Essays in International Monetary Economics

A thesis submitted to the University of Warwick in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Economics

Lucio D'Aguanno

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Declaration

I certify that the thesis I have presented for examination is solely my own work and has not been submitted for a degree at another university.

Summary

This dissertation presents two essays in international monetary economics; the unifying theme is the international dimension of monetary policy. I investigate two issues related to the openness of the economy: (i) the implications of external positions for the conduct of macroeconomic stabilisation policy; (ii) the consequences of monetary unification for social welfare under incomplete international markets. The former subject occupies chapter one; the latter occupies chapter two.

In the first chapter, "Monetary policy and wealth effects with external positions", I develop a two-country DSGE model to study how financial integration affects the international transmission of shocks and the conduct of monetary policy. If the households of each country receive dividends from foreign firms, macroeconomic disturbances are followed by international wealth effects that transfer consumption across countries. The direction of these effects varies across different types of shocks, as these imply different comovements of macroeconomic variables. As a consequence, the choice of the monetary policy mix is shown to rest on the relative importance of different sources of uncertainty.

In the second chapter, "Monetary policy and welfare in a currency union", I explore the welfare cost of abandoning an independent monetary policy to join a currency union, and I investigate what trade gains can outweigh this loss. The consequences of subjecting distinct economies to a single monetary authority are investigated in the context of an open-economy DSGE model with country-specific macroeconomic shocks and incomplete international markets. The dependence of the cost of adopting a single currency on the international synchronisation of business cycles is examined first. Next, the welfare implications of international price misalignments and monetary barriers to trade with separate currencies are considered. Finally, the model is estimated with data from Italy, France, Germany and Spain using standard Bayesian tools. Moderate trade frictions are found to be sufficient for a monetary union to guarantee the same welfare as a regime with national currencies. Under a calibration of these frictions in line with the literature, monetary unification is found to offer positive net welfare gains to all these economies.

Abbreviations

CES: Constant Elasticity of Substitution

CPI: Consumer Price Index

CRRA: Constant Relative Risk Aversion

DSGE: Dynamic Stochastic General Equilibrium

EMU: European Monetary Union

FOC: First Order Condition

GDP: Gross Domestic Product

HPD: Highest Posterior Density

IRF: Impulse Response Function

LCP: Local Currency Pricing

MCMC: Markov Chain Monte Carlo

MH: Metropolis-Hastings

NKM: New Keynesian Model

PCP: Producer Currency Pricing

PPI: Producer Price Index

Chapter 1

Monetary Policy and Wealth Effects with External Positions

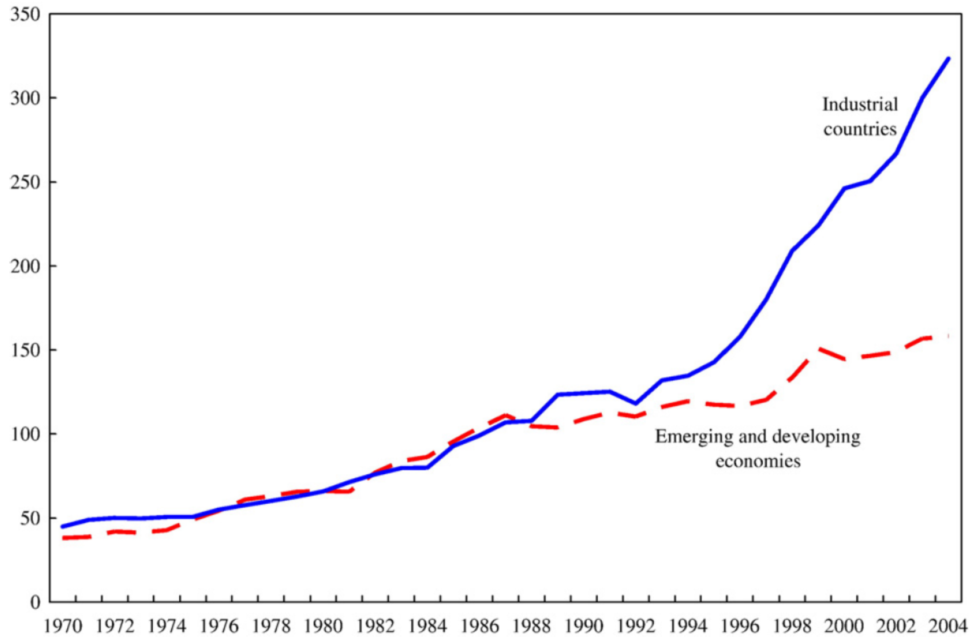
1.1 Introduction

In a series of influential papers, Lane and Milesi-Ferretti (2001, 2006, 2007) collected a large database with estimates of the international investment positions held by over 150 countries, with the purpose of tracking their evolution over the last few decades. They documented a dramatic upsurge of gross cross-border asset and liability holdings since 1970, and interpreted this as a measure of the increase in international financial integration.

Figure 1.1 overleaf is from Lane and Milesi-Ferretti (2007). It shows that the sum of foreign assets and liabilities to gross domestic product (GDP) grew sevenfold for developed economies and threefold for emerging and developing economies over three decades. The authors argued that these external portfolios are large enough for significant international reallocations of wealth to occur when asset prices and exchange rates move. The question then arises as to whether and how this changes the kind of stabilisation policy that monetary authorities should follow.

External positions are not explicitly modelled in the early literature on monetary policy in open economy. Clarida, Galí, and Gertler (2002) and Galí and Monacelli (2005), among others, assume complete international markets for state-contingent securities and exclude the possibility that countries hold claims on each other's output or profits. They conclude that a regime of strict domestic inflation targeting is optimal for both closed and open economies. This "isomorphism" result is rejected in subsequent works by Benigno and Benigno (2003) and Devereux and Engel (2003), which adopt more general preference specifications and different assumptions about pricing; these studies, however, maintain the absence of financial integration.

Figure 1.1: Ratio of sum of external assets and liabilities to GDP for different country groups



Tille (2008) and Devereux and Sutherland (2008) are among the first attempts to introduce external positions into an open-economy dynamic stochastic general equilibrium (DSGE) setting[1]. The former work uses a two-country model with exogenous portfolios of equity and bonds to investigate how alternative cross-country holdings of assets affect international interdependence; it shows that the exact composition of portfolios matters for valuation effects and for the welfare implications of exchange rate movements. The latter work develops a two-country model with endogenous portfolio choice, and explores how its interaction with monetary policy alters the conditions under which the central bank operates; it shows that price stability enhances the risk-sharing properties of nominal bonds against country-specific technology shocks. Neither work, however, discusses monetary policy tradeoffs. Tille (2008) focuses on how international assets and liabilities affect the cross-border transmission of monetary shocks, but does not evaluate alternative monetary policies. Devereux and Sutherland (2008) does study monetary policy explicitly, but it considers an economy where the central bank faces no short-run tradeoff, because productivity and interest-rate shocks are the only sources of uncertainty.

[1] Both models are solved using linear methods.

The objective of the present work is to investigate how the international transmission of macroeconomic shocks is affected by the presence of external positions, with the goal of clarifying how this alters the objective of monetary policy. To this end, I cast the analysis in a two-country DSGE model with monopolistic competition, nominal rigidities and incomplete international asset markets, which I solve using nonlinear methods. Unlike Devereux and Sutherland (2008), I do not study how monetary policy affects the risk-sharing properties of nominally riskless assets; in fact, I work in a Heathcote and Perri (2002) type environment with no international trade in bonds at all. Differently from Tille (2008), I study a cashless economy with traded goods only, and I do not formally focus on nominal exchange rate movements; on the contrary, I assume for simplicity that the two countries use the same currency[2]. In contrast with both Devereux and Sutherland (2008) and Tille (2008), I consider an economy where "cost-push shocks" determine an explicit tradeoff between the different monetary policy goals of inflation and output stability.

I model external positions and their impact on macroeconomic interdependence as follows. Households own exogenously given claims on the profits of domestic and foreign firms, so they are entitled to get some dividend payments from abroad. Exogenous disturbances that affect the profitability of firms cause international shifts of income because they create differentials between the dividends that a country pays to and earns from the rest of the world. As dividends are received in a lump-sum fashion, these spillovers determine wealth effects on households' labour supply that have implications for the dynamics of output, employment and consumption.

I show that the strength of the wealth effects depends on the dimension of external positions, while their direction is tied to the dynamics of firms' profits and depends on the nature of the shocks that are causing the business cycle. More precisely, shocks to productivity and wage markups determine countercyclical wealth effects. As output and profits comove positively under these kinds of disturbances, households receive net dividend payments from abroad when domestic output is low, and make net dividend payments when output is high; this stabilises consumption relative to economic activity. On the contrary, shocks to interest rates and price markups determine procyclical wealth effects. In these cases, output and profits comove negatively, so households are net receivers of dividends when domestic output is high, and net payers when output is low; this exerts a destabilising role on consumption.

[2]Distinct monetary policies can still exist without violating the "impossible trinity" because there is no capital mobility between the two economies.

Whether external positions mitigate or exacerbate the volatility of consumption matters for monetary policy. The output stabilisation objective tends to take higher priority when the volatility of output is large relative to that of consumption, i.e. when wealth effects are countercyclical; a policy of "flexible" inflation targeting is appropriate in this case. Conversely, the stabilisation of output becomes less important when the volatility of output is small relative to that of consumption, i.e. when wealth effects are procyclical; a policy of "strict" inflation targeting tends to emerge as the most desirable policy in this case. This result suggests that external positions can alter the balance between inflation and output stabilisation in either direction, depending on the prevalent type of disturbances in the economy.

The rest of this chapter is organised as follows. Section 2 presents the model with frictionless labour markets and Section 3 solves for its equilibrium. Section 4 discusses international wealth effects and macroeconomic adjustment in the presence of external positions. Section 5 defines my welfare criterion and discusses monetary policy tradeoffs. Section 6 introduces nominal wage rigidities and discusses the effects of wage markup shocks. Section 7 concludes.

1.2 External positions in a two-country model

I consider a two-country DSGE model with incomplete international markets and country-specific goods. Each country is populated by a continuum of measure one of infinitely lived households who get utility from consuming (both domestic and foreign) tradable goods, and disutility from working. Households fully share risk within each country by exchanging a full set of contingent assets, so that attention can be limited to representative agents to save on notation.

International financial markets are incomplete à la Heathcote and Perri (2002): no private asset is available for trade between the countries. Households have access to riskless one-period nominal bonds which cannot be traded across borders; their prices are controlled by the local central banks. The model abstracts from different currencies: the prices of all goods are expressed in the same unit of account. As there is no international capital mobility, countries keep independent monetary policies.

Production in each country takes place in two stages. First, monopolistically competitive producers hire labour and supply a continuum of measure one of intermediate goods, indexed by i; these goods are not traded internationally. Second, perfectly competitive producers adopt a constant elasticity of substitution (CES) technology to aggregate domestically produced intermediates into final consumption goods, denoted $y_{h,t}$ and $y_{f,t}$ respectively; these are freely traded.

Each household owns claims to exogenously fixed[3] portions of the profits of both domestic and foreign firms. These claims represent the external positions in the present setup; as we shall see, the associated dividend payments are an important source of international spillovers of macroeconomic disturbances. The asset structure of the economy under scrutiny emphasises this channel; since it is impossible for households to borrow and lend after uncertainty is realised, the impact of unexpected movements in dividend payments on consumption and employment is exacerbated. Although empirical evidence of internationally integrated markets for debt instruments[4] lends interest to the study of economies where bonds are tradable across borders, this mechanism would be blunted in such environments because households could offset surpluses and shortfalls of dividend earnings to some extent by resorting to intertemporal trade.

1.2.1 Households

Intertemporal problem: utility maximisation Households choose consumption, hours and bonds purchases to maximise their total lifetime utility over an infinite horizon. Their intertemporal problems read

$$\max_{\{c_t, b_{t+1}, n_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t\right)$$

$$\text{s.t. } p_t c_t + q_t b_{t+1} = b_t + w_t n_t + s_h p_{h,t} \Pi_{h,t} + s_f p_{f,t} \Pi_{f,t} + t_t$$

and

$$\max_{\{c_t^*, b_{t+1}^*, n_t^*\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^*, n_t^*\right)$$

$$\text{s.t. } p_t^* c_t^* + q_t^* b_{t+1}^* = b_t^* + w_t^* n_t^* + \left(1 - s_h\right) p_{h,t} \Pi_{h,t} + \left(1 - s_f\right) p_{f,t} \Pi_{f,t} + t_t^*.$$

q_t and q_t^* are the prices of the nominally risk-free one-period bonds that are not internationally traded. t_t and t_t^* are nominal transfers to and from the respective governments.

[3] The fixity of the shares in domestic and foreign firms allows the key international economic linkage of interest (i.e. the spillover of shocks via dividend payments) and its monetary policy implications to be explored most easily, as it impedes confounding adjustments of portfolio holdings over the business cycle and it shuts down the feedback from monetary policy to the composition of portfolios.

[4] Lane (2013) shows that advanced economies hold larger external positions in debt instruments than in equity: the debt-equity ratio of their external assets was about 1.8 in 2010, while that of their external liabilities was about 1.5. For developing countries, the ratios were respectively 3 and 0.5 in the same year.

The household of the home country is entitled to receive exogenously given shares s_h and s_f of the profits of domestic and foreign firms, denoted in real terms as $\Pi_{h,t}$ and $\Pi_{f,t}$ respectively. The foreign household is entitled to receive the remaining shares $1 - s_h$ and $1 - s_f$[5].

Intratemporal problem: consumption allocation Within each period, households choose the bundles of goods that maximise their consumption, defined here as a standard Armington aggregator with elasticity of substitution η and share of imports ζ. Their static problems read

$$\max_{c_{h,t},c_{f,t}} c_t \equiv \left[(1-\zeta)^{\frac{1}{\eta}} \left(c_{h,t}\right)^{\frac{\eta-1}{\eta}} + (\zeta)^{\frac{1}{\eta}} \left(c_{f,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$
$$\text{s.t. } p_t c_t = p_{h,t} c_{h,t} + p_{f,t} c_{f,t}$$

and

$$\max_{c^*_{h,t},c^*_{f,t}} c^*_t \equiv \left[(1-\zeta)^{\frac{1}{\eta}} \left(c^*_{f,t}\right)^{\frac{\eta-1}{\eta}} + (\zeta)^{\frac{1}{\eta}} \left(c^*_{h,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$
$$\text{s.t. } p^*_t c^*_t = p_{f,t} c^*_{f,t} + p_{h,t} c^*_{h,t}.$$

p_t and p^*_t represent the home and foreign consumer price indices (CPIs), i.e. measures of the price levels of all domestically consumed goods:

$$p_t \equiv \left[(1-\zeta)\left(p_{h,t}\right)^{1-\eta} + (\zeta)\left(p_{f,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{1.1}$$

$$p^*_t \equiv \left[(1-\zeta)\left(p_{f,t}\right)^{1-\eta} + (\zeta)\left(p_{h,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{1.2}$$

1.2.2 Firms

Final goods producers Perfectly competitive producers demand local inputs, indexed by i, to make final goods with standard CES technologies[6]:

$$\max_{y_{h,t}(i)} p_{h,t} y_{h,t} - \int_0^1 p_{h,t}(i)\, y_{h,t}(i)\, di \qquad \text{s.t. } y_{h,t} = \left(\int_0^1 y_{h,t}(i)^{\frac{\varepsilon_t-1}{\varepsilon_t}} di\right)^{\frac{\varepsilon_t}{\varepsilon_t-1}},$$

[5]To simplify the exposition, I will focus on parameter configurations where external positions are symmetric ($s_h + s_f = 1$), so that households enjoy equal levels of consumption in the long run.

[6]This functional form specification is fundamental to the model: since intermediate products enter the aggregators symmetrically, using identical quantities of all inputs is the most efficient way to produce any given amount of output. Price dispersion due to nominal rigidities à la Calvo, however, causes the prices of intermediates to differ; this forces final goods producers to use asymmetric quantities of inputs, which reduce output.

$$\max_{y_{f,t}(i)} p_{f,t} y_{f,t} - \int_0^1 p_{f,t}(i)\, y_{f,t}(i)\, di \qquad \text{s.t. } y_{f,t} = \left(\int_0^1 y_{f,t}(i)^{\frac{\varepsilon_t^* - 1}{\varepsilon_t^*}} di \right)^{\frac{\varepsilon_t^*}{\varepsilon_t^* - 1}} .$$

The elasticities of substitution between varieties of intermediates are subject to exogenous innovations that cause cost-push shocks in the goods markets. These disturbances determine fluctuations in the gap between the natural (i.e. flexible-prices) allocation and the efficient one, creating a short-run monetary policy tradeoff; monetary authorities are put in the dilemma of stabilising prices or economic activity.

Intermediate goods producers Monopolistically competitive producers in the home and foreign country make intermediate goods i with the following technologies:

$$y_{h,t}(i) = a_t n_t(i)^{1-\alpha} , \tag{1.3}$$

$$y_{f,t}(i) = a_t^* n_t^*(i)^{1-\alpha} . \tag{1.4}$$

Labour is internationally immobile, so hours are entirely supplied by local households.

The prices of intermediate products are chosen to maximise profits in a Calvo-Yun setting, subject to isoelastic demands by final goods producers. The problems faced by the home and foreign producers are respectively

$$\max_{\bar{p}_{h,t}(i)} \mathbb{E}_t \sum_{\tau=0}^{\infty} \theta_p^\tau q_{t,t+\tau} \left\{ y_{h,t+\tau}(i) \frac{\bar{p}_{h,t}(i)}{p_{h,t+\tau}} - \Psi\left(y_{h,t+\tau}(i)\right) \right\}$$

$$\text{s.t. } \begin{cases} y_{h,t+\tau}(i) = \left(\frac{p_{h,t+\tau}(i)}{p_{h,t+\tau}}\right)^{-\varepsilon_t} y_{h,t+\tau} \\ q_{t,t+\tau} = \beta^\tau \mathbb{E}_t \left(\frac{\lambda_{t+\tau}}{\lambda_t}\right) , \end{cases}$$

and

$$\max_{\bar{p}_{f,t}(i)} \mathbb{E}_t \sum_{\tau=0}^{\infty} \theta_p^\tau q^*_{t,t+\tau} \left\{ y_{f,t+\tau}(i) \frac{\bar{p}_{f,t}(i)}{p_{f,t+\tau}} - \Psi\left(y_{f,t+\tau}(i)\right) \right\}$$

$$\text{s.t. } \begin{cases} y_{f,t+\tau}(i) = \left(\frac{p_{f,t+\tau}(i)}{p_{f,t+\tau}}\right)^{-\varepsilon_t^*} y_{f,t+\tau} \\ q^*_{t,t+\tau} = \beta^\tau \mathbb{E}_t \left(\frac{\lambda^*_{t+\tau}}{\lambda^*_t}\right) . \end{cases}$$

$q_{t,t+\tau}$ and $q^*_{t,t+\tau}$ denote the households' stochastic discount factors for τ periods-ahead *real* payoffs, while θ_p is the index of price stickiness. The $\Psi(\cdot)$ functions represent the real cost of production.

1.2.3 Monetary policy

As mentioned above, the nominal returns on domestic and foreign one-period bonds are certain at the issuing date. They are defined as

$$R_t \equiv \frac{1}{q_t}, \qquad R_t^* \equiv \frac{1}{q_t^*}.$$

These returns are the instruments of monetary policy. The objective of the central banks is to stabilise both prices and economic activity, due to the presence of cost-push shocks[7]. This objective is pursued by setting interest rates according to the following Taylor rules, where R, R^*, π_h and π_f represent the respective nominal interest rate and inflation targets:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_{h,t}}{\pi_h}\right)^{\gamma_\pi} \left(\frac{y_{h,t}}{y_{h,t-1}}\right)^{\gamma_y}\right]^{1-\gamma_R} m_t, \tag{1.5}$$

$$\frac{R_t^*}{R^*} = \left(\frac{R_{t-1}^*}{R^*}\right)^{\gamma_R} \left[\left(\frac{\pi_{f,t}}{\pi_f}\right)^{\gamma_\pi} \left(\frac{y_{f,t}}{y_{f,t-1}}\right)^{\gamma_y}\right]^{1-\gamma_R} m_t^*. \tag{1.6}$$

Notice two features of rules (1.5) and (1.6). First, the relevant inflation measure that monetary authorities control is that of the goods *produced* in their respective countries, rather than those *consumed* there: following Clarida, Galí, and Gertler (2002) and Galí and Monacelli (2005), the interest rate rules have been specified in terms of *PPI inflation rates*

$$\pi_{h,t} \equiv \frac{p_{h,t}}{p_{h,t-1}}, \qquad \pi_{f,t} \equiv \frac{p_{f,t}}{p_{f,t-1}},$$

rather than *CPI inflation rates*

$$\pi_t \equiv \frac{p_t}{p_{t-1}}, \qquad \pi_t^* \equiv \frac{p_t^*}{p_{t-1}^*}.$$

The reason is that inflation and price dispersion entail a resource misallocation on the supply side of the economy in this class of models. Since the law of one price holds continuously, the only price distortion that exists in this environment is that of price dispersion at the level of intermediate goods *within* each country. The

[7]Under these conditions, a policy of *strict* inflation targeting would replicate the flexible prices allocation, not the socially optimal one. Since central banks face the conflicting goals of preventing suboptimal fluctuations in relative prices and avoiding suboptimal fluctuations in output, they must follow *flexible* inflation targeting policies instead.

elimination of producer price inflation is sufficient to eradicate it: see Engel (2011)[8]. Second, the Taylor rules are specified in terms of output *growth* rather than output *gap*, since the former is easily observable in actual practice while the latter is not; this makes them "operational" in the sense of Schmitt-Grohé and Uribe (2007).

1.2.4 Exogenous processes

Productivities follow a vector autoregressive process in logs:

$$\begin{bmatrix} \log a_t \\ \log a_t^* \end{bmatrix} = \begin{bmatrix} \rho_a & \nu_a \\ \nu_a & \rho_a \end{bmatrix} \begin{bmatrix} \log a_{t-1} \\ \log a_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_{a,t} \\ e_{a,t}^* \end{bmatrix}.$$

The off-diagonal elements ν_a represent productivity spillovers and introduce correlation between the two processes. The innovations $e_{a,t}$ and $e_{a,t}^*$ follow orthogonal i.i.d. normal processes:

$$\operatorname{corr}(e_a, e_a^*) = 0.$$

Monetary policy shocks follow an autoregressive process in logs:

$$\begin{bmatrix} \log m_t \\ \log m_t^* \end{bmatrix} = \begin{bmatrix} \rho_m & \nu_m \\ \nu_m & \rho_m \end{bmatrix} \begin{bmatrix} \log m_{t-1} \\ \log m_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_{m,t} \\ e_{m,t}^* \end{bmatrix}.$$

The off-diagonal elements ν_m control international spillovers of monetary policy shocks. The innovations $e_{m,t}$ and $e_{m,t}^*$ follow orthogonal i.i.d. normal processes:

$$\operatorname{corr}(e_m, e_m^*) = 0.$$

The elasticities of substitution between intermediate products follow stochastic processes in logs:

$$\begin{bmatrix} \log\left(\frac{\varepsilon_t}{\bar{\varepsilon}}\right) \\ \log\left(\frac{\varepsilon_t^*}{\bar{\varepsilon}}\right) \end{bmatrix} = \begin{bmatrix} \rho_\varepsilon & \nu_\varepsilon \\ \nu_\varepsilon & \rho_\varepsilon \end{bmatrix} \begin{bmatrix} \log\left(\frac{\varepsilon_{t-1}}{\bar{\varepsilon}}\right) \\ \log\left(\frac{\varepsilon_{t-1}^*}{\bar{\varepsilon}}\right) \end{bmatrix} + \begin{bmatrix} e_{\varepsilon,t} \\ e_{\varepsilon,t}^* \end{bmatrix}.$$

The off-diagonal elements ν_ε represent spillovers of cost-push shocks and introduce correlation between the two processes. The innovations $e_{\varepsilon,t}$ and $e_{\varepsilon,t}^*$ follow orthogonal i.i.d. normal processes:

$$\operatorname{corr}(e_\varepsilon, e_\varepsilon^*) = 0.$$

[8] This is true regardless of the presence of external positions: as argued below, interest rate rules that also target foreign price inflation yield lower social welfare in this enviroment.

1.3 Equilibrium conditions

1.3.1 Households

Intertemporal optimisation The first-order conditions (FOCs) for consumption, saving and labour supply of the home country household, together with the budget constraint[9], are as follows:

$$u_c\left(c_t, n_t\right) = \lambda_t, \tag{1.7}$$

$$q_t = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{p_t}{p_{t+1}} \right), \tag{1.8}$$

$$-\frac{u_n\left(c_t, n_t\right)}{w_t/p_t} = \lambda_t, \tag{1.9}$$

$$c_t + \frac{q_t}{p_t} b_{t+1} = \frac{b_t}{p_t} + \frac{w_t}{p_t} n_t + s_h \frac{p_{h,t}}{p_t} \Pi_{h,t} + s_f \frac{p_{f,t}}{p_t} \Pi_{f,t} + \frac{t_t}{p_t}. \tag{1.10}$$

Analogous FOCs hold for the foreign household: see Appendix E.

Static optimisation The final consumption demands for domestic and imported goods by home and foreign households are respectively

$$c_{h,t} = (1-\zeta) \left(\frac{p_{h,t}}{p_t} \right)^{-\eta} c_t, \qquad c_{f,t} = (\zeta) \left(\frac{p_{f,t}}{p_t} \right)^{-\eta} c_t, \tag{1.11}$$

$$c^*_{f,t} = (1-\zeta) \left(\frac{p_{f,t}}{p^*_t} \right)^{-\eta} c^*_t, \qquad c^*_{h,t} = (\zeta) \left(\frac{p_{h,t}}{p^*_t} \right)^{-\eta} c^*_t. \tag{1.12}$$

1.3.2 Firms

Final goods production The input demand schedules that solve the problems of final goods producers are as follows:

$$y_{h,t}(i) = \left(\frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\varepsilon_t} y_{h,t}, \qquad y_{f,t}(i) = \left(\frac{p_{f,t}(i)}{p_{f,t}} \right)^{-\varepsilon^*_t} y_{f,t}. \tag{1.13}$$

The associated producer price indices (PPIs) implied by perfect competition are

$$p_{h,t} = \left(\int_0^1 p_{h,t}(i)^{1-\varepsilon_t} \, di \right)^{\frac{1}{1-\varepsilon_t}}, \qquad p_{f,t} = \left(\int_0^1 p_{f,t}(i)^{1-\varepsilon^*_t} \, di \right)^{\frac{1}{1-\varepsilon^*_t}}. \tag{1.14}$$

[9] As (1.10) is written in real terms, λ_t should be interpreted as the marginal utility of an additional unit of consumption.

Productivity, employment and aggregate output Combining the production functions (1.3) and (1.4) and the input demands (1.11) and (1.12) with the labour market-clearing conditions

$$n_t = \int_0^1 n_t(i)\, di, \qquad n_t^* = \int_0^1 n_t^*(i)\, di, \tag{1.15}$$

as outlined in Appendix A, we get the exact aggregate production functions:

$$y_{h,t} = a_t \left(\frac{n_t}{d_{h,t}}\right)^{1-\alpha}, \qquad y_{f,t} = a_t^* \left(\frac{n_t^*}{d_{f,t}}\right)^{1-\alpha}. \tag{1.16}$$

The measures of home and foreign *price dispersion* are defined for brevity as

$$d_{h,t} \equiv \int_0^1 \left(\frac{p_{h,t}(i)}{p_{h,t}}\right)^{-\frac{\varepsilon_t}{1-\alpha}} di, \qquad d_{f,t} \equiv \int_0^1 \left(\frac{p_{f,t}(i)}{p_{f,t}}\right)^{-\frac{\varepsilon_t^*}{1-\alpha}} di. \tag{1.17}$$

Intermediate goods production Let us restrict our attention to a symmetric equilibrium where all price resetters in each country face the same problem and therefore choose the same price, denoted respectively $\bar{p}_{h,t}$ and $\bar{p}_{f,t}$. Define optimal relative prices in each country as

$$\tilde{p}_{h,t} \equiv \frac{\bar{p}_{h,t}}{p_{h,t}}, \qquad \tilde{p}_{f,t} \equiv \frac{\bar{p}_{f,t}}{p_{f,t}}. \tag{1.18}$$

As outlined in Appendix D, the optimal price-setting conditions for home firms are

$$g_{h,t}^2 = \mathcal{M}_{p,t} g_{h,t}^1, \tag{1.19}$$

where I have defined recursive auxiliary variables

$$g_{h,t}^2 \equiv y_{h,t}\left(\tilde{p}_{h,t}\right)^{-\varepsilon_t} + \theta_p \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) g_{h,t+1}^2 \left(\frac{\tilde{p}_{h,t}}{\tilde{p}_{h,t+1}}\right)^{-\varepsilon_t} \left(\frac{1}{\pi_{h,t+1}}\right)^{1-\varepsilon_t}, \tag{1.20}$$

$$g_{h,t}^1 \equiv y_{h,t}\frac{mc_{h,t}}{d_{h,t}}\left(\tilde{p}_{h,t}\right)^{\frac{\alpha-1-\varepsilon_t}{1-\alpha}} + \theta_p \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) g_{h,t+1}^1 \left(\frac{\tilde{p}_{h,t}}{\tilde{p}_{h,t+1}}\right)^{\frac{\alpha-1-\varepsilon_t}{1-\alpha}} \left(\frac{1}{\pi_{h,t+1}}\right)^{-\frac{\varepsilon_t}{1-\alpha}}, \tag{1.21}$$

and the desired "frictionless" price markup[10]

$$\mathcal{M}_{p,t} \equiv \frac{\varepsilon_t}{\varepsilon_t - 1}.$$

Analogous pricing conditions can be written for foreign firms: see Appendix E.

1.3.3 Dynamics of aggregate price levels and price dispersion

Given Calvo-Yun pricing, we make use of two facts now: (i) all resetters in each country choose the same prices (symmetric equilibrium); (ii) the distribution of prices among non-resetters at time t corresponds to the distribution of effective prices at time $t-1$ (law of large numbers). These two facts imply that PPIs evolve according to

$$1 = \theta_p \left(\frac{p_{h,t-1}}{p_{h,t}}\right)^{1-\varepsilon_t} + (1-\theta_p)\left(\frac{\bar{p}_{h,t}}{p_{h,t}}\right)^{1-\varepsilon_t},$$

$$1 = \theta_p \left(\frac{p_{f,t-1}}{p_{f,t}}\right)^{1-\varepsilon_t^*} + (1-\theta_p)\left(\frac{\bar{p}_{f,t}}{p_{f,t}}\right)^{1-\varepsilon_t^*}.$$

By the same logic, we can write the price dispersion indices (1.17) recursively:

$$d_{h,t} = \theta_p \left(\frac{1}{\pi_{h,t}}\right)^{-\frac{\varepsilon_t}{1-\alpha}} d_{h,t-1} + (1-\theta_p)\left(\tilde{p}_{h,t}\right)^{-\frac{\varepsilon_t}{1-\alpha}},$$

$$d_{f,t} = \theta_p \left(\frac{1}{\pi_{f,t}}\right)^{-\frac{\varepsilon_t^*}{1-\alpha}} d_{f,t-1} + (1-\theta_p)\left(\tilde{p}_{f,t}\right)^{-\frac{\varepsilon_t^*}{1-\alpha}}.$$

[10] $\mathcal{M}_{p,t}$ is called "frictionless" markup because with $\theta = 0$ (flexible prices) the FOCs become

$$\bar{p}_{h,t} = p_{h,t}\left[\mathcal{M}_{p,t} mc_{h,t}\right]^{\Theta},$$

where the index is defined as in Galí (2008) and is decreasing in both ε and α:

$$\Theta \equiv \frac{1-\alpha}{\varepsilon_t \alpha + 1 - \alpha} \leq 1.$$

Notice that for $\alpha = 0$ (constant returns) we get the following standard markup pricing condition:

$$\bar{p}_{h,t} = \mathcal{M}_{p,t} mc_{h,t} p_{h,t}.$$

1.3.4 Market clearing and the terms of trade

Labour market clearing has been imposed in the calculation of the aggregate production functions. Goods market clearing requires that the following equalities hold:

$$y_{h,t} = c_{h,t} + c^*_{h,t}, \qquad y_{f,t} = c_{f,t} + c^*_{f,t}.$$

Since this is a cashless economy model, nominal variables are not uniquely defined: only real variables are. Therefore, while the absolute price levels $p_{h,t}$ and $p_{f,t}$ cannot be identified, the relative price that clears both markets can be. Such price is called the *terms of trade* and is defined as $s_t \equiv \frac{p_{f,t}}{p_{h,t}}$.

As to the asset markets, the fact that the bonds cannot be traded by the two households implies that their holdings must be zero in equilibrium:

$$b_t = 0, \qquad b^*_t = 0.$$

A summary of the whole set of equilibrium conditions, rearranged in real terms for computational convenience, can be found in Appendix E.

1.4 Equilibrium dynamics

In this section I investigate the dynamic responses of the main macroeconomic variables to technology, monetary and cost-push shocks, and I explore how they differ in the presence and in the absence of external positions.

A conventional CRRA specification is assigned to period utility functions:

$$u(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\varphi}}{1+\varphi}.$$

Table 1.1 overleaf displays the parameterisation adopted for the simulations that follow. Notice that the preferences of home and foreign households over domestic goods and imports are identical in the absence of home bias, i.e. with $\zeta = 1/2$. In this case, the equilibrium adjustment of relative prices to movements in relative output depends directly on the elasticity of substitution between domestic and foreign goods. In the absence of external positions, the Cole and Obstfeld (1991) result applies: if $\eta = 1$, c_t and c^*_t always move one-to-one because terms-of-trade movements neutralise output risks. With external positions, this result does not hold any more: as we will see, c_t and c^*_t move asymmetrically even with a unit Armington elasticity, because some income is shifted across borders by movements in dividend payments, and wealth effects occur.

Table 1.1: Parameterisation of the model

Parameter	Value	Description
α	0	Technology coefficient controlling returns to scale
σ	1	Elasticity of intertemporal substitution
φ	1	Frisch elasticity of labour supply
$\bar{\varepsilon}$	6	Steady-state elasticity of substitution between intermediates
ζ	0.5	Share of consumption allocated to imported goods
η	1	Armington elasticity of international substitution
β	0.99	Subjective discount factor
θ_p	0.66	Price stickiness parameter
γ_R	0.7	Interest rate smoothing parameter in the Taylor rule
γ_π	1.5	Inflation parameter in the Taylor rule
γ_y	3	Output growth parameter in the Taylor rule
ρ_a	0.95	Serial correlation of the log of technology
ρ_m	0.95	Serial correlation of the log of monetary shocks
ρ_ε	0.95	Serial correlation of the log of cost-push shocks
ν_a	0	International spillover of technology shocks
ν_m	0	International spillover of monetary shocks
ν_ε	0	International spillover of cost-push shocks
$\text{std}\,(e_a)$	0.01	Standard deviation of technology shocks
$\text{std}\,(e_m)$	0.01	Standard deviation of monetary shocks
$\text{std}\,(e_\varepsilon)$	0.01	Standard deviation of cost-push shocks

The dynamic properties of the economy without external positions are consistent with those of the standard small-scale New Keynesian model (NKM) discussed in Galí (2008). The impulse response functions (IRFs) of that economy will be shown in what follows as a benchmark for comparison against the economy with external positions.

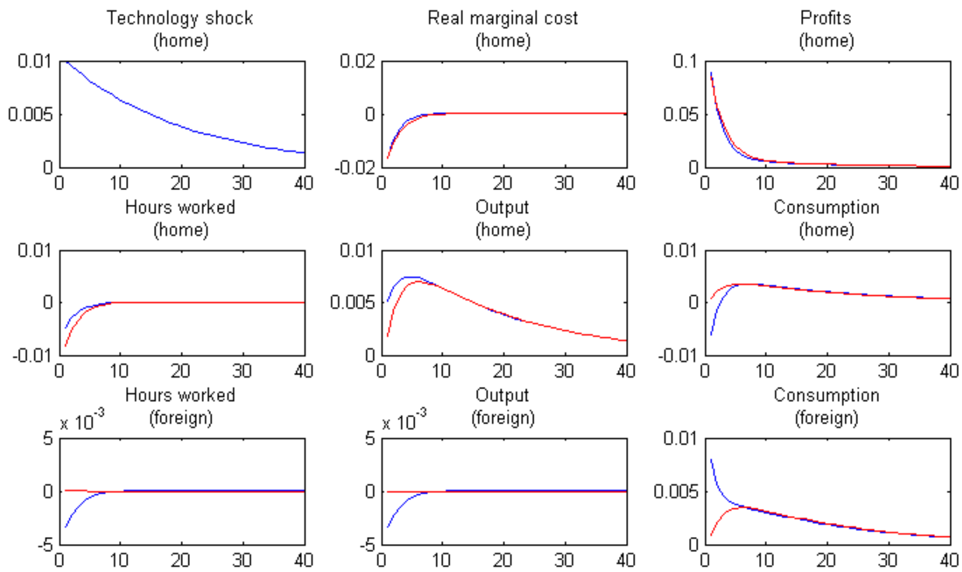
1.4.1 Technology shocks

Figure 1.2 shows that a positive technology shock in the home country causes a decrease in the marginal cost of production and an increase in domestic output. The real profits of home firms jump; as both consumption and leisure are normal goods, the home household reduces the supply of labour.

In the absence of external positions (red lines), home and foreign consumption move one-to-one thanks to the risk-sharing role played by the terms of trade: both households enjoy the larger supply of home goods in equal proportions. Since the foreign household is not entitled to receive any dividends from abroad, there is no wealth effect on labour supply; as a consequence, foreign output is stable[11].

In the presence of external positions (blue lines), firms in the home country pay part of their extra profits in dividends to the foreign household. This creates a *first wealth effect*: the lump-sum component of the foreign household's income jumps, so the household reduces the supply of labour to enjoy more leisure. As a consequence, foreign hours and output fall while the real wage rises. This triggers a *second wealth effect*: since the profits of foreign firms drop, the household of the home country receives smaller dividends from abroad and supplies more labour. For this reason, home output and hours are temporarily higher than in the previous case. Notice that the combined result of these two spillovers is that foreign consumption jumps on impact, while home consumption falls: the presence of external positions implies an international redistribution of resources.

Figure 1.2: Impulse responses to a technology shock with and without external positions (blue vs red)



[11]The calibration with unit elasticities of substitution in Table 1.1 is a knife-edge case where home and foreign products are independent in consumption. For this reason, shocks to the supply of one good do not spillover to the other: see Corsetti and Pesenti (2001).

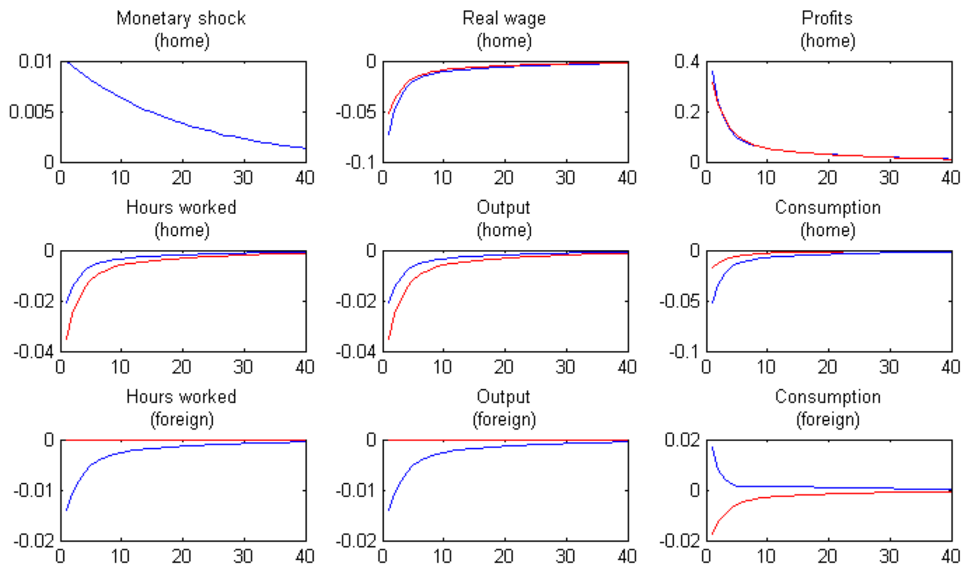
1.4.2 Monetary shocks

Figure 1.3 shows that a monetary policy shock raises the real interest rate in the home country; this exerts a contractionary effect on domestic output and pushes domestic CPI inflation down. Since nominal wages are flexible but prices are not, real wages in the home country fall after the shock, and aggregate profits jump.

In the absence of external positions, home and foreign consumption fall because the total supply of home goods has decreased. As mentioned above, the decline in c_t and c_t^* is symmetric because $\eta = 1$. Since the foreign household is insulated from the dynamics of home profits, there is no change in foreign hours, output and profits.

In the presence of external positions, the two international wealth effects described above come into play again. The foreign household receives higher dividends from abroad and supplies less labour, so foreign output and profits decline. The home household, in turn, receives lower dividends from foreign firms and supplies more labour; as a consequence, home output stays higher than in the previous case. The combined impact of these spillovers is that consumption shows a pronounced fall in the home country and a jump in the foreign country: wealth effects again transfer consumption across borders[12].

Figure 1.3: Impulse responses to a monetary shock with and without external positions (blue vs red)



[12] Notice that total world consumption (not shown here) clearly declines after a contractionary monetary shock, because the supply of home goods drops.

The importance of external positions The magnitude of the output spillovers and the international comovement of consumption following a macroeconomic disturbance depend on the size of the external positions. To uncover this dependence, in Figure 1.4 I plot the behaviour of consumption, output and hours after a contractionary monetary shock for s_h and s_f ranging between 0% and 100%.

Notice that I explore parameter configurations where external positions are symmetric (in the sense that $s_h + s_f = 1$) for simplicity, so that countries pay identical dividends to each other in steady state; this makes their consumption levels equal in the long run. Asymmetric external positions (where one household is entitled to receive a larger portion of total world profits than the other) would not change the nature of the international transmission mechanism from a qualitative point of view, although they would affect the quantitative strength of the two international wealth effects discussed above.

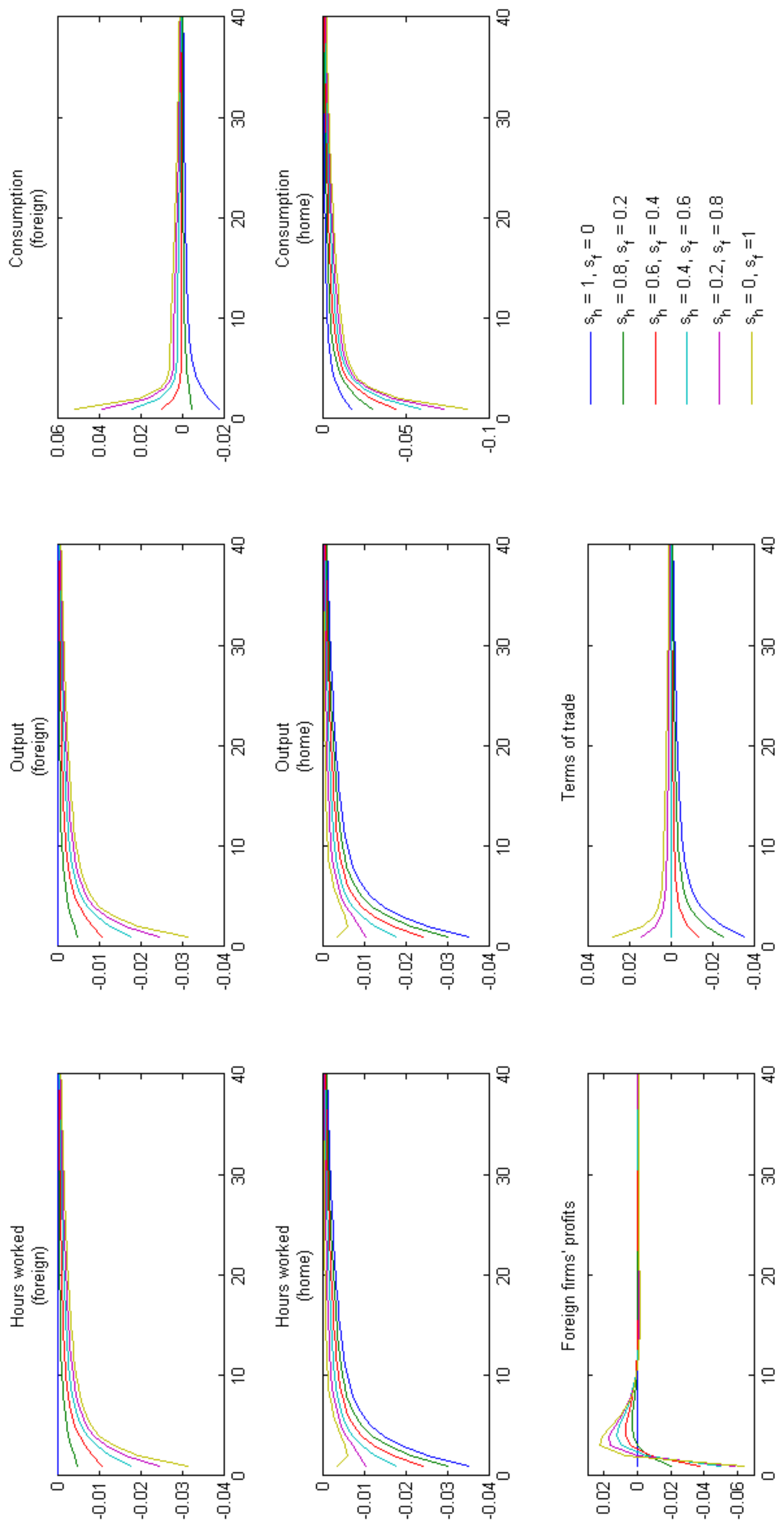
The picture confirms that a monetary shock to the home country leaves foreign output and hours unaffected when external positions are zero. In that case, there is also a perfect correlation between domestic and foreign consumption, thanks to the endogenous risk-sharing role played by the terms of trade. As the external positions get larger, stronger wealth effects are triggered by international dividend payments, and play an increasingly important role in the spillover of macroeconomic disturbances across borders; this shows up in larger movements of foreign output and hours on impact (due to the first wealth effect) and smaller movements in home output and hours (due to the second wealth effect). Since the correlation between domestic and foreign consumption declines accordingly, the degree of international risk sharing is affected negatively by financial integration in this economy[13].

The picture also shows that if the two countries were sufficiently exposed to each other's dividends, the income effects would be large enough to make foreign output actually move more than home output. In that extreme case, the terms of trade of the home country would actually *depreciate* on impact[14].

[13] This is due to the fact that financial integration is assumed to take the form of larger exogenous external positions in this framework, rather than a richer menu of internationally traded assets.

[14] Such a configuration is of limited practical interest given the sizeable home bias in equities documented by Coeurdacier and Rey (2012). The authors calculated that the United States, the Euro Area and the United Kingdom held respectively 77.2%, 56.7% and 54.5% of their equity portfolios in domestic shares in 2008. Since US, Eurozone and UK firms represented only 32.6%, 13.5% and 5.1% of world market capitalisation in that year, this ownership pattern shows a clear bias towards local shares.

Figure 1.4: External positions and the international spillover of monetary shocks

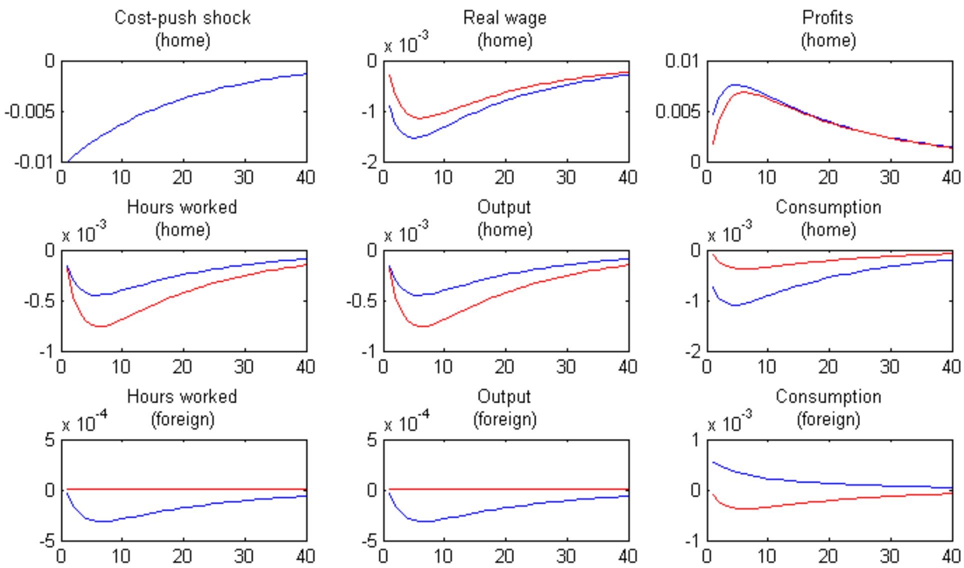


1.4.3 Cost-push shocks

Figure 1.5 shows the impulse responses of selected macroeconomic variables to an adverse cost-push shock in the home country, which takes the form of an exogenous 1% decrease in ε_t.

The cost-push shock temporarily boosts price markups in the home economy, expanding the wedge between the marginal product of labour and the marginal rate of substitution between consumption and leisure. Output, hours and real wages fall, while real profits rise.

Figure 1.5: Impulse responses to a cost-push shock with and without external positions (blue vs red)

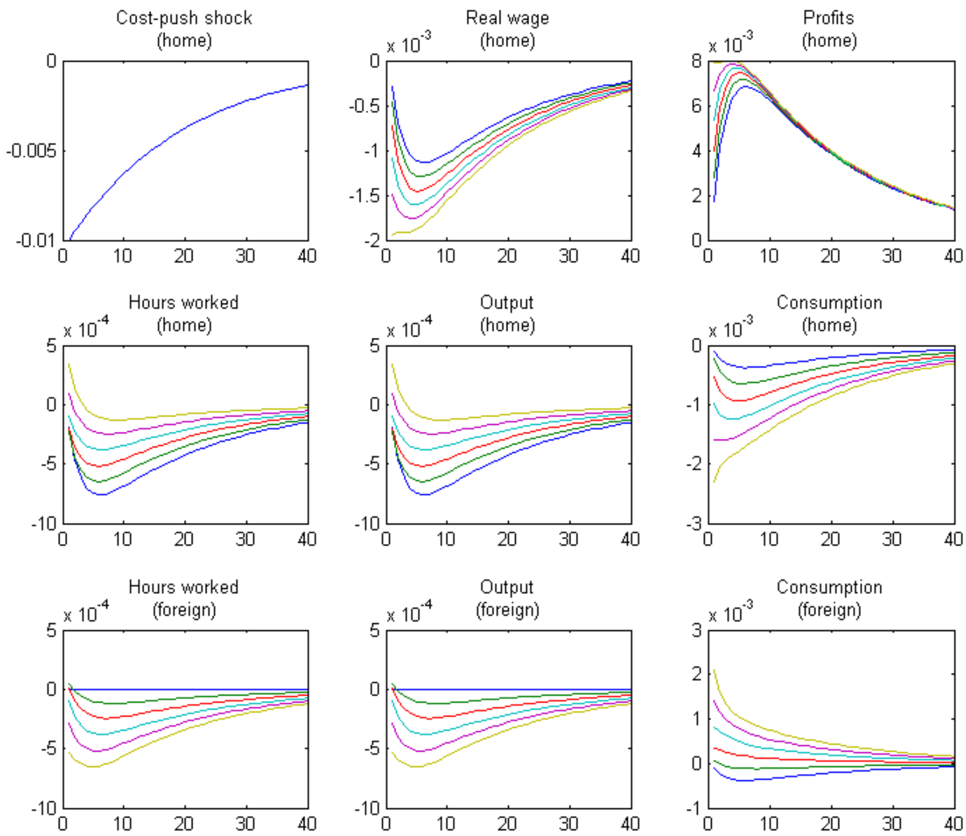


In the presence of external positions (blue lines), additional dividends are paid to the foreign household, triggering a wealth effect that reduces the supply of labour; foreign output and profits fall. This triggers a second wealth effect, because the domestic household in turn receives smaller dividends from abroad and supplies more labour; this makes real wages fall even more in the home country.

In the absence of external positions (red lines), the foreign household is immune to changes in the profits of home firms, so foreign labour supply and output are unaffected by the shock. The extra profits of home firms are received entirely by the home household, who reduces labour supply relative to the previous case. For this reason, domestic real wages stay higher and output stays lower than before.

Figure 1.6 shows how international wealth effects get stronger as external positions increase. The legend is the same as in Figure 1.4, with blue lines representing the economy with no external positions, and yellow lines representing the polar opposite.

Figure 1.6: External positions and impulse responses to a cost-push shock



1.5 Monetary policy and welfare

In this section I study monetary policy within the class of interest rate rules (1.5) and (1.6)[15]. In order to compare the performance of different parameterisations of these rules, I adopt a welfare-based criterion: I search for the Taylor rule parameters vector $(\gamma_R, \gamma_\pi, \gamma_y)$ that maximises the conditional expectation of the total lifetime utility of households, given the current state of the economy.

[15]The identification of the optimal monetary policy is challenging in this environment, because one should solve the so-called "Ramsey problem" of maximising welfare subject to the entire system of (nonlinear) competitive equilibrium conditions of the economy. A characterisation of optimal policy using a linear-quadratic approximation is not pursued for reasons of accuracy, as explained below.

To begin with, I define the welfare of the home and foreign households as follows:

$$V_{h,t} \equiv \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t\right), \qquad V_{f,t} \equiv \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u\left(c_t^*, n_t^*\right). \tag{1.22}$$

These welfare measures must be rearranged recursively for computational reasons. They are then appended to the competitive equilibrium conditions that appear in Appendix E to form the so-called "expanded dynamic system" of the model. A word of caution is necessary about the order of approximation of this system.

I depart from the standard practice of combining a second-order approximation of the welfare function with a first-order approximation of the remaining equilibrium conditions. This approach would be prone to potentially large approximation errors in my incomplete markets framework, because some second-order terms of the welfare functions (1.22) would be ignored while others are included. As shown by Kim and Kim (2003), such a miscalculation can result in spurious welfare rankings.

Following Schmitt-Grohé and Uribe (2007), I compute a second-order accurate solution of the entire expanded system around its nonstochastic steady state. This has two main implications. First, I do not need to make the steady state efficient, so I dispense with the factor-input subsidies financed by lump-sum taxes that are often assumed to induce the perfectly competitive level of employment in steady state. Second, I adopt a recursive representation of the exact nonlinear price dynamics, rather than a New Keynesian Phillips Curve, so I must keep track of the evolution of an additional state variable that measures price dispersion across varieties of intermediate goods.

The identification of the most desirable monetary policy mix involves the comparison of expected lifetime welfare (1.22) across different calibrations of the Taylor rules. Expectations are taken conditional on the initial state of the economy being the competitive equilibrium nonstochastic steady state, which is constant across all monetary regimes; this ensures that the economy starts from the same point in all cases under consideration.

Welfare appears to be decreasing in the interest rate smoothing parameter γ_R, increasing in γ_π, and non-monotonic (concave) in the output reaction coefficient γ_y. The results of numerical work point to the desirability of a regime of *flexible* inflation targeting, where some price stability is traded off against some output stability.

As observed above, the relevant inflation target in this environment is the rate of change of *domestic* goods prices only, as opposed to that of the overall consumer price index. The reason is that the law of one price always holds here, so central banks do not have to target (either directly or through the CPI) the movements of imports prices: these are regarded as efficient.

Furthermore, central banks should engage in inflation targeting uncoordinatedly; prices would become more unstable if both countries reacted to each other's inflation. Because domestic and imported goods are independent in consumption, no cross-border supply spillovers create potential gains from cooperation in this economy, so interest rate feedback rules augmented to respond to foreign inflation à la Clarida et al. (2002) would be welfare-reducing.

In this framework, it is possible to get arbitrarily close to the level of welfare of the nonstochastic economy if central banks adopt Taylor rules with (i) arbitrarily large inflation coefficients, (ii) no inertia in interest rates, and (iii) suitable output coefficients (which depend on the size of external positions here). These facts are well-established in the literature, and can be explained as follows.

First, minimising inflation variations reduces the need to reset prices and helps the economy approach the "natural" or flexible prices allocation. Second, an inertial adjustment of interest rates is unnecessary in a cashless economy, because there is no need to stabilise the opportunity cost of holding money in such an environment. Third, a policy of "leaning against the wind" makes output less volatile (at the cost of some inflation volatility) in the presence of cost-push shocks.

If nominal rigidities were the only friction, central banks would face no monetary policy tradeoff and inflation stabilisation would emerge as the sole objective of monetary policy, regardless of external positions. In that case, the welfare level of the nonstochastic economy would be well approximated by the following "constrained" configuration of the interest rate rules[16]:

$$(\gamma_R, \gamma_\pi, \gamma_y) = (0, 4, 0).$$

In the presence of cost-push shocks, instead, monetary authorities must strike a balance between different goals because the natural level of output varies while its efficient counterpart remains unchanged. In that case, the presence of external positions can tilt the balance towards one objective.

The existence of a monetary policy tradeoff would go unnoticed if the volatility of cost-push shocks was as small as it is in Table 1.1 and Figure 1.5, and strict

[16] Noice that the welfare maximisation problem has no solution here, because the objective function is monotonically increasing in γ_π and the domain of this parameter is unbounded in principle. Following Schmitt-Grohé and Uribe (2007), I rule out Taylor rule parameters larger than 4 on the grounds that they would not be realistic in practice.

inflation targeting would remain the welfare-maximising policy. The monetary policy dilemma becomes visible if the volatility of cost push shocks is as large as $\text{std}(e_\varepsilon) = 0.25$, for instance. A null response to output fluctuations does not maximise welfare any longer under these conditions: central banks can improve upon strict inflation targeting by adopting a policy mix that puts some emphasis on output stability.

As shown in Table 1.2, the output coefficient that maximises welfare in this case tends to stay around one for external positions between zero and fifty percent (which is the most plausible interval in practice), and it steadily declines to zero when the size of external positions gets larger than that. The reason is that the international wealth effects observed in Figures 1.5 and 1.6 operate to make output and hours progressively *less* variable in the face of cost-push shocks; this tends to tilt the balance in favour of inflation stabilisation again.

Table 1.2: External positions and Taylor rule parameters under cost-push shocks

External positions	Optimal $(\gamma_R, \gamma_\pi, \gamma_y)$
0%	$(0, 4, 1)$
25%	$(0, 4, 1)$
50%	$(0, 4, 1)$
75%	$(0, 4, 0.5)$
100%	$(0, 4, 0)$

1.6 Sticky wages

The last section illustrated how wealth effects tend to stabilise output and destabilise consumption in the face of cost-push shocks affecting the production of final goods, if external positions are present. This section shows that the opposite would happen if cost-push shocks affected the labour markets as in Clarida et al. (2002): wealth effects would stabilise consumption and destabilise output[17].

Households no longer choose hours taking the wage as given. On the contrary, they choose their wages and then supply any quantity of labour that satisfies demand. The optimal labour supply conditions (1.9) are replaced by the optimal wage-setting conditions outlined below. Cost-push shocks are modelled as exogenous movements in the elasticity of substitution between different types of labour, along the lines of

[17] Notice that wages are flexible in Clarida et al. (2002): cost-push shocks are assumed to affect the wage markups directly there. Here wages are subject to nominal rigidities as in Schmitt-Grohé and Uribe (2007), instead.

shocks to the elasticity of substitution between varieties of intermediate goods.

1.6.1 Staggered wage setting and the supply of labour

The household of each country is made up of a continuum of workers indexed on the unit interval, each supplying a differentiated labour service j. Since each type of labour is an imperfect substitute for the others, workers can choose their wage now.

Wages are sticky in a Calvo-Yun fashion: in each period, only a fraction $1-\theta_w$ of workers can reset their wage. The rest must keep their existing wage, with no indexation.

Labour packers Firms demand a homogeneous labour service. Perfectly competitive labour packers (i.e. "contractors" or "employment agencies") act as aggregators in each country: they purchase differentiated labour inputs and turn them into a composite labour service. Their goal is to maximise profits subject to a standard CES technology:

$$\max_{n_t(j)} w_t n_t - \int_0^1 w_t(j)\, n_t(j)\, dj \qquad \text{s.t. } n_t = \left(\int_0^1 n_t(j)^{\frac{\psi_t-1}{\psi_t}}\, dj\right)^{\frac{\psi_t}{\psi_t-1}}.$$

The elasticities of substitution between different types of labour follow exogenous processes in logs with orthogonal i.i.d. normal innovations:

$$\begin{bmatrix} \log\left(\frac{\psi_t}{\bar{\psi}}\right) \\ \log\left(\frac{\psi_t^*}{\bar{\psi}}\right) \end{bmatrix} = \begin{bmatrix} \rho_\psi & \nu_\psi \\ \nu_\psi & \rho_\psi \end{bmatrix} \begin{bmatrix} \log\left(\frac{\psi_{t-1}}{\bar{\psi}}\right) \\ \log\left(\frac{\psi_{t-1}^*}{\bar{\psi}}\right) \end{bmatrix} + \begin{bmatrix} e_{\psi,t} \\ e_{\psi,t}^* \end{bmatrix},$$

$$\operatorname{corr}\left(e_\psi, e_\psi^*\right) = 0.$$

These disturbances make wage markups time-varying and cause cost-push shocks to the labour markets. They are sometimes referred to as "labour supply shocks" in the literature, as their effect on aggregate labour supply resembles that of a shock to the preference for leisure.

They generate further fluctuations in the gap between the natural (i.e. flexible-prices and flexible-wages) allocation and its efficient counterpart, exacerbating the tradeoff faced by the monetary authorities. In fact, as argued below, the natural allocation cannot be achieved any longer if both prices and wages are sticky.

The solution to the problem of labour packers is a set of demand schedules for each type of labour:

$$n_t(j) = \left(\frac{w_t(j)}{w_t}\right)^{-\psi_t} n_t. \tag{1.23}$$

The aggregate nominal wage index obtains from a zero-profits condition:

$$w_t = \left(\int_0^1 w_t(j)^{1-\psi_t}\, dj\right)^{\frac{1}{1-\psi_t}}. \tag{1.24}$$

Workers Assuming that utility is separable in labour and consumption, the relevant part of the Lagrangian for the optimal wage setting problem of the workers in the home country is as follows:

$$\max_{\bar{w}_t(j)} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^{\tau}\, u(c_{t+\tau}, n_{t+\tau})$$

$$\text{s.t.} \begin{cases} p_{t+\tau}c_{t+\tau} + q_{t+\tau}b_{t+\tau+1} = b_{t+\tau} + \int_0^1 \bar{w}_t(j)\, n_{t+\tau}(j)\, dj \\ \qquad\qquad +s_h p_{h,t+\tau}\Pi_{h,t+\tau} + s_f p_{f,t+\tau}\Pi_{f,t+\tau} + t_{t+\tau} \\ n_{t+\tau}(j) = \left(\frac{w_{t+\tau}(j)}{w_{t+\tau}}\right)^{-\psi_t} n_{t+\tau}. \end{cases}$$

As outlined in Appendix F, the optimal wage-setting conditions in a symmetric equilibrium are

$$f^1_{h,t} = \mathcal{M}_{w,t} f^2_{h,t},$$

where I have defined the desired "frictionless" wage markup as[18]

$$\mathcal{M}_{w,t} \equiv \left(\frac{\psi_t}{\psi_t - 1}\right)$$

and the recursive auxiliary variables as

$$f^1_{h,t} \equiv (\bar{\mathrm{w}}_t)^{1-\psi_t} (\mathrm{w}_t)^{\psi_t} \lambda_t n_t + \beta\theta_w \mathbb{E}_t \left(\frac{1}{\pi_{t+1}}\right)^{1-\psi_t} \left(\frac{\bar{\mathrm{w}}_t}{\bar{\mathrm{w}}_{t+1}}\right)^{1-\psi_t} f^1_{h,t+1}, \tag{1.25}$$

[18] With flexible wages we obtain the markup wage-setting relationship

$$\bar{\mathrm{w}}_t = mrs_t \left(\frac{\psi_t}{\psi_t - 1}\right),$$

where the real cost of supplying labour is given by the marginal rate of substitution between consumption and leisure:

$$mrs_t \equiv -\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)}.$$

$$f_{h,t}^{2} \equiv -u_n\left(c_t, n_t\right) n_t \left(\frac{\bar{\mathrm{w}}_t}{\mathrm{w}_t}\right)^{-\psi_t} + \beta\theta_w \mathbb{E}_t \left(\frac{1}{\pi_{t+1}}\right)^{-\psi_t} \left(\frac{\bar{\mathrm{w}}_t}{\bar{\mathrm{w}}_{t+1}}\right)^{-\psi_t} f_{h,t+1}^{2}. \tag{1.26}$$

The dynamics of aggregate wages and wage dispersion Applying the Calvo-Yun algebra to the wage index (1.24) and its foreign counterpart, we obtain the laws of motion of wages subject to nominal rigidities:

$$w_t = \left[\theta_w \left(w_{t-1}\right)^{1-\psi_t} + \left(1-\theta_w\right)\left(\bar{w}_t\right)^{1-\psi_t}\right]^{\frac{1}{1-\psi_t}},$$

$$w_t^{*} = \left[\theta_w \left(w_{t-1}^{*}\right)^{1-\psi_t^{*}} + \left(1-\theta_w\right)\left(\bar{w}_t^{*}\right)^{1-\psi_t^{*}}\right]^{\frac{1}{1-\psi_t^{*}}}.$$

Let us define domestic and foreign *nominal wage dispersion* respectively as

$$d_{h,t}^{w} \equiv \int_0^1 \left(\frac{w_t\left(j\right)}{w_t}\right)^{-\psi_t} dj, \quad d_{f,t}^{w} \equiv \int_0^1 \left(\frac{w_t^{*}\left(j\right)}{w_t^{*}}\right)^{-\psi_t^{*}} dj.$$

These state variables evolve according to the laws of motion

$$d_{h,t}^{w} = \theta_w \left(\frac{1}{\pi_{w,t}}\right)^{-\psi_t} d_{h,t-1}^{w} + \left(1-\theta_w\right)\left(\tilde{w}_t\right)^{-\psi_t},$$

$$d_{f,t}^{w} = \theta_w \left(\frac{1}{\pi_{w,t}^{*}}\right)^{-\psi_t^{*}} d_{f,t-1}^{w} + \left(1-\theta_w\right)\left(\tilde{w}_t^{*}\right)^{-\psi_t^{*}},$$

where the domestic and foreign gross *wage inflation* rates have been defined as

$$\pi_{w,t} \equiv \frac{w_t}{w_{t-1}}, \quad \pi_{w,t}^{*} \equiv \frac{w_t^{*}}{w_{t-1}^{*}},$$

and the domestic and foreign *optimal relative wages* are

$$\tilde{w}_t \equiv \frac{\bar{w}_t}{w_t}, \quad \tilde{w}_t^{*} \equiv \frac{\bar{w}_t^{*}}{w_t^{*}}.$$

As outlined in Appendix G, the aggregate output levels are now respectively

$$y_{h,t} = a_t \left(\frac{n_t}{d_{h,t}^{w} d_{h,t}^{p}}\right)^{1-\alpha}, \quad y_{f,t} = a_t^{*} \left(\frac{n_t^{*}}{d_{f,t}^{w} d_{f,t}^{p}}\right)^{1-\alpha}.$$

It is a well-established fact in the literature that the natural allocation cannot be obtained under these conditions, because two distinct sources of inefficiency operate to reduce economic activity: price dispersion and wage dispersion. To restore the level of output that prevails with flexible wages and prices, zero inflation would be needed in both the labour and the goods markets. This, however, would impede the movements of the real wage that are needed to sustain the natural allocation.

Appendix H provides a full list of equilibrium conditions for the economy with sticky wages and prices. Notice that the Taylor rules must prescribe a positive reaction of interest rates to both price and wage inflation now[19].

1.6.2 Equilibrium adjustment to labour supply shocks

I assume that the ψ processes do not spill over across countries, and I set their steady state to coincide with that of the ε processes. The parameters in Table (1.3) below complement those in Table 1.1.

Table 1.3: Additional calibrated parameters

Parameter	Value	Description
θ_w	0.75	Wage stickiness parameter
$\bar{\psi}$	6	Steady-state elasticity of substitution between labour types
ρ_ψ	0.95	Serial correlation of the log of ψ shocks
ν_ψ	0	International spillover of ψ shocks

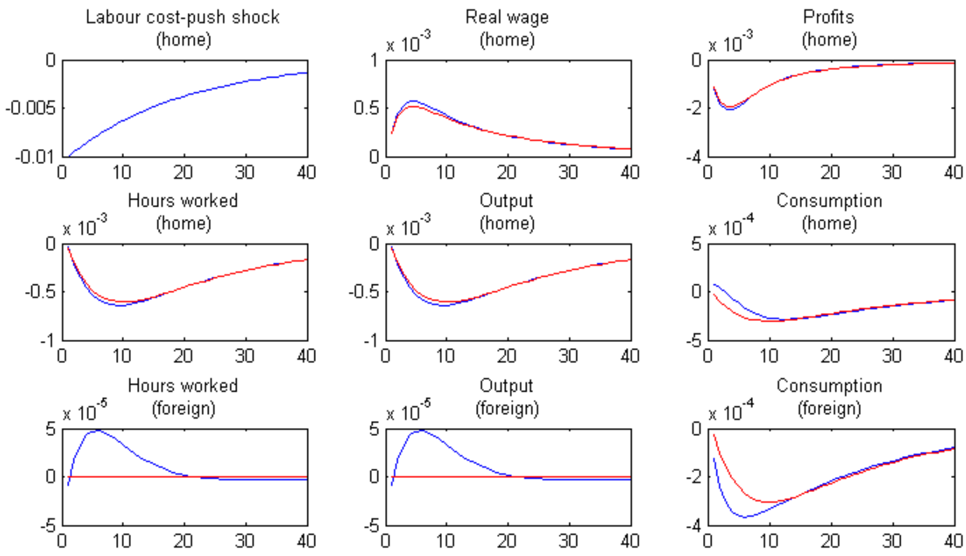
The wage stickiness parameter chosen here implies a four-quarter average duration of wage spells, as opposed to a three-quarter average duration of price spells. Although quick and loose, this textbook calibration captures the fact that wages appear to adjust more sluggishly than prices.

Figure 1.7 displays the IRFs of selected macroeconomic variables to an adverse cost-push shock in the home country's labour market, brought about by a 1% drop in ψ_t. The shock raises real wages and reduces profits in the home country. If external positions are present (blue lines), wealth effects are triggered as usual. If they are absent (red lines), wealth effects do not occur.

The response of real wages is hump-shaped in this economy due to the sluggish adjustment of both nominal wages and prices. As a consequence, aggregate home profits exhibit hump-shaped dynamics too. In the presence of external positions, wealth effects inherit this pattern, and the responses of foreign consumption, hours and output are hump-shaped accordingly.

[19] The Taylor principle requires that the sum of coefficients on all nominal variables exceed one: $\gamma_\pi + \gamma_w > 1$ if $\gamma_y = 0$. A lower sum is sufficient for determinacy when $\gamma_y > 0$.

Figure 1.7: Impulse responses to a labour market cost-push shock with and without external positions (blue and red)



1.6.3 Monetary policy and welfare

If the volatility of cost-push shocks in the labour market is as small as it is in Figure 1.7, that is std $(e_\psi) = 0.01$, the cost of ignoring output fluctuations is small and no clear monetary policy tradeoff emerges: the control of price and wage inflation remains the sole goal of the monetary authority. If cost-push shocks are volatile enough, instead, significant departures of the natural level of output from its efficient counterpart occur, and the stabilisation of economic activity becomes an important additional goal of monetary policy.

Table 1.4 shows what happens to the calibration of the Taylor rules as the size of the external positions varies, under the assumption that std $(e_\psi) =$ std $(e_\varepsilon) = 0.10$. The pattern that emerges is not dissimilar to that observed in Table 1.2. Welfare is monotonically increasing in both γ_π and γ_w, so I constrain both coefficients from exceeding 4. The welfare-maximising γ_y stays constant as external positions range between 0% and 50%, and then declines.

Table 1.4: External positions and Taylor rule parameters under shocks to ψ

External positions	Optimal $(\gamma_R, \gamma_\pi, \gamma_w, \gamma_y)$
0%	$(0, 4, 4, 3)$
25%	$(0, 4, 4, 3)$
50%	$(0, 4, 4, 3)$
75%	$(0, 4, 4, 2.5)$
100%	$(0, 4, 4, 2)$

Whether external positions mitigate or exacerbate the overall volatility of consumption in this model depends on the direction of the wealth effects set into motion by exogenous shocks. This, in turn, depends on the comovement of output and profits, which varies across different types of macroeconomic disturbances.

On one hand, output and profits comove positively after (i) technology shocks and (ii) cost-push shocks in the labour market. External positions exert a stabilising role on consumption in these cases, because they determine *countercyclical* wealth effects: resources are transferred to the foreign country when home output and profits are high, and vice-versa. On the other hand, output and profits comove negatively after (iii) monetary shocks and (iv) cost-push shocks in the goods market. External positions exert a destabilising role on consumption now, because they determine *procyclical* wealth effects: resources are transferred to the foreign country when domestic output is low, because this is associated with high domestic profits.

The output stabilisation objective tends to take lower priority when the volatility of output is small relative to that of consumption; *strict* inflation targeting tends to emerge as the most desirable policy in this case. This is what happens in economies dominated by shocks (iii) and (iv). By the same logic, output stabilisation becomes an important objective of monetary policy in economies dominated by shocks (i) and (ii), where *flexible* inflation targeting tends to be the welfare-maximising policy.

To confirm this intuition, let us increase the relative importance of technology and labour market cost-push shocks by setting

$$\mathrm{std}\,(e_a) = \mathrm{std}\,(e_\psi) = 0.10, \quad \mathrm{std}\,(e_m) = \mathrm{std}\,(e_\varepsilon) = 0.01.$$

Table 1.5 shows that a greater emphasis on output stabilisation does indeed emerge as external positions get larger in this case, because output becomes more volatile than consumption.

Table 1.5: External positions and Taylor rule parameters with larger a and ψ shocks

External positions	Optimal $(\gamma_R, \gamma_\pi, \gamma_w, \gamma_y)$
0%	$(0, 4, 4, 1)$
25%	$(0, 4, 4, 1)$
50%	$(0, 4, 4, 1)$
75%	$(0, 4, 4, 1.5)$
100%	$(0, 4, 4, 1.5)$

Comparing Tables 1.4 and 1.5, we see that in order to draw conclusions about the most appropriate monetary policy mix in the presence of external positions, one must take a stance on the relative importance of the different sources of business cycles. This issue is still the object of an ongoing debate.

Smets and Wouters (2007) investigated the sources of business cycle fluctuations in the United States between 1966 and 2004 in the context of a loglinearised DSGE model with both price and wage stickiness. Their Bayesian methodology pointed to the following posterior mean estimates for the volatilities of the shocks:

$$\text{std}\,(e_a) = 0.45, \quad \text{std}\,(e_m) = 0.24, \quad \text{std}\,(e_\varepsilon) = 0.14, \quad \text{std}\,(e_\psi) = 0.24.$$

If the exogenous processes of the present model were parameterised according to these estimates, the presence of larger external positions would appear to strengthen the case for output stabilisation. The reason for this is intuitive: technology shocks are the overwhelmingly dominant source of output and consumption fluctuations under this calibration[20], so countercyclical wealth effects prevail. As the volatility of consumption is mitigated and that of output is exacerbated by the presence of external positions, the case for flexible inflation targeting is strengthened.

Two notes of caution are in order at this point. First, the Smets and Wouters posterior estimates imply implausibly large volatilities of macroeconomic variables in the context of the present model. More reasonable volatilities would emerge if we calibrated the shocks using the Smets and Wouters priors:

$$\text{std}\,(e_a) = \text{std}\,(e_m) = \text{std}\,(e_\varepsilon) = \text{std}\,(e_\psi) = 0.10.$$

In this case, the welfare-maximising monetary policy mix would appear nearly in-

[20]Domestic technology shocks represent as much as 98% of the variance of output and almost 30% of the variance of consumption in the home country (with the remaining 70% being almost entirely explained by foreign technology shocks).

variant to external positions, due to the flatness of the welfare surface along the γ_y dimension. Second, maximum likelihood estimates of a standard closed economy NKM by Ireland (2004) point to monetary disturbances and cost-push shocks in the goods markets as the main drivers of macroeconomic fluctuations, as opposed to technology shocks. For the reasons explained above, procyclical wealth effects would prevail in this case, and larger external positions would require that central banks adopt weaker output reaction coefficients.

1.7 Concluding remarks

The rise of international financial integration documented by Lane and Milesi-Ferretti (2001, 2006, 2007) has directed attention to the role of external positions in the international transmission of economic shocks. The purpose of the present work is to explore their impact on the conduct of monetary policy, in the context of an explicitly optimisation-based framework.

This chapter shows that if external positions are introduced into an otherwise standard two-country NKM in the form of fixed claims on foreign profits, important international wealth effects on labour supply materialise following technology, monetary and cost-push shocks. These effects reallocate consumption across countries, and cause macroeconomic adjustment to differ from what would be observed in an economy without financial integration.

The key mechanism at work is as follows. As the comovement of aggregate output and profits varies across distinct kinds of disturbances, so does the direction of wealth effects—because these are tied to dividend payments. The procyclicality or countercyclicality of these transfers, in turn, determines whether consumption is stabilised or destabilised relative to output in the presence of external positions. The implication for monetary policy is that central banks should place more emphasis on output stabilisation in the presence of shocks that cause a positive comovement of output and profits (such as shocks to technology or to the wage markup), and less emphasis on such a goal in the presence of shocks that cause a negative comovement of output and profits (such as shocks to interest rates or to the price markup).

Since the choice of the monetary policy mix depends on the relative importance of different macroeconomic disturbances in the presence of external positions, the model stresses the importance of a correct identification of the sources of business cycles for the conduct of monetary policy in a financially integrated world.

Appendices

A - Aggregate production function

Imposing domestic labour market clearing and making use of the production function (1.3) and the demand schedule (1.13) we get

$$\begin{aligned} n_t &= \int_0^1 n_t(i)\, di \\ &= \int_0^1 \left[\frac{y_{h,t}(i)}{a_t}\right]^{\frac{1}{1-\alpha}} di \\ &= \left(\frac{y_{h,t}}{a_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{p_{h,t}(i)}{p_{h,t}}\right)^{-\frac{\varepsilon_t}{1-\alpha}} di. \end{aligned}$$

Define the last integral as in (1.17) and solve for output to get the aggregate production function (1.16).

B - Relative marginal cost

Firm i's real marginal cost and the economy's average real marginal cost are respectively

$$mc_{h,t}(i) = \frac{w_t}{p_{h,t}} \frac{1}{mpn_t(i)}, \qquad mc_{h,t} = \frac{w_t}{p_{h,t}} \frac{1}{mpn_t}.$$

Working out the individual and average marginal products of labour from equations (1.3) and (1.16), we find

$$\begin{aligned} \frac{mc_{h,t}(i)}{mc_{h,t}} &= \frac{mpn_t}{mpn_t(i)} \\ &= \left(\frac{1}{d_{h,t}}\right)^{1-\alpha} \left(\frac{n_t(i)}{n_t}\right)^{\alpha} \\ &= \frac{1}{d_{h,t}} \left(\frac{y_{h,t}(i)}{y_{h,t}}\right)^{\frac{\alpha}{1-\alpha}} \\ &= \frac{1}{d_{h,t}} \left(\frac{p_{h,t}(i)}{p_{h,t}}\right)^{-\frac{\varepsilon_t \alpha}{1-\alpha}}. \end{aligned} \tag{1.27}$$

This equation is useful for expressing the objective functions of price resetters in terms of the economy-wide marginal cost, which facilitates the passage from individual to aggregate price dynamics in the solution of the optimal pricing problem.

An analogous relationship holds abroad:

$$mc_{f,t}(i) = \frac{mc_{f,t}}{d_{f,t}} \left(\frac{p_{f,t}(i)}{p_{f,t}} \right)^{-\frac{\varepsilon_t^* \alpha}{1-\alpha}}.$$

Notice that the real marginal cost becomes constant across firms under constant returns to scale ($\alpha = 0$):

$$mc_{h,t}(i) = \frac{mc_{h,t}}{d_{h,t}}, \qquad mc_{f,t}(i) = \frac{mc_{f,t}}{d_{f,t}}.$$

C - Aggregate profits

Under the production function specification (1.3), the total cost of production of a given variety i in the home country is

$$\begin{aligned}
\Psi(y_{h,t}(i)) &= \frac{w_t}{p_{h,t}} n_t(i) \\
&= \frac{w_t}{p_{h,t}} \left(\frac{y_{h,t}(i)}{a_t} \right)^{\frac{1}{1-\alpha}} \\
\Psi(p_{h,t}(i)) &= \frac{w_t}{p_{h,t}} \left(\frac{y_{h,t}}{a_t} \right)^{\frac{1}{1-\alpha}} \left(\frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\frac{\varepsilon_t}{1-\alpha}}.
\end{aligned}$$

The total aggregate profits of home firms are decreasing in the dispersion of prices:

$$\begin{aligned}
\Pi_{h,t} &\equiv \int_0^1 \Pi_{h,t}(i)\, di \\
&= \int_0^1 y_{h,t}(i) \frac{p_{h,t}(i)}{p_{h,t}} - \Psi(y_{h,t}(i))\, di \\
&= y_{h,t} \int_0^1 \left(\frac{p_{h,t}(i)}{p_{h,t}} \right)^{1-\varepsilon_t} di - \frac{w_t}{p_{h,t}} \int_0^1 n_t(i)\, di \\
&= y_{h,t} - \frac{w_t}{p_{h,t}} \left(\frac{y_{h,t}}{a_t} \right)^{\frac{1}{1-\alpha}} d_{h,t}.
\end{aligned}$$

Likewise, the total cost of production in the foreign country using equation (1.4) is

$$\Psi(p_{f,t}(i)) = \frac{w_t^*}{p_{f,t}} \left(\frac{y_{f,t}}{a_t^*} \right)^{\frac{1}{1-\alpha}} \left(\frac{p_{f,t}(i)}{p_{f,t}} \right)^{-\frac{\varepsilon_t^*}{1-\alpha}}$$

and the total aggregate profits of foreign firms are

$$\begin{aligned}\Pi_{f,t} &\equiv \int_0^1 \Pi_{f,t}(i)\, di \\ &= y_{f,t} - \frac{w_t^*}{p_{f,t}} \left(\frac{y_{f,t}}{a_t^*} \right)^{\frac{1}{1-\alpha}} d_{f,t}.\end{aligned}$$

D - Optimal price setting

By direct substitution of the constraints in the price setter's objective function, the problem becomes

$$\max_{\bar{p}_{h,t}} \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_p \beta)^{\tau} \left(\frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ y_{h,t+\tau} \left(\frac{\bar{p}_{h,t}}{p_{h,t+\tau}} \right)^{1-\varepsilon_t} - \Psi\left(y_{h,t+\tau}(i) \right) \right\}.$$

The FOCs are

$$\begin{aligned}&\mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_p \beta)^{\tau} \left(\frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ (1-\varepsilon_t) \frac{y_{h,t+\tau}}{p_{h,t+\tau}} \left(\frac{\bar{p}_{h,t}}{p_{h,t+\tau}} \right)^{-\varepsilon_t} \right\} \\ &= \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_p \beta)^{\tau} \left(\frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ mc_{h,t+\tau}(i) (-\varepsilon_t) \frac{y_{h,t+\tau}}{p_{h,t+\tau}} \left(\frac{\bar{p}_{h,t}}{p_{h,t+\tau}} \right)^{-\varepsilon_t - 1} \right\}.\end{aligned}$$

We can rewrite these conditions in terms of the economy-wide marginal cost using (1.27):

$$\begin{aligned}&\mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_p \beta)^{\tau} \left(\frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ \frac{y_{h,t+\tau}}{p_{h,t+\tau}} \left(\frac{\bar{p}_{h,t}}{p_{h,t+\tau}} \right)^{-\varepsilon_t} \right\} \\ &= \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_p \beta)^{\tau} \left(\frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ \frac{mc_{h,t+\tau}}{d_{h,t+\tau}} \left(\frac{\varepsilon_t}{\varepsilon_t - 1} \right) \frac{y_{h,t+\tau}}{p_{h,t+\tau}} \left(\frac{\bar{p}_{h,t}}{p_{h,t+\tau}} \right)^{\frac{\alpha - 1 - \varepsilon_t}{1-\alpha}} \right\}.\end{aligned}$$

If we define auxiliary variables

$$g_{h,t}^2 \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_p \beta)^{\tau} \left(\frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ y_{h,t+\tau} \left(\frac{\bar{p}_{h,t}}{p_{h,t}} \right)^{-\varepsilon_t} \left(\frac{p_{h,t}}{p_{h,t+\tau}} \right)^{1-\varepsilon_t} \right\},$$

$$g_{h,t}^1 \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_p \beta)^{\tau} \left(\frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ \frac{mc_{h,t+\tau}}{d_{h,t+\tau}} y_{h,t+\tau} \left(\frac{\bar{p}_{h,t}}{p_{h,t}} \right)^{\frac{\alpha - 1 - \varepsilon_t}{1-\alpha}} \left(\frac{p_{h,t}}{p_{h,t+\tau}} \right)^{-\frac{\varepsilon_t}{1-\alpha}} \right\},$$

we can rewrite the FOCs compactly as

$$g^2_{h,t} = \left(\frac{\varepsilon_t}{\varepsilon_t - 1}\right) g^1_{h,t}.$$

Additional manipulation of these auxiliary variables yields the recursive equations (1.20) and (1.21). Similar conditions hold for the pricing of foreign goods.

E - Equilibrium conditions in real terms

Since nominal variables are not uniquely determined in the present model, we must rewrite the entire system of equilibrium conditions in terms of real variables and relative prices.

Keeping the definitions of optimal relative prices stated in (1.18), we define *real wages* as

$$\mathrm{w}_t \equiv \frac{w_t}{p_t}, \qquad \mathrm{w}^*_t \equiv \frac{w^*_t}{p^*_t},$$

and we rewrite prices in terms of *PPI-to-CPI ratios*:

$$\mathscr{P}_{h,t} \equiv \frac{p_{h,t}}{p_t}, \qquad \mathscr{P}_{f,t} \equiv \frac{p_{f,t}}{p_t}, \qquad \mathscr{P}^*_{h,t} \equiv \frac{p_{h,t}}{p^*_t}, \qquad \mathscr{P}^*_{f,t} \equiv \frac{p_{f,t}}{p^*_t}.$$

Then, we rewrite these as functions of s_t only:

$$\mathscr{P}_{h,t} = \left[(1-\zeta) + (\zeta)(s_t)^{1-\eta}\right]^{\frac{1}{\eta-1}},$$

$$\mathscr{P}_{f,t} = \left[(1-\zeta)\left(\frac{1}{s_t}\right)^{1-\eta} + (\zeta)\right]^{\frac{1}{\eta-1}},$$

$$\mathscr{P}^*_{h,t} = \left[(1-\zeta)(s_t)^{1-\eta} + (\zeta)\right]^{\frac{1}{\eta-1}},$$

$$\mathscr{P}^*_{f,t} = \left[(1-\zeta) + (\zeta)\left(\frac{1}{s_t}\right)^{1-\eta}\right]^{\frac{1}{\eta-1}}.$$

The definitions of the terms of trade and the CPIs are embedded in these four price ratios and need not enter the equilibrium conditions again. The remaining conditions are as follows:

$$c_{h,t} = (1-\zeta)(\mathscr{P}_{h,t})^{-\eta} c_t,$$

$$c_{f,t} = (\zeta)(\mathscr{P}_{f,t})^{-\eta} c_t,$$

$$c_{f,t}^* = (1-\zeta)\left(\mathscr{P}_{f,t}^*\right)^{-\eta} c_t^*,$$

$$c_{h,t}^* = (\zeta)\left(\mathscr{P}_{h,t}^*\right)^{-\eta} c_t^*,$$

$$y_{h,t} = c_{h,t} + c_{h,t}^*,$$

$$y_{f,t} = c_{f,t} + c_{f,t}^*,$$

$$u_c(c_t, n_t) = \lambda_t,$$

$$-\frac{u_n(c_t, n_t)}{\mathrm{w}_t} = \lambda_t,$$

$$q_t = \beta \mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{p_t}{p_{t+1}}\right),$$

$$c_t + \frac{q_t}{p_t}b_{t+1} = \frac{b_t}{p_t} + \mathrm{w}_t n_t + s_h \mathscr{P}_{h,t}\Pi_{h,t} + s_f \mathscr{P}_{f,t}\Pi_{f,t} + \frac{t_t}{p_t},$$

$$u_c(c_t^*, n_t^*) = \lambda_t^*,$$

$$-\frac{u_n(c_t^*, n_t^*)}{\mathrm{w}_t^*} = \lambda_t^*,$$

$$q_t^* = \beta \mathbb{E}_t\left(\frac{\lambda_{t+1}^*}{\lambda_t^*}\frac{p_t^*}{p_{t+1}^*}\right),$$

$$c_t^* + \frac{q_t^*}{p_t^*}b_{t+1}^* = \frac{b_t^*}{p_t^*} + \mathrm{w}_t^* n_t^* + (1-s_h)\,\mathscr{P}_{h,t}^*\Pi_{h,t} + (1-s_f)\,\mathscr{P}_{f,t}^*\Pi_{f,t} + \frac{t_t^*}{p_t^*},$$

$$y_{h,t} = a_t\left(\frac{n_t}{d_{h,t}}\right)^{1-\alpha},$$

$$y_{f,t} = a_t^*\left(\frac{n_t^*}{d_{f,t}}\right)^{1-\alpha},$$

$$mc_{h,t} = \frac{\mathrm{w}_t}{\mathscr{P}_{h,t}}\frac{1}{(1-\alpha)\,a_t}(d_{h,t})^{1-\alpha}(n_t)^{\alpha},$$

$$mc_{f,t} = \frac{\mathrm{w}_t^*}{\mathscr{P}_{f,t}^*}\frac{1}{(1-\alpha)\,a_t^*}(d_{f,t})^{1-\alpha}(n_t^*)^{\alpha},$$

$$d_{h,t} = \theta_p(\pi_{h,t})^{\frac{\varepsilon_t}{1-\alpha}} d_{h,t-1} + (1-\theta_p)(\tilde{p}_{h,t})^{-\frac{\varepsilon_t}{1-\alpha}},$$

$$d_{f,t} = \theta_p(\pi_{f,t})^{\frac{\varepsilon_t^*}{1-\alpha}} d_{f,t-1} + (1-\theta_p)(\tilde{p}_{f,t})^{-\frac{\varepsilon_t^*}{1-\alpha}},$$

$$\tilde{p}_{h,t} = \left[\frac{1-\theta_p(\pi_{h,t})^{\varepsilon_t-1}}{1-\theta_p}\right]^{\frac{1}{1-\varepsilon_t}},$$

$$\tilde{p}_{f,t} = \left[\frac{1-\theta_p\left(\pi_{f,t}\right)^{\varepsilon_t^*-1}}{1-\theta_p}\right]^{\frac{1}{1-\varepsilon_t^*}},$$

$$\pi_{h,t} = \frac{\mathscr{P}_{h,t}}{\mathscr{P}_{h,t-1}}\pi_t,$$

$$\pi_{f,t} = \frac{\mathscr{P}_{f,t}}{\mathscr{P}_{f,t-1}}\pi_t^*,$$

$$g_{h,t}^2 = \mathcal{M}_{p,t}g_{h,t}^1,$$

$$\mathcal{M}_{p,t} = \frac{\varepsilon_t}{\varepsilon_t - 1},$$

$$g_{h,t}^1 = y_{h,t}\frac{mc_{h,t}}{d_{h,t}}\left(\tilde{p}_{h,t}\right)^{\frac{\alpha-1-\varepsilon_t}{1-\alpha}} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_{h,t+1}}\right)^{\frac{\alpha-1-\varepsilon_t}{1-\alpha}}\left(\pi_{h,t+1}\right)^{\frac{\varepsilon_t}{1-\alpha}}g_{h,t+1}^1,$$

$$g_{h,t}^2 = y_{h,t}\left(\tilde{p}_{h,t}\right)^{-\varepsilon_t} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_{h,t+1}}\right)^{-\varepsilon_t}\left(\pi_{h,t+1}\right)^{\varepsilon_t-1}g_{h,t+1}^2,$$

$$g_{f,t}^2 = \mathcal{M}_{p,t}^*g_{f,t}^1,$$

$$\mathcal{M}_{p,t}^* = \frac{\varepsilon_t^*}{\varepsilon_t^* - 1},$$

$$g_{f,t}^1 = y_{f,t}\frac{mc_{f,t}}{d_{f,t}}\left(\tilde{p}_{f,t}\right)^{\frac{\alpha-1-\varepsilon_t^*}{1-\alpha}} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}^*}{\lambda_t^*}\right)\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_{f,t+1}}\right)^{\frac{\alpha-1-\varepsilon_t^*}{1-\alpha}}\left(\pi_{f,t+1}\right)^{\frac{\varepsilon_t^*}{1-\alpha}}g_{f,t+1}^1,$$

$$g_{f,t}^2 = y_{f,t}\left(\tilde{p}_{f,t}\right)^{-\varepsilon_t^*} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}^*}{\lambda_t^*}\right)\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_{f,t+1}}\right)^{-\varepsilon_t^*}\left(\pi_{f,t+1}\right)^{\varepsilon_t^*-1}g_{f,t+1}^2,$$

$$R_t = \frac{1}{q_t},$$

$$R_t^* = \frac{1}{q_t^*},$$

$$\frac{R_t}{\pi/\beta} = \left(\frac{R_{t-1}}{\pi/\beta}\right)^{\gamma_R}\left[\left(\frac{\pi_{h,t}}{\pi_h}\right)^{\gamma_\pi}\left(\frac{y_{h,t}}{y_{h,t-1}}\right)^{\gamma_y}\right]^{1-\gamma_R}m_t,$$

$$\frac{R_t^*}{\pi^*/\beta} = \left(\frac{R_{t-1}^*}{\pi^*/\beta}\right)^{\gamma_R}\left[\left(\frac{\pi_{f,t}}{\pi_f}\right)^{\gamma_\pi}\left(\frac{y_{f,t}}{y_{f,t-1}}\right)^{\gamma_y}\right]^{1-\gamma_R}m_t^*,$$

$$\log m_t = \rho_m \log m_{t-1} + \nu_m \log m_{t-1}^* + e_{m,t},$$

$$\log m_t^* = \rho_m \log m_{t-1}^* + \nu_m \log m_{t-1} + e_{m,t}^*,$$

$$\log a_t = \rho_a \log a_{t-1} + \nu_a \log a^*_{t-1} + e_{a,t},$$

$$\log a^*_t = \rho_a \log a^*_{t-1} + \nu_a \log a_{t-1} + e^*_{a,t},$$

$$\log \varepsilon_t = \rho_\varepsilon \log \varepsilon_{t-1} + \nu_\varepsilon \log \varepsilon^*_{t-1} + e_{\varepsilon,t},$$

$$\log \varepsilon^*_t = \rho_\varepsilon \log \varepsilon^*_{t-1} + \nu_\varepsilon \log \varepsilon_{t-1} + e^*_{\varepsilon,t},$$

$$t_t = q_t b_{t+1} - b_t,$$

$$t^*_t = q^*_t b^*_{t+1} - b^*_t,$$

$$b_t = 0,$$

$$b^*_t = 0,$$

$$\Pi_{h,t} = y_{h,t} - \frac{\mathrm{w}_t}{\mathscr{P}_{h,t}} \left(\frac{y_{h,t}}{a_t} \right)^{\frac{1}{1-\alpha}} d_{h,t},$$

$$\Pi_{f,t} = y_{f,t} - \frac{\mathrm{w}^*_t}{\mathscr{P}^*_{f,t}} \left(\frac{y_{f,t}}{a^*_t} \right)^{\frac{1}{1-\alpha}} d_{f,t},$$

$$V_{h,t} = u\left(c_t, n_t\right) + \beta \mathbb{E}_t V_{h,t+1},$$

$$V_{f,t} = u\left(c^*_t, n^*_t\right) + \beta \mathbb{E}_t V_{f,t+1}.$$

F - Optimal wage setting

Since nominal variables are not uniquely defined, the optimal wage setting problem must be rewritten in terms of real variables.

To begin with, let us write the labour demand schedule (1.23), the aggregate wage index and the household's budget constraint in real terms:

$$n_t\left(j\right) = \left(\frac{\mathrm{w}_t\left(j\right)}{\mathrm{w}_t} \right)^{-\psi_t} n_t,$$

$$\mathrm{w}_t = \left(\int_0^1 \mathrm{w}_t\left(j\right)^{1-\psi_t} di \right)^{\frac{1}{1-\psi_t}},$$

$$\begin{aligned} c_{t+\tau} + \frac{q_{t+\tau}}{p_{t+\tau}} b_{t+\tau+1} =& \frac{b_{t+\tau}}{p_{t+\tau}} + \int_0^1 \frac{\bar{w}_t\left(j\right)}{p_{t+\tau}} n_{t+\tau}\left(j\right) \\ &+ s_h \frac{p_{h,t+\tau}}{p_{t+\tau}} \Pi_{h,t+\tau} + s_f \frac{p_{f,t+\tau}}{p_{t+\tau}} \Pi_{f,t+\tau} + \frac{t_{t+\tau}}{p_{t+\tau}}. \end{aligned} \tag{1.28}$$

Since workers supply any quantity of labour that satisfies demand at the wage charged, the hours worked at time $t+\tau$ by a worker who has been unable to reset his wage since time t are as follows:

$$
\begin{aligned}
n_{t+\tau}(j) &= \left(\frac{\bar{w}_t(j)}{w_{t+\tau}}\right)^{-\psi_t} n_{t+\tau} \\
&= \left(\frac{\bar{\mathrm{w}}_t(j)\prod_{s=1}^{\tau}\frac{1}{\pi_{t+s}}}{\mathrm{w}_{t+\tau}}\right)^{-\psi_t} n_{t+\tau}.
\end{aligned} \tag{1.29}
$$

By direct substitution of (1.29) into (1.28) we obtain the Lagrangian of the wage setting problem:

$$
\begin{aligned}
\mathcal{L} = \mathbb{E}_t \sum_{\tau=0}^{\infty} & (\beta\theta_w)^{\tau} u(c_{t+\tau}, n_{t+\tau}) \\
&+ (\beta\theta_w)^{\tau} \lambda_{t+\tau}\{\frac{b_{t+\tau}}{p_{t+\tau}} + \left(\prod_{s=1}^{\tau}\frac{1}{\pi_{t+s}}\right)^{1-\psi_t} \int_0^1 \bar{\mathrm{w}}_t(j)^{1-\psi_t} n_{t+\tau} (\mathrm{w}_{t+\tau})^{\psi_t} dj \\
&+ s_h \mathscr{P}_{h,t+\tau}\Pi_{h,t+\tau} + s_f \mathscr{P}_{f,t+\tau}\Pi_{f,t+\tau} + \frac{t_{t+\tau}}{p_{t+\tau}} - c_{t+\tau} - \frac{q_{t+\tau}}{p_{t+\tau}} b_{t+\tau+1}\}.
\end{aligned}
$$

Assuming full consumption risk sharing across workers, the cost of supplying work is identical across labour types. Since the labour demand schedule is the same across labour types, we can focus on a symmetric equilibrium where all resetters choose the same wage $\bar{\mathrm{w}}_t$.

The first-order conditions are

$$
\begin{aligned}
&\mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^{\tau} u_n(c_{t+\tau}, n_{t+\tau}) n_{t+\tau} (-\psi_t) (\mathrm{w}_{t+\tau})^{\psi_t} (\bar{\mathrm{w}}_t)^{-\psi_t-1} \left(\prod_{s=1}^{\tau}\frac{1}{\pi_{t+s}}\right)^{-\psi_t} \\
&+ \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^{\tau} \lambda_{t+\tau} \left(\prod_{s=1}^{\tau}\frac{1}{\pi_{t+s}}\right)^{1-\psi_t} n_{t+\tau} (\mathrm{w}_{t+\tau})^{\psi_t} (1-\psi_t) (\bar{\mathrm{w}}_t)^{-\psi_t} = 0.
\end{aligned}
$$

These can be rewritten compactly by means of auxiliary variables:

$$
f_{h,t}^1 \equiv \bar{\mathrm{w}}_t \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^{\tau} \lambda_{t+\tau} \left(\prod_{s=1}^{\tau}\frac{1}{\pi_{t+s}}\right)^{1-\psi_t} n_{t+\tau} \left(\frac{\bar{\mathrm{w}}_t}{\mathrm{w}_{t+\tau}}\right)^{-\psi_t},
$$

$$
f_{h,t}^2 \equiv -\mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^{\tau} u_n(c_{t+\tau}, n_{t+\tau}) \left(\prod_{s=1}^{\tau}\frac{1}{\pi_{t+s}}\right)^{-\psi_t} n_{t+\tau} \left(\frac{\bar{\mathrm{w}}_t}{\mathrm{w}_{t+\tau}}\right)^{-\psi_t},
$$

$$f^1_{h,t} = \left(\frac{\psi_t}{\psi_t - 1}\right) f^2_{h,t}.$$

Additional manipulation yields equations (1.25) and (1.26). Analogous conditions hold abroad.

G - Aggregation under sticky wages: technology, hours and output

The aggregate supply of labour is found by integrating the hours purchased by employment agencies over labour types j:

$$\begin{aligned} n^s_t &= \int_0^1 n_t(j)\, dj \\ &= \int_0^1 \left(\frac{w_t(j)}{w_t}\right)^{-\psi_t} n^d_t dj \\ &= d^w_{h,t} n^d_t. \end{aligned}$$

Notice that n^s_t depends on the aggregation technology adopted by the labour packers, and it includes a first source of inefficiency: nominal wages dispersion.

The aggregate demand for labour, in turn, is found by integrating individual demands for composite labour services over intermediate goods producers i:

$$\begin{aligned} n^d_t &= \int_0^1 n^d_t(i)\, di \\ &= \int_0^1 \left(\frac{y_{h,t}(i)}{a_t}\right)^{\frac{1}{1-\alpha}} di \\ &= \left(\frac{y_{h,t}}{a_t}\right)^{\frac{1}{1-\alpha}} d^p_{h,t}. \end{aligned}$$

n^d_t depends on the aggregation technology adopted by the producers of final goods, and includes a second source of inefficiency: price dispersion across varieties of intermediate goods.

Imposing the equality of labour demand and supply, we obtain the exact relationship between aggregate output, employment and technology in this environment:

$$n_t = d^w_{h,t} \left(\frac{y_{h,t}}{a_t}\right)^{\frac{1}{1-\alpha}} d^p_{h,t}.$$

Accordingly, we make use of the following relative marginal cost relationship to rewrite the optimal price setting problem of the firms in terms of aggregate variables:

$$\frac{mc_{h,t}(i)}{mc_{h,t}} = \frac{1}{d^w_{h,t} d^p_{h,t}} \left(\frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\frac{\varepsilon_t \alpha}{1-\alpha}}.$$

H - Equilibrium conditions under sticky wages

Keeping the definitions of optimal relative prices, real wages and PPI-to-CPI ratios stated in Appendix E, we get the following set of equilibrium conditions written in real terms:

$$\mathscr{P}_{h,t} = \left[(1-\zeta) + (\zeta)(s_t)^{1-\eta} \right]^{\frac{1}{\eta-1}},$$

$$\mathscr{P}_{f,t} = \left[(1-\zeta) \left(\frac{1}{s_t} \right)^{1-\eta} + (\zeta) \right]^{\frac{1}{\eta-1}},$$

$$\mathscr{P}^*_{h,t} = \left[(1-\zeta)(s_t)^{1-\eta} + (\zeta) \right]^{\frac{1}{\eta-1}},$$

$$\mathscr{P}^*_{f,t} = \left[(1-\zeta) + (\zeta) \left(\frac{1}{s_t} \right)^{1-\eta} \right]^{\frac{1}{\eta-1}},$$

$$c_{h,t} = (1-\zeta)(\mathscr{P}_{h,t})^{-\eta} c_t,$$

$$c_{f,t} = (\zeta)(\mathscr{P}_{f,t})^{-\eta} c_t,$$

$$c^*_{f,t} = (1-\zeta) \left(\mathscr{P}^*_{f,t} \right)^{-\eta} c^*_t,$$

$$c^*_{h,t} = (\zeta) \left(\mathscr{P}^*_{h,t} \right)^{-\eta} c^*_t,$$

$$y_{h,t} = c_{h,t} + c^*_{h,t},$$

$$y_{f,t} = c_{f,t} + c^*_{f,t},$$

$$u_c(c_t, n_t) = \lambda_t,$$

$$f^1_{h,t} = \mathcal{M}_{w,t} f^2_{h,t},$$

$$f^1_{h,t} = \bar{\mathrm{w}}_t \left(\frac{\mathrm{w}_t}{\bar{\mathrm{w}}_t} \right)^{\psi_t} \lambda_t n_t + \beta \theta_w \mathbb{E}_t (\pi_{t+1})^{\psi_t - 1} \left(\frac{\bar{\mathrm{w}}_t}{\bar{\mathrm{w}}_{t+1}} \right)^{1-\psi_t} f^1_{h,t+1},$$

$$f^2_{h,t} = -u_n(c_t, n_t) n_t \left(\frac{\bar{\mathrm{w}}_t}{\mathrm{w}_t} \right)^{-\psi_t} + \beta \theta_w \mathbb{E}_t (\pi_{t+1})^{\psi_t} \left(\frac{\bar{\mathrm{w}}_t}{\bar{\mathrm{w}}_{t+1}} \right)^{-\psi_t} f^2_{h,t+1},$$

$$\mathcal{M}_{w,t} = \frac{\psi_t}{\psi_t - 1},$$

$$q_t = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \right),$$

$$c_t + \frac{q_t}{p_t} b_{t+1} = \frac{b_t}{p_t} + \mathrm{w}_t n_t + s_h \mathscr{P}_{h,t} \Pi_{h,t} + s_f \mathscr{P}_{f,t} \Pi_{f,t} + \frac{t_t}{p_t},$$

$$u_c \left(c_t^*, n_t^* \right) = \lambda_t^*,$$

$$f_{f,t}^1 = \mathcal{M}_{w,t}^* f_{f,t}^2,$$

$$f_{f,t}^1 = \bar{\mathrm{w}}_t^* \left(\frac{\mathrm{w}_t^*}{\bar{\mathrm{w}}_t^*} \right)^{\psi_t^*} \lambda_t^* n_t^* + \beta \theta_w \mathbb{E}_t \left(\pi_{t+1}^* \right)^{\psi_t^* - 1} \left(\frac{\bar{\mathrm{w}}_t^*}{\bar{\mathrm{w}}_{t+1}^*} \right)^{1-\psi_t^*} f_{f,t+1}^1,$$

$$f_{f,t}^2 = -u_n \left(c_t^*, n_t^* \right) n_t^* \left(\frac{\bar{\mathrm{w}}_t^*}{\mathrm{w}_t^*} \right)^{-\psi_t^*} + \beta \theta_w \mathbb{E}_t \left(\pi_{t+1}^* \right)^{\psi_t^*} \left(\frac{\bar{\mathrm{w}}_t^*}{\bar{\mathrm{w}}_{t+1}^*} \right)^{-\psi_t^*} f_{f,t+1}^2,$$

$$\mathcal{M}_{w,t}^* = \frac{\psi_t^*}{\psi_t^* - 1},$$

$$q_t^* = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{1}{\pi_{t+1}^*} \right),$$

$$c_t^* + \frac{q_t^*}{p_t^*} b_{t+1}^* = \frac{b_t^*}{p_t^*} + \mathrm{w}_t^* n_t^* + (1 - s_h) \mathscr{P}_{h,t}^* \Pi_{h,t} + (1 - s_f) \mathscr{P}_{f,t}^* \Pi_{f,t} + \frac{t_t^*}{p_t^*},$$

$$y_{h,t} = a_t \left(\frac{n_t}{d_{h,t}^w d_{h,t}^p} \right)^{1-\alpha},$$

$$y_{f,t} = a_t^* \left(\frac{n_t^*}{d_{f,t}^w d_{f,t}^p} \right)^{1-\alpha},$$

$$mc_{h,t} = \frac{\mathrm{w}_t}{\mathscr{P}_{h,t}} \frac{1}{(1-\alpha) a_t} \left(d_{h,}^w d_{h,t}^p \right)^{1-\alpha} (n_t)^\alpha,$$

$$mc_{f,t} = \frac{\mathrm{w}_t^*}{\mathscr{P}_{f,t}^*} \frac{1}{(1-\alpha) a_t^*} \left(d_{f,t}^w d_{f,t}^p \right)^{1-\alpha} (n_t^*)^\alpha,$$

$$d_{h,t}^w = \theta_w \left(\frac{\mathrm{w}_{t-1}}{\mathrm{w}_t} \right)^{-\psi_t} (\pi_t)^{\psi_t} d_{h,t-1}^w + (1-\theta_w) \left(\frac{\bar{\mathrm{w}}_t}{\mathrm{w}_t} \right)^{-\psi_t},$$

$$d_{f,t}^w = \theta_w \left(\frac{\mathrm{w}_{t-1}^*}{\mathrm{w}_t^*} \right)^{-\psi_t^*} (\pi_t^*)^{\psi_t^*} d_{f,t-1}^w + (1-\theta_w) \left(\frac{\bar{\mathrm{w}}_t^*}{\mathrm{w}_t^*} \right)^{-\psi_t^*},$$

$$\frac{\bar{\mathrm{w}}_t}{\mathrm{w}_t} = \left[\frac{1 - \theta_w \left(\pi_{w,t} \pi_t \right)^{\psi_t - 1}}{1 - \theta_w} \right]^{\frac{1}{1-\psi_t}},$$

$$\frac{\bar{\mathrm{w}}_t^*}{\mathrm{w}_t^*}=\left[\frac{1-\theta_w\left(\pi_{w,t}^*\pi_t^*\right)^{\psi_t^*-1}}{1-\theta_w}\right]^{\frac{1}{1-\psi_t^*}},$$

$$d_{h,t}=\theta_p\left(\pi_{h,t}\right)^{\frac{\varepsilon_t}{1-\alpha}}d_{h,t-1}+\left(1-\theta_p\right)\left(\tilde{p}_{h,t}\right)^{-\frac{\varepsilon_t}{1-\alpha}},$$

$$d_{f,t}=\theta_p\left(\pi_{f,t}\right)^{\frac{\varepsilon_t^*}{1-\alpha}}d_{f,t-1}+\left(1-\theta_p\right)\left(\tilde{p}_{f,t}\right)^{-\frac{\varepsilon_t^*}{1-\alpha}},$$

$$\tilde{p}_{h,t}=\left[\frac{1-\theta_p\left(\pi_{h,t}\right)^{\varepsilon_t-1}}{1-\theta_p}\right]^{\frac{1}{1-\varepsilon_t}},$$

$$\tilde{p}_{f,t}=\left[\frac{1-\theta_p\left(\pi_{f,t}\right)^{\varepsilon_t^*-1}}{1-\theta_p}\right]^{\frac{1}{1-\varepsilon_t^*}},$$

$$\pi_{h,t}=\frac{\mathscr{P}_{h,t}}{\mathscr{P}_{h,t-1}}\pi_t,$$

$$\pi_{f,t}=\frac{\mathscr{P}_{f,t}}{\mathscr{P}_{f,t-1}}\pi_t^*,$$

$$g_{h,t}^2=\mathcal{M}_{p,t}g_{h,t}^1,$$

$$g_{h,t}^1=y_{h,t}\frac{mc_{h,t}}{d_{h,t}^w d_{h,t}^p}\left(\tilde{p}_{h,t}\right)^{\frac{\alpha-1-\varepsilon_t}{1-\alpha}}+\theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_{h,t+1}}\right)^{\frac{\alpha-1-\varepsilon_t}{1-\alpha}}\left(\pi_{h,t+1}\right)^{\frac{\varepsilon_t}{1-\alpha}}g_{h,t+1}^1,$$

$$g_{h,t}^2=y_{h,t}\left(\tilde{p}_{h,t}\right)^{-\varepsilon_t}+\theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_{h,t+1}}\right)^{-\varepsilon_t}\left(\pi_{h,t+1}\right)^{\varepsilon_t-1}g_{h,t+1}^2,$$

$$\mathcal{M}_{p,t}=\frac{\varepsilon_t}{\varepsilon_t-1},$$

$$g_{f,t}^2=\mathcal{M}_{p,t}^*g_{f,t}^1,$$

$$g_{f,t}^1=y_{f,t}\frac{mc_{f,t}}{d_{f,t}^w d_{f,t}^p}\left(\tilde{p}_{f,t}\right)^{\frac{\alpha-1-\varepsilon_t^*}{1-\alpha}}+\theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}^*}{\lambda_t^*}\right)\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_{f,t+1}}\right)^{\frac{\alpha-1-\varepsilon_t^*}{1-\alpha}}\left(\pi_{f,t+1}\right)^{\frac{\varepsilon_t^*}{1-\alpha}}g_{f,t+1}^1,$$

$$g_{f,t}^2=y_{f,t}\left(\tilde{p}_{f,t}\right)^{-\varepsilon_t^*}+\theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}^*}{\lambda_t^*}\right)\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_{f,t+1}}\right)^{-\varepsilon_t^*}\left(\pi_{f,t+1}\right)^{\varepsilon_t^*-1}g_{f,t+1}^2,$$

$$\mathcal{M}_{p,t}^*=\frac{\varepsilon_t^*}{\varepsilon_t^*-1},$$

$$R_t=\frac{1}{q_t},$$

$$R_t^* = \frac{1}{q_t^*},$$

$$\frac{R_t}{\pi/\beta} = \left(\frac{R_{t-1}}{\pi/\beta}\right)^{\gamma_R} \left[\left(\frac{\pi_{h,t}}{\pi_h}\right)^{\gamma_\pi} \left(\frac{\pi_{w,t}}{\pi_w}\right)^{\gamma_w} \left(\frac{y_{h,t}}{y_{h,t-1}}\right)^{\gamma_y}\right]^{1-\gamma_R} m_t,$$

$$\frac{R_t^*}{\pi^*/\beta} = \left(\frac{R_{t-1}^*}{\pi^*/\beta}\right)^{\gamma_R} \left[\left(\frac{\pi_{f,t}}{\pi_f}\right)^{\gamma_\pi} \left(\frac{\pi_{w,t}^*}{\pi_w^*}\right)^{\gamma_w} \left(\frac{y_{f,t}}{y_{f,t-1}}\right)^{\gamma_y}\right]^{1-\gamma_R} m_t^*,$$

$$\pi_{w,t} = \frac{\mathrm{w}_{t-1}}{\mathrm{w}_t}\frac{1}{\pi_t},$$

$$\pi_{w,t}^* = \frac{\mathrm{w}_{t-1}^*}{\mathrm{w}_t^*}\frac{1}{\pi_t^*},$$

$$\log m_t = \rho_m \log m_{t-1} + \nu_m \log m_{t-1}^* + e_{m,t},$$

$$\log m_t^* = \rho_m \log m_{t-1}^* + \nu_m \log m_{t-1} + e_{m,t}^*,$$

$$\log a_t = \rho_a \log a_{t-1} + \nu_a \log a_{t-1}^* + e_{a,t},$$

$$\log a_t^* = \rho_a \log a_{t-1}^* + \nu_a \log a_{t-1} + e_{a,t}^*,$$

$$\log \varepsilon_t = \rho_\varepsilon \log \varepsilon_{t-1} + \nu_\varepsilon \log \varepsilon_{t-1}^* + e_{\varepsilon,t},$$

$$\log \varepsilon_t^* = \rho_\varepsilon \log \varepsilon_{t-1}^* + \nu_\varepsilon \log \varepsilon_{t-1} + e_{\varepsilon,t}^*,$$

$$\log \psi_t = \rho_\psi \log \psi_{t-1} + \nu_\psi \log \psi_{t-1}^* + e_{\psi,t},$$

$$\log \psi_t^* = \rho_\psi \log \psi_{t-1}^* + \nu_\psi \log \psi_{t-1} + e_{\psi,t}^*,$$

$$t_t = q_t b_{t+1} - b_t,$$

$$t_t^* = q_t^* b_{t+1}^* - b_t^*,$$

$$b_t = 0,$$

$$b_t^* = 0,$$

$$\Pi_{h,t} = y_{h,t} - \frac{\mathrm{w}_t}{\mathscr{P}_{h,t}}\left(\frac{y_{h,t}}{a_t}\right)^{\frac{1}{1-\alpha}} d_{h,t}^w d_{h,t}^p,$$

$$\Pi_{f,t} = y_{f,t} - \frac{\mathrm{w}_t^*}{\mathscr{P}_{f,t}^*}\left(\frac{y_{f,t}}{a_t^*}\right)^{\frac{1}{1-\alpha}} d_{f,t}^w d_{f,t}^p.$$

Chapter 2

Monetary Policy and Welfare in a Currency Union

2.1 Introduction

The fundamental question of the gains and losses from monetary integration has been of practical importance for decades in Europe[1]. One aspect of monetary unification that has attracted particular attention from economists is the issue of how countries can handle idiosyncratic disturbances and asymmetric business cycles under a common monetary policy[2]. This point is of particular concern to Eurozone members today, as they have surrendered independent interest-rate and exchange-rate policies and are left with a very limited capacity to implement countercyclical fiscal policies[3].

Since the European Monetary Union (EMU) currently lacks both a formal system of interstate insurance (the so-called fiscal or transfer union) and a full degree of internal labour mobility—two crucial elements for the viability of a union according to the optimum currency area theory—its adjustment to macroeconomic shocks is characterised by cross-country heterogeneity. The varied pattern of responses to the recent financial crisis is a case in point. Some Eurozone members have suffered a sharper and longer lasting recession than others: quarterly data from the Federal Reserve Bank of St. Louis show that at the depth of the recession in the first quarter of 2009 real gross domestic product (GDP) fell by 7.4% in Italy, 13.5% in Spain, 5.7%

[1] Corden (1972) and Ingram (1973) are early academic discussions of these themes from a European monetary integration perspective; they have antecedents in the pathbreaking theoretical contributions by Mundell (1961) and McKinnon (1963). Several waves of empirical and theoretical research have followed these studies; a recent review of this literature is Santos-Silva and Tenreyro (2010).

[2] Obstfeld and Peri (1998) and Fatás (1998) are notable contributions to this debate.

[3] See the remarks by Feldstein (2015) and others at the 2015 AEA Annual Meetings session entitled "When will the Euro crisis end?".

in France and 6% in Germany. Seven years after the global financial crisis broke out, output in Portugal, Italy, Greece, Spain and Ireland was still below its pre-crisis level, while this was no longer the case for Germany, as observed by Frankel (2015). Unequal developments in real economic activity are associated with heterogeneous price dynamics: according to Eurostat data, the annual inflation rate was 1% in Austria and -1% in Spain as of February 2016[4], for instance. These differences are problematic from the point of view of the central monetary authority, because they imply adverse cross-country differentials in real interest rates in the face of a uniform nominal interest rate at the union level.

In this chapter I contribute to the debate on the desirability of currency unions by constructing and estimating a dynamic stochastic general equilibrium (DSGE) model that quantifies the cost of abandoning an independent monetary policy; I use this framework to evaluate what reduction in transaction frictions a monetary union should guarantee in order to offset this loss. The distinctive feature of my work is that it considers different dimensions along which an economy with a single currency differs from one with many. In particular, it includes three competing effects of monetary unification: (i) the loss of monetary policy independence caused by the establishment of a unique central bank; (ii) the elimination of the price misalignments associated with nominal rigidities in local currencies; (iii) the expansion of trade enabled by the use of a single currency in international transactions. I model these effects in a unified framework characterised by international heterogeneity and imperfect insurance, and appraise their importance from the point of view of social welfare. I estimate my model of a monetary union with data from Italy, France, Germany and Spain, I evaluate welfare using the utility of households as a criterion, and I run counterfactual scenarios to assess what welfare would be if these countries had separate national currencies and independent monetary authorities. Under a calibration of trade frictions in line with the existing literature, I find that these economies enjoy a higher welfare if they are in a union. I show that this result is given by the nature of their business cycles, which are characterised by fairly positive comovements of the main macroeconomic aggregates, as well as by the strength of the transaction frictions that would affect commerce without a common currency. Finally, I seek the critical amount of trade frictions that equates welfare across alternative monetary arrangements, and I find it to be quite small.

To gain a deeper understanding of the mechanisms that generate the results, let

[4] Eurostat measures the annual rate of inflation as the change in the harmonised index of consumer prices between a given month and the same month of the previous year.

us examine the three effects above more closely and see how they affect welfare. On one hand, giving up the ability to set monetary policy at a national level is costly if business cycles are asynchronous across countries, because separate instruments should be used to stabilise them. The welfare cost of conducting countercyclical policy with a single instrument is a differential measure: it does not only depend on the volatility of macroeconomic shocks (which determines the absolute cost of business cycles), but also on how correlated these shocks are across countries. For this reason, the cost vanishes if business cycles are perfectly symmetric: in that case, the economies behave like one and distinct policy instruments are unnecessary. What makes this welfare cost non-trivial in the presence of imperfectly correlated disturbances is, in general, the lack of international risk sharing and the existence of nominal and real frictions in the economy. This is the first aspect of monetary integration I have identified above. On the other hand, the adoption of a single currency has two advantages. If producers set the prices of their goods in the currency of the buyers, freely floating exchange rates are associated with inefficient short-run price differentials across borders. Introducing a unique currency removes this problem, because it impedes such misalignments and creates a single market where products are sold at a uniform price. This is the second consequence of monetary integration I have determined above. On top of this, the use of a single currency in international commerce eliminates the transaction costs associated with the presence of multiple currencies, and suppresses a monetary barrier to trade; this improves consumption permanently. This is the third and last implication I have indicated above.

The net balance of these effects depends on their quantitative importance for welfare. To assess this, I model them in a fully-optimising setup based on the utility of households. In contrast with existing microfounded studies that analyse different aspects of monetary unification separately[5], I bring them together in a unified framework; this allows me to weigh up the costs and benefits of alternative currency arrangements and examine the conditions under which they are equalised. Differently from earlier works on international monetary regimes, I take a quantitative approach and base my analysis on a model economy whose parameters are

[5] Bacchetta and van Wincoop (2000), Ching and Devereux (2003) and Devereux, Engel, and Tille (2003) are important studies of the consequences of monetary unification based on microeconomic foundations. They focus on its welfare implications based on the effects on trade, risk sharing and price setting respectively. The first two works are based on static models, which are not amenable to estimation; the third one uses an infinite-horizon economy with complete international asset markets and full risk sharing.

estimated with real-world data.

I cast my analysis in a setup with incomplete markets, local currency pricing and monetary barriers to trade. The backbone of my model is an otherwise standard open economy New Keynesian framework with nominal rigidities and monopolistic competition in the spirit of Clarida, Galí, and Gertler (2002). It features two ex-ante identical economies that experience idiosyncratic technology, preference, labour supply and monetary shocks; the problem of ex-post heterogeneity is made explicit through the examination of how welfare is affected by the international comovement of these disturbances. The mechanisms at work in the model have antecedents in the two-region representations of monetary unions by Benigno (2004), Beetsma and Jensen (2005) and Ferrero (2009)[6]. However, while these works consider model economies where union members fully share risk—either via complete asset markets or by specific parameterisations of the elasticities of substitution that guarantee endogenous risk sharing through the terms of trade—I study an environment where international risk sharing is incomplete[7].

One of the most relevant components of my model economy is its asset structure, which follows the characterisation of Benigno (2009). Markets are complete at the level of individual countries, but international trade in financial assets is limited to nominally risk-free bonds. This guarantees that the model has a simple representative agent formulation, while departures from full international risk sharing occur over the business cycle. Unlike Benigno (2009), I compare and rank two different international monetary arrangements. In the first one, separate national currencies exist and monetary policy is controlled by two distinct authorities; these set nominal interest rates on an independent basis, following Taylor-type instrument rules based on country-specific targets. In the second one, a single currency is present and monetary policy is set by one central bank for the whole union, in accordance with a Taylor rule that depends on union-wide objectives. I model both regimes in a cashless economy.

Another important component of the model is the type of price-setting frictions it has. Nominal rigidities are combined with local currency pricing (LCP) à la Engel (2011) or Corsetti, Dedola, and Leduc (2011): firms set prices in the buyers' currency,

[6] Galí and Monacelli (2005) and Forlati (2009) are alternative perspectives where the union is modelled as a continuum of small open economies.

[7] A recent examination of the monetary aspects of currency unions with imperfect insurance and heterogeneity is Bhattarai, Lee, and Park (2015), which draws on the setup laid out by Cúrdia and Woodford (2010). While that work takes the international currency arrangement (i.e. the union) as given and searches for the optimal monetary policy in that environment, I compare the performance of different monetary systems under given, price stability-oriented policies.

and pricing opportunities arrive at random intervals following a conventional Calvo-Yun scheme. In the presence of separate national currencies, domestic and foreign buyers are charged two distinct prices for identical goods; these satisfy the law of one price in the long run, but violate it over the business cycle, causing a misallocation of resources. If a monetary union is in place, instead, the price is unique and the law of one price applies continuously[8].

The last key component of the model is the real structure of the economy. This features full specialisation in production on the part of the two countries (with a standard two-stage manufacturing process in each place) and frictional international trade. The barrier to commerce has a monetary nature, and it only exists in the regime with two currencies; it takes the form of a Lama and Rabanal (2014)[9] type linear "iceberg" shipping cost on imported products. I use this reduced-form feature as a stand-in for the various transaction costs that affect the international exchange of goods when multiple currencies are adopted, as documented empirically by Rose (2000) and Rose and van Wincoop (2001), among others. The effect of this friction is a long-run trade and consumption differential between the union and the monetary independence regime.

In order to quantify the importance of these frictions for social welfare, I follow Schmitt-Grohé and Uribe (2007) and define welfare as the expectation of households' lifetime utility, conditional on the initial state of the economy being the nonstochastic steady state. To assess the relative performance of alternative currency systems, I measure the welfare differential in terms of consumption equivalents; that is, I define the welfare gap as the loss of lifetime consumption that makes households under monetary independence as happy as they would be under a union. A distinctive feature of my work is that I explicitly decompose this gap into the contribution of different frictions, paying special attention to the role of the international correlation of macroeconomic disturbances, or the lack thereof.

My examination of the welfare gap between monetary independence and union starts from a calibrated economy with frictionless trade, producer currency pricing

[8] Engel (2014) revises the main results from the literature on alternative export-pricing specifications and their impact on macroeconomic adjustment and exchange-rate stabilisation. The implications of LCP with sticky prices were first explored in a welfare-based model of monetary policy by Devereux and Engel (2003).

[9] Lama and Rabanal (2014) is a recent attempt to explore the currency area question explicitly from the perspective of monetary policy. The authors use an estimated two-country DSGE model to evaluate the welfare implications of unifying the United Kingdom and the Eurozone under a single currency. Their work is a close relative to mine from a methodological point of view, but it has an emphasis on financial stability issues and unconventional monetary policy measures.

(PCP) and uncertainty only from idiosyncratic technology shocks; it then proceeds with the introduction of one additional friction at a time. I first use the simplified economy to show that inflation and output are more volatile in a monetary union than they are under separate national currencies. I argue that this difference has fundamental welfare implications, and show that the welfare differential between the two regimes vanishes as shocks get perfectly correlated. I then introduce price discrimination back into the model, and show that the inertia of local-currency prices determines inefficient international price misalignments in the face of exchange rate movements. I argue that this friction per se does not alter the welfare ordering of the two regimes, because it leaves the steady state of the economy unaffected and only bites in the short run. Finally, I add trade frictions to the picture, and explain that they reduce imports demand and depress output at all horizons. I argue that this distortion reduces consumption and welfare in the economy with multiple currencies in the long run; this opens up the possibility that households experience a higher welfare with a single currency. The issue of the net balance between competing forces then becomes an empirical one.

My estimation strategy is as follows. I estimate the full-fledged model in its single-currency configuration with quarterly data from Italy, France, Germany and Spain; the estimation is performed on two countries at a time. For each pair, I compute welfare in the monetary union regime and then compare it with a counterfactual scenario where the two countries split and adopt national currencies, keeping identical Taylor rules but basing them on domestic objectives rather than common ones. I carry out the calculation under a calibration of the trade friction in line with Lama and Rabanal (2014), and then study the sensitivity of my results to this specific choice. Adopting standard Bayesian techniques, I find that the welfare gain from monetary integration is positive for these economies: its posterior mean is above 2% of lifetime consumption, if households have no home bias in consumption. The gain is largest for Italy and France, which appear to have the most correlated shocks, and is smallest for Spain and Germany. The result is robust to the introduction of a moderate degree of bias towards the consumption of domestic goods: this is just found to reduce (but not invert) the welfare differential between alternative monetary arrangements.

I conclude with the perspective that a system of separate national currencies could only be desirable if monetary barriers to trade were very small, because in that case there would be little trade gains from introducing a common currency. Based on numerical work, I show that a modest amount of trade frictions is sufficient for the monetary union to guarantee the same social welfare afforded by monetary independence in the present setup. I ascribe this result to the limited cost of business cycle asymmetries for the economies under scrutiny, and argue that the domain of applicability of a single currency could be affected significantly by the addition of further frictions that introduce cross-country heterogeneity along new dimensions.

The rest of this chapter is organised as follows. Section 2 outlines the setup and illustrates its main dynamic properties. Section 3 defines a welfare measure to compare different monetary arrangements, and explores the dependence of the ranking upon the shocks and frictions of the model. Section 4 presents the Bayesian estimation and discusses the results. Section 5 concludes.

2.2 Monetary independence and union in a two-country model

This section is divided in two parts. In the first, I present the optimisation-based setup that I use to assess the welfare benefits of a single currency. In the second, I employ a simplified version of the model to illustrate how macroeconomic adjustment to asymmetric technology shocks differs across alternative currency arrangements.

I base my analysis on a dynamic stochastic general equilibrium model where the world is made up of two countries, h and f, each populated by a continuum of measure one of households with identical preferences. Employing the modeling strategy of Benigno (2009), I assume that households can perfectly pool risks within their respective countries through a full set of state-contingent securities traded locally, but international trade in assets is limited to noncontingent bonds[10]. This asset markets structure is invariant to the international currency arrangement, and is an essential determinant of the welfare cost of business cycles in my framework[11].

I consider two alternative international monetary regimes. In the first regime, called “monetary union”, the two countries share a single currency and monetary policy is controlled by a unique central bank; the bank follows a conventional interest rate rule based on union-wide inflation and output objectives. In the second regime, named “monetary independence”, the countries have separate national currencies and distinct monetary authorities; these follow analogous but independent interest

[10] I keep the Arrow securities implicit for brevity, and restrict attention to the representative agents.

[11] Although the cost of business cycles would exceed that of Lucas (1987) even if international asset markets were complete, because nominal rigidities induce price distortions in this economy, the lack of full insurance amplifies the cost of macroeconomic fluctuations and contributes quantitatively to the tradeoff between different currency arrangements.

rate rules that depend on country-specific policy targets. The economy is cashless, so monetary policy involves the direct control of nominal interest rates; central banks perform this task by choosing the price of nominal bonds with one-period maturity.

Each country specialises in the supply of one good, whose production takes place in two steps. First, monopolistically competitive firms produce a continuum of differentiated goods; price setting is subject to Calvo-Yun nominal rigidities. Second, perfectly competitive firms aggregate locally produced intermediates into final consumption goods, which are traded internationally. Within each country, firms are entirely owned by domestic households.

Firms engage in Engel (2011) type local currency pricing. If different currencies exist, goods are priced in the currency of their destination market; because of price stickiness, nominal exchange rate movements determine short-run international price misalignments. With a single currency, goods are uniformly priced across the union.

The international exchange of goods is affected by frictions of a monetary nature in the regime with separate national currencies, but is free under a monetary union. I model these frictions as "iceberg" shipping costs that cause a fraction of imported goods to be lost in transit between the two countries, along the lines of Lama and Rabanal (2014). This feature is a stand-in for the monetary barriers to trade documented by Rose (2000) and Rose and van Wincoop (2001). The advantage of such a reduced-form specification is that it captures in a compact way the permanent character of the trade gains from monetary unification; these gains represent the key advantage of a currency union over a multicurrency system in my framework[12].

In what follows, I focus on the model with two currencies, which displays the richest notation; I point out the differences with the monetary union along the way.

2.2.1 The model

2.2.1.1 Households

Households make consumption, saving and labour supply decisions. They consume a bundle of domestic and foreign goods, competitively supply indifferentiated labour

[12] It is understood that additional considerations (such as the need to eliminate an inflation bias in monetary policy or the desire to reduce the cost of government borrowing) often support the decision to enter a currency union in actual practice. However, these factors are more important for some economies than for others, and cannot be considered as universally relevant for the choice between different currency systems. Following the optimum currency area theory set forth by Mundell (1961), which reduces the tradeoff between alternative monetary arrangements to the contrast between transaction gains and policy independence losses, I identify the enhancement of trade as the main economic benefit of a monetary union.

services to local producers (whose profits they receive in a lump-sum fashion) and invest in nominally risk-free, one-period pure discount bonds. Households have access to both domestic currency-denominated and foreign currency-denominated bonds, so international borrowing and lending can take place in either currency.

Intertemporal optimisation The representative households of the home and foreign country solve the following dynamic problems:

$$\max_{\{c_t, A_{t+1}, B_{t+1}, n_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \phi_t \frac{n_t^{1+\varphi}}{1+\varphi} \right)$$

$$\text{s.t. } c_t + q_t^A \frac{A_{t+1}}{p_t} + e_t q_t^B \frac{B_{t+1}}{p_t} + \chi q_t^B \left(\frac{e_t B_{t+1}}{p_t} \right)^2 = \frac{A_t}{p_t} + e_t \frac{B_t}{p_t} + \frac{w_t}{p_t} n_t + \frac{p_{h,t}}{p_t} \Pi_{h,t} + \frac{T_t}{p_t}$$

and

$$\max_{\{c_t^*, A_{t+1}^*, B_{t+1}^*, n_t^*\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t^* \left(\frac{(c_t^*)^{1-\sigma}}{1-\sigma} - \phi_t^* \frac{(n_t^*)^{1+\varphi}}{1+\varphi} \right)$$

$$\text{s.t. } c_t^* + \frac{q_t^A}{e_t} \frac{A_{t+1}^*}{p_t^*} + q_t^B \frac{B_{t+1}^*}{p_t^*} + \chi q_t^A \left(\frac{A_{t+1}^*}{e_t p_t^*} \right)^2 = \frac{A_t^*}{e_t p_t^*} + \frac{B_t^*}{p_t^*} + \frac{w_t^*}{p_t^*} n_t^* + \frac{p_{f,t}^*}{p_t^*} \Pi_{f,t} + \frac{T_t^*}{p_t^*}.$$

ξ_t and ξ_t^* are intertemporal preference shocks, while ϕ_t and ϕ_t^* are labour supply shocks. A_t and A_t^* represent each household's holdings of the home currency-denominated asset, while B_t and B_t^* are their respective holdings of the foreign currency-denominated asset. e_t is the nominal exchange rate. Real bond holdings are subject to quadratic costs à la Benigno (2009); these pin down equilibrium portfolios and ensure that bond positions revert to zero in the long run. The costs paid by each household are received by the other through lump-sum transfers, so they do not represent a deadweight loss:

$$\frac{T_t}{p_t} = \left(\frac{e_t p_t^*}{p_t} \right) \chi q_t^A \left(\frac{A_{t+1}^*}{e_t p_t^*} \right)^2, \qquad \frac{T_t^*}{p_t^*} = \left(\frac{p_t}{e_t p_t^*} \right) \chi q_t^B \left(\frac{e_t B_{t+1}}{p_t} \right)^2.$$

The optimality conditions for consumption, saving and labour supply are spelt out in the Appendix.

Intratemporal optimisation Households have utility over consumption of a composite index of domestic and foreign goods, with an imports share parameter ζ and a constant elasticity of substitution η. Their static optimisation problems are respectively

$$\max_{c_{h,t},c_{f,t}} c_t \equiv \left[(1-\zeta)^{\frac{1}{\eta}}(c_{h,t})^{\frac{\eta-1}{\eta}} + (\zeta)^{\frac{1}{\eta}}((1-\tau)\,c_{f,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$
$$\text{s.t. } p_t c_t = p_{h,t}c_{h,t} + p_{f,t}c_{f,t}$$

and

$$\max_{c^*_{h,t},c^*_{f,t}} c^*_t \equiv \left[(1-\zeta)^{\frac{1}{\eta}}\left(c^*_{f,t}\right)^{\frac{\eta-1}{\eta}} + (\zeta)^{\frac{1}{\eta}}\left((1-\tau)\,c^*_{h,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$
$$\text{s.t. } p^*_t c^*_t = p^*_{f,t}c^*_{f,t} + p^*_{h,t}c^*_{h,t},$$

where $p_{h,t}$, $p_{f,t}$, $p^*_{h,t}$ and $p^*_{f,t}$ represent the producer prices of home and foreign goods in each currency. This distinction becomes superfluous with a single currency: in that case, prices are just $p_{h,t}$ and $p_{f,t}$.

A fraction τ of the imported goods are actually lost and do not yield utility. These "iceberg costs" à la Lama and Rabanal (2014) represent frictions associated with the use of different currencies in international commerce; as such, they are null in a currency union and positive in a multicurrency system.

The effective consumer price indices (CPIs) are respectively

$$p_t \equiv \left[(1-\zeta)(p_{h,t})^{1-\eta} + (\zeta)\left(\frac{p_{f,t}}{1-\tau}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

and

$$p^*_t \equiv \left[(1-\zeta)\left(p^*_{f,t}\right)^{1-\eta} + (\zeta)\left(\frac{p^*_{h,t}}{1-\tau}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

The real exchange rate is defined as

$$\mathcal{Q}_t \equiv \frac{e_t p^*_t}{p_t}.$$

The consumption demands associated with these prices are

$$c_{h,t} = (1-\zeta)\left(\frac{p_{h,t}}{p_t}\right)^{-\eta} c_t, \qquad c_{f,t} = (\zeta)(1-\tau)^{\eta-1}\left(\frac{p_{f,t}}{p_t}\right)^{-\eta} c_t,$$

$$c^*_{f,t} = (1-\zeta)\left(\frac{p^*_{f,t}}{p^*_t}\right)^{-\eta} c^*_t, \qquad c^*_{h,t} = (\zeta)(1-\tau)^{\eta-1}\left(\frac{p^*_{h,t}}{p^*_t}\right)^{-\eta} c^*_t.$$

2.2.1.2 Firms

The production of consumer goods takes place in two steps. First, imperfectly competitive firms produce a continuum of measure one of intermediate goods in each country, using technologies that take local labour as their only input. The prices of these goods are set in terms of the currency of the country where they are sold. Second, perfectly competitive firms aggregate intermediates into final products, which are either consumed domestically or exported. I describe each group of producers in the h country; f-country firms have a similar behaviour.

Final goods producers Perfectly competitive producers adopt a constant elasticity of substitution (CES) technology that takes as inputs all varieties of locally produced intermediates and outputs homogeneous final products. Varieties are indexed by i. Separate sets of firms serve the domestic and the foreign market; as they adopt the same technology, the products they make are in fact identical.

The problems faced by the producers of home goods that serve each market are respectively

$$\max_{y_{h,t}(i)} p_{h,t}y_{h,t} - \int_0^1 p_{h,t}(i)\, y_{h,t}(i)\, di \qquad \text{s.t. } y_{h,t} = \left[\int_0^1 y_{h,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

$$\max_{y^*_{h,t}(i)} p^*_{h,t}y^*_{h,t} - \int_0^1 p^*_{h,t}(i)\, y^*_{h,t}(i)\, di \qquad \text{s.t. } y^*_{h,t} = \left[\int_0^1 y^*_{h,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The input demands by these producers are

$$y_{h,t}(i) = \left(\frac{p_{h,t}(i)}{p_{h,t}}\right)^{-\varepsilon} y_{h,t}, \qquad y^*_{h,t}(i) = \left(\frac{p^*_{h,t}(i)}{p^*_{h,t}}\right)^{-\varepsilon} y^*_{h,t},$$

with associated producer price indices (PPIs)

$$p_{h,t} = \left(\int_0^1 p_{h,t}(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}, \qquad p^*_{h,t} = \left(\int_0^1 p^*_{h,t}(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

The distinction between exported and domestically consumed final goods fades in the monetary union, since the underlying intermediate goods are priced in the same currency; in that case, the aggregator function $y^*_{h,t}$ is redundant.

Intermediate goods producers Monopolistically competitive firms hire local labour in each country to produce differentiated intermediate goods i with technologies of the following sort:

$$y_t(i) = z_t n_t(i).$$

z_t and z_t^* denote home and foreign productivities, respectively; these are common to all firms within a country. The quantities produced must satisfy both domestic and foreign demand:

$$y_t(i) = y_{h,t}(i) + y_{h,t}^*(i).$$

Firms face an exogenous probability $1-\theta_p$ of resetting their prices each period. Prices are chosen to maximise discounted profits, subject to isoelastic demand schedules by the producers of final goods. Given the prices, output is demand-determined.

In the presence of two different currencies, firms set a distinct price for each market. The optimal pricing problem faced by producers in the home country is

$$\max_{\bar{p}_{h,t}(i),\, \bar{p}_{h,t}^*(i)} \mathbb{E}_t \sum_{\tau=0}^{\infty} \theta_p^\tau \Lambda_{t,t+\tau} \left\{ \frac{\bar{p}_{h,t}(i)}{p_{h,t+\tau}} y_{h,t+\tau}(i) + e_{t+\tau} \frac{\bar{p}_{h,t}^*(i)}{p_{h,t+\tau}} y_{h,t+\tau}^*(i) - \Psi\left(y_{t+\tau}(i)\right) \right\}$$

$$\text{s.t.} \begin{cases} y_{h,t+\tau}(i) = \left(\frac{p_{h,t+\tau}(i)}{p_{h,t+\tau}}\right)^{-\varepsilon} y_{h,t+\tau} \\ y_{h,t+\tau}^*(i) = \left(\frac{p_{h,t+\tau}^*(i)}{p_{h,t+\tau}^*}\right)^{-\varepsilon} y_{h,t+\tau}^*, \end{cases}$$

where $\Lambda_{t,t+\tau}$ represents the household's stochastic discount factor for τ periods-ahead real payoffs, while the $\Psi(\cdot)$ function is the total real cost of production. The optimal price-setting conditions for the goods that are aimed at the domestic market read

$$g_{h,t}^2 = \mathcal{M}_p g_{h,t}^1,$$

$$g_{h,t}^2 \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_p \beta)^\tau \left(\frac{\lambda_{t+\tau}}{\lambda_t}\right) \left\{ y_{h,t+\tau} \left(\frac{\bar{p}_{h,t}}{p_{h,t}}\right)^{-\varepsilon} \left(\frac{1}{\prod_{s=1}^{\tau} \pi_{h,t+s}}\right)^{1-\varepsilon} \right\},$$

$$g_{h,t}^1 \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_p \beta)^\tau \left(\frac{\lambda_{t+\tau}}{\lambda_t}\right) \left\{ y_{h,t+\tau} \frac{mc_{t+\tau}}{d_{t+\tau}} \left(\frac{\bar{p}_{h,t}}{p_{h,t}}\right)^{-1-\varepsilon} \left(\frac{1}{\prod_{s=1}^{\tau} \pi_{h,t+s}}\right)^{-\varepsilon} \right\},$$

where the desired frictionless markup is defined as

$$\mathcal{M}_p \equiv \frac{\varepsilon}{\varepsilon - 1}.$$

Analogous conditions hold for the goods that are aimed at the export market, as shown in the Appendix; they imply that nominal rigidities and price discrimination interact to cause violations of the law of one price over the business cycle. It should be noted that the law of one price holds in the nonstochastic steady state, because firms are assumed to enjoy the same degree of monopoly power on both markets, so they charge the same markup in the long run[13].

In the presence of a single currency, instead, price setters choose a unique price for both markets and the law of one price holds continuously.

2.2.1.3 Monetary policy

Two different monetary regimes are considered here. In the first, two different currencies exist and the nominal exchange rate is flexible. In the second, only one currency exists.

Monetary independence The national central banks of the two countries control the prices of the bonds denominated in the respective currencies. They set nominal interest rates according to the following rules:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\gamma_y} \left(\frac{e_t}{e_{t-1}}\right)^{\gamma_e}\right]^{1-\gamma_R} m_t, \tag{2.1}$$

$$\frac{R_t^*}{R^*} = \left(\frac{R_{t-1}^*}{R^*}\right)^{\gamma_R} \left[\left(\frac{\pi_t^*}{\pi^*}\right)^{\gamma_\pi} \left(\frac{y_t^*}{y_{t-1}^*}\right)^{\gamma_y} \left(\frac{e_{t-1}}{e_t}\right)^{\gamma_e}\right]^{1-\gamma_R} m_t^*, \tag{2.2}$$

where $R_t = 1/q_t^A$ and $R_t^* = 1/q_t^B$. The m_t and m_t^* terms represent exogenous interest rate shocks. Since firms engage in LCP, monetary authorities target CPI inflation rates:

$$\pi_t \equiv \frac{p_t}{p_{t-1}}, \qquad \pi_t^* \equiv \frac{p_t^*}{p_{t-1}^*}.$$

Taylor rules are specified in terms of real output growth, because the output gap is not observable in practice. They also feature an exchange rate feedback term that prevents nominal exchange rate movements from causing large price misalignments[14]

[13] This condition is known to be violated in actual fact, as persistent deviations from the law of one price are commonly observed due to heterogeneous consumer preferences across markets. As these differences are immune to the choice of the monetary regime, they do not affect the welfare ranking of alternative currency systems; therefore, they can be ignored for the purposes of this model.

[14] Duarte and Obstfeld (2008) argue that the case for the stabilisation of the exchange rate under LCP breaks down in economies with non-traded goods. Their argument, however, depends crucially on the existence of an optimal international risk-sharing arrangement. As shown by De Paoli (2009), prescriptions regarding exchange rate stability tend to be reversed under incomplete asset markets;

and excessive violations of the risk-sharing condition[15]

$$\frac{\lambda_t^*}{\lambda_t} = \frac{e_t p_t^*}{p_t}.$$

Monetary union The central bank of the monetary union adopts a Taylor rule that takes union-wide measures as monetary policy targets:

$$\frac{R_t^u}{R^u} = \left(\frac{R_{t-1}^u}{R^u}\right)^{\gamma_R} \left[\left(\frac{\pi_{u,t}}{\pi_u}\right)^{\gamma_\pi} \left(\frac{y_{u,t}}{y_{u,t-1}}\right)^{\gamma_y}\right]^{1-\gamma_R} m_t^u. \tag{2.3}$$

Union-wide output is the sum of the two countries' GDPs:

$$y_{u,t} = y_t + y_t^*.$$

Inflation in the union is defined as the geometric average of the two GDP inflation rates

$$\pi_{h,t} \equiv \frac{p_{h,t}}{p_{h,t-1}}, \qquad \pi_{f,t} \equiv \frac{p_{f,t}}{p_{f,t-1}},$$

weighted by the sizes of the respective countries:

$$\pi_{u,t} \equiv \frac{p_{u,t}}{p_{u,t-1}} = (\pi_{h,t})^{\frac{1}{2}} (\pi_{f,t})^{\frac{1}{2}}.$$

An endogenous nominal interest rate spread must correct self-fulfilling inflation differentials between the two countries under rule (2.3), in order to rule out unstable solutions:

$$\Omega_t \equiv \frac{R_t^*}{R_t} = \left(\frac{\pi_{f,t}}{\pi_{h,t}}\right)^{\omega},$$

where $\omega \simeq 0$.

2.2.1.4 Exogenous processes

Shocks are common to all households and firms within each country, and follow first-order autoregressive processes in logs. The international spillovers of these shocks are controlled by the parameters ν. Intertemporal preference shocks:

the extension of the present model to non-traded goods and the study of its implications for social welfare under alternative monetary arrangements are left for future work.

[15] Due to the incompleteness of markets, the decentralised equilibrium allocation is inefficient and is supported by suboptimal patterns of international borrowing and lending. This inefficiency also shows up in the form of excess nominal exchange rate volatility, which creates scope for welfare-improving monetary policy intervention.

$$\begin{bmatrix} \log \xi_t \\ \log \xi_t^* \end{bmatrix} = \begin{bmatrix} \rho_\xi & \nu_\xi \\ \nu_\xi & \rho_\xi \end{bmatrix} \begin{bmatrix} \log \xi_{t-1} \\ \log \xi_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_{\xi,t} \\ e_{\xi,t}^* \end{bmatrix}.$$

Labour supply shocks:

$$\begin{bmatrix} \log \phi_t \\ \log \phi_t^* \end{bmatrix} = \begin{bmatrix} \rho_\phi & \nu_\phi \\ \nu_\phi & \rho_\phi \end{bmatrix} \begin{bmatrix} \log \phi_{t-1} \\ \log \phi_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_{\phi,t} \\ e_{\phi,t}^* \end{bmatrix}.$$

Technology shocks:

$$\begin{bmatrix} \log z_t \\ \log z_t^* \end{bmatrix} = \begin{bmatrix} \rho_z & \nu_z \\ \nu_z & \rho_z \end{bmatrix} \begin{bmatrix} \log z_{t-1} \\ \log z_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_{z,t} \\ e_{z,t}^* \end{bmatrix}.$$

Monetary shocks with two currencies:

$$\begin{bmatrix} \log m_t \\ \log m_t^* \end{bmatrix} = \begin{bmatrix} \rho_m & \nu_m \\ \nu_m & \rho_m \end{bmatrix} \begin{bmatrix} \log m_{t-1} \\ \log m_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_{m,t} \\ e_{m,t}^* \end{bmatrix}.$$

Monetary shocks with a single currency:

$$\log m_t^u = \rho_m \log m_{t-1}^u + e_{m,t}^u.$$

The innovations follow i.i.d. normal processes, which may or may not be correlated across countries.

2.2.1.5 Market clearing, price dispersion and output

The GDP of each country is the sum of the goods produced for the domestic market and those intended for exports:

$$y_t = y_{h,t} + y_{h,t}^*, \qquad y_t^* = y_{f,t} + y_{f,t}^*.$$

The market-clearing conditions for these goods are

$$y_{h,t} = c_{h,t}, \qquad y_{h,t}^* = (1-\tau)\, c_{h,t}^* + \tau c_{h,t}^*,$$

$$y_{f,t}^* = c_{f,t}^*, \qquad y_{f,t} = (1-\tau)\, c_{f,t} + \tau c_{f,t}.$$

The market-clearing conditions for labour are

$$n_t = \int_0^1 n_t(i)\, di, \qquad n_t^* = \int_0^1 n_t^*(i)\, di.$$

They can be combined with the demand schedules and the production technologies of individual goods, to yield the exact aggregate production functions of each economy:

$$y_t = \frac{z_t n_t}{d_t}, \qquad y_t^* = \frac{z_t^* n_t^*}{d_t^*}. \tag{2.4}$$

The state variables d_t and d_t^* measure price dispersion at the level of GDP. They are weighted combinations of the price dispersion indices at the PPI level:

$$d_t \equiv x_{h,t} d_{h,t} + x_{h,t}^* d_{h,t}^*, \qquad d_t^* \equiv x_{f,t} d_{f,t} + x_{f,t}^* d_{f,t}^*.$$

The weights equal the shares of domestically consumed and exported output:

$$x_{h,t} \equiv \frac{y_{h,t}}{y_t}, \qquad x_{h,t}^* \equiv \frac{y_{h,t}^*}{y_t}, \qquad x_{f,t} \equiv \frac{y_{f,t}}{y_t^*}, \qquad x_{f,t}^* \equiv \frac{y_{f,t}^*}{y_t^*}.$$

The four indices of price dispersion at PPI level are defined as

$$d_{h,t} \equiv \int_0^1 \left(\frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\varepsilon} di, \qquad d_{h,t}^* \equiv \int_0^1 \left(\frac{p_{h,t}^*(i)}{p_{h,t}^*} \right)^{-\varepsilon} di,$$

$$d_{f,t} \equiv \int_0^1 \left(\frac{p_{f,t}(i)}{p_{f,t}} \right)^{-\varepsilon} di, \qquad d_{f,t}^* \equiv \int_0^1 \left(\frac{p_{f,t}^*(i)}{p_{f,t}^*} \right)^{-\varepsilon} di.$$

Finally, the following market-clearing conditions apply to the internationally traded assets:

$$A_{t+1} = -A_{t+1}^*, \qquad B_{t+1} = -B_{t+1}^*.$$

The laws of motion of the price dispersion indices, the aggregate price levels and the nominal exchange rate are displayed in the Appendix.

2.2.2 Equilibrium adjustment to asymmetric shocks

Before moving on to analyse welfare under alternative international monetary arrangements, I use a simplified version of my model to shed light on how macroeconomic adjustment in a currency union differs from that with two independent monetary policies.

I shut down the disturbances to intertemporal preferences, labour supply and interest rates, and consider an environment where technology shocks represent the only source of uncertainty. I assume that trade is frictionless, so that consumption and output are identical across monetary regimes in the long run. Furthermore, I assume that firms engage in producer currency pricing, so that the law of one price applies: $p_{h,t} = e_t p^*_{h,t}$ and $p_{f,t} = e_t p^*_{f,t}$ at all times.

Table 2.6 in the Appendix displays the calibrated parameters. The time interval of the model is a quarter. I specify identical preferences with unit intertemporal and intratemporal elasticities of substitution. This configuration is known as "macroeconomic independence" in the work of Corsetti and Pesenti (2001), because it implies that domestic and foreign goods are neither complements nor substitutes in consumption; this rules out international supply spillovers. As to the monetary policy block, I calibrate the Taylor rules with identical coefficients across the two monetary regimes to ease the comparison[16].

Let us explore the effects of a positive productivity shock to the home country, under the assumption that this is uncorrelated with foreign productivity. Figure 2.1 shows that inflation rates are less volatile in the regime with two currencies (solid blue lines) than they are in the regime with a single currency (dashed purple lines), because asymmetric disturbances are addressed more effectively by a selective adjustment of home and foreign interest rates. For the same reason, output gaps (not shown) are less volatile in the former regime: the independent movement of local interest rates helps keep home and foreign output levels closer to their respective natural counterparts.

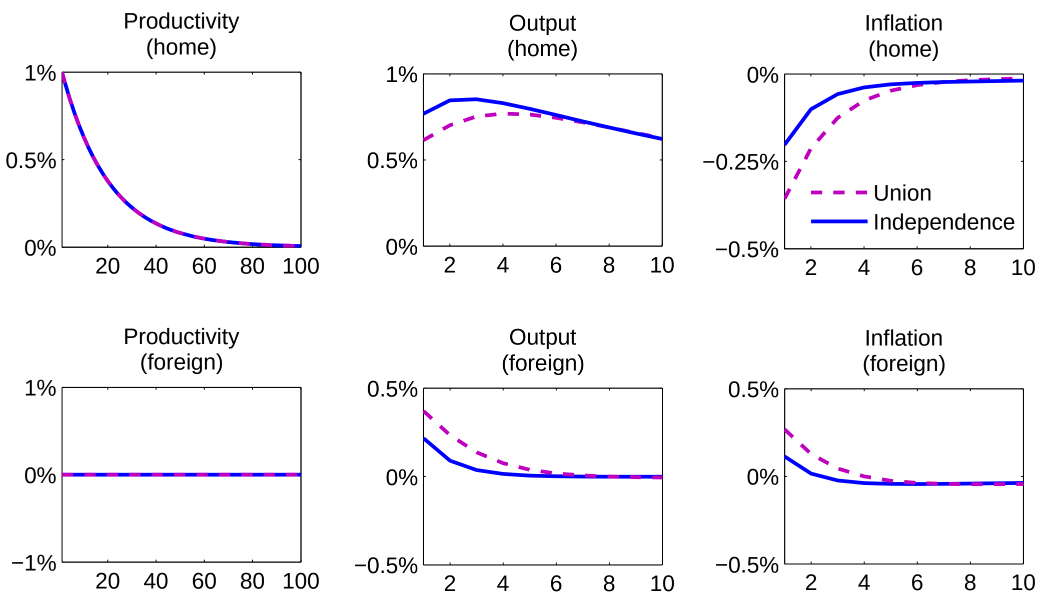
Notice that a positive international comovement of output appears in Figure 2.1, despite the unit elasticities of substitution specified in Table 2.6. This occurs in both monetary regimes, and is due to the response of monetary policy to the shock. Under rules (2.1) and (2.2), the central bank of the home country cuts the nominal interest rate in response to the deflationary effects of the productivity shock. To avoid an excessive movement of the nominal exchange rate, the foreign central bank cuts its interest rate too; this is what causes a jump in foreign inflation and output. Under rule (2.3), the central bank reacts to a local productivity improvement by cutting the nominal interest rate for the whole union; this stimulates foreign activity and inflation. Both variables appear significantly more volatile than in the other regime.

The distance between the impulse response functions (IRFs) under the two regimes depends on the symmetry of the shocks. If z_t and z^*_t were perfectly positively correlated, the IRFs would coincide: the presence of distinct policy instruments does not make a difference when the two economies behave like one. Conversely, if

[16] Interest rate smoothing is muted because there is no need to stabilise the opportunity cost of holding money in a cashless economy. Similarly, the response to output fluctuations is muted in the absence of inefficient cost-push shocks. The inflation and exchange rate coefficients are choosen to guarantee a unique rational expectations equilibrium.

z_t and z_t^* were perfectly negatively correlated, the IRFs would be farther apart: in this case, the economy with separate instruments would display more stability than that with a common currency. The next section explores the consequences of this for social welfare.

Figure 2.1: Impulse responses to a technology shock: monetary independence vs currency union



2.3 Welfare differential between international monetary regimes

I now introduce a welfare metric that allows me to rank alternative monetary regimes.

Following Schmitt-Grohé and Uribe (2007), I define welfare as the conditional expectation of lifetime utility as of time zero, assuming that at this point all state variables equal their steady-state values. The welfare levels associated with *monetary independence* and *currency union* are respectively

$$V_0^{mi} \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^{mi}, n_t^{mi}\right), \qquad V_0^{cu} \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^{cu}, n_t^{cu}\right),$$

where $\{c_t^{mi}\}$, $\{n_t^{mi}\}$, $\{c_t^{cu}\}$ and $\{n_t^{cu}\}$ represent the respective contingent plans for consumption and hours. The welfare cost of abandoning an independent monetary policy and adopting a single currency is measured in terms of foregone consumption, along the lines of Lucas (1987): I define it as the negative subsidy rate λ such that

$$V_0^{cu} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left((1-\lambda)\, c_t^{mi}, n_t^{mi}\right).$$

The welfare gap between these monetary arrangements can be written as

$$V_0^{cu} - V_0^{mi} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u\left((1-\lambda)\, c_t^{mi}, n_t^{mi}\right) - u\left(c_t^{mi}, n_t^{mi}\right)\right].$$

With logarithmic utility, the welfare cost amounts to

$$\lambda = 1 - e^{(1-\beta)\left(V_0^{cu} - V_0^{mi}\right)}. \tag{2.5}$$

2.3.1 Computational method

Due to the frictions that affect the goods and the asset markets, the steady state of the model is inefficient. In order to measure welfare accurately in this setup, I approximate the equilibrium conditions up to second order[17]. My numerical strategy is based on perturbation, which represents a convenient approach to the study of economies with a large number of state variables. The algorithm that I use is Dynare by Adjemian et al. (2011). To address the problem of explosive paths emerging in second-order approximations of this class of models[18], I apply a pruning method due to Kim, Kim, Schaumburg, and Sims (2008).

2.3.2 Shocks, frictions and welfare

In this section, I examine the main determinants of the welfare gap between alternative monetary arrangements. I start from an economy with no trade costs nor international price discrimination; I then introduce these frictions one at a time,

[17] This is needed because first-order methods leave out welfare-relevant terms and incur large approximation errors when the steady state of the economy is Pareto-inefficient, as documented by Kim and Kim (2003).

[18] Higher-order terms of state variables tend to appear in the application of second-order methods without pruning. These terms create "spurious" steady states that are extraneous to the original model; this is problematic because the resulting approximated state-space system is unstable around these points usually, and does not have finite moments. Pruning procedures remove the higher-order terms and preserve only first and second-order terms when the system is iterated forward: the alternative state-space system constructed by these methods achieves stability.

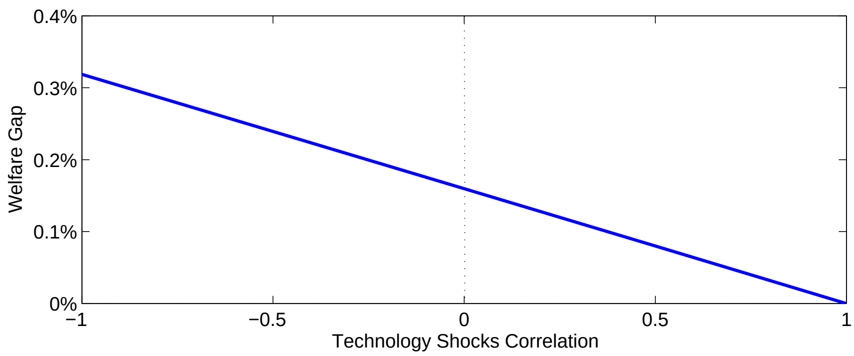
and discuss their welfare implications. I focus on an environment where technology shocks are the only source of uncertainty.

Figure 2.2 plots the welfare gap against the correlation of technology shocks, in an economy where the law of one price holds continuously and international trade is frictionless. The relationship is negative because a stronger comovement of macroeconomic variables reduces the degree of heterogeneity among countries; this decreases the need for region-specific stabilisation policies and the attractiveness of a multicurrency system. The gap goes to zero as $\operatorname{corr}(e_z, e_z^*)$ approaches one; as explained above, the two economies become ex-post identical in this case, and there is no difference between having one or two instruments of monetary policy.

The slope of the welfare gap line depends on the volatility of the exogenous processes. Stronger shocks make the two countries more heterogeneous ex-post, widening the difference between the welfare achievable with separate monetary policy instruments and that achievable with a common one. This effect is proportional to the correlation of productivity disturbances, so it is reflected in a steeper line.

The intercept of the welfare gap line depends on the other macroeconomic disturbances that exist in the economy. The fact that the welfare cost of a monetary union is null when technology shocks are perfectly correlated (as reflected by a zero right intercept in Figure 2.2) is due to the fact that I have shut down the shocks to intertemporal preferences, labour supply and interest rates. If these disturbances were present (and imperfectly correlated), the right intercept of the welfare gap schedule would be positive because the two economies would be heterogeneous even when $\operatorname{corr}(e_z, e_z^*) = 1$[19].

Figure 2.2: Correlation of technology shocks and the welfare cost of a currency union



[19] The right intercept would still be zero if these other disturbances were perfectly positively correlated.

Figure 2.3 shows what happens when we introduce pricing-to-market. The pass-through of nominal exchange rate movements into import prices becomes imperfect when prices are subject to staggered setting in the currency of the destination market. In the short run, identical products are sold at different prices in different places, once converted into a common unit of account; this is inefficient, because it distorts the allocation of demand. The following measures of international price misalignments quantify the departures from the law of one price:

$$m_{h,t} \equiv \frac{e_t p^*_{h,t}}{p_{h,t}}, \qquad m_{f,t} \equiv \frac{e_t p^*_{f,t}}{p_{f,t}}.$$

If nominal rigidities are strong enough, these frictions add an important cost to the regime with national currencies. The monetary union is not affected by these distortions instead, because the presence of a single currency removes the segmentation of final goods markets and impedes price discrimination: goods are uniformly priced across countries. For this reason, the welfare gap between the two regimes shrinks. Since price misalignments materialise over the business cycle and are null in the nonstochastic steady state, the loss of welfare in the multicurrency economy is proportional to the correlation of technology shocks; this is why it affects the slope of the welfare gap schedule.

Figure 2.3: Price discrimination and the welfare cost of a currency union

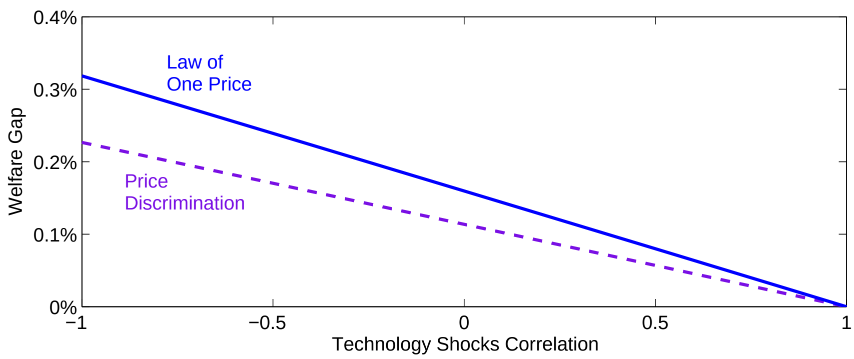
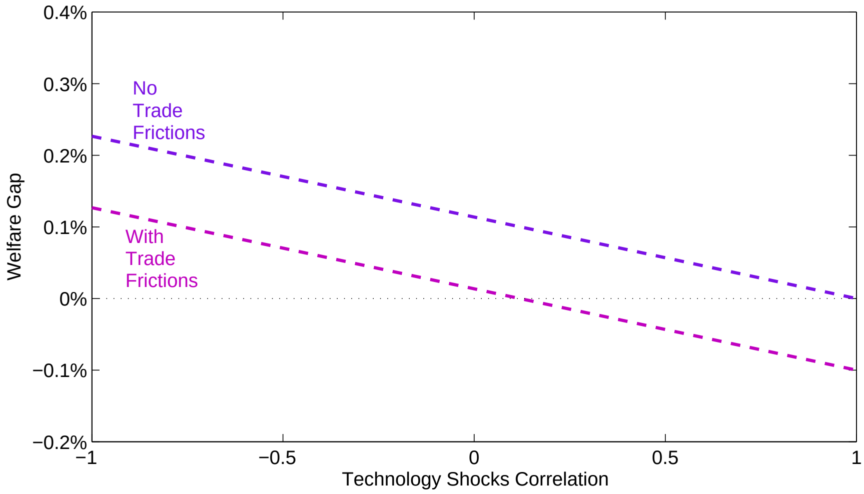


Figure 2.4 illustrates the welfare effect of trade frictions. The presence of monetary barriers to trade reduces the demand for imports, lowering the long-run output level of the economy with multiple currencies relative to that of the union; since this distortion affects the steady state of the economy, it shifts down the entire welfare gap schedule. This affects the welfare ranking between alternative monetary regimes

significantly, because it creates regions of the parameters space where the union Pareto-dominates the system of independent national currencies.

Figure 2.4: Trade frictions and the welfare cost of a currency union



As the position of the welfare gap schedule is closely tied to the quantitative strength of the various shocks and frictions just mentioned, it is difficult to draw conclusions from the present model without letting data discipline its parameters. This is what I do in the next section.

2.4 Bayesian estimation

In this section, the model meets quarterly data from Italy, France, Germany and Spain to be estimated and evaluated using Bayesian tools. In accordance with the structure of the model, the estimations and the subsequent welfare calculations are done on two countries at a time.

My empirical strategy is as follows. For each pair of countries, I estimate the model in its monetary union configuration and compute social welfare under such regime. Next, I simulate the model with separate currencies under the assumption that the estimated parameters are invariant to the monetary arrangement[20]: I assume that

[20] The alternative strategy of separately estimating the model under flexible exchange rates is hindered by data availability issues. The exchange rates of the largest European economies were controlled long before the Euro was established in 1999: a system of semi-pegged exchange rates known as the European Exchange Rate Mechanism was in place from 1979 to 1999. The availability of suitable pre-1979 quarterly data is limited.

the countries keep identical Taylor rules (based on individual rather than union-wide policy objectives), I fix the monetary barriers to trade at a value in line with the existing literature, I compute welfare in this alternative regime and I measure the gap with the previous case. Finally, I let the trade barriers vary and I seek the critical amount of frictions that equates welfare across monetary regimes.

2.4.1 Data

The beginning of the sample is chosen to coincide with the official launch of the Euro in the first quarter of 1999. The end of the sample is the fourth quarter of 2014.

As the model features seven exogenous driving processes, I use seven macroeconomic series as observables: the real GDP, real consumption and CPI inflation series of each country, plus the nominal interest rate of the union (as measured by the Eurozone interbank rate).

The CPI inflation data are obtained from OECD; the source of the remaining series is the FRED archive of the Federal Reserve Bank of St. Louis. The series are in a seasonally adjusted form.

Output and consumption are divided by the GDP deflator, so that they appear in real terms. The series are then turned into per capita terms using population data from Eurostat[21]. Finally, they are detrended by means of a one-sided Hodrick and Prescott filter[22] with smoothing parameter 1600.

2.4.2 Calibrated parameters and priors

I calibrate three sets of parameters: (i) those that have been consistently identified by the literature (e.g. the Calvo parameter on prices), (ii) those that would not be identified from the vector of observable variables chosen here (e.g. the trade cost parameter), and (iii) those that represent simplifying assumptions (e.g. the unit elasticities of intertemporal and trade substitution). These are listed in Table 2.7 in the Appendix.

I run two rounds of estimations: in the first, I assume that households value domestic and imported goods equally; in the second, I allow for some home bias in

[21] Since the frequency of the original population data is annual, I construct population series at a quarterly frequency by interpolation.

[22] The traditional two-sided HP filter takes future values of variables as an input to construct data series. This contradicts the backward-looking structure of the model in state-space form, where the solution today depends only on current and past states.

consumption[23].

I estimate two sets of parameters: (i) the Taylor rule coefficients and (ii) the parameters that control the exogenous processes. I specify the same set of priors before launching all estimations; they are collected in Table 2.8 in the Appendix.

Three parameters are specific to the model with national currencies: these are the trade friction parameter, the international correlation of monetary shocks and the exchange rate coefficient in the Taylor rules. They have no counterpart in the union. I set $\tau = 0.05$ following Lama and Rabanal (2014)[24], I specify $\mathrm{corr}\,(e_m, e_m^*) = 0$, and I fix γ_e at the lowest value that guarantees a unique rational expectations equilibrium. I subsequently check the robustness of my results to changes in these parameters.

2.4.3 Estimation technique

I construct a first-order approximation of the model and its decision rules, so that the likelihood can be generated by Kalman filter projections from the approximated state-space system. This choice is dictated by computational convenience[25].

As the posterior distribution cannot be evaluated analytically, I adopt a standard Metropolis-Hastings (MH) Markov Chain Monte Carlo (MCMC) algorithm to simulate it and produce Bayesian estimates of the parameters. For each round of estimation, I run four parallel chains of the MH algorithm with 500,000 replications each. I set a 50% burn-in period to remove the dependence of the estimates on the parameter vector that initialises the MH algorithm, so the first 250,000 draws are discarded before actually using the posterior simulations.

Tables 2.9 and 2.10 in the Appendix collect the results of the estimations without home bias ($\zeta = 0.5$). Tables 2.11 to 2.16 report the contribution of the different shocks to the variance of the observables at infinite horizon in this configuration. Tables 2.17 and 2.18 in the Appendix summarise the results of the estimations with home bias ($\zeta = 0.35$).

[23] Separate estimations are needed because the share parameter ζ affects the likelihood function generated by the model.

[24] These authors indicate 5 percentage points as a lower bound on the true reduction in transaction costs produced by the introduction of a single currency. This calibration allows me to get a conservative assessment of the gains from monetary unification.

[25] In principle, the construction of the likelihood from a second-order approximation would exploit the nonlinear structure of the model more fully. In practice, the use of a particle filter can be computationally expensive, whereas the difference in terms of point estimates is likely to be small; see Fernández-Villaverde and Rubio-Ramírez (2005).

2.4.4 Welfare results with no home bias

To evaluate the distribution of the welfare gap λ for each pair of countries, I employ the following procedure. First, I take a sample of 1000 Monte Carlo draws from the appropriate joint posterior distribution of estimated parameters; each one provides a parameter vector for solving the model. For each draw, I simulate the model in both monetary configurations and compute welfare. Finally, I calculate the gap in consumption equivalents using equation (2.5).

Table 2.1 reports the results from the economy with no home bias in consumption. The probability densities are displayed in Figure 2.7 in the Appendix.

Table 2.1: Estimates of λ with no home bias

Country pair	Mean	90% HPD interval
Italy and France	-2.55%	[-2.56%, -2.54%]
Italy and Spain	-2.52%	[-2.67%, -2.41%]
France and Spain	-2.52%	[-2.63%, -2.44%]
Italy and Germany	-2.44%	[-2.57%, -2.33%]
France and Germany	-2.42%	[-2.50%, -2.33%]
Spain and Germany	-2.33%	[-2.43%, -2.24%]

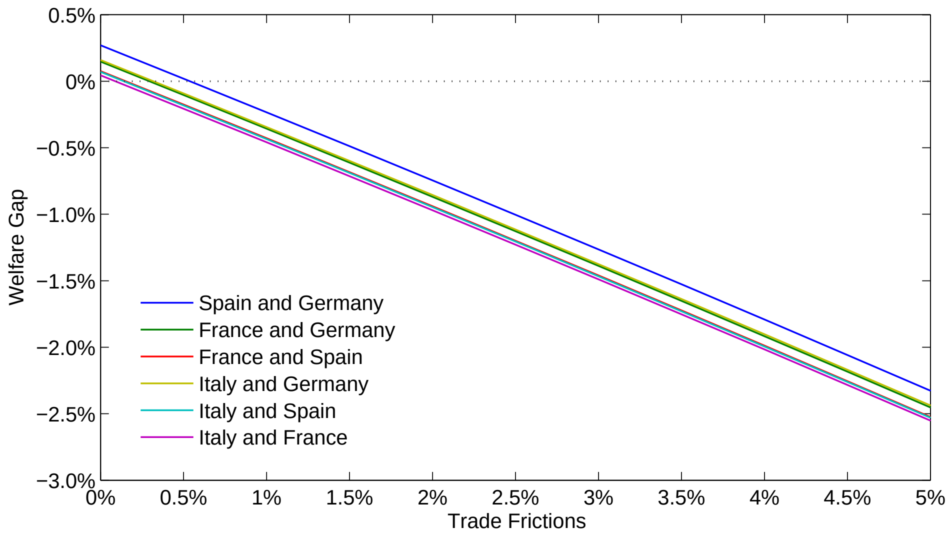
The negative signs on all lambdas mean that all pairs of countries under scrutiny would suffer welfare losses if they adopted separate currencies. The distributions show little dispersion: the highest posterior density (HPD) intervals at a 90% credibility level are quite tight around the respective posterior means.

The countries that stand to lose the most from breaking up the union are Italy and France, while those who stand to lose the least are Germany and Spain. This is consistent with the estimated correlations of shocks displayed in Tables 2.9 and 2.10—particularly those to technology, in the light of the fact that they are responsible for most of the variance of consumption and output in this environment.

The results in Table 2.1 and Figure 2.7 are specific to the configuration chosen for the non-estimated parameters. Among these, three deserve further discussion: they are the exchange rate stabilisation coefficient, γ_e, the correlation between interest rate shocks with separate monetary authorities, $\mathrm{corr}\,(e_m, e_m^*)$, and the iceberg cost, τ. Even though they have no effect on the likelihood of the estimated model (because they are not observable in a monetary union), they affect welfare in the economy with national currencies. To examine the sensitivity of my results to changes in these parameters, I let each vary in isolation while I keep the others fixed[26]. I find that γ_e and $\mathrm{corr}\,(e_m, e_m^*)$ have a negligible impact on the welfare gap: changes in the former leave λ unchanged up to the third decimal digit, while changes in the latter leave it unaffected up to the fourth decimal digit. By contrast, τ is of primary importance: its effects on welfare are as follows.

Figure 2.5 plots the welfare gap between alternative regimes against the intensity of trade frictions. The relationship is negative for all the countries under consideration, because stronger monetary barriers to trade reduce the desirability of adopting separate national currencies. The vertical intercepts on the left side of the diagram are positive because these economies would be better off with independent monetary policies if trade frictions were absent; those on the right side coincide with the posterior means in Table 2.1.

Figure 2.5: Trade frictions and the welfare gap between monetary regimes



As the cost of asymmetric business cycles is small for these economies, moderate amounts of monetary barriers to trade suffice to equate the benefits and the costs of a currency union. Table 2.2 reports the critical amounts of trade frictions that close the welfare gap between monetary regimes for each pair of economies; they correspond to the horizontal intercepts of the welfare gap lines in Figure 2.5. Particularly small frictions appear sufficient for a monetary union between Italy and France to offer the same welfare as a system of national currencies, since the business cycles of these

[26]In these computations, the estimated parameters are fixed at their respective posterior means.

countries are quite strongly correlated; larger frictions seem to be needed to justify a monetary union between Spain and Germany, as they exhibit more asynchronous disturbances and therefore have relatively more to gain from keeping independent monetary policies.

Table 2.2: Trade frictions that equate welfare across regimes without home bias

Country pair	Critical τ
Italy and France	0.09%
Italy and Spain	0.14%
France and Spain	0.15%
Italy and Germany	0.32%
France and Germany	0.36%
Spain and Germany	0.53%

2.4.5 Welfare results with home bias

The importance of the trade frictions is tied to the degree of openness of the two economies, which in turn depends on the weight of imported goods in consumption. Table 2.3 displays the results of welfare calculations based on estimations with a moderate degree of home bias: preferences are calibrated with an imports share of $\zeta = 0.35$ as in Erceg et al. (2009) and Coenen et al. (2009). The associated probability densities are illustrated in Figure 2.8 in the Appendix. The results are qualitatively analogous to those without home bias: all countries experience lower welfare with separate currencies. Two quantitative differences stand out, however: first, the estimated welfare differentials are smaller than before; second, the figures appear to vary more across country pairs.

Table 2.3: Estimates of λ with home bias

Country pair	Mean	90% HPD interval
Italy and France	-1.73%	[-1.75%, -1.71%]
Italy and Spain	-1.61%	[-1.75%, -1.46%]
France and Spain	-1.48%	[-1.59%, -1.32%]
France and Germany	-1.14%	[-1.32%, -0.95%]
Italy and Germany	-1.09%	[-1.32%, -0.95%]
Spain and Germany	-0.92%	[-1.15%, -0.76%]

Figure 2.6 sheds further light on these two effects by helping the reader to visualise the impact of home bias on the relationship between welfare gaps and trade frictions.

Compared to Figure 2.5, the welfare gap lines are flatter and their vertical intercepts are both higher and more spaced out. The slope effect is intuitive: since home bias lowers the volume of imports and exports, it makes the welfare implications of monetary barriers to trade less severe; this lessens the gains from monetary unification. The intercept effect is more interesting: it is due to the fact that the degree of trade openness affects that of international risk sharing. Intuitively, a smaller volume of trade in goods leaves less room for trade in assets; this reduces the amount of insurance that the two countries can achieve[27], exacerbates the welfare cost of business cycles and raises the desirability of distinct monetary policies. Larger welfare gains can be obtained by adopting separate currencies in the absence of trade frictions, as reflected by the higher vertical intercepts on the left side of the diagram. The effect is stronger for the economies that experience the least correlated disturbances: this is why the lines are also more distant from each other now.

Figure 2.6: Trade frictions and welfare with home bias

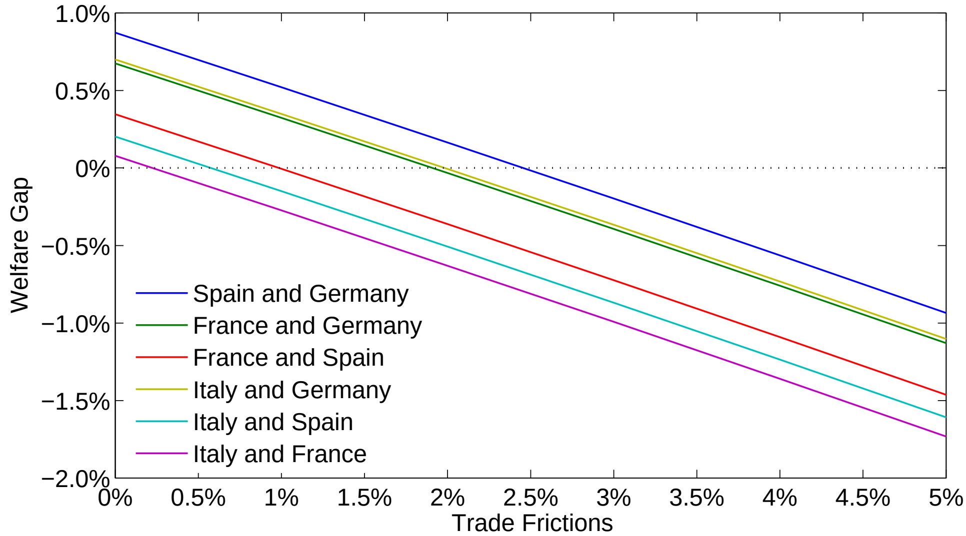


Table 2.4 reports the horizontal intercepts of the welfare gap lines in Figure 2.6. These appear to be more spaced out than in Figure 2.5, because the lower degree of risk sharing magnifies the different business cycle correlations of different economies. Like before, the critical amounts of trade frictions that equate welfare across monetary regimes are smallest for Italy and France and largest for Spain and Germany.

[27] In the extreme case of trade autarky due to a complete home bias ($\zeta = 0$), no risk sharing would be possible at all since countries would find themselves in financial autarky too.

Table 2.4: Trade frictions that equate welfare across regimes with home bias

Country pair	Critical τ
Italy and France	0.22%
Italy and Spain	0.58%
France and Spain	0.99%
Italy and Germany	1.91%
France and Germany	1.98%
Spain and Germany	2.46%

Interestingly, the ranking of welfare differentials in Table 2.3 reflects almost perfectly the ranking of inflation correlations in my sample, as shown in Table 2.5. Since inflation stabilisation is the main objective of monetary policy (as indicated by the estimated Taylor rule coefficients), two countries qualify as good monetary union partners if their inflation processes are sufficiently similar. The only exception to this ordering is the fact that the Germany-Italy pair ranks above the Spain-Germany one, despite having a slightly lower inflation correlation; however, this is due to their much stronger output comovement. Indeed, as economic activity is another important target of monetary policy, countries that display very correlated real GDP processes represent a good match for the purposes of establishing a currency union.

Table 2.5: Correlations of observable inflation and output processes

Country pair	Corr(π_t)	Corr(y_t)
Italy and France	0.86	0.92
Italy and Spain	0.82	0.72
France and Spain	0.81	0.69
France and Germany	0.77	0.91
Italy and Germany	0.66	0.89
Spain and Germany	0.68	0.58

2.5 Concluding remarks

What are the welfare implications of creating a currency union between different sovereign nations? In this chapter, I have revisited this question from the perspective of an open economy DSGE model with asymmetric shocks and imperfect risk sharing. I have emphasised one specific dimension of the tradeoff between alternative monetary arrangements: the conflict between the transactions and efficiency benefits of eliminating national currencies, on one hand, and the costs of abandoning monetary policy independence in the face of asynchronous business cycles, on the other.

Based on numerical work, I have argued that the introduction of a single currency would be welfare-reducing in an economy with local currency pricing and frictionless trade, because it would suppress an instrument of macroeconomic stabilisation and only provide a second-order benefit (that of removing price misalignments across markets). Once monetary barriers to trade are taken into account, however, monetary unification becomes welfare-improving because it eliminates a friction that has first-order effects on the economy.

Estimates from Italy, France, Germany and Spain suggest that these countries enjoy substantial welfare gains from sharing a common currency. The key to these findings is the fact that the welfare cost of imperfectly synchronised business cycles is modest for these economies, so the losses from missing country-specific instruments of monetary policy are easily exceeded by the gains from trade creation.

My modelling strategy ignores a number of important issues in the macroeconomics of monetary unions. It lacks an explicit description of financial intermediation and monetary transmission, it misses taxation and government spending, and it overlooks labour market frictions and unemployment, just to name a few. The introduction of international heterogeneity along these dimensions is likely to have important consequences for macroeconomic adjustment and social welfare in a currency union; it would be interesting to examine how it alters the tradeoff between alternative monetary arrangements and affects the domain of applicability of a single currency within the same type of framework presented here.

Appendix

Notation

It is convenient to write the bond positions of the two households in real terms and express them in their respective currencies:

$$a_{t+1} \equiv \frac{A_{t+1}}{p_t}, \qquad b_{t+1} \equiv e_t \frac{B_{t+1}}{p_t}, \qquad a^*_{t+1} \equiv \frac{A^*_{t+1}}{e_t p^*_t}, \qquad b^*_{t+1} \equiv \frac{B^*_{t+1}}{p^*_t}.$$

It is also useful to define three sets of relative prices. First, the PPI-to-CPI ratios:

$$\mathscr{P}_{h,t} \equiv \frac{p_{h,t}}{p_t}, \qquad \mathscr{P}_{f,t} \equiv \frac{p_{f,t}}{p_t}, \qquad \mathscr{P}^*_{h,t} \equiv \frac{p^*_{h,t}}{p^*_t}, \qquad \mathscr{P}^*_{f,t} \equiv \frac{p^*_{f,t}}{p^*_t}.$$

Second, the optimal relative prices of each good in each currency:

$$\tilde{p}_{h,t} \equiv \frac{\bar{p}_{h,t}}{p_{h,t}}, \qquad \tilde{p}^*_{h,t} \equiv \frac{\bar{p}^*_{h,t}}{p^*_{h,t}}, \qquad \tilde{p}_{f,t} \equiv \frac{\bar{p}_{f,t}}{p_{f,t}}, \qquad \tilde{p}^*_{f,t} \equiv \frac{\bar{p}^*_{f,t}}{p^*_{f,t}}.$$

Third, the relative prices of foreign to domestic goods in each currency:

$$s_t \equiv \frac{p_{f,t}}{p_{h,t}}, \qquad s^*_t \equiv \frac{p^*_{f,t}}{p^*_{h,t}}.$$

Consumption-based real wages in each country are

$$\mathrm{w}_t \equiv \frac{w_t}{p_t}, \qquad \mathrm{w}^*_t \equiv \frac{w^*_t}{p^*_t}.$$

Consumption-based real transfers are

$$t_t \equiv \frac{T_t}{p_t}, \qquad t^*_t \equiv \frac{T^*_t}{p^*_t}.$$

The depreciation of the nominal exchange rate is defined as

$$\Delta e_t \equiv \frac{e_t}{e_{t-1}}.$$

This notation allows us to rewrite the optimality conditions and the market-clearing conditions in a format that is suitable for computation.

Equilibrium conditions

Relative prices:

$$\mathscr{P}_{h,t} = \left[(1-\zeta) + (\zeta)\left(\frac{s_t}{1-\tau}\right)^{1-\eta}\right]^{\frac{1}{\eta-1}},$$

$$\mathscr{P}_{f,t} = \left[(1-\zeta)\left(\frac{1}{s_t}\right)^{1-\eta} + (\zeta)\left(\frac{1}{1-\tau}\right)^{1-\eta}\right]^{\frac{1}{\eta-1}},$$

$$\mathscr{P}^*_{h,t} = \left[(1-\zeta)\left(s^*_t\right)^{1-\eta} + (\zeta)\left(\frac{1}{1-\tau}\right)^{1-\eta}\right]^{\frac{1}{\eta-1}},$$

$$\mathscr{P}^*_{f,t} = \left[(1-\zeta) + (\zeta)\left(\frac{1}{s^*_t(1-\tau)}\right)^{1-\eta}\right]^{\frac{1}{\eta-1}}.$$

Consumption demands:

$$c_{h,t} = (1-\zeta)\left(\mathscr{P}_{h,t}\right)^{-\eta} c_t,$$

$$c_{f,t} = (\zeta)(1-\tau)^{\eta-1}\left(\mathscr{P}_{f,t}\right)^{-\eta} c_t,$$

$$c^*_{f,t} = (1-\zeta)\left(\mathscr{P}^*_{f,t}\right)^{-\eta} c^*_t,$$

$$c^*_{h,t} = (\zeta)(1-\tau)^{\eta-1}\left(\mathscr{P}^*_{h,t}\right)^{-\eta} c^*_t.$$

Market-clearing conditions for goods:

$$y_{h,t} = c_{h,t},$$

$$y^*_{h,t} = (1-\tau)\, c^*_{h,t} + \tau c^*_{h,t},$$

$$y_{f,t} = (1-\tau)\, c_{f,t} + \tau c_{f,t},$$

$$y^*_{f,t} = c^*_{f,t},$$

$$y_t = y_{h,t} + y^*_{h,t},$$

$$y^*_t = y_{f,t} + y^*_{f,t}.$$

Intertemporal optimisation:

$$\lambda_t = \xi_t c_t^{-\sigma},$$

$$\lambda_t = \xi_t \phi_t \frac{n_t^{\varphi}}{\mathrm{w}_t},$$

$$q^A_t = \beta \mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}}\right),$$

$$q^B_t\left(1-2\chi\left(b_{t+1}\right)\right)^{-1} = \beta \mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{\Delta e_{t+1}}{\pi_{t+1}}\right),$$

$$c_t + q^A_t a_{t+1} + q^B_t b_{t+1} + \chi q^B_t\left(b_{t+1}\right)^2 = \frac{a_t}{\pi_t} + \frac{b_t}{\Delta e_t \pi_t} + \mathrm{w}_t n_t + \mathscr{P}_{h,t}\Pi_{h,t} + t_t,$$

$$t_t = \mathcal{Q}_t \chi q^A_t\left(a^*_{t+1}\right)^2,$$

$$\lambda^*_t = \xi^*_t\left(c^*_t\right)^{-\sigma},$$

$$\lambda^*_t = \xi^*_t \phi^*_t \frac{\left(n^*_t\right)^{\varphi}}{\mathrm{w}^*_t},$$

$$q_t^A \left(1 - 2\chi \left(a_{t+1}^*\right)\right)^{-1} = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{1}{\Delta e_{t+1} \pi_{t+1}^*} \right),$$

$$q_t^B = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{1}{\pi_{t+1}^*} \right),$$

$$c_t^* + q_t^A a_{t+1}^* + q_t^B b_{t+1}^* + \chi q_t^A \left(a_{t+1}^*\right)^2 = \Delta e_t \frac{a_t^*}{\pi_t^*} + \frac{b_t^*}{\pi_t^*} + \mathrm{w}_t^* n_t^* + \mathscr{P}_{f,t}^* \Pi_{f,t} + t_t^*,$$

$$t_t^* = \frac{1}{\mathcal{Q}_t} \chi q_t^B \left(b_{t+1}\right)^2.$$

Real exchange rate, relative prices and price misalignments:

$$\mathcal{Q}_t = m_{h,t} \left[\frac{(1-\zeta)\left(s_t^*\right)^{1-\eta} + (\zeta)\left(\frac{1}{1-\tau}\right)^{1-\eta}}{(1-\zeta) + (\zeta)\left(\frac{s_t}{1-\tau}\right)^{1-\eta}} \right]^{\frac{1}{1-\eta}},$$

$$\frac{m_{f,t}}{m_{h,t}} = \frac{s_t^*}{s_t},$$

$$\frac{m_{h,t}}{m_{h,t-1}} = \Delta e_t \frac{\pi_{h,t}^*}{\pi_{h,t}},$$

$$\frac{m_{f,t}}{m_{f,t-1}} = \Delta e_t \frac{\pi_{f,t}^*}{\pi_{f,t}}.$$

Market-clearing conditions for bonds:

$$a_{t+1} = -a_{t+1}^* \mathcal{Q}_t,$$

$$b_{t+1} = -b_{t+1}^* \mathcal{Q}_t.$$

Output, marginal cost, price dispersion and aggregate profits:

$$y_t = \frac{z_t n_t}{d_t},$$

$$y_t^* = \frac{z_t^* n_t^*}{d_t^*},$$

$$mc_t = \frac{\mathrm{w}_t}{\mathscr{P}_{h,t}} \frac{d_t}{z_t},$$

$$mc_t^* = \frac{\mathrm{w}_t^*}{\mathscr{P}_{f,t}^*} \frac{d_t^*}{z_t^*},$$

$$d_{h,t} = \theta_p \left(\pi_{h,t}\right)^{\varepsilon} d_{h,t-1} + (1-\theta_p)\left(\tilde{p}_{h,t}\right)^{-\varepsilon},$$

$$d^*_{h,t} = \theta_p \left(\pi^*_{h,t}\right)^{\varepsilon} d^*_{h,t-1} + (1-\theta_p)\left(\tilde{p}^*_{h,t}\right)^{-\varepsilon},$$

$$d_{f,t} = \theta_p \left(\pi_{f,t}\right)^{\varepsilon} d_{f,t-1} + (1-\theta_p)\left(\tilde{p}_{f,t}\right)^{-\varepsilon},$$

$$d^*_{f,t} = \theta_p \left(\pi^*_{f,t}\right)^{\varepsilon} d^*_{f,t-1} + (1-\theta_p)\left(\tilde{p}^*_{f,t}\right)^{-\varepsilon},$$

$$d_t = x_{h,t} d_{h,t} + x^*_{h,t} d^*_{h,t},$$

$$d^*_t = x_{f,t} d_{f,t} + x^*_{f,t} d^*_{f,t},$$

$$x_{h,t} = \frac{y_{h,t}}{y_t},$$

$$x^*_{h,t} = \frac{y^*_{h,t}}{y_t},$$

$$x_{f,t} = \frac{y_{f,t}}{y^*_t},$$

$$x^*_{f,t} = \frac{y^*_{f,t}}{y^*_t},$$

$$\Pi_{h,t} = y_{h,t} + m_{h,t} y^*_{h,t} - \frac{\mathrm{w}_t}{\mathscr{P}_{h,t}} \frac{y_t}{z_t} d_t,$$

$$\Pi_{f,t} = y^*_{f,t} + \frac{y_{f,t}}{m_{f,t}} - \frac{\mathrm{w}^*_t}{\mathscr{P}^*_{f,t}} \frac{y^*_t}{z^*_t} d^*_t.$$

Price dynamics, inflation and optimal price setting:

$$\tilde{p}_{h,t} = \left[\frac{1-\theta_p \left(\pi_{h,t}\right)^{\varepsilon-1}}{1-\theta_p}\right]^{\frac{1}{1-\varepsilon}},$$

$$\tilde{p}^*_{h,t} = \left[\frac{1-\theta_p \left(\pi^*_{h,t}\right)^{\varepsilon-1}}{1-\theta_p}\right]^{\frac{1}{1-\varepsilon}},$$

$$\tilde{p}_{f,t} = \left[\frac{1-\theta_p \left(\pi_{f,t}\right)^{\varepsilon-1}}{1-\theta_p}\right]^{\frac{1}{1-\varepsilon}},$$

$$\tilde{p}^*_{f,t} = \left[\frac{1-\theta_p \left(\pi^*_{f,t}\right)^{\varepsilon-1}}{1-\theta_p}\right]^{\frac{1}{1-\varepsilon}},$$

$$\pi_{h,t} = \frac{\mathscr{P}_{h,t}}{\mathscr{P}_{h,t-1}}\pi_t,$$

$$\pi^*_{h,t} = \frac{\mathscr{P}^*_{h,t}}{\mathscr{P}^*_{h,t-1}}\pi^*_t,$$

$$\pi_{f,t} = \frac{\mathscr{P}_{f,t}}{\mathscr{P}_{f,t-1}}\pi_t,$$

$$\pi^*_{f,t} = \frac{\mathscr{P}^*_{f,t}}{\mathscr{P}^*_{f,t-1}}\pi^*_t,$$

$$g^2_{h,t} = \mathcal{M}_p g^1_{h,t},$$

$$g^1_{h,t} = y_{h,t}\frac{mc_t}{d_t}\left(\tilde{p}_{h,t}\right)^{-1-\varepsilon} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_{h,t+1}}\right)^{-1-\varepsilon}\left(\pi_{h,t+1}\right)^{\varepsilon} g^1_{h,t+1},$$

$$g^2_{h,t} = y_{h,t}\left(\tilde{p}_{h,t}\right)^{-\varepsilon} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_{h,t+1}}\right)^{-\varepsilon}\left(\pi_{h,t+1}\right)^{\varepsilon-1} g^2_{h,t+1},$$

$$g^{*2}_{h,t} = \mathcal{M}_p g^{*1}_{h,t},$$

$$g^{*1}_{h,t} = y^*_{h,t}\frac{mc_t}{d_t}\left(\tilde{p}^*_{h,t}\right)^{-1-\varepsilon} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\tilde{p}^*_{h,t}}{\tilde{p}^*_{h,t+1}}\right)^{-1-\varepsilon}\left(\pi^*_{h,t+1}\right)^{\varepsilon} g^{*1}_{h,t+1},$$

$$g^{*2}_{h,t} = m_{h,t}y^*_{h,t}\left(\tilde{p}^*_{h,t}\right)^{-\varepsilon} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{\tilde{p}^*_{h,t}}{\tilde{p}^*_{h,t+1}}\right)^{-\varepsilon}\left(\pi^*_{h,t+1}\right)^{\varepsilon-1} g^{*2}_{h,t+1},$$

$$g^{*2}_{f,t} = \mathcal{M}_p g^{*1}_{f,t},$$

$$g^{*1}_{f,t} = y^*_{f,t}\frac{mc^*_t}{d^*_t}\left(\tilde{p}^*_{f,t}\right)^{-1-\varepsilon} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda^*_{t+1}}{\lambda^*_t}\right)\left(\frac{\tilde{p}^*_{f,t}}{\tilde{p}^*_{f,t+1}}\right)^{-1-\varepsilon}\left(\pi^*_{f,t+1}\right)^{\varepsilon} g^{*1}_{f,t+1},$$

$$g^{*2}_{f,t} = y^*_{f,t}\left(\tilde{p}^*_{f,t}\right)^{-\varepsilon} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda^*_{t+1}}{\lambda^*_t}\right)\left(\frac{\tilde{p}^*_{f,t}}{\tilde{p}^*_{f,t+1}}\right)^{-\varepsilon}\left(\pi^*_{f,t+1}\right)^{\varepsilon-1} g^{*2}_{f,t+1},$$

$$g^2_{f,t} = \mathcal{M}_p g^1_{f,t},$$

$$g^1_{f,t} = y_{f,t}\frac{mc^*_t}{d^*_t}\left(\tilde{p}_{f,t}\right)^{-1-\varepsilon} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda^*_{t+1}}{\lambda^*_t}\right)\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_{f,t+1}}\right)^{-1-\varepsilon}\left(\pi_{f,t+1}\right)^{\varepsilon} g^1_{f,t+1},$$

$$g^2_{f,t} = \frac{y_{f,t}}{m_{f,t}}\left(\tilde{p}_{f,t}\right)^{-\varepsilon} + \theta_p\beta\mathbb{E}_t\left(\frac{\lambda^*_{t+1}}{\lambda^*_t}\right)\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_{f,t+1}}\right)^{-\varepsilon}\left(\pi_{f,t+1}\right)^{\varepsilon-1} g^2_{f,t+1},$$

$$\mathcal{M}_p = \frac{\varepsilon}{\varepsilon - 1}.$$

Interest rates and monetary policy with national currencies:

$$R_t = \frac{1}{q_t^A},$$

$$R_t^* = \frac{1}{q_t^B},$$

$$\frac{R_t}{\pi/\beta} = \left(\frac{R_{t-1}}{\pi/\beta}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\gamma_y} (\Delta e_t)^{\gamma_e}\right]^{1-\gamma_R} m_t,$$

$$\frac{R_t^*}{\pi^*/\beta} = \left(\frac{R_{t-1}^*}{\pi^*/\beta}\right)^{\gamma_R} \left[\left(\frac{\pi_t^*}{\pi^*}\right)^{\gamma_\pi} \left(\frac{y_t^*}{y_{t-1}^*}\right)^{\gamma_y} \left(\frac{1}{\Delta e_t}\right)^{\gamma_e}\right]^{1-\gamma_R} m_t^*.$$

Interest rates and monetary policy with a single currency:

$$\frac{R_t^u}{\pi_u/\beta} = \left(\frac{R_{t-1}^u}{\pi_u/\beta}\right)^{\gamma_R} \left[\left(\frac{\pi_{u,t}}{\pi_u}\right)^{\gamma_\pi} \left(\frac{y_{u,t}}{y_{u,t-1}}\right)^{\gamma_y}\right]^{1-\gamma_R} m_t^u,$$

$$\pi_{u,t} = (\pi_{h,t})^{\frac{1}{2}} (\pi_{f,t})^{\frac{1}{2}},$$

$$y_{u,t} = y_t + y_t^*,$$

$$R_t = R_t^u \left(\frac{\pi_{h,t}}{\pi_{u,t}}\right)^{\omega},$$

$$R_t^* = R_t^u \left(\frac{\pi_{f,t}}{\pi_{u,t}}\right)^{\omega}.$$

Exogenous processes:

$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \nu_\xi \log \xi_{t-1}^* + e_{\xi,t},$$

$$\log \xi_t^* = \rho_\xi \log \xi_{t-1}^* + \nu_\xi \log \xi_{t-1} + e_{\xi,t}^*,$$

$$\log \phi_t = \rho_\phi \log \phi_{t-1} + \nu_\phi \log \phi_{t-1}^* + e_{\phi,t},$$

$$\log \phi_t^* = \rho_\phi \log \phi_{t-1}^* + \nu_\phi \log \phi_{t-1} + e_{\phi,t}^*,$$

$$\log z_t = \rho_z \log z_{t-1} + \nu_z \log z_{t-1}^* + e_{z,t},$$

$$\log z_t^* = \rho_z \log z_{t-1}^* + \nu_z \log z_{t-1} + e_{z,t}^*,$$

$$\log m_t = \rho_m \log m_{t-1} + \nu_m \log m_{t-1}^* + e_{m,t},$$

$$\log m_t^* = \rho_m \log m_{t-1}^* + \nu_m \log m_{t-1} + e_{m,t}^*,$$

$$\log m_t^u = \rho_m \log m_{t-1}^u + e_{m,t}^u.$$

Calibration in Section 2.2.2

Table 2.6: Calibration

Parameter	Value	Description
σ	1	Elasticity of intertemporal substitution
φ	1	Frisch elasticity of labour supply
ε	6	Elasticity of substitution between intermediates
ζ	0.5	Share of imports in consumption
η	1	Armington elasticity of substitution
β	0.99	Subjective discount factor
χ	0.005	Bond holdings adjustment cost
θ_p	0.66	Price stickiness parameter
τ	0	Trade cost parameter
γ_R	0	Interest rate smoothing parameter in the Taylor rule
γ_π	1.5	Inflation parameter in the Taylor rule
γ_y	0	Output parameter in the Taylor rule
γ_e	1.5	Exchange rate parameter in the Taylor rule
ω	0.001	Elasticity of interest rates to inflation differentials
σ_{e_z}	0.01	Standard deviation of productivity shocks
σ_{e_m}	0	Standard deviation of monetary shocks
σ_{e_ξ}	0	Standard deviation of preference shocks
σ_{e_ϕ}	0	Standard deviation of labour supply shocks
ρ_z	0.95	Serial correlation of productivity shocks
ρ_m	0.95	Serial correlation of monetary shocks
ρ_ξ	0.95	Serial correlation of preference shocks
ρ_ϕ	0.95	Serial correlation of labour supply shocks
ν_z	0	International spillover of productivity shocks
ν_m	0	International spillover of monetary shocks
ν_ξ	0	International spillover of preference shocks
ν_ϕ	0	International spillover of labour supply shocks
$\text{corr}\left(e_z, e_z^*\right)$	0	International correlation of productivity shocks
$\text{corr}\left(e_m, e_m^*\right)$	0	International correlation of monetary shocks
$\text{corr}\left(e_\xi, e_\xi^*\right)$	0	International correlation of preference shocks
$\text{corr}\left(e_\phi, e_\phi^*\right)$	0	International correlation of labour supply shocks

Estimation: Tables and Figures

Table 2.7: Calibrated parameters

Parameter	Description	Value
σ	Elasticity of intertemporal substitution	1
φ	Frisch elasticity of labour supply	1
ε	Elasticity of substitution between intermediates	6
ζ	Share of imports in consumption	0.5 or 0.35*
η	Armington elasticity of substitution	1
β	Subjective discount factor	0.99
χ	Bond holdings adjustment cost	0.005
θ_p	Price stickiness parameter	0.66
ω	Elasticity of interest rates to inflation differentials	0.001
ν_z	International spillover of productivity shocks	0
ν_m	International spillover of monetary shocks	0
ν_ξ	International spillover of preference shocks	0
ν_ϕ	International spillover of labour supply shocks	0
τ	Trade cost parameter	0.05
$\text{corr}\,(e_m, e_m^*)$	International correlation of monetary shocks	0
γ_e	Exchange rate parameter in the Taylor rule	3

* Estimation without home bias: $\zeta = 0.5$. Estimation with home bias: $\zeta = 0.35$.

Table 2.8: Prior distributions ($\mathcal{N}$ = normal, $\mathcal{G}$ = gamma, $\mathcal{B}$ = beta)

Parameter	Description	Prior distr. (μ, σ)
γ_R	Int. rate smoothing param. in the Taylor rule	$\mathcal{N}$ (0, 0.25)
γ_π	Inflation parameter in the Taylor rule	$\mathcal{N}$ (1.5, 0.25)
γ_y	Output parameter in the Taylor rule	$\mathcal{N}$ (0.5, 0.25)
σ_{e_z}	St. deviation of home productivity shocks	$\mathcal{G}$ (0.05, 0.025)
$\sigma_{e_z^*}$	St. deviation of foreign productivity shocks	$\mathcal{G}$ (0.05, 0.025)
$\sigma_{e_m^u}$	St. deviation of monetary shocks	$\mathcal{G}$ (0.05, 0.025)
σ_{e_ξ}	St. deviation of home preference shocks	$\mathcal{G}$ (0.05, 0.025)
$\sigma_{e_\xi^*}$	St. deviation of foreign preference shocks	$\mathcal{G}$ (0.05, 0.025)
σ_{e_ϕ}	St. deviation of home labour supply shocks	$\mathcal{G}$ (0.05, 0.025)
$\sigma_{e_\phi^*}$	St. deviation of foreign labour supply shocks	$\mathcal{G}$ (0.05, 0.025)
ρ_z	Serial correlation of productivity shocks	$\mathcal{B}$ (0.5, 0.15)
ρ_m	Serial correlation of monetary shocks	$\mathcal{B}$ (0.5, 0.15)
ρ_ξ	Serial correlation of preference shocks	$\mathcal{B}$ (0.5, 0.15)
ρ_ϕ	Serial correlation of labour supply shocks	$\mathcal{B}$ (0.5, 0.15)
$\text{corr}\,(e_z, e_z^*)$	International correlation of productivity shocks	$\mathcal{N}$ (0, 0.25)
$\text{corr}\left(e_\xi, e_\xi^*\right)$	International correlation of preference shocks	$\mathcal{N}$ (0, 0.25)
$\text{corr}\left(e_\phi, e_\phi^*\right)$	International correlation of labour supply shocks	$\mathcal{N}$ (0, 0.25)

Table 2.9: Parameter estimates without home bias, part I

Country pair	Italy and France		Italy and Germany		Italy and Spain	
Parameter	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval
γ_R	0.2780	[0.1147, 0.4448]	0.5115	[0.4032, 0.6241]	0.5152	[0.4074, 0.6279]
γ_π	2.6082	[2.3331, 2.8868]	2.5004	[2.2145, 2.7791]	2.4972	[2.2137, 2.7707]
γ_y	0.1295	[0.0482, 0.2106]	0.1378	[0.0700, 0.2043]	0.3192	[0.1980, 0.4389]
σ_{e_z}	0.0131	[0.0103, 0.0158]	0.0090	[0.0075, 0.0105]	0.0091	[0.0076, 0.0105]
$\sigma_{e_z^*}$	0.0110	[0.0082, 0.0138]	0.0194	[0.0165, 0.0223]	0.0119	[0.0098, 0.0139]
$\sigma_{e_m^u}$	0.0018	[0.0013, 0.0023]	0.0011	[0.0008, 0.0014]	0.0011	[0.0008, 0.0014]
σ_{e_ξ}	0.0015	[0.0007, 0.0023]	0.0408	[0.0224, 0.0586]	0.0437	[0.0241, 0.0636]
$\sigma_{e_\xi^*}$	0.0036	[0.0021, 0.0050]	0.0442	[0.0211, 0.0662]	0.0528	[0.0288, 0.0758]
σ_{e_ϕ}	0.0276	[0.0208, 0.0342]	0.0152	[0.0120, 0.0183]	0.0114	[0.0082, 0.0147]
$\sigma_{e_\phi^*}$	0.0344	[0.0272, 0.0417]	0.0343	[0.0286, 0.0402]	0.0273	[0.0218, 0.0328]
ρ_z	0.9407	[0.9161, 0.9664]	0.9824	[0.9727, 0.9923]	0.9862	[0.9786, 0.9945]
ρ_m	0.9398	[0.9099, 0.9696]	0.9487	[0.9247, 0.9735]	0.9521	[0.9292, 0.9756]
ρ_ξ	0.1493	[0.0508, 0.2421]	0.9732	[0.9600, 0.9867]	0.9738	[0.9608, 0.9870]
ρ_ϕ	0.7583	[0.7061, 0.8145]	0.0486	[0.0153, 0.0799]	0.0611	[0.0202, 0.1003]
$\mathrm{corr}\left(e_z, e_z^*\right)$	0.7795	[0.6126, 0.9663]	-0.3321	[-0.5193, -0.1500]	-0.3394	[-0.5362, -0.1490]
$\mathrm{corr}\left(e_\xi, e_\xi^*\right)$	-0.2386	[-0.6068, 0.1359]	0.3913	[0.0277, 0.7633]	0.3620	[0.0151, 0.7093]
$\mathrm{corr}\left(e_\phi, e_\phi^*\right)$	0.9219	[0.8210, 1.0000]	-0.9622	[-1.0000, -0.9108]	-0.8814	[-1.0000, -0.7114]

Table 2.10: Parameter estimates without home bias, part II

Country pair	France and Germany		France and Spain		Spain and Germany	
Parameter	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval
γ_R	0.5703	[0.4671, 0.6765]	0.5541	[0.4437, 0.6689]	0.7208	[0.6464, 0.7969]
γ_π	2.4830	[2.1965, 2.7755]	2.4720	[2.1805, 2.7611]	2.2821	[1.9773, 2.5971]
γ_y	0.1221	[0.0299, 0.2127]	0.3340	[0.1466, 0.5226]	0.3038	[0.1185, 0.4823]
σ_{e_z}	0.0071	[0.0058, 0.0084]	0.0069	[0.0056, 0.0082]	0.0091	[0.0073, 0.0108]
$\sigma_{e_z^*}$	0.0209	[0.0178, 0.0239]	0.0128	[0.0106, 0.0150]	0.0225	[0.0191, 0.0259]
$\sigma_{e_m^u}$	0.0013	[0.0009, 0.0017]	0.0014	[0.0010, 0.0019]	0.0016	[0.0010, 0.0022]
σ_{e_ξ}	0.0349	[0.0187, 0.0513]	0.0390	[0.0196, 0.0580]	0.0313	[0.0160, 0.0469]
$\sigma_{e_\xi^*}$	0.0432	[0.0202, 0.0655]	0.0478	[0.0263, 0.0699]	0.0367	[0.0134, 0.0597]
σ_{e_ϕ}	0.0139	[0.0108, 0.0171]	0.0088	[0.0058, 0.0118]	0.0177	[0.0135, 0.0220]
$\sigma_{e_\phi^*}$	0.0398	[0.0331, 0.0462]	0.0277	[0.0222, 0.0331]	0.0451	[0.0376, 0.0527]
ρ_z	0.9880	[0.9815, 0.9950]	0.9899	[0.9848, 0.9960]	0.9895	[0.9839, 0.9957]
ρ_m	0.8895	[0.8432, 0.9375]	0.8796	[0.8281, 0.9335]	0.7830	[0.7115, 0.8588]
ρ_ξ	0.9682	[0.9521, 0.9850]	0.9692	[0.9525, 0.9860]	0.9659	[0.9472, 0.9852]
ρ_ϕ	0.0463	[0.0151, 0.0765]	0.0463	[0.0147, 0.0764]	0.0453	[0.0148, 0.0752]
corr (e_z, e_z^*)	-0.5249	[-0.6904, -0.3629]	-0.4754	[-0.6729, -0.2862]	-0.7226	[-0.8387, -0.6096]
corr $\left(e_\xi, e_\xi^*\right)$	0.4379	[0.0635, 0.8280]	0.4044	[0.0480, 0.7667]	0.3088	[-0.0671, 0.6867]
corr $\left(e_\phi, e_\phi^*\right)$	-0.9501	[-1.0000, -0.8803]	-0.7641	[-1.0000, -0.4840]	-0.9659	[-1.0000, -0.9186]

Table 2.11: Unconditional variance decompositions: Italy and France

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	86.95%	0.52%	5.61%	0.67%	2.62%	0.05%	3.57%
y_t^*	4.66%	66.47%	16.08%	7.86%	0.60%	1.26%	3.06%
c_t	40.64%	29.94%	16.69%	5.10%	1.92%	0.66%	5.06%
c_t^*	41.75%	30.90%	14.32%	5.18%	2.04%	0.71%	5.11%
π_t	0.19%	0.04%	0.08%	0.39%	0.06%	0.01%	99.24%
π_t^*	1.09%	1.61%	15.86%	11.63%	0.10%	0.27%	69.44%
R_t^u	0.83%	0.36%	1.00%	1.84%	0.30%	0.04%	95.63%

Table 2.12: Unconditional variance decompositions: Italy and Germany

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	95.73%	0.16%	0.00%	0.00%	0.08%	0.00%	4.03%
y_t^*	6.38%	89.40%	0.00%	0.00%	0.08%	0.00%	4.14%
c_t	36.74%	53.63%	0.00%	0.00%	0.18%	0.00%	9.44%
c_t^*	36.93%	53.52%	0.00%	0.00%	0.18%	0.00%	9.36%
π_t	0.06%	0.01%	0.03%	0.00%	0.03%	0.00%	99.86%
π_t^*	1.00%	0.74%	0.29%	0.08%	0.07%	0.11%	97.72%
R_t^u	0.13%	0.08%	0.26%	0.00%	0.06%	0.00%	99.47%

Table 2.13: Unconditional variance decompositions: Italy and Spain

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	96.73%	0.10%	0.00%	0.00%	0.06%	0.00%	3.11%
y_t^*	1.11%	95.67%	0.00%	0.00%	0.05%	0.00%	3.16%
c_t	48.71%	45.37%	0.00%	0.00%	0.11%	0.00%	5.82%
c_t^*	48.65%	45.45%	0.00%	0.00%	0.11%	0.00%	5.79%
π_t	0.06%	0.01%	0.03%	0.00%	0.03%	0.00%	99.97%
π_t^*	0.51%	0.79%	0.20%	0.07%	0.03%	0.11%	98.29%
R_t^u	0.05%	0.07%	0.23%	0.00%	0.04%	0.00%	99.60%

Table 2.14: Unconditional variance decompositions: France and Germany

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	94.29%	0.11%	0.00%	0.00%	0.05%	0.00%	5.54%
y_t^*	6.13%	88.18%	0.01%	0.00%	0.05%	0.00%	5.64%
c_t	34.57%	52.51%	0.00%	0.00%	0.11%	0.00%	12.80%
c_t^*	34.71%	52.45%	0.01%	0.00%	0.11%	0.00%	12.72%
π_t	0.10%	0.03%	0.07%	0.00%	0.06%	0.00%	99.74%
π_t^*	1.78%	1.27%	0.61%	0.16%	0.12%	0.20%	95.86%
R_t^u	0.28%	0.19%	0.81%	0.00%	0.13%	0.00%	98.59%

Table 2.15: Unconditional variance decompositions: France and Spain

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	95.82%	0.08%	0.00%	0.00%	0.04%	0.00%	4.07%
y_t^*	0.80%	95.04%	0.00%	0.00%	0.04%	0.00%	4.12%
c_t	47.40%	44.95%	0.00%	0.00%	0.07%	0.00%	7.57%
c_t^*	47.36%	45.02%	0.00%	0.00%	0.07%	0.00%	7.55%
π_t	0.14%	0.02%	0.08%	0.00%	0.07%	0.00%	99.68%
π_t^*	1.13%	1.77%	0.52%	0.19%	0.06%	0.02%	96.08%
R_t^u	0.15%	0.24%	0.93%	0.00%	0.12%	0.00%	98.57%

Table 2.16: Unconditional variance decompositions: Spain and Germany

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	83.51%	0.07%	0.01%	0.00%	0.03%	0.00%	16.38%
y_t^*	5.29%	78.07%	0.01%	0.00%	0.02%	0.00%	16.61%
c_t	26.41%	40.93%	0.02%	0.00%	0.06%	0.00%	32.58%
c_t^*	26.49%	40.96%	0.02%	0.00%	0.06%	0.00%	32.47%
π_t	0.12%	0.02%	0.09%	0.00%	0.06%	0.00%	99.71%
π_t^*	1.32%	1.16%	0.60%	0.14%	0.10%	0.16%	96.52%
R_t^u	0.32%	0.31%	2.36%	0.00%	0.12%	0.00%	96.89%

Figure 2.7: Probability density functions of λ without home bias

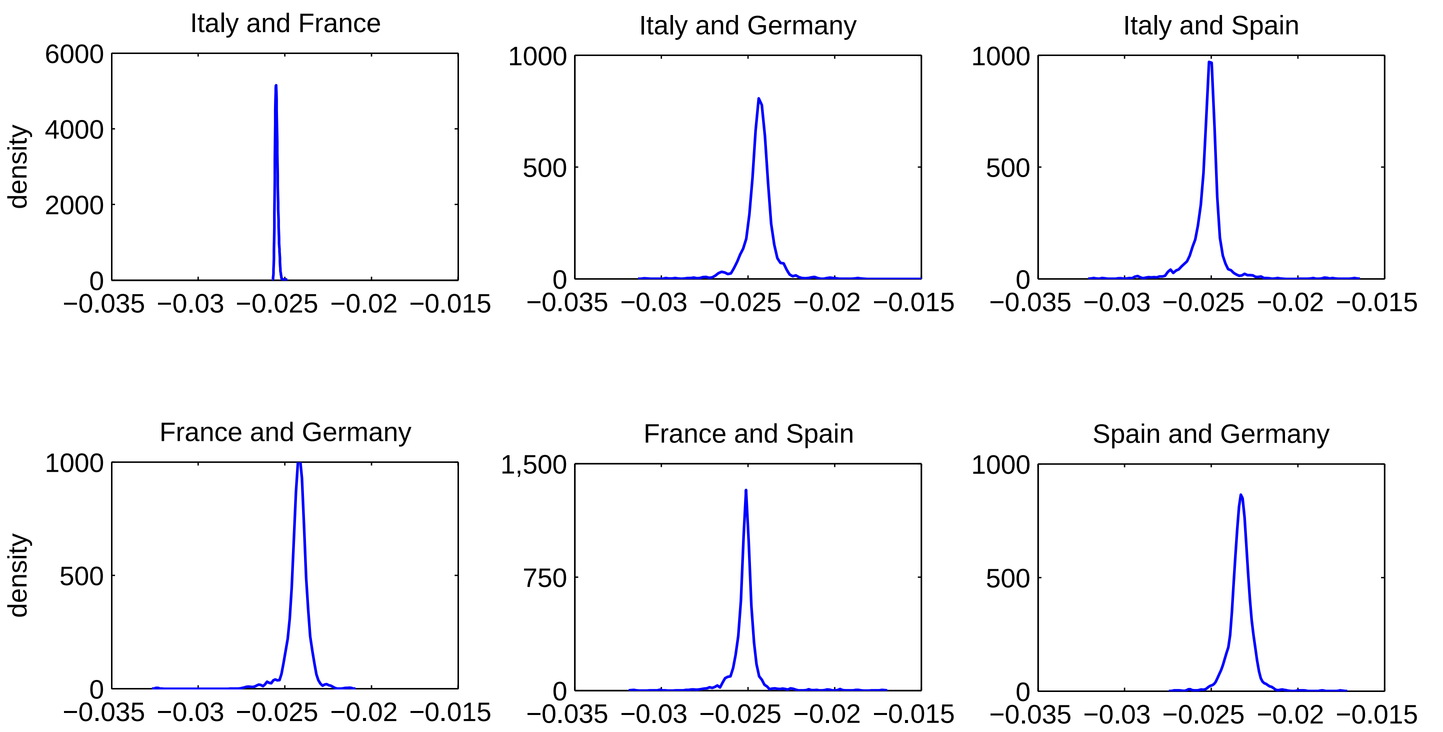


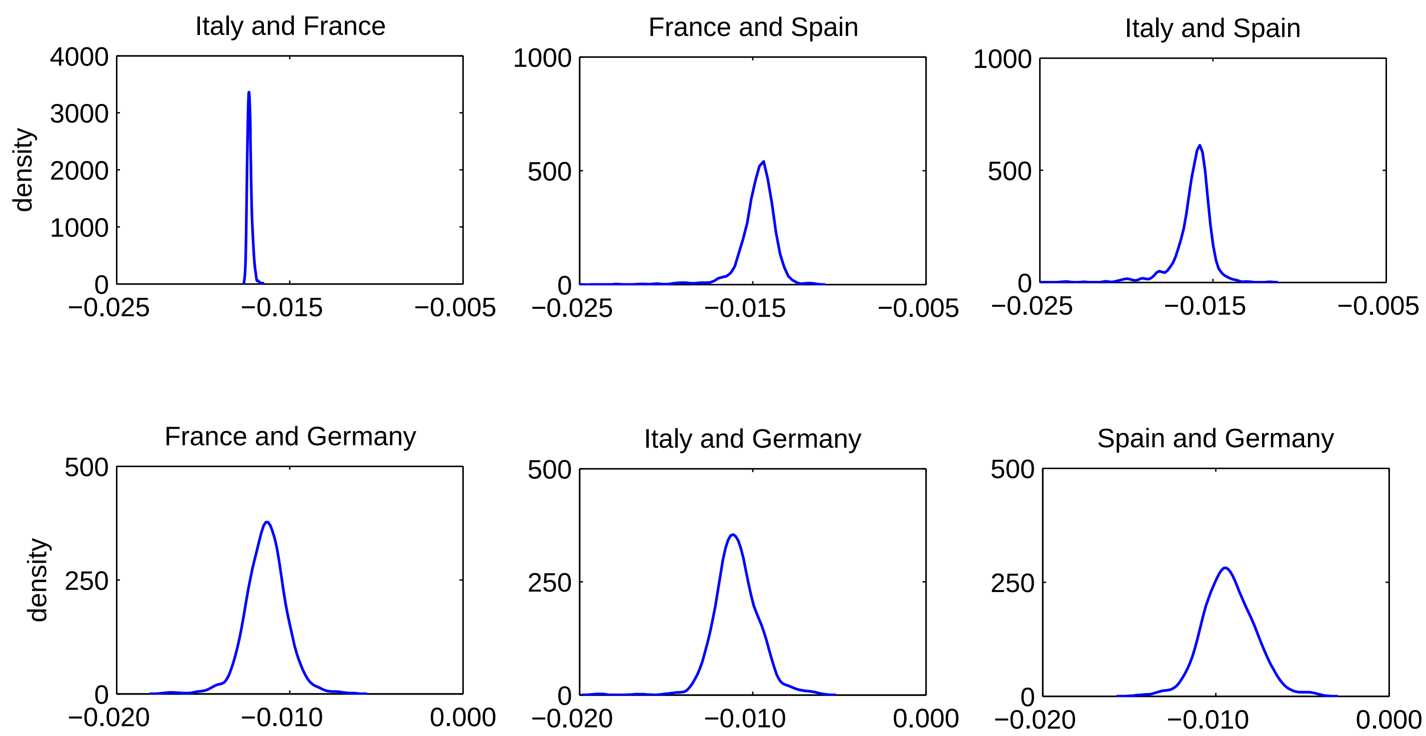
Table 2.17: Parameter estimates with home bias, part I

Country pair	Italy and France		Italy and Germany		Italy and Spain	
Parameter	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval
γ_R	0.3901	[0.2105, 0.5722]	0.6161	[0.4872, 0.7485]	0.5842	[0.4709, 0.7008]
γ_π	1.8971	[1.4634, 2.3248]	2.3544	[2.0545, 2.6613]	2.4430	[2.1485, 2.7304]
γ_y	-0.0141	[-0.1606, 0.1299]	0.2962	[0.1505, 0.4325]	0.1943	[0.0407, 0.3428]
σ_{e_z}	0.0091	[0.0066, 0.0116]	0.0110	[0.0089, 0.0129]	0.0090	[0.0076, 0.0105]
$\sigma_{e_z^*}$	0.0095	[0.0069, 0.0120]	0.0453	[0.0384, 0.0519]	0.0253	[0.0214, 0.0292]
$\sigma_{e_m^u}$	0.0042	[0.0026, 0.0057]	0.0021	[0.0013, 0.0028]	0.0015	[0.0010, 0.0020]
σ_{e_ξ}	0.0030	[0.0019, 0.0041]	0.0321	[0.0156, 0.0486]	0.0373	[0.0198, 0.0549]
$\sigma_{e_\xi^*}$	0.0069	[0.0047, 0.0090]	0.0361	[0.0101, 0.0611]	0.0485	[0.0227, 0.0735]
σ_{e_ϕ}	0.0112	[0.0034, 0.0187]	0.0200	[0.0151, 0.0251]	0.0128	[0.0093, 0.0164]
$\sigma_{e_\phi^*}$	0.0213	[0.0141, 0.0283]	0.0996	[0.0848, 0.1143]	0.0609	[0.0512, 0.0704]
ρ_z	0.9785	[0.9649, 0.9933]	0.9823	[0.9722, 0.9923]	0.9835	[0.9740, 0.9932]
ρ_m	0.5862	[0.4749, 0.6962]	0.8140	[0.7484, 0.8846]	0.8940	[0.8500, 0.9398]
ρ_ξ	0.0904	[0.0234, 0.1545]	0.9659	[0.9463, 0.9859]	0.9701	[0.9548, 0.9863]
ρ_ϕ	0.7873	[0.7276, 0.8478]	0.0324	[0.0101, 0.0528]	0.0439	[0.0136, 0.0727]
$\text{corr}\left(e_z, e_z^*\right)$	0.5346	[0.2512, 0.8214]	-0.3321	[-0.5193, -0.1500]	-0.4526	[-0.6478, -0.2635]
$\text{corr}\left(e_\xi, e_\xi^*\right)$	-0.5459	[-1.0000, -0.1804]	0.3913	[0.0277, 0.7633]	0.2619	[-0.1036, 0.6226]
$\text{corr}\left(e_\phi, e_\phi^*\right)$	0.2509	[-0.1724, 0.6704]	-0.9622	[-1.0000, -0.9108]	-0.9331	[-1.0000, -0.8280]

Table 2.18: Parameter estimates with home bias, part II

Country pair	France and Germany		France and Spain		Spain and Germany	
Parameter	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval
γ_R	0.6537	[0.5076, 0.8009]	0.6131	[0.4833, 0.7520]	0.7558	[0.6618, 0.8476]
γ_π	2.2712	[1.9477, 2.5932]	2.2843	[1.9667, 2.5994]	2.1277	[1.8020, 2.4469]
γ_y	0.3957	[0.1519, 0.6174]	0.3638	[0.0526, 0.7392]	0.7330	[0.3860, 1.0845]
σ_{e_z}	0.0096	[0.0073, 0.0122]	0.0081	[0.0053, 0.0103]	0.0077	[0.0038, 0.0119]
$\sigma_{e_z^*}$	0.0441	[0.0373, 0.0509]	0.0321	[0.0267, 0.0375]	0.0519	[0.0420, 0.0614]
$\sigma_{e_m^u}$	0.0022	[0.0012, 0.0031]	0.0022	[0.0014, 0.0030]	0.0018	[0.0011, 0.0025]
σ_{e_ξ}	0.0257	[0.0125, 0.0391]	0.0269	[0.0129, 0.0403]	0.0290	[0.0147, 0.0435]
$\sigma_{e_\xi^*}$	0.0354	[0.0099, 0.0598]	0.0422	[0.0157, 0.0662]	0.0366	[0.0094, 0.0629]
σ_{e_ϕ}	0.0214	[0.0156, 0.0285]	0.0142	[0.0062, 0.0205]	0.0131	[0.0017, 0.0246]
$\sigma_{e_\phi^*}$	0.0973	[0.0820, 0.1121]	0.0741	[0.0614, 0.0869]	0.1172	[0.0950, 0.1391]
ρ_z	0.9885	[0.9823, 0.9953]	0.9890	[0.9833, 0.9960]	0.9899	[0.9847, 0.9960]
ρ_m	0.6860	[0.6015, 0.7762]	0.7174	[0.6294, 0.8059]	0.7198	[0.6395, 0.8032]
ρ_ξ	0.9615	[0.9387, 0.9843]	0.9636	[0.9431, 0.9855]	0.9619	[0.9402, 0.9843]
ρ_ϕ	0.0331	[0.0104, 0.0551]	0.0357	[0.0107, 0.0593]	0.0321	[0.0097, 0.0532]
$\text{corr}\left(e_z, e_z^*\right)$	-0.7895	[-0.9091, -0.6913]	-0.6688	[-0.8889, -0.3818]	-0.5778	[-0.9327, -0.1255]
$\text{corr}\left(e_\xi, e_\xi^*\right)$	0.1883	[-0.2104, 0.5947]	0.2415	[-0.1486, 0.6182]	0.1573	[-0.2272, 0.5367]
$\text{corr}\left(e_\phi, e_\phi^*\right)$	-0.9674	[-1.0000, -0.9428]	-0.8905	[-1.0000, -0.5543]	-0.6365	[-1.0000, -0.0030]

Figure 2.8: Probability density functions of λ with home bias



Bibliography

Adjemian, S., H. Bastani, M. Juillard, F. Karamé, F. Mihoubi, G. Perendia, J. Pfeifer, M. Ratto, and S. Villemot (2011). Dynare: Reference manual, version 4. *Dynare Working Papers 1.*

Bacchetta, P. and E. van Wincoop (2000). Does exchange-rate stability increase trade and welfare? *American Economic Review 90*(5), 1093–1109.

Beetsma, R. and H. Jensen (2005). Monetary and fiscal policy interactions in a micro-founded model of a monetary union. *Journal of International Economics 67*(2), 320–352.

Benigno, G. and P. Benigno (2003). Price stability in open economies. *Review of Economic Studies 70*(4), 743–764.

Benigno, P. (2004). Optimal monetary policy in a currency area. *Journal of International Economics 63*(2), 293–320.

Benigno, P. (2009). Price stability with imperfect financial integration. *Journal of Money, Credit and Banking 41*(S1), 121–149.

Bhattarai, S., J. W. Lee, and W. Y. Park (2015). Optimal monetary policy in a currency union with interest rate spreads. *Journal of International Economics 96*(2), 375–397.

Ching, S. and M. B. Devereux (2003). Mundell revisited: a simple approach to the costs and benefits of a single currency area. *Review of International Economics 11*(4), 674–691.

Clarida, R. (2007). *G7 Current Account Imbalances - Sustainability and Adjustment.* National Bureau of Economic Research Conference Report.

Clarida, R., J. Galí, and M. Gertler (2002). A simple framework for international monetary policy analysis. *Journal of Monetary Economics 49*(5), 879–904.

Coenen, G., G. Lombardo, F. Smets, and R. Straub (2009). International transmission and monetary policy cooperation. *In ed. Jordi Galí and Mark Gertler (2009).*

Coeurdacier, N. and H. Rey (2012). Home bias in open economy financial macroeconomics. *Journal of Economic Literature 51*(1), 63–115.

Cole, H. and M. Obstfeld (1991). Commodity trade and international risk sharing. *Journal of Monetary Economics 28*(1), 3–24.

Corden, W. M. (1972). Monetary integration. *Princeton Essays in International Finance 93*.

Corsetti, G., L. Dedola, and S. Leduc (2011). Optimal monetary policy in open economies. *In ed. Benjamin Friedmand and Michael Woodford (2011).*

Corsetti, G. and P. Pesenti (2001). Welfare and macroeconomic interdependence. *Quarterly Journal of Economics 116*(2), 421–445.

Cúrdia, V. and M. Woodford (2010). Credit spreads and monetary policy. *Journal of Money, Credit and Banking 42*(6), 3–35.

De Paoli, B. (2009). Monetary policy under alternative asset market structures: The case of a small open economy. *Journal of Money, Credit and Banking 41*(7), 1301–1330.

Devereux, M. B. and C. Engel (2003). Monetary policy in the open economy revisited: Price setting and exchange-rate flexibility. *Review of Economic Studies 70*(4), 765–783.

Devereux, M. B., C. Engel, and C. Tille (2003). Exchange rate pass-through and the welfare effects of the Euro. *International Economic Review 44*(1), 223–242.

Devereux, M. B. and A. Sutherland (2008). Financial globalization and monetary policy. *Journal of Monetary Economics 55*(8), 1363–1375.

Duarte, M. and M. Obstfeld (2008). Monetary policy in the open economy revisited: The case for exchange-rate flexibility restored. *Journal of International Money and Finance 27*(6), 949–957.

Engel, C. (2011). Currency misalignments and optimal monetary policy: A reexamination. *American Economic Review 101*(6), 2796–2822.

Engel, C. (2014). Exchange rate stabilization and welfare. *Annual Review of Economics 6*, 155–177.

Erceg, C., C. Gust, and D. López-Salido (2009). The transmission of domestic shocks in open economies. *In ed. Jordi Galí and Mark Gertler (2009).*

Fatás, A. (1998). Does EMU need a fiscal federation? *Economic Policy 13*(26), 163–203.

Feldstein, M. (2015). Ending the euro crisis. *Journal of Policy Modeling 37*(3), 423–427.

Fernández-Villaverde, J. and J. F. Rubio-Ramírez (2005). Estimating dynamic equilibrium economies: Linear versus nonlinear likelihood. *Journal of Applied Econometrics 20*(7), 891–910.

Ferrero, A. (2009). Fiscal and monetary rules for a currency union. *Journal of International Economics 77*(1), 1–10.

Forlati, C. (2009). Optimal monetary and fiscal policy in the EMU: Does fiscal policy coordination matter? *EPFI CFI Center for Fiscal Policy Working Paper 04.*

Frankel, J. (2015). The euro crisis: Where to from here? *Journal of Policy Modeling 37*(3), 428–444.

Friedman, B. M. and M. Woodford (2011). *Handbook of Monetary Economics.* North-Holland.

Galí, J. (2008). *Monetary Policy, Inflation and the Business Cycle.* Princeton University Press.

Galí, J. and M. Gertler (2009). *International Dimensions of Monetary Policy.* National Bureau of Economic Research.

Galí, J. and T. Monacelli (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies 72*(3), 707–734.

Gourinchas, P. O. and H. Rey (2007). International financial adjustment. *Journal of Political Economy 115*(4), 665–703.

Heathcote, J. and F. Perri (2002). Financial autarky and international business cycles. *Journal of Monetary Economics 49*(3), 601–627.

Ingram, J. C. (1973). The case for European monetary integration. *Princeton Essays in International Finance 98*.

Ireland, P. N. (2004). Technology shocks in the New Keynesian Model. *The Review of Economics and Statistics 86* (4), 923–936.

Kim, J. and S. H. Kim (2003). Spurious welfare reversals in international business cycle models. *Journal of International Economics 60* (2), 471–500.

Kim, J., S. H. Kim, E. Schaumburg, and C. A. Sims (2008). Calculating and using second-order accurate solutions of discrete time dynamic equilibrium models. *Journal of Economic Dynamics and Control 32* (11), 3397–3414.

Lama, R. and P. Rabanal (2014). Deciding to enter a monetary union: the role of trade and financial linkages. *European Economic Review 72*, 138–165.

Lane, P. R. (2013). Financial globalisation and the crisis. *Open Economies Review 24* (3), 555–580.

Lane, P. R. and G. M. Milesi-Ferretti (2001). The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries. *Journal of International Economics 55* (2), 263–294.

Lane, P. R. and G. M. Milesi-Ferretti (2006). The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970-2003. *Journal of International Economics 73* (2), 223–250.

Lane, P. R. and G. M. Milesi-Ferretti (2007). A global perspective on external positions. *In ed. Richard Clarida (2007)*.

Lucas, R. E. (1987). *Models of Business Cycles*. Basil Blackwell.

McKinnon, R. I. (1963). Optimum currency areas. *American Economic Review 53* (4), 717–725.

Mishkin, F. and K. Schmidt-Hebbel (2007). *Monetary Policy under Inflation Targeting*. Central Bank of Chile.

Mundell, R. (1961). A theory of optimum currency areas. *American Economic Review 51* (4), 657–665.

Obstfeld, M. and G. Peri (1998). Regional non-adjustment and fiscal policy. *Economic Policy 13* (26), 205–259.

Rose, A. (2000). One money, one market: the effect of common currencies on trade. *Economic Policy 15*(30), 7–45.

Rose, A. and E. van Wincoop (2001). National money as a barrier to international trade: The real case for currency union. *American Economic Review 91*(2), 386–390.

Santos-Silva, J. and S. Tenreyro (2010). Currency unions in prospect and retrospect. *Annual Review of Economics 2*, 51–74.

Schmitt-Grohé, S. and M. Uribe (2007). Optimal inflation stabilization in a medium-scale macroeconomic model. *In ed. Frederic Mishkin and Klaus Schmidt-Hebbel (2007).*

Smets, F. and R. Wouters (2007). Shocks and frictions in US business cycles: A bayesian DSGE approach. *American Economic Review 97*(3), 586–606.

Tille, C. (2008). Financial integration and the wealth effect of exchange rate fluctuations. *Journal of International Economics 75*(2), 283–294.