

# Nonlinear Control of Single-Phase PWM Rectifiers with Inherent Current-Limiting Capability

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**Abstract**—In this paper, a nonlinear controller with a current-limiting property is proposed to guarantee accurate dc output voltage regulation and unity power factor operation for single-phase PWM rectifiers without the need of a phase-locked-loop (PLL). The proposed current-limiting controller is fully independent of the system parameters and can guarantee asymptotic stability and convergence to a unique solution for the closed-loop system using nonlinear control theory. Without requiring the instantaneous measurement of the grid voltage, a PLL, an external limiter or a saturation unit, the proposed strategy guarantees that the input current of the rectifier will always remain below a given value. An analytic framework for selecting the controller parameters is also presented to provide a complete controller design procedure and it is also proven that the current-limiting property is maintained even when the grid voltage drops. Extensive experimental results are presented to verify the proposed controller when the load changes, the reference dc output voltage changes and the grid voltage drops.

**Index Terms**—PWM rectifiers, nonlinear control, current limit, asymptotic stability, grid voltage dip

## I. INTRODUCTION

AC/DC power converters are widely used in power systems to integrate loads or power sources to the electric grid by operating as a rectifier or an inverter, respectively [1], [2], [3], [4]. Depending on the application, ac/dc converters can be single-phase or three-phase with main tasks the accurate dc bus voltage regulation and power factor correction (PFC). For rectifiers, the PFC can be achieved by controlling the input current to be in phase with the input voltage and has been extensively studied in the literature [5], [6], [7], [8], [9], [10], [11], [12].

AC/DC converters are inherently nonlinear systems due to their switching operating function. Among these devices, the single-phase full-bridge or H-bridge rectifier represents a common PFC converter operating in pulse-width-modulating (PWM) mode and its model can be generalized in the three-phase converter case [9], [13], [14], [15], [16]. Therefore, several researchers have developed control strategies for single-phase rectifiers to achieve both dc output voltage regulation

and unity power factor operation. The most commonly used method includes a cascaded structure where an outer loop is used to control the dc output voltage and an inner current loop is used to control the input current to be in phase with the input voltage. In the traditional control methods, a Proportional-Integral (PI) controller is included into the outer loop and the inner current controller usually consists of a hysteresis current method [5], [17], [18], [19]. Additionally, the cascaded control structure can be combined with intelligent techniques such as fuzzy control to incorporate a sensorless design, as described in [20]. The traditional control methods have a simple structure and are effective in practice but they lack from a rigorous stability theory for the complete closed-loop system. Although for rectifier applications, boost-type PFC rectifier can be used instead of full-bridge rectifiers [21], the efficiency of these converters is significantly reduced and they result in higher power losses, especially for high-power applications [22].

Due to the nonlinear dynamic model of the converter, which can be obtained using the average analysis [23], [24], passivity-based control represents a powerful tool and can be effectively applied to achieve both control tasks and guarantee global asymptotic stability of the closed-loop system [7], [12], [15]. Since the accurate knowledge of the load is required in this case, adaptive passivity-based structures have been developed to cope with the load uncertainty [25], [26], [12]. However, the control scheme still depends on the rest of the system parameters, i.e. the inductor, the capacitor and the measurement of the grid voltage. These parameters might not be accurately known a priori or might change during the system operation. Furthermore, more complicated loads can be connected at the rectifier output, (e.g. nonlinear, power converter-fed loads), which will increase the complexity of the model and consequently the controller design.

Furthermore, in practice, except from the requirement of an asymptotically stable closed-loop system solution, the input current should be maintained below a maximum value at all times for protection purposes. Hence, the development of current-limited rectifiers has been an active area of research for several decades [27], [28], [29], [30], [31]. Current limitation should be maintained at all times, especially during transients, load changes and input voltage sags, since these cases can be catastrophic for the rectifier as mentioned in [32], [33]. Although current limitation can be achieved with the advanced passivity-based methods under accurate knowledge of the system parameters, the traditional control techniques that are parameter-free require external current limiters, protection circuits or saturation units in the control design to limit the input current [34], [35], [36], [37], [38]. Several approaches

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also inherit a switching behavior between the normal operation and the current-limiting operation when the grid voltage drops [36], [39]. However, the use of saturation units can lead to undesired oscillations in several applications and asymptotic stability cannot be guaranteed, mainly due to integrator windup [32], [38]. Although anti-windup techniques can be inserted into the control design, traditional anti-windup methods lack from a rigorous stability analysis and modern anti-windup methods require the knowledge of the plant to guarantee stability [40], [41], [42]. Hence, it is a challenge to design a parameter-free controller with a simple and unified structure (no switching between control algorithms) that guarantees nonlinear stability of the closed-loop system and a given limit for the input current even under grid voltage sags.

In this paper, the single-phase full-bridge rectifier is investigated and a nonlinear controller that achieves accurate dc output voltage regulation, unity power factor operation and closed-loop stability with a limited input current, is developed. The proposed current-limiting nonlinear controller is fully independent of the system parameters, has a simple structure that leads to an easy implementation and achieves PFC at the input of the rectifier, which practically corresponds to PFC for the complete system. Using nonlinear Lyapunov theory, the controller operation is investigated and based on the input-to-state stability theory [43], closed-loop system stability in the sense of boundedness and eventually asymptotic convergence to a desired solution are proven. Particularly, for a given maximum RMS value  $I_{max}$  of the input current, the controller parameters can be suitably selected to guarantee that the input current will always remain bounded below this given value. This imposes a significant advantage compared to the existing parameter-free control techniques, since the current-limiting function is embedded into the original control structure, no external limiters or switching operation are required, leading to a continuous-time controller with the same dynamics that allows the investigation of stability. Moreover, only an initial estimation of the RMS value of the grid voltage (which is known in practice) is required, while the instantaneous measurement of the grid voltage or an additional PLL are not needed, thus further simplifying the controller implementation. The current-limiting capability is guaranteed even in the cases where the input voltage varies or rapidly drops, extending the proposed controller performance to both normal and abnormal operations of the grid, i.e. during grid faults. Extensive experimental results are provided for the single-phase full-bridge rectifier to illustrate the proposed approach and verify its current-limiting capability under load changes, reference dc output voltage changes and input voltage sags.

The paper is organized as follows: In Section II, the dynamic model of the rectifier is obtained and the main problem addressed in this paper is formulated. In Section III, the current-limiting controller is proposed and analyzed. Closed-loop system stability is proven, an analytic framework for selecting the controller parameters is presented and the controller performance is extended to the cases of input voltage variations. In Section IV, experimental results are provided to verify the proposed controller performance and finally, in Section V, some conclusions are drawn.

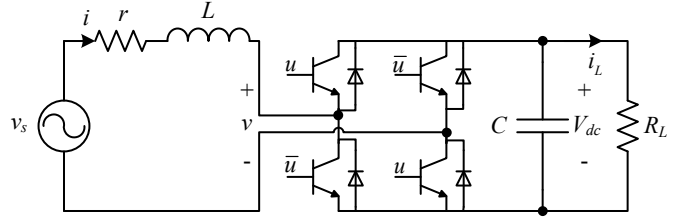


Figure 1. Single-phase full-bridge rectifier

## II. PROBLEM FORMULATION

Consider a single-phase full-bridge rectifier feeding a resistive load as shown in Fig. 1. The converter consists of a boosting filter inductor  $L$  with a small parasitic resistance  $r$ , a dc capacitor  $C$  and 4 switches, whose switching signals are obtained from a PWM circuit taking values in the finite set  $\{-1, 1\}$ . Although the filter can be of different types (e.g.  $LCL$ ) to achieve better harmonic attenuation for the grid current, here an  $L$  filter is considered for simplicity, since as it is explained below, at this stage, the current-limiting property and the stability are the main goals in this paper and not the power quality improvement. Parameter  $R_L$  represents the load resistance,  $i$  is the inductor current,  $V_{dc}$  is the dc output voltage,  $v$  is the converter input voltage and  $v_s$  is the single-phase grid voltage of the form  $v_s = \sqrt{2}V_s \sin \omega t$ , where  $V_s$  is the RMS grid voltage and  $\omega$  is the grid angular frequency.

Using average model analysis [44], the nonlinear dynamic model of the system can be obtained using the Kirchhoff laws and the power equivalence of the converter as:

$$L \frac{di}{dt} = -ri - uV_{dc} + v_s \quad (1)$$

$$C \frac{dV_{dc}}{dt} = ui - \frac{V_{dc}}{R_L}, \quad (2)$$

where the control input  $u = \frac{v}{V_{dc}}$  represents the continuous-time duty-ratio signal of the rectifier bounded in the range  $[-1, 1]$ , which is fed to the PWM generator to result in the discrete signals of the switching elements, while the grid voltage  $v_s$  represents an external uncontrolled input.

For system (1)-(2), the main tasks are to achieve accurate dc output voltage regulation and unity power factor operation. The average value of the dc output voltage  $\bar{V}_{dc}$  should be always regulated at a given reference value  $V_{dc}^{ref}$ . The value of  $\bar{V}_{dc}$  can be obtained from  $V_{dc}$  with a low-pass filter which rejects the second-order harmonics. In practice, the average value of the dc output voltage is calculated using a low-pass filter for  $V_{dc}^2$  and then taking the square root of the result [45]. For the unity power factor operation, the current  $i$  should be in phase with the grid voltage  $v_s$ . In many applications of ac/dc converters (in the rectifier or inverter mode), power factor is also considered at the input of the converter [1], i.e., the current  $i$  to be in phase with the converter voltage  $v$ , since in most cases the filter inductor does not cause a significant phase shifting between the two voltages  $v_s$  and  $v$ .

Opposed to the traditional control methods [5], [17], [18] which lack from a rigorous nonlinear stability analysis and the advanced nonlinear controllers [7], [12], [15], [25], [26] that

depend on the system parameters and the load dynamics, in this paper, the existence of a control structure for rectifiers is investigated that achieves both tasks and incorporates all of the following properties:

- 1) Complete independence from the system and load parameters;
- 2) Nonlinear closed-loop system stability with a given current limit;
- 3) Simple structure, based on the dynamics and the sensors required for the implementation.

### III. PROPOSED CONTROLLER DESIGN AND STABILITY ANALYSIS

#### A. Current-limiting nonlinear controller

Taking into account all of the controller properties mentioned in the previous section, the following current-limiting nonlinear controller is proposed:

$$u(t) = \frac{v(t)}{\bar{V}_{dc}(t)} = \frac{w(t)i(t)}{V_{dc}(t)}, \quad (3)$$

where  $w$  represents a virtual resistance which changes according to the nonlinear dynamic equations

$$\dot{w} = c \left( \bar{V}_{dc} - V_{dc}^{ref} \right) w_q^2 \quad (4)$$

$$\dot{w}_q = -\frac{c(w-w_m)w_q}{\Delta w_{max}^2} \left( \bar{V}_{dc} - V_{dc}^{ref} \right) - k \left( \frac{(w-w_m)^2}{\Delta w_{max}^2} + w_q^2 - 1 \right) w_q, \quad (5)$$

with  $c$ ,  $k$ ,  $w_m$ ,  $\Delta w_{max}$  being positive constants. Note that for the implementation of (3), the current  $i$  should be the average value (sinusoidal) of the actual inductor current and can be obtained using a low-pass filter that rejects the higher harmonics (switching ripples), while  $V_{dc}$  represents the actual output voltage which includes the second-order harmonics.

From the controller structure, it becomes clear that the proposed controller is fully independent of the system parameters and does not require the instantaneous measurement of the grid voltage  $v_s$  or a PLL. When the average dc output voltage  $\bar{V}_{dc}$  is regulated at the reference value  $V_{dc}^{ref}$ , then the controller parameter  $w$  is regulated at a constant value  $w^e$ , since  $\dot{w} = 0$  from (4), and consequently (3) becomes

$$v(t) = w^e \cdot i(t) \quad (6)$$

which shows that the input voltage of the converter  $v$  is in phase with the current  $i$  and therefore unity power factor is achieved. As a result, both control tasks of the converter can be accomplished. In order to investigate whether this operation is possible, the controller dynamics are further analyzed.

For system (4)-(5), consider the Lyapunov function

$$W = \frac{(w-w_m)^2}{\Delta w_m^2} + w_q^2. \quad (7)$$

Its time derivative becomes

$$\dot{W} = -2k \left( \frac{(w-w_m)^2}{\Delta w_m^2} + w_q^2 - 1 \right) w_q^2. \quad (8)$$

Therefore, if the initial conditions are chosen  $w_0 = w_m$  and  $w_{q0} = 1$ , then  $\dot{W}(t) = 0 \Rightarrow W(t) = W(0) = 1, \forall t \geq 0$  implying that  $w$  and  $w_q$  start and stay thereafter on the ellipse  $W_0$ :

$$W_0 = \left\{ w, w_q \in \mathbb{R} : \frac{(w-w_m)^2}{\Delta w_m^2} + w_q^2 = 1 \right\}, \quad (9)$$

as shown in Fig. 2, which means that  $w \in [w_{min}, w_{max}] = [w_m - \Delta w_m, w_m + \Delta w_m]$ . Note that the same operation is obtained for any initial conditions  $w_0$  and  $w_{q0}$  defined on the ellipse  $W_0$ , i.e. satisfying

$$\frac{(w_0 - w_m)^2}{\Delta w_m^2} + w_{q0}^2 = 1, \quad (10)$$

with  $w_{q0} > 0$ . Hence, one can choose accordingly the parameters  $w_m$  and  $\Delta w_m$  in order for  $w_{min} > 0$  at all times, i.e., it should be

$$w_m > \Delta w_m > 0, \quad (11)$$

which leads to  $w(t) > 0, \forall t \geq 0$  and makes sense since it represents a virtual resistance in a rectifier application. The given bounds for the state  $w$  are important for limiting the current under a given maximum value as it will be analytically explained in the stability analysis described in Subsection III-B.

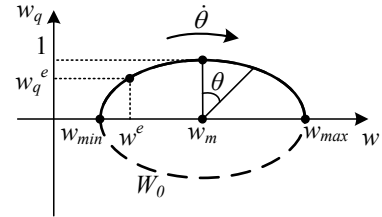


Figure 2. Controller states operation on ellipse  $W_0$

When the controller states operate on the ellipse  $W_0$ , the controller dynamics (4)-(5) become

$$\dot{w} = c \left( \bar{V}_{dc} - V_{dc}^{ref} \right) w_q^2 \quad (12)$$

$$\dot{w}_q = -\frac{c(w-w_m)w_q}{\Delta w_{max}^2} \left( \bar{V}_{dc} - V_{dc}^{ref} \right). \quad (13)$$

For system (12)-(13), consider the following transformation

$$w - w_m = \Delta w_m \sin \theta \quad (14)$$

$$w_q = \cos \theta \quad (15)$$

which results after some simple calculations that

$$\dot{\theta} = \frac{c w_q \left( \bar{V}_{dc} - V_{dc}^{ref} \right)}{\Delta w_m}. \quad (16)$$

Expression (16) shows that the controller states  $w$  and  $w_q$  will move on the ellipse  $W_0$  with angular velocity given by (16). Therefore, if  $\bar{V}_{dc} \rightarrow V_{dc}^{ref}$ , then  $\dot{\theta} \rightarrow 0$  and both controller states will stop moving and converge to their steady-state values  $w^e$  and  $w_q^e$  respectively, as shown in Fig. 2. This makes possible the convergence of the complete converter system, since if  $w$  and  $w_q$  pass the desired equilibrium point during

the transient response, the angular velocity will change sign and the states will oscillate around the equilibrium point.

Additionally, the controller states will move exclusively on the upper semi-ellipse of  $W_0$  (Fig. 2), for initial conditions  $w_0$  and  $w_{q0} > 0$  that satisfy (10). The reason is that if  $w$  and  $w_q$  try to reach the horizontal axis, then  $w_q \rightarrow 0$  and as a result from (16),  $\dot{\theta} \rightarrow 0$  which forces the controller states to slow down independently from the difference  $\bar{V}_{dc} - V_{dc}^{ref}$ . This fact prohibits the existence of an oscillating behavior (limit cycle) for the controller dynamics themselves, i.e., they will never continuously travel around the ellipse  $W_0$ . In addition, since  $w_q \rightarrow 0$  results in  $\dot{w} \rightarrow 0$  in (4), this means that the integration slows down near the limits, i.e. when  $w \rightarrow w_{min}$  or  $w \rightarrow w_{max}$ , and hence the proposed controller does not suffer from integrator windup issues. It should be mentioned that instead of the control dynamics (4)-(5) one can implement the proposed controller using (14), (15) and (16), and result in the same behavior since the two representations are equivalent.

### B. Closed-loop system stability

From the previous analysis, it is clear that the proposed controller is able to achieve both precise output voltage regulation and unity power factor. However, in order to accomplish both tasks, closed-loop system stability should be guaranteed at all times. Since  $w_m > \Delta w_m > 0$  and  $w_0, w_{q0}$  satisfy (10), then  $w$  and  $w_q$  are bounded with  $w \in [w_{min}, w_{max}]$ , where  $w_{min} > 0$ . By substituting (3) into the original system (1)-(2), it yields

$$L \frac{di}{dt} = -(r + w)i + v_s \quad (17)$$

$$C \frac{dV_{dc}}{dt} = \frac{wi^2}{V_{dc}} - \frac{V_{dc}}{R_L} \quad (18)$$

which is still a nonlinear system but (17) can be investigated as a time-varying system with  $w \in [w_{min}, w_{max}]$ , where  $w_{min}, w_{max} > 0$ .

Now, for system (17), consider the Lyapunov function

$$V = \frac{1}{2}Li^2 \quad (19)$$

with time derivative

$$\begin{aligned} \dot{V} &= -(r + w)i^2 + v_s i \\ &\leq -(r + w_{min})i^2 + v_s i < 0, \quad \forall |i| > \frac{|v_s|}{r + w_{min}} \end{aligned} \quad (20)$$

which proves that system (17) is input-to-state stable (ISS) [43] and since the grid voltage is assumed to have a constant amplitude (stiff grid), then the inductor current will be bounded for all  $t \geq 0$ . Then, the remaining dynamics (18) can be written as

$$\frac{1}{C} \frac{dV_{dc}^2}{dt} = -\frac{V_{dc}^2}{R_L} + wi^2 \quad (21)$$

which is a first-order differential equation of  $V_{dc}^2$  with input  $wi^2$ . This system is bounded-input bounded-state stable and since  $w$  and  $i$  are proven to remain bounded, then  $V_{dc}^2$  is bounded and consequently  $V_{dc}$  is bounded. Therefore, the closed-loop system solution  $\begin{bmatrix} i(t) & V_{dc}(t) & w(t) & w_q(t) \end{bmatrix}^T$  will remain bounded for all  $t \geq 0$ .

Although the boundedness of the closed-loop system is proven by considering a single load resistor  $R_L$ , it also holds true for any strictly dissipative load connected at the output of the rectifier with input  $V_{dc}$  and output the load current  $i_L$ , suitably extending the proposed controller application to rectifiers with more complicated loads (e.g. nonlinear, power converter-fed loads [7]). Taking into account the parasitic elements of the converter, the proof directly follows since for a strictly dissipative load there exists  $V_l(q) \geq 0$  and  $\psi(q) > 0$ , such that  $\dot{V}_l \leq V_{dc}i_L - \psi(q)$ , where  $q = [i_L \ q_1 \ \dots \ q_{n-1}]^T \in R^n$  is the load state vector [43].

As a result, a maximum bound for the rectifier and load states can be always guaranteed with the proposed strategy. However, in practice a very important issue for the rectifier operation is to guarantee a given limit for the input current below a certain value. Since  $i$  is an ac signal, it is required for its RMS value  $I$  to remain below a given maximum value  $I_{max}$ . This also corresponds to a maximum allowed power of the system given as  $P_{max} = V_s I_{max}$  (since unity power factor is achieved). According to (20) and taking into account that  $v_s = \sqrt{2}V_s \sin \omega t$ , it is proven that

$$|i| \leq \frac{\sqrt{2}V_s}{r + w_{min}}, \quad \forall t \geq 0, \quad (22)$$

if initially  $i(0)$  satisfies the above inequality, indicating that  $i$  introduces an ultimate bound, since according to the ISS property, the derivative of the Lyapunov function (19) is negative outside of this area. Inequality (22) can be expressed using the RMS value of the current as

$$I \leq \frac{V_s}{r + w_{min}}, \quad (23)$$

because (22) is satisfied for all  $t \geq 0$ . Since it is required that  $I \leq I_{max}$  at all times, then the controller parameter  $w_{min}$  can be chosen as

$$w_{min} = \frac{V_s}{I_{max}} - r \approx \frac{V_s}{I_{max}}, \quad (24)$$

since the inductor resistance  $r$  is usually considered very small and can be neglected. As a result, by selecting  $w_{min}$  according to (24), then  $I(t) < I_{max}, \forall t \geq 0$ . It should be mentioned that due to the neglected parasitic resistance  $r$  and the small phase shifting of the filter inductor, the actual current will be limited to a slightly lower value than  $I_{max}$  but as it is already analytically proven, it will be  $I(t) < I_{max}, \forall t \geq 0$ . Thus, in practice, a slightly larger  $I_{max}$  can be selected to determine  $w_{min}$  in (24).

Hence, the proposed controller can achieve an inherent current-limiting property for the rectifier without additional limiters or switching the controller operation. The controller remains continuous-time and allows the investigation of stability using nonlinear systems theory.

Now assume there exists a pair  $(w^e, w_q^e)$  for which  $w^e \in [w_{min}, w_{max}]$  corresponding to  $\bar{V}_{dc} = V_{dc}^{ref}$ . Then the current equation becomes

$$L \frac{di}{dt} = -(r + w^e)i + v_s \quad (25)$$

which represents a linear resistive-inductive  $RL$  circuit with resistance  $r + w^e$ . For a given  $w^e > 0$ , system (25) asymptotically converges to a unique sinusoidal solution  $i(t)$ , since the system has a negative pole ( $-\frac{r+w^e}{L}$ ) and the input  $v_s$  is sinusoidal with a constant amplitude and frequency. Using the average values and the power equivalence between ac and dc sides, at the steady state there is

$$\frac{(\bar{V}_{dc}^e)^2}{R_L} = w^e (I^e)^2$$

where  $\bar{V}_{dc}^e$  is the steady-state value of  $\bar{V}_{dc}$  and  $I^e$  is the RMS value of the steady-state solution  $i(t)$  of (25), which results in

$$\bar{V}_{dc}^e = I^e \sqrt{R_L w^e}. \quad (26)$$

Hence, the steady-state value of the average dc output voltage  $\bar{V}_{dc}^e$  is unique and according to the controller dynamics can only be  $V_{dc}^{ref}$ . Since  $V_{dc}(t) > 0$  (rectifier operation), then the solution  $V_{dc}(t)$  of the output voltage is also unique.

However, if  $\frac{(V_{dc}^{ref})^2}{R_L} > P_{max}$  where  $P_{max} = V_s I_{max}$  due to the unite power factor, i.e., if  $V_{dc}^{ref}$  is chosen very large or the load  $R_L$  changes to a small value, there will not exist a feasible  $w^e$  inside the range  $[w_{min}, w_{max}]$  corresponding to the desired solution. In this case,  $w$  will continuously decrease (since  $\dot{\theta} < 0$  from  $\bar{V}_{dc} - V_{dc}^{ref} < 0$  and (16)) until it reaches the minimum value  $w_{min}$  and the input current will reach the maximum value  $I_{max}$ . Note that  $w \rightarrow w_{min}$  corresponds to  $w_q \rightarrow 0$  which leads the angular velocity  $\theta \rightarrow 0$  according to (16). This means that  $\bar{V}_{dc}$  will reach a value  $\bar{V}_{dc}^e \neq V_{dc}^{ref}$  for which  $\frac{(\bar{V}_{dc}^e)^2}{R_L} = P_{max}$  holds true. Therefore, even if by mistake or due to unpredicted errors the reference value  $V_{dc}^{ref}$  increases above the maximum allowed value (corresponding to the maximum allowed power  $P_{max}$ ), the previous analysis still applies and the closed-loop system will guarantee the current-limiting property.

The above analysis implies that there always exists  $w^e \in [w_{min}, w_{max}]$  at the steady state, corresponding to a unique solution  $[i(t) \ V_{dc}(t) \ w(t) \ w_q(t)]^T$  for the closed-loop system. However, the physical limitations of the rectifier should be considered for achieving the current-limiting property. Since the rectifier represents a boost power electronic device, the dc output voltage introduces a minimum limit, which assuming sinusoidal PWM operation, results in  $\sqrt{2}V_s$ . By neglecting the parasitic resistance of the inductor and taking into account the power equivalence, it results in

$$P_{max} = V_s I_{max} = \frac{\bar{V}_{dc}^2}{R_L} \geq \frac{2V_s^2}{R_L},$$

which defines the allowed range of the load resistance

$$R_L \geq \frac{2V_s}{I_{max}}. \quad (27)$$

This is a limitation of the rectifier since for a smaller load resistance, the input current will increase since the current can flow through the diodes independently from the control design. By taking into consideration all of these properties, asymptotic convergence to a unique solution can be proven as shown below.

Particularly, if (27) is satisfied for the load, then the closed-loop system states are bounded and a current-limiting property  $I < I_{max}$  is achieved. As explained in the previous analysis, in this case there exists  $w^e \in [w_{min}, w_{max}]$  corresponding to a unique solution of the closed-loop system. Since  $w$  and  $w_q$  operate exclusively on  $W_0$ , i.e., given from (12)-(13), then for a sufficiently small  $c > 0$  in the controller design, the closed-loop system can be investigated as a two time-scale system with slow dynamics (12)-(13) and fast dynamics (17)-(18) as described in [43]. The fast current and voltage dynamics (17)-(18) (with respect to the controller dynamics) are investigated using the frozen parameter  $w$ . As in the case of (25), the current dynamics asymptotically converge to a unique solution depending on the frozen parameter  $w$ . Since  $w$  is proven to remain bounded inside the given range  $w \in [w_{min}, w_{max}]$ , then asymptotic stability of the solution  $i(t, w)$  holds uniformly in  $w$ , which is sinusoidal since the system represents a typical  $RL$  circuit with positive resistance  $r + w$  (see equation (25)). Consequently, the voltage dynamics (21) asymptotically converge to a solution  $V_{dc}(t, w)$  uniformly in the frozen parameter  $w$  since it can be viewed as a linear system with state  $V_{dc}^2$ , input  $wi^2$ , which has a negative real pole, i.e.  $-\frac{2}{CR_L}$ . Then, taking into account (26), it yields

$$\bar{V}_{dc}(w) = I^e(w) \sqrt{R_L w}. \quad (28)$$

As a result, (28) introduces the boundary layer of the system. In this way, the slow controller dynamics (12)-(13) become

$$\dot{w} = c \left( I^e(w) \sqrt{R_L w} - V_{dc}^{ref} \right) w^2 \quad (29)$$

$$\dot{w}_q = -\frac{c(w - w_m)w_q}{\Delta w_{max}^2} \left( I^e(w) \sqrt{R_L w} - V_{dc}^{ref} \right), \quad (30)$$

This represents a stable second-order autonomous system in the sense of boundedness which, according to the analysis presented in Subsection III-A, cannot have a periodic solution on the ellipse of  $W_0$ . Additionally, no chaotic solution exists for (29)-(30) based on the Poincare-Bendixon theorem [46] and as a result the controller states  $w$  and  $w_q$  will asymptotically converge to one of the equilibrium points: i)  $(w^e, w_q^e)$  corresponding to  $\bar{V}_{dc}^e = I^e(w^e) \sqrt{R_L w^e} = V_{dc}^{ref}$ , ii)  $(w_{min}, 0)$ , or iii)  $(w_{max}, 0)$ , since they represent the possible positive limit points of system (29)-(30) inside the bounded range depending on the value of  $V_{dc}^{ref}$  [43, Lemma 4.1]. As a result, for a sufficiently small  $c > 0$ , the nonlinear closed-loop system (17)-(18), (4)-(5) asymptotically converges to a unique solution  $[i(t) \ V_{dc}(t) \ w^e \ w_q^e]^T$ , satisfying the current-limiting property  $I < I_{max}$  [43]. As previously explained, the steady-state value  $\bar{V}_{dc}^e$  satisfies  $\bar{V}_{dc}^e = V_{dc}^{ref}$  when  $\frac{(V_{dc}^{ref})^2}{R_L} \leq P_{max}$  or  $\bar{V}_{dc}^e = \sqrt{V_s I_{max} R_L} < V_{dc}^{ref}$  when  $\frac{(V_{dc}^{ref})^2}{R_L} > P_{max}$ .

Since the controller parameter  $c$  should not be chosen very high, an analytic framework for defining its value along with the rest of the controller parameters is required to be obtained. This is described in the following subsection.

### C. Controller parameters selection

Having defined  $w_{min}$  from (24), the rest of the controller parameters should be also suitably designed as follows:

*Parameter k:* As it has been seen from (5), the parameter  $k$  is multiplied by the term  $\frac{(w-w_m)^2}{\Delta w_m^2} + w_q^2 - 1$ , which is zero on the ellipse  $W_0$ . Hence, the role of  $k$  is to make the controller dynamics of  $w_q$  robust with respect to external disturbances or calculation errors since if  $w$  and  $w_q$  are disturbed from  $W_0$  for any reason, they will quickly return to their initial trajectory. Therefore,  $k$  should be chosen sufficiently large in order for the controller states  $w$  and  $w_q$  to be quickly attracted and stay on the desired ellipse.

*Parameters  $w_m$  and  $\Delta w_m$ :* The ellipse  $W_0$  defines the maximum and minimum value of  $w$  which are  $w_{max} = w_m + \Delta w_m$  and  $w_{min} = w_m - \Delta w_m$ , respectively. Parameter  $w_{min}$  is chosen from (24) for a given maximum value  $I_{max}$  of the current. In the same framework,  $w_{max}$  corresponds to a minimum input current value  $I_{min}$  from the expression

$$w_{max} \approx \frac{V_s}{I_{min}} \quad (31)$$

since the RMS value of  $v$  is approximately equal to the RMS value of the grid voltage  $V_s$ , due to the negligible voltage drop on the inductor. For a given constant load  $R_L$ , there exists a minimum current  $I_{min}$  since the rectifier represents a boost power electronic device with  $\bar{V}_{dc} \geq \sqrt{2}V_s$  (for sinusoidal PWM). Therefore,  $I_{min}$  can be calculated as

$$I_{min} = \frac{2V_s}{R_L}. \quad (32)$$

However, (32) depends on the load  $R_L$  and if the load changes, the current might reach lower values. In practice, the controller should be able to operate for any load and even in the case of no load connected to the output. Since in the case of no load, a small current of mA or  $\mu A$  still flows through the converter due to the parasitic elements of the system, i.e. the inductor, the capacitor and the switches,  $I_{min}$  can be chosen relatively small to cover all load cases. It should be noted that in a common control operation of a rectifier, when the current drops to very small values, the PWM is turned OFF and the converter operates as a diode rectifier since there is practically no current measured to define the power factor. Therefore, parameter  $w_{max}$  is calculated from (31) for a relatively small current  $I_{min}$ .

Now, taking into account (24) and (31), the controller parameters  $w$  and  $\Delta w_m$  are obtained as

$$w_m = \frac{w_{max} + w_{min}}{2} = \frac{V_s}{2} \left( \frac{1}{I_{min}} + \frac{1}{I_{max}} \right), \quad (33)$$

$$\Delta w_m = \frac{w_{max} - w_{min}}{2} = \frac{V_s}{2} \left( \frac{1}{I_{min}} - \frac{1}{I_{max}} \right). \quad (34)$$

*Parameter c:* To define a framework for choosing the value of  $c$ , a worst case scenario is considered where  $w$  starts from  $w_{max}$  and reaches the minimum value  $w_{min}$  at the steady state, by operating on the upper semi-ellipse of  $W_0$ . In this case, the system starts from a minimum output voltage  $V_{dc}^{initial}$  and reaches a maximum voltage  $\bar{V}_{dc}^e = \sqrt{V_s I_{max} R_L}$  (depending on  $I_{max}$ ), i.e. there is a maximum difference

$\Delta V_{dc}^{max} = |V_{dc}^{initial} - \bar{V}_{dc}^e|$ . If one assumes that  $t_s$  is the settling time needed for  $w$  and  $w_q$  to travel the whole upper semi-ellipse of  $W_0$ , which corresponds to an arc with central angle of  $\pi rad$ , with an angular velocity  $\dot{\theta}$ , then in the worst case the angular velocity will be  $\frac{\pi}{t_s} rad/sec$  (if assumed constant and equal to its maximum value). On this trajectory, the second controller state  $w_q$  is always less or equal to 1. Then, one can define the maximum angular velocity  $\dot{\theta}_{max}$  (where  $w_q = 1$  and  $|\bar{V}_{dc} - V_{dc}^{ref}| = \Delta V_{dc}^{max}$ ) to be equal to  $\frac{\pi}{t_s} rad/sec$  as

$$\dot{\theta}_{max} = \frac{c \Delta V_{dc}^{max}}{\Delta w_m} = \frac{\pi}{t_s}. \quad (35)$$

Then parameter  $c$  is obtained as

$$c = \frac{\pi \Delta w_m}{t_s \Delta V_{dc}^{max}} \quad (36)$$

for a maximum difference  $\Delta V_{dc}^{max}$  required and a given settling time  $t_s$ . In practice, since the angular velocity decreases as soon as  $\bar{V}_{dc}$  approaches  $V_{dc}^{ref}$  and also  $w_q \leq 1$ , then parameter  $c$  can take larger values, or in other words the settling time  $t_s$  can be chosen much smaller than the original value. Expression (36) just provides a starting value of  $c$  for a smooth response. Then  $c$  can be increased until a satisfactory response is achieved.

After selecting the controller parameters, the proposed current-limiting controller can be implemented as shown in Fig. 3, where it is clear that no PLL or instantaneous measurement of the grid voltage is required for the implementation of the controller, opposed to the traditional techniques. This significantly simplifies the implementation of the proposed controller and increases the reliability of the system. It should be noted that a low-pass filter is added at the measurement of the output voltage  $V_{dc}$  to remove the second-order harmonics and a phase-lead low-pass filter is added at the measurement of the current  $i$  to remove the switching ripples and also apply a small phase-shifting (if needed) in order to obtain unity power factor at the whole system instead of the input of the rectifier, i.e., to cancel the small phase shifting caused by the inductor  $L$  [1].

### D. Controller performance under grid voltage variations

Although it is proven that the proposed controller can limit the current when unrealistic values of  $V_{dc}^{ref}$  are applied, one of the most challenging tasks is to limit the current under variations of the grid voltage and especially under voltage dips. According to the ISS analysis, it is proven that  $I < I_{max}$  when  $w_{min}$  is selected according to (24). In this case, the grid voltage is considered stiff and  $V_s = V_n$ , where  $V_n$  is the rated RMS voltage. If it is assumed that the grid voltage introduces variations, i.e.  $V_s \in [0, V_{max}]$ , where  $V_{max}$  is the maximum value of the RMS grid voltage, then following the same ISS analysis and taking into account (22)-(23), it yields that

$$I \leq \frac{V_{max}}{r + w_{min}}, \quad \forall t \geq 0, \quad (37)$$

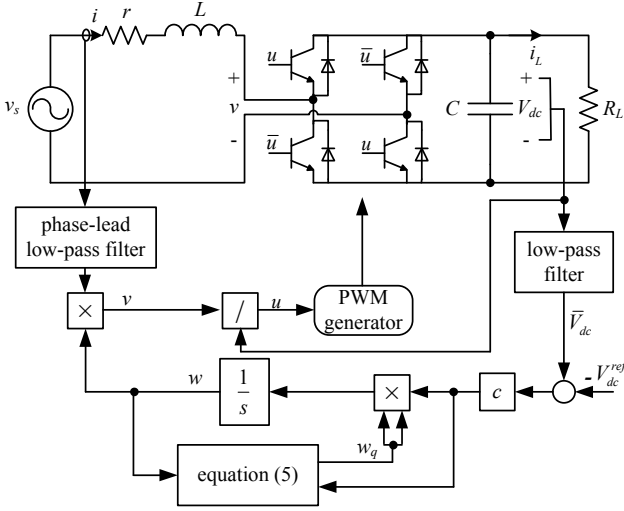


Figure 3. Implementation of the proposed current-limiting controller

as long as initially the RMS value of the current satisfies the above inequality. Hence, by selecting

$$w_{min} = \frac{V_{max}}{I_{max}} \quad (38)$$

then  $I(t) < I_{max}, \forall t \geq 0$ , which guarantees the current-limiting property even when the grid voltage varies. This includes the case where a voltage dip occurs. In order to calculate the maximum current during a voltage dip, consider the case where  $w_{min}$  is selected according to (38) and suddenly a  $p \times 100\%$  percentage drop occurs at the grid voltage, where  $0 \leq p \leq 1$ , i.e. the RMS grid voltage becomes  $(1-p)V_s$ , where  $V_s$  was the original value of the grid voltage before the fault. Then according to (23) there is

$$I \leq \frac{(1-p)V_s}{r + \frac{V_{max}}{I_{max}}} < (1-p) \frac{V_s}{V_{max}} I_{max} < (1-p) I_{max}. \quad (39)$$

Inequality (39) implies that the current will be limited below a lower value depending on the percentage of the voltage dip. This is due to the fact that the measurement of the grid voltage is not used for the controller design, which significantly simplifies its implementation. Nevertheless, in any case, the input current will be lower than  $I_{max}$  as required to protect the rectifier. Note that the same controller can be extended to applications with an  $LCL$  filter instead of an  $L$  filter, where the capacitor voltage remains in the range  $[0, V_{max}]$ , depending on the grid voltage and the filter parameters.

#### IV. EXPERIMENTAL VALIDATION

In order to verify the efficiency of the proposed controller, a single-phase full-bridge rectifier with a load resistor  $R_L$  operating under the proposed current-limiting controller described in Fig. 3 was experimentally investigated. A switching frequency of 19kHz was used for the PWM operation and the proposed controller was implemented using the TMS320F28335 DSP with a sampling frequency of 16kHz. The system and the controller parameters are given in Table I. Because of the limitations of the input voltage level for

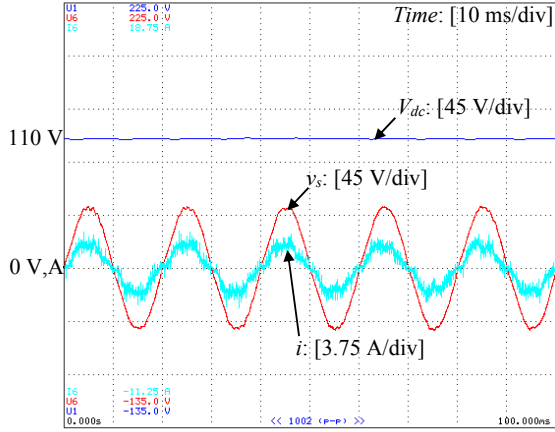
Table I  
SYSTEM AND CONTROLLER PARAMETERS

Parameters	Values	Parameters	Values
$L$	2.2 mH	$I_{max}$	3 A
$r$	0.5 $\Omega$	$I_{min}$	1 mA
$C$	1650 $\mu$ F	$k$	100
$R_L$	100 ~ 320 $\Omega$	$t_s$	0.4 s
$V_s$	36 V	$\Delta V_{dc}^{max}$	50 V
$\omega$	100 $\pi$ rad/s	$w_0$	60 $\Omega$

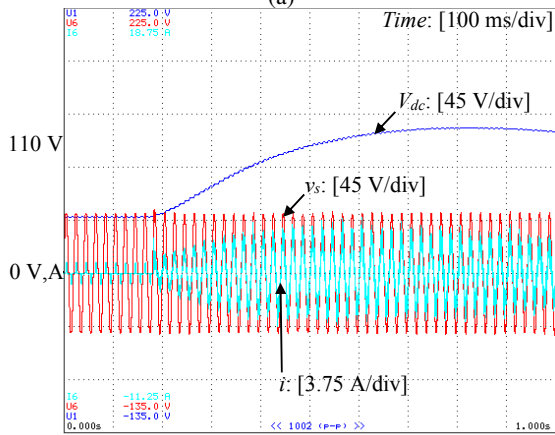
the experimental setup, a 36 V RMS input voltage was used due to the popularity of 110 V/36 V transformers. For the controller implementation, the low-pass filter  $\frac{1}{0.01s+1}$  was used at the measurement of the dc output voltage to reject the second-order harmonics and the phase-lead low-pass filter  $\frac{31(0.06s+1)}{(0.003s+1)(s+270)}$  was used at the measurement of the inductor current to remove the switching ripples and apply a small phase shifting to cancel the effect of the filter inductor. This is commonly used in power converter control applications when a feed-forward term is introduced at the control signal [1]. Note that different types of filters can be used for both measurements (e.g. hold filter for the dc output voltage) but the above filters were considered for simplicity.

##### A. Operation with normal grid

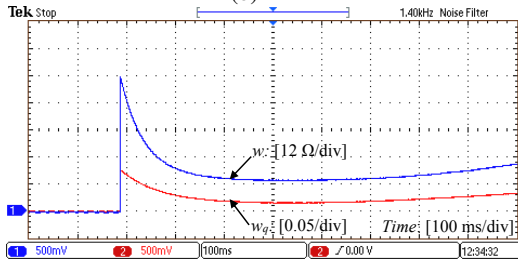
Initially, the system operates as a diode rectifier with  $V_s = 36$  V and  $R_L = 320 \Omega$ , and the controller is not active. Fig. 4(b) shows the transient response when the controller is enabled with  $V_{dc}^{ref} = 110$  V, which corresponds to a typical voltage level in ac or dc power applications. The dc output voltage increases and smoothly converges to the reference value after a short transient, while the unity power factor is maintained during the whole operation. Note that the transient response can be faster if the capacitance  $C$  is reduced, but this will increase the second-order ripples of the output voltage. The smooth transient is due to the proposed controller which applies a varying resistance starting from  $w_0 = 60 \Omega$  and reduces while moving on the desired ellipse  $W_0$ , as shown in Fig. 4(c) and 4(d). The initial condition of  $w_q$  was defined as  $w_{q0} = \sqrt{1 - \frac{(w_0 - w_m)^2}{\Delta w_m^2}} = 0.073$  according to (10). Hence, the proposed strategy additionally offers a soft start-up solution of the closed-loop system. Then, the load suddenly changes from 320  $\Omega$  to 220  $\Omega$ , and the output voltage is regulated at the reference value after a short transient as shown in Fig. 5(b). The transient response of the controller states is shown in Fig. 5(c) and it is verified in Fig. 5(d) that they exclusively operate on the desired ellipse  $W_0$ . The steady-state responses of the output voltage, the grid voltage and the inductor current are shown in Fig. 5(a). It is clear that almost unity power factor is achieved (over 0.98 measured which is acceptable in practice) and the output voltage is regulated at the desired value  $V_{dc}^{ref} = 110$  V with small second-order harmonics caused by the full-bridge rectifier operation. Due to the limitation of the experimental setup, although the current waveforms have been obtained from a power analyzer, the current measurement used for the controller implementation was provided from inside the power module, which reduces the



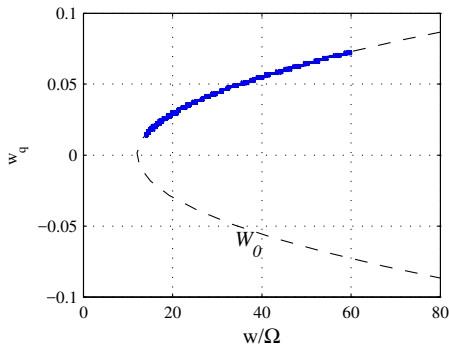
(a)



(b)

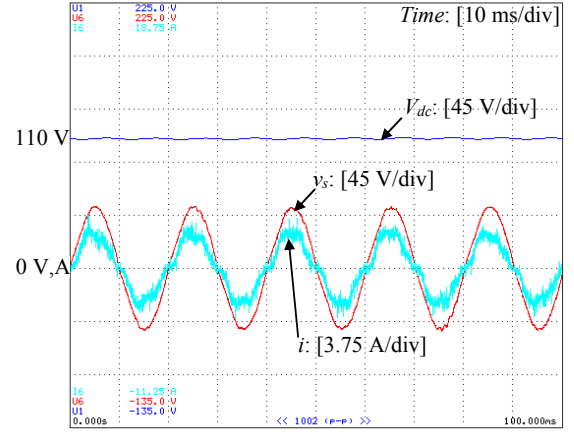


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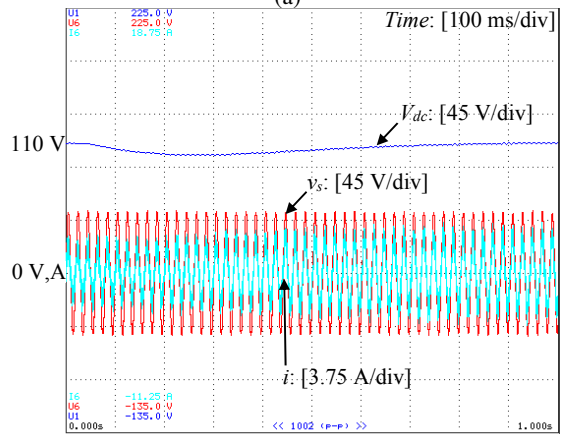


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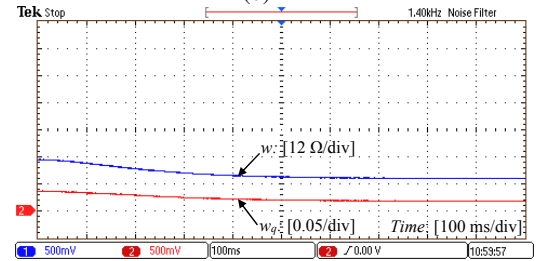
Figure 4. Experimental results of the proposed current-limiting controller under normal operation when the controller is enabled with  $V_{dc}^{ref} = 110$  V and  $R_L = 320 \Omega$ : (a) steady-state response of the system states, (b) transient response of the system states, (c) transient response of the controller states, (d)  $w - w_q$  plane.



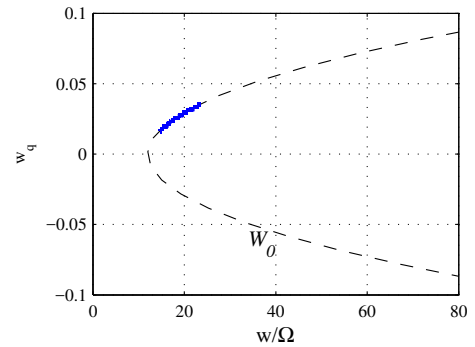
(a)



(b)



(c)



(d)

Figure 5. Experimental results of the proposed current-limiting controller under normal operation when the load changes from  $320 \Omega$  to  $220 \Omega$  with  $V_{dc}^{ref} = 110$  V: (a) steady-state response of the system states, (b) transient response of the system states, (c) transient response of the controller states, (d)  $w - w_q$  plane.

accuracy of the measurement and has an impact on the power quality. Note that different current measurement units, filter

design or PWM techniques that improve the total harmonic distortion of the current can be applied using the proposed



controller [47], [48], [49], and are currently investigated.

In order to verify the current-limiting property of the controller, the reference output voltage changes from 110 V to 140 V, when the load resistor is 220  $\Omega$ . However, as it becomes clear from Fig. 6(a), the output voltage is regulated at around 120 V because the current tries to violate the maximum limit. Although the limit of the current was set to  $I_{max} = 3$  A, the RMS value of the steady-state current was measured at 2.2 A which is slightly less than  $I_{max}$ , since as mentioned in Subsection III-B  $w_{min}$  is calculated from (24) where the parasitic resistance  $r$  of the inductor  $L$  was neglected and the power factor is slightly less than 1. However, this still results in  $I < I_{max}$  which is desired. In practice,  $I_{max}$  can be chosen slightly higher for the selection of  $w_{min}$  to cope with this issue. According to the transient response of the controller states (Fig. 6(c)),  $w \rightarrow w_{min}$  and  $w_q \rightarrow 0$  as expected at the limit of the current. This is also shown from the controller state trajectory on the ellipse  $W_0$  on the  $w - w_q$  plane, in Fig. 6(d), verifying the theory developed in the paper.

To further validate the current limitation, while the reference dc output voltage is kept constant at  $V_{dc}^{ref} = 110$  V, the load changes from 320  $\Omega$  to 100  $\Omega$ , which is a larger change than the one described in Fig. 5. This causes the input current to increase and be limited again at 2.2 A. This leads to a drop of the output voltage from 110 V to 82 V, as shown from the transient and the steady-state responses of the system states in Fig. 7(b) and Fig. 7(a), respectively. Hence, the proposed controller automatically reduces the output voltage to protect the rectifier from large currents. The controller states  $w$  and  $w_q$  tend to  $w_{min}$  and 0, respectively, while operating exclusively on  $W_0$ , as shown in Fig. 7(c) and Fig. 7(d).

### B. Operation under grid voltage dips

In order to further test the current-limiting capability of the proposed controller, two scenarios of voltage dips at the grid voltage are investigated. The desired output voltage is again set at 110 V and the load resistance is  $R_L = 220 \Omega$  for the whole operation. Initially, the RMS grid voltage drops from 36 V to 30 V, which corresponds to a 17% voltage drop, i.e.  $p = 0.17$ . According to the analysis of Subsection III-D, the current will be limited to a lower value corresponding to 83% of the maximum current. As shown in Fig. 8(b), the current increases as the voltage drops and its RMS value is limited at 1.82 A, as verified from the steady state response of Fig. 8(a). Since the experimental results of Fig. 6 have indicated that the given controller limits the current at a maximum value of 2.2 A, then the analysis of Subsection III-D proves that the current should be limited below  $0.83 \times 2.2 \text{ A} = 1.83 \text{ A}$ , which is the case. The output voltage drops to a lower value to maintain the power equivalence. The controller states  $w$  and  $w_q$  are regulated at their minimum values  $w_{min}$  and 0, respectively, as shown in Fig. 8(c), and their trajectory stays on  $W_0$ , as shown in Fig. 8(d).

Finally, the RMS of the grid voltage suddenly drops from 36 V to 23 V (36% voltage drop) and the results are shown in Fig. 9. The transient response is illustrated in Fig. 9(b) where the output voltage drops and is regulated to a value lower than

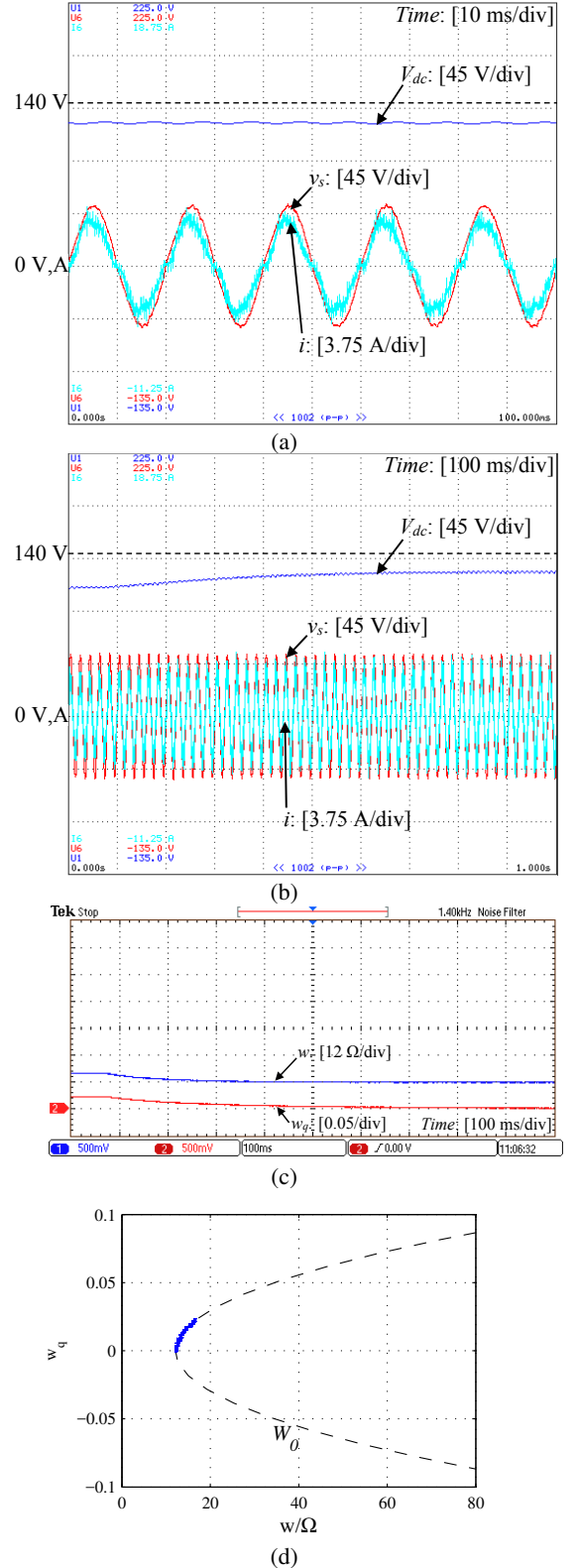


Figure 6. Experimental results of the proposed controller reaching the current limit when the  $V_{dc}^{ref}$  changes from 110 V to 140 V (current limiting activated with  $\bar{V}_{dc} \rightarrow 120$  V): (a) steady-state response of the system states, (b) transient response of the system states, (c) transient response of the controller states, (d)  $w - w_q$  plane.

the reference (around 77 V) because the input current is limited at 1.38 A, as expected from the theory ( $0.64 \times 2.2 \text{ A} = 1.4 \text{ A}$ ).

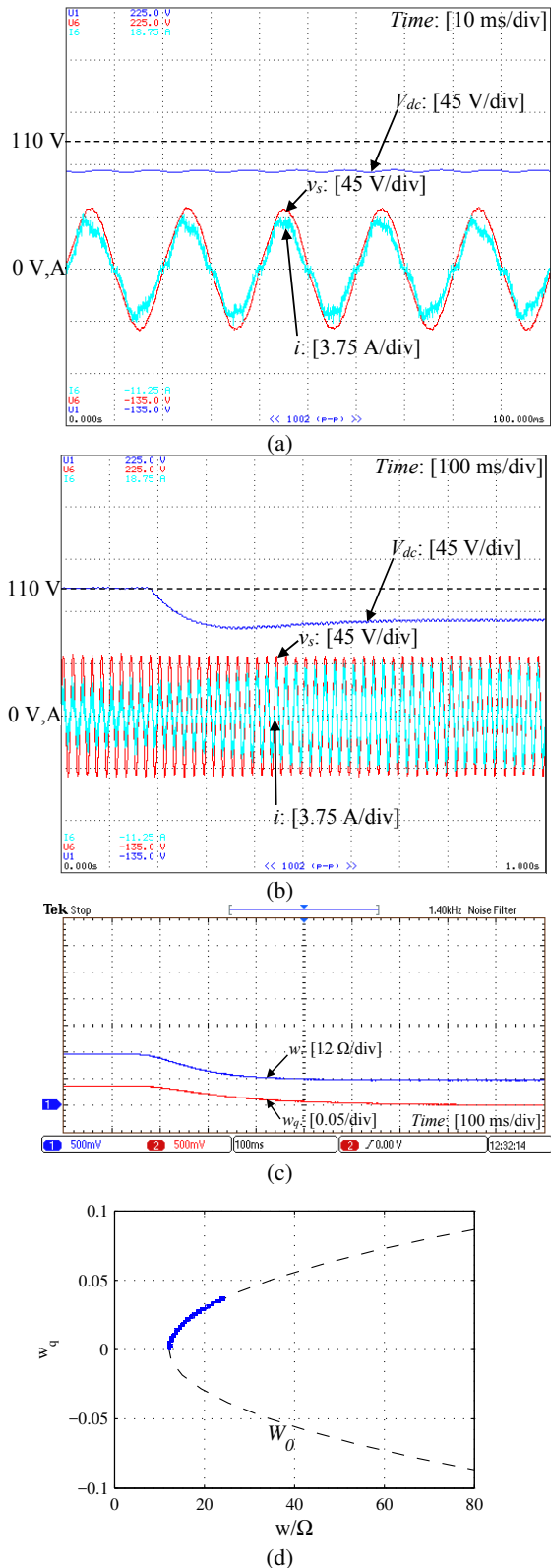


Figure 7. Experimental results of the proposed controller reaching the current limit when the load changes from  $320\ \Omega$  to  $100\ \Omega$  with  $V_{dc}^{ref} = 110\ \text{V}$  (current limiting activated with  $V_{dc} \rightarrow 82\ \text{V}$ ): (a) steady-state response of the system states, (b) transient response of the system states, (c) transient response of the controller states, (d)  $w - w_q$  plane.

This is clearly shown from the steady-state response of Fig. 9(a). As in the previous case, the controller states  $w$  and

$w_q$  converge to  $w_{min}$  and 0, respectively, while moving on the desired ellipse (Fig. 9(c) and Fig. 9(d)). As a result, it is verified that even when voltage dips occur at the grid, the proposed controller maintains the input current below a maximum value without requiring the measurement of the grid voltage, sag detection mechanisms or additional protection devices.

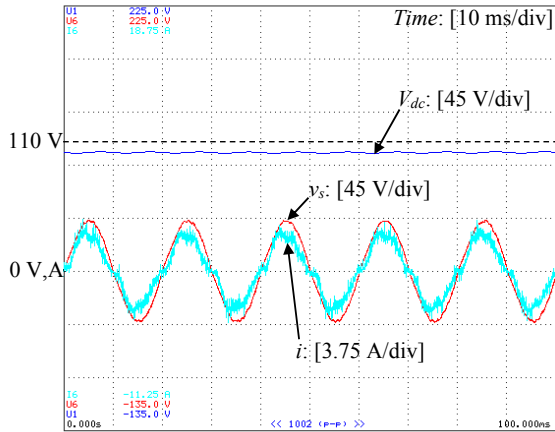
## V. CONCLUSIONS

In this paper, a nonlinear controller with an inherent current-limiting capability was proposed for single-phase rectifiers. The developed strategy guarantees nonlinear asymptotic stability and convergence to a unique solution at all times, while achieving the main tasks of the rectifier operation, i.e., accurate output voltage regulation and unity power factor operation. An analytic description of the controller parameters selection was provided to guarantee that the input current will be limited below a given value during transients even if the grid voltage varies. Opposed to the existing control techniques, the proposed current-limiting controller is fully independent from the system parameters and does not require a PLL or the instantaneous measurement of the grid voltage, leading to a simplified implementation. Extensive experimental results were provided to support the theoretical background of the proposed approach and verify its effective operation.

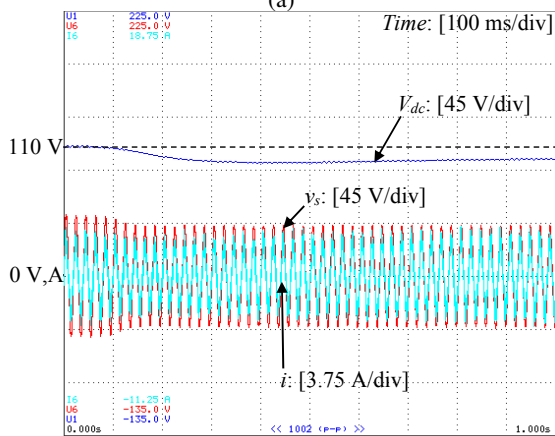
It is worth noting that the use of a positive dynamic virtual resistance in the proposed controller structure can guarantee the required stability but restricts the proposed controller application only to rectifiers and not to inverters. Hence, this represents a simplified control approach for rectifier-fed passive loads. Further investigation is required to obtain a generic structure that can be applied to both types of ac/dc converters with different operating conditions (e.g. constant power, constant current loads) and satisfy some additional practical limitations (e.g. saturation of the control input) with an improvement of the power quality. These issues represent interesting topics for future research.

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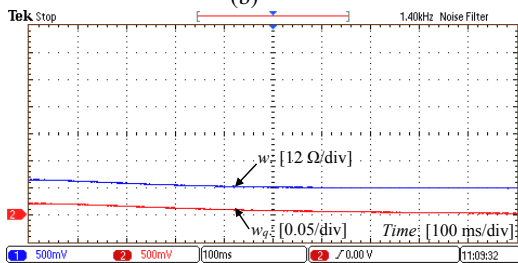
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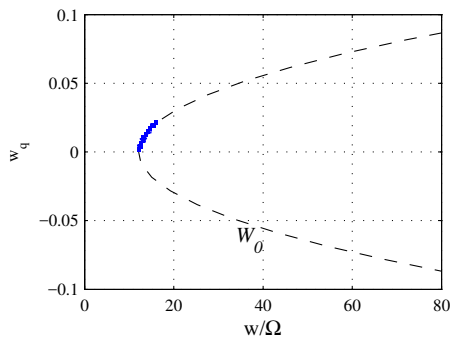
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(b)

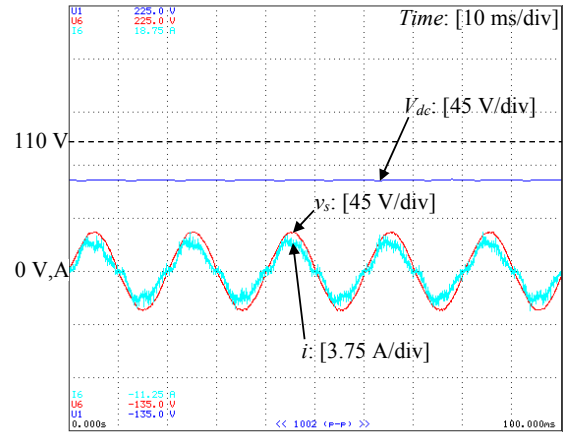


(c)

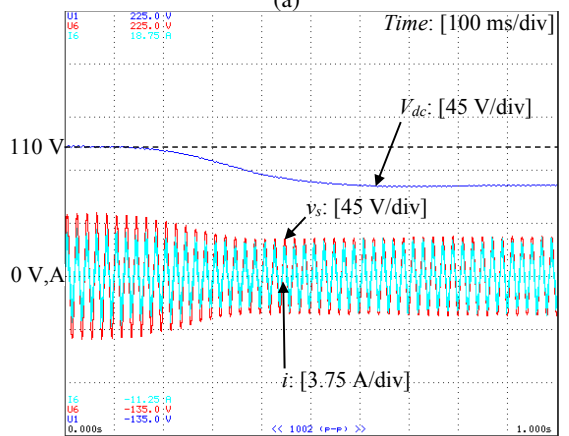


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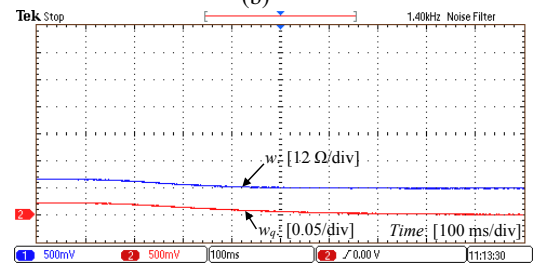
Figure 8. Experimental results of the proposed current-limiting controller when the RMS input voltage changes from 36 V to 30 V (current limiting activated with  $\bar{V}_{dc} \rightarrow 100$  V): (a) steady-state response of the system states, (b) transient response of the system states, (c) transient response of the controller states, (d)  $w - w_q$  plane.



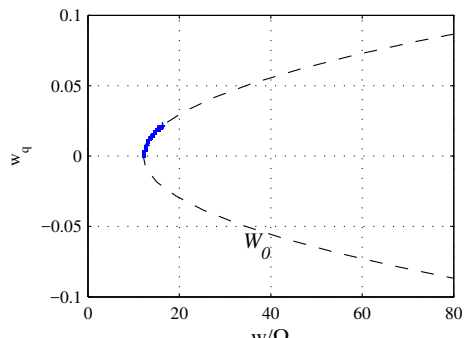
(a)



(b)



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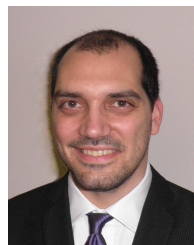
(d)

Figure 9. Experimental results of the proposed current-limiting controller when the RMS input voltage changes from 36 V to 23 V (current limiting activated with  $\bar{V}_{dc} \rightarrow 77$  V): (a) steady-state response of the system states, (b) transient response of the system states, (c) transient response of the controller states, (d)  $w - w_q$  plane.

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