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An EOQ Model for MRO Customers under Stochastic Price to Quantify Bullwhip Effect for the Manufacturer

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Abstract

Motivated by a particular multinational cutting-tools manufacturer, we extend the traditional economic order quantity (EOQ) model for maintenance-repair-and-overhaul (MRO) customers under stochastic purchase price and use it to show how price variance leads to bullwhip effect for the MRO manufacturer despite constant consumption by the customer. Our extension of the EOQ model is based on two assumptions that are reasonable for MRO customers: (a) customer consumption rate of the product is constant; and (b) the customer places each order when the inventory level drops to a pre-specified level (say, zero). We determine the customer's optimal ordering quantity in closed form expressions, which enables us to examine the impact of sales price variance on the variance in the orders the customer places on the manufacturer, thus creating a pricing-induced bullwhip effect. We then extend our analysis to multiple products and multiple customer segments and discuss ways for the manufacturer to mitigate the variance in the customer's orders.

Keywords: EOQ model; stochastic price; bullwhip effect; order variance; transactional pricing; MRO sector; multiple products; multiple customer segments

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1 Introduction

Pricing has typically been studied as a strategic topic in the operations literature and day-to-day transactional pricing, say, by way of discounts in orders, has been largely ignored. However, variable discounts across sales transactions is a common phenomenon in Business-to-Business (B2B) transactions that is well documented in the practitioner literature (cf. Marn and Rosiello, 1992; Marn, Roegner and Zawada, 2003; Sodhi and Sodhi, 2005; Kotler, Rackham and Krishnaswamy 2006). Such variable price discounting, sometimes seen as 'hockey-stick' unit sales whereby sales personnel seek to make their month-end sales quotas by offering customers big discounts have been recognized as causing order variance (Lee, Padmanabhan and Whang 1997); however, the impact of sales price fluctuations on the variance in the customer's orders has not been quantified analytically.

Although we are motivated by the setting at a particular multinational cutting-tools manufacturer, we seek general results for the maintenance-repair-and-overhaul (MRO) sector. Taking the setting to be of constant consumption, as with an MRO customer, we extend the EOQ model for the customer's orders incorporating stochastic price and show how this implies order variance for the MRO manufacturer thus showing how price variance causes bullwhip effect.

In this paper, we provide *closed-form* expressions capturing the impact of such transaction-level varying sales prices on the variance in the customer's orders even when the customer has constant consumption. We start with the base case with a manufacturer having a single product and one customer whose consumption rate is constant and who orders when inventory goes to zero. These assumptions are reasonable for MRO customers in general including cutting tools. The reader is referred to Erlenkotter (1990, 2013) and Khan et al. (2011) for comprehensive reviews of the evolution of the EOQ model and analysis since Ford Whitman Harris presented the EOQ model for the first time in 1913 (Harris, 1913). While the customer seeks to determine her optimal order quantity so as to minimize long-run average costs including the fixed ordering cost, we extend the traditional EOQ model to capture the fact that the customer faces uncertain varying sales prices offered by the manufacturer through variable discounts or surcharges. We then obtain closed-form expressions for "variance" of the customer's orders for this base case. Next we extend the results to the MRO manufacturer having multiple products and multiple customer segments. Moreover, we discuss the managerial implications of our analysis and ways to mitigate the customer's order variance.

Our contribution is twofold: *One*, we add to the EOQ literature by extending the EOQ model for a customer facing stochastic purchase price. Sana (2011) has analyzed the production quantity and the selling price of a firm who faces price-dependent random demand using the EOQ setting while Sana (2012) has examined a more general problem based on the newsboy problem setting. In contrast to Sana's (2011; 2012) customer demand as "exogenous", we take customer's demand on the manufacturer to be "endogenous" in that it is derived from the customer's rational purchasing behavior having observed varying prices in the past. Netessine and Tang (2009) have compiled articles dealing with endogenous customer demand in the operations management literature.

Two, we contribute to the bullwhip-effect literature by quantifying the impact of sales price variance on the customer's order variance (cf. Lee et al. 1997; Ozelkan and Cakanyildirim 2009; Hamister and Suresh 2008). In the business-to-consumer (B2C) literature promotions of staple products like shampoo only increases the shampoo manufacturer's bullwhip effect without changing the consumer's consumption of shampoo. In this context, Ho et al. (1998) focused on *mean* effects pertaining to a shopper's rational shopping behavior for a single product by way of expected purchase quantity and expected shopping frequency. However, our focus is on the *variance* for the manufacturer (not the customer) and moreover, we take into account multiple products and multiple customer segments.

Managerial implications of our work are as follows. (1) To reduce the customer's order variance, manufacturers should aim to reduce not only the variance in the transaction prices but also the fixed ordering cost for its customers. (2) Even though increasing market share by increasing the number of customers can reduce the coefficient of variation of the customer orders, the marginal benefit of this effect diminishes quickly with increasing number of customers. (3) Managers should avoid invoice-level discounting and choose new segments of customers carefully because market heterogeneity increases the variance of customer orders.

This paper is organized as follows. Section 2 provides the details of the particular cutting tools manufacturing company that motivates our work along with the MRO sector as a whole. In Section 3, we present our base model that examines the rational purchasing behavior minimizing long-term total of purchasing, holding and ordering costs of a single customer with constant consumption, and ordering from a manufacturer who offers variable prices. Section 4 extends the base model to the case when the manufacturer sells multiple products. Section 5 extends the base model to multiple customer segments, each with its own consumption rate. A discussion of the results to the company's situation follows in

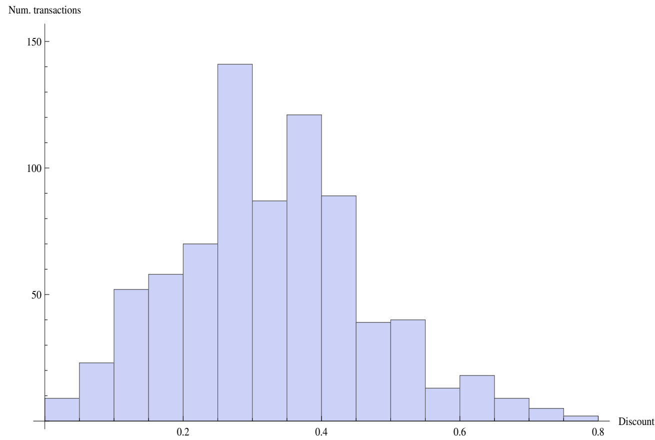

Figure 1: Discounts across 776 transactions in a quarter on products in the same family

Section 6 and we conclude with some ideas for future research.

2 A Cutting-Tools Manufacturer and the MRO Sector

Our work is motivated by a global manufacturer of cutting tools whose customers include original-equipment-manufacturer (OEM) auto companies at one extreme and small-scale service shops at the other. The company has about 6,000 basic Stock Keeping Units (SKU) of cutting tools. Moreover, customized tooling for particular customers increase the number of products to over 30,000 SKUs at any given time.

Fluctuating prices. As is common in most B2B transactions, the company allows its salespeople to provide transaction-specific discounts on list prices to customers. It also plans promotions and adjusts standard discounts for large customers on a regular basis. The company knows that the "realized" prices (i.e., after the transaction discount) of its various products, both at the SKU level and at the family level, vary considerably across transactions as do SKU-specific unit sales. Based on our analysis for some SKUs, we find, for instance, that when the coefficient of variation of prices across all customers for a week was 0.09, the coefficient of variation of the unit sales was 0.46. Indeed, the variation in discounts for SKUs in the same family varies from 2% to 80 % (Figure 1).

Transaction sales prices fluctuate due to multiple reasons: The marketing department plans various promotions such as "buy three drills, get the sleeve free" generally for its smaller customers; similar volume-based price discounts apply for bigger customers as well globally. There are also "local" promotional initiatives" at the country-level. Each salesperson can offer different discounts and that too at the level of an individual transaction. System-programmed rules for discounting also create price variations with the application of multiple discounting rules: a customer gets a discount based on its category, with a larger customer getting a larger discount. Sales personnel also used a quantity-specific discount schedule for different product families. Customers negotiate further discounts on individual products to try and have each cutting tool at the same per piece price even though list prices vary quite a bit. There were also single-use transactional discounts offered at customers' request to close deals. In practice, there are price discounts at different levels: item-level, invoice level, customer-level (Marn and Rosiello, 1992). Finally, dramatically fluctuating prices of specialized steels and other raw materials require the company to occasionally impose temporary surcharges (negative discounts) to pass at least some of the increased costs to customers.

The impact of price fluctuations on manufacturing operations. According to the director of Pricing in the company, these price fluctuations not only diluted profitability, but also caused unit sales to fluctuate which in turn lead to scheduling problems in production, with resulting delays leading to further discounts. Given the constant usage of the products, unit sales fluctuation could result in longer replenishment intervals, which can trigger more price discounts. Also, the variance of unit sales could lead to wrong pricing decisions. For example, a downward trend across a few weeks raised arguments for lowering the price (or do another promotion) despite the argument that customers who bought more in one week would buy less in subsequent weeks.

As such, we were motivated to seek to quantify the impact of the price variance on the variance of manufacturer's orders (and hence on the manufacturer's operations).

Assumptions. Given the nature of use of cutting tools by the manufacturer's customers, we sought empirically-justifiable simplifications by way of two modeling assumptions for the customers, the same as those for the standard EOQ model:

1. *Customer's consumption is constant.* Customers of this company such as auto OEMs consume cutting tools at a constant rate regardless of the mix of car models they are producing at any time. Their consumption of cutting tools is independent of the

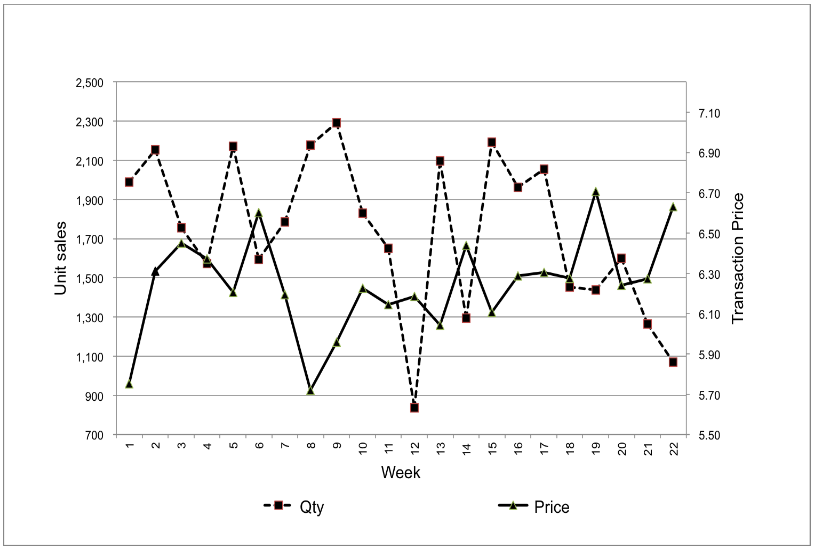

Figure 2: Price and quantity for a particular SKU over time, aggregated for transactions in the week

purchasing price because the opportunity cost of a cutting tool being unavailable is much higher than its purchase price.

2. *Customers make "planned" purchases triggered by zero inventory.* For the larger of the manufacturer's customers, the ERP systems create orders with 'standard' quantities based on zero inventory projection, assuming constant consumption. Hence, for our model, we assume the purchase of cutting tools is triggered only by zero inventory. (However, our analysis can be extended to the case when the re-order point is a constant different from zero.)

Based on these two assumptions, customers could simply order pre-specified quantities on a regular schedule using the EOQ calculation if prices were constant, and the manufacturer would face stable orders from customers. But because prices are variable, and weekly prices and weekly unit sales for the manufacturer are negatively correlated: for instance, -0.54 in one case and -.52 in another. Figure 2 reflects this for a particular SKU. Therefore, we shall extend the EOQ model to incorporate price variability by assuming that all customers are 'rational' (and risk-neutral): they seek to minimize the long-run average cost including the cost of purchasing, holding and ordering when determining the order size *at the time of the purchase*.

2.1 The MRO Sector

The above assumptions can be made for other customers of *consumable* MRO products (except spare parts or manufacturing equipment). The first EOQ assumption of *constant consumption* by MRO customers holds for the same reason as it does for the cutting tools company. Customers of MRO products such as light bulbs in offices, lubricant cleaning solution, and stationery do not vary their consumption rate and consume these products as part of ongoing operations. The focus is to reduce the customer's ordering and inventory costs, while keeping availability high.

The wide variety of MRO products is why the second EOQ assumption of *planned purchases* holds for customers of this sector as a whole. The assumption is justified because customers do not, and indeed cannot, monitor prices continuously for all their MRO products, nor do they want to have more inventory than what they need given their predictable consumption of these products.

Despite its size and significance, not much has been written in the operations literature about the MRO industry as regards procurement (Gelderman et al. 2008) and even less about pricing in this sector. The huge variety of products entailed in MRO purchasing as well as the relatively small unit and dollar volumes distinguish MRO purchasing from direct purchasing (van Weele, 2010: p.7-8). The MRO industry is significant in sales volume, and by some estimates and depending on what is included, about US $ 800 million. The market size of office stationeries in the US alone was $42 billion in 2011 according to industry research firm Ibisworld, and the commercial airlines-focused MRO was $42 billion for 2011 worldwide according to Oliver Wyman, a consultancy while the military aviation MRO spend was $52 billion that year according to Kaufmann et al. (2007).

The industry has more than five million product types (Sawhney 2004) sold by a huge variety of sellers who are manufacturers or distributors or both. For one automotive OEM alone, Lee and Blancas (2006) mention more than 1,200 MRO suppliers. W.W.Grainger sells products ranging from adhesives (glues, tapes, etc.), electrical (cables, batteries, electrical wires), fasteners (nails, screws, hooks, etc.), janitorial supplies (cleaning equipment and cleaning supplies), lighting (light bulbs), and many other product types.

Manufacturers and distributors cater to wide-ranging customer segments. For instance, W.W. Grainger's customer base comprises facilities maintenance professionals from: (1) government offices, schools and correctional institutions; (2) heavy manufacturing customers in petro/chemicals, lumber, primary metals and rubber industries; (3) light manufacturing customers in food and beverage processing plants and pharmaceutical companies; (4) retail/wholesale customers such as grocery stores; and (5) contracting firms in maintaining and repairing existing facilities.

The MRO sector is also distinct from the broad B2B sector in that 'fixed' order costs are important to reduce (Gunasekaran et al. 2009). Although individual product families are a miniscule part, a customer may still view the total MRO spend as being problematically high. With tens of thousands of products, ordering costs and inventory costs become significant. Indeed, the advent of the Internet and e-commerce in the late 1990s fueled electronic ordering to reduce ordering costs (Croom, 2000; Kaplan and Sawhney, 2000; Gelderman et al., 2008). Kaplan and Sawhney (2000) describe different types of B2B e-marketplaces including 'MRO hubs' such as Grainger.com and BizBuyer.com. Muylle and Croon (2004) describe the efforts of a distributor of mechanical supplies, the Baudaoin Group; Sawhney (2004) discusses the efforts of an MRO manufacturer, a division of Rockwell; Mukund and Radhika (2003) do so for an MRO customer, motorcycle manufacturer Harley Davidson.

3 Base Model

Our base model captures the purchasing behavior of a risk-neutral customer, say, someone from the purchasing department who has to order the single consumable MRO product (as assumed) that his company consumes at a known constant rate r per period. He orders when the inventory drops to zero, incurring a fixed ordering cost $K > 0$ for order-specific activities such as issuing the order, receiving against it, and processing the payment.

While the customer has the knowledge about the distribution of discounts from past experience, he learns about or negotiates the product price p only at the time of purchase, a reasonable assumption for MRO products. Upon observing p, he decides on the purchase quantity $Q(p)$. (Even if the order is created by the customer's ERP system as may be the case for larger customers, the order quantity can be modified subsequently.) For ease of exposition, we assume the manufacturer fulfills the order instantly from inventory. The customer continues to consume the product at rate r and orders again when the product runs out. *The customer's decision is therefore a variant of the EOQ model with price uncertainty.*

The observed price is modeled as one of S (discounted) pricing 'scenarios', where each price scenario s is realized with probability π_s. For each scenario s, the (discounted) price is p_s. Taking μ to be the average price and σ^2 to be the variance of the (discounted) transaction price, we have

$$\mu = \sum_{s=1}^{S} \pi_s \, p_s,$$
$$\sigma^2 = \sum_{s=1}^{S} \pi_s (p_s - \mu)^2 \tag{3.1}$$

For the cutting-tools manufacturer, the mean discount for the product family in Figure 1 is 32.76 %, and the standard deviation is 13.85 % (in the same customer category). Multiplying these with the respective list prices of the different SKUs in this family provides the values of μ and σ for these SKUs.

3.1 Notation

For the remainder of this paper, we shall use the following notation for the base model. For various extensions of the base model, the notation is the same or extends that of the base model.

Parameters r: customer consumption rate per unit time

h: unit holding cost per unit time

s: price scenario, $s = 1, \cdots, S$

μ: expected selling price

σ^2: variance of selling price

K: fixed ordering cost

$\hat{K}$: adjusted ordering cost, where $\hat{K} = K - \frac{r}{2h}\sigma^2$

Variables

Q_s^*: optimal purchase quantity under price scenario s

t_s^*: elapsed time until next purchase incident

D_i: manufacturer's unit sales in period i

3.2 The Customer's Order

The customer seeks to minimize his long-term average "relevant" cost per unit time. The customer's purchases Q_s units when the observed price is p_s incurring a fixed ordering cost K and paying a unit holding cost h per unit time (e.g., warehouse storage fee). Because his consumption rate is r, his next purchase incident will occur when the inventory runs out after time $t_s = Q_s/r,$ with r being constant.

The customer's *total relevant cost* associated with an order at any time period under price scenario s comprises: (1) the fixed ordering cost K; (2) the purchasing cost p_sQ_s associated with purchase quantity Q_s; and (3) the inventory cost $h(\frac{Q_s}{2})(\frac{Q_s}{r})$. Because the next order will occur after time $t_s = Q_s/r$, the average relevant cost per unit time until the next order is given by

$$\frac{K + p_sQ_s + h(\frac{Q_s}{2})(\frac{Q_s}{r})}{(\frac{Q_s}{r})}$$

By modeling each purchasing incident as a *renewal*, we can apply renewal theory (e.g., Ross 1980) to show that the expected relevant cost per unit time $R(Q_1, \ldots, Q_S)$, for any given purchasing policy $(Q_1, \ldots, Q_S)$, satisfies:

$$R(Q_1, \ldots, Q_S) \quad = \quad \frac{K + \sum_{s=1}^{S}[\pi_s p_s Q_s + \pi_s h_s \frac{Q_s^2}{2 \cdot r}]}{\sum_{s=1}^{S}[\pi_s \frac{Q_s}{r}]}. \tag{3.2}$$

Ho et al. (1998) show that the function $R(Q_1, .., Q_s, ..., Q_S)$ is jointly pseudo-convex (Avriel 1976) in $(Q_1, .., Q_s, .., Q_S)$ and thus obtain the optimal purchase policy $(Q_1^*, .., Q_s^*, ..., , Q_S^*)$ that minimizes the expected relevant cost per unit time $R(Q_1, .., Q_s, ..., Q_S)$. Also, Ho et al. (1998) proved the following proposition:

Proposition 1 *The customer's optimal purchasing policy* $(Q_1^*, .., Q_s^*, ..., , Q_S^*)$ *can be expressed as:*

$$Q_s^* \quad = \quad \sqrt{\frac{2\hat{K}r}{h}} - \frac{r}{h}(p_s - \mu). \tag{3.3}$$

and the minimum expected relevant cost per unit time $\overline{R}^* = R(Q_1^*, .., Q_s^*, ..., , Q_S^*)$ *is given by:*

$$\overline{R^*} \quad = \quad \mu r + \sqrt{2\hat{K}rh} \tag{3.4}$$

where $\hat{K}$ *is the 'adjusted' ordering cost:*

$$\hat{K} \quad = \quad K - \frac{r}{2h}\sigma^2. \tag{3.5}$$

Thus, a rational customer will buy more than (or less than) the reference purchase quantity $\sqrt{2\hat{K}r/h}$ when the observed price p_s is lower than (or higher than) the average price μ.[1] This helps explain the (discounted) price-quantity relationship we observe in Figure 2. Furthermore, $\hat{K}$ increases in K and decreases in σ^2, so Proposition 1 reveals that the optimal purchasing quantity Q_s^* and the minimum expected relevant cost $\overline{R}^*$ increase with the fixed ordering cost K and decrease with increasing price variability σ^2.

3.3 Order Variance for the Manufacturer

Upon observing the transaction price p_s, the customer purchases Q_s^* units. Hence, by using (3.3) and by noting the "elapsed time until next purchase incident" (or length of the ordering cycle) is $t_s^* = \frac{Q_s^*}{r}$. we get:

Lemma 1 *The optimal order cycle of a rational customer satisfies the following properties:*

$$t_s^* = \sqrt{\frac{2\hat{K}}{hr}} - \frac{1}{h} \cdot (p_s - \mu), \tag{3.6}$$

$$t^* = E(t_s^*) = \sqrt{\frac{2\hat{K}}{hr}}, \tag{3.7}$$

$$Var(t_s^*) = \frac{\sigma^2}{h^2}, \tag{3.8}$$

$$E([t_s^*]^2) = Var(t_s^*) + [E(t_s^*)]^2 = \frac{2K}{hr} \tag{3.9}$$

Proof: See Appendix for all proofs.

When the price variance is zero, the order-cycle variance is also zero and we are back to the standard EOQ situation. When price variance is not zero, each purchase incident is a "renewal" of a stochastic process (Ross, 1980) because the purchase incident occurs only when the inventory drops to zero. Also, we can view the elapsed time t_s^* as the "'length of the renewal cycle" after the customer observed the realized price is p_s during the purchasing

[1] In the event when $\sqrt{\frac{2\cdot\hat{K}\cdot r}{h}} - \frac{r}{h} \cdot (p_s - \mu) < 0$, Q_s^* could be negative. However, because the function R given in (3.2) is pseudo-convex in $(Q_1, \cdots, Q_S)$, it is optimal to truncate those negative Q_s^* to a minimal positive level, say, 1. We assume that K is sufficiently large so that $\hat{K} > 0$ and $Q_s^* > 0$.

incident. By applying a well-known result in renewal theory (Feller (1968)), the steady state probability that a purchase incident occurs during any time period is given by

$$\frac{1}{E(t_s^*)} = \frac{1}{t^*}$$

The unit time period can be taken as being small enough that a purchasing incident either occurs once or not all in this period – indeed this is the case when the unit of time is a week for the manufacturer in question – so that the average time t^* given in (3.7) between successive orders is greater than 1; i.e.,

$$t^* = E(t_s^*) = \sqrt{\frac{2\hat{K}}{hr}} \geq 1$$

Hence, for any given time period i and given price p_s, the unit sales D_i for the manufacturer are:

$$D_i = \begin{cases} 0 & \text{with prob. } (1 - \frac{1}{t^*}) \\ Q_s^* & \text{with prob. } \frac{1}{t^*} \end{cases} \tag{3.10}$$

where t^* is given in (3.7). By considering (3.10) along with Lemma 1, we get:

Proposition 2 *In steady state, the expected sales and the variance of the sales observed by the manufacturer in any time period i are:*

$$\begin{aligned} E(D_i) &= r, \\ Var(D_i) &= r^2(\frac{Var([t_s^*])}{t^*} + t^* - 1). \end{aligned} \tag{3.11}$$

Therefore, when $t^* \geq 1$, and $Var([t_s^*]) > 0$, then the manufacturer will suffer order variance, i.e., $Var(D_i) > 0$ even though the customer's consumption is constant. Note however that $Var(D_i) \neq 0$ even when price variance is zero. This is because orders are lumpy in discrete time – some periods have positive sales other periods are zero whenever t^* exceeds one. So we must consider the *incremental* increase in variance of unit sales caused by the price variance. By considering Proposition 2, we get:

Corollary 1 *In steady state, the variance of the sales $Var(D_i)$ is decreasing in K for $K \in (\frac{r}{2h}\sigma^2,\ 2\frac{r}{2h}\sigma^2]$ and increasing in K for $K \in (2 \cdot \frac{r}{2h}\sigma^2, \infty)$. Moreover, $Var(D_i)$ is convex in K over the range $(\frac{r}{2h}\sigma^2,\ 4\frac{r}{2h}\sigma^2)$ and concave in K over the range $[4\frac{r}{2h}\sigma^2, \infty)$.*

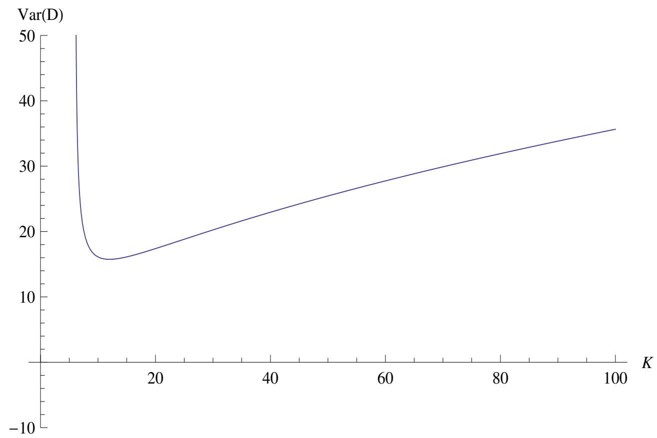

Figure 3: Unit-sales variance and the fixed ordering cost K with $h = 1, r = 1.5$, and $\sigma = 5$

We know from the standard EOQ model, and hence its extension here, that the customer will purchase more in any time period as K increases, i.e., Q_s^* given in (3.3) and t_s^* are increasing in K), the sales variance $Var(D_i)$ increases in K when the effect of the latter dominates the transaction price variance. However, Corollary 1 reveals an unexpected result when the transaction price variance dominates the fixed cost, the sales variance $Var(D_i)$ would actually decrease as the fixed ordering cost K is increasing over the range $(\frac{r}{2h}\sigma^2, 2\frac{r}{2h}\sigma^2)$ (Figure 3).

This is because when the ordering cost K is small, the effect of price variance σ^2 dominates the unit sales variance $Var(D_i)$. Because $t^* = E(t_s^*) = \sqrt{\frac{2\hat{K}}{hr}} \geq 1$, one can check from (3.5) that $K \geq \frac{r}{2h}\sigma^2 + \frac{hr}{2}$. In this case, when price variance is sufficiently high, say, when $\sigma^2 > h^2$, $\frac{r}{2h}\sigma^2 + \frac{hr}{2} < 2\frac{r}{2h}\sigma^2$. Therefore, for any $K \in (\frac{r}{2h}\sigma^2 + \frac{hr}{2},\ 2\frac{r}{2h}\sigma^2]$, Corollary 1 reveals that the variance of unit sales $Var(D_i)$ is decreasing in K when K lies within this interval. This suggests that, when the price variance is high, the manufacturer can decrease the unit sales variance by actually increasing the customer's ordering cost K. However, for most of the range of reasonable values for the various parameters, σ^2 is small relative to K and $Var(D_i)$ is increasing and concave in K while $K \geq \frac{r}{2h}\sigma^2 + \frac{hr}{2}$.

To isolate the effect of ordering cost K and price variance σ^2 on the order variance $Var(D_i)$, consider the case with constant prices so that $\sigma^2 = 0$. It is easy to check from (3.11) that $Var(D_i|\sigma^2 = 0) = r^2(\sqrt{\frac{2K}{hr}} - 1)$. Essentially, $Var(D_i|\sigma^2 = 0)$ is the sales variation caused by the ordering cost K alone that causes lumpiness in the orders for the manufacturer. Formally, we have:

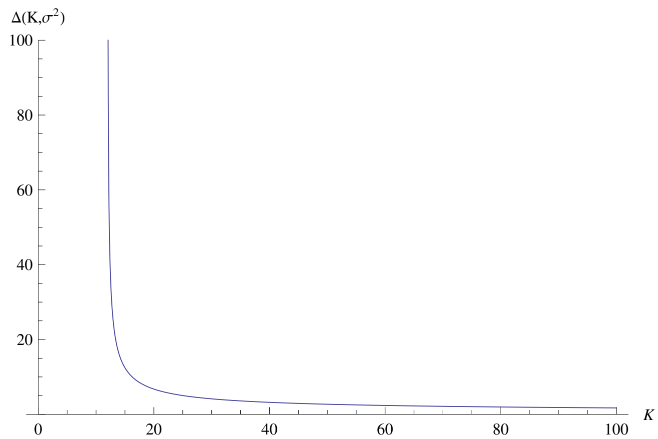

Figure 4: Incremental variance and the fixed ordering cost K when $h = 1, r = 1.5$, and $\sigma = 5$.

Corollary 2 *For any given price variance σ^2, the* incremental *sales variance $\Delta(K, \sigma^2)$ caused by the price variance decreases in K and $\Delta(K, \sigma^2) = 0$ as $K \to \infty$, where*

$$\Delta(K, \sigma^2) = Var(D_i) - Var(D_i | \sigma^2 = 0) = r^2 \sqrt{\frac{2K}{hr}} \left(\sqrt{\frac{K}{K - \frac{r}{2h}\sigma^2}} - 1 \right) \geq 0, \qquad (3.12)$$

Corollary 2 has the following implications. When K is small, price variance σ^2 plays a significant role in the order variance $Var(D_i)$ so that it is much higher than the order variance $Var(D_i | \sigma^2 = 0)$; i.e., $\Delta(K, \sigma^2)$ is large. However, as the ordering cost K increases, the effect of K dominates the effect of σ^2 so that the gap between $Var(D_i)$ and $Var(D_i | \sigma^2 = 0)$ becomes smaller as K increases. Therefore, when K is small – as both manufacturers and other sellers seek to have for their customers – the firm should consider reducing price variance σ^2 to reduce the variance of unit sales $Var(D_i)$. However, when K is large, the firm should focus on reducing K instead of price variance σ^2 (Figure 4).

4 Extending to Multiple Products

We now extend our base model by including multiple products, especially as many MRO manufacturers like our cutting-tools manufacturer have thousands or even tens of thousands of SKUs. Suppose the manufacturer sells two MRO products – this is easily extended to any number of products – that the customer consumes at a constant rate r_i, for $i = 1, 2$.

When selling two products under varying prices, we specify the retail price of each product in each time period according to a set of S pricing 'scenarios', where each price scenario s occurs with probability π_s so that the unit price of the products is (p_{1s}, p_{2s}). Obviously, $\sum_{s=1}^{S} \pi_s = 1$. Let μ_i be the average price and let σ_i^2 be the variance of the price for product i as per (3.1).

The customer needs to decide on a purchase policy over time, given the 'joint' ordering cost K is independent of the purchase quantity of either product or indeed the number of products, While there are different types of purchase policies that the customer may adopt, we examine two extremes, an "uncoordinated" policy and a "coordinated" one. (We shall use the superscript (u) to denote the uncoordinated purchase policy and superscript (c) to denote the coordinated purchase policy.)

Both policies apply to varying degree for the customers of the manufacturer in our context. A large number of MRO products are typically purchased in "an uncoordinated and decentralized manner" (Croom, 2000). On the other hand, for cutting tools in particular, orders may be coordinated by large customers' ERP systems.

4.1 Uncoordinated Policy

Under the uncoordinated purchase policy, the customer purchases either product independently of each other, i.e., determines the optimal order cycle for one product without taking the order cycle of the other product into consideration. This allows us to use the same analysis presented for the base case with one product to determine the optimal order cycle $t_{is}^{(u)}$ for each product i. Specifically, we can apply (3.6) to show that

$$t_{is}^{(u)} = \sqrt{\frac{2\hat{K}}{hr_i}} - \frac{1}{h}(p_{is} - \mu_i), \quad \text{for } i = 1, 2. \tag{4.1}$$

Given the optimal order cycle under the uncoordinated policy, we can retrieve the optimal purchase quantity for each product i by letting $Q_{is} = r_i \cdot t_{is}^{(u)}$ for $i = 1, 2$.

Because the purchase behavior of each product is independent of the other product, we can apply the expression given in (3.11) to show that, the total sales variance in any time period i under the 'uncoordinated' policy denoted by $Var^u(D_{i1} + D_{i2})$, can be expressed as:

$$Var^{(u)}(D_i) = r_1^2\left(\sqrt{\frac{2K^2}{hr_1(K - \frac{r_1}{2h}\sigma_1^2)}} - 1\right) + r_2^2\left(\sqrt{\frac{2K^2}{hr_2(K - \frac{r_2}{2h}\sigma_2^2)}} - 1\right) \tag{4.2}$$

The total sales variance $Var^{(u)}(D_i)$ given in (4.2) possesses the same structure as the sales variance $Var(D_i)$ given in (3.11) as in the single product case so the total sales variance $Var^{(u)}(D)$ exhibits the same properties as presented in Proposition 2.

4.2 Coordinated Policy

Under the coordinated policy, the customer purchases all products during each purchasing incident so that both products' inventory must "run out at the same time". This "equal runout time" policy, proposed by Karmarkar (1981) has been used as a heuristic for scheduling multiple items on a single machine (cf. Karmarkar, 1981; Federgruen, 1984).[2] However, Karmarkar (1981) used this concept for scheduling production of different products with fixed selling price while we use this concept to determine the purchasing cycle of different products with uncertain purchasing prices. More importantly, we are examining how price variance and covariance across multiple products affect their unit sales. To our knowledge, the joint impact of pricing of multiple products on their respective unit sales has not been examined thus far in the operations management literature.

Under the coordinated policy, the order cycle at each purchase incident depends only on the observed price scenario s when the observed unit price is (p_{1s}, p_{2s}). Therefore, the customer must purchase quantities of both products ensuring they will run out simultaneously at time t_s so that $Q_{is} = r_i t_s$ for $i = 1, 2$.

By using the fact that $Q_{is} = r_i t_s$ under the coordinated policy, we can determine the customer's total relevant cost associated with a purchase incident under price scenario s in terms of the order cycle t_s. Considering again the fixed cost K, the purchasing cost, and the inventory cost as part of the total relevant cost incurred once for an order cycle of length t_s, the *average relevant cost per unit time* for the customer under the coordinated policy $R_c(t_1, t_2, \cdots, t_S)$ is:

$$R_c(t_1, \ldots, t_S) = \frac{K + \sum_{s=1}^{S}[\pi_s(p_{1s}r_1 + p_{2s}r_2)t_s + \pi_s h(\frac{r_1 t_s}{2}\frac{r_1 t_s}{r_1} + \frac{r_2 t_s}{2}\frac{r_2 t_s}{r_2})]}{\sum_{s=1}^{S}[\pi_s t_s]} \quad (4.3)$$

[2]Besides the coordinated policy, one could consider a "nested" policy so that the order cycle of a product is a 'power of 2' of the order cycle of the other product. The power-of-2 policy has been examined by Roundy (1989) as a heuristic for solving the classic multi-item lot sizing problem.

The function R_c being jointly pseudo-convex in $(t_1, \cdots, t_S)$, we can determine the optimal order cycle policy $(t_1^*, \ldots, t_S^*)$ that minimizes the expected relevant cost per unit time $R_c(t_1, \ldots t_S)$. Once we determine the optimal order cycle t_s^*, we can retrieve the corresponding optimal purchase quantity $Q_{is}^* = r_i t_s^*$ for $i = 1, 2$, and $s = 1, \cdots, S$. In preparation, let

$$\begin{aligned} a_s &= p_{1s} r_1 + p_{2s} r_2, \quad \text{for } s = 1, \cdots, S, \\ b &= h\left(\frac{r_1 + r_2}{2}\right) \end{aligned} \tag{4.4}$$

Also, let ρ be the correlation between the price of product 1 and product 2 over time so that the covariance between p_{1s} and p_{2s} is equal to $\rho\sigma_1\sigma_2$. By using (4.4), we can define:

$$a = E_s(a_s) = r_1\mu_1 + r_2\mu_2, \tag{4.5}$$

$$\sigma_m^2 = Var(a_s) = r_1^2\sigma_1^2 + r_2^2\sigma_2^2 + 2\rho r_1 r_2 \sigma_1 \sigma_2, \text{ and} \tag{4.6}$$

$$\hat{K}_m = K - \frac{\sigma_m^2}{4b}. \tag{4.7}$$

We now present the optimal order cycle t_s^* that minimizes the customer's average relevant cost per unit time under the coordinated policy $R_c(t_1, t_2, \cdots, t_S)$ as given in (4.3).

Proposition 3 *Under the coordinated policy with equal runout time for both products at each purchase incident, the customer's optimal order cycle $t_s^{(c)}$ satisfies:*

$$t_s^{(c)} = \sqrt{\frac{\hat{K}_m}{b}} - \frac{1}{2b}(a_s - a) \tag{4.8}$$

where a_s and b are given in (4.4), a is given in (4.5), and $\hat{K}_m$ is given in (4.7).

By using the optimal order cycle t_s^* given in Proposition 3, we can retrieve the optimal purchase quantity for each product as:

$$Q_{is}^{(c)} = r_i \cdot t_s^* = r_i \left[\sqrt{\frac{\hat{K}_m}{b}} - \frac{1}{2b} \cdot (a_s - a)\right], \quad \text{for } i = 1, 2. \tag{4.9}$$

The optimal purchase quantity $Q_{is}^{(c)}$ possesses the same structure as the optimal purchase order quantity Q_s^* in the base model as shown in (3.3) as can be seen from (4.4), (4.5), and (4.7). Similarly, the optimal order cycle under the coordinated policy $t_s^{(c})$ has the same structure as the optimal order cycle t_s^* in the base model as shown in (3.6). By using (4.5), (4.6), and (4.7), we can prove the following result.

Corollary 3 *Under the coordinated policy, the optimal order cycle $t_s^{(c)}$ given in (4.8) satisfies:*

$$t^{(c)} = E(t_s^{(c)}) = \sqrt{\frac{\hat{K}_m}{b}}, \tag{4.10}$$

$$Var(t_s^{(c)}) = \frac{\sigma_m^2}{4b^2}, \quad \textit{and} \tag{4.11}$$

$$E([t_s^{(c)}]^2) = Var(t_s^{(c)}) + [E(t_s^{(c)})]^2 = \frac{K}{b}. \tag{4.12}$$

We can use Corollary 3 to determine the manufacturer's sales variance under the coordinated policy. Because the purchase incident occurs only when the inventory of both products drops to zero, each purchase incident is a "renewal". Under the coordinated policy, the length of each renewal cycle after the customer observed the realized price is $(p_{1s},\ p_{2s})$ is equal to $t_s^{(c)}$ that is given in (4.8). As in the base model, the steady state probability that a purchase incident occurs during any time period is given by $1/E[t_s^{(c)}] = 1/t^{(c)}$ following from (4.10). Hence, for any given time period i and particular price scenario s, the total sales D_i observed by the manufacturer is given by:

$$D_i = \begin{cases} 0 & \text{with prob. } (1 - \frac{1}{t^{(c)}}), \\ Q_{1s}^{(c)} + Q_{2s}^{(c)} & \text{with prob. } \frac{1}{t^{(c)}}, \end{cases}$$

where $Q_{is}^{(c)}$ is given in (4.9). By considering the total sales D_i as stated above, we get:

Proposition 4 *In steady state, the expected value and the variance of the manufacturer's total unit sales in any time period i is:*

$$E^{(c)}(D_i) = (r_1 + r_2),$$
$$Var^{(c)}(D_i) = (r_1 + r_2)^2 \cdot (\frac{E([t_s^{(c)}]^2)}{t^{(c)}} - 1) = (r_1 + r_2)^2 (\sqrt{\frac{K^2}{\hat{K}_m \cdot b}} - 1) \tag{4.13}$$

We can make three observations. First, from (4.13) it follows that the total sales variance $Var^{(c)}(D_i)$ possesses the same functional form as the sales variance $Var(D_i)$ given in (3.11)

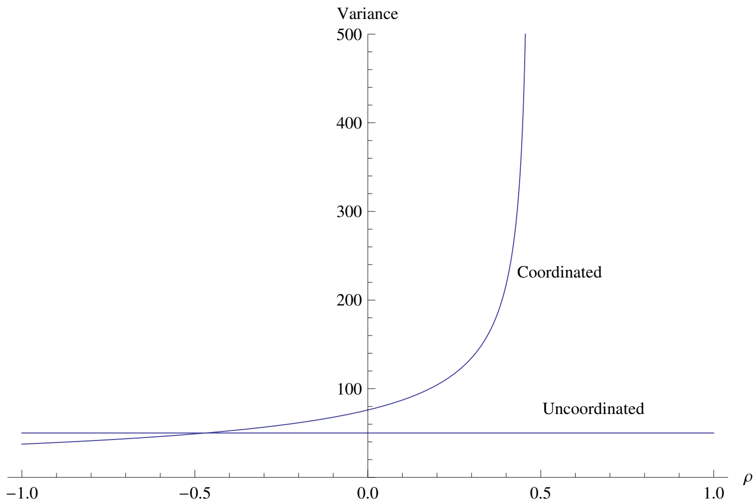

Figure 5: Variance of the total unit sales (across both products) with coordinated and uncoordinated policies against correlation of the prices of the two products for specific values of $r = 2.5, K = 20, h = 1.5$, and $\sigma = 3.5$.

as in the single product case. Therefore, the total sales variance $Var^{(c)}(D_i)$ satisfies the same properties as those for the single product case presented in Section 3.

Second, it follows from (4.7) that $\hat{K}_m = K - \sigma_m^2/4b$ decreases with increasing σ_m^2 and from (4.6) σ_m^2 increases with increasing ρ. Therefore, Proposition 4 implies that the manufacturer's order variance $Var^{(c)}(D_i)$ also increases with increasing ρ. This in turn implies that the manufacturer can reduce the sales variance by 'coordinating' the pricing between products 1 and 2 so that the resulting correlation coefficient is as low as possible, say, $\rho < 0$. Therefore, even when the individuals products' prices vary so that $\sigma_1^2 > 0$ and $\sigma_2^2 > 0$, the manufacturer should 'coordinate' the prices between products 1 and 2 with $\rho < 0$ to lower the variance of total unit sales.

Finally, we compare the total sales variance under the coordinated policy and under the uncoordinated policy. By comparing $Var^{(c)}(D_i)$ and $Var^{(u)}(D_i)$ given in (4.13) and (4.2), we obtain the following result:

Corollary 4 *When $r_1 = r_2 = r$ and $\sigma_1^2 = \sigma_2^2 = \sigma^2$, $Var^{(c)}(D_i) > Var^{(u)}(D_i)$ if and only if $K > (1-\rho)\frac{r}{2h}\sigma^2$.*

Corollary 4 suggests that, when the ordering cost K is sufficiently large or when $\rho > 0$,

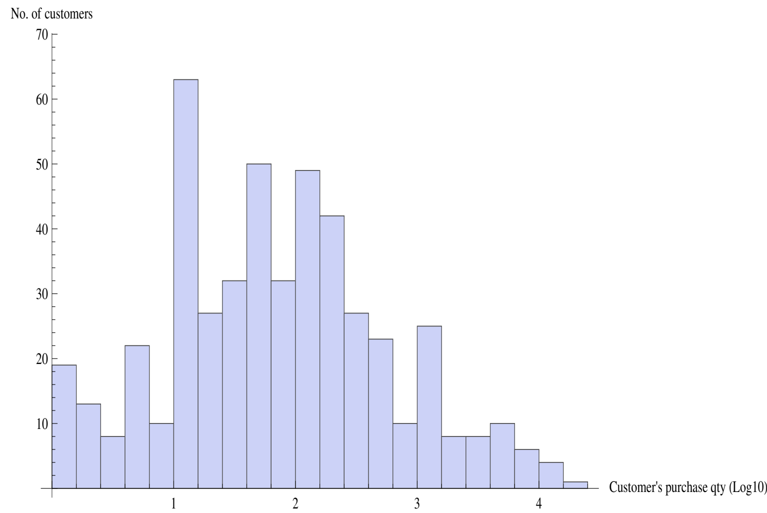

Figure 6: Distribution of logarithm of unit sales of a product to 489 different customers in a particular quarter

the total sales is more 'lumpy' under the coordinated policy (because the customer purchases both products each time). Consequently, total order variance is higher under the coordinated policy. However, when K is sufficiently small or when $\rho < 0$, the opposite is true. In other words, when the pricing between the two products are negatively correlated, the total order quantity evens out because the purchase quantity of one product is low while that of the other is (likely) high. Hence, the total sales is less 'lumpy' under the coordinated policy when the pricing across products is negatively correlated. This observation implies that the manufacturer can reduce the variance of total unit sales by keeping the correlation of prices across products low, preferably negative even when it cannot reduce the individual product's price variance (see Figure 5).

5 Extending to Heteregenous Customer Segments

The cutting tools manufacturer, and indeed any MRO manufacturer, has a wide range of customers. Unit sales for individual products vary dramatically across these customers as can be seen from the example in Figure 6 pertaining to total quarterly unit sales of a particular product family to 489 different customers; we took the logarithm because of the enormously wide range from 1 to 18,000 units sold.

As such, we need to extend our base model to the case of multiple customer segments $j = 1, 2, ...$ with different consumption rates r_j and different ordering costs K_j. *Each customer segment has N_j customers.* As with the base case, there is only a single product. For notational convenience, we present the two-segment case. Let us first examine the purchasing behavior of a customer in segment j. Using the same approach as that in Sections 2.1 and 2.2, we can show that Proposition 1 and Lemma 1 continue to hold. Specifically, during a purchasing incident, each customer in segment j will purchase Q_{js} upon observing price p_s and will order again after his inventory drops to zero. Hence, the purchase quantity Q^*_{js} can be obtained from (3.3) by substituting r with r_j and $\hat{K}$ from (3.5) with $\hat{K}_j$ so that

$$Q^*_{js} = \sqrt{\frac{2\hat{K}_j r_j}{h}} - \frac{r_j}{h}(p_s - \mu), \tag{5.1}$$

where

$$\hat{K}_j = K_j - \frac{r_j}{2h}\sigma^2, \text{ for } j = 1, 2. \tag{5.2}$$

By noting that the time until the next purchase is equal to t^*_{js} , where $t^*_{js} = Q^*_{js}/r_j$, we can apply Lemma 1 to show that:

$$t^*_j = E(t^*_{js}) = \sqrt{\frac{2\hat{K}_j}{hr_j}}, \tag{5.3}$$

$$E([t^*_{js}]^2) = Var(t^*_{js}) + [E(t^*_{js})]^2 = \frac{2K_j}{hr_j}. \tag{5.4}$$

By using the renewal theory as described in Section 3.1, the steady state probability that each customer in segment j will order in any time period is given by $\frac{1}{E(t^*_{js})} = \frac{1}{t^*_j}$. By assuming that the purchasing behavior of a customer is independent of other customers, the total number of customers in segment j who place an order at any time period i, denoted by n_{ij}, follows a binomial distribution; i.e., $n_{ij} \sim Binomial(N_j, \frac{1}{t^*_j})$. Hence, for any given price p_s and given number of customers from each segment n_{ij}, the sales D_i observed by the manufacturer in any time period i is given as:

$$(D_i \mid p_s, n_{i1}, n_{i2}) = \sum_{j=1}^{2} n_{ij} Q^*_{js}, \tag{5.5}$$

where $n_{ij} \sim Binomial(N_j, \frac{1}{t^*_j})$, t^*_j is given in (5.3), and Q^*_{js} is given in (5.1).

By taking conditional expectation and conditional variance and by using the same approach as presented in the proof of Proposition 2, we obtain:

Proposition 5 *In steady state, the expected value and the variance of unit sales observed by the manufacturer in any time period are as follows:*

$$E(D_i) = \sum_{j=1}^{2} N_j r_j$$

$$Var(D_i) = \sum_{j=1}^{2} \left[\left(\frac{N_j}{t_j^*}\right)^2 \frac{r_j^2 \sigma^2}{h^2} + N_j \; r_j^2 (\frac{1}{t_j^*}) \left(1 - \frac{1}{t_j^*}\right) \frac{2K_j}{hr_j} \right]$$

$$= \sum_{j=1}^{2} \left[\frac{N_j^2 r_j^3 \sigma^2}{2\hat{K}_j h} + N_j r_j^2 \left(\sqrt{\frac{2K_j^2}{hr_j \hat{K}_j}} - \frac{K_j}{\hat{K}_j} \right) \right] \tag{5.6}$$

where $\hat{K}_j$ is given in (5.2).

To verify that the multiple customer-segment model generalizes the base model, consider the case when there is only 1 customer (i.e., $j = 1$, $N_j = 1$, $r_j = r$, and $\hat{K}_j = \hat{K}$). Then, we can show that $Var(D_i)$ given in (5.6) can be simplified to the equivalent expression in (3.11). Consequently, $Var(D_i)$ is increasing in the price variance σ^2 and that the general properties as exhibited in Corollaries 1 and 2 continue to hold. Specifically, by differentiating $Var(D_i)$ given in (5.6) with respect to K, we can show that the variance of the manufacturer's unit sales is decreasing in K when K is below a certain threshold but increasing in K beyond this threshold.

To examine the impact of the j'th segment size N_j, it is easy to check from (5.6) that $Var(D_i)$ is increasing in N_j when $\sqrt{\frac{2K_j^2}{hr_j \hat{K}_j}} > \frac{K_j}{\hat{K}_j}$, i.e., when $K_j > \frac{r_j}{2h}\sigma^2 + \frac{hr_j}{2}$. Hence, we can conclude that the manufacturer's sales variance is increasing in N_j when K is sufficiently large or when the consumption rate r_j is sufficiently small.

In addition, consider a special case with equal number of customers in each segment, i.e., $N_1 = N_2 = N$. Then we can show that, when the ordering cost $K_j > \frac{r_j}{2h}\sigma^2 + \frac{hr_j}{2}$ for the customer segments $j = 1, 2$, the manufacturer's order variance increases with segment size N, while the coefficient of variation $\frac{\sqrt{Var(D_i)}}{E(D_i)}$ decreases with N. *The fact that the coefficient of variation decreases implies there is some benefit to be gained growing sales or capturing market share.*

With different-sized segments, consider $N_1 = (1 - \alpha)N$ and $N_2 = (1 + \alpha)N$ where $\alpha \in (0, 1)$ represents a measure of segment-size heterogeneity, given some "typical" size N. In similar vein, consider $r_1 = (1-\beta)r$ and $r_2 = (1+\beta)r$ with $\beta \in (0, 1)$ as a consumption-rate

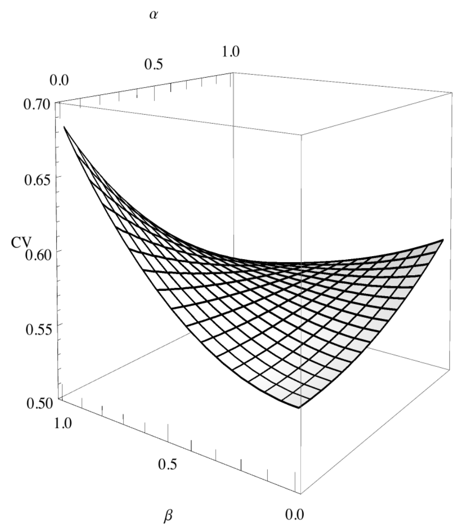

Figure 7: Coefficient of variation (CV) for unit sales against segment-size heterogeneity (α) and consumption-rate heterogeneity (β) for selected values of $N = 15, r = 1.6, h = 1, K = 50$, and $\sigma = 2.5$.

heterogeneity for some "typical" consumption rate r. Then we can consider the coefficient of variation $\sqrt{Var(D_i)}/E[D_i]$ as a function of α, β and N respectively for some particular values of r, h, K and σ. Figure 7 shows how the coefficient of variation (CV) varies with segment-size heterogeneity α and consumption-rate heterogeneity β for certain values of the other parameters, suggesting that the CV is more sensitive to consumption-rate heterogeneity (β) than to segment-size heterogeneity (α) at least for this example. The example also shows how the CV decreases with increasing average segment size N (Figure 7).

We also observe asymptotic behavior of CV with increasing number of customers in each segment, at least for specific values of $\alpha = \beta = 0.5$ (Figure 8). This suggests the benefits in terms of the reduction of CV with increasing number of customers in each market segment taper off. Therefore, to reduce the CV, the manufacturer would eventually have to target price variance to reduce the variance of unit sales.

6 Conclusions

We extended MRO customer's use of EOQ ordering – quite suitable given the constant consumption and ordering when the inventory goes below a threshold level – to incorporate

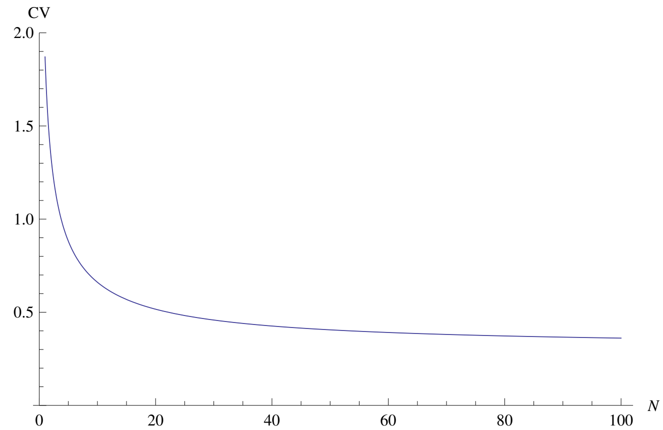

Figure 8: Coefficient of variation against the "base" customer size segment N, with specific values of $\alpha = 0.5, \beta = 0.5, r = 1.6, K = 50, h = 1$ and $\sigma = 2.5$.

stochastic purchase price. Using this for the customer's orders, and motivated by the setting at a particular cutting tools manufacturer, we examined how MRO manufacturers' unit sales are affected by the variance in the transactional prices they charge their customers. The customers determine both the order size and the order interval *endogenously* to minimize the long-run average cost of purchasing, holding and ordering. We provided closed-ended analytical results and also take into account the company's multiple products and multiple consumer segments.

To position our paper in the literature, unlike traditional EOQ models focusing on the purchasing/production quantities for one entity, we considered both the customer who satisfies the EOQ assumptions as well as the manufacturer who supplies this customer, given stochastic prices. For the customer's decision, we extended the EOQ model to deal with stochastic purchase price and obtained closed-form expressions (Proposition 1 and Lemma 1). We were then able to analyze the impact of variable transaction prices on the orders that the manufacturer gets (Proposition 2). By doing so, we examined the implications of an MRO manufacturer's pricing on its unit sales (in terms of expected value and variance) determined "endogenously" by the customer's rational purchasing behavior seeking to minimize long-term average cost of purchasing, holding inventory, and ordering.

The closed-form expressions we obtain by extending the EOQ model for the MRO *customer* to include the variance of the procurement price allows us to infer the following for the MRO *manufacturer*: (1) The manufacturer's price variance has a significant impact on the variance of the manufacturer's unit sales even though the customer's consumption is constant. (2) For multiple products, discounts or other promotional pricing should be uncoordinated or, even better, be negatively correlated so that when the effective price of one product is low, that for the other should remain high. Invoice-level discounts, i.e., all products being discounted by the same amount in any transaction are therefore a bad idea. (3) Market heterogeneity increases unit-sales variance for the manufacturer, both by way of different number of customers in the different market segments and by way of the market segments having different consumption rates. This reinforces that idea that greater variety for the same volume may be expected to lead to greater process inefficiency. (4) Although gaining market share decreases the coefficient of variation of these unit sales, however, the effect tapers off rather quickly. (5) The *ordering cost* aquires a special significance; indeed, the manufacturer can decrease its own unit-sales variance in all of these cases by reducing the customer's ordering cost.

Here are some ideas for future work: (1) We could extend this work to the manufacturer's rate of production. For instance, we could consider the situation where production can be assumed to be constant as for instance in the process industry. (2) We could introduce a competing manufacturer in the situation we considered in this paper especially in relation to the tradeoffs of increasing unit sales versus increasing the variance (while decreasing the coefficient of variation as discussed above). (3) Holding costs play an important role as we can see from the closed-end expressions we have obtained. However, we have not tied these to either the purchase price or to the different ordering schemes like VMI that the manufacturer may offer the customer, with implications for the customer. (4) Although the results are motivated by the MRO sector, our results may provide the incremental variance due to price variance for the broad B2B sector, over and above the variance due to fluctuations in the customers' own demand that we had assumed constant in this paper – see Sodhi and Tang (2011) in this regard.

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7 Appendix: Proofs

Proof of Lemma 1: These results follow immediately from Proposition 1. We omit the details. ■

Proof of Proposition 2: First, observe that $Q_s = rt_s^*$ under any scenario s with price p_s. Hence, $E(D_i|p_s) = \frac{Q_s^*}{t^*} = r\frac{t_s^*}{t^*}$, and the first statement follows immediately. Next, compute the conditional variance of D_i

$$\begin{aligned} Var(D_i|p_s) &= (0 - r\frac{t_s^*}{t^*})^2(1 - \frac{1}{t^*}) + (rt_s^* - r\frac{t_s^*}{t^*})^2\frac{1}{t^*} \\ &= r^2 t_s^{*2}\frac{1}{t^*}(1 - \frac{1}{t^*}) \end{aligned}$$

By using the conditional variance formula, $Var(D_i) = E_s(Var(D_i|p_s)) + Var_s(E(D_i|p_s))$ and by applying (3.7) and (3.9), we obtain the second statement after some algebra. ■

Proof of Corollary 1: We prove the first statement by differentiating $Var(D_i)$ given in (3.11) with respect to K. By considering (3.8) and (3.7) along with the fact that $\hat{K} = K - \frac{r}{2h}\sigma^2$, it can be shown after some algebra that:

$$\frac{dVar(D_i)}{dK} = r^2\sqrt{\frac{2}{hr}}\left(\sqrt{\frac{1}{(K - \frac{r}{2h}\sigma^2)}}\right)\frac{(K - 2\cdot\frac{r}{2h}\sigma^2)}{2(K - \frac{r}{2h}\sigma^2)^2}$$

Similarly, we can prove the second statement by considering second derivatives of $Var(D_i)$ with respect to K. We omit the details. ■

Proof of Corollary 2: By considering (3.11), (3.8) and (3.7) and by simplifying the terms, it can be shown that (after some algebra):

$$\begin{aligned} Var(D_i) - Var(D_i|\sigma^2 = 0) &= r^2(\frac{Var([t_s^*])}{t^*} + t^* - 1) - r^2(\sqrt{\frac{2K}{hr}} - 1) \\ &= r^2\sqrt{\frac{2K}{hr}}\cdot(\sqrt{\frac{K}{\hat{K}}} - 1) \end{aligned}$$

Because $\Delta(K,\sigma^2) = Var(D_i) - Var(D_i|\sigma^2 = 0) = r^2\cdot\sqrt{\frac{2K}{hr}}\cdot(\sqrt{\frac{K}{\hat{K}}} - 1)$, it suffices to show that $f(K)$ is decreasing in K and converges to 0 as $K \to \infty$, where $f(K) = K^{0.5}(K^{0.5}\hat{K}^{-0.5} - 1)$. Differentiating $f(K)$ with respect to K, we obtain

$$\frac{df(K)}{dK} = 0.5K^{-0.5}\hat{K}^{-1.5}(K^{0.5} - \hat{K}^{0.5})(\hat{K} - K - K^{0.5}\hat{K}^{0.5})$$

after some algebra. As $\hat{K} < K$ for any $\sigma^2 > 0$, we can conclude that $\frac{df(K)}{dK} < 0$. Second, by noting that $f(K)$ converges to 0 as $K \to \infty$, we have proven our result. ■

Proof of Proposition 3: Use (4.4) to rewrite the objective function $R_c(t_1, \cdots, t_S)$ given in (4.3) as:

$$R_c(t_1, \cdots, t_S) = \frac{K + \sum_{s=1}^{S}[\pi_s(a_s t_s + b t_s^2)]}{\sum_{s=1}^{S} \pi_s t_s}$$

We can obtain the optimal t_s^* by considering the first-order condition and using the fact that R_c is jointly pseudo-convex in $(t_1, \cdots, t_S)$. First, we differentiate R_c given above with respect to t_s and consider the first-order conditions associated with t_s, for all scenarios $s = 1, ..., S$. It is easy to show that under the first order conditions t_s satisfies:

$$t_s = t_1 + \frac{1}{2b}(a_1 - a_s), \quad \text{for } s = 1, \cdots, S$$

where a_1, a_s and b are given in (4.4). Next, substituting t_s into R_c, we can rewrite R_c as a function of t_1. By letting $x = t_1 + \frac{1}{2b}(a_1 - a)$, where a is given in (4.5) and by transforming the decision variable t_1 to x, we can simplify R_c further as a function of x, After some algebra, one can show R_c can be simplified as:

$$R_c(x) = \frac{\hat{K}_m + ax + bx^2}{x}$$

where $\hat{K}_m$ is given in (4.7). In this case, the optimal x^* that minimizes $R_c(x)$ is given as: $x^* = \sqrt{\widehat{K}_m/b}$. Because $x = t_1 + \frac{1}{2b}(a_1 - a)$, we can retrieve the optimal t_1. Given t_1, we can use t_s given above to obtain (4.8). ■

Proof of Corollary 3: The proof follows from (4.5), (4.6), and (4.7) and we omit the details. ■

Proof of Proposition 4: The proof follows the same approach as that for Proposition 2 by taking the conditional expectation and using the conditional variance formula. ■

Proof of Corollary 4: When $r_1 = r_2 = r$ and $\sigma_1^2 = \sigma_2^2 = \sigma^2$, we can apply (4.7), (4.13) and (4.2) to show that $Var^{(c)}(D_i)$ under coordination and $Var^{(u)}(D_i)$ without coordination can be simplified as

$$Var^{(c)}(D_i) = 2r^2(\sqrt{\frac{4K^2}{hr\left(K - \frac{r}{2h}\sigma^2(1+\rho)\right)}} - 1)$$

$$Var^{(u)}(D_i) = 2r^2(\sqrt{\frac{2K^2}{hr(K - \frac{r}{2h}\sigma^2)}} - 1)$$

Hence, $Var^{(c)}(D_i) > Var^{(u)}(D_i)$ if and only if $2/(K - \frac{r}{2h}\sigma^2(1+\rho)) > 1/(K - \frac{r}{2h}\sigma^2)$. We obtain the desirable result by rearranging the terms. ■

Proof of Proposition 5: First, observe that $Q_{js} = r_j t^*_{js}$ under any realized price p_s. Hence, by taking the conditional expectation of $(D_i|p_s, n_{i1}, n_{i2})$ given in (5.5), we get $E(D_i) = \sum_{j=1}^{2} N_j \frac{1}{t^*_j} \cdot r_j t^*_j$ and we obtain the first statement. Next, by using the conditional variance formula, $Var(D_i) = E_s(Var_{n_{ij}}(D_i|p_s, n_{ij})) + Var_s(E_{n_{ij}}(D_i|p_s, n_{ij}))$ and by applying (5.3) and (5.4), we obtain the second statement after some algebra. ■