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# Local Sparse Reconstructions of Doppler Frequency using Chirp Atoms

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***Abstract*—The paper considers sparse reconstruction of Doppler and microDoppler time-frequency (TF) signatures of radar returns of moving targets from limited or incomplete data. The typically employed sinusoidal dictionary, relating the windowed compressed measurements to the signal local frequency contents, induces competing requirements on the window size. In this paper, we use chirp dictionary for each window position to relax this adverse window length-sparsity interlocking. It is shown that local frequency reconstruction using chirp atoms better represents the approximate piece-wise chirp behavior of most Doppler TF signatures. This enables the utilization of longer windows for accurate time-frequency representations. Simulation examples are provided demonstrating the superior performance of local chirp dictionary over its sinusoidal counterpart.**

## I. Introduction

Nonstationary signals arise in a broad class of active sensing modalities, including sonar, radar, and ultrasound. They are the preferred type of smart jamming and also characterize many passive sensing problems, such as speech and electromyographic recordings [1]- [4]. In particular, nonstationarity underlines Doppler and micro Doppler signals which represent radar returns from moving target [5]-[8]. Time-frequency signal representations (TFSRs) reveal the signal local structure which changes with time. In so doing, they enable separations of nonstationary signals that are mixed in both time and frequency domains, where traditional windowing and filtering based approaches fail to capture or distinguish between individual signal components. TFSRs are commonly obtained using linear basis signal decomposition [9], [10], and quadratic time-frequency distributions (QTFDs), generally referred to as Cohen's class [11], [12]. The latter have their roots in the nonparametric Wigner-Ville distribution (WVD).

QTFDs are defined by two-dimensional (2D) kernels which convolve the WVD for interference reduction. The reduced-interference distribution (RID) kernels act on preserving the true signal power terms, referred to as auto-terms, and eliminating, or at least considerably attenuating, the undesired cross-terms. Cross-terms represent false power concentrations and are generated from the data bilinear lag products underlying QTFDs. It has been analytically shown that missing and randomly sampled nonstationary signals give rise to artifacts in both the time-frequency domain and the ambiguity domain [13]- [15]. These artifacts clutter the signal components and hide pertinent signal structure, including the instantaneous frequencies. Efforts and attempts to use traditional RID kernels to reduce the type of clutter induced by missing samples along with mitigations of signal cross-terms have proven both unsuccessful and ineffective.

In compressive sensing, a sparse representation of a signal is projected onto a much lower dimensional measurement space. This leads, in general, to decreasing the data-acquisition requirements from time, logistic, and hardware complexity perspectives. It is then possible to record a small number of linear measurements of a signal and then reconstruct the complete set of all samples. The required number of observations is slightly more than the signal sparsity level but much less than the signal dimension. Although vastly applied in many applications, little considerations have been given to CS and sparse reconstructions of nonstationary signals.

Owing to their instantaneous narrowband characteristics and power concentrations, the signatures of a large class of nonstationary signals occupy small regions in the TF domain. This property casts these signals as sparse in the joint-variable representations and has recently invited sparse signal reconstruction and compressive sensing (CS) techniques to play an important and fundamental role in TF signal analysis and processing, especially with compressed observations [16]-[18]. This role would depend on the signal local sparsity level. For most single and multicomponent FM signals, local reconstruction of TF signatures from few random observations is deemed to outperform global signal reconstructions, which deals with a much broader signal bandwidth. i.e., weaker sparsity.

One of the most straightforward sparse reconstructions of local signal frequency characteristics is achieved by applying a sliding window, reminiscent of the STFT [17], [19]. Using partial Fourier basis, one can proceed to apply Greedy algorithms or convex optimization techniques to find the sparsest frequency contents that describe the observations within the time window. This approach involves sinusoidal dictionary that relates the windowed compressed observations to their local sparse frequencies. However, in many situations, the nonstationary signal frequency law is more properly approximated by piece-wise second order polynomials than fixed frequency sinusoids. In this case, a chirp dictionary, in lieu of sinusoidal dictionary, is better suited for sparse reconstruction problems dealing with FM signals. Further, compared to reconstruction techniques using parameterized atoms [20], which also directly operate on the data, the proposed chirp-based local frequency sparse technique does not assume any specific signal structure and, as such, is able to maintain its desirable performance for a wide class of nonstationary signals. Chirp dictionary have another two important attractions, namely, 1) They have been shown to satisfy the restricted isometry property (RIP) for perfect reconstruction similar to Gaussian and Bernoulli random dictionaries, 2) They enjoy fast implementation and lead to rapid signal recovery [21].

Section II of this paper formulates the problem and presents the sinusoidal and chirp dictionaries. It delineates the difference in the two dictionaries and shows how to construct the chirp dictionary for the underlying time-frequency representation problem. This section also proposes an average to be performed at each time-frequency point to deal with variations in chirp parameters constructed from overlapping windows. Section III focuses on the RIP associated with the chirp dictionary and provides the lower bound on the number of observations for exact recovery. Section IV includes simulations results. The conclusions are given in Section V.

## II. Problem Formulations

Consider an arbitrary continuous-time non-stationary signal $x_c(t)$, which consists of $K$ components:

$$x_c(t) = \sum_{k=1}^{K} A_k(t) \exp\left(j\phi_k(t) + v_c(t)\right), \quad 0 \le t < T \tag{1}$$

where $A_k(t)$ and $\phi_k(t)$ are the time- varying positive amplitude and phase of the $k^{\text{th}}$ component, $v_c(t)$ is an additive white noise, and $T$ is the total observation interval. It is assumed that the phase time- variations are much faster than those of amplitudes. The continuous-time instantaneous frequency (IF) of the $k^{\text{th}}$ component is defined as:

$$F_k(t) = \frac{1}{2\pi}\frac{d\phi_k(t)}{dt} \tag{2}$$

We assume that it is known a priori that the absolute IFs do not exceed $F_{\max}$ i.e. $|F_k(t)| \le F_{\max}$. We also assume that the IFs do not vary abruptly but rather smoothly over time, which is a reasonable assumption in many applications including radar.

To avoid aliasing, the continuous-time signal is first passed through a low-pass filter to remove out-of-band noise, and then sampled with a rate $F_s \ge 2F_{\max}$. The discrete-time signal is:

$$\begin{aligned} x(n) &= \sum_{k=1}^{K} A_k(nT_s)\exp(j\phi_k nT_s) + v(n), \\ n &= 0, 2, ..., N-1 \end{aligned} \tag{3}$$

where $T_s = 1/F_s$ is the sampling period, $x(n)$ and $v(n)$ are the discrete-time version of $x_c(t)$ and $v_c(t)$, and $N = \lfloor T/T_s \rfloor$. The cut-off frequency of the low-pass filter is chosen to be equal to $F_s$ so that the samples of $v(n)$ can be assumed uncorrelated.

The proposed approach builds on the local approximation of each signal component as a chirp. That is, by dividing the observation time interval into (possibly overlapping) time windows of a judiciously chosen duration, $T_w$, the discrete-time signal over each window is approximated by:

$$\begin{aligned} x_m(n) \approx &\sum_{k=1}^{K} C_{k,m} \exp\left\{ j2\pi \left[ \alpha_{k,m}\frac{n^2}{2F_s^2} + \beta_{k,m}\frac{n}{F_s} \right] \right\} \\ &+ v_m(n) \quad 0 \le n < N_w - 1 \end{aligned} \tag{4}$$

where $m$ is the window index, $C_{k,m}$, $\alpha_{k,m}$ and $\beta_{k,m}$ are the complex amplitude, the chirp rate, and the initial frequency of the $k^{\text{th}}$ component/chirp over the $m^{\text{th}}$ window, $x_m(n) = x(mL+n)$ and $v_m(n) = v(mL+n)$, with $L$ being the shift between two consecutive windows in terms of number of samples, and $N_w = \lfloor T_w/T_s \rfloor$.

Since $|F_k(n)| \le F_{\max}$, the initial frequency $|\beta| \le F_{\max}$, and frequency change in a period of $T_w$ cannot exceed $F_{\max}$, thus chirp rate $\alpha$ has range value:

$$\alpha \in [-F_{\max}F_s/N_w,\ F_{\max}F_s/N_w] \tag{5}$$

The parameter space of interest is (see Figure 1):

$$\begin{aligned} &\mathbf{\Omega} = \{(\alpha, \beta) \text{ such that} \\ &|\alpha| \le, F_{\max}F_s/N_w, |\beta| \le F_{\max} \text{ and } |\alpha T_w + \beta| \le F_{\max}\} \end{aligned} \tag{6}$$

The discrete dictionary, to be used in CS, is designed by

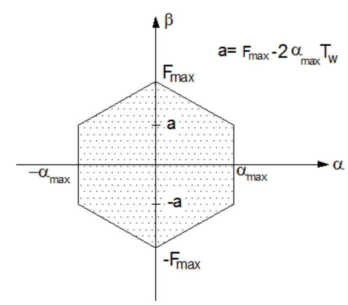



Fig. 1. 2D value space $\mathbf{\Omega}$ of $\alpha, \beta$.

uniformly sampling the 2D parameter space $\mathbf{\Omega}$. Let $I$ denote the total number of chirp rate values in the discrete dictionary. For the $i^{\text{th}}$ chirp rate value in the dictionary, which we denote as $\tilde{\alpha}_i$, let $\tilde{\beta}_{i,j}$ denote the corresponding possible values for the initial frequency, where $j = 1, ..., J_i$. Since the shape of the parameter space $\mathbf{\Omega}$ is not rectangular, the $J_i$'s are not all equal.

By performing a sparse component analysis within each window, we can track the time-variations of the chirp parameters of each component (i.e. chirp rate, initial frequency and complex amplitude), thus estimating arbitrary IFs.

In vector form, the signal over the $m^{\text{th}}$ window can be expressed as:

$$\mathbf{X}_m = \mathbf{\Psi}\mathbf{S}_m + \mathbf{V}_m \tag{7}$$

where $\mathbf{X}_m = [x_m(0), ..., x_m(N_w - 1)]^T$, $\mathbf{V}_m = [v_m(0), ...v_m(N_w-1)]^T$, $\mathbf{S}_m$ ia a $K$- sparse amplitude vector of length $\sum_{i=1}^{I} J_i$, and the dictionary matrix, $\mathbf{\Psi}$, is defined as:

$$\begin{aligned} &\mathbf{\Psi} = [\mathbf{\Psi}_1, \mathbf{\Psi}_2, ..., \mathbf{\Psi}_I] \\ &\mathbf{\Psi}_i = [\psi_{i,1}, \psi_{i,2}, ..., \psi_{i,J_i}] \\ &\psi_{i,j}|_n = \exp\left( j2\pi(\tilde{\alpha}_i\frac{n^2}{2F_s^2} + \tilde{\beta}_{i,j}\frac{n}{F_s}) \right); \\ &i = 1, ..., I, j = 1, ..., \end{aligned} \tag{8}$$

Since $K < N_w \ll \sum_{i=1}^{I} J_i$, solving for $\mathbf{S}_m$ in equation (7) becomes a sparse recovery (or CS) problem, which can be solved by:

$$\hat{\mathbf{S}}_m = \arg\min \|\mathbf{S}_m\|_1 \quad s.t. \quad \|\mathbf{X}_m - \mathbf{\Psi}\mathbf{S}_m\|_2^2 \leq \epsilon \qquad (9)$$

where $\|\|_1, \|\|_2$ denotes $L_1$, $L_2$ norms respectively, $\epsilon$ is the noise level. The solution for Equation (9) can be obtained by greedy algorithm such as Orthogonal Matching Pursuit (OMP) or linear programming [22], [23]. The proposed method is basically using a chirp dictionary, and select the atoms which best match the local structure of the signal, which is similar to Matching Pursuit algorithm. However, sparse reconstruction considers the sparsity level of the signal, as well as minimum observations required for exact recovery.

In addition to employing different dictionaries, the process of obtaining the final signal time-frequency signatures is also different for sinusoidal and chirp atoms. In the case of sinusoidal atoms, or dictionary, the sparse reconstruction algorithm, whether it is OMP or convex optimization, returns the local frequency contents, which are referred to the center point of the sliding window, similar to the generation of spectrograms. On the other hand, for the case of chirp dictionary, the chirp parameters returned by the sparse reconstructions describes the segment of the data captured by the window and, as such, represent the local signal behavior over the entire window extent, and not only the center point. Since overlapping windows generate overlapping chirps, some averaging process is in order and must be performed to render unique answers at each time sample. In essence, for every time-frequency point $(t, f)$, we sum all the magnitudes of reconstructed chirps provided by all corresponding sliding windows which include the time sample, $t$. In so doing, any chirp anomaly will be de-emphasized, whereas accurate frequency representations of the underlying signal will be strengthened. All time-frequency points having summed magnitude smaller than a certain threshold are ignored and will not be considered. In the simulation section, we demonstrate the advantages of performing the proposed time-frequency averaging over the case of no averaging, where we display the results by just overlaying the reconstructed chirps in the time-frequency domain.

## III. RIP ANALYSIS OF $\mathbf{\Psi}$

In this section, we examine the RIP associated with the chirp dictionary used in the previous section. Similar to the work in [21], we consider the bounds on the eigenvalues of the outer product of the dictionary matrix. We show that these bounds compete with those of Gaussian random dictionaries and as such lead to the same conditions on sparsity and compressed observations. The analysis follows closely that of [21] but differs in the final results due to differences in dictionary structure. Let $Q = \sum_{i=1}^{I} J_i$ and $\mathbb{Q} = \{1, ..., Q\}$. The structure of matrix $\mathbf{\Psi}$ is described in Eq. 8. The matrix $\mathbf{\Psi}$ would allow exact recovery of the original $K$- sparse input $\mathbf{S}_m$ by $l_1$ minimization if it has restricted isometry constant $\delta_{2K}$ satisfying the condition [24]:

$$\delta_{2K} < 1. \qquad (10)$$

Let $\Gamma \subset \mathbb{Q}$, where $\text{card}(\Gamma) \leq 2K$, and let $\mathbf{\Psi}_\Gamma$ denote the matrix formed by the columns of $\mathbf{\Psi}$ indexed by the subset $\Gamma$. From [24], we have:

$$(1 - \delta_{2K}) \leq \lambda_{min} \leq \lambda_{\max} \leq (1 + \delta_{2K}) \qquad (11)$$

where $\lambda_{\max}$, $\lambda_{min}$ are the maximum and minimum eigenvalues of $\mathbf{\Psi}_\Gamma^H \mathbf{\Psi}_\Gamma$. Based on Eq.10 and Eq. 11, if $\mathbf{\Psi}_\Gamma^H \mathbf{\Psi}_\Gamma$ has eigenvalues in the range $(0, 2)$ for all subsets $\Gamma$, then $\mathbf{\Psi}$ enables exact recovery of the original $K$- sparse input $\mathbf{S}_m$ by $l_1$ minimization .

Since the chirp dictionary $\mathbf{\Psi}$ is deterministic, the above requires checking all $\binom{Q}{2K}$ possible $\Gamma$ to find $\delta_{2K}$, which can be a computationally formidable problem. According to [26], a Gaussian random matrix $\mathbf{G} \in \mathbb{C}^{N_w \text{x} Q}$ with entries of zero mean and variance $1/N_w$ can satisfy $\delta_{2K} < 1$ with number of measurements $O(K \log(Q/K))$. Therefore, we will compare the bounds on the eigenvalues of $\mathbf{\Psi}_\Gamma^H \mathbf{\Psi}_\Gamma$ with those of $\mathbf{G}_\Gamma^H \mathbf{G}_\Gamma$. In the simulations, $N_w = 50$, $Q = 16512$ and 100000 random realizations of subset $\Gamma$ are used to estimate the eigenvalue statistics. The simulations are repeated for different cardinalities of subset $\Gamma$. Figure 2 shows the bounds (sample mean + 3 STD for the maximum eigenvalue and sample mean -3 STD for the minimum eigenvalue) for both chirp and Gaussian dictionaries.

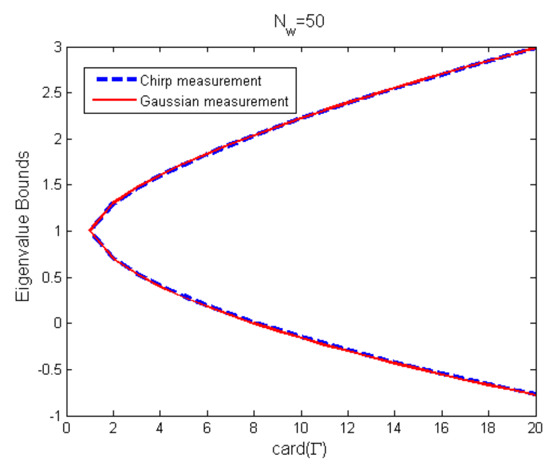



Fig. 2. Eigenvalue bounds of $\mathbf{\Psi}_\Gamma^H \mathbf{\Psi}_\Gamma$ and $\mathbf{G}_\Gamma^H \mathbf{G}_\Gamma$

Figure 2 shows that the bound for the two dictionaries are very close to each other. This implies that $\mathbf{\Psi}$ can also satisfy condition $\delta_{2K} < 1$ with high probability if the minimum number of observations is $O(K \log(Q/K))$.

It is noteworthy that if the signal is a chirp, the sparsity level $K$ of windowed signal $\mathbf{S}_m$ is constant irrespective of the window size $N_w$. Therefore, in the case of missing samples, we can increase the window size to obtain enough observations for exact recovery. On the other hand, if the sinusoid dictionary is used, a larger window length directly results in less sparsity. Thus, chirp signals benefit from chirp dictionaries when it comes to sparse reconstruction.

## IV. SIMULATION RESULTS

This section demonstrates the performance of local reconstruction with the chirp dictionary and sinusoid dictionary when applied to two different types of signals. In order to verify the proposed approach, firstly, we sample the data at

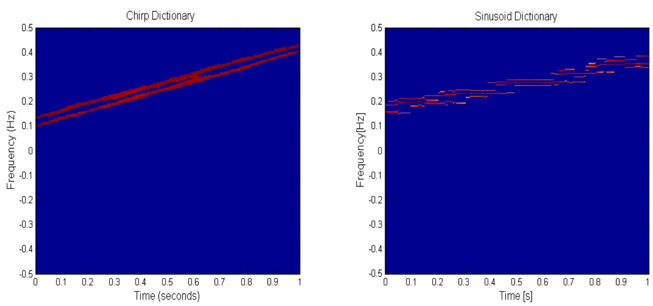
Nyquist rate, and then randomly discard some of the samples. In the two cases, the sampling frequency is $F_s = 256Hz$, the total signal length is $N = 256$, and only 50% of the data is used to estimate the instantaneous frequencies. When computing the TF representation, a rectangular window is used.

In the first example, the signal consists of two closely-parallel chirps. Its discrete-time version is expressed as:

$$\begin{aligned} x(n) &= \exp\left\{ j2\pi[(0.1F_s)\frac{n}{N} + (0.3F_s)\frac{n^2}{2N^2}] \right\} \\ &+ \exp\left\{ j2\pi[(0.13F_s)\frac{n}{N} + (0.33F_s)\frac{n^2}{2N^2}] \right\} + v(n) \end{aligned} \tag{12}$$

where $n = 0, 1, ..., N-1$. The signal-to-noise ratio is set to $SNR = 10$dB. To capture enough data to resolve the two chirps, the window size is set to a large value, $N_w = 90$. Sparsity level is assumed to be $K = 5$. The result in Figure 6.b shows the failure of local reconstruction using the sinusoid dictionary due to lack of sparsity in frequency. In contrast, when the chirp dictionary is used, the sparsity remains constant, irrespective of the window size $N_w$. The two chirps are clearly resolved as evident from Figure 6.a.

In the next example, we consider a three component- signal,



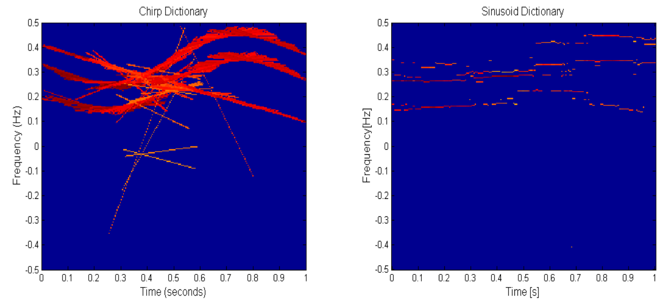

(a) (b)

Fig. 3. Local reconstruction of a two- component signal when 50 % of data is missing using (a) Chirp atoms (b)Sinusoid atoms

which is expressed as:

$$\begin{aligned} x(n) = &\exp\left\{ j[(0.1F_s)\cos(2\pi\frac{n}{N}) + 2\pi(0.25Fs)\frac{n}{N}] \right\} \\ &+ \exp\left\{ j[(0.1F_s)\cos(2\pi\frac{n}{N}) + 2\pi(0.35Fs)\frac{n}{N}] \right\} \\ &+ \exp\left\{ j2\pi[(0.4F_s)\frac{n}{N} - (0.3F_s)\frac{n^2}{2N^2}] \right\} + v(n) \end{aligned} \tag{13}$$

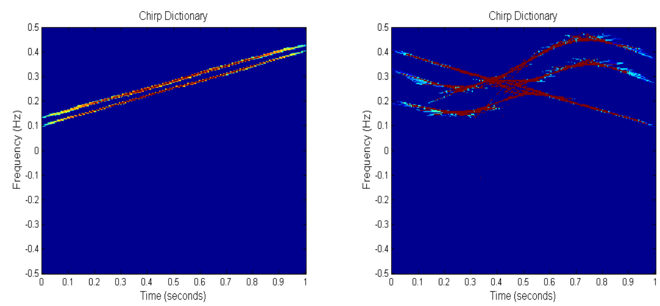
where $n = 1, 2, ..., N-1$. The signal is thinned by discarding 50% of data samples. Sparsity level is assumed to be $K = 5$, $SNR = 30$dB. The higher $SNR$ is necessary if no average is used. The signal in this case can be approximated by piecewise chirp, thus too large of a window length would result in incorrect local signal frequency structure. On the other hand, a smaller window size would not guarantee enough observations. The window size is $N_w = 70$. The result in Figure 4 once again shows that the reconstruction using sinusoid atoms fails to resolve the signal, whereas the chirp dictionary yields a desirable TF representation. The TF representation using the




(a) (b)

Fig. 4. Local reconstruction of a three- component signal when 50 % of data missing using (a) Chirp atoms (b)Sinusoid atoms

chirp dictionary gets even better when the proposed averaging of Section II is applied. With the same parameters used in the above examples, the effect of averaging is shown in Figure 5.



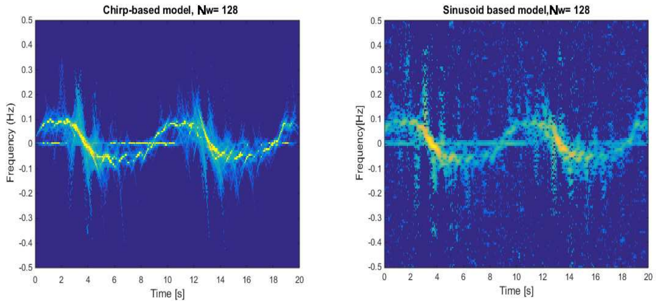

(a) (b)

Fig. 5. Averaging local reconstruction using chirp dictionary of (a) Two-component signal (b)Three component signal

In the third simulation, we use the data from human gait radar returns. The data has 20000 samples with $F_s = 1000$. The data is first uniformly sampled at Nyquist rate, and then randomly taken 50% of samples. Sparsity level is assumed to be $K = 30$. Rectangular and Hanning window are employed when Chirp and Sinusoid dictionary are used, respectively. The result in Figure 6 shows that Chirp atoms are more suitable for the torso and limbs' microDoppler presentations.

## V. Conclusion

The accurate piece-wise chirp approximations to the time-frequency signature of many Doppler and microDoppler signals motivate the use of chirp dictionary for sparse reconstruction of the signal local frequency structure under full and incomplete data. Compared to a sinusoidal dictionary, the chirp dictionary attempts to relax the inverse relationship between the local frequency sparsity and the length of the observation window. This relationship is synonymous with




(a) (b)

Fig. 6. Local reconstruction of a real signal returned from a human gail when 50 % of data is missing using (a) Chirp atoms (b)Sinusoid atoms

sinusoidal dictionaries when applied to FM signals. The OMP Greedy algorithm was applied to compare the performance of the two dictionaries under different Doppler signatures. For sliding windows, the chirp dictionary outperformed the sinusoidal dictionary, irrespective of the window size. Unlike the sinusoidal dictionary, where OMP results are referenced to the window mid-point, the chirp dictionary-based reconstruction produces the chirp parameters best fitting of the entire windowed data. These parameters may differ when moving by one or more samples. As such, an average of the chirp dictionary reconstructions corresponding to overlapping windows was performed at each time-frequency point, enabling improved time-frequency signal representations compared to the non-averaging case.

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