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Essays on the Modelling of S&P 500 Volatility

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Thesis Submitted for the Degree of Doctor of Philosophy in Finance

City University
Cass Business School
Department of Finance

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Contents

Abbreviations

ACF	Autocorrelation Function
ADF	Augmented Dickey-Fuller Unit Root Test
AIC	Akaike Information Criterion
APARCH	Asymmetric Power Autoregressive Conditional Heteroskedasticity
ARCH	Autoregressive Conditional Heteroskedasticity
ARFIMA	Autoregressive Fractionally Integrated Moving Average
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Avereage
ATM	At-the-Money
BDF	Brock, Dechert and Scheinkman Independence Test
BHHH	Berndt, Hall, Hall and Hausman Algorithm
BPI	Binomial Path Independence
BS	Black & Scholes Model
CBOE	Chicago Board Options Exchange
CBOT	Chicago Board of Trade
CDF	Cumulative Distribution Function
CME	Chicago Merchantile Exchange
CMG	Cameron-Martin-Girsanov
CUBS	City University Business School
DIVF	Dynamic Implied Volatility Function
DVF	Deterministic Volatility Function
DF	Dickey-Fuller Unit Root Test
DTB	Deutsche Terminborse
EGARCH	Exponential GARCH

EGARCH-M	Exponential GARCH in Mean
EMS	European Monetary System
ERM	Exchange-Rate Mechanism
EWMA	Exponentially Weighted Moving Average
FIGARCH	Fractional Integrated GARCH
FIEGARCH	Fractional Integrated EGARCH
FTSE-100	Financial Times Stock Exchange 100
GARCH	Generalised ARCH
GARCH-M	GARCH in mean
GBM	Geometric Brownian Motion
GED	Generalised Error Distribution
GMLE	Guassian quasi-MLE
GMM	Generalised Method of Moment
HKSE	Hong Kong Stock Exchange
HIS	Hang Seng INdex
HMSE	Heteroskedasticity-adjusted MSE
HW	Hull & White Model
IID	Independent & Identically Distributed
IMM	International Money Market
ISD	Implied Standard Deviation
ITM	In-the-Money
IV	Implied Volatility
IVF	Implied Volatility Function
KS	Kolmogorov-Smirnov statistic
LEAPS	Long-term Equity Anticipation Securities

LHS	Left-Hand Side
LIFFE	London Financial Futures Exchange
LL	Logarithmic Loss
LLR	Log-Likelihood Ratio
LM	Larange-Multipler Test
LR	Likelihood-Ratio Test
LRNVR	Locally Risk-Neutral Valuation Relationship
LTCM	Long Term Capital Management
MA	Moving Average
MAE	Mean-Absolute Error
MAPE	Mean-Abolute Percent Error
MMEO	Mean-Mixed Error (over prediction)
MMEU	Mean-Mixed Error (under prediction)
MONEP	Marché des Options Négociables de Paris
MSE	Mean-Square Error
NARCH	Nonlinear ARCH
NLS	Nonlinear Least-Square
NYSE	New York Stock Exchange
OLS	Ordinary Least Squares
OTC	Over-the-Counter
OTM	Out-of-the-Money
PIDE	Partial Intergro-Differential Equation
PDE	Partial Differential Equation
PDF	Probability Density Function
PHLX	Philadelphia Exchange

QMLE	Quasi-Maximum Likelihood Estimation
RHS	Right-Hand Side
RMAE	Root Mean Absolute Error
RMAPE	Root Mean Absolute Percent Error
RMSE	Root Mean Squared Error
RND	Risk-Neutral Distribution
S&P 100	Standard & Poor 100
S&P 500	Standard & Poor 500
SACF	Sample Autocorrelation Function
SBC	Schwarz Bayesian Critereon
SDE	Stochastic Differential Equation
SPSE	Sum of Price Square Error
SV	Stochastic Volatility
SWARCH	Switching ARCH
TS-GARCH	Taylor's and Schwert's GARCH
VAR	Vector Autoregressive Model
VIX	CBOE's Volatility Index
VOLAX	DTB's Volatility Index Futures
VX1	MONEP's 31-day Short-Term Implied Volatility Index
WISD	Weighted Implied Standard Deviation

Important Symbols

β	Elasticity Requirement
C	Call Option
$\chi^2(m)$	m^{th} degree Chi-statistics
$D(\ )$	Any Distribution
δ	Correlation for Wiener Processes
H_0	Null Hypothesis
H_1	Alternative Hypothesis
h_t	Conditional Variance based on information up to time t-1
$I(0)$	Covaraince Stationary Process
$I(1)$	Non-Stationary Process
I_t	Information set up to time t
L	Lag Operator
μ	Distributional Mean
$N(\ )$	Normal Distribution
P	Put Option
$Q^2(m)$	Ljung-Box Statistics at lag m for Serial Correlation
$Q(m)$	Box-Pierce Statistics at lag m for Serial Correlation
$\rho(m)$	Sample Autocorrelation Function at lag m
σ_t	Stochastic Volatility
$T(\ )$	Student-t Distribution
W	Wiener Process or Brownian Motion

Tables

Figures

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Declaration

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Abstract

This dissertation studies the patterns of term-structure of implied volatility and examines the performance of different specifications of time-series and options-based volatility forecasting models under the influence of the observed market biases. Our research is based primarily upon the use of S&P 500 data for the period 1982-2002. There are three self-contained but seemingly related projects in this dissertation. The objectives of this research are: 1) to characterise the term-structure of implied volatility; 2) to compare the performance of asymmetric power ARCH and EGARCH models; 3) to evaluate the forecasting performance of time-series and options-based variance swap valuation models.

The observed market anomalies in the term-structure of implied volatility of S&P 500 futures options are investigated between 1983 and 1998. Term-structure evidence indicates that short-term options are most severely mispriced by the Black-Scholes formula. We find evidence that option prices are not consistent with the rational expectations under a mean-reverting volatility process. In addition, skewness premiums results show that the degrees of anomalies in the S&P 500 options market have been gradually worsening since around 1987. As correlation may be responsible for skewness, our diagnostics suggest that leverage and jump-diffusion models are more appropriate for capturing the observed biases in the S&P 500 futures options market.

Sixteen years of daily S&P 500 futures series are employed to examine the performance of the APARCH models that use asymmetric parameterisation and power transformation on conditional volatility and its absolute residual to account for the slow decay in returns autocorrelations. No evidence can be found supporting the relatively complex APARCH models. Log-likelihood ratio tests confirm that power transformation and asymmetric parameterisation are not effective in characterising the returns dynamics within the context of APARCH specifications. Furthermore, results of a 3-state regime-switching model support the notion that the performance of conditional volatility models is prone to the state of volatility of the returns series. In addition, AIC statistics stipulate that EGARCH is best in "noisy" periods whilst GARCH is the top performer in "quiet" periods. Overall, aggregated rankings for the AIC metric show that the EGARCH model is best. Options-based volatility trading exercises also reveal that EGARCH and GARCH can generate statistically significant ex-ante profit in one out of four sample periods after transactions costs. When considering a stochastic volatility model, there seems to be little incentive to look beyond a simple model which allows for volatility clustering and a leverage effect.

The volatility forecasting performance of different specifications of time-series and options-based variance swap valuation models on the S&P 500 index is evaluated from three months before to after the 9/11 attacks. By far, the option-based Demeterfi et al. (1999) variance swap valuation framework is the most popular tool to price variance swaps. This framework stipulates that pricing a variance swap can be viewed as an exercise in computing the weighted average of the implied volatility of the options required even under the influence of volatility skew. Our research design offers a comprehensive empirical study of the relative merits of competing option pricing models. Based on results from six carefully chosen contract days, we illustrate that implied models may overpredict future variance and underperform time-series models. The reasons could be: 1) the implied strategy was originally developed for hedging; 2) implied volatility is predominantly a monotonically decreasing function of maturity and therefore options-based strategy cannot produce enough variance term-structure patterns; 3) distributional dynamics implied by option parameters is not consistent with its time-series data as stipulated by the maximum likelihood estimation of the square-root process. Future research needs to use a larger sample set in order to establish a more statistically significant result to justify our findings. Until then we have a strong reservation about the use of Demeterfi et al. methodology for variance forecasting.

To My Parents

CHAPTER 1 Introduction to the Study

"Learning without thought is labour lost; thought without learning is perilous."

— Confucius

1.1 Introduction

This dissertation is a quantitative study whose primary objective is to investigate the performance of different specifications of time-series and options-based volatility forecasting models under the influence of the observed market biases in the S&P 500 markets. Our research work is based primarily upon the use of futures, futures on options and index options data for the period 1982-2002. This first chapter of the dissertation introduces the background of the study, specifies the problems of the study and describes its significance. The chapter concludes by outlining the structure of the dissertation.

1.2 Background of the Study

Volatility of the underlying asset price is the primary determinant of option prices and many related derivatives instruments. An option pricing model that does not properly capture the evolution of volatility processes can give rise to option prices that do not agree well with prices observed in the market and can also reduce investor's ability to hedge risk. The Black-Scholes option pricing model is commonly used to price a wide range of options contract. However, its erratic empirical behaviour is well documented, a phenomenon known as "volatility smile" (e.g. MacBeth et al., 1979; Rubinstein, 1985; Bollerslev et al., 1992). Contrary to the basic assumptions of the Black-Scholes formula, implied volatility exhibits both smile effects and term-structure patterns. Many market factors such as the leverage effect, taxing, industrial cycles, serial correlated news arrival, market psychology etc have played very crucial roles in causing these observed biases in the marketplace. As a result, normal distribution is not adequate to specify the returns dynamics and researchers have yet to deal with fat-tails and excess kurtosis which form the basis of smile effects. Below we will briefly discuss the three areas of interest in this dissertation, namely, the term-structure of volatility, modelling of volatility and volatility derivatives.

1.2.1 Term-Structure of Volatility

The modelling of the term-structure of implied volatility has been discussed by many researchers, e.g. Rubinstein (1985), Stein (1989), Diz and Finucane (1993), Heynen, Kemna, and Vorst (1994) and Xu and Taylor (1994). Rubinstein (1985) documented that implied volatility of exchange traded call options between August 1976 and August 1978 exhibited a systematic pattern with respect to different maturities and exercise prices. Rubinstein's most intriguing result was that the direction of bias changed signs between sub-periods, implying that skewness of the risk-neutral density changed over time. Subsequently, numerous efforts have been made to investigate the mean-reverting process and term-structure of implied volatility. Stein (1989) pioneered the examination of the term-structure of the average at-the-money options' implied volatility using two maturities on S&P 100 index options. By using a mean-reverting volatility model, evidence suggested that long-maturity options tended to "overreact" to changes in the implied volatility of short-maturity options because investors had a systematic tendency to overemphasise recent data at the expense of other information when making projections. This result was disputed by Diz and Finucane (1993) following their analysis of similar S&P 100 index data. The term-structure of implied volatility has also been discussed by Heynen, Kemna and Vorst (1994). Basing their results upon Duan (1995), Heynen et al. derived the term-structures of implied volatility for EGARCH, GARCH and a mean-reverting stochastic model in a similar way to Stein (1989). Only two values of time-to-maturity were investigated and Heynen et al. concluded that EGARCH gave the best description of asset prices of the term-structure of implied volatility. Xu and Taylor (1994) also studied at-the-money currency options and used a mean-reverting volatility model to establish relationships between long- and short-term expectations of implied volatility for any number of maturity *T*. Xu et al.'s model could explain the time-varying crossovers of implied volatility at different maturities but it did not emphasise the effects of volatility smile. Surprisingly little research has been done on the properties and evolution of implied volatility. Past research has mainly focused on "fitting" a theoretical option model to the observed biases in a particular options market from an arbitrarily short span of data for at-the-money contracts. Since the term-structure of implied volatility reflects the time-varying market expectations of asset volatility over different time horizons, it is imperative to focus on a single market and gain a thorough understanding of its behaviour.

1.2.2 Modelling of Volatility

Since the late 1980's many researchers have developed alternative option-pricing models in order to cope with the observed term-structure biases in the equity market. The latest one-factor implied models such as Derman and Kani (1994), Rubinstein (1994) and Dupire (1994) have created specifications that can implicitly model volatility as a deterministic function of time. However the major setback for "implied" methods is that they all require substantial "engineering" efforts to calibrate their lattice structures. These complex models are usually reserved for the valuation of exotic options and are seldom used for volatility forecasts. On the other hand, a more structural approach to improving the forecasting performance is to model volatility as a time-varying stochastic variable. Whilst stochastic models such as Hull and White (1987), Johnson and Shanno (1987), Scott (1987) and Stein and Stein (1991) provide another means to capture smile effects, many problems limit the use of these stochastic volatility models. The main problem associated with stochastic volatility models is that volatility is not a traded asset and is therefore unobservable. Besides, estimations of continuous-time models' parameters are problematic as real-world data are recorded at discrete intervals.

Following the path-breaking paper by Engle (1982), an alternative literature has focused on discrete-time autoregressive conditional heteroskedasticity (ARCH) models. The development of ARCH models is driven by three regularities of equity returns: 1) equity returns are strongly asymmetric, e.g. negative returns are followed by larger increases in volatility than equally large positive returns; 2) equity returns are fat-tailed; 3) equity returns are persistent (persistence refers to volatility clustering). This class of discrete-time models hypothesises that both smile effects and term-structure patterns can be explained by allowing the underlying asset's volatility to obey a stochastic process. There is a voluminous literature suggesting that discrete time-varying volatility models are practical and insightful. The usefulness of ARCH modelling is such that volatility is predictable and readily implemented. ARCH models assume the presence of a serially correlated news arrival process and require only the use of past data. As such, ARCH models allow conditional variance to change over time as a function of past conditional variance and error, whilst leaving unconditional variance constant. Most of the early research efforts focused on conditional models that imposed symmetry on the conditional variance structure. In response to criticisms that the symmetric model may not be appropriate for modelling stock returns volatility, more recent research has considered other features such as leverage effects, power transformation etc in the variance equation. There are, indeed, so many conditional volatility models in the finance literature that it is cumbersome to provide a comprehensive survey of them all.

Recently, the topics of long memory and persistence have attracted considerable attention in terms of the second moment of an asset returns process. The development of long-memory models is based on the observations of the so-called "stylised facts". For example, Ding et al. (1993) invented the APARCH models that used the Box-Cox transformation on conditional variance and its absolute residual to account for the slow decay of autocorrelations in the returns process. Subsequently, many researchers have also developed different specifications for the long-memory process (e.g. Baille, 1996; Bollerslev et al., 1996; Ding et al., 1996). Several papers have given the impression that their models are capable of accounting for empirical features such as volatility clustering and leptokurtosis in the distribution of returns. Despite the huge amount of effort researchers has put into modelling volatility, it is clear that empirical issues remain unexplored for many of these more "elaborate" models.

1.2.3 Volatility Derivatives

Until now the conventional instruments for implementing a volatility hedge remain rather crude. The most widely accepted way of speculating on volatility is usually achieved through the purchase of European call and put options. Traditional techniques such as delta hedging strategy always focus on the reduction of delta-risk. Once the underlying index moves, however, a delta-neutral trade can become long or short delta. Rehedging becomes necessary to maintain a delta-neutral position as the market moves. Since transaction and operational costs generally prohibit continuous rehedging, residual exposure of the underlying ultimately arises from options-based volatility strategies. Despite the fact that there has been an increased interest in volatility products since the late 1990's, little research has been directed towards to the development of volatility derivatives. The first theoretical paper to value volatility derivatives is by Grünbichler et al. (1996). Grünbichler et al. presented a simple but technically complicated framework that used the equilibrium approach within which specific closed-form solutions for volatility futures and option prices were derived within a mean-reversion framework. Later, Gupta (1997) and Engle et al. (1998) discussed the issues related to the hedging of volatility. Subsequently, Andersen and Andreasen (1999), Rolfes and Henn (1999), Chriss and Morokoff (1999), Demeterfi et al. (1999), Brenner et al. (2000), Brockhaus and Long (2000), Heston and Nandi (2000b), Howison et al. (2001), Little and Pant (2001), Carr and Madan (1999, 2002), Javaheri et al. (2002) and Théoret et al. (2002) also researched volatility derivatives, but the amount of research invested in volatility products still pales in comparison with other well-studied exotic derivatives products such as barrier and Asian options. Volatility risk has yet to be dealt with so that investors and traders can directly express their views on future volatility.

The arrival of variance swaps offers an opportunity for traders to take synthetic positions in volatility and hedge volatility risk. They were first introduced in 1998 in the aftermath of the Long Term Capital Management (LTCM) melt down when implied stock index volatility levels rose to unprecedented levels. These variance swap contracts are mostly based on equity indices and they were originally designed to be a replacement for traditional options-based volatility strategies such as straddle or hedged call/put options. Despite its name, a variance swap is actually an over-the-counter forward contract whose payoff is based on the realised volatility of a stated equity index. Their payoff at expiration is equal to:

$$(\sigma_R^2 - K_{vol}^2) * N$$

where N is the notional amount of the swap in some currency units per annualised variance point, σ_R and K_{vol} are the realised stock volatility over the life of the contract (n days) quoted in annual term, i.e. $\frac{F}{n}\sum_{i=0}^{n-1}\left(\frac{S_{i+1}-S_i}{S_i}\right)^2$, and the fixed annualised volatility delivery price, respectively. F is the appropriate annualisation factor.

Since a variance swap provides pure exposure on future volatility levels, it is considered a cleaner bet on volatility than options-based strategy. It allows counterparties to exchange cash-flows – floating variance for fixed variance. Counterparties can use a variance swap to speculate the spread between future realised (floating) and implied (fixed) volatility, or to hedge the volatility exposure of other positions or businesses. According to Curnutt (2000), some of the possible strategies using variance swaps are:

i) Speculating a directional view that implied volatility is too high or too low relative to anticipated realised volatility because 1) volatility follows a mean-reverting process. In this model, high volatility decreases and low volatility increases; 2) there is a negative correlation between volatility and stock or index level. The volatility stays high after large downward moves in the market; 3) volatility increases with the risk and uncertainty;

ii) Implementing a view that the implied volatility in one equity index is mispriced relative to the implied volatility in another equity index;

iii) Trading volatility on a forward basis by purchasing a variance swap of one expiration and a variance swap of another expiration.

Institutional users such as hedge funds are attracted to own variance swap, especially when their portfolios are naturally short vega, as an alternative to using options to take on or hedge volatility exposure. By far, the model developed by Demeterfi et al. (1999) is the most popular tool to price variance swaps but, surprisingly, no research has ever considered using market data to test for its usefulness. This framework stipulates that pricing a variance swap can be viewed as an exercise in computing the weighted average of the implied volatility of the options required even under the influence of volatility skew. Therefore information embedded in option prices is used directly without having to be filtered through the underlying time-series. As long as the movement of the underlying asset is continuous, the pricing and hedging of variance contracts is completely independent of the choice of the volatility process.

1.3 The Problem Statement

This dissertation investigates the performance of different specifications of time-series and options-based volatility forecasting models under the influence of the observed market biases. In order to present the results in a meaningful and manageable manner, three self-contained but interrelated projects are included in this dissertation. In this section we will state the objectives for each of the three projects separately. We end the section by noting the delimitations of the study.

1.3.1 Objectives of the First Research Project

Chapter 3, entitled *"A report on the Properties of the Term-Structure of S&P 500 Implied Volatility"*, is a descriptive study. It examines the empirical behaviour of S&P 500 futures option's implied volatility using daily data from 1983 through 1998. We consider this research work one of the most extensive empirical studies of S&P 500 implied volatility term-structure in literature to date. The primary objectives are:

i) To observe, characterise and analyse the patterns of the term-structure of implied volatility in the S&P 500 marketplace;

ii) To investigate whether option prices are in line with the rational expectations hypothesis under a mean-reverting volatility assumption;

iii) To identify what types of option models would be consistent with the observed moneyness biases in the S&P 500 options market.

Intermediate results obtained in Chapter 3 can also help facilitate our research efforts in modelling volatility in Chapters 4 and 5.

1.3.2 Objectives of the Second Research Project

Chapter 4, entitled *"An Empirical Comparison of APARCH Models"*, investigates the performance of APARCH models (Ding et al., 1993) that can potentially account for the slow decay in returns autocorrelations using daily S&P 500 futures series from 1983 through 1998. The use of the APARCH framework is convenient to evaluate different model specifications because log-likelihood-based statistics can be used to directly test for the robustness of many nested models[1]. Our primary objectives are:

i) To check whether the unrestricted APARCH model is a good description of the process driving volatility by investigating the significance of asymmetric parameterisation and power transformation within the context of APARCH specifications using log-likelihood ratio tests;

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[1] See appendix A.1 for these nested models.

ii) To provide evidence that the in-sample performance of asymmetrical and symmetrical conditional volatility models are prone to the state of volatility by using a 3-state regime switching volatility conditional model to separate high and low volatility states;

iii) To compare the in-sample performance of EGARCH (Nelson, 1991) with APARCH models based on aggregate AIC statistics;

iv) To illustrate the quality of different conditional volatility forecasts by predicting the one-step ahead changes of implied volatility and conducting ex-ante (out-of-sample) S&P 500 straddle trading exercises.

1.3.3 Objectives of the Third Research Project

The title of Chapter 5 is "*Empirical Performance of Alternative Variance Swap Valuation Models*". The model developed by Demeterfi et al. (1999) is the most popular tool to price variance swaps, but surprisingly, no research has ever considered using market data to test for its usefulness in forecasting volatility. The pricing of variance swap can be viewed as the market consensus of expected future variance. Chapter 5 examines different specifications of time-series and options-based variance swap models' volatility forecasting performance on the S&P 500 index from June 2001 to November 2001. After the terrorist attacks on September 11, 2001, the longer-termed forward variance has become more volatile than the shorter-termed forward variance. Based on six well-selected contract days, we design the three-, six- and nine-month variance swap contracts for each contract day and analyse them by evaluating different specifications of implied and time-series models at different points in time. Our primary goals are:

i) To present a complete picture of how each generalisation of the benchmark Black-Scholes model can really improve the volatility forecasting performance of variance swaps and whether each generalisation is consistent between in- and out-of-sample results;

ii) To explore whether there is any systematic difference in volatility forecasting performance between time-series and options-based variance swap valuation models. It is intended to investigate whether options-based models, which are forward-looking, are capable of outperforming discrete-time processes, which use only historical information, in predicting future variance.

1.3.4 Delimitations of the Study

Volatility models and their forecasts are of interest to many types of economic agents, e.g. options traders require asset volatility to price options whilst portfolio managers need volatility forecasts to access risks of their portfolio. Having the ability to estimate volatility more accurately than others means that one could have more success from trading activities. Given the changing nature of volatility term-structure in the marketplace, it is important for us to focus on a single market and gain a thorough understanding of its behaviour. If the term-structure of implied volatility shows any specific pattern then some models, such as stochastic volatility models or autoregressive heteroskedasticity models, may be used to account for these imperfections in the market. In this dissertation, we have opted for the use of S&P500 market data. The S&P 500 index is capitalisation-weighted, representing the market value of all outstanding common shares of the 500 large-capitalisation firms listed in the U.S.A. This is of importance to investors because S&P 500 products are one of the most liquid contracts in the financial world. Liquidity is the ability of a market to efficiently absorb the execution of large purchases and sales. It is a key component to attracting investors and ensuring a market's success. In fact, the S&P 500 index has long been the benchmark by which professionals measure portfolio performance and its immense size guarantees that S&P 500 products are ideal as a hedging tool.

1.4 The Significance of the Study

We will explain the research significance for each of the self-contained projects individually.

1.4.1 Significance of the First Research Project

Chapter 3 studies the observed market anomalies in the S&P 500 futures options market. The term-structure of implied volatility reflects the time-varying market expectations of asset volatility over different time horizons. Despite the extensive investigation and the evidence accumulated thus far on the term-structure of implied volatility, no past study has ever considered a large empirical study of the S&P 500 implied volatility term-structure. Prior to this research, past papers have always examined the term-structure of implied volatility only for particular at-the-money contracts. The purpose of Chapter 3 is to fill this gap in the literature by utilising all available daily S&P 500 futures option prices from the inception of S&P 500 futures option in March 1983 to December 1998. Although descriptive in nature, we extend previous term-structure work in several ways:

i) The new aspect of this research is that we define relative implied volatility as implied volatility normalised by its corresponding at-the-money implied volatility for each maturity group. The use of relative implied volatility allows the measurement of relative degrees of anomaly in the implied volatility term-structure across a broad moneyness range;

ii) Our sample period is more extensive, making the results more statistically reliable.

Our research is of importance to institutional investors because S&P 500 products are one of the most liquid contracts in the financial world and their immense size guarantees that they are ideal as a hedging tool. If the term-structure of implied volatility shows any specific patterns then some models, such as stochastic volatility models or GARCH-type models, may be more suitable to make adjustments for market imperfections that cannot be explained by the Black-Scholes formula. These adjustments could be important even for small levels of predictability, especially for longer maturity options.

1.4.2 Significance of the Second Research Project

In Chapter 4 we compare the performance of the asymmetric power ARCH (Ding et al., 1993) models with the EGARCH (Nelson, 1991) model. The existing literature favours some rather complex volatility specifications but usually their empirical performance is little explored. Since the development of long-memory models in the early 1990's, there has been little research about the significance of their specifications. In Chapter 4 we investigate the importance of power transformation and asymmetric parameterisation within the context of APARCH specifications. The consequence of this research is not only significant to discrete-time finance but also potentially meaningful for continuous-time stochastic volatility literature. Whilst the research on discrete and continuous-time models has evolved independently, many continuous-time models can be thought of as the limits of GARCH-type processes. For example, Nelson (1991) showed that EGARCH(1,1) converged to the Wiggins model (1987) in continuous time limit. Moreover, Duan (1997) also proved that most of the existing bivariate diffusion models that had been used to model asset returns volatility could be represented as limits of a family of GARCH models. If it can be shown that there is not much to gain from moving beyond a more parsimonious discrete specification such as EGARCH or GARCH, there seems to be little incentive to look beyond a simple bivariate stochastic model which allows for volatility clustering and a leverage effect such as the Hull-White model (1987) or the Heston model (1993).

1.4.3 Significance of the Third Research Project

Chapter 5 inspects the pricing performance of options-based and time-series variance swap valuation models on the S&P 500 index. Variance swap is an exciting new product that immunises traders' exposure into the ups and downs of volatility. It is getting more popular because it is one of the very few financial products to allow traders speculate on future volatility levels. The Demeterfi et al. (1999) variance swap pricing methodology has been widely accepted by practitioners but little tested and scrutinised. Regrettably, no empirical studies have ever used any market data to investigate the pricing performance of variance swap valuation models. This research presents the first of any known attempts to use market data to evaluate the effectiveness of the Demeterfi et al. framework. In particular, it represents the first study on variance swaps under alternative time-series and competing option pricing models. It is also not known whether and by how much each option model will improve variance swap pricing. Since implied volatility can be regarded as the market's expectation of future realised volatility, the implication of any poor variance forecasting by options-based models is such that practitioners and academicians alike may need to look for a way to integrate historical and market information in a composite option pricing model.

1.5 Organisation of the Dissertation

The structure of this dissertation is as follows. Chapter 1 is the introduction. Chapter 2 reviews the literature. The rest of this dissertation is divided into three self-contained but interrelated projects and each project is accompanied by an abstract. Chapter 3 characterises the term-structure of S&P500 implied volatility and examines empirical issues relating to rational and distributional hypotheses in the S&P 500 futures options market. Chapter 4 compares the in-sample performance of the APARCH with EGARCH models in different volatility regimes using sixteen years of daily S&P 500 futures series. It also assesses the quality of different statistical criteria and conducts a preference-free approach to select the best out-of-sample model. Chapter 5 adopts a financial engineering approach to evaluate the performance of different time-series and options-based variance swap valuation models on the S&P500 index under the influence of term-structure biases found in Chapters 3 and 4. Chapter 6 summarises and discusses the results and suggests directions for future research.

CHAPTER 2 Review of the Literature

This chapter will review the literature on issues related to option pricing, as a means of providing an intellectual background for the present dissertation. It will examine both the theoretical and empirical studies in these areas, giving special emphasis in volatility. The chapter organises the review by examining the six aspects of finance literature: option pricing theories, conditional and stochastic volatility models, implied methodology, market anomalies and diagnostic tests.

2.1 Option Pricing Theories

2.1.1 No-Arbitrage Approach

The Black-Scholes option pricing formula (Black & Scholes, 1973) relates the price of an option to the underlying asset price, the volatility of the return of the underlying asset, dividend yield, interest rate and strike price. The main assumptions that Black-Scholes proposed were the following:

i) Markets are frictionless, efficient and complete;

ii) Constant interest rate[2] and volatility;

iii) Portfolio rebalanced continuously;

iv) No-arbitrage and trades are self-financing;

v) The underlying asset S follows geometric Brownian motion[3] (GBM).

The underlyinng asset dynamics is given by:

$$dS = \mu S dt + \sigma S dW$$

[2] The original Black-Scholes paper assumes a constant interest rate. But this assumption can be relaxed and no-arbitrage can still be applied as long as interest rate is deterministic.

[3] Unlike arithmetic Brownian, geometric Brownian motion does not allow the underlying asset to become negative, a property that is consistent to limited liability of stock ownership (Samuelson 1965).

where the percentage change from t to $t+dt$ is normally distributed with mean μdt and variance $\sigma^2 dt$; W is the Wiener process, and μ and σ are the instantaneous return drift and volatility, respectively.

2.1.1.1 Black-Scholes Formula

According to the Black-Scholes assumptions, one can apply Ito's lemma[4] to show that it is possible to create a synthetic hedged portfolio $\upsilon(S,t)$ that consists of a long position in stock and a short position in option. If rebalanced continuously, this hedged position can be achieved independent of stock price movements and its instantaneous return drift. The discrete-time version of the diffusion model is given by:

$$\Delta S = \mu S \Delta t + \sigma S \Delta W \qquad (1)$$

The above discrete-time relationship involves a small approximation. It assumes that the drift and variance rate of S remain constant in a very short discrete time period. In addition, the change in the value of the option is governed by the stochastic differential equation (SDE) that satisfies the Ito's lemma:

$$\Delta C = \left[C_S \mu S + C_t + \frac{1}{2} C_{SS} \sigma^2 S^2 \right] \Delta t + C_S \sigma S \Delta W \qquad (2)$$

where C_t is the first partial derivative of option price with respect to time, and C_S and C_{SS} the first and second partial derivative of option price with respect to stock price, respectively. In a hedged portfolio of C_S shares long and one call option short, the change of value of this portfolio in a small discrete time period is:

$$\Delta \upsilon(S,t) = C_S \Delta S - \Delta C \qquad (3)$$

Substituting (1) and (2) into (3), one gets:

$$\Delta \upsilon(S,t) = -C_t \Delta t - \frac{1}{2} C_{SS} \sigma^2 S^2 \Delta t \qquad (4)$$

[4] See Hull (2000) pp.235-236 for the derivation of Ito's Lemma.

Since the increments of the portfolio are dependent on the same source of underlying uncertainty, it is possible to form a risk-free portfolio in discrete time. Under no-arbitrage[5] condition, the return earned on it in a short discrete period must equal risk free rate r so that:

$$\Delta \upsilon(S,t) = r\upsilon(S,t)\Delta t \qquad (5)$$

Substituting (5) into (4) one can write the following SDE:

$$rC = rC_S S + C_t + \frac{1}{2}C_{SS}\sigma^2 S^2 \qquad (6)$$

If $\Theta = \frac{\partial C}{\partial t}$, $\Delta = \frac{\partial C}{\partial S}$ and $\Gamma = \frac{\partial^2 C}{\partial S^2}$, equation (6) can be rewritten as:

$$rC = r\Delta S + \Theta + \frac{1}{2}\Gamma\sigma^2 S^2 \qquad (7)$$

The most striking feature of the Black-Scholes derivation is that equations (6) and (7) are independent of instantaneous stock return μ; one only needs to know the risk-free rate in order to backout the option price *C*. By transforming (6) into an equivalent heat transfer problem in physics, it can be solved analytically subject to boundary conditions. The European call formula is expressed as follows[6]:

$$C(S,t) = SN(d_1) - Xe^{-rt}N(d_2)$$

$$d_1 = \frac{\ln(\frac{S}{X}) - (r - d + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

where $N(\cdot)$ is the cumulative distribution function, r the risk-free rate, $N(d_1)$ the hedge parameter, σ the volatility, d the dividend yield, X the strike price, S the stock price and T the maturity.

In the Black-Scholes options model, prices are always a non-decreasing function of the volatility. Furthermore, stock price is distributed log-normally. The distribution of change in ln(*S*) between time 0 and *T* is given by:

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[5] The no-arbitrage theorem simply states that two equivalent assets must not be sold for different prices.

[6] Wilmott (1997) give precise details in solving equation (6).

$$d\ln(S) = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dW$$

$$\ln\left(\frac{S_T}{S_0}\right) \sim N\left[\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right]$$

$$\ln(S_T) \sim N\left[\ln(S_0) + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right]$$

The distributional result implies that the expected continuously compound return for ln(*S*) is $\left(\mu - \frac{\sigma^2}{2}\right)$ per year whereas $\frac{\Delta S}{S}$ is distributed as $N(\mu\Delta t, \sigma\sqrt{\Delta t})$. Under the real probability measure, the expected forward price and its instantaneous value at time *T* are given by:

$$E(S_T) = S_0 e^{\mu T}$$

$$S_T = S_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}\varepsilon_t}, \; \varepsilon_T \sim N(0,1)$$

It is widely noted that option prices are not priced off the real measure but risk-neutral measure. According to Merton (1976), option prices rely on put-call parity to enforce the internal consistency of option pricing. The put-call parity is a no-arbitrage condition which shows that the value of a European call option with a certain exercise price and exercise date can be deduced from the value of a European put option with the same exercise price and date, and vice versa. For a non-dividend paying stock, this relationship is given by:

$$C - P = S - Xe^{-rt}$$

If put-call parity is violated, arbitrage will arise. Note that put-call parity is true regardless whether the asset price distribution is log-normal. It does not, however, hold for American options. The Black-Scholes formula can be rearranged such that a stock option at any instant can be thought of a weighted portfolio of risky stock and riskless zero-coupon bonds:

$$C(S,t) = e^{-rt}[SN(d_1)e^{rt} - XN(d_2)]$$

$N(d_2)$ can be interpreted as the probability that the option will be exercised in a risk-neutral world whereas $XN(d_2)$ is the strike price times the probability that the strike price will be paid. Accordingly, the term $SN(d_1)e^{rt}$ is the expected value of a variable that equals S_T if $S_T > X$ and zero otherwise in a risk-neutral world.

2.1.1.2 Other Variations

There are many variations of the Black-Scholes model – so many that it is cumbersome to provide a comprehensive survey of it. For instance, Merton (1973) derived the Black-Scholes formula independently based on a three-asset riskless hedge model. Merton's model had the advantage such that interest rate was taken to be deterministic. Merton's paper also developed a set of restrictions for the rational pricing of European and American options without making any distributional assumption and gave the solutions to perpetual American call and put options. Other prominent option pricing models include the jump-diffusion model by Merton (1976), the futures option model by Black (1976), the compound option formula by Geske (1979), the American option pricing model by Barone-Adesi and Whaley (1986), the stochastic volatility model by Hull and White (1987) etc. This list is by no means exhaustive. The use of any particular model should be judged on its own merits.

2.1.1.3 Implied Volatility

In the Black-Scholes formula all but the volatility parameter σ is observable. Historical data may be used to estimate σ, however, many other techniques could also be employed to approximate σ (e.g. Brenner & Subrahmanyam, 1988; Bharadia et al., 1995; Corrado & Miller, 1996). On ther other hand, one may observe the market price of the option and invert the Black-Scholes formula to determine σ. This market's assessment of the underlying asset's volatility, which reflects the average volatility over the remaining life of the option, is known as implied volatility. Its calculation is usually accomplished by using Newton-Raphson method, which uses the first derivative of the option price with respect to σ, $\frac{\partial C}{\partial \sigma}$[7], to speed up convergence. According to Figlewski (1989a), the implied volatility of an option will represent the equilibrium between supply and demand. It is generally believed that prices in the market reflect all available information affecting the value of a contract. In principal, implied volatility gives a direct reading of the market's future volatility estimate. If implied

[7] This is referred to vega. Calculating vega from the Black-Scholes model may seem strange because Black-Scholes equation assumes that volatility is constant. It would be theoretically and conceptually more correct to calculate vega from a model where volatility is assumed to be stochastic.

volatility is low compared to volatility forecast, a trader will prefer to buy options, and vice versa.

2.1.2 Martingale Approach

The essence of the Martingale approach is to change the probability measure so as to make the discounted stock a Martingale, therefore making its drift zero. The option price can be expressed as the discounted value of the expected cash-flow under the risk-neutral measure. Furthermore, the Black-Scholes formula can also be obtained from the Martingale approach. The following sub-sections discuss the underlying concept of the probabilistic approach and illustrate how it can be used to solve for option prices[8].

2.1.2.1 Underlying Concepts

The Black-Scholes formula can be derived via the probabilistic approach. Mathematicians have known for a while that to be random is not necessarily to be without some internal structure in non-random ways. The central theme of the probabilistic approach demonstrates that the arbitrage-justified contingent claim is the expectation of the discounted claim under one special measure Q under which the discounted underlying process is also a Martingale. Under the Martingale measure Q, derivatives can be valued with the risk-free rate via no-arbitrage, where the risk-free rate is readily available in the market. Thus the real measure P which the underlying follows is irrelevant. The necessity to have a new measure Q in place for asset valuation can best be illustrated in the following example. Suppose an analyst would like to calculate the price of an asset. One way to do this is to exploit the relation:

$$E_t^P\left[\frac{1}{(1+R_t)}S_{t+1}\right]=S_t$$

by calculating the expectation on the LHS. By doing this requires a knowledge of the distribution of R_t which requires knowing the risk premium μ where risk-free rate $r_t = R_t - \mu$. Yet it is usually difficult to obtain the risk premium before knowing the asset

[8] The materials used in this section for the probabilistic approach are extracted from Baxter and Rennie (1996) and Neftci (1996).

price. On the other hand, it might be easier to transform the mean of R_t without having to use the risk premium. If one can find a new probability measure Q without having to use the risk premium such that:

$$E_t^Q\left[\frac{1}{(1+r_t)}S_{t+1}\right]=S_t$$

where it can be very useful for calculating the asset price.

The above illustration implies that there is a separation of process and measure and only the size and interrelation of its underlying movements affects the prices of derivatives, but the probabilities of achieving them does not. For example, the forward contract on stock maturing at time T may not be enforceable[9], but the fair price of the forward contract is $S_t\exp(rT)$, which does not depend on the expected value of the stock under its real measure.

2.1.2.2 Discrete-Time Process

The use of probabilistic approach includes the concepts of Martingale, filtration F_i, stock and bond processes. A stock process S is a Martingale with respect to an arbitrary measure P and a filtration F_i if:

$$E_P(S_j \mid F_i)=S_i$$

for all $i\leq j$. That means the process S has no drift under P, no bias up or down in its value under the expectation operator E_P.

A filtration F_i is the history of the stock up until tick-time i; filtration fixes a history of choices or paths. The conditional expectation operator $E_Q(\cdot\mid F_i)$ extends the idea of expectation to two parameters – a measure Q under a history F_i. The measure Q tells us which "probabilities" to use in determining path-probability and thus the expectation whilst the filtration serves to take expectations from later starting point rather than along the whole of a path from time zero. Coupled with the use of binomial representation theorem, a no-

[9] The expected stock price from the Kolmogorov's strong law is: $S_0\exp(\mu+0.5\sigma^2)$

arbitrage, self-financing hedging strategy can be constructed to price contingent claim in a binomial environment. Given a binomial tree model with a stock *S* and bond *B*, then (ϕ_i, φ_i) is a self-financing strategy to construct a contingent claim *X* if:

i) Both (ϕ_i, φ_i) are known by time *i*-1;

ii) The change in value *V* of the portfolio defined by the strategy obeys the difference equation: $\Delta V_i = \phi_i \Delta S_i + \varphi_i \Delta B_i$ where ΔS_i and ΔB_i are the changes in *S* and *B* from time $i-1$ to i, respectively;

iii) $\phi_T S_T + \varphi_T B_T \equiv X$, the final claim.

Binomial representation theorem assures that $\phi_{i+1} S_i + \varphi_{i+1} B_i = \phi_i S_i + \varphi_i B_i$. Thus at any time *i*, the value of a claim *X* maturing at date *T* is $B_i E_Q(B_T^{-1} X \mid F_i)$. It is also noted that both $B_T^{-1} X$ and $B_i^{-1} S_i$ are *Q*-Martingales.

2.1.2.3 Continuous-Time Process

The discrete models are only a rough approximation to the way that prices actually move. The binary choice of a single jump "up" or "down" only becomes more important as the ticks get closer and closer. In a continuous process, values can be expressed in arbitrarily fine fractions and they cannot make instantaneous jumps. Two special tools are used for manipulating stochastic processes:

i) If $dX_t = \mu_t dt + \sigma_t dW_t$ and $f(X_t)$ is twice differentiable then $d(f(X_t) = (\sigma_t f'(X_t))dW_t + (\mu_t f'(X_t) + \frac{1}{2}\sigma_t^2 f''(X_t))dt$

ii) If $dX_t = \mu_t dt + \sigma_t dW_t$ and $dY_t = \upsilon_t dt + \rho_t dW_t$ then $d(X_t Y_t) = X_t dY_t + Y_t dX_t + \sigma_t \rho_t dt$

i) is referred Ito's formula. Its most immediate use is to generate SDE's from a functional expression for a process. ii) is the product rule. It is noted that the final term on the LHS in ii) is actually $dX_t dY_t$ (following from $(dW_t)^2 = dt$), marking the difference between Newtonian and stochastic calculus.

The above equations are a manipulation of differentials of Brownian motion, not a manipulation of measure. W_t is not strictly a Brownian motion in its own right, but a

Brownian motion with respect to some measure P, a P-Brownian motion. One important tool for manipulation of measure is the Radon-Nikodym derivative. The Radon-Nikodym derivative of Q with respect to P, i.e. $\frac{dQ}{dP}$, exists only if two measures P and Q are equivalent and operate on the same sample space and agree on what is possible, i.e. $P(A) > 0 \Leftrightarrow Q(A) > 0$, where A is any event in the sample space. One can only uniquely define $\frac{dQ}{dP}$ and $\frac{dP}{dQ}$ if P and Q are equivalent and Q could be extracted from P and $\frac{dQ}{dP}$, and vice versa. For example, $E_P(X) = E_Q\left(\frac{dQ}{dP}X\right)$ if $\frac{dQ}{dP}$ exists. To price a contingent claim X up to time t given F_s where $t > s$, the procedures are as follows:

i) $\varsigma_t = E_P\left(\frac{dQ}{dP} \mid F_t\right)$

ii) $E_Q(X_T) = E_P\left(\frac{dQ}{dP}X_T\right)$, for all claims knowable by time T.

iii) $E_Q(X_t \mid F_s) = \varsigma_s^{-1}E_P(\varsigma_t X_t \mid F_s), s \le t \le T$

The last condition implies that $E_Q(X_t) = E_P(\varsigma_t X_t)$. i) gives an idea of the amount of change of measure $\frac{dQ}{dP}$ so far up to time t along the current path whereas $\varsigma_t\varsigma_s^{-1}$ in iii) represents the amount of change of measure from time s to time t. All the measure changes on Brownian motion can do is to change the drift.

If one considers W_t a P-Brownian motion and defines $\frac{dQ}{dP} = \exp\left(-\gamma W_T - \frac{1}{2}\gamma^2 T\right)$ for some time horizon T, under the results of the moment generating function:

$$E_Q(\exp(\theta W_T)) = \exp\left(-\theta\gamma T + \frac{1}{2}\theta^2 T\right) \sim N(-\gamma T, T)$$

Therefore the marginal distribution of the Brownian motion under Q is also normal with variance T and mean $-\gamma T$. Furthermore, if $\tilde{W}_t = W_t + \gamma t$ then:

i) $E_Q(\exp(\theta \tilde{W}_t)) = \exp\left(\frac{1}{2}\theta^2 t\right), \tilde{W}_t \sim N(0,t)$ under Q

ii) $E_Q(\exp(\theta(\tilde{W}_{t+s}-\tilde{W}_t))\,|\,F_t)=\exp\left(\frac{1}{2}\theta^2 t\right)$, $(\tilde{W}_{t+s}-\tilde{W}_t)\sim N(0,t)$

In order to make a process Martingale, the drift of a non-Martingale process needs to be changed. The Cameron-Martin-Girsanov theorem changes the drift of a process. The Cameron-Martin-Girsanov (CMG) theorem states that if W_t is a P-Brownian motion and γ_t is a F-previsible process satisfying the boundary condition $E_P(\exp(\frac{1}{2}\int_0^T \gamma_t^2 dt))<\infty$, then there exists a measure Q such that:

i) Q is equivalent to P, i.e. $P_P(dz)>0$ *if and only if* $P_Q(dz)>0$ for some interval dz

ii) $\frac{dQ}{dP}=\exp\left(-\int_0^T \gamma_t dW_t-\frac{1}{2}\int_0^T \gamma_t^2 dt\right)$

iii) $\tilde{W}_t=W_t+\int_0^t \gamma_s dt$

Condition iii) implies that W_t is a drifting Q-Brownian motion with drift $-\gamma_t$ at time t. If X_t is defined to be the exponential Brownian motion with SDE:

$$dX_t = X_t(\sigma dW_t + \mu dt)$$

where W_t is a P-Brownian motion. Applying the CMG with $\gamma_t=\left(\frac{\mu-\nu}{\sigma}\right)$ that satisfies the boundary condition $E_P(\exp(\frac{1}{2}\int_0^T \gamma_t^2 dt))<\infty$, there exists a new measure Q such that $\tilde{W}_t=W_t+t\left(\frac{\mu-\nu}{\sigma}\right)$ is a Q-Brownian motion. This means that the differential of X under Q is:

$$dX_t = X_t(\sigma d\tilde{W}_t + \nu dt)$$

which gives X the drift ν (usually it is the risk-free rate) but the volatility process remains the same.

2.1.2.4 Self-Replicating Strategy

In continuous-time finance, a stochastic process M_t is a Martingale with respect to a measure P if $E_P(M_t | F_s) = M_s, s \leq t$. The Martingale representation theorem states that if M_t is a Q-Martingale with volatility σ_t and if N_t is also a Q-Martingale, then there exists an F-previsible process ϕ such that $\int_0^T \phi_t^2 \sigma_t^2 dt < \infty$ and the process N_t can be written as:

$$N_t = N_0 + \int_0^t \phi_s dM_s$$

The above equation implies that $dN_t = \phi_t dM_t$, which is driftless; hence $E(\int_0^t \phi_t dM_t) = 0$ is unpredictable. For example, if $dN_t = \sigma_t N_t dM_t$ for some F-previsible process σ_t, then $E\left(\exp(\frac{1}{2}\int_0^T \sigma_s^2 ds) \right) < \infty \Leftrightarrow N_t$ is a Martingale and the solution to this SDE is:

$$N_t = N_0 \exp\left(\int_0^t \sigma_s dM_s - \frac{1}{2}\int_0^t \sigma_s^2 ds \right)$$

With the help of mathematical tools – Ito, CMG, and the Martingale representation theorem, one needs a self-financing property to replicate a contingent claim X which consists of a stock and a riskless cash account bond. Suppose there are a riskless bond B and a risky security S with volatility σ_t, and a claim X on events up to time T. A replicating strategy for X is a self-financing portfolio $(\phi, \varphi) \Leftrightarrow dV_t = \phi_t dS_t + \varphi_t dB_t$ such that

$$\int_0^T \sigma_t^2 \phi_t^2 dt < \infty, V_T = \phi_T S_T + \varphi_T B_T = X$$

This replicating strategy enforces the law of one price and prevents arbitrage from arising in the market. Because (ϕ, φ) is self-financing and the portfolio is worth X at time T guaranteed, the bought derivative and sold portfolio would safely cancel at time T, and no extra money is required between times t and T, i.e. $\phi_t S_t + \varphi_t B_t = \phi_{t+1} S_{t+1} + \varphi_{t+1} B_{t+1}$. Under the assumption of the Black-Scholes model for a continuously tradable stock and bond, the no-arbitrage price for a final claim X at time T is given by:

$$V_t = B_t E_Q(B_T^{-1} X | F_t) = e^{-r(T-t)} E_Q(X | F_t)$$

where Q is the Martingale measure for the discounted stock $B_t^{-1}S_t$ and r the risk-free interest rate. The following steps summarise the procedures to construct a solution for the Black-Scholes model:

i) $B_t = \exp(rt)$, $S_t = \exp(\mu dt + \sigma dW)$; r, μ, σ are the riskless interest rate, stock drift and stock volatility, respectively. If the stock follows geometric Brownian motion, $S_t = \exp(\mu dt + \sigma dW_t)$, then $Y_t = \log(S_t)$ follows a simple drifting Brownian motion $Y_t = \mu dt + \sigma dW_t$. By applying Ito's lemma one can write down the SDE for $S_t = \exp(Y_t)$ as $dS_t = \sigma S_t dW_t + (\mu - r + \frac{1}{2}\sigma^2)S_t dt$

ii) Invoke CMG and find a Q measure such that the discounted stock process $Z_t = B_t^{-1}S_t$ is a Q-Martinagle. The SDE's are:

$dZ_t = Z_t(\sigma dW_t + (\mu - r + \frac{1}{2}\sigma^2)dt) \Leftrightarrow dZ_t = \sigma Z_t d\tilde{W}$. The drift is $(\mu - r + \frac{1}{2}\sigma^2)/\sigma$ under W.

iii) Convert the discounted claim from a random variable into a process by taking the conditional expectation under Q, i.e. $E_t = E_Q(B_T^{-1}X \mid F_t)$.

iv) If both E_t, Z_t are Q-Martingales, the Martingale representation theorem states that:

$E_t = E_0 + \int_0^t \phi_t dZ_t \Leftrightarrow dE_t = \phi d_t dZ_t$ where ϕ_t is previsible. In addition, a self-financing portfolio can be formed if one holds ϕ_t units of the stock and $\varphi_t = E_t - \phi_t Z_t$ units of the bond at time t such that $dV_t = \varphi_t dS_t + \kappa_t dB_t$ is equivalent to $dE_t = \phi_t dZ_t$.

v) The no-arbitrage price of the claim X at time t is given by: $V_t = B_t E_Q(B_T^{-1}X \mid F_t) = e^{-r(T-t)}E_Q(X \mid F_t) = B_t E_t$

Because of risk premium, $E^P\left(e^{-rt}S_t \mid S_u, u < t\right) > e^{-ru}S_u$. It is also noted that the measure Q is not the measure which makes the stock a Martingale, but the measure that makes the discounted stock a Martingale, and the arbitrage price of the claim is the expectation under Q of the discounted claim. This means that under the new measure the drift of the stochastic differential $d(e^{-rt}X)$ is zero. Therefore, the price for a European call option at time t which expires at time T with strike k is $C(S_t, t) = e^{-r(T-t)}E_Q(\max(S_T - k))$, and the solution for the Black-Scholes model under the Martingale framework is:

$$C(S_t,t) = S_t N\left(\frac{\log(\frac{S_t}{k}) + (r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right) - ke^{-r(T-t)}N\left(\frac{\log(\frac{S_t}{k}) + (r-\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)$$

2.1.2.5 Kolomogorov Equation

Next suppose that $V_t = V(S_t,t)$. Since $dS_t = \sigma S_t d\tilde{W}_t + rS_t dt$, then Ito's lemma gives:

$$dV_t = d(V(S_t,t)) = \left(\sigma S_t \frac{\partial V}{\partial S}\right)d\tilde{W} + \left(rS_t \frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t}\right)dt$$

Substituting $dV_t = \phi_t dS_t + \varphi_t dB_t, dB_t = rB_t dt$ into the above equation:

$$dV_t = (\sigma S_t \phi_t)d\tilde{W}_t + (rS_t\phi_t + r\varphi_t B_t)dt$$

Matching the volatility and drift terms in the above equations, the hedge parameter is found to be:

$$\phi_t = \frac{\partial V}{\partial S}$$

where the partial differential equation for V is:

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV + \frac{\partial V}{\partial t} = 0$$

This PDE, coupled with the boundary conditions that $V(S,T)$ must satisfy, gives another way of solving the Black-Scholes option pricing model. In addition, the above PDE can be derived from the Kolomogorov backward equation. The backward equation describes the way in which the conditional probability distribution of the stock price, $F(S_T,T \mid S_t,t)$, is altered as a function of time, t. The backward equation for the diffusion is given by:

$$\frac{1}{2}\sigma^2 S^2 F_{SS} + \mu S F_S + F_t = 0$$

where $F(S_T,T \mid S,t)$ must satisfy the above equation for all (S_T,T). In a risk-neutral world, $\mu = r$, the risk-free rate. If $V(S,T)$ can be assumed to be a function of only S, i.e. $h(S)$, such that $V(S,T) = e^{-r(T-t)}E(h(S_T) \mid S_t)$, the Black-Scholes PDE can again be derived by substituting $V(S,T)$ into the Kolomogorov backward equation. The significance of this approach is that one can solve the option valuation problem only for those cases where the conditional probability distribution of the terminal stock value is known (Cox et al., 1976).

2.1.2.6 Market Price of Risk

The sole purpose of the above discussions is to make the discounted process a Martingale. A process is tradable if its discounted price is a Martingale under the Martingale measure Q. Assume there are two tradable risky securities S_t^1, S_t^2 such that:

$$dS_t^i = S_t^i(\sigma_i dW_t + \mu_i dt),\ i = 1,2$$

If they are tradable, the discounted prices of S_t^1, S_t^2 need to be Martingales under the same measure Q. If $B_t = \exp(rt)$, then $\tilde{W}_t = W_t + \left(\frac{\mu_i - r}{\sigma_i}\right)t$ for $i = 1,2$ must be a Q-Brownian motion. This is true only if $\frac{\mu_1 - r}{\sigma_1} = \frac{\mu_2 - r}{\sigma_2} = \gamma$. γ is called the market price of risk and can be interpreted as the extra return over riskless rate per unit of risk. The intuition is that for two processes to be tradable in the same market, they must have the same market price of risk. The market price of risk is also the drift change of the underlying P-Brownian motion given by CMG. This resulting Q-measure makes one possible to convert asset prices discounted by the risk-free rate from sub-Martingales into Martingales. All expected returns equal the riskless rate r under Q and all claims become equal to their expected payoffs under Q discounted by the risk-free rate. Choosing a particular market price of risk is also referred to as defining the probability measure.

2.1.2.7 Summary of the Martingale Approach

We have just illustrated that the Martingale approach implies the same PDE's utilised by the PDE methodology. The difference is that in the Martingale approach, the PDE is a consequence of risk-neutral asset price, whereas in the PDE method, one begins with the PDE's to obtain risk-free prices. The Black-Scholes formula can be obtained from either approach.

2.2 Conditional Heteroskedastic Models

2.2.1 Underlying Concepts

We will briefly review the difference between conditional and unconditional moments before discussing the conditional heteroskedasticity models.

2.2.1.1 Random Walk

Suppose r_t follow a random walk $r_t = r_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0,\sigma^2)$. This process can be rewritten as:

$$r_t = r_0 + \sum_{i=1}^{t} \varepsilon_i$$

Taking the first and second moment of the above equation, the unconditional mean and variance are given by:

$$E(r_t) = r_0$$
$$V(r_t) = t\sigma^2$$

Consequently, a random walk has a constant unconditional mean but a time varying unconditional variance. Its conditional mean and variance are:

$$E(r_t \mid r_{t-1}) = r_{t-1}$$
$$V(r_t \mid r_{t-1}) = E(r_t - E(r_t \mid r_{t-1}))^2 = E(r_{t-1} + \varepsilon_t - E(r_t \mid r_{t-1}))^2 = \sigma^2$$

Whilst the unconditional variance of a random walk model tends to infinite at time increases, the conditional variance is constant.

2.2.1.2 Skewness and Kurtosis

Suppose the returns $r_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$. The first four moments of returns are given by:

$$\kappa_1 = E(r)$$
$$\kappa_2 = E(r - \kappa_1)^2$$
$$\kappa_3 = E(r - \kappa_1)^2$$
$$\kappa_4 = E(r - \kappa_1)^4 - 3\kappa_2^2$$

Statistically, skewness and kurtosis are defined as:

$$\gamma_1 = \frac{\kappa_3}{(\kappa_2)^{3/2}} \quad \textit{Skewness}$$

$$\gamma_2 = \frac{\kappa_4}{(\kappa_2)^2} \quad \textit{Kurtosis}$$

Skewness is the "shape" of a probability distribution". Skewness in returns of financial assets can arise from many sources. In particular, It can be induced through asymmetric risk preferences in investors. A negative skewness in returns can be viewed as the phenomenon where, after returns have been standardised by subtracting the mean, a negative returns of a given magnitude have a higher probability than a positive returns of the same magnitude, and vice versa. On the other hand, kurtosis describes the "tallness" of a probability distribution. Probability density functions with values of kurtosis less than 3 are called platkurtic (short-tailed), and those with values greater than 3 are called leptokurtic (long-tailed). If kurtosis equal to 3, then it is mesokurtic. Normal-distribution has kurtosis and skewness equal to 3 and 0, respectively.

2.2.1.3 Unconditional and Conditional Variances

Traditionally volatility is estimated using historical time-series. An unbiased estimate of the variance rate per day using the most recent *m* observations is given by:

$$\sigma_n^2 = \frac{1}{m-1}\sum_{i=1}^{m} \varepsilon_{n-i}{}^2$$

where $\varepsilon_{n-i} = (r_{n-i} - \bar{r})$, $\bar{r} = \frac{1}{m}\sum_{i=1}^{m} r_{n-i}$.

The problem associated with the above unconditional variance estimate is that it gives equal weight to all $(r_{n-i} - \bar{r})^2$. Given the objective is to monitor the current level of volatility, it is not inappropriate to give more weight to more recent data, thus giving rise to the exponentially weighted moving average (EWMA) model. The EWMA model is the most basic type of conditional heteroskedasticity model. It is given by:

$$h_t^2 = \lambda h_{t-1}^2 + (1-\lambda)\varepsilon_{t-1}^2$$

This volatility forecast depends on the most recent estimate on volatility as well as observations on changes in the variable ε_{t-1}. The conditional volatility forecast can be rewritten as:

$$h_t^2 = (1-\lambda)\sum_{i=1}^{m}\lambda^{i-1}\varepsilon_{t-i}^2 + \lambda^m h_0^2$$

For a large m, the second term on the RHS of the equation can be ignored. The weights for the residue square decline at rate λ. The EWMA approach is designed to track changes in volatility. Investment bank J.P. Morgan uses the EWMA model with $\lambda = 0.94$ for updating daily volatility estimates in its RiskMetrics database[10].

2.2.2 Autoregressive Conditional Heteroskedasticity Models

2.2.2.1 ARCH Model

Time-varying variance models can explain nonlinear dependence and leptokurtosis. A very substantial econometric literature has focused on discrete time autoregressive conditional heteroskedasticity (ARCH) models, following the path-breaking paper by Engle (1982). The univariate ARCH models consist of two equations: 1) the conditional mean equation describes the observed data as a function of other variables plus an error term; 2) the conditional variance equation specifies the evolution of the conditional variance of the error from the conditional mean equation. An ARCH(p) process with normal-distribution is modelled as follows:

$$r_t = g(x_{t-1};a) + \varepsilon_t$$
$$\varepsilon_t = h_t e_t \qquad e_t \sim N(0,1) \qquad \varepsilon_t \mid I_{t-1} \sim N(0,h_t^2)$$
$$h_t^2 = \alpha_0 + \sum\nolimits_{i=1}^{p}\alpha_i\varepsilon_{t-i}^2$$
$$\alpha_0 > 0, \alpha_i \geq 0$$
$$\sum\nolimits_{i=1}^{p}\alpha_i \leq 1, \qquad E(\varepsilon_t \mid I_{t-1}) = E(\varepsilon_t) = 0,\ h_t^2 = E(\varepsilon_t^2 \mid I_{t-1}^2) = V(r_t \mid I_{t-1})$$

where a is the parameter vector, h_t the conditional volatility, r_t the log return of an asset and $g(x_{t-1};a)$ the function that constitutes the conditional mean; x_{t-1} could be exogenous.

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[10] See Hull pp.372 for details.

According to Bollerslev et al. (1992), failure to model the fat-tailed property can lead to invalid estimates of standard errors. One important feature of ARCH model is its ability to model excess kurtosis. The kurtosis for ARCH(1) is:

$$\gamma_2 = 3\frac{(1-\alpha_1^2)}{(1-3\alpha_1^2)}$$

which is greater than 3, the kurtosis coefficient of the normal distribution. Therefore, the ARCH(1) process has tails heavier than the normal distribution. A popular alternative to the use of normal distribution for modelling shocks/residuals in the context of ARCH is t-distribution. The t-distribution is specified as follows:

$$\varepsilon_t = h_t (v - 2/v)^{1/2} \omega_t$$
$$E(\omega_t) = 0$$
$$V(\omega_t) = (v-2)/v$$

where $\omega_t \sim i.i.d.\ student$ with v degrees of freedom.

The t-distribution is a mixture of normal distributions having different variances. It is useful for modelling financial series because normal-distribution may not be adequate to fully account for the excess kurtosis (fat-tailed) of financial data. As the degree of freedom v goes to infinity, it includes the normal distribution as a limiting case. One variation of ARCH models is the ARCH-in–Mean (ARCH-M) model. In the ARCH-M model introduced by Engle, Lilien and Robins (1987), the conditional mean is an explicit function of the conditional volatility:

$$r_t = g(x_{t-1}; h_t; a) + \varepsilon_t$$

where a is the parameter vector, x_{t-1} is exogenous, i.e. lag values of r_t. In this model, an increase of the conditional variance will increase/decrease the conditional mean, depending on the sign of the partial derivative of $g(x_{t-1}; h_t; a)$. This model is ideally suited to handling tradeoff between risk and expected return. The most common functional form of $g(x_{t-1}; h_t; a)$ involves linear or logarithmic functions of h_t or h_t^2. Finally, it is noted that an ARCH model becomes a EWMA model when $\alpha_i = (1-\lambda)\lambda^{i-1}$.

2.2.2.1.1 Implications

The usefulness of ARCH modelling is such that volatility is predictable. In addition, they are very simple to implement and able to account for several empirical features like volatility clustering and leptokurtosis in the distribution of returns. ARCH models assume the presence of a serially correlated news arrival process. Consequently, h_t is a random variable that depends upon recent information[11]. As such, ARCH models allow the conditional variance to change over time as the weighted average of innovation/past errors, whilst leaving unconditional variance constant. The order of the lag p determines the length of time for which a shock persists in conditioning the variance of subsequent errors. The larger the value of p the longer the episodes of volatility will tend to be. The use of information of previous period I_{t-1} should produce volatility forecast more accurately than unconditional variance does. Although the shock to the conditional mean ε_t is uncorrelated, it does not imply that they are independent, i.e. $\text{cov}(\varepsilon_t^d, \varepsilon_{t-1}^d) \neq 0$. It is also noted that the shock to variance, $\varepsilon_t^2 - h_t^{\ 2}$, is serially uncorrelated innovation and may be considered as variance surprise. Finally, the 90% and 95% one-step conditional prediction intervals are $\pm 1.64 h_t$ and $\pm 1.96 h_t$, respectively.

2.2.2.1.2 Maximum Likelihood Estimation

Maximum likelihood estimates for ARCH models are generally used to jointly estimate the returns and conditional variance processes. The log-likelihood function is given by:

$$r_t = g(x_{t-1}; a) + \varepsilon_t$$
$$L(\theta) = \sum_{t=1}^{T} [\log f(\varepsilon_t h_t^{\ -1}) - \log(h_t)]$$

where θ is the vector of unknown parameters in the model and the conditional density function. The objective of maximum likelihood estimation is to maximise $L(\theta)$, i.e. the probability of having the observed data under a priori a distributional assumption. The likelihood function typically assumes that the conditional density is Gaussian, so that the

[11] These models assume that second moments are time dependent.

logarithmic likelihood of the sample is simply the sum of the individual normal conditional densities:

$$\log f(\varepsilon_t h_t^{-1}) = -0.5\log(2\pi) - 0.5\,\varepsilon_t^2 / h_t^2$$

2.2.2.2 GARCH Model

Engle (1982) found that a large lag p was required in ARCH to model financial series. This would necessitate estimating a large number of parameters subject to inequality restrictions. As a result, Bollerslev (1986) extended ARCH by allowing the conditional variance of the innovation to depend on lagged innovations and its lagged conditional variance. This process is called generalised ARCH (GARCH). The GARCH(p,q) model is given by:

$$r_t = g(x_{t-1};a) + \varepsilon_t$$
$$\varepsilon_t = h_t e_t \qquad e_t \sim N(0,1) \qquad \varepsilon_t \mid I_{t-1} \sim N(0,h_t)$$
$$h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}^2$$
$$\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0$$
$$\lambda = \sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j$$

where λ is usually between zero and one but may be bigger than or equal to one for higher order models.

2.2.2.2.1 Implications

GARCH models allow for clustering of periods with high and low volatility. A GARCH(p,q) is analogous to an ARMA(p,q) representation. It reverts to a long-run mean and is leptokurtic. Both ARCH and GARCH impose restrictions on coefficients to ensure a positive variance[12]. If q=0, the process reduces to a ARCH(p) process. In addition, the GARCH(p,q) process can be approximated to any degree of accuracy by a stationary ARCH(j) for a sufficiently large value of j. Furthermore, both ARCH and GARCH models are symmetric models. Compared to ARCH, GARCH allows for a longer memory and a more flexible lag structure and may therefore be justified as a more parsimonious description. Finally, the

[12] For ARCH(p) and GARCH(1,1) the Bollerslev inequality constraints (non-negativity of parameters) are sufficient. A set of more relaxing constraints was derived by Nelson and Cao (1992), which allows for some negative parameters in estimation for higher order GARCH.

EWMA model can be viewed as a particular case of GARCH(1,1) where $\alpha_0 = 0, \alpha_i = 1-\lambda, \beta = \lambda$.

The use of GARCH models is widespread. The GARCH(1,1) specification has proven to be an adequate representation for most financial time series. In order to understand the nature of persistence in variance under the GARCH(1,1) model one can write it as follows:

$$h_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2$$
$$h_t^2 = \alpha_0 + \lambda h_{t-1}^2 + \alpha_1 v_{t-1}^2$$
$$v_{t-1}^2 = \varepsilon_{t-1}^2 - h_{t-1}^2$$
$$\lambda = (\alpha_1 + \beta_1)$$

where v_t is serially uncorrelated with mean zero. The parameters of GARCH are meaningful. α_1 can be viewed as a "news" coefficient, with a higher value implying that recent news has a greater impact on price changes. Engle and Bollersleve (1986) shows that conditional kurtosis of a distribution of multi-step returns depends upon α_1. Higher α_1 implies higher conditional kurtosis and the coefficient of kurtosis is $\kappa = 6\alpha_1^2(1-\beta_1^2 - 2\alpha_1\beta_1 - 3\alpha_1^2)^{-1}$, which is leptokurtic.

Just as α_1 reflects the impact of recent news, β_1 can be thought of as reflecting the impact of "old news", picking up the impact of news which arrived before yesterday (Antoniou and Holmes, 1995). If one believes that "old news" will have less impact on today's price changes, then β_1 should fall relative to α_1. By repeatingly substituting v_t^2 into the conditional variance equation and eliminating h_{t-1}^2, one can express the unconditional variance as:

$$h_t^2 = \sigma^2 + \alpha_1(v_{t-1}^2 + \lambda v_{t-2}^2 + \lambda v_{t-1}^2 + \ldots)$$
$$\sigma^2 = \frac{\alpha_0}{1-\lambda}$$

The above expressions make clear the dependence of the persistence of volatility shocks v_t^2 on the sum of the GARCH parameters, λ. If $\lambda \to 1$ from below the effects of past shocks on current variance become stronger. For $\lambda = 1$, the process is said to be integrated in variance or IGARCH (Engle and Bollerslev, 1986). In this case, shocks do not decay over time and unconditional variance does not exist. This extreme behaviour of the IGARCH process may reduce its attractiveness for asset pricing because IGARCH assumptions could make the pricing for long-term contracts very sensitive to the initial conditions. The GARCH(1,1) model can also be written as:

$$h_t^2 = (1-\lambda)\sigma^2 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2$$
$$h_{t+k}^2 - \sigma^2 = \alpha_1(\varepsilon_{+kt-1}^2 - \sigma^2) + \beta_1(h_{t+k-1}^2 - \sigma^2)$$
$$E(h_{t+k}^2) = \sigma^2 + (\alpha_1 + \beta_1)^k (h_t^2 - \sigma^2)$$

If $(\alpha_1 + \beta_1) < 1$, the *k*-day forecast will be stable as *k* increases. This variance forecasts exhibits mean reversion with a reversion level of its unconditional variance and a reversion rate of $(1-\lambda)$. The expected future variance equation shows that when the current volatility is above the long-term volatility, the GARCH(1,1) model estimates a downward-sloping volatility term structure. When the current volatility is below the long-term volatility, it estimates an upward-sloping volatility term structure. Finally, the estimate of the volatility for valuing the *N*-day option is given by:

$$\frac{1}{N}\sum_{k=0}^{N-1} E(h_{t+k}^2) = \sigma^2 + (h_{t+1}^2 - \sigma^2)\frac{1-(\alpha_1+\beta_1)^N}{N(1-\alpha_1-\beta)}$$

2.2.2.3 EGARCH Model

The ARCH and GARCH models impose symmetry on the conditional variance structure which may not be appropriate for modelling and forecasting stock returns volatility. The exponential GARCH (EGARCH) model was invented by Nelson (1991) in response to the criticisms that the stock returns were negatively correlated with changes in return volatility. EGARCH considers asymmetry in the variance equation. The EGARCH(1,1) specification can be modelled as follows:

$$r_t = g(x_{t-1}; a) + \varepsilon_t$$
$$\varepsilon_t = h_t e_t \qquad e_t \sim N(0,1) \qquad \varepsilon_t \mid I_{t-1} \sim N(0, h_t)$$
$$\log h_t^2 = \omega + \lambda_1 z_{t-1} + \lambda_2(|z_{t-1}| - (2/\pi)^{0.5}) + \beta \log h_{t-1}^2$$

where $z_t = \dfrac{\varepsilon_t}{h_t}$ is the normalised residual.

2.2.2.3.1 Implications

This model accommodates the asymmetric relation between stock returns and volatility changes. A negative λ_1 implies that a negative shock increases the conditional variance; it measures the sign effect. An estimated positive λ_2 indicates that a shock greater than $(2/\pi)^{0.5}$ also increases the conditional variance; it measures the size effect. The degree of asymmetry or skewness can be measured by the absolute value of the ratio $\delta = \frac{-1+\lambda_1}{1+\lambda_1}$. In other words, it can be said that a negative standardised innovation (bad news) increases volatility δ times more than a positive standardised innovation of an equal magnitude. The use of logarithms also means that parameters can be negative without the variance becoming negative. Thus it is not necessary to restrict parameter values to avoid negative variances as in the ARCH and GARCH models. The estimate of the volatility for valuing the N-day option is given by Heynen and Kat (1994):

$$\frac{1}{N}\sum_{k=0}^{N-1} E(h_{t+k}^2) =$$

$$\frac{\sigma^2}{NC} + \sum_{k=1}^{N} h_{t+1}^{2\beta^{K-1}} * \exp\left[\frac{-(\omega - \lambda_2\sqrt{\frac{2}{\pi}})\beta^{k-1}}{1-\beta}\right] * \exp\left[\frac{-0.5(\lambda_1^2+\lambda_2^2)\beta^{2(k-1)}}{1-\beta^2}\right] * C_k(\beta,\lambda_1,\lambda_2)$$

where

$$\sigma^2 = \exp\left[\frac{\omega - \lambda_2\sqrt{\frac{2}{\pi}}}{1-\beta} + \frac{1*(\lambda_1^2+\lambda_2^2)}{2*(1-\beta^2)}\right] * C(\beta,\lambda_1,\lambda_2)$$

$$C(\beta,\lambda_1,\lambda_2) = \Pi_{m=0}^{\infty}[F_m(\beta,\lambda_1,\lambda_2) + F_m(\beta,-\lambda_1,\lambda_2)]$$

$$F_m(\beta,\lambda_1,\lambda_2) = N[\beta^m(\lambda_1+\lambda_2)] * \exp(\beta^{2m}\lambda_1\lambda_2)$$

$$C_1 = 1, C_k = \Pi_{m=0}^{k-2}[F_m(\beta,\lambda_1,\lambda_2 + F_m(\beta,-\lambda_1,\lambda_2)], k \geq 2$$

As an alternative to using t- or normal distribution, Nelson (1991) employed a generalised error distribution (GED) with the EGARCH model:

$$f(\varepsilon_t \mid I_{t-1}) = \frac{v\exp(-\frac{1}{2}\mid\frac{\varepsilon}{\lambda}\mid^{v})}{\lambda 2^{(1+v^{-1})}\Gamma(v^{-1})}$$

where $\Gamma(\cdot)$ is the gamma function, $\lambda = [2^{-(2/v)}\Gamma(1/v)/(3/v)]^{0.5}$ and v is the degree of freedom. The GED encompasses distributions with tails both thicker and thinner than the normal and includes the normal as a special case. If $v = 2$ this produces a normal density, whilst $v > (<) 2$ is more thin- (fat-) tailed than a normal. Generally it is believed that EGARCH is better than ARCH/GARCH in volatility modelling because EGARCH incorporates leverage effects in its model.

2.2.3 Long Memory and Asymmetric Models

2.2.3.1 Underlying Concepts

Recently, the topics of long memory and persistence have attracted considerable attention in terms of the second moment of a process. The presence of long memory can formally be defined as the persistence of observed autocorrelations. If the quantity:

$$\lim_{n\to\infty}\sum_{j=-n}^{n}\mid\rho_j\mid$$

is non-finite then the process possesses long memory. Consequently, the autocorrelations exhibit persistence that are neither consistent with an I(1) process nor an I(0) process.

2.3.3.1.1 Stylised Facts

The development of long-memory is based on the observations of the so-called "stylised facts". Ding, Granger and Engle (1993) investigated the long memory property of daily S&P 500 returns from 1928 to 1991 and established a few "stylised facts" which held for a large number of financial series. They found that: 1) significant and positive sample autocorrelations for squared returns and the power transformation of the absolute return $\mid r\mid^{d}$ [13] at up to 2,500 lags with a series of 17,054 observations. Their rate of decay was slower than exponential, i.e. the autocorrelation function decreased fast at the beginning and then

[13] d is a positive real number.

decreased very slowly and remained significantly positive so that $corr(|r_t|,|r_{t-k}|) > corr(r_t^2, r_{t-k}^2)$; 2) it was possible that returns were serially uncorrelated but was dependent; 3) the long memory property could be mainly attributed to the pre-war period and the market had retained long memory of extraordinary events like the great depression in 1929. This property was most pronounced when $d \approx 1$ for stock returns. This is termed as 'Taylor effects'. Ding and Granger (1996) latter found that the long memory property was strongest when $d \approx 1/4$ for foreign exchange rate returns.

2.3.3.1.2 Inadequacy of GARCH-type Models for Long-Run Effects

Ding and Granger also studied the autocorrelation functions for the IGARCH(1,1) process. They consider following set of equations:

$$GARCH(1,1) \quad h_t^2 = \alpha_0 + \alpha_1 \varepsilon_t^2 + \beta_1 h_t^2$$
$$If\ \alpha_1 + \beta_1 = 1 \quad \equiv IGARCH(1,1)$$

$$GARCH(1,1)\ Autocorrelation: \rho_1 \approx \alpha_1 + \frac{1}{3}\beta_1, \rho_k \approx (\alpha_1 + \frac{1}{3}\beta_1)(\alpha_1 + \beta_1)^{k-1}$$

$$IGARCH(1,1)\ Autocorrelation: \rho_k = \frac{1}{3}(1+2\alpha_1)(1+2\alpha_1^2)^{-k/2}\ for\ \alpha_1 \neq 0$$

The autocorrelations for GARCH(1,1) decreases exponentially. Interestingly, the autocorrelation function for IGARCH(1,1) is also exponentially decreasing. Thus the IGARCH(1,1) is not persistent in volatility at all in the sense that the autocorrelation function for ε_t^2 dies out exponentially. These results are very counterintuitive. The explanation of these findings is that a shock may permanently affect the "expectation" of a future conditional variance process, but it does not permanently affect the "true" conditional variance itself. A simple example below will illustrate this situation:

$$\varepsilon_t = e_t h_t, e_t \sim N(0,1), h_t^2 = \varepsilon_{t-1}^2$$
$$E_t(\varepsilon_{t+k}^2) = E_t(h_{t+k}^2) = \varepsilon_t^2$$
$$h_{t+k}^2 \to 0\ as\ k \to \infty$$

In this case, the real impact of a shock will converge to zero whilst the expectation of the conditional variance depends on past shocks. Therefore GARCH-type model are inadequate to account for the long memory property. In fact, they are more appropriate to use for modelling of short-run effects.

2.2.3.2 ARFIMA Model

According to Baillie (1996), the extent of shock persistence in financial data such as index is consistent with a stationary process, but where the autocorrelations take far longer to decay than the exponential rate associated with the ARMA class. An important class of discrete-time long memory process is the Autoregressive Fractionally Integrated Moving Average (ARFIMA(*p*,*d*,*q*)) model:

$$\phi(L)(1-L)^d(r_t-\mu)=\theta(L)\varepsilon_t$$

where d denotes the fractional differencing parameter and L is the lag operator.

When $d=1$ it is an ARMA process. All the roots of $\varphi(L)$ and $\theta(L)$ lie outside the unit circle and ε_t is the white noise. The r_t process for $d \neq 0$ is said to be I(d). For $-0.5<d<0.5$, this process is covariance stationary, whilst $d>0$ the process is long memory. For $0.5 \leq d < 1$, the process is no longer stationary but retains its mean-reverting characteristics. For $d=1$, the process becomes a standard unit root process, which implies complete persistence. The long-run characteristic is captured by the fractional differencing parameter d. For high lag k, the autocorrelations of the ARFIMA process is given by:

$$\rho_k \approx ck^{2d-1}$$

where $c>0$ and the autocorrelations exhibit a hyperbolic decay. The ARFIMA process can be used for prediction:

$$r_t=\sum_{j=1}^{\infty}\pi_j r_{t-j}+\varepsilon_t$$

where $\pi=(1-L)^d\varphi(L)\theta(L)^{-1}$. Since the ARFIMA process is not compatible with any finite-dimensional state space representation, there is no readily available solution to the truncation problem associated with using this autoregressive representation for prediction. In addition, the identifiability of high order ARFIMA models is often problematic. Li (2002) managed to apply the ARFIMA model for forecasting of currency volatility and the only parameter to be estimated was d. Coupled with the use of 5-minute data and over-the-counter options, Li found that historical volatility provided better prediction about future realised volatility than implied volatility at horizons ranging from one month to six months.

There are other more complex long-memory models, e.g. N-component GARCH by Ding et al. (1996), fractionally-integrated GARCH (FIGARCH) by Baillie et al. (1996) and fractionally-integrated EGARCH (FIEGARCH) by Bollerslev et al. (1996). Those models are largely theoretical and require the use of beta and gamma distributions to estimate the autoregressive parameters. Although Bollerslev et al. were able to demonstrate some application of fractionally-integrated EGARCH on pricing S&P 500 call options, laborious estimation procedures have essentially made them undesirable for practical applications.

2.2.3.3 News Impact Curve

In the 1993 study by Engle and Ng, Engle and Ng considered the mapping between h_t and ε_{t-1}, terming this the "news impact curve". This curve is very useful in describing asymmetry in the $\varepsilon_{t-1} \Leftrightarrow h_t$ space. In this paper, Engle et al. pointed out that two broad decisions needed to be made: about the "shift/position" and "rotation/shape" of such a curve. The mapping framework is given by:

$$\varepsilon_t = e_t h_t$$
$$f(e_{t-1}) = | e_{t-1} - b | - c(e_{t-1} - b)$$
$$h_t = \alpha_0 + \alpha_1 h_{t-1}\{| e_{t-1} - b | - c(e_{t-1} - b)\} + \beta_1 h_t$$

where $-\infty < b < \infty$ and $-1 \le c \le 1$.

The parameter b controls the magnitude and direction of a shift in the $e_{t-1} \Leftrightarrow h_t$ space whilst c rotates the curve and produces the "rotations". If one draws the mapping of the above example in the $e_{t-1} \Leftrightarrow f(e_t)$ space, the following is observed:

i) $c = 0$. If $b > 0$ the news impact curve is shifted to the left and one will obtain asymmetry that matches the stylised facts of stock return volatility: for negative shocks, volatility rises more than for equally large positive shocks. This effect is most pronounced for small shocks. For extreme large shocks, the asymmetric effect becomes negligible;

ii) $b = 0$. If $c > 0$ the news impact curve rotates clockwise by changing the slopes of the curve on either side of the origin: negative shocks create more volatility. If $c < 0$ the curve moves counter-clockwise and positive shocks create more volatility. The size of asymmetric effect relative to the total response is constant;

iii) $b \neq 0, c \neq 0$. In this case, c will not just cause a pure rotation of the curve. The slopes are different on either side of the curve around the origin b.

The "news impact curve" classification has allowed researchers to understand more about the impact of individual parameters on volatility shocks. It is noted that ARCH and GARCH processes have impact curves that are symmetric around zero, whereas EGARCH is asymmetric around zero.

2.2.3.4 APARCH Models

The imposition of a quadratic mapping between the history of ε_t and h_t may be too restrictive. Ding, Granger and Engle (1993) invented the Asymmetric Power ARCH (APARCH) model, which nests many popular conditional variance models as special cases[14]. The APARCH models impose a Box-Cox power transformation on the conditional standard deviation process and its absolute residuals. The APARCH(p,q) model is given by:

$$\varepsilon_t = e_t h_t$$

$$h_t^\delta = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j h_{t-i}^\delta$$

where $\delta \geq 0$ and the parameters α_i and γ_i control the responses of h_t^δ to $(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta$.

By inspecting the autocorrelation function, one can understand why APARCH models are used for modelling of long memory effects. If $E[|e_t|^\delta] = 1, E[|e_t|^{2\delta}] = \xi$ and $\gamma_1 = 0$ and that $E[|\varepsilon_t|^\delta]$ exists, it follows that:

$$\begin{aligned} \rho_1 &= corr(|\varepsilon_t|^\delta, |\varepsilon_{t-1}|^\delta) \\ &= \alpha + \frac{\beta}{\xi} - \beta\frac{\xi - 1}{\xi}\left[\frac{\xi(1-\alpha-\beta)(1+\alpha+\beta)}{1-(\xi\alpha^2+\beta^2+2\alpha\beta)} - 1\right]^{-1} \end{aligned}$$

where the autocorrelation function of ε_t^2 for the APARCH(1,1) process is:

[14] These models are shown in appendix A.1.

$$\rho_k = \rho_1(\alpha_1 + \beta_1)^{k-1}$$

If $E[|\varepsilon_t|^\delta]$ is unbounded and if $\alpha+\beta<1$ and $\xi\alpha^2+\beta^2+2\alpha\beta>1$ the autocorrelation can be approximated by:

$$\rho_k \approx \left[\alpha_1 + \left(1-\frac{2}{\xi}\right)\beta_1\right](\alpha_1+\beta_1)^{k-1}$$

When $\alpha+\beta=1$ and $\alpha>0$, it becomes a IGARCH(1,1) in $|\varepsilon_t|^\delta$ and the autocorrelation function is:

$$\rho_k \approx \frac{1+(\xi-1)\alpha}{\xi}[1+(\xi-1)\alpha^2]^{-k/2}$$

In any cases, autocorrelations of APARCH decreases exponentially, not hyperbolically. Ding et al. results showed that the estimated power δ was 1.43 and its asymmetric parameter γ equal to –0.373, which suggested significant long memory and leverage effect did exist in S&P 500 returns.

2.2.3.5 *Hentschel Framework*

In the 1995 study by Hentschel, Hentchel developed a framework that could nest most of the existing family of GARCH models, including the models in the APARCH family. The nesting of these models was accomplished by properly choosing the "degree" of transformation. This framework can be written as:

$$\varepsilon_t = e_t h_t$$
$$f_i(e_t) = |e_t - b_i| - c(e_t - b_i)$$
$$\frac{h_t^\delta - 1}{\delta} = \alpha_0 + \sum_{i=1}^{p}\alpha_i h_{t-i}^\delta f_i^v(e_{t-i}) + \sum_{j=1}^{q}\frac{h_{t-1}^\delta - 1}{\delta}$$

where $-1 \le c \le 1$ and $v>0$. δ controls the shape of the transformation and $\delta \ge 0$. If $\delta > 1$, the transformation of δ is convex; otherwise it is concave. The parameter v serves to transform $f(\cdot)$. If $v>1$, this transformation is convex; if $0<v<1$, the transformation is concave. In addition, the APARCH model is the special case of $v=\delta, b=0, |c|\le 1$. Figure 1 classifies all of the nested models within Hentschel's framework.

Figure 1: Hentschel's Framework

δ	ν	b	c	Models
0	1	0	Free	EGARCH
1	1	Free	|c|≤1	TS-GARCH
2	2	0	0	GARCH
2	2	Free	0	Nonlinear-Asymmetric GARCH
2	2	0	Free	GJR GARCH
Free	δ	0	0	Nonlinear ARCH
Free	δ	0	|c|≤1	APARCH

Using the S&P 500 returns data, Hentschel found that: 1) $\delta \approx 1.5$ when $v = \delta$; 2) $\delta \approx 1.1$ when δ, v were free; 3) c was neither statistically nor economically significant; 4) $\delta \approx v$ and it was between one and two; 5) small shocks made more contributions to volatility, but not large shocks. Furthermore, the "shifting" of news impact curve was the dominating factor in modelling asymmetry. As a result, the presence of b was more significant than c; 6) GARCH produced higher volatility than the EGARCH or the freely estimated models.

2.2.3.6 *Other Asymmetric Models*

There are many other asymmetric volatility models in the finance literature – so many that it is cumbersome to provide a comprehensive survey of it. Most of them try to mimic the "shift" and "rotation" effects in volatility. Some of the more well-known models are discussed below:

i) TS-GARCH: $\delta = v = 1, \gamma_i = c = 0$. Taylor modelled the conditional standard deviation as a linear combination of past conditional variances in 1986 and Schwert (1989) modelled the conditional standard deviation as a linear function of lagged absolute residuals. The TS-GARCH model combines both of Taylor's and Schwert's models that use the lagged conditional variances and absolute residuals. The TS-GARCH is given by $h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}) + \sum_{j=1}^{q} \beta_j h_{t-i}$. A related model is the QGARCH model of Sentana (1991): $h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i (\varepsilon_{t-i} - \gamma_i)^2 + \sum_{j=1}^{q} \beta_j h_{t-i}^2$. Strictly speaking this model produces a symmetric curve around γ, but with no rotations.

ii) GJR: $\delta = v = 2, c = 0$. The model proposed in Glosten, Jagannathan and Runkle (1993) is an extension of the GARCH model that takes account of asymmetric effect on volatility. This model can be expressed as follows:

$h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \; \varepsilon_{t-i}{}^2 + \sum_{j=1}^{q} \beta_j h_{t-i}^2 + \sum_{i=1}^{p} \lambda_i \; S_{t-i}^- \varepsilon_{t-i}{}^2$ where $S_{t-i}^- = 1 \; if \; \varepsilon_{t-i} < 0, \; S_{t-i}^- = 0$ otherwise. This model can also be written into the following form[15]: $h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^{q} \beta_j h_{t-i}^2$

iii) TARCH: $\delta = v = 1, \beta_j = c = 0$. Zakian (1990) suggested a conditional standard deviation of the form $h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i^+ \varepsilon_{t-i}^+ - \sum_{j=1}^{p} \alpha_j^- \varepsilon_{t-j}^-$ where $\varepsilon_t^+ = \max(\varepsilon_t, 0)$, $\varepsilon_t^- = \min(\varepsilon_t, 0)$, $\alpha_0 \geq 0, \alpha_i^+ \geq 0, \alpha_i^- \geq 0$. Zakian referred to this formulation as a threshold ARCH (TARCH) model because the coefficient of ε_t changed when it crossed the threshold of zero. When $\varepsilon_{t-i} > 0$, the conditional standard deviation is linear in ε_{t-i} with slope α_i^+ and when $\varepsilon_{t-i} < 0$, the conditional standard deviation is linear in ε_{t-j} with slope $-\alpha_j^-$. This model can also be written as:

$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})$ where $\alpha_0 \geq 0, \alpha_i \geq 0$.

iv) NARCH: $\delta = v, \gamma_i = \beta_j = 0$. Higgins and Bera (1992) proposed a nonlinear ARCH (NARCH) model, which still requires non-negativity restrictions, but includes linear ARCH as a special case and log ARCH as a limiting case. The NARCH model is written as $h_t^\delta = \alpha_0 + \sum_{i=1}^{p} \alpha_i (\varepsilon_{t-i})^\delta$ where $\alpha_i \geq 0, \delta \geq 0$. The NARCH model can be regarded as a Box-Cox power transformation applied to ε_{t-i}. When $\delta = 2$, it is an ARCH(p). As $\delta \to 0$ it becomes the log ARCH model:

$\log(h_t) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \log(\varepsilon_{t-i})^2$

2.2.4 GARCH Option Models

The use of GARCH is not just limited to volatility forecasting and modelling. Recently, there has been a lot of attention on using GACH models to price options. Because early GARCH models did not allow for option pricing along the lines of Black and Scholes, i.e.

incompleteness due to discrete trading, researchers had mainly focused on Monte-Carlo simulation or approximation of GARCH option models. For example, Myers and Hanson (1993) studied option prices of soybean futures from CBOE from 1988 through 1990. Myers et al. compared performance of different models based on Monte-Carlo simulation and closed-form approximation of GARCH models[16]. They reported that the GARCH option pricing approach clearly estimated option prices better than the standard Black's model did with historical volatility in terms of root-mean-square-error. Myers et al. suggested that it was the constant variance assumption, rather the normality assumption, which represented the biggest deficiency in Black's model of pricing commodity options. In the study of Kansas City wheat futures, Kang and Brorsen (1995) also conducted a Monte-Carlo study and compared performance between the asymmetric GARCH-t model that had incorporated the day-of-the-week and time-to-maturity effects in the conditional variance equation with the Black's formula. In out-of-sample prediction, the GARCH-t model predicted actual option premiums more accurately than Black's model for deep in-the-money call and put options and deep out-of-the-money put options in terms of root-mean-square-error. The Monte-Carlo simulation results confirmed Hull and White's findings (1987) that differences between Black's model and the GARCH-t model increased as time to maturity increased and the Black's model overpriced close-to-the-money options.

Since the mid-90's, many researchers have began their efforts in reconciling the differences between discrete-time and continuous-time models. Duan (1995, 1997) was first to derive the GARCH option pricing model and its corresponding delta formula based on equilibrium-type arguments concerning the utility function of a representative agent. The advantage of this model is that European options can be evaluated by the risk-neutral valuation method. This model uses the locally risk-neutral valuation relationship (LRNVR) to price options which can implicitly account for volatility smile. This GARCH option model is a function of the risk premium embedded in the underlying asset. Thus the locally risk-neutral valuation relationship does not "eliminate" risk. This contrasts with the standard preference-free option pricing model. With the new pricing measure Q, the GARCH option model is specified as:

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[15] See Ding and Granger (1993) for its derivation.

[16] The closed-form model is approximation and assumes that distribution of price change is approximately normal.

$$\ln\left[\frac{S_t}{S_{t-1}}\right] = r - \frac{1}{2}h_t + \xi_t,\ \xi_t \mid \phi_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i (\xi_{t-i} - \lambda\sqrt{h_{t-i}})^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$

This model introduces correlation between lagged asset and conditional variance. Under this measure, the underlying asset is leptokurtic. Thus the GARCH option pricing model is capable of reflecting the changes in the conditional volatility of the underlying asset in a parsimonious manner and may be able to explain some systematic biases associated with the Black-Scholes model. After "locally neutralised", the discounted asset price becomes Martingale. In addition, formulae for terminal asset price and the option delta are available. This implies that the GARCH options can be evaluated by the risk-neutral valuation method, i.e. $C(S,t) = e^{-(T-t)r} E^{Q}[\max(S_T - K, 0) \mid \phi_t]$, because the expected rate of return under the new measure Q is no longer equal to $e^{(r+\lambda\sqrt{h_t})}$. But it is known that this model still fails to capture volatility smiles for short-dated options, which is perhaps better explained by jump-type models for the stock price process.

Kallsen and Taqqu (1998) bridged the GARCH discrete-time setting to a no-arbitrage continuous-time setting and demonstrated that the completeness of the market holds for a broad class of GARCH-type models. The basic idea of this continuous time extension of GARCH-type models was to maintain a constant volatility during an interval formed by two integer dates. Kallsen et al. derived the same GARCH pricing formula as Duan but they did not agree on the hedging formula. Later, Garcia and Renault (1998) proposed a stochastic volatility model which also ensured the validity of Duan's results. Garcia et al. concluded that stochastic volatility models and GARCH-type option pricing models were not as far apart as originally believed.

Heston and Nandi (1998) presented the necessary mappings to approximate the parameters of the continuous-time option pricing model on the basis of the parameters of the discrete-time GARCH model. A parameter that related to the expected risk premium of the asset did appear in this option formula, however, option prices were not at all sensitive to the risk premium parameter. The advantage of this model is that option prices can be easily computed by closed-form solutions using the formula of Heston but its disadvantage is such that the same Wiener process drives both asset returns and variance under the risk-neutral measure.

Heston and Nandi (2000a) presented a closed-form solution for options and hedge ratios when variance of the spot asset followed a GARCH(*p*,*q*) process and was correlated with asset returns. This discrete-time GARCH model with a single lag converged to Heston's (1993) continuous-time stochastic volatility model as the observation interval shrank but its variance was driven by two perfectly correlated Wiener processes. Empirical results showed that this GARCH option pricing model was superior to the ad hoc Black-Scholes model of Dumas et al. (1998) that used a separate implied volatility for each option to fit the smile on S&P 500 index options.

2.2.5 Other Developments

The conditional volatility approach is a popular tool in modern risk management, partly because of its vast literature and also its simplicity in implementation. Since Engle (1982) developed the autoregressive conditional heteroskedasticity (ARCH) model, numerous models have emerged in literature. We will discuss a few important research papers to illustrate the developments in this section.

Engle and Lee (1993) invented a component GARCH model for stock market volatility which could be decomposed into a permanent and a transitory component. Engel et al. found that leverage effect in the stock market was mainly a temporary behaviour of the volatility process. This component model could be written as a GARCH(2,2) process so a regular GARCH(1,1) was only a single dynamic component of this conditional variance model. Engle et al. found that this component model was successful in describing the effect of the "October 1987 Crash" on stock volatility changes.

Duan (1997) proposed an augment GARCH process which encompassed many popular GARCH specifications as special cases. In the diffusion limit the augment GARCH process was shown to contain many existing bivariate diffusion models such as Hull and White (1987), Wiggins (1987), Scott (1987), Stein and Stein (1991) and Heston (1993a). The augmented GARCH process is widely used as a direct approximation to the stochastic volatility models in option pricing.

2.3 Stochastic Volatility Models

2.3.1 Underlying Concepts

Financial researchers have modelled volatility as if it were behaving in a random way, building models of stochastic volatility. Stochastic volatility models allow volatility to be driven by a separate random process. They can possibly fit in the gap for the inadequacy of the ARCH/GARCH models by allowing the following features in their models:

i) Volatility term-structure patterns

ii) Mean-Reversion

iii) Correlation between volatility and asset returns

2.3.1.1 Wiener Process

The $W_t : t \geq 0$ is a P-Wiener process if and only if:

i) W_t is continuous and $W_0 = 0$.

ii) $W_t \sim N(0,t)$

iii) $W_{s+t} - W_s \sim N(0,t)$ and is independent of F_s, the history of the process up to time s.

It is important to note that W_t is continuous everywhere but it is differentiable nowhere.

2.3.1.2 Stochastic Process

A stochastic process S is a continuous process $(S_t : t \geq 0)$ such that S_t can be written as:

$$S_t = S_0 + \int_0^t \sigma_s dW_s + \int_0^t \mu_s ds$$

where σ, μ are random $F-$previsible processes such that $\int_0^t (\sigma_s^2 + |\mu_s|)ds$ is finite for all times t with probability 1. The differential form of the above stochastic process can also be written as:

$$dS_t = \sigma_t dW_t + \mu_t dt$$

The behaviour of S_t fluctuates around a straight line with slope μ_t. The size of σ_t determines the extent of the fluctuations around this line. In particular, these fluctuations do not become larger as time passes. Given σ_t, μ_t and S_0, the process S is unique.

2.3.1.3 Stochastic Differential Equation

In the special case when σ and μ depend on W only through S_t, such as $\sigma_t = \sigma(S,t)$, the stochastic differential equation (SDE) for S is given by:

$$dS_t = \sigma(S_t,t)dW_t + \mu(S_t,t)dt$$

Regrettably, there are few soluble SDE's. One of them is geometric Brownian motion. The SDE for geometric Brownian motion is $dS_t = S_t(\sigma dW_t + \mu dt)$. This setup gives asset prices that fluctuate randomly around an exponential trend. Its solution is:

$$S_t = S_0 \exp((\mu - \frac{1}{2}\sigma^2)t + \sigma W_t)$$

2.3.1.4 Ornstein-Uhlenbeck Process

The stochastic process σ_t is random and not observable. One of the most studied and celebrated continuous-time stochastic volatility models is the Ornstein-Uhlenbeck process:

$$\begin{aligned} dS_t / S_t &= \alpha dt + \sigma dW_1 \\ d(\ln\sigma) &= \lambda(\xi - \ln\sigma)dt + \gamma dW_2 \\ dW_1 dW_2 &= \delta dt \end{aligned}$$

where δ is the correlation for the Wiener processes W_1 and W_2, ξ the long-term mean and λ the speed of the mean-reverting process.

The continuous models are intrinsic in understanding theoretical finance. This model introduces a correlation in the formulation of volatility process. In practical world, however, stocks or commodities are traded discretely. The discrete-time models are approximations of their continuous counterparts. The discrete-time model of the corresponding continuous process is:

$$\ln(S_t) = \ln(S_{t-1}) + \mu + \sigma_{t-1}U_t$$
$$\ln(\sigma_t) = \alpha + \phi[\ln(\sigma_{t-1}) - \alpha)] + \theta\eta_t$$

where U_t and η_t are bivariate normal with correlation δ. According to Taylor (1994), the lagged volatility σ_{t-1} in the mean equation of the discrete-time process is the Euler approximation of its continuous time counterpart. However, it can be argued that a more natural simplification would be:

$$\ln(S_t) = \ln(S_{t-1}) + \mu + \sigma_t U_t$$

Therefore, the main difference between the ARCH model and discrete-time model is that the ARCH models' innovations depend on the past information set I_{t-1} whilst in the case of discrete-time stochastic volatility models, they are independent of the returns history I_{t-1}. The ARCH models tell that past information can be used to predict the future but the discrete-time stochastic volatility models imply that this information is irrelevant for future volatility.

2.3.2 Hull-White Model

Continuous time stochastic volatility models endogenise the volatility patterns and may be used directly in valuation and hedging. They are largely theoretical and usually their applications are computational intensive. A well-known stochastic volatility model is the Hull-White model (1987). This model is based upon the following continuous-time process:

$$dS = \phi S dt + \sigma S dW_1$$
$$d\sigma^2 = \mu(\kappa - \sigma^2)dt + \xi\sigma^2 dW_2$$

where W_1, W_2 are Wiener processes and ϕ, μ, κ, ξ are constant.

This model stipulates that the variance rate has a drift to pull it back to a level κ at rate μ. ξ is the volatility of the volatility and it is possible to estimate ξ by examining the changes in volatility implied by option prices. Since volatility is not a traded asset, it is not possible to form a hedge portfolio that eliminates all the risk. If W_1 *and* W_2 are not correlated so that volatility is not correlated with stock price and the volatility is uncorrelated with aggregate consumption (zero systematic risk, no risk preferences, i.e. constant risk premia), then the

Hull-White price is the mean Black-Scholes price, evaluated over the conditional distribution of average variance:

$$C = \int_0^{\infty} c(\bar{V}) g(\bar{V}) d\bar{V}$$

where $\bar{V}$ is the average value of the variance rate, c is the Black-Scholes price and g is the probability distribution of $\bar{V}$ in a risk-neutral world. Furthermore, Hull and White derived an analytic solution for European-style option based on Taylor-series expansion. In this case, the Hull-White model can be written as a combination of Black-Scholes solution with adjustment terms. Their main empirical result was that different "asymmetric" patterns could be generated by changing the μ, ξ and the sign of correlation parameters. Hull and White concluded that longer-term near-the-money European call options had lower implied than did shorter-term options. Finally, it is noted that the GARCH(1,1) model can be written as a discrete-time approximation to the diffusion process of the Hull-White model.

Hull and White attempted to use their model to explain Rubinstein's (1985) findings on the term-structure of implied volatility. But Rubinstein's results from comparing implied volatility were not consistent across different times to maturity. It was necessary to posit in the Hull-White model that, from one year to the next, the correlation between stock prices and the associated volatility reversed sign. No reason could be found to justify such a change of sign.

2.3.3 Johnson-Shanno Model

Johnson and Shanno (1987) applied an equilibrium approach to derive an option pricing model and attempted to explain that changing the sign of correlation between volatility and return processes was responsible for the switch in exercise biases in Rubinstein's 1985 results. The Johnson-Shannon model model is given by:

$$dS = Sdt + \sigma S^{\alpha} dZ \qquad (\alpha > 0)$$
$$d\sigma = \mu_P \sigma dt + \sigma_P \sigma^{\beta} dZ_P \qquad (\beta \geq 0)$$

Johnson and Shannon assumed there existed a traded asset J that had the same random term as the variance of the stock:

$$dJ = \mu_J J dt + \sigma_J J^{\beta} dZ_P$$

Thus a risk-free hedge could be formed by longing one share of J and shorting $\left(\frac{\partial C}{\partial P}\right)^{-1}$ of option. Johnson et al. used Monte-Carlo simulation to solve for a numerical solution and found that their model could account for some term-structure of the implied volatility for the out-of-the-money call options. Johnson and Shannon concluded that: 1) they could not assert that the switch in bias in Rubinstein's paper was caused by an upward shift in correlation; 2) they could not point to any macroeconomic event that would indicate a change of correlation in Rubinstein's study period of 1976-1978.

2.3.4 *Stein-Stein Model*

Stein and Stein (1991) derived a closed-form option-pricing solution via inverse Fourier transformation. Stein and Stein (1991) formulated the evolution of stochastic volatility based on the Ornstein-Uhlenbeck process:

$$dS_t = \alpha S_t dt + \sigma S_t dW_1$$
$$d\sigma = -\delta(\sigma - \kappa)dt + \theta dW_2$$

where dW_1, dW_2 are uncorrelated.

The Stein-Stein model is more general than that of the Hull-White because it does not rely on Taylor-series expansion to solve explicitly for the option price. Simulations suggest that this model exhibits a U-shape as the strike price was varied. However, this model has the disadvantage that it cannot capture skewness effects that arise from returns-volatility correlation. Nevertheless, the way Stein et al. derived the solution via Fourier transformation opened a new way for researchers to look for more complex stochastic volatility models.

2.3.5 *Heston Model*

Heston (1993a) derived a closed-form solution for the price of a European-style option on an asset with its variance followed the Ornstein-Uhlenbeck process. This is the first stochastic volatility model with closed-form solution that can account for correlation between volatility and asset returns. The Heston model is given by:

$$dS = \phi S dt + \sigma S dW_1$$
$$d\sigma^2 = \kappa[\theta - \sigma^2]dt + \xi \sigma dW_2$$

where κ is the speed which σ^2 reverts to its long-term mean θ. As opposed to the Hull-White model where risk premium was zero, Heston specified a volatility risk premium that was proportional to the variance: $\lambda(S,\sigma^2,t) = \lambda\sigma^2$. Using Ito's lemma and standard arbitrage arguments, Heston (1993a, 1993b) showed that the price of a European call was given by:

$$c(S,\sigma,t) = Sp_1 - KB(t,T)p_2$$

where p_1, p_2 and $B(t,T)$ are the conditional probabilities that can be calculated from formulas, and the price of a pure discount bond at time t with maturity of T, respectively[17].

Heston's model has the advantage that it allows arbitrary correlation between volatility and asset returns. It can link any type of bias to the dynamics of the spot price and the distribution of spot returns. Heston suggested that this model might be able to explain some option biases that changed through time by Rubinstein (1985). In addition, the Heston model can possibly incorporate stochastic interest rates in pricing formula. Heston found that: 1) correlation between volatility and the spot price was necessary for explaining skewness and strike price biases; a positive correlation results in high variance when the spot asset rises and this spreads the right tail of the probability density relative to the left tail, and vice versa; 2) skewness in the distribution of spot returns affected the pricing of in-the-money options relative to out-of-the-money options. Without this correlation, it is generally known that stochastic volatility only changes the kurtosis through ξ.

2.3.6 Merton Model

Apart from the Wiener process, researchers have also tried other processes to model risks. One of the pioneer works was by Merton (1976). Merton suggested a model where the asset price had jumps superimposed upon a geometric Brownian motion. In this seminal paper, Merton used two different sources to represent risks: 1) Wiener process to model daily news and risks that come randomly from the market and are diversifiable; 2) Poison process to

[17] The details can be found in the Heston's (1993a) appendix.

describe jumps/shocks that capture the arrival of important news and are non-diversifiable. This model can be described by the following SDE:

$$dS = (a - \lambda k)Sdt + \sigma SdW + \lambda dq$$

where the parameter a is the instantaneous expected return on the stock, σ the instantaneous volatility of the returns, λ the rate of arrival, dW the Wiener process and dq the Poisson process.

It is important to note that the size of Poisson outcomes does not depend on the infinitesimal interval *dt*. Instead, the probabilities associated with the outcomes are only a function of *dt*. The size of Brownian motion gets smaller as *dt* approaches zero. The Black-Scholes model can be written as a special case of the Merton model when $\lambda = 0$. Due to the non-diversifiable risks presented in this model, no-arbitrage argument cannot be invoked to price options. The jump-diffusion model can give rise to fatter left and right tail than the Black-Scholes model and is consistent with the implied volatility patterns observed for currency options. In a study of stochastic volatility and jump-diffusion models, Bakshi et al. (1997) showed that some new stochastic models had improved pricing performance relative to the Black-Scholes formula, but there was also evidence to suggest that the benefits derived from these mathematical parameterisations used for option pricing were not in proportion with the complexity of the models. Nevertheless, the Merton model has successfully inspired many researchers to seek for alternative stochastic processes to price options.

2.3.7 Other Developments

Since Rubinstein (1985) documented the observed implied volatility patterns in relation to moneyness, many researchers had tried to use the two-dimension diffusion models to account for these biases. We have selected a few of them for discussions.

In a study of stohcastic volatility option pricing model, Scott (1987) assumed that volatility risk could be diversified away and changes in volatility were uncorrelated with the stock return. This study used the equilibrium asset pricing model to derive solutions for the continuous time diffusion process. Scott'solution was similiar to those of Hull-White model that the solution was integral of the Black-Scholes formula and the distribution function for the variance of the stock price. Scott computed option prices via Monte-Carlo simulations and found that the model was marginally better than the Black-Scholes model at explaining actual option prices.

Lo and Wang (1995) investigated the effect of predictability of asset return on option prices under the Ornstein-Uhlenbeck process. Even though predictability was typically induced by the drift, which did not enter the option pricing formula under the no-arbitrage framework, Lo et al. showed that predictability was linked to the parameters that did enter the Black-Scholes option pricing formula. In addition, Lo et al. constructed an adjustment for predictability to the Black-Scholes formula and demonstrated that this adjustment could be important even for small levels of predictability, especially for longer maturity options.

Gesser and Poncet (1997) compared the performance of the Hull-White model and the Heston model using twenty days of at-the-money dollar-mark forward option data. Gesser et al. found that the Heston model was superior to the Hull-White model because 1) correlation was allowed between volatility and asset returns; 2) the market price of volatility risk was not constant but proportional to the variance in the Heston's model. Gesser et al. also pointed out that the Hull-White model's poor performance was possibly caused by the low-order Taylor-series approximation that Hull and White used in the derivation process. Despite its success in accurately reproducing term-structure of volatility and minimising volatility fitting errors, the Heston model still failed to reproduce smile convexities as observed in foreign exchange market.

Nandi (1998) studied how the incorporation of a non-zero correlation between asset returns and volatility impacted pricing and hedging in the Heston model. The data that Nandi used were 126 days of S&P 500 index in 1992. The unobservable instantaneous volatilities were estimated jointly with other time invariant parameters using generalised method of moment to avoid any potential inconsistency in the estimation process. Nandi found that the Black-Scholes model outperformed the zero correlation version of the Heston model in terms of pricing. However, the non-zero correlation version of the Heston model outperformed the Black-Scholes model, both in terms of out-of-sample pricing and hedging. Nandi acknowledged that future research could be directed towards developing simpler stochastic models that were easier to estimate.

Corrado and Su (1998) used the Hull-White model to study the stochastic process implied by the S&P 500 index options. Corrado and Su's paper provided evidence that observed option prices on the S&P 500 index corresponded to a mean-reverting stochastic volatility process, where return volatility was strongly negatively correlated with changes in stock index levels. Corrado et al. also showed that a stochastic volatility option pricing model provided a significant improvement over the Black-Scholes model in out-of-sample assessment.

Madan et al. (1998) used the variance gamma process to price European options that allowed for skewness and excess kurtosis in a risk-neutral framework. In contrast to traditional Brownian motion, the variance-gamma process is a pure jump process with an infinite arrival rate of jumps. This process has finite variation and a random time change that can be written as the difference of two increasing processes each giving separately the market up and down moves. Closed-form solutions for European options were derived and the new option valuation formula nested the Black-Scholes model as a special case. Maden et al. demonstrated that the Black-Scholes model could be rejected in favour of the variance-gamma model.

Das and Sundaram (1999) derived closed-form solutions for the conditional and unconditional skewness and kurtosis of two classes of important models: stochastic volatility with mean-reversion and Poisson jump-diffusion processes. Das et al. found that each model exhibited some term-structure patterns that were fundamentally inconsistent with those observed in the market and neither class of models constituted an adequate explanation of the empirical evidence. Furthermore, this study showed that jump-diffusions could only generate realistic and sharp implied volatility smile at short maturities but not at long maturities. In contrast, stochastic volatility models were not capable of generating high levels of skewness and kurtosis at short maturities under "reasonable" parameterisations but the smile did not flatten out appreciably as maturity increased. Das et al. found that a variety of implied volatility patterns were possible for at-the-money options under stochastic volatility models and they concluded that stochastic volatility models were better than jump-diffusion models.

Overall, stochastic models take into account some of the characters of volatility. This allows in part the explanation of the "volatility smile". But many problems limit the use of stochastic volatility models. First, volatility is not a traded asset. No traded asset is instantaneously perfectly correlated with volatility so it is not possible to build a hedge portfolio to eliminate volatility risk. Thus it is impossible to price options by no-arbitrage techniques without introducing as an exogenous parameter the market price of volatility risk. Second, estimations of several non-observable parameters using maximum likelihood method are not valid in cases discussed above because stock returns are dependent over time and joint distribution for a sample of observations would be very difficult to derive. Third, one usually has to make questionable assumption that asset returns and volatility is uncorrelated[18]. Fourth, closed-form solution usually does not exist for solving of these two-dimensional partial differential equations and requires the use of Monte-Carlo simulation as well as advanced econometric and numerical techniques, which are computationally demanding. Last, there is no systematic way to determine the changing sign and magnitude of correlation, which is important in generating smile convexities. The factors mentioned above make it very challenging to evaluate more complex products.

2.4 Implied Methodology

2.4.1 Underlying Concepts

Implied methodology refers to the methods to exploit information about the distribution of the future asset from the options market. The major innovation that implied models offer is the direct gain of market information embedded in traded option prices without having to be filtered through the underlying asset's properties. Many studies have shown that options contain information not found in the underlying time-series that is useful for predicting future volatility (e.g. Chiracs and Manster, 1978; Day and Lewis, 1992; Lamoureux and Lastrapes, 1993). There are two major approaches in extracting market information in the implied framework: 1) the direct approach makes assumptions about the distribution of the risk-neutral distribution; 2) the indirect or implied approach does not make any distributional assumptions and accepts observed options are priced consistently but not necessary correctly. Neither approach makes any assumptions about the stochastic process of the underlying asset price but implied methods are proven to be more general because any given risk-neutral distribution is consistent with many different stochastic processes.

The primary reason for using market information is the existence of the observed options' biases. The volatility smile curve indicates that market participants make more complex assumptions than geometric Brownian motion about the path of the underlying asset price.

[18] Some assume that the volatility risk is not priced.

Consequently, market participants attach different probabilities to terminal values of the underlying asset price than those that are consistent with a log-normal distribution. The extent of the convexity of the smile curve indicates the degree to which the market risk-neutral distribution function differs from the Black-Scholes' constant volatility assumption. Any variations in the shape of the smile curve are mirrored by corresponding changes in the slope and convexity of the call pricing function[19]. In particular, the more convex the smile curve, the greater the probability the market attaches to extreme outcomes for the asset price. This causes the market risk-neutral distribution function to have "fatter tails" than with a log-normal density function. Moreover, the sign of the slope in the volatility smile curve also reflects the skew of the market risk-neutral distribution: a positively (negatively) sloped implied volatility smile curve results in a risk-neutral distribution that is more (less) positively skewed than the log-normal risk-neutral distribution that would result from a flat smile curve.

2.4.2 Direct Approach

The direct approach corresponds to the way market information is explicitly extracted from options market. The risk-neutral distribution functions are usually assigned a priori according to the "beliefs" of the researcher. Since risk-neutral and true distributions will be equal only if investors are truly risk-neutral, or if risk in the underlying security is not priced, the risk-neutral distribution embedded in option prices is usually different from that of the actual distribution. From the pricing perspective, risk-neutral distribution are sufficient statistics in an economic sense – they summarise all relevant information about preferences and business conditions for purposes of pricing financial securities.

2.4.2.1 Breeden-Litzenberger Method

Breeden and Litzenberger (1978) were first to show that the second partial derivative of the call pricing function with respect to the exercise price is directly proportional to the risk-neutral distribution function. The slope and convexity of the smile curve could be translated into probability space to reveal the market's implied risk-neutral distribution function for the asset price. Since observed option prices are only available at discretely spaced intervals

[19] See Bahra (1997) for a more detailed discussion on these issues.

rather than being continuous, some approximation for the second derivative is necessary and more than one implied distribution could be implied, depending on the approximation chosen.

Shimko (1993) derived an analytic expression for the probability density functions under the parabolic implied volatility assumption by fitting a quadratic relationship between implied volatility and exercise price. The Black-Scholes formula was then used to invert the smoothed volatility into option prices, thus allowing the application of Breeden et al.'s results straightforwardly. However, Shimko's extrapolation procedure, which grafted log-normal tails onto the observable part of the implied risk-neutral distribution function, was that it arbitrarily assigned a constant volatility structure to the smile outside of the traded strike range. Therefore it was not always possible to ensure a smooth transition for the observable part of the distribution to the tails. In addition, nothing in the Shimko's approach could prevent negative probabilities.

Malz (1997) used the volatility function technique to access the risk-neutral distribution of exchange rates. The estimate of the volatility smile was parameterised by the traded straddle, strangle and risk-reversal option prices so it did not require the construction of a cubic spline function or regression on implied volatilities. Unlike Shimko, Malz did not make special allowances for the tails and allowed the fitted curve to cover the entire range of delta, hence the entire support for the probability density function. Malz concluded that this method leaded to smoother estimates of the risk-neutral distribution and more accurate volatility estimates for wing options.

2.4.2.2 Multi-Log-Normality Method

The use of log-normal density function has also received a great deal of attention. Using the framework of Ritchey (1990), Melick and Thomas (1997) constructed implied distributions using the multi-log-normal method. Melick et al. applied this framework to options on crude oil futures with three log-normal functions. Bahra (1997) reviewed various techniques for estimating the risk-neutral distribution function of an underlying asset price from the prices of options and derived the two-log-normal framework for estimating the risk-neutral distribution using observed market data. Subsequently, Dinenis et al. (1998) and Gemill et al. (1999) also used this two-log-normal framework to study the "usefulness" of events embedded in currency options.

In similar spirits to Bahra (1997), Dinenis et al. (1998) investigated the implied risk-neutral distribution around the exit of Sterling in 1992 and Gemill et al. (1999) studied the FTSE 100 index options over the 1987-1997 period. Dinenis et al. suggested that the two-log-normal framework was able to provide critical information in regard to the exit of Sterling whilst Gemill et al. found that although the two-log-normal model fitted the data significantly better than the Black-Scholes model, the out-of-sample performance was only marginally better. Gemill et al. also tested the "usefulness" of their model during elections and a number of market crashes. Despite options are forward-looking instruments, Gemill et al. concluded that implied distributions did not anticipate various market crashes under study and suggested that risk-neutral distribution could only help in telling a "market story" during elections.

Later, Campa et al. (1998) studied implied exchange rate distributions of European Monetary System cross-rates using three smoothing methods: implied binomial, two-log-normal and cubic spline approaches. Campa et al. found that risk-neutral distributions fluctuated widely from week to week without apparent reason. They stipulated that the two-log-normal distribution might impose too rigid a structure on the resultant risk-neutral distribution and argued that the two-log-normal approach made little economic sense.

2.4.2.3 Approximating the Risk-Neutral Density Distribution

Another vital development in recovering risk-neutral distribution is specialized to the problem of option valuation where the underlying security distribution, if not log-normal, can be approximated by a log-normally distributed random variable. Jarrow and Rudd (1982) were first to derive a theoretical framework to include the influence of skewness and kurtosis in pricing option. Their idea was motivated from the fact that a large class of valuation problems where the underlying distribution was itself a convolution of other distributions. In such situations, partial information concerning the underlying distribution may be known (e.g. its moments may be tabulated) but the distribution function itself may be so complex as to prevent direct integration. Jarrow et al. adjusted the Black-Scholes formula by approximating the true distribution with log-normal distribution and the resulting option pricing equation could be viewed as a linear combination of the Black-Scholes solution plus some adjustment terms that accounted for the discrepancies between skewness and kurtosis of the log-normal distribution and the true distribution. Later, Corrado and Su (1997) used Jarrow et al.'s method to investigate the S&P 500 index option market and found that the volatility smile was effectively flattened. Corrado et al. concluded that skewness and kurtosis added to the Black-Scholes formula significantly improved accuracy and consistency for pricing deep in-the-money and out-of-the-money options. Following Jarrow et al.'s footstep, Rubinstein (1998) applied Edgeworth expansion directly to discretise risk-neutral distribution and valued options in conjunction with the method of implied binomial tree. Investor's opinions about skewness and kurtosis could be introduced to the risk-neutral distribution and this model could also be used to value American as well as exotic options.

2.4.3 Indirect Approach

Indirect/implied approach employs the no-arbitrage condition to price options. The use of implied approach is motivated by the "beliefs" that both exotic and vanilla instruments should be priced based on the same set of information and therefore they are expected to deviate consistently from the theoretically correct prices by a similar amount. Consequently, traded European call and put options can be used to hedge the more complicated over-the-counter instruments even if the products included in the hedge may not be correctly priced. Since Breeden and Litzenberger (1978) demonstrated that risk-neutral distributions could be recovered from options by pricing butterfly spreads and expressed as the second derivative of the call option price with respect to the exercise price, recent developments have considered implied tree models that incorporate observed volatility structures into the option pricing process[20]. Methods of incorporating the volatility smile into tree-based models have been suggested by Longstaff (1990), Rubinstein (1994), Derman and Kani (1994) and Dupire (1994) for European options. The following sections discusses different types of implied tree models.

2.4.3.1 Implied Tree Assumptions

The basic assumption for implied tree model is that risk-neutral distribution assumes a specific functional form and the stochastic process followed by the stock price S in a risk-neutral world is governed by:

$$dS = rSdt + S\sigma(S,t)dz$$

[20] Jackworth (1999) and Flamouris (2001) provide a good review for the development of implied models.

The above diffusion equation is closely related to the original Black-Scholes model except that local volatility $\sigma(S,t)$ is no longer constant but depends on stock price and time. It is important to note that no functional form is prescribed for local volatility in the implied tree technique. Instead, special rules are developed for deducing the risk-neutral path probabilities, Arrow-Debreu prices[21] , and transition probabilities for stock price movements in the tree from one time level to the next in such a way that the market prices of options can be reproduced with the tree used in a no-arbitrage fashion. Thus, given N different states, the time t price of a contingent claim expiring at time T is given by:

$$\begin{aligned}\Pi(t) &= \sum_{s=1}^{N} V(s)p(s) \\ &= \sum_{s=1}^{N} V(s)e^{-r(T-t)} \frac{p(s)}{e^{-r(T-t)}} \\ &= \sum_{s=1}^{N} V(s)e^{-r(T-t)}\pi(s)\end{aligned}$$

where V is the payoff function, p the Arrow-Debreu price and $\pi(s)$ sums to one. $\pi(s)$ can be viewed as the risk-neutral probability. When the state space is continuous, the price of a contingent claim is derived by integrating the payoff over the risk-neutral density function of the underlying asset and then discounting at the risk-free rate:

$$\Pi(t) = e^{-r(T-t)} \int_0^{\infty} V(s)f(s)ds$$

where f is the risk-neutral density function.

2.4.3.2 Rubinstein Model

Given a set of option prices at maturity, Longstaff (1990) assumed a uniform probabiltiy distribution between strike prices at end nodes and used them to price options. Subsequent research by Rubinstein (1994) found that Longstaff's method could frequently produce negative probabilities. Rubinstein started with a priori distribution and built a binomial tree backward from options at a single expiration. Final probabilities were extracted by a

[21] The Arrow-Debreu price is the discounted expected price of a security at a particular state that pays one unit of currency assuming other states pay nothing. The disadvantage of adapting this methodology is that only

nonlinear minimisation routine and a set of terminal risk neutral probabilities was assigned to the logarithmically equidistantly spaced final nodes. Stock prices were extracted by backward induction until the origin coincided with the spot price. Rubinstein demonstrated that the recovered terminal risk-neutral distribution exhibited a very bumpy behaviour. The disadvantages of this approach were that: 1) it depended on the assumption of binomial path independence and 2) the end risk-neutral distribution assumed no functional form[22] although local volatility could be easily determined by using the above backwards recursive solution procedure.

In the 1996 study by Jackerth and Rubinstein, Jackerth et al. smoothed the risk-neutral distribution by considering alternative optimisation specifications and found that crash was more likely than it was under the assumption of log-normality. Since Rubinstein's implied tree required only one period of option prices at expiration for European option evaluation, investor's biases could easily be introduced to change the terminal distribution of stock returns to enhance pricing flexibility. Later, Rubinstein (1995) demonstrated another implied tree that could be used to back out risk-neutral probabilities with dividend payout. Parameters could be solved by working backwards recursively from the end of the tree and this tree could be used for American options.

2.4.3.3 Dupire Model Model

Dupire (1994) used the forward Fokker-Planck equation to derive a continuous time solution that relates option prices and local volatility:

$$\sigma^2(K,T) = \frac{qC + \frac{\partial C}{\partial T} + (r-q)K\frac{\partial C}{\partial K}}{\frac{1}{2}K^2\frac{\partial^2 C}{\partial K^2}}$$

where T is maturity, q the dividend yield, r the risk-free rate, C the call price and K the strike price.

European options can be priced because an call or a put option can be expressed as a linear combination of the constituent payoffs at each terminal state.

[22] It is thus difficult to use it for hedging because it does not describe the underlying asset's dynamics.

Implied volatility varies with time to expiration and strike. In contrast, local volatility implies a variation with future index level and time and behaves much like the instantaneous volatility. Dupire's idea was to extract implied distribution and to construct the whole diffusion process that was consistent with the market prices. The above relationship, however, is not so universal since it holds only because it will satisfy a specific set of strike prices and maturities. In addition, Dupire's method was limited to call options; put option prices can only be extracted from put-call parity but these are not market prices. In order to back out the local volatility function, the above formula requires the use of the first and second derivatives. Zou and Derman (1996) applied the Edgeworth expansion method to approximate the second derivative. The first derivative $\frac{\partial C}{\partial T}$ could be obtained by interpolating the volatility term-structure. It turned out that the second derivative was a probability density function and could be obtained by suitable approximations of which the errors had well-defined meaning.

2.4.3.4 Derman-Kani Model

Derman and Kani (1994) improved and extended Rubinstein's model by exploiting all market information in traded European options. Unlike Rubinstein's approach, Derman et al.'s tree did not assume any a priori distribution. Their objective was to construct a tree that was consistent with the observed option prices at all maturities so that $\sigma(S,t)$ could be deduced numerically and exotic options priced consistently under no-arbitrage conditions. Since there were not enough traded maturities and strikes at each node, option prices were frequently interpolated or extrapolated of the existing options' set. This tree was rather sensitive to the interpolation and extrapolation method and required adjustments to avoid arbitrage violation. Later, Chriss (1996b) improved Derman et al.'s methodology and presented an iterative procedure to solve the problem of extracting implied information from American options.

The Derman-Kani and Dupire implied methodologies are conceptually similar. Derman et al. used both call and put options of all striking prices and maturities available on a given underlying asset whilst Dupire used only call options. Dupire assumed a trinomial tree and set risk-free rate equal to zero whilst Derman et al. fitted a binomial tree. Both trees were built in forward fashion and the nodes at each time step were determined by option prices expiring at that time step. Furthermore, no binomial path-independence (BPI) assumption was required, thus eliminating the need of equal path probabilities for all paths leading to the same ending node. They were both able to capture not only the smile, but also its term structure, which was crucial for accurate pricing of American and path-dependent derivatives. Usually over-the-counter or exotic products such as lookback and barrier options are priced via the Derman-Kani methodology.

2.4.4 Other Developments

In the following sub-sections, we will discuss the latest research developments for the implied methodogy.

2.4.4.1 Direct Approach

There are other approaches that make direct assumptions about the distribution of risk-neutral distribution, for example, Malz (1996) assumed a specific jump-diffusion model in order to extract the risk-neutral distribution for the realignment probabilities of the pound sterling in European Monetary System. On the other hand, non-parametric methods are preferred when one has no idea about what type of probability density function or process should be used. Aït-Sahallia and Lo (1998) proposed an arbitrage-free, semi-parametrical kernel regression model that required no need for choosing any a prior distribution for the risk-neutral distribution and no parametric restrictions on the underlying asset's price dynamics.. Unlike the implied binomial tree, which is an attempt to obtain the risk-neutral distribution that comes closest to correctly pricing the existing options at a single point in time, the kernel model is an attempt to estimate the risk-neutral distribution as a function of certain economic variables and use many cross sections of option prices. This method requires few assumptions other than smoothness of the function to be estimated and regularity of the data used to estimate it. Besides being able to capture skewness and kurtosis, it is shown to be robust to the potential misspecification of any given parametric pricing formula. However, Aït-Sahallia et al.'s approach is very data intensive, generally requiring several thousand data points for a reasonable level of accuracy.

2.4.4.2 Indirect Approach

Derman, Kani and Chriss (1996) presented a trinomial tree which claimed to have fitted the observed prices better than the Derman-Kani binomial model with two more degrees of freedom. Later, Derman, Kani and Zou (1996) illustrated the way to apply the market's consensus for local volatility deduced from the spectrum of available Black-Scholes implied volatility. Three "rules of thumb" were derived to describe the correct hedge ratio Δ and the relationship between local and implied volatility according to strike prices and index levels. In a study of pricing options using the lattice model, Jackwerth (1997) generalised Rubinstein's model[23] through the introduction of a simple arbitrary weight function. In choosing a piecewise linear weight function, each kink in the weight function allowed one to match the market price of one option and the connecting segments to give structure to the remaining tree. The tree was constructed in a backward fashion and governed by this weight function and had advantages over Rubinstein's that it was guaranteed to have nodal probabilities everywhere positive and below one. In addition, it was able to accommodate any kind of options, e.g. European, American or exotic, with different times to expiration.

2.5 Factors Influencing Option Pricing

If the Black-Scholes model were correct, options that are only differ by strike prices would all have exactly the same implied volatility. In actual markets, however, option prices are affected by supply and demand, taxes, transaction costs, price discreteness, constraints on margin purchases and short sales of the stock etc and they are not necessarily priced according to the Black-Scholes formula. Furthermore, stock returns may not be continuous and discontinuous process like jump-diffusion process may be able to account for the abnormal events that actually observed in the underlying market. Consequently, the assumption that the underlying process is log-normal, as assumed by Black-Scholes formula, will no longer be valid. Altogether, these factors give rise to huge discrepancies for options of same maturity but different strike prices, a phenomenon known as volatility smile[24].

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[23] Rubinstein's model is a special case of Jackwerth's with a linear weight function.

[24] If the options are written on a stock or a stock index, then for data after 1987 crash, it has been found that implied volatility tend to be higher for out-of-the-money puts (in-the-money calls) and lower for in-the-money puts (out-of-the-money calls), than the Black-Scholes model would predict.

2.5.1 Underlying Concepts

2.5.1.1 Observed Biases

MacBeth and Merville (1979) studied options of six common stocks traded on CBOE between December 1975 and December 1976. MacBeth et al. observed that the implied volatility on equity options tended to decline as the exercise price was higher and that in-the-money options with a short time to expiration had implied volatility greater than those with a longer time to expiration. MacBeth et al. also documented that out-of-the-money options with shorter maturities to have implied volatility somewhat lower than longer-maturity out-of-the-money options. MacBeth et al. concluded that the Black-Scholes formula over-priced out-of-the-money options and under-priced in-the-money options. The extent to which Black-Scholes model under-priced (over-priced) an in-the-money (out-of-the-money) option increased with the extent to which the option was in-the-money (out-of-the-money), and decreased as the time to expiration decreased.

Of the many studies that documented the shortcomings of Black-Scholes formula, perhaps the most systematic and complete was that of Rubinstein (1985). Rubinstein examined matched pairs of call option transactions from the Berkeley Options Database to conduct non-parametric tests of the Black-Scholes null hypothesis that implied volatility exhibited no systematic differences across strike prices or time to maturity for otherwise identical options. If deviations from the Black-Scholes model were white noise, the option with the lower strike price would have a higher implied volatility for about half the observations. Rubinstein found that implied volatility tended to be higher for out-of-the-money puts (in-the-money calls) and lower for in-the-money puts (out-of-the-money calls) than the Black-Scholes model would predict. In addition, results were statistically significant but changed across sub-sample periods, indicating systematic deviations from the Black-Scholes model existed but the pattern of deviations varied over time. Rubinstein did not attempt to model these observed biases but suggested that a composite model was necessary to capture these abnormalities.

2.5.1.2 Historical Volatility versus Implied Volatility

Some studies questioned whether volatility forecasts should be based on historical data, implied volatility or some combination of two. The early literature found implied volatility was better at forecasting volatility than estimators based on historical data. In a study of information content in options, Canina and Figlewski (1993) investigated the ability of implied volatility of S&P 100 index options to forecast actual volatility. Canina et al. found that implied volatility had no explanatory power but that estimates of historical volatility could explain some of the realised volatility and concluded that the implied volatility poorly forecasted actual volatility. Later, Jorion (1995) found that implied volatility outperforming historical time-series in a foreign exchange study. Subsequent research has generally supported that implied volatility is the better predictor, but results have been mixed. The debate is still open and no general conclusion can be drawn. Various weighted-average techniques for calculating implied volatility have also been suggested recently, but empirical evidence suggests that the near-the-money option is as good as a weighted average at predicting volatility (Mayhew 1995).

There also appears to have a term-structure of volatility in the options market. For example, when historical volatility is above its mean, there is a great likelihood that it will decline and when historical volatility is below its mean, there is a great likelihood that it will increase. This is called the mean-reverting property. Moreover, the longer the maturity the greater the likelihood that the volatility of the underlying contract will return to its mean. Consequently, there is a tendency for the implied volatility of long-term options to remain closer to the mean volatility of an underlying contract than the implied volatility of short-term options. Thus over long periods of time, historical volatility of the underlying contract will be the dominant force affecting implied volatility. Over short periods of time, however, many other factors can play a role. If the market foresees events which could cause the underlying asset to become more volatile, anticipation of these events might cause implied volatility to change in ways that are not necessarily consistent with historical volatility. In summary, any future event which could have unexpected consequences can have a profound effect on implied volatility. What is certain, however, is that the Black-Scholes assumption of constant volatility is an invalid one.

2.5.1.3 Time-series Properties

After the 1987 market crash, the Black-Scholes model has been proven to have many deficiencies and its accuracy depends on the statistical behaviour up to the first four moments of the underlying asset returns. Particularly, the stock returns distributions seem to exhibit fatter tails towards the left side of the distribution than the symmetric normal distribution does, giving more weight to the probability of future downwards underlying movements. Since the market crash in 1987, many researchers have realised the importance of being able to correctly model skewness, which is a function of the second and third moments of a time-series and kurtosis. Skewness describes the "shape" whilst kurtosis is the "tallness/flatness" of a probability distribution and it can be viewed as the clustering of volatility. Because Black-Scholes formula assumes that volatility is uncorrelated with asset returns, it cannot capture important skeweness effects that arise from such correlation. As a result, the out-of-money options have typically higher implied volatility than at-the-money and in-the-money options.

2.5.2 Overreaction Hypothesis

Since implied volatility provides vital market information about asset pricing, researchers have become more interested in the consistency of implied volatility with historical data. De Bondt and Thaler (1985) tested whether the "overreaction" hypothesis was predictive on NYSE stocks from 1926 through 1982. De Bondt et al. found that the "overreaction" effects were asymmetric and suggested that most individuals tended to overweight recent information and underweight prior data, i.e. investors seemed to attach disproportionate importance to short-run economic developments. Later, Stein (1989) pioneered the examination of the term-structure of the average at-the-money options' implied volatility using two maturities on S&P 100 index options. Evidence suggested that long-maturity options tended to "overreact" to changes in the implied volatility of short-maturity options because investors had a systematic tendency to overemphasise recent data at the expense of other information when making projections. This result was disputed by Diz and Finucane (1993) following their analysis of similar S&P 100 index data. The term-structures of implied volatility had also been discussed by Heynen, Kemna and Vorst (1994). Basing their results upon Duan (1995), Heynen et al. constructed GARCH-type-pricing models for the relation between short- and long-term implied volatility with three different assumptions of stock return volatility behavior, i.e., mean-reverting, GARCH and EGARCH models. This paper concluded that EGARCH(1,1) was best to describe asset prices and the term-structure of options' implied volatility. In addition, Heynen et al. showed that longer-term implied volatility was consistent with forecasts of average volatility in their model, thus rejecting Stein's results that traders overreacted with the arrival of new data. Xu and Taylor (1994) also investigated the term-structure of implied volatility implied by the nearest-the-money options on four Philadelphia Stock Exchange currency options using data from 1985 to 1989. Any number of maturities could be studied in this study but Xu et al.'s simple model could only permit three shapes for a graph of volatility expectations, i.e. constant, monotonic increasing or decreasing as a function of maturity. Xu et al. found that the implied volatility term-structure was significant, and there were frequent crossovers between 15-day and long-term expectations. Consequently, the slope of the term-structure often changed. Xu et al. concluded that volatility shocks were transitory and therefore the currency options market did not overreact.

2.5.3 Information Content

2.5.3.1 Evidence Supporting the Significance of Implied Volatility

Many researchers have studied asymmetry of stock market volatility in the past (e.g. Fama,1965; Officer, 1973; French, 1980). Beginning in the mid 1970's, a number of studies had investigated the information content of observed option prices. They include Latane and Rendleman (1976), Galai (1977), Chiras and Manaster (1978), Schmalensee and Trippi (1978) – in addition to many others. Latane and Rendleman (1976) examined options of 24 companies on CBOE and reported that the weighted average implied volatility was a better predictor of future volatility than historical estimates. Latane et al. concluded that the weighted implied volatility could be used to identify under- and over-priced options. Later, Galai (1977) studied volatility estimates of 32 stocks traded on CBOE from April 1973 to October 1974. Results of ex-post and ex-ante trading experiments indicated that the market did not seem perfectly efficient to market makers and Galai also showed that the Black-Scholes formula was able to differentiate between over- and under-priced options. In a study of 23 company stocks traded on CBOE between June 1873 and April 1975, Chiras and Manaster (1978) reported that implied volatility inferred from option prices had been better predictors of standard deviations of future stock returns than historical estimates. Schmalensee and Trippi (1978) also found implied volatility to be better predictor of future stock price variability than past variability of the underlying security.

2.5.3.2 Evidence Against the Significance of Implied Volatility

Some studies found that implied volatility did not content any useful information. Gemill (1986) compared a wide range of weighting schemes using data from the London Traded Options Market. Gemill found that implied volatility forecasts were only marginally better than the historic-based forecast and better ex-ante forecasts could be obtained by linearly adjusting implied volatility and historic estimates based upon past observations. Randolph et al. (1990) addressed several questions concerning S&P 500 futures volatility based on two December futures contracts. Randolph et al. found that implied volatility did not appear to be a useful predictor of upcoming changes in volatility when observed on a daily basis. Kumar and Shastri (1990) tested the information content of non-dividend paying call options implied in option premia. Kumar et al. reported that no abnormal profits could be made from this information. Therefore, the option market's assessment of the stock prices contained no extra information in regard to stock market prices.

2.5.3.3 Other Developments

Other results are mixed. Baroni-Adesi and Morck (1991) tested whether monthly observed option prices predicted ex-post calculated option prices efficiently on S&P 100. Baroni-Adesi et al. reported that implied volatility consistently over-estimated ex-post observed index variability. Baroni-Adesi et al. also found that observed option prices appeared to incorporate good predictions of index variability over the remaining life of the option before the 1987 crash but its predictive power was less impressive after the crash. Day and Lewis (1992) used weekly prices of call options on the S&P 100 index to study the relative forecasting power of implied volatility versus historical data by adding implied volatility as an explanatory variable in GARCH and EGARCH models. Day et al. found that for the OEX options, both implied volatility and historical data contained incremental information about future volatility. However, Day et al. could not make any statement concerning the relative information content of GARCH forecasts and implied volatility. Lamoureux and Lastrapes (1993) also performed an analysis[25] similar to Day and Lewis with daily data on individual at-the-money stock options. Their hypothesis was that if markets were informationally efficient, then information

[25] The authors examined a class of stochastic volatility option pricing models represented by Hull and White (1987) in which volatility risk is unpriced.

available at the time market prices were set could not be used to predict actual return variance better than the variance forecast embedded in the option price, which represented the subjective expectation of the market. Lamoureux et al.'s findings showed that implied volatility tended to underpredict realised volatility whilst forecasts of variance from past returns contained relevant information not contained in the forecast constructed from implied volatility.

Other important studies on the subject of information content include Beckers (1981) and Jorioin (1995). Beckers (1981) studied CBOE and NYSE call options and proposed a simple ad hoc procedure to adjust the implied volatility calculations for dividend payments. By using a simple regression model, Beckers concluded that most of the relevant information was reflected in at-the-money options. Jorion (1995) conducted an excellent study on the information content and one-day predictability of implied volatility derived from CME currency futures and options from 1985 to 1992. Jorion found that implied volatility had some useful information for next-day volatility. Although implied volatility was an estimate, results indicated that options provided informative forecasts of future volatility that were superior to those of time-series models such as GARCH(1,1) and MA(20).

2.5.4 Negative Relationship Between Returns and Volatility

2.5.4.1 Evidence Supporting Leverage Effect as Sole Explanation for Asymmetries

Black (1976) was first to observe the "leverage effect" for individual stocks. Black hypothesised that large declines in equity would raise the debt-to-equity ratio so a negative shock to stock returns would generate more volatility than a positive shock of equal magnitude, and vice versa. This argument suggested that one could expect the volatility of equity to be a decreasing function of price. Later, Cox and Ross (1976) proposed the constant elasticity of variance model. In this model, the stock price volatility was proportional to $\frac{1}{S^{\alpha}}, 0 \leq \alpha \leq 1$. Cox et al. assumed that fixed costs had to be met regardless of the firm's operating performance and it had the effect of increasing volatility when the stock price declined, and vice versa. This model was consistent with the pattern of implied volatility observed for equity options. In a study of weekly implied volatility of six low dividend-yields common stocks traded on CBOE from April 1974 to May 1975, Schmalensee and Trippi (1977) found evidence against the hypothesis that implied volatility was unforecastable. Schmalensee et al. concluded that implied volatility was very sensitive to the direction of movement of the stock price, generally rising when the stock price fell. Gesky (1979) viewed the equity in a levered firm as a call option on the value of the firm, *V*, with its strike price equal to the face value of outstanding debt, *A*. Thus an option on stock of the firm that expired earlier than the debt maturity could be regarded as an option on an option on *V*. Gesky model posited that if the volatility of *V* and that the amount of debt *A* were constant, the volatility of the stock would be negatively correlated with *V*. This pattern was broadly consistent with the implied volatility observed for equity options. In the 1986 study by Chance, Chance used both transaction prices and bid-ask prices of the first four months of S&P 100 call options in 1984 to examine the behaviour of implied volatility across exercise prices and expirations. Chance found that implied volatility tended to decline with higher exercise prices.

2.5.4.2 Evidence Against Leverage Effect as Sole Explanation for Asymmetries

Some studies stated that leverage effects could not be the sole explanation for the negative relation between returns and volatility. French et al. (1987) examined the relation between excess monthly returns on common stocks and predictable volatility of S&P 500 from January 1928 through December 1984. French et al. constructed monthly variance estimates by averaging the squared daily returns. Results showed that the expected market risk premium was positively related to the predictable volatility of stock returns. In addition, French et al. found evidence that unexpected stock market returns were negatively related to the unexpected change in the volatility of stock returns. However, French et al. concluded that leverage was probably not the sole explanation for the negative relation between stock returns and volatility. Another explanation of "leverage effect" concerns what Rubinstein (1994) called "crashophobia". This study stated that traders were concerned about another crash similar to that experienced in October 1987 so traders priced options accordingly. There also appeared that the option-implied probability distribution for a stock price had fatter left tail than the probability distribution calculated from empirical data on stock market returns. Market dynamics might also be responsible for the observed asymmetries in the market. Antoniou et al. (1998) used the Glosten et al. (1993) conditional volatility model to examine the impact of futures trading of six countries on stock index volatility. Antoniou et al.'s results suggested that the onset of futures trading had had a major effect on the dynamics of the stock market. This evidence was inconsistent with the leverage effect being the sole explanation for asymmetries. Antoniou et al. suggested that market dynamics was a much better explanation than leverage alone.

2.5.5 Persistency of Volatility Shocks

2.5.5.1 Evidence Against Persistency of Volatility Shocks

Poterba and Summers (1986) evaluated the changing risk premium hypothesis and examined the influence of changing stock market volatility on the level of stock prices when both volatility and risk premia followed an AR(1) process. Basing their results upon impulse response analysis, Poterba et al reported that shocks to volatility decayed rapidly and therefore could affect required returns for only very short intervals. They concluded that shocks in volatility through their influence on investors' risk premia were not persistent and would not have any substantial effect on stock market values. Later, Schwert (1990) analysed the behaviour of stock return volatility using S&P 500 daily data from 1885 through 1988. Schwert used a 22-order autoregression model to remove autoregressive and seasonal effects from daily data and found that stock volatility rose and fell faster around October 19, 1987 than historical evidence would imply. Most importantly, Schwert found that implied volatility was lower than those from predictions of the regression model. The lower levels of implied volatility were an indication that volatility was not persistent and traders could expect volatility to return to lower levels soon.

2.5.5.2 Structural Change as Explanation of Persistency

A common finding when the GARCH model is applied to high frequency asset price data is that shocks to variance are strongly persistent. In an examination of 30 randomly selected common stock daily return data from January 1, 1963 through Novemeber 13, 1979, Lamoureux and Lastrapes (1990) found that it might be misleading to take full account of strong persistence, i.e. IGARCH behaviour, in GARCH literature. The high persistence in variance in daily stock returns data was due to time-varying GARCH parameters. The time-varying GARCH parameters were the results of time-varying unconditional mean. Lamoureux et al. allowed for deterministic or structural shifts in the unconditional variance of the stochastic process and argued that such shifts, if unaccounted for, might bias upward GARCH estimates of persistence in variance. The GARCH process was also used by Engle and Mustafa (1992) to study S&P 500 options and their implied conditional volatility. The GARCH(1,1) model indicated very strong persistence of the conditional variances from the observed option prices, and the degrees of persistence of volatility shocks implied by options on the S&P500 closing prices was found to be similar to that estimated from historical data on the index itself. However, the GARCH model exhibited weak persistence of conditional volatility and a low half-life of volatility following the October 1987 crash. This evidence suggested that the option market participants favoured a model which implied structural change of conditional variance. In a 1997 study by Pericli and Koutmos, Pericli et al. examined the impact of introduction of futures on both the conditional mean and the conditional variance by including structural dummy variables in the EGARCH model. Pericli et al.'s findings suggested that there had been significant structural changes in the distribution of returns in the S&P 500 the period following the flexible exchange rate regime. In contrast to the results of Antoniou and Holmes (1995) for the U.K. market, Pericli et al. concluded that the introduction of index futures and options had produced no structural changes on volatility.

2.5.5.3 Identifying Structural Breaks

According to Lamoureux et al. (1990), the longer the sample period the higher the probability that structural shifts will be present. Ignoring simple structural shifts in unconditional volatility can lead to the spurious appearance of extremely strong persistence in variance. An explanation of high persistence, i.e. the sum of AR and MA parameters of GARCH model is close to one, might also be due to instability of unconditional variance in the samples. Including dummy variables to account for regime changes diminishes the degree of ARCH/GARCH persistence. The difficulty associated with inclusion of dummy variables is that it is extremely easy to falsely use inappropriately timed dummy variables.

Hamilton's (1988, 1989) method of estimating non-stationary time series may prove to be productive to develop means of identifying the timing of structural shifts. The motivation for using this model comes from the high degree of estimated persistence in volatility observed after fitting the ARCH-type models. The regime-switching models seek to capture discrete shifts in the behaviour of financial variables by allowing the parameters of the underlying data-generating process to take on different values in different time periods. Later, Hamilton (1990) studied a regime-switching model with constant moments in each regime and estimated parameters based on the maximum log-likelihood function of the probability of switching regimes. Hamilton (1994) simplified the estimation procedures by reformulating the problem in terms of the probability of being in a particular regime, conditional on observable information. Hamilton and Susmel (1994) and Cai (1994) considered switching ARCH (SWARCH) models in which the conditional variance was selected from a number of possible ARCH processes which depended upon the state that eventuated. Such SWARCH models have been applied to stock returns by Hamilton and Suamel (1994) and to U.S. Treasury bill yields by Cai (1994). These Markov models were able to identify multiple documented "structural breaks" without using inappropriately timed dummy variables.

2.5.6 Market Efficiency

The usual way to measure the performance of a volatility prediction model is to assess its ability to predict future volatility. As volatility is unobservable, however, there is no natural metric for measuring the accuracy of any particular model. Realised rates of return, though, allow us to test the efficacy of variance-driven option prices and provide a test for market efficiency with respect to volatility forecasts.

2.5.6.1 Volatility Trading

Black and Scholes (1972) first tested market efficiency of CBOT options market from 1966 to 1969. Using daily data, Black et al. found that profit opportunities vanished after taking account of transactions costs. Later, Galai (1977) examined horizontal delta-neutral spreads and hedges, Bhattacharya (1983) also looked at vertical delta-neutral spreads, and Chiras and Manaster (1978) reported on both types. All three studies used a Black-Scholes call model on data from the mid 1970's and found profits that seemed to be abnormal, yet these authors were reluctant to claim that riskless arbitrage profits existed. Bhattacharya revised the hedge ratio every two weeks for a variable holding period of the spread position. Note that neither Galai nor Chiras et al. revised their hedge ratios over time. This was more problematic for the latter, as their holding period was one month compared to Galai's single day.

At a later stage, researchers have started using volatility forecasting models to access market efficency. Harvey and Whaley (1992) analysed S&P 100 index option-market efficiency using an implied volatility measure as a proxy for conditional volatility. Basing their results upon a regression model, Harvey et al. provided evidence that S&P 500 index's call and put implied volatility changes were predictable in a statistical sense in both in- and out-of-sample analyses. However, Harvey et al. found that, after transaction costs, a trading strategy based upon out-of-sample volatility changes did not generate economic profits and maintained that S&P 100 index option market was allocationally efficient. In a study of trading one-day hypothetical NYSE European straddles, Engle, Hong, Kan and Noh (1993) proposed an elegant trading framework to assess profits from options trading for competing volatility forecasting algorithms and compared them in a simulated market. Since straddle was delta-neutral, there was no need to hedge them. Furthermore, straddle prices were relatively insensitive to dividend payouts. Engle et al. found that abnormal profits earned by the GARCH forecast model were economically significant and dominated those earned by other time-series volatility forecast models such as AR(1), ARMA(1,1) and moving average of squared daily returns. Noh, Engle and Kane (1994) also compared the forecasting ability of implied volatility of S&P 500 European options with that of a GARCH model by trading straddles of maturity longer than 15 days and nearest to the money each day. Noh et al. found that the GARCH forecast method returned a greater profit than the rule based on implied volatility regression model. Welch et al. (1995) devised a comprehensive spreading strategy to arbitrage over- and under-valued calls on the CBOE. Welch et al. used a variable revision procedure and a variable holding period to reduce risk in delta-neutral spreads and discovered that trading of vertical spreads were most profitable. These spreads were profitable after commissions except for public traders after the crash of October 1987; they were more profitable and more numerous before the crash in October 1987.

2.6 Common Diagnostic Tests

The objective of this section is to introduce the basic diagnostic tests used in this dissertation. A thorough examination of the econometrics is beyond the scope of this dissertation. We recommend Hamilton (1994) for a proper treatment of time-series issues. Mills (1993), Enders (1994) and Gujarati (1996) have also an excellent discussion in applied econometric.

2.6.1 Test for Stationarity

Non-stationarity of a financial series is a common phenomenon and it is natural for researchers to investigate whether there is a unit root associated with the log prices of a financial asset, i.e., whether such a series, defined as r_t, is $I(1)$ or not. The standard method of testing for a unit root is the Dickey-Fuller (DF) test. Consider the following AR(1) equation:

$$r_t = \alpha + \beta r_{t-1} + u_t$$

We may test $H_0 : \beta = 1$ *vs.* $H_1 : \beta < 1$ using the t-ratio from the regression of Δr_t on r_{t-1}. The critical values follow the statistics derived by Dickey and Fuller. Augmented Dickey-Fuller test, which includes the lags of Δr_t in the regression, is another popular choice. Note that care must be taken when applying the augmented DF test because different critical values are used for different assumptions of a series, i.e. $\alpha = 0$ or $\alpha \neq 0$ (with or without trend).

2.6.2 Test for Independence

It is sometimes important to know whether a series is independent or not. A time series model is considered as adequate if the residuals are distributed as i.i.d. series. A series cannot be independent if the coefficients of its autocorrelation function (ACF) are non-zero. Instead of testing on the coefficients of the ACF, many sophisticated tests for independence have been developed. The BDS statistic proposed by Brock, Dechert and Scheinkman (1996) is used to test for independence by focusing upon estimated marginal and joint densities. Consider two random variables X and Z. They are independent if:

$$f_x(X) f_z(Z) = f_{xz}(X, Z)$$

where $f(\cdot)$'s are the density functions of the random variables. Next define:

$$v_t = E(f_x(X_t) f_z(Z_t) - f_{xz}(X_t, Z_t))$$

The BDS test can be regarded as testing if the sample mean of v_t is zero. It is still possible that $E(v_t) = 0$ but that the random variables are not independent. In the BDS test of independence of a random variable X, Z is simply the lagged value of X. The densities need to be estimated with kernel estimation. BDS showed that if a time-series is i.i.d. then its BDS statistic is asymptotically standard normal-distributed. According to Pagan (1996), the BDS test is likely to be robust to heteroskedasticity, but not to serial correlation.

2.6.3 Test for Normality

In theory, returns are assumed to be normal but that many studies have shown that financial time-series exhibit densities which have tails that are fatter than the normal and have much higher peaks than the normal around zero, e.g. Homaifar and Helms (1990). Most tests of normality focus upon higher order moments of r_t:

$$E(r_t^3) = 0$$
$$E(r_t^4) = 3[E(r_t^2)]^2$$

The above statistics check whether there is skewness or excess kurtosis in the data. On the other hand, the most widely used tests are:

$$\psi_1 = T^{-1}\sum_{t=1}^{T} 1/\sqrt{6\hat{\sigma}^6}$$
$$\psi_2 = \sum_{t=1}^{T} (1/\sqrt{24\hat{\sigma}^8})(\varsigma_t^4 - 3\hat{\sigma}^2\varsigma_t^2)$$

where $\hat{\sigma}^2$ is the estimate of variance and $\varsigma_t = r_t - \bar{r}_t$.

In applying the above test statistics, one may need to adjust these statistics by accounting for dependence in r_t^k. Rather than focusing upon the higher order moments of returns, it is sometimes more useful to obtain a plot of the density for r_t by using non-parametric estimation methods and to concentrate on certain of the characteristics that stand out from such a visual inspection. An easy way to estimate the density non-parametrically is to use a kernel based estimator:

$$\hat{f}(r) = (1/Th)\sum_{t=1}^{T} K\left(\frac{r_t - r}{h}\right)$$

where K is a kernel and h is the window width for a Gaussian kernel, $h = 0.9\hat{\sigma}_x T^{-1/5}$.

One popular normality test is the Kolmogorov-Smirnov test. The Kolmogorov-Smirnov statistic is defined as:

$$KS = \left[\sqrt{n} - 0{,}01 + \frac{0.85}{\sqrt{n}} \right] D_n$$

$$D_n = \sup \left[| F_n(x) - \hat{F}(x) | \right]$$

where

n = total number of data points

$\hat{F}(x) =$ the normal distribution

$$F_n(x) = \frac{N_i}{n}$$

$N_i = the\ number\ of\ X_i\text{'} less\ than\ x$

The critical values of the Kolmogorov-Smirnov test have been tabulated using Monte-Carlo simulation. When the value of the test statistics is greater than the critical value, then the null hypothesis of normality should be rejected.

2.6.4 Hypothesis Tests for Dependence

Standard likelihood-ratio (LR) procedures may be used to test the hypothesis that no ARCH effects are present in a time-series:

$$H_0 : \alpha_1 = \alpha_2 ... = \alpha_q = 0$$

But the numerical estimation required under the ARCH alternative makes that a rather tedious approach (needs both restricted and unrestricted models). Instead, the Lagrange-Multiplier (LM) approach, which requires estimation only under the null, is preferable. Engle (1982) proposed a simple LM test for ARCH under the assumption of conditional normality that involved only a least-squares regression of squared residuals on an intercept and lagged squared residuals. Under the null of no ARCH effects, $T * R^2$ from that regression is asymptotically distributed as $\chi^2(q)$ where q is the number of lagged squared residuals in the regression, T is the number of observations and R^2 is the coefficient of determination from the regression. But the underlying assumption of conditional normality for the LM test is too restrictive. Thus, less formal diagnostics are often used, such as the sample autocorrelation function of squared residuals.

McLeod and Li (1983) developed the Q^2-statistic for nonlinear serial dependence, following the suggestion that the autocorrelation function of the squares of a time series can be useful for identifying the bilinear type nonlinear time-series. For the time-series ε_t, its Q^2-statistic is given by:

$$Q^2(m) = T(T+2)\sum_{\tau=1}^{m}\frac{\rho(\tau)^2}{T-\tau}$$

$$\rho(k) = \sum_{t=1}^{T-k}(\varepsilon_t^2 - u^2)(\varepsilon_{t+k}^2 - u^2)\Big/\sum_{t=1}^{T}(\varepsilon_t^2 - u^2)^2$$

$$u^2 = \sum_{t=1}^{T}\varepsilon_t^2 / T$$

where $\rho(\tau)$ is the sample autocorrelation function of ε_t^2.

Another similar test is the Box-Pierce test, which is formulated as:

$$Q^*(m) = T\sum_{\tau=1}^{m}\rho(\tau)^2$$

where $\rho(\tau)$ is the sample autocorrelation function of ε_t.

Under the null hypothesis of no autocorrelation in the squared values of the time-series up to lag *m*, the asymptotic distribution of the Q^2 and Q^* statistics are asymptotically distributed as $\chi^2(m)$. If the null for Q^2 is rejected, then nonlinear dependence, such as GARCH, may be present. Note that in the Q^2 test ε^2 are used instead of ε. This is based on the belief that investigation of the autocorrelations of the power transformation of the residuals reveals more information about higher independence of residuals. If the null for Q^* statistic is rejected, then linear dependence may be present. A variant of Box-Pierce test statistic is the Ljung-Box statistic. Ljung-Box statistic has the exact formula of the Q^2-statistic except the time series being investigated is ε_t and it is likely to perform better than the Box-Pierce in small samples.

After determining the parameters for a ARCH-type model, it is often of interest to test the null hypothesis that the standardised residuals, $\varepsilon_t^* = \frac{\varepsilon_t}{h_t} \mid I_{t-1}$, are conditional homoskedastic. The idea is that a model can be judged by how well it removes autocorrelation from ε_t. Therefore, if the model is correctly specified, ε_t^* should behave as white noise. The various diagnostic checks that are commonly used include testing the normality of ε_t^* and considering the sample autocorrelations of ε_t^{*2}. In the large sample Guassian white-noise case,

$$\hat{\rho}(\tau) \overset{i.i.d.}{\sim} N\left[0, \frac{1}{T}\right], \tau = 1,2,...$$

$$\hat{Q}(m) = T(T+2)\sum_{\tau=1}^{m} \frac{1}{(T-\tau)} \hat{\rho}(\tau)^2 \sim \chi^2(m)$$

where $\hat{\rho}(\tau)$ denotes the sample autocorrelation at lag τ.

2.7 Summary

In this literature review we have covered many issues relating to conditional heteroskedasticity models, stochastic models and deterministic implied models. Whilst deterministic volatility models produce term-structure effects, stochastic volatility can simultaneously explain these patterns as well as skewness and kurtosis effects. As far as term-structure patterns are concerned, it seems appropriate to use mainly at-the-money volatility to derive empirical estimates of the diffusion parameter of the volatility process.

Research in implied option pricing is expanding fast. All of the theories that were initially used exploited information from the underlying asset's time-series. The implied models, however, suggest that information embedded in option prices should be used directly without having to be filtered through the underlying asset's properties. The criteria for the goodness of a distribution more often are the fit it provides to the observed option prices and less frequently its ability to forecast the statistical properties of future data.

Despite extensive research, there still remains no general agreement as to how to condition such stochastic and implied models for the asymmetric nature of stock return volatility. Conditional heteroskedasticity volatility models seem to be more mature and robust for researchers to forecast volatility.

CHAPTER 3 A Report on the Properties of the Term-Structure of S&P 500 Implied Volatility

Abstract

This chapter examines the observed market anomalies in the term-structure of implied volatility of S&P 500 futures options between 1983 and 1998. Rigorous filtering procedures are applied to remove uninformative options records and we analyse in excess of 250,000 option prices in a span of sixteen years. Prior to this research, past papers have always examined the term-structure of implied volatility only for particular at-the-money contracts. The new aspect of this research is that we define relative implied volatility as implied volatility normalised by its corresponding at-the-money implied volatility for each maturity group. Consequently, each option group's relative implied volatility depends on the level of the at-the-money implied volatility and therefore implied volatility term-structure can be investigated. Contrary to the basic assumptions of the Black-Scholes formula, implied volatility exhibits both smile effects and term-structure patterns. Term-structure evidence reveals that smile effects are strongest for short-term options, indicating that short-term options are most severely mispriced by the Black-Scholes formula. Furthermore, at-the-money implied volatility is fitted to a harmonic model. Specific properties of time-series behaviour of implied volatility for different maturity groups are characterised. In addition, we find evidence that option prices are not consistent with the rational expectations under a mean-reverting volatility process. Finally, observed option prices are used to judge whether moneyness biases are consistent with the skewness of the risk-neutral distribution derived from any specific distributional hypothesis. Skewness premiums results agree with the term-structure analysis that the degrees of anomalies in the S&P 500 options market have been gradually worsening since around 1987. As correlation may be responsible for skewness, our diagnostics suggest that leverage and jump-diffusion models are more appropriate for capturing the observed biases in the S&P 500 futures options market. The intermediate results obtained in this chapter are complementary to Chapters 4 and 5 which apply different modelling techniques to account for the observed term-structure biases in the S&P 500 options market.

3.1 Introduction

3.1.1 Background of the Study

Accurate valuation of options or related derivatives requires the understanding of the dynamics of implied volatility. Surprisingly, little research has been conducted into the properties and evolution of implied volatility. The modelling of the term-structure of implied volatility has been discussed by many researchers, e.g. Rubinstein (1985), Stein (1989), Diz and Finucane (1993), Heynen, Kemna, and Vorst (1994) and Xu and Taylor (1994). Rubinstein (1985) documented that implied volatility of exchange traded call options between August 1976 and August 1978 exhibited a systematic pattern with respect to different maturities and exercise prices. Rubinstein's most intriguing result was that the direction of bias changed signs between sub-periods, implying that skewness of the risk-neutral density changed over time. Subsequently, numerous efforts have been made to investigate the mean-reverting process and term-structure of implied volatility. Stein (1989) pioneered the examination of the term-structure of the average at-the-money options' implied volatility using two maturities on S&P 100 index options. By using a mean-reverting volatility model, evidence suggested that long-maturity options tended to "overreact" to changes in the implied volatility of short-maturity options because investors had a systematic tendency to overemphasise recent data at the expense of other information when making projections. This result was disputed by Diz and Finucane (1993) following their analysis of similar S&P 100 index data. The term-structure of implied volatility has also been discussed by Heynen, Kemna and Vorst (1994). Basing their results upon Duan (1995), Heynen et al. derived the term-structures of implied volatility for EGARCH, GARCH and a mean-reverting stochastic model in a similar way to Stein (1989). Only two values of time-to-maturity were investigated and Heynen et al. concluded that EGARCH gave the best description of asset prices of the term-structure of implied volatility. Xu and Taylor (1994) also studied at-the-money currency options and used a mean-reverting volatility model to establish relationships between long- and short-term expectations of implied volatility for any number of maturity T. Xu et al.'s model could explain the time-varying crossovers of implied volatility at different maturities but it did not emphasise the effects of volatility smile. Insofar past research has mainly focused on "fitting" a theoretical option model to the observed biases in a particular options market from an arbitrarily short span of data for at-the-money contracts. Since the term-structure of implied volatility reflects the time-varying market expectations of asset volatility over different time horizons, it is imperative to focus on a single market and gain a thorough understanding of its behaviour.

3.1.2 The Problem Statement

This chapter examines the empirical behaviour of S&P 500 futures option's implied volatility using daily data from 1983 through 1998. We consider this research work one of the most comprehensive empirical studies of S&P 500 implied volatility term-strucutre in literature to date. Our primary objective is to observe, characterise and analyse the patterns of the term-structure of implied volatility in the S&P 500 marketplace. Particularly, we focus our study on at-the-money implied volatility and identify specific properties for its term-structure. The second objective is to investigate whether option prices are in line with the rational expectations hypothesis under a mean-reverting volatility assumption. The final objective in this research is to identify what types of option models would be consistent with the observed moneyness biases in the S&P 500 options market. Intermediate results obtained in Chapter 3 can also help facilitate our research efforts in modelling volatility in Chapters 4 and 5.

3.1.3 The Significance of the Study

The term-structure of implied volatility reflects the time-varying market expectations of asset volatility over different time horizons. Despite the extensive investigation and the evidence accumulated thus far on the term-structure of implied volatility, no past study has ever considered a large empirical study of the S&P 500 implied volatility term-structure. Prior to this research, past papers have always examined the term-structure of implied volatility only for particular at-the-money contracts. The purpose of this chapter is to fill this gap in the literature by utilising all available daily S&P 500 futures option prices from the inception of S&P 500 futures option in March 1983 to December 1998. Although descriptive in nature, we extend previous term-structure work in several ways:

i) The new aspect of this research is that we define relative implied volatility as implied volatility normalised by its corresponding at-the-money implied volatility for each maturity group. The use of relative implied volatility allows the measurement of relative degrees of anomaly in the implied volatility term-structure across a broad moneyness range;

ii) Our sample period is more extensive, making the results more statistically reliable.

Our research is of importance to institutional investors because S&P 500 products are one of the most liquid contracts in the financial world and their immense size guarantees that they are ideal as a hedging tool[26]. If the term-structure of implied volatility shows any specific patterns then some models, such as stochastic volatility models or GARCH-type models, may be more suitable to make adjustments for market imperfections that cannot be explained by the Black-Scholes formula. These adjustments could be important even for small levels of predictability, especially for longer maturity options.

[26] It is the second most liquid options traded on CME after currency options.

3.1.4 Organisation

The remainder of this chapter is organised as follows. Section 3.2 describes how to construct the term-structure of implied volatility. Section 3.3 introduces the dataset. Section 3.4 examines the properties of the S&P 500 implied volatility term-structure. Section 3.5 summarises the results.

3.2 Methodology

This study is descriptive and uses a number of empirical techniques to characterise the term-structure of implied volatility of the S&P 500 options market. The major contribution that permits us to study the term-structure of implied volatility is that we use relative implied volatility to organise the implied volatility term-structure data. This section gives special emphasis to the construction of relative implied volatility. In addition, we explain the reasons for using S&P 500 futures options in this study as opposed to S&P 500 spot options. Finally, we discuss the methods and strategies used in our term-structure analysis.

3.2.1 Relative Implied Volatility

This study examines the observed market anomalies in the term-structure of implied volatility of S&P 500 futures options between 1983 and 1998. Since the term-structure of volatility is time-varying, one of the challenges in studying S&P 500 futures options is to find a consistent and meaningful way to compare implied volatility for different maturity and moneyness groups. There are many ways to define options' moneyness. It is a common practice for researchers to use either $F-X$ or F/X to represent moneyness. $F-X$ is the intrinsic value of a call. It is an absolute measure of deviation of an option price from a particular strike. Thus the value $F-X$ does not manifest itself as a common measure to compare options with different underlying and strikes, i.e. options on the S&P 500 futures index of different maturity months and strikes. Conversely, the F/X ratio is a more flexible measure of moneyness. This ratio readily allows options of different strikes under the same underlying to be compared but it only deals contracts with the same maturity, i.e. options on the S&P 500 futures index of different strikes. Recently, Figleski (2002) classified options in a relative way by moneyness as a function of how many standard deviations, in terms of $\sigma\sqrt{T}$, that the strike price was away from the current asset price, where σ was the Black-Scholes implied volatility and T was time to maturity for the option. This formulation has the advantage that the probability an option in a given category that will end up in the money at expiration is largely independent of volatility or option maturity. This moneyness definition, however, is not appropriate for the investigation of the term-structure of S&P 500 futures options' implied volatility because it implies that long-maturity options will cover a much larger moneyness range than short-maturity options even when the term-structure of volatility is flat.

In order to compare implied volatility of different strikes and maturities, we partition the data into 30 sub-groups, i.e. six moneyness and five maturity groups. Contracts are aggregated over maturity ranges and moneyness groups. For each group, the overall implied volatility is then calculated in terms of its average implied volatility. The moneyness ratio ranges from 0.75 to 1.25+ with increment of 0.1. This discretisation of moneyness groups allows allow a thorough examination of smile effects. Table 1 shows the time-to-maturity and moneyness partitioning in this study. For example, group 1 consists of all call/put contracts traded in the market with $0.75 \leq F/X < 0.85$ and $21 \leq T \leq 70$. Next each moneyness group within a particular maturity group is normalised by its corresponding at-the-money maturity group to adjust for maturity effects. Hence, the relative at-the-money implied volatilities for the groups in the highlighted cells in table 2 are normalised to one. This approach is indeed similar in spirit to Rosenberg (1999), which formulated the deterministic implied volatility function through an explicit specification of the at-the-money implied volatility.

Table 1: Time-to-Maturity and Moneyness Groups

Maturity / $(F/X)_i$	21-70	71-120	121-170	171-220	221+
0.75-0.85	1	2	3	4	5
0.85-0.95	6	7	8	9	10
0.95-1.05	11	12	13	14	15
1.05-1.15	16	17	18	19	20
1.15.1.25	21	22	23	24	25
1.25+	26	27	28	29	30

Table 2: Normalised Data Groups

Maturity / Groups	21-70	71-120	121-170	171-220	220+
I	1	2	3	4	5
II	6	7	8	9	10
III	11	12	13	14	15
IV	16	17	18	19	20
V	21	22	23	24	25
VI	26	27	28	29	30

3.2.2 Futures Options versus Spot Index Options

This study investigates the term-structure of S&P 500 implied volatility by employing all available daily option prices from the inception of S&P 500 futures option in 1983 to 1998. From a theoretical viewpoint, implied volatilities of S&P 500 futures options and S&P 500 spot index options are not identical. However, there are several advantages in using the S&P 500 futures options versus S&P 500 spot index options. First, since both futures contracts and futures options are traded on the same CME trading floor, the daily futures options database contains settlement prices which reflects market conditions at the close of trading for each contract. The underlying futures contract is often closed out prior to delivery so that the exercise of the futures option does not usually lead to delivery of the underlying asset. Thus index futures tend to entail lower transactions costs than spot index, leading to a more efficient and liquid market that can more accurately reflect the consensus of investors. Second, in theory S&P 500 index and its futures display very similar characters because arbitrage conditions force the S&P 500 futures to mimic the spot index. Therefore, it is reasonable to expect the volatility of futures prices to be similar to the volatility of spot prices. Third, according to Ramaswamy et al. (1985) and Natenberg (1995), there is very little early exercise premium in S&P 500 futures options. Consequently, S&P 500 futures options and S&P 500 index options are almost identical. Last, the use of options on futures contracts avoids the complication of incorporating dividend information into the option pricing model because futures prices already contain the market's assessment of dividend payout over the life of the futures contract. Therefore, providing that futures contracts and options expire at the same time, implied volatilities on S&P 500 futures options and S&P 500 index options are almost identical.

3.2.3 Strategies

As a descriptive study, the research reported here examines the observed market anomalies in the term-structure of implied volatility of S&P 500 futures options between 1983 and 1998. Prior to this research, past papers have always examined the term-structure of implied volatility only for particular at-the-money contracts. The data are examined using several strategies. First, exclusionary restrictions are applied to remove uninformative options records from the S&P 500 futures options database. Rigorous filtering procedures are applied to filter our options records and they are described in full detail in section 3.3.4. Second, the Black-Scholes implied volatilities are calculated by employing the quadratic approximation approach developed by Barone-Adesi and Whaley (1987). The term-structure of relative implied volatility is then constructed following the procedures outlined in sections 3.2.1 and 3.3.4 for the period 1983-1998 for the five maturity groups. The evolution of the S&P 500 term-structure of relative implied volatility for each maturity group is then graphically inspected and its general patterns and properties are deduced. Third, we focus our study on at-the-money implied volatility, giving special emphasis to the analysis of the shortest-maturity at-the-money option groups. Two-sample t-statistics are used to investigate the variability of the at-the-money implied volatility term-structure. Furthermore, a simple harmonic model is employed to study the movements between different at-the-money maturity groups, and specific properties of time-series behaviour of implied volatility for different maturity groups are observed. Fourth, we consider whether the implied volatility term-structure of the S&P 500 options market is consistent with the rational expectations hypothesis under a mean-reverting volatility model developed by Stein (1989). Fifth, we apply the skewness premiums technique developed by Bates (1991, 1997) to judge whether moneyness biases are consistent with the skewness of the risk-neutral distribution derived from any specific distributional hypothesis. Since options exist only for specific exercise prices, we construct skewness premiums by interpolating implied volatility for desired exercise prices from a cubic spline fitted through the shortest-maturity implied volatility of call and put options from 1983 to 1998.

3.2.4 Summary of the Methodology

Sections 3.2.1-3 have illustrated the methods and techniques used for the analysis of the S&P 500 implied volatility term-structure. It should be emphasised that this descriptive study requires the use of many numerical techniques. The next section describes the S&P 500 data used in this report.

3.3 Data Description

3.3.1 S&P 500 Futures and Futures Options

The option data used in this study are American options on S&P 500 index futures. S&P 500 futures began trading on March 21, 1982, and options on S&P500 futures commenced the following year on January 23, 1983. The options and futures data are obtained from the Futures Industry Institute covering all reported daily trades and quotes of CME from January 28, 1983 through December 31, 1998. S&P 500 futures options are based on the price of S&P 500 futures but not the underlying S&P 500 index. Upon exercise, a call (put) futures option holder merely acquires a long (short) futures position with a futures price equal to the exercise price of the option and the option holder's account shows an unrealised gain based on the strike and settlement price of the futures contract. The settlement price is calculated as the average of the highest and lowest transaction prices in the last 30 seconds of trading, which reflects market conditions at the close of trading for each contract. Since this research focuses on the volatility of returns resulting from underlying economic factors rather than from the market's microstructure, therefore, price information is restricted to settlement returns[27].

3.3.2 Contract Specifications

CME futures and options trade side-by-side in the same market. They are also open and close at the same time. Because of the low cost of transacting between the two markets, options and

[27] For instance, Randolph et al. (1990) used daily settlement prices on two S&P 500 futures contracts for their study; Bahra (1997) argued that the bias due to asynchronous data could be reduced significantly by using exchange settlement prices rather than intra-day quotes; Rosenberg (1999) used settlement prices on S&P 500 futures and options; Ederington and Guan (2002) used settlement prices to calculate the implied volatility of S&P 500 futures options.

futures prices are likely to be highly synchronised, which alleviates the problem of non-synchronous quotes afflicting markets such as S&P 100 index options. S&P 500 futures have a cash settlement at expiration in December, March, June, and September. Trading of S&P 500 index futures and its options opens at 8:30 a.m. and closes at 3:15 p.m. U.S. Central Time. S&P 500 index futures contracts are extremely liquid and are frequently used by investors for portfolio hedging. The size of one futures contract is $250 multiplied by the index level, where each index point (10 ticks or 100 basis points) is worth $250. The minimum move in the futures price is 0.1 point and this is worth $25.

S&P 500 index futures options expire on the same day as the underlying futures contracts[28]. Since 1987, intra-quarterly options have been introduced to offer at least six shorter-term call and put options for traders. Serial month options only exercise into the next nearest futures contract month, for instance, January and February options exercise into March futures contract, April and May options exercise into June futures contract. Thus on the third Friday of the serial month at 3:15 p.m., options will settle based on the prices of the quarterly futures contracts. In addition, a one-point change in the S&P 500 futures option premium represents the same dollar value of a one-point change in the S&P 500 futures. Furthermore, the set of options contracts available for a given maturity depends upon the past price movements of the stock market during the history of that maturity of option. Strike price increments are generally integers divisible by 25, although strikes that are integers divisible by 5 and 10 may be added.

3.3.3 Approximating Implied Volatility for American Options

Option on index futures is analogous to a stock providing a continuous dividend yield where the dividend yield is equal to the domestic risk-free rate. Because S&P 500 futures option is American and its risk-free rate always positive, there is some chance that it will be optimal to exercise an option early. Thus American futures options are worth more than their European counterparts and put-call parity does not hold. Since there is not any analytic solution available for evaluating American futures options, this study employs the quadratic approximation approach developed by Barone-Adesi et al. (1987) to calculate implied

[28] They usually expire on the third Friday of the delivery month.

volatility[29]. This technique uses interval subdivision method to backout the implied volatility, which can guarantee convergence to a unique solution and is considered more accurate and computationally efficient than finite-difference or binomial method. Although quadratic approximation may not be very accurate for long-maturity options, it is still found to be an efficient and reliable method for short- and moderate-maturity options. A thorough examination of numerical method for the American option problem is beyond the scope of this dissertation. We recommend Ju et al. (1999) for a detailed discussion of efficacy of different approximation techniques for American options.

3.3.4 Filtering

The use of high quality options data is important to the integrity of any credible research. Following Rubinstein (1985), several exclusionary restrictions are applied to remove uninformative options records from our database:

i) Time to maturity fewer than 21 calendar days;

ii) Implied volatility < 4% and > 90%;

iii) Options with F/X ratio less than 0.75;

iv) $C < F - X$ and $P < X - F$;

v) Options with premia ≤ 0.01 index point;

vi) Non-Traded options.

Criterion i) is used to eliminate options with extreme short maturities as their implied volatilities behave erratically. Criterion ii) excludes those unreasonable options records with extreme implied volatilities resulted from our approximations. Criterion iii) removes extreme deep in-the-money put options and deep out-of-the-money call options that may introduce biases to our calculations, as they are very sensitive to a small change in the option prices. Criterion iv) states that American options cannot be less than their intrinsic values, otherwise a riskless arbitrage could arise. Criterion v) is used to exclude options for which the necessarily discrete market prices are likely to distort calculations of implied volatility. Criterion vi) eliminates artificial trading behaviour by floor traders to influence their margin requirements. Before filtering, there were 305,260 call and 354,173 put records with 254 and

[29] We sincerely thank Giovanni Barone-Adesi for making the quadratic approximation program available.

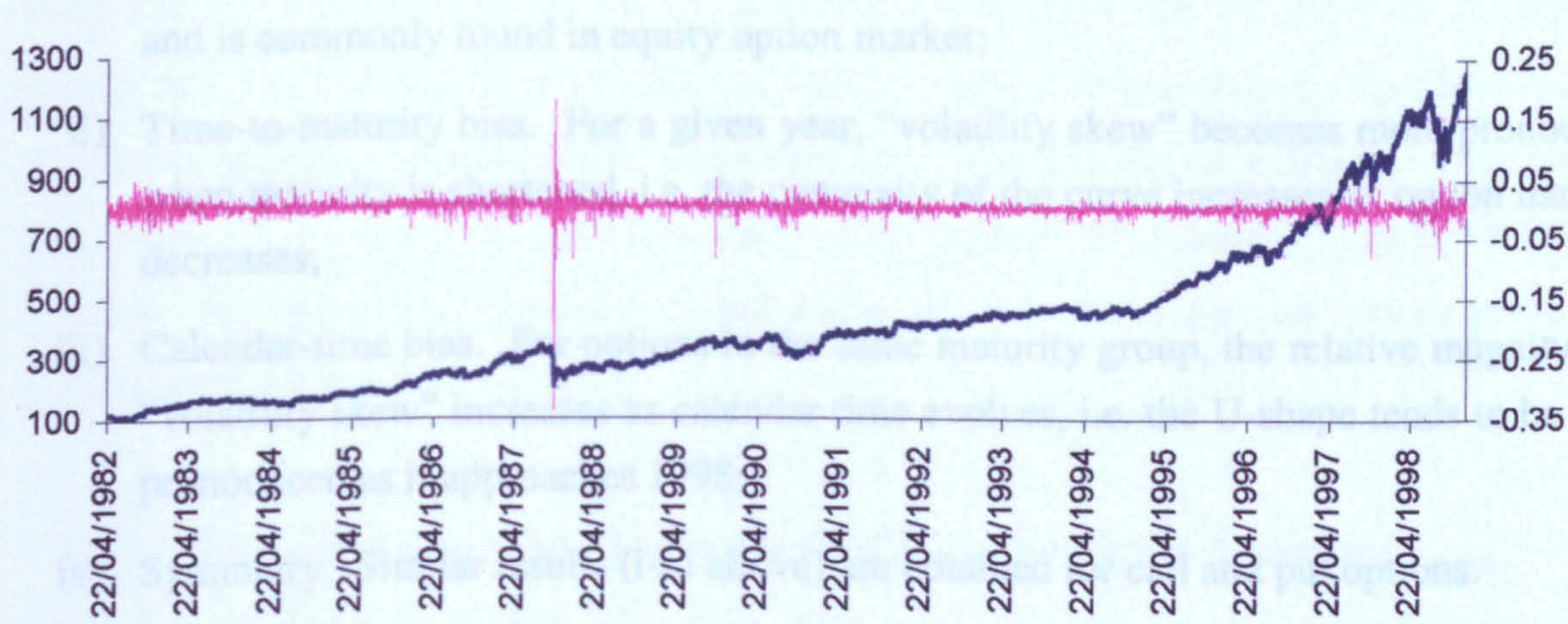
249 strikes for call and put options, respectively. After applying the above filter rules, there are 99,494 call and 149,442 put options with 247 and 234 strikes in our database.

3.4 Results and Analysis

3.4.1 Financial and Political Events for 1983-1998

This study utilises the full history of S&P 500 futures option prices traded on CME for the construction of the relative term-structure of implied volatility. We begin this study by inspecting figure 2, which plots the S&P 500 futures series and its log-returns for the period 1983-1998. A preliminary investigation of figure 2 reveals that the entire return series could be divided into different states of volatility. For instance, the stock market crashed in October 19, 1987, the "Gulf War" in January 1990; the Asian Financial Crisis and default of the Russian Debt Market in 1997 and 1998 all leaded to the increase in the returns volatility. Causal inspection of figure 2 finds that there is considerable volatility clustering as soon as returns volatility jumps. Consequently, it is plausible that returns volatilities are predictable.

Figure 2:S&P500 Futures & Returns: 1983-1998



3.4.2 Properties of Implied Term-Structure

The term-structure of relative implied volatility is constructed following the procedures outlined in sections 3.2.1 and 3.3.4. As a proxy for the risk-free interest rate, we use daily middle rates on U.S. Treasury bills from Datastream matching maturity closest to the expiration date of the options. There are certain improvements that distinguish our research from previous studies:

i) The sample period is more extensive than previous studies. For example, Rubinstein's study used intra-day option records of merely 30 stocks traded on CBOE from August 23 through August31, 1978 whilst we employ sixteen years of options data here;

ii) Call and put options are investigated here whilst most studies used only call options;

iii) The moneyness range is larger. For example, Stein (1989), Diz and Finucane (1993), Heynen, Kemma, and Vorst (1994), and Xu and Taylor (1994) only investigated the term-structure of the at-the-money contracts.

Figures 3 to 12 chronicle the evolution of the term-structure of relative implied volatility for call and put options arranged in five maturity groups over 1983-1998. For ease of comparison, all figures are displayed using the same scaling factor. On inspection of graphical evidence we observe several properties for the implied volatility term-structure of S&P 500 futures options:

i) Moneyness bias. For a given maturity group, the further away is moneyness from the at-the-money region, the more pronounced is the bias, i.e. the lowest implied volatility always occurs near at-the-money regions and the magnitude of bias is generally higher in low strike prices than in high strike prices. This finding is termed "volatility skew" and is commonly found in equity option market;

ii) Time-to-maturity bias. For a given year, "volatility skew" becomes more pronounced when maturity is shortened, i.e. the convexity of the curve increases as option maturity decreases;

iii) Calendar-time bias. For options in the same maturity group, the relative magnitude of "volatility skew" increases as calendar time evolves, i.e. the U-shape tends to be more pronounced as it approaches 1998;

iv) Symmetry. Similar results (i-iii above) are obtained for call and put options.

3.4.2 Interpretation of the Implied Term-Structure Results

Results i) and ii) in section 3.4.2 are in line with general literature[30] concerning moneyness and maturity biases. Observed irregularities in relative implied volatility constitute strong

[30] See Rubinstein (1985) and Canina et al. (1993).

evidence against the hypothesis that the Black-Scholes' implied volatility is the market's fully rational volatility forecast. The U-shape can be the result of: 1) illiquid market; 2) non-normality returns distribution. Bid-ask spread in illiquid market is typically huge for out-of-the money options and this can artificially introduce high volatility to out-of-the-money options, forming the basis for "volatility skew". But perhaps the more credible reason responsible for the observed U shape is non-normality in the returns data. The "volatility skew" could also be a result of active use of portfolio insurance policies to protect investors' portfolios, thus creating a surging demand for out-of-the money put options and driving up their prices and volatility. Our term-structure evidence also shows that the convexity of relative implied volatility of longer-term options is relatively insensitive to evolution of calendar time. Thus smile effects are strongest for short-term options, indicating that short-term options are the most severely mispriced by the Black-Scholes formula and present perhaps the greatest challenge to any alternative option pricing models.

Result iii) provides an important description of the evolution of "smile effects" in the term-structure of the S&P 500 market. Strong evidence supports the notion that implied volatility has been getting more skewed as calendar time evolves. Moreover, the relative degrees of anomalies decrease as term-to-maturity lengthens. Once again this evidence suggests that the Black-Scholes formula severely misprices short-term options. On the other hand, result iv) reveals that implied volatility of call options in a given in-the-money (out-of-the-money) category is quite similar to implied volatility of put options in the opposing out-of-the-money (in-the-money) category, which is generally true regardless of sample period or term-to-maturity. Such similarities in pricing structure exist between call and put options mainly due to the working of the put-call parity.

3.4.3 Characters of At-the-Money Implied Volatility Term-Structure

Having inferred the general properties of the relative implied volatility term-structure of S&P 500 futures options in section 3.4.2, this section focuses on characterising at-the-money implied volatility. The reasons for studying at-the-money options are: 1) at-the-money options are more liquid; 2) at-the-money options are less contaminated by microstructure data problems. Consequently, it is generally believed that the Black-Scholes implied volatility is empirically indistinguishable from most stochastic and conditional volatility models when the options are at-the-money and have short times to expiration. For the above reasons, we investigate three important features of the at-the-money implied volatility term-structure: 1) variability; 2) mean-reverting property; 3) term-structure consistency.

3.4.3.1 Variability of Implied Volatility

The Black-Scholes formula assumes there is a constant implied volatility term-structure. The two-sample t-test is employed to investigate the variability of at-the-money implied volatility term-structure for the period 1983-1998. Tables 3 and 4 give the two-sample t-statistics for equal means but unequal variances for the average implied volatility of the at-the-money call and put options, i.e. $0.95<F/X<1.05$. The two-sample test is defined as follows:

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

$$t = \frac{X_1 - \bar{X}_2}{\sqrt{s_1^2 / N_1 + s_2^2 / N_2}}$$

where $N_1, N_2, X_1, \bar{X}_2, s_1^2, s_2^2$ are the sample sizes, sample means and variances for sample groups 1 and 2, respectively.

When this test is performed in the highlighted areas in tables 3 and 4, we reject the null hypothesis that the two sample means are equal at 95% confidence interval where sample 1 is the 21-70 at-the-money call (put) option group. It is also evident in tables 3 and 4 that the term-structure of at-the-money implied volatility is more variable in the 1990's than the 1980's. Statistically speaking, the systematic divergences of the term-structure can be traced back to 1987 since when the magnitudes of t-values have become significantly and systematically higher. This high variability could be a result of frequent crossovers of implied volatility at different maturities such that a longer-maturity's implied volatility and a shorter-term option' implied volatility move in opposite direction.

Table 3: t-statistics for equal means but unequal variances for at-the-money calls

Sample 1: 21-70 Call Options

Year / Time-to-maturity	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98
71-120	2.4	2	1.1	-3.5	9.7	3.8	-0.9	-0.9	-0.7	-7.1	-9.6	-10.1	-3.8	-8.8	-1.4	2.8
121-170	-3.9	1.7	1.6	0.7	10.6	-2.9	-7	2.4	-5.1	-18.6	-21.5	-19.5	-10	7.9	4.5	-5.2
171-220	-0.2	0.3	-0.5	1.2	15.4	1.3	-2.8	2.6	2.2	-12.5	-20.6	-24.6	-8.5	-7	0.9	-6.3
220+	-0.7	6.3	1.4	3.7	0.8	-1.3	-1.8	1	-5.4	-24	-30.6	-27.9	-20.5	-2	3.6	-4.7

Table 4: t-statistics for equal means but unequal variances for at-the-money puts

Sample 1: 21-70 Put Options

Year / Time-to-maturity	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98
71-120	1.6	2.1	0.4	-3.6	7.2	4.7	2.4	0.8	2.9	-4.8	-7.0	-9.8	-3.8	-5.9	-1.4	0.8
121-170	-3.0	3.0	0.9	0.5	13.0	-2.1	-6.1	2.9	-3.3	-19.6	-13.4	-13.0	-10.0	7.9	-2.4	-0.1
171-220	0.9	4.7	0.5	0.3	13.5	2.1	-1.8	4.8	0.0	-11.5	-13.8	-14.8	-8.5	-2.0	-2.1	-2.8
220+	----	1.5	1.0	3.6	1.5	-1.2	-1.9	8.0	-3.3	-20.2	-22.5	-18.8	-20.5	2.3	-1.5	-1.7

3.4.3.2 Mean-Reversion of Implied Volatility

The mean-reversion property is perhaps the most popular and uncontested assumption of modelling volatility. Many researchers have modelled volatility as a mean-reverting process, e.g. Hull and White (1987), Nelson (1991), Stein and Stein (1991), Heston (1993) and Bakshi et al. (1997). Figures 13 and 14 plot the least squares fit through the average call and put implied volatility of the nearest maturity group, 21-70, for a sixth-order polynomial and a line. The fitted curves clearly illustrate that implied volatility has a linear and a harmonic components. The flattened linear components for both call and put options are consistent with the mean-reverting property of stationary volatility processes, i.e. volatility will always tend towards the long-term unconditional mean volatility. In order to characterise the patterns of the at-the-money implied volatility term-structure, we employ a simple harmonic model to analyse the movements between different at-the-money maturity groups:

$$\alpha + \beta * \sin(\omega * t + \theta)$$

where α is the intercept and can be interpreted as the long-term expectation of mean implied/unconditional volatility, β is the amplitude or intensity of fluctuation, ω is the angular frequency and θ is the phase shift in radians which is used to adjust the time lag .

Figures 15 and 16 display the least squares fit through the average implied volatilities of the 21-70 call and put option groups for the harmonic model. Graphical inspection of figures 15 and 16 again shows that the divergence of at-the-money implied volatilities between different maturity groups starts in 1987 and becomes more pronounced in the 1990's. Furthermore, it is evident in figures 15 and 16 that the term-structure of S&P 500 implied volatility frequently inverts so the slope of the term-structure often changes.

Curve-fitting results for call and put implied volatilities are presented in tables 5 and 6. Several observations can be drawn in regard to the results from tables 5 and 6:

i) Observations from α indicate that put options have a higher long-term expectation of implied volatility than call options;

ii) Observations from β imply that put options have a larger magnitude of fluctuation than call options. Furthermore, the shorter the maturity, the larger the β;

iii) Put options have a slightly higher angular frequency ω and a more negative phases than call options in each maturity;

iv) Longer maturity options appear to have a faster rate of change of implied volatility ω.

Result i) provides evidence that put options command a higher premium than call options in each maturity group, which is consistent with Black's leverage effect. A possible explanation for these results is that purchase of S&P 500 futures is a convenient and inexpensive form of portfolio insurance. Thus excess buying pressure of front-month put options may cause prices to increase, resulting in higher puts' implied volatilities. Furthermore, average call and put implied volatilities mean-revert to their long-term mean of 16% and 16.8%, respectively. That is to say that when implied volatility is above its long-term mean level, the implied volatility of an option should be decreasing in the time to expiration, and vice versa. Result ii) demonstrates that shorter maturity options are more variable than longer maturity options. In addition, the variation of put options' implied volatility is higher than call options. The amplitude parameter β can be interpreted as the volatility of implied volatility. Consequently, the 21-70 put option group can be viewed as the most volatile option group.

Result iii) indicates that put options have a higher frequency ω and a more negative phase parameter θ than call options in each maturity. Therefore, put options are perceived to be leading call options. It can be viewed as evidence that put options are more "responsive" to the arrival of new information. Result iv) states that longer-term options have a faster rate of change of implied volatility because ω tends to be a monotonically increasing function of maturity[31], suggesting that longer-term options probably react too "rapidly" to the arrival of new information relative to shorter-term options.

Table 5: Curve-fitting estimations for Average Call Implied Volatility from 1983-1998

Maturity	21-70	71-120	121-170	171-220	220+	All Calls
α	0.15761	0.15739	0.16087	0.15976	0.16207	0.15971
β	0.04503	0.04012	0.03846	0.03618	0.03938	0.03908
ω	0.57206	0.59379	0.61965	0.62885	0.65104	0.61659
θ	-1.30832	-1.54263	-1.92601	-1.8948	-2.28744	-1.82570

Table 6: Curve-fitting estimations for Average Put Implied Volatility from 1983-1998

Maturity	21-70	71-120	121-170	171-220	220+	All Puts
α	0.16779	0.16558	0.16782	0.16527	0.17252	0.16804
β	0.04790	0.04213	0.0418	0.04043	0.03566	0.04061
ω	0.60780	0.63181	0.65038	0.64877	0.7142	0.64912
θ	-1.6960	-1.9531	-2.2889	-2.1746	-2.9762	-2.1957

3.4.3.3 Consistency of Implied Volatility Tem-Structure

In section 3.4.3.2, the result shows that longer-term options possibly react too "rapidly" to the arrival of new information relative to shorter-term options. This section considers whether the term-structure of implied volatility of the S&P 500 options market is consistent with rational expectations hypothesis under a mean-reverting volatility process.

[31] An exception is the the 171-220 puts which is slightly slower the 121-170 puts.

Basing the results upon a continuous-time mean-reverting volatility process, Stein (1989) derived the following theoretical relationship between the implied volatilities on options of two maturities:

$$d\sigma_t = -\alpha(\sigma_t - \bar{\sigma})dt + \beta\sigma_t dW$$

$$E_t(\sigma_{t+j}) = \bar{\sigma} + \rho^j(\sigma_t - \bar{\sigma})$$

$$i_t = \frac{1}{T}\int_{j=0}^{T}(\bar{\sigma} + \rho^j(\sigma_t - \bar{\sigma}))dj = \bar{\sigma} + \frac{\rho^T - 1}{T\ln\rho}(\sigma - \bar{\sigma})$$

$$\frac{(i_t^d - \bar{\sigma})}{(i_t^n - \bar{\sigma})} = \frac{T(\rho^k - 1)}{K(\rho^T - 1)} = \beta(K,T,\rho)$$

where $0 < \rho = e^{-\alpha} < 1$ is the weekly mean-reversion parameter, $\bar{\sigma}$ the long-term mean level of instantaneous volatility, $i_t^n(T)$ the nearby implied volatility with time to expiration T and $i_t^d(K)$ the distant implied volatility with time to expiration $K > T$.

Under Stein's continuous-time AR(1) setup, implied volatility is mean-reverting. This structure also hypothesises that the implied volatility of a longer-term option should move by less than one percent in response to a one-percent move in the implied volatility of a shorter-term option. Consequently $\beta(K,T,\rho)$ can be thought of as an elasticity relationship - given a movement in nearby implied volatility i_t^n, there should be a smaller movement in distant implied volatility i_t^d. The boundary condition for this elasticity requirement is as follows:

$$0 < \beta(K,T,\rho) < 1,\ for\ 0 < \rho < 1,\ 0 < T < K$$

This condition imposes a stringent constraint on how the term-structure of implied volatility can change. Using two daily S&P 100 implied volatility series from December 1983 to September 1987, Stein found that elasticity turned out to be larger than suggested by the AR(1) structure, indicating that long maturity options might have attached disproportionate importance or risk premiums to changes in short-maturity options.

In our analysis the elasticity relationship is directly testable by substituting i_t^n, i_t^d and $\bar{\sigma}$ into $\beta = \frac{(i_t^d - \bar{\sigma})}{(i_t^n - \bar{\sigma})}$. The nearby implied volatility i_t^n is calculated using the shortest at-the-money group. The distant implied volatility i_t^d is calculated using one of the longer-dated option groups, i.e. 71-120, 121-170, 171-220 or 220+. In addition, we use the averaged expectations of implied volatility for all call and put options in tables 5 and 6 as a proxy for the long-run mean level of instantaneous volatility $\bar{\sigma}$. Whilst using annual data is not the most technically rigorous way to investigate the elasticity relationship, nevertheless it should still shed some light on the consistency of the implied volatility term-structure because of the extensive span of our dataset. Our estimated $\bar{\sigma}$ is 16.39% whilst a similar average historical volatility estimated from S&P 500 index daily returns by Zhang and Shu (1999) is 15.87%. Results in tables 7 and 8 show that there are times when the empirically estimated $\beta(K,T,\rho)$ can depart significantly from the theoretical elasticity requirements. The highlighted areas in tables 7 and 8 identify a number of maturity groups that are not bounded within a reasonable range over the period 1983-1998. This empirical evidence demonstrates that $\beta(K,T,\rho)$ is very variable and the boundary restriction is frequently violated. Notably, these violations are most pronounced for the longest maturity group, 220+. In addition to Stein (1989), Bates (1996) and Bakshi et al. (1997) questioned whether the volatility process implied by traded options was consistent with the properties implied in its time-series. Whilst not mathematically rigorous, estimated β's provide evidence that option prices are inconsistent with the rational expectations under a mean-reverting volatility process.

Table 7: $\beta(K,T,\rho)$ for Calls

Nearby Options: 21-70 Call Options

Year / Distant Groups	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98
71-120	1.44	1.08	1.02	1.56	0.71	0.84	0.96	1.06	0.92	0.87	0.92	0.87	0.98	0.80	1.03	0.87
121-170	0.27	1.09	1.03	0.90	0.45	1.20	0.52	0.81	0.04	0.63	0.80	0.75	0.90	1.22	0.83	1.24
171-220	0.92	1.02	0.99	0.80	0.45	0.90	0.79	0.69	1.44	0.62	0.77	0.64	0.90	0.73	0.96	1.42
220+	0.62	1.75	1.05	-0.72	0.09	1.73	0.69	0.84	-0.46	0.37	0.65	0.55	0.78	0.91	0.85	1.23

Table 8: $\beta(K,T,\rho)$ **for Puts**

Nearby Options: 21-70 Put Options

Year / Distant Groups	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98
71-120	1.44	1.08	1.02	1.56	0.71	0.84	0.96	1.06	0.92	0.87	0.92	0.87	0.98	0.80	1.03	0.87
121-170	0.27	1.09	1.03	0.90	0.45	1.20	0.52	0.81	0.04	0.63	0.80	0.75	0.90	1.22	0.83	1.24
171-220	0.92	1.02	0.99	0.80	0.45	0.90	0.79	0.69	1.44	0.62	0.77	0.64	0.90	0.73	0.96	1.42
220+	0.62	1.75	1.05	-0.72	0.09	1.73	0.69	0.84	-0.46	0.37	0.65	0.55	0.78	0.91	0.85	1.23

3.4.4 Option Pricing Under Asymmetric Processes

Having examined many important features of the term-structure of implied volatility in sections 3.4.2 and 3.4.3, our goal in this section is to investigate what types of models would be consistent with the observed biases in the S&P 500 futures options market. We apply the skewness premiums technique developed by Bates (1991, 1997) to inspect S&P 500 futures options' pricing irregularities during 1983-1998.

3.4.4.1 Skewness Premiums

3.4.4.1.1 Underlying Concepts

Bates (1991,1997) demonstrated that asymmetries of the risk–neutral distribution embedded in an American options could be examined by using relative prices of out-of-the-money call and put options, and thereby judged the merits of alternative distributional hypotheses. This technique hypothesises that if the underlying asset price follows geometric Brownian motion as in the case of the Black-Scholes formula, the $x\%$ out-of-the-money call options should be approximately $x\%$ more expensive than the $x\%$ out-of-the-money European put options. With market asymmetry, however, skewed distributions create systematic divergences. Consequently, one could use the observed prices of call and put options to judge whether the $x\%$ rules are consistent with the skewness of the risk-neutral distribution derived from any specific distributional hypothesis - an exercise roughly comparable to looking at moneyness biases. For example, a perceived market crash will lead to out-of-the-money put options on S&P 500 futures being priced higher than out-of-the-money call options, indicating that it is more likely for put options to finish in the money than call options. The $x\%$ skewness is defined as the percentage deviation of $x\%$ out-of-the-money call prices from $x\%$ put prices:

$$SK(x) = c(F,T;X_c)/p(F,T;X_p) - 1$$

where $X_p = [F/(1+x)] < F < X_c = [F(1+x)], x > 0,$ and F is the underlying forward price for American futures options.

For American options on futures, the skewness premium has the following properties for the distributions regardless of the maturity of the options if at-the-money skewness premiums are approximately equal to zero:

i) $0\% \le SK(x) \le x\%$ for

1) Arithmetic and geometric Brownian motion;
2) Standard constant elasticity volatility processes;
3) Benchmark stochastic volatility and jump-diffusion processes;

ii) $SK(x) < 0\%$ only if

1) Volatility of returns increases as the market falls, or
2) Negative jumps are expected under the risk-neutral distribution;

iii) $SK(x) > x\%$ if and only if

1) Volatility of returns increases as the market rises, or
2) Positive jumps are expected under the risk-neutral distribution.

3.4.4.1.2 Data Construction

CME's settlement records are again used for the skewness premiums analysis. The sample period begins from the inception of S&P 500 futures option in March 1983 to December 1998. Three exclusionary restrictions are applied to the data:

i) Only contracts of a single maturity are considered for any day, namely, contracts with maturities between 21-70 days. Longer maturities are too thinly traded and shorter maturities are too near maturity to contain useful information;

ii) Exclude non-traded options to eliminate artificial trading behaviour;

iii) At least five strikes for call options and five strikes for put options are required everyday to enhance the quality of interpolations.

Since options exist only for specific exercise prices, skewness premiums cannot be implemented directly. In contrast to the methodology employed by Bates[32], we interpolate implied volatility for desired exercise prices from a cubic spline fitted through the implied volatility of call (put) options. As a proxy for the risk-free interest rate, we use daily middle rates on U.S. Treasury bills from Datastream matching maturity closest to the expiration date of the options. Option prices with the desired strikes are obtained by inserting the interpolated volatility into the Baroni-Adesi et al.'s (1987) American option pricing model. The filtering restrictions result in the data from 1,600 days being used out of a total of 3,789 records.

3.4.4.2.2 Results of Distributional Hypothesis

Skewness premiums from March 1983 to December 1998 for $x = 0\%$ and $x = 4\%$ are given in figures 17 and 18, respectively. Theoretically, at-the-money call and put options should be priced identically, yielding a skewness premium value of 0%, which is in fact largely observed except at the inception of S&P 500 options in 1983-1984. Over 1983-1984, the 0% skewness premium fluctuates randomly in the range of $\pm 8\%$. From 1985-1998, 0% skewness premium remains around zero. The 4% skewness premium plot, however, implies that volatility of returns is largely negatively correlated to the futures price and out-of-the-money put options are priced consistently higher than out-of-the-money call options. The 4% skewness premium shown in figure 18 indicates gradual downward shifts over time in skewness. In 1983, the premium is typically negative and in excess of the 4% benchmark. Between 1984 and 1985, the premium is largely positive and less than 4%, suggesting that the observed prices are consistent with the Black-Scholes formula. Starting from the late 1986, strong assessments of downside risk begin emerging and growing more negatively until the middle of 1994 and then slowly returning to a less negative level and stabilising around 1998.

Skewness premiums show that the S&P 500 market has been systemically pricing away the Black-Scholes formula since 1986. These biases are substantial and persistent even during the early years and are accompanied by an increasingly negative premiums since around 1987.

[32] Options prices were interpolated from a cubic spline fitted through the ratio of options prices to futures prices in Bates' study.

The fluctuations in the sign and magnitude of skewness premium in figure 18 imply that one needs models of time-varying skewness to complement the log-normal distribution. The skewness premium technique cannot identify which process would best fit the observed options but negative skewness premiums suggest that stochastic volatility processes with a large negative correlation between volatility and market shocks or jump-diffusion processes could best fit the observed option prices. Whilst more broad-reaching in this analysis, our investigation accords with and does not contradict Bates' (1997) investigation of the S&P 500 market.

3.5 Summary

This study is descriptive research and we have employed many models and techniques to investigate the S&P 500 implied volatility term-structure. Since we have anaylsed in excess of 250,000 option prices over a 16-year period, inferences drawn from this research must not be viewed as tentative. Contrary to the basic assumption of the Black-Scholes formula, implied volatility exhibits both smile effects and term-structure patterns. We have demonstrated that the term-structure of S&P 500 implied volatility follows some patterns:

i) Implied volatility tends towards a long-term mean of about 16%;

ii) Put options have higher premiums and a larger range of fluctuation than call options;

iii) Short-maturity options are more volatile than long-maturity options.

Smile effects are found to be strongest for short-term options, indicating that short-term options are the most severely mispriced by the Black-Scholes formula and therefore present the greatest challenge to any alternative option pricing models. Basing our results upon a harmonic model, we find the rate of change of put implied volatility is faster than call's, thus providing a basis to argue that put options are more "responsive" to a change of market sentiment. Furthermore, we report there is evidence that options prices are not consistent with the rational expectations under a mean-reverting volatility assumption. Finally, skewness premiums agree with the term-structure results in section 3.4.3.1 that S&P 500 moneyness biases have been progressively worsening since around 1987. Results of the negative 4% skewness premiums demonstrate that volatility of returns increases as the market falls. As correlation may be responsible for skewness, our diagnostics agree with Bates (1997) and suggest that leverage (stohcastic volatility processes with a large negative correlation between volatility and market shocks) and jump-diffusion models with negative-mean jumps are more recommended for capturing the observed biases in S&P 500 futures options market

Figure 3: Call Maturity = 21 – 70 Days

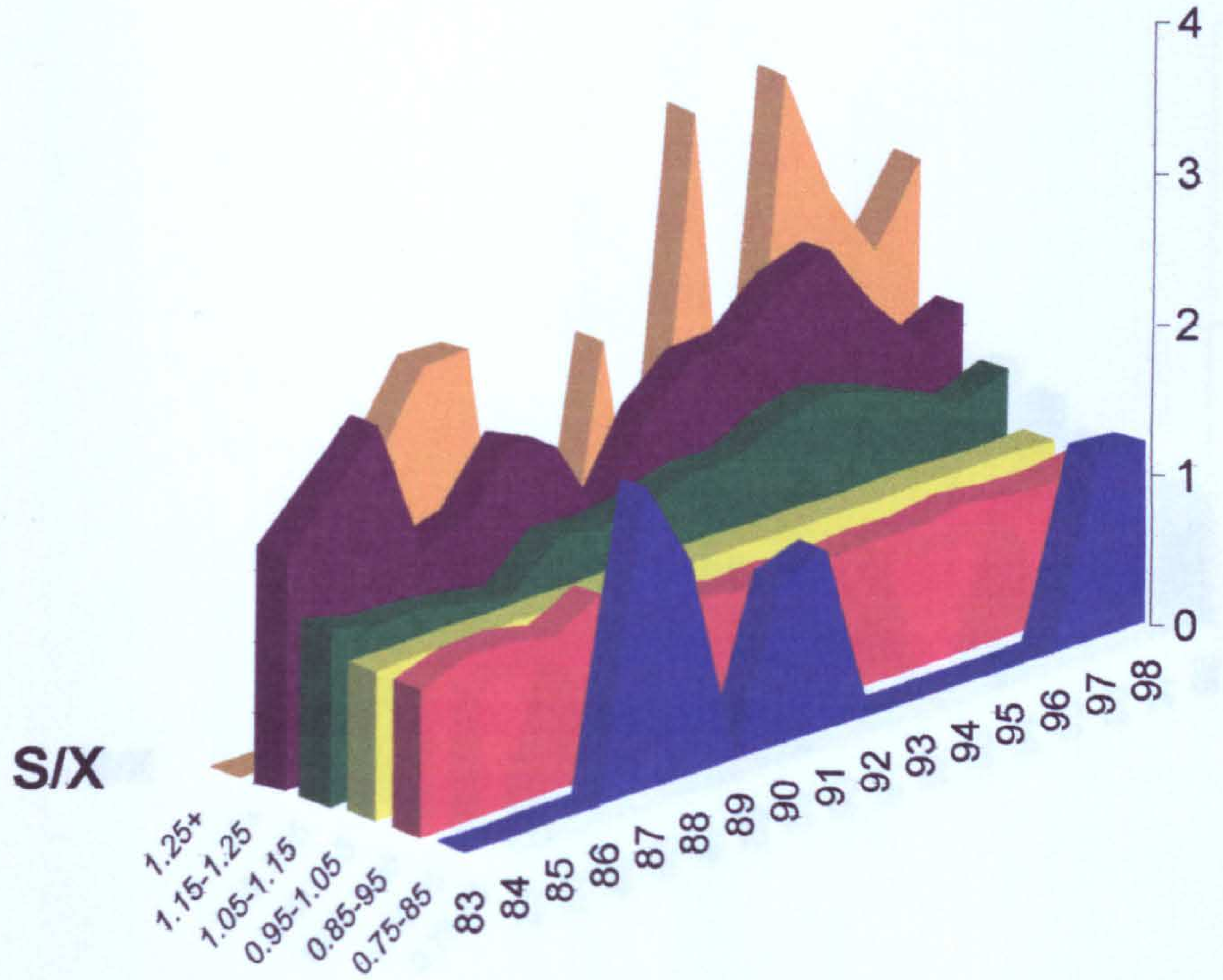


Figure 4: Call Maturity = 71 - 120 Days

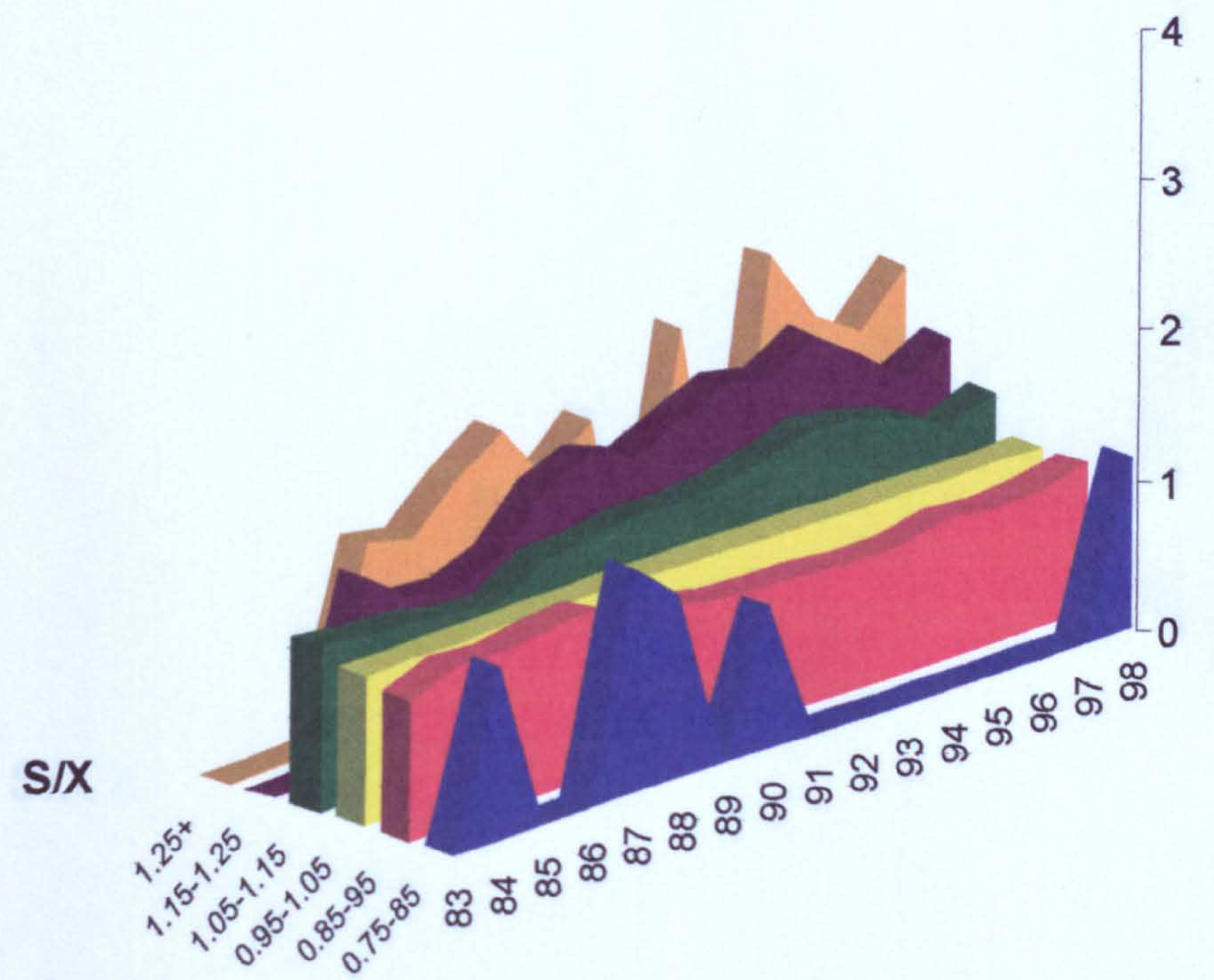


Figure 5: Call Maturity = 121 - 170 Days

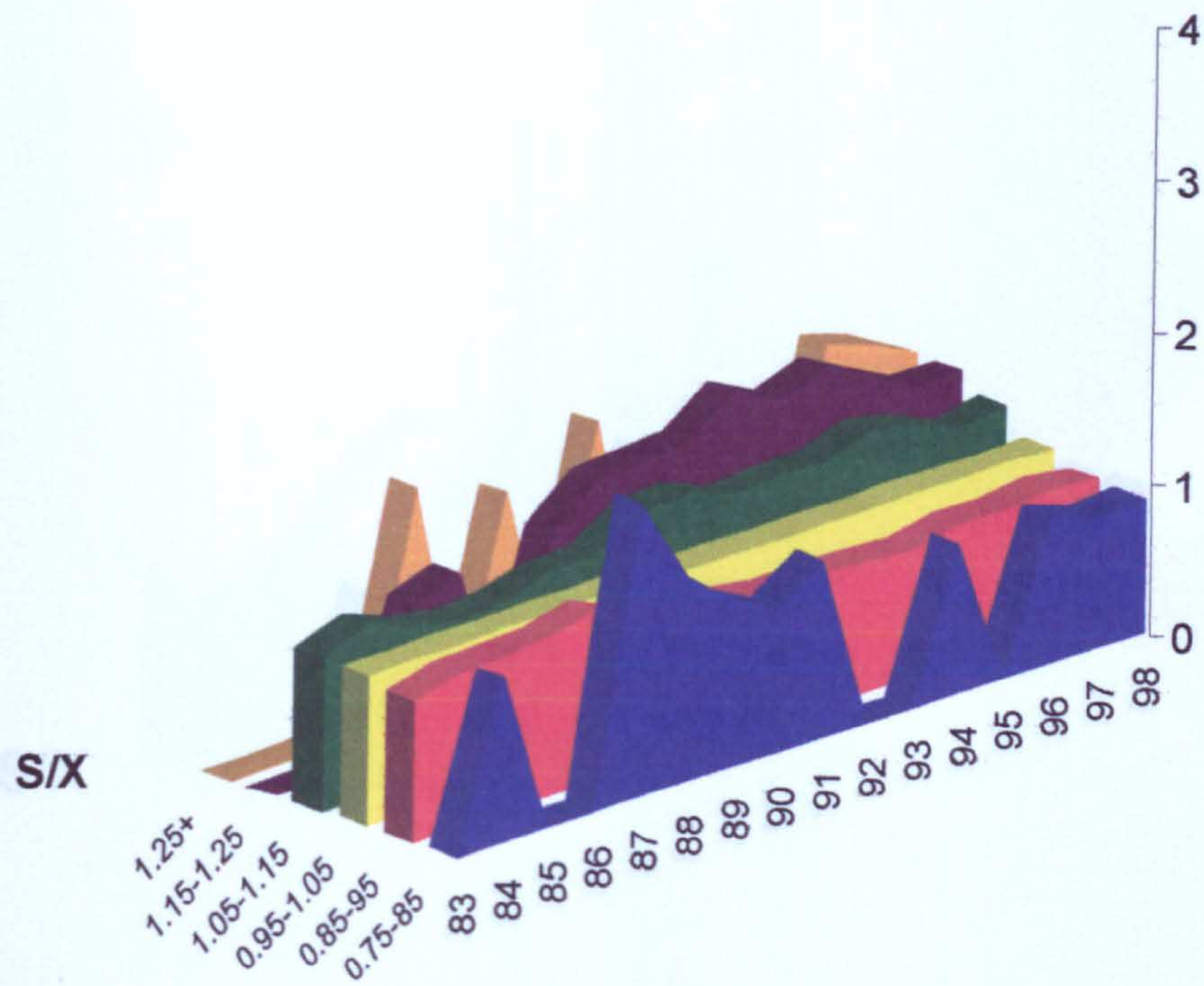


Figure 6: Call Maturity = 171 - 220 Days

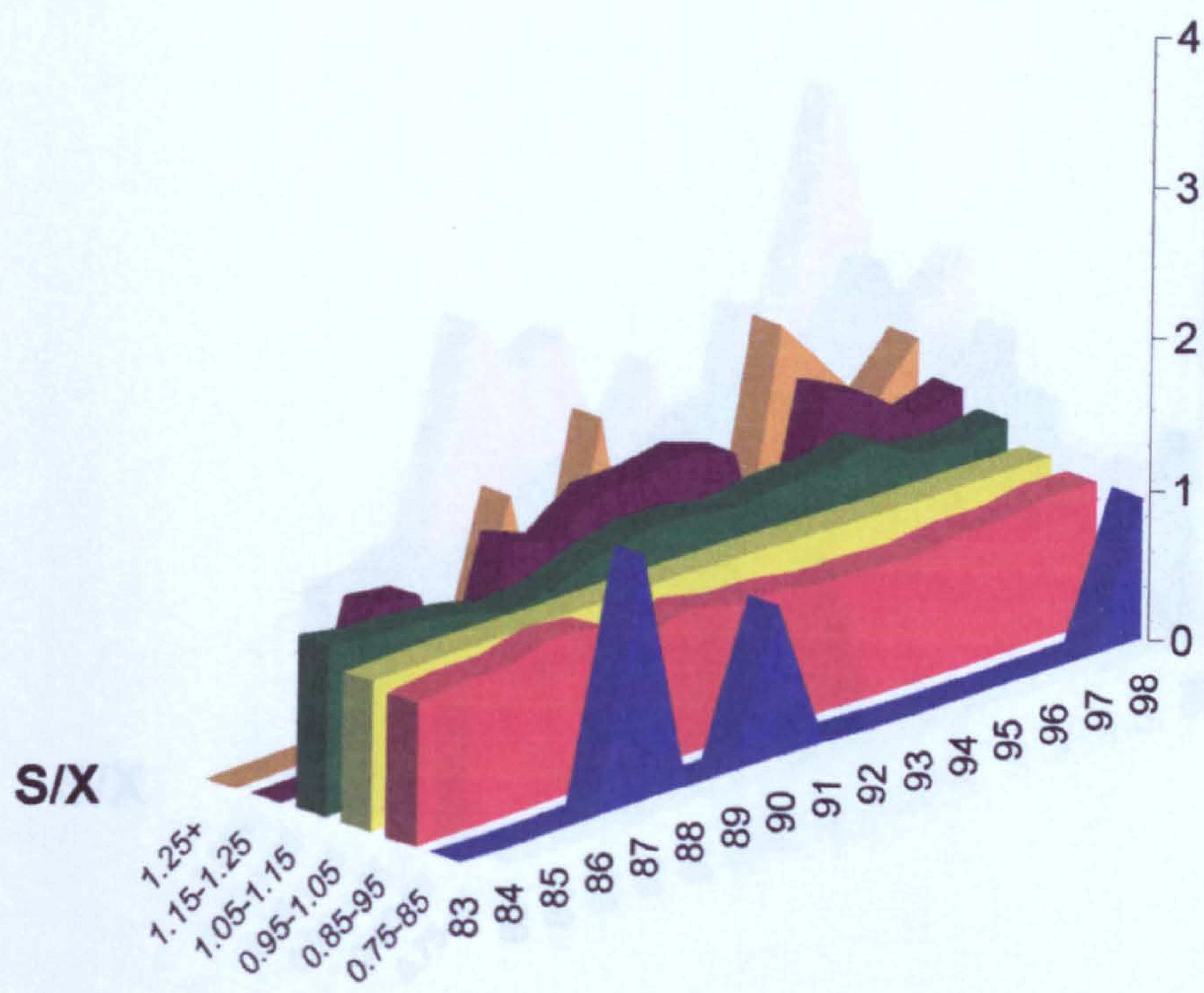


Figure 7: Call Maturity = 221+ Days

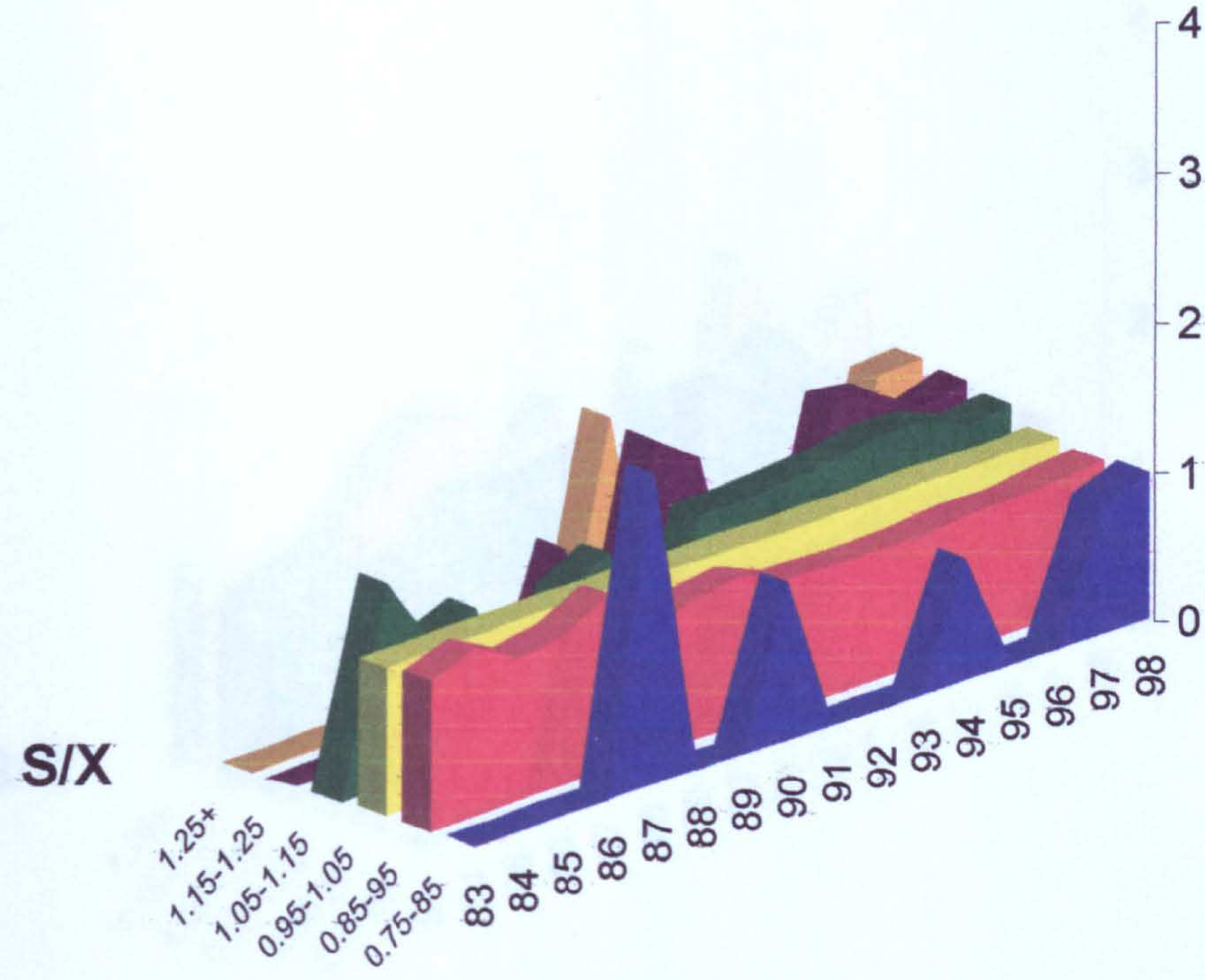


Figure 8: Put Maturity = 21 - 70 Days

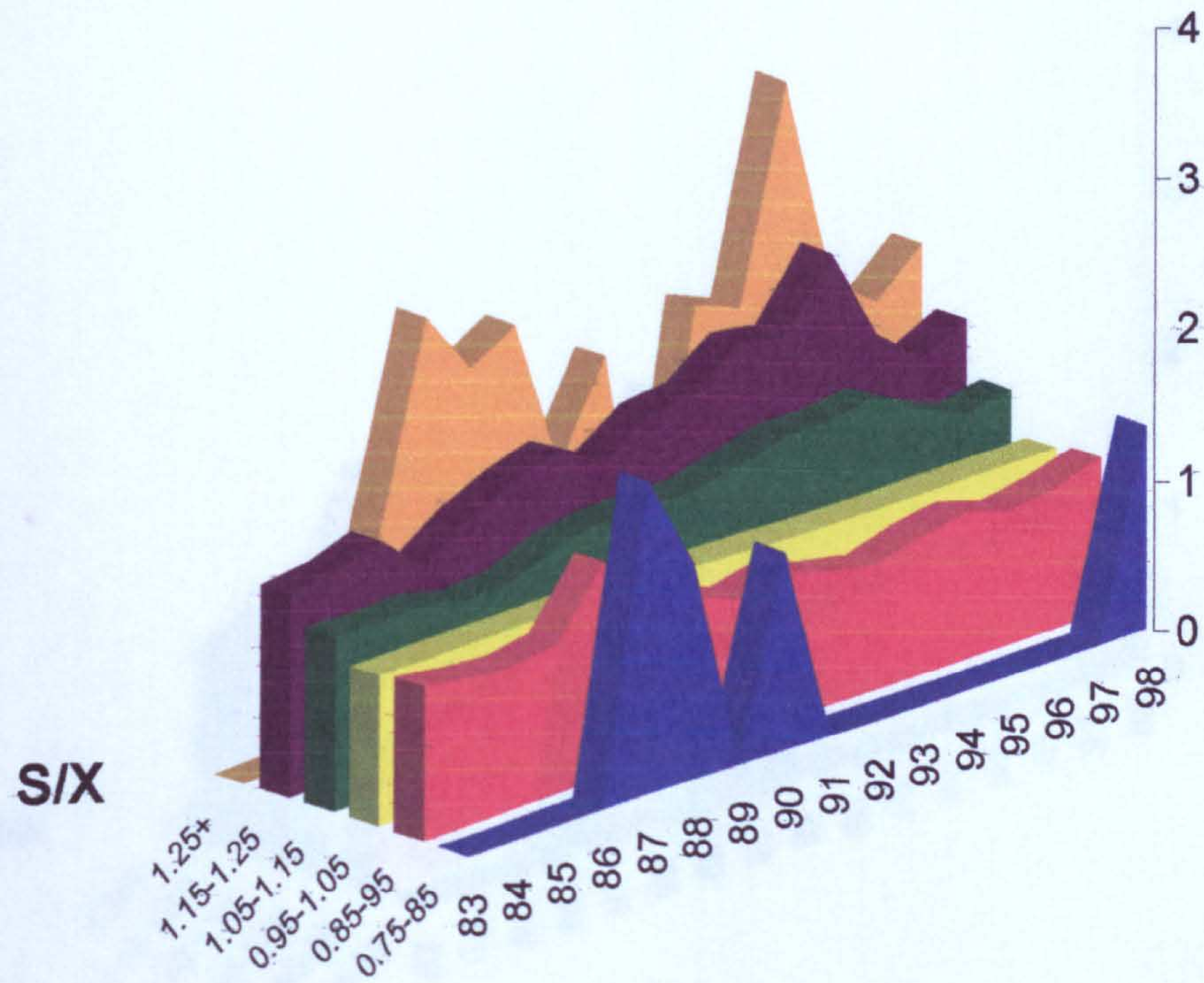


Figure 9: Put Maturity = 71 - 120 Days

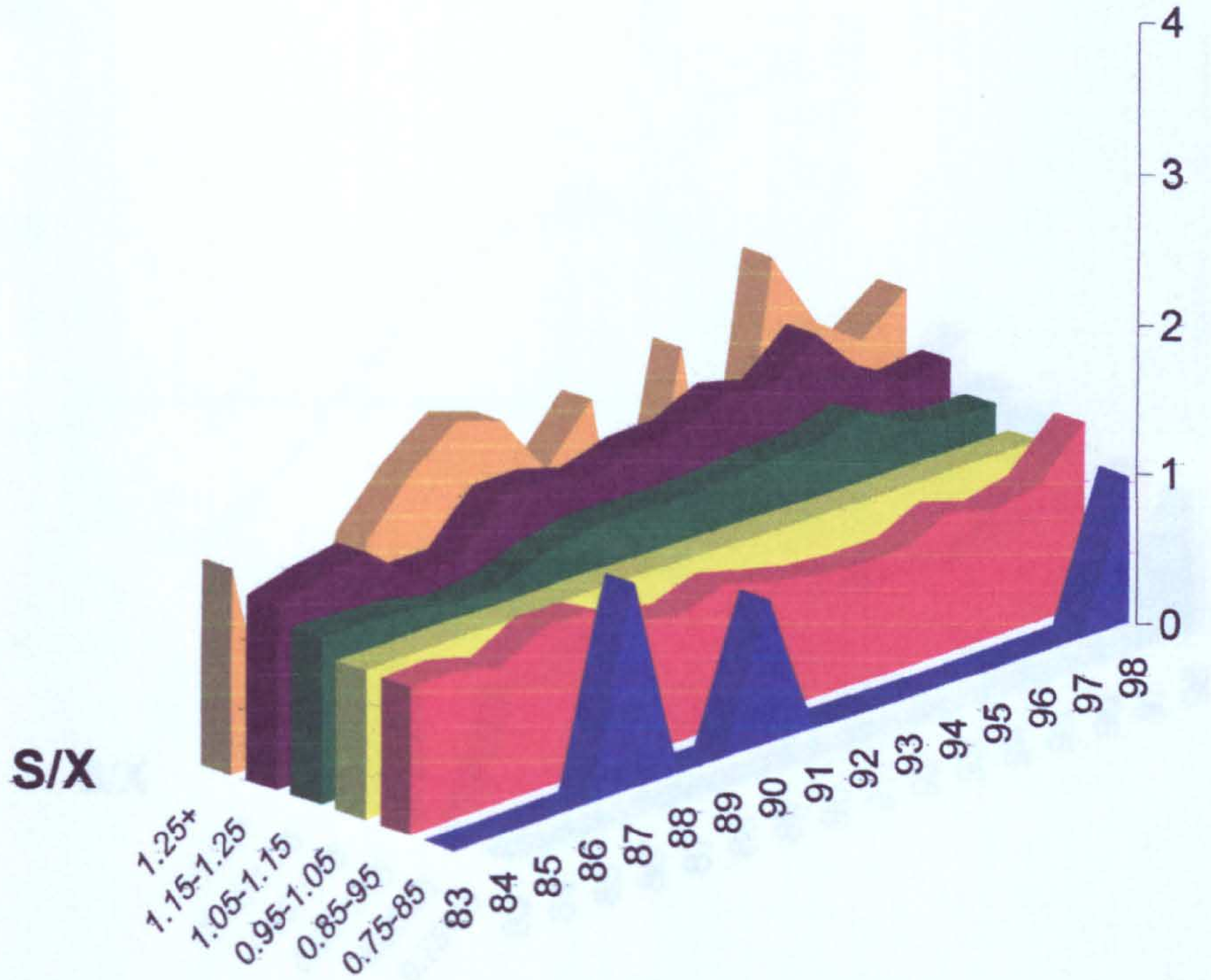


Figure 10: Put Maturity = 121 - 170 Days

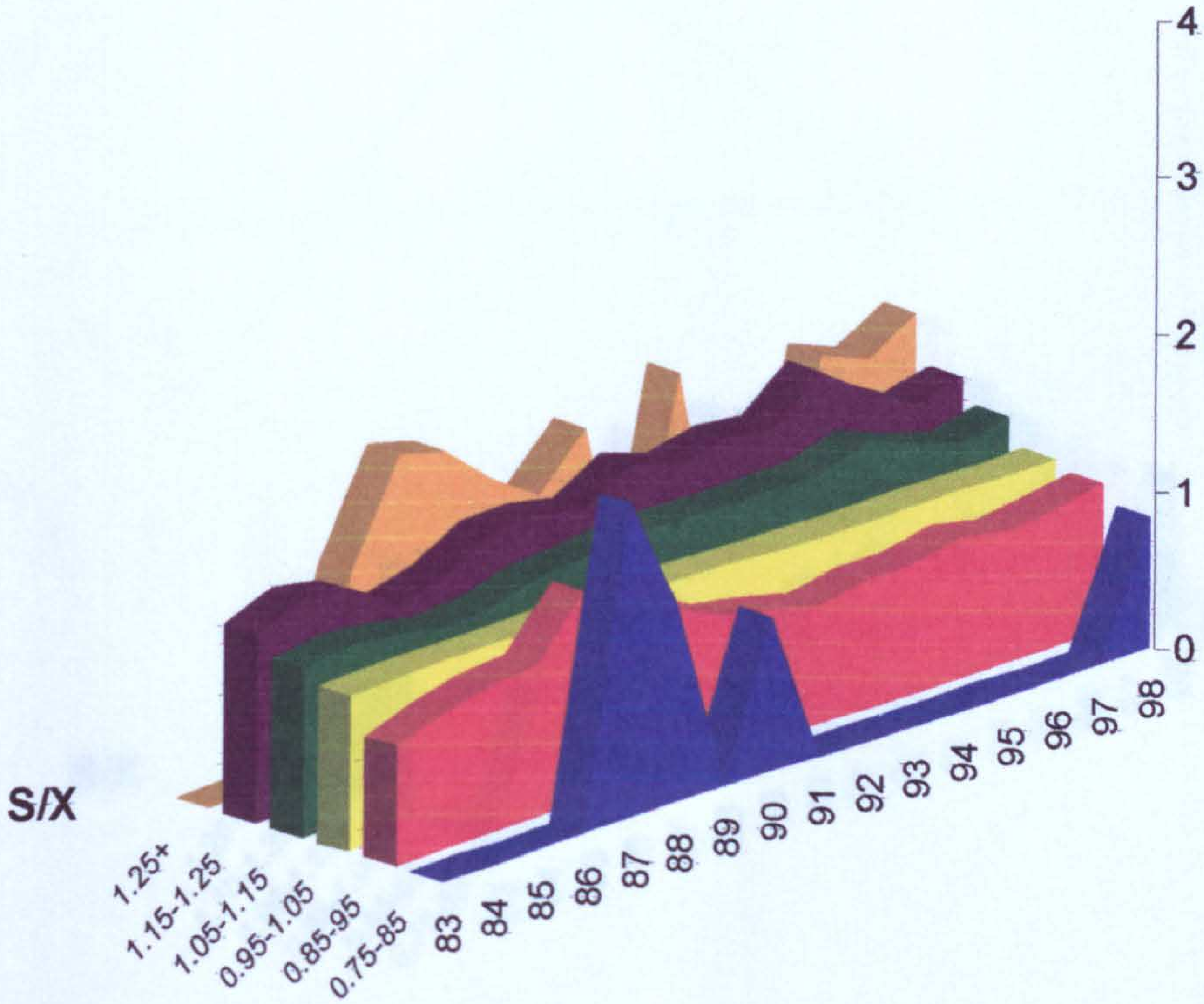


Figure 11: Put Maturity = 171 - 220 Days

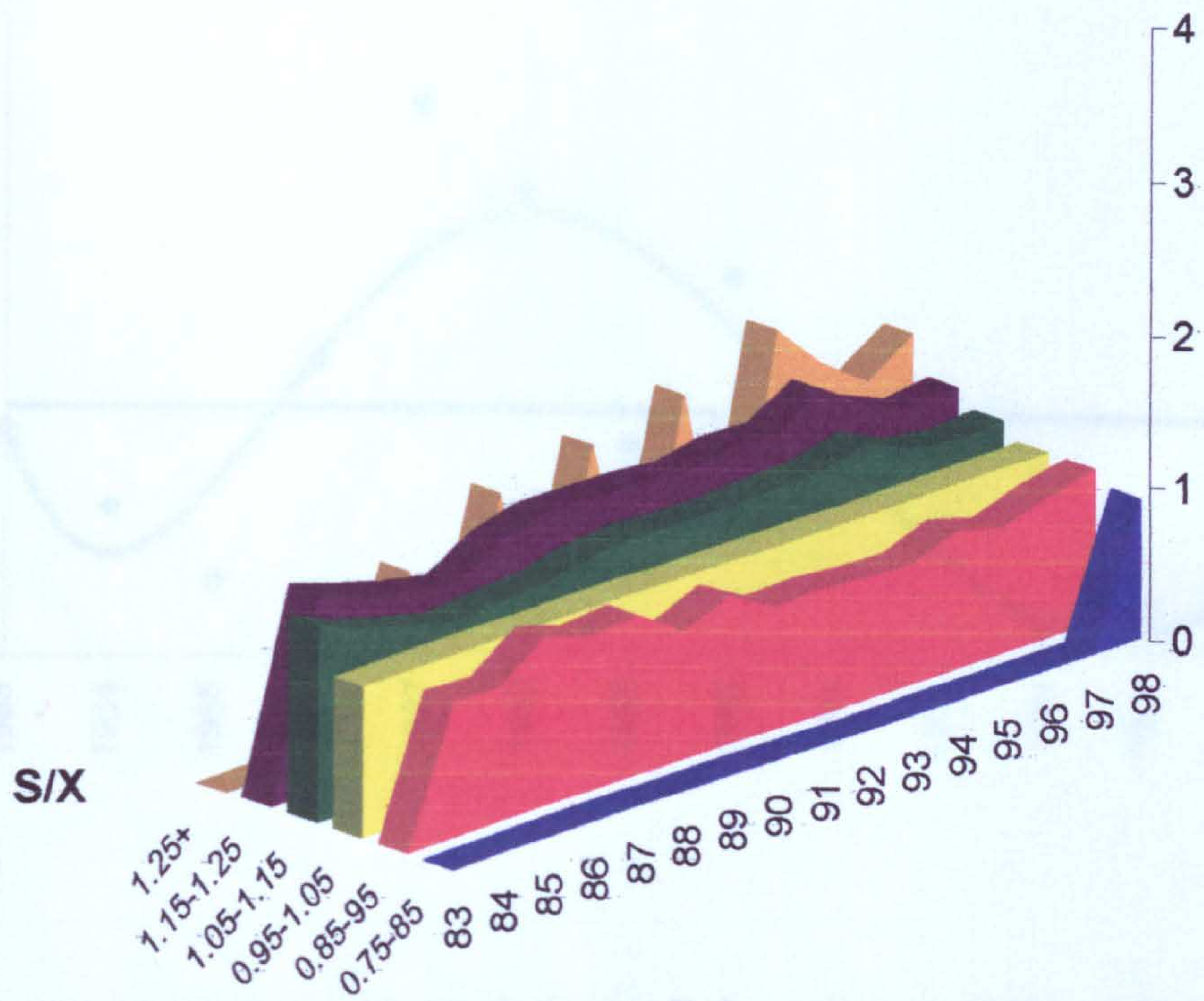


Figure 12: Put Maturity = 221+ Days

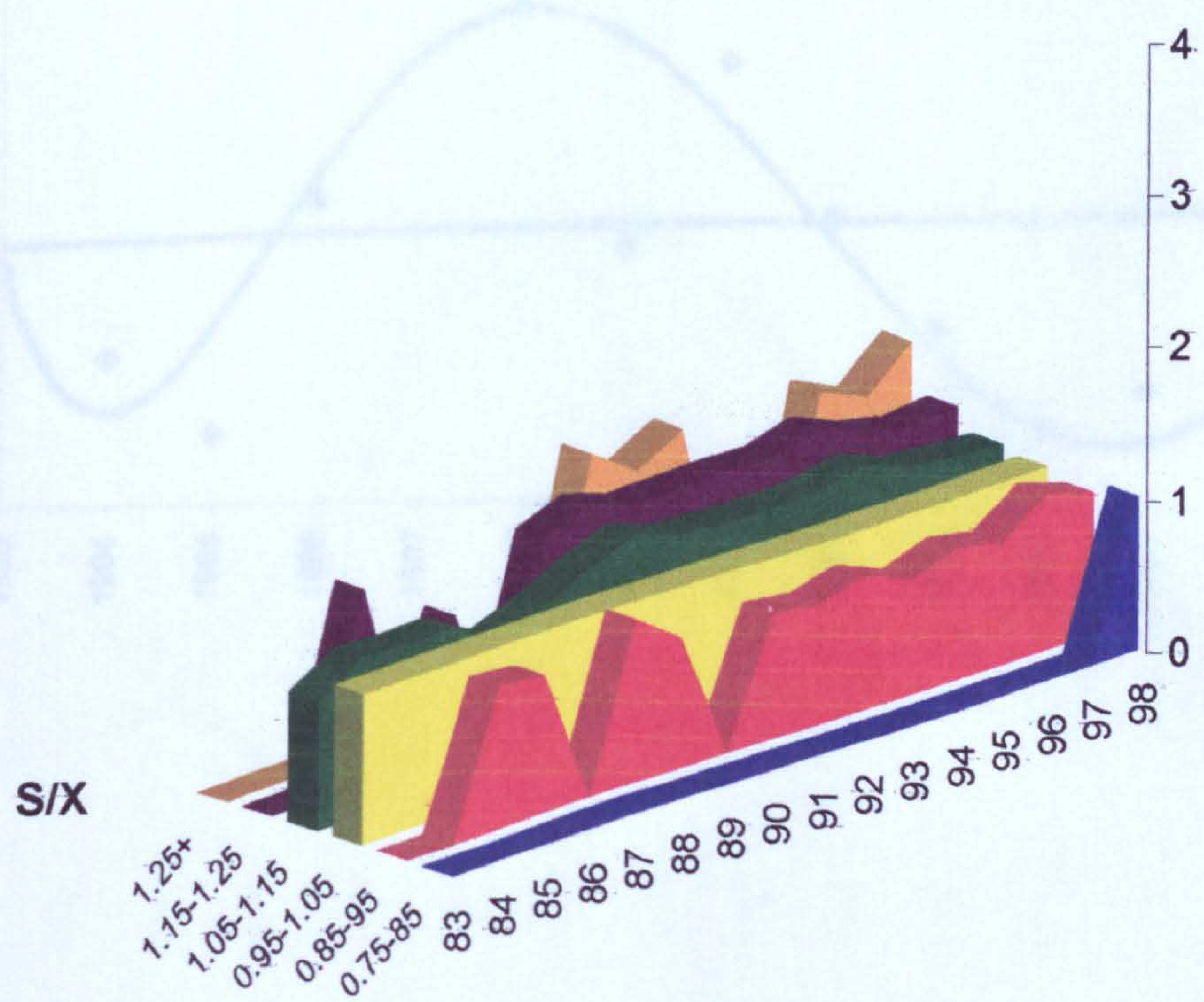


Figure 13: 21-70 Calls with Sixth-Order Polynomial and Linear Trend

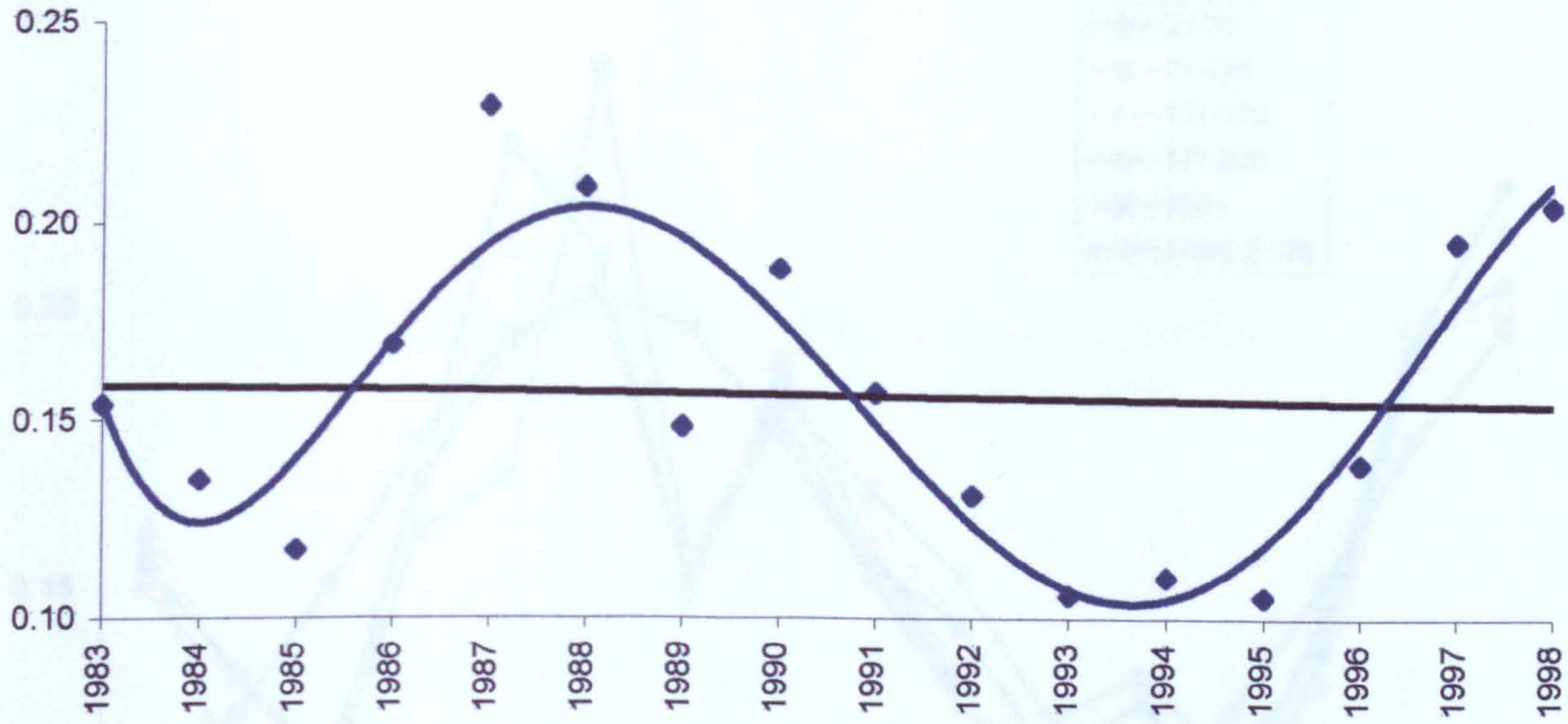


Figure 14: 21-70 Puts with Sixth-Order Polynomial and Linear Trend

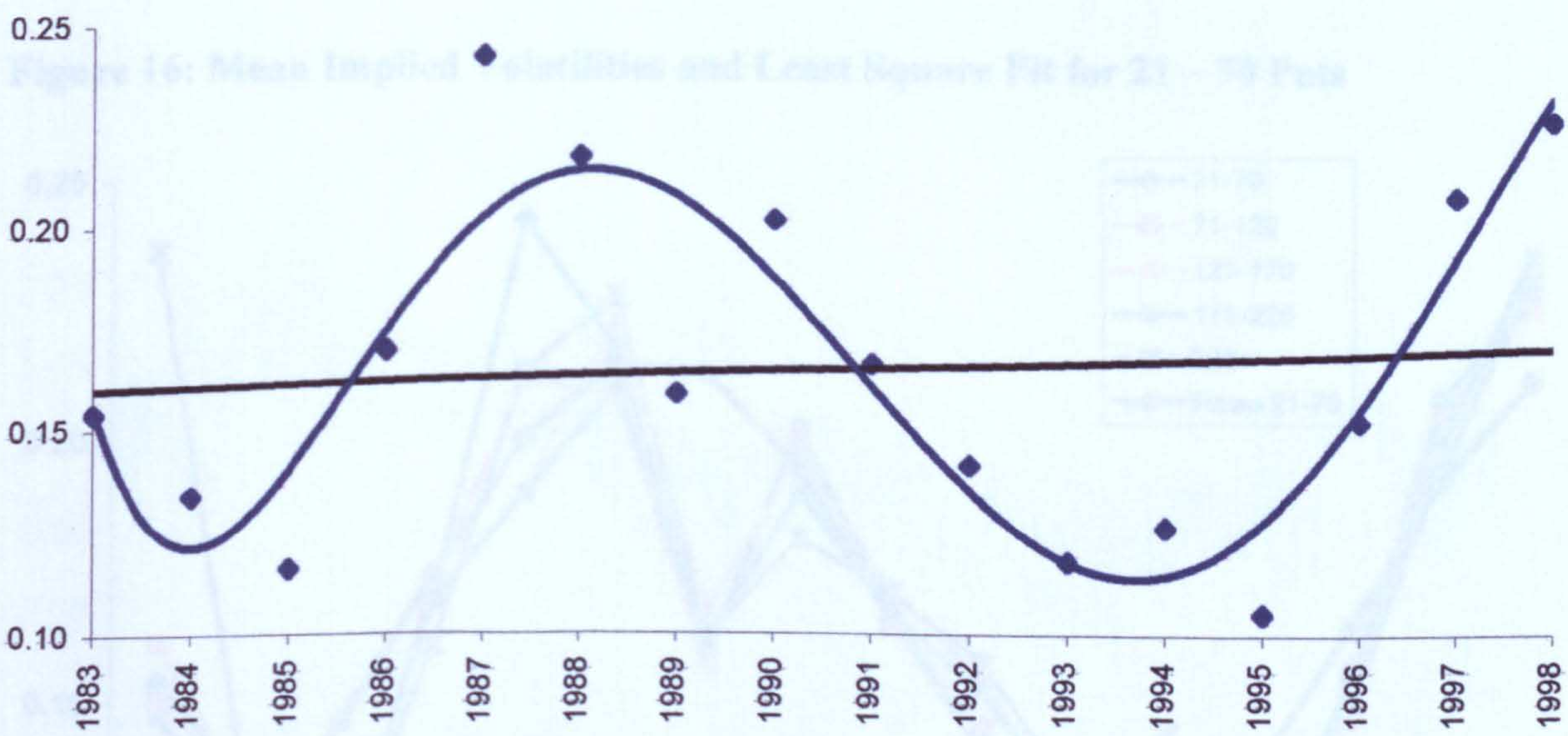


Figure 15: Mean Implied Volatilities and Least Squares Fit for 21 – 70 Calls

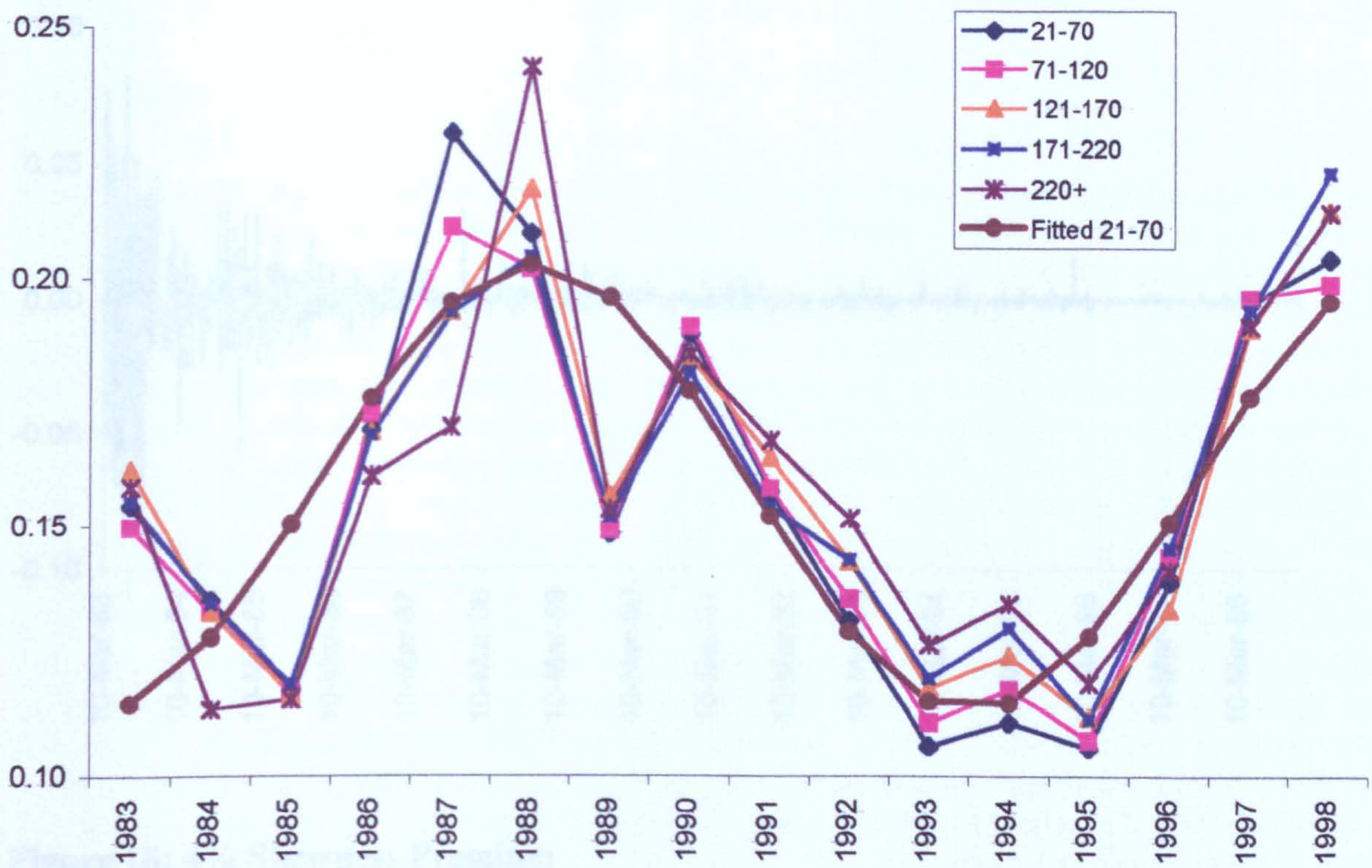


Figure 16: Mean Implied Volatilities and Least Square Fit for 21 – 70 Puts

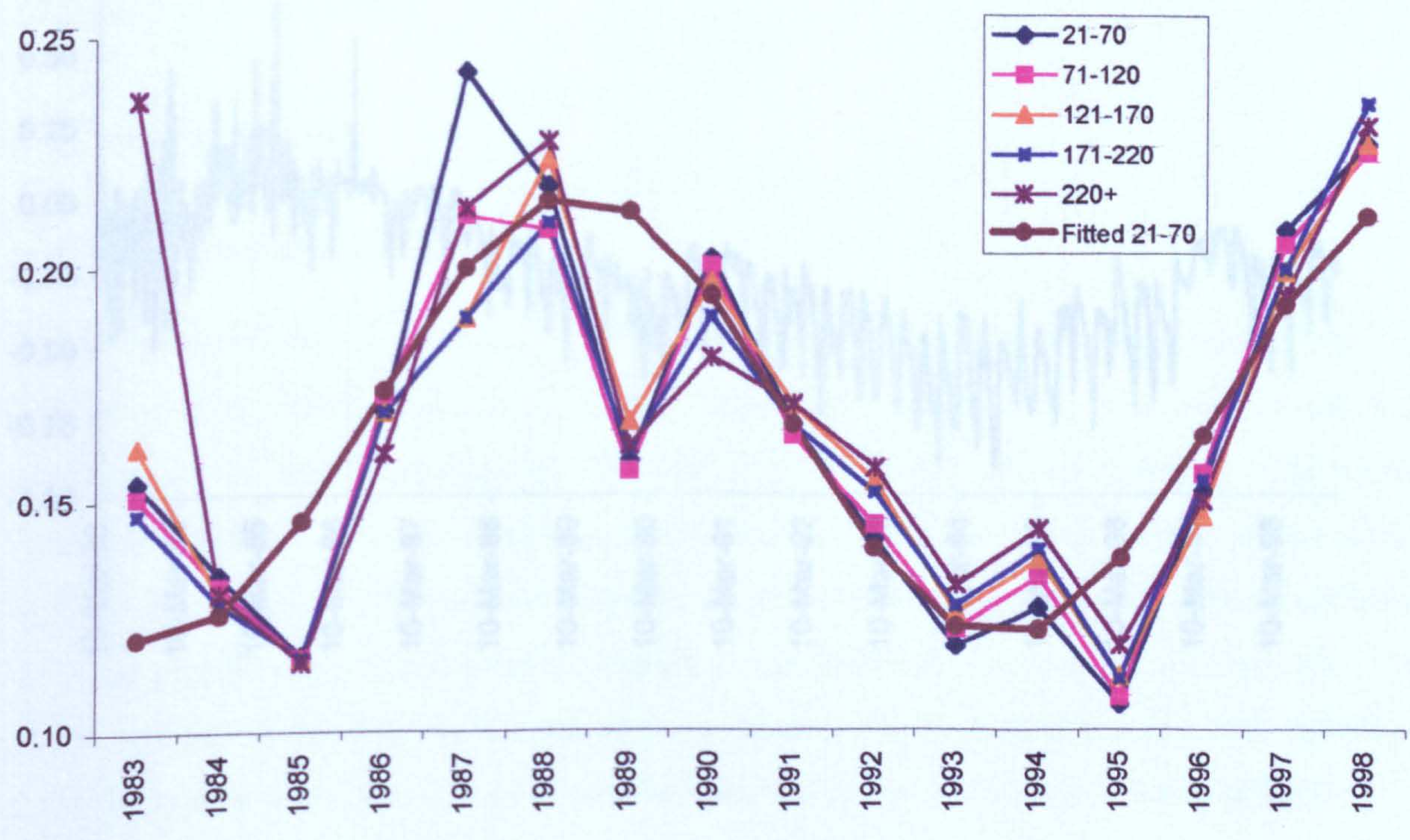


Figure 17: 0% Skewness Premium

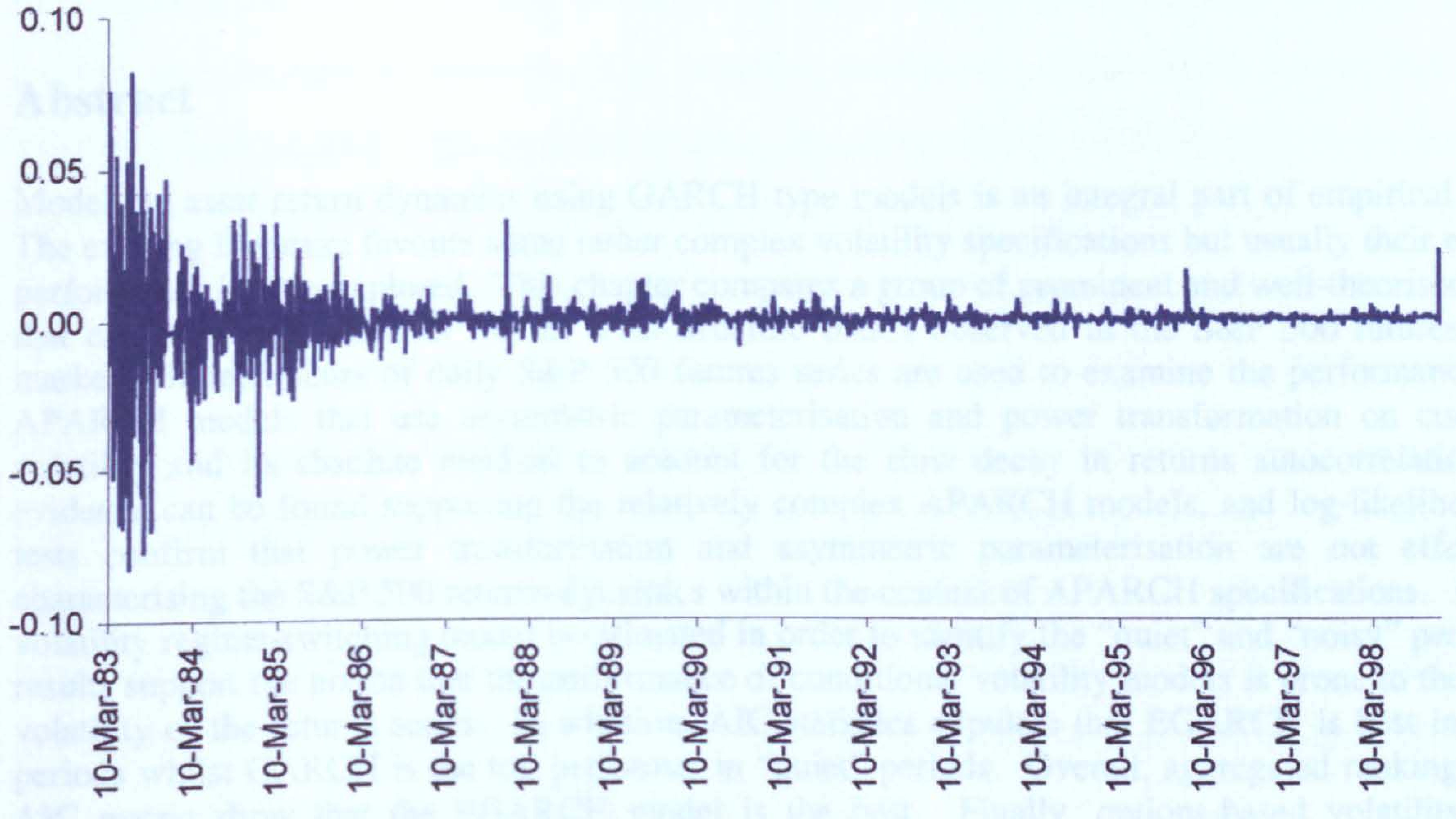
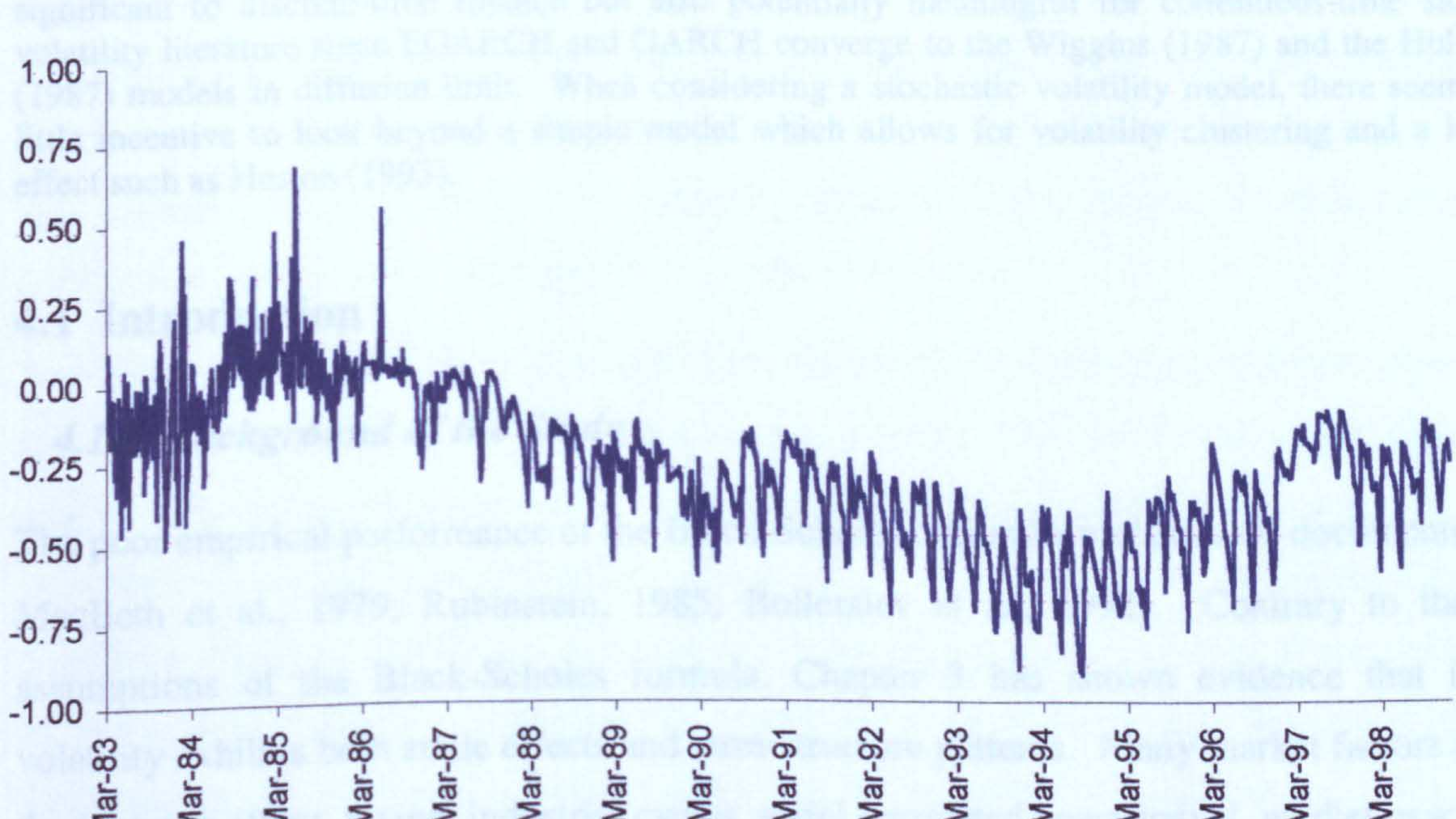


Figure 18: 4% Skewness Premium



CHAPTER 4 An Empirical Comparison of APARCH Models

Abstract

Modelling asset return dynamics using GARCH-type models is an integral part of empirical finance. The existing literature favours some rather complex volatility specifications but usually their empirical performance is little explored. This chapter compares a group of prominent and well-theorised models that can potentially account for the term-structure biases observed in the S&P 500 futures options market. Sixteen years of daily S&P 500 futures series are used to examine the performance of the APARCH models that use asymmetric parameterisation and power transformation on conditional volatility and its absolute residual to account for the slow decay in returns autocorrelations. No evidence can be found supporting the relatively complex APARCH models, and log-likelihood ratio tests confirm that power transformation and asymmetric parameterisation are not effective in characterising the S&P 500 returns dynamics within the context of APARCH specifications. A 3-state volatility regime-switching model is estimated in order to identify the "quiet" and "noisy" periods and results support the notion that the performance of conditional volatility models is prone to the state of volatility of the returns series. In addition, AIC statistics stipulate that EGARCH is best in "noisy" periods whilst GARCH is the top performer in "quiet" periods. Overall, aggregated rankings for the AIC metric show that the EGARCH model is the best. Finally, options-based volatility trading exercises reveal that EGARCH and GARCH can generate statistically significant ex-ante profit in one out of four sample periods after transactions costs, however, it also exposes the insufficiency of a delta-neutral hedge in the event of large market moves. The consequence of this research is not only significant to discrete-time finance but also potentially meaningful for continuous-time stochastic volatility literature since EGARCH and GARCH converge to the Wiggins (1987) and the Hull-White (1987) models in diffusion limit. When considering a stochastic volatility model, there seems to be little incentive to look beyond a simple model which allows for volatility clustering and a leverage effect such as Heston (1993).

4.1 Introduction

4.1.1 Background of the Study

The poor empirical performance of the Black-Scholes option formula is well documented (e.g. MacBeth et al., 1979; Rubinstein, 1985; Bollerslev et al., 1992). Contrary to the basic assumptions of the Black-Scholes formula, Chapter 3 has shown evidence that implied volatility exhibits both smile effects and term-structure patterns. Many market factors such as the leverage effect, taxing, industrial cycles, serial correlated news arrival, market psychology etc have played very crucial roles in causing these observed biases in the marketplace. As a result, normal distribution is not adequate to specify the returns dynamics and researchers have yet to deal with fat-tails and excess kurtosis which form the basis of smile effects.

Following the path-breaking paper by Engle (1982), an alternative literature has focused on discrete-time autoregressive conditional heteroskedasticity (ARCH) models. The development of ARCH models is driven by three regularities of equity returns: 1) equity returns are strongly asymmetric, e.g. negative returns are followed by larger increases in volatility than equally large positive returns; 2) equity returns are fat-tailed; 3) equity returns are persistent; persistence refers to the volatility clustering. This class of discrete-time models hypothesises that both smile effects and term-structure patterns, as evidenced in Chapter 3, can be explained by allowing the underlying asset's volatility to obey a stochastic process.

There is a voluminous literature suggesting that discrete time-varying volatility models are practical and insightful. The usefulness of ARCH modelling is such that volatility is predictable and readily implemented. ARCH models assume the presence of a serially correlated news arrival process and require only the use of past data. As such, ARCH models allow conditional variance to change over time as a function of past conditional variance and error, whilst leaving unconditional variance constant.

Most of the early research efforts focused on conditional models that imposed symmetry on the conditional variance structure. In response to criticisms that symmetric model may not be appropriate for modelling stock returns volatility, more recent research has considered other features such as leverage effects, power transformation etc in the variance equation. There are, indeed, so many conditional volatility models in the finance literature that it is cumbersome to provide a comprehensive survey of them all.

Recently, the topics of long memory and persistence have attracted considerable attention in terms of the second moment of an asset returns process. The development of long-memory models is based on the observations of the so-called "stylised facts"[33]. For example, Ding et al. (1993) invented the APARCH models that used the Box-Cox transformation on conditional variance and its absolute residual to account for the slow decay of autocorrelations in the returns process. Subsequently, many researchers have also developed different specifications for the long-memory process (e.g. Baille, 1996; Bollerslev et al., 1996; Ding et al., 1996). Several papers have given the impression that their models are capable of accounting for empirical features such as volatility clustering and leptokurtosis in the

[33] See section 2.2.3 for discussion of the long-memory process.

distribution of returns. Despite the huge amount of effort researchers have put into modelling volatility, it is clear that empirical issues remain unexplored for many of these more "elaborate" models.

4.1.2 The Problem Statement and Hypotheses

This chapter investigates the in-sample performance of APARCH models (Ding et al., 1993) that can potentially account for the slow decay in returns autocorrelations using daily S&P 500 futures series from 1983 through 1998. The use of the APARCH framework is convenient to evaluate different model specifications because log-likelihood-based statistics can be used to directly test for the robustness of many nested models (see appendix A.1). The main hypotheses used in this project are that:

i) If the APARCH specification is a good description of the process driving volatility, then hypothesis tests can be applied to reject the nested models in favour of the less restricted models:

$$H_0 : restricted\ APARCH\ \text{mod}\,els$$
$$H_1 : less\ \ restricted\ APARCH\ \text{mod}\,el$$

ii) If structural change of volatility can have an influence on the performance of conditional heteroskedastic models, then asymmetric models should have better performance than symmetric models in high volatility state, and vice versa.

In this chapter our goals are:

iii) To investigate the effectiveness of asymmetric parameterisation and power transformation within the context of APARCH specifications using log-likelihood ratio tests;

iv) To provide evidence that the in-sample performance of asymmetrical and symmetrical conditional volatility models are prone to the state of volatility by using a 3-state regime switching volatility conditional model to separate high and low volatility states;

v) To compare the in-sample performance of EGARCH (Nelson, 1991) with APARCH models based on aggregate AIC statistics;

vi) To illustrate the quality of different conditional volatility forecasts by predicting the one-step ahead changes of implied volatility and conducting ex-ante (out-of-sample) S&P 500 straddle trading exercises.

4.1.3 The Significance of the Study

The existing literature favours some rather complex volatility specifications but usually their empirical performance is little explored. Since the development of long-memory models in the early 1990's, there has been little research about the significance of their specifications. In this chapter we investigate the importance of power transformation and asymmetric parameterisation within the context of APARCH specifications. The consequence of this research is not only significant to discrete-time finance but also potentially meaningful for continuous-time stochastic volatility literature. Since there is a direct linkage between discrete GARCH-type models and bivariate diffusion models, if it can be shown that there is not much to gain from moving beyond a more parsimonious discrete specification such as EGARCH or GARCH, there seems to be little incentive to look beyond a simple stochastic model which allows for volatility clustering and a leverage effect such as the Hull-White model (1987) or the Heston model (1993).

4.1.4 Organisation

The remainder of this chapter is organised as follows. We discuss the in- and out-of-sample performance criteria, introduce the APARCH models and explain our experiment design in section 4.2. Section 4.3 describes the S&P 500 dataset. Section 4.4 presents estimation results and evaluates the in- and out-of-sample performance of different conditional volatility models under different statistical and economic metrics. Section 4.5 summaries the results.

4.2 Methodology

This study uses several econometric methods to evaluate the in-sample performance of a group of well-theorised conditional volatility models in the S&P 500 market. The data used for estimations are drawn from the S&P 500 futures and its options markets from the period 1983-1998. In this section we first review the criteria and methods that are used in comparing conditional volatility models in this project. We will then discuss strategies that are used in analysing our results. Finally, we will explain the models used in carrying out the study, giving special emphasis to the APARCH models.

4.2.1 Performance Criteria

A few performance metrics are used in this project to measure the in- and out-of-sample performance of different conditional volatility models:

i) Log-likelihood tests, which are based on the maximum likelihood estimation numbers, are employed to test for the effectiveness of APARCH features;

ii) AIC metric, which penalises the use of less parsimonious models, is used to select the best in-sample model in each sub-period;

iii) The best overall in-sample model is chosen by the use of aggregate AIC ranking, which is defined as sum of the rank for each model in each sub-period. The lower, the better;

iv) The only criterion used for evaluation of out-of-sample performance of conditional volatility models is profit and loss. The performance of volatility predictors is evaluated based on their ability to predict volatility changes and generate ex-ante profits from trading nearest-the-money S&P 500 straddles in four non-overlapping out-of-sample periods. The higher is the rate of returns per trade, the better is the model.

4.2.2 Analytical Procedures

This chapter uses many numerical and econometrical techniques to measure in- and out-of-sample performance of different time-series volatility models. They are carried out using the following procedures:

i) Construct the S&P 500 futures series by rolling over sixty-eight S&P 500 futures contracts for the period 1983-1998. The issues relating to the rollovers of futures contracts are described in full detail in section 4.3.1;

ii) Partition the constructed time-series into four non-overlapping segments. The partitioning of the data is motivated by the observation that the series do not exhibit homogeneous behaviour over the entire 16-year period;

iii) Estimate the parameters of APARCH models and apply likelihood-based statistics to assess the relative performance of different models in each sub-period. The APARCH framework provides a general specification of the volatility dynamics that nests many well-known models, and log-likelihood ratio tests can be used to directly test for the robustness of different model specifications. Consequently, the effectiveness of asymmetric parameterisation and power transformation can be examined within the context of APARCH specifications;

iv) Study how structural change of volatility can have an influence on the performance of asymmetrical and symmetrical conditional volatility models. To support the hypothesis that performance of asymmetric and symmetric models are prone to the state of the samples, a three-state regime-switching model developed by Hamilton and Susmel (1994) is employed to identify any structural breaks in volatility of the S&P 500 futures series from 1983 through 1998. This regime-switching model stipulates that conditional variance is selected from a number of possible ARCH processes which depends upon the state that eventuates;

v) Repeat the above analysis with the inclusion of EGARCH. In addition to the log-likelihood inferences, we explore the ability of additional statistical error functions that allow for asymmetry in the loss functions of investors to track the in-sample forecasting performance of conditional volatility models. The statistical error functions used in this study are listed in appendix B.2;

vi) Conduct out-of-sample straddle trading exercises to illustrate the quality of various conditional volatility forecasts.

4.2.3 Conditional Volatility Models

4.2.3.1 APARCH Specification

The APARCH (Ding et al., 1993) family is an ideal specification to study the long-memory process and conditional volatility in general since it can nest many popular models in a common framework. Adopting these specifications therefore allows one to investigate the performance of a number of existing specifications whilst keeping the empirical analysis manageable. Seven models are included as special cases: APARCH, ARCH, GARCH, TSGARCH-I, TSGARCH-II, GJR and TARCH. Appendix A.1 shows the functional forms of these nested APARCH models. A more general framework which can also nest a number of GARCH-type models, including the models in the APARCH family, is given by Hentschel (1995)[34]:

[34] See section 2.2.3.5 for details.

$$\varepsilon_t = e_t h_t$$
$$f_i(e_t) = | e_t - b_i | - c(e_t - b_i)$$
$$\frac{h_t^\delta - 1}{\delta} = \alpha_0 + \sum_{i=1}^{p} \alpha_i h_{t-i}^\delta f_i^v(e_{t-i}) + \sum_{j=1}^{q} \frac{h_{t-1}^\delta - 1}{\delta}$$

where $-1 \le c \le 1$, $v > 0$ and $\delta > 0$.

According to Engle and Ng (1993)[35], the b parameter controls the magnitude and direction of a shift in the $e_{t-1} \Leftrightarrow h_t$ space whilst c produces the "rotations". δ controls the shape of the transformation and if $\delta > 1$ the transformation is convex; otherwise it is concave. The APARCH model is a special case of $v = \delta, b = 0, | c | \le 1$:

$$\varepsilon_t = e_t h_t$$
$$h_t^\delta = \alpha_0 + \sum_{i=1}^{p} \alpha_i (| \varepsilon_{t-i} | - c_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j h_{t-i}^\delta$$

Using S&P 500 returns data, some of Hentschel's important results are: 1) $\delta \approx 1.5$ when $v = \delta$; 2) the c parameter was neither statistically nor economically significant in the model; 3) small shocks made more contributions to volatility, but not large shocks. The "shifting" of news impact curve was the dominating factor in modelling asymmetry. As a result, the presence of b was more significant than c. The autocorrelation function for APARCH(1,1) was derived by Ding and Granger (1996):

$$\xi = E | e_t |^\delta$$
$$\rho_1 = \alpha_1 + \frac{\beta_1}{\xi} - \beta_1 \frac{\xi - 1}{\xi} \left[\frac{\xi(1 - \alpha_1 - \beta_1)(1 + \alpha_1 + \beta_1)}{1 - (\xi\alpha^2 + \beta_1^2 + 2\alpha_1\beta)} - 1 \right]^{-1}$$
$$\rho_k = \rho_1 (\alpha_1 + \beta_1)^{k-1}$$

It is noted that autocorrelations of APARCH models decrease exponentially, not hyperbolically. Ding et al.'s results showed that the estimated power δ was 1.43 and its asymmetric parameter c equal to –0.373, which suggested significant long memory and leverage effects did exist in S&P 500 returns.

[35] The reader is referred to section 2.2.3.3 for details of Engle and Ng (1993).

4.2.3.2 Lag Structure of APARCH Models

A substantial simplification in comparing models can be made if one imposes a fixed lag structure by restricting the nested models to the order of $p = q = 1$[36]. Moreover, Pagan and Schwert (1990) showed that low-order GARCH models could fit stock return volatility extremely well. Therefore, the benefit of including additional parameters beyond $p + q = 2$ is very small. In most applications, the special case that $p = q = 1$ is found to be suffice (e.g. Akgiray, 1989; Bollerslev et al., 1992; Lamoureux and Lastrapes, 1990; Poon and Taylor, 1992; Engle, 1993; Taylor, 1994; Kang and Brorsen, 1995; Antoniou and Holmes, 1995; Jorion, 1995; Antoniou et al., 1998; Duan and Wei, 1999). Throughout this study, APARCH(1,1) is the unrestricted model. In addition to APARCH(1,1), we will present estimates for a well-known but non-nested asymmetric models, EGARCH (Nelson, 1991)[37] to complement our analysis.

4.2.3.3 EGARCH

The exponential GARCH (EGARCH) model was invented by Nelson (1991) in response to the criticisms that the stock returns were negatively correlated with changes in return volatility. EGARCH considers asymmetry in the variance equation. The EGARCH(1,1) specification can be modelled as follows:

$$r_t = g(x_{t-1}; a) + \varepsilon_t$$
$$\varepsilon_t = h_t e_t \qquad e_t \sim N(0,1) \qquad \varepsilon_t \mid I_{t-1} \sim N(0, h_t)$$
$$\log h_t^2 = \omega + \lambda_1 z_{t-1} + \lambda_2 (\mid z_{t-1} \mid - (2/\pi)^{0.5}) + \beta \log h_{t-1}^2$$

where $z_t = \dfrac{\varepsilon_t}{h_t}$ is the normalised residual.

A negative λ_1 implies that a negative shock increases the conditional variance; it measures the sign effect. An estimated positive λ_2 indicates that a shock greater than $(2/\pi)^{0.5}$ also increases the conditional variance; it measures the size effect. This model accommodates the asymmetric relation between stock returns and volatility changes. The degree of asymmetry

[36] With the exception of ARCH and TARCH in which p=1, q=0.

[37] Since EGARCH is not nested within APARCH, they cannot be compared with the log-likelihood test.

or skewness can be measured by the absolute value of the ratio $\delta = \frac{-1+\lambda_1}{1+\lambda_1}$. In other words, it can be said that a negative standardised innovation (bad news) increases volatility δ times more than a positive standardised innovation of an equal magnitude. The use of logarithms also means that parameters can be negative without the variance becoming negative. Therefore, it is not necessary to restrict parameter values to avoid negative variances as in the ARCH and GARCH models.

4.2.4 Summary of the Methodology

Sections 4.2.1-3 have reviewed the performance criteria, models and strategies used for the comparison of performance of EGARCH and APARCH models. It should be noted that the purpose of this study is to: 1) investigate the effectiveness of asymmetric parameterisation and power transformation; 2) study the impact of structural change of volatility on the performance of asymmetrical and symmetrical models. We assess the performance of our models both in- and out-of-sample. The use of out-of-sample trading is primarily intended to illustrate the usefulness of our conditional volatility forecasts. The next section discusses the construction and partitioning of the S&P 500 returns series in this study.

4.3 Data Description

The dataset comprises of daily settlement prices of S&P 500 futures and its options for the period from 1983 through 1998. We use the same options data described in Chapter 3 and apply several filters to the options data that are identical to those outlined in section 3.3.4. We refer the reader to Chapter 3 for the contract specifications of S&P 500 futures and its options.

4.3.1 Rollover of S&P 500 Futures Contracts

In order to investigate volatility forecastability, a futures series is required. Sixty-eight futures contracts are studied between January 1983 through December 1998. Because the maximum life span of S&P 500 futures contract is two years, a continuous series of nearby daily futures prices needs to be constructed. It is well-known that the rollovers of futures contracts can generate significant biases in the various time-series properties of the artificial price series, depending upon the rollover method chosen. The necessary decisions involved in rolling over contracts include:

i) The point in time at which the current contract is rolled to the next;

ii) The adjustment of price level of the contract upon rollover.

According to Ma et al. (1992), it appears that different conclusions can be drawn from the empirical results estimated from time-series generated from different contract rollover methods. Moreover, most of the differences in results cannot be predicted. While adjusting the differential price levels at rollover dates reduces volatility, some artificial but drastic measurement errors are created if returns are computed using the adjusted price series. The magnitude of the measurement error can get so large that the direction and size of the biases from the different rollover methods is lost. Two subtle problems arise when the price levels are differenced multiple times over different ranges of the time series:

i) The level-adjust procedure effectively replaces the large positive/negative daily price changes at the rollover dates with zero price changes. Consequently, the variance and serial correlation estimates may be biased;

ii) Negative price syndrome. Futures prices can become negative if they are differenced multiple times.

There is no "best" method to rollover contracts. Despite the fact that rollover methods are potentially problematic, a long time-series has to be constructed to provide enough degrees of freedom for any meaningful statistical inference. We avoid rolling over at the delivery date since it almost always generates excessive volatility. During the maturity months, the nearby futures prices are rolled over by the daily prices of first deferred contract. The rollover is occurred on the first trading day of the maturity month. Following this method, the futures dataset contains 4,046 observations.

4.3.2 Partitioning and Descriptive Statistics for Time-Series

The S&P 500 futures time-series constructed in section 4.3.1 are divided into four non-overlapping periods: 1983-1986, 1987-1990, 1991-1994 and 1995-1998. The partitioning of the data is motivated by the observation that the series do not exhibit homogeneous behaviour over the entire 16-year period. The S&P 500 futures series is I(1) in each of these periods. First differences in logs of S&P 500 futures levels are employed to calculate returns. Each period contains about 1,000 observations. Tables 9-11 show the descriptive statistics for r, r^2 and $|r|$. The Dickey-Fuller test rejects the null hypothesis that there is a unit root in the full sample and each of the sub-periods. The Jarque-Bera statistic also rejects the null hypothesis that r, r^2 or $|r|$ is normal in the full sample and all of its sub-periods.

The standard deviation of 1987-1990's return is 0.017467, which is highest among all sub-periods. Skewness is negative in all periods except in 1991-1994, which is slightly positive. Therefore, it is more likely to have negative than positive returns. Excess kurtosis is 179 for the entire series. Excess kurtosis in sub-periods 1983-1986, 1987-1990, 1991-1994 and 1995-1998 are 2.5, 148.8, 3.1 and 8.2, respectively. The 1987-1990 return series has the most negative skewness and most positive excess kurtosis. Our preliminary statistics posit that futures returns are fat-tailed and not normal. Figures 19-33 are the sample autocorrelation plots for r, r^2 and $|r|$ with their 95% confidence intervals $+/-\frac{1.96}{\sqrt{N}}$. Figures 19, 25 and 31 show that there are some small negative low-order return autocorrelations in 1983-1998, 1987-1990 and 1995-1998. In addition, Ljung-Box statistics for r in table 9 are significant in 1983-1998, 1987-1990 and 1995-1998, which also suggest that r's are serial correlated. An inspection of their corresponding autocorrelation plots, however, show that r's are not related to many lags – an indication of short memory. This suggests that volatility in the distant future is insensitive to current information in sub-periods.

Ljung-Box statistics for r^2 in table 10 are significant in all periods except in 1983-1986. Figures 20, 23, 26, 29 and 32 are correlograms for r^2. It is evident from figures 23 and 29 that r^2 in 1983-1986 and 1991-1994 do not contain many lags of memory. Table 11 shows the descriptive statistics for $|r|$. Although returns themselves contain little serial correlation, there is substantially more correlation in absolute returns. Ljung-Box statistics for $|r|$ are significant in all periods except in 1983-1986. On inspection of figures 21, 27 and 33, we observe that $|r|$ can have autocorrelations as high as 70 lags; they decay slowly and remain significant until around 70, 15 and 75 lags in 1983-1998, 1987-1990 and 1995-1998, respectively - an evidence of long memory. Figures 20, 26, and 32 also display significant and large positive peaks for r^2 in the first few lags of autocorrelations in 1983-1998, 1987-

1990 and 1995-1998. However, they decay very rapidly and disappear completely within 10 lags.

4.3.2.1 Summary of Descriptive Statistics

A number of observations can be drawn from the descriptive analysis in this section:

i) Returns are not independent, although they are likely to be uncorrelated;

ii) Transformation of returns, i.e. $|r|$ and r^2 are more predictable. These two series have "longer memory" than returns;

iii) 1983-1998, 1987-1990, 1995-1998 are statistically more "noisy" and correlated;

iv) 1983-1986 and 1991-1994 are statistically more "quiet" and less correlated.

4.4 Results & Analysis

This section discusses our empirical results for the in-sample and out-of-sample tests. First, we present the results from maximum likelihood estimation of the parameters for APARCH models. Second, we investigate how structural change of volatility can have an influence on the performance of asymmetrical and symmetrical conditional volatility models. Third, we extend the in-sample analysis by introducing more loss functions and an additional asymmetrical conditional volatility model. Fourth, we evaluate the performance of different conditional volatility models by conducting option trading experiments in a number of out-of-sample periods.

4.4.1 Rationale for AR(1) Return Process

According to Koutmos and Tucker (1996), the serial correlation displayed in the S&P 500 futures series could be a result from thin trading of some stocks, non-synchronous measurement of the component stock prices of the index or rollover of futures contracts. To remove the autocorrelations in the daily S&P 500 series, an AR(1) process is used to formulate the conditional mean returns. This AR(1) return process is given by:

$$r_t = a_0 + a_1 r_{t-1} + \varepsilon_t$$

where $\varepsilon_t = v_t h_t$, v_t ~ i.i.d. student-t and h_t is the conditional volatility.

The AR(1) process simply states that returns are first order autocorrelated. Among others, Akgiray (1989), Hamilton (1989,1994), Heynen and Kat (1994) and Bracker et al. (1999) also suggested the use of AR(1) in modelling the conditional mean equation. In practice, it is not uncommon to model returns using MA(1) (e.g. Poon and Taylor, 1992; Ding et al., 1993). MA(1) was fit to the S&P 500 series but AR(1) is proven to be more suitable across all periods. Since the primary objective is to select a consistent conditional volatility model rather than studying the market microstructure of the S&P 500 index, other specifications for conditional mean are not considered here[38].

Empirical evidence frequently shows that normal distribution is not sufficient to remove fat tails from the empirical distribution of asset returns. Since non-normal distributions usually achieve better results than the normal density, all models are estimated with t-distribution. The Berndt, Hall, Hall and Hausman (1974) algorithm is used to obtain parameter estimates and maximises the log-likelihood function in GAUSS program. In addition, the gradient is calculated numerically. Our parameter estimates are insensitive to various initial conditions for our sample, making it likely that global maxima are achieved.

4.4.2 In-Sample Analysis: Maximum Likelihood Estimations of APARCH Parameters

Section 4.4.2 has two goals: 1) to apply log-likelihood ratio tests to test for the "effectiveness" of APARCH features, i.e. power transformation and asymmetric parameterisation; 2) to investigate the in-sample performance of different APARCH models based on log-likelihood statistics. Summary for maximum likelihood estimations is presented in section 4.4.2.3.

The parameter estimates for the seven nested APARCH models in appendix A.1 are obtained by maximisation of the log-likelihood function. The general APARCH framework is given by:

[38] For example, intraweek effects, such as Monday effect, are not to be studied. Mixon (2002) also argued that 80-90% of the variation in implied volatilities in short-dated S&P 500 options could be explained by contemporaneous return.

$$r_t = a_0 + a_1 r_{t-1} + \varepsilon$$
$$h_t^{\delta} = \alpha_0 + \alpha_1 (|\varepsilon_{t-1}| - \gamma_1 \varepsilon_{t-1})^{\delta} + \beta_1 h_{t-1}^{\delta}$$

Tables 12-16 present the estimates of APARCH models and their Akaike Information Criterion (hereafter AIC)[39] and log-likelihood (hereafter LL) statistics for 1983-1998, 1983-1986, 1987-1990, 1991-1994 and 1995-1998, respectively. The 12th order Ljung-Box statistics for $\frac{\varepsilon_t}{h_t}$ and $\frac{\varepsilon_t^2}{h_t^2}$ are shown in table 17. Table 18 shows the model rankings for the AIC metric in each of the periods.

4.4.2.1 In-Sample Results from Maximum Likelihood Estimations

A number of observations can be drawn from tables 12-18. They are reported as follows:

i) The intercept parameter a_0 in the conditional mean equation is positive but it is significant only in 1983-1998, 1987-1990 and 1995-1998. The estimated AR(1) term, a_1, is negative and significant only in 1983-1998, suggesting returns are likely to be first order correlated in the long run. These results also suggest that the returns process is a white noise process in 1983-1986 and 1995-1998;

ii) An observed trend for ARCH and TARCH in tables 12-16 is that they have the lowest AIC and LL statistics in all sample periods. In addition, the 12th order Lung-Box statistics for $\frac{\varepsilon_t}{h_t}$ and $\frac{\varepsilon_t^2}{h_t^2}$ in table 17 show that ARCH and TARCH are extremely poor in capturing the first order and ARCH effects. AIC and LL statistics confirm that ARCH and TARCH are inferior models compared to other members of the APARCH family in every sub-sample. Because of this poor performance, one can safely disregard the significance of ARCH and TARCH models;

iii) APARCH(1,1) has the highest LL statistics in the full sample and its sub-periods. This is not a surprising result since APARCH(1,1) is the least parsimonious model within the APARCH framework;

iv) The power parameter δ for APARCH(1,1) is estimated to be 0.9981 for the entire 16-year period, which is not significantly different from one. This result is in line with the "Taylor effect" property. The sub-periods' results are mixed. The power

[39] AIC=LLR-p where p is the number of parameters.

parameter δ is significant and close to one in 1987-1990 and 1995-1998 but it is close to two in 1983-1986 and insignificant in 1991-1994;

v) The asymmetric parameter γ_1 is positive and significant for APARCH in 1983-1998, 1987-1990 and 1995-1998[40] but it is insignificant in 1983-1986 and 1991-1994;

vi) Model rankings for the AIC metric in table 18 indicate that the asymmetrical APARCH and TSGARCH-II models are the top performers in 1983-1998, 1987-1990 and 1995-1998;

vii) Symmetrical GARCH ranks first for the AIC metric in 1983-1986 and 1991-1994, respectively. However, GARCH is only ranked fifth by the AIC metric in 1987-1990 and 1995-1998.

4.4.2.2 Are APARCH Specifications Effective?

Estimation results obtained in section 4.4.2.1 lead us to cast doubt on the performance of APARCH specifications in sub-periods because GARCH is found to have outperformed other more complex models in 1983-1986 and 1991-1994. In addition, estimated δ and γ_1 of APARCH(1,1) are often insignificant in sub-periods. In this section we use log-likelihood ratio tests[41] (hereafter LLR) to examine the effectiveness of the power and asymmetric parameters within the context of APARCH in sub-periods.

4.4.2.2.1 LLR Test: Is Power Transformation Effective?

The Box-Cox power transformation is one of the most distinguishing features of APARCH specifications. The power parameter δ is believed to be responsible for the long decay in the autocorrelation function but is it effective in improving in-sample fit? This question is answered by conducting the following hypothesis tests:

$$H_0: GJR \ \ vs \ \ H_1: APARCH \qquad \sim \chi^2(1)$$
$$H_0: TSGARCH-II \ \ vs \ \ H_1: APARCH \qquad \sim \chi^2(1)$$

---

[40] These results are consistent with Black's observation that negative shocks are weighed more heavily than positive shocks in modelling volatility.

[41] The idea behind the LLR test is that if the a priori restrictions are valid, the restricted and unrestricted log-likelihood values should not be different. Formally if model A, having *n* parameters, is nested within model B, having *m* parameters, and the true parameters are within the parameter space defined by model A, then $2[\ln(L_B)-\ln(L_A)]$ approximately follows a χ^2 distribution with (*m-n*) degrees of freedom.

The following results are obtained for the hypothesis tests:

i) LLR tests can only reject GJR against APARCH(1,1)[42] at the 5% level in the entire sample, but not in any sub-periods;

ii) LLR tests cannot reject TSGARCH-II (when $\delta = 1$) at the 5% level against APARCH in either the full sample or any sub-periods.

Hence, it suggests that the incorporation of a free power parameter is less significant in sub-periods. The fact that LLR test cannot reject TSGARCH-II in favour of APARCH also signifies that the usefulness of power transformation in long sample is questionable.

4.4.2.2.2 LLR Test: Is Asymmetric Parameterisation Effective?

Another fixture of APARCH specifications is what Engle and Ng (1993) term as "rotation" when studying the "news impact curve"[43], in which γ_1 is responsible for this "rotation" effect within the APARCH framework. The use of the asymmetric parameter γ_1 is supposed to help capture leverage effects in the underlying asset but is this asymmetric parameterisation effective? The following hypothesis tests are conducted to test for the usefulness of asymmetric parameterisation:

$$H_0: GARCH \;\; vs \;\; H_1: GJR \qquad \sim \chi^2(1)$$
$$H_0: TSGARCH-I \;\; vs \;\; H_1: TSGARCH-II \qquad \sim \chi^2(1)$$
$$H_0: GARCH \;\; vs \;\; H_1: APARCH \qquad \sim \chi^2(2)$$

The results are:

i) LLR tests can only reject GARCH against GJR[44] in full sample at the 5% level, but not in other sub-periods except in 1995-1998.

ii) LLR tests can only reject TSGARCH-I ($\gamma_1 = 0$) against TSGARCH-II in full sample at the 5% level, but not in other sub-periods except in 1995-1998;

iii) LLR tests cannot reject GARCH against APARCH[45] at 5% level in 1983-1986, 1987-1990 and 1991-1994.

---

[42] APARCH(1,1) becomes GJR(1,1) when its power parameter δ =2.

[43] See sections 2.2.3.3 and 4.2.1 for details of Engle and Ng (1993).

[44] GJR(1,1) becomes GARCH(1,1) when its asymmetric parameter $\gamma_1 = 0$.

[45] APARCH(1,1) becomes GARCH(1,1) when $\delta = 2$ and $\gamma_1 = 0$.

Based on the above hypothesis tests, we conclude that the effectiveness of asymmetric parameterisation in sub-periods is questionable within the APARCH framework. These results are in agreement with Hentschel's (1995) findings that γ_1 (rotation effect) is neither statistically nor economically significant.

4.4.2.3 Discussions for APARCH In-Sample Results

Due to the complexity of our experiment design, it is necessary to restate the results of sections 4.4.2.1-2:

i) Asymmetrical models such as TSGARCH-II and APARCH exhibit superior performance in the full period, 1987-1990 and 1995-1998 in terms of AIC and LLR statistics. The estimated power parameter, δ, is close to one. Therefore, it is not surprising that the performance of TSGARCH-II is as robust as APARCH;

ii) Symmetrical GARCH outperforms other APARCH models during 1983-1986 and 1991-1994 in terms of the AIC metric;

iii) Results from LLR tests question the usefulness of the power transformation in modelling conditional volatility within the context of APARCH in both short and long samples;

iv) Results from LLR tests cast doubt on the effectiveness of incorporating the leverage effect within the context of APARCH in sub-periods;

v) ARCH and TARCH models, which do not incorporate any lag conditional volatility, underperform every model all the times.

In addition, table 18 also shows that the rankings across models are mixed in different sample periods. Our results indicate that asymmetrical models such as TSGARCH-II and APARCH tend to provide better fit in 1983-1998, 1987-1990 and 1995-1998 whilst symmetrical GARCH is favoured in 1983-1986 and 1991-1994. There is no evidence to confirm that there is a single model that will remain robust in every sub-period.

4.4.3 Are Conditional Volatility Models Prone to the State of Volatility?

In this section we investigate whether structural change of volatility can have an influence on the performance of asymmetrical and symmetrical conditional volatility models.

4.4.3.1 Student-t SWARCH(3,2)-L Model

As mentioned earlier, the partitioning of the data is motivated by the observation that the futures series do not exhibit homogeneous behaviour over the entire 16-year period. To support the hypothesis that performance of asymmetric and symmetric models are prone to the state of the samples, a 3-state student-t Markov-switching ARCH-L(2) developed by Hamilton and Susmel (1994) is employed to model the S&P 500 futures series from 1/1/1983 to 31/12/1998 and identify any structural breaks in volatility.

The specification of a leveraged student-t 3-state, second order-ARCH Markov-switching model is given by:

$$
\begin{aligned}
r_t &= \alpha + \phi r_{t-1} + u_t \\
u_t &= \sqrt{g_{s_t}} \cdot \tilde{u}_t \\
\tilde{u}_t &= h_t \cdot v_t \\
h_t^2 &= \lambda_0 + \lambda_1 \tilde{u}_{t-1}^2 + \lambda_2 \tilde{u}_{t-2}^2 + \xi \cdot d_{t-1} \cdot \tilde{u}_{t-1}^2
\end{aligned}
$$

where v_t~i.i.d. student-t with unit variance and v degrees of freedom, $s_t = 1,2,3$, $g_1 = 1$ when $s_t = 1$, $g_2 = k$ when $s_t = 2$, and $g_3 = l$ when $s_t = 3$, $l > k > 0$, $d_{t-1} = 1$ if $\tilde{u}_{t-1} \leq 0$, $d_{t-1} = 0$ if $\tilde{u}_{t-1} > 0$ and $-1 < \xi < 1$.

This switching model postulates the existence of an unobserved state variable, denoted s_t, that takes on the value of one, two or three. This variable characterises the "state" or "regime" that the process r_t is in at date *t*. When $s_t = 1$, the observed r_t is presumed to have been drawn from a low volatility state, when $s_t = 2$ r_t is presumed to have been drawn from a mid volatility state, whereas when $s_t = 3$ r_t is presumed to have been drawn from a high volatility state. The transition probabilities for the Markov chain for evolution of the unobserved state variable is written as:

$$
P = \begin{bmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \\ p_{13} & p_{23} & p_{33} \end{bmatrix}
$$

where $i = j = 1,2,3$, $p_{i,j} = \Pr ob(s_t = j \mid s_{t-1} = i)$, the transition probability from state *i* at time *t*-1 to state *j* at time *t*. The process for s_t is presumed to depend on past realisations of return

r and state s only through s_{t-1}. The inference about the particular state the process is in at date t using the full sample of observations T can be used to construct the "smoothed probability", $p(s_t \mid r_T, r_{T-1}, \ldots, r_{-3})$.

4.4.3.2 Detecting Structural Breaks in S&P 500 Futures Series

The methods developed in Hamilton (1989) are used to estimate parameters for S&P 500 futures from 28/4/1982 to 31/12/1998 and make inferences about the unobserved regimes[46]. The estimated student-t SWARCH-L(3,2) specification with its standard errors are given by:

$$r_t = 0.06551 + 0.03369 r_{t-1} + u_t$$
$$\quad (0.01203) \quad (0.01587)$$

$$g_1 = 1, g_2 = 2.51152, g_3 = 9.74334$$
$$\qquad\qquad (0.18513) \qquad (1.34630)$$

$$h_t^2 = 0.40248 + 6.68390 \cdot 10^{-4} \tilde{u}_{t-1}^2 + 0.025455 \tilde{u}_{t-2}^2 + 0.078354 d_{t-1} \cdot \tilde{u}_{t-1}^2$$
$$\quad (0.029049) \quad (0.01105) \qquad (0.01755) \qquad (0.03246)$$

$$v = 4.91416$$
$$\quad (0.40128)$$

$$P = \begin{bmatrix} 0.99242 & 0.00251 & 0.01594 \\ 0.00373 & 0.99493 & 0.01600 \\ 0.00375 & 0.00255 & 0.96801 \end{bmatrix}$$

4.4.3.2.1 Interpretations of Estimated SWARCH(3,2)-L Parameters

All coefficients, except λ_1 and λ_2, are significant. In addition, returns exhibit significant serial correlation. Although the t-ratio of 1.4507 for λ_2 is insignificant at the 5% level, its t-value is not completely out of line. The degree of freedom is 4.9142, which is far apart from being normal. The conditional variance in states 2 and 3 are estimated to be 2.512 and 9.743 times as large as in state 1, implying a subtle break of volatility in regimes 2 and 3. ξ is positive and significant at the 5% level, suggesting that leverage effects do play an important role in our data.

[46] We use GAUSS program downloaded from Hamilton's website at UCSD to estimate this model. We are thankful to Jame Hamilton's generosity.

The high transition probabilities (diagonal of *P*) indicate that if the system is in either state 1, 2 or 3, it is likely to remain in that state. Figure 34 plots the daily S&P 500 returns and smoothed probabilities for state 3, the ultra high volatility regions. States 1 and 2 are combined to display the milder volatility regions in figure 35. A 50% horizontal line is drawn in order to determine a switch of volatility state. Five short periods of high-volatility episodes can be identified characterising 4/1982-12/1982, 3/1987-4/1987, 10/1987-1/1988, 8/1990-11/1990 and the twin-peak region between 10/1997-11/1997 and 7/1998-10/1998. At the inception of S&P 500 futures market in 1982, the market is extremely volatile. The October 1987 crash is likely to be responsible for the observed high volatility in 10/1987-1/1988. The market is judged to have been in the high-volatility state in the second half of 1990 because of the Gulf War. The surges of volatility in 10/1997-11/1997 and 7/1998-10/1998 coincide with the timing of the Asian Financial Crisis and Russian Debt Moratorium, respectively. The origin of the 3/1987-4/1987 cannot be identified with any documented macroeconomic event.

4.4.3.3 Implications of Results from SWARCH(3,2)-L Model

The student-t SWARCH-L(3,2) results confirm that:

i) 1987-1990 and 1995-1998 are in the high-volatility state;

ii) 1983-1986 and 1995-1998 are in a more "subdued" state;

iii) Student-t SWARCH-L(3,2) is able to capture a number of economically important features of the data which may not otherwise be captured by standard conditional volatility models;

The student-t SWARCH-L(3,2) result has not only validated our assumption that asymmetric models such as TSGARCH-II and APARCH are more appropriate for volatile samples (symmetric models such as GARCH are more appropriate for less volatile periods), but also lent credibility to our finding in sections 3.4.3 and 3.4.4 that the S&P 500 market has started behaving more volatile and asymmetrically since 1987. Finally, the multiple volatility breakpoints in S&P 500 futures series support the contention that perhaps there is no single APARCH model is rich enough to allow thorough assessment of asymmetry and structural effect at the same time.

4.4.4 Additional In-Sample Analysis: EGARCH and Statistical Loss Functions

Evidence in sections 4.4.2 and 4.4.3 demonstrates that: 1) asymmetrical (symmetrical) models are superior to symmetrical (asymmetrical) models in more (less) volatile sample periods; 2) it is ineffective to incorporate power transformation and asymmetric parameterisation within the context of APARCH specifications; 3) notably, multiple structural breaks in the S&P 500 futures series imply that no single APARCH model is rich enough to model volatility in the presence of asymmetry and structural change at the same time.

In this section we extend our analysis by including a popular asymmetrical EGARCH (Nelson, 1991) model. In addition to the likelihood–based inferences, we also explore the ability of eight additional statistical error functions that allow for symmetry/asymmetry in the loss functions of investors to track the in-sample performance of the conditional models.

4.4.4.1 Inclusion of EGARCH

The EGARCH specification is not nested within the APARCH framework but it is important to study the performance of EGARCH with APARCH models because EGARCH is a more parsimonious specification which converges to the Wiggins model in diffusion limit. A EGARCH(1,1) can be written as:

$$\varepsilon_t = h_t v_t$$
$$\log h_t^2 = \alpha_0 + \alpha_1 z_{t-1} + \gamma_1 (|z_{t-1}| - (2/\pi)^{0.5}) + \beta_1 \log h_{t-1}^2$$

where $z_t = \frac{\varepsilon_t}{h_t}$, v_t ~i.i.d. student-t with unit variance and v degrees of freedom.

Estimates of the EGARCH model are displayed with APARCH models in tables 12-16. The following are observed:

i) All α_1 of EGARCH are negative, indicating that a negative shock increases the conditional volatility;

ii) All estimates of γ_1 for EGARCH are positive and significant, suggesting that a shock greater than $(2/\pi)^{0.5}$ also increases the conditional volatility;

iii) Negative α_1 and positive γ_1 are consistent with Black's leverage effect in equity returns;

iv) All $|\beta_1| < 1$, meaning that the EGARCH process is stationary in each sub-period;

v) $|\beta_1|$ is significantly smaller during 1991-1994 and 1995-1998, which suggests that the persistence of volatility clustering is relatively limited in the second half of the samples.

4.4.4.1.1 In-Sample Results for EGARCH

Due to excess kurtosis and negative skewness in the S&P 500 futures returns series, the prior expectation is that asymmetrical models provide a better fit to the "noisy" periods as opposed to symmetrical models, and vice versa. Model ranking for AIC in table 19 shows that:

i) EGARCH is the best model in terms of AIC in 1983-1998 and the pre-defined "noisy" sub-periods in 1987-1990 and 1995-1998;

ii) GARCH remains best model in terms of AIC in the pre-defined "quiet" sub-periods in 1983-1986 and 1991-1994;

iii) EGARCH is only ranked fifth and fourth in the pre-defined "quiet" sub-periods in 1983-1986 and 1991-1994.

iv) Overall EGARCH is best in terms of aggregate AIC score, followed by TSGARCH-II and GJR whilst GARCH and APARCH are tied in fourth.

Apparently our prior expectation that asymmetrical (symmetrical) models provide a better fit to the noisy (quiet) periods is upheld. In addition, these results demonstrate that EGARCH is a more consistent and capable than APARCH models to capture asymmetries in the S&P 500 market.

4.4.4.1.2 Discussion for In-Sample Results based on AIC

Our results indicate that the APARCH model performs poorly in the S&P 500 market. On the basis of the AIC metric, we find that EGARCH and GARCH provide the best in-sample fit for the S&P500 data in different sub-periods. The results obtained from the in-sample analysis are not unanticipated since the EGARCH model measures both sign and size effects: a negative α_1 implies that a negative shock increases the conditional variance (sign effect); an estimated positive λ_1 indicates that a shock greater than $(2/\pi)^{0.5}$ also increases the conditional variance (the size effect). Thus the EGARCH model is able to accommodate a more complex asymmetric relation between stock returns and volatility changes.

4.4.4.1.3 Plausible Explanation for the Poor Performance of APARCH

The APARCH model was originally designed to model the long memory property inherited in the power transformation of absolute returns. There is, indeed, little evidence for long-memory in sub-periods as evidenced in the autocorrelations plots in figures 23, 24, 26, 27, 29, 30, 32 and 33 for r^2 and $|r|$. Therefore, it is not surprising that the APARCH model is not able to outperform EGARCH and GARCH even in a large sample.

4.4.4.2 Inclusion of Alternative Statistical Loss Functions

In the previous analysis models are selected using the likelihood-based inferences such as the AIC metric and LLR test. AIC and LLR statistics use information inferred from maximisation of log-likelihood functions to give an indication of goodness-of-fit of the model estimated, which may deviate from the results of other more meaningful loss functions. The log-likelihood statistic selects the most appropriate model by maximising the probability of having the observed data given that the functional form of the probability density function is pre-determined. The AIC criterion, in turn, chooses the most parsimonious model by using information from the log-likelihood function plus a penalty adjustment involving the number of estimated parameters. These criteria are subject to distributional assumption. Therefore, it is also very important to examine the ability of other distributional-free loss functions to track the in-sample performance of conditional volatility models.

4.4.4.2.1 Procedures for Calculating In-Sample Statistical Errors

In order to make complete our analysis, eight additional statistical loss functions are included in the in-sample study and their functional forms are shown in appendix A.2. They are:

i) Mean-square error (MSE);

ii) Mean absolute error (MSE);

iii) Mean-absolute percent error (MAPE);

iv) Mean-mixed error which penalises under-predictions (MMEU);

v) Mean-mixed error which penalises over-predictions (MMEO);

vi) Logarithmic loss function (LL);

vii) Heteroskedasticity-adjusted mean-square error (HMSE);

viii) Guassian quasi-maximum likelihood function (GMLE).

MSE, MAE and MAPE are symmetrical loss functions whereas MMEO, MMEU, LL, HMSE and GMLE are asymmetrical loss functions. Asymmetrical loss functions are included here because investors do not necessarily attribute equal importance to both over- and under-predictions of volatility of similar magnitude. The Performance of a conditional volatility prediction model judged by its ability to predict future ex post volatility. Following Bracker and Smith (1999), the procedures in measuring the alternative in-sample statistical errors are:

i) Estimate the structural parameters for the whole sample and each sub-period in our sample, i.e. 1983-1998, 1983-1986, 1987-1990, 1991-1993 and 1995-1998;

ii) Use ε_t as a volatitliy proxy esimated from the structural mean equation at day t:

$$r_t = a_0 + a_1 r_{t-1} + \varepsilon_t$$

iii) Calculate the statistical error statistics according to table A.2 where T is the number of observations per period and h_t is the ex-post predicted conditional volatility at day t.

4.4.4.2.2 Results for Alternative Statistical Loss Functions

Table 21 exhibits the in-sample model rankings for MMEO and MMEU. Table 22 shows the in-sample rankings of models under HMSE, GMLE and LL statistics. Table 23 displays the in-sample rankings for MSE, MAE and MAPE. Table 24 is the aggregated rankings for all statistical loss functions. Results from tables 21-23 reveal that:

i) Alternative model rankings are highly sensitive to the statistics used to assess the accuracy of the forecasts; In the case of EGARCH, it is ranked first by MAE but is the worst model according to MSE and MAPE statistics;

ii) It is interesting to realise the exceptional performance of models ranked by some statistical loss functions that are previously identified as poor models in the AIC analysis. For example, the ARCH and TARCH models are ranked last by log-likelihood inference statistics such as AIC, LL and GMLE but they are the best models according to the MMEU statistic.

4.4.4.2.3 Comments on Results for In-Sample Statistical Loss Functions

Our results show that no single model is clearly superior under alternative statistical criteria. Consequently, it is not sensible to evaluate forecasting performance with only a single statistical loss function. As suggested by Li (2002), these confusing results could be introduced by the way volatility proxy was constructed using squared returns. Furthermore, the forecasting performance of different conditional volatility models may as well depend on the specific asset class under consideration. The question remains what criteria should one use to judge the superiority of any volatility forecast.

4.4.5 Out-of-Sample Analysis: Trading S&P 500 Straddles

4.4.5.1 Background

Whilst in-sample investigations provide useful insights into volatility forecasting performance, models are selected only on the basis of ex-post information. For practical forecasting purposes, the predictive ability of these models needs to be examined out-of-sample. Given the conflicting results of the competing statistical loss functions in section 4.4.4.2, it is recommended that the choice of error measure should depend on the ultimate usage of the forecasting procedure, i.e. the utility function of the user of the forecast. The AIC metric might be more appropriate for selecting models when there is a given distributional assumption. In the context of option trading, however, a call option buyer being concerned with over-predictions, would prefer the MMEO statistic[47].

4.4.5.2 Volatility Forecasting Models

The purpose of this section is to use an out-of-sample preference-free approach to illustrate the quality of different forecasting models by predicting the one-step ahead changes of implied volatility and conducting ex-ante S&P 500 straddle trading. The models under investigation in this section are:

i) EGARCH(1,1)
ii) GARCH(1,1)
iii) ARCH(1)
iv) A two-stage predictor of conditional volatility.

[47] See Brailsford and Faff (1996) for details.

The forecasting models employed in our trading experiments are selected primarily based upon the results obtained in section 4.4.4. They range from naive models to the moderately complex class of ARCH models. Unlike other ARCH-type models that simultaneously estimate parameters from both the conditional mean and variance equations, a two-stage regression model updates them independently. A two-stage predictor models conditional volatility by first calculating the proxy for conditional volatility and then fitting a standard AR(1) model for this proxy. It is given by:

$$r_t = a_0 + \varsigma_t$$
$$|\varsigma_t| = \alpha_0 + \alpha_1 |\varsigma_{t-1}|$$

where $\varepsilon_t = v_t h_t, v_t \sim i.i.d.\ normal$ and h_t is the conditional volatility. $|\varsigma_t|$ is a proxy for expected future volatility.

4.4.5.3 Trading Methodology

As volatility is unobservable, there is no natural metric for measuring the accuracy of any particular model. Realised returns, however, allow one to test the performance of volatility-driven option trades and provide a test for market efficiency with respect to volatility forecasts. Many studies have used realised profits as a yardstick to assess the forecasting performance of conditional volatility models, e.g. Engle et al. (1993) and Noh et al. (1994)[48]. This section evaluates the performance of different volatility forecasting models by assessing whether profits can be generated from trading nearest-the-money[49] straddles on S&P 500 futures with shortest remaining times to maturity[50].

4.4.5.3.1 Why Trading Delta-Neutral Straddles?

According to Becker et al. (1991), the advantages of the use of nearest-the-money options are:

i) It reduces the non-synchronous data problem because they have the greatest liquidity and represent accurate measures of ex-ante market volatility;

[48] See section 2.5.6.1 for review of volatility trading.

[49] In the real world with limited supply of options we are more likely to trade nearest-the-money straddles.

[50] It corresponds to region 11 in our database. They are nearest-the-money options with maturities between 21 and 70 days.

ii) The purchase (sell) of a straddle is a simple strategy established as a volatility trade when a trader has a bullish (bearish) outlook on the volatility of the underlying futures;

iii) Nearest-the-money call and put options have deltas close to 0.5 and –0.5, respectively, giving straddle a combined position of nearly zero[51]. A near-zero delta would mean that a small change of the underlying futures in either direction would have little or no impact on the option price, thus making straddle essentially a volatility trade.

4.4.5.3.2 Trading Assumptions

A few assumptions are needed in order to rationalise our trading strategy. They are as follows:

i) Conditional volatility is a reasonable proxy for at-the-money implied volatility. This is not unreasonable since many studies have found that the Black-Scholes implied volatility is empirically indistinguishable from most stochastic and conditional volatility option pricing models when options are at-the-money and have short times to expiration;

ii) The changes in implied volatility is predictable in a statistical sense (e.g. Harvey et al., 1991, 1992; Noh et al., 1994; Fleming et al., 1995; Bilson, 2002) but not the level of implied volatility, and profits depend on correct forecasts of the directional change of the underlying futures' volatility;

iii) Within this study, it is noted that the forecasting horizon matches the investment horizon, but not the remaining maturity of the straddles. The use of shortest-maturity straddles should mitigate the impact of the maturity mismatch problem;

iv) The forecasts made on week t for week t+1 are weekly instantaneous volatility that tends towards the short-term weekly mean volatility. An anticipated gain results from the expected tendency of options to increase or decrease in volatility.

4.4.5.3.3 Trading Strategy

The following procedures explain how we conduct our trading exercises of S&P 500 straddles in this study:

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[51] Alternatively, a long call and a short futures contracts with an appropriate hedge ratio can achieve a delta-zero position too. However, the hedge ratio is not necessary to be integer and we cannot trade fractional contracts of futures, making the delta-neutral position more difficult to obtain. Moreover, it requires a bigger investment in margins than a pure option hedge (Wood et al. 1987).

i) S&P 500 futures time-series constructed in section 4.3.1 are divided into four successive non-overlapping sub-periods of four years, i.e. 1983-1986, 1987-1990, 1991-1994 and 1995-1998;

ii) The last two years in each of the sub-periods, i.e. 1985-1986, 1989-1990, 1993-1994, 1997-1998, are reserved for out-of-sample evaluation purposes;

iii) Each volatility predictor forms a trading opinion by estimating a one-step ahead forecast on each week during out-of-sample periods;

iv) For each Wednesday of an out-of-sample period, we select the shortest-maturity straddle whose exercise price is closest to the current futures level;

v) At the end of Wednesday's trading on each week t, conditional volatility estimates are obtained from processing the most recent returns data up to and including week $t-1$;

vi) The coefficient estimates are then applied to the information available on week t to form forecasts of the volatility change for week $t+1$;

vii) If volatility is predicted to increase (decrease) from week t to week $t+1$, a straddle is purchased (sold);

viii) Once a straddle position is obtained at week t, the trade will be reversed at week $t+1$;

ix) We assume that options can be sold and purchased at daily settlement prices, and actual settlement prices on CME's futures options are used to compute the profit and loss;

x) Sample size is increased by one as most recent data become available and each model's parameters are re-estimated on every successive Wednesday over the remaining of the out-of-sample periods, therefore successive weekly estimates of volatility on week t can be calculated recursively using only returns information before week t.

In addition, we assume that there is no margining requirement and agents are free to sell short. Each agent invests $100 and trades the nearest-to-the-money contract. When a straddle is sold, the agent is allowed to invest the proceeds plus $100 in a risk-free asset. Two general cases are considered: 1) without transactions costs; 2) with transactions costs; we assume that a straddle trade costs of 25 basis point ($250*0.25=$62.5) for both legs in commissions. The rate of return on buying straddles is computed as follows:

$$RR_t = \frac{100}{(C_{t-1}+P_{t-1})}(C_t + P_t - C_{t-1} - P_{t-1}) + 100 * I * r_f - \frac{100}{(C_{t-1}-P_{t-1})} * TC$$

where C_t and P_t are call and put prices at time t, respectively. I is either 0 when the trade is a buy or equal to 1 when the trade is a sell to allow agents accumulate interest in their accounts. TC is the transactions costs that can take on either 0 (no transactions costs) or 0.25 point (with transactions costs); r_f is the risk-free rate. This method of calculating the rate of returns is indeed identical to the one used by Noh et al. (1994).

4.4.5.3.4 Why Not Other Trading Strategy?

Whilst it may be argued that one can buy/sell straddle if forecasted volatility is above/below implied volatility, we must point out that this trading strategy assumes implicitly that implied volatility is forecastable. However, our main trading assumption in this study is less stringent and only requires the forecastability of the directional change of implied volatility, but not the level. In addition, multi-period ahead forecast must be formed to match the maturity of the straddle in order to make the aforementioned trading strategy workable. Among others, Noh, Engle and Kane (1994) used this approach to trade straddle and found that the GARCH model was able to return profits. Whilst this trading strategy is definitely rational, trading for the directional change of volatility is a more flexible strategy and we feel that that there is no unique way to devise trading signals.

4.4.5.4 Trading Database

The dataset comprises of weekly settlement prices of S&P 500 futures options for the period from 1983 through 1998. The same options and futures databases constructed in Chapters 3 and 4 are used for the out-of-sample trading experiment. We refer the reader to sections 3.3 and 4.3 for their contract specifications.

4.4.5.4.1 Weekly Straddles

S&P 500 index futures options are American and expire on the same day as the underlying futures contracts. The futures and option price data are Wednesday's[52] settlement prices from CME. When a holiday occurs on Wednesday, Tuesday's observation is used in its place. The size of one futures contract is $250 multiplied by the index level and each index point is worth $250. The minimum move in the futures price is 0.1 point or $25. A one-point change in

S&P 500 futures option premium represents the same dollar value of a one-point change in the S&P 500 futures. As a proxy for the risk-free interest rate, we use daily middle rates on U.S. Treasury bills from Datastream matching maturity closest to the expiration date of the options.

4.4.5.4.2 Weekly Time-Series Statistics

The S&P 500 futures time-series constructed in section 4.3.1 are divided into four successive non-overlapping sub-periods of four years, i.e. 1983-1986, 1987-1990, 1991-1994 and 1995-1998. Results from the Dickey-Fuller test rejects the null hypothesis that there is a unit root in each of the four sub-periods. Skewness is negative for all sub-periods[53], suggesting that weekly return is more likely to be negative. Excess kurtosis for 1983-1986, 1987-1990, 1991-1994 and 1995-1998 are 0.850, 7.919, 1.411 and 2.081, respectively. Ljung-Box statistics for r up to 10 lags are insignificant for all sub-periods, meaning that returns are not autocorrelated. In addition, correlograms for r also confirm that serial correlation is insignificant in any sub-periods.

These preliminary statistics posit that weekly returns are less leptokurtic and autocorrelated (closer to normally distributed) than daily returns amid weekly returns are more negatively skewed. This finding is consistent with the consensus that the longer the interval over which returns are calculated, the lesser is the autocorrelation. Consequently, it is not necessary to remove any first-order autocorrelation from the return series.

4.4.5.5 Results of Trading At-the-Money Straddles

4.4.5.5.1 Preliminary Statistics for Directional Trading Signals

This section aims to demonstrate that our four predictors produce very different buy/sell signals at times.

Table 25 shows the correlations for the out-of-sample directional trading signals[54] generated from different volatility prediction models in each sub-period. Table 26 also exhibits some basic statistics for the forecasts of volatility changes. The mix of low values of positive and

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[52] Wednesdays are chosen because few holidays fall on Wednesdays.

[53] They are –0.242, -1.627, -0.019 and –0.842 for 1983-1986, 1987-1990, 1991-1994 and 1995-1998, respectively.

negative correlations coefficients on out-of-sample buy and sell signals in 1985-1986 confirms that at the inception of the S&P 500 options market volatility predictors produce very mixed opinions on their one-step ahead forecasts.

After 1985-1986, however, all correlations coefficients are large and positive, indicating that our predictors have become more likely to agree with each other on issuing the same buy or sell signal. One plausible explanation for this dramatic change of forecasting behaviour is that it is caused by the increase of returns autocorrelations after the 1987 crash. It suggests that our conditional models are capable of picking up "volatility clustering" or "memory", thus making different volatility predictors to produce similar forecasts. Table 26 also shows that standard deviations of volatility changes have become significantly larger since 1987. The min/max statistics indicate that the ARCH model is more likely to produce high estimates and therefore over-predict volatility changes. In contrary, the GARCH model is likely to have smaller estimates and under-predict volatility.

Tables 27-30 present the before-transactions-costs statistics for each volatility predictor for all sub-periods. The average maturity of straddles in 1985-1986 is 0.198 year (ten weeks). Since the introduction of serial contracts in 1987, it has been reduced to around 0.12 year (six weeks) in 1997-1998. Although strike price increments are generally integers divisible by five, futures level raises from 139 to 1245.15 during the entire sample period. Therefore, straddles are closer to delta-neutral towards the end of the sample period and standard deviations of their delta also have decreased steadily from 0.107 in 1985-1986 to 0.021 in 1997-1998. In addition, the descriptive statistics show that call prices have increased from 5.403 in 1985-1986 to 27.349 points in 1997-1998. Correspondingly, put prices have also raised from 5.502 to 27.267 points. Furthermore, results show that the ARCH model is very keen to produce buy signals, issuing the highest number of buys in three of out four sub-periods.

In contrast, both the EGARCH and GARCH models prefer selling than buying but GARCH can be perceived as even more willing to short, issuing 99 sells versus 5 buys in 1985-1986. Finally, our descriptive statistics show that the two-stage regression model is as likely to buy

[54] The signal is 1 when it is a buy and –1 when it is a sell.

as to sell in all sub-periods. Our analysis suggests that the four predictors under study are indeed quite different at times.

4.4.5.5.2 Profit and Loss: Trading At-the-Money Straddles

Before Transaction Costs and No Delta Filter

Without transactions costs, the EGARCH model has the highest rates of return per trade in 1985-1986 and 1989-1990, respectively. In 1993-1994, the EGARCH model ranks second after the GARCH model. The EGARCH model is second to the ARCH model in 1997-1998. Before transactions costs, profits can be made in 1985-1986 and 1993-1994, although only EGARCH and GARCH models can produce statistically significant returns at t-ratios of 1.66 and 2.48 in 1993-1994[55], respectively. Trading results also indicate that no predictor can make any profit in 1989-1990, and ARCH is the only model that is successful in earning profit in 1997-1998.

Before Transactions Costs and 3% Delta Filter

In the results discussed thus far, data are unfiltered. A more rational trading approach is to exercise our strategy only when nearest-the-money straddles are close to delta-neutral. Consequently, a filtering rule is applied to remove trades that do not satisfy put-call-futures parity[56] by trading straddles with absolute delta less than or equal to 3%. Tables 31-34 report the before-transactions-costs statistics with a $\pm 3\%$ delta filter for each volatility predictor in 1985-1986, 1989-1990, 1993-1994 and 1997-1998, respectively. Under this filter, the number of transactions in 1985-1986, 1989-1990, 1993-1994 and 1997-1998 are traded in only 16.3%, 30.1%, 19.4% and 54.4%, respectively. Tables 31-34 also show that standard deviations of straddles' delta have been reduced dramatically. After applying this filter, the EGARCH model has the highest rates of return per trade in three out of four out-of-sample periods, i.e. 1985-1986, 1989-1990 and 1993-1994. Although all predictors succeed in making profits in 1993-1994, only the EGARCH and GARCH models can produce statistically significant returns at t-ratios of 2.22 and 2.16. It is also noted that GARCH fails to make any profits in all sub-periods except in 1993-1994. Finally, both the ARCH and two-stage regression

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[55] Since the t-ratio of return from trading straddle are assumed to be independent, the t-ratio is computed as a ratio of mean to standard deviation divided by the square root of the number of observations.

[56] From practical point of view it means the European put-call-futures parity. See Fung and Fung (1997).

models make losses in three out of four out-of-sample periods, i.e. 1985-1986, 1989-1990 and 1997-1998 although ARCH still remains first in 1997-1998.

Before Transaction Costs and 3% Delta Filter (excluding one spurious point)

Curiously, the performance of the EGARCH model is second to a simple ARCH model in 1997-1998. A careful scrutiny of our options data from 1997-1998 reveals that a "spurious" trade made between 22/10/1997 and 29/10/1997 is indeed very erratic. The price of this straddle has increased from 40.3 to 59.85 points within a week. During the same time period, its delta has decreased from 0.0097 to –0.6535. The timings of this "spurious" trade coincide with the surge of volatility at height of the Asian Financial Crisis identified in section 4.4.3.2. During 20-23 of October 1997, the Hong Kong stock market suffers its heaviest losses ever, shedding nearly a quarter of its value in four days. A week later on 27 October of 1997, Asian jitters spill over on to world stock markets. The Dow Jones index plunges 554 points, its largest single-day point loss ever[57]. Therefore, it is not unreasonable to assume that a prudent trader would exercise extreme caution in such a chaotic trading environment. After removing this questionable data point, we find that the EGARCH model is first in terms of rate of returns in 1997-1998. Table 35 exhibits the before-transactions-costs statistics for 1997-1998 with the $\pm 3\%$ delta filter after the removal of this questionable data point. These results also show that the EGARCH model is the only profitable predictor in 1997-1998.

After Transaction Costs and 3% Delta Filter (excluding one spurious point)

None of the profits reported in our trading strategies thus far have attempted to account for the effects of transaction costs. With transactions costs of 25 basis points for both legs, the profits are dramatically reduced although rankings between predictors remain in the same orders. The summary statistics for after-transactions-costs with a $\pm 3\%$ delta filter are given in tables 36-39 for each volatility predictor for the periods 1985-1986, 1989-1990, 1993-1994 and 1997-1998, respectively. No predictors can earn any profits in 1985-1986, 1989-1990 and 1997-1998. In addition, EGARCH and GARCH have the first and second highest rate of returns per trade in each sub-period, respectively. After transactions costs, all predictors from 1993-1994 have positive rates of return but only EGARCH and GARCH can generate returns that significantly exceed transactions costs at t-ratios of 1.49 and 1.45, respectively. This

[57] Source from Tudor, G. (2000)

argument is supported by figure 36, which shows the cumulative rate of return from straddle trading of agents using EGARCH, GARCH, ARCH and a two-stage regression model with transactions costs and a $\pm 3\%$ delta filter in 1993-1994.

4.4.5.5.3 Trading Summary

We report that EGARCH produces the highest rate of returns per trade in every sub-period. In addition, EGARCH and GARCH can generate statistical significant ex-ante profit after transactions costs. Therefore, we cannot deny that there are certain degrees of inefficiency and predictability in the S&P 500 market. Finally, our trading experiments also reveal the presumption of using delta-neutral trade to create a risk-free portfolio is not practical in the event of large index movements. A new derivatives instrument is needed to allow traders and investors speculate on volatility more directly and efficiently.

4.5 Summary

This chapter compares the performance of a group of well-theorised conditional volatility models that can potentially account for the term-structure biases observed in the S&P 500 futures options market. Sixteen years of daily S&P 500 futures series are used to examine the performance of the APARCH models that use asymmetric parameterisation and power transformation on conditional volatility and its absolute residual to account for the slow decay in returns autocorrelations. Our results are:

i) No evidence can be found supporting the relatively complex APARCH models. Log-likelihood tests confirm that asymmetric parametersiation and power transformation are not effective in characterising the S&P 500 returns dynamics within the context of APARCH specifications;

ii) Results from the 3-state volatility regime-switching model supported the notion that the performance of conditional volatility models is prone to the state of volatility of the returns series. Furthermore, log-likelihood based statistics stipulate that the EGARCH model is best in "noisy" periods whilst GARCH is the top performer in "quiet" periods;

iii) Overall, aggregate rankings for the AIC criterion show that the EGARCH model is the best performer;

iv) In-sample results show that it is not sensible to evaluate forecasting performance with only a single statistical loss function;

v) Out-of-sample results demonstrate that the EGARCH model outperforms GARCH, and both of them can generate statistically significant ex-ante returns in one out of four sample periods;

vi) Trading experiments also reveal that the presumption of using delta-neutral trade to create a risk-free portfolio is not practical in the event of large index movements.

Our findings are not only significant to discrete-time finance but also potentially meaningful for continuous-time stochastic volatility literature because continuous-time stochastic volatility models can be thought of as the limits of ARCH-type process. Nelson (1991), for instance, showed that EGARCH(1,1) converged to a specific bivarate diffusion model in continuous time limit. Moreover, Duan (1997) also proved that most of the existing bivariate diffusion models that had been used to model asset returns volatility could be represented as limits of a family of GARCH models. When considering a stochastic volatility model, there seems to be little incentive to look beyond a simple model which allows for volatility clustering and a leverage effect such as Heston (1993).

Table 9: Descriptive Statistics for *r*

	1983-1998	1983-1986	1987-1990	1991-1994	1995-1998
DF stat.	-65.65442 [.000]	-32.64347 [0.000]	-32.16886 [.000]	-32.61263 [.000]	-34.76218 [.000]
Maximum	0.177493	0.037518	0.177493	0.042612	0.056547
Minimum	-0.337004	-0.056886	-0.337004	-0.036987	-0.077621
Mean	0.000573	0.000530	0.000308	0.000329	0.000981
Std. Dev.	0.011845	0.009347	0.017467	0.007111	0.010881
Skewness	-5.279559	-0.077911	-0.6452279	0.211765	-0.532956
Kurtosis-3	179.218	2.470201	148.7873	3.068323	8.15034
Q(10)	79.085 [.000]	10.925 [0.363]	59.226 [.000]	8.9945 [.532]	19.038 [.040]
Jarque-Bera stat.	5432196 [.000]	257.5546 [.000]	939563.7 [.000]	404.9461 [.000]	2848.995 [.000]
#. Obs.	4045	1009	1011	1013	1012

Table 10: Descriptive Statistics for r^2

	1983-1998	1983-1986	1987-1990	1991-1994	1995-1998
DF stat.	-58.38758 [.000]	-31.20798 [.000]	-29.287 [.000]	-31.23613 [.000]	-23.41297 [.000]
Maximum	0.113572	0.003236	0.113572	0.001816	0.006025
Minimum	0.000000	0.000000	0.000000	0.000000	0.000000
Mean	0.000141	8.76E-05	0.000305	5.06E-05	0.000119
Std. Dev.	0.001883	0.000185	0.003739	0.000114	0.000374
Skewness	55.24303	7.173269	28.14637	7.123234	10.05873
Kurtosis-3	3269.947	92.15109	836.6669	80.24314	132.6307
Q(10)	342.93 [.000]	7.7053 [.720]	83.672 [.000]	26.486 [0.003]	225.80 [.000]
Jarque-Bera stat.	1.80E+09 [.000]	365663.5 [.000]	29621473 [.000]	280344.5 [.000]	758814.9 [.000]
#. Obs.	4045	1009	1011	1013	1012

Table 11: Descriptive Statistics for |*r*|

	1983-1998	1983-1986	1987-1990	1991-1994	1995-1998
DF stat	-49.4674 [.000]	-32.33748 [.000]	-23.33977 [.000]	-30.55499 [.000]	-25.51959 [.000]
Maximum	0.337004	0.056886	0.337004	0.042612	0.077621
Minimum	0.000000	0.000000	0.000000	0.000000	0.000000
Mean	0.007115	0.006877	0.009016	0.005122	0.007449
Std. Dev.	0.009484	0.006349	0.01496	0.004940	0.007988
Skewness	13.43799	1.903316	12.66372	2.074516	3.159958
Kurtosis-3	394.0034	6.027266	245.3357	7.04718	17.11600
Q(10)	1332.2 [.000]	6.5947 [.763]	425.59 [.000]	42.258 [.000]	319.56 [.000]
Jarque-Bera stat.	26285933 [.000]	2136.489 [.000]	2562509 [.000]	2822.777 [.000]	14037.23 [.000]
#. Obs.	4045	1009	1011	1013	1012

Table 12: Estimated Parameters for 1983 – 1998

	a_0	a_1	α_0	α_1	β_1	γ_1	δ	ν	AIC	LL
APARCH	0.000566	-0.03114	0.000141	0.06138	0.941286	0.526122	0.998113	4.434411	15761.93	15769.93
	(4.677)	(-2.106)	(1.214)	(5.697)	(75.975)	(4.804)	(5.894)	(12.687)	~	~
ARCH	0.000737	-0.03317	9.70E-05	0.153383	0	0	2	3.320262	15587.27	15592.27
	(5.994)	(-2.135)	(14.294)	(4.368)	~	~	~	(16.534)	~	~
GARCH	0.000695	-0.03375	1.34E-06	0.049727	0.938481	0	2	4.23194	15738.65	15744.65
	(5.955)	(-2.342)	(1.774)	(3.826)	(52.682)	~	~	(-12.384)	~	~
TSGARCH-I	0.000658	-0.03649	8.78E-05	0.053548	0.952648	0	1	4.25791	15749.37	15755.37
	(5.727)	(-2.431)	(2.714)	(5.529)	(104.44)	~	~	(12.928)	~	~
TSGARCH-II	0.000566	-0.03114	0.00014	0.061366	0.941271	0.525712	1	4.434428	15762.93	15769.93
	(4.757)	(-2.104)	(2.566)	(5.657)	(76.55)	(5.041)	~	(12.675)	~	~
GJR	0.000633	-0.03057	2.02E-06	0.054233	0.921746	0.284769	2	4.331409	15745.49	15752.49
	(5.37)	(-2.093)	(1.44)	(3.625)	(30.715)	(2.301)	~	(12.432)	~	~
TARCH	0.000671	0.000952	0.009245	0.183006	0	0.325711	1	3.333928	15583.02	15589.02
	(5.389)	(0.061)	(26.528)	(6.128)	~	(2.503)	~	(16.409)	~	~
EGARCH	0.00057	-0.03069	-0.13268	-0.05228	0.985006	0.107852	~	4.451695	15764.73	15771.73
	(4.815)	(-2.109)	(-3.426)	(-3.618)	(234.657)	(6.892)	~	(12.52)	~	~

Bracketed values are t-statistics. Estimates are heterskedastic-consistent.

Table 13: Estimated Parameters for 1983 - 1986

	a_0	a_1	α_0	α_1	β_1	γ_1	δ	ν	AIC	LL
APARCH	0.000407	-0.04941	3.52E-07	0.014898	0.9741247	0.16749	2.178932	5.017256	3898.194	3906.194
	(1.551)	(-0.903)	(0.424)	(1.907)	(123.372)	(0.782)	(4.259)	(6.565)	~	~
ARCH	0.000483	-0.05408	9.00E-05	0.001955	0	0	2	4.507424	3893.338	3898.338
	(1.865)	(-2.026)	(12.453)	(0.096)	~	~	~	(7.05)	~	~
GARCH	0.000428	-0.04742	7.49E-07	0.0165	0.9750014	0	2	5.007661	3899.795	3905.795
	(1.667675)	(-1.718)	(1.685)	(2.81693)	(154.2846)	~	~	(6.542)	~	~
TSGARCH-I	0.0004117	-0.05029	8.209E-05	0.0215354	0.975268	0	1	5.026743	3898.736	3904.736
	(1.585108)	(-1.817)	(1.742406)	(2.934989)	(182.8809)	~	~	(6.601)	~	~
TSGARCH-II	0.0004106	-0.05032	8.22E-05	0.02167	0.975159	0.013533	1	5.027212	3897.737	3904.737
	(1.583079)	(-1.817)	(1.7025)	(2.657)	(155.341)	(0.049)	~	(6.599)	~	~
GJR	0.000404	-0.04946	8.15E-07	0.01671	0.973805	0.174299	2	5.020589	3899.159	3906.159
	(1.55)	(-1.805)	(1.545)	(2.674)	(123.374)	(0.743)	~	(6.556)	~	~
TARCH	0.000435	-0.04454	0.009393	0.007904	0	0.970857	1	4.64424	3892.246	3898.246
	(1.448344)	(-1.564)	(10.7684)	(0.068146)	~	(0.042)	~	(6.371)	~	~
EGARCH	0.000409	-0.05023	-0.08268	-0.0017	0.990892	0.043779	~	5.02453	3897.766	3904.766
	(1.56)	(-1.812)	(-2.209)	(-0.104)	(255.939)	(2.152)	~	(-6.602)	~	~

Bracketed values are t-statistics. Estimates are heterskedastic-consistent.

Table 14: Estimated Parameters for 1987 - 1990

	a_0	a_1	α_0	α_1	β_1	γ_1	δ	ν	AIC	LL
APARCH	0.0007678	-0.01373	0.000197	0.052397	0.907472	0.999977	1.207423	4.306788	3726.837	3734.837
	(2.614)	(-0.455)	(0.695)	(1.921)	(26.506)	(3190.562)	(3.861)	(4.289)	~	~
ARCH	0.000971	-0.00766	0.000155	0.225647	0	0	2	2.988148	3686.546	3691.546
	(3.299)	(-0.243)	(5.43)	(2.445)	~	~	~	(8.533)	~	~
GARCH	0.001011	-0.02473	6.52E-06	0.054998	0.904233	0	2	3.410253	3719.677	3725.677
	(3.666)	(-0.869)	(2.402)	(2.128)	(28.557)	~	~	(7.059)	~	~
TSGARCH-I	0.001001	-0.02564	0.000353	0.080954	0.916725	0	1	3.387015	3720.844	3726.844
	(3.7)	(-0.842)	(2.572)	(3.674)	(42.842)	~	~	(7.231)	~	~
TSGARCH-II	0.0006397	-0.00529	0.0007522	0.0688826	0.8814309	0.999969	1	4.4949707	3726.737	3733.737
	(1.976)	(-0.166)	(1.959)	(3.273)	(19.239)	(3.475)	~	(4.304)	~	~
GJR	0.0008948	-0.01923	8.097E-06	0.023289	0.891383	0.999989	2	3.554544	3722.337	3729.337
	(3.116)	(-0.656)	(2.033)	(1.609)	(17.918)	(87592)	~	(6.992)	~	~
TARCH	0.0009182	-0.02058	0.0118337	0.223759	0	0.246172	1	2.939039	3683.215	3689.215
	(3.092)	(-0.688)	(10.073)	(3.562)	~	(1.096)	~	(8.587)	~	~
EGARCH	0.000787	-0.01675	-0.30163	-0.10033	0.965752	0.088584	~	3.709069	3728.323	3735.323
	(2.721)	(-0.575)	(-2.446)	(-3)	(69.669)	(2.769)	~	(6.666)	~	~

Bracketed values are t-statistics. Estimates are heterskedastic-consistent.

Table 15: Estimated Parameters for 1991 - 1994

	a_0	a_1	α_0	α_1	β_1	γ_1	δ	ν	AIC	LL
APARCH	0.00018733	-0.03565	0.00021738	0.04896233	0.95205305	0.652948	0.833591	4.965615	4214.578	4222.578
	(0.178212)	(-0.225)	(0.016675)	(2.083582)	(19.28324)	(0.101)	(0.071)	(2.437)	~	~
ARCH	0.00025	-0.04634	5.34e-05	0.018235	0	0	2	3.863223	4196.9138	4201.914
	(1.308)	(-1.633)	(10.394)	(0.611)	~	~	~	(7.745)	~	~
GARCH	0.000291	-0.04045	3.13E-07	0.022724	0.971259	0	2	4.612193	4215.5078	4221.5078
	(1.584)	(-1.421)	(1.2435)	(2.701)	(91.463)	~	~	(6.992)	~	~
TSGARCH-I	0.000295	-0.04005	5.57E-05	0.032423	0.968355	0	1	4.654995	4213.82	4219.82
	(1.615)	(-1.385)	(1.178)	(2.617)	(72.492)	~	~	(7.034)	~	~
TSGARCH-II	0.00019703	-0.0345026	9.204E-05	0.04826	0.95216	0.5692	1	4.78296	4215.415	4222.415
	(0.99)	(-1.146)	(0.92)	(2.212)	(35.001)	(2.362)	~	(6.598)	~	~
GJR	0.0002504	-0.035908	5.72E-04	0.0260212	0.9672727	0.1833364	2	4.6910017	4215.287	4222.287
	(1.314)	(-1.218)	(0.993)	(2.258)	(59.952)	(1.173)	~	(6.757)	~	~
TARCH	0.0002314	-0.037529	0.007081	0.0613219	0	0.9998242	1	3.8394792	4197.853	4203.853
	(1.689)	(-1.279)	(18.581)	(1.418)	~	(1.608)	~	(7.908)	~	~
EGARCH	0.000204	-0.0344068	-0.115	-0.047245	0.0880778	0.98783	~	4.7737504	4215.234	4222.234
	(1.050)	(-1.172)	(-1.467)	(-1.628)	(3.191)	(123.025)	~	(-6.650)	~	~

Bracketed values are t-statistics. Estimates are heterskedastic-consistent.

Table 16: Estimated Parameters for 1995 - 1998

	a_0	a_1	α_0	α_1	β_1	γ_1	δ	ν	AIC	LL
APARCH	0.0009922	-0.025204	0.00018022	0.0881192	0.905517	0.93282292	1.0807832	5.4712962	3908.896	3916.896
	(2.913)	(-0.834)	(0.380)	(3.350)	(30.266)	(3.061)	(1.904)	5.912	~	~
ARCH	0.0013928	-0.0388864	9.7761E-05	0.26389	0	0	2	3.3453302	3845.595	3850.595
	(5.316)	(-1.115)	(7.099)	(3.088)	~	~	~	8.770	~	~
GARCH	0.0013589	-0.036126	2.1412E-06	0.10834	0.88298	0	2	4.71171	3894.692	3900.692
	(5.639)	(-1.176)	(1.425)	(3.148)	(21.909)	~	~	6.445	~	~
TSGARCH-I	0.0012924	-0.0440017	0.00017552	0.10675351	0.9061453	0	1	4.6827234	3895.085	3901.085
	(5.395)	(-1.316)	(1.620)	(3.841)	(32.606)	~	~	6.558	~	~
TSGARCHII	0.00096578	-0.025781	0.00025989	0.09147315	0.90682132	0.91296746	1	5.488194	3909.87	3916.87
	(3.830)	(-0.853)	(1.867)	(4.105)	(32.635)	(5.031)	~	5.847	~	~
GJR	0.0011561	-0.02253	2.703E-06	0.0472384	0.8817252	0.9999938	2	5.2413098	3907.309	3914.309
	(4.8195)	(-0.729)	(1.876)	(3.476)	(24.7765)	(3154.289)	~	5.9759	~	~
TARCH	0.0012787	-0.050463	0.00900586	0.26091579	0	0.44469103	1	3.4152999	3849.191	3855.191
	(4.976)	(-1.592)	(13.515)	(4.377)	~	(2.481)	~	8.904	~	
EGARCH	0.00097706	-0.024591	-0.253921	-0.148073	0.161526	0.972035	~	5.448343	3910.398	3917.398
	(3.811)	(-0.816)	(-2.216)	(-4.599)	(4.726)	(79.070)	~	-5.827	~	~

Bracketed values are t-statistics. Estimates are heterskedastic-consistent.

Table 17: 12th order Ljung-Box statistics for $\frac{\varepsilon_t}{h_t}$ and $\frac{\varepsilon_t^2}{h_t^2}$

	1983-1998	1983-1986	1987-1990	1991-1994	1995-1998
APARCH					
Q_{12}	14.2359 0.2859	10.3005 0.5896	8.6629 0.7314	16.9122 0.1529	12.6628 0.394
Q_{12}^2	12.8218 0.3821	5.0733 0.9555	3.3533 0.9925	7.7935 0.801	6.3095 0.8997
ARCH					
Q_{12}	29.2377 0.0036	13.9869 0.3015	29.1073 0.0038	14.987 0.2421	17.3921 0.1354
Q_{12}^2	418.1956 0.000	9.864 0.6279	99.3827 0.000	23.9331 0.0208	85.17 0.000
GARCH					
Q_{12}	16.4104 0.1732	9.8438 0.6297	10.973 0.5312	19.0669 0.0869	15.2964 0.2256
Q_{12}^2	8.3566 0.7567	5.3111 0.9468	2.6837 0.9974	8.0623 0.7802	7.6012 0.8155
TSGARCH-I					
Q_{12}	15.9829 0.192	10.1537 0.6025	10.3789 0.5828	18.3103 0.1066	14.704 0.258
Q_{12}^2	22.3838 0.0334	5.4356 0.9418	4.4615 0.9736	9.3794 0.6702	11.1127 0.5193N
TSGARCH-II					
Q_{12}	14.2362 0.2859	10.1624 0.6017	8.864 0.7145	16.9914 0.1499	12.6711 0.3934
Q_{12}^2	12.7818 0.3851	5.4226 0.9424	3.9355 0.9846	7.4777 0.8245	6.1887 0.9063
GJR					
Q_{12}	15.549 0.2128X	10.2391 0.595	9.6854 0.6435	18.6708 0.0968	12.9384 0.3735
Q_{12}^2	5.7647 0.9275	5.066 0.9557	3.1856 0.9941	7.0684 0.8531	7.0385 0.8551
TARCH					
Q_{12}	28.1106 0.0053	13.5663 0.3293	34.7099 0.0005	14.6532 0.261	18.0688 0.1136
Q_{12}^2	550.9643 0.000	9.675 0.6444	140.4172 0.000	24.0214 0.0202	83.6644 0.000
EGARCH					
Q_{12}	14.2566 0.2846	10.1609 0.6018	8.8706 0.7139	17.226 0.1413	12.5406 0.4033
Q_{12}^2	8.2834 0.7626	5.4073 0.943	2.7761 0.9969	7.1652 0.8465	5.7963 0.926

The p-values are reported in italic.

Table 18: Model Rankings for the AIC Metric (Excluding EGARCH)

	1983-1998	1983-1986	1987-1990	1991-1994	1995-1998
	AIC	AIC	AIC	AIC	AIC
APARCH	1	4	1	4	2
ARCH	6	6	6	7	7
GARCH	5	1	5	1	5
TSGARCH-I	3	3	4	5	4
TSGARCH-II	1	5	2	2	1
GJR	4	2	3	3	3
TARCH	7	7	7	6	6

Table 19: Model Rankings for AIC Statistics (Including EGARCH)

	1983-1998	1983-1986	1987-1990	1991-1994	1995-1998
	AIC	AIC	AIC	AIC	AIC
APARCH	2	4	2	5	3
ARCH	7	7	7	8	8
GARCH	6	1	6	1	6
TSGARCH-I	4	3	5	6	5
TSGARCH-II	2	6	3	2	2
GJR	5	2	4	3	4
TARCH	8	8	8	7	7
EGARCH	1	5	1	4	1

Table 20: Aggregated Rankings for AIC Statistics (Including EGARCH)

	Score	Rank
APARCH	*14*	*4*
ARCH	*30*	*7*
GARCH	*14*	*4*
TSGARCH-I	*19*	*6*
TSGARCH-II	*13*	*2*
GJR	*13*	*2*
TARCH	*30*	*7*
EGARCH	*11*	*1*

Note: Score is the sum of the rank for each model in each sub-period.

Table 21: Model Rankings for MMEU and MMEO Criteria

	1983-1998		1983-1986		1987-1990		1991-1994		1995-1998	
	MMEU	MMEO	MMEU	MMEO	MMEU	MMEO	MMEU	MMEO	MMEU	MMEO
APARCH	8	2	3	6	7	2	8	1	6	3
ARCH	2	8	1	8	2	7	2	7	1	8
GARCH	4	5	4	4	3	6	3	6	3	6
TSGARCH-I	5	6	7	1	4	4	4	5	5	5
TSGARCH-II	7	3	8	2	8	1	7	2	8	1
GJR	3	4	5	5	5	5	5	4	4	4
TARCH	1	7	2	7	1	8	1	8	2	7
EGARCH	6	1	6	3	6	3	6	3	7	2

Table 22: Model Rankings for HMSE, GMLE and LL Criteria

	1983-1998			1983-1986			1987-1990			1991-1994			1995-1998		
	HMSE	GMLE	LL	HMSE	GMLE	LL	HMSE	GMLE	LL	HMSE	GMLE	LL	HMSE	GMLE	LL
APARCH	5	2	1	8	2	6	5	1	2	6	2	5	2	2	2
ARCH	1	7	7	1	8	8	1	8	8	1	5	7	8	8	7
GARCH	7	5	5	6	3	1	8	6	6	7	6	3	6	6	4
TSGARCH-I	8	6	6	4	5	4	7	5	5	3	7	4	5	5	6
TSGARCH-II	4	1	2	5	6	2	3	2	1	4	1	2	1	1	1
GJR	2	3	3	7	1	3	4	4	3	8	4	6	4	4	5
TARCH	3	8	8	2	7	7	2	7	7	2	8	8	7	7	8
EGARCH	6	4	4	3	4	5	6	3	4	5	3	1	3	3	3

Table 23: Model Rankings for MSE, MAE and MAPE Criteria

	1983-1998			1983-1986			1987-1990			1991-1994			1995-1998		
	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE	MSE	MAE	MAPE
APARCH	3	2	4	5	6	1	2	3	3	3	2	5	2	3	2
ARCH	8	6	5	7	8	4	8	8	4	7	7	7	7	8	6
GARCH	5	7	2	1	4	3	4	6	2	4	5	1	5	6	4
TSGARCH-I	6	4	6	2	2	6	5	4	6	5	4	4	6	5	5
TSGARCH-II	2	3	3	3	3	5	1	2	5	2	6	2	1	2	3
GJR	4	8	1	4	5	2	3	7	1	1	3	3	3	4	1
TARCH	1	5	7	6	7	7	6	5	7	6	8	6	4	7	7
EGARCH	7	1	8	8	1	8	7	1	8	8	1	8	8	1	8

Table 24:Aggregated Rankings for Statistical Loss Functions

	MSE	MAE	MAPE	MMEU	MMEO	LL	HMSE	GMLE
	Rank (Score)	Rank (Score)	Rank (Score)	Rank (Score)	Rank (Score)	Rank (Score)	Rank (Score)	Rank (Score)
APARCH	3 (12)	3 (14)	3 (11)	6 (24)	3 (12)	4 (15)	6 (21)	1 (7)
ARCH	7 (29)	8 (31)	5 (21)	1 (6)	7 (30)	7 (30)	1 (11)	7 (29)
GARCH	4 (14)	6 (21)	2 (10)	3 (13)	6 (22)	3 (14)	8 (27)	5 (21)
TSGARCH-I	5 (18)	4 (15)	5 (21)	5 (20)	4 (15)	6 (19)	5 (19)	6 (22)
TSGARCH-II	1 (7)	2 (13)	4 (15)	8 (31)	1 (6)	1 (6)	2 (13)	2 (10)
GJR	2 (11)	5 (19)	1 (7)	4 (19)	5 (18)	5 (17)	7 (23)	3 (13)
TARCH	6 (22)	7 (27)	7 (27)	1 (6)	7 (30)	7 (30)	2 (13)	7 (29)
EGARCH	8 (31)	1 (4)	8 (32)	7 (25)	2 (11)	2 (13)	4 (17)	3 (13)

Note: Score is the sum of the rank for each model in each sub-period.

Table 25: Correlations Between Out-of-Sample Buy and Sell Signals

	1985-1986				1989-1990			
	EGARCH	GARCH	ARCH	2-STAGE	EGARCH	GARCH	ARCH	2-STAGE
EGARCH	1.000	.167	-.070	.064	1.000	.406	.304	.264
GARCH		1.000	-.058	-.054		1.000	.758	.7578
ARCH			1.000	.017			1.000	.84453
2-STAGE				1.000				1.000
	1993-1994				1997-1998			
	EGARCH	GARCH	ARCH	2-STAGE	EGARCH	GARCH	ARCH	2-STAGE
EGARCH	1.000	.595	.468	.480	1.000	.622	.154	.394
GARCH		1.000	.702	.677		1.000	.151	.490
ARCH			1.000	.942			1.000	.454
2-STAGE				1.000				1.000

Table 26: Statistics for Forecasts of Volatility Changes

	1985-1986				1989-1990			
	EGARCH	GARCH	ARCH	2-STAGE	EGARCH	GARCH	ARCH	2-STAGE
#.samples	104	104	104	104	103	103	103	103
Max	31.643	3.862	25.187	8.521	87.7453	60.325	112.367	209.584
Min	-29.058	-1.675	-.455	-7.213	-30.678	-17.235	-48.265	-61.292
Mean	.806	-.008	.353	-.098	2.263	1.058	2.044	9.551
Std. dev	.579	3.228	9.275	.1274	21.747	13.126	20.898	48.297
	1993-1994				1997-1998			
	EGARCH	GARCH	ARCH	2-STAGE	EGARCH	GARCH	ARCH	2-STAGE
#.samples	103	103	103	103	103	103	103	103
Max	107.833	81.722	166.999	125.45	80.161	64.820	62.662	34.029
Min	-8.565	-26.559	-60.505	-50.472	-17.288	-7.476	-24.824	-23.583
Mean	1.079	1.258	2.524	2.160	1.269	1.020	.192	.286
Std. dev	14.265	12.685	24.325	21.268	14.799	10.430	10.158	9.933

Table 27: Before-transactions-costs Statistics for 1985-1986 without Filter

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	0.606881	-0.85174	0.497811	-1.30702
Std. R. of Returns	13.32615	13.3071	13.51409	13.26873
#. of Trades	104	104	101	104
Ave. Delta	0.013777	0.013777	0.01441	0.013777
Std. Delta	0.10788	0.10788	0.108375	0.10788
Ave. Maturity	0.198419	0.198419	0.199023	0.198419
Ave. Call Price	5.402885	5.402885	5.442574	5.402885
Std. Calls	2.309928	2.309928	2.331023	2.309928
Ave. Put Price	5.501923	5.501923	5.527228	5.501923
Std. Puts	2.149022	2.149022	2.163903	2.149022
# of Buys	45	5	52	54
# of Sells	59	99	49	50

Table 28: Before-transactions-costs Statistics for 1989-1990 without Filter

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	-0.12772	-2.45341	-2.5796	-3.34284
Std. R. of Returns	14.71959	14.49996	14.47757	14.31479
#. of Trades	103	103	103	103
Ave. Delta	0.025737	0.025737	0.025737	0.025737
Std. Delta	0.077366	0.077366	0.077366	0.077366
Ave. Maturity	0.11443	0.11443	0.11443	0.11443
Ave. Call Price	7.540777	7.540777	7.540777	7.540777
Std. Calls	2.016813	2.016813	2.016813	2.016813
Ave. Put Price	7.399515	7.399515	7.399515	7.399515
Std. Puts	1.937485	1.937485	1.937485	1.937485
# of Buys	38	41	50	50
# of Sells	65	62	53	53

Table 29: Before-transactions-costs Statistics for 1993-1994 without Filter

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	2.279404	3.350164	1.714102	2.004451
Std. R. of Returns	13.94236	13.72455	14.0219	13.98359
#. of Trades	103	103	103	103
Ave. Delta	0.011027	0.011027	0.011027	0.011027
Std. Delta	0.075565	0.075565	0.075565	0.075565
Ave. Maturity	0.11443	0.11443	0.11443	0.11443
Ave. Call Price	6.85	6.85	6.85	6.85
Std. Calls	1.560276	1.560276	1.560276	1.560276
Ave. Put Price	6.892233	6.892233	6.892233	6.892233
Std. Puts	1.337941	1.337941	1.337941	1.337941
# of Buys	31	38	53	52
# of Sells	72	65	50	51

Table 30: Before-transactions-costs Statistics for 1997-1998 without Filter

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	-0.02342	-1.03321	0.753204	-0.96787
Std. R. of Returns	11.60207	11.55187	11.58106	11.55768
#. of Trades	103	103	103	103
Ave. Delta	0.026539	0.026539	0.026539	0.026539
Std. Delta	0.021121	0.021121	0.021121	0.021121
Ave. Maturity	0.121851	0.121851	0.121851	0.121851
Ave. Call Price	27.27524	27.27524	27.27524	27.27524
Std. Calls	7.941751	7.941751	7.941751	7.941751
Ave. Put Price	27.53689	27.53689	27.53689	27.53689
Std. Puts	8.314578	8.314578	8.314578	8.314578
# of Buys	39	31	48	48
# of Sells	64	72	55	55

Table 31: Before-transactions-costs Statistics for 1985-1986 with ±3% Delta Filter

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	0.342188	-0.32483	-2.41412	-4.01786
Std. R. of Returns	13.90462	13.90696	13.67675	13.26126
#. of Trades	17	17	17	17
Ave. Delta	0.0012	0.0012	0.0012	0.0012
Std. Delta	0.016493	0.016493	0.016493	0.016493
Ave. Maturity	0.227881	0.227881	0.227881	0.227881
Ave. Call Price	6.123529	6.123529	6.123529	6.123529
Std. Calls	1.947283	1.947283	1.947283	1.947283
Ave. Put Price	6.732353	6.732353	6.732353	6.732353
Std. Puts	2.304136	2.304136	2.304136	2.304136
# of Buys	10	1	11	7
# of Sells	7	16	6	10

Table 32: Before-transactions-costs Statistics for 1989-1990 with ±3% Delta Filter

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	1.170446	-0.62096	-2.03609	-2.21947
Std. R. of Returns	13.79986	13.82599	13.6767	13.64618
#. of Trades	31	31	31	31
Ave. Delta	-0.0042	-0.0042	-0.0042	-0.0042
Std. Delta	0.016332	0.016332	0.016332	0.016332
Ave. Maturity	0.11445	0.11445	0.11445	0.11445
Ave. Call Price	7.320968	7.320968	7.320968	7.320968
Std. Calls	1.778987	1.778987	1.778987	1.778987
Ave. Put Price	8.006452	8.006452	8.006452	8.006452
Std. Puts	2.087851	2.087851	2.087851	2.087851
# of Buys	17	16	19	20
# of Sells	14	15	12	11

Table 33: Before-transactions-costs Statistics for 1993-1994 with ±3% Delta Filter

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	5.28826	5.189391	3.114228	3.825862
Std. R. of Returns	10.68355	10.73501	11.54099	11.31548
#. of Trades	20	20	20	20
Ave. Delta	-0.00638	-0.00638	-0.00638	-0.00638
Std. Delta	0.018011	0.018011	0.018011	0.018011
Ave. Maturity	0.123425	0.123425	0.123425	0.123425
Ave. Call Price	7.05	7.05	7.05	7.05
Std. Calls	1.208087	1.208087	1.208087	1.208087
Ave. Put Price	7.5975	7.5975	7.5975	7.5975
Std. Puts	1.462377	1.462377	1.462377	1.462377
# of Buys	6	6	9	8
# of Sells	14	14	11	12

Table 34: Before-transactions-costs Statistics for 1997-1998 with ±3% Delta Filter

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	0.061182	-1.07436	0.483998	-1.72362
Std. R. of Returns	12.42948	12.3787	12.42263	12.30109
#. of Trades	56	56	56	56
Ave. Delta	0.012262	0.012262	0.012262	0.012262
Std. Delta	0.01277	0.01277	0.01277	0.01277
Ave. Maturity	0.122945	0.122945	0.122945	0.122945
Ave. Call Price	26.02054	26.02054	26.02054	26.02054
Std. Calls	6.334112	6.334112	6.334112	6.334112
Ave. Put Price	27.26696	27.26696	27.26696	27.26696
Std. Puts	6.569587	6.569587	6.569587	6.569587
# of Buys	19	16	30	24
# of Sells	37	40	26	32

Table 35: Before-transactions-costs Statistics for 1997-1998 with ±3% Delta Filter (Excluding One Data Point)

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	0.942586	-0.2136	-0.38922	-0.87467
Std. R. of Returns	10.63213	10.66822	10.66245	10.63067
#. of Trades	55	55	55	55
Ave. Delta	0.012309	0.012309	0.012309	0.012309
Std. Delta	0.012883	0.012883	0.012883	0.012883
Ave. Maturity	0.123686	0.123686	0.123686	0.123686
Ave. Call Price	26.13364	26.13364	26.13364	26.13364
Std. Calls	6.335168	6.335168	6.335168	6.335168
Ave. Put Price	27.39	27.39	27.39	27.39
Std. Puts	6.564702	6.564702	6.564702	6.564702
# of Buys	19	16	29	24
# of Sells	36	39	26	31

Table 36: After-transactions-costs Statistics for 1985-1986 with ±3% Delta Filter

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	-1.91925	-2.58628	-4.67556	-6.2793
Std. R. of Returns	13.95439	13.69692	13.72358	13.20722
#. of Trades	17	17	17	17
Ave. Delta	0.0012	0.0012	0.0012	0.0012
Std. Delta	0.016493	0.016493	0.016493	0.016493
Ave. Maturity	0.227881	0.227881	0.227881	0.227881
Ave. Call Price	6.123529	6.123529	6.123529	6.123529
Std. Calls	1.947283	1.947283	1.947283	1.947283
Ave. Put Price	6.732353	6.732353	6.732353	6.732353
Std. Puts	2.304136	2.304136	2.304136	2.304136
# of Buys	10	1	11	7
# of Sells	7	16	6	10

Table 37: After-transactions-costs Statistics for 1989-1990 with ±3% Delta Filter

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	-0.57337	-2.36478	-3.7799	-3.96328
Std. R. of Returns	13.7521	13.9163	13.75325	13.72006
#. of Trades	31	31	31	31
Ave. Delta	-0.0042	-0.0042	-0.0042	-0.0042
Std. Delta	0.016332	0.016332	0.016332	0.016332
Ave. Maturity	0.11445	0.11445	0.11445	0.11445
Ave. Call Price	7.320968	7.320968	7.320968	7.320968
Std. Calls	1.778987	1.778987	1.778987	1.778987
Ave. Put Price	8.006452	8.006452	8.006452	8.006452
Std. Puts	2.087851	2.087851	2.087851	2.087851
# of Buys	17	16	19	20
# of Sells	14	15	12	11

Table 38: After-transactions-costs Statistics for 1993-1994 with ±3% Delta Filter

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	3.530293	3.431423	1.35626	2.067894
Std. R. of Returns	10.60153	10.69246	11.54567	11.30632
#. of Trades	20	20	20	20
Ave. Delta	-0.00638	-0.00638	-0.00638	-0.00638
Std. Delta	0.018011	0.018011	0.018011	0.018011
Ave. Maturity	0.123425	0.123425	0.123425	0.123425
Ave. Call Price	7.05	7.05	7.05	7.05
Std. Calls	1.208087	1.208087	1.208087	1.208087
Ave. Put Price	7.5975	7.5975	7.5975	7.5975
Std. Puts	1.462377	1.462377	1.462377	1.462377
# of Buys	6	6	9	8
# of Sells	14	14	11	12

Table 39: After-transactions-costs Statistics for 1997-1998 with $\pm 3\%$ Delta Filter (Excluding One Data Point)

	EGARCH	GARCH	ARCH	2-STAGE
Rate of Returns	0.447472	-0.70872	-0.88434	-1.36978
Std. R. of Returns	10.63932	10.65304	10.66224	10.63684
#. of Trades	55	55	55	55
Ave. Delta	0.012309	0.012309	0.012309	0.012309
Std. Delta	0.012883	0.012883	0.012883	0.012883
Ave. Maturity	0.123686	0.123686	0.123686	0.123686
Ave. Call Price	26.13364	26.13364	26.13364	26.13364
Std. Calls	6.335168	6.335168	6.335168	6.335168
Ave. Put Price	27.39	27.39	27.39	27.39
Std. Puts	6.564702	6.564702	6.564702	6.564702
# of Buys	19	16	29	24
# of Sells	36	39	26	31

Figure 19: Autocorrelations for *r* (1983-1998)

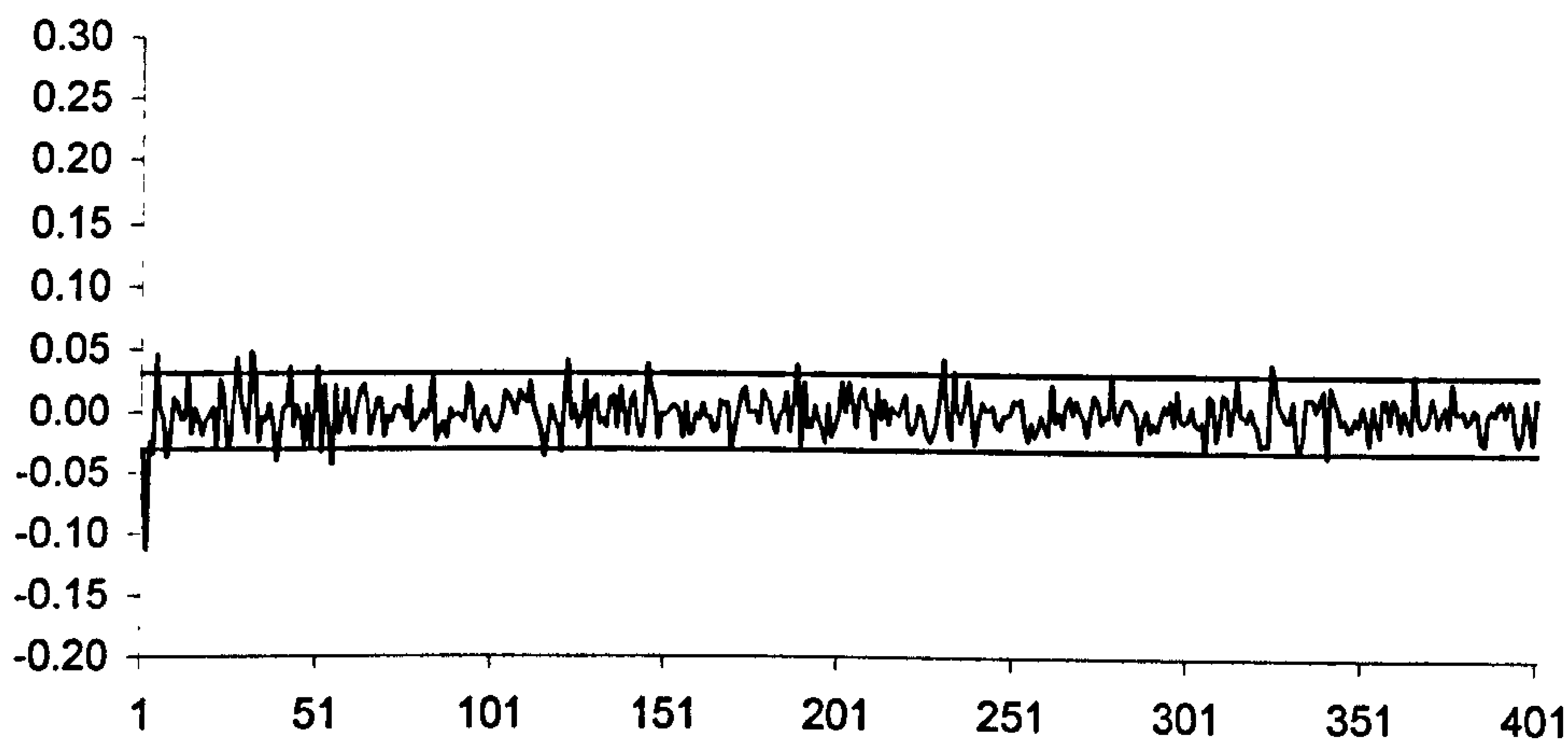


Figure 20: Autocorrelations for r^2 (1983-1998)

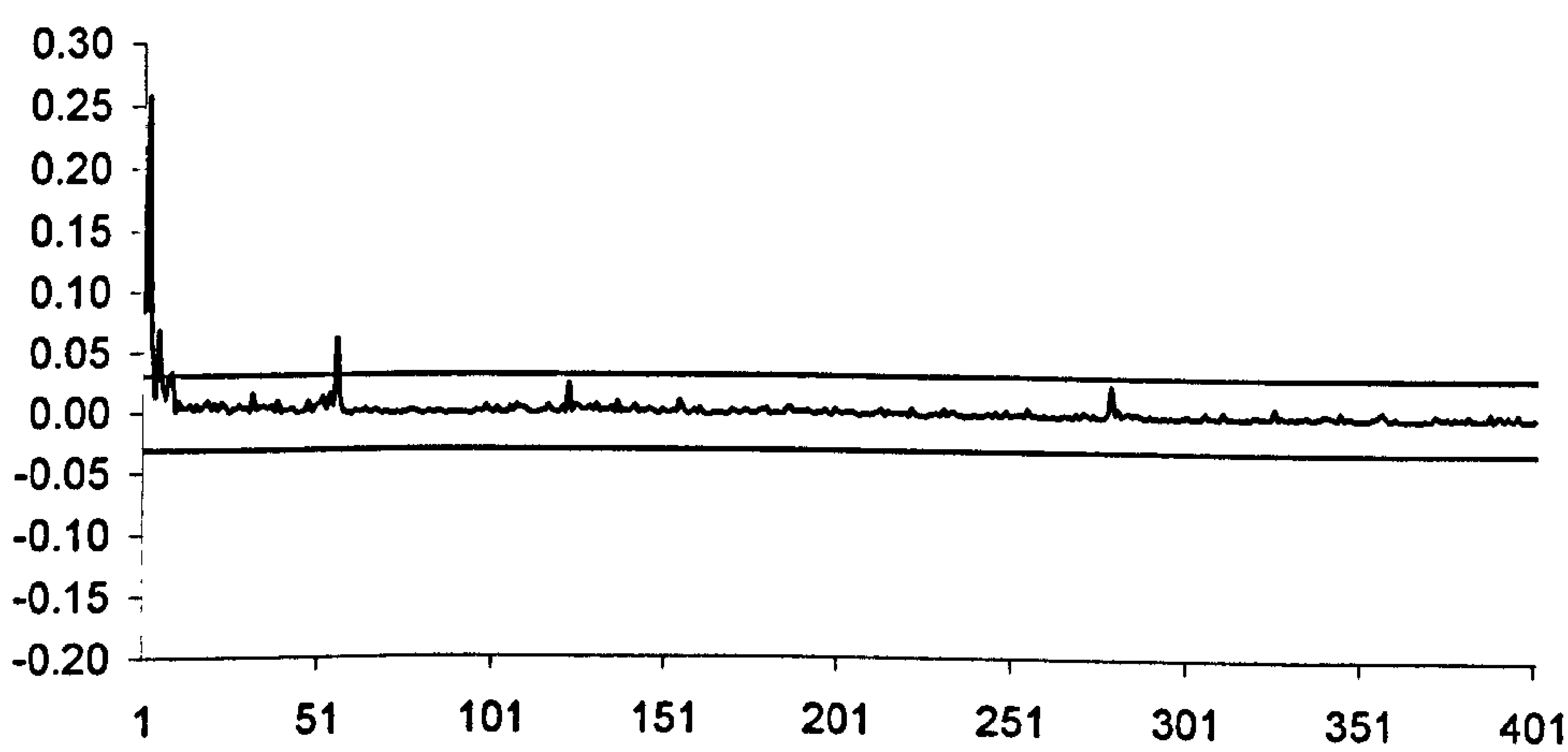


Figure 21: Autocorrelations for |*r*| (1983-1998)

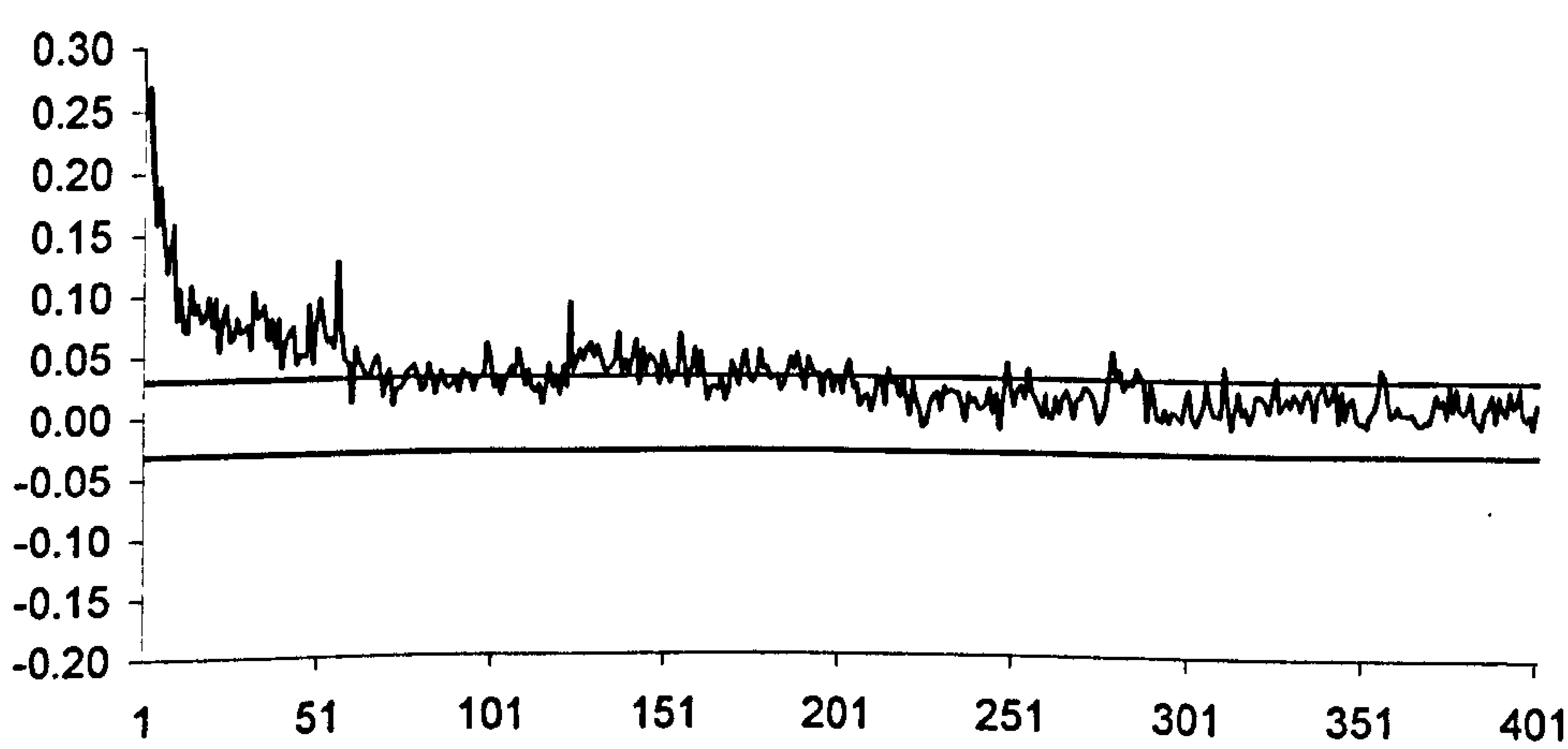


Figure 22: Autocorrelations for *r* (1983-1986)

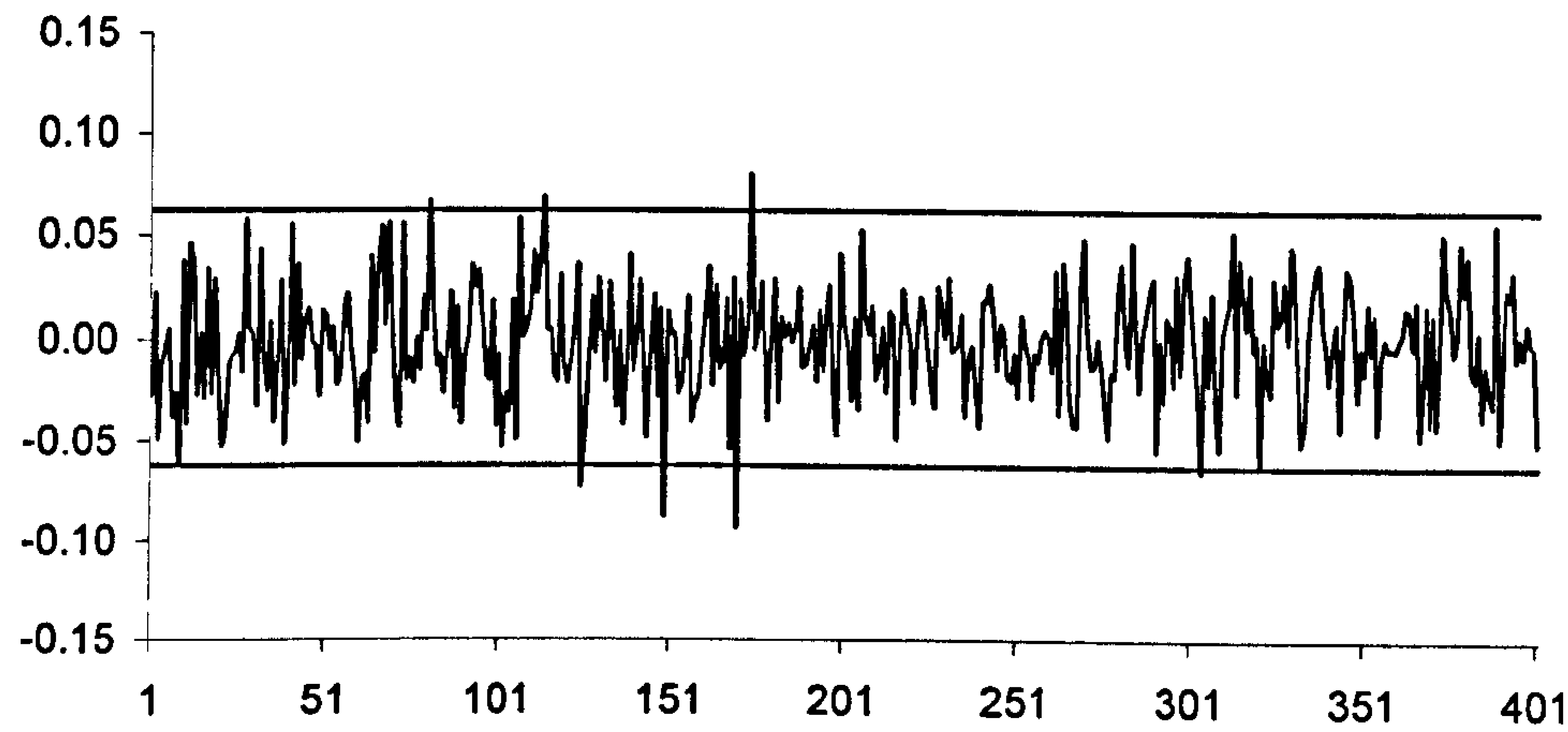


Figure 23: Autocorrelations for r^2 (1983-1986)

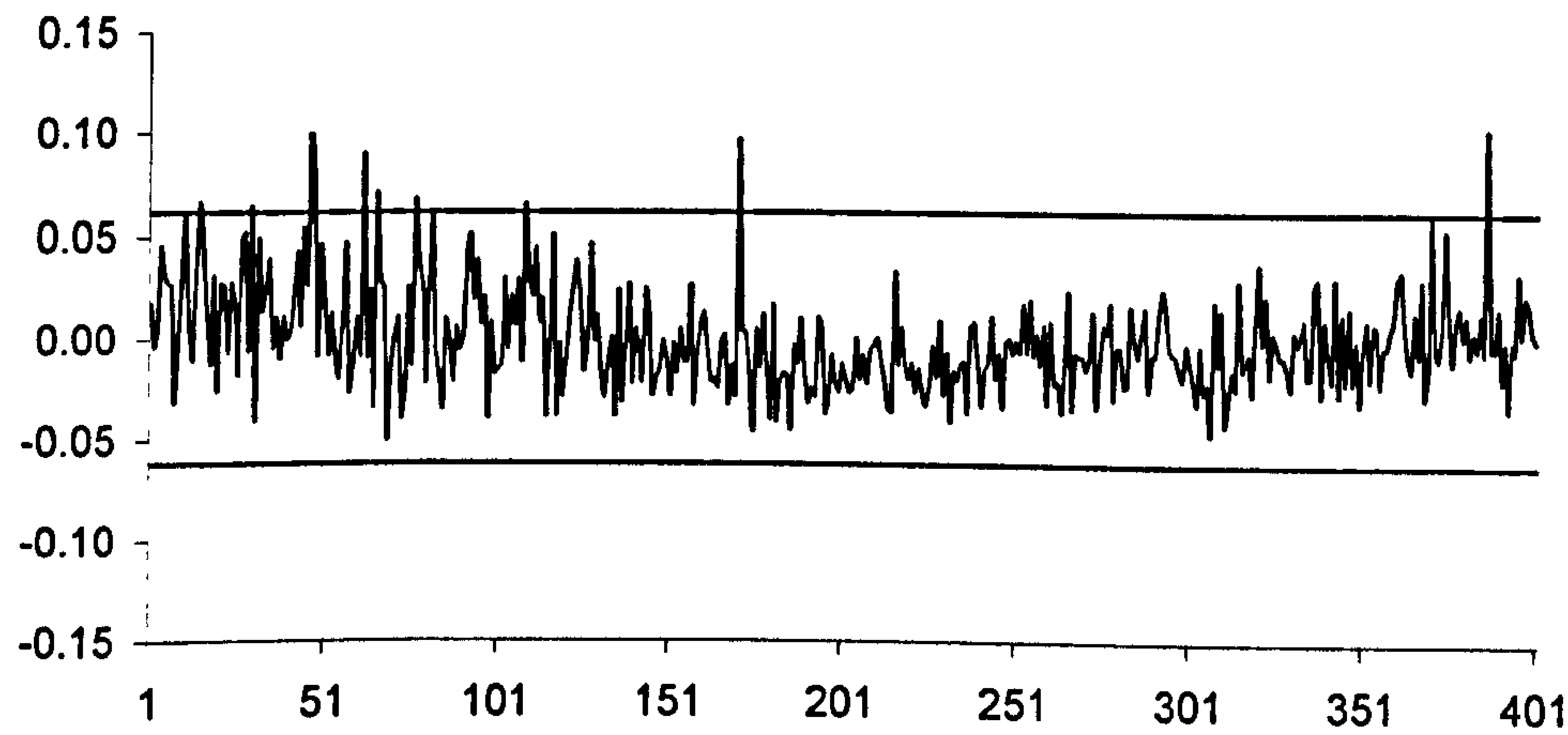


Figure 24: Autocorrelations for |*r*| (1983-1986)

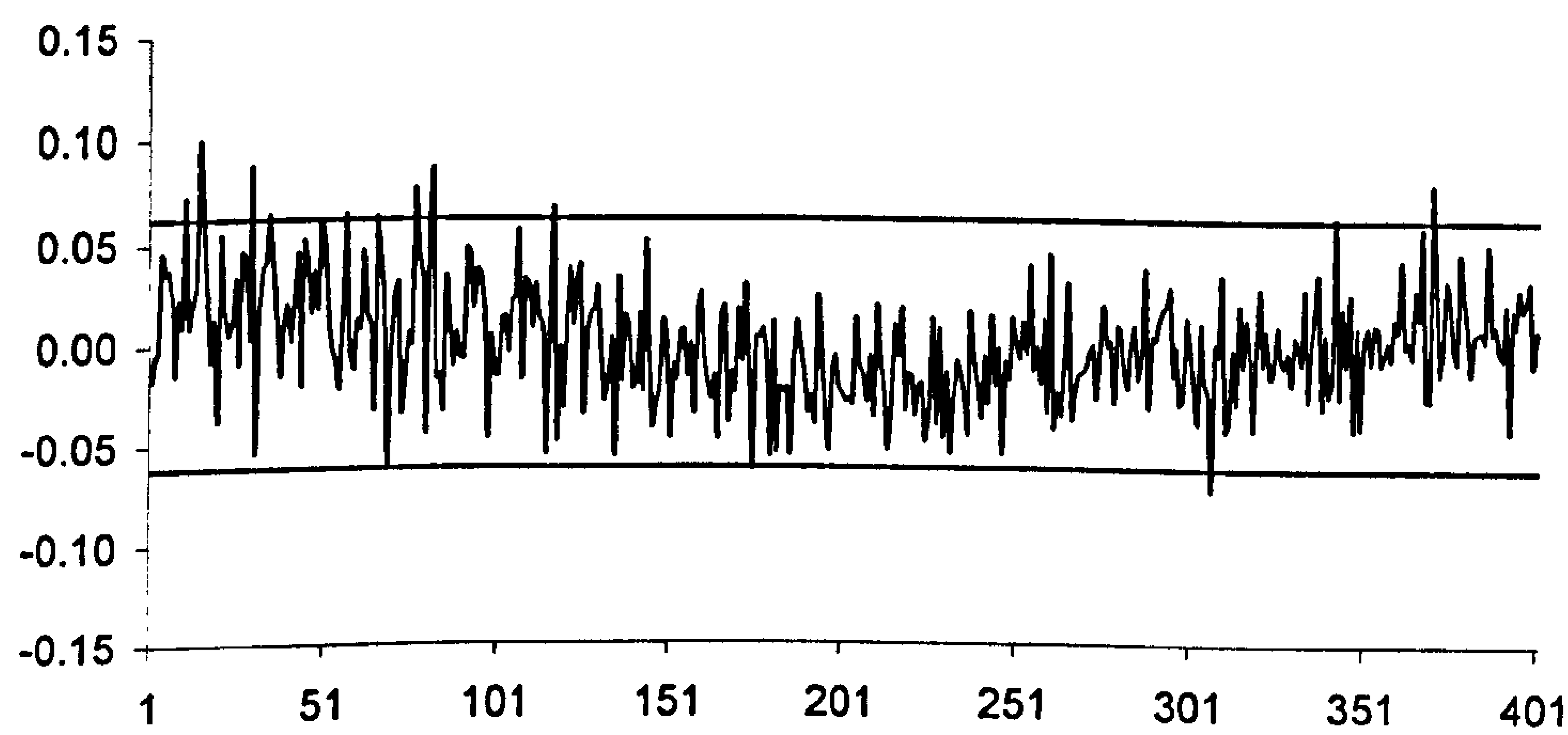


Figure 25: Autocorrelations for *r* (1987-1990)

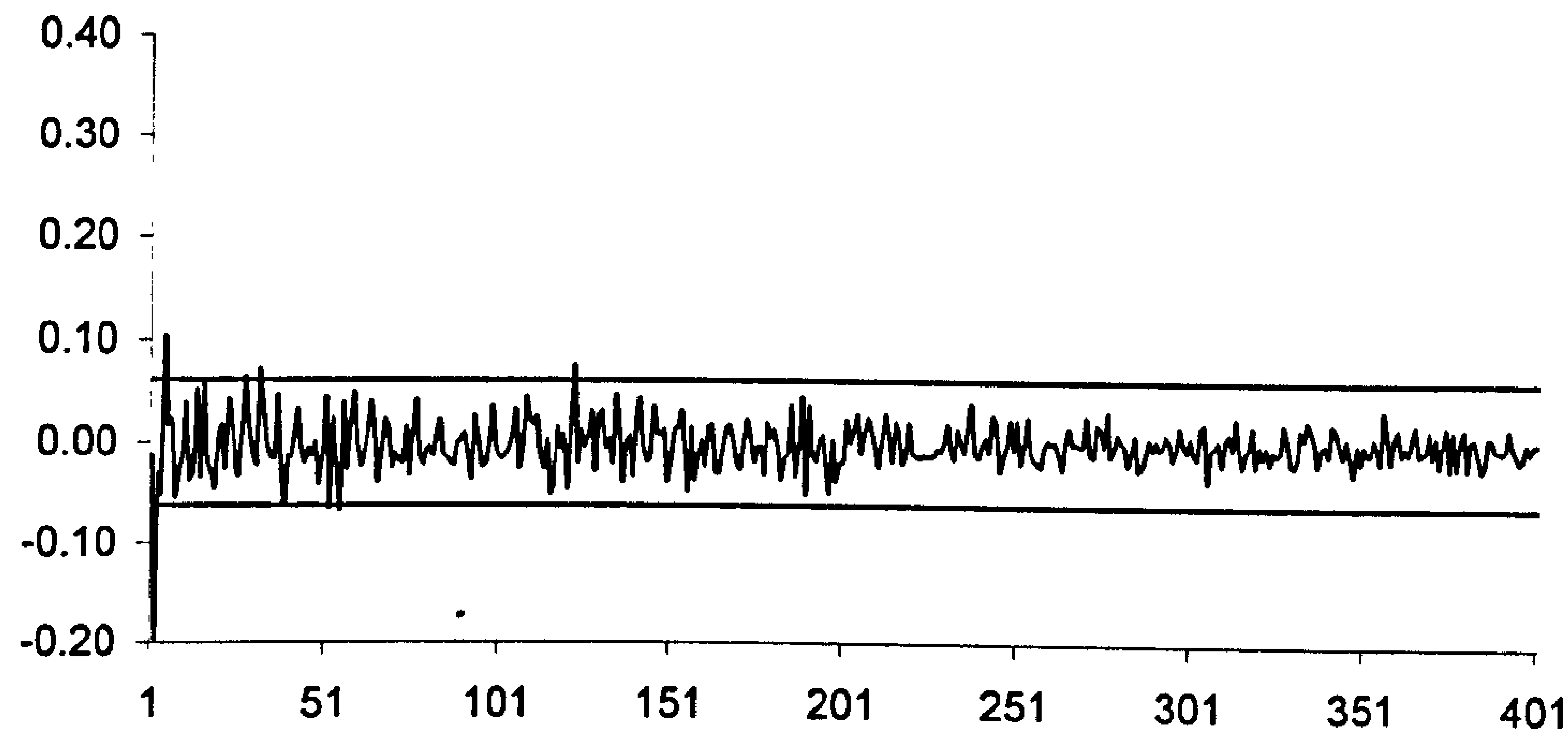


Figure 26: Autocorrelations for r^2 (1987-1990)

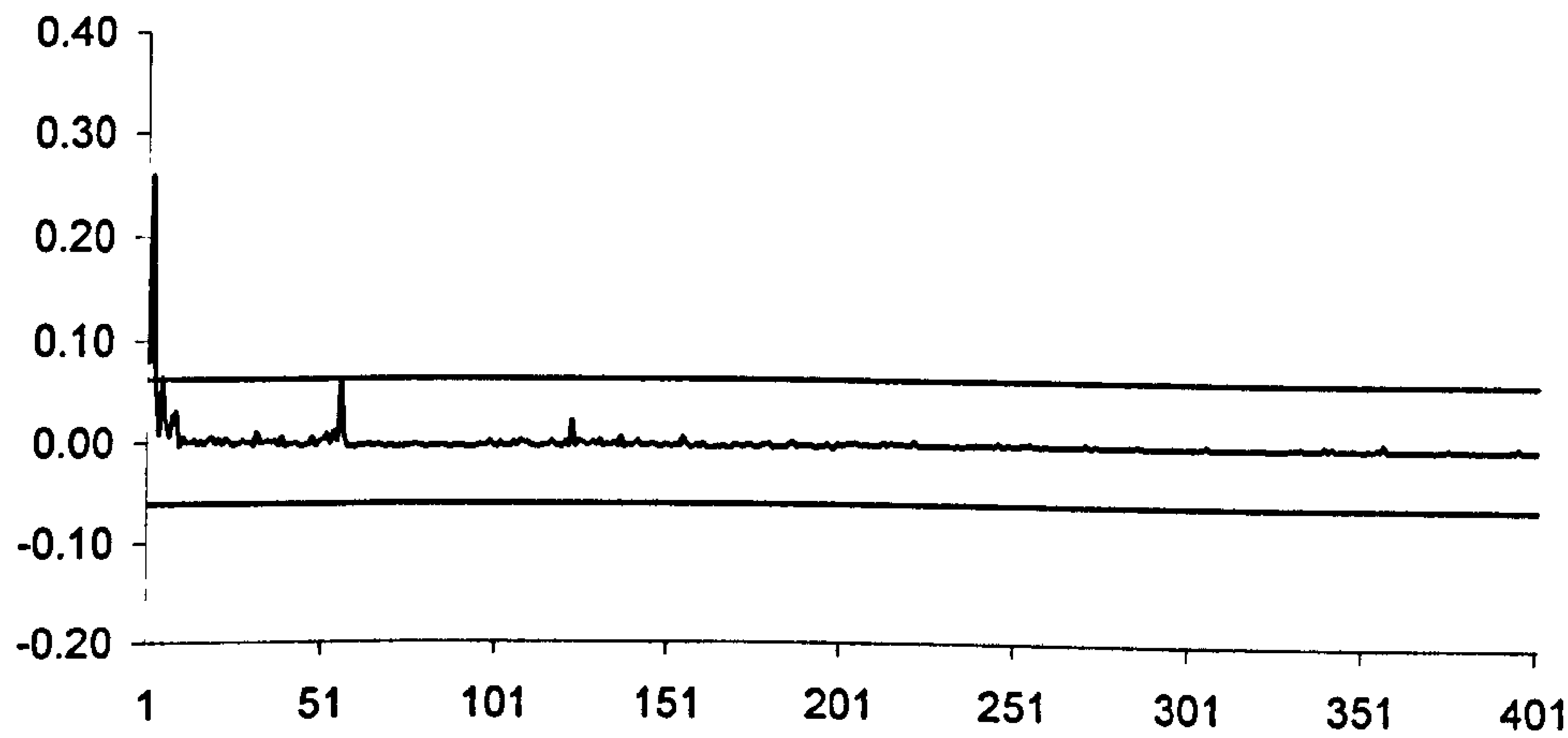


Figure 27: Autocorrelations for $|r|$ (1987-1990)

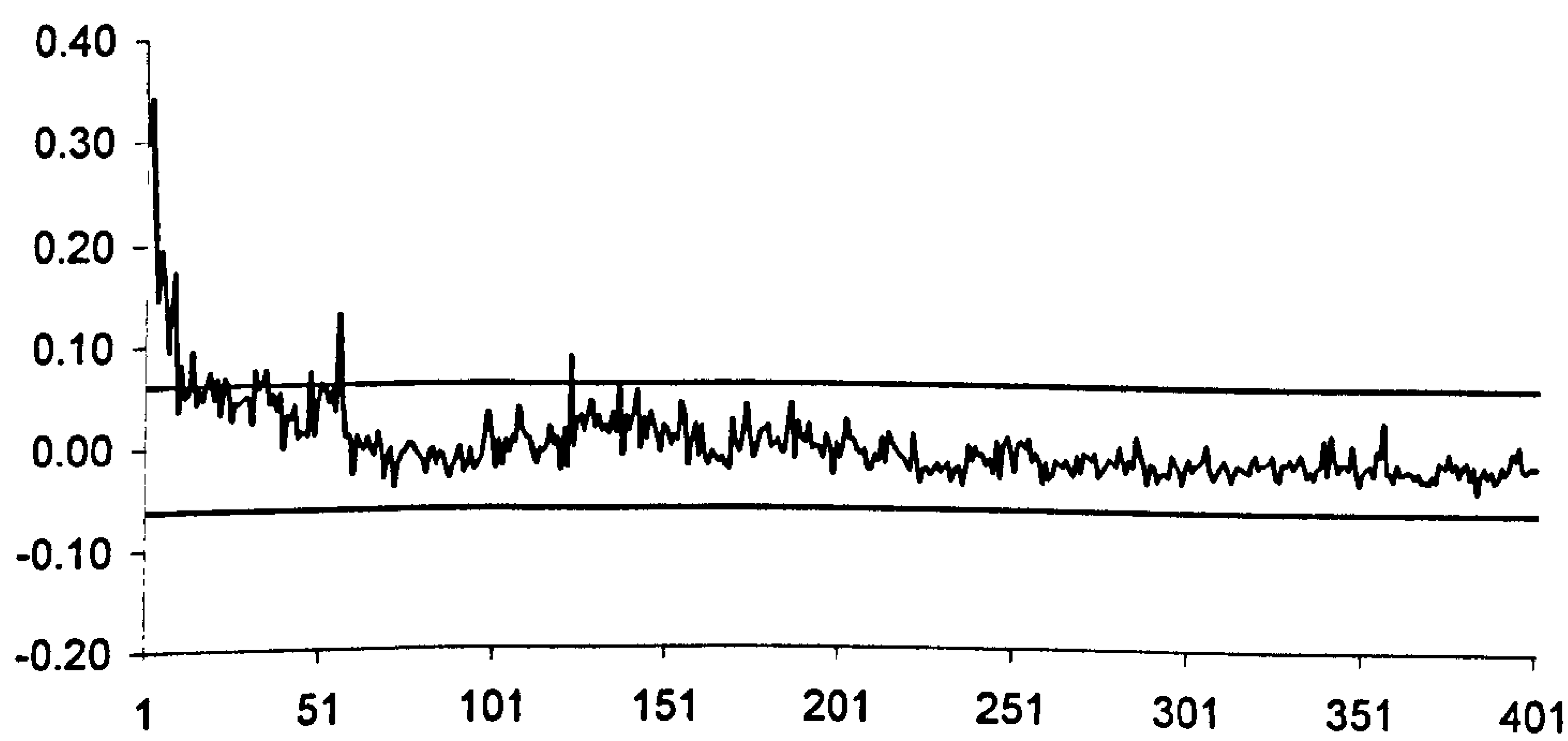


Figure 28: Autocorrelations for *r* (1991-1994)

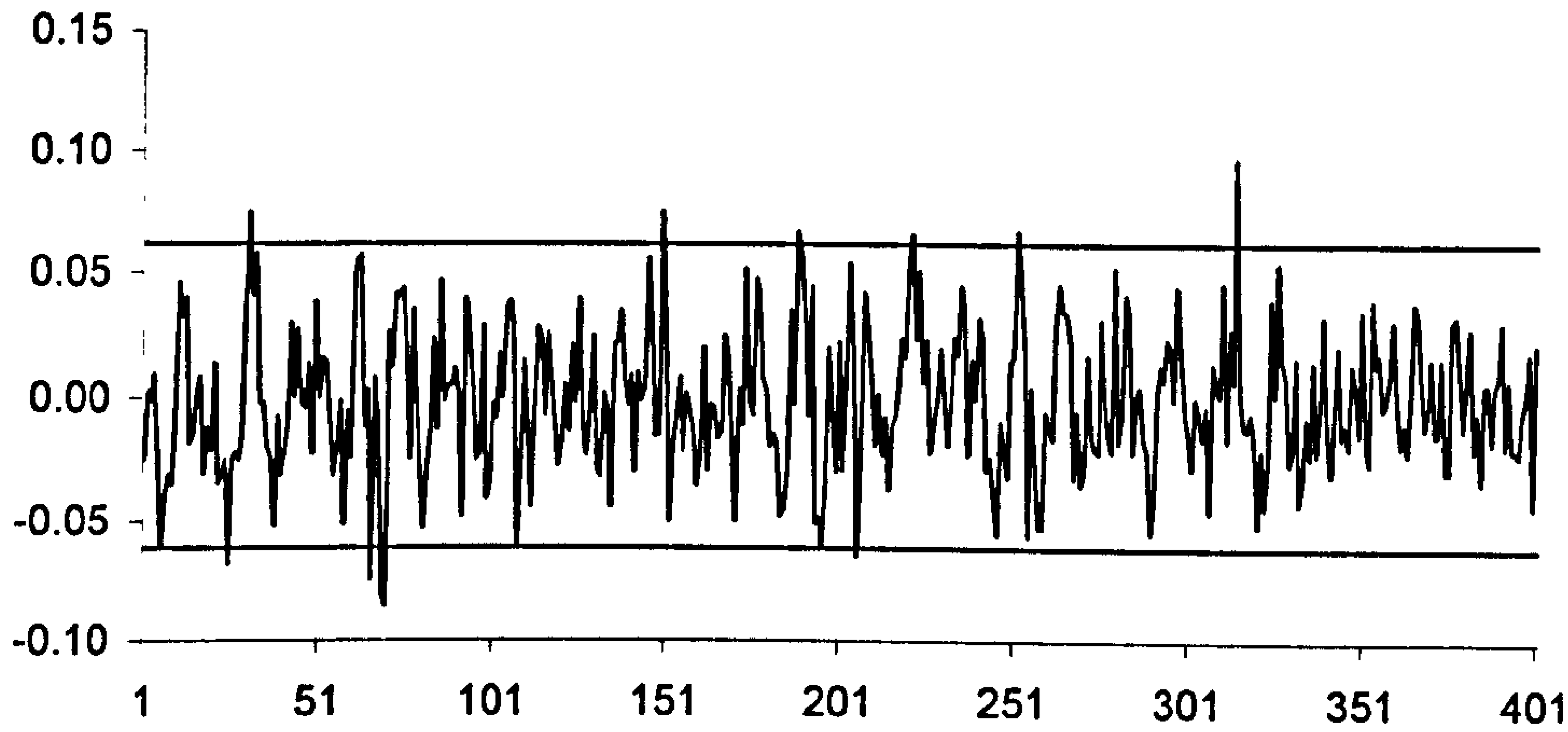


Figure 29: Autocorrelations for r^2 (1991-1994)

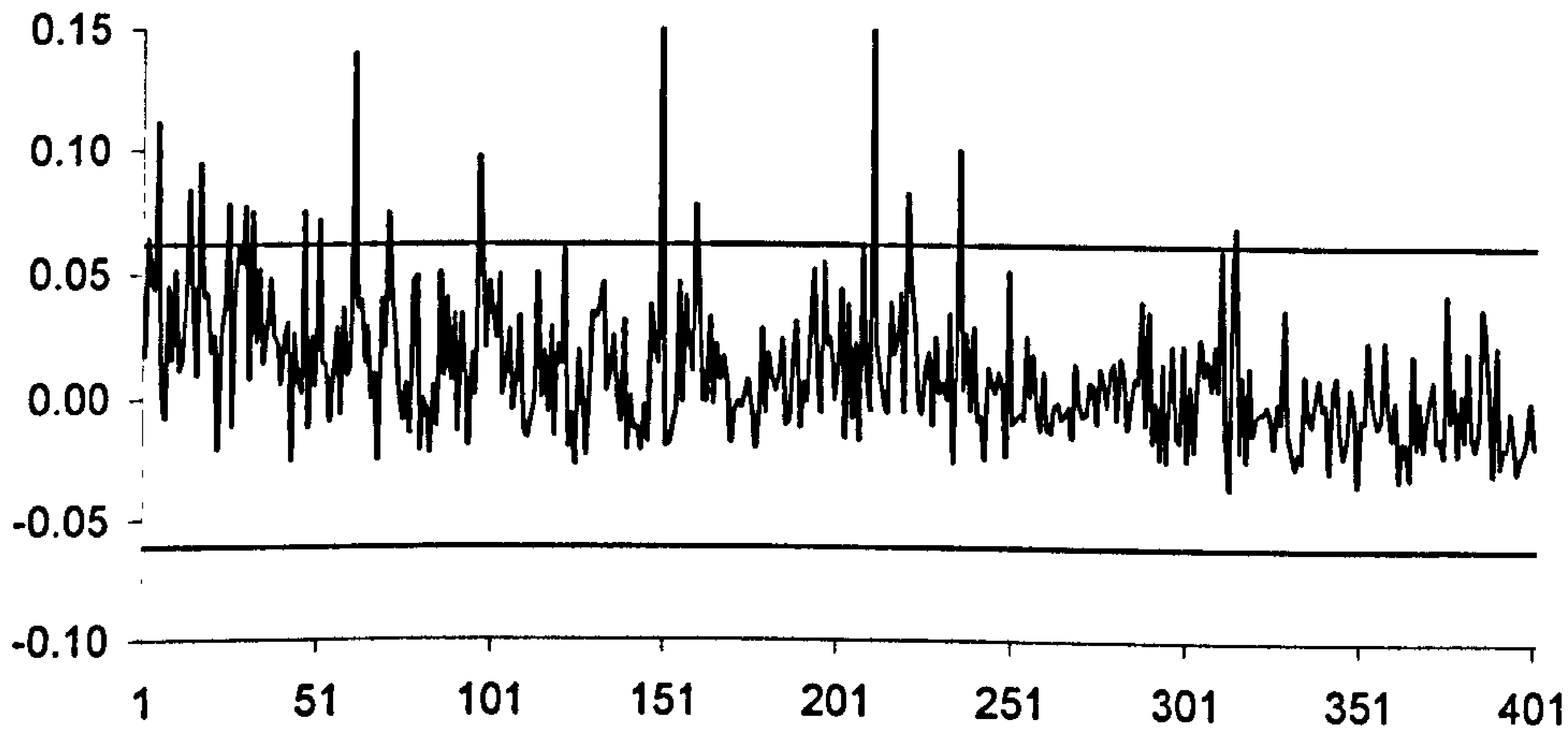


Figure 30: Autocorrelations for |*r*| (1991-1994)

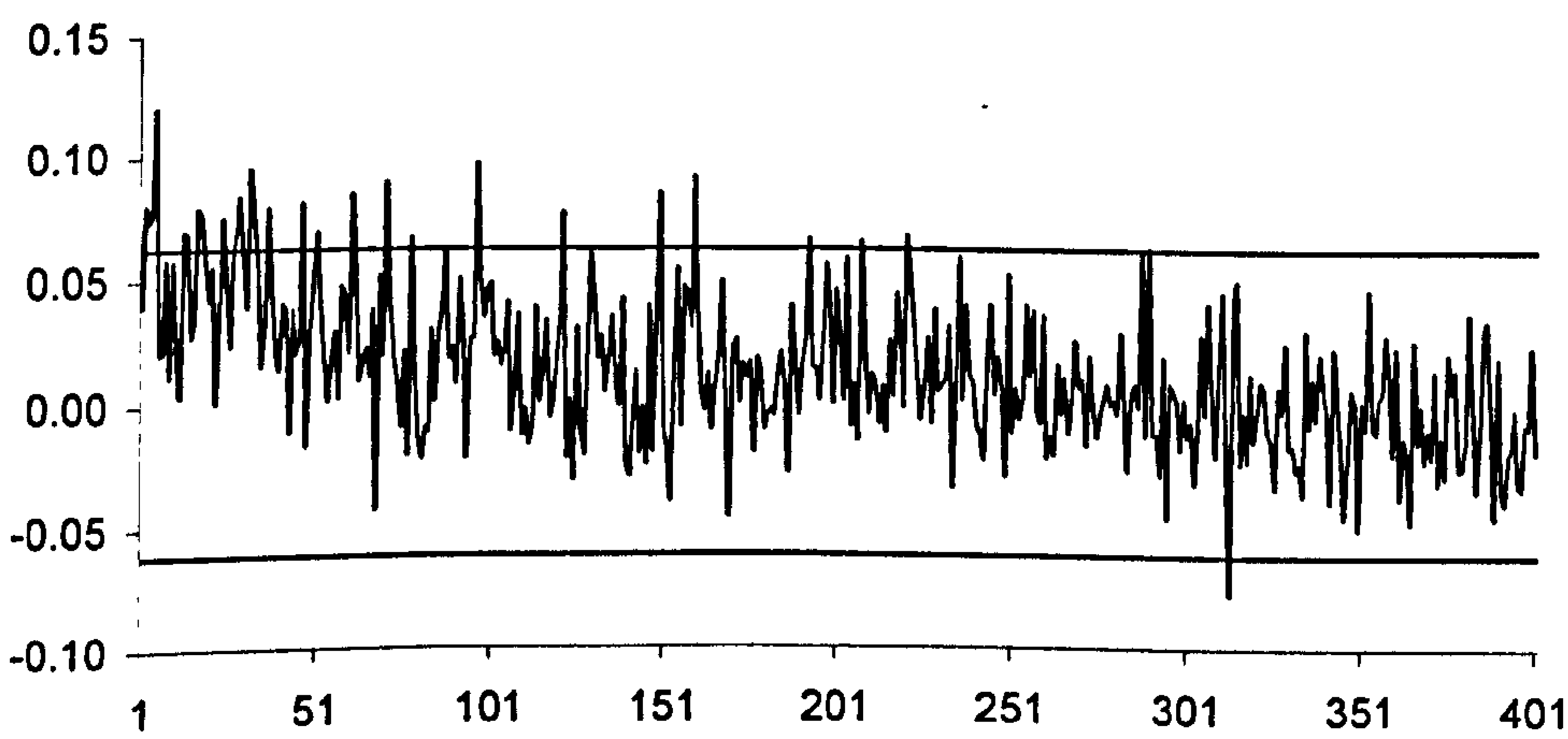


Figure 31: Autocorrelations for *r* (1995-1998)

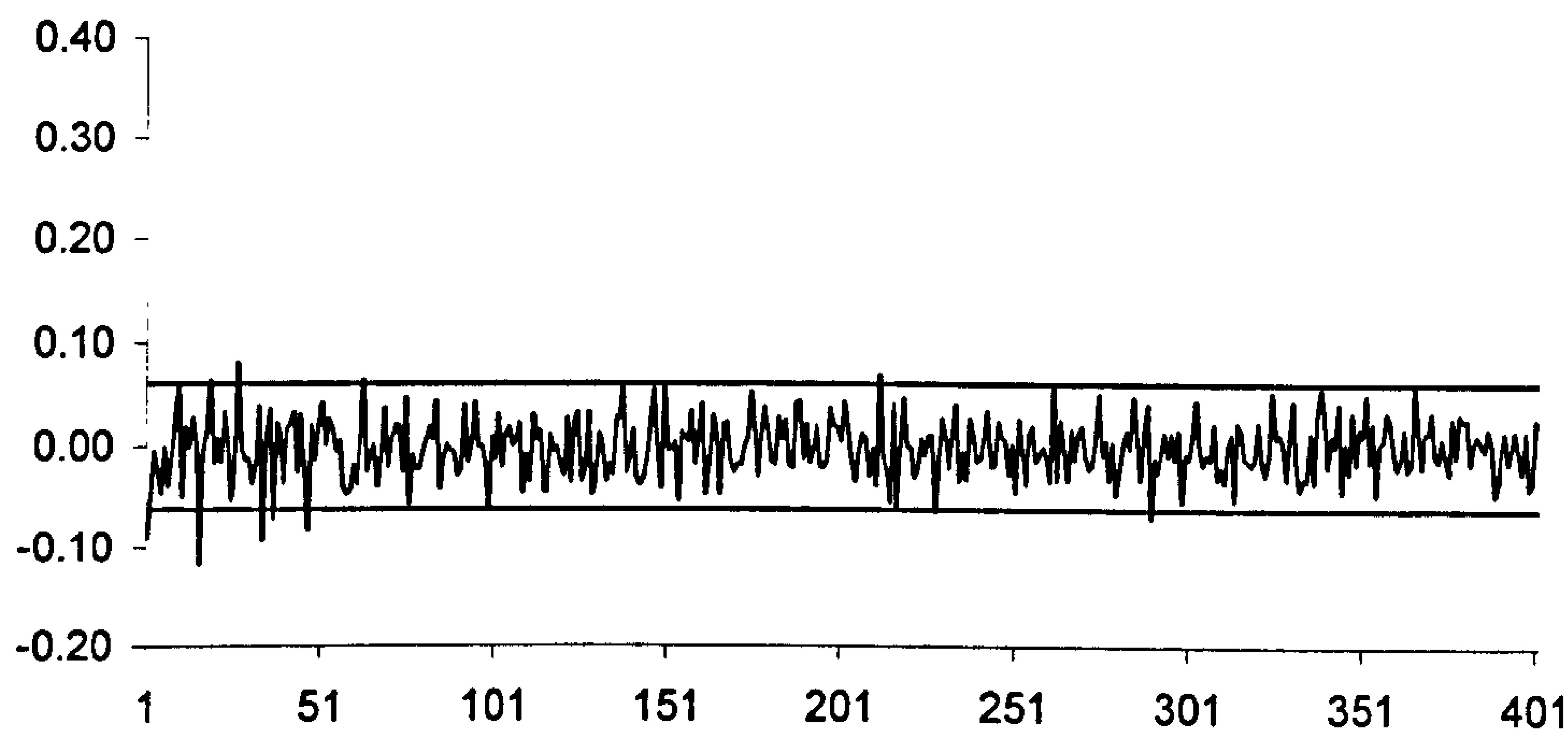


Figure 32: Autocorrelations for r^2 (1995-1998)

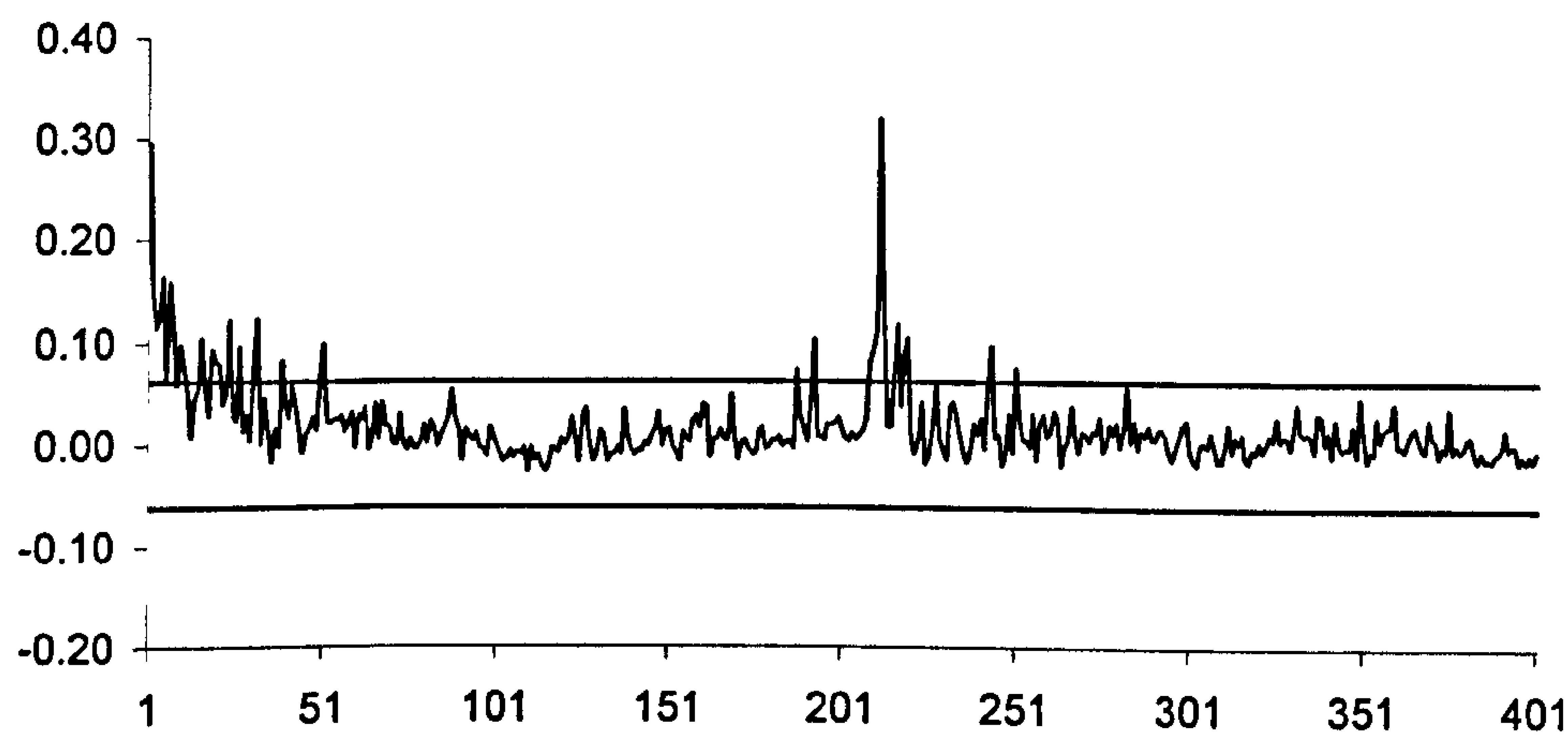


Figure 33: Autocorrelations for |*r*| (1995-1998)

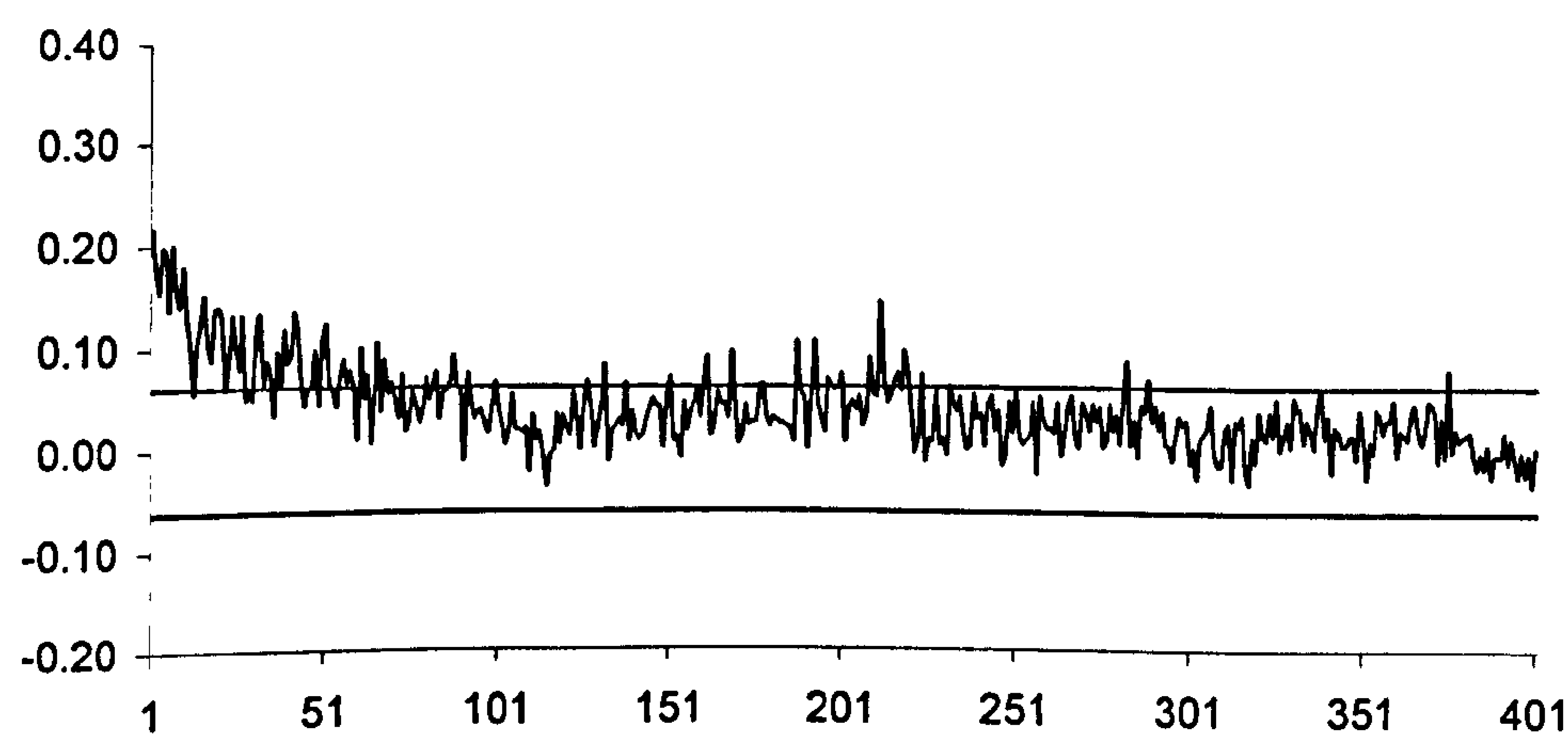


Figure 34: 3-State *SWARCH-L(3,2)*: High Volatility Regions

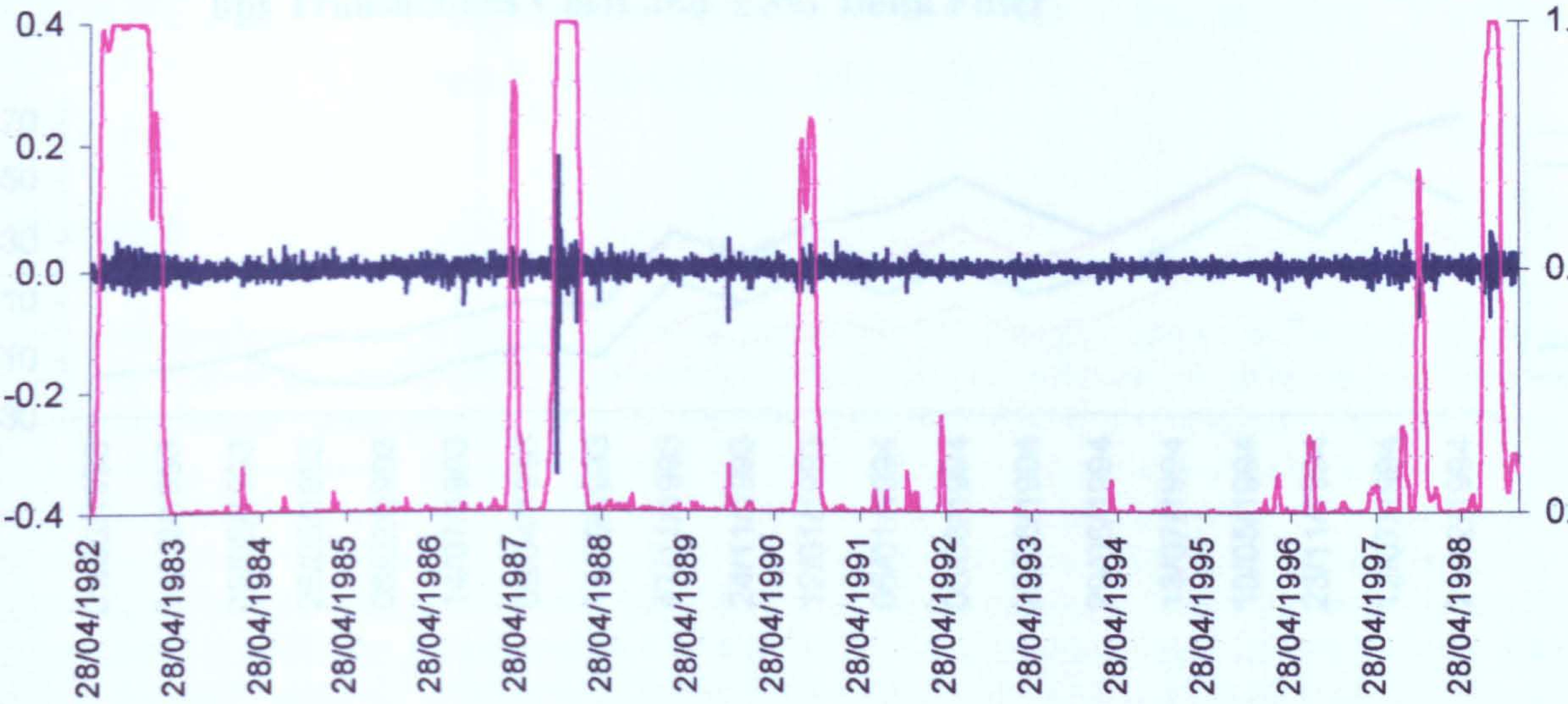


Figure 35: 3-State *SWARCH-L(3,2)*: Low Volatility Regions

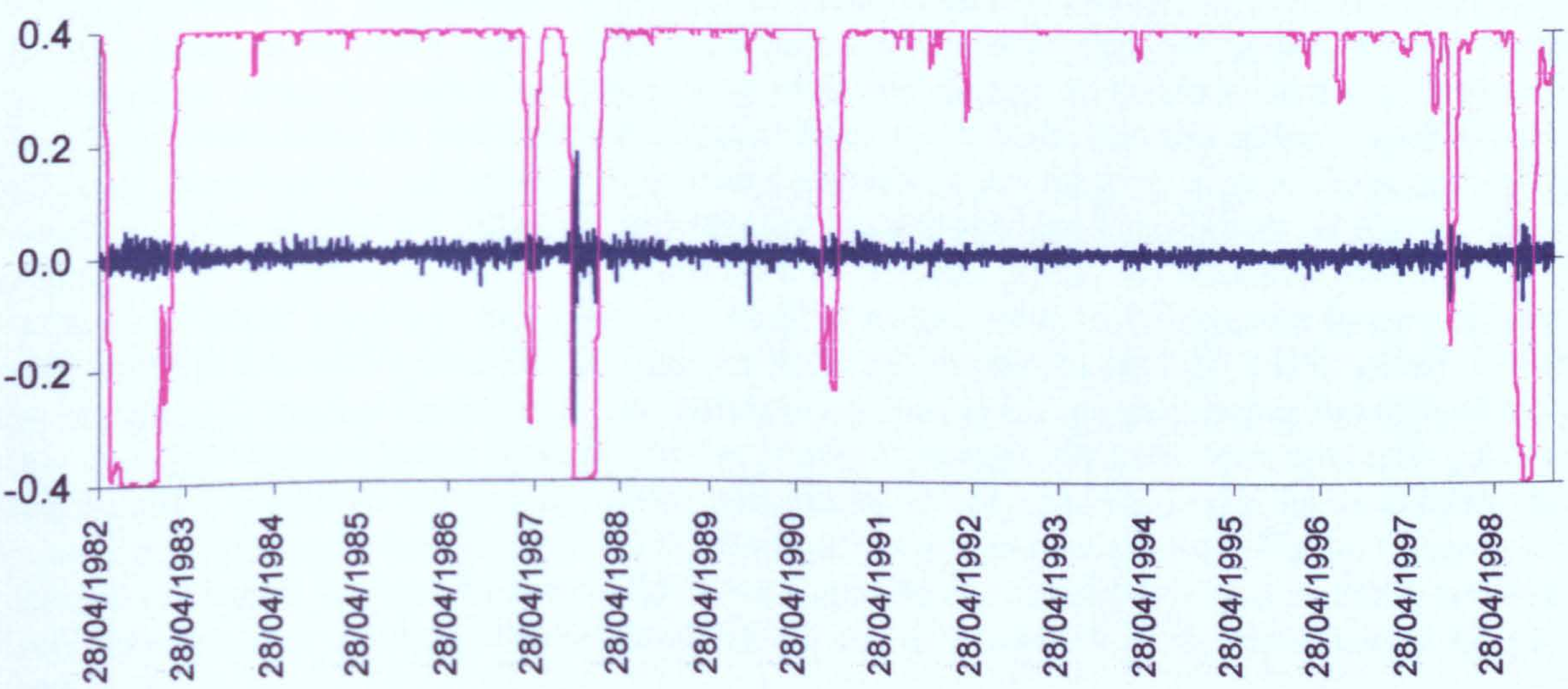
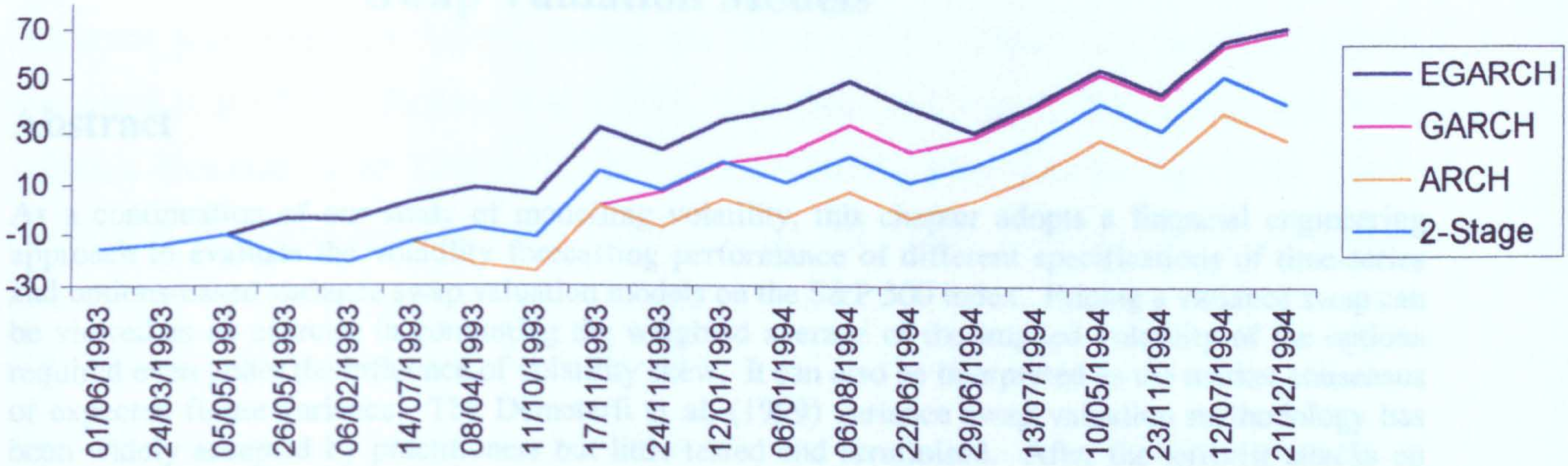


Figure 36: Cumulative Rate of Return From Straddles Trading (1993-1994) With 25 bps Transactions Costs and $\pm 3\%$ Delta Filter



CHAPTER 5 Empirical Performance of Alternative Variance Swap Valuation Models

Abstract

As a continuation of our study of modelling volatility, this chapter adopts a financial engineering approach to evaluate the volatility forecasting performance of different specifications of time-series and options-based variance swap valuation models on the S&P 500 index. Pricing a variance swap can be viewed as an exercise in computing the weighted average of the implied volatility of the options required even under the influence of volatility skew. It can also be interpreted as the market consensus of expected future variance. The Demeterfi et al. (1999) variance swap valuation methodology has been widely accepted by practitioners but little tested and scrutinised. After the terrorist attacks on September 11, 2001, the longer-termed forward variance has become more volatile than the shorter-termed forward variance. This research presents the first of any known attempts to use market data to evaluate the effectiveness of the Demeterfi et al. framework. It contributes to this nascent literature by analysing the three-, six- and nine-month variance swap contracts from June 2001 to November 2001 using different specifications of implied and time-series models. Our research design is rich enough to admit a number of prominent models including: 1) ad hoc Black-Scholes model; 2) stochastic volatility model; 3) jump-diffusion model; 4) local volatility model; 5) EGARCH; 6) GARCH variance swap model. We aim to find out whether using more complex option pricing models to accommodate observed market anomalies is an effective strategy to improve variance forecastability. Based on results from six well-selected contract days, we illustrate that the options-based framework, although more capable of incorporating many stylised facts, may be a poor forecaster of future variance. Just as forward interest rates are not necessarily good predictors of futures rates, the arbitrage-free based Demeterfi et al. framework is not necessarily an effective predictor of future variance. Results from our data show that implied models tend to overpredict future variance and underperform time-series models. The reasons could be: 1) implied strategy was originally developed for hedging; 2) implied volatility is predominantly a monotonically decreasing function of maturity and therefore options-based strategy cannot produce enough variance term-structure patterns; 3) distributional dynamics implied by option parameters is not consistent with its time-series data as stipulated by the maximum likelihood estimation of the square-root process. Future research need to use a larger sample set in order to establish a more statistically significant result to clarify our findings. Until then we have a strong reservation about the use of Demeterfi et al. methodology for variance forecasting.

5.1 Introduction

5.1.1 Background of the Study

Despite the fact that there has been an increased interest in volatility products since the late 1990's, little research has been directed towards to the development of volatility derivatives. The first theoretical paper to value volatility derivatives is by Grünbichler et al. (1996). Grünbichler et al. presented a simple but technically complicated framework that used the equilibrium approach within which specific closed-form solutions for volatility futures and option prices were derived within a mean-reversion framework. Later, Gupta (1997) and Engle et al. (1998) discussed the issues related to the hedging of volatility. Subsequently, Andersen and Andreasen (1999), Rolfes and Henn (1999), Chriss and Morokoff (1999), Demeterfi et al. (1999), Brenner et al. (2000), Brockhaus and Long (2000), Heston and Nandi (2000b), Howison et al. (2001), Little and Pant (2001), Carr and Madan (1999, 2002), Javaheri et al. (2002) and Théoret et al. (2002) also researched volatility derivatives, but the amount of research invested in volatility products still pales in comparison with other well-studied exotic derivatives products such as barrier and Asian options.

Until now the conventional instruments for implementing a volatility hedge remain rather crude. The most widely accepted way of speculating on volatility is usually achieved through the purchase of European call and put options. Traditional techniques such as delta hedging strategy always focus on the reduction of delta-risk. In Chapter 4 we have demonstrated the insufficiency of a delta-neutral hedge in the event of large market moves. Once the underlying index moves, however, a delta-neutral trade can become long or short delta. Rehedging becomes necessary to maintain a delta-neutral position as the market moves. Since transaction and operational costs generally prohibit continuous rehedging, residual exposure of the underlying ultimately arises from options-based volatility strategies. It is clear that even though options have the effect of adjusting the volatility profile of a portfolio, it also induces additional exposure to the underlying and other market factors. Thus volatility risk has yet to be dealt with so that investors and traders can directly express their views on future volatility.

5.1.1.1 New Way of Trading: Variance Swap

The arrival of variance swaps offers an opportunity for traders to take synthetic positions in volatility and hedge volatility risk. They were first introduced in 1998 in the aftermath of the Long Term Capital Management (LTCM) melt down when implied stock index volatility levels rose to unprecedented levels. These variance swap contracts are mostly based on equity indices and they were originally designed to be a replacement for traditional options-based volatility strategies such as straddle or hedged call/put options. Over the past few years, variance swaps have grown into a sizeable market[58]. Despite its name, a variance swap is actually an over-the-counter forward contract whose payoff is based on the realised volatility of a stated equity index. Their payoff at expiration is equal to:

$$(\sigma_R^2 - K_{vol}^2) * N$$

where N is the notional amount of the swap in some currency units per annualised variance point, σ_R and K_{vol} are the realised stock volatility over the life of the contract (n days) quoted in annual term, i.e. $\frac{F}{n}\sum_{i=0}^{n-1}\left(\frac{S_{i+1}-S_i}{S_i}\right)^2$, and the fixed annualised volatility delivery price, respectively. F is the appropriate annualisation factor.

5.1.1.2 Usage of Variance Swap

Since a variance swap provides pure exposure on future volatility levels, it is considered a cleaner bet on volatility than an options-based strategy. It allows counterparties to exchange cash-flows – floating variance for fixed variance. Counterparties can use variance swap to speculate the spread between future realised (floating) and implied (fixed) volatility, or to hedge the volatility exposure of other positions or businesses. According to Curnutt (2000), some of the possible strategies using variance swaps are:

i) Speculating a directional view that implied volatility is too high or too low relative to anticipated realised volatility because 1) volatility follows a mean-reverting process. In this model, high volatility decreases and low volatility increases; 2) there is a negative correlation between volatility and stock or index level. The volatility stays high after large downward moves in the market; 3) volatility increases with the risk and uncertainty;

ii) Implementing a view that the implied volatility in one equity index is mispriced relative to the implied volatility in another equity index;

iii) Trading volatility on a forward basis by purchasing a variance swap of one expiration and a variance swap of another expiration.

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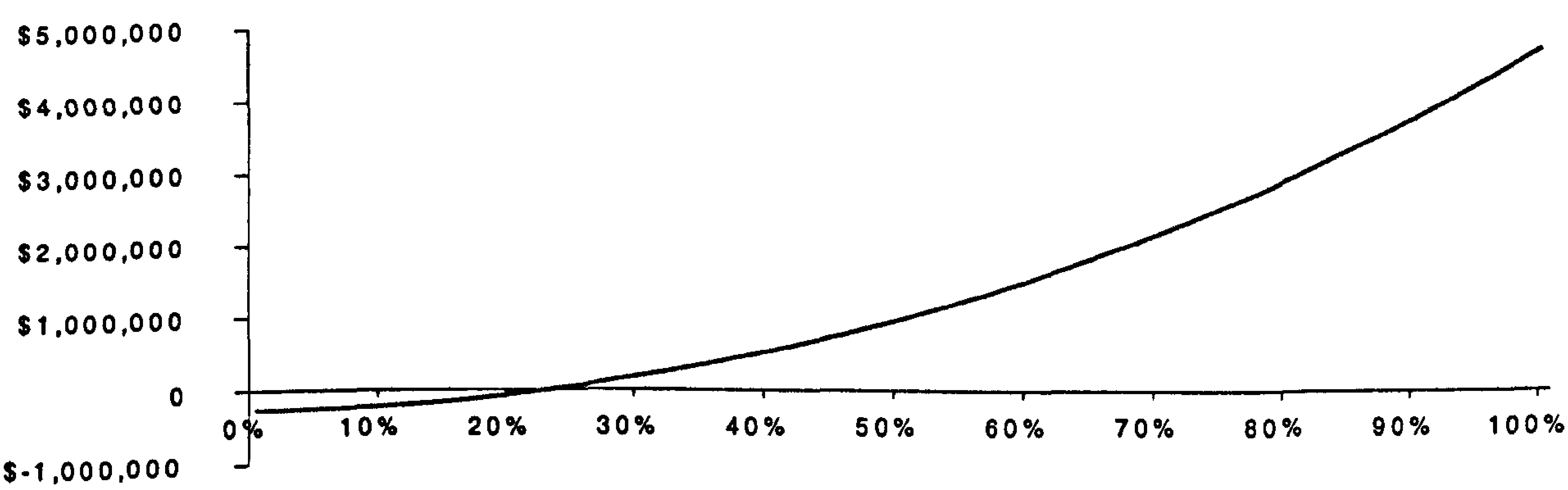
[58] *Capital Markets News*, Federal Bank of Chicago, March 2001.

5.1.1.3 Variance Swap Example

The following example illustrates to the reader how variance swap really works: using the S&P 500 as the underlying index, a volatility level of $K_{vol} = 23\%$ is fixed for one year. This corresponds to a nominal variance of 5.29%. Counterparty B agrees to pay Counterparty A a notional amount of US$5,000,000 for each percentage point of realised variance point above 5.29% and Counterpary A agrees to pay Counterparty B US$5,000,000 per variance point below this value. In this case, the notional value of the contract, or fixed leg payment, is US$26,450,000. Suppose realised volatility (variance) of S&P 500 during this time period turned out to be 43% (18.49%). The payoff to the party that receives variance is US$5,000,000 x (18.49% - 5.29%), or US$660,000. If realised volatility were 3%, the payoff to the party that pays volatility would be US$5,000,000 x (0.09% - 5.29%), or a loss of only US$260,000. Figure 37 illustrates the payoff of a long variance swap under different levels of realised volatility. Its payoff is nonlinear in volatility. This means, for instance, that a one percent deviation of realised volatility above the price has a different (larger) payoff than a one percent deviation of volatility below the delivery price.

Figure 37:Volatility vs. Variance Swap Payoffs – Long



The maturity of variance swap contracts can run from three months to five or even seven years, although most trades occupy around the one- year spectrum[59]. The primary cost associated with variance swaps is the bid/ask spread, which is approximately double the spread in the straddle market. Their bid/ask spreads on a S&P 500 variance swap range from one variance point for a one-year contract to two variance points for a longer-maturity

[59] See Mehta (1999) for further details.

contract. Institutional users such as hedge funds are attracted to own variance swap, especially when their portfolios are naturally short vega, as an alternative to using options to take on or hedge volatility exposure.

5.1.2 The Problem Statement and Hypotheses

The model developed by Demeterfi et al. (1999) is the most popular tool to price variance swaps, but surprisingly, no research has ever considered using market data to test for its usefulness. This chapter examines different variance swap models' performance on the S&P 500 index from June 2001 to November 2001. After the terrorist attacks on September 11, 2001, the longer-termed forward variance has become more volatile than the shorter-termed forward variance. We analyse the three-, six- and nine-month variance swap contracts by evaluating different specifications of implied and time-series models at different points in time. The underlying hypotheses of this project are that if options-based Demeterfi et al. (1999) framework is mathematically correct then:

i) Each generalisation of the benchmark Black-Scholes model should be able to improve the volatility forecastability of the options-based pricing model;

ii) If option prices are indeed representative of their underlying time-series and forward-looking then the forecastability of options-based variance swap models should be superior to their time-series counterparts.

In this study our goals are:

i) To present a complete picture of how each generalisation of the benchmark Black-Scholes model can really improve the variance forecastability of variance swaps and whether each generalisation is consistent between in- and out-of-sample results;

ii) To investigate whether there may be any systematic difference in variance forecasting performance between time-series and options-based variance swap valuation models. It is intended to explore whether options-based models, which are forward-looking, are capable of outperforming discrete-time processes, which use only historical information, in predicting future variance.

5.1.3 The Significance of the Study

The Demeterfi et al. (1999) variance swap pricing methodology has been widely accepted by practitioners but little tested and scrutinised. Regrettably, no empirical studies have ever used any market data to investigate the pricing performance of variance swap valuation models. This research presents the first of any known attempts to use market data to shed light on the variance forecastability of variance swap valuation models under alternative time-series and competing option pricing models. Since implied volatility can be regarded as the market's expectation of future realised volatility, the implication of any poor variance forecastability by options-based models is such that practitioners and academicians alike may need to look for a way to integrate historical and market information in a composite option pricing model.

5.1.4 Organisation

The remainder of this chapter is organised as follows. In section 5.2 we review the methodology and models. Section 5.3 introduces the dataset. Section 5.4 discusses the calibration procedures/results and analyses the empirical findings. Section 5.5 summaries the results.

5.2 Methodology

This section discusses the approaches and models used for volatility forecasting. We first review the criteria used in judging the variance forecastability of different time-series and options-based variance swap models. We then outline the implied framework for variance swap developed by Demeterfi et al. (1999). This methodology exclusively uses traded options to forecast variance. Subsequently, we discuss different option pricing models that can account for observed market anomalies in the S&P 500 options market. Finally we illustrate the time-series approach in forecasting variance.

5.2.1 Performance Criteria

The variance forecastability of different variance swap models are evaluated in the following ways:

i) In-sample analysis. In view of option-pricing, it refers to the ability of each generalisation of Black-Scholes option model to fit the call option data and produce the least pricing error. Sum of price square error (SPSE) is used to judge whether one option model is better than the other on each contract day;

ii) Out-of-sample analysis. It compares the variance forecastability of all six time-series and options-based models. The criteria used in selecting the best model is mean square error (MSE), and aggregate MSE ranking[60] is applied to evaluate the overall performance of each time-series and options-based variance swap model for each of the three maturity months, i.e. three-, six- and nine-month contracts;

iii) Consistency of options-implied distributional dynamics and time-series properties. Maximum likelihood estimation of a square-root process is used in order to identify potential inconsistency between options-implied dynamics and time-series data by looking into the estimated structural parameters.

It should be noted that our results are based on the use of eighteen well-designed variance contracts between June 2001 and December 2001. Although our sample is small and sample periods are overlapping, we point out that the price fitting and variance forecastability of the options-based models are insensitive to the choice of sample periods because options are supposed to be forward-looking and do not use historical data.

5.2.2 The Options-based Variance Swap Framework

The original Black-Scholes model assumes that volatility is constant or deterministic, but recently many researchers have developed option pricing models that recognise the stochastic nature of volatility, e.g. Hull and White (1987), Heston (1993a). New financial engineering techniques have also made it possible to explore volatility trading in a more sophisticated manner. The idea behind these innovations is that volatility can be hedged without having to worry about its future volatility level. Whaley (1993) was among first to advocate the use of volatility futures and its options on CBOE. Consequently, the VIX, which indicates the level of the at-the-money implied volatility on S&P 100, was created in 1993 for CBOE[61]. Whaley

[60] Aggregate rank is defined as the sum of the rank for each model in each sub-period.

[61] The MONEP created the VX1 and VX6 indexes in October 1997. On January 19, 1998, the Deutsche Terminborse (DTB) became the first exchange in the world to list volatility futures based on an underlying equity index of implied volatility when it launched the VOLAX futures. Readers are referred to Werner and Roth (1998) for details on VOLAX contracts.

pointed out that such products were free of price risk and could be used to hedge volatility. Trading volumes in these contracts, however, have been low. According to Neuberger (1994), volatility futures was potentially subject to manipulation. As a result, Neuberger addressed this concern by designing the log-contract to provide an accurate and flexible volatility hedge. Since then log-contract has become an indispensable component for volatility research.

5.2.2.1 Log-Contract

Neuberger (1994) demonstrated that by dynamically hedging the log-contract against a static futures position it was possible to engineer the future profit or loss as an exact linear function of the realised quadratic variation. This result is not dependent on any assumption that returns are generated by a Brownian diffusion process, or that volatility is constant. The "fair price" of the log-contract at any time can be shown as:

$$L_t = \log(F_t) - \frac{1}{2}\sigma_R^2(T-t)$$

where F_t is the futures price at time *t*, σ_R the constant future realised volatility and *T* the maturity. The value of the contract at time *T* is $L_T = \log(F_T)$.

The "fair price" is the direct result of dynamically hedging the log-contract with appropriate amounts of futures contracts until maturity. The delta for log-contract is equal to $1/F_t$ and independent of volatility. If traders' view on volatility is $K_{vol} \neq \sigma_R$, the value of log-contract will not be the "fair price". In this case, the present value of the profit and loss of this hedging strategy over the life of the contract can be shown as:

$$\frac{1}{2}(K_{vol}^2 - \sigma_R^2) * T$$

where σ_R is the realised volatility over the life of the contract and K_{vol} the volatility implied in the price of the log-contract at time 0.

By dynamically hedging the log-contract, it is clear that one can replicate the cash-flows of variance swap and gamble on volatility. But even though log-contract is a powerful mathematical tool to hedge volatility, it is only a hypothetical tool. In addition to the availability of log-contract, Neubeger's results are also conditional on the feasibility of forming discrete and dynamical hedges. Nevertheless, Neuberger's work has greatly facilitated the introduction of volatility derivatives, such as volatility futures, options and swaps.

In order to understand the mechanics of log-contract, one can take a Taylor-series expansion of the logarithm of the price up to second-order derivatives which gives:

$$\log S_{i+1} - \log S_i = \frac{S_{i+1} - S_i}{S_i} - \frac{1}{2}\left(\frac{S_{i+1} - S_i}{S_i}\right)^2$$

Summing both sides of the above equation over the total number of days n in the contract and rearranging terms, one obtains:

$$\sum_{i=0}^{n-1}\left(\frac{S_{i+1} - S_i}{S_i}\right)^2 = 2\log\frac{S_0}{S_n} + 2\sum_{i=0}^{n-1}\frac{S_{i+1} - S_i}{S_i}$$

The LHS is the floating leg of variance swap, which can be replicated by holding a derivative with payoff equal to the first term of the RHS – a log-contract, and a forward position – the second term of the RHS. Thus the sole concern in setting the delivery price of variance swap is to engineer the cash-flows on the RHS, in particular the log payoff.

5.2.2.2 Demeterfi et al. Framework

Since log-contract is non-traded and requires dynamic hedging in order to replicate the cash-flows of variance swap, it is not a "direct" bet on variance/volatility. In order to provide a direct and forward exposure on volatility, Demeterfi et al. (1999) developed a formal and rigorous theoretical framework for the pricing of variance swaps. This study showed that the future level of volatility could be inferred from the prices of traded options of the underlying asset and thereby derivatives on volatility be valued. Demeterfi et al. initially focused on the replication of the delivery price under the Black-Scholes framework with deterministic volatility. Since variance swap is a forward contract on variance, the delivery price must make the swap of zero value initially. Under the assumptions of zero interest rates and dividend yields, Demeterfi et al. proved that a constant vega, υ, could be obtained by owning a portfolio of infinite call and put options weighted inversely by their square of strikes, K^2. Figures 38-41 show how the BS υ's vary with stock price S for portfolios consisting of different number of call options weighted inversely by K^2.

Figure 38: Vega of Individual Strikes: 80, 100, 120

Figure 39: Sum of the Vega contributions of Individual Strikes: 80, 100, 120

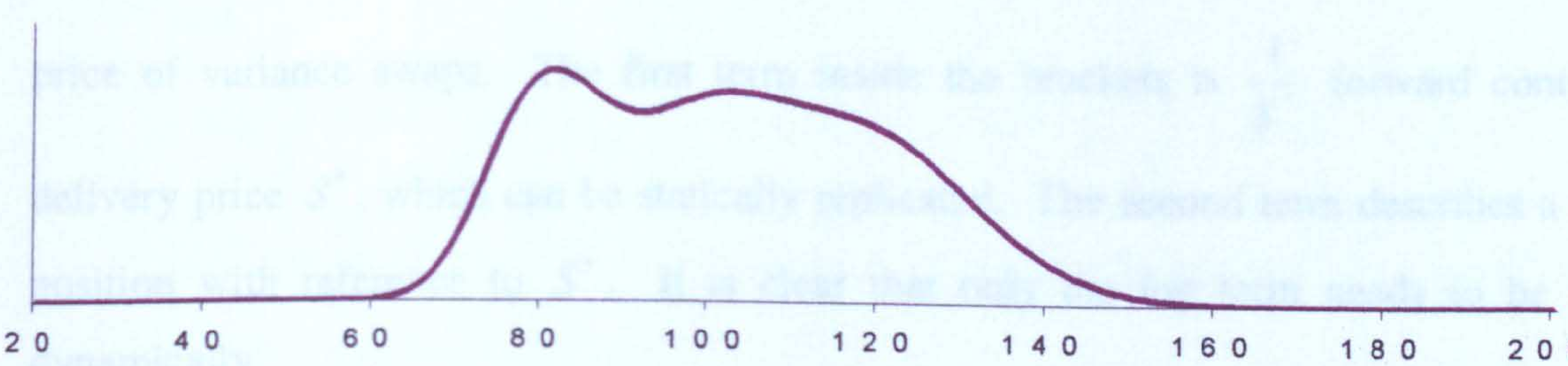


Figure 40: Vega of Individual Strikes: 60 to 140 spaced 10 apart

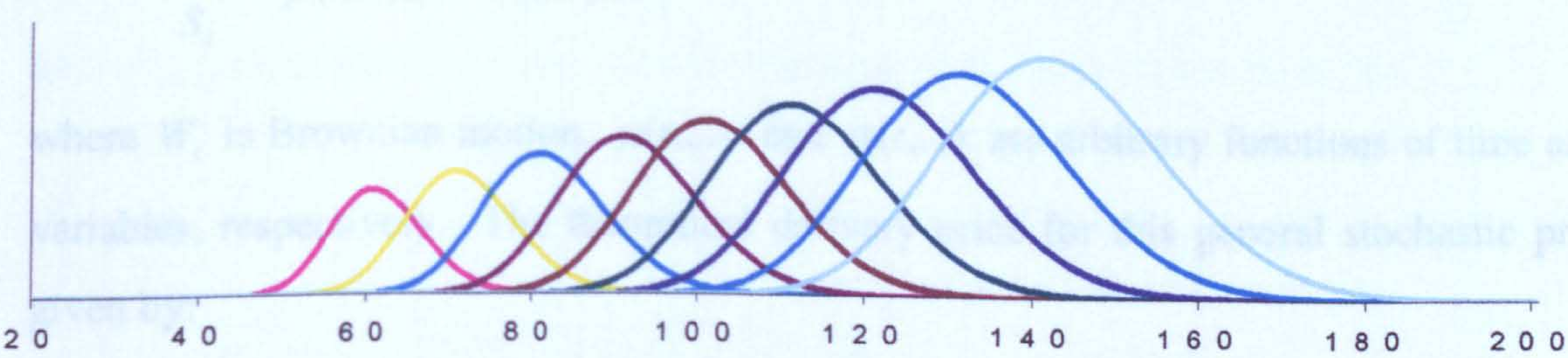
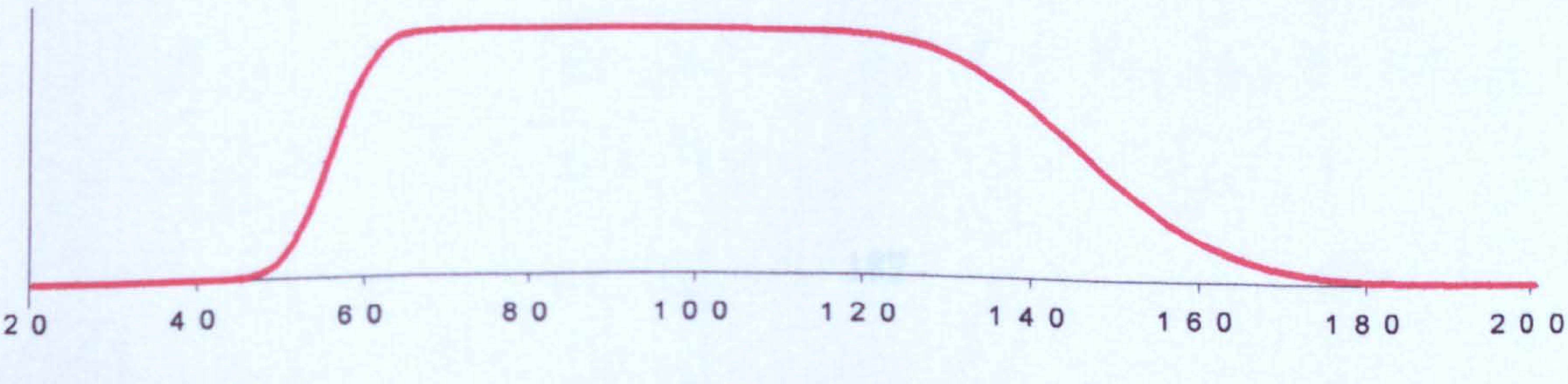


Figure 41: Sum of the Vega contributions of Individual Strikes: 60 to 140 spaced 10 apart



5.2.2.2.1 Derivation of Demterfi et al. Framework

To obtain an initial exposure of a unit of currency per volatility point squared, this portfolio at time 0 can be constructed as follows:

$$\Pi_0 = \frac{2}{T}\left[\frac{S_0 - S^*}{S^*} - \log(\frac{S_0}{S^*})\right] + \sigma_I^2$$

where S^* is usually the at-the-money forward stock or spot level and σ_I^2 is the view or estimate of future realised variance from traders.

The hedging of the above portfolio is similar to that of log-contract: if the realised variance turns out to have been σ_R^2 the net payoff on the dynamically hedged position until expiration will be equal to $(\sigma_R^2 - \sigma_I^2)$. The terms inside the squared parenthesis are values of the "fair" price of variance swaps. The first term inside the brackets is $\frac{1}{S^*}$ forward contract with delivery price S^*, which can be statically replicated. The second term describes a short log position with reference to S^*. It is clear that only the log term needs to be rehedged dynamically.

Demeterfi et al. also relaxed some Black-Scholes assumptions and derived the diffusive solutions for the delivery price conditional on no jump. The asset price evolution is given by:

$$\frac{dS_t}{S_t} = \mu(t,...)dt + \sigma(t,...)dW_t$$

where W_t is Brownian motion, $\mu(t,...)$ and $\sigma(t,...)$ are arbitrary functions of time and other variables, respectively. The theoretical delivery price for this general stochastic process is given by:

$$V = \frac{1}{T}\int_0^T \sigma^2$$
$$K_{vol}^2 = E[V]$$
$$K_{vol}^2 = \frac{1}{T}E\left[\int_0^T \sigma^2(t,...)dt\right]$$
$$d(\log S_t) = \left[\mu - \frac{1}{2}\sigma^2\right]dt + dZ_t$$
$$\therefore \frac{dS_t}{S_t} - d(\log S_t) = \frac{1}{2}\sigma^2 dt$$
$$K_{vol}^2 = \frac{2}{T}E\left[\int_0^T \frac{dS_t}{S_t} - \log\frac{S_T}{S_0}\right]$$

$$E\left[\int_0^T \frac{dS_t}{S_t}\right] = rT,\ \log\frac{S_T}{S_0} = \log\frac{S_T}{S^*} + \log\frac{S^*}{S_0}$$
and
$$-\log\frac{S_T}{S^*} = -\frac{S_T - S^*}{S^*} + \int_0^{S^*}\frac{1}{K^2}Max(K - S_T,))dK + \int_{S^*}^{\infty}\frac{1}{K^2}Max(S_T - K,))dK$$
$$\therefore K_{vol}^2 = \frac{2}{T}\left(rT - \left(\frac{S_0}{S^*}e^{rT} - 1\right) - \log\frac{S^*}{S_0} + e^{rT}\int_0^{S^*}\frac{1}{K^2}P(K)dK + e^{rT}\int_{S^*}^{\infty}\frac{1}{K^2}C(K)dK\right)$$

where $C(K)$ and $P(K)$ denote the current fair value of a European call and a European put of strike struck at K that mature at time T with risk-free interest rate r and some arbitrary boundary S^* separating actively traded out-of-the money call and put options. On the basis of a piecewise linear approximation to this payoff for a finite set of call and put strikes, $K_{i,C}$ and $K_{i,P}$, respectively, the appropriate option portfolio weights are given by:

$$g(S_T) = \frac{2}{T}\left(\frac{S_T - S^*}{S^*} - \log\frac{S_T}{S^*}\right)$$
$$w(K_{i,C}) = \frac{g(K_{i+1,C}) - g(K_{i,C})}{K_{i+1,C} - K_{i,C}} - \sum_{j=0}^{i-1} w(K_{j,C}) \qquad \textit{for calls}$$
$$w(K_{i,P}) = \frac{g(K_{i+1,P}) - g(K_{i,P})}{K_{i,P} - K_{i+1,P}} - \sum_{j=0}^{i-1} w(K_{j,P}) \qquad \textit{for puts}$$

where the order of the strikes is:

$$K_{i-1,P} < ... < K_{3,P} < K_{2,P} < K_{1,P} < S^* = K_0 < K_{1,C} < K_{2,C} < K_{3,C} < ... < K_{i-1,C}$$

Appendix B.7 summarises the procedures to calculate the "fair" delivery price. The above "adjusted" weights guarantee that option payoffs will always exceed or match the value of log-contract. Clearly the essence of this derivation is that log payoff can be decomposed into a portfolio consisting of a forward contract and out-of-the-money call and put options[62]. This approach to the fair value of future variance is the most rigorous from a theoretical point of view and makes fewer assumptions than the initial intuitive treatment. From a hedging perspective, it makes precise the intuitive notion that implied volatility can be regarded as the market's expectation of future realised volatility. Most importantly, it provides a direct connection between the market cost of options and the strategy for capturing future realised volatility, even when there is an implied volatility skew and the simple Black-Scholes formula is invalid. From a practical perspective, traders may express views on volatility using variance swaps without having to delta hedge.

5.2.2.2.2 Implementation Issues with Demeterfi et al. Framework

Few issues merit our attention in pricing variance swap using Demeterfi et al. framework. First, since log() payoffs are not traded in the marketplace, one will have to approximate them with traded European options in a limited strike range. Because these strikes cannot exactly duplicate such cash-flows, they will capture less than the true realised variance. According to Little and Pant (2001), this reduction is greater for the longer-maturity swaps. Second, the asset price may fail to remain diffusive. When asset price displays jumps, the impact of jumps on the pricing and hedging of volatility derivatives is significant and it can cause the strategy to capture a quantity that is not the true realised variance. To fully implement a replication strategy for variance swaps, one needs price continuity and a consistent stochastic volatility model for options. Finally the above analysis is based upon approximating the discretely sampled variance used in the contract terms of most variance swaps by a continuously sampled variance. Whilst this approximation can be expected to provide very reasonable estimates for short-dated variance swaps when the sampling is frequent, they may not perform well with less frequent sampling for longer periods. We refer the reader to Chriss and Morokoff (1999) for practical risk management issues in regard to variance swaps.

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[62] See Carr and Madan (2002) for its derivation.

Despite Demeterfi et al. framework is not perfect, it remains an essential component for the variance swap valuation exercises in this research.

5.2.3 Option Models for Variance Swaps

In a study of finding an arbitrage-free framework for pricing of volatility derivatives, Carr et al. (2002) found that as long as the movement of the underlying asset is continuous[63], the pricing and hedging of variance contracts is completely independent of the choice of the volatility process. Carr et al. showed that model-independent prices of variance swaps could be inferred from the market prices of European-style vanilla options. Therefore, pricing a variance swap can be viewed as an exercise in computing the weighted average of the implied volatility of the options required to replicate the swap even under the influence of volatility skew. That is, the delivery price is set so as to reflect the aggregate cost in terms of the implied volatility of the hedge portfolio.

However, results in Chapters 3 and 4 demonstrate that the term-structure of implied volatility is pronounced in the S&P 500 marketplace. In addition to many studies, Rubinstein (1985, 1994) also documented evidence that implied volatility tended to rise for deep in-the-money and out-of-the-money options. The presence of skews, smiles and, to a lesser degree, term-structures violates the most basic assumptions of the Black-Scholes model and makes it necessary to revisit the concept of pricing and hedging of vanilla options. In order to accommodate market reality, it is necessary to extend the Black-Scholes model in a meaningful fashion. In particular, one needs to generate lepotokurtic distributions via a stochastic process for the spot and possible some additional hidden variables. The main difficulty is that there are many models and processes that can be used for this purpose and their relative merits and drawbacks partly depend on a specific problem at hand.

The 1990's witnessed several important developments in order to describe smile effects. For instance, Dupire (1994), Derman & Kani (1994) and Rubinstein (1994) developed the deterministic smile models. An alternative approach would be to consider the volatility as another stochastic variable, and there is growing evidence to support this hypothesis. Merton (1973) derived the first European option pricing solution for the jump-diffusion model;

[63] There is not an equivalent framework for asset that follows a jump-diffusion process.

subsequently, Ball and Torous (1985) and Bates (1991) confirmed that jump component could explain some of the empirically observed mispricing in options market. Bates (1996), Bakshi et al. (1997), Anderson & Andreassen (1999) and many others also invented stochastic jump-diffusion models. More general stochastic volatility models were developed by Hull and White (1987), Johnson and Shanno (1987), Scott (1987), Wiggins (1987), Stein et al. (1991), Ball and Roma (1994) and Schöbel and Zhu (1999). This list is by no means exhaustive.

The models developed by most of the above research papers require either the use of Monte-Carlo simulation or numerical solution of a two-dimensional parabolic partial differential equation, which is computationally intensive to implement. Too often, option models are chosen ad hoc, for instance, on the grounds of their tractability and solvability. Finding a meaningful theoretical framework and implementing it in practice remains a major challenge to practitioners and academicians alike. In the following sub-sections we will explain what types of option pricing models are selected for the pricing of variance swaps.

5.2.3.1 Stochastic Volatility Models

5.2.3.1.1 Justification for the Stochastic Volatility Approach

Diffusion models assume that volatility is, like the underlying asset, a continuous random variable. This is so-called the time-state-dependent approach. There are many reasons why we should model volatility as a diffusive process. For example, it could simply represent estimation uncertainty, or it could arise as a friction from transaction costs, or it could simulate non-Gaussian (heavy-tailed) returns distributions, or it could simulate leverage effect and capture volatility as a stationary, mean-reverting process. Bakshi et al. (2000) suggested that one-dimensional diffusion models were inadequate to explain pricing inconsistency observed in S&P 500 options. After controlling for time-decay and market microstructure factors, Bakshi et al. stipulated that if one had to introduce another state variable that affected option prices, this second stochastic process would be volatility. Zhang and Shu (2002) also provided evidence that stochastic volatility models outperformed the Black-Scholes model significantly in almost all moneyness-maturity groups. In other words, stochastic volatility is a far-reaching extension of the Black-Scholes' log-normal model, describing a much more complex market.

However, parameter estimation and stability of the estimates in time presents the major mathematical and practical challenge in using the stochastic volatility model. Without a formula for option prices under a particular stochastic volatility model, estimating the risk-neutral parameters is computationally intensive. Many questionable models are often chosen so that there is a closed-form solution, and this usually means taking the volatility to be independent of the Brownian motion driving the underlying asset price, whereas common experience suggests that a negative correlation exists, for instance, between stock index and volatility. Furthermore, the relatively poor performance of some of these models in capturing the observed implied volatility surface (see Das and Sundaram, 1999), as well as their difficult calibrations and inherent market incompleteness, make them unattractive to both academicians and practitioners. Consequently, pricing of options in the presence of stochastic volatility is difficult and seldom can be done analytically.

5.2.3.1.2 Heston Model

Recent research has shown that allowing for correlation as a free parameter can explain many observed market anomalies. Rubinstein (1994) discovered that the local volatility of stock index was negatively correlated with the level of the index. In a pure diffusive model, this negative skewness can only be achieved through a negative correlation between returns and volatility. In addition, Nandi (1998) found that accounting for correlation between returns and volatility in the stochastic volatility model substantially improved the mispricing of out-of-the-money options when compared to both the zero correlation version of the stochastic volatility model and the widely used Black-Scholes model. Since Heston (1993a) invented the Fourier approach to option pricing under stochastic volatility, the study of stochastic volatility models has become much easier. This approach permits a closed-form solution for European options and at the same time allows a non-zero risk premium for volatility as well as an arbitrary correlation between asset returns and volatility. One can also use the information contained in a long time-series or the options market to calibrate model parameters in an in-sample context and thereafter compute out-of-sample option prices.

The most important feature of Heston model is that it can account for correlation between volatility and asset returns. Correlation between volatility and asset returns is necessary to generate skewness and skewness in the distribution of asset returns and it affects the pricing of in-the-money options relative to out-of-the-money options. Without this correlation, increasing the volatility of volatility of stochastic volatility only increases the kurtosis of asset returns, which in turn only affects the pricing of near-the-money versus far from-the-money options. Since options are usually traded near-the-money and the Black-Scholes formula produces option prices virtually identical to the stochastic volatility models for at-the-money options, this explains some of the empirical support for the use of stochastic volatility model.

The stochastic volatility model used in our variance swap pricing exercises is Heston's stochastic volatility option pricing model, in which volatility is correlated with the underlying asset. The variance process is modelled as a square-root process with mean-reversion. The Heston model is nested within Bakshi et al. (1997) framework. It is given as follows:

$$dS(t) = rdt + \sqrt{V_t}dW_s$$
$$dV_t = (\theta_v - \kappa_v V_t)dt + \sigma_v\sqrt{V_t}dW_v$$

where r is the constant spot interest rate; V_t is the diffusion component of returns variance conditional on no jump occurring; W_s and W_v are each a standard Brownian motion with correlation $Cov[dW_s, dW_v] = \rho dt$; $\kappa_v, \theta_v / k_v$ and σ_v are respectively the speed of adjustment, long-run mean, and variation coefficient of the diffusion process V_t.

The solution for the above set of formulas is based on the idea that whilst the probability that the underlying asset price is greater (less) than the strike price cannot be expressed analytically, the corresponding characteristic function can indeed be described analytically. For a European call option written on the stock with strike price K and maturity T, its time t price is given by:

$$C(t,T) = S_t * \Pi_t(t,T;S,r,V_t) - K * B(t,T)\Pi_2(t,T;S,r,V_t)$$

where $B_t(t,T)$ is the zero-coupon bond that pays a unit of currency in T-t periods. The price of a European put can be obtained from the put-call parity. Since Ramaswamy and Sundaresan (1985), Scott (1993) and Bakshi et al. (1997) found that the stochastic interest rate model did not significantly improve the performance of the Black-Scholes model, we will not consider the stochastic interest rate model in this study. Therefore, $B_t(t,T)$ is reduced to $e^{-r(T-t)}$. Given the characteristic functions f_j^{sv}'s, the conditional probability density functions Π_1 and Π_2 can be recovered from inverting the respective characteristic functions as in Heston (1993), Bates (1996) and Pan (2002):

$$\Pi_j(t,T;S_t,r,V) = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left[\frac{e^{-i\phi \ln(K)} f_j^{sv}(t,T,S_t,r,V_t;\phi)}{i\phi}\right] d\phi$$

for $j=1,2$. The characteristic functions are given in appendix C.1.

5.2.3.2 Jump-Diffusion Models

5.2.3.2.1 Justification for the Jump-Diffusion Approach

The explanation that volatility smile is the sole consequence of time-state-dependent or diffusive local volatility is far from common intuition, and it has become increasingly clear that the assumptions underlying the pure diffusive approach are not particularly realistic. It is a well-known fact that the pure diffusion model overprices long-term options and cannot take account of the strong smile effects exhibited by short-term options. In addition, many studies have showed that modelling jump component can improve option pricing performance. For example, Jorion (1988) discovered that there was evidence of jump component in equities and foreign exchange even explicit allowance was made for possible conditional heteroskedasticity. The importance of introducing a jump component in modelling stock price dynamics had also been noted in Bates (1996, 2000) and Bakshi et al. (1997) who stated that pure diffusion-based models had difficulties in explaining smile effects, particularly in short-term option prices. Bakshi et al. concluded that the Poisson-type jump components in jump-diffusion models could be used to address these concerns. In addition, Madan et al. (1998) introduced a pure jump process with a random time change for European options and found that the Black-Scholes model could be rejected in favour of the variance-gamma model. Furthermore, empirical investigations of time-series conducted by Carr et al. (2000) indicated that stock index dynamics was essentially devoid of a diffusion component. Carr et al. stated that risk-neutral processes for indices and stocks tended to be pure jump processes of infinite activity and finite variation. Moreover, Lipton (2001) advocated the use of models that took into account local jumps and stochastic features of the volatility dynamics for pricing and risk management of foreign exchange options. Finally, using Bates's (2000) model with time-varying jump-risk premia, Pan (2002) found that the stochastic jump model dominated pure diffusion models. Pan concluded that introducing volatility-risk premia in addition to jump-risk premia would not result in any significant improvement in the goodness of fit.

Qualitatively, jump-diffusion models produce distributions of returns that are mixtures of normal distributions and do have attractive leptokurtic features, at least for short maturities. The jump model can capture some types of crash phenomena, e.g. stock market crashes, 9/11-type events, currency devaluation etc. The jump-diffusion asset dynamics can be modelled as the resultant of two components:

i) The continuous part which is a reflection of new information that has a marginal impact on the underlying asset;

ii) The jump part which is a reflection of important news that has an instantaneous, non-marginal impact on the underlying asset.

The jump parameter allows better tracking of volatility by accounting for sudden changes in volatility that accompanies upward or downward movements in the asset. It gives the model an extra dimension of flexibility in valuing options across different strikes. Such models also imply an inverse relationship between option maturity and the magnitude of skewness, with little skewness for long-maturity options.

However, the use of Demeterfi et al. framework is based on the approximation of $\log\left(\frac{S_T}{S_0}\right)$ payoff when stock prices do not jumps. When stock prices do jump, log-contract can no longer capture realised volatility[64]. This is because $\log\left(\frac{S_T}{S_0}\right)$ can be replicated by an infinite number of weighted market call and put options only when the sample path of the underlying process is continuous. Given the shortcomings of pure diffusion models, the extension to include jumps in pricing options is well motivated. Although the use of Demeterfi et al. framework requires the underlying process to be pure diffusive, it would be pedantic to completely ignore its validity simply because its sample path may not be strictly continuous. To highlight the "impact" of non-continuous asset dynamics on variance swap pricing, we will apply the jump-diffusion model to the Demeterfi et al. framework whilst maintaining all other assumptions made by the original analysis. We emphasise that even though this strategy is not perfectly consistent on a scientific basis, it may demonstrate any possible pricing improvements over the classic time-state-dependent approach.

5.2.3.2.2 Bakshi et al. Model

We adopt the closed-form jump-diffusion option model developed by Bakshi et al. (1997) for the jump-analysis. Following Baskshi et al., this risk-neutral jump-diffusion setup is rich enough to admit many known variations[65] of the Black-Scholes model as special cases including: 1) Black-Scholes model: $\lambda = 0$ and $\theta_v = \kappa_v = \sigma_v = 0$; 2) Heston model: $\lambda = 0$. The jump-diffusion model is given by:

$$dS(t) = (r - \lambda\mu_j)dt + \sqrt{V_t}dW_s + J_t dq_t$$
$$dV_t = (\theta_v - \kappa_v V_t)dt + \sigma_v \sqrt{V_t}dW_v$$

where r is the constant spot interest rate, λ is the frequency of jumps per year; V_t is the diffusion component of return variance conditional on no jump occurring; W_s and W_v are each a standard Brownian motion with correlation $Cov[dW_s, dW_v] = \rho dt$; J_t is the percentage jump size conditional on a jump occurring that is log-normally, identically, and independently distributed over time with unconditional mean μ_J. The standard deviation of $\ln(1+J_t)$ is σ_J; q_t is a Poisson jump counter with intensity λ so that $P(dq_t = 1) = \lambda dt$ and $P(dq_t = 0) = 1 - \lambda dt$; $\kappa_v, \theta_v / k_v$ and σ_v are respectively the speed of adjustment, long-run mean, and variation coefficient of the diffusion, V_t.

The advantage of modelling volatility as a square-root process is that volatility never becomes negative. For a European call option written on the stock with strike price K and maturity T, its time t price is given by:

$$C(t,T) = S_t * \Pi_1(t,T;S,r,V_t) - K * B(t,T)\Pi_2(t,T;S,r,V_t)$$

[64] See Demeterfi et al. (1999) for details.

[65] Note that we simplify the Bakshi et al. model by eliminating the stochastic interest part.

Given the characteristic functions f_j^{svj}'s, conditional probability density functions Π_1 and Π_2 can be recovered from inverting the respective characteristic functions as in Heston (1993), Bates (1996) and Pan (2002):

$$\Pi_j(t,T;S_t,r,V) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi\ln(K)} f_j^{svj}(t,T,S_t,r,V_t;\phi)}{i\phi}\right] d\phi$$

for $j = 1,2$, with the characteristic functions f_j^{svj}. The characteristic functions are given in appendix C.2. The price of a European put can be obtained from the put-call parity. The total return variance can be decomposed into two components:

$$\frac{1}{dt} Var_t\left(\frac{dS_t}{S_t}\right) = V_t + V_{J,t}$$

where $V_{J,t} = \frac{1}{dt} Var_t(J_t dq_t) = \lambda(\mu_J^2 + (e^{\sigma_J^2} - 1)(1+\mu_J)^2)$ is the instantaneous variance of the jump component.

5.2.3.3 Local Volatility Models

5.2.3.3.1 Justification for the Local Volatility Approach

The local volatility model, also known as deterministic volatility function, is the most natural extension to the Black-Scholes model in which the volatility term can be formulated as a function of asset level and time. The local volatility model assumes that asset level and time are the dominant contribution to smile effects. In theory, the constant Black-Scholes implied volatility for an option maturing at time t can be formulated as weighted average of local volatility $\sigma(S,t)$ before time t. Consequently, one can extract the market's consensus for future local volatility from a spectrum of available market options as quoted by the implied Black-Scholes volatility.

Dupire (1994) was first to show how to uniquely derive the local volatility function given market option prices with all strikes and maturities are available. Dupire's continuous-time result has been supplemented by a number of discrete-time numerical methods. For example, Longstaff (1990), Rubinstein (1994), Derman and Kani (1994), Derman, Kani and Chriss (1996) and Chriss (1996a) fit the volatility smiles through careful manipulation of the local branching probabilities in implied binomial or trinomial tree framework. These "implied" methods assume the existence of a complete spanning set of European call option prices, which, in practice, requires the use of extrapolation and interpolation of the available market option prices. They offer a relatively straightforward approach fitting the volatility smile, but suffers from a number of setbacks: 1) tree methodology needs extensive "engineering" treatment to infer probabilities because negative transition probabilities are not allowed; 2) trees such as Derman-Kani use options at each time interval. Bad probabilities occur frequently and lead to extremely erratic convergence behaviour. The reader is referred to section 2.4 for a detailed survey of the "implied" methodology.

Whereas the implied-tree is primarily based on a discretisation of the asset price process, the finite-difference approach focuses on developing a discrete-time model by discretising the fundamental no-arbitrage partial differential equation. The application of finite-difference scheme to the volatility smile problem has been studied by many authors, e.g. Lagnado and Osher (1997), Andersen and Brotherton-Ratcliffe (1998), Coleman et al. (1999), Chryssanthakopoulos (2001) and Little and Pant (2001). Whilst somewhat more complicated to evaluate and calibrate, however, finite-difference scheme is shown to exhibit much better stability and convergence properties than implied-trees because finite-difference method does not involve explicit adjustments of branching probabilities and allows for independent prescription of the stock- and time-partitioning. The explicit finite-difference scheme can also be shown to be similar to a trinomial tree, however, it is commonly acknowledged that implicit or Crank-Nicolson schemes is unconditionally stable whilst explicit schemes are not[66].

5.2.3.3.2 One-Factor Model

The inspiring research by Breeden et al. (1978) stated that the risk-neutral probability distributions could be recovered from European-style options by pricing butterfly spreads, and therefore expressed as the second derivative of the call option price with respect to the exercise price. Based upon the Breeden et al's results, Dupire (1994) showed how one could relate the partial derivatives of standard European options to local volatility function. The idea behind Dupire's method is to extract implied distribution and construct the whole

[66] Zvan et al. (1998) deal with the necessary conditions to avoid spurious oscillations.

diffusion process that is consistent with the market observed prices. In the risk-neutral world, Dupire's local volatility model is assumed to evolve according to the following one-factor continuous-time diffusion model:

$$\frac{dS}{S} = ((r(t) - q(t))dt + \sigma(S,t)dW$$

where $r(t)$ is the risk-neutral drift, $q(t)$ is the dividend yield, and dW is a Wiener process. Given a continuum of traded European calls with different strikes and maturities, Breeden et al. found that:

$$p(S,t;K,T) = e^{r_f T} \frac{\partial^2 C_{KT}}{\partial K^2}$$

where $p(S,t;K,T)$ is the conditional probability density function and C_{KT} is the current market value of an option with strike price K and maturity T at asset level S and time t; r_f is the constant risk-free rate. In the continuous-time limit when risk-free rate and dividend are constant, and $\frac{\partial^2 C_{KT}}{\partial^2 K^2} \neq 0$, $\sigma(K,t)$ is completely determined from the volatility smile. At time t and strike K, Dupire relates option prices to $\sigma(K,t)$ as follows[67]:

$$\sigma(K,T) = 2\left(\frac{\frac{\partial C_{KT}}{\partial T} + qK\frac{\partial C_{KT}}{K} + (r_f - q)C_{KT}}{K^2 \frac{\partial^2 C_{KT}}{\partial K^2}} \right)$$

The major advantage of the above one-factor continuous model, as compared to the jump-diffusion or stochastic model, is that no non-traded source of risk such as the jump or stochastic volatility is introduced. In addition, the first derivative of the European call or put option price with respect to the strike price is proportional to the relevant risk-neutral tail probability whilst its second derivative is proportional to the conditional probability density. Given there are enough strike prices, the patterns of implied volatility across different strike prices can uniquely identify the shape of the risk-neutral density and distribution. Consequently, the completeness of this one-factor diffusion model allows for arbitrage pricing and hedging.

5.2.3.3.3 Coleman et al. Approach

Dupire's continuous-time results have been supplemented by a number of finite-difference methods. For example, Zou and Derman (1997) applied the "pseudo-analytical" method to extract local volatility surface by approximating the derivatives of options prices with respect to the strike levels and maturity using Edgeworth expansion for the pricing of lookback options. Andersen et al. (1998) illustrated how to construct the stable finite-difference lattice to extract local volatility consistent with the equity option volatility smile and term-structure of interest rate using implicit and Crank-Nicholson lattices[68]; Andersen et al. also demonstrated its application by pricing down-and-out knock-out options. In addition, Coleman et al. (1999) developed a Crank-Nicholson scheme to "optimise" local volatility surface by introducing "smoothness" in the Black-Scholes PDE discretisation process.

In this study we adopt the spline functional approach of Coleman et al.(1999) to directly construct the local volatility surface and price variance swap via finite-difference method. In additional to Coleman et al., Little et al. (2001) also approximated a variance swap by using the Crank-Nicholson method in an extended Black-Scholes framework that was based on a cleverly decomposition of a two-dimensional problem into the solving of a set of one-dimensional Black-Scholes partial differential equations. At a glance, Little et al.'s method seems to be attractive because this finite-difference model directly prices a variance swap based on a discretely sampled variance and allows for the incorporation of local volatility. Besides computationally intensive, the major deficiencies of Little et al.'s setup are: 1) one has to make an assumption of the underlying asset process; 2) local volatility is assumed to be exogenous and therefore requiring the use of a separate method to extract and incorporate volatility smile. This is in contrast to the assumption-free Demeterfi et al. model that only requires the implied volatilities for different maturities in order to value a variance swap, and therefore, we will not consider the Little et al.'s methodology here.

Coleman et al.'s method solves for the local volatility function by directly discretising the no-arbitrage partial differential equation using the finite-difference method. Given S_{init}, r, q and $\sigma(S,t)$ and under the no-arbitrage condition, the option value must satisfy the Black-Scholes

[67] See also pp. 8-10 of Andersen and Brotherton-Ratcliffe (1998) for a detailed derivation of this formula.

partial differential equation for every price of the asset level and for every time from starting time to the expiry given by Merton (1973):

$$
\begin{aligned}
&\frac{\partial C}{\partial t}+(r-q)S\frac{\partial C}{\partial S}+\frac{1}{2}\sigma(S,t)^2S^2\frac{\partial^2 C}{\partial^2 S}=rC \\
&\lim_{s\to+\infty}\frac{\partial C(S,t)}{\partial S}=e^{-q(T-t)}, \qquad t\in[0,T] \\
&C(0,t)=0, \qquad t\in[0,T] \\
&C(S,T)=\max(S_T-K,0)
\end{aligned}
$$

where $C(S,t)$ denotes the option value of an underlying asset with an arbitrary strike at K and expiry at T, $t\in[0,T]$. The boundary conditions for the upper (u) and lower (l) spatial boundaries are:

$$
\frac{\partial^2 C}{\partial S^2}\Big|_{S=u}=\frac{\partial^2 C}{\partial S^2}\Big|_{S=l}=0
$$

Before applying finite-difference method to calculate option prices, $\sigma(S,t)$ needs to be approximated. Due to lack of market option price data, i.e. non-continuum of strikes, this can be regarded as a well-known but ill-posed function approximation problem from a finite dataset with a nonlinear observation functional. Therefore, there are an infinite number of solutions to the problem given a set of the market option price data. To tackle this problem, the Coleman et al. model introduces "smoothness" to facilitate accurate approximation of the local volatility function from a finite set of data. The Coleman et al. model assumes that the underlying asset follows a one-factor diffusion model and incorporates bicubic spline in the choice of parameterisation. After choosing the number of spline knots and their placement, $\sigma(S,t)$ can be represented by an interpolating spline with a fixed end condition. The spline knots uniquely construct $\sigma(S,t)$ and the knots are determined by solving a constrained nonlinear optimisation problem to match the market option prices, therefore effectively turning it into an inverse spline minimisation problem with respect to local volatility at the spline knots. The local volatility calibration procedures are summarised as follows:

i) Assume there are m observed option closing prices $C_j \quad j=1,\ldots,m$

[68] Andersen et al. (1998) found that explicit finite difference method was not well-behaved in the fitting of the volatility smile.

ii) Choose p spine knots $\{s_i, t_i\}_{i=1}^{p}$ with corresponding local volatility $\sigma_i^* = \sigma(s_i, t_i)$

iii) Define an interpolating spline $c(s_i, t_i) = \sigma_i^*$ $i = 1,...p$

iv) Let $\hat{C}_j(c(S,t;\sigma^*)) = \hat{C}(c(S,t;\sigma^*), K_j, T_j)$, $j = 1,...,m$

v) Given p spline knots, $(s_1,t_1),...(s_p,t_p)$, solve for the p-vector σ^* by minimising the objective function:

$$\underset{\sigma^*}{MIN}\, f(\sigma^*) = \frac{1}{2}\sum_{j=1}^{m}(\hat{C}_j(c(S,t;\sigma^*)) - C_j)^2 \text{ subject to } l < \sigma^* < u$$

In contrast to Dupire (1994), Coleman et al. do not emphasise on the matching of the market option price data. The objective is to reconstruct as smooth as possible the local volatility function $\sigma^*(S,t)$. This way, the local volatility surface possesses certain properties a priori, namely, smoothness and better chance of convergence. The approximation of σ^* in the above minimisation process requires the evaluation of European options $\hat{C}$. This inverse minimisation problem can only be computed numerically via a tree method or a finite-difference approach.

Several issues merit our attention in this inverse minimsation problem. First, to construct a spline efficiently via finite-difference method, the spline knots should be placed in a rectangular mesh covering the asset-time space D[69]. Second, the number of spline knots is to be no greater than the number of option prices ($p \leq m$) in order not to allow too many degrees of freedom in approximating $\sigma(S,t)$. Under mild assumptions, the Coleman et al. approach corresponds to a monotonically decreasing sequence of objective function values and guarantees convergence, i.e. $\sigma_k^* \rightarrow \sigma^*, k = 1,...,\infty$. l, u are the lower and upper bounds that can be imposed on the local volatility at the knots. In addition, both traded European call/put options may be used to calibrate the spline approximation to the local volatility function $\sigma^*(S,t)$. A thorough examination of finite-difference method is beyond the scope of this dissertation. We recommend Andersen et al. (1998) for a proper treatment of the implicit infinite difference approach to extract local volatility surface consistent with smile effects. Wilmott et al. (1993), Tsiveriotis and Chriss (1998) and Little and Pant (2001) have also a

[69] In general, we have no a priori knowledge of D within which the volatility values are significant for pricing available options.

good discussion in pricing options under the one-dimensional Black-Scholes PDE environment.

5.2.3.4 Ad hoc Black-Scholes Model

In light of the Black-Scholes model's moneyness and maturity biases, researchers and especially practitioners have always tried to find ways to "live with the smile". One of the proposed ways, whilst arguably ad hoc, is to estimate and use an "implied volatility matrix". This formulation is also termed as "practitioner Black-Scholes". We adopt Dumas et al. (1998) methodology and construct an ad hoc Black-Scholes model in which each option has its own implied volatility depending on the strike price K and time-to-maturity T. Dumas et al. observed that the Black-Scholes implied volatility for S&P 500 options tended to have a parabolic shape and argued that quadratic forms for volatility function were suffice to parameterise implied volatility. Specifically we use the functional form:

$$\sigma_{IV}(K,\tau) = a_0 + a_1 K + a_2 K^2 + a_3 \tau + a_4 \tau^2 + a_5 K\tau$$

where σ_{IV} is implied volatility using the Black-Scholes formula for an option of strike K and time-to-maturity τ.

This formulation is not only internally inconsistent with the Black-Scholes assumptions but also generates option prices which are not subject to demand and supply forces and violates local no-arbitrage conditions, and therefore potentially erroneous. But Dumas et al. did show that the implied binomial tree or the deterministic volatility models of Derman and Kani (1994), Dupire (1994) and Rubinstein (1994) underperformed the ad hoc Black-Scholes model in terms of out-of-sample options valuation errors in the S&P 500 index option market. Furthermore, the ad hoc Black-Scholes model is very different from the local volatility approach. The local volatility approach models the smile effects with the spots and strikes whereas the ad hoc Black-Scholes prices depend on moneyness alone. This regression-based ad hoc strategy, although naive, is definitely more challenging and flexible than using the local volatility model for pricing variance swaps. Comparing the ad hoc Black-Scholes strategy to the local volatility and stochastic volatility with/without jump strategies should therefore yield insights on their relative efficacies in terms of forecasting volatility and valuing options.

5.2.4 Time-Series Models for Forecasting Variance

The models discussed so far for pricing variance swaps includes only the implied models – stochastic volatility with/without jump models and local volatility model under the Demeterfi et al. assumption-free framework. Although interesting, these approaches do largely depend on a very limited number of option strikes to infer the prices of an entire continuum of options of every strike and maturity on the underlying asset.

5.2.4.1 Justification for the Conditional Volatility Approach

On the other hand, time-series models can be used to directly approximate the delivery price for a variance swap. Indeed discrete-time GARCH-type processes can be linked to bivariate diffusion models, and vice versa. For example, Nelson (1991) showed that EGARCH processes converged weekly to a specific stochastic volatility bivariate diffusion model. More recently, Duan (1997) generalised these results and brought the largely separate GARCH and bivariate diffusion literatures together. Duan showed that most of the existing bivariate diffusion models that had been used to model asset returns and volatility could be represented as limits of a family of GARCH models. Despite the fact that the time-series approach ignores the direct modelling of volatility smile effect and uses only historical information, there may still be some advantages in implementing heteroskedastic models using the vast literatures on numerical procedures for GARCH-type models.

5.2.4.2 GARCH-Variance Swap

An alternative way for pricing variance swaps is to use stochastic volatility models that are in good agreement with historical time series, such as the GARCH-variance swap (GARCH-VS) model invented by Javaheri et al. (2002). The GARCH-VS model uses the Ornstein-Uhlenbeck process to model for its variance in continuous time:

$$dv = k(\theta - v)dt + \gamma v dW$$

where k is the speed of mean reversion, θ is the long-term mean reversion level, γ is the volatility of the volatility and dW is Brownian motion.

Javaheri et al. originally used a partial differential equation approach to determine the first two moments of realised variance and approximate the expected realised volatility. Expected realised variance is evaluated by determining the first moment of realised variance in a discrete and continuous context. It uses the discrete time process GARCH(1,1) to calculate expected realised variance $E\left[I=\frac{1}{T}\int_0^T v(t)dt\right]$. The structural parameters estimated from the GARCH(1,1) environment were derived by Engle and Mezrich (1995) and Javaheri et al. (2002) as follows:

$$
\begin{aligned}
&\varepsilon_t = h_t u_t, \qquad u_t \sim i.i.d. \\
&r_t = \varepsilon_t \qquad\quad \varepsilon_t \sim N(0, h_t) \\
&h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 \\
&\theta = \frac{V}{dt} \\
&k = \frac{1-\alpha_1-\beta_1}{dt}
\end{aligned}
$$

where α_1, β_1 are the autoregressive parameters of GARCH(1,1), $V = \frac{\alpha_0}{(1-\alpha_1-\beta_1)}$ is the unconditional variance, h_t is the conditional volatility, T is time to maturity and v is the instantaneous variance to the last observation in the GARCH(1,1) model. The expected delivery price of a variance swap can be written as:

$$E\left[\frac{1}{T}\int_0^T v(t)dt\right] = \left[\theta * (T + \frac{(e^{-k*T}-1)}{k}) + \frac{1}{k}(1-e^{-k*T}) * v\right] / T$$

Since the GARCH-VS model has a closed-form solution for variance swap valuation, one can easily use the time-series information in the history of asset prices to estimate model parameters as in Théoret (2002).

5.2.4.3 EGARCH Simulations

In Chapter 4 we have shown evidence that EGARCH is the best model and outperforms GARCH in both in- and out-of-sample tests. In addition to the GARCH-VS model, we include the EGARCH model (Nelson, 1991) and analyse its performance by directly calculating the delivery prices of variance swaps from its simulated sample paths. The EGARCH(1,1) is given by:

$$\varepsilon_t = h_t v_t$$
$$r_t = \varepsilon_t$$
$$\log h_t^2 = \alpha_0 + \alpha_1 z_{t-1} + \gamma_1 (|z_{t-1}| - (2/\pi)^{0.5}) + \beta_1 \log h_{t-1}^2$$

where $z_t = \frac{\varepsilon_t}{h_t}$, v_t ~i.i.d. student-t with unit variance and v is the degrees of freedom. Given a set of structural parameter $(\alpha_0, \alpha_1, \gamma_1, \beta_1)$, we simulate the N-step return data with 100,000 replications in order to calculate the delivery prices for variance swaps, where N is number of trading days.

5.2.5 Summary of the Methodology

Sections 5.2.1-4 have explained the performance criteria, methods, models and underlying hypotheses used in this study for variance swap valuation. It should be emphasised that our study uses both time-series and options-based variance swap models to investigate the S&P 500 index market. Our research is designed to include the results from the best models shown in Chapters 3 and 4 including: 1) ad hoc Black-Scholes model; 2) stochastic volatility model; 3) jump-diffusion model; 4) local volatility model; 5) EGARCH; 6) GARCH variance swap model. The next section presents the S&P 500 data used in this study.

5.3 Data Description

5.3.1 Specifications and Filtering

The dataset comprises of the daily closing prices of the S&P 500 index for the period from June 1999 through December 2001. The option prices used in this study are S&P 500 call options[70] traded on CBOE. We obtain the closing bid/ask option prices traded on CBOE for the third Friday's from the three months before to after September 11, 2001. These options are European and settled for a cash amount equal to 100 times the difference between index level and strike price. Similar option data were formerly used by Rubinstein (1985), Bakshi et al. (1997) and Nandi (1998).

[70] The S&P 500 index is a value-weighed index. S&P 500 index options are traded on CBOE whilst S&P 500 index futures options used in Chapters 3 and 4 are traded on CME.

S&P 500 index options expire on Saturday immediately following the third Friday of the expiration month. There are three near-term expiration months followed by three additional months from the March-quarterly cycle, i.e. March, June, September and December. Our option database is supplied by an option specialist[71]. Following the lead of Bakshi et al. (1997), several exclusion filters are applied to remove uninformative options records from our database:

i) Options with less than six days to expiration may induce liquidity-related biases and they are excluded from the sample;

ii) Price quotes lower than \$0.375 are eliminated to mitigate the impact of price discretenesss on option valuation;

iii) Quotes not satisfying the arbitrage restriction: $C(t,T) \geq \max(0, S - D - K * e^{-r(T-t)})$ are taken out of the sample;

iv) Options with no open interest are not included because of liquidity problem.

5.3.1.1 Dividends

S&P 500 index options are chosen because these are the second most active index options market in the U.S. and, in terms of open interest in options, they are the largest. In contrast to S&P100 index options, there are no wild card features that can complicate the valuation process. It is also easier to hedge S&P 500 index options because there is a very active market for S&P 500 futures. In fact, it is one of the best markets for testing a European option valuation model[72]. As many of the stocks in the S&P 500 index pay dividends, there is a need to obtain the ex-dividend spot index level. We collect daily cash dividends for the S&P 500 index from Bloomberg from June 2001 to December 2002[73]. We arrive at the present value of the dividends and subtract it from the current index level in order to obtain the dividend-exclusive S&P 500 index that is used as input into the option models. The ex-dividend spot index level is:

[71] Option data are provided by *ivolatility.com* in New York.

[72] Refer to Rubinstein (1994) for more details.

[73] The calculation of ex-dividend spot level requires the use of up to 18 months of future dividends to make adjustments on its index level.

$$S_{ex-dividend}(t) = S_{close}(t) - \sum_{i=1}^{T-t} e^{-i*r_i} D_{t+i}$$

where D_{t+i} is the actual dividend in the future, S_{close} is the closing index price, r_i is the continuously constant risk-free compounded rate corresponding to i periods to expiration from day t calculated from interpolated U.S. Treasury yields provided by the U.S. Treasury Department. Implied volatility is computed by applying the Newton-Raphson method to the Black-Scholes call option formula:

$$C(t,T) = S_{ex-devidend}(t)N(d_1) - Xe^{-r(T-t)}N(d_2)$$
$$d_1 = \frac{\ln[S_{ex-dividend}(t)/X] + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$
$$d_2 = d_1 - \sigma\sqrt{T-t}$$

5.3.1.2 Calibration Using Call Options

Few issues merit our attention when using the call options database for option models' calibration. First, we have demonstrated in section 3.4.3 that the implied volatility of call options in a given in-the-money (out-of-the-money) category are quite similar to the implied volatility of put options in the opposing out-of-the-money (in-the-money) category regardless of sample period or term-to-expiration. For a fixed term-to-expiration, call and put options imply the same U-shaped volatility pattern across strike prices. Such similarities in pricing structure existing between call and put options mainly due to the working of the put-call parity, and it is this link that makes call and put options of the same strike price and the same expiration exhibit similar levels of mispricing. Second, Bakshi et al. (1997) used S&P 500 put options to estimate the parameters of some stochastic models and found that results were qualitatively similar. Because of these two reasons, only call options are used to calibrate the ad hoc Black-Scholes, stochastic volatility with/without jump and local volatility models. We argue that basing our calibrations to follow solely on results obtained from S&P 500 call options should not present a biased picture of the candidate models. After applying the exclusionary criteria to the data, the average number of options available on each day is 100.

5.3.2 Financial and Political Events

The decision to value the June-November 2001 variance swap contracts is neither incidental nor arbitrary. Many significant global macroeconomic and political events occurred during the 2001-2002 period. For example, the September 11 terrorist attacks in New York, U.S. led war in Afghanistan, weak global economic growth, corporate scandals such as Tyco, U.S. investment banks' scandals, the collusion between Enron and its auditor Arthur & Andersen, bankruptcies, e.g. United Airlines, US Airways and WorldCom, worldwide bursting of technology, media and telecoms bubble, E.U. enlargement, circulation of Euros, surging oil price, possible war against Iraq and the Israel-Palestine conflicts in the Middle East have all conspired to spook markets.

On the equity side, the global market was extremely volatile and depressing during the 2000-2002 period. In the U.S. there were more than 186 bankruptcies recorded with $368 billion in assets collapsed in 2002. Tokyo finished 2002 with a 19 percent decline in the Nikki 225 average. The market sank to a 19-year low in November 2002 and suffered losses over nine consecutive trading days in December 2002, its longest losing streak for 11 years. In 2002 the European bourses suffered their worst year since 1974 with a fall of 22.1 percent in the MSCI Europe index. Germany had also lost almost 35 percent as hopes for a recovery were frustrated in 2002. On Wall Street the Dow Jones index had plummeted 17 percent during 2002, its worst performance for 28 years. The technology weighted NASDAQ composite had done even worse with a fall of 32 percent. London's FTSE 100 plunged 25 percent in 2002. In December 2002 the FTSE 100 index extended a losing streak into eight consecutive sessions, its longest sequence of falls since its inception in 1984. Cumulative losses for the FTSE World index since the start of 2000, after the bursting of the technology, media and telecoms bubble, totalled 43 percent. The 2000-2002 period was the worst three-year performance since 1929-1931 when world markets fell 58.8 percent. By comparison, world markets lost 39 percent in 1973 and 1974 at the height of the world oil shock. Investors had indeed endured a turbulent ride over 2000-2002.

5.3.3 Descriptive Statistics for Call Options and S&P 500 Index in 1999-2002

Basic statistics for S&P 500 index option data are shown in table 40. Table 40 reveals that average implied volatility is notably higher in post-September 11 period. It is also evident from figure 42 that returns cluster in time. Appendices B.1-B.6 exhibit the specifications and various option input parameters for our daily option contracts. Causal inspection of appendices B1-B6 reveals that: 1) lower strikes have a higher implied volatility; 2) volatility smile is more pronounced for near-term options. These results agree with the "stylised" fact presented in Chapter 3.

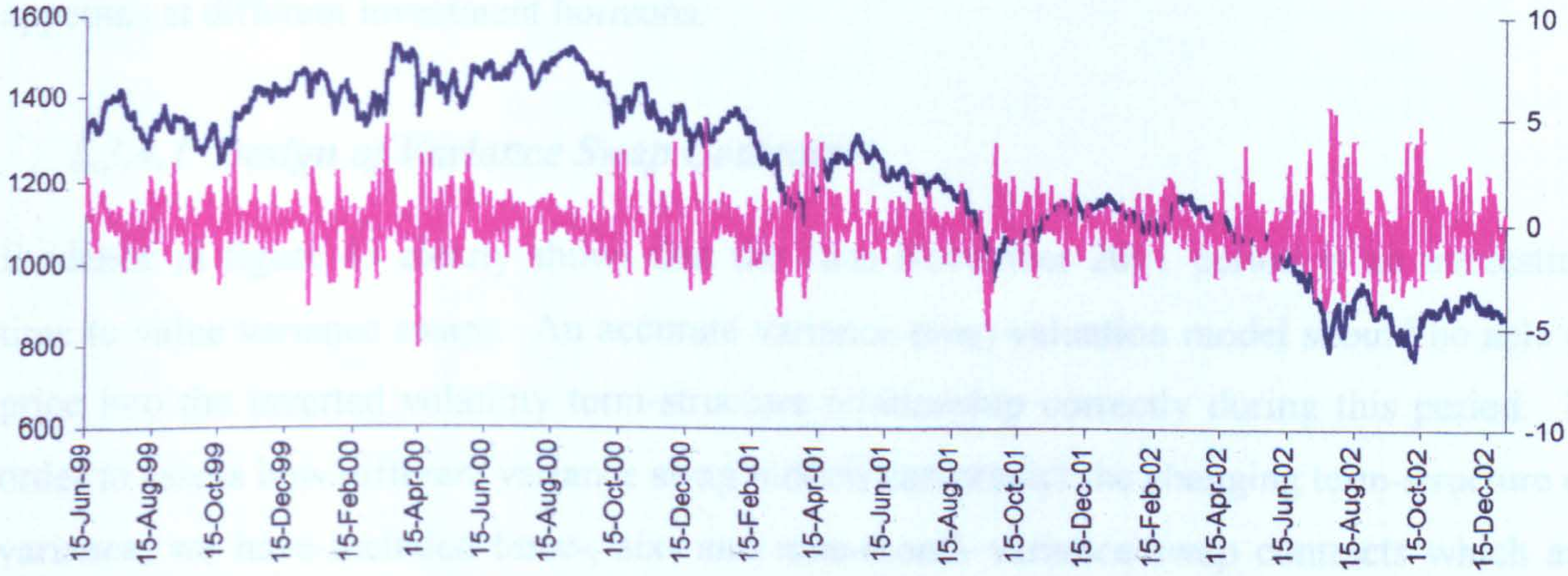
Table 40: Basic Statistics for S&P 500 Index Options

	6/15/2001	7/20/2001	8/17/2001	9/21/2001	10/19/2001	11/16/2001
#. of Options	131	89	78	117	83	106
Mean Call Price	72.457 (87.751)	58.135 (62.02)	68.471 (81.118)	28.852 (42.526)	53.868 (66.467)	89.913 (98.478)
Strike Range	800-1900	1025-1900	800-1900	800-1900	800-1700	700-1700
#. of strikes	40	33	35	41	32	50
Mean Maturity	0.61 (0.4389)	0.5531 (0.4466)	0.6085 (0.4455)	0.4983 (0.4056)	0.5633 (0.386)	0.4444 (0.3597)
Mean Imp. Vol.	0.2094 (0.0481)	0.1853 (0.0213)	0.1981 (0.051)	0.2856 (0.06707)	0.2287 (0.04624)	0.2363 (0.08588)

Table 41: Descriptive Statistics for *r*

	Full Period 16/06/1999-31/12/2002	Pre-9/11 15/06/1999-10/09/2001	Post 9/11 17/09/2001-31/12/2002
DF stat.	-29.85260 [.000]	-23.5922p [.000]	-17.82220 [.000]
Maximum	.055732	.048884	.055732
Minimum	-.060052	-.060052	-.050468
Mean	-0.0004325	-0.000299	-0.00066424
Std. Dev.	.014137	.013112	.015778
Skewness	.16579	0.073602	.26741
Kurtosis-3	1.22370	1.40510	0.83187
Q(10)	5.42500 [.861]	10.18850 [.424]	5.93000 [.821]
Jarque-Bera stat.	59.74210 [.000]	47.06940 [.000]	13.28510 [.0013]
#. Obs.	892	505	326

Figure 42: S&P 500 index and Returns: 1999-2002



5.3.4 Contract Specifications

Since variance swaps are not traded on organised markets, contract terms such as maturity, annualisation factor etc are negotiable. Investment banks quote daily delivery prices for their counterparties for various maturities running from three months to two year. Figure 43 plots the future realised[74] three-, six- and nine-month variance[75] from September 1999 to March 2002. During these periods, average returns are close to zero. Table 41 shows that Ljung-Box statistics up to the 10th order are not significant, which suggest that returns are not serial correlated. Both skewness and excess kurtosis are slightly positive, but Jarque-Bera test statistics reject the null hypothesis that returns are normal in all intervals.

Descriptive statistics in table 41 indicate that returns in the pre- and post-9/11 periods are statistically similar. But a close inspection of realised forward variance in figure 43 reveals that S&P 500 index's variance process displays a mean-reverting property. In addition, the realised forward variance spread between the 3- and 9-month contracts has been widening since the 9/11 attacks. It is also evident in figure 43 that realised forward variances have inverted at different maturities after September 11, 2001, i.e. the longer-termed forward

[74] Readers should not be confused it with the smoothing average approach. Our results represent what the variances that would have been obtained if we had entered the variance swap trades on a particular day.

[75] Variances are calculated by summing the arithmetic returns and the mean of returns is assumed to be zero. Annualisation factor is 252 and observation frequency is daily.

variance has become more volatile than the shorter-termed forward variance. September 11, 2001 has indeed served as a reflection point where investors have clearly changed their risk appetites at different investment horizons.

5.3.4.1 Design of Variance Swap Contracts

Evidence in figure 43 clearly shows that the June-November 2001 period is an interesting time to value variance swaps. An accurate variance swap valuation model should be able to price into the inverted volatility term-structure relationship correctly during this period. In order to assess how different variance swap models can predict the changing term-structure of variance, we have included three-, six- and nine-month variance swap contracts which are compatible with the International Money Market (IMM) rulebook[76]. The specifications for the three-, six- and nine-month variance swap contracts are shown in table 42. It is noted that our variance swap contracts always begin on the third Fridays and end on the Thursdays prior to the third Fridays of the maturity month. For example, the start and end dates for the three-month June 2001 variance swap contract correspond to the inception of the June 2002 S&P 500 futures contract and the last trading day of the September 2001 S&P 500 futures contract on CBOE, respectively.

Figure 43: Realised Forward Variances

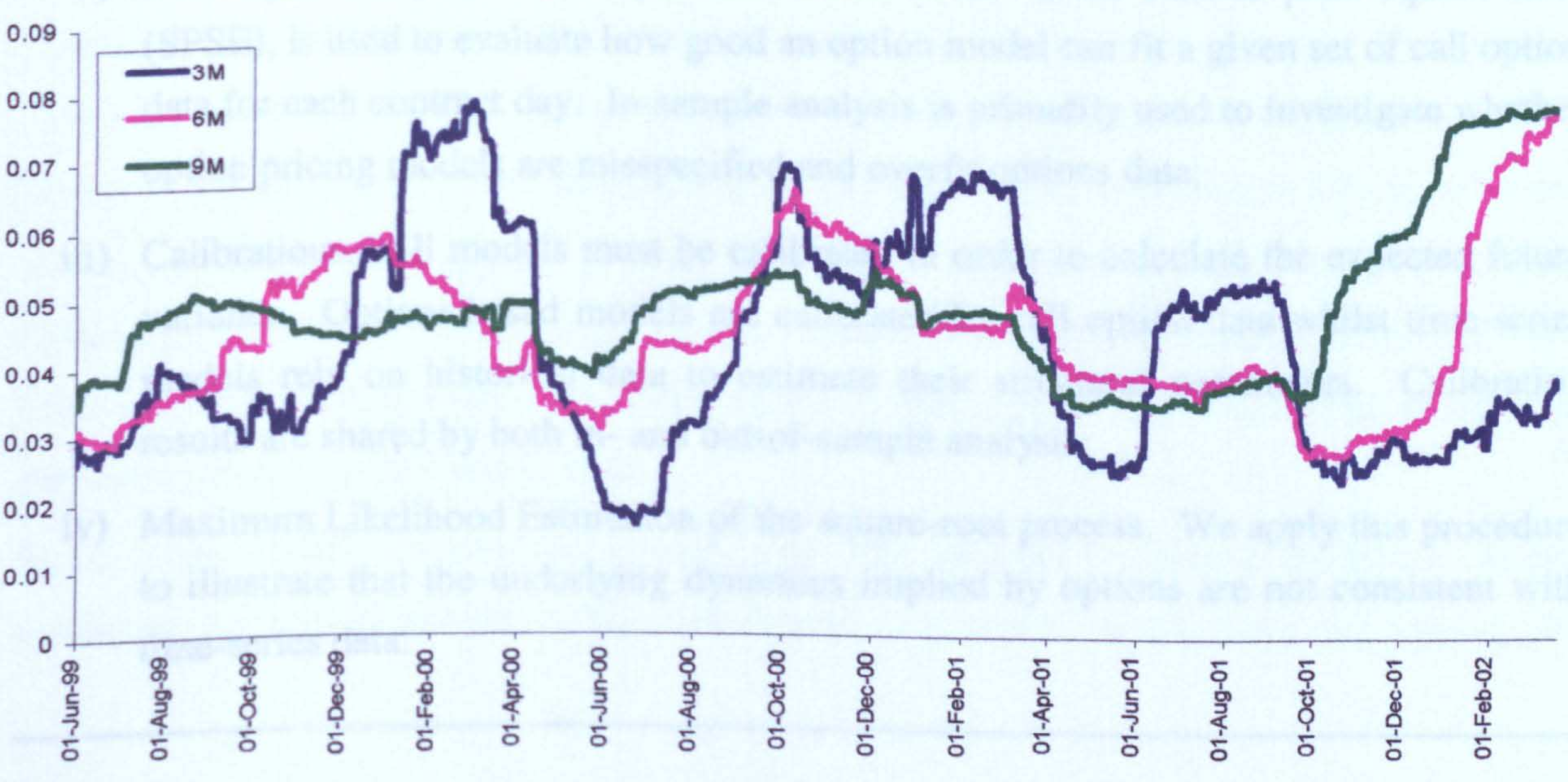


Table 42: Contract Specifications for Variance Swaps

Pre-9/11	June 2001		July 2001		August 2001	
Maturity	Start	End	Start	End	Start	End
3-Month	15/06/01	20/09/01	20/7/01	18/10/01	17/08/01	15/11/01
6-Month	15/06/01	20/12/01	20/7/01	17/01/02	17/08/01	14/02/02
9-Month	15/06/01	14/03/02	20/7/01	18/04/02	17/08/01	16/05/02
Post-9/11	September 2001		October 2001		November 2001	
Maturity	Start	End	Start	End	Start	End
3-Month	21/09/01	20/12/01	19/10/01	17/01/02	16/11/01	14/02/02
6-Month	21/09/01	14/03/02	19/10/01	18/04/02	16/11/01	16/05/02
9-Month	21/09/01	20/06/02	19/10/01	18/07/02	16/11/01	15/08/02

5.4 Results & Analysis

Six variance swap models are investigated to determine the quality of variance forecastability the models deliver. In this section we carry out the following analytic procedures to compare the variance forecasting performance of various time-series and options-based variance swap models:

i) Out-of-Sample Analysis. The out-of-sample error criterion is judged by MSE tests. Each model's performance is based on the aggregate ranking for each of the three contract months, i.e. 3M, 6M and 9M;

ii) In-Sample Analysis. In-sample test, which relies on the sum of price square error (SPSE), is used to evaluate how good an option model can fit a given set of call option data for each contract day. In-sample analysis is primarily used to investigate whether option pricing models are misspecified and overfit options data;

iii) Calibrations. All models must be calibrated in order to calculate the expected future variance. Options-based models are calibrated by call option data whilst time-series models rely on historical data to estimate their structural parameters. Calibration results are shared by both in- and out-of-sample analysis;

iv) Maximum Likelihood Estimation of the square-root process. We apply this procedure to illustrate that the underlying dynamics implied by options are not consistent with time-series data.

[76] We thank Philipp Jokisch for contributing to this idea.

We explain our calibration procedures in section 5.4.1. In- and out-of-sample results are reported and analysed in sections 5.4.2.2, and maximum likelihood estimation of the square-root process is conducted in section 5.4.2.3.

5.4.1 Calibration Procedures

Sections 5.4.1.1-4 discuss the econometric and numerical methods that are used for calibrations of the ad hoc Black-Scholes, stochastic volatility (Heston, 1993), stochastic volatility with jump (Bakshi et al., 1997), local volatility (Coleman et al., 1999), EGARCH (Nelson, 1991) and GARCH-VS (Javaheri et al., 2002) models.

5.4.1.1 Calibrations for Stochastic Volatility with/without Jump

Estimation of stochastic processes on discrete-time data is difficult. Since volatility is not directly observable, many parameter estimation methods have relied either on time-series analysis of volatility proxies such as conditional volatility or on cumbersome econometric techniques such as Scott (1987) and Wiggins (1987) using moment matching procedures[77]. Instead of estimating parameters from the underlying asset return data, we imply out the parameters of the stochastic models from the cross-section of observed option prices using all actively traded call option prices as in Bakshi et al. (1997). A major disadvantage with the "implied" methodology is that it is lack of a formal statistical theory. This approach is to assume that the market uses a wide range of data and information to determine the structural parameters of the risk-neutral underlying asset and variance processes. The primary advantage of using market option prices for parameter estimations, however, is that it "gauges" the sentiments of the marketplace by using the information inferred from the cross-section of the market option prices, information that essentially is forward looking. Consequently, volatility smiles translate into unique values for the volatility of volatility and asset-volatility correlation in a stochastic volatility model, and into unique jump distribution parameters in a jump-diffusion model.

Since closed-form solutions are available for our selected stochastic models, a natural candidate for the estimation of the risk-neutral parameters, which enter the pricing and

[77] Both Scott and Wiggins found that the parameter estimates were sensitive to the moments which they fitted.

hedging formula, is a non-linear least squares (NLS) procedure involving minimisation of the sum of squared errors between the models and market prices. For the full stochastic volatility/jump-diffusion model, Φ is the set of stochastic volatility and jump parameters: $\Phi = \{\rho, \kappa_v, \theta_v, \sigma_v, \lambda, \mu_J, \sigma_J\}$. The first four are the parameters of the stochastic volatility model whilst the remaining three are jump parameters. The following steps summarise our calibration procedures:

i) Collect N call options on the S&P 500 index on the same day, for N greater than or equal to one plus the number of parameters to be estimated. For $n=1,...,N$ and $\Phi = \{\rho, \kappa_v, \theta_v, \sigma_v, \lambda, \mu_J, \sigma_J\}$, let $\hat{C}_n(t,T_n,K_n)$ be observed price and $C_n(t,T_n,K_n)$ its model price. For each n, define:

$$\varepsilon_n(V_t, \Phi) = \hat{C}_n(t,T_n,K_n) - C_n(t,T_n,K_n)$$

ii) Choose Φ and instantaneous volatility V_t to minimise the following objective function:

$$SSE(t) = \underset{\Phi}{MIN} \sum_{n=1}^{N} \varepsilon_n^2(V_t, \Phi)$$

An alternative objective function, the percentage error, which can be obtained by dividing dollar errors by the underlying index price, may be used to estimate implied parameters. This is a sensible metric because option prices are theoretically non-stationary but option-asset price ratios are stationary under most hypothesised processes. However, this metric would lead to a more favourable treatment of cheaper options, e.g. out-of-the-money options at the expense of in-the-money and long-term options. Based on the above considerations, we have chosen to adopt the SSE approach. The computer program MATLAB is employed to implement the option pricing formulas and minimisation routines. Among others, Bates (1995,1996) and Bakshi et al. (1997) have also applied this technique for similar purposes.

5.4.1.2 Calibrations for Local Volatility Model

We apply finite-difference method in MATLAB using a trust-region optimisation algorithm (Coleman et al., 1999) with a partial differential equation (PDE) approach[78] to directly solve for local volatility $\sigma(S,t)$. The Black-Scholes partial differential equation is discretised along the *S*-dimension with log-spacing. Crank-Nicholson finite-difference method is used for solving the Black-Scholes partial differential equation because it improves the stability and convergence of the finite-difference algorithm. Given any σ^*, the bicubic spline $c(s,t;\sigma^*)$ with the variational end condition[79] is computed and evaluated using the functions in the MATLAB SPLINE TOOLBOX. We use a uniformly spaced mesh with $N \times M$ grid points in the pre-determined rectangular region $[0, f * S_{init}] \times [0, \tau]$, where τ is the maximum maturity in the market option data and *f* is the range parameter for which the local volatility is significant for pricing. The discretisation scheme is given by:

$$S_i = (1/f) * S_{init} + i * \Delta S, \qquad i = 0,\ldots,M-1$$
$$t_j = j\frac{\tau}{M-1}, \qquad j = 0,\ldots,N-1$$
$$\Delta S = [f * S_{init} - (1/f) * S_{init}]/(M-1)$$

We use backwards difference to approximate $\frac{\partial C}{\partial t}$ and central difference to approximate $\frac{\partial^2 C}{\partial S^2}, \frac{\partial C}{\partial S}$. The resulting system is tridiagonal and can be solved by MATLAB at each time step using row reduction rather than matrix inversion, i.e. the LU decomposition method. Starting from $j = M-1$ for which time the terminal condition is known and progressing backwards through time, we successively solve for the $j-1$ option values until $j = 1$, which gives time-zero option values along the *S*-dimension. In addition, the boundary conditions $\frac{\partial^2 C}{\partial S^2}|_{S=U} = \frac{\partial^2 C}{\partial S^2}|_{S=L} = 0$ are incorporated into the finite difference scheme by setting their central difference approximation to zero. Further descriptions of finite-difference method go

[78] We sincerely thank Demetri Chryssanthakopoulos for making printed copies of his codes available.

[79] This is a MATLAB option to ensure that second derivatives are zero.

beyond the scope of this study but the reader is referred to Chapter 2 of Press et al. (1992) and Andersen et al. (1998) for a more thorough investigation of the implementation issues.

5.4.1.2.1 Trust-Region Reflective Quasi-Newton Method

Bicubic spline is the most important element in the implementation of finite-difference method. Its parameterisation is determined by solving a constrained non-linear optimisation problem to match the market option prices as closely as possible. Andersen et al. (1998) suggested that bicubic splines might suffer from the drawback that smoothness was only guaranteed in the S-direction. The reader is referred to Dierckx (1995) for discussions of more sophisticated spline interpolation schemes that are smooth in both T- and S-directions. The "csape" and "fnval" functions available within the MATLAB Spline Toolbox are used for the construction of natural bicubic splines to ensure that $\sigma(K,T)$ and its partial derivatives $\frac{\partial\sigma}{\partial T}, \frac{\partial\sigma}{\partial K}, \frac{\partial^2\sigma}{\partial^2 K}$ are well behaved.

The built-in MATLAB Optimisation Toolbox function for non-linear least squares minimisation is "lsqnonlin". Through the MATLAB Optimisation Toolbox function "optimset" we select the options: Large Scale Algorithm ON, Jacobian OFF, and Function Tolerance 1×10^{-3}. Pre-conditioned Conjugate Gradient is left to the default value of zero. These settings refer to, respectively, the "trust-region reflective quasi-Newton" method proposed by Coleman et al. (1999).

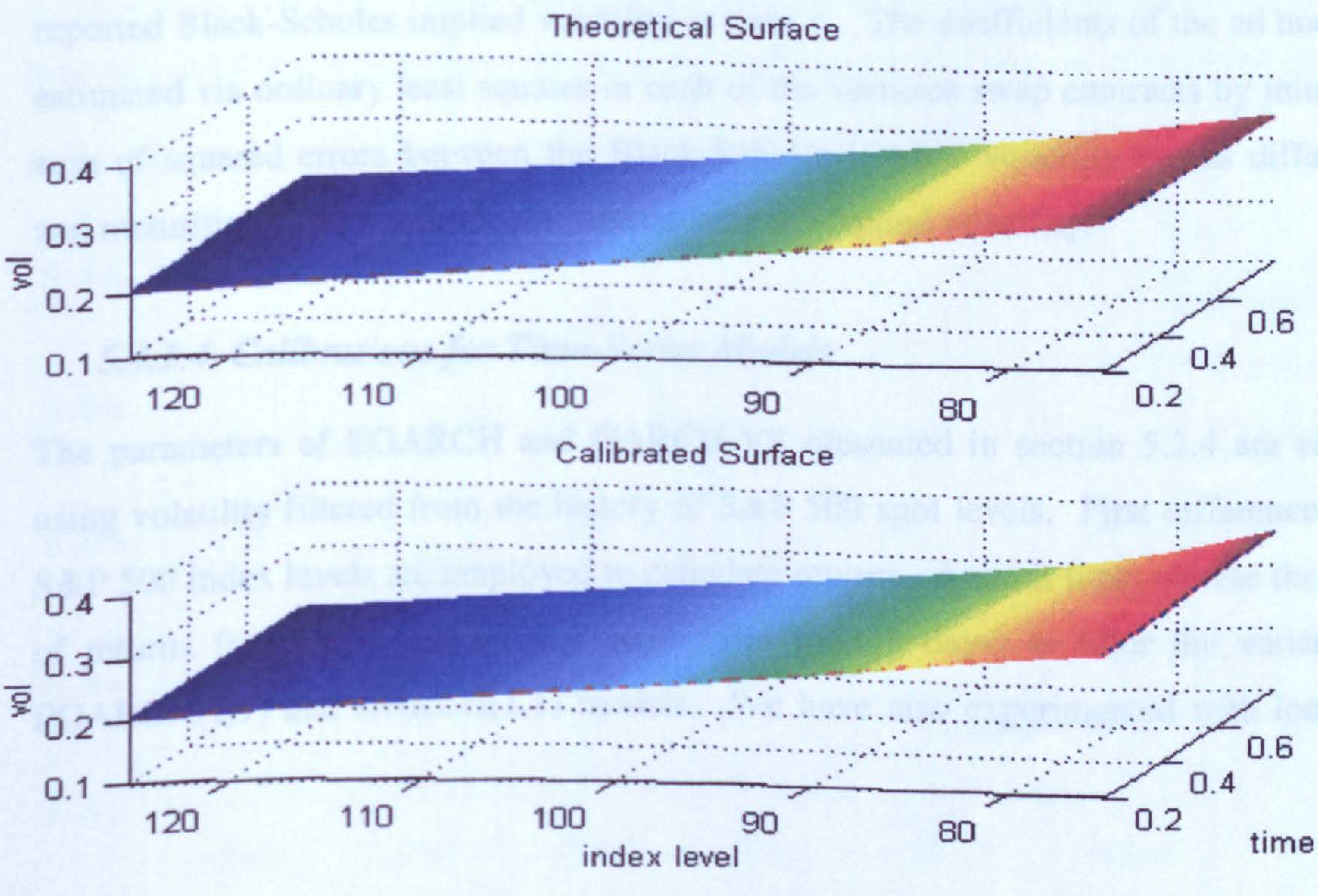
5.4.1.2.2 Calibrations for Absolute Diffusion Process

In order to demonstrate the effectiveness of the Coleman et al. method in reconstructing the local volatility surface, we consider the case where volatility is inversely proportional to index price. In this example, the underlying is assumed to follow an absolute diffusion process:

$$\frac{dS_t}{S_t} = \mu(S_t,t)dt + \frac{\alpha}{S_t}dW_t$$

Analytic formula for European options of the absolute diffusion process is available (see Cox and Ross, 1976). Since the local volatility surface is known a priori, we have chosen to set the market European option call prices equal to values provided by the finite-difference routines. We set $\alpha = 25$ and let the initial stock index be $S_{init} = 100$, risk-free interest rate $r = 4\%$ and dividend rate $q = 1\%$. We consider twenty-four European call options on the underlying following the above absolute diffusion process. Call options are equally spaced with strike prices K=[75:10:125] and maturities T=[0.2:0.2:0.8]. The discretisation parameters for asset steps and time steps are set as M=200 and N=50, respectively. The lower and upper bounds for the local volatility at the $K \times T$ knots are $l_i = -1$ and $u_i = 1$ for i=1,2,3...24. We let the number of spline knots p equal to the number of options $m = 24$ and calibrate the spline knots equidistantly on the grid $\left[\frac{S}{f} : S * f\right]$ where the range parameter $f = 2$. The initial volatility values at the spline knots are specified as 0.2. The optimisation method requires five iterations and the computed optimal objective function is 7.877×10^{-6}. With an average pricing error of 2.712×10^{-4} index point, the Crank-Nicholson method excellently reproduces actual call option pricing across the full range of strikes. Figure 44 demonstrates the accuracy of this local volatility reconstruction. The local volatility reconstruction is excellent. Indeed our methodology can reliably reconstruct the local volatility surface in the region $[75,125] \times [0.2,0.8]$.

Figure 44:Calibrated Local Volatility Surfaces for Absolute Diffusion Process



5.4.1.2.3.. Finite-Difference Settings

Having verified that our algorithm accurately reproduces the volatility smile, we now turn to the pricing of variance swaps. We choose the number of knots $p < m$, where $p = 72 < m$. The spline knots are placed uniformly between the endpoints of these intervals, with twelve knots along the *S*-dimension and six knots along the *T*-dimension. Cubic splines are fit to all *T* columns of the (*S* - *T*)-space and then a second cubic spline is fit along the *S* direction. The local volatility surface has been calibrated over a set $R:[(1/f)*S_{init}, f*S_{init}]\times[0,\tau]$ of the (*S* - *T*) space, where τ is the maximum maturity. We choose $M = 200$ asset steps and $N = 50$ time steps for the PDE discretisation and the range parameter f is set to 2 in order to accommodate the maximum level of strikes. The constant dividend yield is set to equal to 1.46%, which is the average yield over 2001-2002 obtained from Bloomberg[80]. We have proxied the constant interest rate to be 2.31% using average one-year U.S. Treasury yields during the period studied. A summary of the parameters and settings for the problem is provided in appendix D.1

5.4.1.3 Calibrations for Ad Hoc Black-Scholes Model

Following Dumas et al. (1998) and Heston and Nandi (2000), we estimate the volatility function $\sigma(X,\tau)$ in section 5.2.3.4 by fitting the deterministic volatility function to the reported Black-Scholes implied volatility at time τ. The coefficients of the ad hoc model are estimated via ordinary least squares in each of the variance swap contracts by minimising the sum of squared errors between the Black-Scholes implied volatility across different strikes and maturities and the model's functional form of implied volatility.

5.4.1.4 Calibrations for Time-Series Models

The parameters of EGARCH and GARCH-VS presented in section 5.2.4 are estimated by using volatility filtered from the history of S&P 500 spot levels. First differences in logs of S&P 500 index levels are employed to calculate returns. At each time, we use the time-series of returns from the previous two years (504 trading days) to filter the variance for the EGARCH(1,1) and GARCH(1,1) models. We have also experimented with longer filtered intervals such as three or four years for estimations. The results, however, are similar, perhaps due to the strong mean reversion in variance. Given a set of structural parameter $(\alpha_0, \alpha_1, \gamma_1, \beta_1)$, we simulate the *N*-step return data using the EGARCH model. The Monte-Carlo values reported are based on simulating 100,000 paths with a time step of $\Delta t = \frac{1}{252}$. The diffusion parameters k, V, θ and the expected delivery price of the GARCH-VS are calculated based on the formulas described in section 5.2.4.1.

5.4.2 Empirical Results

Figure 45 confirms the changing nature of the term-structure of realised variances in the variance swap contracts on the S&P 500 index: long-term realised variance have gone up significantly after September 11, 2001 whilst realised variances for shorter swaps have tended to decline. A convenient way to examine the deviation between the Black-Scholes model price and market price is to plot the Black-Scholes implied volatility as a function of the exercise price. Figure 46 validates the usual findings in numerous studies that implied volatility tends to vary across exercise prices, with implied volatility higher for in-the-money options and flattens out monotonically as maturity increases. The substantially smaller magnitudes of the pre-9/11 smiles relative to the post-9/11 smiles is also evident in figure 46. In view of forecasting variance, an accurate variance swap forecasting model should not only take account of the smile effects but also the changing term-structure of variance correctly.

Figure 45: Future Realised Variances for 3M, 6M & 9M Variance Swap

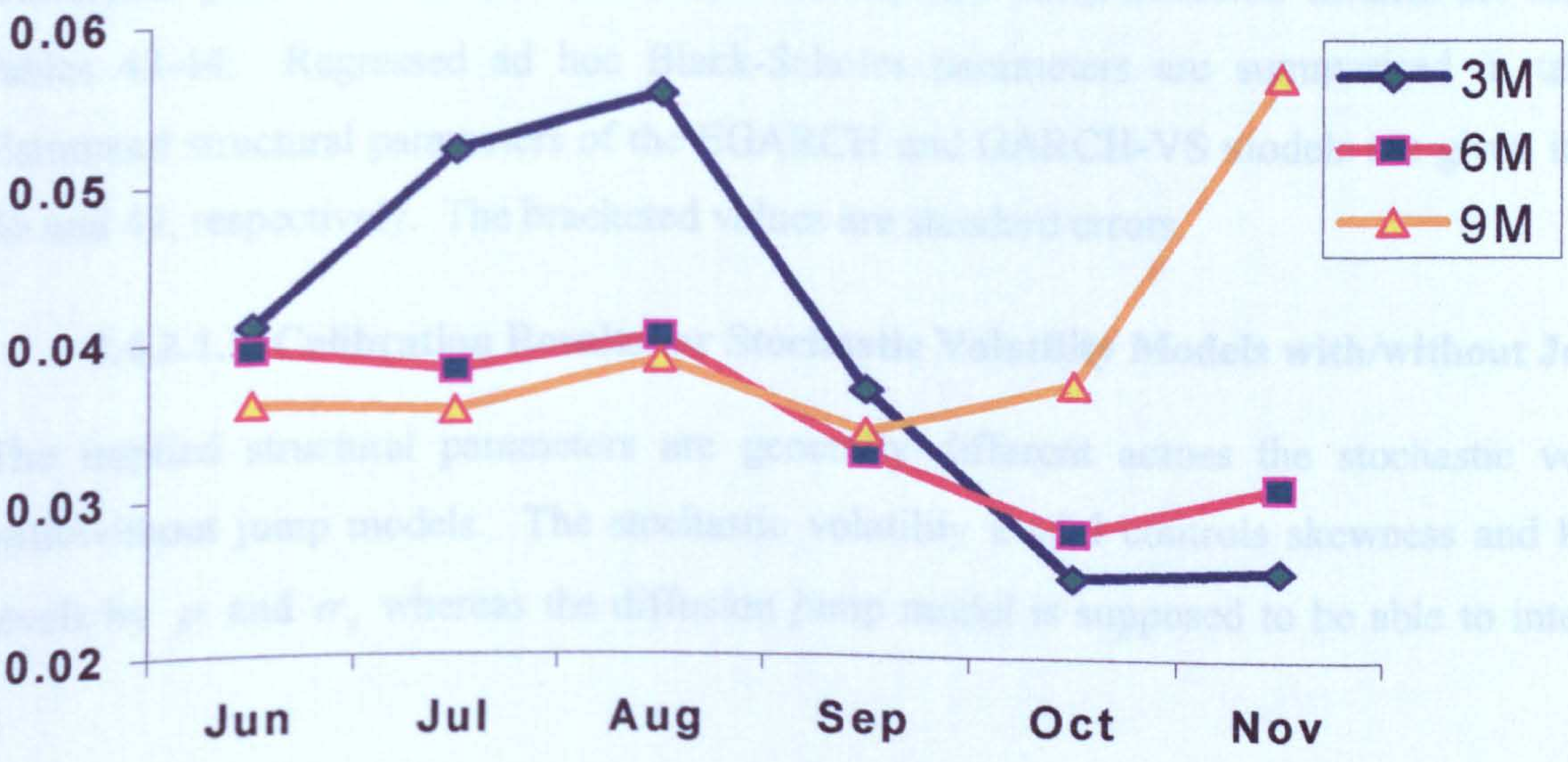
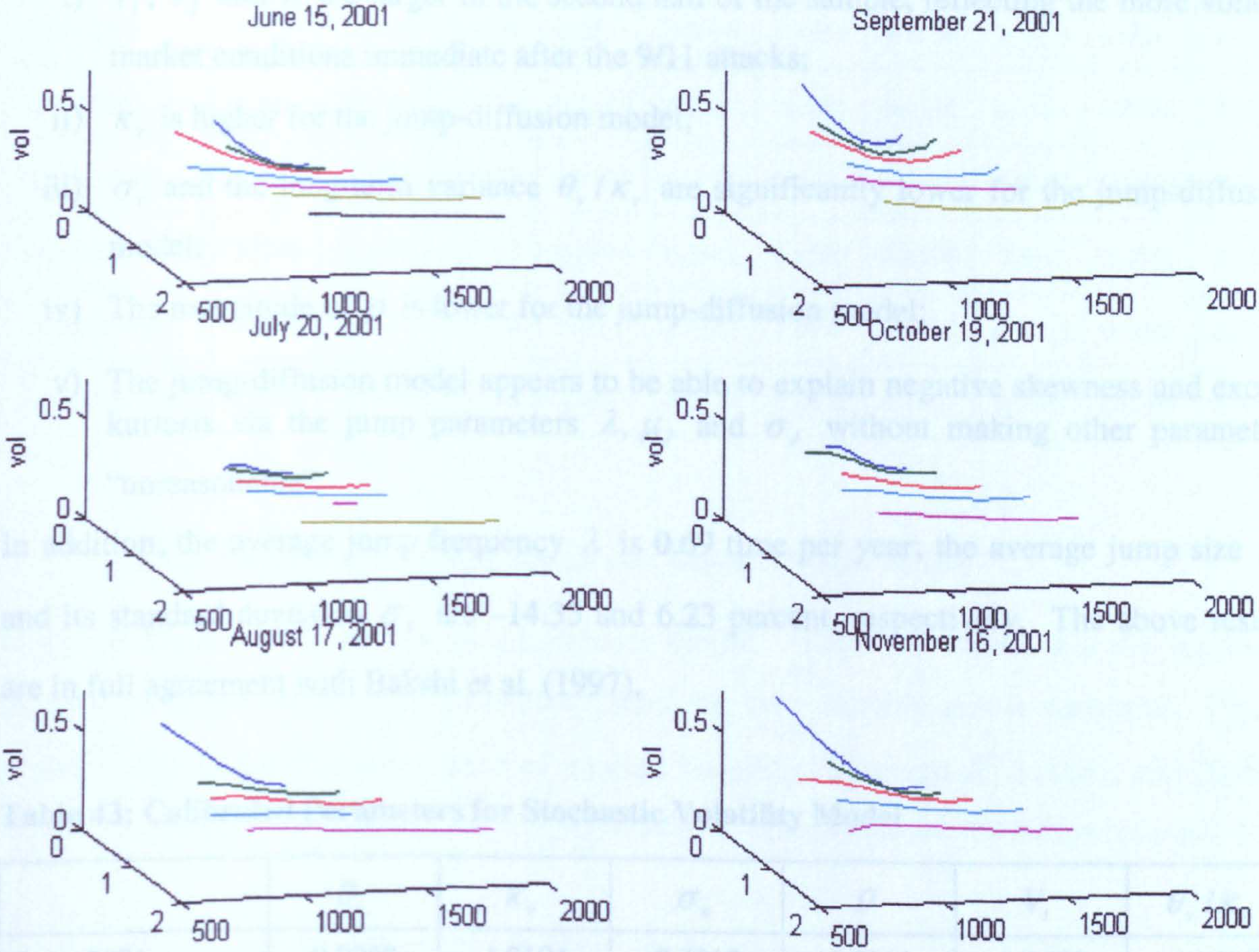


Figure 46: Term-Structure of Implied Volatility



5.4.2.1 Calibration Results for Options-based Models

Calibrated parameters of the stochastic volatility and jump-diffusion models are shown in tables 43-44. Regressed ad hoc Black-Scholes parameters are summarised in table 45. Estimated structural parameters of the EGARCH and GARCH-VS models are given in tables 46 and 47, respectively. The bracketed values are standard errors.

5.4.2.1.1 Calibration Results for Stochastic Volatility Models with/without Jumps

The implied structural parameters are generally different across the stochastic volatility with/without jump models. The stochastic volatility model controls skewness and kurtosis levels by ρ and σ_v whereas the diffusion jump model is supposed to be able to internalise more negative skewness and higher kurtosis. A number of observations can be drawn from tables 43-44:

i) V_t, V_J and λ are larger in the second half of the sample, reflecting the more volatile market conditions immediate after the 9/11 attacks;

ii) κ_v is higher for the jump-diffusion model;

iii) σ_v and the long-term variance θ_v / κ_v are significantly lower for the jump-diffusion model;

iv) The magnitude of ρ is lower for the jump-diffusion model;

v) The jump-diffusion model appears to be able to explain negative skewness and excess kurtosis via the jump parameters λ, μ_J and σ_J without making other parameters "unreasonable".

In addition, the average jump frequency λ is 0.69 time per year; the average jump size μ_J and its standard deviation σ_J are –14.35 and 6.23 percent, respectively. The above results are in full agreement with Bakshi et al. (1997).

Table 43: Calibrated Parameters for Stochastic Volatility Model

	θ_v	κ_v	σ_v	ρ	V_t	θ_v / κ_v
June-2001	0.0989	1.9194	0.4219	-0.7011	0.0482	0.0515
July-2001	0.0757	1.9360	0.3104	-0.6485	0.0378	0.0391
August-2001	0.0818	2.2232	0.3271	-0.7135	0.0467	0.0368
September-2001	0.2136	3.3672	1.3677	-0.6388	0.1770	0.0634
October-2001	0.1547	3.5877	0.5816	-0.6505	0.0845	0.0431
November-2001	0.1209	3.0570	0.5246	-0.6358	0.0565	0.0396

Table 44: Calibrated Parameters for Stochastic Volatility with Jump Model

	θ_v	κ_v	σ_v	ρ	V_t	λ	m_J	σ_J	V_J	θ_v / κ_v
June-2001	0.0711	4.2926	0.1812	-0.5333	0.0366	0.4589	-0.1836	0.1439	0.0219	0.0166
July-2001	0.0684	1.9683	0.2850	-0.7293	0.0347	0.4884	-0.0191	0.0827	0.0034	0.0348
August-2001	0.0421	5.9795	0.0231	0.5747	0.0354	0.6491	-0.1892	0.0261	0.0235	0.0070
September-2001	0.1166	3.1058	1.6002	-0.6294	0.1643	0.6808	-0.1578	7.8e-7	0.0170	0.0375
October-2001	0.0749	5.5933	0.7492	-0.4159	0.0722	1.0116	-0.1438	0.0659	0.0242	0.0134
November-2001	0.0245	4.5700	0.3216	-0.1037	0.0359	0.8581	-0.1679	0.0553	0.0260	0.0054

5.4.2.1.2 Calibration Results for ad hoc Black-Scholes Model

Next we focus on the ad hoc Black-Scholes model. Table 45 shows that the regressed parameters of the ad hoc Black-Scholes model are very variable across contracts. This is probably due to the changing nature of implied volatility. Average R^2 is 0.85, and Durbin-Watson's statistics cannot reject the null hypothesis that residuals are not autocorrelated.

Table 45: Estimated Parameters for Ad Hoc Black-Scholes Model

	a_0	a_1	a_2	a_3	a_4	a_5	R^2	DW Stat
June-2001	9.044E-01 (5.789E-02)	-7.775E-04 (9.343E-05)	1.895E-07 (3.960E-08)	-1.948E-01 (3.608E-02)	1.689E-02 (1.114E-02)	1.217E-04 (3.032E-05)	0.7975	p=0.376
July-2001	5.476E-01 (3.133E-02)	-4.099E-04 (5.070E-05)	1.021E-07 (2.080E-08)	-6.174E-02 (1.358E-02)	1.470E-02 (3.911E-03)	2.931E-05 (1.138E-05)	0.9266	p=911
August-2001	8.026E-01 (8.163E-02)	-6.223E-04 (1.373E-04)	1.206E-07 (6.080E-08)	-3.013E-01 (5.197E-02)	3.178E-02 (2.068E-02)	1.845E-04 (4.228E-05)	0.7404	p=0.85
September-2001	1.123E+00 (6.978E-02)	-1.113E-03 (1.260E-04)	4.011E-07 (5.880E-08)	-2.619E-01 (4.7145E-02)	1.250E-01 (2.004E-02)	-7.369E-07 (4.343E-05)	0.8524	p=0.436
October-2001	8.287E-01 (4.201E-02)	-7.017E-04 (7.327E-05)	1.862E-07 (3.290E-08)	-2.553E-01 (2.362E-02)	4.645E-02 (1.119E-02)	1.264E-04 (2.193E-05)	0.9438	p=0.619
November-2001	1.387E+00 (9.328E-02)	-1.469E-03 (1.670E-04)	4.207E-07 (7.700E-08)	-5.254E-01 (6.293E-02)	4.402E-02 (3.703E-02)	3.471E-04 (5.663E-05)	0.848	p=0.520

5.4.2.1.3 Calibration Results for EGARCH and GARCH Variance Swap Models

On the other hand, it appears that the evolutions of the estimated parameters of the EGARCH and GARCH-VS models are more stationary as compared to the implied parameters of the stochastic models. Based on the statistical results of the estimated parameters in tables 46 and 47, EGARCH seems to be more capable than GARCH to describe the underlying returns dynamics. Negative and significant α_1's also indicate that EGARCH is able to capture asymmetry in returns.

Table 46: Estimated Parameters for EGARCH

	α_0	α_1	γ_1	β_1
June-2001	-0.4255 (0.1901)	-0.1839 (0.0358)	0.05622 (0.0381)	0.9518 (0.022)
July-2001	-0.4562 (0.2369)	-0.1734 (0.0379)	0.07421 (0.0406)	0.9485 (0.0274)
August-2001	-0.4499 (0.2409)	-0.1789 (0.0388)	0.07508 (0.0401)	0.9493 (0.0278)
September-2001	-0.4397 (0.2332)	-0.1823 (0.0382)	0.0732 (0.0398)	0.9504 (0.027)
October-2001	-0.5201 (0.2561)	-0.1831 (0.0413)	0.0823 (0.0424)	0.9412 (0.0297)
November-2001	-0.3742 (0.2744)	-0.1869 (0.0407)	0.0669 (0.0488)	0.9583 (0.0318)

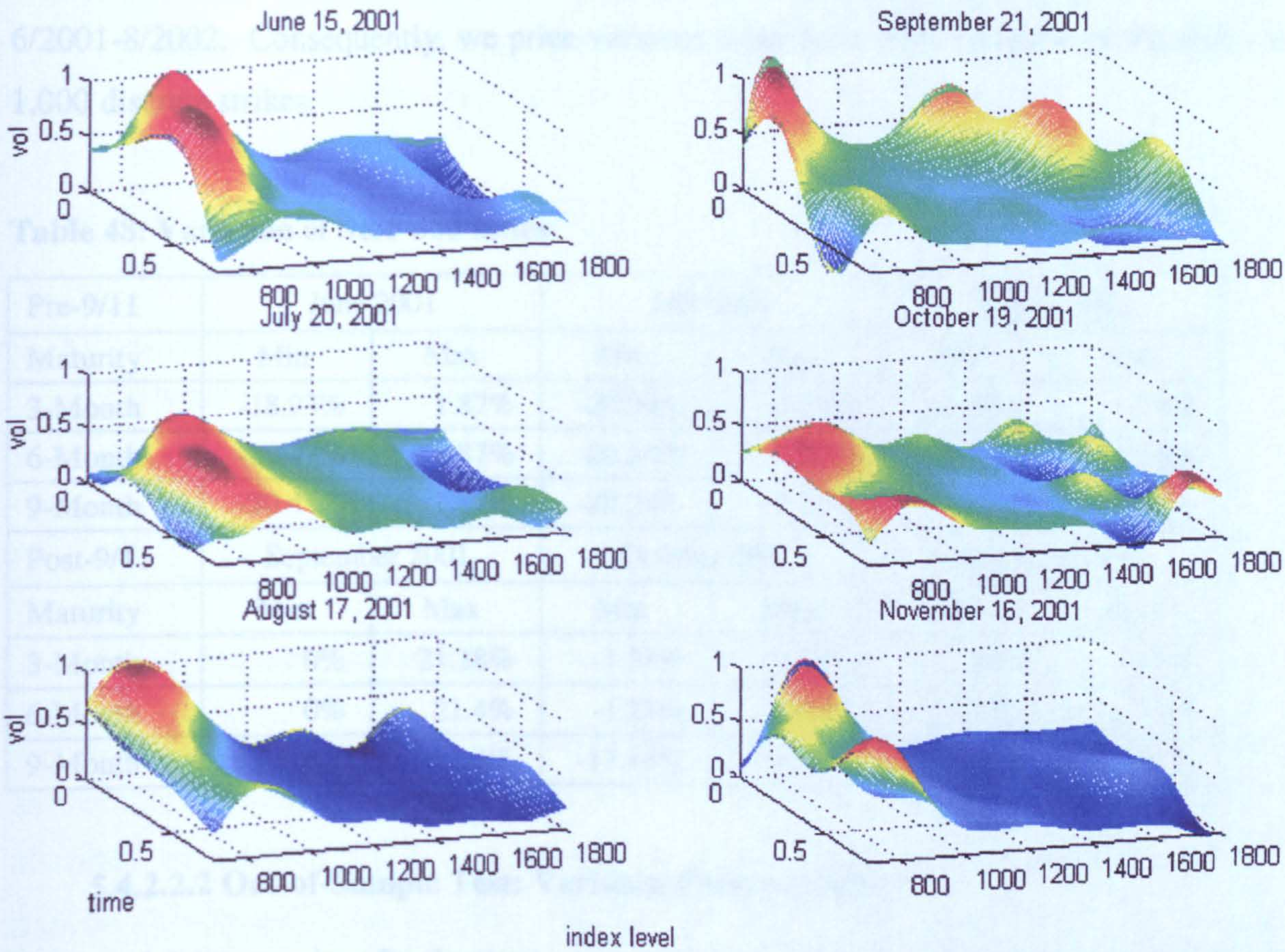
Table 47: Estimated Parameters for GARCH-Variance Swap

	α_0	α_1	β_1
June-2001	1.23e-05 (1.68e-05)	0.0921 (0.0681)	0.8411 (0.1480)
July-2001	1.27e-05 (1.85e-05)	0.0899 (0.0705)	0.8415 (0.1594)
August-2001	1.24e-05 (1.98e-05)	0.0941 (0.0812)	0.839 (0.1764)
September-2001	1.58e-05 (1.62e-05)	0.1175 (0.0723)	0.8041 (0.1365)
October-2001	1.804e-05 (1.58e-05)	0.1234 (0.0697)	0.7845 (0.1294)
November-2001	1.903e-05 (1.50e-05)	0.1277 (0.0651)	0.7755 (0.1196)

5.4.2.1.4 Calibration Results for Local Volatility Model

Last, we turn our attention to the local volatility model. Figure 47 displays the calibrated local volatility surfaces from each of the six sets of option prices. . Notably the variation in local volatility is greater than the variation in implied volatility that produced it in figure 46. For skewed option markets, this behaviour is consistent with the Zou et al.'s (1997) heuristic rule that local volatility varies with the index level about twice as rapidly as implied volatility varies with strike. This result leads us to believe that our calibration procedures can reliably reconstruct the local volatility surfaces. However, the highly variable shape of the local volatility surfaces is potentially problematic because it implies that future local volatility smiles will be very different from today's. The typical downward-sloping volatility smile is, to a large extent, driven by the fear of rapid downward movements of the underlying index. The local volatility approach typically predicts that future volatility will tend to flatten out and disappear over time. This prediction, however, is clearly at odds with market reality that the volatility smile tends to be quite stationary over time.

Figure 47: Calibrated Local Volatility Surfaces



5.4.2.2 Variance Swap Forecasting Results

5.4.2.2.1 Implementation Issues for Options-based Variance Swap Model

The variance estimator is given by: $\frac{F}{n}\sum_{i=0}^{n-1}\left(\frac{S_{i+1}-S_i}{S_i}\right)^2$

where n is the number of trading days and the annualisation factor, F, is set to 252; S is the closing price of the S&P 500 index. Moreover, the sample mean is assumed to be zero.

One major concern for using the Demeterfi et al. framework is the "wing effect", which refers to the implementable low and high strike prices for replicating the hedged portfolio. The range of strikes attributable to this strategy can be chosen by focusing on the central region where there is sufficient liquidity. Clearly, judgement is required in determining the range of strikes[81]. Table 48 shows variations of the S&P 500 index during the life-span of the corresponding variance swap contracts. It has a range between –29.94% and 21.4% in 6/2001-8/2002. Consequently, we price variance swap for a 30% variation of the index with 1,000 discrete strikes.

Table 48: Variation of S&P 500 Index

Pre-9/11	June 2001		July 2001		August 2001	
Maturity	Min	Max	Min	Max	Min	Max
3-Month	-18.93%	1.87%	-20.24%	0.82%	-16.88%	1.98%
6-Month	-20.47%	1.87%	-20.24%	0.82%	-16.88%	1.98%
9-Month	-20.47%	1.87%	-20.24%	0.82%	-16.88%	1.98%
Post-9/11	September 2001		October 2001		November 2001	
Maturity	Min	Max	Min	Max	Min	Max
3-Month	0%	21.18%	-1.28%	9.23%	-5.14%	2.97%
6-Month	0%	21.4%	-1.28%	9.23%	-7.83%	2.97%
9-Month	0%	21.4%	-17.88%	9.23%	-29.94%	2.97%

5.4.2.2.2 Out-of-Sample Test: Variance Forecastability

Estimated delivery prices for the three-, six- and nine-month variance swap contracts for each of the six variance swap models are given in table 49; realised future variance are also shown in the second column of the same table. Table 50 reports the aggregate mean-square price errors (MSPE) for the three-, six- and nine-month contracts; aggregate model rankings for three-, six- and nine-month contracts are bracketed and displayed in the same table.

Based on results from eighteen variance swap contracts, a number of observations can be drawn from tables 49 and 50:

i) Aggregate MSPE ranking of the models from table 50 is robust across maturities;

ii) Conditional heteroskedastic models outperform options-based models in predicting variance with GARCH-VS ranked first in all maturities. More strikingly, we find that

[81] We thank Tom Ley for this invaluable comment.

even a naive EGARCH simulation can deliver less forecasting errors than the highly sophisticated options-based pricing models;

iii) The jump-diffusion model is similar to the stochastic volatility model in producing variance forecasts. Adding a jump component to a stochastic volatility model serves to increase variance in short maturity but does not seem to enhance variance forecastability;

iv) The local volatility model underperforms ad hoc Black-Scholes model in making variance forecasts;

v) All models predominately overpredict variance. On average there is a 81% chance that any variance swap model will overprice future variance;

vi) The amount of overpricing is more manifest in the aftermath of the 9/11 attacks;

vii) Last and most importantly, we observe that the options-based variance swap pricing models cannot produce enough variance term-structure patterns.

5.4.2.2.3 Comments on Out-of-Sample Results

Although our sample is small, it is still puzzling to see there is such a large discrepancy between options-based and time-series models in terms of variance forecastability. One plausible explanation for the disappointing performance of the options-based pricing framework concerns with the fact that the Demeterfi et al. methodology was originally developed for hedging. The time-series methods use historical information to price variance swaps and could be different from the expectations about the future evolution of the asset price that are embedded in option prices. Theoretically option prices should summarise all relevant information regarding expected future volatility whereas the time-series approach can exploit only the subset of that information inferrable from the past history of stock index prices.

Table 49: Delivery Prices for 3M, 6M and 9M Variance Swap Contracts

Contracts	Realised Variance	EGARCH	GARCH-VS	Ad Hoc BS	Local Volatility	Stochastic Volatility.	Jump-Diffusion
Jun-2001 3M	0.04093	0.05052	0.03573	0.05704	0.07840	0.04860	0.05400
Jun-2001 6M	0.03939	0.04761	0.04114	0.05269	0.07036	0.04761	0.04937
Jun-2001 9M	0.03599	0.04671	0.04282	0.04873	0.06616	0.04633	0.04660
Jul-2001 3M	0.05225	0.04212	0.03532	0.03917	0.05498	0.03802	0.03817
Jul-2001 6M	0.03819	0.04224	0.04100	0.03788	0.05604	0.03770	0.03769
Jul-2001 9M	0.03568	0.04244	0.04289	0.03669	0.05398	0.03701	0.03693
Aug-2001 3M	0.05583	0.04580	0.03503	0.05344	0.06946	0.04431	0.04857
Aug-2001 6M	0.04030	0.04419	0.04076	0.04589	0.06019	0.04196	0.04237
Aug-2001 9M	0.03878	0.04355	0.04274	0.03976	0.05910	0.03985	0.03905
Sep-2001 3M	0.03702	0.07539	0.04076	0.13005	0.13874	0.12703	0.12880
Sep-2001 6M	0.03291	0.06063	0.04550	0.10088	0.10017	0.09670	0.09656
Sep-2001 9M	0.03432	0.05425	0.04747	0.08109	0.08297	0.07843	0.07761
Oct-2001 3M	0.02514	0.04324	0.04063	0.07396	0.08366	0.06959	0.07152
Oct-2001 6M	0.02798	0.04324	0.04501	0.06220	0.06390	0.05914	0.05810
Oct-2001 9M	0.03743	0.04309	0.04648	0.05340	0.06411	0.05260	0.05131
Nov-2001 3M	0.02566	0.03114	0.04106	0.06467	0.06323	0.05107	0.05302
Nov-2001 6M	0.03089	0.03548	0.04539	0.05182	0.05292	0.04643	0.04612
Nov-2001 9M	0.05703	0.03709	0.04680	0.04151	0.05570	0.04308	0.04215

Table 50: Aggregate Mean-Square Price Errors and Model Rankings for 3M, 6M and 9M Variance Swap Contracts

		EGARCH	GARCH-VS	Ad Hoc BS	Local Volatility	Stochastic Volatility	Jump-Diffusion
3M	MSE	0.00213	0.00124	0.01300	0.01678	0.01112	0.01174
	Rank	(2)	(1)	(5)	(6)	(3)	(4)
6M	MSE	0.00112	0.00067	0.00644	0.00797	0.00535	0.00530
	Rank	(2)	(1)	(5)	(6)	(4)	(3)
9M	MSE	0.00101	0.00047	0.00285	0.00474	0.00248	0.00240
	Rank	(2)	(1)	(5)	(6)	(4)	(3)

5.4.2.2.4 In-Sample Fit for Option Pricing Models

Given the implied framework is supposed to provide a forward-looking means to "gauge" market sentiment, it is important to understand why the options-based Demeterfi et al. framework has such a poor variance forecasting performance. Table 51 reports the sum of price square error (SPSE) for each of the option models for each contract day. A few observations are in order:

i) The SPSE is successively lower as we extend from the ad hoc Black-Scholes to the stochastic volatility models with/without jump and local volatility model;

ii) The local volatility model has the lowest SPSE in all contract days whilst allowing jumps to occur reduces the SPSE further over the stochastic volatility model;

iii) Overall, modelling for skewed and leptokurtic distributions via the relaxed Black-Scholes specifications further enhances the model's ability to fit option prices. But the finding that the local volatility model does not improve variance forecastability over the ad hoc Black-Scholes model is highly surprising, especially given the local volatility model's excellent in-sample pricing performance.

The above observations suggest that a flexible but theoretically inconsistent model may dominate in-sample fit but has much less predictive power for predicting future variance, which implies that a misspecified model achieves good in-sample results by overfitting the options data.

Table 51: In-Sample Fit (SPSE) for Option pricing Models

	Ad Hoc BS	Local Volatility	Stochastic Volatility	Jump-Diffusion
JUNE-2001	995.3221	35.0748	178.2048	81.7872
JULY-2001	51.4489	2.2035	25.1238	23.8116
AUGUST-2001	554.6932	5.9678	120.7058	32.8119
SEPTEMBER-2001	759.4753	16.7763	170.2324	106.6719
OCTOBER-2001	197.8162	0.5986	67.0353	13.2934
NOVEMBER-2001	2079.2547	8.3802	238.1859	42.5544

5.4.2.3 Consistency with the Time-series Properties of Volatility

Based on our limited sample, we have demonstrated that in-sample fit of daily option prices is progressively better as we extend from the ad hoc Black-Scholes to the stochastic volatility models with/without jump and local volatility model. By far our evidence shows that:

i) Incorporating stochastic volatility and jumps to the option model does not lead to a superior performance to the GARCH-type models in terms of forecasting future variance but it does contribute to a better in-sample fitting;

ii) The mean-square error based ranking of the local volatility model is in sharp contrast with the ranking obtained based on the in-sample fit of option prices; hence there may be an issue of overfitting.

A possible interpretation of these results is that the local volatility model does not properly relate its volatility surface to the path-dependent dynamics of volatility. Since the sole source of variations of options under the local volatility model is the underlying index, option prices, regardless of maturity and moneyness, must perfectly co-vary with each other and with the underlying asset. This potentially imposes a stringent restriction on option price dynamics. In the next section our goal is to investigate whether option prices are consistent with its underlying dynamics.

5.4.2.3.1 CIR Square-Root Process

Basing the results upon Cox et al's (1985) stochastic interest rate model, Bates (1996) developed an econometric method for testing the consistency of the distribution implied in option prices with its time-series properties. Bakshi et al. (1997) applied this test to the S&P 500 index and found that the stochastic volatility with/without jump models were misspecified because the volatility of volatility σ_v was too high. But Bakshi et al.'s study only tested for the consistency of its implied structural parameters with the evolution of option prices. Their results were potentially problematic because volatility implied by the stochastic option prices were used as a surrogate for true volatility. Consequently, little was known whether the implausible structural parameters were caused by misspecification of the models, or by problems with the estimation procedure. The question remains open whether the distribution implied by option prices is the same as that directly observed from market asset price.

In this section we directly investigate the consistency of implied distributional assumption with the evolution of underlying index price. Following Bates (1996), when volatility risk-premium is proportional to the conditional variance V_t, the transition density of $y = 2cV_{t+\Delta t}$ conditional on V_t for a square-root process is noncentral $\chi^2(4\theta_v/\sigma_v^2, 2cV_t e^{-\kappa_v \Delta t})$, where $c^{-1} = 0.5\sigma_v^2(1-e^{-\kappa_v \Delta t})/\kappa_v$. The transition density of $\ln(V_{t+\Delta t}/V_t)$ is given by:

$$P(\ln(V_{t+\Delta t}/V_t) = \frac{e^{-0.5(e^z+\Lambda)}(e^z)^{0.5\nu}}{2^{0.5\nu}} \sum_{j=0}^{\infty} \frac{(0.25e^z\Lambda)^j}{\Gamma(0.5\nu + j)j!}$$

where $\nu = 4\theta_v/\sigma_v^2$, $\Lambda = 2cV_t e^{-\kappa_v \Delta t}$ and $e^z = 2cV_{t+\Delta t}$.

5.4.2.3.2 Results of Maximum Likelihood Estimation

Maximum likelihood estimates of the parameters θ_v, κ_v and σ_v using historical time-series are shown in table 52. Average values of the implied structural parameters for the stochastic volatility and jump-diffusion models in tables 43 and 44 are also presented in the same table for ease of comparison.

Table 52: Estimated & Implied Structural Parameters

	θ_v	κ_v	σ_v	ρ	MLE Value
30-Day Historical Volatility	0.12117 (3.221e-3)	1.35228 (0.63628)	0.32318 (3.486e-4)	-0.21600	-602.0167
Weighted 30-Day BS Implied Volatility	0.26997 (7.038e-3)	3.22071 (1.53316)	0.41141 (5.715e-4)	-0.76250	-557.5923
Implied Stochastic Volatility Model	0.12427	2.68185	0.58888	-0.66470	N.A.
Implied Jump-Diffusion Model	0.06627	4.25158	0.52672	-0.30615	N.A.

Estimation procedures for CIR process are:

i) ρ is estimated by calculating correlation between volatility changes and index returns;

ii) We use a weekly 30-day historical volatility series as a proxy for the true volatility[82];

iii) Since Bakshi et al. (1997) reported that implied instantaneous volatility was on average less than 0.5 percent apart among the Black-Scholes and the stochastic

[82] The time-series comprises of the weekly observations for the period from June 1999 through December 2001.

volatility models, we have also estimated the parameters for a vega-weighted Black-Scholes implied volatility series. This series is constructed by averaging the vega-weighted Black-Scholes 30-day call and put implied volatility.

Since options are priced off the risk-neutral process but not the true process, parameters estimated from true and risk-neutral distributions can be different. Because of volatility and jump risk premiums, only θ_v, σ_v and ρ are directly comparable in table 52[83]. Four observations are in order:

i) Option-implied σ_v's are significantly higher than its historical estimate, although it is not as high a level as it was suggested by Bakshi et al. (1997), who found a 300% difference between the "true" and "implied" parameters;

ii) Estimated historical κ_v, which is 1.352, is consistent with Das et al. (1999) assertion that plausible values for κ_v are in a neighbourhood of unity. Furthermore, higher estimates for option-implied κ_v confirm that volatility risk-premium is significantly positive;

iii) Estimated correlation between index returns and historical volatility is significantly different from the correlations implied by the 30-day Black-Scholes implied volatility and the stochastic option models. Based on an EGARCH specification for equity-return dynamics, Nelson (1991) gave an estimate of –0.12 for the correlation between returns and changes in the true volatility, which is closer to our historical time-series estimates;

iv) The results for θ_v's are mixed and we cannot draw any consistent observation to explain our finding.

Nevertheless, it appears indisputable that the distributional dynamics implied by option prices and its underlying index are not consistent.

5.5 Summary

This chapter has emphasised the empirical implications of forecasting variance using conditional heteroskedastic approaches and an arbitrage-free options-based variance swap framework in the period from three months before to after the 9/11 attacks. The exercises are

[83] See Bates (1996) for a detailed explanation of risk-neutral versus true distributions.

carried out by employing the latest time-series and option pricing models in finance literature to generate skewness and kurtosis in returns distributions. The Demeterfi et al.(1999) variance forecasting framework has been examined from a practical perspective and we have understood some of its properties and limitations. During the six contract days from three-months before to after the 9/11 terrorist attack, we show that the Demterfi et al. framework overpredicts future variance and that time-series forecasting models have a smaller MSPE. In addition, we illustrate that options-based models cannot predict the directional changes of the 3M, 6M and 9M future variance.

Our results are in direct violation to the underlying hypotheses that:

i) Each generalisation of the benchmark Black-Scholes model should be able to improve the volatility forecastability of the options-based pricing model;

ii) If option prices are indeed representative of their underlying time-series and forward-looking then the forecastability of options-based variance swap models should be superior to their time-series counterparts.

In particular, we cast doubt on the usefulness of the local volatility model as a forecasting tool because it has the best in-sample fitting result but worst volatility forecastability. We observe that using a more flexible and sophisticated option pricing model may improve the in-sample fitting of option prices but not necessarily forecastability of future variance. In our view, it is important that an accurate variance forecasting model should not only take account of the smile effects but also the changing term-structure of variance (crossovers) correctly. Therefore we have a strong reservation about the effectiveness of forecasting future variance through log-contract replications

In summary, we provide some evidence in small sample that there is inconsistency in volatility forecasting performance between options-based models and time-series models from three months before to after the 9/11 terrorist attacks. The reasons could be: 1) the Demeterfi et al. framework was originally developed for hedging and its strategy can only guarantee that the replicating portfolio will have the same payoff at maturity as the variance swap regardless of the actual path taken by the index. Our finding and arguments are consistent with Bakshi and Kapadia (2003) that a negative volatility risk premium suggests an equilibrium where index options act as a hedge to the downside risk, therefore making investors to pay a higher price (implied volatilty) to hold options in their portfolio than its price when volatility is not priced. Rather than predicting future variance, the delivery price probably only reflects the costs of replication; 2) implied volatility is largely a monotonically decreasing function of maturity and therefore the options-based strategies cannot produce enough variance term-structure patterns; 3) distributional dynamics implied by option parameters is not consistent with its time-series data as stipulated by the maximum likelihood estimation of the square-root process. Results of maximum likelihood estimation of a square-root process also suggest that option models may rely on implausible levels of correlation and, to a lesser extent, volatility variation to rationalise the observed option prices. In particular, the high magnitude of negative correlation in option prices, which generates excessive levels of negative skewness, could be the culprit for the observed strike price biases in the equity-index market.

Finally, although the forecast periods are overlapping, we must point out that this will only affect the forecasting performance of time-series models. Options-based variance swap models are supposed to be forward-looking and therefore insensitive to the choice of sample periods. A larger sample group is indeed required in order to draw a more consistent and statistically significant conclusion about the superiority of time-series variance forecasting models. Until then we have a strong reservation about the use of Demeterfi et al. methodology for volatility forecasting

CHAPTER 6 Summary, Discussion and Suggestions for Further Research

6.1 Introduction

As an aid to the reader, this final chapter of the dissertation restates the research problems in this study. The major sections of this chapter summarise and discuss the results. The final section makes recommendation for future research.

6.2 Statement of the Problem

Throughout the first project (Chapter 3) we examine the empirical behaviour of S&P 500 futures option's implied volatility using daily data from 1983 through 1998. The primary objective is to observe, characterise and analyse the patterns of the term-structure of implied volatility in the S&P 500 marketplace. The second objective is to investigate whether option prices are in line with the rational expectations hypothesis under a mean-reverting volatility assumption. The final objective in this work is to identify what types of option models would be consistent with the observed moneyness biases in the S&P 500 options market.

In the second project (Chapter 4) we investigate the performance of APARCH models that can potentially account for the slow decay in returns autocorrelations using daily S&P 500 futures series from 1983 through 1998. The objectives are:

i) To investigate the effectiveness of asymmetric parameterisation and power transformation within the context of APARCH specifications;

ii) To study the impact of structural change of volatility on the performance of asymmetrical and symmetrical conditional volatility models;

iii) To compare the performance of EGARCH (Nelson, 1991) with APARCH models;

iv) To explore the ability of different symmetrical and asymmetrical statistical loss functions to track the in-sample forecasting performance of conditional volatility models;

v) To assess the quality of different conditional volatility forecasts by conducting ex-ante straddle trading exercises.

In continuation of our study of modelling volatility, the third project (Chapter 5) evaluates the volatility forecasting performance of time-series and options-based variance swap valuation models on the S&P 500 index. The primary goal is to present a complete picture of how each generalisation of the benchmark Black-Scholes model can really improve the volatility forecasting performance of variance swaps and whether each generalisation is consistent between in- and out-of-sample results. The second goal is to investigate whether there is any systematic difference in performance between time-series and options-based variance swap valuation models. It is intended to explore whether options-based models, which are forward-looking, are capable of outperforming discrete-time processes, which use only historical information, in predicting future variance.

6.3 Summary of the Results

In this research we have examined many empirical issues relating to the modelling of volatility from both the options market and time-series perspectives. The results are summarised in this section.

The first project (Chapter 3), entitled "*A Report on the Properties of the Term-Structure of S&P 500 Implied Volatility*", analyses the term-structure of implied volatility using S&P 500 futures and its options data from 1983 to 1998. Contrary to the basic assumption of the Black-Scholes formula, implied volatility exhibited both smile effects and term-structure patterns. Term-structure analysis revealed that: 1) implied volatility tended towards a long-term mean of about 16%; 2) put options had higher premiums and a larger range of fluctuation than call options; 3) short-maturity options were more volatile than long-maturity options. Results from harmonic analysis showed that put options were more "responsive" to a change of market sentiment than call options. In addition, smile effects were found to be strongest for short-term options, indicating that short-term options were most severely mispriced by the Black-Scholes formula and therefore presented the greatest challenge to any alternative option pricing models. Furthermore, evidence suggested that option prices were not consistent with the rational expectations under a mean-reverting volatility assumption. We also conducted a distributional test to find out whether observed moneyness biases were consistent with the skewness of the risk-neutral distribution derived from any specific distributional hypothesis. The 4% skewness premiums results agreed with the term-structure analysis that the degrees of anomalies in the S&P 500 options market had been gradually worsening since around 1987. As correlation might be responsible for skewness, our diagnostics suggested that leverage and jump-diffusion (with negative-mean jumps) models were more appropriate for capturing the observed biases in the S&P 500 futures options market.

Based upon the skewness premiums results in Chapter 3 that a leverage model was suitable to model the observed market anomalies, the second project (Chapter 4), entitled "*An Empirical Comparison of APARCH Models*", compares a group of well-theorised conditional volatility models that can potentially account for the observed term-structure biases in the S&P 500 futures options market. Sixteen years of daily S&P 500 futures series were used to examine the performance of the APARCH models that used asymmetric parameterisation and power transformation on conditional volatility and its absolute residual to account for the slow decay in returns autocorrelations. No evidence could be found supporting the relatively complex APARCH models. Log-likelihood ratio tests also confirmed that asymmetric parameterisation and power transformation were not effective in characterising the S&P 500 returns dynamics within the context of APARCH specifications. In addition, a 3-state volatility regime-switching model was used to detect the "quiet" and "noisy" periods and provide evidence that the performance of conditional volatility models was prone to the state of volatility of the return series. The AIC metric showed that EGARCH was best in "noisy" periods whilst GARCH was the top performer in "quiet" periods. In an effort to rate the performance of different conditional volatility models, aggregated rankings were used to determine the best overall model. Aggregated rankings for the AIC metric showed that EGARCH was the best model. We also attempted to apply additional statistical criteria that allowed for symmetry/asymmetry in the loss functions of investors to select the best volatility forecasting model, but results were mixed. In-sample results showed that no single model was clearly superior. Since it was not sensible to evaluate forecasting performance with only a single statistical loss function, we evaluated the performance of volatility predictors based on their ability to predict volatility changes and generate ex-ante profits from trading nearest-the-money S&P 500 straddles in four two-year out-of-sample periods. Out-of-sample results demonstrated that the EGARCH model outperformed GARCH and both of them could generate statistically significant ex-ante returns in one out of four sample periods. Therefore, there were certain degrees of inefficiency in the S&P 500 futures options market. Finally, our trading experiments also revealed that the presumption of using delta-neutral trades to create risk-free portfolios was not practical in the event of large index movements. We concluded that a new derivatives instrument was needed to allow traders and investors speculate on volatility more directly and efficiently.

Motivated by the findings in Chapter 4 that traditional options-based volatility trading strategies were vulnerable to large market moves, the third project (Chapter 5), entitled "*Empirical Performance of Alternative Variance Swap Valuation Models*", evaluated the volatility forecasting performance of different specifications of time-series and options-based variance swap valuation models on the S&P 500 index in the period from three months before to after the 9/11 attacks. Our exercises were carried out by employing the latest time-series and option pricing models in finance literature to generate skewness and kurtosis in returns distribution. Based on results from six well-chosen contract days from three-months before to after the 9/11 terrorist attack, we showed that the Demterfi et al. framework overpredicted future variance and that time-series forecasting models have a smaller MSE. In addition, we illustrated options-based models could not predict the directional changes of 3M, 6M and 9M future variance. We observed that using a more flexible and sophisticated option pricing model might improve the in-sample fitting of option prices but not forecastability of future variance. Finally, results from maximum likelihood estimation of the square root process suggested that the high magnitude of negative correlation in option prices, which generated excessive levels of negative skewness, might be responsible for the observed strike price biases in the S&P 500 index market.

6.4 Discussions of the Results

This dissertation is a quantitative study whose primary objective is to investigate the performance of different specifications of time-series and option-based volatility forecasting models under the influence of the observed market biases. Our research is based primarily upon the use of S&P 500 data for the period 1982-2002. It contains three self-contained but seemingly related projects. This section discusses the implications and anticipated (less anticipated) findings in this study as well as the relationship of the current research to previous research.

6.4.1 Term-Strucutre of Implied Volatility

The study started with a graphical inspection of the term-structure of implied volatility. Term-structure analysis revealed that implied volatility systematically followed some predictable patterns, which implied that models such as stochastic volatility models and conditional heteroskedastic models could be used to account for the inefficiency in the market. The anticipated results included the finding of moneyness and maturity biases. Observed irregularities in relative implied volatility constituted strong evidence against the hypothesis that the Black-Scholes' implied volatility was the market's fully rational volatility forecast. The U-shape could be the result of: 1) illiquid market; 2) non-normality returns distribution. Bid-ask spread in illiquid markets was typically huge for out-of-the money options and this could artificially introduce high volatility to out-of-the-money options, forming the basis for "volatility skew". But perhaps the more credible reason responsible for the observed U shape was non-normality in the returns data. The "volatility skew" could also be a result of active use of portfolio insurance policies to protect investors' portfolios, thus creating a surging demand for out-of-the money put options and driving up their prices and volatility. Our term-structure evidence also showed that the convexity of relative implied volatility of longer-term options was relatively insensitive to evolution of calendar time. Thus smile effects were strongest for short-term options, indicating that short-term options were the most severely mispriced by the Black-Scholes formula and presented perhaps the greatest challenge to any alternative option pricing models, which agreed with the results in Das et al. (1999).

The term-structure evidence also supported the notion that implied volatility had been getting more skewed as calendar time evolved, thus stressing the increasing importance of using leverage models to characterise skewness properly. Moreover, the relative degrees of anomalies decreased as term-to-maturity lengthened. Once again this evidence suggested that the Black-Scholes formula severely mispriced short-term options. In addition, evidence also revealed that the implied volatility of call options in a given in-the-money (out-of-the-money) category was quite similar to implied volatility of put options in the opposing out-of-the-money (in-the-money) category, which was generally true regardless of sample period or term-to-maturity. Such similarities in pricing structure existed between call and put options mainly due to the working of the put-call parity. Other more regular results in Chapter 3 included: 1) put options commanded a higher premium than call options in each maturity group, which was consistent with Black's leverage effect. A possible explanation of these results was that purchase of S&P 500 futures was a convenient and inexpensive form of portfolio insurance. Thus excess buying pressure of front-month put options might cause prices to increase, resulting in higher puts' implied volatilities. Furthermore, average call and put implied volatilities mean-reverted to their long-term mean of 16% and 16.8%, respectively. That was to say that when implied volatility was above its long-term mean level, the implied volatility of an option would have to be decreasing in the time to expiration, and vice versa; 2) implied volatility of shorter maturity options were more variable than longer maturity options; 3) variation of put options' implied volatility was higher than call options. The latter result could be viewed as evidence that put options were more "responsive" to the arrival of new information. Less anticiplated, though, were the findings that option prices might not be consistent with the rational expectations hypothesis under an AR(1) process. In addition to Stein (1989), Bates (1996) and Bakshi et al. (1997), the results from elasticity requirements questioned whether the volatility process implied by traded options was consistent with the properties implied in its time-series. Given that the Black-Scholes' assumption of constant volatility was so poorly violated, it was not surprising that the 4% skewness premiums results recommended the use of a leverage model or a jump-diffusion model to capturing the observed market biases.

6.4.2 Conditional Heteroskedastic Models

In order to find a time-series model that could take account of the observed term-structure biases in Chapter 3, the performance of the APARCH and EGARCH models were compared by using different types of in-sample criteria. Likelihood-based statistics questioned the rationale of using the more complex APARCH models. In particular, the use of asymmetric parameterisation and power transformation were shown to be ineffective within the APARCH specifications. According to the AIC metric, EGARCH and GARCH were top models in "noisy" and "quiet" periods, respectively. Overall, EGARCH was the best model based on aggregated AIC rankings. Since EGARCH and GARCH converged to some specific stochastic volatility diffusion models in continuous limit, these results indicated that there was little incentive to look beyond a simple stochastic model which allowed for volatility clustering and a leverage effect such as Heston (1993). These findings are in full agreement with Christoffersen et al. (2002), which based their analysis upon evaluating the in- and out-of-sample MSE on option prices to determine the best model specifications. Christoffersen et al. also pointed out that more might be gained from changing the specification of other fundamental building blocks of the stochastic models, such as jump. Additional statistical criteria, which allowed for symmetry/asymmetry in the loss functions of investors to select the best volatility forecasting model, were also used to determine the best model, but results were mixed. Given the conflicting ranking results from different statistical loss functions, we proceeded to use an economic criterion - trading nearest-the-money straddles, to measure the out-of-sample performance of the EGARCH and GARCH models. We evaluated the forecasting performance of different volatility forecasting models by assessing whether profits can be generated from trading weekly nearest-the-money straddles on S&P 500 futures with shortest remaining times to maturity based on out-of-sample forecasts of volatility changes. As ex-ante volatility predictions a priori did not take account of future unexpected events, it was not anticipated that there would be much difference in the out-of-sample performance of different volatility predictors. We found EGARCH and GARCH were able to make ex-ante profits in one of four two-year out-of-sample periods. In addition, EGARCH had the highest rate of returns per trade in all sub-periods and therefore the best economic value in the S&P 500 futures options market. Our findings reinforced the idea that volatility changes were predictable and GARCH-type models might be able to make adjustments for market imperfections that could not be explained by the Black-Scholes formula. Finally, our trading experiments also revealed that the presumption of using delta-neutral trades to create risk-free portfolios was not practical in the event of large index movements, and large changes needed to be correctly predicted at critical dates when there were potential profit available. Many studies have demonstrated the problems associated with delta-neutral trading strategies (e.g. Boyle et al., 1980; Leland, 1985; Figlewski et al., 1994). In general, the longer it takes to reverse a delta-neutral trade, the more exposure it has to volatility fluctuations. But dynamically hedging and rebalancing the position once a day until expiration would be so prohibitively expensive that it is impractical even for an option market marker (see Figlewisk, 1989b). Rebalancing less frequently can reduce costs, but risk increases. Therefore, we reached the verdict that a new derivatives instrument would be needed to allow traders speculate on volatility.

6.4.3 Time-series and Options-based Variance Forecasting Models

The skewness premiums analysis conducted in Chapter 3 indicated that jump-diffusion and leverage models were best for capturing observed term-structure biases in the S&P 500 market. In addition, results in Chapter 4 suggested that EGARCH and GARCH were adequate to model time-series behaviour of the S&P 500 market whilst a continuous model such as Heston (1993) was ideal for option pricing. Furthermore, volatility trading experiments showed that the presumption of delta-neutrality was unrealistic. Motivated by these findings, Chapter 5 evaluated the volatility forecasting performance of different specifications of time-series and options-based variance swap valuation models on the S&P 500 index. Our research was designed to include the results from the best models shown in Chapters 3 and 4 including: 1) EGARCH; 2) GARCH variance swap model; 3) stochastic volatility model; 3) jump-diffusion model; 5) local volatility model; 6) ad hoc Black-Scholes model. Based on our limited sample, we showed that the Demterfi et al. framework overpredicts future variance and that time-series forecasting models have a smaller MSE. In particular, the local volatility model, which was the least parsimonious specification, had the best in-sample fitting performance but worst variance forecasting performance. We illustrated that the use of more flexible and sophisticated option pricing models within the context of the Demeterfi et al. framework might not be able to improve the performance of variance swap pricing. These findings had brought two major questions to our attention. First, since options-based models overpriced future variance by a huge margin, we asked whether option prices were consistent with time-series properties? Maximum likelihood estimation of the square-root process confirmed that the distributional dynamics implied by option prices not consistent with its underlying index. The implication of this finding was that academicians and practitioners alike would have to look for a way to integrate historical and market information in a composite option pricing model. The second question was why did local volatility perform so poorly in forecasting future variance given its excellent in-sample pricing performance? One explanation was that a flexible but theoretically inconsistent model might dominate in-sample fit but had much less predictive power for predicting future variance, which implied that a misspecified model achieves good in-sample results by overfitting the options data. As Bakshi et al. (2002) pointed out, the poor performance of one-factor models, such as the local volatility model, could also be a result of the monotonicity property and perfect correlation property that imposed a stringent constraint on how option prices could change with the underlying asset price. Therefore extreme caution must be taken when using the options for volatility forecasting.

6.4.4 Final Comment

Finally, a note has to be made in regard to the use of S&P 500 data in this dissertation. Although the S&P 500 market data were employed throughout our analysis, we must stress that our findings are not likely to be market specific. It is important for investors to understand the S&P 500 market because S&P 500 products are one of the most liquid contracts in the financial world. However, we expect results obtained in this dissertation can be generalised to other markets as well.

6.5 Recommendations for Further Research

This dissertation has provided new insights into modelling volatility but also raised many new questions. The following areas are recommended for additional research.

Firstly, there is an urgent need to establish a consensus on whether option prices, which are forward-looking, should be used for forecasting purposes. Recent studies such as Dumas et al. (1998) and Gemmill et al. (1999) cast doubt on the usefulness of option prices as forecasting tools. As the latter points out, options only react to crucial events but they do not predict them. According to Flamouris (2001), the criterion for the goodness of a implied distribution more often was the fit it provides to the observed option prices and less frequently its ability to forecast the statistical properties of future data. Given the fact that the main advantage of using options-based methodologies is the no-arbitrage pricing of exotic and vanilla products, perhaps further research should first consider work along the lines of hedging performance of options-based variance swap models.

Secondly, one might want to repeat our variance forecasting experiments using a larger sample set. This will allow one to infer a statistically significant result to conclude whether time-series models outperform option-based models in forecasting volatility. In addition, forecast periods should be non-overlapping so the sample does not consist of dependent observations[84].

Thirdly, it would be of interest to extend the work on the overreaction hypothesis in Chapter 3. Since we restricted our investigation to testing the rational expectation hypothesis of Stein (1989) using aggregated data, further analysis can be performed on daily data using fixed maturity series to check whether option prices are really consistent with the AR(1) specification.

Fourthly, an interesting extension of our work on model rankings in Chapter 4 would be to compare the options-based results using alternative economic criteria. For instance, Lopez (1995) presented the idea of transforming volatility forecasts into probability forecasts. However, one should be cautious when using these metrics as it is not clear whose utility function they reflect (Orakcioglu, 2000). Alternative trading approaches are also possible, but with certain caveats. For example, although it does not seem practicable, one can trade options on daily basis. One can also experiment with the size of the data window when estimating the structural parameters, or match the forecasting horizon to the remaining maturity of the straddle. All these amendments may lead to different results.

Fifthly, special attention should be given to the incorporation of jumps into the delivery prices of variance swap within a theoretically consistent framework. Currently, the dominant Demeterfi et al. framework requires price continuity and a consistent stochastic volatility model for options to price variance swaps. As maturity gets longer one would expect more jumps to occur. Since the evidence in Chapter 5 shows that the replication of log-contracts through traded options is not an effective way to forecast future variance, another possible extension would be to conduct Monte-Carlo simulations on log-contracts.

Next, a reasonable improvement in estimating structural parameters of stochastic volatility models in Chapter 5 can be achieved by using more advanced econometric techniques such as moment matching procedures employed by Scott (1987) and Wiggins (1987). Unlike maximum likelihood estimation method used in Bakshi et al. (1997) and this study, moment matching procedures do not assume a priori a distribution. Thus it can offer an alternative

[84] We thank Roy Batchelor for pointing out the problems associated with Chapter 5 of this dissertation.

view of finding out whether option prices are consistent with the time-series properties of the underlying asset.

Finally, and most importantly, we must change our view that volatility is not a tradable asset. For example, the MONEP created the VX1 and VX6 indexes in October 1997; on January 19, 1998, the Deutsche Terminborse (DTB) became the first exchange in the world to list volatility futures based on an underlying equity index of implied volatility when it launched the VOLAX futures. Recent advances in financial engineering have also developed a number of ways to trade volatility contracts (see Howison et al., 2001). Yet it is difficult to conduct research on these volatility contracts because their existence is largely at the development stage and there is no liquid market to test any potential models. Therefore, the investigation of modelling more complex volatility products such as options must be postponed until more OTC data are available.

EPILOGUE

Volatility is a timeliness subject. It is one of the core concepts of financial theory, especially in modern portfolio theory, risk management and option pricing. The past two decades have witnessed an explosion of volatility models, both in option pricing and forecasting, in order to take account of the imperfections displayed in options market and time-series. As part of this research, I have implemented, applied and scrutinised many volatility models from a practical perspective. I believe that recent studies have attached too much weight to theory and financial research is frequently devoid of financial logic and argument. In this dissertation, I have tried to strike a balance between practicalities and technicalities whilst not scarifying any academic vigour. I believe that more immediate question does not lie in the realm of more complex models but in checking out their market performance in terms of forecasting, trading and pricing. Finally, I stress that analytical skills are as important as mathematical skills, and studying finance is as much an art as a science.

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APPENDICES

A.1 APARCH Models

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$$
$$\varepsilon_t = s_t e_t \qquad e_t \sim N(0,1)$$
$$\alpha_0 > 0, \delta \geq 0,$$
$$\alpha_i \geq 0, i = 1, \ldots, p,$$
$$-1 < \gamma_i < 1, i = 1, \ldots, p,$$
$$\beta_j \geq 0, j = 1, \ldots, q$$

Model 1: APARCH (Base Model)

$$s_t^{\sigma} = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^{\delta} + \sum_{j=1}^{q} \beta_j s_{t-j}^{\delta}$$

Model 2: ARCH

$$\delta = 2, \gamma_i = 0, \beta_j = 0$$
$$s_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}{}^2$$

Model 3: GARCH

$$\delta = 2, \gamma_i = 0$$
$$s_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}{}^2 + \sum_{j=1}^{q} \beta_j s_{t-j}^2$$

Model 4: Taylor & Schwert's GARCH – I

$$\delta = 1, \gamma_i = 0$$
$$s_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i |\varepsilon_{t-i}| + \sum_{j=1}^{q} \beta_j s_{t-j}$$

Model 5: Taylor & Schwert's GARCH – II

$$\delta = 1$$
$$s_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}) + \sum_{j=1}^{q} \beta_j s_{t-j}$$

Model 6: GJR

$$\delta = 2$$
$$s_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^{q} \beta_j s_{t-j}^2$$

Model 7: TARCH

$$\delta = 1, \beta_j = 0$$
$$s_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})$$

A.2 In-Sample Model Selection Criteria

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left[\hat{\sigma}_t^2 - \sigma_t^2\right]^2$$ *Mean – Square Error*

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|\hat{\sigma}_t^2 - \sigma_t^2\right|$$ *Mean – Absolute Error*

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\frac{\left|\hat{\sigma}_t^2 - \sigma_t^2\right|}{\sigma_t^2}$$ *Mean – Absolute Percent Error*

$$MME(U) = \frac{1}{T}\left[\sum_{t=1}^{O}\left|\hat{\sigma}_t^2 - \sigma_t^2\right| + \sum_{t=1}^{U}\sqrt{\left|\hat{\sigma}_t^2 - \sigma_t^2\right|}\right]$$ *Mean – Mixed Error (under prediction)*

(Brailsford and Faff, 1996)

$$MME(O) = \frac{1}{T}\left[\sum_{t=1}^{U}\left|\hat{\sigma}_t^2 - \sigma_t^2\right| + \sum_{t=1}^{O}\sqrt{\left|\hat{\sigma}_t^2 - \sigma_t^2\right|}\right]$$ *Mean – Mixed Error (over prediction)*

(Brailsford and Faff, 1996)

$$LL = \frac{1}{T}\sum_{t=1}^{T}\left[\ln(\hat{\sigma}_t^2) - \ln(\sigma_t^2)\right]^2$$ *Logarithmic Loss*

(Pagan and Schwert, 1990)

$$HMSE = \frac{1}{T}\sum_{t=1}^{T}\left[\frac{\sigma_t^2}{\hat{\sigma}_t^2} - 1\right]^2$$ *Heteroskedasticity – adjusted MSE*

(Bollerslev and Ghysels, 1994)

$$GMLE = \frac{1}{T}\sum_{t=1}^{T}\left[\ln(\hat{\sigma}_t^2) + \frac{\sigma_t^2}{\hat{\sigma}_t^2}\right]$$ *Guassian quasi – MLE*

(Bollerslev et al., 1994)

B.1 June 15, 2001 Call Options

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1214.35	995	227.4	225.4	226.4	0.4356	07/21/01	0.0959	3.5200	0.6479	1
1214.35	1005	217.5	215.5	216.5	0.4198	07/21/01	0.0959	3.5200	0.6479	1
1214.35	1075	150	148	149	0.3361	07/21/01	0.0959	3.5200	0.6479	22
1214.35	1100	126.5	124.5	125.5	0.3084	07/21/01	0.0959	3.5200	0.6479	277
1214.35	1125	103.7	101.7	102.7	0.2833	07/21/01	0.0959	3.5200	0.6479	209
1214.35	1150	82	80	81	0.2611	07/21/01	0.0959	3.5200	0.6479	257
1214.35	1175	62.2	60.2	61.2	0.2439	07/21/01	0.0959	3.5200	0.6479	29
1214.35	1200	44.2	42.2	43.2	0.2257	07/21/01	0.0959	3.5200	0.6479	1882
1214.35	1225	29.3	27.3	28.3	0.2112	07/21/01	0.0959	3.5200	0.6479	2443
1214.35	1250	17.3	16.3	16.8	0.1986	07/21/01	0.0959	3.5200	0.6479	9026
1214.35	1275	9.9	8.9	9.4	0.1927	07/21/01	0.0959	3.5200	0.6479	11371
1214.35	1280	8.4	7.4	7.9	0.1881	07/21/01	0.0959	3.5200	0.6479	618
1214.35	1285	7.6	6.6	7.1	0.1890	07/21/01	0.0959	3.5200	0.6479	3283
1214.35	1300	5.1	4.4	4.75	0.1874	07/21/01	0.0959	3.5200	0.6479	6717
1214.35	1325	2.35	1.9	2.125	0.1824	07/21/01	0.0959	3.5200	0.6479	4321
1214.35	1350	1	0.9	0.95	0.1819	07/21/01	0.0959	3.5200	0.6479	10642
1214.35	1375	0.7	0.4	0.55	0.1905	07/21/01	0.0959	3.5200	0.6479	4452
1214.35	1025	202.3	200.3	201.3	0.3217	08/18/01	0.1726	3.5200	0.8969	10
1214.35	1175	73.3	71.3	72.3	0.2331	08/18/01	0.1726	3.5200	0.8969	781
1214.35	1200	56.3	54.3	55.3	0.2216	08/18/01	0.1726	3.5200	0.8969	953
1214.35	1250	29.3	27.3	28.3	0.2024	08/18/01	0.1726	3.5200	0.8969	3831
1214.35	1275	19.8	18.3	19.05	0.1965	08/18/01	0.1726	3.5200	0.8969	2083
1214.35	1300	12.8	11.3	12.05	0.1905	08/18/01	0.1726	3.5200	0.8969	2993
1214.35	1325	7.5	6.8	7.15	0.1851	08/18/01	0.1726	3.5200	0.8969	614
1214.35	1350	4.5	3.8	4.15	0.1824	08/18/01	0.1726	3.5200	0.8969	1039
1214.35	1375	2.7	2	2.35	0.1810	08/18/01	0.1726	3.5200	0.8969	2534
1214.35	1400	1.5	1.05	1.275	0.1799	08/18/01	0.1726	3.5200	0.8969	1719
1214.35	1425	0.9	0.45	0.675	0.1794	08/18/01	0.1726	3.5200	0.8969	164
1214.35	800	424.2	422.2	423.2	0.4289	09/22/01	0.2685	3.5222	0.9658	4715
1214.35	1050	186	184	185	0.2777	09/22/01	0.2685	3.5222	0.9658	267
1214.35	1100	142.5	140.5	141.5	0.2548	09/22/01	0.2685	3.5222	0.9658	2158
1214.35	1125	122.2	120.2	121.2	0.2454	09/22/01	0.2685	3.5222	0.9658	167
1214.35	1150	102.7	100.7	101.7	0.2350	09/22/01	0.2685	3.5222	0.9658	4169
1214.35	1200	68.7	66.7	67.7	0.2196	09/22/01	0.2685	3.5222	0.9658	9200
1214.35	1225	54.1	52.1	53.1	0.2119	09/22/01	0.2685	3.5222	0.9658	3077
1214.35	1240	46.7	44.7	45.7	0.2095	09/22/01	0.2685	3.5222	0.9658	434
1214.35	1250	41.5	39.5	40.5	0.2053	09/22/01	0.2685	3.5222	0.9658	17406
1214.35	1260	37.1	35.1	36.1	0.2031	09/22/01	0.2685	3.5222	0.9658	407
1214.35	1275	31.1	29.1	30.1	0.1999	09/22/01	0.2685	3.5222	0.9658	6423
1214.35	1285	27.3	25.3	26.3	0.1970	09/22/01	0.2685	3.5222	0.9658	1104
1214.35	1300	22.2	20.2	21.2	0.1928	09/22/01	0.2685	3.5222	0.9658	8441
1214.35	1325	15.5	14	14.75	0.1887	09/22/01	0.2685	3.5222	0.9658	5339
1214.35	1350	10.5	9.5	10	0.1857	09/22/01	0.2685	3.5222	0.9658	11196
1214.35	1375	6.5	6	6.25	0.1804	09/22/01	0.2685	3.5222	0.9658	10587
1214.35	1400	4.4	3.7	4.05	0.1792	09/22/01	0.2685	3.5222	0.9658	7051
1214.35	1425	2.85	2.2	2.525	0.1777	09/22/01	0.2685	3.5222	0.9658	4733
1214.35	1450	1.7	1.25	1.475	0.1753	09/22/01	0.2685	3.5222	0.9658	5744
1214.35	1475	1.05	0.6	0.825	0.1729	09/22/01	0.2685	3.5222	0.9658	4431
1214.35	1500	0.9	0.45	0.675	0.1806	09/22/01	0.2685	3.5222	0.9658	2766

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1214.35	1525	0.65	0.2	0.425	0.1816	09/22/01	0.2685	3.5222	0.9658	3482
1214.35	800	430.7	428.7	429.7	0.3021	12/22/01	0.5178	3.5493	1.0005	7
1214.35	900	336.3	334.3	335.3	0.2849	12/22/01	0.5178	3.5493	1.0005	1663
1214.35	950	290.6	288.6	289.6	0.2748	12/22/01	0.5178	3.5493	1.0005	2
1214.35	995	250.7	248.7	249.7	0.2652	12/22/01	0.5178	3.5493	1.0005	1680
1214.35	1025	224.4	222.4	223.4	0.2558	12/22/01	0.5178	3.5493	1.0005	1462
1214.35	1050	203.6	201.6	202.6	0.2509	12/22/01	0.5178	3.5493	1.0005	252
1214.35	1100	163.5	161.5	162.5	0.2388	12/22/01	0.5178	3.5493	1.0005	1264
1214.35	1150	126.8	124.8	125.8	0.2275	12/22/01	0.5178	3.5493	1.0005	4531
1214.35	1175	109.8	107.8	108.8	0.2216	12/22/01	0.5178	3.5493	1.0005	1328
1214.35	1200	94.3	92.3	93.3	0.2169	12/22/01	0.5178	3.5493	1.0005	7006
1214.35	1225	79.5	77.5	78.5	0.2108	12/22/01	0.5178	3.5493	1.0005	4005
1214.35	1250	66.7	64.7	65.7	0.2069	12/22/01	0.5178	3.5493	1.0005	7606
1214.35	1275	54.9	52.9	53.9	0.2022	12/22/01	0.5178	3.5493	1.0005	3996
1214.35	1300	44.1	42.1	43.1	0.1965	12/22/01	0.5178	3.5493	1.0005	15712
1214.35	1325	35.2	33.2	34.2	0.1924	12/22/01	0.5178	3.5493	1.0005	8315
1214.35	1350	27.7	25.7	26.7	0.1887	12/22/01	0.5178	3.5493	1.0005	6407
1214.35	1375	21.3	19.8	20.55	0.1855	12/22/01	0.5178	3.5493	1.0005	1534
1214.35	1400	16	14.5	15.25	0.1813	12/22/01	0.5178	3.5493	1.0005	12452
1214.35	1425	12	10.5	11.25	0.1784	12/22/01	0.5178	3.5493	1.0005	3918
1214.35	1450	8.8	7.8	8.3	0.1766	12/22/01	0.5178	3.5493	1.0005	9054
1214.35	1475	6.5	5.5	6	0.1747	12/22/01	0.5178	3.5493	1.0005	83
1214.35	1500	4.7	4	4.35	0.1736	12/22/01	0.5178	3.5493	1.0005	12551
1214.35	1525	3.4	2.7	3.05	0.1720	12/22/01	0.5178	3.5493	1.0005	777
1214.35	1550	2.35	1.9	2.125	0.1707	12/22/01	0.5178	3.5493	1.0005	5363
1214.35	1575	1.7	1.25	1.475	0.1698	12/22/01	0.5178	3.5493	1.0005	145
1214.35	1600	1.4	0.95	1.175	0.1728	12/22/01	0.5178	3.5493	1.0005	10306
1214.35	1650	0.8	0.35	0.575	0.1723	12/22/01	0.5178	3.5493	1.0005	4125
1214.35	1675	0.6	0.15	0.375	0.1709	12/22/01	0.5178	3.5493	1.0005	525
1214.35	1025	238.5	236.5	237.5	0.2446	03/16/02	0.7479	3.5401	1.0018	28
1214.35	1050	218.5	216.5	217.5	0.2406	03/16/02	0.7479	3.5401	1.0018	1433
1214.35	1100	180.1	178.1	179.1	0.2314	03/16/02	0.7479	3.5401	1.0018	191
1214.35	1125	162.4	160.4	161.4	0.2281	03/16/02	0.7479	3.5401	1.0018	289
1214.35	1150	145.2	143.2	144.2	0.2238	03/16/02	0.7479	3.5401	1.0018	748
1214.35	1175	128.8	126.8	127.8	0.2194	03/16/02	0.7479	3.5401	1.0018	19
1214.35	1200	113.2	111.2	112.2	0.2146	03/16/02	0.7479	3.5401	1.0018	2384
1214.35	1225	98.6	96.6	97.6	0.2100	03/16/02	0.7479	3.5401	1.0018	238
1214.35	1250	85.4	83.4	84.4	0.2062	03/16/02	0.7479	3.5401	1.0018	622
1214.35	1275	73.3	71.3	72.3	0.2027	03/16/02	0.7479	3.5401	1.0018	3160
1214.35	1300	62.2	60.2	61.2	0.1989	03/16/02	0.7479	3.5401	1.0018	4015
1214.35	1325	52.2	50.2	51.2	0.1952	03/16/02	0.7479	3.5401	1.0018	9
1214.35	1350	43.6	41.6	42.6	0.1923	03/16/02	0.7479	3.5401	1.0018	1001
1214.35	1375	35.7	33.7	34.7	0.1884	03/16/02	0.7479	3.5401	1.0018	441
1214.35	1400	28.8	26.8	27.8	0.1846	03/16/02	0.7479	3.5401	1.0018	7239
1214.35	1425	23.2	21.2	22.2	0.1817	03/16/02	0.7479	3.5401	1.0018	297
1214.35	1450	18.2	16.7	17.45	0.1788	03/16/02	0.7479	3.5401	1.0018	1703
1214.35	1475	14.2	12.7	13.45	0.1757	03/16/02	0.7479	3.5401	1.0018	16
1214.35	1500	11	10	10.5	0.1741	03/16/02	0.7479	3.5401	1.0018	869
1214.35	1600	3.7	3	3.35	0.1670	03/16/02	0.7479	3.5401	1.0018	365
1214.35	1050	237.3	234.3	235.8	0.2403	06/22/02	1.0164	3.5379	1.0019	1
1214.35	1100	200.7	197.7	199.2	0.2331	06/22/02	1.0164	3.5379	1.0019	102
1214.35	1150	166.9	163.9	165.4	0.2262	06/22/02	1.0164	3.5379	1.0019	1350
1214.35	1200	136	133	134.5	0.2192	06/22/02	1.0164	3.5379	1.0019	2788
1214.35	1250	108.2	105.2	106.7	0.2119	06/22/02	1.0164	3.5379	1.0019	1577

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1214.35	1300	84.2	81.2	82.7	0.2055	06/22/02	1.0164	3.5379	1.0019	4395
1214.35	1350	64.1	61.1	62.6	0.1999	06/22/02	1.0164	3.5379	1.0019	4656
1214.35	1400	47.4	44.4	45.9	0.1942	06/22/02	1.0164	3.5379	1.0019	6940
1214.35	1450	33.9	30.9	32.4	0.1883	06/22/02	1.0164	3.5379	1.0019	6897
1214.35	1500	23.1	20.1	21.6	0.1814	06/22/02	1.0164	3.5379	1.0019	5974
1214.35	1550	14.7	13.2	13.95	0.1758	06/22/02	1.0164	3.5379	1.0019	851
1214.35	1600	9.4	8.4	8.9	0.1719	06/22/02	1.0164	3.5379	1.0019	6076
1214.35	1650	6.1	5.1	5.6	0.1692	06/22/02	1.0164	3.5379	1.0019	2849
1214.35	1700	3.7	3	3.35	0.1660	06/22/02	1.0164	3.5379	1.0019	5639
1214.35	1750	2.2	1.75	1.975	0.1636	06/22/02	1.0164	3.5379	1.0019	1120
1214.35	1800	1.35	0.9	1.125	0.1612	06/22/02	1.0164	3.5379	1.0019	4132
1214.35	1850	0.85	0.4	0.625	0.1591	06/22/02	1.0164	3.5379	1.0019	1150
1214.35	1900	0.6	0.15	0.375	0.1589	06/22/02	1.0164	3.5379	1.0019	3644
1214.35	1100	233.5	230.5	232	0.2290	12/21/02	1.5151	3.7772	1.0019	64
1214.35	1150	201.1	198.1	199.6	0.2238	12/21/02	1.5151	3.7772	1.0019	1785
1214.35	1200	171	168	169.5	0.2184	12/21/02	1.5151	3.7772	1.0019	4897
1214.35	1250	143.2	140.2	141.7	0.2126	12/21/02	1.5151	3.7772	1.0019	4127
1214.35	1300	118.4	115.4	116.9	0.2073	12/21/02	1.5151	3.7772	1.0019	3116
1214.35	1350	96.3	93.3	94.8	0.2022	12/21/02	1.5151	3.7772	1.0019	1367
1214.35	1400	77.5	74.5	76	0.1979	12/21/02	1.5151	3.7772	1.0019	3569
1214.35	1450	60.6	57.6	59.1	0.1923	12/21/02	1.5151	3.7772	1.0019	3783
1214.35	1500	46.4	43.4	44.9	0.1868	12/21/02	1.5151	3.7772	1.0019	3740
1214.35	1550	35.1	32.1	33.6	0.1823	12/21/02	1.5151	3.7772	1.0019	1667
1214.35	1600	26.3	23.3	24.8	0.1786	12/21/02	1.5151	3.7772	1.0019	4587
1214.35	1650	18.6	17.1	17.85	0.1748	12/21/02	1.5151	3.7772	1.0019	1060
1214.35	1700	13.2	11.7	12.45	0.1709	12/21/02	1.5151	3.7772	1.0019	2046
1214.35	1800	6.4	5.4	5.9	0.1653	12/21/02	1.5151	3.7772	1.0019	1200
1214.35	1900	3.1	2.4	2.75	0.1621	12/21/02	1.5151	3.7772	1.0019	6500

B.2 July 20, 2001 Call Options

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1210.85	1050	164	162	163	0.2144	08/18/01	0.0767	3.5300	0.8520	362
1210.85	1075	139.9	137.9	138.9	0.2388	08/18/01	0.0767	3.5300	0.8520	3
1210.85	1100	116.1	114.1	115.1	0.2341	08/18/01	0.0767	3.5300	0.8520	660
1210.85	1125	93.2	91.2	92.2	0.2280	08/18/01	0.0767	3.5300	0.8520	326
1210.85	1150	71.4	69.4	70.4	0.2177	08/18/01	0.0767	3.5300	0.8520	1237
1210.85	1175	51.7	49.7	50.7	0.2090	08/18/01	0.0767	3.5300	0.8520	3451
1210.85	1200	34.3	32.3	33.3	0.1967	08/18/01	0.0767	3.5300	0.8520	4437
1210.85	1225	21	19.5	20.25	0.1911	08/18/01	0.0767	3.5300	0.8520	14228
1210.85	1250	11	10	10.5	0.1812	08/18/01	0.0767	3.5300	0.8520	10578
1210.85	1275	5.4	4.7	5.05	0.1782	08/18/01	0.0767	3.5300	0.8520	15186
1210.85	1300	2.7	2	2.35	0.1797	08/18/01	0.0767	3.5300	0.8520	15988
1210.85	1325	1	0.7	0.85	0.1756	08/18/01	0.0767	3.5300	0.8520	4888
1210.85	1050	170.3	168.3	169.3	0.2357	09/22/01	0.1726	3.5300	1.0870	279
1210.85	1100	124.9	122.9	123.9	0.2233	09/22/01	0.1726	3.5300	1.0870	2150
1210.85	1125	103.6	101.6	102.6	0.2166	09/22/01	0.1726	3.5300	1.0870	644
1210.85	1150	83.8	81.8	82.8	0.2108	09/22/01	0.1726	3.5300	1.0870	4212
1210.85	1190	55.4	53.4	54.4	0.1990	09/22/01	0.1726	3.5300	1.0870	483
1210.85	1200	49.6	47.6	48.6	0.1984	09/22/01	0.1726	3.5300	1.0870	14007
1210.85	1210	43.6	41.6	42.6	0.1948	09/22/01	0.1726	3.5300	1.0870	7358
1210.85	1225	35.5	33.5	34.5	0.1903	09/22/01	0.1726	3.5300	1.0870	27100
1210.85	1240	28.5	26.5	27.5	0.1867	09/22/01	0.1726	3.5300	1.0870	4860
1210.85	1250	24.7	22.7	23.7	0.1861	09/22/01	0.1726	3.5300	1.0870	21295
1210.85	1275	15.8	14.3	15.05	0.1798	09/22/01	0.1726	3.5300	1.0870	6893
1210.85	1285	13.3	11.8	12.55	0.1789	09/22/01	0.1726	3.5300	1.0870	1202
1210.85	1300	9.6	8.6	9.1	0.1755	09/22/01	0.1726	3.5300	1.0870	10994
1210.85	1325	5.6	4.9	5.25	0.1726	09/22/01	0.1726	3.5300	1.0870	10914
1210.85	1350	3.3	2.6	2.95	0.1715	09/22/01	0.1726	3.5300	1.0870	12910
1210.85	1375	1.8	1.35	1.575	0.1705	09/22/01	0.1726	3.5300	1.0870	9773
1210.85	1400	1.2	0.75	0.975	0.1751	09/22/01	0.1726	3.5300	1.0870	6684
1210.85	1425	0.7	0.25	0.475	0.1735	09/22/01	0.1726	3.5300	1.0870	4933
1210.85	1450	0.45	0.3	0.375	0.1838	09/22/01	0.1726	3.5300	1.0870	5829
1210.85	1025	208.8	206.8	207.8	0.2219	12/22/01	0.4219	3.5300	1.2069	1462
1210.85	1050	187.3	185.3	186.3	0.2204	12/22/01	0.4219	3.5300	1.2069	700
1210.85	1060	178.8	176.8	177.8	0.2190	12/22/01	0.4219	3.5300	1.2069	450
1210.85	1100	145.7	143.7	144.7	0.2111	12/22/01	0.4219	3.5300	1.2069	1708
1210.85	1150	108.6	106.6	107.6	0.2041	12/22/01	0.4219	3.5300	1.2069	4545
1210.85	1175	91.8	89.8	90.8	0.2001	12/22/01	0.4219	3.5300	1.2069	1411
1210.85	1200	76.4	74.4	75.4	0.1961	12/22/01	0.4219	3.5300	1.2069	7525
1210.85	1250	49.6	47.6	48.6	0.1863	12/22/01	0.4219	3.5300	1.2069	8715
1210.85	1275	38.2	36.2	37.2	0.1801	12/22/01	0.4219	3.5300	1.2069	4858
1210.85	1300	29.4	27.4	28.4	0.1770	12/22/01	0.4219	3.5300	1.2069	16318
1210.85	1325	21.5	20	20.75	0.1723	12/22/01	0.4219	3.5300	1.2069	8751
1210.85	1350	15.9	14.4	15.15	0.1700	12/22/01	0.4219	3.5300	1.2069	6892
1210.85	1375	11.6	10.1	10.85	0.1681	12/22/01	0.4219	3.5300	1.2069	2822
1210.85	1400	8.2	7.2	7.7	0.1670	12/22/01	0.4219	3.5300	1.2069	14456
1210.85	1425	5.3	4.6	4.95	0.1628	12/22/01	0.4219	3.5300	1.2069	4269

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1210.85	1450	3.6	2.9	3.25	0.1609	12/22/01	0.4219	3.5300	1.2069	9344
1210.85	1475	2.35	1.9	2.125	0.1598	12/22/01	0.4219	3.5300	1.2069	112
1210.85	1500	1.75	1.3	1.525	0.1618	12/22/01	0.4219	3.5300	1.2069	13476
1210.85	1525	1.15	0.7	0.925	0.1597	12/22/01	0.4219	3.5300	1.2069	785
1210.85	1550	0.85	0.4	0.625	0.1606	12/22/01	0.4219	3.5300	1.2069	5218
1210.85	1575	0.6	0.15	0.375	0.1593	12/22/01	0.4219	3.5300	1.2069	155
1210.85	1025	221.6	219.6	220.6	0.2103	03/16/02	0.6521	3.5482	1.2115	28
1210.85	1050	201	199	200	0.2089	03/16/02	0.6521	3.5482	1.2115	1433
1210.85	1100	162.1	160.1	161.1	0.2050	03/16/02	0.6521	3.5482	1.2115	202
1210.85	1125	143.6	141.6	142.6	0.2016	03/16/02	0.6521	3.5482	1.2115	339
1210.85	1150	126.3	124.3	125.3	0.1988	03/16/02	0.6521	3.5482	1.2115	750
1210.85	1175	110	108	109	0.1957	03/16/02	0.6521	3.5482	1.2115	19
1210.85	1200	94.8	92.8	93.8	0.1925	03/16/02	0.6521	3.5482	1.2115	2982
1210.85	1225	80.2	78.2	79.2	0.1877	03/16/02	0.6521	3.5482	1.2115	757
1210.85	1250	67.6	65.6	66.6	0.1849	03/16/02	0.6521	3.5482	1.2115	1433
1210.85	1275	56.2	54.2	55.2	0.1819	03/16/02	0.6521	3.5482	1.2115	3782
1210.85	1300	46	44	45	0.1786	03/16/02	0.6521	3.5482	1.2115	3889
1210.85	1325	37.1	35.1	36.1	0.1753	03/16/02	0.6521	3.5482	1.2115	340
1210.85	1350	29.4	27.4	28.4	0.1719	03/16/02	0.6521	3.5482	1.2115	973
1210.85	1375	23.2	21.2	22.2	0.1695	03/16/02	0.6521	3.5482	1.2115	444
1210.85	1400	17.7	16.2	16.95	0.1667	03/16/02	0.6521	3.5482	1.2115	7715
1210.85	1425	13.5	12	12.75	0.1643	03/16/02	0.6521	3.5482	1.2115	294
1210.85	1450	10.1	9.1	9.6	0.1628	03/16/02	0.6521	3.5482	1.2115	1965
1210.85	1475	7.4	6.4	6.9	0.1601	03/16/02	0.6521	3.5482	1.2115	84
1210.85	1500	5	4.7	4.85	0.1576	03/16/02	0.6521	3.5482	1.2115	788
1210.85	1600	1.5	1.05	1.275	0.1551	03/16/02	0.6521	3.5482	1.2115	464
1210.85	1325	54.7	52.7	53.7	0.1786	06/22/02	0.9205	3.5805	1.2117	1010
1210.85	1425	25.8	23.8	24.8	0.1690	06/22/02	0.9205	3.5805	1.2117	1
1210.85	1100	212.9	209.9	211.4	0.2031	12/21/02	1.4192	3.7535	1.2117	64
1210.85	1150	180.4	177.4	178.9	0.2002	12/21/02	1.4192	3.7535	1.2117	1980
1210.85	1200	150.4	147.4	148.9	0.1963	12/21/02	1.4192	3.7535	1.2117	5218
1210.85	1250	123.1	120.1	121.6	0.1918	12/21/02	1.4192	3.7535	1.2117	4484
1210.85	1300	98	95	96.5	0.1857	12/21/02	1.4192	3.7535	1.2117	3846
1210.85	1350	77.1	74.1	75.6	0.1815	12/21/02	1.4192	3.7535	1.2117	1867
1210.85	1400	59.1	56.1	57.6	0.1767	12/21/02	1.4192	3.7535	1.2117	4417
1210.85	1450	44.1	41.1	42.6	0.1718	12/21/02	1.4192	3.7535	1.2117	4930
1210.85	1500	32.9	29.9	31.4	0.1689	12/21/02	1.4192	3.7535	1.2117	3740
1210.85	1550	24.3	21.3	22.8	0.1666	12/21/02	1.4192	3.7535	1.2117	1717
1210.85	1600	16.3	14.8	15.55	0.1623	12/21/02	1.4192	3.7535	1.2117	6711
1210.85	1650	10.9	9.9	10.4	0.1588	12/21/02	1.4192	3.7535	1.2117	1065
1210.85	1700	7.2	6.2	6.7	0.1553	12/21/02	1.4192	3.7535	1.2117	2071
1210.85	1800	3.5	2.8	3.15	0.1546	12/21/02	1.4192	3.7535	1.2117	12350
1210.85	1900	1.55	1.1	1.325	0.1525	12/21/02	1.4192	3.7535	1.2117	6540

B.3 August 17, 2001 Call Options

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1161.95	800	365.6	363.6	364.6	0.5364	09/22/01	0.0959	3.4525	0.6392	5774
1161.95	1025	145.6	143.6	144.6	0.3187	09/22/01	0.0959	3.4525	0.6392	15
1161.95	1050	121.8	119.8	120.8	0.2897	09/22/01	0.0959	3.4525	0.6392	284
1161.95	1100	78.2	76.2	77.2	0.2563	09/22/01	0.0959	3.4525	0.6392	2369
1161.95	1125	58.4	56.4	57.4	0.2389	09/22/01	0.0959	3.4525	0.6392	1633
1161.95	1150	41.1	39.1	40.1	0.2244	09/22/01	0.0959	3.4525	0.6392	5930
1161.95	1190	18.5	17.5	18	0.1980	09/22/01	0.0959	3.4525	0.6392	9816
1161.95	1200	15	14	14.5	0.1962	09/22/01	0.0959	3.4525	0.6392	27223
1161.95	1210	12.4	11	11.7	0.1961	09/22/01	0.0959	3.4525	0.6392	10767
1161.95	1225	8	7.3	7.65	0.1896	09/22/01	0.0959	3.4525	0.6392	30525
1161.95	1240	5.1	4.6	4.85	0.1852	09/22/01	0.0959	3.4525	0.6392	5130
1161.95	1250	3.8	3.3	3.55	0.1835	09/22/01	0.0959	3.4525	0.6392	30757
1161.95	1275	1.95	1.5	1.725	0.1851	09/22/01	0.0959	3.4525	0.6392	9960
1161.95	1285	1.4	0.95	1.175	0.1829	09/22/01	0.0959	3.4525	0.6392	1232
1161.95	1300	0.7	0.35	0.525	0.1747	09/22/01	0.0959	3.4525	0.6392	21666
1161.95	900	275.2	273.2	274.2	0.2664	12/22/01	0.3452	3.3486	0.9878	1730
1161.95	950	229.4	227.4	228.4	0.2656	12/22/01	0.3452	3.3486	0.9878	2
1161.95	995	189.1	187.1	188.1	0.2541	12/22/01	0.3452	3.3486	0.9878	1931
1161.95	1025	163.4	161.4	162.4	0.2466	12/22/01	0.3452	3.3486	0.9878	1468
1161.95	1050	142.8	140.8	141.8	0.2400	12/22/01	0.3452	3.3486	0.9878	700
1161.95	1060	134.9	132.9	133.9	0.2378	12/22/01	0.3452	3.3486	0.9878	450
1161.95	1100	103.1	101.1	102.1	0.2204	12/22/01	0.3452	3.3486	0.9878	2257
1161.95	1150	70.2	68.2	69.2	0.2092	12/22/01	0.3452	3.3486	0.9878	4897
1161.95	1175	55.6	53.6	54.6	0.2016	12/22/01	0.3452	3.3486	0.9878	1757
1161.95	1200	43.3	41.3	42.3	0.1964	12/22/01	0.3452	3.3486	0.9878	9020
1161.95	1225	32.7	30.7	31.7	0.1907	12/22/01	0.3452	3.3486	0.9878	7039
1161.95	1250	24	22	23	0.1854	12/22/01	0.3452	3.3486	0.9878	13786
1161.95	1275	17.1	15.6	16.35	0.1816	12/22/01	0.3452	3.3486	0.9878	7011
1161.95	1300	12.2	10.7	11.45	0.1792	12/22/01	0.3452	3.3486	0.9878	17208
1161.95	1325	8.2	7.2	7.7	0.1763	12/22/01	0.3452	3.3486	0.9878	9602
1161.95	1350	5.5	4.8	5.15	0.1748	12/22/01	0.3452	3.3486	0.9878	8906
1161.95	1375	3.6	2.9	3.25	0.1722	12/22/01	0.3452	3.3486	0.9878	2816
1161.95	1400	2.1	1.65	1.875	0.1680	12/22/01	0.3452	3.3486	0.9878	15650
1161.95	1425	1.4	0.95	1.175	0.1677	12/22/01	0.3452	3.3486	0.9878	4464
1161.95	1450	0.95	0.5	0.725	0.1674	12/22/01	0.3452	3.3486	0.9878	9393
1161.95	1475	0.65	0.4	0.525	0.1711	12/22/01	0.3452	3.3486	0.9878	122
1161.95	900	283.4	281.4	282.4	0.2293	03/16/02	0.5753	3.3390	1.0046	18
1161.95	1025	177.4	175.4	176.4	0.2269	03/16/02	0.5753	3.3390	1.0046	28
1161.95	1050	157.7	155.7	156.7	0.2219	03/16/02	0.5753	3.3390	1.0046	1433
1161.95	1100	120.7	118.7	119.7	0.2111	03/16/02	0.5753	3.3390	1.0046	209
1161.95	1125	104.2	102.2	103.2	0.2071	03/16/02	0.5753	3.3390	1.0046	339
1161.95	1150	88.6	86.6	87.6	0.2023	03/16/02	0.5753	3.3390	1.0046	750
1161.95	1175	74.5	72.5	73.5	0.1980	03/16/02	0.5753	3.3390	1.0046	24
1161.95	1200	61.3	59.3	60.3	0.1927	03/16/02	0.5753	3.3390	1.0046	4023
1161.95	1225	49.8	47.8	48.8	0.1883	03/16/02	0.5753	3.3390	1.0046	1394
1161.95	1250	40.1	38.1	39.1	0.1849	03/16/02	0.5753	3.3390	1.0046	3302

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1161.95	1275	30.8	28.8	29.8	0.1786	03/16/02	0.5753	3.3390	1.0046	3032
1161.95	1300	24.3	22.3	23.3	0.1769	03/16/02	0.5753	3.3390	1.0046	3853
1161.95	1325	18.5	17	17.75	0.1745	03/16/02	0.5753	3.3390	1.0046	1826
1161.95	1350	13.7	12.2	12.95	0.1707	03/16/02	0.5753	3.3390	1.0046	2874
1161.95	1375	9.9	8.9	9.4	0.1682	03/16/02	0.5753	3.3390	1.0046	459
1161.95	1400	7.1	6.1	6.6	0.1653	03/16/02	0.5753	3.3390	1.0046	8375
1161.95	1425	4.9	4.2	4.55	0.1627	03/16/02	0.5753	3.3390	1.0046	294
1161.95	1450	3.4	2.7	3.05	0.1602	03/16/02	0.5753	3.3390	1.0046	1951
1161.95	1475	2.3	1.85	2.075	0.1589	03/16/02	0.5753	3.3390	1.0046	84
1161.95	1500	1.7	1.25	1.475	0.1593	03/16/02	0.5753	3.3390	1.0046	869
1161.95	1600	0.45	0.4	0.425	0.1642	03/16/02	0.5753	3.3390	1.0046	921
1161.95	850	337.8	335.8	336.8	0.1964	06/22/02	0.8438	3.3713	1.0054	286
1161.95	1325	32.7	30.7	31.7	0.1777	06/22/02	0.8438	3.3713	1.0054	1010
1161.95	1425	12.1	10.6	11.35	0.1640	06/22/02	0.8438	3.3713	1.0054	1
1161.95	900	316.4	313.4	314.9	0.2113	12/21/02	1.3425	3.4859	1.0054	552
1161.95	950	276.9	273.9	275.4	0.2134	12/21/02	1.3425	3.4859	1.0054	1
1161.95	1100	170.1	167.1	168.6	0.2035	12/21/02	1.3425	3.4859	1.0054	64
1161.95	1150	139.5	136.5	138	0.1981	12/21/02	1.3425	3.4859	1.0054	2622
1161.95	1200	112.5	109.5	111	0.1934	12/21/02	1.3425	3.4859	1.0054	5801
1161.95	1225	99.8	96.8	98.3	0.1903	12/21/02	1.3425	3.4859	1.0054	251
1161.95	1250	87.9	84.9	86.4	0.1871	12/21/02	1.3425	3.4859	1.0054	4534
1161.95	1300	67.7	64.7	66.2	0.1825	12/21/02	1.3425	3.4859	1.0054	4125
1161.95	1350	50.7	47.7	49.2	0.1776	12/21/02	1.3425	3.4859	1.0054	2016
1161.95	1400	36.8	33.8	35.3	0.1725	12/21/02	1.3425	3.4859	1.0054	4418
1161.95	1450	26.2	23.2	24.7	0.1682	12/21/02	1.3425	3.4859	1.0054	5431
1161.95	1500	17.5	16	16.75	0.1642	12/21/02	1.3425	3.4859	1.0054	4338
1161.95	1550	11.8	10.3	11.05	0.1607	12/21/02	1.3425	3.4859	1.0054	1577
1161.95	1600	7.5	6.5	7	0.1571	12/21/02	1.3425	3.4859	1.0054	6753
1161.95	1650	4.9	4.2	4.55	0.1557	12/21/02	1.3425	3.4859	1.0054	1070
1161.95	1700	3.2	2.5	2.85	0.1538	12/21/02	1.3425	3.4859	1.0054	4214
1161.95	1800	1.15	0.7	0.925	0.1481	12/21/02	1.3425	3.4859	1.0054	12361
1161.95	1900	0.6	0.15	0.375	0.1487	12/21/02	1.3425	3.4859	1.0054	6540

B.4 September 21, 2001 Call Options

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
965.8	800	179.2	175.2	177.2	0.6323	10/20/01	0.0767	2.1200	0.8605	167
965.8	900	91.6	87.6	89.6	0.4811	10/20/01	0.0767	2.1200	0.8605	177
965.8	975	40.5	36.5	38.5	0.3980	10/20/01	0.0767	2.1200	0.8605	75
965.8	995	30.5	27	28.75	0.3832	10/20/01	0.0767	2.1200	0.8605	2448
965.8	1010	24.4	20.4	22.4	0.3714	10/20/01	0.0767	2.1200	0.8605	4085
965.8	1020	20	17	18.5	0.3617	10/20/01	0.0767	2.1200	0.8605	2322
965.8	1025	18	15.3	16.65	0.3562	10/20/01	0.0767	2.1200	0.8605	4395
965.8	1030	16.9	13.9	15.4	0.3563	10/20/01	0.0767	2.1200	0.8605	694
965.8	1040	12.1	10.2	11.15	0.3324	10/20/01	0.0767	2.1200	0.8605	761
965.8	1050	10.5	9.1	9.8	0.3391	10/20/01	0.0767	2.1200	0.8605	15188
965.8	1060	9.1	7.1	8.1	0.3382	10/20/01	0.0767	2.1200	0.8605	1174
965.8	1070	7.7	5.7	6.7	0.3381	10/20/01	0.0767	2.1200	0.8605	566
965.8	1075	5.9	5.1	5.5	0.3280	10/20/01	0.0767	2.1200	0.8605	2050
965.8	1080	6	4.6	5.3	0.3342	10/20/01	0.0767	2.1200	0.8605	1075
965.8	1090	5	3.6	4.3	0.3337	10/20/01	0.0767	2.1200	0.8605	453
965.8	1100	3.8	2.8	3.3	0.3293	10/20/01	0.0767	2.1200	0.8605	8756
965.8	1125	2.15	1.5	1.825	0.3272	10/20/01	0.0767	2.1200	0.8605	5810
965.8	1150	1.45	0.8	1.125	0.3337	10/20/01	0.0767	2.1200	0.8605	8718
965.8	1175	0.9	0.25	0.575	0.3309	10/20/01	0.0767	2.1200	0.8605	7376
965.8	1200	1	0.3	0.65	0.3671	10/20/01	0.0767	2.1200	0.8605	7665
965.8	850	141.3	137.3	139.3	0.4562	11/17/01	0.1534	2.1747	1.3828	17
965.8	900	101.9	97.9	99.9	0.4152	11/17/01	0.1534	2.1747	1.3828	2
965.8	950	67.5	63.5	65.5	0.3772	11/17/01	0.1534	2.1747	1.3828	11
965.8	995	42.6	38.6	40.6	0.3479	11/17/01	0.1534	2.1747	1.3828	784
965.8	1025	28.9	24.9	26.9	0.3252	11/17/01	0.1534	2.1747	1.3828	1539
965.8	1050	20.4	17.4	18.9	0.3153	11/17/01	0.1534	2.1747	1.3828	648
965.8	1075	14.3	11.3	12.8	0.3066	11/17/01	0.1534	2.1747	1.3828	1043
965.8	1100	8.5	7.5	8	0.2951	11/17/01	0.1534	2.1747	1.3828	1165
965.8	1125	6	5.1	5.55	0.2969	11/17/01	0.1534	2.1747	1.3828	2038
965.8	1150	3.9	2.9	3.4	0.2912	11/17/01	0.1534	2.1747	1.3828	645
965.8	1175	2.65	1.75	2.2	0.2913	11/17/01	0.1534	2.1747	1.3828	1313
965.8	1200	2	1.1	1.55	0.2965	11/17/01	0.1534	2.1747	1.3828	3997
965.8	1225	1.55	0.65	1.1	0.3019	11/17/01	0.1534	2.1747	1.3828	2729
965.8	1250	1.25	0.35	0.8	0.3081	11/17/01	0.1534	2.1747	1.3828	5140
965.8	1275	1.05	0.15	0.6	0.3151	11/17/01	0.1534	2.1747	1.3828	99
965.8	1300	1.2	0.3	0.75	0.3441	11/17/01	0.1534	2.1747	1.3828	1466
965.8	1325	1.05	0.15	0.6	0.3520	11/17/01	0.1534	2.1747	1.3828	174
965.8	800	190.6	186.6	188.6	0.4384	12/22/01	0.2493	2.2495	1.6462	1784
965.8	900	111.2	107.2	109.2	0.3743	12/22/01	0.2493	2.2495	1.6462	2155
965.8	950	77.9	73.9	75.9	0.3461	12/22/01	0.2493	2.2495	1.6462	9
965.8	975	63.6	59.6	61.6	0.3340	12/22/01	0.2493	2.2495	1.6462	3
965.8	1025	38.5	34.5	36.5	0.3045	12/22/01	0.2493	2.2495	1.6462	5877
965.8	1050	29	25.3	27.15	0.2937	12/22/01	0.2493	2.2495	1.6462	7097
965.8	1060	26.2	22.2	24.2	0.2914	12/22/01	0.2493	2.2495	1.6462	455
965.8	1075	21.6	18.6	20.1	0.2873	12/22/01	0.2493	2.2495	1.6462	3926
965.8	1100	15.8	12.8	14.3	0.2798	12/22/01	0.2493	2.2495	1.6462	4371

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
965.8	1150	8	6.7	7.35	0.2746	12/22/01	0.2493	2.2495	1.6462	8558
965.8	1175	5.5	4.1	4.8	0.2680	12/22/01	0.2493	2.2495	1.6462	4721
965.8	1200	3.6	2.6	3.1	0.2632	12/22/01	0.2493	2.2495	1.6462	15940
965.8	1225	2.6	1.7	2.15	0.2636	12/22/01	0.2493	2.2495	1.6462	7937
965.8	1250	1.95	1.3	1.625	0.2682	12/22/01	0.2493	2.2495	1.6462	19339
965.8	1275	1.6	0.7	1.15	0.2698	12/22/01	0.2493	2.2495	1.6462	6948
965.8	1300	1.5	0.6	1.05	0.2816	12/22/01	0.2493	2.2495	1.6462	21389
965.8	1325	1.25	0.35	0.8	0.2855	12/22/01	0.2493	2.2495	1.6462	10051
965.8	1350	1.05	0.15	0.6	0.2886	12/22/01	0.2493	2.2495	1.6462	8999
965.8	1400	0.9	0.25	0.575	0.3137	12/22/01	0.2493	2.2495	1.6462	15367
965.8	900	125.7	121.7	123.7	0.3189	03/16/02	0.4795	2.3326	1.8399	31
965.8	950	94	90	92	0.3016	03/16/02	0.4795	2.3326	1.8399	54
965.8	995	69.4	65.4	67.4	0.2866	03/16/02	0.4795	2.3326	1.8399	289
965.8	1025	55.9	51.9	53.9	0.2794	03/16/02	0.4795	2.3326	1.8399	509
965.8	1050	45.8	41.8	43.8	0.2725	03/16/02	0.4795	2.3326	1.8399	2300
965.8	1075	37.1	33.1	35.1	0.2663	03/16/02	0.4795	2.3326	1.8399	71
965.8	1100	28.7	24.7	26.7	0.2560	03/16/02	0.4795	2.3326	1.8399	1225
965.8	1125	22.2	19.2	20.7	0.2511	03/16/02	0.4795	2.3326	1.8399	1191
965.8	1150	17.4	14.4	15.9	0.2472	03/16/02	0.4795	2.3326	1.8399	1352
965.8	1175	13.5	10.5	12	0.2433	03/16/02	0.4795	2.3326	1.8399	1291
965.8	1200	10	8	9	0.2403	03/16/02	0.4795	2.3326	1.8399	6416
965.8	1225	7.8	5.8	6.8	0.2388	03/16/02	0.4795	2.3326	1.8399	1598
965.8	1250	5.9	4.5	5.2	0.2385	03/16/02	0.4795	2.3326	1.8399	4369
965.8	1275	4.4	3	3.7	0.2351	03/16/02	0.4795	2.3326	1.8399	3082
965.8	1280	3.9	2.9	3.4	0.2338	03/16/02	0.4795	2.3326	1.8399	2
965.8	1300	3.3	2.3	2.8	0.2353	03/16/02	0.4795	2.3326	1.8399	4352
965.8	1325	2.55	1.65	2.1	0.2353	03/16/02	0.4795	2.3326	1.8399	2003
965.8	1350	2.05	1.15	1.6	0.2361	03/16/02	0.4795	2.3326	1.8399	3928
965.8	1375	1.7	0.8	1.25	0.2379	03/16/02	0.4795	2.3326	1.8399	800
965.8	1400	1.3	0.65	0.975	0.2395	03/16/02	0.4795	2.3326	1.8399	9296
965.8	1425	1.1	0.2	0.65	0.2363	03/16/02	0.4795	2.3326	1.8399	294
965.8	1450	0.95	0.05	0.5	0.2376	03/16/02	0.4795	2.3326	1.8399	2006
965.8	1500	0.9	0.25	0.575	0.2592	03/16/02	0.4795	2.3326	1.8399	1162
965.8	850	174.1	170.1	172.1	0.3031	06/22/02	0.7479	2.4342	1.8602	116
965.8	950	109.5	105.5	107.5	0.2792	06/22/02	0.7479	2.4342	1.8602	6
965.8	995	85.2	81.2	83.2	0.2682	06/22/02	0.7479	2.4342	1.8602	451
965.8	1050	60.6	56.6	58.6	0.2567	06/22/02	0.7479	2.4342	1.8602	1471
965.8	1100	42.3	38.3	40.3	0.2453	06/22/02	0.7479	2.4342	1.8602	1796
965.8	1125	35.3	31.3	33.3	0.2417	06/22/02	0.7479	2.4342	1.8602	105
965.8	1150	28.8	24.8	26.8	0.2366	06/22/02	0.7479	2.4342	1.8602	4348
965.8	1175	23	20	21.5	0.2326	06/22/02	0.7479	2.4342	1.8602	160
965.8	1250	11.6	9.6	10.6	0.2235	06/22/02	0.7479	2.4342	1.8602	6911
965.8	1300	7.6	5.6	6.6	0.2208	06/22/02	0.7479	2.4342	1.8602	6893
965.8	1325	5.8	4.4	5.1	0.2190	06/22/02	0.7479	2.4342	1.8602	1016
965.8	1350	4.7	3.3	4	0.2183	06/22/02	0.7479	2.4342	1.8602	5425
965.8	1400	2.85	1.95	2.4	0.2166	06/22/02	0.7479	2.4342	1.8602	14585
965.8	1425	2.3	1.4	1.85	0.2160	06/22/02	0.7479	2.4342	1.8602	325
965.8	1450	1.9	1	1.45	0.2160	06/22/02	0.7479	2.4342	1.8602	9377
965.8	1500	1.25	0.35	0.8	0.2133	06/22/02	0.7479	2.4342	1.8602	8297
965.8	1525	1.05	0.15	0.6	0.2125	06/22/02	0.7479	2.4342	1.8602	310
965.8	900	165.1	159.1	162.1	0.2692	12/21/02	1.2466	2.6133	1.8624	568
965.8	950	135.1	129.1	132.1	0.2595	12/21/02	1.2466	2.6133	1.8624	1

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
965.8	995	111.3	105.3	108.3	0.2522	12/21/02	1.2466	2.6133	1.8624	254
965.8	1050	87.1	81.1	84.1	0.2464	12/21/02	1.2466	2.6133	1.8624	1877
965.8	1100	67.4	61.4	64.4	0.2386	12/21/02	1.2466	2.6133	1.8624	2217
965.8	1150	50.5	44.5	47.5	0.2297	12/21/02	1.2466	2.6133	1.8624	2886
965.8	1200	36.9	30.9	33.9	0.2214	12/21/02	1.2466	2.6133	1.8624	5798
965.8	1225	31.7	25.7	28.7	0.2187	12/21/02	1.2466	2.6133	1.8624	253
965.8	1250	26.8	20.8	23.8	0.2151	12/21/02	1.2466	2.6133	1.8624	4593
965.8	1300	18.5	15.5	17	0.2123	12/21/02	1.2466	2.6133	1.8624	4077
965.8	1350	13.7	10.7	12.2	0.2110	12/21/02	1.2466	2.6133	1.8624	2007
965.8	1400	9.1	7.1	8.1	0.2066	12/21/02	1.2466	2.6133	1.8624	4870
965.8	1450	6	4.6	5.3	0.2029	12/21/02	1.2466	2.6133	1.8624	5516
965.8	1500	4.4	3	3.7	0.2027	12/21/02	1.2466	2.6133	1.8624	4738
965.8	1550	3.7	2.55	3.125	0.2092	12/21/02	1.2466	2.6133	1.8624	1577
965.8	1600	2.6	1.7	2.15	0.2083	12/21/02	1.2466	2.6133	1.8624	6531
965.8	1650	2.7	1.8	2.25	0.2207	12/21/02	1.2466	2.6133	1.8624	1070
965.8	1700	2.3	1.4	1.85	0.2247	12/21/02	1.2466	2.6133	1.8624	4214
965.8	1800	1.4	0.5	0.95	0.2237	12/21/02	1.2466	2.6133	1.8624	12361
965.8	1900	1	0.2	0.6	0.2282	12/21/02	1.2466	2.6133	1.8624	6545

B.5 October 19, 2001 Call Options

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1073.5	900	176.8	174.8	175.8	0.3676	11/17/01	0.0767	2.2900	0.9318	18
1073.5	950	130.3	128.3	129.3	0.3564	11/17/01	0.0767	2.2900	0.9318	1846
1073.5	1025	66.7	64.7	65.7	0.3087	11/17/01	0.0767	2.2900	0.9318	3616
1073.5	1050	48.7	46.7	47.7	0.2913	11/17/01	0.0767	2.2900	0.9318	8617
1073.5	1075	33.5	31.5	32.5	0.2764	11/17/01	0.0767	2.2900	0.9318	10462
1073.5	1100	20.7	19.2	19.95	0.2590	11/17/01	0.0767	2.2900	0.9318	16096
1073.5	1125	12.1	10.6	11.35	0.2481	11/17/01	0.0767	2.2900	0.9318	10061
1073.5	1150	6.5	5.5	6	0.2415	11/17/01	0.0767	2.2900	0.9318	10757
1073.5	1175	3.2	2.5	2.85	0.2354	11/17/01	0.0767	2.2900	0.9318	5605
1073.5	1200	1.3	1.2	1.25	0.2314	11/17/01	0.0767	2.2900	0.9318	9984
1073.5	1225	0.7	0.4	0.55	0.2315	11/17/01	0.0767	2.2900	0.9318	5318
1073.5	800	277.4	275.4	276.4	0.3604	12/22/01	0.1726	2.2364	1.4118	2902
1073.5	900	184	182	183	0.3492	12/22/01	0.1726	2.2364	1.4118	3092
1073.5	950	140.1	138.1	139.1	0.3277	12/22/01	0.1726	2.2364	1.4118	1361
1073.5	995	103.7	101.7	102.7	0.3082	12/22/01	0.1726	2.2364	1.4118	9588
1073.5	1025	81	79	80	0.2910	12/22/01	0.1726	2.2364	1.4118	10857
1073.5	1050	64.1	62.1	63.1	0.2790	12/22/01	0.1726	2.2364	1.4118	16233
1073.5	1060	57.8	55.8	56.8	0.2740	12/22/01	0.1726	2.2364	1.4118	6559
1073.5	1100	36.2	34.2	35.2	0.2566	12/22/01	0.1726	2.2364	1.4118	23692
1073.5	1150	17.4	15.9	16.65	0.2394	12/22/01	0.1726	2.2364	1.4118	17883
1073.5	1175	11.6	10.1	10.85	0.2338	12/22/01	0.1726	2.2364	1.4118	6475
1073.5	1300	1.1	0.65	0.875	0.2248	12/22/01	0.1726	2.2364	1.4118	21540
1073.5	1325	0.75	0.3	0.525	0.2263	12/22/01	0.1726	2.2364	1.4118	9694
1073.5	900	195	193	194	0.2859	03/16/02	0.4027	2.1778	1.8128	51
1073.5	950	154.4	152.4	153.4	0.2755	03/16/02	0.4027	2.1778	1.8128	1079
1073.5	995	120.5	118.5	119.5	0.2635	03/16/02	0.4027	2.1778	1.8128	1774
1073.5	1025	100.2	98.2	99.2	0.2569	03/16/02	0.4027	2.1778	1.8128	1639
1073.5	1050	84.3	82.3	83.3	0.2499	03/16/02	0.4027	2.1778	1.8128	7888
1073.5	1075	69.9	67.9	68.9	0.2434	03/16/02	0.4027	2.1778	1.8128	1879
1073.5	1100	56.4	54.4	55.4	0.2352	03/16/02	0.4027	2.1778	1.8128	12447
1073.5	1125	45.1	43.1	44.1	0.2296	03/16/02	0.4027	2.1778	1.8128	2128
1073.5	1150	35.2	33.2	34.2	0.2235	03/16/02	0.4027	2.1778	1.8128	5713
1073.5	1175	27.1	25.1	26.1	0.2186	03/16/02	0.4027	2.1778	1.8128	2059
1073.5	1200	20	18.5	19.25	0.2130	03/16/02	0.4027	2.1778	1.8128	12048
1073.5	1225	14.7	13.2	13.95	0.2085	03/16/02	0.4027	2.1778	1.8128	2792
1073.5	1250	10.3	9.3	9.8	0.2040	03/16/02	0.4027	2.1778	1.8128	5990
1073.5	1275	7.3	6.3	6.8	0.2006	03/16/02	0.4027	2.1778	1.8128	3026
1073.5	1280	7	6	6.5	0.2016	03/16/02	0.4027	2.1778	1.8128	3
1073.5	1300	5	4.3	4.65	0.1980	03/16/02	0.4027	2.1778	1.8128	5090
1073.5	1325	3.5	2.8	3.15	0.1961	03/16/02	0.4027	2.1778	1.8128	2003
1073.5	1350	2.3	2	2.15	0.1952	03/16/02	0.4027	2.1778	1.8128	4697
1073.5	1375	1.65	1.2	1.425	0.1940	03/16/02	0.4027	2.1778	1.8128	805
1073.5	1400	1.1	0.65	0.875	0.1912	03/16/02	0.4027	2.1778	1.8128	8865
1073.5	1425	0.8	0.35	0.575	0.1909	03/16/02	0.4027	2.1778	1.8128	269
1073.5	1450	0.65	0.2	0.425	0.1936	03/16/02	0.4027	2.1778	1.8128	2006
1073.5	850	248.1	246.1	247.1	0.2635	06/22/02	0.6712	2.2282	1.8690	121

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1073.5	950	168.7	166.7	167.7	0.2529	06/22/02	0.6712	2.2282	1.8690	24
1073.5	995	136.8	134.8	135.8	0.2455	06/22/02	0.6712	2.2282	1.8690	2849
1073.5	1050	101.4	99.4	100.4	0.2341	06/22/02	0.6712	2.2282	1.8690	3390
1073.5	1100	74.1	72.1	73.1	0.2245	06/22/02	0.6712	2.2282	1.8690	1767
1073.5	1125	62.5	60.5	61.5	0.2206	06/22/02	0.6712	2.2282	1.8690	560
1073.5	1150	51.8	49.8	50.8	0.2158	06/22/02	0.6712	2.2282	1.8690	5458
1073.5	1200	34.7	32.7	33.7	0.2084	06/22/02	0.6712	2.2282	1.8690	6395
1073.5	1250	22.1	20.1	21.1	0.2014	06/22/02	0.6712	2.2282	1.8690	8208
1073.5	1300	13.6	12.1	12.85	0.1970	06/22/02	0.6712	2.2282	1.8690	7132
1073.5	1325	10.3	9.3	9.8	0.1947	06/22/02	0.6712	2.2282	1.8690	1118
1073.5	1350	7.8	6.8	7.3	0.1921	06/22/02	0.6712	2.2282	1.8690	5471
1073.5	1400	4	3.3	3.65	0.1849	06/22/02	0.6712	2.2282	1.8690	12348
1073.5	1425	2.95	2.25	2.6	0.1829	06/22/02	0.6712	2.2282	1.8690	329
1073.5	1450	2.15	1.7	1.925	0.1826	06/22/02	0.6712	2.2282	1.8690	8702
1073.5	1500	1.2	0.75	0.975	0.1805	06/22/02	0.6712	2.2282	1.8690	8287
1073.5	1525	0.9	0.45	0.675	0.1793	06/22/02	0.6712	2.2282	1.8690	320
1073.5	1550	0.7	0.25	0.475	0.1786	06/22/02	0.6712	2.2282	1.8690	3458
1073.5	1600	0.65	0.2	0.425	0.1898	06/22/02	0.6712	2.2282	1.8690	6212
1073.5	900	228.6	224.6	226.6	0.2362	12/21/02	1.1699	2.4130	1.8768	568
1073.5	950	193.1	189.1	191.1	0.2337	12/21/02	1.1699	2.4130	1.8768	152
1073.5	995	163.4	159.4	161.4	0.2299	12/21/02	1.1699	2.4130	1.8768	2801
1073.5	1050	129.9	125.9	127.9	0.2231	12/21/02	1.1699	2.4130	1.8768	2677
1073.5	1100	103.3	99.3	101.3	0.2173	12/21/02	1.1699	2.4130	1.8768	4743
1073.5	1150	80.4	76.4	78.4	0.2119	12/21/02	1.1699	2.4130	1.8768	2692
1073.5	1200	60.9	56.9	58.9	0.2061	12/21/02	1.1699	2.4130	1.8768	7779
1073.5	1225	52.4	48.4	50.4	0.2030	12/21/02	1.1699	2.4130	1.8768	253
1073.5	1250	45.1	41.1	43.1	0.2008	12/21/02	1.1699	2.4130	1.8768	4646
1073.5	1300	32.4	28.4	30.4	0.1951	12/21/02	1.1699	2.4130	1.8768	4476
1073.5	1350	21.8	19.8	20.8	0.1901	12/21/02	1.1699	2.4130	1.8768	2210
1073.5	1400	15	13	14	0.1863	12/21/02	1.1699	2.4130	1.8768	6233
1073.5	1450	9.5	8.5	9	0.1821	12/21/02	1.1699	2.4130	1.8768	5648
1073.5	1475	7.6	6.6	7.1	0.1800	12/21/02	1.1699	2.4130	1.8768	15
1073.5	1500	6.1	5.1	5.6	0.1782	12/21/02	1.1699	2.4130	1.8768	4782
1073.5	1550	3.7	2.9	3.3	0.1740	12/21/02	1.1699	2.4130	1.8768	1677
1073.5	1600	2.3	1.8	2.05	0.1725	12/21/02	1.1699	2.4130	1.8768	6443
1073.5	1650	1.55	1.05	1.3	0.1721	12/21/02	1.1699	2.4130	1.8768	1070
1073.5	1700	1	0.5	0.75	0.1700	12/21/02	1.1699	2.4130	1.8768	3574

B.6 November 16, 2001 Call Options

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1138.65	700	440.8	438.8	439.8	0.6922	11/16/01	0.0959	2.0147	0.9460	189
1138.65	800	341.5	339.5	340.5	0.5605	11/16/01	0.0959	2.0147	0.9460	2832
1138.65	850	291.9	289.9	290.9	0.4930	11/16/01	0.0959	2.0147	0.9460	1
1138.65	900	242.6	240.6	241.6	0.4346	11/16/01	0.0959	2.0147	0.9460	3093
1138.65	910	232.8	230.8	231.8	0.4240	11/16/01	0.0959	2.0147	0.9460	9
1138.65	950	193.7	191.7	192.7	0.3791	11/16/01	0.0959	2.0147	0.9460	1388
1138.65	960	184.1	182.1	183.1	0.3709	11/16/01	0.0959	2.0147	0.9460	1068
1138.65	970	172.5	172.4	172.45	0.3369	11/16/01	0.0959	2.0147	0.9460	2906
1138.65	980	164.8	162.8	163.8	0.3493	11/16/01	0.0959	2.0147	0.9460	156
1138.65	990	155.2	153.2	154.2	0.3384	11/16/01	0.0959	2.0147	0.9460	1123
1138.65	995	150.5	148.5	149.5	0.3345	11/16/01	0.0959	2.0147	0.9460	10175
1138.65	1010	136.3	134.3	135.3	0.3192	11/16/01	0.0959	2.0147	0.9460	7596
1138.65	1025	122.4	120.4	121.4	0.3059	11/16/01	0.0959	2.0147	0.9460	12530
1138.65	1050	99.8	97.8	98.8	0.2842	11/16/01	0.0959	2.0147	0.9460	18344
1138.65	1060	91	89	90	0.2754	11/16/01	0.0959	2.0147	0.9460	10750
1138.65	1070	82.3	80.3	81.3	0.2660	11/16/01	0.0959	2.0147	0.9460	6
1138.65	1080	74.2	72.2	73.2	0.2602	11/16/01	0.0959	2.0147	0.9460	2963
1138.65	1090	65.9	63.9	64.9	0.2504	11/16/01	0.0959	2.0147	0.9460	6423
1138.65	1095	62	60	61	0.2468	11/16/01	0.0959	2.0147	0.9460	1338
1138.65	1100	58.1	56.1	57.1	0.2426	11/16/01	0.0959	2.0147	0.9460	29062
1138.65	1110	50.7	48.7	49.7	0.2353	11/16/01	0.0959	2.0147	0.9460	512
1138.65	1115	47.6	45.6	46.6	0.2352	11/16/01	0.0959	2.0147	0.9460	1952
1138.65	1120	44.1	42.1	43.1	0.2312	11/16/01	0.0959	2.0147	0.9460	3347
1138.65	1140	31	29	30	0.2137	11/16/01	0.0959	2.0147	0.9460	6608
1138.65	1150	25.2	23.9	24.55	0.2077	11/16/01	0.0959	2.0147	0.9460	31822
1138.65	1160	21	19.5	20.25	0.2059	11/16/01	0.0959	2.0147	0.9460	1653
1138.65	1175	14.8	13.3	14.05	0.1974	11/16/01	0.0959	2.0147	0.9460	9606
1138.65	1300	0.6	0.25	0.425	0.1980	11/16/01	0.0959	2.0147	0.9460	21895
1138.65	900	244.8	242.8	243.8	0.3566	11/16/01	0.1726	1.9825	1.3711	2
1138.65	950	197.3	195.3	196.3	0.3272	11/16/01	0.1726	1.9825	1.3711	122
1138.65	1050	107.4	105.4	106.4	0.2647	11/16/01	0.1726	1.9825	1.3711	1085
1138.65	1075	87.4	85.4	86.4	0.2521	11/16/01	0.1726	1.9825	1.3711	3829
1138.65	1100	68.9	66.9	67.9	0.2400	11/16/01	0.1726	1.9825	1.3711	7780
1138.65	1125	52.2	50.2	51.2	0.2280	11/16/01	0.1726	1.9825	1.3711	9967
1138.65	1150	37.4	35.4	36.4	0.2149	11/16/01	0.1726	1.9825	1.3711	7697
1138.65	1175	25.7	23.7	24.7	0.2054	11/16/01	0.1726	1.9825	1.3711	1981
1138.65	1200	16.3	14.8	15.55	0.1959	11/16/01	0.1726	1.9825	1.3711	3582
1138.65	1225	9.8	8.8	9.3	0.1894	11/16/01	0.1726	1.9825	1.3711	1559
1138.65	1250	5.6	4.9	5.25	0.1846	11/16/01	0.1726	1.9825	1.3711	5556
1138.65	1300	1.65	1.2	1.425	0.1785	11/16/01	0.1726	1.9825	1.3711	26
1138.65	1350	0.6	0.15	0.375	0.1789	11/16/01	0.1726	1.9825	1.3711	203
1138.65	750	393.2	391.2	392.2	0.3154	11/16/01	0.3260	1.9835	1.8099	32
1138.65	900	249.4	247.4	248.4	0.2936	11/16/01	0.3260	1.9835	1.8099	51
1138.65	950	203.7	201.7	202.7	0.2779	11/16/01	0.3260	1.9835	1.8099	1078
1138.65	975	181.5	179.5	180.5	0.2690	11/16/01	0.3260	1.9835	1.8099	2
1138.65	995	164.3	162.3	163.3	0.2627	11/16/01	0.3260	1.9835	1.8099	2113

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1138.65	1025	139.3	137.3	138.3	0.2524	11/16/01	0.3260	1.9835	1.8099	1818
1138.65	1050	119.7	117.7	118.7	0.2451	11/16/01	0.3260	1.9835	1.8099	8661
1138.65	1075	100.8	98.8	99.8	0.2359	11/16/01	0.3260	1.9835	1.8099	3924
1138.65	1100	83.4	81.4	82.4	0.2278	11/16/01	0.3260	1.9835	1.8099	13324
1138.65	1125	67.1	65.1	66.1	0.2184	11/16/01	0.3260	1.9835	1.8099	7102
1138.65	1150	53.2	51.2	52.2	0.2120	11/16/01	0.3260	1.9835	1.8099	8548
1138.65	1175	40.5	38.5	39.5	0.2037	11/16/01	0.3260	1.9835	1.8099	3118
1138.65	1200	29.7	27.7	28.7	0.1953	11/16/01	0.3260	1.9835	1.8099	16011
1138.65	1225	21.2	19.7	20.45	0.1897	11/16/01	0.3260	1.9835	1.8099	3922
1138.65	1250	14.9	13.4	14.15	0.1852	11/16/01	0.3260	1.9835	1.8099	11200
1138.65	1275	10.3	9.3	9.8	0.1833	11/16/01	0.3260	1.9835	1.8099	3051
1138.65	1280	9.3	8.3	8.8	0.1811	11/16/01	0.3260	1.9835	1.8099	602
1138.65	1300	6.7	5.7	6.2	0.1784	11/16/01	0.3260	1.9835	1.8099	6105
1138.65	1325	4.2	3.5	3.85	0.1750	11/16/01	0.3260	1.9835	1.8099	2241
1138.65	1350	2.7	2	2.35	0.1727	11/16/01	0.3260	1.9835	1.8099	4966
1138.65	1375	1.6	1.15	1.375	0.1704	11/16/01	0.3260	1.9835	1.8099	789
1138.65	1400	1.25	0.8	1.025	0.1755	11/16/01	0.3260	1.9835	1.8099	8718
1138.65	1425	0.9	0.45	0.675	0.1770	11/16/01	0.3260	1.9835	1.8099	269
1138.65	1450	0.65	0.2	0.425	0.1776	11/16/01	0.3260	1.9835	1.8099	1986
1138.65	850	303.6	301.6	302.6	0.2564	11/16/01	0.5945	2.1262	1.9409	121
1138.65	950	215.1	213.1	214.1	0.2437	11/16/01	0.5945	2.1262	1.9409	28
1138.65	995	178	176	177	0.2350	11/16/01	0.5945	2.1262	1.9409	2670
1138.65	1050	136.3	134.3	135.3	0.2248	11/16/01	0.5945	2.1262	1.9409	3398
1138.65	1075	118.7	116.7	117.7	0.2196	11/16/01	0.5945	2.1262	1.9409	1226
1138.65	1100	102.4	100.4	101.4	0.2150	11/16/01	0.5945	2.1262	1.9409	5919
1138.65	1125	86.9	84.9	85.9	0.2092	11/16/01	0.5945	2.1262	1.9409	3837
1138.65	1150	73.2	71.2	72.2	0.2051	11/16/01	0.5945	2.1262	1.9409	5729
1138.65	1200	48.8	46.8	47.8	0.1942	11/16/01	0.5945	2.1262	1.9409	8264
1138.65	1250	30.7	28.7	29.7	0.1856	11/16/01	0.5945	2.1262	1.9409	6669
1138.65	1300	18.4	16.9	17.65	0.1803	11/16/01	0.5945	2.1262	1.9409	7224
1138.65	1325	14	12.5	13.25	0.1778	11/16/01	0.5945	2.1262	1.9409	1669
1138.65	1350	10	9	9.5	0.1740	11/16/01	0.5945	2.1262	1.9409	5481
1138.65	1400	5.1	4.4	4.75	0.1689	11/16/01	0.5945	2.1262	1.9409	12147
1138.65	1425	3.7	3	3.35	0.1676	11/16/01	0.5945	2.1262	1.9409	329
1138.65	1450	2.75	2.05	2.4	0.1672	11/16/01	0.5945	2.1262	1.9409	8703
1138.65	1500	1.35	0.9	1.125	0.1651	11/16/01	0.5945	2.1262	1.9409	8537
1138.65	1525	1.1	0.65	0.875	0.1675	11/16/01	0.5945	2.1262	1.9409	320
1138.65	1550	0.8	0.35	0.575	0.1662	11/16/01	0.5945	2.1262	1.9409	3465
1138.65	1600	0.6	0.15	0.375	0.1722	11/16/01	0.5945	2.1262	1.9409	6212
1138.65	800	362.5	359.5	361	0.2031	11/16/01	1.0932	2.4733	1.9580	1
1138.65	900	276.8	273.8	275.3	0.2195	11/16/01	1.0932	2.4733	1.9580	568
1138.65	950	237.4	234.4	235.9	0.2206	11/16/01	1.0932	2.4733	1.9580	1700
1138.65	995	203.5	200.5	202	0.2176	11/16/01	1.0932	2.4733	1.9580	4077
1138.65	1050	164.7	161.7	163.2	0.2118	11/16/01	1.0932	2.4733	1.9580	3602
1138.65	1100	132.5	129.5	131	0.2054	11/16/01	1.0932	2.4733	1.9580	7891
1138.65	1150	104.9	101.9	103.4	0.2011	11/16/01	1.0932	2.4733	1.9580	3436
1138.65	1200	80.4	77.4	78.9	0.1953	11/16/01	1.0932	2.4733	1.9580	10029
1138.65	1225	69.5	66.5	68	0.1922	11/16/01	1.0932	2.4733	1.9580	311
1138.65	1250	59.6	56.6	58.1	0.1892	11/16/01	1.0932	2.4733	1.9580	6659
1138.65	1300	42.4	39.4	40.9	0.1824	11/16/01	1.0932	2.4733	1.9580	5822
1138.65	1350	29.6	26.6	28.1	0.1774	11/16/01	1.0932	2.4733	1.9580	2160
1138.65	1400	18.9	17.4	18.15	0.1715	11/16/01	1.0932	2.4733	1.9580	6950

S	X	Bid	Ask	Mid	BS Imp. Vol.	Exp.	Maturity	Yield (%)	Dis. Div.	Open Interest
1138.65	1450	12.2	10.7	11.45	0.1672	11/16/01	1.0932	2.4733	1.9580	5670
1138.65	1475	9.5	8.5	9	0.1654	11/16/01	1.0932	2.4733	1.9580	14
1138.65	1500	7.9	6.9	7.4	0.1658	11/16/01	1.0932	2.4733	1.9580	4788
1138.65	1550	4.7	4	4.35	0.1622	11/16/01	1.0932	2.4733	1.9580	1591
1138.65	1600	2.8	2.1	2.45	0.1588	11/16/01	1.0932	2.4733	1.9580	6376
1138.65	1650	1.6	1.15	1.375	0.1566	11/16/01	1.0932	2.4733	1.9580	1070
1138.65	1700	1	0.55	0.775	0.1552	11/16/01	1.0932	2.4733	1.9580	3568

B.7 Theoretical Delivery Price for Demeterfi et al. Variance Swap Model

The delivery price is given by:

$$K_{vol}^2 = \frac{2}{T}\left(rT - \left(\frac{S_0}{S^*} e^{rT} - 1 \right) - \log\frac{S^*}{S_0} + e^{rT}\int_0^{S^*} \frac{1}{K^2} P(K)dK + e^{rT}\int_{S^*}^{\infty} \frac{1}{K^2} C(K)dK \right)$$

The appropriate option portfolio weights for a finite set of call and put strikes, $K_{i,C}$ and $K_{i,P}$ are given by:

$$g(S_T) = \frac{2}{T}\left(\frac{S_T - S^*}{S^*} - \log\frac{S_T}{S^*} \right)$$

$$w(K_{i,C}) = \frac{g(K_{i+1,C}) - g(K_{i,C})}{K_{i+1,C} - K_{i,C}} - \sum_{j=0}^{i-1} w(K_{j,C}) \qquad \textit{for calls}$$

$$w(K_{i,P}) = \frac{g(K_{i+1,P}) - g(K_{i,P})}{K_{i,P} - K_{i+1,P}} - \sum_{j=0}^{i-1} w(K_{j,P}) \qquad \textit{for puts}$$

C.1 Characteristic Functions For SV Model

$$f_1^{SV} = \exp\left\{\begin{array}{l}\left\{-\frac{\theta_v}{\sigma_v^2}\left[2\ln\left(1-\frac{[\zeta_v-\kappa_v+(1+i\phi)\rho\sigma_v](1-e^{-\zeta_v\tau})}{2\zeta_v}\right)\right]\right. \\ \qquad -\frac{\theta_v}{\sigma_v^2}[\zeta_v-\kappa_v+(1+i\phi)\rho\sigma_v]\tau+i\phi r\tau+i\phi\ln[S(t)]\Big\} \\ \qquad +\frac{i\phi(i\phi+1)(1-e^{-\zeta_v\tau})V(t)}{2\zeta_v-[\zeta_v-\kappa_v+(1+i\phi)\rho\sigma_v](1-e^{-\zeta_v\tau})}\end{array}\right\}$$

$$f_2^{SV} = \exp\left\{\begin{array}{l}-\frac{\theta_v}{\sigma_v^2}\left[2\ln\left(1-\frac{[\zeta_v^*-\kappa_v+i\phi\rho\sigma_v](1-e^{-\zeta_v^*\tau})}{2\zeta_v^*}\right)\right] \\ \qquad -\frac{\theta_v}{\sigma_v^2}[\zeta_v^*-\kappa_v+i\phi\rho\sigma_v]\tau+i\phi r\tau+i\phi\ln[S(t)] \\ \qquad +\frac{i\phi(i\phi-1)(1-e^{-\zeta_v^*\tau})V(t)}{2\zeta_v^*-[\zeta_v^*-\kappa_v+i\phi\rho\sigma_v](1-e^{-\zeta_v^*\tau})}\end{array}\right\}$$

$$\zeta_v = \{[\kappa_v-(1+i\phi)\rho\sigma_v]^2 - i\phi(i\phi+1)\sigma_v^2\}^{1/2}$$

$$\zeta_v^* = \{[\kappa_v-i\phi\rho\sigma_v]^2 - i\phi(i\phi-1)\sigma_v^2\}^{1/2}$$

C.2 Characteristic Functions For SVJ Model

$$f_1^{SVJ} = \exp\left\{ \begin{array}{l} \left\{-\frac{\theta_v}{\sigma_v^2}\left[2\ln\left(1-\frac{[\zeta_v-\kappa_v+(1+i\phi)\rho\sigma_v](1-e^{-\zeta_v\tau})}{2\zeta_v}\right)\right]\right. \\ \qquad -\frac{\theta_v}{\sigma_v^2}[\zeta_v-\kappa_v+(1+i\phi)\rho\sigma_v]\tau+i\phi r\tau+i\phi\ln[S(t)] \\ \qquad +\frac{i\phi(i\phi+1)(1-e^{-\zeta_v\tau})V(t)}{2\zeta_v-[\zeta_v-\kappa_v+(1+i\phi)\rho\sigma_v](1-e^{-\zeta_v\tau})} \\ \qquad +\lambda(1+\mu_J)\tau[(1+\mu_J)^{i\phi}e^{(i\phi/2)(1+i\phi)\sigma_J^2}-1]-\lambda i\phi\mu_J\tau \end{array} \right\}$$

$$f_2^{SVJ} = \exp\left\{ \begin{array}{l} -\frac{\theta_v}{\sigma_v^2}\left[2\ln\left(1-\frac{[\zeta_v^*-\kappa_v+i\phi\rho\sigma_v](1-e^{-\zeta_v^*\tau})}{2\zeta_v^*}\right)\right] \\ \qquad -\frac{\theta_v}{\sigma_v^2}[\zeta_v^*-\kappa_v+i\phi\rho\sigma_v]\tau+i\phi r\tau+i\phi\ln[S(t)] \\ \qquad +\frac{i\phi(i\phi-1)(1-e^{-\zeta_v^*\tau})V(t)}{2\zeta_v^*-[\zeta_v^*-\kappa_v+i\phi\rho\sigma_v](1-e^{-\zeta_v^*\tau})} \\ \qquad +\lambda\tau[(1+\mu_J)^{i\phi}e^{(i\phi/2)(i\phi-1)\sigma_J^2}-1]-\lambda i\phi\mu_J\tau \end{array} \right\}$$

$$\zeta_v = \left\{[\kappa_v-(1+i\phi)\rho\sigma_v]^2 - i\phi(i\phi+1)\sigma_v^2\right\}^{1/2}$$

$$\zeta_v^* = \left\{[\kappa_v-i\phi\rho\sigma_v]^2 - i\phi(i\phi-1)\sigma_v^2\right\}^{1/2}$$

D.1 MATLAB Optimisation Toolbox Settings

Trust-Region Reflective Quasi-Newton Method

Asset Range Factor	f=2
Local Volatility Knots	p<m
Asset Knots	12
Time Knots	6
Lower Volatility Bound	u=-1
Upper Volatility Bound	l=1
Function Tolerance	1×10^{-3}
Asset Levels	M=200
Time Levels	N=50
PCG Bandwidth	0