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Minimum Sensitivity Based Robust Beamforming with Eigenspace Decomposition

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Abstract An enhanced eigenspace-based beamformer (ESB) derived using the minimum sensitivity criterion is proposed with significantly improved robustness against steering vector errors. The sensitivity function is defined as the squared norm of the appropriately scaled weight vector and since the sensitivity function of an array to perturbations becomes very large in the presence of steering vector errors, it can be used to find the best projection for the ESB, irrespective of the distribution of additive noises. As demonstrated by simulation results, the proposed method has a better performance than the classic ESBs and the previously proposed uncertainty set based approach.

Keywords Eigenspace · robust beamformer · minimum sensitivity.

1 Introduction

The standard Capon beamformer (SCB) chooses the weight vector by minimising the array output power subject to a look direction constraint [1,2], assuming that the steering vector of the signal of interest (SOI) is known exactly [3]. However, this assumption may not be valid due to factors such as direction-of-arrival (DOA) mismatch error, array calibration error, local scattering, near-far spatial signature mismatch and finite sample effect [3–17]. Whenever this happens, the output signal-to-interference-plus-noise ratio (SINR) of the SCB degrades significantly.

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In order to improve the robustness of the SCB against steering vector errors, many methods have been proposed, such as diagonal loading (DL) [18] and norm-constrained Capon beamforming (NCCB) [9, 19]. Based on the uncertainty set of the steering vector, several robust beamformers were recently proposed [3–5, 7, 12].

As a classic beamforming method, the eigenspace-based beamformer (ESB) can be used for robust beamforming in the presence of steering vector errors and finite sample effect [8]. Since the actual steering vector lies within the signal-plus-interference (SI) subspace, the idea of ESB is to project the presumed steering vector onto the SI subspace to give a better estimation of the real steering vector of the SOI. However, the ESB suffers from severe performance degradation if the dimension of the SI subspace, i.e., the number of sources, can not be estimated correctly [11, 12]. Two well-known methods for estimating the number of sources are the Akaike information criterion (AIC) and the minimum description length (MDL). However, experimental evidence shows that either the AIC or the MDL can not give satisfactory estimation results for situations with a small sample size and a low signal-to-noise ratio (SNR) [20].

When the dimension of the SI subspace is obtained correctly, the resultant steering vector will lead to minimum sensitivity of the ESB to steering vector errors. However, if the SI subspace is overestimated or underestimated, there will be a large error in the projected steering vector. In such a case, the beamformer's sensitivity can become very large [1, 9, 12]. Based on the minimum sensitivity criterion, we here develop a novel method to estimate the dimension of the SI subspace to improve the robustness of the ESB. For an M-sensor array, M steering vectors can be obtained by projecting the presumed steering vector onto M estimates of the SI subspace. Then, the corresponding M weight vectors can be calculated using the M different steering vectors. We then compare the sensitivity values of the M weight vectors, and choose the weight vector with the minimum value for robust adaptive beamforming.

This paper is organized as follows. In Section 2, a review of the Capon beamformer and the eigenspace-based beamformer is provided.The proposed method is introduced in Section 3. Simulation results are presented in Section 4 and conclusions are drawn in Section 5.

2 Background

Consider an M-sensor uniform linear array (ULA) with an adjacent sensor spacing d. The received data at the nth snapshot can be expressed as

$$\mathbf{x}[n] = [x_1[n], \cdots, x_M[n]]^T = \sum_{i=1}^{P} \mathbf{a}(\theta_i) s_i[n] + \mathbf{n}[n] \tag{1}$$

where $[\cdot]^T$ denotes the transpose operation, $x_m[n]$ is the received data at the mth sensor, $s_i[n]$ is the ith source, $\mathbf{n}[n]$ is the additive noise with a covariance matrix $\mathbf{R}_n$, P is the number of impinging signals, and

$$\mathbf{a}(\theta_i) = [1, e^{-j2\pi d \sin(\theta_i)/\lambda}, \cdots, e^{-j2\pi(M-1)d\sin(\theta_i)/\lambda}]^T \tag{2}$$

is the $M \times 1$ steering vector of the ith signal with a direction θ_i, with λ denoting the signal wavelength.

Assume that all impinging signals and noise are uncorrelated with each other. Then the covariance matrix can be expressed as

$$\begin{aligned}\mathbf{R}_{xx} &= E[\mathbf{x}[n]\mathbf{x}[n]^H] \\ &= \sum_{i=1}^{P} \sigma_i^2 \mathbf{a}(\theta_i)\mathbf{a}(\theta_i)^H + \mathbf{R}_n = \sum_{m=1}^{M} \gamma_m \mathbf{u}_m \mathbf{u}_m^H \end{aligned} \tag{3}$$

where $E[\cdot]$ is expectation, $[\cdot]^H$ represents the Hermitian transpose, σ_i^2 is the power of $s_i[n]$, γ_m denotes the mth eigenvalue corresponding to the mth eigenvector $\mathbf{u}_m$ of $\mathbf{R}_{xx}$. Note that $\gamma_1 \geq \gamma_2 \geq \cdots \gamma_P > \gamma_{P+1} \geq \cdots \geq \gamma_M$. In practice, $\mathbf{R}_{xx}$ is replaced by the sample covariance matrix

$$\hat{\mathbf{R}}_{xx} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{x}[n]\mathbf{x}^H[n] \tag{4}$$

where N is the number of snapshots.

Without loss of generality, we assume that the first signal is the SOI. Then the Capon beamformer is obtained by solving the following optimisation problem:

$$\min_{\mathbf{w}} \mathbf{w}^H \hat{\mathbf{R}}_{xx} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \hat{\mathbf{a}} = 1 \tag{5}$$

where $\mathbf{w}$ is the $M \times 1$ complex weight vector and $\hat{\mathbf{a}}$ is the presumed SOI steering vector. The solution is given by [1]

$$\mathbf{w} = \frac{\hat{\mathbf{R}}_{xx}^{-1}\hat{\mathbf{a}}}{\hat{\mathbf{a}}^H \hat{\mathbf{R}}_{xx}^{-1}\hat{\mathbf{a}}}. \tag{6}$$

Since the actual steering vector lies within the SI subspace, we can project the presumed steering vector onto the SI subspace, resulting in the following ESB estimate

$$\mathbf{a} = \mathbf{U}_s \mathbf{U}_s^H \hat{\mathbf{a}} \tag{7}$$

where $\mathbf{U}_s = [\mathbf{u}_1, \cdots, \mathbf{u}_D]$ is the estimate of the SI subspace, with D being the estimated number of sources. Since $\mathbf{a}$ is the projection of $\hat{\mathbf{a}}$ onto the SI subspace, it is straightforward that the distance between the estimated $\mathbf{a}$ and the actual steering vector $\bar{\mathbf{a}}$ is shorter than that between $\hat{\mathbf{a}}$ and $\bar{\mathbf{a}}$. Based on (6) and (7), the weight vector of the ESB can be expressed as

$$\mathbf{w} = \frac{\mathbf{U}_s \text{diag}[\gamma_1, \cdots, \gamma_D]^{-1} \mathbf{U}_s^H \hat{\mathbf{a}}}{\hat{\mathbf{a}}^H \mathbf{U}_s \text{diag}[\gamma_1, \cdots, \gamma_D]^{-1} \mathbf{U}_s^H \hat{\mathbf{a}}}. \tag{8}$$

3 Proposed Method

In the presence of steering vector errors, we have the following relationship between the presumed steering vector $\hat{\mathbf{a}}$ and the true steering vector $\bar{\mathbf{a}}$

$$\bar{\mathbf{a}} = \hat{\mathbf{a}} + \boldsymbol{\delta} \tag{9}$$

where $\boldsymbol{\delta}$ is an unknown complex vector that describes the effect of steering vector distortions. We assume that the norm of the steering vector distortion $\boldsymbol{\delta}$ can be bounded by a known constant:

$$\|\boldsymbol{\delta}\| \leq \varepsilon \tag{10}$$

where $\|\cdot\|$ denotes the Euclidean norm and ε is the upper bound on the norm of the steering vector distortion $\boldsymbol{\delta}$. Define the array response for the presumed steering vector as $\mathbf{w}^H\hat{\mathbf{a}}$, then the maximum deviation of the array response from $\mathbf{w}^H\hat{\mathbf{a}}$ for $\bar{\mathbf{a}}$ is

$$\max_{\|\boldsymbol{\delta}\|\leq\varepsilon} |\mathbf{w}^H\bar{\mathbf{a}} - \mathbf{w}^H\hat{\mathbf{a}}| = \varepsilon\|\mathbf{w}\|. \tag{11}$$

Clearly, (11) increases with the norm of the beamformer weight vector $\mathbf{w}$. So we can define the sensitivity function of an array to perturbations by the squared norm of the weight vector [1,9]

$$\hat{\mathbf{T}}_{se} = \|\mathbf{w}\|^2. \tag{12}$$

As the norm of the weight vector increases, the beamformer's sensitivity increases, leading to a high sidelobe level and possible cancellation of the desired signal. To mitigate this problem, the NCCB method imposes a norm constraint on the weight vector, and [12] proposes to calculate the Capon beamformer with the minimum sensitivity to model errors by minimising the norm of the adaptive weights, considering the uncertainty set for the signal steering vector.

In order to avoid an arbitrarily low sensitivity achieved by scaling the beamformer's weight vector without changing the output SINR performance, it is important to define the beamformer's sensitivity as the squared norm of the scaled weight vector $\tilde{\mathbf{w}} = \mathbf{w}/(\hat{\mathbf{a}}^H\mathbf{w})$, which satisfies $\tilde{\mathbf{w}}^H\hat{\mathbf{a}} = 1$ [12]. This leads to the definition of the beamformer's sensitivity as [1,9,12]

$$\mathbf{T}_{se} = \frac{\|\mathbf{w}\|^2}{|\mathbf{w}^H\hat{\mathbf{a}}|^2}. \tag{13}$$

Here, we employ the beamformer's sensitivity defined in (13) instead of the one in (12) to develop the proposed method. The idea is to find the SI subspace which gives the lowest sensitivity, which corresponds to an ESB with the highest robustness to steering vector errors compared to the ESBs computed by the wrong SI subspace.

Let us define M estimates of the SI subspace as

$$\mathbf{U}_k = [\mathbf{u}_1, \cdots, \mathbf{u}_k], \ k = 1, \cdots, M. \tag{14}$$

We then obtain M estimated steering vectors by projecting the presumed steering vector $\hat{\mathbf{a}}$ onto the M different estimates of the SI subspace $\{\mathbf{U}_k\}_{k=1}^M$ as

$$\mathbf{a}_k = \mathbf{U}_k\mathbf{U}_k^H\hat{\mathbf{a}}, \ k = 1, \cdots, M. \tag{15}$$

Using (8) with $\hat{\mathbf{a}}$ replaced by $\mathbf{a}_k$, the corresponding M weight vectors are given by

$$\mathbf{w}_k = \frac{\mathbf{U}_k \mathrm{diag}[\gamma_1, \cdots, \gamma_k]^{-1}\mathbf{U}_k^H\mathbf{a}_k}{\mathbf{a}_k^H\mathbf{U}_k \mathrm{diag}[\gamma_1, \cdots, \gamma_k]^{-1}\mathbf{U}_k^H\mathbf{a}_k}, \ k = 1, \cdots, M. \tag{16}$$

Then the M resultant beamformers' sensitivity value can be obtained by

$$\mathbf{T}_{se}(k) = \frac{\|\mathbf{w}_k\|^2}{|\mathbf{w}_k^H \hat{\mathbf{a}}|^2}, \ k = 1, \cdots, M. \tag{17}$$

Note that if the exact information of the dimension of the SI subspace can be obtained, the calculated steering vector by projecting the presumed steering vector onto the estimate of the SI subspace will be very close to the actual steering vector. That is, even if there is a large steering vector error, the projected one is still a good estimate of the true steering vector and the calculated optimum weight vector will give an effective beamforming result, with its response to the desired signal very close to the desired response, which means the norm of the resultant weight vector will be small according to (11) and therefore, such a beamformer will have a very low sensitivity. Hence, the one with the minimum value $\mathbf{T}_{se}(k)$ will be chosen as the final solution to the robust beamforming problem.

4 Simulations

In this section, simulations are performed to study the performance of the proposed method compared with the SCB, the uncertainty set based robust Capon beamforming (RCB) method in [3], the maximally robust Capon beamformer (MRCB) of [12], and two cases of the ESB, which use the AIC and MDL algorithms for estimating the number of sources, respectively. Note that the MRCB employs a Newton-like algorithm to search for the optimum solution and therefore has a higher complexity than the proposed method.

We consider a ULA with $M = 10$ sensors and half-wavelength spacing between adjacent sensors. The SOI arrives from $\theta_1 = 0^\circ$. Two interfering signals with interference-to-noise ratio (INR) of 30 dB impinge on the array from the directions -40° and 30°, respectively. The array is steered toward the direction $\bar{\theta}_1 = \theta_1 + \Delta\theta$, where $\Delta\theta$ is the DOA mismatch error. Both gain and phase errors are also considered and the actual steering vector can be written as $\bar{\mathbf{a}}(\theta) = \Gamma \mathbf{a}(\theta)$, where $\Gamma = \text{diag}[1, \alpha_1 e^{-j\psi_1}, \cdots, \alpha_{M-1} e^{-j\psi_{M-1}}]$ is the diagonal matrix holding the calibration errors, with α_k and ψ_k standing for the amplitude and phase errors, respectively. We further assume that the amplitude and phase errors have a uniform distribution: $\alpha_k \in [0.8, 1.2]$ and $\psi_k \in [-\pi/100, \pi/100]$. Note that Γ changes from run to run while remaining constant for all snapshots. The uncertainty level for the RCB method is set to $\varepsilon = 4$. The SNR and INR at each sensor are defined by

$$\text{SNR} = 10 \log_{10}(\sigma_1^2/\sigma^2) \tag{18}$$

$$\text{INR}_i = 10 \log_{10}(\sigma_i^2/\sigma^2), \ i = 2, 3 \tag{19}$$

where the power level σ^2 is adjusted to give the desired SNR and INR. The output SINR is defined by

$$\text{SINR} = 10 \log_{10} \frac{\sigma_1^2 |\mathbf{w}^H \bar{\mathbf{a}}(\theta_1)|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}} \tag{20}$$

where $\mathbf{R}_{i+n}$ is the interference-plus-noise covariance matrix. All results are averaged over 100 independent simulation runs.

4.1 Example 1

In the first example, the additive noises are spatially and temporally white with variance $\sigma^2 = 1$. The DOA mismatch is $\Delta\theta = 3^\circ$. With SNR $= 5$ dB and $N = 100$, we first show the change of the sensitivity value of (17) in Fig. 1. Clearly the minimum value of the sensitivity function has been obtained when the estimated dimension of the SI subspace is correct.

In Fig. 2, we consider the resultant beam pattern of the beamformers. Two cases of ESB with the estimated dimension of the SI subspace $\beta = 2$ and $\beta = 4$ are considered. We can see that the SOI is considered to be an interference by the SCB, and hence, the SOI is cancelled by the SCB. When the dimension of the SI subspace is underestimated ($\beta = 2$) or overestimated ($\beta = 4$), the main beams of the ESBs of $\beta = 2$ and $\beta = 4$ are distorted, which means they do not work at all in these scenarios. On the other hand, the SOI is preserved by the RCB, MRCB and the proposed method. Moreover, the proposed method and the MRCB point their main beam to the desired look direction rather than the presumed one, while the RCB method has its main beam towards the presumed look direction.

Fig. 3 shows the output SINR of the beamformers versus the number of snapshots N for SNR $= 5$ dB, where the proposed method, the RCB, and the MRCB have all provided sufficient robustness and achieved a high output SINR. As shown, the output SINR of the SCB degrades significantly with a DOA mismatch of 3°. With a lower sample size, neither the AIC-based nor the MDL-based ESBs can give a good result. This is because the estimated dimensions of the SI subspace based on the MDL and AIC algorithms are not correct due to the finite sample effect. As the number of snapshots increases, the MDL algorithm can give a correct estimation result, while the MDL-based ESB has achieved the same performance as the proposed one and better than that of the AIC-based ESB.

Fig. 4 shows the output SINR of the beamformers versus input SNR when the number of snapshots is $N = 50$ and $N = 200$, respectively. With the same set of parameters as in Fig. 4, Fig. 5 shows the corresponding source number estimation result of the MDL-based, AIC-based and the proposed methods. It can be seen from Fig. 4 that neither of the AIC-based and MDL-based ESBs can give a satisfactory result when the number of snapshots is $N = 50$, because they tend to give a wrong dimension estimation of the SI subspace for a small sample size, as shown in Fig. 5. Additionally, at a low SNR, unlike the source number overestimation case, which has less impact on the performance of the ESB (this could be explained as follows: when the SNR is very low, the Capon beamformer would become less sensitive to steering vector errors as this equivalent to a large diagonal loading factor; so an overestimation of the SI subspace can at least reduce the steering vector error by some degree and therefore have less impact on the performance of the beamformer.), the underestimation of sources significantly degrades its performance. This can be observed for the case with $N = 200$, where the number of sources is underestimated by the AIC and MDL algorithms at low SNR, leading to severely degraded performance for both beamformers.

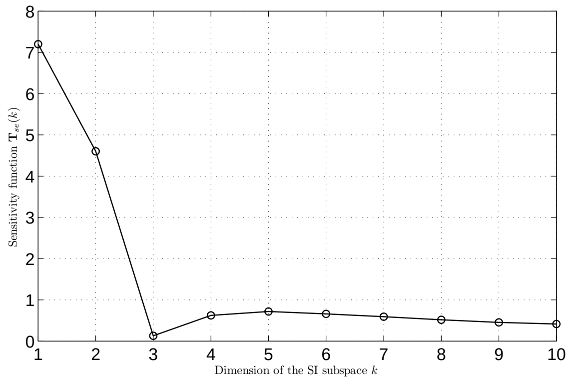


Fig. 1 Sensitivity function versus the estimated dimension of the SI subspace.

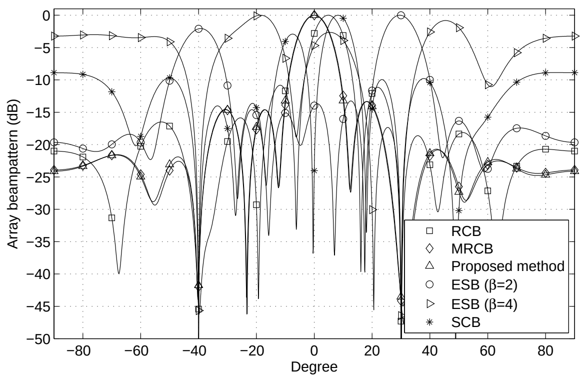


Fig. 2 The resultant beampattern of the beamformers.

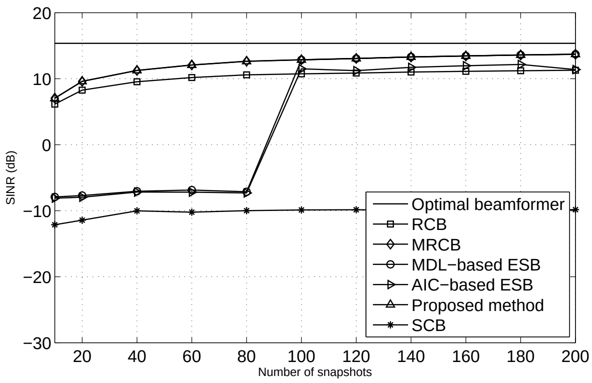
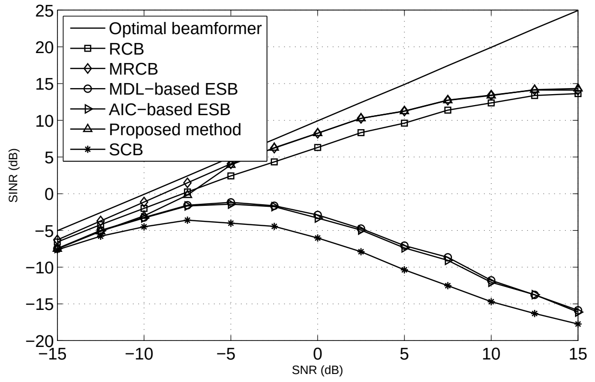
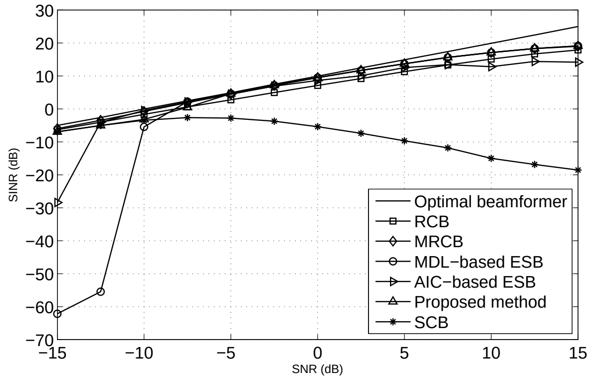


Fig. 3 Output SINR of the beamformers versus the number of snapshots.



(a) N=50



(b) N=200

Fig. 4 Output SINR of the beamformers versus input SNR.

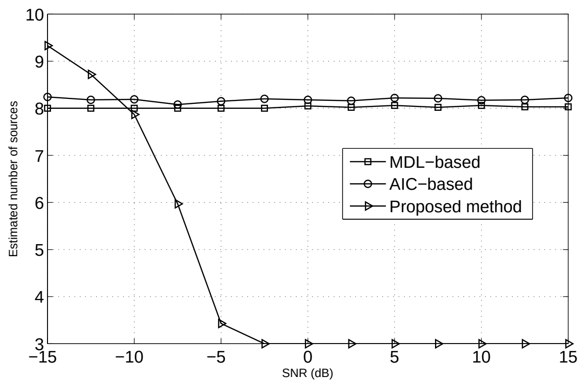
4.2 Example 2

In the second example, two noise scenarios are considered: 1) spatially correlated noise; 2) spatially nonuniform white noise. The covariance matrix of the correlated noise is assumed to have its ijth element given by

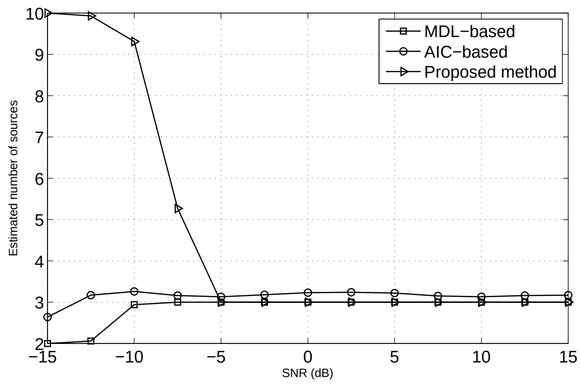
$$[\mathbf{R}_n]_{i,j} = \sigma^2 0.5^{|i-j|},\ 1 \le i, j \le M \ , \tag{21}$$

while the spatially nonuniform white noise is assumed to have its covariance matrix given by $\mathbf{R}_n = \sigma^2 \text{diag}(1, 2, 0.8, 5, 3, 4, 1, 1, 7, 3)$. The other parameters remain the same as in the first example.

In source number estimation, the additive noise is usually assumed to be white and have a uniform power distribution over the sensors. However, in practice, this assumption is rarely true. As a result, the performance of most source number estimation algorithms will degrade significantly in the presence of spatially correlated or nonuniform noise. Fig. 6 shows the output SINR of the beamformers versus the number of snapshots for SNR = 5 dB. Clearly, unlike the spatially white noise environment, as shown in Fig. 3, the performance of both the MDL-based and AIC-based ESBs has not improved much as the number of snapshots increases under either spatially correlated noise or spatially nonuniform white noise.



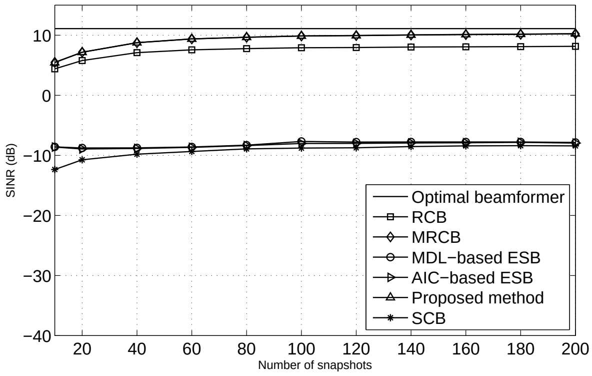
(a) N=50



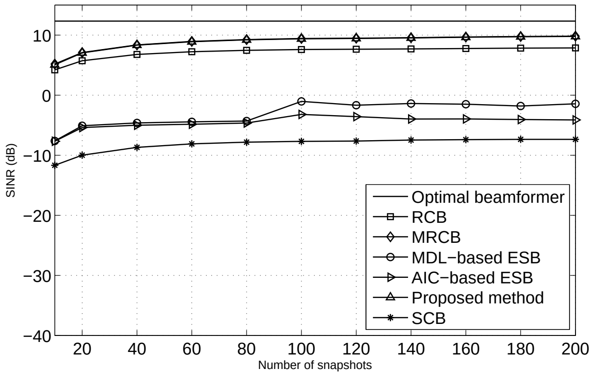
(b) N=200

Fig. 5 Source number estimation versus input SNR.

Figs. 7 and 8 show the output SINR of the beamformers as a function of input SNR for $N = 50$ and $N = 200$, respectively. With the same parameters as in Figs. 7 and 8, Figs. 9 and 10 show the corresponding source number estimation result of the MDL-based, AIC-based and the proposed methods. It can be clearly seen from Figs. 7 and 8 that the beamformers considered have similar performance when the input SNR is small; however, the performance of the SCB, the AIC-based and MDL-based ESBs degrade dramatically as the input SNR increases. On the other hand, the performance of the proposed method, the RCB, and the MRCB remain satisfactory for the whole range of input SNR. As shown in Figs. 9 and 10, the MDL-based and AIC-based algorithms can not give a correct result for the full range of input SNR considered, while the proposed method has provided the right result when input SNR is larger than about -5 dB for the first scenario and about 2.5 dB for the second scenario. Therefore, we see clearly from Figs. 7 and 8 that both the AIC-based and the MDL-based ESBs have failed to give a sensible performance in terms of output SINR; on the other hand, the proposed method consistently outperforms the other methods for moderate or high input SNR.



(a) Noise scenario 1)



(b) Noise scenario 2)

Fig. 6 Output SINR of the beamformers versus the number of snapshots.

5 Conclusions

To improve the robustness of the classic eigenspace-based adaptive beamformer, a minimum sensitivity criterion has been introduced to aid the selection of the required signal subspace for projecting the presumed steering vector, leading to a beamformer with maximum robustness among all of the M SI subspace choices. Simulation results have shown that the proposed method consistently outperforms the existing MDL-based and AIC-based ESBs and the previously proposed RCB method. In particular, it does not rely on the usual assumption for the additive noise being white and uniform across all sensors, and gives a satisfactory result when both the AIC-based and the MDL-based ESBs fail.

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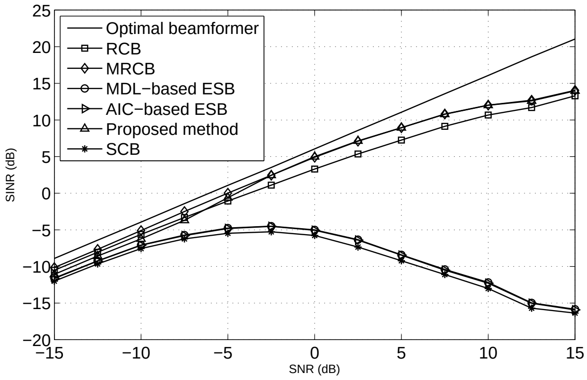
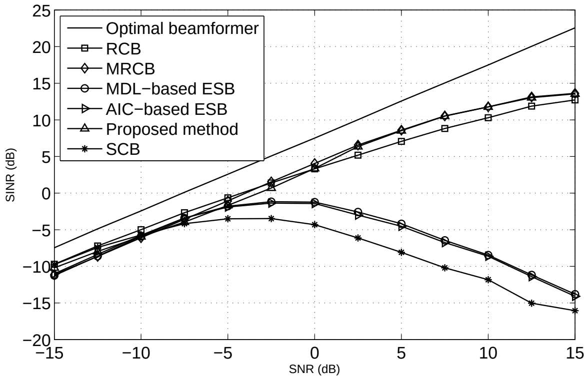


(a) Noise scenario 1)



(b) Noise scenario 2)

Fig. 7 Output SINR of the beamformers versus input SNR for N=50.

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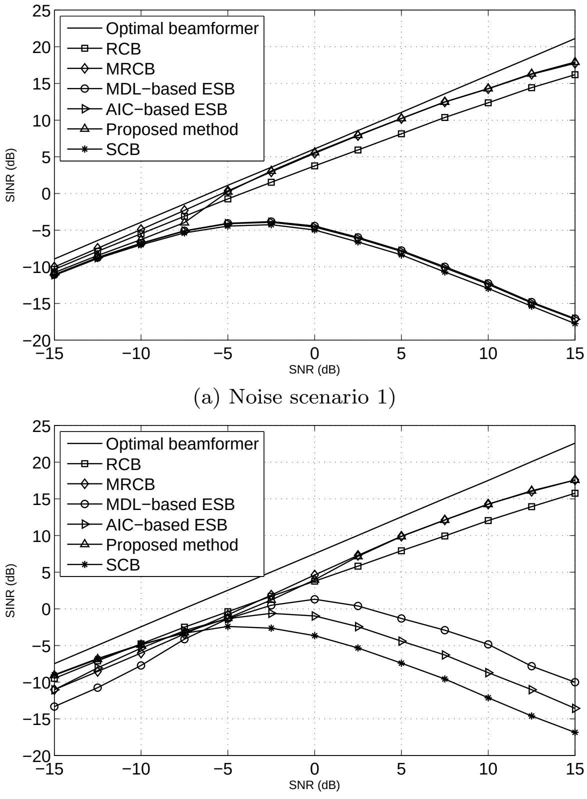

(a) Noise scenario 1)

(b) Noise scenario 2)

Fig. 8 Output SINR of the beamformers versus input SNR for N=200.

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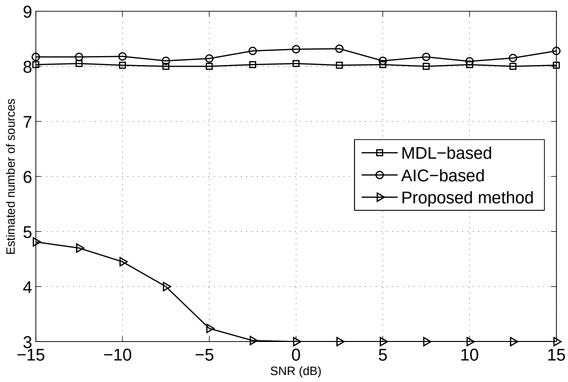
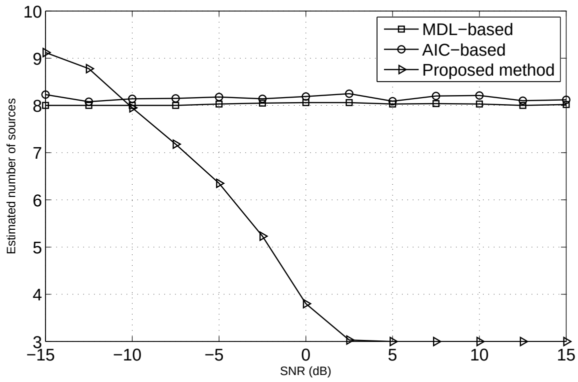
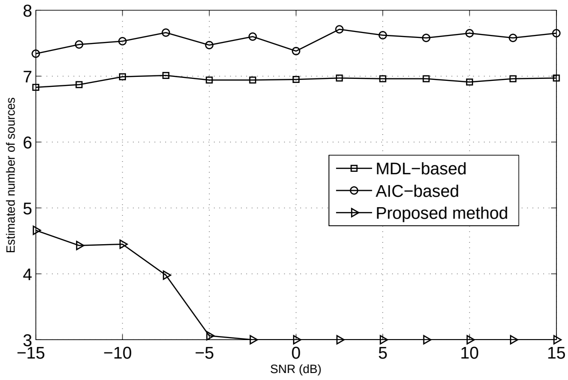


(a) Noise scenario 1)

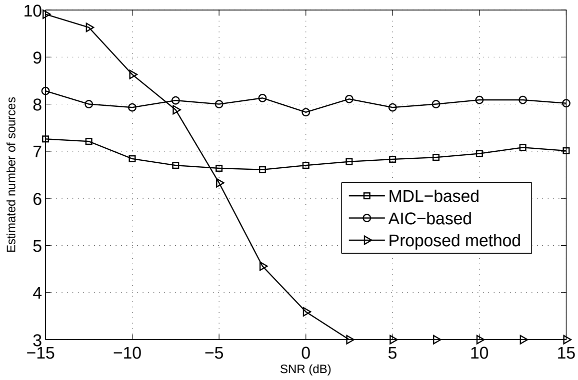


(b) Noise scenario 2)

Fig. 9 Source number estimation versus input SNR for N=50.



(a) Noise scenario 1)



(b) Noise scenario 2)

Fig. 10 Source number estimation versus input SNR for N=200.