Long Range and Long Duration Underwater Localization using Molecular Messaging

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***Abstract*—In this paper, we tackle the problem of how to locate a single entity with an unknown location in a vast underwater search space. In under-water channels, traditional wave-based signals suffer from rapid distance- and time-dependent energy attenuation, leading to expensive and lengthy search missions. In view of this, we investigate two molecular messaging methods for location discovery: a Rosenbrock gradient ascent algorithm, and a chemical encoding messaging method. In absence of explicit diffusion channel knowledge and in presence of diffusion noise, the Rosenbrock method is adapted to account for the blind search process and allow the robot to recover in areas of zero gradient. The two chemical methods are found to offer attractive performance trade-offs in complexity and robustness. Compared to conventional acoustic signals, the chemical methods proposed offers significantly longer propagation distance (1000km) and longer signal persistence duration (months).**

***Index Terms*—under-water communications, molecular communications, localization, chemical noise**

I. Introduction

Terrestrial long-range wireless communication systems have operated successfully on land, offering a variety of broadcast and multi-cast services. Reliable wireless communication systems usually have knowledge of (i) the distance or location area of the receiver, and (ii) the channel for successful long term radio planning and real-time dynamic transmission adjustments. However, challenges remain in scenarios where the transmitters and receivers are separated by a long distance, have no knowledge of each others' location areas, and little knowledge is available about the propagation channel dynamics. This is especially the case in search and rescue services (e.g. for locating an underwater crashed object such as a submarine or an aircraft). Such a localization problem has two distinctive characteristics: (i) a hidden transmitter (the crashed object), and (ii) absence of receivers in the vicinity of the transmitter. We call this the hidden transmitter and absent receiver (HTAR) problem, as detailed in our previous paper [1]. The transmitter blindly broadcasts a distress signal, in the hope that receivers can detect it. Knowledge of either where the transmitter is, or presence of the receiver in the vicinity of the transmitter within a set time frame would solve the localization problem. The time frame constraint arises from the finite energy of the transmitter as well as other reasons.

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A. Review of Underwater Systems

In underwater environments, the propagation of wave based and molecular signals can be slow and hence both the distance- and time-dependent dimensions are considered. Current *black box* and other underwater communications utilize acoustic waves to transmit information in the form of 10ms sharp pulses on a 37.5 kHz carrier frequency. A typical battery supply can last up to 30 days with proposals for 90 days for future systems. The current receiver technologies (180dB and 1μPa) can reliably detect the signal at a range of 5km (normal conditions) and 7km (good conditions) [2]. The fundamental problem with all wave-based communications is that once the signal pulse is transmitted, the pulse's energy decays with propagation distance over time. There is both a finite distance (approx. 10 to 30km) and time (approx. a few seconds after the last transmission), beyond which the receiver cannot reliably receive the signal. Therefore, the absence of receivers in the reception zone during the short transmission time period will lead to the loss of transmitters' location. In order to solve this time-constrained HTAR problem, the transmitter must send messages that can persist for a long period and over long distances.

B. Review of Chemical Messaging

Originally, we proposed to encode the location of the transmitter inside the chemical composition of biomolecules [1], and allow the molecules to diffuse across oceans. This only then requires the receiver to chemically detect and decode the chemical structure to reveal the location of the transmitter. It was found that unlike acoustic communications, molecular communications provide a viable solution to solve the HTAR problem. Information molecules are able to diffuse long distances ($\sim$ 1000km) and achieve long endurance for detection ($\sim$ years). However, the bio-molecules suffer from high complexity in encoding and rapid biological degradation in sea water. This leads to a rapid decay in concentration and a shortened detection distance and signal persistence time. Hence, in this paper, we propose the chemical gradient localization as an alternative low complexity solution. Unlike the previous solution, the transmitter releases a non-biological chemical tracer, which contains no embedded information. A search robot is used to seek out the transmitter through a gradient descent approach.

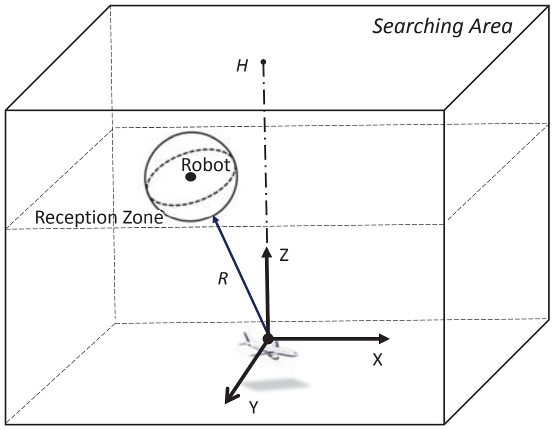

Fig. 1. Illustration of Underwater Diffusion Model.

C. Related Work and Contribution

Animals and insects use olfaction to trace the location of the odour source for foraging or reproductive activities. The problem of finding the source of the odour plumes is known as Chemical Plume Tracing (CPT). Odour plumes are created when odour molecules are released from their source and taken away by a combination of diffusion and random turbulent flow caused by temperature gradients. This combined process can be modeled on a macroscopic scale as diffusion with empirical diffusivity parameters that reflect the random turbulent flow [3], [4]. The basic approach for CPT is to calculate a concentration gradient with subsequent plume tracing based on gradient ascent. However, gradient-based algorithms are only feasible in environments where flow can be approximated by diffusion (low Reynolds numbers), resulting in a chemical concentration field that is reasonably well defined by a continuous function with a peak near the source [5], [6]. Existing researches on localization based on chemical gradients have largely considered a homogeneous diffusion environment where the diffusivity D is a constant and the search space is a plane [7], [8]. This assumption is valid for small volumes of search space or on a single plane (constant depth). However, the diffusivity will vary significantly in a vast ocean [3], creating potential **zones of zero gradient**. In this paper, we improve over previous under water gradient localization methods by proposing a multi-stage gradient algorithm that can recover from zones of zero gradient. A search robot with a chemical sensor is employed to search for the crashed object, similar to [9]–[11]. The advantages of this method over the chemical information carrier method in [1] are: (i) no longer necessary to embed the location information in the chemical, and (ii) possible to employ a simple non-organic tracing chemical that doesn't suffer from degradation in sea water.

The rest of the paper is organised as follows. In Section II, we define the diffusion process in the oceans, in particular we discuss the heterogeneous diffusivity characteristics and noise models. In Sections III, we present the proposed gradient localization method. In Section IV, we compare the performance between underwater communication systems and analyze the trade-off between detection probability and robustness.

II. System Model

A. Diffusion Channel

In this section, we present the underwater diffusion channel and the receiver sensitivity definition. We consider underwater diffusion in the context of oceans for the HTAR problem, e.g., an aircraft or a submarine has crashed into the ocean and sunk to a certain depth. The underwater diffusion propagation model is shown in Fig.1. We assume that the molecules used are of the same density as water and the vertical forces exerted to the molecules are entirely related to diffusion and ocean currents. The propagation process of molecules released at the origin can be modeled by solving Fick's laws of diffusion. In oceans, the rate of diffusion is **non-isotropic**. If the molecules are released at the time instant $t = 0$, the impulse response (hitting probability density function) ϕ at a given point (x, y, z) of a hemisphere is:

$$\phi(x, y, z, t) = \frac{2\exp\left(-\frac{x^2}{4D_x t} - \frac{y^2}{4D_y t} - \frac{z^2}{4D_z t}\right)}{(4\pi t)^{3/2}\sqrt{D_x D_y D_z}}, \quad (1)$$

where D_x, D_y and D_z are the diffusivities of x, y, z directions respectively. Since the diffusivity is non-isotropic in oceans, we consider specific ocean diffusivity values found in [3]. For a depth of 3-5km, the diffusivity is constant for the horizontal and vertical directions, i.e., $D_x = D_y$ and D_z are approximately constants.

We assume the crashed object has a transmitter that releases molecules continuously for a time period T at a constant magnitude M. We consider the input molecular signal $x(t)$ can be modelled as a rectangular pulse with magnitude M and pulse width T given as: $x(t) = M[u(t) - u(t-T)]$, where $u(t)$ is the Heaviside function. The channel output without noise can be calculated as the convolution of the input signal $x(t)$ and the channel response in Eq.1 given as

$$\begin{aligned} p(t) &= x(t) * \phi(x, y, z, t) \\ &= \frac{M}{2\pi DR}\left[\operatorname{erfc}\left(\frac{R}{2\sqrt{Dt}}\right) - \operatorname{erfc}\left(\frac{R}{2\sqrt{D(t-T)}}\right)\right], \end{aligned} \quad (2)$$

where D is the equivalent diffusion coefficient given as $D = \left(D_x D_y D_z\right)^{\frac{1}{3}}$, R is the equivalent molecular propagation distance given as $R = \frac{1}{D}(D_x D_y z^2 + D_x D_z y^2 + D_y D_z x^2)^{1/2}$ and erfc () is the complementary error function (see Appendix A).

B. Detection and Noise

The mobile robot's receiver will detect an instantaneous signal $p(t)$ given in Eq. 2, and exploit it for gradient based localization. Unlike most molecular signaling channels, the absence of channel knowledge and synchronization means that the peak signal is not relevant. In sensing chemical concentrations, the receiver's sensitivity is defined by the Limit of Detection (LOD) value. The LOD which is defined as the quantity of compound that gives a signal intensity that is a factor of 3 greater than the standard deviation of the background signal [12]. The unit of LOD is *parts-per notion* which is a set of pseudo units to describe small values of miscellaneous dimensionless quantities. For a given receiver sensitivity LOD threshold, we define the *Arrival Time for Detection* as the total time which the molecule needs to diffuse in the environment until the molecule concentration exceeds the threshold.

In terms of noise, existing communication research has focused on counting noise [13], which is approximately Gaussian distributed. However, when we consider molecule motion in an ocean, we are more interested in the dominant LOD and the background chemical noise. It has been shown that the background chemical noise is Gaussian distributed [14] $\sim \mathcal{N}\left(0, (\frac{\text{LOD}}{3})^2\right)$. By transferring the units in parts-per notation [ppq] to concentration [molecules/m^3] [1], the calculation of time-varying instantaneous SNR is given as:

$$\text{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}} = \frac{|p(t)|^2}{\sigma^2} = \frac{9|p(t)|^2}{\text{LOD}^2}. \quad (3)$$

III. Chemical Gradient Localization

Inspired by animal's method of locating objects (e.g., prey) using smell, we propose a search-and-rescue robot that homes in on the chemical emitted by a crashed object. Existing localization methods using chemical concentration gradients have largely considered a homogeneous diffusion environment where the diffusivity D is a constant in all directions. This assumption is valid for small volumes of search space or on a single plane but not for a vast ocean where the diffusivity in different directions will vary significantly [3] creating potential zones of **near zero gradient**. For example, in Fig. 2, an emitter located at $[0, 0, 0]$ emits a rectangular pulse with magnitude M of chemicals and after a certain time the diffusion varies at various depths and distances. The rate of diffusion on a plane is much faster than across planes [3], and hence the robot may find a viable gradient at a deeper depth (smaller z value) but not enough gradient at lower regions. In this paper, we improve over previous under water gradient localization methods by proposing a multi-stage gradient algorithm that can recover from zones of zero gradient.

A. Multi-Stage Gradient Algorithm

To operate in a non-isotropic diffusion environment and without an explicit function of the gradient, we propose a Rosenbrock gradient based search method [16] as detailed in Algorithm 1 and described below as a two-stage process. Each stage is a general process that comprises of multiple robot steps.

[1] We assume the molecules has a similar molar mass of 200 in [15]. Thus the transferring is given as LOD $= 1\text{ppq} = 10^{-3}\text{ppt} = 10^{-3} \times 1\text{ng/L} = 10^{-12}\text{g/L} = \frac{10^{-12}}{200} \times 6 \times 10^{23} \times 10^3\text{molecules/m}^3 = 3 \times 10^{12}\text{molecules/m}^3$

- **1. blind search stage:** In this initialization stage we have to find a feasible searching area where there is sufficient chemical gradient to initiate the correct start direction. The robot begins at position $\boldsymbol{P}_0$.
- **2. repetitive search stage:** the robot searches in the sea according to the Rosenbrock method. The robot waits a certain time period (stays stationary) before it re-calculates a new search direction vector. If the new search vector is in agreement with previous vector, the robot increases the waiting period by α-fold. If not, it increases the waiting period by β-fold. The starting waiting period is given by $\boldsymbol{e}$. Each step will remember the location of previous calculation $\boldsymbol{P}_s$ and update it with the new location $\boldsymbol{P}_e$. Once the Rosenbrock searching process stops: 1) it stops in the vicinity of the transmitter and the robot surfaces to report its location and the whole searching process terminates successfully; or 2) it is still far away but can not find a gradient to action upon. In this case, the robot stops and we set it to a new start point $(x_n, y_n, z_n + h)$ and start another Rosenbrock searching process with the blind search stage detailed above. We repeat this step until the robot reaches the crashed object.

The reason we use the Rosenbrock algorithm is that, in the considered scenario the location of the transmitter is unknown so that we cannot obtain the analytic function of the molecular concentration varying along the locations. Therefore, although we have formulated the target location problem as an optimization problem, we cannot obtain the analytic objective function. Other gradient based schemes e.g. the conjugate gradient algorithm and the Newton algorithm which are all premised on the analytic gradient function, cannot be applied in our scenario. Therefore, we have to use the numerical search scheme to obtain the optimum point on the objective function. Rosenbrock algorithm is specially designed for such a realistic problem. For example, in our simulation, the robot will obtain various molecular concentration values from the hidden objective function during both blind search step and repeat search step. Based on the receipted concentration values, the proposed Rosenbrock search algorithm can construct their approximate gradient so that to trace the hidden object.

Fig. 2 shows an example of the robot movement path in finding the transmitter. Each numbered point represents the algorithm re-calculating a new movement vector. At point 0, the algorithm performs blind search to find the correct initial vector and travels along it for a period of $\boldsymbol{e}$. Then it reaches point 1, whereby the algorithm has verified that the new direction agrees with the previous one and increases the travel duration

Algorithm 1 Rosenbrock gradient search algorithm.

function SEARCHINGTRANSMITTER($\boldsymbol{P}_0$)

  Initialise $\alpha \leftarrow 2$; $\beta \leftarrow -0.5$; $\boldsymbol{e} \leftarrow \mathbf{1}$;

  $\boldsymbol{\xi}_1 \leftarrow \begin{bmatrix}1\\0\\0\end{bmatrix}$; $\boldsymbol{\xi}_2 \leftarrow \begin{bmatrix}0\\1\\0\end{bmatrix}$; $\boldsymbol{\xi}_3 \leftarrow \begin{bmatrix}0\\0\\1\end{bmatrix}$;

  Blind Search $\boldsymbol{\xi}_s \leftarrow$ INITIALDIRECTION($\boldsymbol{P}_0$);

  $\boldsymbol{P}_s \leftarrow \boldsymbol{P}_0$;

  repeat ▷ Start searching

    $\boldsymbol{P}_e \leftarrow$ ROSENBROCKSEARCHING($\boldsymbol{P}_s$, α, β, $\boldsymbol{e}$, $\boldsymbol{\xi}_s$);

    $\boldsymbol{P}_s \leftarrow \boldsymbol{P}_e + \begin{bmatrix}0 & 0 & h\end{bmatrix}$;

    $\boldsymbol{\xi}_s \leftarrow \begin{bmatrix}\boldsymbol{\xi}_1 & \boldsymbol{\xi}_2 & -\boldsymbol{\xi}_3\end{bmatrix}$;

  until Receiver robot stops in vicinity of the transmitter, i.e., $\boldsymbol{P}_e \in \mathcal{P}$

  return $\boldsymbol{P}_e$;

end function

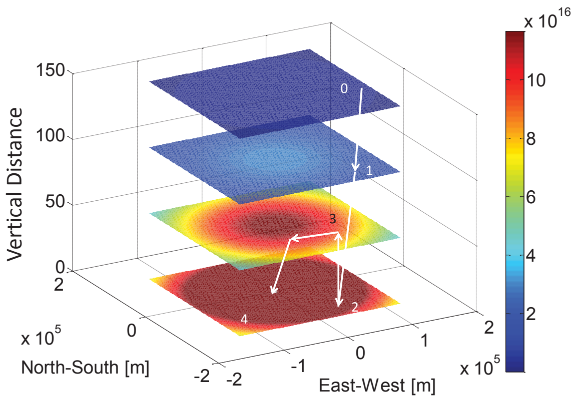

Fig. 2. Plot of chemical concentration (molecules per m^3) at various distances away from source for $t = 347$days

by a factor of α. Upon reaching point 2, the algorithm discovers that no viable gradient can be found and the algorithm performs a reset. At the reset, it surfaces $h = 50$m upwards to find a gradient (slower rate of diffusion) at point 3, whereby it proceeds to find the crash object at point 4. The reason why the robot moves upwards is because the horizontal planar diffusion is less progressive (likelihood of sharper gradient) at lower depths of the ocean. The *number of search iterations* is defined as the number of distinctive stages it took for the robot to find the transmitter (4 in the case of Fig. 2). A *repeat stage* is defined as the robot losing the gradient and repeating the first part of the algorithm to regain the gradient (1 occurrence in the case of Fig. 2). The *total trace distance* is defined as the total distance travelled

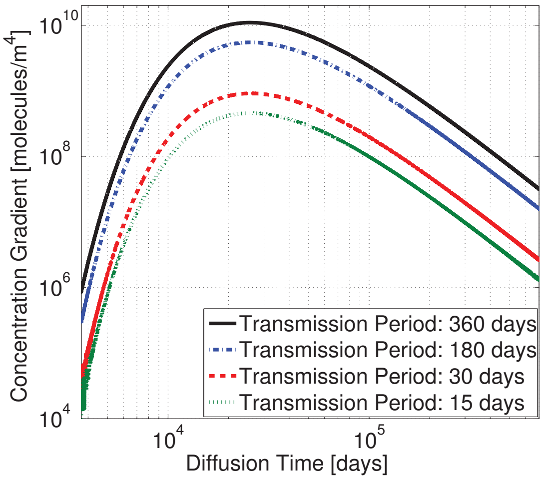

Fig. 3. Plot of chemical concentration gradient (molecules per m^4) for various transmission durations at the transmitter. The receiver is at x =100km, y =100km, z =1000m away from the transmitter.

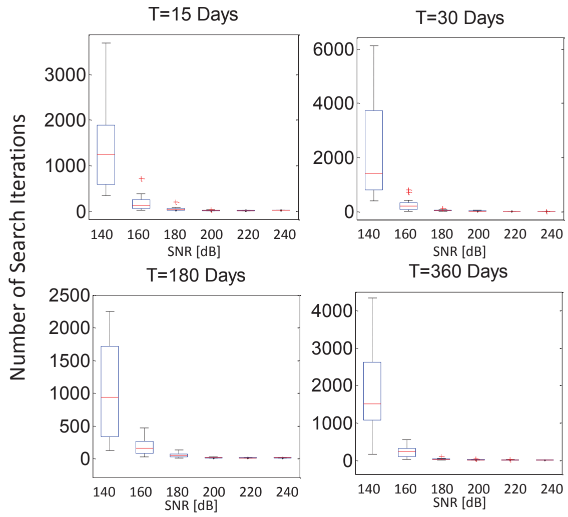

Fig. 4. Plot of search iterations as a function SNR. The receiver starts at x =100km, y =100km, z =1000m away from the transmitter.

by the search robot (including any repeat stages).

B. Performance Results

We now evaluate the algorithm's robustness in three areas: (i) the effect that the transmission duration has on the concentration gradient, (ii) the amount of ambient chemical noise in the ocean, and (iii) the starting location of the robot. The parameters used in the analysis and numerical simulations are found in Table.I.

1) Robust Concentration Gradient: In the first part, we show that unlike wave-based communications, the gradient of molecular concentration is not strongly affected by the transmission period. In acoustic wave communications, the power of the acoustic signal will decay to below noise level after less than 1 minute of propagation. As shown in Fig. 3, not only will there be a significant gradient ($\frac{\partial p(t)}{\partial R}$ [molecules per m^4]) after several years, but also the shape of the gradient doesn't vary between a transmission period of T =15 days, and T=360 days.

2) SNR Threshold (based on LOD): In Section II, we defined the macroscopic ocean noise as a function of the LOD, and the SNR is defined in Eq. (3). The main effect of noise is to cause false gradients and causing incorrect search direction decisions. Fig. 4 shows the simulation for total number of search iterations as a function of the SNR. Monte-Carlo simulations were performed for each SNR value ranging from 140dB to 240dB, for different transmission durations T. It is found that if the SNR falls below a threshold value of 140dB, the number of search iterations required grows rapidly. For an SNR value of 200 or over, the algorithm is robust enough to always find the hidden object with approximately 24 search iterations. The results also reinforce the earlier claim that increasing the transmission duration doesn't significantly affect the number of search iterations' sensitivity to noise.

TABLE I
THE DEFINITION OF PARAMETERS WITH THE SIMULATION VALUES IN THE MODEL

Parameters	Values
Transmitted Molecule Magnitude (M)	1 [mol/s]
Transmission Period (T)	30 [days]
Sea Depth (H)	5 [km]
Reception Zone Radius (r)	1 [m]
Limit of Detection (LOD)	1 [ppq]
Depth Adjustment (h)	50 [m]
Horizontal Distance (x, y)	$0 - 1000$ [km]
Vertical Distance (z)	$0 - 2000$ [m]
Horizontal Diffusivity (D_x, D_y)	250 [m^2/s]
Vertical Diffusivity (D_z)	4.5×10^{-5} [m^2/s]

3) Starting Location: In terms of where the robot initially starts, we consider how the number of search iterations vary in according to the horizontal distance x, y and the vertical distance z. In Fig. 5(a) and (b), we show the total number of search iterations (steps) for different (a) horizontal distance (where vertical distance fixed at 1000m) and (b) vertical distance range (where horizontal distance fixed at 100km). The results show

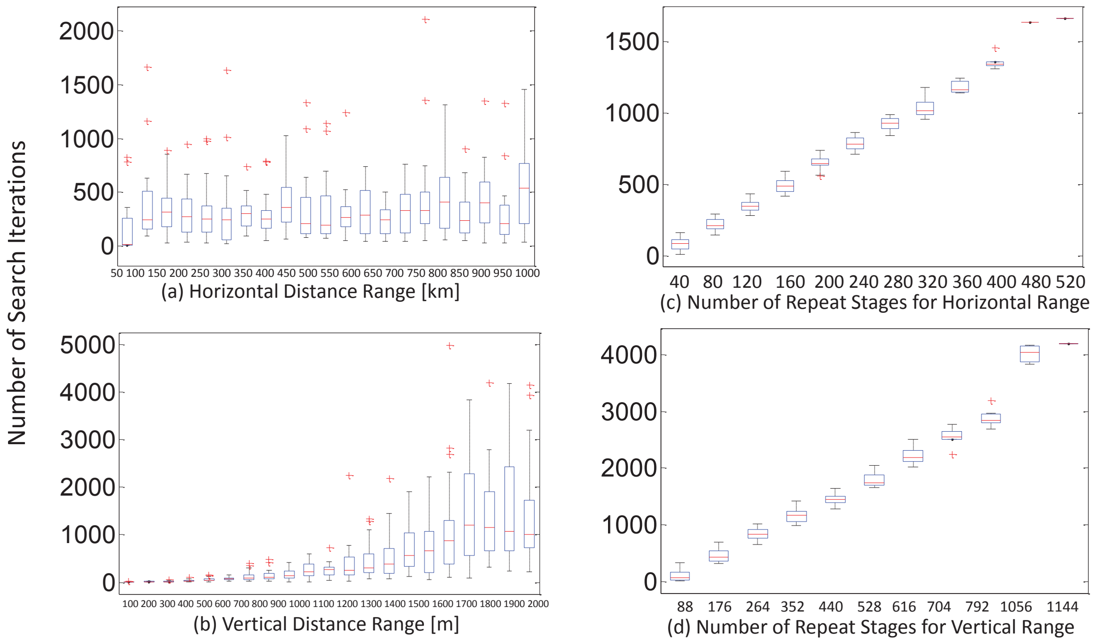

Fig. 5. Box plot of number of search iterations as a function of the: (a) horizontal distance, (b) vertical distance. Box plot of search iterations as a function of the number of repeat stages for: (c) horizontal distance range, (d) vertical distance range.

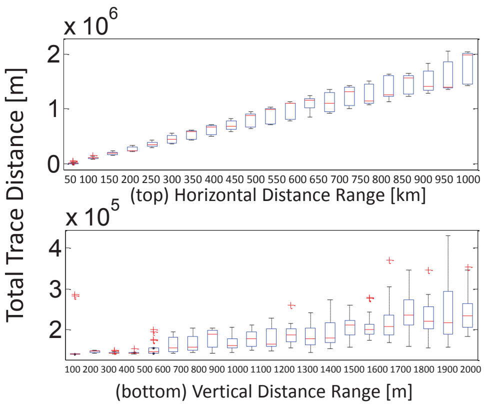

Fig. 6. Boxplot of total trace distance travelled by the robot as a function of (top) horizontal distance range and (bottom) vertical distance range.

that the number of search iterations is largely uncorrelated with the horizontal distance, but rises significantly for a vertical distance of over 1300m. In Fig. 5(c) and (d), we show the number of search iterations (steps) for different number of repeat stages (as a result of different horizontal and vertical distances). The results show that the search iterations are positively correlated with the repeat stages at both distances. A repeat stage can significantly increase the search distance of the robot, and we show this next. In Fig. 6, we show the total trace distance as a function of the horizontal and vertical distance. The results show that the total trace distance is positively correlated with both the vertical and horizontal distance. In terms of the outliers, they are more exasperated for the vertical distance due to the existence of a high number of repeat stages described above.

IV. Performance Comparison

This section compares the performance of the chemical gradient localization method proposed in this paper with chemical information carrier method in [1] and conventional communication systems (i.e., acoustic and optical). In particular, we focus on the energy attenuation and latency performance metrics.

A. Molecular Communication

1) Chemical Gradient Localization (CGL): The proposed method in this paper is based on a gradient ascend localization method using the chemical gradient in oceans. The received molecular energy can be considered as the total number of molecules accumulated over time. We consider a narrow pulse transmission and an infinite

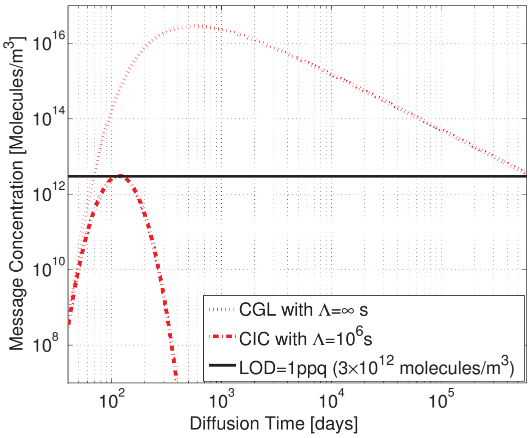

Fig. 7. Plot of chemical concentration for: (1) chemical gradient localization (CGL) method with receiver LOD and (2) chemical information carrier (CIC) method with molecular decay rates.

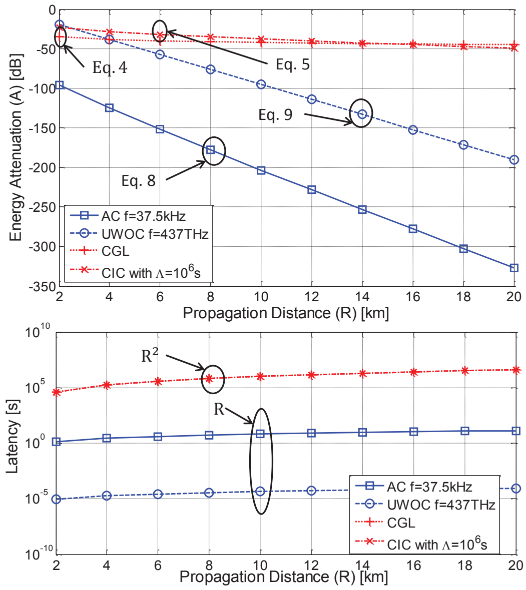

Fig. 8. Plot of energy attenuation as function of propagation distance (top) and plot of latency as function of propagation distance (bottom) for acoustic, wireless optical, chemical gradient localization and chemical information carrier

time reception. The energy at a CGL receiver can be expressed by integrating Eq.1 over time:

$$E_{\text{CGL}} = \int_0^\infty \phi(R,t)\mathrm{d}t = \frac{M}{2\pi DR} \tag{4}$$

Therefore, using Eq. 4, the energy attenuation for chemical gradient localization is $\propto \frac{1}{R}$, and the time delay to peak amplitude is $\propto R^2$ [1].

2) Chemical Information Carriers (CIC): In the chemical information carrier method [1], biological components (peptides and N-linked glycan), which are employed as sufficiently complex chemical information carriers, will be detected as food by the bacteria in the oceans. Thus, the information components can be damaged during the propagation in the ocean. Accordingly, we consider that CIC molecules have a life expectancy with a *molecule degradation*, which can be modelled as an exponential distribution [17], and the concentration with degradation $\phi(t)$ is modeled as $\phi(t) = \phi_0 \exp(-\lambda t)$, where: $\lambda = \frac{\ln(2)}{\Lambda_{1/2}}$. ϕ_0 is the non-degraded concentration found in Eq. (1), λ is the rate of degradation, and $\Lambda_{1/2}$ is the corresponding half-life of the message molecule. Therefore, the energy of CIC is given as

$$\begin{aligned} E_{\text{CIC}} &= \int_0^\infty \phi(R,t)\exp(-\lambda t)\,\mathrm{d}t \\ &= \frac{M}{4\pi DR}\left[1.84\exp(-R\sqrt{\frac{\lambda}{D}}) + 0.16\exp(R\sqrt{\frac{\lambda}{D}})\right]. \end{aligned} \tag{5}$$

The time delay to peak amplitude delay is the same as CGL.

Fig. 7 shows the results of chemical concentration as a function of diffusion time for the two methods under consideration at an equivalent distance $R = 20$km. It is found that the concentration of CIC is very sensitive to molecule half-life time $\Lambda_{1/2}$, but the receiver will immediately know the location after decoding the chemical message. On the other hand, CGL does not suffer from molecular degradation because no information needs to be encoded in bio-molecules. The detection period is sufficient long (above LOD threshold), but the search robot has to wait a long time before the concentration reaches above receiver sensitivity, and wait a further longer time before the robot can locate the transmitter after many search iterations. In summary, CIC provide the exact location information for the receiver so that once one information carrier is captured and decoded, the location can be found. However, the complexities of assembling the carriers and the carrier life expectancy need to be carefully considered. CGL, on the other hand, doesn't suffer from the complexity and degradation issues, but is sensitive to the noise in the ocean environment and the need for waiting a longer period in order to initialize the search start.

B. Conventional Communication

In Fig. 8, we show the energy attenuation and latency as a function of propagation distance for both molecular and conventional systems (see Appendix B). Fig. 8 (top) show that conventional systems suffer faster energy attenuation than molecular communications. This means rescue nodes are far more likely to detect molecular messages at long distances than either acoustic or optic signals. Fig. 8 (bottom) shows the latency (the peak pulse's arrival time) increases quadratically for molecular communications, whereas acoustic and optical waves' latency increase linearly. This means the nodes that expect molecular signals need to wait significantly longer.

We summarize the difference between the 4 underwater communication metrics as follows: optical wireless has a high data rate (Mbits/s), but the free-space transmission distance is strictly limited to a few metres underwater. Acoustic systems are widely used for search and rescue nowadays due to its low complexity and reasonable detection distance (10-30km). However, when the search space is large and the search duration is prolonged, acoustic systems can not reliably allow the rescuers to find the hidden crashed object (as it is in the MH370 case). The persistence time of the signal after transmission is a few seconds, indicating that the search time is limited by the battery life of the system. Compared to the aforementioned conventional systems, both of the chemical methods have the advantages of higher signal persistence time (months to years) and longer propagation distance (1000km or more).

V. Conclusion

In this paper, we tackle an underwater rescue problem of trying to locate a single hidden object. Current state-of-the-art acoustic systems can be received up to 30km away and each signal pulse persists for only a few seconds to a minute after transmission. Typical plane crash search radius can be up to 500km and the search duration can take from several months to years. The rapid energy attenuation of acoustic waves in underwater channels leads to expensive and lengthy long range search and recovery missions.

Therefore, we are motivated to propose a chemical based signalling method. Previously, we proposed to embedd the location information inside the chemical composition of bio-molecules. These suffered rapid degradation in the ocean as a food source for bacteria. As a result, an alternative method is proposed in this paper, whereby no information is embedded in the chemical molecules. Instead, the molecules serve as a chemical concentration gradient to aid a robot to find the transmitter through gradient ascent. An adapted Rosenbrock algorithm is implemented to achieve the blind search process. A reset element is employed to account for the heterogeneous diffusivity characteristic of oceans, which can cause regions of zero gradient. The key discovery is that the transmission duration is not important, but the Limit of Detection (LOD) is important for minimizing the search duration. This means one would want to design a molecular transmitter that can transmit high concentration for a shorter time interval..

In comparing the two methods, it is found that: the previously proposed chemical messaging method is more complex, but is reliable against diffusion channel variations in the ocean channel; whereas the gradient ascent method proposed in this paper has a low implementation complexity but is sensitive to the the Limit of Detection and chemical noise. In summary, both chemical messaging methods offer superior performances in search distance and duration when compared against conventional acoustic systems.

Appendix

A. Channel Step Response

We consider the Laplace transform of the channel impulse response $\phi(R, D, t)$ is:

$$\mathcal{L}_t\left[\frac{2}{(4\pi Dt)^{3/2}}\exp\left(-\frac{R^2}{4Dt}\right)\right] = \frac{e^{-R\sqrt{\frac{s}{D}}}}{2\pi DR}. \quad (6)$$

Therefore, a step response with delay τ is an inverse Laplace transform of $\exp(-\tau s)/s \times$ Eq. 6:

$$S\left(R, t, \tau\right) = \frac{M}{2\pi DR}\mathrm{erfc}\left(\frac{R}{2\sqrt{D(t-\tau)}}\right). \quad (7)$$

B. Conventional System Propagation in Under-Water Channel

1) Acoustic Communication (AC): In an underwater acoustic channel over a propagation distance R [km], the propagation channel's energy attenuation A [dB] is statistically characterised by [18]:

$$A_{\mathrm{AC}}(R, f_a) = k10\log_{10}(R) + R(a(f_a)) + 10\log_{10}(A_0), \quad (8)$$

where A_0 is a constant, and k is an acoustic spreading factor (typically 2). The function $a(f_a)$ characterizes the absorption coefficient which is a function of frequency f_a [kHz]. The time delay to peak amplitude's arrival is $\propto R$.

2) Underwater Wireless Optical Communication (UWOC): In an underwater wireless optical channel over a propagation distance R [km], the attenuation of the light beam in water can be quantified with beam light attenuation coefficient $c(v/f_o)$ in [19]:

$$A_{\text{UWOC}}(f_o, R) = \exp\left(-c\left(\frac{v}{f_o}\right)R\right), \qquad (9)$$

where f_o is the frequency of the light beam, v is the speed of light in sea water. The time delay to peak amplitude's arrival is $\propto R$.

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