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# Optimized Limit for Polarimetric Calibration of Fully-Polarized SAR Systems

J. Chen, W. Guo, Z. Li and W. Liu

The optimized limit for polarimetric calibration of fully-polarized SAR systems is derived by establishing an error model as a function of cross-talk, channel imbalance and system noise. Compared to noise equivalent sigma zero (NESZ), the polarimetric error below the optimized limit is too small to affect the signal of cross-polarized channel. Thus, polarimetric calibration could be relaxed or even ignored in this case. With the backscatter model, optimized limits for cross-talk and channel imbalance at X, C and L-bands are presented, respectively. Moreover, when ignoring channel imbalance, the limit for cross-talk is given in a quantitative way. These results are very useful in practice, allowing significant reduction in calibration cost.

**Introduction:** Spaceborne polarimetric synthetic aperture radar (SAR) systems, such as TerraSAR-X, RADARSAT-2 and ALOS-PALSAR are of great importance to various geophysical research and applications [1-2], where a major limiting factor is the accuracy of the measured scattering matrix affected by cross-talk and channel imbalance. Thus, polarimetric calibration is crucial for obtaining accurate measurements via fully-polarized-SAR (FP-SAR) [3-4]. Moreover, although system noise also has a strong impact on SAR image quality, it is not considered in existing polarimetric calibration process. In fact, for many SAR systems, the error caused by system noise is much larger than that of polarimetric error, and polarimetric calibration accuracy does not need to be very high. For systems with large NESZ, polarimetric calibration can even be ignored to reduce cost, without affecting the system performance much.

In this letter, the optimized limit for polarimetric calibration of FP-SAR systems is derived based on the error model established with cross-talk, channel imbalance and system noise. Since the error model is related to waveband, results at different bands are then presented. Simulations are performed to evaluate the theoretical results for polarimetric calibration based on several existing SAR platforms.

**Error model:** The measured scattering vector  $\mathbf{M}$  for a fully-polarized SAR is given by [5]:

$$\mathbf{M} = \begin{bmatrix} M_{HH} \\ M_{HV} \\ M_{VH} \\ M_{VV} \end{bmatrix} = \begin{bmatrix} 1 & \delta_4 & \delta_2 & \delta_2\delta_4 \\ \delta_3 & f_2 & \delta_2\delta_3 & f_2\delta_2 \\ \delta_1 & \delta_1\delta_4 & f_1 & f_1\delta_4 \\ \delta_1\delta_3 & f_2\delta_1 & f_1\delta_3 & f_1f_2 \end{bmatrix} \cdot \begin{bmatrix} S_{HH} \\ S_{HV} \\ S_{VH} \\ S_{VV} \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} \quad (1)$$

where  $S_{HH}$ ,  $S_{HV}$ ,  $S_{VH}$  and  $S_{VV}$  are elements of the true scattering matrix;  $\delta_1$  and  $\delta_2$  stand for cross-talks on receive, while  $\delta_3$  and  $\delta_4$  for cross-talks on transmission;  $f_1$  and  $f_2$  are channel imbalances for transmission and reception, respectively;  $N_1$ ,  $N_2$ ,  $N_3$  and  $N_4$  denote the system (thermal) noise terms with independent and identical Gaussian distributions.

Ignoring the higher order cross-talk terms, (1) can be simplified to:

$$\begin{bmatrix} M_{HH} \\ M_{HV} \\ M_{VH} \\ M_{VV} \end{bmatrix} \approx \begin{bmatrix} S_{HH} + \delta_4 S_{HV} + \delta_2 S_{VH} + N_1 \\ \delta_3 S_{HH} + f_2 S_{HV} + f_2 \delta_2 S_{VV} + N_2 \\ \delta_1 S_{HH} + f_1 S_{VH} + f_1 \delta_4 S_{VV} + N_3 \\ f_2 \delta_1 S_{HV} + f_1 \delta_3 S_{VH} + f_1 f_2 S_{VV} + N_4 \end{bmatrix} \quad (2)$$

Thus, the covariance matrix of the measured scattering vector  $\mathbf{M}$  is given by:

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \\ C_{41} & C_{42} & C_{43} & C_{44} \end{bmatrix} \quad (3)$$

where  $C_{ij} = \langle M_i \cdot M_j^* \rangle$ , with  $M_i$  being the  $i$ -th element of  $\mathbf{M}$ .

Substituting (2) into (3), the covariance matrix can be expressed as the sum of the true scattering matrix and some distortion/error matrices. The maximum likelihood estimate (MLE) of the cross-polarized term  $C_{c-p}$  is given by [6]:

$$C_{c-p} = \frac{(C_{22} + C_{23} + C_{32} + C_{33})}{4} \quad (4)$$

Combining (2) and (4), with the assumption of reflection symmetry,  $C_{c-p}$  can be further simplified as:

$$C_{c-p} = \sigma_{HV} + (|f|^2 - 1)\sigma_{HV} + |\delta|^2 \sigma_{HH} + |f|^2 |\delta|^2 \sigma_{VV} + 2|\delta|^2 f^* A e^{j\theta} + \frac{\sigma_N}{2} \quad (5)$$

where  $\sigma_{HV} = \langle S_{HV} \cdot S_{HV}^* \rangle$ ,  $\sigma_{HH} = \langle S_{HH} \cdot S_{HH}^* \rangle$ ,  $\sigma_{VV} = \langle S_{VV} \cdot S_{VV}^* \rangle$ ,  $A e^{j\theta} = \langle S_{HH} \cdot S_{VV}^* \rangle$ ,  $\sigma_N = \langle N_i \cdot N_i^* \rangle$ . Without loss of generality, we assume  $\delta = \delta_i$ ,  $i=1,2,3,4$  and  $f_1 = f_2 = f$ .  $\sigma_N$  represents equivalent noise power i.e. NESZ. According to [6],  $A e^{j\theta}$  takes its maximum as  $\sqrt{\sigma_{HH} \sigma_{VV}}$  when  $\theta = \pi$ .  $\sigma_{HV}$ ,  $\sigma_{HH}$  and  $\sigma_{VV}$  denote the backscattering coefficients of the target, which can be calculated according to Ulaby's terrain backscatter model as [7]:

$$\sigma_{**} = P_1 + P_2 \exp(-P_3 \theta) + P_4 \cos(P_5 \theta + P_6) \quad (6)$$

where the subscript “\*\*” represents HH, HV or VV,  $P_i$  is the coefficient in Ulaby's model, and  $\theta$  is incidence angle.

From (6), the maximum error power  $\sigma_{error}$  resulting from cross-talk, channel imbalance and system noise for cross-polarized channels is:

$$\sigma_{error} = (|f|^2 - 1)\sigma_{HV} + |\delta|^2 \sigma_{HH} + |f|^2 |\delta|^2 \sigma_{VV} + 2|\delta|^2 |f| \sqrt{\sigma_{HH} \sigma_{VV}} + \frac{\sigma_N}{2} \quad (7)$$

where the first four terms are polarimetric errors, and the last one is system noise. Based on (7), a relative error index  $R$  is then defined, representing the influences of polarimetric and system errors:

$$R = \frac{(|f|^2 - 1)\sigma_{HV} + |\delta|^2 \sigma_{HH} + |f|^2 |\delta|^2 \sigma_{VV} + 2|\delta|^2 |f| \sqrt{\sigma_{HH} \sigma_{VV}}}{\sigma_{HV} + \frac{\sigma_N}{2}} \quad (8)$$

**Optimized limit for polarimetric calibration:** According to (8), if a maximum value is set for  $R$  for a specific application (such as 20%), the upper bound of cross-talk and channel imbalance can be derived as the optimized limit for polarimetric calibration, below which the polarimetric error of cross-polarized channel is negligible. However, it is noteworthy that here the upper bound of cross-talk and channel imbalance is coupled.

If channel imbalance is ignored ( $f=1$ ), cross-talk would be the main polarimetric error factor in (8). Then, the optimized limit of cross-talk is given by:

$$|\delta|_{op}^2 = \frac{R\sigma_{HV} - \frac{\sigma_N}{2}}{\sigma_{HH} + \sigma_{VV} + 2\sqrt{\sigma_{HH} \sigma_{VV}}} \quad (9)$$

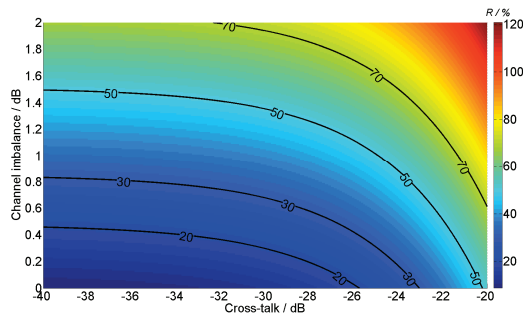
**Experimental results and discussions:** Soil and rock surface is selected as typical terrain targets for analyzing the optimized limit of polarimetric calibration. The simulation parameters are listed in Table 1.

**Table 1:** Experiment parameters.

Parameter	Value		
Waveband	X	C	L
NESZ (dB)	-26	-28	-34
Intensity of cross-talk (dB)	-20 ~ -40		
Intensity of channel imbalance (dB)	0 ~ 2		
Typical incidence angle (°)	30		

Firstly, the optimized limit of polarimetric calibration is analyzed in detail considering both cross-talk and channel imbalance. During simulation, a different NESZ is chosen for different waveband consistent with the actual situation. Fig. 1 shows the result of X-band,

and the curves represent the upper bound of cross-talk and channel imbalance with different relative error, below which polarimetric error can be ignored. The optimized limit varies with the relative error index  $R$ . Table 2 shows the maximum limits for X, C and L-band SAR satellites at  $R=20\%$ .

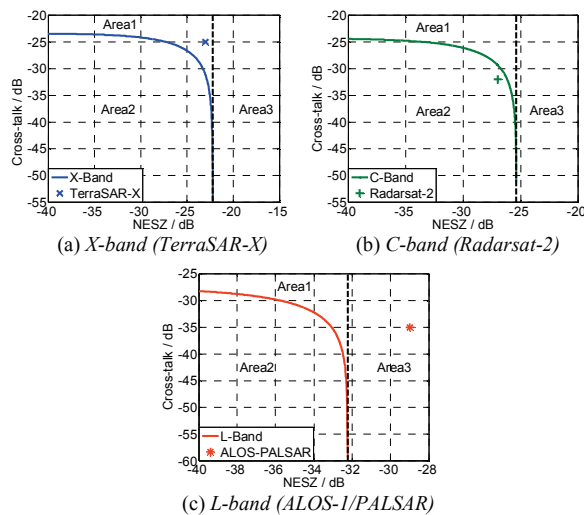


**Fig. 1** Optimized limit of cross-talk and channel imbalance at X-band.

**Table 2:** Maximum limits of cross-talk and channel imbalance at different wavebands at  $R=20\%$ .

Waveband	cross-talk (dB)	channel imbalance (dB)
X	-25.74	0.46
C	-27.71	0.36
L	-32.22	0.23

Ignoring channel imbalance, the optimized limit on cross-talk at the specified NESZ level can be derived using (9) at  $R=20\%$ . Fig. 2 shows the optimized limits for cross-talk as a function of NESZ at different waveband, with markers indicating the actual cross-talk and NESZ values for TerraSAR-X, Radarsat-2 and ALOS-PALSAR. If cross-talk accuracy is smaller than the limit (See Area 2 in Fig. 2), the cross-talk calibration requirement is achievable. However, it is not for parameters in Area 1 because cross-talk accuracy is larger than the limit. In Area 3, the error caused by NESZ has exceeded the established relative error, thus achievable for cross-talk calibration requirement.



**Fig. 2** Optimized limit for cross-talk as a function of system noise level at  $R=20\%$  (Area 2 denotes the achievable parameters for polarimetric calibration, while Area 1 and Area 3 for unrealistic parameters.)

The parameters of TerraSAR-X, Radarsat-2 and ALOS-PALSAR are used to illustrate the result. NESZ of TerraSAR-X (at X-band) is between  $-19\text{dB}$  and  $-26\text{dB}$  with an average value of  $-23\text{dB}$ , and cross-talk is  $-25\text{dB}$  [8]. Referring to Fig. 2, cross-talk is larger than the optimized limit, thus cross-talk calibration should be considered. For Radarsat-2 (at C-band), NESZ is between  $-22\text{dB}$  and  $-31\text{dB}$  with an average of  $-27\text{dB}$  and cross-talk is  $-32\text{dB}$  [9]. According to Fig. 2, the error for cross-talk could be ignored. While NESZ and cross-talk of

ALOS-PALSAR are about  $-29\text{dB}$  (an average from  $-24\text{dB}$  to  $-34\text{dB}$ ) and  $-35\text{dB}$ , respectively [10], in most cases cross-talk calibration is not needed because NESZ is larger than the cutting off value  $-32.3\text{dB}$  shown in Fig. 2, which has made the relative error exceed the limit.

**Conclusion:** The error model of cross-polarized channels as a function of polarimetric error and system noise has been established, based on which optimized limits for polarimetric calibration of FP-SAR systems at X, C and L-bands are proposed as thresholds for ignoring calibration. By ignoring the channel imbalance error, the optimized limit for cross-talk was then derived. These results provide very useful guidance in practice, allowing significant calibration cost reduction while still meeting the design requirements of the system. As an example, our analysis has shown that with relative error index  $R=20\%$ , polarimetric calibration (without channel imbalance) for TerraSAR-X should be considered in general, while negligible for Radarsat-2 and ALOS-PALSAR.

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