model.

Following (3.7), some prior coefficients B_0 are restricted to zero on lags. The prior for coefficient matrix $B_0(j)$ has therefore the form given as

$$B_0(j) = \begin{pmatrix} B_{11}^0(j) & 0 \\ B_{21}^0(j) & B_{22}^0(j) \end{pmatrix},$$

and incorporates the belief that the coefficients of matrix $B_{12}^0(j)$ for all $j = 1, ..., p$ are close to zero. To ensure that these restrictions are also fulfilled for the posterior, so that the appropriate parameters stay close to zero, it is necessary to set the elements of the prior variance H matrix belonging to these coefficients very close to zero. For the remaining coefficients, regarding the first lag $j = 1$, the prior mean on its own lag is set equal to 0.95, *e.g.,* the diagonals of matrices $B_{11}(1)$, $B_{22}(1)$ and $B_{33}(1)$ equals 0.95. For all other elements of matrix $B(1)$, the elements are set to be zero. The vector $\tilde{C}$ is a zero vector and matrix $B(j)$, $j = 2, ...p$, is a zero matrix. The elements of the prior variance H correspond to all the coefficients except those for $B_{12}(j)$, $B_{13}(j)$ and $B_{23}(j)$, which are set to be sufficiently large that these coefficients are mainly determined within the model. To summarise, H is a $((m \times (m \times p + 1)) \times (m \times (m \times p + 1)))$ diagonal matrix, with near-zero elements for coefficients which are believed to be zero, and large elements for the remaining coefficients. The details for a model with three countries are given in Appendix 3.B.

Gibbs Sampling

To carry out the Bayesian inference, I use a Gibbs sampling procedure, which is a posterior Markov chain Monte Carlo (MCMC) simulation mechanism. Gibbs sampling is a numerical method that uses a great many draws from a conditional distribution to approximate joint and marginal posterior distribution for B and Σ. The reason for using Gibbs sampling to calculate the marginal density is that analytical methods are either unavailable or very complicated.[11]

The Gibbs algorithm iterates M times and produces draws for B and Σ. Each iteration requires sampling from the conditional posterior distribution, which after the burn-in draws are discarded converges to the marginal distribution. Samples from the beginning of the chain, the first J draws are discarded to remove the influence of starting values. Once draws from the posterior distribution are obtained, I implement a structural analysis to ensure that the sign restrictions hold. Appendix 3.C shows the convergence of the algorithm via recursive mean plots.

The Gibbs algorithm is given as follows.

1. Set the priors for coefficient matrix $p\left(vec\left(B\right)\right) \sim N\left(vec\left(B_0\right), H\right)$ and for the variance - covariance matrix $p\left(\Sigma\right) \sim IW\left(\bar{S}, \alpha\right)$ as described above, and the starting values obtained from OLS estimation.

[11] Another option to approximate marginal distribution is by using LaPlace Approximation, see e.g. by Mackowiak (2006).

2. Sample conditional posterior distribution of B, the first coefficient vector $vec(B_1)$, with variance V^* and mean M^* as given in (3.16).

3. Given $vec(B_1)$, draw variance-covariance matrix Σ_1 from Inverse Wishart distribution.

4. Compute a matrix $\tilde{A}$, such that $\tilde{A}\tilde{A}' = \Sigma$ using a Cholesky and QR decomposition according to (3.4).

5. Identify the signs on $\tilde{A}$. If they satisfy the sign conditions, matrix $\tilde{A}$ will be used for further analysis, if not this step is repeated.

6. Repeat 1-6 M times to obtain $vec(B_1), ..., vec(B_M)$, $\Sigma_1, ...\Sigma_M$ and burnt the first J iterations. Use the remaining last $M-J$ iterations to approximate the marginal posterior distribution, the posterior mean and variance.

I set $M = 50000$ iterations of which the first $J = 45000$ are discarded and keep $M - J$ draws to use for further inference. First, it is worth mentioning that the $\tilde{A}$ matrix is not unique. That is, it is possible to find different $\tilde{A}$ matrices that satisfy the sign restrictions. One of the options to deal with this is to draw $\tilde{A}$ matrix 100 times and choose the one closest to the median. This is the matrix, which I use for analysing the impulse response functions, FEVD and historical decomposition.

3.3 Empirical Analysis and Results

In this section, I describe the results of my analysis. As representatives of the EEC, I selected the Czech Republic, Hungary, Poland and the Slovak Republic. These countries have similar characteristics and underwent similar development paths after the their Soviet-imposed regimes collapsed. During the 1990s, their economies were characterised by a period of privatisation and adopted floating exchange rates regimes. As they undertook market reforms they also competed with each other for foreign direct and indirect investments. They all initially experienced rapid GDP growth and in the last two decades, developed from being classified as emerging markets to fully industrialised parts of the European Union.

An important question is how many variables should be included in the VAR model. As in Mackowiak (2007), I use a small scale model with three domestic variables for each country. Regressions are run for the period 1995 - 2012 for Hungary and Poland, from 1996 to 2012 for the Czech Republic and from 1997 to 2012 for the Slovak Republic. Because of the limited number of observations, I restrict the analysis only to the most important macroeconomic data such as GDP growth, CPI inflation and the nominal exchange rate for each country, and run a constant, rather than time varying, BVAR. Similar to the data chosen in Chapter 2, the source of the data is Datastream and the details on each of the particular time series are given in Appendix 3.A. All data, except of the interest rate, are either in logarithms or log differences and aggregated to quarterly values; furthermore, GDP and the price index data series are in seasonally adjusted.

Following Canova (2005) and Mackowiak (2006), I use the VAR model described in Section 3.2 for each EEC separately, in combination with either US or German variables, or both. The literature generally opts for two lags for quarterly data, which fits well with Mackowiak (2006), who estimates the model using monthly data with six lags as an optimum. Given my quarterly data, both criteria, the Akaike and Swarz confirm that VAR(2) estimation fits best.

I run three groups of estimations. First, I estimate the impact of a US monetary shock directly on the EEC macroeconomic variables. The goal here is to assess the direct impact of US monetary policy shocks on EEC markets. This estimation is analogous to the one from Kim (2001), who shows the impact of this shock on G6 countries. Second, I compare this impact with the direct impact of the Deutsche Bundesbank's interest rate (after 2001, the ECB's). This estimation is parallel to that from Mackowiak (2006), who claims that, since Germany is by far the most important trade partner for all of the countries included in the estimation (with export shares ranging from 25 to 30 percent), the innovation in German monetary policy should play a major rule for EEC. Finally, the third group of estimations analyse the impact of a US monetary shock on EEC controlling for Germany. Henceforth, the three estimations are in short referred to as: 1) direct US monetary shock (US_EEC); 2) direct German monetary shock (GER_EEC); and 3) US monetary shock with control for German variables (US_GER_EEC).

The impulse responses for the three groups of estimations are given by the median response function for the domestic variables for 12 periods, due to an increase in the interest rate of the large economy by one standard deviation point, and are displayed in a posterior 68% band extracting the 16th and 84th percentile of the simulated impulse response distribution. The impulse response functions for the estimations are presented in Figures 3.1-3.3. It is significant that the pattern of the impulse responses are similar for all the three groups of estimations: the monetary shock generated abroad is followed by a decrease in the GDP growth and a depreciation of the domestic currency in all EEC. The impact on CPI inflation is, however, ambiguous.

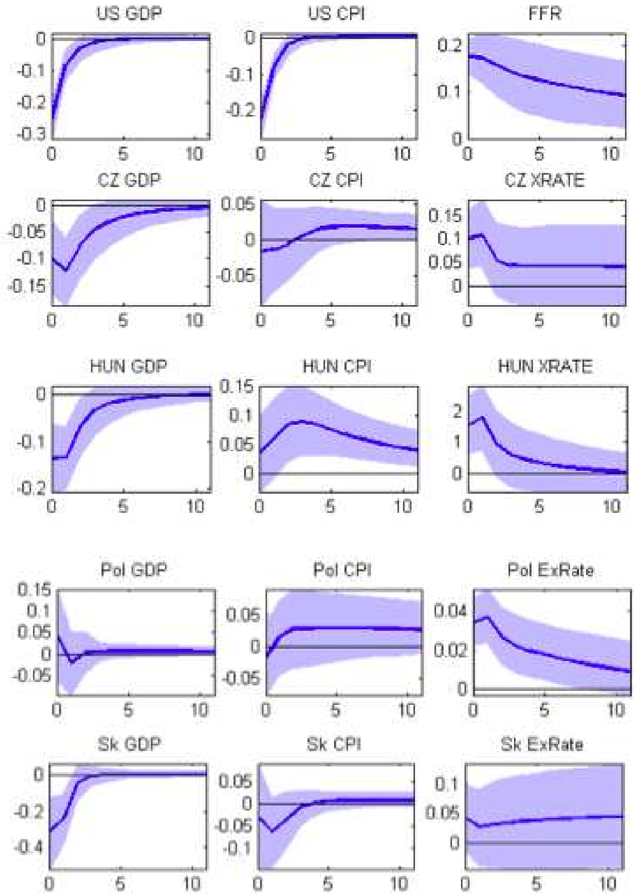

Figure 3.1: Dynamic Effect of a US monetary shock on EEC macroeconomic variables

Figure 3.1 illustrates the direct impact of the US monetary policy shock on the EEC. In my sample, the income absorption effect is the weakest in Poland (the largest of the EEC), where the GDP growth recovers fully after only three periods (less than one year). Conversely, the biggest effect is on the Slovak Republic (the smallest of the EEC), where the impact is as big as on the US GDP rate of growth itself. A contractionary monetary policy leads unambiguously to the appreciation of the dollar relative to all the other currencies in the model. This is in line with the theoretical predictions, and due to the fact that the investors are willing to invest more in US bonds, thereby causing an increase in demand for US dollars. The effect on CPI inflation is ambiguous for two reasons. On the one hand, the slow down in the domestic activity causes the prices to decrease. On the other hand, the depreciation of the domestic currency increases import prices, which generate an increase in the domestic CPI inflation. In my impulse responses, the second effect is clearly stronger in Hungary, but may also dominate in Poland.

The impulse responses in Figure 3.2 show the direct impact of German (later, European) monetary shock on EEC variables. Similar to the first estimation, here the effect on GDP growth in Poland is lowest and in the Slovak Republic it is strongest. On the contrary, in all countries, except for the Slovak Republic, GDP growth may increase after a short period (half a year), showing that after a while the income absorption effect may be dominated by the expenditure switching effect. There is no such a positive effect on Slovak output, which is consistent with the fact that the exchange rate is not allowed to depreciate since Slovakia is a member of the Eurozone and therefore only the income absorption effect takes place.

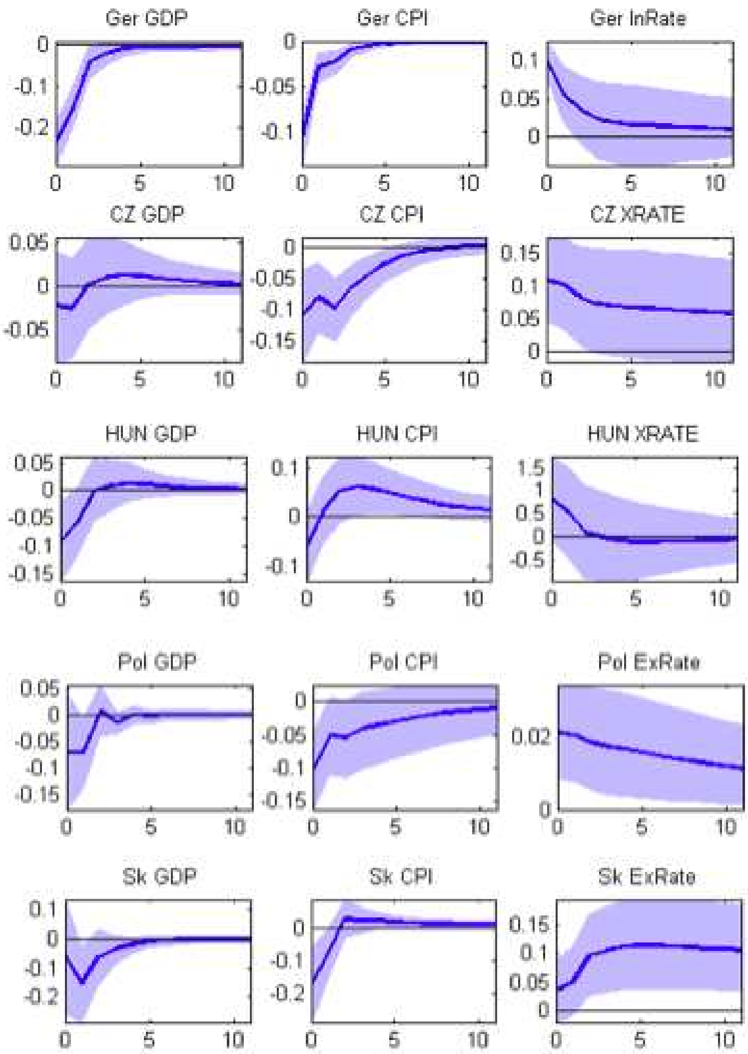

Figure 3.2: Dynamic effect of a ECB monetary shock on EEC macroeconomic variables.

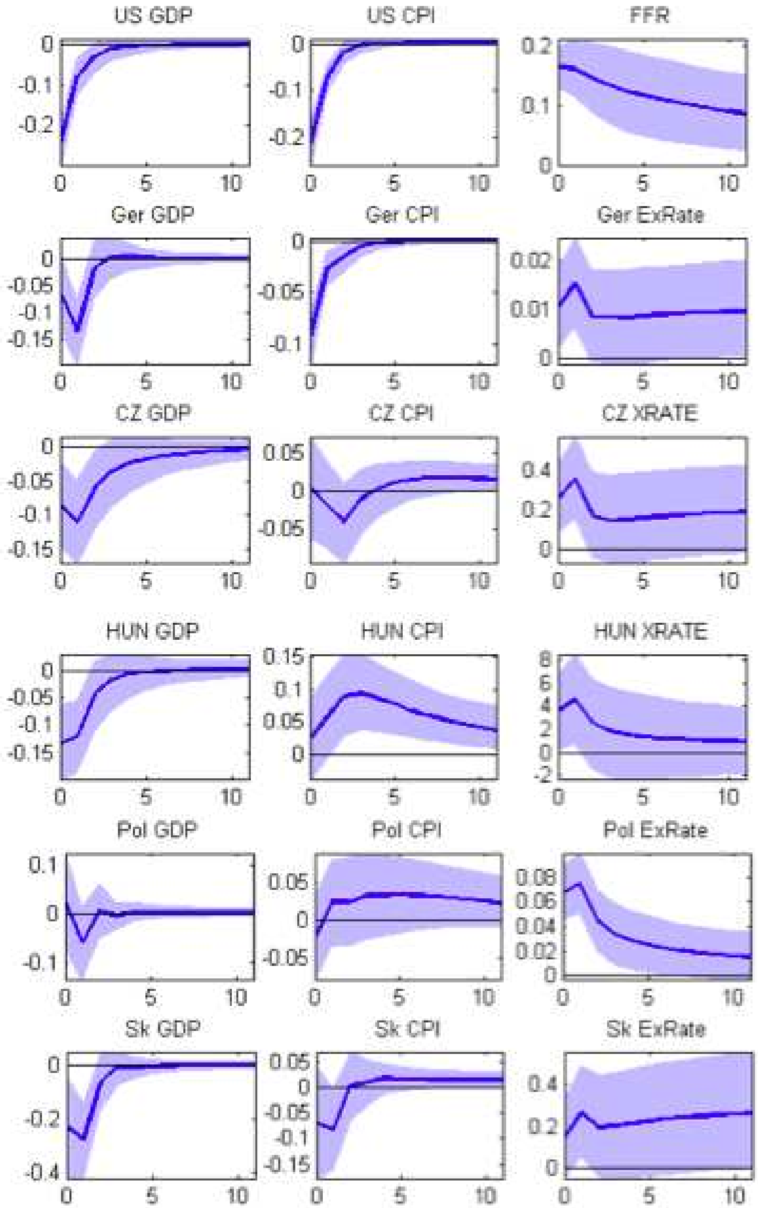

Figure 3.3: Dynamic effect of a US monetary shock on German and EEC macroeconomic variables

Figure 3.3 illustrates the impact of US monetary shock and its effect on Germany as well as on other countries (though Germany is considered a large economy relative to the EEC). The model is detailed in Appendix 3.B. The relevant impulse responses show that an unanticipated increase in the Federal Funds rate leads to a contraction in US macroeconomic variables as well as in those of all other countries. However, adding German macroeconomic variables into the model does not alter the reaction of the EEC variables to the innovation in the Federal Funds rate. Furthermore, comparing the result with the one from second estimation, it is clear that German GDP growth and inflation react similarly to the unanticipated increase in Federal Funds rate than to its own shock.

To summarise, three findings can be identified from my analysis. First, an exogenous contractionary monetary shock reduces output growth in all EEC significantly (except for Poland), regardless the origin of the shock. Second, the effect of the German (later, ECB's) shock on EEC GDP growths is smaller and dies out quicker than the one generated in the United States. Third, both exogenous monetary shocks induce a depreciation in the domestic currency and have an ambiguous effect on domestic inflation.

Tables 3.1 and 3.2 report the median share of the FEVD for forecast horizons of 1 quarter (refer to as the short-run), 4 quarters (1 year, the medium run) and 12 quarters (3 years, the long-run). Although the contribution of the German shock is higher in the short run, after three years, the contribution of US shocks and German shocks are of similar size for both the EEC output growth as well as inflation.

Table 3.1 compares the FEVD for the CPI inflation for all three groups of

US_EEC

	CZ CPI	Hun CPI	Pol CPI	SK CPI
1	0.75	0.83	0.60	0.63
4	3.59	1.97	1.41	3.10
12	6.98	5.97	3.14	4.38

Ger_EEC

	CZ CPI	Hun CPI	Pol CPI	SK CPI
1	5.60	1.37	0.91	0.65
4	9.60	2.40	2.00	2.91
12	12.93	7.41	4.63	4.59

US_Ger_EEC

	CZ CPI	Hun CPI	Pol CPI	SK CPI
1	1.33	0.64	0.52	0.60
4	6.20	1.75	1.31	2.96
12	10.98	5.68	3.23	4.38

Table 3.1: Forecasting Error Variance Decompositions (FEVDs) for CPI inflation

estimations, and shows that the German monetary policy shock explains more of the CPI inflation for all countries than its US counterpart, especially in the short run. The difference is large, especially for the Czech Republic (although, when controlling for Germany, the difference dies out in the long run). Generally, in the long run, the US monetary shock accounts for 3 to 7 percent of the variability of the CPI inflation and when I control for the effect from for Germany, it explains

US_EEC

	CZ CPI	Hun CPI	Pol CPI	SK CPI
1	13.77	7.20	7.05	0.76
4	22.48	14.58	8.12	10.48
12	22.88	16.29	8.78	11.09

Ger_EEC

	CZ CPI	Hun CPI	Pol CPI	SK CPI
1	21.28	11.40	4.30	12.57
4	19.56	10.42	5.55	17.20
12	24.18	13.16	6.74	18.10

US_Ger_ECC

	CZ CPI	Hun CPI	Pol CPI	SK CPI
1	12.91	6.12	8.40	0.77
4	19.78	12.12	9.26	10.14
12	19.80	12.94	9.94	10.61

Table 3.2: Forecasting Error Variance Decompositions (FEVDs) for GDP growth

up to 11 percent. The German (later, ECB's) shock explains mostly the Czech inflation, in the long run up to 13 percent. Generally, the exogenous monetary policy shocks explains more of the inflation in the Czech Republic and less of it in Poland. Table 3.2 shows that a sizeable fraction of the variation in real GDP growth can be attributed to external monetary policy shocks. The US generates higher variation in Hungarian and Polish GDP, even when controlled for Germany, whereas the Czech and the Slovak Republic are the countries most exposed to the German (later, ECB's) monetary shock. In general, the exogenous monetary shocks explain more of the GDP the variation than of the CPI inflation in a 12-period horizon.

What would it happen in the absence of any shock but those generated by monetary policy? The historical decomposition shows the contribution of the monetary policy shock to the endogenous variables, and therefore the overall effects of the exogenous monetary policy shock in specific periods. Figures 3.4-3.6 show the detrended variables (represented by the blue line) and its decomposition in the structural shocks to the data, where the red (dark) bars measure the contribution of the monetary policy shock for the estimated model for the period 2005-2012 for all the three groups of estimations. By looking at the specific period, the US monetary shock plays a significant role in explaining the GDP growth in the Czech Republic and Hungary, and less in Poland and the Slovak Republic. The Slovak GDP growth is better explained by the German (later, ECB's) shock. Again, this is consistent with the Slovak Republic joining the Eurozone in 2009. Although the contribution of the exogenous monetary policy shock is relatively small, there are some sub-periods, *i.e.* during the recession, in which these shocks are significant. For example, the bottom-left panel of Figure 3.4 shows clearly that the recession in Poland was driven by the US shock. Similar but weaker results are found for the other countries as well.

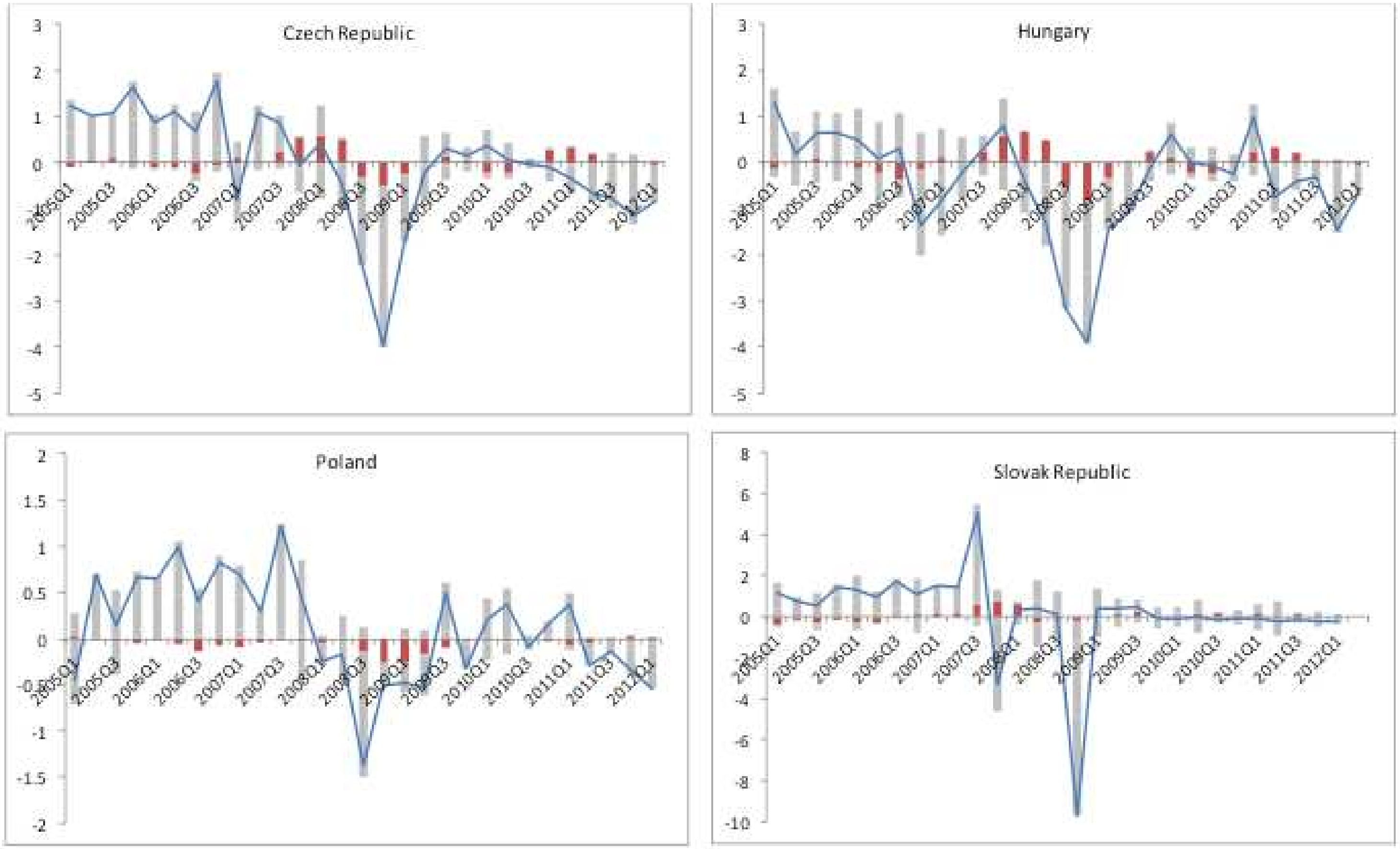

Figure 3.4: Contribution of a US monetary policy shock to the EEC GDP Growth

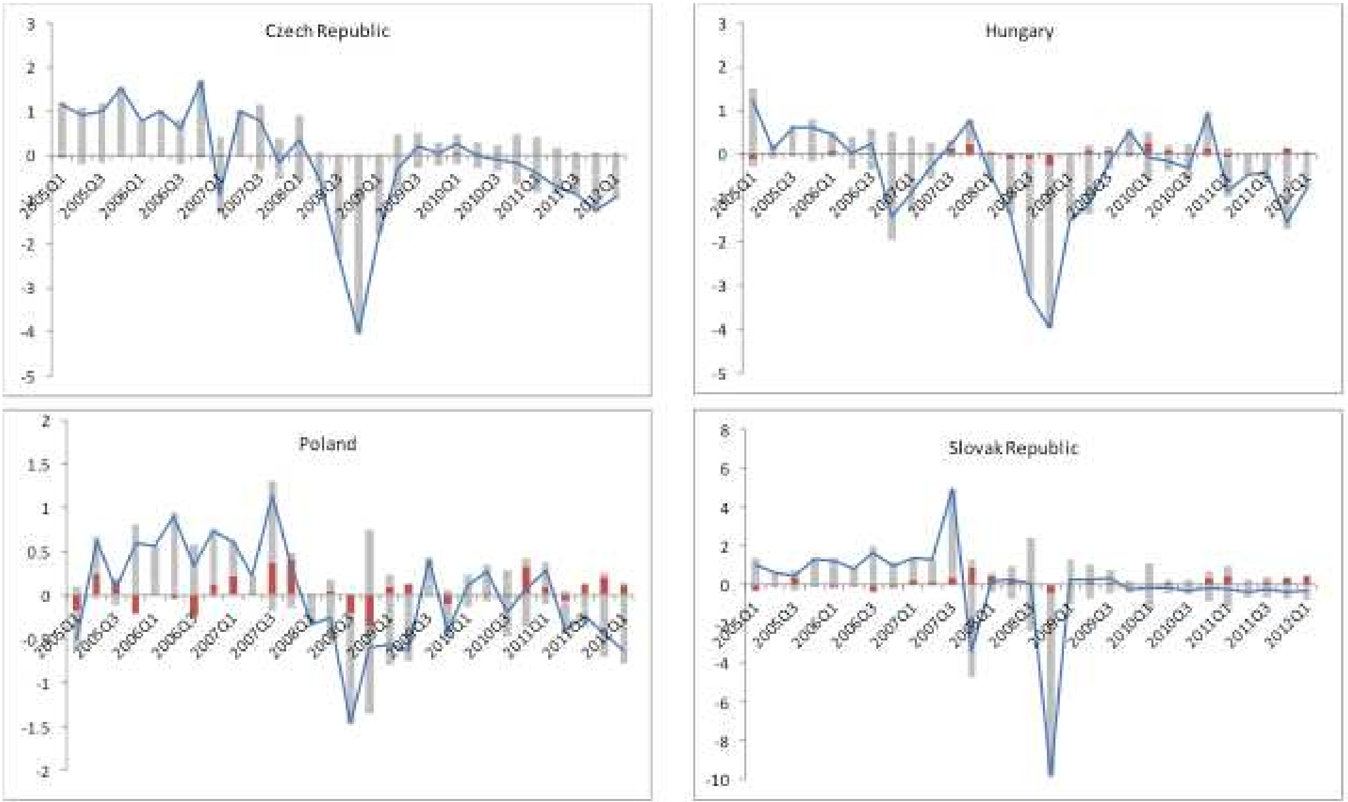

Figure 3.5: Contribution of a ECB monetary policy shock to the EEC GDP Growth

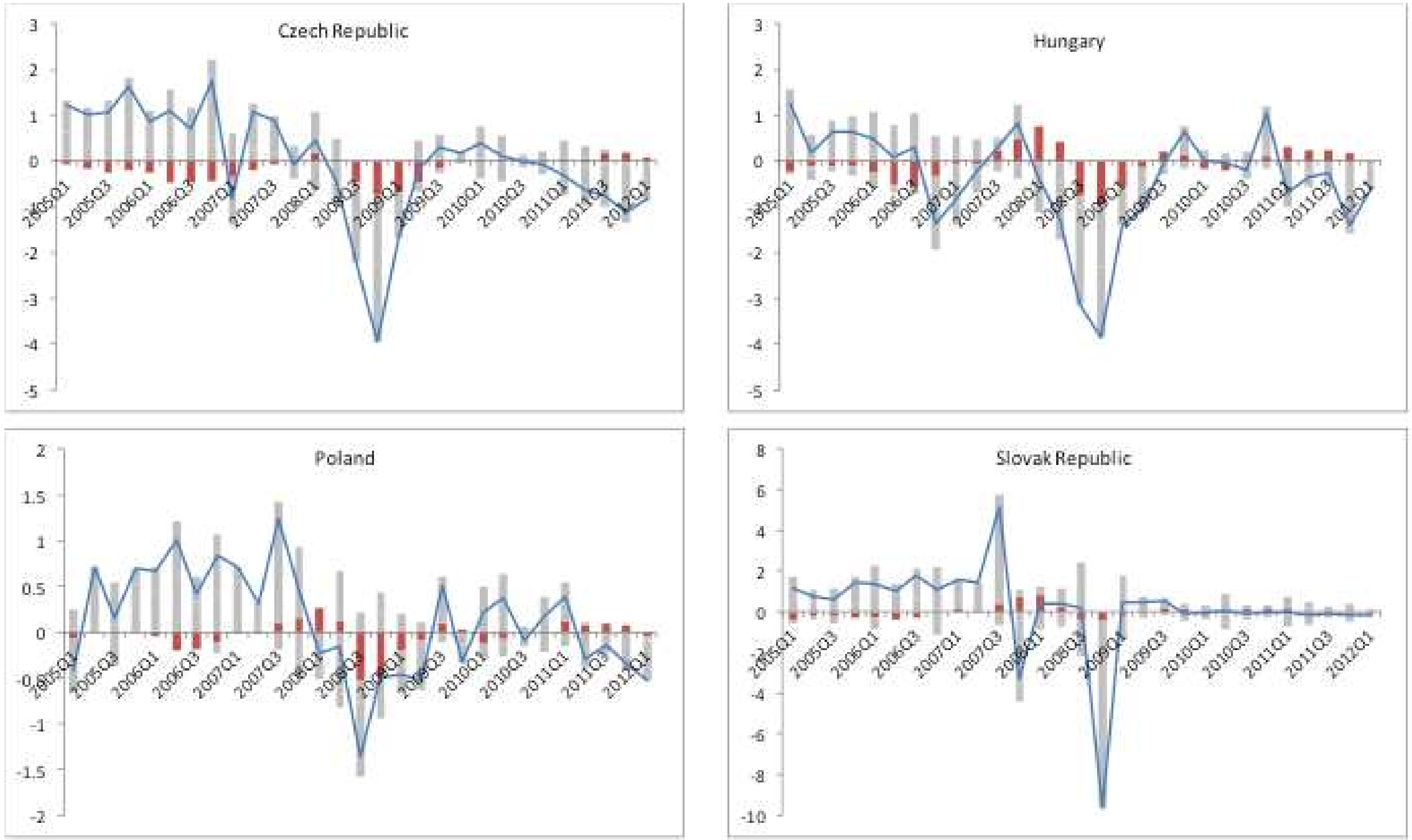

Figure 3.6: Contribution of a US monetary policy shock to the German and EEC GDP Growth

3.4 Reconciling the DSGE and VAR Responses

The results in the form of impulse response functions discussed in the last section, which follow from my VAR estimation, can be compared with those from Chapter 2. There, using a DSGE estimation, I demonstrated that a foreign contractionary policy shock leads to an increase in domestic output and inflation. On the contrary, the VAR estimation suggests that a foreign contractionary shock leads to a decrease in output, with an ambiguous effect on inflation.

According to the mainstream theory, represented by the Dornbush-Mundell-Fleming (DMF) model, under a flexible exchange rate regime, a monetary contraction in a large economy, represented by an increase in the interest rate, has two contradictory effects on the variables of a small open economy (SOE): the expenditure switching effect and the income absorption effect.

The expenditure switching effect leads to a depreciation of the SOE currency. This is caused by the appreciation of the large country's currency, which leads to a worsening in its current account due to the fall in exports and the rise in imports. The effect on the small open economy trade balance is positive, since its exports increase and imports decrease as its currency depreciates. As a result, SOE output increases.

By contrast, the income absorption effect leads to a decrease in SOE output. According to this effect, a monetary contraction in the large economy leads to lower output, due to a drop in its domestic consumption and investment. This leads to a decrease in imports, and thereby to an improvement in the large economy's trade balance. Since these imports represents the small open economy's exports, the trade balance of the latter worsens, leading to a decline in its GDP.

The findings described in the previous section point out that the income absorption effect dominates. This is consistent with the results in Kim (2001), who shows that an expansion in the US monetary policy leads to a boom in G6 countries. The findings are also in line with the theory of Betts and Devereux (1999). These authors argue that, if exports are priced in the foreign currency (and imports in the domestic one), then no expenditure switching effect happens, thus the income absorption effect naturally drives the economic dynamics. The appreciation in the domestic currency worsens the terms of trade in both the small and the large economy, and outputs of both countries decrease proportionally.

The results of DSGE estimation discussed in Chapter 2 are suggestive of the expenditure switching effect overweighting the income absorption effect. The reason is twofold. On the one hand, currency invoicing is neglected, hence the expenditure switching effect plays a role in the determination of the equilibrium dynamics. On the other hand, my estimations suggest that the small open economy central bank is at most mildly concerned with exchange rate targeting. In this latter case, the SOE currency is bound to depreciate, and this leads to an increase in SOE output. For the findings of my VAR analysis to hold, by contrast, the SOE central bank must be strictly targeting the exchange rate, or committed to a peg with the large economy's currency. In this case, the income absorption effect would dominate its expenditure switching counterpart.[12]

Further investigation of the dichotomy between the two results may become an interesting topic for further research. All the more so if one considers this

[12] To assess whether the DMF theory could rationalise the model developed in chapter 2, I have also run the relevant estimations using a higher and tighter prior on parameter ϕ_S. In this case, the estimated output decreases (*i.e.*, in the opposite direction relative to the benchmark estimation). It should be stressed, however, that comparing the marginal data densities of the two models suggest that lower value of ϕ_S fits the data better.

issue in conjuction with the central claim raised by Betts and Devereux, *i.e.*, that the currency invoicing becomes a critical point in explaining the dynamics of the macroeconomic variables. In this respect, investigating the effect of currency invoicing on the monetary policy transmission mechanism from the US to the EEC, with particular regard to the degree of exchange rate tightening, appears to be a promising subject for future research.

My line of reasoning also appears to find indirect support elsewhere in the literature. Benati and Surico (2009), for instance, argue that structural VAR models impose minimal restrictions on the data and may not account for expectations. As such, these models may thus not be able to pick up the monetary policy reaction functions very accurately. In particular, they may be unable to correctly pin down the targets of the central bank, such as the exchange rates in the case at hand. This argument may explain the difference, between VAR and DSGE estimation, in the response by the SOE central bank to the large economy monetary policy shock.

3.5 Concluding Remarks

This chapter investigated the impact of US monetary policy shock on four Eastern European Countries, namely the Czech Republic, Hungary, Poland and the Slovak Republic, using a SVAR methodology. The structural VAR process is identified using two types of restrictions. First, I introduce sign restrictions to ensure that a contractionary monetary policy shock in the large economy causes a decrease both in its inflation and output. Second, I impose zero restrictions on the channels feeding back from the small open economy to the large economy, in order to guarantee that the economic variables of the former has no influence on those of the latter.

I find that a contractionary monetary policy in the large economy significantly reduces output growth in all EEC, independently of whether the large economy is represented by the US or Germany. In particular, US monetary policy appears to influence EEC macroeconomic variables at least as much as its German (later, ECB's) counterpart, even after controlling for the indirect effect of the former through German macroeconomic variables.

For future research, it would be interesting to extend the analysis by including more endogenous variables, such as the real exchange rate, the current account balance and other trade data. When dealing with these extensions, it would be preferable to use a FAVAR method, which is more suitable for large scale models with small numbers of observations.

3.A Data

For the analysis, Datastream was a source for following data:

- As an indicator of monetary policy shock:
  - US Money market rate - federal funds rate (USI60B..)
  - Day to Day money market rate monthly average (BDSU0101R)

  - Exchange rate, used in percentage logarithm values
  - German Mark to US $ (USWGMRK)
  - Czech Koruna to US $ (USCZECK)
  - Hungarian Forint to US $ (USHUNGF)
  - Polish Zloty to US $ (USPOLZL)
  - Slovak Koruna to US $ (SXUSDSP)

The FRED database was used as a source for following time series:

- As a measure of aggregate price level, seasonally adjusted and in the first difference of the logarithm values
  - Consumer Price Index of All Items in United States (USACPIALLQINMEI)
  - Consumer Price Index of All Items in Germany (DEUCPIALLQINMEI)
  - Consumer Price Index: All Items for the Czech Republic (CZECPIALLMINMEI)

- Consumer Price Index: All Items for Hungary (HUNCPIALLMINMEI)
- Consumer Price Index: All Items for Poland (POLCPIALLMINMEI)
- Consumer Price Index: All Items for the Slovak Republic (SVKCPIALLQINMEI)

• As a measure of real GDP activity, seasonally adjusted and in the first difference of the logarithm values

  - Real Gross Domestic Product for US (GDPC96)
  - Current Price Gross Domestic Product in Germany (DEUGDPNQDSMEI)
  - GDP Implicit Price Deflator in Germany (DEUGDPDEFQISMEI)
  - Current Price Gross Domestic Product in Czech Republic(CZEGDPNQDSMEI)
  - GDP Implicit Price Deflator in Czech Republic (CZEGDPDEFQISMEI)
  - Current Price Gross Domestic Product in Hungary (HUNGDPNQDSMEI)
  - GDP Implicit Price Deflator in Hungary (HUNGDPDEFQISMEI)
  - Current Price Gross Domestic Product in Poland (POLGDPNQDSMEI)
  - GDP Implicit Price Deflator in Poland (POLGDPDEFQISMEI)
  - Current Price Gross Domestic Product in Slovak Republic (SVKGDPNQDSMEI)
  - GDP Implicit Price Deflator in Slovak Republic (SVKGDPDEFQISMEI)

3.B Model with US, German and EEC data

In the particular case of three countries, *e.g.*, the US, Germany and a (domestic) EEC like the Czech Republic, the matrix $B(j)$ can be written as

$$B(j) = \begin{pmatrix} B_{11}(j) & 0 & 0 \\ B_{21}(j) & B_{22}(j) & 0 \\ B_{31}(j) & B_{32}(j) & B_{33}(j) \end{pmatrix}, \tag{3.18}$$

where $B_{12}(j)$, $B_{13}(j)$ and $B_{23}(j)$ are zero matrices with $m \times (m \times p + 1)$ parameters, meaning that EEC variables have impact on neither German nor the US economy, and where $B_{31}(j)$ and $B_{32}(j)$ respectively give the direct impact of US and German variables on the EEC. The first line represents US economy.

The identification scheme has the following form

$$\begin{pmatrix} \epsilon\left\{FFR^{US}\right\} \\ \epsilon\left\{\Delta GDP^{US}\right\} \\ \epsilon\left\{\Delta CPI^{US}\right\} \\ \epsilon\left\{\Delta GDP^{G}\right\} \\ \epsilon\left\{\Delta CPI^{G}\right\} \\ \epsilon\left\{XR^{G}\right\} \\ \epsilon\left\{\Delta GDP^{EEC}\right\} \\ \epsilon\left\{\Delta CPI^{EEC}\right\} \\ \epsilon\left\{XR^{EEC}\right\} \end{pmatrix} = \begin{pmatrix} + & . & . & 0 & 0 & 0 & 0 & 0 & 0 \\ - & . & . & 0 & 0 & 0 & 0 & 0 & 0 \\ - & . & . & 0 & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & 0 & 0 & 0 \\ . & . & . & . & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & 0 \\ . & . & . & . & . & . & . & . & . \end{pmatrix} \begin{pmatrix} e\left\{FFR^{US}\right\} \\ e\left\{\Delta GDP^{US}\right\} \\ e\left\{\Delta CPI^{US}\right\} \\ e\left\{\Delta GDP^{G}\right\} \\ e\left\{\Delta CPI^{G}\right\} \\ e\left\{XR^{G}\right\} \\ e\left\{\Delta GDP^{EEC}\right\} \\ e\left\{\Delta CPI^{EEC}\right\} \\ e\left\{XR^{EEC}\right\} \end{pmatrix}.$$

For VAR(2), the model has the following form

$$Y_t = B_1 Y_{t-1} + B_2 Y_{t-2} + C + \epsilon_t.$$

The prior mean for $vec\,(B_0)$ is set to be equal 0.95 for coefficients on own first lags and equal zero on all other remaining coefficients. The VAR(2) model under the prior can be written as

$$\begin{pmatrix} Y_t^{US} \\ Y_t^{G} \\ Y_t^{EEC} \end{pmatrix} = \begin{pmatrix} diag(0.95) & 0 & 0 \\ 0 & diag(0.95) & 0 \\ 0 & 0 & diag(0.95) \end{pmatrix} \begin{pmatrix} Y_{t-1}^{US} \\ Y_{t-1}^{G} \\ Y_{t-1}^{EEC} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} Y_{t-2}^{US} \\ Y_{t-2}^{G} \\ Y_{t-2}^{EEC} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \epsilon_t^1 \\ \epsilon_t^2 \\ \epsilon_t^3 \end{pmatrix},$$

where Y_t^{US} is a 3×3 matrix of US variables, the interest rate, GDP growth and CPI inflation, Y_t^G and Y_t^{EEC} are 3×3 matrices of German and EEC variables respectively, namely the GDP growth, CPI inflation and nominal exchange rate.

Assuming 9 endogenous variables, the prior variance matrix H is a 171×171 diagonal matrix, where diagonal elements are set close to zero for coefficients restricted to zero and large for the remaining coefficients. In particular, with reference to the part of the matrix H corresponding to either matrix $B\,(j)$, $j = 1, 2$ as given by (3.18), the elements are all given a very high value (10 000) except for those corresponding to $B_{12}(j)$, $B_{13}(j)$ and $B_{23}(j)$, which are set very low (1/10.000) to impose the prior strictly.

3.C Convergence of the Gibbs Sampler

This appendix illustrates the convergence of the Gibbs sampling algorithm for the direct US monetary shock model for $M = 50000$ and $J = 45000$. Following Canova (2007) and Blake and Mumtaz (2012), I examine the recursive mean of the retained $M - J$ draws. The draws from the conditional posterior distributions converge to the marginal posterior distribution if the recursive mean of the retained draws is stationary without any trend.

Figures 3.7 to 3.9 display the recursive means for the variance-covariance matrix Σ and the matrix of regressor B for the Czech Republic for all three models. It is easy to see that the recursive mean for each variance and covariance in Σ fluctuates around its mean with no trend. Similarly, for each version of the model, all regression coefficients from matrix B are stable for the retained iterations, which implies that the number of iterations and retained draws is sufficient to approximate the marginal posterior distribution

For the remaining countries, the figures look similar and are available from the author upon request.

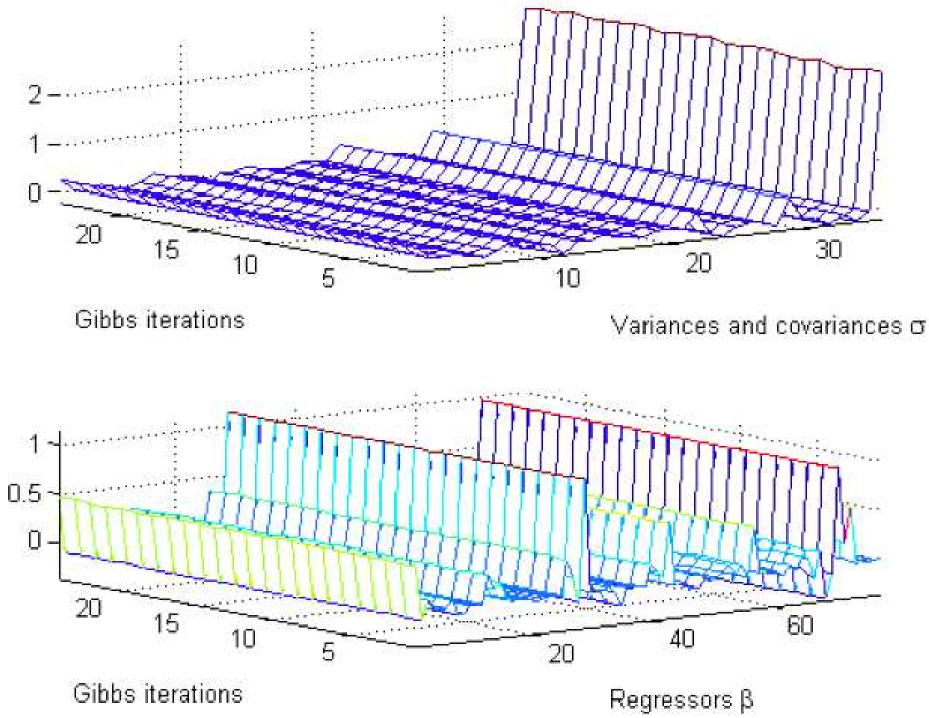

Figure 3.7: Recursive means of the retained Gibbs draws for Σ matrix (above) and B matrix (below) for (US_Czech Republic) model.

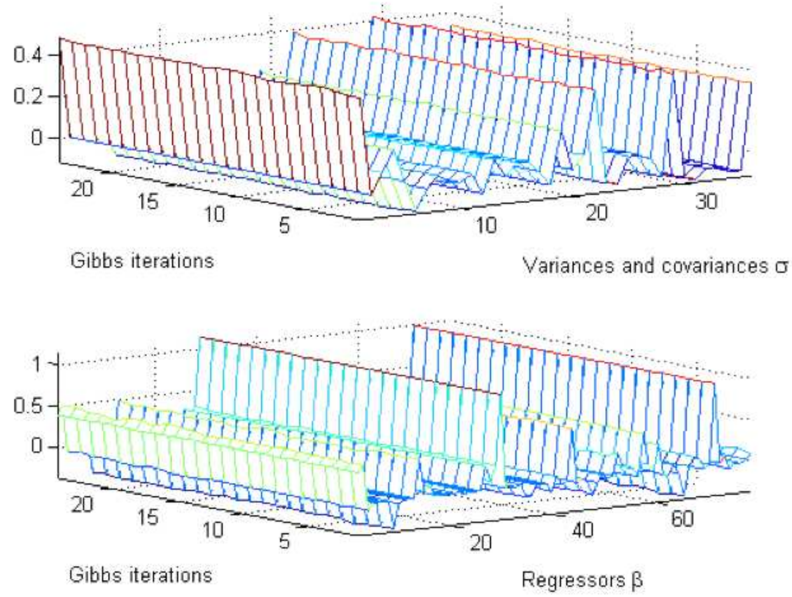

Figure 3.8: Recursive means of the retained Gibbs draws for Σ matrix (above) and B matrix (below) for (Ger_Czech Republic) model.

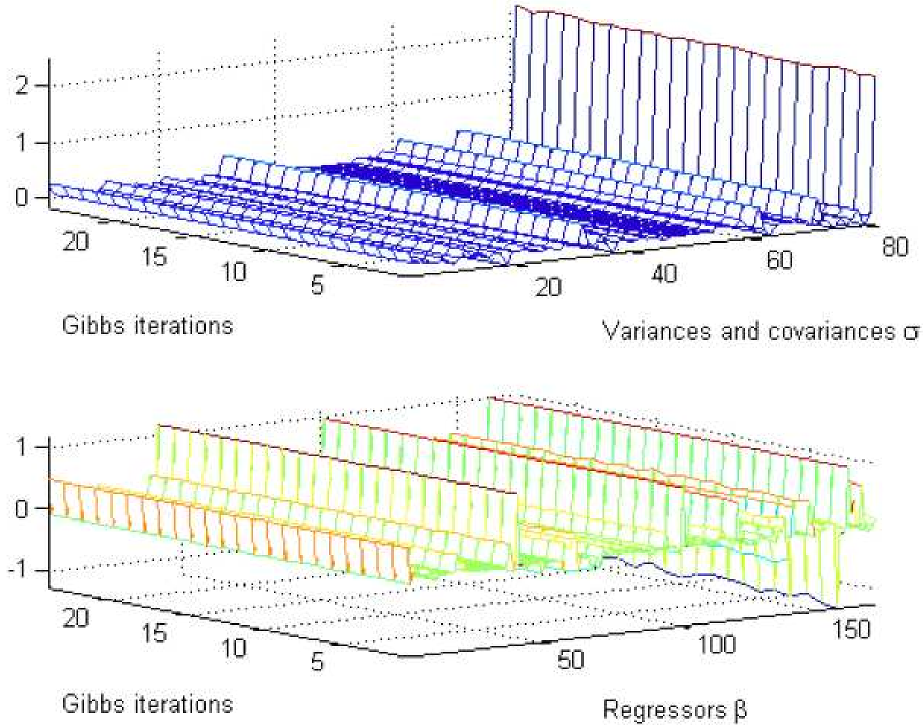

Figure 3.9: Recursive means of the retained Gibbs draws for Σ matrix (above) and B matrix (below) for (US_Ger_Czech Republic) model.

Conclusion

This thesis represents a contribution to the field of monetary policy. The theoretical contribution is illustrated in Chapter 1. I show that a non-zero steady state inflation may have a positive impact on a social welfare. Analysing simple and optimal monetary policy rules in a model that considers backward looking as well as forward looking agents. I find that the welfare loss that a Hamiltonian social planner obtains in the steady state depends on the level of inflation. In particular, a positive (and sufficiently low) inflation may be beneficial for the economy, as the negative effect due to a rise in the price dispersion may be more than offset by the positive effect that nominal rigidities generate on labour supply.

In the second chapter, I studied the performance and characteristics of simple monetary policy rules using a two-countries model. I performed Bayesian inference, where the novel feature was the inclusion of real unit labour costs for a better measurement of the Phillips curve. I demonstrated that a simple monetary policy rule mimicking an optimal rule generates the best outcome and that the non-zero inflation part included in the Phillips curve improves the model fit significantly. To analyse how the model performs in the context of a small open economy, I used data for three Eastern European Countries, namely the Czech Republic, Hungary, Poland. Using a posterior odds test, I find evidence that the central banks of all these countries target a PPI inflation instead of CPI inflation, contrary to what is usually assumed in the empirical literature. I also find that in the case of a SOE, the model with a non-zero steady state inflation performs substantially better.

The third chapter investigated the impact of a US monetary policy shocks on four Eastern European Countries, the Czech Republic, Hungary, Poland and the Slovak Republic, using a SVAR methodology with economic interpretable restrictions. I find that US monetary policy influences EEC macroeconomic variables at least as much as its German (later, ECB's) counterpart, even when controlling for the indirect effect of the former through German macroeconomic variables.

My research might be extended in a number of directions. For instance, in the first chapter, the analysis has been conducted with reference to a stochastic environment that is generated by a single shock to TFP. An obvious extension of the model would consider a cost push shock as an alternative to the stochastic TFP. A preliminary study of this case, however, leads to results very similar to those discussed in this chapter, with the exception that inflation inertia appears to have a relatively higher impact on the dynamics of the endogenous variables. Chapter 1 analysed an economy that is closed. It is natural to pose the question of how the results would change if the economy were allowed to trade in international markets.

A natural extension to the second chapter would be to investigate the effect of currency invoicing on the monetary policy transmission mechanism from Germany to the EEC, with particular regard to the degree of exchange rate tightening. Finally, with regard to the third chapter, it would be interesting to extend my analysis by including more endogenous variables, such as real exchange rate, current account and trade data. In this case, it is preferable to use a FAVAR method, which is more suitable for large scale models with small numbers of observations.

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