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Orthogonal Frequency Division Multiplexed Quantum Key Distribution in The Presence of Raman Noise

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ABSTRACT

In this paper, we investigate the performance of orthogonal frequency division multiplexed quantum key distribution (OFDM-QKD) in an integrated quantum-classical wavelength-division-multiplexing system. The presence of an intense classical signal alongside the quantum one generates Raman background noise. Noise reduction techniques should, then, be carried out at the receiver to suppress this crosstalk noise. In this work, we show that OFDM-QKD enables efficient filtering, in time and frequency domains, making it an attractive solution for the high-rate links at the core of quantum-classical networks.

Keywords: Quantum key distribution, orthogonal frequency division multiplexing, crosstalk, Raman scattering

1. INTRODUCTION

Quantum key distribution (QKD) is an attractive candidate for providing security in future communications networks. Despite various advancements in theoretical and practical aspects of QKD, its widespread adoption requires overcoming some practical challenges. One major requirement, to make QKD technologies cost effective, is the transmission of quantum and data signals on the same infrastructure.[1,2] In particular, wavelength-division-multiplexing (WDM) is a promising technique that enables the integration of quantum and classical signals. Simultaneous transmission of quantum and classical signals in this scheme will, however, lead to new sources of noise that can adversely affect the operation of quantum channels. In particular, it has been shown that Raman scattering is the dominant source of background noise in such hybrid systems.[3] The conventional approach to reducing the effect of such a crosstalk noise on QKD channels is to apply filtering in time and frequency domains.[4,5] In this work, we use the inherent filtering in an orthogonal frequency division multiplexed (OFDM) QKD system[6] to further reduce this crosstalk noise. In the ideal implementation, this would remove any source of noise orthogonal to the intended signal, and it would enhance the secret key generation rate of the QKD channel.

The major challenge in the presence of a classical channel alongside a quantum one is the Raman scattered photons generated by the nonlinear effects in the fiber.[7] The Raman noise generated by data channels may overlap with the frequency band of the quantum channels, increasing the noise level at QKD receivers. It is then crucial to employ efficient filtering techniques to suppress this background noise. In recent experiments,[4,5] narrow-band filters (NBFs) have been used at the quantum receiver to mitigate this crosstalk in the frequency domain. Furthermore, minimal time-gating at the detectors has also been employed. Another technique to reduce the Raman noise is to use power control schemes to minimize the launched power in the classical channels.[4] All put together, these conventional solutions can to some extent alleviate the data-channel-induced background problem and enable simultaneous transmission of QKD and data channels over the same fiber. However, in order to achieve optimal filtering, only one time-frequency mode corresponding to the QKD signal should pass through the filters. This may require ultra narrow-band filters, matching to the bandwidth of QKD signals, which may be challenging to acquire in practice.

Here, we propose to use OFDM-QKD, as an effective technique to optimally filter the background noise in hybrid quantum-classical WDM systems. OFDM-QKD exploits the orthogonality between its subchannels to multiplex them efficiently. This way, not only the available bandwidth is efficiently used, but also we can even separate orthogonal signals overlapping in the frequency domain. This is not possible by simple spectral filtering used in WDM systems. In this approach, the subcarriers are multiplexed by the use of an optical circuit that performs inverse Fourier transform in the optical domain. Each subcarrier is then extracted by an appropriate all-optical OFDM decoder at the receiver node.[6] This technique offers two advantages. First, because of the spectral efficiency of OFDM, the total key rate per unit of bandwidth would be enhanced.[6] Secondly, because OFDM decoders separate the intended signal from any other signal orthogonal to

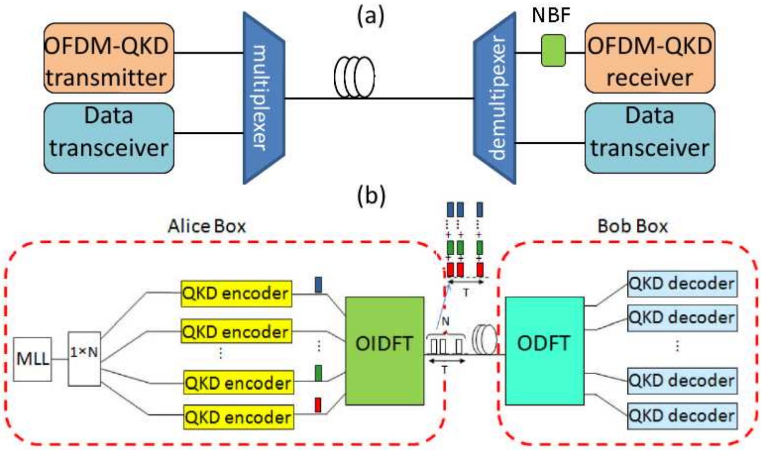

Figure 1: (a) OFDM-QKD system multiplexed with one classical channel in a WDM setup. (b) OFDM-QKD system:[6] A train of short pulses generated by a Mode-locked laser (MLL) is split into N paths. The OFDM symbol is generated by multiplexing the output pulses of the QKD encoders by the OIDFT circuit. The OFDM symbol consists of a series of pulses, each being a superposition of pulses from different inputs. At the receiver, an ODFT circuit demultiplexes the subcarriers.

it, the effective filtering offered by OFDM-QKD is optimal. In this paper, we study the effect of such optimal filtering on the performance of a QKD system—prone to Raman-induced background noise—and compare our results with the ones reported in recent experiments.[4,5]

In the following, we describe the OFDM-QKD system in Sec. II. In Sec. III, the secret key rate of our OFDM-QKD system in the presence of Raman noise is obtained. Some numerical results are presented in Sec. IV. Finally, we conclude the paper in Sec. V.

2. SYSTEM DESCRIPTION

We consider a WDM system consisted of one quantum band and a classical data channel, as shown in Fig. 1 (a). The quantum band is consisted of N subchannels multiplexed by the OFDM technique. We refer to the node that hosts OFDM-QKD transmitter by the Alice node, and the other node is referred to by the Bob node. We assume that the quantum and classical channels are both in the C-band and, respectively, use wavelengths λ_q and λ_d. We consider the effect of the forward channel, when Alice is sending some classical data to Bob, separately from the backward channel, when Bob is transmitting classical data to Alice.

One major problem with the transmission of classical and quantum signals on the same optical fiber is the noise induced on quantum channels because of the Raman scattered light from classical channels.[7] As the spectrum of Raman noise overlaps with the frequency band of the quantum channel, it cannot completely be filtered out. Hence, effective noise reduction techniques at the receiver are required. We assume that a data laser with optical power I is used at the transmitter of the classical channel. If the classical transmitter is on the Alice side, it would generate forward scattered light at the QKD receiver. If it is on the Bob's side, it will generate backward Raman scattered light on Bob's detectors. The Raman noise power at the quantum receiver, for forward and backward scattering is, respectively, given by[3]

$$I_R^f = I e^{-\alpha L} L \rho(\lambda_d, \lambda_q) \Delta\lambda, \tag{1}$$

$$I_R^b = I \frac{(1 - e^{-2\alpha L})}{2\alpha} \rho(\lambda_d, \lambda_q) \Delta\lambda, \tag{2}$$

where $\Delta\lambda$ is the bandwidth of the quantum receiver, and $\rho(\lambda_d, \lambda_q)$ is the Raman cross section (per unit of fiber length and bandwidth). In the above equations α and L represent fiber attenuation coefficient (per unit of fiber length) and fiber length, respectively. Then, the corresponding average number of Raman photons, at the Bob's QKD detectors, is given by

$$\mu_R^f = \frac{\eta_d I_R^f \lambda_q T_d}{hc}, \tag{3}$$

$$\mu_R^b = \frac{\eta_d I_R^b \lambda_q T_d}{hc}, \tag{4}$$

where T_d and η_d are, respectively, the width of the gate interval and the quantum efficiency of the photodetectors, c is the speed of light, and h is the Planck's constant. As we show in Sec. III, this background photon count would adversely affect the key rate of QKD channels.

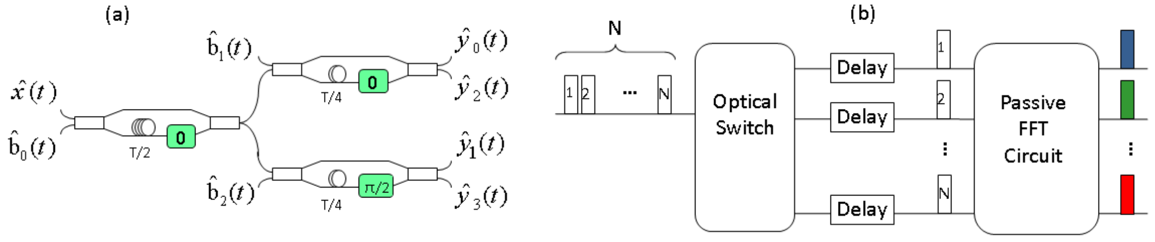

Figure 2: (a) OIDFT circuit for $N = 4$. It consists of two stages of MZI. (b) The OFDM decoder implemented by an optical switch followed by appropriate delays and a passive DFT circuit.[6]

Conventionally, the impact of the Raman noise on the quantum channel is reduced by filtering techniques, both in time and frequency domains. According to (1) and (2), the Raman noise power is directly proportional to the bandwidth of the quantum receiver, $\Delta\lambda$. One can then reduce the Raman crosstalk noise by using NBFs, as shown in Fig. 1(a). Furthermore, based on (3) and (4), one can use time-gating at the detectors of the QKD decoders to further reduce the average number of Raman photons. Another technique is to reduce the launch power of the classical signal, I, as much as possible. To this aim, one can set this parameter to the value that matches the receiver sensitivity corresponding to a desired bit error rate (BER). In recent experiments,[4,5] NBFs with bandwidths as low as 70 GHz and 15 GHz have been used, and the time-gate interval has been 100-ps long. While such filtering techniques have shown to be effective in reducing the Raman background noise,[4,5] they have some limitations in practical implementations. For instance, further reduction in the bandwidth of the NBFs requires us to employ ultra narrow-band filters, which may not be without its practical challenges. The time-gate interval is also limited by certain specification of the single-photon detectors, such as their time jitter and bandwidth.

In this paper, we use OFDM-QKD[6] in a hybrid quantum-classical WDM setup to maximally remove the Raman crosstalk noise, and, at the same time, achieve a high secret key generation rate. This technique enables N QKD subchannels to be multiplexed into N orthogonal modes. The orthogonality between these modes is guaranteed by the condition $\Delta f = 1/T$, where Δf is the frequency separation between the subcarriers associated with each subchannel, and T represents the OFDM symbol duration. Note that, in the bandpass regime, the effective bandwidth associated with a signal with a temporal width T is roughly given by $2/T$. This implies that the spectrum associated with different subchannels in an OFDM setting could be overlapping. Such spectrally overlapping signals cannot be separated using conventional filtering techniques in WDM. Instead, in optical OFDM systems, the subchannels are multiplexed by means of an all-optical circuit that performs inverse discrete Fourier transform (IDFT), and, correspondingly, they can be demultiplexed by the discrete Fourier transform (DFT) operation. Put together, OFDM offers a spectrally efficient way of using the available bandwidth, while it can remove the noise coming from orthogonal spaces to our modes of interest.

The OFDM-QKD system, considered in this paper, is shown in Fig. 1 (b). At the OFDM-QKD transmitter, Alice uses N QKD encoders to prepare her key bits in parallel. We assume that the efficient decoy-state phase-encoded BB84 protocol is used in each QKD subchannel.[8] The output optical pulses are multiplexed by the optical IDFT (OIDFT) circuit. We denote the width of the optical pulses generated by the mode-locked laser (MLL) by T_p, where $T_p \simeq T/N$. Defining $\hat{a}_k$ as the annihilation operator corresponding to the spatial mode at the output of the k^{th} QKD encoder, the output operator of the OFDM-QKD transmitter is, then, given by[6]

$$\hat{x}(t) = \frac{1}{N}\sum_{l=0}^{N-1} \hat{c}_l\, p(t - l\frac{T}{N}), \tag{5}$$

where $p(t)$ is the pulse shape of the MLL's output and $\hat{c}_l = \sum_{k=0}^{N-1} \hat{a}_k e^{j2\pi kl/N}$ is the l^{th} temporal mode at the output of the OIDFT circuit. As can be seen, the relation between $\hat{c}_l$'s and $\hat{a}_k$'s is similar to the IDFT operation. This is why IDFT and

DFT operations are used to, respectively, multiplex and demultiplex OFDM signals. Our previous work provides a full quantum analysis of such an OFDM-QKD setup.[6]

While we still need some spectral filtering for our OFDM-QKD system, such requirements could be considerably milder than that of an equivalent system that only relies on conventional filtering. The subchannel extraction for the OFDM receiver takes place in two steps. First, we need to spectrally filter the entire OFDM symbol from the signals in neighboring channels. This can be done by a conventional NBF with a bandwidth $W \simeq N\Delta f \simeq {T_p}^{-1}$. This filter will act on the entire OFDM symbol, and would let all the within subchannels pass through. For instance, for $T_p = 10$ ps, W roughly corresponds to a 1-nm-wide filter, which is commonly used in conventional optical communications systems. The ultra narrow-band QKD subchannels are then separated by the OFDM decoder using the orthogonality of the OFDM subchannels. In fact, the OFDM decoder will serve the role of an ultra narrow-band filter, which may be otherwise hard to implement on its own. The price we pay here is a more complicated receiver structure that requires accurate time-gating with a resolution corresponding to T_p.

Another advantage of OFDM-QKD is its higher rate per unit of bandwidth. The spectral efficiency of OFDM-QKD is achieved by letting the subchannels overlap in the frequency domain, while maintaining the orthogonality condition $\Delta f = 1/T$. The conventional filtering methods used in the WDM scheme is, then, not applicable here. Instead, the OFDM decoder separates the orthogonal signals. This makes OFDM-QKD an optimal technique in terms of spectral efficiency. As compared to a single-channel QKD system with a pulse width T_p, the OFDM-QKD would use almost half the bandwidth to offer the same total rate.

The OFDM-QKD system shown in Fig. 1 (b) is implemented by means of circuits that perform OIDFT and optical DFT (ODFT) operations. An example of the OIDFT circuit for $N = 4$ is depicted in Fig. 2 (a). This circuit is consisted of multiple Mach-Zehnder interferometers (MZIs) with appropriate phase shift and delay parameters. As for the ODFT circuit, we assume that the OFDM decoder shown in Fig. 2 (b) is used at the OFDM-QKD receiver.[6] In this decoder, the operation of serial to parallel conversion is done by an active optical switch followed by appropriate delays. Then, a passive DFT circuit, composed of beam splitters and phase shifters performs the DFT operation.

In the following sections, we investigate the performance of our OFDM-QKD system in the presence of Raman noise. We present an analysis of the secret key generation rate, and provide numerical results to support our theoretical findings.

3. KEY RATE ANALYSIS

In this section, we analyse the secret key generation rate of our OFDM-QKD system in the presence of Raman noise. We assume that the efficient decoy-state BB84 protocol is used to generate secure keys.[8] We denote the average number of photons per QKD subchannel at the output of the Alice box, for the main signal state, by μ. The secret key rate per transmitted pulse in each subchannel, at the limit of an infinitely long key, is lower bounded by $\max[0, P(\mathrm{Y}_0)]$, where[9]

$$P(\mathrm{Y}_0) = Q_1(1 - h(e_1)) - f Q_\mu h(E_\mu). \tag{6}$$

Here, $h(p) = -p\log_2 p - (1-p)\log_2(1-p)$ is the binary entropy function and f denotes the error correction inefficiency. In the above equation, Q_μ, E_μ, Q_1, and e_1 represent the overall gain, the QBER, the gain of the single photon state, and the error rate of the single photon state, respectively. The overall gain, Q_μ, and the quantum bit error rate (QBER), E_μ, are respectively given by

$$\begin{aligned} Q_\mu &= 1 - (1 - \mathrm{Y}_0)e^{-\eta\mu}, \\ E_\mu &= (\mathrm{Y}_0/2 + e_d(1 - e^{-\eta\mu}))/Q_\mu, \end{aligned} \tag{7}$$

while the gain and the error rate of the single photon state are, respectively, as follows:

$$\begin{aligned} Q_1 &= \mathrm{Y}_1 \mu e^{-\mu}, \\ e_1 &= (\mathrm{Y}_0/2 + e_d\eta)/\mathrm{Y}_1. \end{aligned} \tag{8}$$

Here, Y_0 represents the probability of any detector clicks without having any transmitted photons, which can stem from the detectors' dark count or external background noise, and Y_1 is the yield of a single photon state. Furthermore, the parameters e_d and η denote the probability of phase instability and the total transmissivity of the link, respectively. With the repetition period of the QKD signal denoted by T_s, the secret key rate per subchannel is given by

$$R_{ch} = \max[0, P(\mathrm{Y}_0)/T_s], \tag{9}$$

where

$$\mathrm{Y}_0 = 1 - (1 - (p_{\mathrm{dc}} + p_R))^2. \tag{10}$$

In the above equation, $p_{\mathrm{dc}} = \gamma_{\mathrm{dc}} T_d$, where γ_{dc} denotes the photodetectors dark count rate. Furthermore, p_R represents the average number of Raman photons, given by $p_R = \mu_R^f$ and $p_R = \mu_R^b$ for forward and backward data channels, respectively.

4. NUMERICAL RESULTS

In this section, we investigate the performance of the QKD channels in the WDM system shown in Fig. 1 (a). We assume that the quantum channel is centered at 1550 nm. As for the classical channel, we consider two different wavelengths, namely 1570 nm and 1590 nm. We assume that the fiber attenuation coefficient, α, is 0.2 dB/km. To implement the power control method[4] described in Sec. II, we set the launch power of data lasers to $I = 10^{(-3.45+\alpha L/10)}$ mW. This corresponds to -36.8 dBm receiver sensitivity, corresponding to a $BER < 10^{-9}$, after considering 2.3 dBm of safety margin.[4] The nominal values used for the system parameters are listed in Table I. These parameters are chosen based on practical considerations.[6,9]

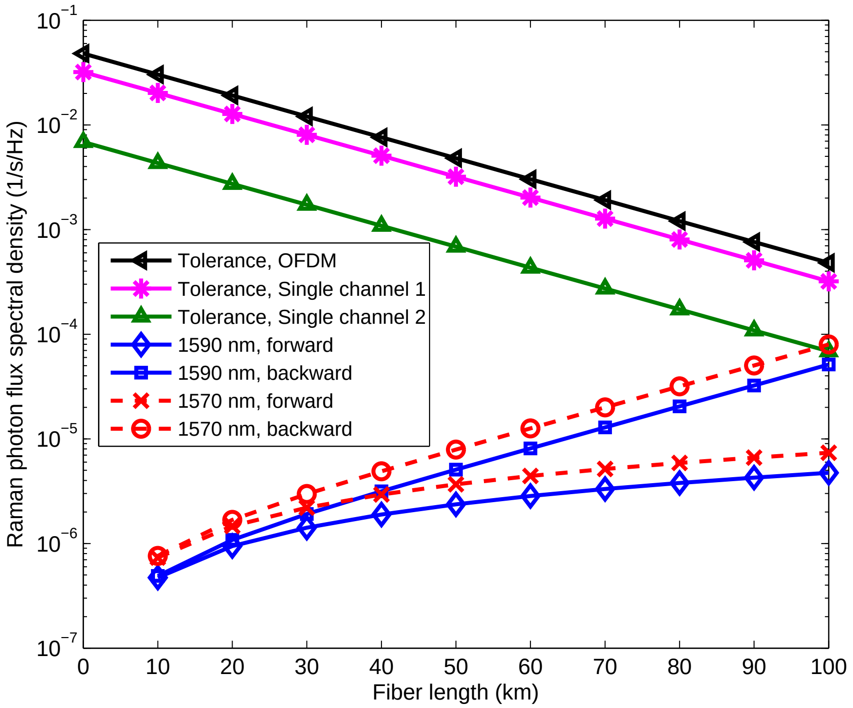

Figure 3: Raman-noise tolerance for the OFDM-QKD system and single channel 1 and single channel 2 schemes versus distance. The tolerance level is defined as the point where the Raman noise is 10 times lower than the received quantum signal. The Raman noise level generated by two forward and backward classical channels at wavelengths 1570 nm and 1590 nm are also shown.

Figure 3 shows the Raman noise tolerance level versus distance for our OFDM-QKD system, and compares it with the actual Raman noise level for forward and backward classical channels at two different wavelengths. The tolerance level is defined as the Raman noise level that is 10 dB weaker than that of the quantum signal.[4] This roughly corresponds to the threshold value of quantum bit error rate (QBER) at which no secret key can be generated. We compare our results with the ones obtained from a single QKD channel, reported in recent experiments,[4,5] with 100 ps of time gating. We consider

Table 1: Nominal values for system parameters

Parameter	Value
Average number of photons per signal pulse	0.48
Quantum Efficiency, η_d	0.3
Receiver dark count rate, γ_{dc}	1E-7 ns^{-1}
Error correction inefficiency, f	1.22
Phase stability error, e_d	0.03
Laser pulse repetition interval, T_s	250 ps
OFDM symbol duration, T	100 ps
Pulse width, T_p	11.5 ps
Number of subcarriers, N	8
Time gate of single channel	100 ps
Bandwidth of NBF for single channel	15, 70 GHz

two cases of spectral filtering for the single QKD channel, considering two different values of 15 GHz and 70 GHz for the bandwidth of the NBF. These two cases are referred to in this section by “single channel 1” and “single channel 2”, respectively.[4,5] As can be seen in Fig. 3, in terms of the tolerance level to the Raman noise, our OFDM-QKD system outperforms both schemes with only a single QKD channel by 1.77 dB (for single channel 1) and 8.45 dB (for single channel 2). This shows that our OFDM-QKD system can mitigate Raman background noise more effectively.

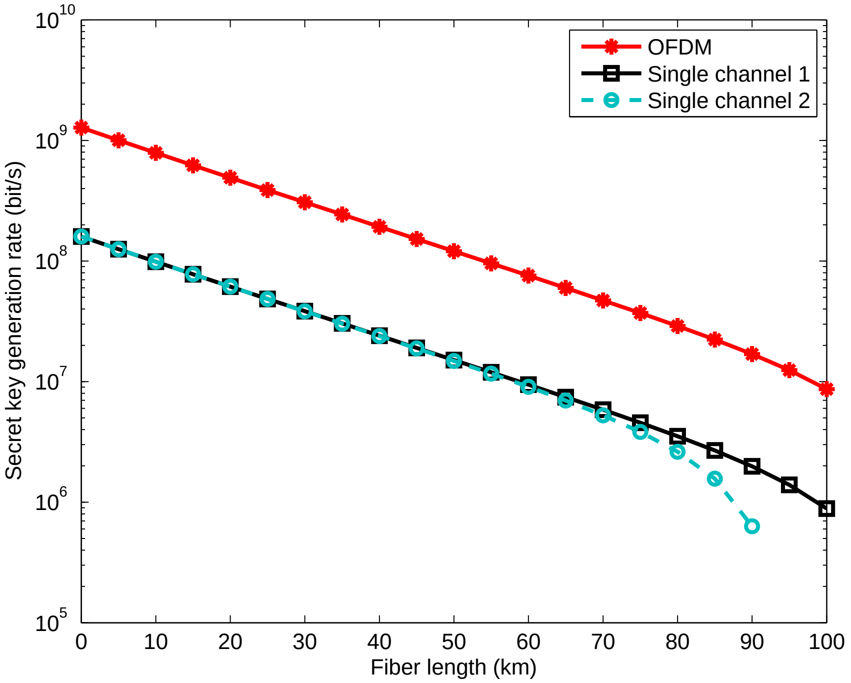

Figure 4: Secret key generation rate for the OFDM-QKD setup, single QKD channel 1, and single QKD channel 2, versus distance. The classical channel is assumed to be at 1590 nm, in the backward direction.

To further investigate the effect of optimal filtering in our OFDM-QKD setup, we compare the total secret key generation rate with the one achieved in single channel 1 and single channel 2 schemes. In all cases, the repetition rate for the QKD channel is assumed to be 4 GHz. We assume that one classical channel in the backward direction (from Bob to Alice), centered at 1590 nm, is multiplxed with the quantum channel. The total secret key generation rate for all cases is depicted in Fig. 4. It can be seen that the OFDM-QKD system provides higher rate per WDM channel, compared to the single QKD channel. There is nearly one order of magnitude improvement in the rate. This is mainly because of multiplexing 8 channels within one symbol period. In principle, one can get a higher rate from the single-QKD-channel cases as well, if instead of 100-ps-long pulses they use shorter pulses and higher repetition rate. But, even in that case, in terms of spectral efficiency, they would not beat the OFDM system. From Fig. 4, it can also be concluded that, because of its better noise reduction feature, the optimal filtering in the OFDM-QKD extends the maximum secure distance for the QKD operation.

5. CONCLUSIONS

In this paper, we considered an OFDM-QKD system, integrated with one classical channel using WDM techniques. We then examined the influence of Raman background noise generated by the classical signal on the QKD operation. We showed that OFDM-QKD would enable optimal simultaneous time-frequency filtering. This property was shown to provide improvement over previous noise reduction techniques, and it would enhance the total secret key rate per WDM QKD channel.

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REFERENCES

1. N. A. Peters, P. Toliver, T. E. Chapuran, R. J. Runser, S. R. McNown, C. G. Peterson, D. Rosenberg, N. Dallmann, R. J. Hughes, K. P. McCabe, J. E. Nordholt, and K. T. Tyagi, "Dense wavelength multiplexing of 1550 nm QKD with strong classical channels in reconfigurable networking environments," *New J. Phys.* **11**, p. 045012, April 2009.
2. T. E. Chapuran, P. Toliver, N. A. Peters, J. Jackel, M. S. Goodman, R. J. Runser, S. R. McNown, N. Dallmann, R. J. Hughes, K. P. McCabe, J. E. Nordholt, C. G. Peterson, K. T. Tyagi, L. Mercer, and H. Dardy, "Optical networking for quantum key distribution and quantum communications," *New J. Phys.* **11**, p. 105001, Oct. 2009.
3. P. Eraerds, N. Walenta, M. Legre, N. Gisin, and H. Zbinden, "Quantum key distribution and 1 Gbps data encryption over a single fibre," *New Journal of Physics* **12**(6), p. 063027, 2010.
4. K. Patel, J. Dynes, I. Choi, A. Sharpe, A. Dixon, Z. Yuan, R. Penty, and A. Shields, "Coexistence of high-bit-rate quantum key distribution and data on optical fiber," *Physical Review X* **2**(4), p. 041010, 2012.
5. K. Patel, J. Dynes, M. Lucamarini, I. Choi, A. Sharpe, Z. Yuan, R. Penty, and A. Shields, "Quantum key distribution for 10 Gb/s dense wavelength division multiplexing networks," *Applied Physics Letters* **104**(5), p. 051123, 2014.
6. S. Bahrani, M. Razavi, and J. A. Salehi, "Orthogonal frequency-division multiplexed quantum key distribution," *IEEE J. Lightwave Technol.* **33**, pp. 4687–4698, Dec 2015.
7. G. P. Agrawal, *Nonlinear fiber optics*, Academic press, 2007.
8. H.-K. Lo, H.-F. Chau, and M. Ardehali, "Efficient quantum key distribution scheme and a proof of its unconditional security," *Journal of Cryptology* **18**(2), pp. 133–165, 2005.
9. M. Razavi, "Multiple-access quantum key distribution networks," *Communications, IEEE Transactions on* **60**(10), pp. 3071–3079, 2012.