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Cut-off period for slow magnetoacoustic waves in coronal plasma structures

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ABSTRACT

Context. There is abundant observational evidence of longitudinal compressive waves in plasma structures of the solar corona, which are confidently interpreted in terms of slow magnetoacoustic waves. The uses of coronal slow waves in plasma diagnostics, as well as analysis of their possible contribution to coronal heating and the solar wind acceleration require detailed theoretical modelling.
Aims. We investigate the effects of obliqueness, magnetic field and non-uniformity of the medium on the evolution of long-wavelength slow magnetoacoustic waves guided by field-aligned plasma non-uniformities, also called tube waves. Special attention is paid to the cut-off effect due to the gravity stratification of the coronal plasma.
Methods. We study the behaviour of linear tube waves in a vertical untwisted straight field-aligned isothermal plasma cylinder.We apply the thin flux tube approximation, taking into account effects of stratification caused by gravity. The dispersion due to the finite radius of the flux tube is neglected. We analyse the behaviour of the cut-off period for an exponentially divergent magnetic flux tube filled in with a stratified plasma. The results obtained are compared with the known cases of the constant Alfven speed and the pure acoustic wave.
Results. We derive the wave equation for tube waves and reduce it to the form of the Klein–Gordon equation with varying coefficients, which contains explicitly the cut-off frequency. The cut-off period is found to vary with height, decreasing significantly in the low-beta plasma as well as in the plasma with the beta of the order of unity. The depressions in the cut-off period profiles can affect the propagation of longitudinal waves along coronal plasma structures towards the higher corona and form coronal resonators.

Key words. Magnetohydrodynamics (MHD) – Waves – Sun: corona – Methods: analytical

1. Introduction

The abundant observational evidence for magnetohydrodynamic (MHD) wave processes operating in the solar corona motivates the development of precise theoretical models. The importance of studies of coronal waves and oscillations is traditionally associated with such fundamental solar physics problems as coronal heating and the solar wind acceleration. The understanding of MHD waves can also give insight into the nature of coronal mass ejections and flares as the wave behaviour may reflect their triggering mechanisms (e.g., Vršnak & Cliver 2008). Of special importance are seismological applications of wave studies, which provide the valuable and sometimes unique information about important parameters of the solar coronal plasma (Stepanov et al. 2012; De Moortel & Nakariakov 2012).

Observations in Extreme Ultraviolet (EUV) spectral lines show the presence in the corona of MHD waves of different types and periods. In particular, the longitudinal along the field, propagating EUV disturbances observed in various solar plasma structures, e.g., in coronal loops and plumes, (e.g., DeForest & Gurman (1998); Berghmans & Clette (1999); De Moortel et al. (2000), see also De Moortel (2009) for review) are interpreted in terms of slow magnetoacoustic waves (e.g., Ofman et al. (2000); Nakariakov et al. (2000); Afanasyev & Nakariakov (2015), see Nakariakov (2006) for review). Their periods are found to range from several minutes up to several tens of minutes. For example, wave trains of 3 and 5 min periods were observed in coronal loops (De Moortel et al. 2002). The longer longitudinal waves (10–12 min) were detected in polar plumes (Ofman et al. 1997). Yuan et al. (2011) revealed the long-period waves of about 29, 53 and 75 min periods in the lower corona. Recently, Bakunina et al. (2013) and Chorley et al. (2010) reported on the long-period oscillations in radio band in sunspot magnetospheres.

With respect to long-period slow magnetoacoustic waves, the solar atmosphere is an essentially dispersive medium. Apart from the dispersion connected with the field-aligned filamentation (Zajtsev & Stepanov 1975; Edwin & Roberts 1983), the dispersion effects appear also due to the gravitational stratification, and result in the cut-off effect for harmonic waves as well as spreading and appearance of the trailing oscillatory wake for broadband pulses. The dispersive behaviour of acoustic waves propagating vertically upwards in the stratified atmosphere was demonstrated by Lamb (1932). Wave equations governing the wave propagation and evolution can be reduced to the Klein–Gordon (KG) equation, which explicitly contains the cut-off frequency.

The behaviour of long-period longitudinal waves in the solar atmosphere in terms of acoustic waves was very intensively investigated several decades ago (see, e.g., Sutmann et al. 1998,

⋆ Visiting Fellow at CFSA, University of Warwick, UK and references therein). However, one should take into account the magnetic nature of waves as well as the field-aligned structuring of the coronal plasma (e.g., Van Doorsselaere et al. 2008). In such a case, the slow magnetoacoustic mode manifests itself as a tube mode due to the wave-guiding conditions within plasma structures. Due to the field-aligned non-uniformity, the local wave vector is essentially oblique with respect to the magnetic field. In low-β plasmas, the speed of propagation of tube waves (tube speed) is only slightly different from the sound speed. The effect becomes much more pronounced in plasmas with $\beta \sim 1$. In this context long-period tube waves in solar plasma structures were analysed by Defouw (1976); Roberts & Webb (1978, 1979); Rae & Roberts (1982); Hasan & Kalkofen (1999); Musielak & Ulmschneider (2003). **The authors were mainly aimed at the case of photospheric flux tubes expanding into the chromosphere.** They obtained the KG equation describing the tube wave evolution in presence of gravitational stratification, and estimated the cut-off frequency. However, in those studies, the authors considered the case of exponentially divergent magnetic flux tube with the characteristic scale height equal to twice the density scale height, and therefore they analysed the particular case of a constant Alfvén speed. As a result, they obtained the KG equation with constant coefficients, also assuming the constant sound speed. In a real solar atmosphere, the divergence of magnetic flux tubes may not be linked that way to the density scale height, and the Alfvén speed profile may be varying along the magnetic field. Thomas (1982) considered the cut-off frequency in the stratified atmosphere with the horizontal magnetic field. Roberts (2006) raised a new formulation of the problem for longitudinal waves and considered the stratified atmosphere penetrated by a uniform vertical magnetic field.

In this paper, we consider the propagation of long-period slow magnetoacoustic waves in coronal plasma structures, taking into account the arbitrary variation (within our **simplifying** assumptions) of the magnetic field. We utilise the thin flux tube approximation in order to derive the KG equation for tube waves and obtain the cut-off period profiles. The paper is organised as follows. In Section 2 the wave equation for tube waves is derived. In Section 3 we reduce the wave equation to the KG form, obtain the cut-off frequency, and analyse its behaviour under the coronal conditions. Section 4 contains the discussion of results obtained and concluding remarks.

2. Wave equation for tube waves

We consider azimuthally symmetric linear slow magnetoacoustic waves in a vertical magnetic flux tube without steady plasma flows. Assuming that the wavelength is much longer than the radius of the tube, we apply the traditional thin flux tube approximation (e.g., Roberts & Webb (1978), see also Zhugzhda (1996)), using the cylindrical coordinates (r, φ, z). The axisymmetric flux tube is assumed to be straight, untwisted and non-rotating, and filled in with the plasma of a constant temperature. The plasma inside the flux tube is stratified by gravity in the vertical direction along the tube axis. We neglect dispersion effects caused by the finite radius of the flux tube (e.g., Zhugzhda 1996) and the finite scale of the wave localisation outside the flux tube (e.g., Roberts 1985; Zhugzhda & Goossens 2001), therefore analysing sufficiently long waves. The governing set of equations is

$$
\begin{aligned}
\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z}\right) &= -\frac{\partial p}{\partial z} - \rho g,\\
\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial z} &= 0,\\
p + \frac{B^2}{8\pi} &= p_{\mathrm{T}}^{\mathrm{ext}},\\
\frac{\partial B}{\partial t} + u\frac{\partial B}{\partial z} + 2Bv &= 0,\\
\frac{\partial \rho}{\partial t} + 2\rho v + \frac{\partial}{\partial z}(\rho u) &= 0,\\
p &= p(\rho, s),
\end{aligned} \tag{1}
$$

where B is the longitudinal component of the magnetic field, v is the radial derivative of the radial component of the plasma velocity, u is the longitudinal component of the plasma velocity, ρ, p and s are the plasma density, pressure, and specific entropy, respectively, and g is the gravity acceleration. The last equation is the equation of state. Eqs. (1) take into account the wave-guiding nature of the wave propagation, perturbations of the magnetic field, and gravity stratification of the plasma. The external total pressure $p_{\mathrm{T}}^{\mathrm{ext}}$ is assumed to be constant and therefore we do not consider disturbances in the external medium, concentrating on waves inside the flux tube. This assumption is justified if the characteristic wave speed inside the flux tube is lower than the propagation speeds of waves in the external medium (see Roberts & Webb 1979; Zhugzhda 1996).

Eqs. (1) are linearised in standard manner. Small perturbations of the equilibrium state (indicated by the subscript 0) are introduced as follows:

$$
\begin{aligned}
&\rho = \rho_0 + \rho_1, \quad p = p_0 + p_1, \quad s = s_0 + s_1,\\
&B = B_0 + B_1, \quad v = v_1, \quad u = u_1.
\end{aligned} \tag{2}
$$

Perturbations of the azimuthal components of the plasma velocity and magnetic field are assigned to be zero since we do not consider the torsional mode in this study. Note that we omit the subscript 1 for the velocity components in the remainder of the paper. The equilibrium values depend on the vertical coordinate z.

By substituting Eqs. (2) into Eqs. (1) and restricting our attention to linear terms only, and also eliminating the variables s_1, p_1 and v, we have

$$
\begin{aligned}
\frac{B_0}{4\pi}\frac{\partial B_1}{\partial t} + c_0^2\frac{\partial \rho_1}{\partial t} - \frac{p_0}{R_{\mathrm{gas}}}(\gamma - 1)\,u\frac{ds_0}{dz} &= 0,\\
\rho_0\frac{\partial u}{\partial t} + \rho_1 g - \frac{B_0}{4\pi}\frac{\partial B_1}{\partial z} - \frac{B_1}{4\pi}\frac{dB_0}{dz} &= 0,\\
\rho_0\frac{\partial B_1}{\partial t} + \rho_0\frac{dB_0}{dz}u - B_0\frac{\partial \rho_1}{\partial t} - B_0\frac{d\rho_0}{dz}u - B_0\rho_0\frac{\partial u}{\partial z} &= 0,
\end{aligned} \tag{3}
$$

where $c_0^2 = \gamma p_0/\rho_0$ is the equilibrium sound speed, R_{gas} is the gas constant, and γ is the adiabatic index, which come from the equation of state.

By differentiating Eqs. (3) with respect to t and z and collecting terms with variable u and with its derivatives, we obtain the wave equation

$$\frac{\partial^2 u}{\partial t^2} - c_T^2 \frac{\partial^2 u}{\partial z^2} + \left(\frac{1}{B_0}\frac{dB_0}{dz} c_T^2 \frac{V_A^2 - c_0^2}{V_A^2 + c_0^2} + \gamma g \frac{c_T^4}{c_0^4} \right) \frac{\partial u}{\partial z} + \left[c_T^2 \frac{1}{B_0}\frac{d^2 B_0}{dz^2} + c_T^2 \frac{1}{B_0^2}\left(\frac{dB_0}{dz}\right)^2 \frac{c_0^2 - V_A^2}{V_A^2 + c_0^2} + \frac{1}{B_0}\frac{dB_0}{dz}\frac{2 g c_T^2}{V_A^2 + c_0^2} + \frac{1}{B_0}\frac{dB_0}{dz}\gamma g \frac{c_T^2}{c_0^2}\left(\frac{1}{\gamma} - \frac{c_T^2}{c_0^2}\right) + N^2 + \frac{c_T^2}{c_0^2}\frac{g}{H}\left(\frac{1}{\gamma} - \frac{c_T^2}{c_0^2}\right) \right] u = 0, \quad (4)$$

where we have introduced the Brunt–Väisälä (buoyancy) frequency N, tube speed c_T, and density scale height H as

$$N^2 = g\left(\frac{1}{\gamma p_0}\frac{dp_0}{dz} - \frac{1}{\rho_0}\frac{d\rho_0}{dz}\right), \; c_T = \frac{c_0 V_A}{\sqrt{c_0^2 + V_A^2}}, \; \frac{1}{H} = -\frac{1}{\rho_0}\frac{d\rho_0}{dz}, \quad (5)$$

respectively, and where $V_A = B_0/\sqrt{4\pi\rho_0}$ is the equilibrium Alfvén speed.

Eq. (4) describes the dynamics of longitudinal waves in thin magnetic flux tubes. Let us consider the limiting cases for plasma conditions inside the flux tube. In the infinite magnetic field limit, $(V_A/c_0 \to \infty)$, the tube speed c_T becomes equal to the sound speed c_0. In this case, plasma motions in the wave become exactly longitudinal, and the tube wave degenerates into the plane pure acoustic wave. Thus, the effect of the plasma structuring vanishes and Eq. (4) takes the form of the well-known wave equation for acoustic waves in a stratified isothermal medium (see, e.g., Sutmann et al. 1998)

$$\frac{\partial^2 u}{\partial t^2} - c_0^2 \frac{\partial^2 u}{\partial z^2} + \gamma g \frac{\partial u}{\partial z} = 0, \quad (6)$$

provided that the vertical gradient of the magnetic field, dB_0/dz, is limited (i.e. not tending to infinity), which is the case, e.g., for a cylindrical flux tube with the constant magnetic field.

For a thin flux tube with a finite constant magnetic field, $B_0 =$ const, we have

$$\frac{\partial^2 u}{\partial t^2} - c_T^2 \frac{\partial^2 u}{\partial z^2} + \gamma g \frac{c_T^4}{c_0^4}\frac{\partial u}{\partial z} + \left[N^2 + \frac{c_T^2}{c_0^2}\frac{g}{H}\left(\frac{1}{\gamma} - \frac{c_T^2}{c_0^2}\right)\right] u = 0. \quad (7)$$

After some algebra, Eq. (7) completely coincides with Eq. (3.9) of Roberts (2006).

3. Klein–Gordon form of the wave equation and the cut-off period

Eq. (4) can be reduced to the form of the KG equation. Let us denote the coefficients in front of $\partial u/\partial z$ and u in Eq. (4) by K_1 and K_2 respectively, and introduce the new variable U as

$$u = e^{\Psi(z)} U(z), \qquad \Psi' = \frac{K_1}{2 c_T^2}, \quad (8)$$

where

$$K_1 = \frac{1}{B_0}\frac{dB_0}{dz} c_T^2 \frac{V_A^2 - c_0^2}{V_A^2 + c_0^2} + \gamma g \frac{c_T^4}{c_0^4}.$$

Henceforth, the dash means differentiation with respect to z. By substituting Eq. (8) into Eq. (4), we obtain the KG equation for the variable U

$$\frac{\partial^2 U}{\partial t^2} - c_T^2 \frac{\partial^2 U}{\partial z^2} + \omega_u^2 U = 0, \quad (9)$$

where the squared cut-off frequency, ω_u^2, is

$$\omega_u^2 = -c_T^2(\psi')^2 - c_T^2\Psi'' + K_1\Psi' + K_2 = \frac{c_T^2}{4}\left(\frac{1}{B_0}\frac{dB_0}{dz}\frac{V_A^2 - c_0^2}{V_A^2 + c_0^2} + \frac{1}{H}\frac{c_T^2}{c_0^2}\right)^2 - \frac{c_T^2}{2}\left(\frac{1}{B_0}\frac{dB_0}{dz}\frac{V_A^2 - c_0^2}{V_A^2 + c_0^2} + \frac{1}{H}\frac{c_T^2}{c_0^2}\right)' + K_2, \quad (10)$$

and

$$K_2 = c_T^2 \frac{1}{B_0}\frac{d^2 B_0}{dz^2} + c_T^2 \frac{1}{B_0^2}\left(\frac{dB_0}{dz}\right)^2 \frac{c_0^2 - V_A^2}{V_A^2 + c_0^2} + \frac{1}{B_0}\frac{dB_0}{dz}\frac{2 g c_T^2}{V_A^2 + c_0^2} + \frac{1}{B_0}\frac{dB_0}{dz}\gamma g \frac{c_T^2}{c_0^2}\left(\frac{1}{\gamma} - \frac{c_T^2}{c_0^2}\right) + N^2 + \frac{c_T^2}{c_0^2}\frac{g}{H}\left(\frac{1}{\gamma} - \frac{c_T^2}{c_0^2}\right).$$

Eq. (10) gives the explicit expression for the cut-off frequency with an account of the wave-guiding nature of the wave propagation and magnetic field variation. **Note that we refer to the quantity expressed by Eq. (10) as "cut-off frequency", however, it depends on the vertical coordinate z and therefore its role is not as clear as for the simpler acoustic case of an isothermal medium. The quantity in Eq. (10) corresponds to the cut-off frequency in a local approximation (see also Section 4).**

In the limit of the infinite magnetic field, from Eq. (10) we have the usual cut-off frequency for acoustic waves

$$\omega_u^2 = \left(\frac{c_0}{2H}\right)^2, \quad (11)$$

if the gradient of the magnetic field, $\partial B_0/\partial z$, is limited. In the case of constant magnetic field, the cut-off frequency coincides with that obtained for the isothermal plasma from Roberts (2006)

$$\omega_u^2 = \frac{c_T^2}{4H^2}\left(\frac{c_T}{c_0}\right)^4 - \frac{c_T^2}{2H}\frac{\left(c_T^2\right)'}{c_0^2} + N^2 + \frac{c_T^2}{c_0^2}\frac{g}{H}\left(\frac{1}{\gamma} - \frac{c_T^2}{c_0^2}\right). \quad (12)$$

In the case of exponentially divergent magnetic flux tube, with the characteristic scale height for the magnetic field equal to twice the density scale height, the Alfvén speed is constant, and we have for the constant cut-off frequency

$$\omega_u^2 = N^2 + \frac{c_T^2}{H^2}\left(\frac{3}{4} - \frac{1}{\gamma}\right)^2, \quad (13)$$

which is consistent with results by Defouw (1976); Roberts & Webb (1978); Rae & Roberts (1982); Musielak & Ulmschneider (2003). In the limit of the infinite magnetic field, Eq. (13) differs from the cut-off frequency for pure acoustic waves propagating vertically upwards, Eq. (11). The reason for this appears to be the divergence of the magnetic flux tube **as well as its elasticity due to the magnetic nature. The contribution of the geometry and elasticity of the tube to the cut-off effect was discussed in the review by Roberts (1991)**.

Let us analyse the behaviour of the cut-off frequency given by Eq. (10). We consider the exponentially divergent magnetic flux tube, $B_0 = B_{00}\exp(-z/L)$, with different values of the magnetic field scale height, L, in order to demonstrate the main differences from the pure acoustic case as well as from the case of the constant Alfvén speed, which are generally used for the interpretation of observations. The number density is assumed to be barometric, $n_0 = n_{00}\exp(-z/H)$, where $n_{00} = 5 \times 10^8$ cm^{-3},

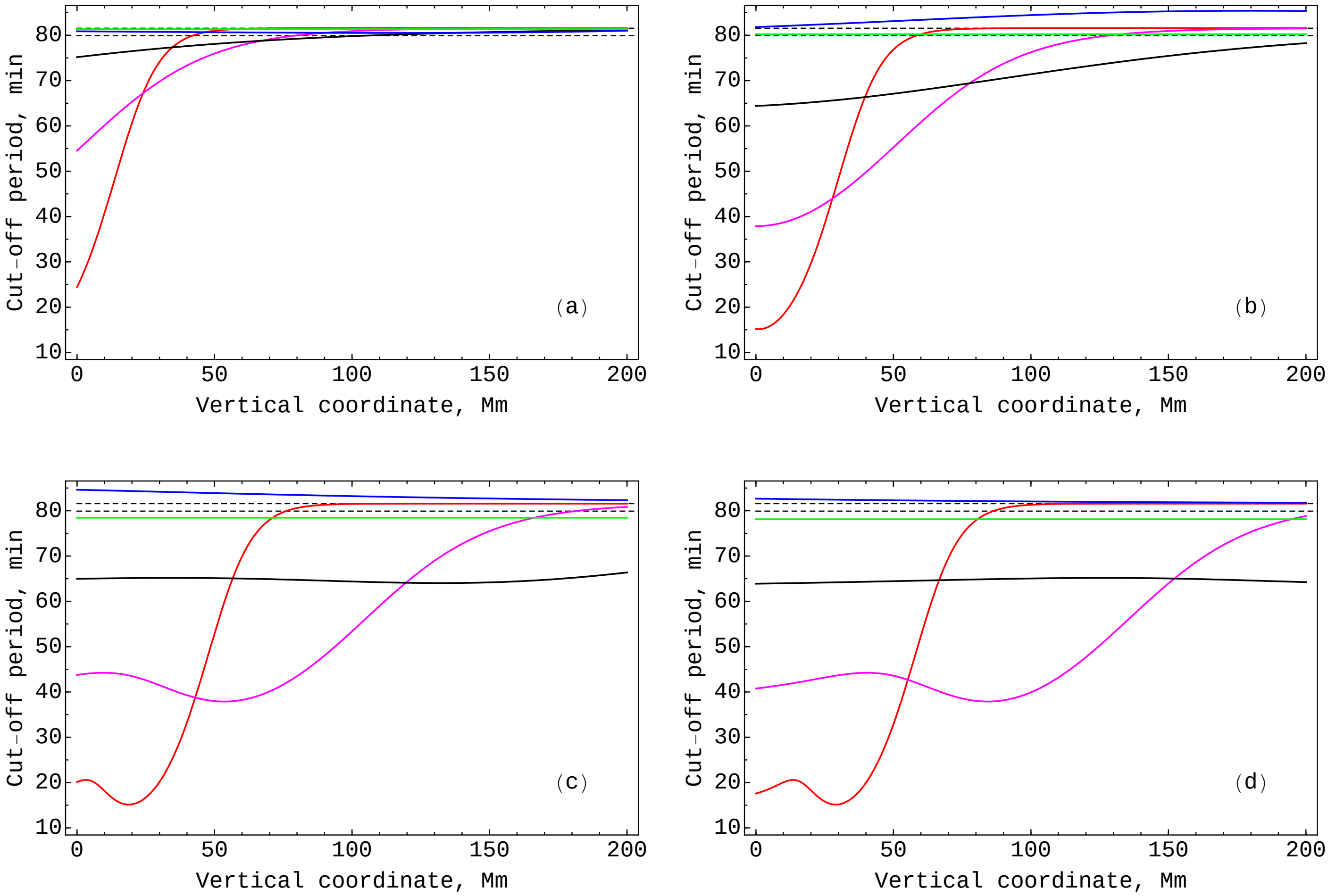

Fig. 1. Dependence of the cut-off period for longitudinal waves in an exponentially divergent magnetic flux tube on the vertical coordinate z. The colours correspond to different values of the magnetic field scale height, $L = 0.2H$ (*red*), $0.5H$ (*purple*), H (*black*), $2H$ (*green*), $5H$ (*blue*). The dashed lines mark the acoustic cut-off period (*lower*) and Brunt–Väisälä period (*upper*). Different panels show the cut-off periods for the high (*a*), of the order of unity (*b*), low (*c*) and very low (*d*) plasma β in the solar corona (see the corresponding values of the Alfvén speed and sound speed in the text).

$H = 2R_{\rm gas}T/\widetilde{m}M_{\rm H}g$ is the density scale height, $T = 1.4 \times 10^6$ K is the plasma temperature, $M_{\rm H}$ **is** the molar mass of hydrogen, $\widetilde{m} = 1.27$ **is** the average atomic weight of an ion, $\gamma = 5/3$.

For convenience, we plot the cut-off period that is $2\pi/\omega_{\rm u}$. Figure 1 shows the behaviour of the cut-off period with height. Different panels correspond to different values of the magnetic field, B_{00}: (*a*) $B_{00} = 0.5$ G, $V_{\rm A} = 43$ km/s; (*b*) $B_{00} = 1.5$ G, $V_{\rm A} = 130$ km/s; (*c*) $B_{00} = 5.0$ G, $V_{\rm A} = 433$ km/s; (*d*) $B_{00} = 10$ G, $V_{\rm A} = 866$ km/s; the sound speed is constant for all the panels, $c_0 = 175$ km/s. **Case (*a*) is likely to correspond too low Alfvén speed for coronal conditions, however, we should include it into consideration to provide the analysis of the high-β plasma. Such plasma conditions may be in polar plumes at higher heights.**

In the case of weakly divergent magnetic flux tube in comparison with the density falloff, $L = 5H$, (*blue* curve in Fig. 1), the difference in the cut-off period is quite insignificant, reaching as much as about 6% in the plasma with $\beta \sim 1$. For the coronal conditions ($\beta \lesssim 1$), the cut-off period for waves in the weakly divergent tube is higher than the acoustic cut-off period and buoyancy period marked by dashed lines in the Figure.

The case of $L = 2H$ (*green* curve) is the case of the constant Alfvén and sound speeds. The cut-off period is also constant, and its value slightly varies depending on the plasma β, as seen in Fig. 2.

For $L = H$ (*black* curve in Fig. 1) in the low-β plasma, the cut-off period keeps almost constant, however, its value is below acoustic cut-off period by about 20%. In the plasma with $\beta \sim 1$, the cut-off period grows with height, approaching the acoustic cut-off period above the height of about 200 Mm.

Of interest is the behaviour of the cut-off period in strongly divergent magnetic flux tubes with $L = 0.2H$ and $L = 0.5H$ (*red* and *purple* curves respectively). The cut-off period is decreased down to 20–40 min at the base of the corona, and then grows up to the acoustic cut-off period value. In the low-β conditions, the cut-off period profiles have the local maxima lower in the corona, seen in Fig. 1, e.g., for waves of the periods of 15–20 min for the $L = 0.2H$ case and of about 40 min for the $L = 0.5H$ case. Besides, the quite extensive (up to the height of about 50–100 Mm) plateaus of the reduced cut-off period exist in the low-β cases.

4. Discussion and Conclusions

The behaviour of waves observed in the solar corona can be considerably affected by the transverse non-uniformity of the plasma. In this study, we have constructed the mathematical

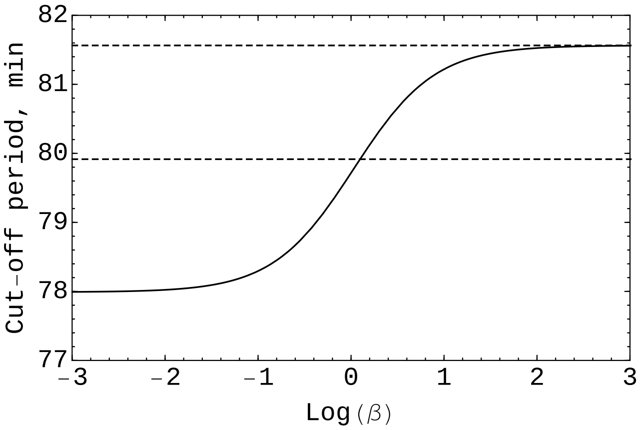

Fig. 2. Dependence of the cut-off period on plasma β for the case of a constant Alfvén speed inside the flux tube ($L = 2H$). The dashed lines mark the acoustic cut-off period (*lower*) and Brunt–Väisälä period (*upper*).

model describing the dynamics of longitudinal waves in vertical field-aligned coronal plasma structures. We have derived the wave equation (Eq. (4)) that could be rewritten as the Klein–Gordon equation (Eq. (9)), which allow one to consider the propagation and evolution of linear long-wavelength slow magnetoacoustic waves in thin magnetic flux tubes filled in with an isothermal stratified plasma.

The main purpose of the study was to investigate the effect of obliqueness of the slow MHD waves on their evolution in the stratified atmosphere. Waves with the wavelengths much greater than the transverse spatial size of the waveguide are essentially oblique. In contrast with plane acoustic waves traditionally used in modelling longitudinal waves in coronal plasma structures, oblique slow magnetoacoustic waves perturb not only the plasma velocity along the field and the density, but also the transverse components of the velocity and the magnetic field. For clarity, long-wavelength slow magnetoacoustic waves guided by field-aligned plasma non-uniformities are referred to as tube waves.

To analyse the propagation of tube waves in divergent flux tubes, we have used the thin flux tube approximation. Despite the cases of the strong exponential divergence of the tubes, the method appears to be valid for our analysis. Indeed, it is possible to obtain simple estimates in the frameworks of the exponential model. The transverse size of coronal plasma structures at the base of corona is of the order of 1 Mm. At a height of two or four characteristic scale heights for the magnetic field, L, the transverse size of a plasma structure grows as much as e or e^2 times respectively, and therefore it keeps to be much less (or less, at least) then the wavelength of coronal longitudinal disturbances under study. The latter value can be estimated, e.g., as 60–120 Mm for the sound speed of 200 km s^{-1} and the periods ranging in 300–600 s.

The stratification of the plasma due to gravity results in the dispersive cut-off effect for harmonic longitudinal waves. For uniform profiles of the characteristic wave propagation speeds, waves with the periods shorter than the cut-off period can propagate freely upwards, while those with longer periods exponentially decay. Thus, the cut-off effect prevents the propagation of longer waves towards the higher corona. The analysis of the Eq. (10) has shown that in general the cut-off period varies with the height, decreasing significantly in the low-β plasma as well as in the plasma with $\beta \sim 1$. The nature of the varying cut-off period is not as simple as in the case of uniform characteristic wave speeds. Our results are obtained in a local approximation and are valid for sufficiently smooth profiles of the characteristic wave propagation speeds. The depressions in the cut-off period profiles can affect the same way the propagation of longer longitudinal waves along coronal plasma structures and prevent their penetration higher in the corona. However, it should be noted that such effect appears to be considerable only for sufficiently extensive plateaus of the reduced cut-off period, while through the quite narrow regions of the reduced cut-off period, waves may tunnel (e.g., see the discussion in Verwichte et al. 2006). On the other hand, in the low-β conditions, the local maxima in the cut-off period profiles lower in the corona imply the possible existence of coronal resonators for longitudinal waves. The detailed analysis of this effect by solving numerically the obtained wave equation, Eq. (4), as well as the study of the evolution of waves propagating upwards should be addressed in a dedicated study.

The effect of the depression of the cut-off period is found to be significant for strongly divergent magnetic flux tubes. The strong divergence of coronal plasma structures was identified in observations. In particular, Deforest et al. (1997) found the super-radial expansion of coronal plumes. Besides, observational studies showed the importance of the geometrical factor on the evolution of waves. For instance, Marsh et al. (2011) pointed out that the area divergence had the dominant effect over thermal conduction on oscillations with longer periods (12 min), which travel along cool loops.

The reduced cut-off period in the corona can be responsible for the appearance of longer period waves and oscillations observed in the corona. In the case of a uniform profile of the characteristic wave propagation speed, after the perturbation of the coronal plasma by a broadband pulse, the wave front propagates upwards, while the plasma behind it oscillates at the cut-off frequency (see, e.g. Lamb 1932; Suematsu et al. 1982; Sych et al. 2012). In a stratified medium, such oscillations at the local cut-off frequency, which are often called trailing oscillatory wakes, could be considered for the interpretation of longer-period compressive waves observed in the corona. Indeed, compressive oscillations with the periods longer than three and five minutes are often observed in the corona, For example, Sych & Nakariakov (2008) detected 15-min oscillations above sunspots. The study by Ofman et al. (1997) implies the possible fluctuations on longer timescales (20-50 min) high above the limb (1.9–2.45 solar radii). Miyamoto et al. (2014) detected quasi-periodic density disturbances with the period ranging from 100 to 2000 s at 1.5–20.5 solar radii. The analysis of spectroscopic observations revealed propagating disturbances of about 14.5 min period in a coronal hole (Gupta et al. 2012). Krishna Prasad et al. (2012) detected 12–25 min periodicities in off-limb fan loop-structures, on-disk plume-like structures, and in the polar plume/interplume regions. Waves of 12, 17 and 22-min periods were found in the loop fans of active regions (Krishna Prasad et al. 2014). Thus, the abundant detection of long-period compressive waves and oscillations in the corona could be attributed to the effect of the slow magnetoacoustic cut-off frequency in field-aligned plasma non-uniformities.

To summarise, we briefly specify the basic results obtained in this study:

– We have constructed the model describing the dynamics of longitudinal waves in vertical, field-aligned coronal plasma structures.
– We have derived the equation describing the propagation of linear long-wavelength slow magnetoacoustic waves in mag-

netic flux tubes filled in with a stratified plasma of constant temperature.
- The cut-off period for longitudinal waves is found to vary with height, decreasing significantly in the low-β plasma as well as in the plasma with β of the order of unity.
- The depressions in the cut-off period profiles can prevent the propagation of longitudinal waves along coronal plasma structures towards the higher corona if the periods are longer than these cut-off values that can be as short as fifteen or twenty minutes.
- The long-period (e.g., 15–60 min) oscillations observed in the solar corona can be the trailing oscillatory wakes induced by broad-band perturbations.

The effect of the significant decrease in the cut-off frequency for slow magnetoacoustic waves in the presence of field-aligned non-uniformities of the stratified plasma, demonstrated in this paper, requires a detailed follow-up study.

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