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Guo, X, Wu, J, Sun, H et al. (2 more authors) (2016) Timetable coordination of first trains in urban railway network: A case study of Beijing. Applied Mathematical Modelling, 40 (17-18). pp. 8048-8066. ISSN 0307-904X

https://doi.org/10.1016/j.apm.2016.04.004

Please quote this paper as: Guo, X., Wu, J., Sun, H., Liu, R. and Gao, Z. (2016) Timetable coordination of first trains in urban railway network: a case study of Beijing. **Applied Mathematical Modelling**. In press.

Timetable coordination of first trains in urban railway network: a case study of Beijing

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Abstract

A model of timetable coordination of first trains in urban railway networks, based on the importance of lines and transfer stations, is proposed in this paper. A sub-network connection method is developed, and a mathematical programming solver is utilized to solve the suggested model. A simple test network and a real network of Beijing railway network are modeled to verify the effectiveness of our suggested model. Results demonstrate that the proposed model is effective in improving the transfer performance in that they reduce the connection time significantly.

Keywords: Departure time; Timetable coordination; First trains; Urban railway network

1. Introduction

There is an increasing development worldwide for urban railway network (URN) as an effective transportation mode to alleviate traffic congestion in cities. The denser an URN is, the more convenient it becomes to the travelers. However, having more lines and stations to an URN increases the complexity of timetable optimization for the system. What's more, the earlier the departure times for first trains, the higher operation cost to the URN. There are therefore trade-offs to be made between travelers who want short transfer waiting time and operators who want to minimize operational costs. Trade-offs are also to be made between different departure times for different lines, such that the overall transfer connection times are small. This is considered as the first train timetabling coordination problem.

Generally, timetable optimization is to design a schedule which can help transportation authorities to maximize their service level (such as minimizing transfer time, maximizing transfer

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accessibility), or to minimize some generalized cost of a combination of the above. There are many studies focusing on the transfer time, and optimization models are proposed to design or adjust a timetable. For example, Jansen et al. (2002) applied Tabu search method to adjust the dispatching times of trains on a route to synchronize the timetable by minimizing passenger transfer time. Cevallos and Zhao (2006) aimed to change an existing timetable by considering the coordination between lines. In their paper, the objective was to reduce the waiting time at the transfer stations. Chen and Wang (2007) proposed a method for calculating a reasonable departure time by decreasing the waiting time at transfer stations during the day. Wong et al. (2008) presented a mixed-integer-programming optimization model for schedule synchronization problem which minimizes the transfer waiting times of all passengers. They applied the method to the Mass Transit Railway of Hong Kong. Shafahi and Khani (2010) proposed two mixed integer programming models to minimize the total waiting time at transfer stations. Yang et al. (2012) considered the optimization of energy consumption and travel time as the objective based on a coasting control method. Wu et al. (2014) proposed a timetable synchronization optimization model to optimize passengers' waiting time while limiting the waiting time equitably over all transfer stations in Beijing railway network. Nayeem et al. (2014) proposed two algorithms on minimizing the waiting time and the number of transfers simultaneously.

Other researchers have concentrated on the aspect of the generalized cost to design the optimized timetable. Yan and Chen (2002) developed a model for intercity timetable setting. The model is formulated as a mixed integer multiple commodity network flow problem. Zhao and Zeng (2008) proposed a model to minimize passengers' transfer cost and presented a heuristic method to optimize transit network planning. In the study, the transfer cost is separated into walking time between stops, the waiting time at transfer stations and transfer penalty time. Meanwhile, simultaneous approach of optimal passenger cost and timetabling of transit systems has only been superficially explored, the synchronization between schedules and operational status is still to be resolved. Gallo et al. (2011) examined the frequency optimization problem under the assumption of elastic demand in a regional metro system. The objective of the model is to minimize the generalized cost which combines of transit user costs, car user costs, operator costs and external costs. Sun et al. (2014) formulated three optimization models to design a capacitated demand-sensitive peak and off-peak timetables.

There have been studies in dynamical re-scheduling in response to real-time information to enhance the service quality of URN. Taniguchi and Shimamoto (2004) presented a dynamic vehicle scheduling model that incorporates real-time information using variable travel time. Dynamic traffic simulation was utilized to update travel time. Vansteenwegen and Oudheusden (2006) proposed a linear programming model considering delay time in the actual operation. They aimed to compute the ideal buffer times for each connection, which was subsequently used in the linear program model for re-scheduling. Yan et al. (2006) developed a scheduling model which considers stochastic demand. They applied a simulation technique, coupled with link-based and path-based routing strategies, to develop two heuristic algorithms to solve the problem. Niu and Zhou (2013) developed integer programming models to optimize train timetables in a heavily congested urban rail corridor. Based on time-dependent, origin-to-destination trip records from an automatic fare collection system, a nonlinear optimization model was designed to solve the problem on a realistic sized corridor.

In timetabling problem, several inputs are necessary, e.g., service time of day, departure time for the first train, departure time for the last train and schedule for during-the-day operation. Most of the existing literatures on the subject of timetabling for URN have been concerned with the ‘normal’ operation during the day, when the service can be considered infinite and there is not a start or an end of the service. Scheduling for during-the-day operation is different to that for the first or the last trains. For during-the-day operation, the high service frequencies naturally reduce the connection time at transfer stations. All trains can connect to the feeder trains or be connected by other trains and within a reasonably short period of time. For example, at transfer station (Fig.1), for passengers from the q' train in line l' transferring to connecting q train in line l, their maximum connection time tends to be the headway of line l. During the peak period, when transit frequencies are high, Chakroborty (2003) demonstrated that missing a connection only increases transfer connection time by a relatively short interval. On the other hand, during off-peak period, Yan and Chen (2002) argued that when transit frequencies are low, missing a connection means long waiting times and the absence of synchronization may even discourage people from using public transport. In other words, it is important to study the synchronous timetable in off-peak hour.

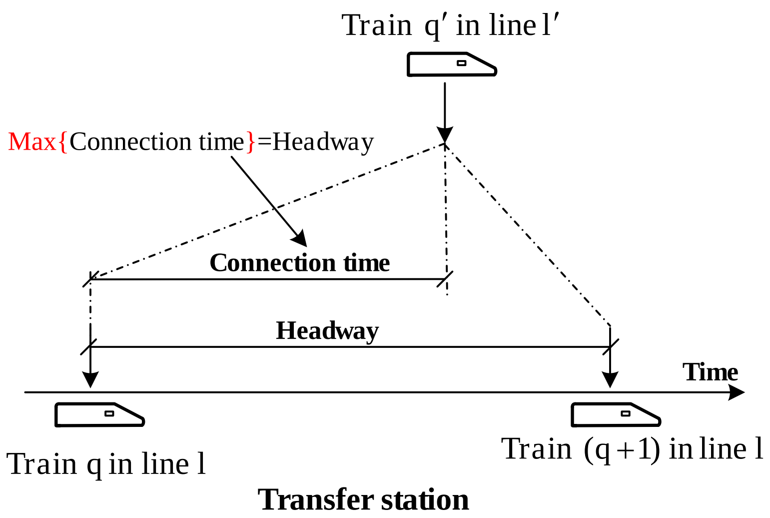

Fig. 1. The connecting trains in normal operation trains.

The first train timetabling problem which occurs in the morning off-peak hour becomes ever more important with the expansion of URN. The first train indicates the first operating train in each line every day. Passengers usually have to transfer to the other line(s) to complete their travel within the network. Therefore they are more concerned with service connectivity and transfer coordination. Trade-offs need to be made between passengers’ perspective and operator’s perspective to set the departure times for first trains within reasonable cost, without causing excessive long connection time at any transfer station in the URN. To illustrate the problem, we assume that the first train in line l' has to connect to the first train in line l in a transfer station (as illustrated in Fig.2). An unbalanced departure time of first train will lead to the follow situation: the departure time of first train in line l is much later than the first train in line l', thus the first train can make successful transfer in line l' to connecting trains in line l. The connecting trains in line l can be the first train, the second train, etc. and the shortest connection time is several times longer than headway in line l. Because of no train ahead of the first train in line l, and adjustment the headways towards the line to achieve the best synchronization state is useless for the whole performance of the system.

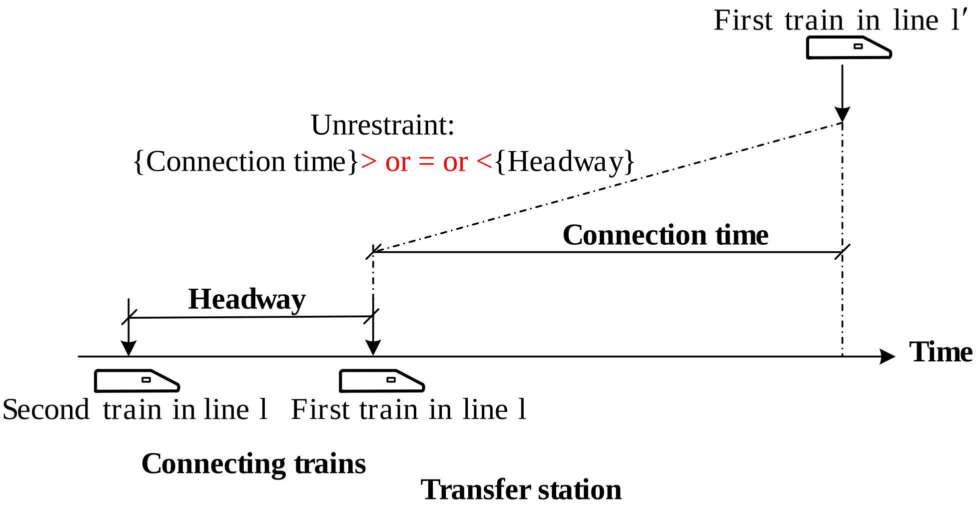

Fig. 2. The connecting trains in the first train problem.

Taking the Beijing URN in Fig. 3 for instance, six first trains depart from vehicle depots in three bi-directional lines (Line 4, Line 5 and Line 10). We present in the Table 1 the current first train connection times. In Table 1, ‘4 up to 10 up’ means the first up train of line 4 can connect the first up train of line 10. Similarly, ‘4 up to 10 down’ means the first up train of line 4 can connect the first down train of line 10. The first train running in the up direction of line 4 arrives at HDHZ station at 5:42:00 am, and the first train in the up train direction of Line 10 departs at 6:31:00 am. It takes passengers 5 minutes to walk from line 4 to line 10. As a result, the transfer connection time is 44 minutes which are a long time for passengers to wait. In another example, the first train running in the down direction of line 5 arrives at HN station at 5:12:05 am, and the first train in the down train direction of line 10 departs at 5:14:00 am. It takes passengers 2 minutes to transfer from line 5 to line 10. Thus, the connecting train is just leaving when the passengers come to the platform and they even can see the train leaving the platform. Therefore, we should also avoid this situation that when passengers miss the connecting train for a few minutes.

Table 1.

Transit planning process for the first trains in HDHZ station and HN station

Station	Transfer	Transfer walking time (s)	Arriving train	First connecting train	waiting (s)
HDHZ	4 up to 10 up	300	5:42:00	6:31:00	44 min
	10 down to 4 up	260	4:58:00	5:42:00	40 min 20 sec
	10 down to 4 down	260	4:58:00	5:09:11	6 min 51 sec
	4 down to 10 up	300	5:09:11	6:31:00	76 min 49 sec
HN	5 up to 10 up	120	5:45:51	6:15:00	27 min 51 sec
	5 down to 10 up	120	5:12:05	6:15:00	60 min 53sec

	5 down to 10 down	120	5:12:05	5:14:00	——
	10 down to 5 up	120	5:14:00	5:45:51	29 min 51 sec

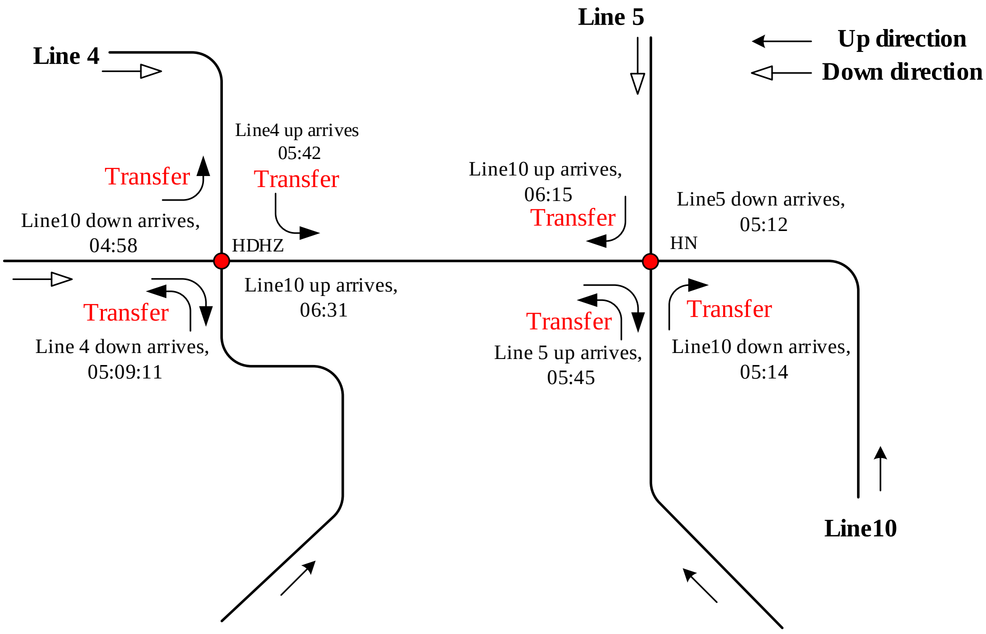

Fig. 3. The connecting first trains in a subset of the Beijing railway network.

Table 2 presents a snapshot of the connection time for first trains at some of the key transfer stations in the entire Beijing railway network. It shows that the connection time for some of these lines is in hours, which are way beyond expectation. Such extremely long connection time for first trains will clearly lead to low network accessibility and to discourage passengers from riding urban railway transit.

Table 2.

A snapshot of the connection time for the first trains in Beijing railway network

Station	**Number of transfer directions**				**The connection time (h)**
	1	2	3	4	
HDHZ	4 up to 10 up	4 up to 10 down	4 down to 10 up	4 down to 10 down	3.10
ZCL	10 up to 13 up	10 up to 13 down	10 down to13 up	10 down to 13 down	2.83
BTC	10 up to 8 up	10 up to 8 down	10 down to 8 up	10 down to 8 down	2.37
DZM	2 up to 13 up	2 up to 13 down	2 down to 13 up	2 down to 13 down	2.16
HN	10 up to 5 up	10 up to 5 down	10 down to 5 up	10 down to 10 down	2.16

In addition to minimize transfer time, timetabling is also to formulate reasonable headway, running time and dwell time so as to coordinate the departure times of trains at transfer stations. However, there are important differences in system characteristics between the normal during-the-day operation and the first trains. For the first trains, for example, the capacity of the trains is considered to be sufficiently high relative to the demand for such early morning services. Secondly, the running time between any two stations and transfer time at station can be fixed because there is no expected delay due to congestion. Thus, the train operation can be implemented strictly according to the train operation diagram. Thirdly, there are upper and lower bounds as to when the departure times of first trains can be scheduled (due to the constraints of the day and the required operating time of the line). Last but not least, there is an unbalanced distribution of passenger inflows between the up direction and the down direction for the first trains in the morning when passengers are more likely to transfer from the suburbs to the downtown areas. So there are directions where transfer stations and lines are more important than the other directions.

Scheduling for the first and last trains has only recently begun to draw research interests. Xu and Zhang (2008) proposed a multi direction transfer model for first and last train scheduling. Depending on the characteristics of passenger flow in the morning and evening, they presented the method to calculate the departure time's domains of the first and last trains. Zhou et al. (2013) presented a coordination optimization model on first trains' departure times to minimize passengers' total waiting time at origins and transfer waiting time for the first connecting trains. Chun et al. (2014) put forward a dynamic passenger volume distribution method according to the generalized travel cost. Then a connection optimization model of last train departure time is built to increase accessible passenger volume and reduce passengers' transfer waiting time of all origin and destination (OD) pairs for last trains. Kang et al. (2014) established a last-train network transfer model for Beijing URN to maximize passenger transfer connection headways, which reflect last-train connections and transfer waiting time. Kang and Zhu (2015) proposed a first train coordination model, while Kang et al. (2015) constructed an optimization model to minimize the running time and dwell time and to maximize the average transfer redundant time and network transfer accessibility of last trains.

Table 3.

Literature on timetabling for the three different schedule types

<table>
<tr><th>Scheduling type</th><th>Objective</th><th>Selected references</th></tr>
<tr><td rowspan="5">During-the-day operation</td><td>Minimize travel time</td><td>Nachtigall and Voget (1997); Jansen et al. (2002); Cevallos and Zhao (2006); Chen and Wang (2007); Wong et al. (2008) ; Shafahi and Khani (2010); Yang et al. (2012); Wu et al. (2014); Nayeem et al. (2014); Ibarra-Rojas and Rios-Solis (2012); Chakroborty (2003); Castillo et al. (2011); Castillo et al. (2015)</td></tr>
<tr><td>Minimize cost</td><td>Yan and Chen (2002); Zhao and Zeng (2008); Gallo et al. (2011); Li et al. (2013) ; Sun et al. (2014)</td></tr>
<tr><td>Dynamic re-scheduling</td><td>Taniguchi and Shimamoto (2004); Vansteenwegen and Oudheusden (2006); Yan et al. (2006); Niu and Zhou (2013)</td></tr>
<tr><td>maximize company profits</td><td>Caprara et al. (2013); Yaghini et al. (2011)</td></tr>
<tr><td>Minimize train delay</td><td>Li et al. (2013)</td></tr>
<tr><td rowspan="3">Last train operation</td><td>Maximize transfer accessibility</td><td>Xu and Li (2012); Kang et al. (2014)</td></tr>
<tr><td>Maximize transfer connection headways</td><td>Kang et al. (2015); Zhou et al. (2013)</td></tr>
<tr><td>Minimize transfer time</td><td>Chun et al. (2014); Xu and Zhang (2008)</td></tr>
</table>

First train operation	Coordinate departure times of first trains	Xu and Zhang (2008); Zhou et al. (2013); Kang and Zhu (2015)

Table 3 summarizes the key literatures for the three schedule types of URN: the normal during-the-day operation, the first and the last train operations. It can be seen that the objectives of timetable optimization among the different scheduling types are quite distinct; the differences are also highlighted by the system characteristics in Table 4.

Table 4.
The characteristics of the three schedule types in URN

Characteristics	During-the-day operation	The first train operation	The last train operation
Sufficient train capacity	May not be in rush hour	Yes	Yes
Passenger flow consideration	Yes	No	No
Successful passenger transfer	Yes	Yes	Yes or No
Transfer accessibility	High	Low	Low
Consideration of line coordination	No	Yes	Yes
Connection time	short	long	Long

Thus far, studies on first train scheduling have been limited and none has distinguished the importance of lines and transfer stations in relation to transfer demand. All lines and transfer stations have been considered as equally important. In a large URN, there is generally an un-even distribution of demand, especially for first trains, which places different weight on the utilization of different lines and at different transfer stations. To fill this gap, in this paper, we propose a first train timetabling optimization model with explicit consideration of the importance of lines and transfer stations.

2. Ti0metable coordination model of first trains

2.1. Assumptions

To facilitate the model formulation, several assumptions are made throughout the paper. They are listed below.

Assumption 1. The capacity of the first trains can meet the passengers' travel demand according to the actual data statistics of passenger travel OD flow volume. Therefore, the effects of passenger flow on the timetable coordination are not considered.

Assumption 2. The running time between any two stations and transfer time at the transfer station are given. The running time is derived in advance by operators, based on the speed of the train and the length of the line section. The transfer time utilized in the actual case study is obtained by a field survey and data processing.

Assumption 3. The upper and lower bounds of the departure times of first trains are specified by operators. To start the service too early or too late will have an impact on the cost or performance of the railway system directly. The bounds are given according to the practical experience of operators.

2.2. Symbol notations

The following lists the notations used in our first train transfer optimization model.

Network variables:

L : the set of lines, $l \in L$, $L = \{l \mid l = 1, 2, \ldots\ldots m\}$, where m is the total number of lines in the URN, there are as many lines in this network as there are sets of transfer stations generally

S : the set of transfer stations in the network, $S = \{S_l \mid S_l = S_1, S_2, \ldots\ldots S_m\}$;

S_l : the set of transfer stations in line l, $S_l = \{s_l^g \mid s_l^g = s_l^1, s_l^2, \ldots\ldots s_l^{n_l}\}$, where n_l is the total number of transfer stations in line l;

$S_{ll'}$: the set of transfer stations from line l to line l', $S_{ll'} = \{s_{ll'}^j \mid s_{ll'}^j = s_{ll'}^1, s_{ll'}^2, \ldots\ldots s_{ll'}^q\}$, where q is the intersection number of line l and line l';

Z : the set of all stations in the URN, $Z = \{z_l \mid z_l = z_1, z_2, \ldots\ldots z_m\}$;

Z_l : the set of stations in line l, $z_l^k \in Z_l$, $Z_l = \{z_l^k \mid z_l^k = z_l^1, z_l^2, \ldots\ldots z_l^p\}$, where p is the total number of stations in line l;

$T_{ll'}^{sj}$: the transfer walking time at transfer station $s_{ll'}^j$ from line l to line l'.

H_l : the headway in line l;

$R_l^{z^{k-1}z^k}$: the running time of first train from station z_l^{k-1} to the adjacent station z_l^k in line l;

$DW_l^{z^k}$: the dwell time of first train at station z_l^k in line l;

Decision variables:

$D_l^{z^k}$: the departure time of first train at station z_l^k in line l;

$A_l^{z^k}$: the arrival time of first train at station z_l^k in line l;

$C_{ll'}^{sj}$: the connection time at station $s_{ll'}^j$ for passengers who transfer from line l to first connecting train in line l' successfully;

2.3. Importance of line and transfer station

For first train coordination, the major concern lies not in the total passenger transfer time in this period, but that no passengers should have to wait excessively long for their transfer. The directions of travel of the demand for first-trains (mostly from residential to work areas), rather than the absolute passenger volume, are more important factors to consider. For this reason, we define the importance of a station/line' connectivity in a URN. We introduce the concept of importance degrees to describe the connectivity of lines and transfer stations.

2.3.1. Importance of line

The importance of line l is affected by four topological properties of a URN: the number of transfer stations δ_{l1}, the number of connection lines δ_{l2}, the number of stations δ_{l3} excluding the transfer stations, and the overall length δ_{l4} of the line. Applying the multi-criteria decision method, we define the importance of line as a weighted product of these structural factors:

$$\beta_l = \delta_{l1}^{\gamma_1} \times \delta_{l2}^{\gamma_2} \times \delta_{l3}^{\gamma_3} \times \delta_{l4}^{\gamma_4} \quad (1)$$

where γ_1, γ_2, γ_3 and γ_4 represent the relative weights of importance of the four criteria, and $\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 = 1.0$. The values of these weights are drawn from expert experience.

The weighted product model of (1) has the property that all four contributing factors are benefit criteria, in that the higher the values are, the more importance they bring to the line. For example, the addition of a new transfer station to the line will attract not only more passengers using the line, but also passengers from other stations and lines. In addition, the four factors are all indispensable components of the line importance, e.g., if the number of transfer stations is zero, the line's importance as far as train coordination is concerned, will also be zero.

2.3.2. Importance of transfer station

In this study, according to the geographic position of a station, we consider that a URN can be divided into two areas: the downtown area (the inner, dashed area in Fig. 4) and the suburb area (the outside area and the rest of the network in Fig. 4). Transfer stations in each area have distinctly different importance degrees.

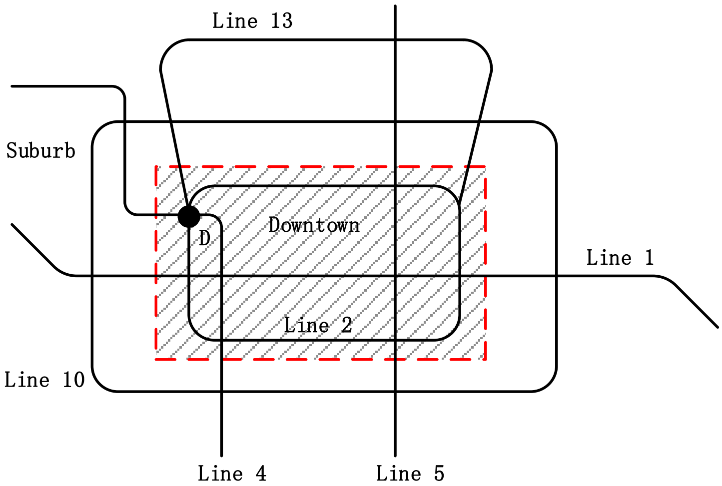

Fig. 4. Sub-networks in URN.

The importance of a transfer station is determined primarily by the importance of the lines it is connected to. In addition, we rank the importance of a station by its relative location in the URN: whether it is in the downtown or the suburb area, and whether it is connected to the most important line in the URN. Using the same multi-criteria analysis method, we formulate the station importance as:

$$\alpha_{s_{ll'}^{j}} = \left(\theta_{s_{ll'}^{j}} \times \varepsilon_1 + (1-\theta_{s_{ll'}^{j}}) \times \varepsilon_2 + \lambda_{s_{ll'}^{j}} \times \varepsilon_3\right) \times \prod_{l=1}^{c_{s_{ll'}^{j}}} \beta_l \qquad (2)$$

where $c_{s_{ll'}^{j}}$ is the number of lines connected to station $s_{ll'}^{j}$, ε_1 and ε_2 are the importance value for a transfer station in the downtown area and the suburban area respectively, and ε_3 is an importance value associated being on the most important line. $\theta_{s_{ll'}^{j}}$ and $\lambda_{s_{ll'}^{j}}$ are 0-1 integer variables. If the station belongs to the downtown area $\theta_{s_{ll'}^{j}}=1$; otherwise, $\theta_{s_{ll'}^{j}}=0$. Likewise, if a station is connected to the most important line in the URN, $\lambda_{s_{ll'}^{j}}=1$; otherwise, $\lambda_{s_{ll'}^{j}}=0$.

2.4. Problem formulation and model properties

Scheduling of first trains in a URN can be formulated as a transfer optimization model. In this model, the objective is to minimize total connection time at transfer stations coupled with the importance degree of the station. Generally, it is expected that the transfer demand are low in the suburban area than that in the downtown area because of the lack of choices of other lines to take in the suburban areas. This is especially the case for the first trains. The key is to give priority to minimize the connection time in lines with higher degree and at more important stations. The proposed importance degrees can assist in dealing with this problem effectively, with less important stations and lines in the suburb making a negligible contribution to the total connection time. Thus, the objective of the first train optimization problem can focus on the stations or lines which have high importance degree.

For each line l, the arriving time $A_l^{z^s}$ and the departure time $D_l^{z^s}$ at the station z_l^s can be calculated according to the departure time at the starting station z_l^0, accumulative running time and dwell time from the starting station to the current transfer station, and the dwell time at the current station (see Fig. 5). This is represented in Eq. (3) and Eq. (4). $D_l^{z^0}$ is the departure time of first train in line l at the starting station z_l^0.

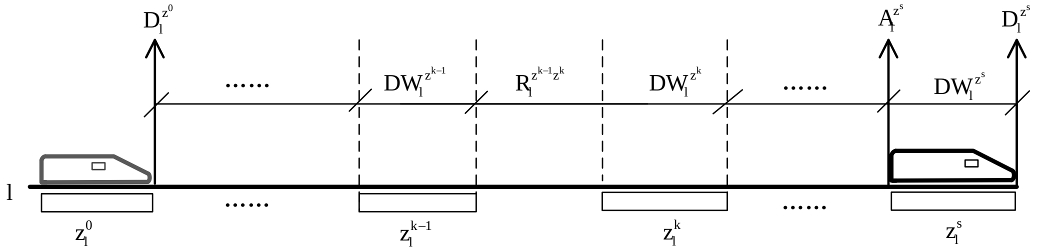

Fig. 5. The calculating progress of arriving time and departure time at station z_l^s.

$$A_l^{z^s} = D_l^{z^0} + \sum_{k=1}^{s} R_l^{z^{k-1}z^k} + \sum_{k=1}^{s-1} DW_l^{z^k} \tag{3}$$

$$D_l^{z^s} = A_l^{z^s} + DW_l^{z^s} \tag{4}$$

Let us consider a group of passengers transferring from line l to line l' at transfer station $s_{ll'}^j$ which is the same as station z_l^k in line l and station $z_{l'}^{k'}$ in line l' (see Fig. 6).

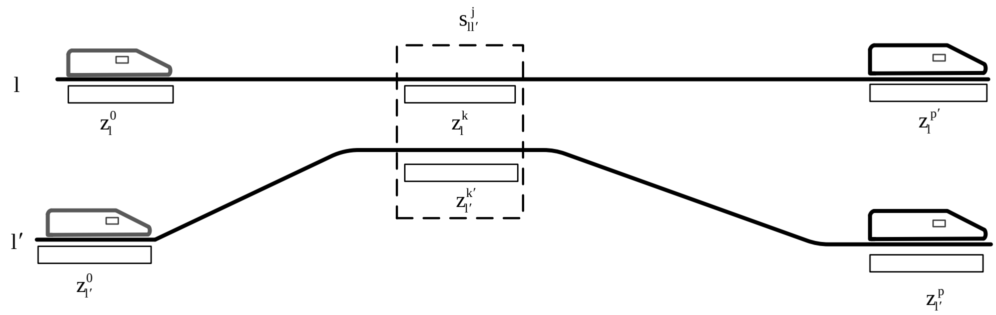

Fig. 6. The transfer station $s_{ll'}^j$.

Thus the successful transfer connection time from line l to line l' at the transfer station $s_{ll'}^j$ can be described with the following formulation:

$$C_{ll'}^{sj} = D_{l'}^{s_{ll'}^{j}} - (A_{l}^{s_{ll'}^{j}} + T_{ll'}^{sj}) \tag{5}$$

The minimum connection time is longer than the transfer walking time which can ensure the successful transfers, and then the total connection time from all transfer stations becomes:

$$CT = \sum_{l' \in L} \sum_{l \in L} \Big(\sum_{s_{ll'}^{j} \in S_{ll'}} C_{ll'}^{sj} \Big) \tag{6}$$

In most previous literatures on scheduling for normal during-the-day operations, the objective function is usually to minimize the total passenger waiting time where the number of transfer passengers is explicitly considered. Giving our Assumption 1 on the relatively low demand for first trains, in this paper, we focus on minimizing the connection time between first trains. In fact, a major novelty of our model is to apply the importance degrees of lines and stations in the objective function for optimizing first train coordination. Our objective can be formulated as follows:

$$f = \min \sum_{l' \in L} \sum_{l \in L} \sum_{s_{ll'}^{j} \in S_{ll'}} (\alpha_{s_{ll'}^{j}} \times \beta_{l} \times \eta_{ll'}^{sj} \times C_{ll'}^{sj}) \tag{7}$$

$$\text{s.t.} \quad A \le D_{l}^{z^{0}} + \sum_{k=1}^{s} R_{l}^{z^{k-1} z^{k}} + \sum_{k=1}^{s-1} DW_{l}^{z^{k}} \le B \tag{8}$$

$$A_{l}^{z^{k-1}} < D_{l}^{z^{k-1}} < A_{l}^{z^{k}} < D_{l}^{z^{k}} \tag{9}$$

$$C_{ll'}^{sj} = D_{l'}^{s_{ll'}^{j}} - (A_{l}^{s_{ll'}^{j}} + T_{ll'}^{sj}) \tag{10}$$

$$M \times (\eta_{ll'}^{sj} - 1) \le C_{ll'}^{sj} < M \times \eta_{ll'}^{sj} \tag{11}$$

The objective function (7) follows a multi-criteria formulation of the contributing factors to transfer costs: the importance degree of the station at which a transfer happens, the importance degree of the line from the transfer is made, and the transfer cost. The product within Eq. (7) is a measure of the cost-importance of an individual transfer at a station. The objective function sums the individual cost-importance measures of all transfer directions, and presents then a cost-importance measure of the whole network. Constraint (8) means that the departure time of first train at any stations cannot be earlier than A and later than B, where A and B are constants. The transfer walking time between two lines within a transfer station is fixed and given. It includes the time of passenger getting off a vehicle, walking to another vehicle and getting on. Constraint (9) sets the timing order for the arrival and departure times in stations z_{l}^{k} and z_{l}^{k-1}. Constraint (10) ensures that missing a connection is prohibited. A binary variable $\eta_{ll'}^{sj}$ is introduced. For all lines and stations, where M is a sufficiently large positive number. Eq. (11) states that, if $\eta_{ll'}^{sj}$ =1 when passengers succeed in transferring, then $0 \le C_{ll'}^{sj} < M$. On the other hand, if passengers fail to transfer, $C_{ll'}^{sj} < 0$ and $\eta_{ll'}^{sj}$ =0, then $-M \le C_{ll'}^{sj} < 0$.

Here we present a Mixed Inter Linear Programming (MILP) model for the timetabling problem of Eq. (7). To find an effective solution, we analyze the mathematical properties of the proposed model.

Property 1. According to constraints (8)-(10), the feasible domains of $D_l^{z^k}$ and $A_l^{z^k}$ can be bounded by time windows, which are expressed by the following expressions:

$$A_l^{z^k} \in \left[\max\left\{A, A_l^{z^{k-1}}\right\}, \min\left\{B, A_l^{z^p}\right\}\right] \tag{12}$$

$$D_l^{z^k} \in \left[\max\left\{A, D_l^{z^{k-1}}\right\}, \min\left\{B, D_l^{z^p}\right\}\right]. \tag{13}$$

Proof. The procedures for obtaining the departure time window are described as below. When $A_l^{z^k} = A$, according to the constraint (8) and Eq. (3), $A \le D_l^{z^0} + \sum_{k=1}^{s} R_l^{z^{k-1}z^k} + \sum_{k=1}^{s-1} DW_l^{z^k} = A_l^{z^k}$, thus $k = 0$. The earliest arrival time in line l is A. The latest arrival time in line l is $A_l^{z^p}$. If $A_l^{z^k} = B$, the latest arrival time in line l is $A_l^{z^p} \ge B$. However, the constraint (8) bounds $A_l^{z^p} \le B$. Thus, $A_l^{z^p} = B$ and $k = p$, the station is the last one in line l. Therefore, we can obtain Expressions (12) bounding the arrival time in line l, and Expressions (13) bounding the departure time $D_l^{z^k}$.

Property 2. Let N be an integer and determined by $N = \max\left\{\left[\frac{D_l^{z^0} - D_{l'}^{z^0}}{H_{l'}}\right], 0\right\}$. Here, the symbol [] represents the integer portion of the argument. Thus, N represents the number of trains that has operated in line l' before the start of service in line l. The lower and upper bounds of the first connection time between the first trains l' and line l are $T_{ll'}^{sj}$ and $\left\{(N-1)\times H_{l'} + H_l\right\}$, respectively.

Proof. We can easily obtain the lower bound of valid connection time between first trains is $T_{ll'}^{sj}$. According to constraint (10), when the two trains arrive the station at the same time, that is $D_{l'}^{s_{ll'}^j} = A_l^{s_{ll'}^j}$. Then we can get $C_{ll'}^{sj} = T_{ll'}^{sj}$. Assume that the first train of line l just leaving when the $N-th$ train of line l' arrive the transfer station, thus the connection time between $N-th$ train of line l' and first train of line l is H_l. Therefore, the connection time between the first trains of these two lines is $\left\{(N-1)\times H_{l'} + H_l\right\}$. If the $N-th$ train of line l' can make a connection with the first train of line l, the connection time between the first trains in two lines is $\left\{(N-1)\times H_{l'} + \phi\right\}$, where $\phi < H_l$. Thus, we obtain the upper bounds of first connection time form first train in line l' to first train in line l is $\{(N-1)\times H_{l'}+H_l\}$.

2.5 Sub-networks Connection Method (SCM)

In this paper, the timetabling problem belongs to the NP-hard class (Ibarra-Rojas and Rios-Solis, 2012; Kang et al., 2015). Therefore, a mathematical programming solver is selected to solve the model ensuring that the operation managers can obtain a solution within a reasonable amount of time. Many of the model variables and constraints are closely related to the topology of the rail network and planning period in the proposed model. These can be calculated prior to conducting the optimization process. To further improve the efficiency of the solution method, we present below a pre-processing method to reduce the computation time. Here we describe the Sub-network Connection Method (SCM) below and show how to derive the departure times of first trains at starting stations. Then, a mathematical programming solver, CPLEX Solver is used to solve the suggested model. Additionally, we compare the performances of the CPLEX Solver with artificial intelligence algorithms in the first train timetabling problem.

Step1. Divide sub-networks

According to the network layout, the URN is divided into several sub-networks denoted by $R=\{r\in R, r\mid r=1,2\}$, where r represents the number of sub-networks. In most cases, the URN is divided into the downtown area and the suburban area.

Step2. Choose the benchmark line

For each sub-network, the first step is to choose benchmark line and the base station. A benchmark line, according to the principle of preference theory, is the line that has the maximum number of connection lines. This is most likely to be found in the first layer. A base station is the transfer station that has the maximum number of transfer passengers. In the example shown in Fig. 3, line 2 will be the benchmark line and the transfer station D will be the base station. In our model formulation, the benchmark line and the base station are the key factors. We calculate departure times and arrival times of all stations in the sub-network r by using the departure time at the benchmark line. Namely, the departure time at benchmark line is the initial time stamp.

Step3. Calculate departure times at stations in the benchmark line

According to the departure time at the base station in different directions (up and down), we can calculate the departure times at all stations in the benchmark line with Eq. (4).

Step4. Calculate the departure times at transfer stations in the first layer

In the first layer, we take the order of the lines' importance degree as the computation sequence. Then, we calculate the departure times at transfer stations in other lines by using the departure time at the base station. To ensure the connection of both directions, the departure time at transfer station is chosen on the later of up direction time and down direction time.

Step5. Calculate the departure times at lines which belong to the first layer

In the first layer, we determine the departure times at all stations according to the departure times at transfer stations calculated in step 4.

Step6. Calculate the departure times at lines which belong to the second layer

Choosing the departure times at transfer stations connecting the two layers is crucial to calculate the departure times at all stations in the second layer. The departure times at key transfer stations are then used as initial values to calculate the departure times at the lines in the second layer.

Step7. Verify

Check departure times at transfer stations. If it is in the reasonable time range, repeat Step 5 and Step 6. Otherwise, we choose the benchmark line and the base station again and perform Steps 3-7 until the departure times at all transfer stations are reasonable. The reasonable time range can stipulate in the subway operating company. Here, the reasonable time means that the departure times of first trains must fit in a consolidated standard. For example, in the Beijing subway, the departure times of first trains shall not be earlier than 04:00 and not later than 05:30. The procedure of SCM is illustrated in Fig. 7.

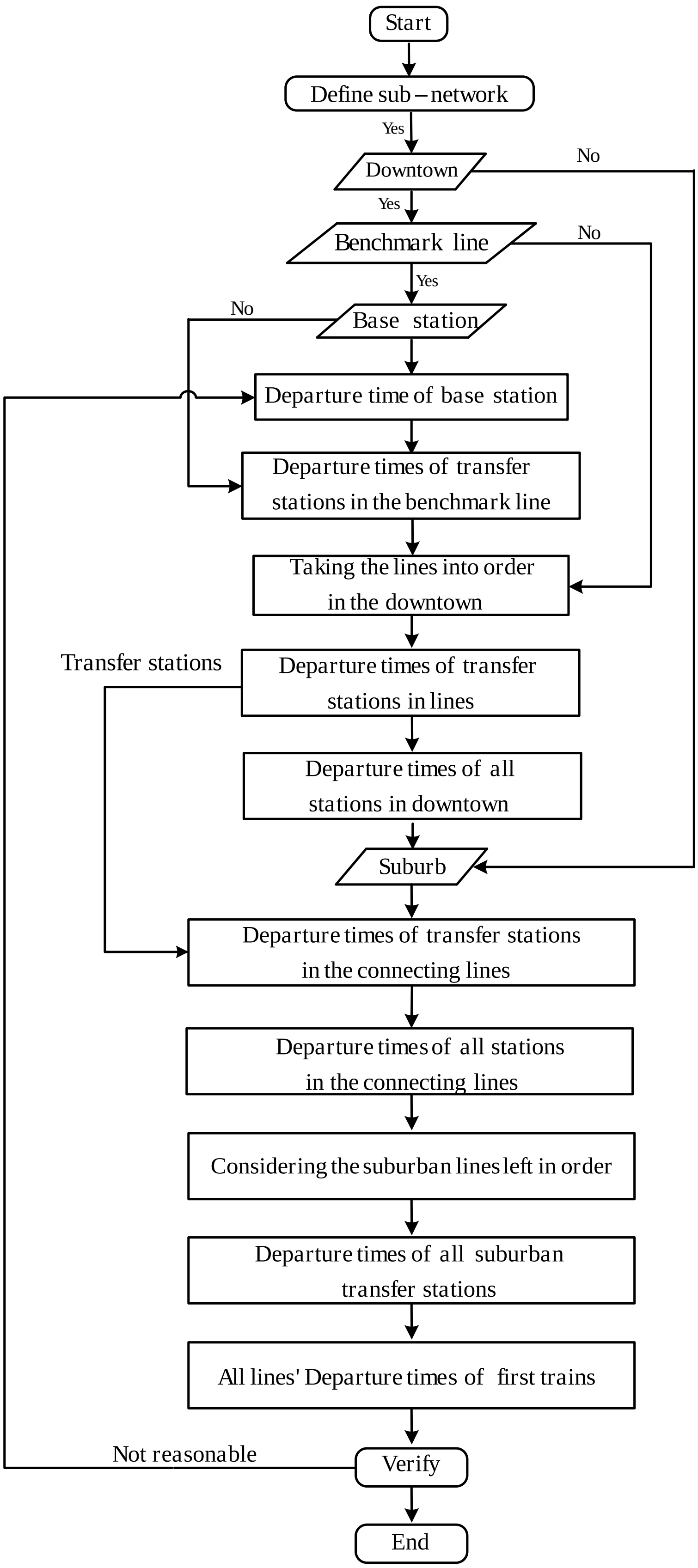

Fig. 7. The flow diagram of SCM.

3. Case study

3.1 A simple URN

3.2.1. Network parameters

In this section, we illustrate the workings of our proposed algorithm through a small test network (see Fig. 8) with three lines and five transfer stations. For this simple network, we did not need to divide it into two sub-networks. We follow the SCM procedure to calculate the initial departure times in each line. The initial departure times at all transfer stations are shown in Table 5. Both the average transfer time and the dwell time are assumed to be 0.5 minute.

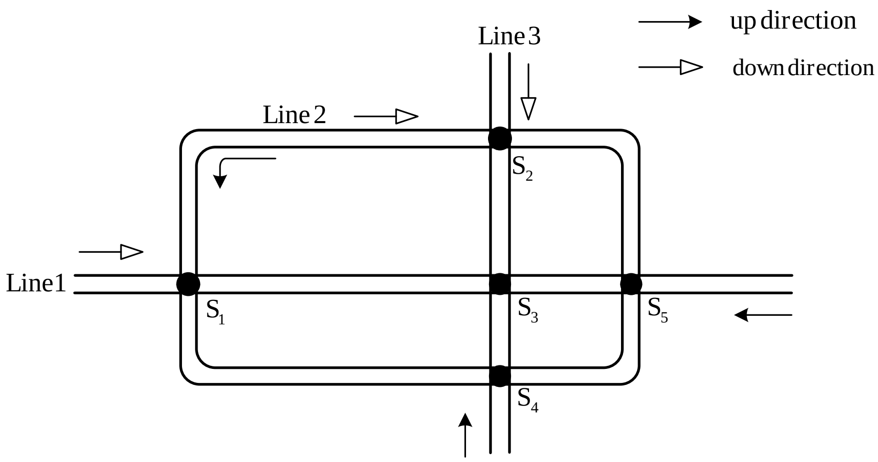

Fig. 8. A simple test transit network.

Table 5.
Initial departure times for the test network

Line	Direction	S_1	S_2	S_3	S_4	S_5
Line 1	Up	6:04	——	6:02	——	6:00
	Down	6:01	——	6:03	——	6:05
Line 2	Up	6:05	6:13	——	6:06	6:10
	Down	6:11	6:03	——	6:10	6:06
Line 3	Up	——	6:19	6:17	6:15	——
	Down	——	6:04	6:06	6:08	——

3.2.2. Optimization results

The experiments are tested using CPLEX Solver 12.5 on a personal computer with an Intel Core i3, 2.13 GHz CPU and 4GB RAM. We consider the up and down directions of a line in Fig. 8 network as separate lines in our model. The transfer directions include the directions from the suburban area to the downtown area. Applying the CPLEX Solver, we obtain the optimized total connection time of 2160s. Table 6 shows the timetable after optimization for this numerical example.

Table 6.

Departure times at the test network after optimization

Line	**Direction**	S_1	S_2	S_3	S_4	S_5
Line 1	Up	6:04	——	6:02	——	6:00
	Down	6:00	——	6:02	——	6:04
Line 2	Up	6:05	6:13	——	6:06	6:10
	Down	6:10	6:02	——	6:9	6:05
Line 3	Up	——	6:14	6:12	6:10	——
	Down	——	6:08	6:10	6:12	——

As mentioned above, the departure time in line 2 is chosen as the benchmark. Therefore, departure times in other lines can be obtained with the suggested model. In addition, connection time for all transfer stations is shown in Table 7. The results show that the total connection time is decreased by 12.3%.

Table 7.

The connection time at transfer stations (min)

Station	**Transfer direction**	**Connection time**		**Improved Value/min**
		Before Optimization	**After Optimization**	
S_1	1 down to 2 up	4	1	3
	1 up to 2 down	7	6	1
	1down to 2 down	10	10	0
	1 up to 2 up	1	1	0
S_2	2 down to 3 up	12	12	0
	2 down to 3 down	1	6	-5
	3 down to 2 up	9	5	4
	2 up to 3 up	2	1	1
S_3	1 down to 3 up	14	10	4
	1 up to 3 down	4	8	-4
	1 down to 3 down	3	8	-5
	1 up to 3 up	15	10	5
S_4	2down to 3up	9	1	8
	2 up to 3 down	2	6	-4
	3 down to 2 down	2	3	-1
	2 up to 3 up	13	4	9
S_5	1 down to 2 up	5	6	-1
	1 up to 2 down	6	5	1

1 down to 2 down	1	1	0
1 up to 2 up	10	10	0
Total connection time	130	114	16

3.2.3. Comparison of solution methods

In this section, we compare the performance of using CPLEX Solver with other alternative optimization methods in solving the first train timetabling coordination problem. Three other intelligent algorithms examined are: simulated annealing (SA), genetic algorism (GA) and Particle Swarm Optimization (PSO). The tests are all conducted on the simple network shown in Fig. 8. The performances of the four optimization results are presented in Table 8. Two conclusions can be put forward here.

(1) All methods reach similar optimized results (in terms of objective function values). However, the CPU times are different. It took CPLEX Solver 0.45s to obtain the optimal solution, while for GA, SA and PSO, the CPU times are 10.61s, 44s and 132s respectively.
(2) All intelligent algorithms should test the parameters to get the more accurate solutions, the value of parameters have direct influence on the optimal results. The CPLEX Solver is not necessary to test parameters.
(3) All methods improve the objective function from 6600s in the original timetable to 2160s. However, the CPLEX Solver reaches the optimal solution much faster and more effective than the other three methods.

Table 8.
Results of first train scheduling by different methods

Method	**CPU (s)**	**Iterations**	**Objective**	**Departure time**	
				Upper	**Lower**
Original	-------	-------	6600	6:00	6:19
SA	44	726	2160	6:02	6:14
GA	10.61	58	2160	6:03	6:11
PSO	132.17	300	2160	6:00	6:19
CPLEX	0.45	-------	2160	6:00	6:11

3.2 Beijing railway network

3.3.1. Network description

In order to verify the proposed model and solution algorithm, this paper takes Beijing railway network as a case study, which has 16 lines, 261 stations. (See Fig. 13). All the transfer stations have been marked with black dot. The downtown area of this URN is marked by the dashed area.

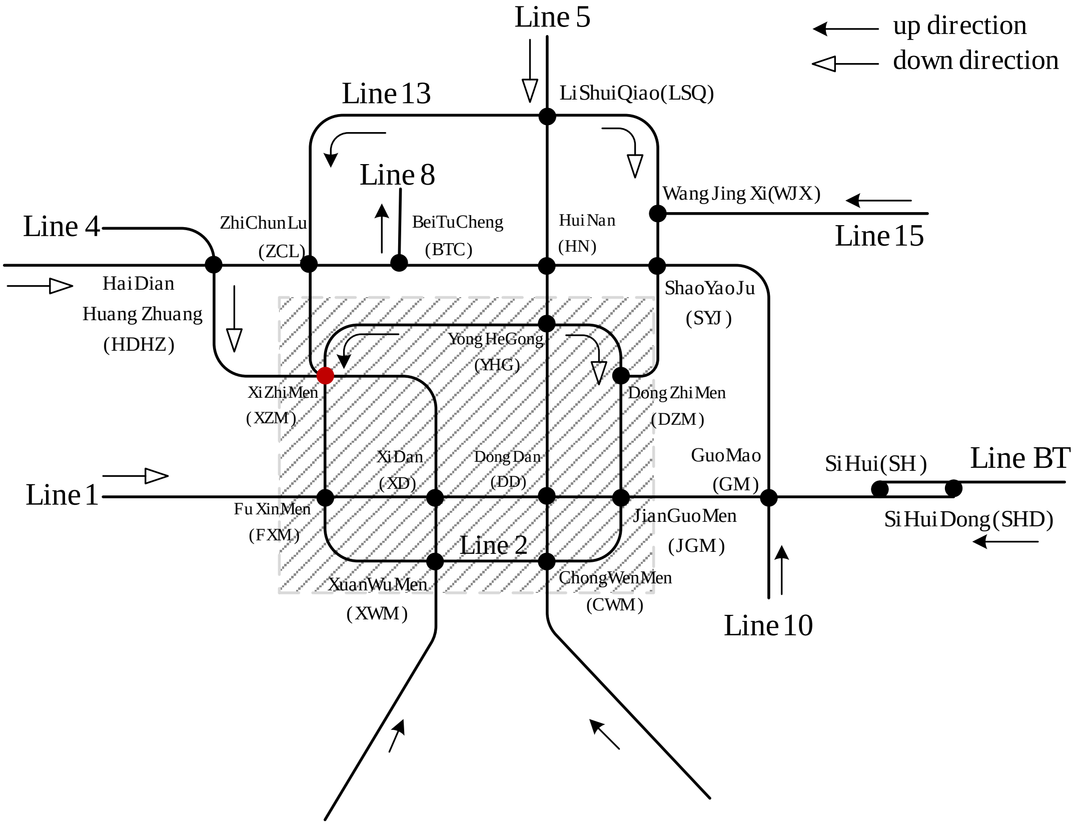

Fig. 13. Beijing railway network.

3.3.2. Line importance

According to Eq. (1), the importance of line in Beijing railway network are calculated and shown in Table 9. The parameters are given as $\gamma_1 = 0.4$, $\gamma_2 = 0.2$, $\gamma_3 = 0.3$, $\gamma_4 = 0.1$. The expert knowledge suggests that: (1) the number of transfer station is the most important factor as it determines the passengers' accessibility especially in the large scale network; (2) the number of stations is more important than the line length because more stations will transport more passengers. The numbers of stations, the number of transfer stations, the number of connection lines and overall length should pertain to the same order of magnitude. Thus, the line length is represented as length/kilometers and based on real data obtained from the geographic information database of Beijing metro network.

Table 9.
The importance of line in Beijing railway network

Line	The number of transfer stations δ_1	The number of stations (Removing transfer stations) δ_2	The number of connection lines δ_3	Length/km δ_4	The line importance
Line 1	7	16	5	31.04	8.7

Line 2	7	11	4	23	7.3
Line 4	5	19	5	28	7.8
Line 5	6	17	5	27.6	8.1
Line 8	2	8	2	7.168	3.0
Line 10	7	15	5	24.6	8.4
Line 13	8	8	7	40.5	9.0
Line BT	2	11	1	17.2	2.8

As line 2 is the only line belonging to the downtown area, it is set as the benchmark line. Then ordering line importance from largest to smallest, we get: line 13, line 1, line 10, line 5, line 4, line 8, and line BT.

3.3.3 Transfer station importance

From Fig. 13, we can easily obtain the value $c_{s_{ll'}^{j}}$, i.e. the number of connection lines for each station. Except XZM station which is connected to three lines, all other stations are connected to just two lines. Then setting the importance variables $\varepsilon_1 = 0.3$, $\varepsilon_2 = 0.2$, $\varepsilon_3 = 0.5$, we can calculate from Eq. (2) the importance of transfer stations in Beijing railway network and the results are shown in Fig. 14. It is found that XZM station is the most important transfer station in Beijing railway network.

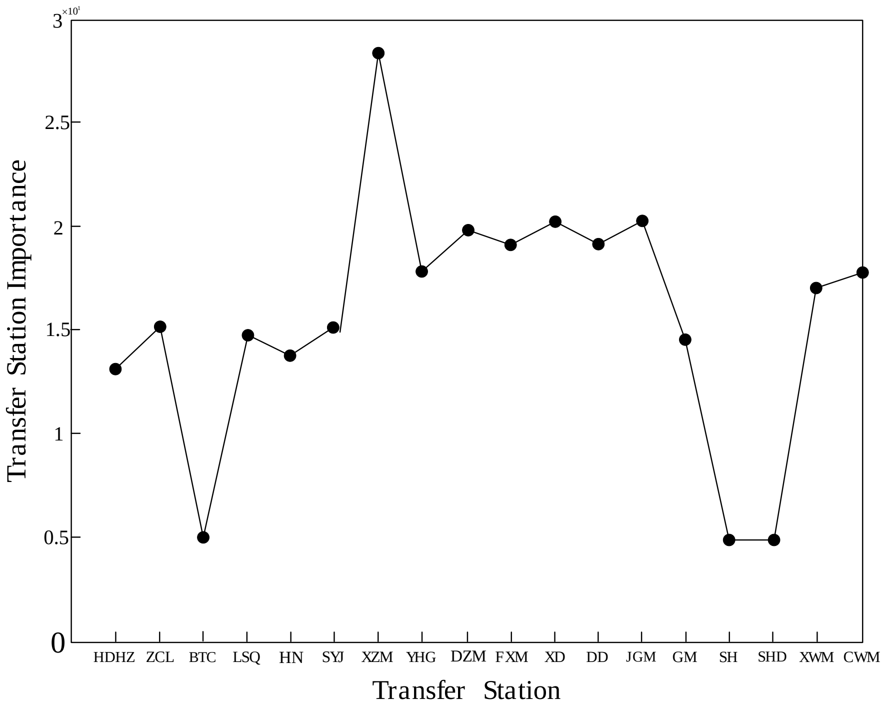

Fig. 14. Importance value of transfer stations in Beijing railway network.

3.3.4. Initial solution

Following the MSC procedure, we calculate the departure times at lines according to their order of importance, and select a transfer station which has the largest importance degree as a base station in benchmark line. For Beijing railway network, we select XZM station as a base station to calculate the departure time in line 2. The initial departure times of first trains are shown in Table 10.

Table 10.

Initial departure times of first trains in Beijing railway network

Line	Line 1	Line 2	Line 4	Line 5	Line 8	Line 10	Line 13	Line BT
UP	4:42	5:10	5:06	5:12	4:58	4:28	5:41	5:38
DOWN	5:11	5:04	5:05	4:41	5:55	6:00	5:52	4:45

3.3.5. The system performance of Beijing railway network

Optimized results of Beijing railway network

The upper bounds of departure times are 6:00 and lower bounds of departure times are 4:30. The optimized departure times are listed in Table 11.

Table 11.

The optimal departure times of first trains in all lines

Line	Line 1	Line 2	Line 4	Line 5	Line 8	Line 10	Line 13	Line BT
UP	04:53	05:18	05:23	05:14	05:44	05:13	05:10	05:46
DOWN	05:20	05:19	05:00	04:52	05:26	05:15	05:00	05:16

The total connection time

Table 12 shows the departure times in Beijing railway network, and the parts of connection time before and after optimization are given in Table 13. After optimization, the selected stations' total connection time is 12521 seconds (see Table 13). Compared to the 26174 seconds in the current timetable (see Table 13), the optimized results reduce the selected stations' total connection time by 11172 seconds or 43%. For the whole Beijing railway network, the improvement is 44% (from 93941 seconds to 53078 seconds). The results indicate that the proposed model is effective in solving the first train timetabling problem.

Table 12.

The departure times of first trains in actual operation of Beijing railway network

Line	Line 1	Line 2	Line 4	Line 5	Line 8	Line 10	Line 13	Line BT
UP	05:09	05:09	05:10	05:19	06:02	05:39	05:00	05:49

DOWN	05:05	05:03	05:00	04:59	05:18	04:53	05:00	05:19

Table 13.

The connection time before and after optimization at selected transfer stations

Station	Transfer	Transfer walking time (s)	The connection time before optimization (s)	The waiting time after optimization (s)	Improvement (s)
HDHZ	4 up to 10 up	300	2940	300	2640
	10 down to 4 up	260	2635	2450	185
	10 down to 4 down	260	666	300	366
	4 down to 10 up	300	4909	2450	2459
HN	10 up to 5 up	120	1977	1582	395
	5 down to 10 up	120	3773	456	3317
	5 down to 10 down	120	115	313	-198
	10 down to 5 up	120	1911	439	1472
FXM	1 up to 2 up	210	1257	539	718
	2 up to 1 down	180	775	274	501
	1 down to 2 down	180	354	325	29
	1 up to 2 down	180	——	210	——
GM	10 down to 1 up	300	61	966	-905
	10 up to 1 up	260	1148	1250	-102
	1 down to 10 up	260	962	667	295
	10 up to 1 down	300	2691	——	——
	Total connection time		26174	12521	11172

Minimizing *"Just Missed"*

The concept of "just missed" describes the situation where the connecting train is just leaving when the passengers come to the platform (Kang et al., 2014). This situation should be avoided if at all possible. If the connection time is less than the transfer walking time, a count of "just missed" is registered. Table 14 presents the improvement on "just missed" in the optimized results. It shows that the optimized schedule is effective in removing all just-missed.

Table 14.

Comparison of original timetable and optimized timetable in "Just Missed", parts of Beijing railway

Station	Transfer	Transfer walking time (s)	Connection time (s)		Just Missed	
			Original	Optimized	Original	Optimized
SYJ	13 up to 10 up	270	260	1111	1	0
XZM	4 down to 2 down	420	199	435	1	0

DD	1 down to 5 up	230	81	526	1	0
DD	1 up to 5 down	230	24	230	1	0
JGM	2 down to 1 up	240	126	240	1	0
GM	10 down to 1 up	300	61	966	1	0
HN	5 down to 10 down	120	115	313	1	0

The influence to subsequent trains

The service time and headway are two important indicators to illustrate the system performance in the timetable. The influence of first train timetable optimization for subsequent train operations in URN system can be evaluated by these two indicators.

(1) The service time

To ensure that the vehicle maintenance and equipment maintenance, scheduling should not change the length of non-service time. Table 15 reveals the service time of the actual operation timetable and the optimized timetable utilized the proposed model. There is almost no change in the length of service time by the proposed first train scheduling model. The rate of service time change is 0.11%, suggesting that the timetable has a minimal impact on the service time while making improvements in connection times and avoiding just-missed.

Table 15.

Comparison of operation time in Beijing railway network

Operation time	**Line 1**		**Line 2**		**Line 4**		**Line 5**	
	Up	**Down**	**Up**	**Down**	**Up**	**Down**	**Up**	**Down**
Original timetable	17:46	18:10	17:51	18:21	17:28	17:20	17:52	17:49
Optimized timetable	18:04	17:54	17:41	18:05	17:14	17:19	17:57	17:55
Operation time	**Line 8**		**Line 10**		**Line 13**		**Line BT**	
	Up	**Down**	**Up**	**Down**	**Up**	**Down**	**Up**	**Down**
Original timetable	17:08	17:02	17:55	17:17	17:42	18:45	16:53	18:03
Optimized timetable	17:25	16:53	18:20	16:54	17:31	18:44	16:55	18:05

(2) The mean headway and headway variance

We utilize headway distributions as an indicator to measure the impact of the first train departures on subsequent trains. Table 16 denotes the departure times of lines in actual operation of Beijing railway network, for the first six trains of the line.

Table 16.

The departure times of trains in actual operation of Beijing railway network

Line	**Direction**	**Original departure time**					
		1st train	**2nd train**	**3rd train**	**4th train**	**5th train**	**6th train**
Line 1	**Up**	5:09:00	5:14:50	5:22:50	5:29:20	5:35:20	5:41:20
	Down	5:05:00	5:10:30	5:13:30	5:17:30	5:20:00	5:25:30
Line 2	**Up**	5:09:00	5:16:06	5:24:06	5:31:36	5:38:06	5:44:36

	Down	5:03:00	5:14:12	5:22:12	5:28:12	5:36:12	5:44:12
Line 4	**Up**	5:10:00	5:25:00	5:35:00	5:43:00	5:51:00	5:59:00
	Down	5:00:00	5:15:00	5:25:00	5:35:00	5:42:16	5:49:32
Line 5	**Up**	5:19:00	5:25:00	5:31:00	5:37:00	5:42:00	5:47:00
	Down	4:59:00	5:04:00	5:09:00	5:14:00	5:19:00	5:24:00
Line 8	**Up**	6:02:00	6:10:30	6:18:00	6:25:30	6:33:00	6:40:30
	Down	5:18:00	5:26:30	5:34:00	5:41:30	5:49:00	5:56:30
Line 10	**Up**	5:39:00	5:46:32	5:55:36	6:04:40	6:13:44	6:22:48
	Down	4:53:00	4:57:32	5:02:04	5:06:36	5:11:08	5:15:08
Line 13	**Up**	5:00:00	5:08:00	5:15:30	5:22:30	5:28:27	5:36:30
	Down	5:00:00	5:10:00	5:19:00	5:30:15	5:31:00	5:42:45
Line BT	**Up**	5:49:00	5:57:00	6:03:00	6:09:00	6:15:00	6:21:00
	Down	5:19:00	5:26:00	5:32:00	5:38:00	5:44:00	5:50:00

Table 17 lists the optimized timetable which has the scheduled departure times of first trains is as proposed by the optimization model. It is worth mentioning that headways of subsequent trains are invariant. Comparison with the original timetable, the light typeface represent invariant departure times, and the bold typeface represents the optimized departure times.

Table 17.

Optimized departure times of trains in Beijing railway network. The ones in bold mark the new trains following the optimization.

Line	**Direction**	**Optimized departure time**					
		1st train	**2nd train**	**3rd train**	**4th train**	**5th train**	**6th train**
Line 1	**Up**	**4:53:00**	**5:01:00**	5:09:00	5:14:50	5:22:50	5:29:20
	Down	5:20:00	5:25:30	5:31:00	5:36:30	5:42:00	5:47:30
Line 2	**Up**	**5:18:00**	5:24:06	5:31:36	5:38:06	5:45:06	5:52:06
	Down	**5:19:00**	5:22:12	5:28:12	5:36:12	5:44:12	5:52:12
Line 4	**Up**	**5:23:00**	5:35:00	5:43:00	5:51:00	5:59:00	6:07:00
	Down	5:00:00	5:15:00	5:25:00	5:35:00	5:42:16	5:49:32
Line 5	**Up**	**5:14:00**	5:19:00	5:25:00	5:31:00	5:37:00	5:42:00
	Down	**4:52:00**	4:59:00	5:04:00	5:09:00	5:14:00	5:19:00
Line 8	**Up**	**5:44:00**	**5:52:00**	6:02:00	6:10:30	6:18:00	6:25:30
	Down	5:26:30	5:34:00	5:41:30	5:49:00	5:56:30	6:04:00
Line 10	**Up**	**5:13:00**	**5:28:00**	5:39:00	5:46:32	5:55:36	6:04:40
	Down	5:15:08	5:22:30	5:40:00	5:47:30	5:56:30	6:05:30
Line 13	**Up**	**5:10:00**	5:15:30	5:22:30	5:28:27	5:36:30	5:42:30
	Down	5:00:00	5:10:00	5:19:00	5:30:15	5:31:00	5:42:45
Line BT	**Up**	**5:46:00**	5:57:00	6:03:00	6:09:00	6:15:00	6:21:00
	Down	**5:16:00**	5:26:00	5:32:00	5:38:00	5:44:00	5:50:00

We use train working diagrams to further illustrate the changes of first train scheduling. There are three situations as illustrated in Figs. 15-17. The first situation is showed in Fig. 15. The x-coordinate indicates the departure times of trains along line 1, and the y-coordinate indicates the stations in line 1. The black thin lines denote the trains in the original timetable and the red heavy lines represent the adding trains in the optimized timetable. It can be seen that the first train departs earlier in the optimized timetable than the original timetable. There are two trains added in the optimized timetable and the trains after depart according to the original timetable.

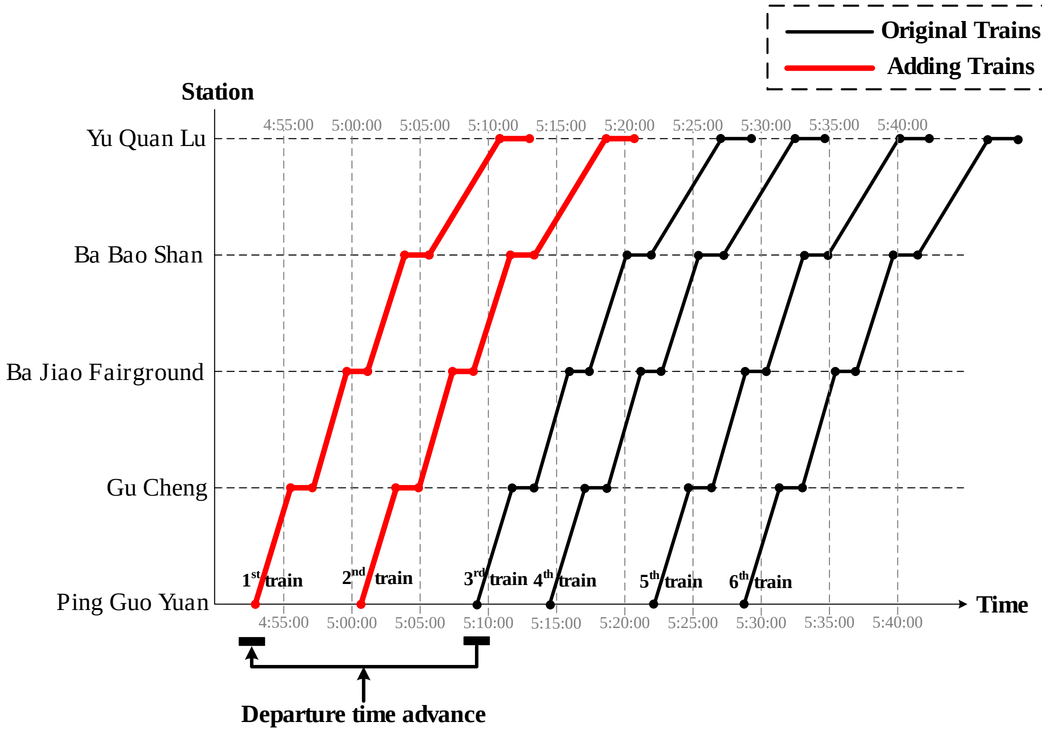

Fig. 15. Sketch train working diagram in line 1 up direction.

The second situation illustrated in Fig. 16, shows that the first four trains in the original timetable were cancelled, so the departure time of the first train is postponed. The optimized timetable suggests that it is not necessary to schedule these four trains, which can lead to significant cost savings.

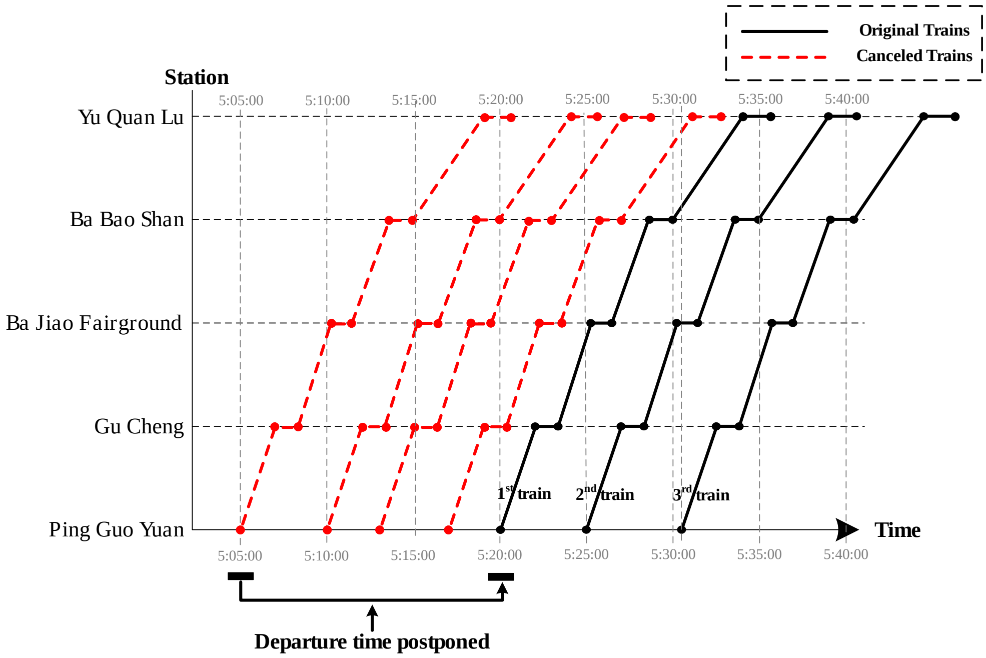

Fig. 16. Sketch train working diagram in line 1 down direction.

The third situation is shown in Fig. 17, where both adding trains and canceling trains happened in the optimized timetable compared to the original timetable. In this example, two trains are canceled and one train is added but at a later departure time, resulting in overall cost savings to the operators.

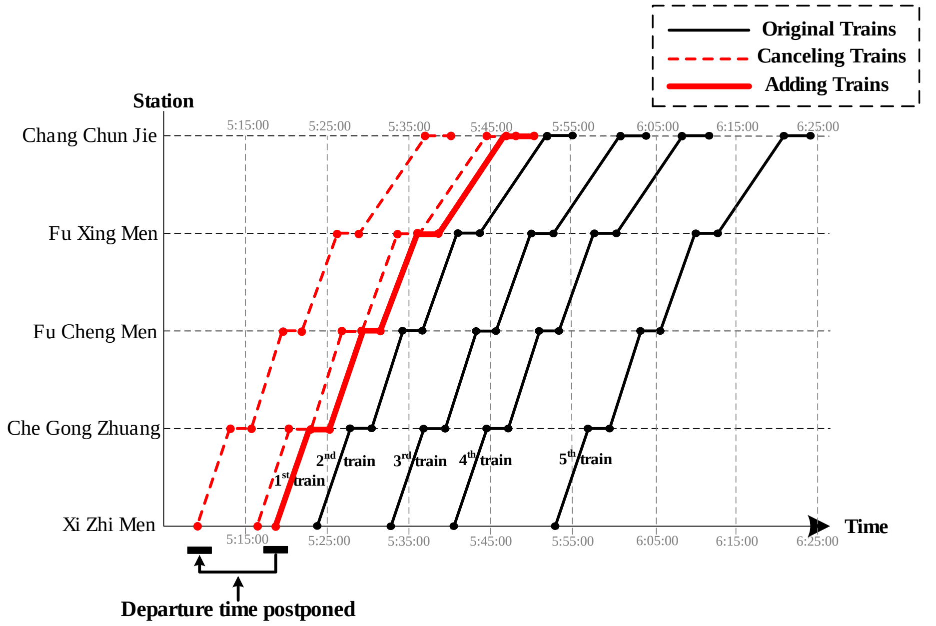

Fig. 17. Sketch train working diagram in line 2 up direction.

The mean and variance for the headway of the first six trains are presented in Table 18. The overall rate of change in the mean headway is 0.7% and in the headway variance is 1.2%, suggesting that the departure times of first trains optimized by the proposed model has minimal impact on subsequent trains.

Table 18.

Comparisons of headway between original timetable and optimized timetable

Line	Direction	Original timetable: The mean headway (hour)	Original timetable: Headway variance	Optimized timetable: The mean headway (hour)	Optimized timetable: Headway variance
Line 1	Up	0:06:28	0.049128	0:07:16	0.044759
	Down	0:04:06	0.046735	0:05:30	0.052004
Line 2	Up	0:07:07	0.049496	0:06:49	0.051953
	Down	0:08:14	0.048375	0:06:38	0.051643
Line 4	Up	0:09:48	0.051811	0:08:48	0.055046
	Down	0:09:54	0.04887	0:09:54	0.04887
Line 5	Up	0:05:36	0.051896	0:05:36	0.050172
	Down	0:05:00	0.045339	0:05:24	0.043664
Line 8	Up	0:07:42	0.067496	0:08:18	0.061574
	Down	0:07:42	0.052562	0:07:42	0.055092
Line 10	Up	0:08:46	0.059737	0:10:20	0.052914
	Down	0:04:26	0.043378	0:10:06	0.05301

Line 13	**Up**	0:07:18	0.046774	0:06:30	0.049262
	Down	0:08:33	0.04753	0:08:33	0.04753
Line BT	**Up**	0:06:24	0.062301	0:07:00	0.061929
	Down	0:06:12	0.052136	0:06:48	0.051797
Average		0:07:05	0.0512	0:07:34	0.0518

The influence to Transfer passengers

The mean and variance for the headway of the first six trains are presented in Table 18. The overall rate of change in the mean headway is 0.7% and in the headway variance is 1.2%, suggesting that the departure times of first trains optimized by the proposed model has minimal impact on subsequent trains.

4. Conclusion

The first train problem becomes an important issue with the expansion of urban railway networks. Passengers usually have to transfer to other line(s) to complete their journal within a URN. The coordination of first trains is important because extremely long connection time for first trains will lead to low network accessibility and discourage passengers from riding urban railway transit. On the other hand, the earlier the departure times for first trains, the higher operation cost to the URN. There are therefore trade-offs to be made between travelers who want make a good coordination between first trains so that they can transfer smoothly and operators who want to minimize operational costs.

In this paper, a timetable coordination optimization model of the first trains' departure time is proposed while minimizes the connection time based on the importance of transfer stations and lines in URN. The CPLEX Solver is combined with a practical method of SCM to solve this problem. To verify the effectiveness of the proposed model, a case study of Beijing railway network is performed. The result shows that the total transfer connection time is significantly reduced and the just-missed situations avoided.

For further research, we suggest that the extra travelling time should be considered as a non-deterministic factor in research of transfer optimization. The first train groups problem (This was a timetabling problem involving not only the first train, but consecutive trains in the morning period) that are compatible with passenger volume can be calculated by considering the transfer coordination. In addition, in real life operations, parameters are difficult to calibrate due to the complexities of the network structure and the line characteristics. More empirical work is obviously required. Finally, a tolerance level can be considered for train departure time to increase the robustness of the schedule, the tolerance level represents the bounds of first train groups' departure time to ensure a successful transfer, crucially, no redundancy trains.

Acknowledgments

This paper is partly supported by China National Funds for Distinguished Young Scientists (71525002), the National Basic Research Program of China (2012CB725400), NSFC (71322102), Research Foundation of State Key Laboratory of Rail Traffic Control and Safety (RCS2016ZT002), and the UK Rail Safety and Standards Board (Project RSSB-T1071). In addition, we would like to thank the editor and reviewers for their valuable and helpful comments and suggestions, which greatly improved this paper.

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