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# A farewell to brake reaction times? Kinematics-dependent brake response in naturalistic rear-end emergencies

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**Abstract:** Driver braking behavior was analyzed using time-series recordings from naturalistic rear-end conflicts (116 crashes and 241 near-crashes), including events with and without visual distraction among drivers of cars, heavy trucks, and buses. A simple piecewise linear model could be successfully fitted, per event, to the observed driver decelerations, allowing a detailed elucidation of when drivers initiated braking and how they controlled it. Most notably, it was found that, across vehicle types, driver braking behavior was strongly dependent on the urgency of the given rear-end scenario's kinematics, quantified in terms of visual looming of the lead vehicle on the driver's retina. In contrast with previous suggestions of brake reaction times (BRTs) of 1.5 s or more after onset of an unexpected hazard (e.g., brake light onset), it was found here that braking could be described as typically starting less than a second after the kinematic urgency reached certain threshold levels, with even faster reactions at higher urgencies. The rate at which drivers then increased their deceleration (towards a maximum) was also highly dependent on urgency. Probability distributions are provided that quantitatively capture these various patterns of kinematics-dependent behavioral response. Possible underlying mechanisms are suggested, including looming response thresholds and neural evidence accumulation. These accounts argue that a naturalistic braking response should not be thought of as a slow reaction to some single, researcher-defined "hazard onset", but instead as a relatively fast response to the visual looming cues that build up later on in the evolving traffic scenario.

**Keywords:** Rear-end crashes, reaction time, kinematics, visual looming, deceleration

## 1 Introduction

When the driver of a vehicle is suddenly faced with an unexpected, critical risk of collision, how does he or she respond? If evasive maneuvering is applied, when does it begin? How is it carried out?

Conclusive answers to these questions have been a long-standing objective of traffic safety research, and have a range of implications: In the design of roads, vehicles, or vehicle support systems for safety and automation, quantitative models of driver behavior can be very directly applied, for example in system algorithms or in computer simulations of crashes (e.g., Perel, 1982; Fambro et al. 2000a; MacAdam, 2001; Brännström et al., 2010; Markkula, 2015). In the broader study of traffic safety, the way one thinks about drivers' emergency responses can also be important in more subtle ways, for example by shaping design of experiments and subsequent interpretations of results, or by guiding one's analysis of actual crashes to understand their causation (e.g., Naing et al., 2009; Engström et al., 2013b), sometimes for purposes of litigation (e.g., Maddox and Kiefer, 2012).

The driver's reaction time (RT) is a concept that traffic safety researchers have repeatedly made use of in models, when designing studies, and when analyzing driver behavior close to crashes. The RT usually represents the time duration from the appearance of a potential hazard, such as a lead vehicle's brake lights activating, until the driver under study initiates some form of evasive response (Society of Automotive Engineers, 2015). Especially for braking responses, there is a considerable literature measuring brake reaction times (BRTs) and how they are influenced by factors such as driver age, gender, cognitive load, situation urgency, number of stimuli for the driver to consider, warnings, and so on (see for example the studies by Barrett et al., 1968; Olson and Sivak, 1986; Fambro et al., 1998; McGehee et al., 1999; Lee et al., 2002; Jurecki and Stanczyk, 2009, 2014; Fitch et al., 2010; Ljung Aust et al., 2013; and the reviews by Olson, 1989; Green, 2000; Muttart, 2003, 2005).

Green’s much-cited review (2000) aimed to determine typical RT values for different driving conditions. Expectancy was identified as the major factor determining BRT, with estimated values of 0.70-0.75 s for fully anticipated events, 1.25 s for unexpected but common events such as brake light onsets, and 1.5 s for surprise events such as sudden path intrusions. These canonical, situation-independent, BRT values drew criticism from Summala (2000), who pointed to evidence that BRTs for highly unexpected events can, if the traffic scenarios in question are sufficiently urgent, decrease to 1 s or lower. Similar dependencies between situation kinematics (the relative motion of involved road users, in terms of distances, speeds, etc.) and BRT have been reviewed by Muttart (2003, 2005) and have also been demonstrated in more recent test track and driving simulator studies (Jurecki & Stanczyk, 2009, 2014; Engström, 2010; Ljung Aust et al., 2013). However, a detailed, large-scale analysis is still outstanding, especially for naturalistic (i.e. real-traffic) emergencies.

As for what happens beyond the point of brake onset, it has been reported from both controlled and naturalistic studies that drivers will often, but not always, show maximum deceleration levels close to their vehicle’s limits on the given road (McGehee et al., 1999; Fambro et al., 2000b; Lee et al., 2007). From some controlled studies, there are also reports of progressive or step-wise ramping up towards these maximum levels (Prynne and Martin, 1995; Fambro et al., 2000b; Lee et al., 2002). Again, a detailed, quantitative account of emergency braking control is lacking, especially for naturalistic data.

This paper presents time-series analyses of situation kinematics and driver braking behavior observed in naturalistic rear-end crashes and near-crashes, continuing from the work by Victor et al. (2015, pp. 76-84). They showed, for one set of naturalistic passenger car data, that when visually distracted drivers looked back to the road to find a rear-end collision threat, the time delay before they exhibited any discernible physical reaction to the situation was strongly kinematics-dependent. Here, these results are extended by including (1) not only driver physical reaction but also actual measured deceleration behavior, (2) events without any off-road eye glances, and (3) an additional data set of recorded events that includes truck and bus drivers in addition to car drivers.

It will be described here how drivers’ deceleration behavior in the studied events varied markedly with situation kinematics, in certain rather specific manners, across data sets and vehicle types. Statistical-level descriptions of this variability, potentially useful in quantitative approaches to traffic safety, will be provided. Possible psychological mechanisms behind the observed behaviors will be discussed, and it will also be argued that the findings make the concept of a “brake reaction time” seem inadequate as a means for describing and understanding driver behavior in surprise emergencies.

## 2 Method

### 2.1 Data sets

The naturalistic events analyzed here came from two different sources: passenger car events from the Second Strategic Highway Research Program (SHRP 2), and passenger car, heavy truck, and bus events from the Analysis of Naturalistic External Datasets (ANNEXT) project. Table 1 provides an overview of the number of events per data set and vehicle type. In the remainder of this paper, the truck and bus events will be combined and treated together.

**Table 1.** Number of naturalistic rear-end events by data set and subject vehicle type.

| | SHRP 2 | ANNEXT | | | |
|---|---|---|---|---|---|
| | Passenger car | Passenger car | Heavy truck | Bus | Total |
| Crashes | 46 | 26 | 28 | 16 | 116 |
| Near-crashes | 211 | 11 | 11 | 8 | 241 |

Within SHRP 2, the world's largest naturalistic driving study to date was carried out, collecting over 80 million kilometers of driving data from instrumented cars driven by 3147 drivers across six sites in the US. As noted above, the present paper describes analyses building on those by Victor et al. (2015), comprising 46 crashes and 211 near-crashes; more specifically all of the critical events in the SHRP 2 database that were categorized as being of rear-end type (Scenarios 22-26 in the taxonomy by Najm and Smith, 2007) at the time of data extraction (spring of 2014).

ANNEXT was a pilot project between Lytx, Chalmers University of Technology and AB Volvo, which selected and annotated naturalistic crashes and near-crashes, originally recorded by Lytx as part of a behavior-based safety program for commercial fleets. The present paper uses the 100 rear-end events collected by the ANNEXT project; 77 events from the US and 23 events from Africa (South Africa, Nigeria, Zambia, and Zimbabwe). These events were selected using the following criteria: (1) The speed of the subject vehicle should be higher than 15 km/h at the start of the evasive maneuver or the moment of crash impact (thus excluding minor low-speed crashes), (2) the driver of the subject vehicle should not be wearing sunglasses, and (3) the lead vehicle should remain in the same lane from the beginning of the event until the crash (thus excluding cut-in events).

For both SHRP 2 and ANNEXT, candidate events were identified using various triggers, such as acceleration thresholds. In SHRP 2, candidate events were also identified by Automatic Crash Notification algorithms running in the vehicles, incident button presses by the participating drivers, and reports by the organizations that performed the data collection. In both projects, human video reviewers made the final judgment on whether the captured event was a true crash ("any contact […] with an object […] at any speed in which kinetic energy is measurably transferred or dissipated"; Victor et al., 2015, p. 20), a near-crash ("any circumstance that requires a rapid, evasive maneuver […] that approaches the limits of the vehicle capabilities. As a general guideline, subject vehicle braking greater than 0.5 g or steering input that results in a lateral acceleration greater than 0.4 g to avoid a crash constitutes a rapid maneuver"; ibid.), or neither.

For further details on the SHRP 2 and ANNEXT data sets, including driver demographics and other descriptive variables, see (Victor et al. 2015, pp. 31-40) and (Engström et al., 2013b).

The data variables used in the present analyses were:

- Manually annotated time point of first discernible physical reaction of the subject vehicle's driver to the collision threat ("including body movement, posture, a change in facial expression, a movement of the leg toward the brake", Victor et al., 2015, p. 27; see also McGehee and Carsten, 2010, for further insight into these types of physical reactions to critical traffic events).
- Manually annotated time-series of the eye glance behavior of the subject vehicle's driver, detailing whether gaze was directed toward the road ahead or not (using the "Eyes on Path" definition on p. 26 of Victor et al., 2015), as well as whether eyes were closed or open. For the SHRP 2 dataset these annotations were made by two annotators separately to increase reliability; see (Klauer et al., 2010; p. 18) for more details on the adopted procedure.
- Manual annotation of the evasive maneuver applied by the subject vehicle's driver, here reduced to the following categories: braking; steering; braking and steering; no maneuver.
- Digitally recorded time-series of subject vehicle longitudinal speed and acceleration.

- Manually annotated time-series of the forward-facing video, especially the width of the lead vehicle in the video images. These width annotations were used to estimate optical quantities relating to the visual looming of the lead vehicle (Lee, 1976; see also further below), as well as lead vehicle speed and distance between subject vehicle and lead vehicle. For details on the data extraction and processing procedures to derive these measures, see (Bärgman et al., 2013) and (Victor et al, 2015, pp. 21-29).

The SHRP 2 data were available at 10 Hz, and the ANNEXT data at 4 Hz. However, in all ANNEXT events and some SHRP 2 events the longitudinal speed data were in practice updated only at the 1 Hz provided by the on-board GPS receiver.

## 2.2 Event time-series data extraction

For the purpose of the present analyses, which have a specific focus on the longitudinal acceleration signal, the start and end points of individual events were defined as follows:

The event start point was set to 6 s before collision (crashes) or 6 s before the time of minimum time-to-collision (TTC, near-crashes), for events with no off-road glances during this 6 s interval. For events with at least one off-road glance in the same interval, the event start point was set to just after the last off-road glance before collision/minimum TTC, disregarding any glances that were judged to be part of the emergency response itself: Specifically, in 17 events, the last annotated glance before collision/minimum TTC occurred after an annotated physical reaction or a clearly identifiable defensive deceleration;  these glances were interpreted as being in response to the critical situation (e.g. to check for escape routes or to brace for impact) rather than being part of causing the situation. For these events, the start point was set to just after the next to last off-road glance, or to 6 s before collision/minimum TTC if there were no other off-road glances in that time interval.

For crashes, the event end point, after which no data were included, was set to 0.3 s before collision in order to exclude any part of the sharp acceleration pulse at impact; for the ANNEXT crashes with 4 Hz data, this meant in practice that the final included data point was 0.5 s before collision. For near-crashes, the event end point was set to 0.5 s after minimum TTC, since it was found that drivers generally maintained their maximum deceleration for at least this long.

A number of events were excluded from analysis altogether, for not matching the scope of the present analyses: In 11 events, the drivers still had their eyes off the road at collision or minimum TTC, so there was no event start point as defined above, and thus no longitudinal acceleration data to analyze. In 13 events, the last instance of the driver not looking at the road ahead was an eye closure rather than an off-road glance, possibly signaling driver drowsiness, and these were excluded since the aim here was to address only events with either a clear visual distraction or where the driver's eyes were on the road for the full 6 s before collision/min TTC. In one event, the subject vehicle was the struck rather than the striking vehicle. Nine additional events were excluded due to various problems with the recorded data; see Appendix A for full details.

These exclusions left a data set of 323 events, with 99 crashes (62 passenger car, 37 truck/bus) and 224 near-crashes (210 passenger car, 14 truck/bus). Note, however, that many of the various analyses performed on the data imposed even further requirements on availability of specific recorded signals, etc. These requirements will be described as the analyses in question are introduced below; the exact impacts of the various inclusion criteria are detailed in Appendix A.

## 2.3 Acceleration model fitting

For each event where driver braking had been coded by the annotator, and where there were at least 0.5 s of data between the event start and end points as defined above, a fitting was made of the piecewise linear model of acceleration shown in Figure 1, to the acceleration data recorded for the subject vehicle. The model assumes an initial constant level of acceleration $a_0$, a time of brake onset[1] $t_B$ at which

[1] Throughout this paper, "brake onset" refers to themodel parameter $t_B$, i.e. the time at which the subject vehicle's driver is estimated to have begun braking. If the brake onset of the lead vehicle is intended, this is instead referred to as "lead vehicle brake light onset" or similar.

acceleration starts decreasing linearly with a rate $j_B$ (the jerk), and a final constant level of acceleration $a_1$. Parameter-fitting was conducted by exhaustively searching a uniformly spaced grid $a_0 \in \{-0.2, -0.195, \ldots, 0.2\}$ g, $j_B \in \{-7, -6.75, \ldots, 0\}$ m/s$^3$, $a_1 \in \{-1, -0.95, \ldots, 0\}$ g, and $t_B$ in 0.1 s steps between the start and end points of the event as defined above. For each event, the model parameterization with the least total square deviation between model and observation was identified. By definition, this is also the model parameterization with the largest coefficient of determination $R^2$ (Fields, 2009), used here as a measure of goodness of model fit, again per event:

$$R^2 = 1 - \frac{\sum_{k=1}^{n} (\hat{x}_k - x_k)^2}{\sum_{k=1}^{n} (x_k - \bar{x})^2}, \tag{1}$$

where $x_k$ is the observed longitudinal acceleration at time step $k$ (out of a total of $n$ samples in the event), $\hat{x}_k$ is the piecewise linear model acceleration at the same time step, and $\bar{x}$ is the average observed acceleration in the event.

If a range of different $j_B$ values yielded the same $R^2$ for an event, because full deceleration ramp-up[2] occurred between two consecutive samples, $j_B$ was chosen as the maximum value in the range (but these events were not used for studying $j_B$ itself; see Section 3.4). For the 4 Hz ANNEXT da possible that a range of $t_B$, fitted at 10 Hz, all yielded the same $R^2$; in these cases $t_B$ was chos middle point of the interval (see the ANNEXT crash example to the left in Figure 3).

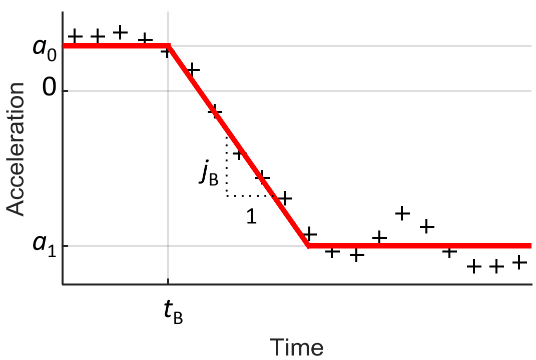


**Figure 1.** The piecewise linear model of acceleration (the red line), fitted to some example data points (the plus signs).

## 2.4 Analyses of kinematics-dependence

The metric adopted for quantifying situation kinematics was inverse tau, $\tau^{-1} = \dot{\theta} / \theta$, the ratio between the lead vehicle's optical expansion rate $\dot{\theta}$ ("theta-dot") on the driver's retina, and its optical size $\theta$ (Lee, 1976). Inverse tau is a visually available estimate of inverse TTC (Lee, 1976), and thus increases as the potential collision draws nearer. Furthermore, it has often been suggested that inverse tau and similar quantities play an important role in determining drivers' responses to obstacles and collision threats (Lee, 1976; Kiefer et al., 2003, 2005; Fajen, 2005, 2008; Kondoh et al., 2008, 2014; Jurecki & Stanczyk, 2009, 2014).

A number of analyses were carried out to test for dependencies between situation kinematics, in terms of $\tau^{-1}$, and various measures of driver behavior. Since the involved data were typically heteroscedastic (the variance in the driver behavior measure changing with $\tau^{-1}$), standard linear correlation and regression methods, maximizing $R^2$, would not allow for statistical testing. Instead, Spearman's non-parametric rank correlation test was used throughout. For illustration purposes, the approximate slopes and intercepts of the dependencies were also calculated using so-called robust linear regression, with reduced sensitivity to outliers, using the MATLAB function robustfit with default settings (The MathWorks MATLAB Release 2012b).

[2] Throughout this paper, the term "deceleration ramp-up" is used to describe the gradual increase of deceleration towards a positive maximum, whereas the figures show acceleration decreasing to a negative minimum. The alternative term "acceleration ramp-down" is avoided, since it could be taken to signify a decreasing positive acceleration.

For six of the ANNEXT crash events, there was a manual effort to restore missing $\tau^{-1}$ values in the last moments before crash, where the needed annotations had not been made because the lead vehicle was partly outside the forward camera's field of view. In these events, a cubic spline was fitted to the last five samples (1.25 s) of available $\tau^{-1}$, and extrapolated either one sample ahead (0.25 s, two events), two samples ahead (0.5 s, three events), or three samples ahead (0.75 s, one event), to obtain an estimate of $\tau^{-1}$ at the end of the driver's last off-road glance. In practice, these six extrapolations affected only the results on physical reaction timing (Section 3.1 below); for all other analyses the events in question were excluded because they were either less than 0.5 s long, or were not sufficiently well fitted by the acceleration model (see Section 3.2).

In addition, exploratory analyses were carried out to test in what ways the results would change if, instead of using $\tau^{-1}$ to quantify situation kinematics, one used (1) the optical expansion rate $\dot{\theta}$ (cf. Lamble et al., 1999; Maddox and Kiefer, 2012), or (2) the quantity $v/\tau$, with $v$ being the longitudinal speed of the own vehicle (cf. Kiefer et al., 2003, 2005; Fajen, 2005, 2008; Treiber and Kesting, 2013; Kusano et al., 2015).

### 2.5 Describing the observed behavior with probability distributions

To provide a more detailed description of the observed driver deceleration behaviors, parametric probability distributions were fitted for the model parameters of the piecewise linear deceleration model, in some cases after appropriate transformations. These probability distributions can be highly useful in quantitative approaches to traffic safety, but are not central to the main arguments of this paper. Therefore, the full method description, and some intermediate results and notes on usage have been placed in Appendix B.

## 3 Results

Below, the results reported in Victor et al. (2015) on physical reaction timing in the SHRP 2 data set will first be reiterated and extended with the ANNEXT data. Next, it will be described to what extent the piecewise linear model was able to describe the naturalistic deceleration behavior. Finally, the obtained model fits will be analyzed with respect to deceleration onset timing ($t_B$), deceleration ramp-up ($j_B$), and maximum deceleration ($a_1$), and summarized by means of probability distributions. When full details on number of excluded events per inclusion criteria for a figure are not given in the text below, they are available in Appendix A.

### 3.1 *Timing of drivers' physical reactions*

Figure 2 shows that the general patterns of behavior observed by Victor et al. (2015) in the SHRP 2 data set were also present in the ANNEXT data. In the figure, $\tau_{\mathrm{ELG}}^{-1}$ denotes the kinematical urgency of the situation, in terms of the magnitude of visual looming, confronting the drivers when they looked back to the road after the end of the last off-road glance (ELG). In other words, Figure 2 shows only events in which there was at least one off-road glance in the last six seconds before collision/minimum TTC. Note the rather sharp demarcation at approximately $\tau_{\mathrm{ELG}}^{-1}$ = 0.2 s$^{-1}$, separating events into what Victor et al. (2015) referred to as eyes-on-threat and eyes-off-threat events. In the eyes-off-threat events, the lead vehicle looming had reached $\tau_{\mathrm{ELG}}^{-1}$ = 0.2 s$^{-1}$ or more by the time drivers looked back to the road. In these events, visually discernible physical reactions (such as changes in posture or facial expression; see Section 2.1) occurred quickly, almost always less than 1 s after ELG, with further decreases with increasing kinematical urgency. This correlation was statistically significant for both car and truck/bus crash events. In contrast, in eyes-on-threat events (with $\tau_{\mathrm{ELG}}^{-1}$ < 0.2 s$^{-1}$), physical reactions were much slower, almost always occurring later than 1 s after ELG, and scattering more or less uniformly up toward 6 s (the maximum possible value given how these data were extracted).

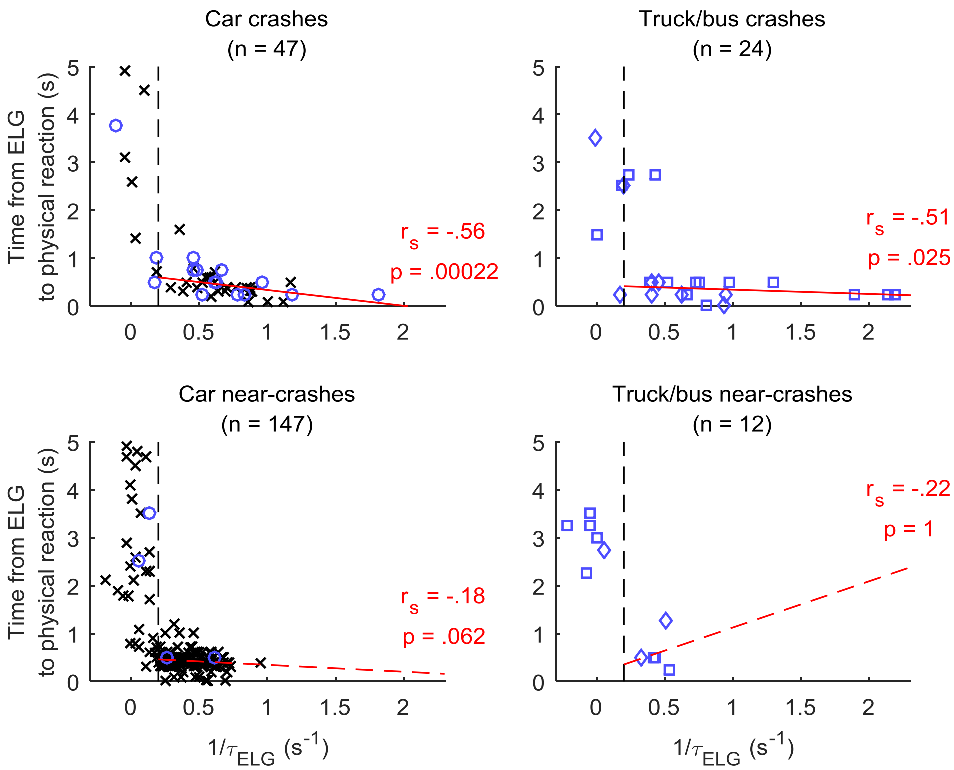


**Figure 2.** Time from the end of the last off-road glance (ELG) to the first observable physical reaction of the driver to the collision threat, as a function of $\tau^{-1}$ at ELG. Black crosses and blue circles in the left panels show car events from the SHRP 2 and ANNEXT data sets, respectively. Blue squares and diamonds in the right panels show truck and bus events, respectively. The black dashed lines show $\tau_{\mathrm{ELG}}^{-1}$ = 0.2 $s^{-1}$, separating events defined as eyes-on-threat and eyes-off-threat, respectively. The $r_s$ and p values were obtained using the Spearman rank correlation method, for the data points with $\tau_{\mathrm{ELG}}^{-1} \geq 0.2$ $s^{-1}$. The red lines are for illustration purposes only, showing the outcome of robust linear regression; solid for Spearman rank correlations with p < .05, dashed otherwise.

## 3.2 Acceleration model fits

Driver braking, either alone or in combination with steering, was the annotated evasive maneuver in 309 of the 323 extracted events (96 %). Steering alone was annotated in four cases (1 %) and ten cases (3 %) were annotated as complete non-reactions, all of them crashes.

Figure 3 shows that for most events with annotated braking, the observed deceleration data were closely approximated by the piecewise linear model. Obtaining $R^2$ values of .90 or above for the shortest fitted data durations of 0.5 s, with very few data points (as mentioned above, events with shorter durations than 0.5 s were excluded from fitting), is not that impressive, but the scatter plots in Figure 3 indicate that similarly good fits were also obtained when the durations of fitted data were longer, up to the maximum of 6.5 s. From qualitative inspection of example fits such as those shown in Figure 3, it was deemed that when $R^2 \geq .70$, which was the case for 282 of the 301 model-fitted events (94 %), all of the fitted model parameters $t_B$, $j_B$, and $a_1$ said something meaningful about the observed driver behavior. Consequently, only these events were included in the analyses described below (again, see Appendix A for a complete overview of the inclusion/exclusion process). The exact value of this $R^2$ cut-off is not crucial; a sensitivity analysis showed that changing it to e.g. $R^2 \geq .80$ did not alter any of the conclusions of this paper.

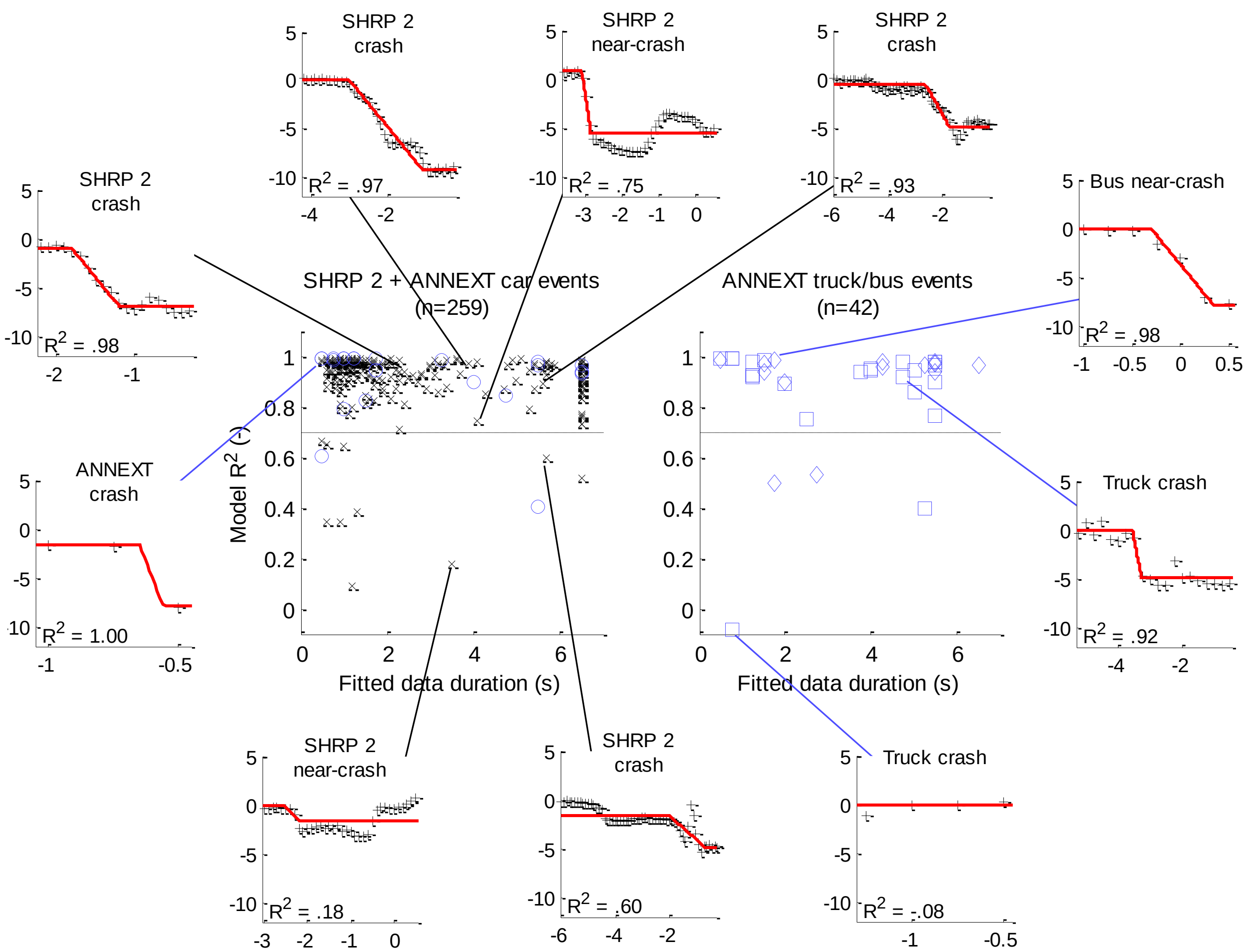


**Figure 3.** Outcome of fitting the acceleration model to the naturalistic data, in events where driver braking had been annotated. The two panels in the middle illustrate the variation in the duration between event start and end points (as defined in the text) as well as the goodness-of-fit ($R^2$) of the model when fitted to each event separately. Symbols as in Figure 2. The dashed line shows $R^2 = 0.70$. Three events with model $R^2 < -0.1$ are not visible (1 SHRP 2 event, 1 truck event, 1 bus event; note that negative $R^2$ values can occur here if the best-fitting model is a constant that, because of the grid search fitting method, does not quite match the observed average deceleration in the event). The surrounding plots show example events, with time in seconds on the x axis, acceleration in m/s$^2$ on the y axis, black plus signs for the observed data, and red lines for the fitted models.

For those events which the model did not fit so well, this generally seemed to be due to one of the following reasons: (1) the driver letting go of the brakes before the minimum TTC was reached in a near-crash (bottom left example in Figure 3), (2) atypical acceleration patterns due to rapid pedal movements or possible sensor limitations (bottom center example), or (3) the acceleration signal showed no signs of driver braking, despite its having been coded by the annotator (bottom right example).

In a number of events, on both sides of the $R^2$ cut-off, there were signs of step-wise deceleration ramp-up (bottom center, top left, and top right examples).

## 3.3 Deceleration onset timing

Figure 4 shows an analysis very similar to that shown in Figure 2, but instead of time from ELG to physical reaction, it shows time from ELG to actual brake onset, as estimated by the $t_B$ parameter in the fitted acceleration models. In addition to the events excluded by the $R^2 \geq .70$ criterion mentioned above, 13 events in which $t_B$ was fitted to the very first sample in the event were also excluded here, since it was not clear from such a fit whether deceleration ramp-up really started at that point, or was already ongoing since previously.

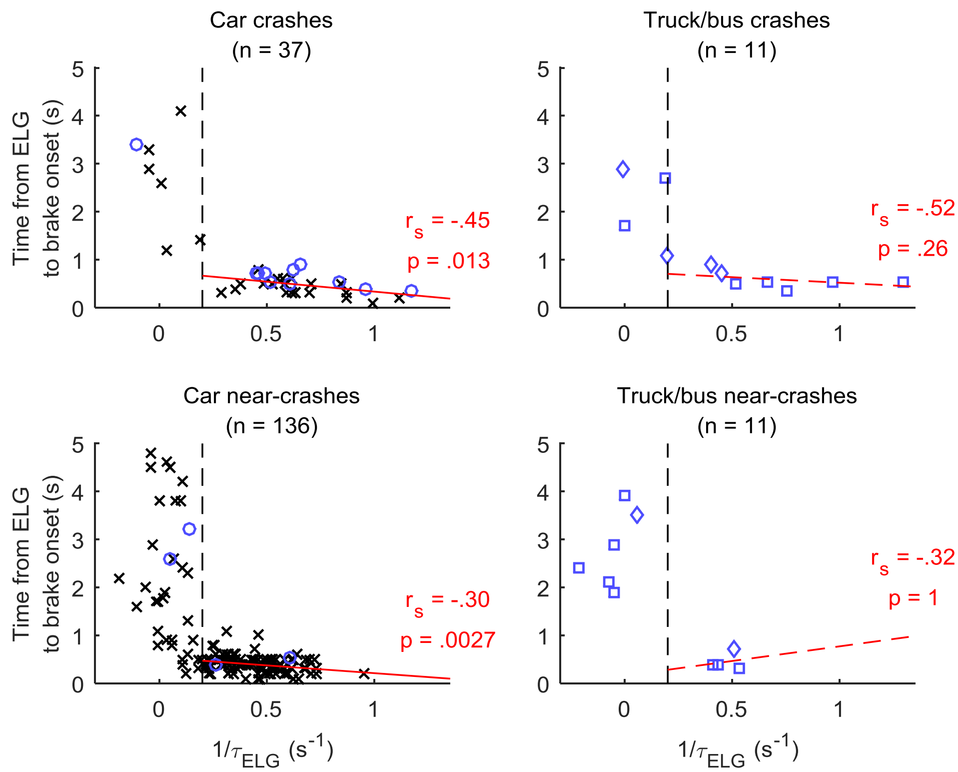


**Figure 4.** Time from the end of the last off-road glance (ELG) until brake onset, as a function of $\tau^{-1}$ at ELG. Symbols as in Figure 2.

Interestingly, except for the correlation for truck/bus crashes with $\tau_{\mathrm{ELG}}^{-1} \geq 0.2$ $s^{-1}$ becoming non-significant due to the lower number of included events, and the opposite occurring for the car near-crashes, all of the general patterns of behavior seen in Figure 2 persist in Figure 4. This similarity suggests a close correspondence between the drivers' physical reactions and their actual brake applications. Indeed, as shown in Figure 5, out of the 192 events for which both physical reaction and brake onset could be analyzed (all of the events in Figure 4 except three events where no physical reaction had been annotated), brake onset occurred within ± 0.5 s of the physical reaction in 176 (92 %) of cases. Given that the correlation between $\tau_{\mathrm{ELG}}^{-1}$ and time to response for car near-crashes was significant for brake onset (Figure 4) but only marginally significant for physical reactions (Figure 2), it could be suspected that the relationship between physical reactions and brake onset might be less clear for these events. However, the distribution shown in Figure 5 stays virtually identical if including only car near-crashes.

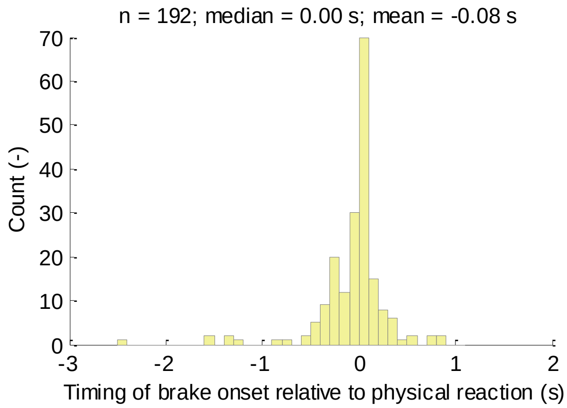


**Figure 5.** Time of brake onset relative to the time of driver physical reaction, with positive values indicating that brake onset occurred after the physical reaction.

Figure 6 shows the same data as in Figure 4, but with time of reaction on the y axis now measured relative to the point in time $t_{0.2}$ when the driver first saw looming of magnitude $\tau^{-1} \geq 0.2$ $s^{-1}$. Note that for eyes-off-threat events, this point in time coincides exactly with the end of the last glance, so for these events Figure 4 and Figure 6 are identical. Also added in Figure 6 are the events without any off-road glances, which can be sensibly graphed here, but could not in Figure 2 or Figure 4 since the time of reaction in those figures is relative to the last glance. Of the 142 eyes-off-threat events in Figure 6, brake onset came within one second after $t_{0.2}$ in 141 cases (99 %), and the average was 0.42 s after $t_{0.2}$. For the 122 eyes-on-threat events, brake onset came within one second after $t_{0.2}$ in 80 cases (66 %); brake onset came before $t_{0.2}$ in 20 cases (16 %), and more than one second after it in the remaining 22 cases (18 %).

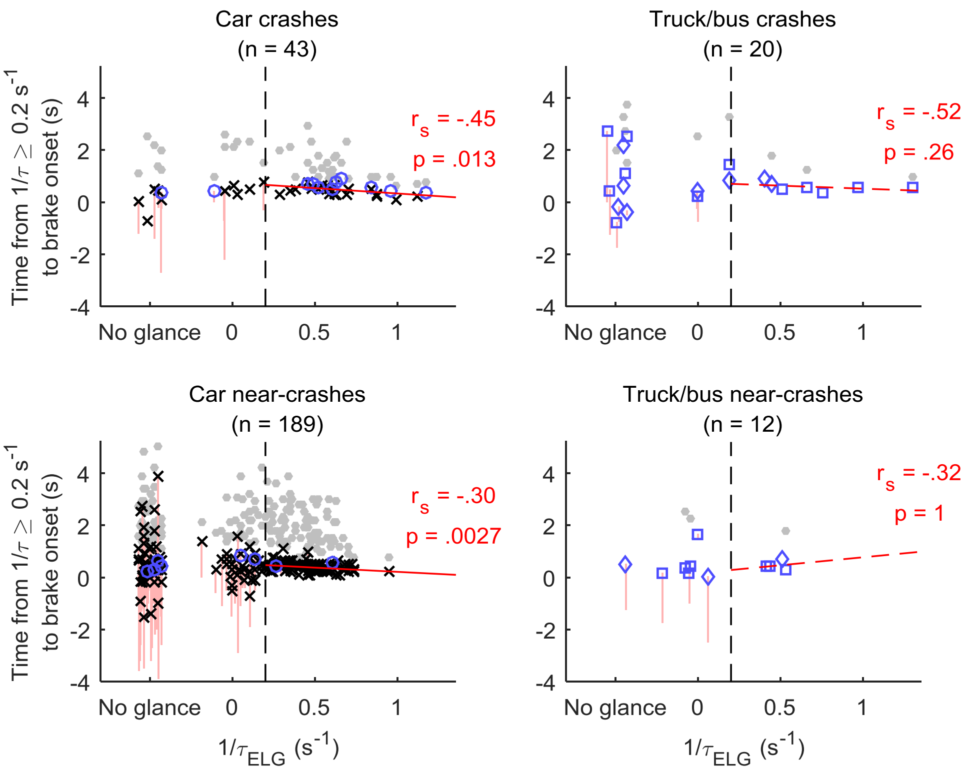


**Figure 6.** Brake onset timing relative to situation kinematics. The data in Figure 4 is repeated here, but offset so that time is counted from $t_{0.2}$, the first time the driver saw $\tau^{-1} \geq 0.2$ s$^{-1}$, i.e., the y axis shows the time from $t_{0.2}$ to brake onset (which can be negative, so the term "reaction time" does not work well here). Also included here are the events without any off-road glance (with slight random scatter along the x axis for legibility). The red traces indicate the duration of the lead vehicle's last brake light activation, for events where there was at least one brake light onset while the driver had the eyes on the road. The gray dots show estimated times of non-reaction collisions, i.e. when collision would have occurred had the driver not braked at all, for those events where such an estimation was feasible.

Figure 6 also shows the estimated times at which collision would have occurred had the subject vehicle driver not applied emergency braking, instead maintaining the constant acceleration $a_0$ after time $t_B$. Note that these times of non-reaction collision could not be estimated in some cases where lead vehicle annotations were lacking toward the end of the event (see Appendix B.1 for further details).

Of the 264 events in Figure 6, the lead vehicle's brake lights were active throughout the entire event in 155 cases (59 %). For the 90 events (34 %) where there was at least one brake light onset during the event, a red, vertical trace has been added, extending down to the point in time when the last brake light onset occurred. Figure 7 provides another view of these 90 events, in the form of distributions of brake onset timing relative to brake light onset and situation kinematics, respectively. In the remaining 19 events (7 %), the lead vehicle brake lights were off throughout.

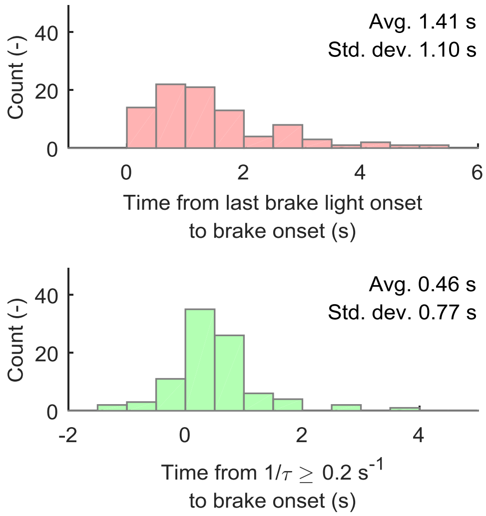


**Figure 7.** Distributions of brake onset timing relative to lead vehicle brake light onset (top), and relative to $t_{0.2}$, the first time the driver saw a $\tau^{-1}$of at least 0.2 $s^{-1}$ (bottom). Note that in both panels the x axes have the same scale, so the variability can be compared visually even though the points from which time is counted (the zeros on the x axes) differ between the panels.

## 3.4 Deceleration ramp-up

Figure 8 shows that deceleration ramp-up was faster when the situation at brake onset was more urgent, i.e.the fitted $j_B$ values were more negative at higher values of $\tau_B^{-1}$, the inverse tau at brake onset (i.e., at time $t_B$). This correlation was statistically significant in all four data subsets except for the truck/bus near-crashes. The included events are the same as in Figure 6, but with the added requirement (see Appendix A) of having at least one intermediate sample of data between the last sample with acceleration $a_0$ and the first one with $a_1$; otherwise jerk fitting is not reliable since a range of $j_B$ values will all fit the data equally well.

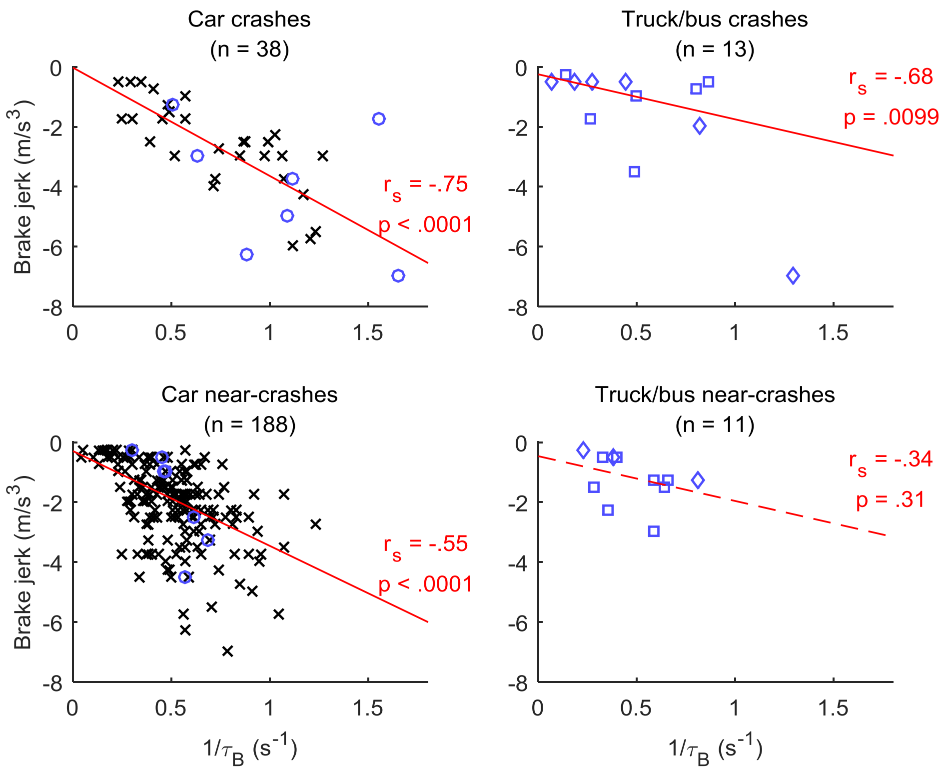


**Figure 8.** Brake jerk as a function of $\tau^{-1}$ at the time when the subject vehicle's driver started braking. Symbols and regression lines as in previous figures. Three data points are not visible in the plots: one car crash with $\tau_B^{-1}$ = -0.02 s$^{-1}$ ($j_B$ = -0.25 m/s$^3$), and one truck/bus crash and one car near-crash with large $\tau_B^{-1}$ values of 2.49 s$^{-1}$ and 7.26 s$^{-1}$, respectively ($j_B$ values of -4.25 m/s$^3$ and -3.50 m/s$^3$).

### 3.5 Maximum deceleration

Figure 9 shows that if there were any correlations between situation kinematics at the time when the driver started ramping up deceleration ($\tau_B^{-1}$), and the subsequent maximum deceleration $a_1$, then for crashes these were weak ($r_s$ close to zero) and not statistically significant. For near-crashes, these correlations were stronger, reaching statistical significance for the car data. Again, the included events here are the same events as in Figure 6, but now with the added requirement that the $a_1$ plateau was fitted as starting at least one sample before the end of the extracted event, to exclude cases where deceleration ramp-up might not have been completed (again, see Appendix A).

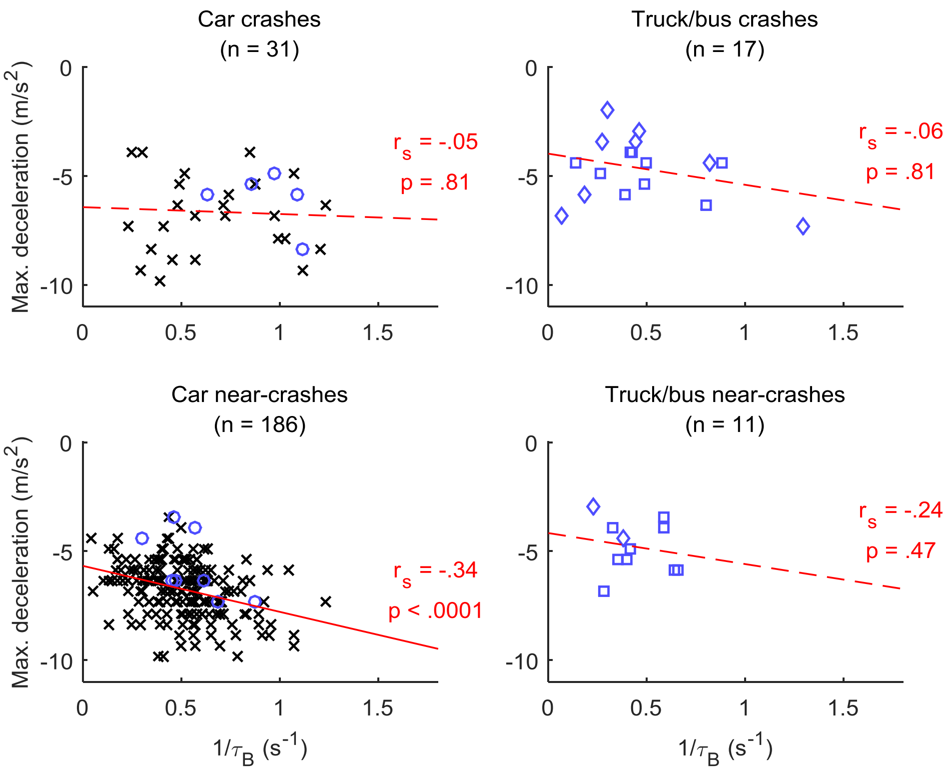


**Figure 9.** Maximum deceleration as a function of $\tau^{-1}$ at the time when the subject vehicle's driver started braking. Symbols and regression lines as in previous figures. Two data points are not visible in the plots: one car crash with $\tau_{\mathrm{B}}^{-1}$ = -0.02 s$^{-1}$ ($a_1$ = -5.40 m/s$^2$), and one car near-crash with $\tau_{\mathrm{B}}^{-1}$ = 7.26 s$^{-1}$ ($a_1$ = -7.36 m/s$^2$).

## 3.6 Probability distributions describing the observed deceleration behavior

Above, various patterns of behavioral variability across kinematical situations have been described. Figure 10 provides a quantitative description of this variability, in the form of probability distributions for deceleration onset timing, deceleration ramp-up, and maximum deceleration; in some cases using transformed quantities, introduced below.

Deceleration onset timing is described in Figure 10 in terms of the dimensionless quantity $\alpha_{\mathrm{B}}$, defined as follows:

$$\alpha_{\mathrm{B}} = \frac{t_{\mathrm{B}} - t_{0.2}}{t_{\mathrm{C}} - t_{0.2}}, \quad (2)$$

where $t_{\mathrm{B}}$ is the time of deceleration onset, $t_{0.2}$ is the time at which the driver first sees visua exceeding $\tau^{-1} = 0.2\ \mathrm{s}^{-1}$ as mentioned above, and $t_{\mathrm{C}}$ the time at which a non-braking drive collided. In other words, $t_{\mathrm{B}} - t_{0.2}$ is the time to brake onset (rather than a "brake reaction time") from $t_{0.2}$, and $\alpha_{\mathrm{B}} = 0$ for a brake onset exactly at $t_{0.2}$, and $\alpha_{\mathrm{B}} = 1$ for a brake onset exactly at collision. The motivation behind this mathematical transformation is that brake onset timing variability (both averages and standard deviations) scales roughly with time left to collision – as the gray dots in Figure 6 ($t_{\mathrm{C}}$) spread higher up above the x axis ($t_{0.2}$), so do the black crosses ($t_{\mathrm{B}}$) – making $\alpha_{\mathrm{B}}$ largely kinematics-independent. In Appendix B, the interested reader can verify that the transformation in Equation (2) indeed does cause most traces of kinematics-dependence to disappear. The only major dependence left to consider is that among eyes-off-threat events the deceleration onsets always occur after $t_{0.2}$, such that $\alpha_{\mathrm{B}} > 0$, and not before it, whereas in eyes-on-threat events negative $\alpha_{\mathrm{B}}$ can also occur. Therefore, as shown in Figure 10, separate probability distributions were fitted for these two classes of event; a normal distribution for eyes-on-threat, and a log-normal one for eyes-off-threat. It should be noted that $\alpha_{\mathrm{B}}$ could only be estimated for a small number of truck/bus events (11 and 6, for eyes-on-threat and eyes-off-threat events, respectively), so the truck/bus $\alpha_B$ distributions are shown here mainly for completeness, rather than for them to necessarily be used in applications.

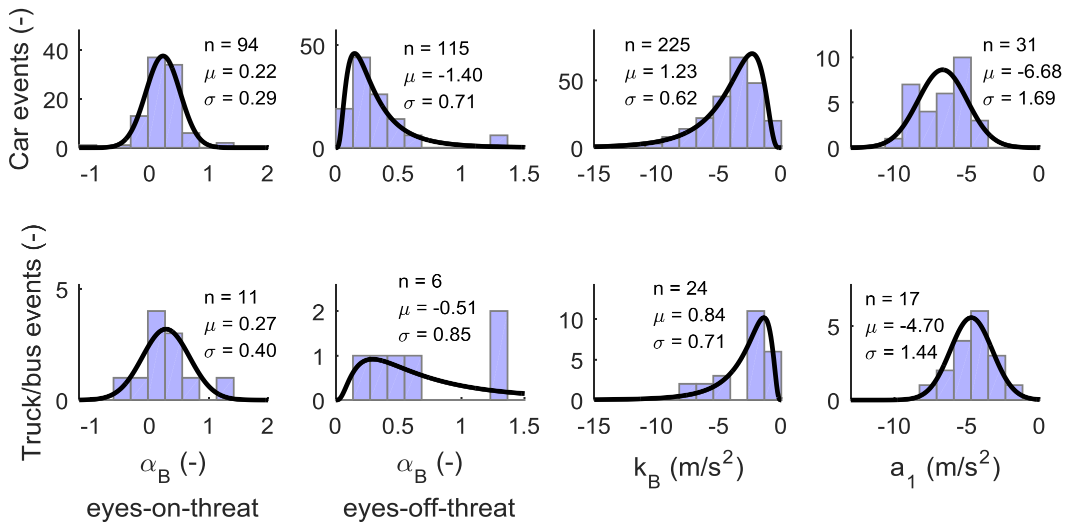


**Figure 10.** Observed distributions (blue bars) of brake onset timing $\alpha_B$, brake jerk gain $k_B$, and maximum deceleration $a_1$, as well as maximum-likelihood fits of probability distributions for the same quantities (black lines). Events where drivers were annotated as non-braking are also included, graphed at $\alpha_B = 1.25$ but fitted as $\alpha_B > 1$. For full details, see Appendix B.

Deceleration ramp-up variability is captured by introducing the very simplest type of linear dependency that one might suggest based on Figure 8:

$$j_B = k_B \tau_B^{-1}, \tag{3}$$

where $k_B$ is thus a linear gain which translates inverse tau at brake onset into the brake jerk $j_B$. Again, refer to Appendix B to see that this gain $k_B$ is only weakly kinematics-dependent, if at all, a 10 it can be seen that its distribution is well described as a log-normal one, mirrored to the n of the number line.

The quantity adopted for describing maximum deceleration is the same $a_1$ used above in the text, fitted to crash events only. Again, see Appendix B for details.

## 3.7 Alternative measures of kinematical urgency

During the approach up to a potential rear-end collision, $\tau^{-1}$, $v/\tau$, and $\dot{\theta}$ all behave similarly; all increase faster as the impact draws nearer (assuming constant speeds, $v/\tau \propto \tau^{-1}$, and while there is still some time left to the collision $\tau^{-1} \approx 1/\mathrm{TTC}$ and $\dot{\theta} \propto 1/\mathrm{TTC}^2$), and all three measures become bounded close to the impact (since $\theta$ cannot grow arbitrarily large). Indeed, the results presented above in this paper were all found to be largely unaffected by the choice of kinematical measure, however with some caveats for $v/\tau$.

The clear cut-off between eyes-on-threat and eyes-off-threat type reactions as seen in Figures 2 and 4 (for $\tau^{-1} \approx 0.2\ \mathrm{s}^{-1}$), was observable also at $\dot{\theta} \approx 0.02\ \mathrm{rad\ s}^{-1}$ and $v/\tau \approx 2\ \mathrm{m/s}^2$, with most eyes-on-threat reactions again occurring within a second after these thresholds. However, for $v/\tau$ this cut-off was somewhat less clear (more fast reactions below the threshold and slow reactions above it) than for the other measures, and the spread in time of reactions around the threshold was also wider (cf. Figure 7; the standard deviation of 0.77 s in the lower panel changes to 0.83 s with $\dot{\theta}$, and to 0.98 s with $v/\tau$). The pattern of decreasing times to reactions for increasing severity in eyes-off-threat events (Figure 4) was also present for all measures, but was not statistically significant for $v/\tau$. The results on deceleration control (Figures 8 and 9) were similar across all three measures, although for $\dot{\theta}$ the $j_B$ regression line had a more pronounced non-zero intercept of about 1 m/s$^2$ (suggesting a constant term in the $\dot{\theta}$ counterpart to Equation 3), and for $v/\tau$ this intercept seemed to differ somewhat between event types.

# 4 Discussion

The present analyses have provided several novel insights into driver braking behavior in emergency situations. One important finding is that drivers who returned their eye gaze to the forward direction at some point before the crash almost always applied their brakes in response to the collision threat (96 % of all events, 90 % if considering only crashes). Previously, based on crash statistics from police reports, it has been proposed that it is fairly common for crash-involved drivers to not attempt any evasive maneuver at all (Wiacek and Najm, 1999; Kusano and Gabler, 2012). The present results suggest that, at least for rear-end conflicts, these reports of non-maneuvering might be attributable not so much to drivers being highly unresponsive to collision threats, but rather to inopportune off-road glances lasting all the way up to the collision itself, and possibly also to methodological difficulties of correctly identifying some late or moderate braking attempts after the fact, without the type of detailed time-series data records that have been used here.

The most important new finding in this paper is that deceleration behavior was highly kinematics-dependent. The observed rich diversity of behavior across different kinematical situations is clearly not at all captured by the type of situation-independent probability distributions for BRT, jerk, and maximum deceleration frequently assumed in previous work, including by the present authors themselves (see the literature review by Markkula et al. 2012, as well as more recent examples by Van Auken et al., 2011; Kusano and Gabler, 2012; Markkula, 2013; Bärgman et al., 2015; van Noort et al., 2015) It is interesting to consider what psycho-motor mechanisms might produce these behavioral phenomena. Before doing so, however, a more methodologically oriented discussion is needed.

## 4.1 Can selection bias explain the observed patterns of brake timing?

One concern that can be raised regarding the data sets used here is that they only included crashes and near-crashes, and not any less severe events. Especially with respect to the eyes-on-threat events, one might ask whether the observed driver reactions, generally occurring only after visual looming had reached levels of about $\tau^{-1} = 0.2\ \mathrm{s}^{-1}$, and at higher $\tau_{\mathrm{B}}^{-1}$ values than are typical for routine driving (Kiefer et al., 2003, 2005; Kusano et al., 2015), might not be due to a type of selection bias: Perhaps, there were many drivers who reacted earlier in similar situations, but who were, because of their faster reactions, able to avoid the collision with sufficiently non-severe maneuvering that the event didn't even register as a near-crash, thus excluding themselves from the present analyses?

It should be acknowledged that this type of phenomenon may well to some extent be biasing the distributions of driver behavior observed here, putting more emphasis on behaviors that are more likely to result in near-crashes and crashes. However, at least two arguments can be made for why this type of bias is not likely to be the main generator behind the observed kinematics-dependencies.

First, it should be noted that while selection bias could conceivably account for the scarcity of reactions below $\tau^{-1} = 0.2$ s$^{-1}$, it is less clear how it could explain that most reactions seem to happen shortly after this threshold. If the reactions observed here are merely the tail end of some more conventional BRT distribution, then why do these tail end reactions fall off so sharply soon after $\tau^{-1} = 0.2$ s$^{-1}$ in Figure 7?

Second, the reaction timing patterns observed here can actually be used to predict the BRT results from previous controlled studies, where the discussed type of selection bias is guaranteed to be absent. Figure 11 shows, on the x axis, the observed average BRT in four different driving simulator studies, all using the same general type of rear-end scenario (a passenger car overtakes, then brakes unexpectedly while the driver has his/her eyes on the road ahead), but with different parameter settings in terms of speeds, time headways, and lead vehicle decelerations; see Appendix C for details. The y axis of Figure 11 shows the corresponding BRT predictions using the eyes-on-threat $\alpha_{\mathrm{B}}$ distribution (Figure 10). As can be seen, this reaction timing model, derived to capture the kinematics-dependencies in the naturalistic data, explains 73 % of the variability in average BRT from the controlled studies. This finding suggests that BRTs are affected similarly by kinematics in the naturalistic and controlled data sets, which, in turn, provides a strong argument that the kinematics-dependencies observed in the naturalistic data are not merely artifacts arising from selection bias. These re-analyses of previous simulator study data are pursued in more detail in (Engström and Markkula, in prep.).

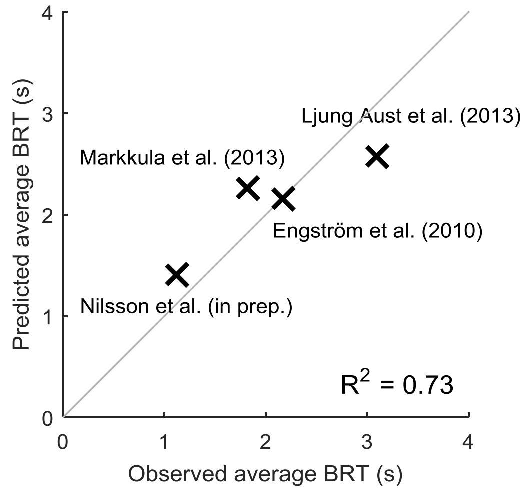


**Figure 11.** Comparisons of average brake reaction time (BRT) observed in four different simulator studies, to the average BRTs that one would predict based on the naturalistic kinematics-dependencies presented in this paper.

## 4.2 Possible mechanisms underlying the observed behavior

### 4.2.1 Deceleration timing from reactions to lead vehicle brake lights?

If the observed patterns of deceleration timing are not artifacts, what causes them? One common assumption in the driver behavior literature is that drivers will generally react, after some BRT, to the sight of lead vehicle brake lights (Liebermann et al., 1995; Shinar et al., 1997; Smith et al., 2005; Salvucci and Beltowska, 2008; Ratcliff and Strayer, 2013). However, such an account is not very helpful for explaining the present observations.

To begin with, brake lights clearly cannot help explain the patterns of variability in brake onset timing for that majority of events in which the lead vehicle brake lights were either active throughout the entirety of the subject vehicle driver's last forward glance (59 %), or not active at all (7 %). Furthermore, as illustrated in Figures 6 and 7, among the remaining minority of events (34 %) in which one or more brake light onsets occurred during the last forward glance, the subject vehicle driver's own brake onset more often than not occurred more than a second later than the last brake light onset, and even up to five or six seconds later. With time differences of, say, three seconds or longer, it seems questionable to talk of the drivers' braking as being in reaction to the brake lights. For intermediate time differences, of one to two seconds, the situation is more ambiguous, and we will return to this type of situation further below.

### 4.2.2 Deceleration timing from looming thresholds?

Overall, a much more powerful explanation for the present observations can be obtained from response threshold models of brake timing, which postulate that drivers initiate braking once some visual cue, for example $\tau^{-1}$, reaches a certain threshold value (Lee, 1976; Kiefer et al, 2003, 2005; Flach et al., 2004; Fajen, 2005, 2008; Wada et al., 2009; Treiber et al., 2013). Previously, such models have mainly been studied in the context of routine driving, but the results presented here, for example in Figure 6, suggest that a threshold-based model also works very well for describing behavior in surprise rear-end emergencies: In eyes-on-threat events, few drivers responded before visual looming reached $\tau^{-1} = 0.2\ \mathrm{s}^{-1}$ (or $\dot{\theta} = 0.02\ \mathrm{rad/s}$, or $v/\tau = 2\ \mathrm{m/s^2}$), and most drivers responded within a second after reaching this threshold[3]. In eyes-off-threat events, where the driver looked back to the road with this threshold already being surpassed, the times to response were even shorter, averaging at 0.42 s.

It is interesting to note that the speed-dependent quantity $v/\tau$ seemed less useful for predicting time of brake onset than $\tau^{-1}$ or $\dot{\theta}$. It is well established that something like a threshold on $v/\tau$ might be determining brake onset timing in routine driving (Kiefer et al., 2003, 2005; Treiber et al., 2013; Kusano et al., 2015), which makes sense given that, at least for stationary obstacles, $v/\tau$ can be used to estimate the deceleration required to avoid collision (Fajen, 2005). However, the present results tentatively suggest that in critical situations, drivers do not factor in their own speed into their braking decision.

In addition to response thresholds, a related but different type of threshold often discussed in the literature is the looming detection threshold, the minimum threshold at which a driver can at all detect visual looming, most often measured in terms of $\dot{\theta}$. Under controlled circumstances, with expected looming stimuli, one typically finds this threshold to be around 0.003 rad/s (Lamble et al., 1999), but it has been suggested that, for unexpected collision threats in naturalistic conditions, it might rise as high as 0.02 or 0.03 rad/s (Maddox and Kiefer, 2012), similar to the 0.02 rad/s $\dot{\theta}$ cut-off observed here. Whether one prefers to think of this cut-off as a detection threshold, at which drivers start perceiving the threat, or a response threshold, at which they start responding to it, comes down to how one thinks about the underlying psychological and biological mechanisms.

Indeed, the lack of any underlying theory can be seen as a general limitation of these threshold models. Thresholds can roughly fit the average brake onsets observed here, but why at these specific levels? If what is at play here is a detection threshold, why is it higher in these naturalistic surprise situations than in the laboratory? Similarly, if what is being observed here is the effect of a response threshold, why do the responses occur at higher $\tau^{-1}$ (lower TTC), on average, in these surprise emergencies than typical values for routine driving behavior? (Kiefer et al., 2003, 2005; Kusano et al., 2015) Furthermore, beyond average behavior, the threshold models neither capture nor explain the observed variability in behavior: Why do eyes-on-threat brake onsets differ so widely between events? And why do eyes-off-threat responses occur faster with increasing kinematical urgencies? The next section introduces a type of model which provides tentative answers to all of these questions.

#### 4.2.3 Deceleration timing from accumulation of looming evidence?

From laboratory experiments in psychology and neuroscience, there is wide support for the hypothesis that stimulus-driven action timing is determined by noisy accumulation of sensory evidence, up to a threshold at which the action occurs (Gold and Shadlen, 2007; Purcell et al., 2010; Ratcliff and Van Dongen, 2011). Markkula (2014) adopted this general idea in a framework for driver control modeling, and proposed that drivers' decisions to apply deceleration are based on the accumulation of various types of stimuli, including – but not limited to – visual looming. As schematically illustrated in Figure 12, there can also be other, anticipatory perceptual cues providing evidence either for the possible need for deceleration (such as lead vehicle brake lights or an upcoming intersection), or against the need for deceleration (such as a lead vehicle's turn indicator signaling a lane change, or an upcoming traffic light shifting from red to green). Figure 12 provides a schematic illustration.

[3] Furthermore, and interestingly, even better fits for the simulator data reanalysis in Figure 11 can be obtained with a $\dot{\theta}$ threshold model, somewhat modified compared to what is predicted by the naturalistic data studied here (Engström and Markkula, in prep.).

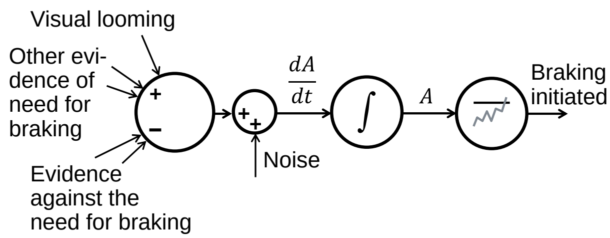


**Figure 12.** An evidence accumulation account of brake timing. The rate of change of a quantity A depends on various sources of evidence for or against the need of braking, and braking is initiated once A exceeds a threshold. (Adapted from Markkula, 2014.)

First of all, note that according to this type of account, something like a lead vehicle brake light certainly can trigger a driver braking response. If there are other anticipatory cues that converge to signal a probable need for deceleration—such as may often be the case in routine driving—a brake light onset could be precisely what pushes the accumulator above its threshold. However, as discussed above, in the near-crashes and crashes studied here, this seems not to have been the case in general.

Figure 13 illustrates how the evidence accumulation model proposed by Markkula (2014) describes the response process in a rear-end situation, in which a brake light onset has not triggered an immediate braking response. First, consider panel (a), and note how visual looming grows faster as the collision draws nearer. The time axis in the figure has been divided into three approximate stages of approach.

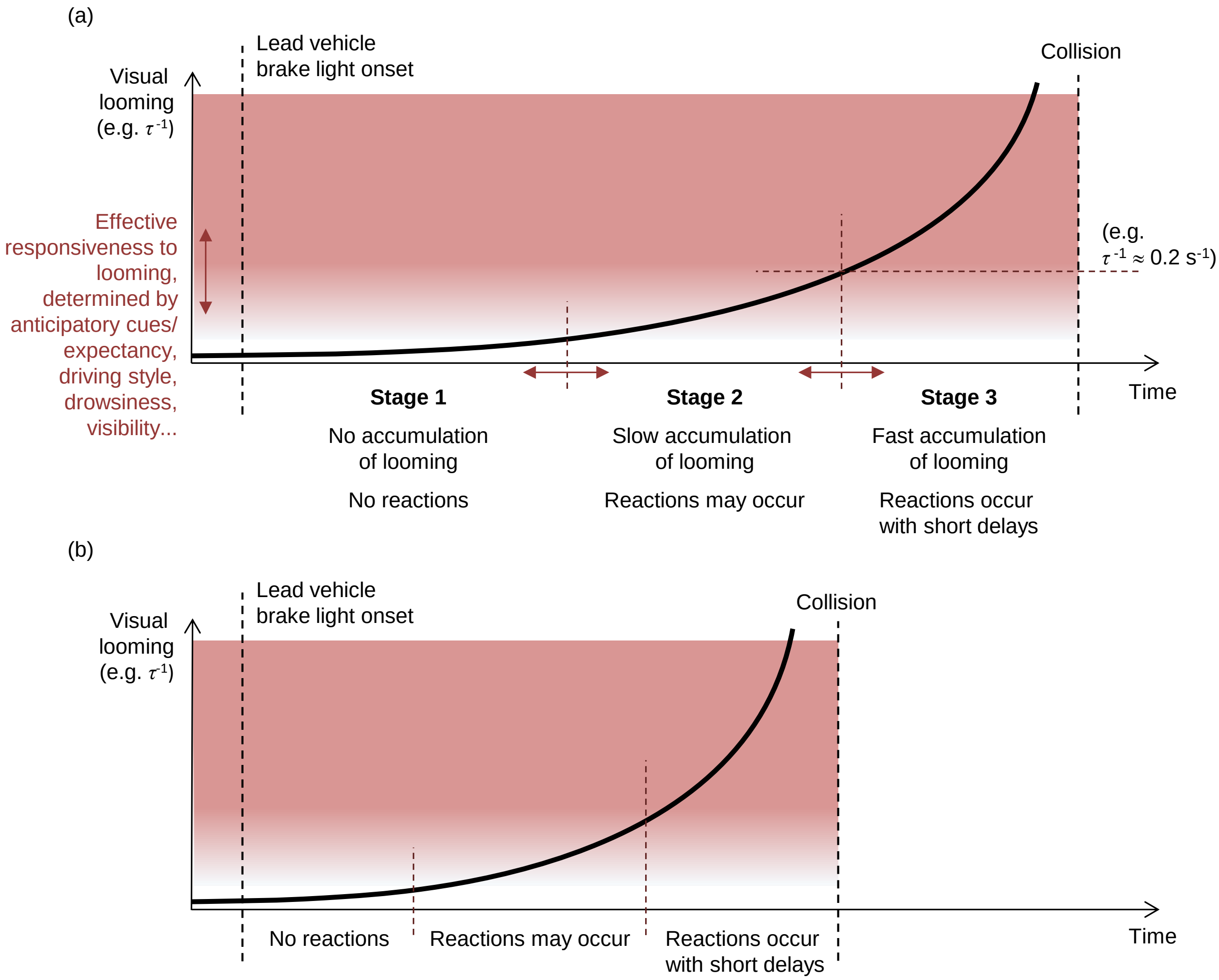


**Figure 13.** A visualization of the process of responding to a braking lead vehicle, as suggested by the evidence accumulation account of Markkula (2014). In the depicted hypothetical situations, lead vehicle brake light onset in itself was not enough to trigger a driver response. Compared to panel (a), panel (b) shows a situation where the potential collision comes sooner after brake light onset, due, for example, to a shorter initial headway, or a stronger lead vehicle deceleration.

A driver who looks back toward the road in the first stage, or does not look away from the road at all, will nevertheless not react at all during this stage, because looming cues are still too weak to trigger any evidence accumulation. Instead, the driver may react during the second stage, but if so with a large variability between drivers and events since the signal-to-noise ratio, of the still weak looming sensory input versus natural fluctuations in neural activity, is still rather low. The same is true for a driver who looks back toward the road ahead during the second stage itself. However, if the driver does not react during the second stage, or looks back toward the road only in the third and final stage of approach, where the large looming cues accumulate quickly, reactions will occur without much further delay, and even faster when the collision is more imminent.

In other words, this type of account provides a coherent explanation for the patterns of reaction timing shown in Figures 2, 4, and 6 of this paper, both in terms of the roughly threshold-like average onsets and the kinematics-dependent variability, larger for longer times left to collision.

Note that the driver's effective responsiveness to looming, illustrated in Figure 13 by the color gradient in the y direction, is assumed to depend on a number of factors. These factors include expectancy (i.e., anticipatory cues, cf. Figure 12), driver states such as drowsiness (see Ratcliff and Van Dongen, 2011), driving style (such as aggressiveness), and visibility conditions. If the responsiveness gradient is moved vertically (for example, upwards to signify reduced responsiveness, perhaps because of reduced expectancy), the transitions between the three stages of approach will move horizontally (in the same example, rightwards to yield later reactions). Specifically, the results presented here suggest that in many of the recorded events, the driver- and situation-dependent responsiveness to looming was such that the approximate transition between slow and fast accumulation, and thus between the second and third stages of approach, occurred at around $\tau^{-1} = 0.2\ \mathrm{s}^{-1}$ (or, roughly equivalently, $\dot{\theta} = 0.02\ \mathrm{rad/s}$; cf. Section **Error! Reference source not found.**). Furthermore, note that with different kinematics, for example in a higher-urgency situation such as depicted in panel (b) of Figure 13, the progression of approach stages will be different, with associated effects on response timing. Thus, the evidence accumulation account can explain the dependencies of reaction timing both on expectancy, as stressed by Green (2000)[4], and on situation urgency, as stressed by Summala (2000).

Beyond evidence accumulation, another mechanism that could be invoked to help explain the very fast brake responses in eyes-off-threat situations, is the perception of looming in the visual periphery (Lamble et al., 1999). Such looming perception could allow drivers to initiate evidence accumulation and foot movements even before shifting eye gaze to the road ahead. To fully enable this type of analysis, more exact data on gaze targets than the present manual annotations would be desirable (something which would of course also strengthen the overall analyses presented here).

#### 4.2.4 Deceleration ramp-up from repeated expectation violations?

The modeling framework by Markkula (2014) might also provide some insight regarding the present results on emergency deceleration control. Notably, a braking model simulation in (Markkula, 2014) predicted the same type of roughly linear ramp-up to a constant maximum that has been observed here. According to that model, this behavior arises because drivers apply intermittent brake pedal adjustments, with magnitudes that scale with how much the visual looming deviates from the looming the driver is expecting to see. Such a strategy can be well adapted to routine driving without necessarily generalizing well to more severe situations, in which visual looming will not disappear in response to braking. This violation of the driver's expectations leads, in the model, to further brake pedal depressions, and the roughly linear type of ramp-up observed here, with larger jerks for more severe situations (cf. Figure 8).

#### 4.2.5 Maximum decelerations from threat being averted and driver/vehicle limits?

With regards to the maximum attained deceleration levels, many closed-loop models of driver braking (see e.g. the review by Markkula et al., 2012), would predict the type of maximum deceleration plateaus that have been observed here. According to these models, in a near-crash this plateau would occur because the collision threat had been averted, and naturally so at larger maximum decelerations in more urgent situations. This type of model can therefore explain the observed increases in maximum deceleration with increased kinematical urgency in near-crashes (Figure 9, bottom panels; $r_s \approx 0.2 – 0.3$, statistically significant for the car data). The lack of a similar correlation for the crash data (Figure 9, top panels; $r_s \approx 0 – 0.1$, non-significant) could be explained by the possibility that the maximum deceleration plateau occurred for another reason in these events: the reaching of some maximum allowable deceleration, dictated either by the driver or by the vehicle on the given road surface.

[4] Interestingly, but so far only anecdotally: For those eyes-on-threat events where brake onset occurred very late, i.e., a long time after $\tau^{-1} = 0.2\ \mathrm{s}^{-1}$, the video recordings often show situations where normal traffic expectations are severely violated, for example a slower lead vehicle signaling to leave the motorway but then instead staying in lane and stopping, or a lead vehicle stopped on a ramp from which the subject vehicle driver is trying to merge onto a motorway. In the terms of the evidence accumulation account, the turn indicator and the context of being in a merging situation, respectively, provide strong evidence against the decision to brake, making the following driver highly non-responsive to the lead vehicle's looming.

### 4.3 The brake reaction time concept

The presented empirical data, as well as the possible underlying mechanisms proposed to explain them, all suggest that it is inadequate to think in terms of BRTs when considering surprise emergencies. Traditionally, BRT has been thought of as mainly a property of the driver, relating to the time it takes for the brain to process perceptual information and trigger a motor response producing a pedal depression. By contrast, the present results point to the need to consider emergency braking reactions as strongly determined by the environmental context, more specifically the evolving situation kinematics and visual looming. This finding indicates that in the general case, there may not be any well-defined, discrete event which the driver can be said to be reacting to.

In effect, what has been said here suggests a new type of interpretation of the typical BRT values reported in the literature for surprise emergencies, such as Green's (2000) 1.5 seconds: Rather than driver braking in these events being slow reactions to some researcher-defined "hazard onset", such as brake lights coming on, these reactions may be better thought of as rather fast responses to the visual looming cues that build up later on. Under this interpretation, BRT values observed under surprise emergency conditions mainly measure how quickly the critical traffic scenario under study develops in time after the "hazard onset". This point is nicely illustrated by the driving simulator results shown in Figure 11 (to be further elaborated in Engström and Markkula, in prep), in which the average BRTs for the same basic scenario vary between 1 and 3 seconds depending only on how kinematically urgent the scenario is tuned to be.

Also Green (2009) has pointed out, with reference to Fajen and Devaney (2006), that it is not always clear at what point an upcoming potential obstacle becomes a hazard that the driver has to respond to. The results presented here suggest that this ambiguity is present not just in some rare special cases, but rather that it is an inherent property of any rear-end conflict.

In sum, it is generally not clear when to set the BRT stopwatch running. To make matters worse, the kinematics-dependency in deceleration ramp-up observed here (Figure 8) also makes it somewhat unclear exactly when to stop the BRT measurement. Much of the literature on BRT focuses primarily on the timing of brake onset, and disregards the subsequent braking control (perhaps because of a tacit assumption that ramp-up is always fast). However, if a maximum deceleration is not reached until after two seconds or more, when should one judge braking to have really started? This ambiguity may be more of a practical concern than a conceptual one, but it can nevertheless hamper the interpretation and cross-study comparison of BRT values.

Note that the problem of undefined start and end points for the BRT interval is especially pronounced in the eyes-on-threat type of rear-end conflicts, where the driver looks forward while visual looming starts increasing. In the eyes-off-threat events studied here, especially those with $\tau_{\mathrm{ELG}}^{-1}$ well above 0.2 $s^{-1}$, it seems more sensible to speak of a BRT, from the end of the last glance until the start of deceleration ramp-up (which also tends to be rather sharp at the correspondingly large $\tau_{\mathrm{B}}^{-1}$). However, these BRTs, typically well below 1 s in duration, were nevertheless kinematics-dependent (shorter for more urgent situations), and again, the closer one gets to $\tau_{\mathrm{ELG}}^{-1}$ = 0.2 $s^{-1}$, the more ambiguous the BRT measurement starts to seem.

### 4.4 Passenger cars versus trucks and buses

Interestingly, the general patterns of braking behavior were very similar between drivers of passenger cars and heavier, commercial vehicles. The main difference was that heavy vehicle braking was slower in ramp-up, and reached lower final magnitudes (see, for example, Figure 10). This is in line with existing data on braking performance (Heusser, 1991, Dunn et al., 2012). Even though modern trucks and buses can, in theory and under optimal circumstances, be capable of braking magnitudes that approach those of passenger cars, actual heavy vehicle braking capability depends markedly on brake maintenance (Radlinski, 1982) as well as on the type of heavy vehicle (Dunn et al., 2012). Another factor to consider is that heavy vehicle drivers may wish to avoid large accelerations, due to passengers who may be unbelted, trailers risking jack-knifing, fragile goods, etc.

It should also be noted that while passenger car data were available both at 4 Hz and 10 Hz, heavy vehicle data were available only at the lower sample rate, something which made the inclusion criteria adopted here leave rather few heavy vehicle events for some analyses.

## 5 Conclusions

The model-based analyses of naturalistic rear-end near-crashes and crashes presented here have provided a number of novel insights into driver emergency braking: (1) Drivers who looked toward the collision threat before impact did, with few exceptions, initiate defensive braking. (2) Brake onset almost always occurred within ±0.5 s of a visually discernible physical reaction by the driver to the collision threat. (3) Crucially, brake onset timing defied description in terms of a single value or distribution of BRT: Responses were very fast when drivers looked back to the road late in the situation (brake onset on average 0.42 s after last glance in eyes-off-threat events), but in events where the driver had eyes on the arising threat, times to brake onset from the last off-road glance or lead vehicle brake light activation were long and highly variable. (4) Regardless of eye glance behavior, brake onsets could instead be better understood as coming in response to the developing situation kinematics, probably mediated by visual looming: Brake onset most often occurred within a second after the driver first saw visual looming above the approximate threshold of 0.2 $s^{-1}$ for $\tau^{-1}$ (0.02 rad/s for $\dot{\theta}$; 2 $m/s^2$ for $v/\tau$, although the latter measure seemed somewhat less predictive of brake timing), with even faster reactions in more urgent situations. (5) These overall patterns of brake timing variability could be explained by positing underlying mechanisms for evidence accumulation. Furthermore, (6) after brake onset, deceleration could be well described as a linear ramp-up followed by a constant maximum deceleration, where (7) the rate of ramp-up was faster in more urgent situations. (8) The maximum deceleration did not vary with situation kinematics in crashes, presumably reflecting vehicle or driver limits on acceleration, but (9) maximum deceleration was kinematics-dependent in near-crashes, presumably because the drivers did not continue to brake harder once the collision threat had been averted.

It has been proposed here that, although highly prevalent in traffic safety research, BRT may not be a meaningful measure of driver behavior in surprise emergencies. Emergency braking seems to be determined more by the gradual build-up of kinematical urgency and visual looming, than by the discrete onset of some well-defined hazardous event, processed slowly and kinematics-independently in the driver’s head before triggering a braking response. This distinction is also important from the perspective of trying to understand how crashes happen and how best to prevent them; rather than highlighting slow reactions as an important causal factor, the present findings reinforce the importantly different notion that late reactions can be due to drivers having failed to react to early, anticipatory cues (for example, brake lights), forcing them to respond to looming cues instead, as a last resort (Engström et al., 2013a).

If one does choose to use the BRT concept to describe behavior in surprise emergencies, great care should be taken when comparing BRT values reported from different studies using different critical scenarios. Specifically, a given observed effect of an experimental manipulation on BRT in one scenario may not generalize well to other scenarios. For example, it has been previously reported that cognitive load (Engström, 2010) and collision warnings (Lee et al., 2002; Ljung Aust et al., 2013) both interact with scenario urgency in their effects on measured BRT.

Quantitative models have also been proposed here, capturing the observed variability in emergency braking behavior without making use of the BRT concept. These models could be applied in a variety of contexts where BRT-based models have been previously used, such as road design, support system algorithms, and support system evaluation; it is an interesting open question to what extent doing so would change any previous conclusions in these areas. An especially relevant challenge for future research is to generalize the present results to situations where a driving support system has issued a collision warning. Current methods for estimating the safety benefits of such systems assume BRT-type driver responses to the warning (Van Auken et al., 2011; Kusano and Gabler, 2012; Erbsmehl and Schebdat, 2015; van Noort et al., 2015), whereas, as mentioned above, evidence from simulator studies

(Lee et al., 2002; Ljung Aust et al., 2013) suggest that driver responses remain kinematics-dependent also in the presence of warnings[5].

The results presented here also seem relevant to current research on self-driving vehicles, where much effort is being spent on understanding the driver's response process when suddenly brought back into the control loop, for example because of a collision risk (Gold et al., 2013; Louw et al., 2015; Zeeb et al., 2015). What has been presented here points to a possible deeper understanding of how drivers make use of their perceptual input in critical situations, in terms of various perceptual mechanisms acting on visual looming information. Such an understanding merits further pursuit in controlled studies, and should apply regardless of whether drivers are in, or are just about to resume, control over their vehicle.

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[5] In the evidence accumulation framework, the warning could be thought of as providing another piece of evidence for the need to braking. If so, then brake onsets in scenarios with warnings should follow the same overall kinematics-dependencies summarized in Figure 10, but with a shift towards earlier onset timings. Absent further empirical research, a conservative approach to safety benefits estimation would be to use the distributions provided here, and assume that warnings only affect eye movement behavior.

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## Appendix A: Event inclusion

Figure A1 illustrates the process of event inclusion, from the initial data set to the various figures in this paper. For those figures that are not represented in Figure A1, inclusion is described in detail in the main text.

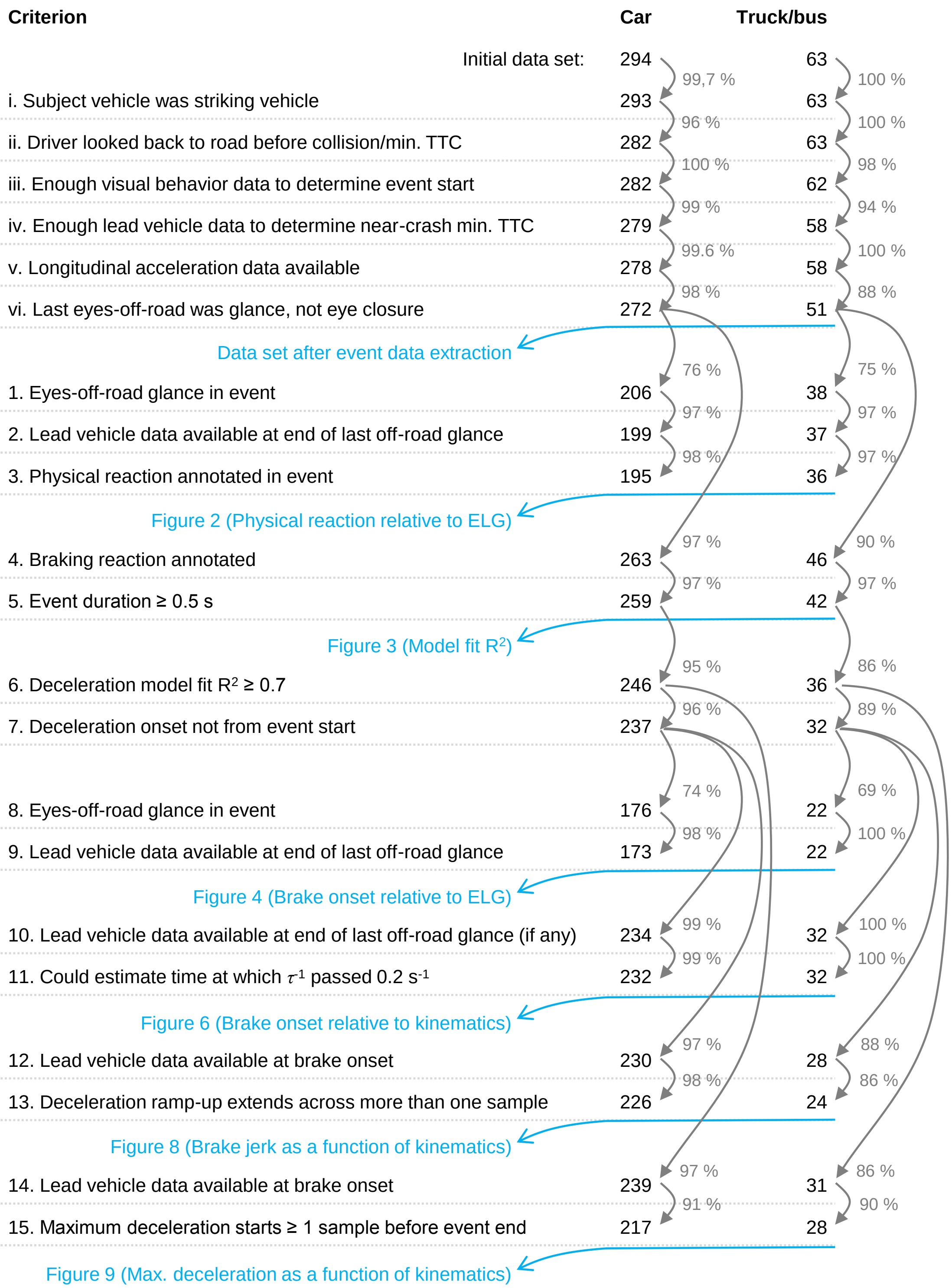


**Figure A1.** Event inclusion. To understand what inclusion criteria were applied in order to select the events for a particular figure in the paper, one can for example start where the figure is mentioned above, and work backwards up along the arrows to the top. ELG = end of last off-road glance.

## Appendix B: The probability distributions

This appendix provides method details and intermediate results for arriving at the probability distributions provided in Figure 10 of the main text, as well as brief notes on the possible use of these distributions.

### B.1 Data transformation

Figure B1 repeats the same reaction timing data as in Figure 6, transformed to show the quantity $\alpha_{\mathrm{B}}$ defined in Equation (2), and with the exclusion of 48 events (30 car; 18 truck/bus) where it was not possible to estimate the time $t_{\mathrm{C}}$ of non-reaction collision, typically because of missing lead vehicle width annotations in the later parts of the event. In the full data set of 323 extracted events, complete non-maneuvering by the driver was annotated in ten cases, and Figure B1 shows five of these; the five others lacked sufficient lead vehicle annotations to estimate $\tau_{\mathrm{ELG}}^{-1}$.

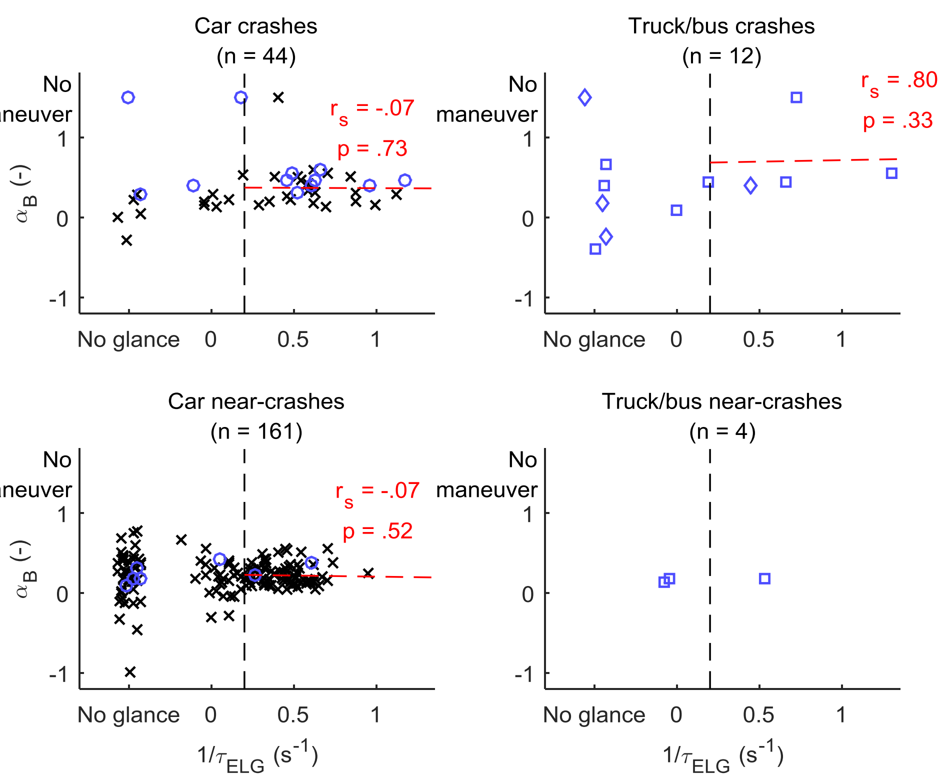


**Figure B1.** Brake onset timing, expressed using the kinematics-scaled quantity $\alpha_{\mathrm{B}}$. Symbols and regression lines as in similar figures in the main text. Two points are not visible: One car near-crash without off-road glance, with $\alpha_{\mathrm{B}} = -5$, and one truck near-crash with $\tau_{\mathrm{ELG}}^{-1} = -0.75\ \mathrm{s}^{-1}$, $\alpha_{\mathrm{B}} = 0.34$.

Figure B2 repeats the same brake jerk data as in Figure 8, but transformed to show the quantity $k_{\mathrm{B}}$, calculated from the observed $j_{\mathrm{B}}$ and $\tau_{\mathrm{B}}^{-1}$ as suggested by Equation (3), i.e. $k_{\mathrm{B}} = j_{\mathrm{B}}/\tau_{\mathrm{B}}^{-1}$.

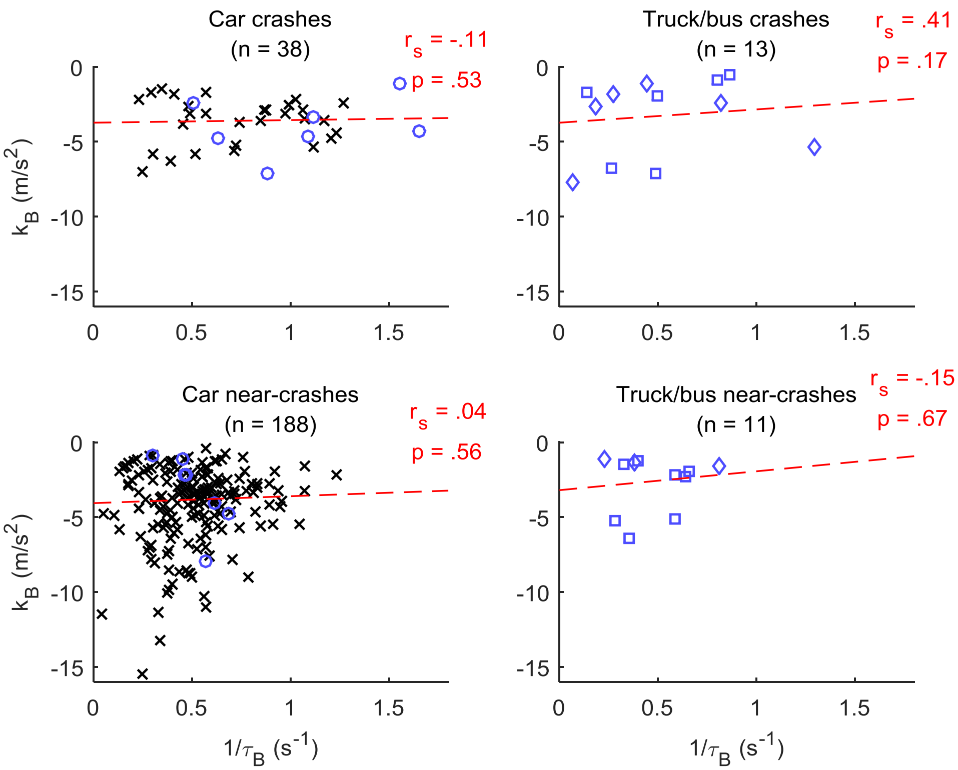


**Figure B2.** Brake jerk, expressed using the kinematics-scaled quantity $k_{\mathrm{B}}$. Symbols and regression lines as in previous figures. The same three data points that were not visible in Figure 8 are also not visible here, for the same reasons.

## B.2 Parameter-fitting

Maximum likelihood probability distributions were fitted for each of the model parameters $\alpha_{\mathrm{B}}$, $k_{\mathrm{B}}$, and $a_1$. The $a_1$ and eyes-on-threat $\alpha_{\mathrm{B}}$ distributions were fitted as normal,

$$P(X = x) = p_{\mathrm{N}}(x) \equiv \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

with $X = a_1$ or $X = \alpha_{\mathrm{B}}$. The eyes-off-threat $\alpha_{\mathrm{B}}$ distribution was fitted as log-normal,

$$P(\alpha_{\mathrm{B}} = x) = p_{\mathrm{LN}}(x) \equiv \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}},$$

and the $k_{\mathrm{B}}$ distribution as a log-normal distribution mirrored to cover negative values, i.e. $P(k_{\mathrm{B}} = x) = p_{\mathrm{LN}}(-x)$.

When fitting the $\alpha_{\mathrm{B}}$ and $k_{\mathrm{B}}$ distributions, all events shown in Figures B1 and B2 were included, with the motivation that including behavior from both near-crashes and crashes will give as complete as possible a view of actual behavior variability in surprise rear-end emergencies. One eyes-on-threat car near-crash outlier with $\alpha_{\mathrm{B}} = -5$ (due to a questionably small $t_{\mathrm{C}} - t_{0.2}$) was excluded. Also excluded was one car crash with a small negative $\tau_{\mathrm{B}}^{-1}$.

As discussed in Section 4.2.4 of the main text, the near-crash $a_1$ data showed correlations with situation kinematics that were probably related to drivers aborting their deceleration ramp-up once the collision threat had been averted. Properly accounting for this phenomenon by including the near-crash data in the distribution fits would require something like a joint distribution for $\alpha_{\mathrm{B}}$, $k_{\mathrm{B}}$, and $a_1$. Here, the $a_1$ distributions were instead fitted to only the crash data in Figure 9, thus presumably capturing the variability of maximum decelerations at all allowable by the driver and the vehicle in these surprise emergencies.

For the $k_{\mathrm{B}}$ and $a_1$ distributions, parameter-fitting was performed using standard closed form expressions (e.g. observed mean as maximum likelihood estimate of actual mean of a normal distribution). For the $\alpha_{\mathrm{B}}$ distributions, also the ten non-braking events were included in the fittings, treated as $\alpha_{\mathrm{B}} > 1$, so these distributions were instead fitted by calculating the actual model likelihoods, in a parameter grid search with 0.01 precision for all distribution parameters. The five above-mentioned non-braking events which were excluded from Figure B1 due to non-estimable $\tau_{\mathrm{ELG}}^{-1}$, were found to all have a very short time (≤ 0.5 s) from ELG to actual crash, and could therefore confidently be treated as eyes-off-threat events.

B.3 Usage notes

The first thing to note about the $\alpha_{\mathrm{B}}$ distributions and the way they have been fitted, is that they are intended to capture not only timing of braking, but also whether or not the driver applies the brakes at all ($\alpha_{\mathrm{B}} \leq 1$ versus $\alpha_{\mathrm{B}} > 1$). This type of view of non-braking, as "too late braking", fits well with the evidence accumulation account presented in Section 4.2.3 (see also Markkula et al., 2013; pp. 1273-1274). In practice, actual non-braking was observed in the present data somewhat more often than what is predicted by the fitted distributions. For example, for the car eyes-off-threat events, the distribution predicts $P(\alpha_{\mathrm{B}} > 1) = 0.024$ whereas the observed fraction is 6/115 = 0.052. There are two things to note here: First, the adopted inclusion criteria for the non-reaction events were less strict than for the events with braking, presumably making non-braking somewhat overrepresented in Figure 10. Second, it is possible that some very late brake reactions may have been coded by annotators as non-reactions, since the effects on vehicle accelerations may have been small. Consistent with this idea, it can be noted in Figure 10 that no $\alpha_{\mathrm{B}}$ values close to but below one were observed, and for the car eyes-off-threat events one gets, for example, $P(\alpha_{\mathrm{B}} > 0.8) = 0.049$.

Another thing to note is that Equation (3) allows arbitrarily large values for $j_{\mathrm{B}}$. To prevent biologically infeasible deceleration ramp-ups in very critical situations with large $\tau_{\mathrm{B}}^{-1}$, one could introduce a ceiling, for example $j_{\mathrm{B}} \leq 8\ \mathrm{m/s^3}$ (cf. Figure 8).

Furthermore, it should be noted that even if the collision has already been averted the model proposed here will continue to ramp up deceleration to $a_1$, interpretable as the maximum deceleration allowed by the vehicle or driver (as discussed above). This type of model behavior is probably acceptable in most applications. If it is not, the most straightforward model extension would be the addition of a separate mechanism for aborting ramp-up, for example triggered once visual looming falls below some threshold, preferably empirically identified.

Also note that no probability distributions have been fitted for the initial acceleration parameter $a_0$, since it has been assumed here that in most applications this parameter can either be set to zero, be defined by the simulated traffic scenario itself, or be known from a specific naturalistic event in a what-if simulation (e.g. Bärgman et al., 2015).

In quantitative approaches to traffic safety, the models and distributions proposed here should provide a significant step forward compared to prevailing BRT-based approaches. However, as discussed in Section 4.1, it should be acknowledged that the lack of less critical events in the studied data sets mean that the distributions in Figure 10 may to some extent be over-representing such traffic situations (e.g., low road friction), driver states (e.g., drowsiness), and driver behaviors (e.g., slow deceleration ramp-up) that are more prone to lead to near-crashes and crashes, potentially biasing distributions towards later reactions and weaker deceleration responses. One interesting avenue for future work would be to try to appropriately fill in these probability distributions with driver behavior data from less critical scenarios.

## Appendix C: Explaining brake reaction times in previous simulator studies

The four simulator studies referenced in Figure 11 of this paper all used the same basic driving scenario, originally proposed by Engström et al. (2010): A faster vehicle overtakes the participant's vehicle, and then, after reaching a certain time headway, starts decelerating for no apparent reason. Table C1 lists the specific scenario parameters used in the four studies, as well as the observed average BRTs for unexpected exposure. Note that in the case of the (Markkula et al., 2013) study, four conditions (low and high driving experience, two different experiments) have been averaged across, weighted by number of participants per condition. In all studies, the participants drove without any secondary tasks, except in the (Ljung Aust et al., 2013) study, where lead vehicle deceleration begun while drivers were looking at an in-vehicle display. In the present analysis, only the "long headway" scenario from that study was included, which in practice was of the eyes-on-threat type, at least for the majority of drivers: On average, the participants looked back to the road ahead 1.49 s after lead vehicle deceleration onset, whereas $\tau^{-1} = 0.2\ \mathrm{s}^{-1}$ occurred at about 1.95 s. In other words, all four scenarios included here can be considered eyes-on-threat scenarios.

To obtain Figure 11, these scenarios were recreated in computer simulation, and average brake onset timings were predicted using the eyes-on-threat $\alpha_{\mathrm{B}}$ model for passenger cars, from Figure 10 of this paper. See (Engström and Markkula, in prep.) for extended analyses of this type.

**Table C1.** Parameters of subject vehicle (SV) and lead vehicle (LV) kinematics in a rear-end scenario shared across four previous simulator studies, as well as observed average brake reaction times (BRTs).

| Original reference | SV type | SV speed (km/h) | LV speed before deceleration (km/h) | Time headway before LV deceleration (s) | LV deceleration (g) | Observed average BRT (s) |
|---|---|---|---|---|---|---|
| Engström et al. (2010) | Car | 70 | 80 | 1.5 | 0.51 | 2.17 |
| Ljung Aust et al. (2013) | Car | 90 | 90 | 2.5 | 0.55 | 3.10 |
| Markkula et al. (2013) | Truck | 80 | 80 | 1.5 | 0.35 | 1.82 |
| Nilsson et al. (in prep.) | Car | 80 | 80 | 1.3 | 0.6 | 1.11 |