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The dynamics of deferred decision ☆

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ABSTRACT

Decision makers are often unable to choose between the options that they are offered. In these settings they typically defer their decision, that is, delay the decision to a later point in time or avoid the decision altogether. In this paper, we outline eight behavioral findings regarding the causes and consequences of choice deferral that cognitive theories of decision making should be able to capture. We show that these findings can be accounted for by a deferral-based time limit applied to existing sequential sampling models of preferential choice. Our approach to modeling deferral as a time limit in a sequential sampling model also makes a number of novel predictions regarding the interactions between choice probabilities, deferral probabilities, and decision times, and we confirm these predictions in an experiment. Choice deferral is a key feature of everyday decision making, and our paper illustrates how established theoretical approaches can be used to understand the cognitive underpinnings of this important behavioral phenomenon.

1. Introduction

Cognitive models provide a powerful, theoretically constrained approach to studying preferential decision making (Busemeyer & Johnson, 2004; Newell & Bröder, 2008). These models formally describe the psychological mechanisms underlying choice, and in doing so are able to explain a variety

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of behavioral findings, including decoy effects, reference dependence, anchoring effects, and risky choice effects (Bhatia, 2013, 2014; Bogacz, Usher, Zhang, & McClelland, 2007; Busemeyer & Townsend, 1993; Diederich, 1997; Glöckner & Betsch, 2008; Pleskac & Busemeyer, 2010; Rangel & Hare, 2010; Roe, Busemeyer, & Townsend, 2001; Stewart, Chater, & Brown, 2006; Trueblood, Brown, & Heathcote, 2014; Usher & McClelland, 2004). For this reason, cognitive models are rapidly replacing traditional utility-based approaches as desirable theoretical tools for understanding preferential choice behavior (see Oppenheimer & Kelso, 2015 for a discussion).

Theories of decision making within the cognitive tradition typically make predictions about choice probabilities, decision times, attention to external information or information stored in memory, and judgments of confidence. These are some of the most important behavioral, cognitive, and metacognitive outcomes in a decision, and modeling these outcomes is necessary in order to characterize the choice process. That said, many existing theories of decision making are incomplete. They are largely unable to capture the causes and consequences of choice deferral, that is, the decision to disengage from the choice task without selecting any available options (but see Busemeyer, Johnson, & Jessup, 2006; Jessup, Veinott, Todd, & Busemeyer, 2009; White, Hoffrage, & Reisen, 2015). The failure to decide is a fundamental feature of everyday preferential decision making. Most consumer, financial, health, food, and entertainment choices are not forced, and decision makers can often wait to make the choice at a later point in time, or even completely avoid the choice in favor of the status quo or default.

The importance of deferral as a decision outcome was recognized by Tversky and Shafir (1992) who showed that the probability of choice deferral reduces in the presence of dominated decoys. Since then a large literature in psychology and marketing has attempted to characterize the determinants of choice deferral, and the consequences of allowing choice to be deferred (see Anderson, 2003; Chernev, Böckenholt, & Goodman, 2015; Scheibehenne, Greifeneder, & Todd, 2010 for reviews). This work has established that the likelihood of choice deferral depends not only on dominance relations, but also on variables such as option desirability, attribute commonality, and attribute alignability (e.g. Chernev, 2005; Chernev & Hamilton, 2009; Dhar, 1997; Dhar & Sherman, 1996; Gourville & Soman, 2005; White & Hoffrage, 2009; White et al., 2015; also Tversky & Shafir, 1992). Additionally the mere presence of deferral as a feasible outcome in the choice task can affect the relative choice probabilities of the available options, and reverse certain behavioral effects (Dhar & Simonson, 2003).

Formally modeling choice deferral involves a departure from the assumption of forced choice, which is standard in cognitive decision making research. Besides this, it fits very cleanly into the general decision modeling paradigm. Many existing models of decision making already make explicit predictions regarding variables such as dominance, desirability, attribute commonality, and attribute alignability; variables that also characterize the determinants and consequences of choice deferral. It may be possible to modify one of these models to successfully predict key findings regarding choice deferral.

We find that this is indeed the case. In this paper, we study the properties of a deferral-based time limit, initially suggested by Jessup et al. (2009). This mechanism applies to sequential sampling models, for which it generates deferral when a decision threshold is not crossed by a particular time. In the first part of the paper we implement this time limit in Bhatia's (2013) associative accumulation model (AAM), which serves as a convenient back-end model for studying the relationship between deferral and the various features of the choice set. Using the choice options and parameter values assumed in Bhatia (2013) we find that the proposed mechanism is able provide a parsimonious explanation for eight different existing behavioral effects regarding choice deferral. AAM is not the only back-end model that is able to account for these effects, and we show that a more restricted variant of AAM, a leaky competitive accumulator (LCA) model (Usher & McClelland, 2001) can capture four of these effects (and indeed, that these four effects emerge from the assumptions AAM adopts from LCA).

Additionally, our assumption of a deferral time limit within a sequential sampling model makes strong, general predictions regarding decision times, and their relationship with choice and deferral probabilities. These predictions are largely independent of the specific sequential sampling model used to specify the accumulation process, and thus hold for AAM, LCA, and a number of related models. In the second half of the paper we develop a behavioral task to test these predictions. In this experiment subjects make choices both with and without the option to defer, thus allowing us to make the necessary within subject comparisons. Our results from this experiment show that the deferral time limit is successful in describing decision times and their relationship with choice deferral, and therefore presents a powerful approach to modifying models within the sequential sampling framework to incorporate choice deferral. Additionally, by illustrating a robust relationship between deferral probability and decision time, this experiment highlights the need for a dynamic model of choice deferral rather than one that is purely static.

Although there have been prior exploratory attempts at studying choice deferral using cognitive models (Busemeyer et al., 2006; Jessup et al., 2009; White et al., 2015), this paper is the first to use this approach to provide a comprehensive analysis of a large number of existing deferral-based effects, and the first to test and confirm a predicted relationship between choice deferral and decision time. In doing so, this paper extends the descriptive scope of the cognitive decision modeling framework to include one of the most studied and most important effects in preferential choice research. It also sheds light on the cognitive mechanisms that underlie behavior in everyday settings, in which choice is not forced, and shows that a cohesive theoretical account of this behavior is in fact possible.

2. Choice deferral effects

Consider a decision maker who, one evening, turns on the television to watch a movie. In this setting he finds that he has a choice between an action movie and a documentary. This choice is not forced: The decision maker can disengage from the task and defer his choice, with the intention to resume the decision at a later time, wait to come across other movies, or even forego watching a movie (Anderson, 2003; Chernev et al., 2015; Dhar & Sherman, 1996; Iyengar & Lepper, 2000; Jessup et al., 2009; Scheibehenne et al., 2010; Tversky & Shafir, 1992).

How do the options available in the choice task—in the above case, an action movie and a documentary—influence the probability of choice deferral? Conversely, how does the ability to defer choice affect the underlying choice probabilities of these options? Most current cognitive decision models do not consider *free choice* tasks, in which deferral is a feasible outcome. However these questions have been tackled experimentally, in tasks in which decision makers are given two or more options along with the ability to not make the decision (usually in the form of a response claiming that they cannot decide or that they wish to search for more information). In this section we will summarize eight experimental findings on the determinants and consequence of choice deferral that cognitive decision models should be able to capture.

Before we do this, however, it is useful to precisely define the choice setting we are considering. Choice deferral is typically studied in multi-attribute choice tasks, in which available choice options are defined on various decision attributes. The overall desirability, or value, of an option is proportional to the total desirability of its attributes, and the goal of the choice task is to select the choice option with the highest desirability. In the above example, an action movie could be seen as having high amounts of the "exciting" attribute whereas a documentary could be seen as having high amounts of the "informative" attribute. Formally, we can represent each of these choice options as a vector of M attributes, $\boldsymbol{x_i} = (x_{i1}, x_{i2}, \ldots, x_{iM})$. Here, x_{ij} is a scalar that represents the amount of attribute j in choice option i, with x_{ij} assumed to be greater than or equal to zero. If the first two attributes are exciting and informative, an action movie could be represented using a vector (10, 0,...) and a documentary could be represented using a vector (0, 10,...). Unless stated otherwise, we will assume that all attributes in the examples used in this paper are equally valuable.

Any particular choice set can be written as $\{\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N}\}$, where N is the total number of available options. In settings where deferral is not allowed, and the choice is forced, we can write the probability of choosing an option $\boldsymbol{x_i}$ as $\Pr(\boldsymbol{x_i} | \boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N})$. Likewise, in settings where deferral is allowed, and the choice is free, we can write the probability of choosing an option $\boldsymbol{x_i}$ as $\Pr(\boldsymbol{x_i} | \boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N}, \mathrm{D})$, and the probability of deferring choice as $\Pr(\mathrm{D} | \boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N}, \mathrm{D})$. Fig. 1 represents a hypothetical choice space with choice options defined on two desirable attributes, and Table 1 summarizes the attribute values of the options in this figure.

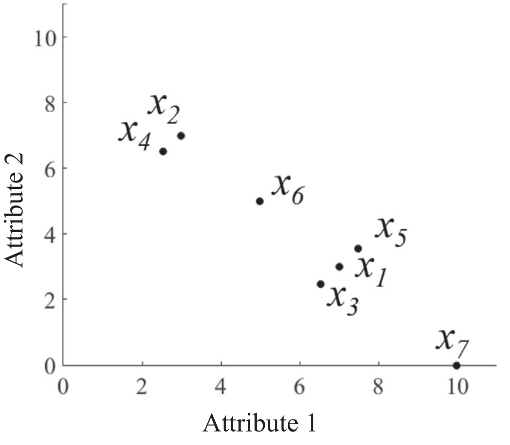

Fig. 1. A collection of choice options defined on two desirable attributes. Deferral probabilities are a function of the specific choice options offered to decision makers (Effects 1–5). Additionally, allowing decision makers to defer choice can affect the underlying choice probabilities of these options (Effects 6–8).

Table 1
Summary of choice options used in examples.

Option	Attribute			
	1	2	3	4
$\boldsymbol{x_1}$	7	3		
$\boldsymbol{x_2}$	3	7		
$\boldsymbol{x_3}$	6.5	2.5		
$\boldsymbol{x_4}$	2.5	6.5		
$\boldsymbol{x_5}$	7.5	3.5		
$\boldsymbol{x_6}$	5	5		
$\boldsymbol{x_7}$	10	0		
$\boldsymbol{x_8}$	7	7	3	0
$\boldsymbol{x_9}$	7	7	0	3
$\boldsymbol{x_{10}}$	14	0	1.5	1.5
$\boldsymbol{x_{11}}$	0	14	1.5	1.5
$\boldsymbol{x_{12}}$	10	10	3	0
$\boldsymbol{x_{13}}$	10	10	0	3
$\boldsymbol{x_{14}}$	7	7	3	0
$\boldsymbol{x_{15}}$	7	3	7	0
$\boldsymbol{x_{16}}$	0	3	7	7

2.1. Effect 1: dominance

Perhaps the earliest finding on the determinants of choice deferral in free choice pertains to the presence of a dominated option. Particularly Tversky and Shafir (1992) found that choices between two options were less likely to lead to deferral if one of the options was superior to the other on all attributes, even when the values of the options were held constant. This is related to the asymmetric dominance effect (Huber, Payne, & Puto, 1982), which many existing cognitive models have attempted to capture (see also Effect 7). In Fig. 1, $\boldsymbol{x_1} = (7, 3)$ dominates $\boldsymbol{x_3} = (6.5, 2.5)$ but not $\boldsymbol{x_4} = (2.5, 6.5)$. Thus, even though $\boldsymbol{x_3}$ and $\boldsymbol{x_4}$ have the same desirability, the dominance effect predicts a higher probability of deferral when the decision maker is asked to choose between $\boldsymbol{x_1}$ and $\boldsymbol{x_4}$ compared to $\boldsymbol{x_1}$ and $\boldsymbol{x_3}$, that is, $\Pr(D|\boldsymbol{x_1}, \boldsymbol{x_4}, D) > \Pr(D|\boldsymbol{x_1}, \boldsymbol{x_3}, D)$.

2.2. Effect 2: absolute desirability

In addition to dominance, Tversky and Shafir (1992) documented a second effect: Choices between highly valuable options were less likely to lead to deferral than choices between relatively less valuable options, controlling for the differences in the values of the two options. In Fig. 1, $\mathbf{x_1} = (7, 3)$ and $\mathbf{x_2} = (3, 7)$ are equally desirable, $\mathbf{x_3} = (6.5, 2.5)$ and $\mathbf{x_4} = (2.5, 6.5)$ are also equally desirable, and additionally $\mathbf{x_1}$ and $\mathbf{x_2}$ are more desirable than $\mathbf{x_3}$ and $\mathbf{x_4}$. In this setting, the absolute desirability effect suggests a higher probability of deferral when deciding between $\mathbf{x_3}$ and $\mathbf{x_4}$ than between $\mathbf{x_1}$ and $\mathbf{x_2}$, that is $\Pr(D|\mathbf{x_3}, \mathbf{x_4}, D) > \Pr(D|\mathbf{x_1}, \mathbf{x_2}, D)$. Empirical evidence in support of this effect has also been documented by Chernev and Hamilton (2009) and White and coauthors (White & Hoffrage, 2009; White et al., 2015).

2.3. Effect 3: relative desirability

The relative desirability of available options has also been shown to affect the probability of choice deferral. Particularly, Dhar (1997) has found that reducing the differences in the values of the available options can increase the incidence of choice deferral, so that choice is especially likely to be deferred when the available options are equally desirable. In Fig. 1, there is no difference in the average desirability of $\mathbf{x_1} = (7, 3)$ and $\mathbf{x_2} = (3, 7)$, or the average desirability of $\mathbf{x_4} = (2.5, 6.5)$ and $\mathbf{x_5} = (7.5, 3.5)$ (both sets of choice pairs have an average amount of 5 units in each attribute per option). However, these pairs of options differ in terms of their relative desirability. The values of $\mathbf{x_4}$ and $\mathbf{x_5}$ are substantially different (with $\mathbf{x_5}$ being much more desirable than $\mathbf{x_4}$), whereas the values of $\mathbf{x_1}$ and $\mathbf{x_2}$ are identical. In this setting, the relative desirability effect predicts a higher probability of deferral in the decision between $\mathbf{x_1}$ and $\mathbf{x_2}$ compared to the decision between $\mathbf{x_4}$ and $\mathbf{x_5}$, that is $\Pr(D|\mathbf{x_1}, \mathbf{x_2}, D) > \Pr(D|\mathbf{x_4}, \mathbf{x_5}, D)$.

It is important to note that the effect of the relative desirability of options can be strong enough to overpower the effect of the absolute desirability of options, outlined above. For example, Dhar (1997) has found that replacing a desirable option in a choice set with an undesirable option can in fact decrease the incidence of deferral, despite the fact that this replacement reduces the average desirability of the choice set. White and coauthors (White & Hoffrage, 2009; White et al., 2015) provide additional evidence in support of this effect.

2.4. Effect 4: common and unique attributes

It is possible to increase the incidence of choice deferral even if dominance, absolute desirability, and relative desirability are kept constant. This has been documented by Dhar and Sherman (1996) who found that deferral is more likely in choices with common good but unique bad attributes, compared to equivalent choices with common bad but unique good attributes. To illustrate this, consider options $\mathbf{x}_8 = (7, 7, 3, 0)$, $\mathbf{x}_9 = (7, 7, 0, 3)$, $\mathbf{x}_{10} = (14, 0, 1.5, 1.5)$, $\mathbf{x}_{11} = (0, 14, 1.5, 1.5)$, where attributes 1 and 2 are desirable, and attributes 3 and 4 are undesirable. Here, $\mathbf{x_8}$ and $\mathbf{x_9}$ have common amounts of the desirable attribute, but unique amounts of the undesirable attributes. In contrast, $\mathbf{x_{10}}$ and $\mathbf{x_{11}}$ have common amounts of the undesirable attributes, but unique amounts of the desirable attributes. Besides these differences, these choice pairs are identical in terms of both their absolute desirability and relative desirability (all four options have a total of 10 units of desirable attributes and 3 units of undesirable attributes). In this setting, the common and unique attributes effect would predict that deferral would be more likely in the choice between $\mathbf{x_8}$ and $\mathbf{x_9}$ compared to the choice between $\mathbf{x_{10}}$ and $\mathbf{x_{11}}$, that is, $\Pr(D|\mathbf{x_8}, \mathbf{x_9}, D) > \Pr(D|\mathbf{x_{10}}, \mathbf{x_{11}}, D)$. The options discussed here are summarized in Table 1.

Note that it is not the case that common attributes are ignored in choices involving deferral. Increasing the amount of desirable common attributes so as to increase the overall values of the available options reduce the incidence of deferral, as documented by Nagpal et al. (2011). This is consistent with Effect 2, described above. For options $\mathbf{x_{12}} = (10, 10, 3, 0)$, and $\mathbf{x}_{13} = (10, 10, 0, 3)$, this effect would predict that $\Pr(D|\mathbf{x_8}, \mathbf{x_9}, D) > \Pr(D|\mathbf{x_{12}}, \mathbf{x_{13}}, D)$.

2.5. Effect 5: alignability

A related determinant of choice deferral involves the alignability of the attributes in the available choice options (Gentner & Markman, 1997). A number of researchers have found that individuals place a higher weight on an attribute if it is alignable across the choice options, that is, if it is present in multiple choice options, compared to if it is non-alignable or unique across the choice options (Markman & Medin, 1995; Nowlis & Simonson, 1997). This can lead to choice reversals across choice sets and other related choice set effects (Kivetz & Simonson, 2000; see also Bhatia, 2013 for a review).

Alignability has been shown to strongly affect the probability of choice deferral (Gourville & Soman, 2005). Particularly, choice deferral is more likely in choice sets with multiple non-alignable attributes compared to choice sets with alignable attributes. To illustrate this, consider options $\boldsymbol{x}_{14} = (7, 7, 3, 0)$, $\boldsymbol{x}_{15} = (7, 3, 7, 0)$, and $\boldsymbol{x}_{16} = (0, 3, 7, 7)$. These are variants of the options in Effect 4, except for the fact that all of the four attributes are desirable. Now, in this setting, attribute 1 is alignable across options $\boldsymbol{x}_{14}$ and $\boldsymbol{x}_{15}$ but not across options $\boldsymbol{x}_{14}$ and $\boldsymbol{x}_{16}$ (besides this difference, these choice pairs are identical in terms of both their absolute desirability and relative desirability). The alignability effect thus states that deferral should be more likely in the choice between $\boldsymbol{x}_{14}$ and $\boldsymbol{x}_{16}$ compared to the choice between $\boldsymbol{x}_{14}$ and $\boldsymbol{x}_{15}$, that is, $\Pr(D|\boldsymbol{x}_{14}, \boldsymbol{x}_{16}, D) > \Pr(D|\boldsymbol{x}_{14}, \boldsymbol{x}_{15}, D)$. Chernev (2003, 2005) and Greifeneder, Scheibehenne, and Kleber (2010) document additional evidence supporting this effect. The options discussed here are summarized in Table 1.

2.6. Effect 6: extreme options

The effects discussed thus far illustrate how the probability of deferral is affected by the composition of the choice set. Now we will consider how allowing for the possibility of deferral can alter the choice probabilities of the existing options in the choice set. This has been empirically examined by Dhar and Simonson (2003) who found that allowing for deferral disproportionally reduces the choice probability of an all-average choice option, with moderate amounts of all attributes, compared to an extreme option, with large amounts of only one attribute. When choices are represented as in Fig. 1, the extreme options effect implies that the probability of choosing the all-average option $\boldsymbol{x}_6 = (5, 5)$ over the extreme option $\boldsymbol{x}_7 = (10, 0)$ is lower when decision makers are given the option to defer choice, relative to when deferral is not a possibility. If we write the relative choice probability of choosing $\boldsymbol{x}_i$ over $\boldsymbol{x}_{i'}$ in a certain choice set C as $\mathrm{Rel}(\boldsymbol{x}_i, \boldsymbol{x}_{i'}|C) = [\Pr(\boldsymbol{x}_i|C) - \Pr(\boldsymbol{x}_{i'}|C)]/[\Pr(\boldsymbol{x}_i|C) + \Pr(\boldsymbol{x}_{i'}|C)]$, then this effect implies that $\mathrm{Rel}(\boldsymbol{x}_6, \boldsymbol{x}_7|\boldsymbol{x}_6, \boldsymbol{x}_7) > \mathrm{Rel}(\boldsymbol{x}_6, \boldsymbol{x}_7|\boldsymbol{x}_6, \boldsymbol{x}_7, D)$.

2.7. Effect 7: asymmetric dominance

Another consequence of allowing deferral relates to the asymmetric dominance effect. This is the finding that the relative choice probability of an option increases with the introduction of a novel option (a decoy), that it, but not its competitor, dominates (Huber et al., 1982; Wedell & Pettibone, 1996). Again note that in Fig. 1, $\boldsymbol{x}_3 = (6.5, 2.5)$ is dominated by $\boldsymbol{x}_1 = (7, 3)$ but not by $\boldsymbol{x}_2 = (3, 7)$. Using the above notation, this effect implies that $\mathrm{Rel}(\boldsymbol{x}_1, \boldsymbol{x}_2|\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) > \mathrm{Rel}(\boldsymbol{x}_1, \boldsymbol{x}_2|\boldsymbol{x}_1, \boldsymbol{x}_2)$, that is $\boldsymbol{x}_1$ should be more likely to be chosen over $\boldsymbol{x}_2$ when $\boldsymbol{x}_3$ is part of the choice set.

The asymmetric dominance effect is typically studied without the possibility of deferral. Dhar and Simonson (2003) have however run experiments in which decision makers are presented with the standard asymmetric dominance effect options but are also given the possibility to defer choice. In these settings Dhar and Simonson find that free choice increases the asymmetric dominance effect, that is $\mathrm{Rel}(\boldsymbol{x}_1, \boldsymbol{x}_2|\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, D) - \mathrm{Rel}(\boldsymbol{x}_1, \boldsymbol{x}_2|\boldsymbol{x}_1, \boldsymbol{x}_2, D) > \mathrm{Rel}(\boldsymbol{x}_1, \boldsymbol{x}_2|\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) - \mathrm{Rel}(\boldsymbol{x}_1, \boldsymbol{x}_2|\boldsymbol{x}_1, \boldsymbol{x}_2)$.

2.8. Effect 8: compromise

The final consequence of allowing deferral relates to the compromise effect. This is the finding that the relative choice probability of an option increases with the addition of a decoy that makes the option appear as a compromise (Simonson, 1989; Simonson & Tversky, 1992). In Fig. 1, $\boldsymbol{x}_7 = (10, 0)$ has more of attribute 1 and less of attribute 2 than both $\boldsymbol{x}_1 = (7, 3)$ and $\boldsymbol{x}_2 = (3, 7)$. Due to this, $\boldsymbol{x}_1$

can be seen as being a compromise between x_2 and x_7. In this setting the compromise effect predicts that Rel(x_1, x_2|x_1, x_2, x_7) > Rel(x_1, x_2|x_1, x_2), that is x_1 should be more likely to be chosen over x_2 when x_7 is part of the choice set.

As with the asymmetric dominance effect, Dhar and Simonson (2003) have run experiments in which decision makers are presented with the standard compromise effect options but are also allowed the possibility to defer choice. In contrast to the asymmetric dominance effect, Dhar and Simonson find that free choice reduces the compromise effect, that is Rel(x_1, x_2|x_1, x_2, x_7) − Rel(x_1, x_2|x_1, x_2) > Rel(x_1, x_2|x_1, x_2, x_7, D) − Rel(x_1, x_2|x_1, x_2, D).

2.9. Choice overload

There is one effect closely related to the above, that we will not directly consider in this paper. This is the choice overload effect, which states that decision makers are less likely to make a choice when they are offered a large choice set compared to a moderate or small choice set (Iyengar & Lepper, 2000). Scheibehenne et al. (2010) have argued that this effect does not emerge on aggregate, whereas Chernev et al. (2015) have claimed that although there may not be a main effect of the size of the choice set on choice deferral, adding options to the choice set can nonetheless increase the probability of deferral, when these options alter the dominance relations, mean desirability, relative desirability, and attribute commonality and alignability of the choice set. As Chernev et al.'s argument in support of the choice overload effect suggests that the effect is merely a combination of Effects 1–5, we will not be considering choice overload separately in this paper.

3. A model of choice deferral effects

The above effects illustrate a systematic relationship between the decision to defer choice, and the composition of the choice set. With our movie choice example, these effects suggest that choices between the movies would be less likely to be deferred if the decision maker was deciding between an action movie and an inferior but similar movie (such as a second, less enjoyable action movie), instead of the initial action movie and documentary. Likewise choice should be less likely to be deferred if the two available movies were both highly desirable, or if, one movie was seen as being much more desirable than the other. Finally, we could alter the incidence of choice deferral by varying the attribute overlap between the movies.

Likewise the presence or absence of deferral can affect underlying choice probabilities between the available options. If the decision maker was forced to make a choice between the available movies, we would observe a higher choice probability of choosing an all-average movie compared to an extreme movie. Similarly, the incidence of the asymmetric dominance effect would reduce, but the incidence of the compromise effect would increase, compared to a free choice setting in which deferral was allowed.

3.1. Modeling choice deferral

The above effects show that deferral related behavior is sensitive to both the values and the attribute compositions of the available options. Nearly all cognitive models of multi-attribute choice are able to make predictions regarding these variables, suggesting that formally modeling free choice does not require a drastic move away from existing frameworks.

Indeed there have been prior attempts to integrate a deferral mechanism into decision field theory (DFT) (Busemeyer & Townsend, 1993; Roe et al., 2001). DFT is a dynamic model of preferential choice, which assumes that attribute values are sampled sequentially and stochastically, and accumulated into preferences over the time course of the decision process. Sequential sampling models are commonly used to study low-level decisions, such as perceptual and lexical decisions (Link & Heath, 1975; Nosofsky & Palmeri, 1997; Ratcliff, 1978; Ratcliff, Gomez, & McKoon, 2004; Usher & McClelland, 2001). They are both biologically plausible and have insightful statistical interpretations (Bogacz, Brown, Moehlis, Holmes, & Cohen, 2006; Gold & Shadlen, 2007). Additionally the models are able to make rigorous quantitative predictions regarding not only choice probabilities but also decision times, confidence, and related decision outcomes (Bhatia, 2013, 2014; Busemeyer & Townsend, 1993; Diederich, 1997; Johnson & Busemeyer, 2005; Krajbich, Armel, & Rangel, 2010; Pleskac & Busemeyer, 2010; Roe et al., 2001; Trueblood et al., 2014; Tsetsos, Chater, & Usher, 2012; Usher & McClelland, 2004). By using a sequential sampling mechanism, DFT presents a theoretically desirable approach to modeling preferential choice.

Jessup et al. (2009) have assumed that choice is deferred within a DFT model, if the decision is not made by a certain time. They have shown that this time limit-based extension to DFT can explain the choice overload effect; that is, it can, in certain settings, lead to increased choice deferral in the presence of multiple choice options.

Busemeyer et al. (2006) also present an alternate DFT-based model of deferral. This model assumes that the possibility of choice deferral is processed as just another choice option, and that choice is deferred when this option is chosen. Busemeyer et al. (2006) show that that this assumption can explain the increase in the asymmetric dominance effect and decrease in the compromise effect, in the presence of deferral (Effects 7 and 8).

White et al. (2015) have also presented a formal model of deferral. Their model is not based in the sequential sampling and accumulation framework, but instead assumes that decision makers evaluate choice options in two stages, with one stage involving evaluations of absolute desirability and the other stage involving evaluations of relative desirability. Choice can be deferred if the absolute desirabilities of the options or the relative desirabilities of the options fail to reach a threshold. With these assumptions, White et al.'s model is able to explain Effects 2 and 3.

None of these models explain all of the effects outlined above. The DFT-based models of Jessup et al. (2009) and Busemeyer et al. (2006), for example, do not attempt to explain deferral's relationship with dominance, absolute option desirability, relative option desirability, attribute overlap, or the effect of choice deferral on the choice probability of extreme options (Effects 1–6). Additionally, Jessup et al.'s (2009) model does not attempt to explain deferral's relationship with the asymmetric dominance and compromise effects (Effects 7 and 8). Similarly White et al.'s (2015) two-stage model does not attempt to explain deferral's relationship with dominance, attribute overlap, or the effect of choice deferral on various choice probabilities (Effects 1 and 4–8).

That said, the models outlined above provide many valuable insights regarding the ways in which deferral could be formalized. Their inability to account for all the effects discussed in this paper may stem not from their specific assumptions regarding the mechanisms underlying deferral, but rather the back-end models onto which these mechanisms are attached. Thus, for example, Jessup et al.'s (2009) time limit mechanism could in fact be a good way of formalizing choice deferral within a sequential sampling framework, and could potentially explain Effects 1–8 when implemented within a different sequential sampling model.

3.2. Modeling choice set dependence

We first attempt to account for the above eight effects using Jessup et al.'s (2009) time limit mechanism. In order to test whether this mechanism can be used to explain Effects 1–8, which pertain to a relationship between deferral and the composition of the choice set, we implement this time limit within the associative accumulation model (AAM) (Bhatia, 2013). Like DFT, AAM is a sequential sampling and accumulation theory of preference which assumes the values of attributes are stochastically and dynamically aggregated into preferences. AAM differs from existing accumulation models of preferential choice in suggesting that the representation and retrieval of information about the available options plays a key role in the decision. Particularly, decision makers are assumed to use a simple two-layer neural network to store the relationships between the options and their attributes. As in related models of semantic representation and object representation, the connections between options and attributes capture the associations between these options and attributes. These associations are assumed to be equal to the amount of an attribute that each option has, so that options that have high amounts of a certain attribute also have stronger associative connections with the attribute. Activating an option by including it in the choice set, activates its component attributes, and subsequently affects the probability with which these attributes are attended to and aggregated into preferences. This is in contrast to models such as multi-alternative decision field theory, multi-attribute leaky competitive accumulation, and multi-alternative linear ballistic accumulation, which assume that attribute activation and sampling is independent of the composition of the choice set (Roe et al., 2001; Trueblood et al., 2014; Usher & McClelland, 2004).

As an example of AAM's attribute activation property, consider again our choice between the movies. AAM predicts that the probability of thinking of an attribute is increased in the presence of choice options that contain large amounts of that attribute. Thus decision makers who have to choose between an action movie and a documentary should be more likely to attend to the documentary's key attributes (e.g. informativeness) compared to decision makers who are not given the documentary as an available option. Likewise adding a new type of movie, say a horror movie, would lead to increased attention towards horror related attributes, relative to the initial setting with just the action movie and documentary.

This attribute activation property can be seen as a type of weighting-bias. That said, while this bias acts to increase the influence of attributes that are present in large quantities in one alternative or are present in multiple alternatives, it does not necessarily lead to a change in judgments of subjective attribute importance (e.g. those studied in Wedell & Pettibone, 1996). It may very well be the case that explicit judgments of attribute importance diverge from those that would be predicted by AAM, as suggested in Wedell and Pettibone's work.

Additionally, the weighting-type biases proposed by AAM diverge from other types of weighting biases which are associated with changes to the choice set, such as manipulations of the range of an attribute (e.g. Ariely & Wallsten, 1995; Huber et al., 1982). Thus it is possible to increase the attention to an attribute in the AAM model while keeping range constant. This is why, for example, frequency decoys can bias preferences (Wedell, 1991): AAM is able to predict this (Bhatia, 2013), but a model relying solely on range-weighting cannot. A similar point holds with phantom dominating decoys (e.g. Pettibone & Wedell, 2000) and symmetrically dominated range decoys (Wedell, 1991). Finally, evidence for the types of attentional biases predicted by AAM has been documented by Bhatia (2014) and is also observed in experiments on reference dependence (see Bhatia, 2013 for details).

While attending to attributes associated with the available options seems to be efficient, it can lead to certain types of inconsistency. Particularly, adding or removing options from the available choice set can alter attribute sampling probabilities and subsequently reverse choice. This dependence between choice, and the options that are available in the decision, allows AAM to explain a large range of findings regarding choice set dependence, such as the asymmetric dominance and compromise effects, alignability and conflict effects, less is more effects, and reference point effects (see Bhatia, 2013 for more details). In this sense AAM could serve as a convenient back-end process with which the properties of the deferral time limit can be tested.

3.3. A combined approach

In the first half of this paper, we will implement the insights of Jessup et al. (2009) using AAM as a back-end model, and assume that choice is deferred if the accumulators do not cross the decision threshold by a certain time limit. Recall that we can represent an available option as a vector of M attributes, $\boldsymbol{x_i} = (x_{i1}, x_{i2}, \ldots, x_{iM})$. AAM assumes that the associative connection between a choice option, i, and an attribute, j, is simply equal to the amount of the attribute in the choice option, x_{ij}. The probability of sampling an attribute is given by the relative strength of association of the attribute with the choice set. For an attribute j, in a choice set with N available options, this sampling probability can be written as:

$$w_j = \frac{\sum_{i=1}^{N} [x_{ij}] + a_0}{\sum_{k=1}^{M} \left(\sum_{i=1}^{N} [x_{ik}] + a_0 \right)}$$

Here a_0 is a constant that determines the strength of the associative bias. As a_0 increases, the associative bias in AAM is reduced. At $a_0 = \infty$, each attribute is equally likely to be sampled, and decisions are choice set-independent. Overall, the above equation implies that an attribute is more likely to be attended to if it is strongly associated with multiple options in the choice set.

Once an attribute is sampled, AAM assumes that its value in every available option is calculated and added to the accumulating preferences. The value of attribute j in option i can be written as $V_j(x_{ij})$, where V_j is a positive and increasing function if the attribute is desirable and a negative and decreasing function if the attribute is undesirable. Preferences are also subject to gradual leakage, captured by parameter d, lateral inhibition, captured by parameter l, and a zero mean noise with standard deviation σ, captured by parameter ε. If attribute j is sampled at time t, then the preference for option i can be written as:

$$P_i(t) = d \cdot P_i(t-1) - l \cdot \sum_{k \neq i} P_k(t-1) + V_j(x_{ij}) + \varepsilon_i(t-1)$$

Finally, an upper threshold Q determines both the option that is chosen, and the time at which the decision is made. If an option $\boldsymbol{x_i}$ crosses Q at time t then $\boldsymbol{x_i}$ is chosen and t is the decision time. In forced choices where deferral is not an option, the decision terminates only after some option has crossed Q.

What happens in free choices, where deferral is allowed? As in Jessup et al. (2009) we assume that choice is deferred if a time limit T is crossed without the decision having been made at an earlier time period. Hence in choices with the possibility of deferral, some option is chosen if it crosses Q before T, and choice is deferred if this event does not happen before T. An illustration of a typical choice is presented in Fig. 2.

It is important to note that the model presented here involves some simplifications. Firstly we assume that the activation function for $P_i(t)$ is linear, and that activation can fall below zero. A non-linear activation function with a lower bound at zero is more biologically plausible. However as we will only be considering desirable options with minimal levels of lateral inhibition, a lower bound on our preference activation will not be necessary. All of the results discussed in this paper emerge with the piecewise linear activation function used in Usher and McClelland (2001, 2004).

Secondly, we assume that choice is made using only a single (upper) threshold. Some prior work has assumed the possibility of a lower threshold, which is useful for capturing choice option elimination (e.g. Roe et al., 2001). Again, however, we will only be considering desirable options with minimal levels of lateral inhibition, and so a lower rejection threshold will not be necessary. All of the results discussed in this paper emerge with the negative rejection threshold used in Roe et al. (2001).

Finally, we have assumed that the deferral time limit is deterministic. This is a tremendous simplification. If this was actually the case then we would observe a censored time distribution in the presence of deferral, which is unrealistic. Here we have a single fixed T only for expositional convenience. In later sections, which examine decision time predictions in more detail, we will assume the possibility of a variable T.

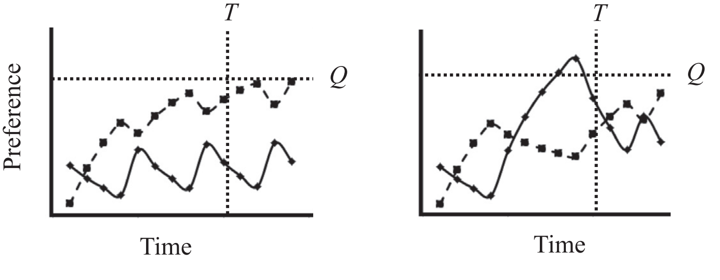

Fig. 2. An illustration of the deferral time limit in a choice involving two options. The first option to cross the threshold Q is chosen. If deferral is allowed, and if neither of the two accumulators cross Q before the deferral time limit T, then choice is deferred. In the first panel choice would be deferred if deferral was allowed, and the option corresponding to the dashed line would be chosen if deferral was not allowed. In the second panel the option corresponding to the solid line would be chosen both if deferral was allowed and if it wasn't.

3.4. Properties of the deferral time limit

According to the proposed approach, choice is deferred if the accumulators for the available options do not cross the decision threshold before the time limit. In this sense, the time limit T is a vertical boundary corresponding to deferral (just like the decision threshold Q is a horizontal boundary corresponding to choice). T is most likely to be crossed if the rate of accumulation towards Q is particularly slow. This implies that the probability of deferring choice can be changed by altering the speed at which preferences are accumulated, with faster accumulation leading to reduced deferral. AAM predicts that attribute attention, and subsequently the rate of accumulation, can change if existing options are modified, suggesting that a time limit mechanism for deferral, using AAM as a back-end model, may be able to account for dependence of deferral on the dominance relations, absolute and relative desirability, and attribute overlap in the choice set (Effects 1–5).

Another implication of the proposed deferral mechanism is that it introduces a type of time dependence to the choice task. By stopping the choice task at time T, choice deferral disproportionately detracts from the choice probabilities of options that become desirable only later on in the decision. If deferral was not a possibility, these options would increase in their relative preference strength after T, and eventually be chosen. Since deferral is allowed, this cannot happen, and the decision often terminates in deferral instead of the selection of these options. This implies that allowing for the possibility of choice deferral can disproportionately alter the choice probabilities of available options, if the preferences for these options increase at different rates over time. Indeed allowing for the possibility of choice deferral can even reverse choice probabilities of options if modal threshold crossing probabilities before T and after T vary. The dynamics of AAM are sophisticated enough to generate these types of time dependencies, suggesting that a time limit mechanism for deferral, implemented with AAM, may be able to account for findings regarding the effect of deferral on underlying choice probabilities (Effects 6–8).

3.5. Other sequential sampling models

The assumptions of sequential sampling of attributes, and decay and inhibition in preferences, outlined above, are not unique to the associative accumulation model. Rather they are derived from existing dynamic models of decision making, notably the leaky competitive accumulator (LCA) (Usher & McClelland, 2001) and multi-alternative decision field theory (MDFT) (Roe et al., 2001). The novelty of AAM is in its assumption of associative attribute sampling, according to which the choice set can influence which attributes are accumulated during the decision process. This mechanism is controlled by the parameter a_0, and can be turned off by setting $a_0 = \infty$. In this case the above setting would reduce to a leaky competitive accumulator which sequentially samples and accumulates attribute values with leakage and inhibition, but one whose attribute sampling probabilities do not vary as a function of the choice set. Note that Usher and McClelland (2004) present a multi-attribute extension of their LCA model, which features leakage, but differs in terms of other properties, relative to the proposed AAM model. In the remainder of the paper our reference to the LCA model will pertain to the 2001 version of the model. Any references to the 2004 multi-attribute extension will be explicitly labeled as such.

It is important to note that changing the composition of the choice set or the possibility of deferral can influence choices in the above model, even if the associative mechanism is inactive. For example, the changes in accumulation caused by modifying the desirability of an alternative in the choice set, do not only emerge because these changes can alter attribute sampling probabilities they also emerge because this type of change affects values $V_j(x_{ij})$ which are aggregated into preferences. In this light, we will be examining which of the deferral effects we wish to describe require the full AAM model, and which of them can also be obtained with the above AAM specification, but with very large values of a_0, in which the associative component of the model is absent and model is reduced to a leaky competitive accumulator. We will also consider setting $I = 0$, thereby further simplifying the LCA model to have just leaky racing accumulators without inhibition (leakage itself does not alter our predictions and thus for simplicity we will avoid further simplifying this model into a race accumulator without leakage).

In addition to AAM there are also a number of additional approaches that build on the multiattribute leaky competitive accumulator framework to attempt to explain findings such as asymmetric dominance and compromise effects. These include the MDFT model (Roe et al., 2001) which assumes distant-dependent inhibition and the multi-attribute leaky competitive accumulator (MALCA) (Usher & McClelland, 2004) which allows for loss aversion-based inter-attribute comparisons alongside leaky competitive accumulation. A more recent sequential model that does not assume sequential sampling of attributes or decay and leakage in preferences is the multi-attribute linear ballistic accumulator (Trueblood et al., 2014). All three of these models could be modified to include a deferral time limit, as we have done with the AAM model above. For parsimony this paper will not explicitly test the properties of these deferral-based extensions of MDFT, MALCA, or MLBA, but these properties, and their ability to explain existing deferral effects, will be discussed in detail in later sections.

4. Explaining deferral effects

In this section, we will show how the time dynamics of preference accumulation in the model outlined above relate to the causes and consequences of choice deferral. Simulations will use model parameters specified in Bhatia (2013): we will set $d = 0.8$, $\sigma = 0.05$, $P_i(0) = 0$ for all i, $V_j(x_{ij}) = x_{ij}^{0.5}$ for all j. We will vary a_0, so as to examine the dependence of our predictions on the associative mechanisms proposed by AAM, with values of $a_0 = 0$ corresponding to settings where this mechanism is strongly active, and $a_0 = \infty$ corresponding to settings where this mechanism is absent and the model mimics a leaky competitive accumulator. $a_0 = 10$ is the parameter value used in Bhatia (2013). We will also assume that $l = 0.01$, while occasionally limiting $l = 0$ in conjunction with $a_0 = \infty$, in order to test the predictions of a further simplified version of the above model in in which attribute sampling without associative biases aggregates preferences in (leaky) racing accumulators. Lastly we will assume that the vertical and horizontal thresholds T and Q are equidistant from the origin and that both are equal to 10, which is the total attribute value of the core choices that we are considering (see Fig. 1), though we will occasionally vary these parameters to examine the robustness of our effects. Each simulation will be repeated 10,000 times, and displayed responses will be averaged over these trials. Choice options used in Bhatia (2013) will be the basis of these simulations. These options are displayed in Fig. 1.

Note that our use of Bhatia's (2013) parameters and choice options indicates that all of the results discussed in the following sections are fully compatible with those discussed in Bhatia (2013), and that the deferral-based extension of AAM can provide a simultaneous account of the results from both papers using a single set of parameter values (the very weak amount of lateral inhibition assumed here, which is absent in Bhatia (2013), does not alter Bhatia's (2013) findings). Additionally, by restricting ourselves to previously used options and parameter values, and setting $T = Q = x_{11} + x_{12} = x_{21} + x_{22} = 10$, we have minimized the amount of theoretical flexibility involved in applying our model to new effects. Our ability to explain these effects (shown in the coming sections) despite our restrictive parameter assumptions suggests that our approach's predictions regarding deferral are fairly robust.

That said, some of the predictions of the model can be improved by tweaking these options and parameters. For example, with the above setup, our model predicts relatively high deferral rates. This could be remedied by having a slightly higher value of T or d, or having slightly lower values of Q and x_{ij}. As this part of the paper aims primarily to provide a qualitative account of choice deferral effects, we will ignore these quantitative considerations, and limit ourselves to the options and parameters specified above.

4.1. Effect 1: dominance

The deferral time limit, with AAM as a back-end process, predicts a reduced probability of choice deferral with dominated options. Recall that AAM assumes that attribute attention is proportional to the association of the attributes with the available options, so that attributes that are present in multiple options are also associated with multiple options, and thus receive a higher attentional weight. If one option dominates another, they typically share the same primary attribute, and this attribute is highly likely to be sampled. For example, in the choice set {$\boldsymbol{x_1}$, $\boldsymbol{x_3}$} in Fig. 1, attribute 1 is highly present in both options, and is thus the attribute decision makers most frequently attend to. This means that the preference for option $\boldsymbol{x_1}$—the most desirable option, which is also the option that is the strongest on attribute 1—increases and crosses a threshold quickly, and that choice is subsequently unlikely to be deferred.

When a dominated option is replaced with an equally desirable non-dominated option, there is a dispersion in the sampling probabilities of the underlying attributes, as attributes associated with the novel, non-dominated option are now more likely to be sampled. Thus, in our example, if $\boldsymbol{x_3}$ is replaced with $\boldsymbol{x_4}$, decision makers are relatively more likely to sample attribute 2. This reduces the rate of accumulation for all choice options (including the most desirable option, $\boldsymbol{x_1}$) increasing the probability that thresholds are not crossed by the deferral time limit.

Consider, for example, the choice options presented in Fig. 1: $\boldsymbol{x_1}$ = (7, 3), $\boldsymbol{x_3}$ = (6.5, 2.5) and $\boldsymbol{x_4}$ = (2.5, 6.5). When we implement our model with the parameters listed in the previous section, we find that the sampling probability of attribute 1 in the set {$\boldsymbol{x_1}$, $\boldsymbol{x_3}$} is 0.61, whereas the sampling probability of attribute 1 in the set {$\boldsymbol{x_1}$, $\boldsymbol{x_4}$} is 0.50. Subsequently the expected increase in the preference for $\boldsymbol{x_1}$, in each time period, in the set {$\boldsymbol{x_1}$, $\boldsymbol{x_3}$}, is 2.28, whereas the equivalent increase in preference in the set {$\boldsymbol{x_1}$, $\boldsymbol{x_4}$} is 2.18. As a result of this $\boldsymbol{x_1}$ is less likely to cross the threshold before the time limit in the set {$\boldsymbol{x_1}$, $\boldsymbol{x_4}$} compared to the set {$\boldsymbol{x_1}$, $\boldsymbol{x_3}$}, and we obtain Pr(D|$\boldsymbol{x_1}$, $\boldsymbol{x_4}$, D) = 0.92 > Pr(D|$\boldsymbol{x_1}$, $\boldsymbol{x_3}$, D) = 0.82.

This dependence of attribute sampling on the composition of the choice set is a function of the AAM parameter a_0, and cannot be predicted by a restricted LCA type model in which the associative mechanisms of AAM are inactive. Particularly, for high values of a_0, attribute attention is independent of the choice set, and the above dominance biases disappear. This is shown in Fig. 3, which plots the difference in the deferral probabilities in the choice set {$\boldsymbol{x_1}$, $\boldsymbol{x_4}$} compared to the choice set {$\boldsymbol{x_1}$, $\boldsymbol{x_3}$}, that is, Pr(D|$\boldsymbol{x_1}$, $\boldsymbol{x_4}$, D) – Pr(D|$\boldsymbol{x_1}$, $\boldsymbol{x_3}$, D). Positive values in this figure correspond to the emergence of the dominance effect described above. As can be seen in Fig. 3, this effect is likely to emerge when a_0 is small. This effect disappears as a_0 increases and the strength of the associative mechanisms in AAM weaken. Fig. 3 also plots the difference in choice probabilities of $\boldsymbol{x_1}$ in the choice set {$\boldsymbol{x_1}$, $\boldsymbol{x_4}$} compared to the choice set {$\boldsymbol{x_1}$, $\boldsymbol{x_3}$}, that is, Pr($\boldsymbol{x_1}$|$\boldsymbol{x_1}$, $\boldsymbol{x_4}$, D) – Pr($\boldsymbol{x_1}$|$\boldsymbol{x_1}$, $\boldsymbol{x_3}$, D). It shows that $\boldsymbol{x_1}$ is more likely to be

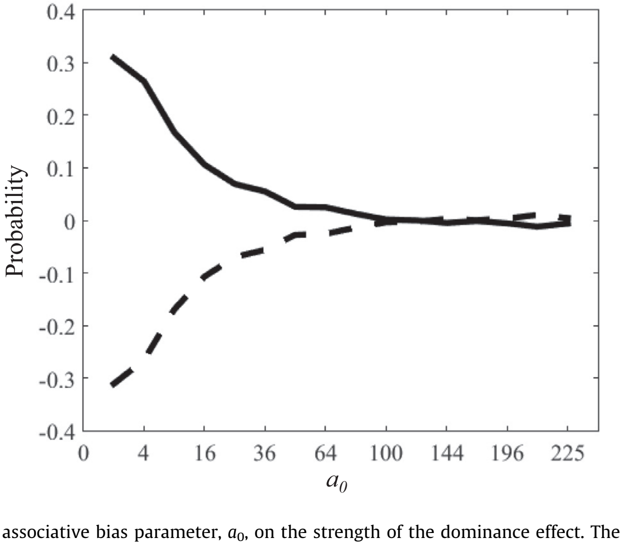

Fig. 3. The effect of the associative bias parameter, a_0, on the strength of the dominance effect. The solid line represents the difference in the deferral probability in the set {$\boldsymbol{x_1}$, $\boldsymbol{x_4}$} compared to the set {$\boldsymbol{x_1}$, $\boldsymbol{x_3}$}, whereas the dashed line represents the difference in the choice probability of $\boldsymbol{x_1}$ in the set {$\boldsymbol{x_1}$, $\boldsymbol{x_4}$} compared to the set {$\boldsymbol{x_1}$, $\boldsymbol{x_3}$}.

selected if it dominates its competitor when a_0 is small, compared to if it is large. This is consistent with the explanation for the asymmetric dominance effect in Bhatia (2013) (see also Effect 7).

Additionally note that the effect described here is not sensitive to $T = Q = 10$, and can emerge with different values for these parameters. Indeed, setting $T = 12$ can in fact lead to more reasonable deferral probabilities, that more closely match those obtained in Tversky and Shafir (1992). Particularly, with $T = 12$ and $Q = 10$, we obtain $\Pr(D|\boldsymbol{x_1}, \boldsymbol{x_4}, D) = 0.56 > \Pr(D|\boldsymbol{x_1}, \boldsymbol{x_3}, D) = 0.45$.

Finally, the overall effect of different dominated vs. non-dominated options on deferral is illustrated in Fig. 4. Here we fix one of the options as $\boldsymbol{x_1} = (7, 3)$. The shade of each point (a, b) in this figure represents the deferral probability generated by the choice set $\{\boldsymbol{x_1}, (a, b)\}$, using the parameter values outlined above. Lighter shades indicate a higher deferral probability. As can be clearly seen in the figure, keeping $a + b$ fixed, the deferral probability is lower when (a, b) is dominated by $\boldsymbol{x_1}$ compared to when it isn't. Note that Fig. 4 also displays a number of additional interesting patterns. For example, the dominance effect in deferral seems to share some of the properties of the asymmetric dominance effect in choice. Particularly, frequency decoys that are dominated by an option on its primary dimension lead to higher deferral than range decoys that are dominated by an option on its secondary dimension. This is similar to the effect of range vs. frequency decoys documented by Huber et al. (1982) and predicted by Bhatia (2013) and related models. Additionally the probability of deferral seems to be decreasing both in the extremity and in the absolute desirability of (a, b), so that the highest rates of deferral are observed for non-dominated placements of (a, b) which are relatively undesirable and have moderate amounts of the two attributes. Some of these patterns will be discussed in detail in subsequent sections.

4.2. Effect 2: absolute desirability

Fig. 4 indicates that the deferral time limit may be able to capture more than just the dominance effect. For example, the fact that deferral probabilities increase as the value of $\boldsymbol{x_4}$ decreases suggests that our approach can also account for the effect of the absolute desirability of the available options on the probability of choice deferral. Some simple analysis shows that this is indeed the case. Even though the proposed model features lateral inhibition, accumulation in our model is not completely relativistic: higher rates of accumulation and subsequently higher preferences can be obtained by increasing the absolute desirability of available options while keeping their relative desirability con-

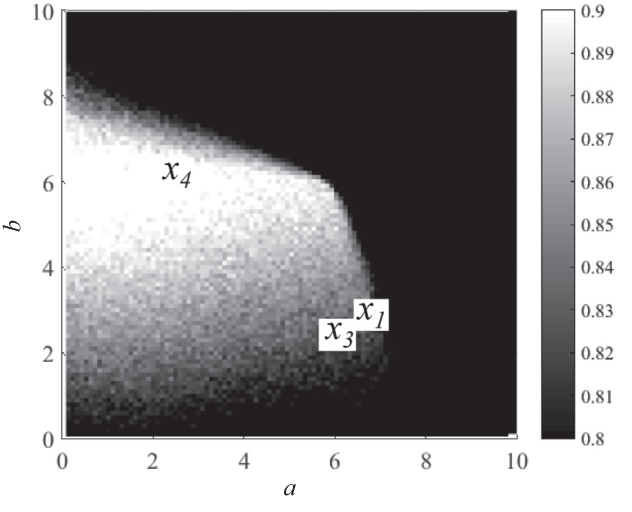

Fig. 4. The effect of varying placements of a choice option on deferral probability. The shade of each point (a, b) in this figure represents the deferral probability generated by the choice set $\{\boldsymbol{x_1}, (a, b)\}$.

stant. Due to the deferral-based time limit, deferral is less likely to occur when accumulation rates are high, and is thus less likely to occur when the choice set contains highly desirable items. Indeed using the choices outlined in Fig. 1, we find that choice is deferred more frequently in the set $\{\boldsymbol{x_3}, \boldsymbol{x_4}\}$, compared to the set $\{\boldsymbol{x_1}, \boldsymbol{x_2}\}$, with $\Pr(D|\boldsymbol{x_3}, \boldsymbol{x_4}, D) = 0.97 > \Pr(D|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = 0.85$. Additionally, as expected, we obtain $\Pr(\boldsymbol{x_3}|\boldsymbol{x_3}, \boldsymbol{x_4}, D) = \Pr(\boldsymbol{x_4}|\boldsymbol{x_3}, \boldsymbol{x_4}, D)$ and $\Pr(\boldsymbol{x_1}|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = \Pr(\boldsymbol{x_2}|\boldsymbol{x_1}, \boldsymbol{x_2}, D)$.

As mentioned above, Fig. 4 does display this property. However, as this figure confounds both dominance and relative desirability (discussed below), it is not sufficient by itself to conclusively establish the effect of absolute desirability on choice. Fig. 5 remedies this problem. It displays the probability of deferral in the choice sets $\{k_1 \cdot (7, 3), k_2 \cdot (3, 7)\}$, where k_1 and k_2 are varied in increments of 0.01 from 0.5 to 1.5. Each point in this figure captures the probability of deferring choice for corresponding coordinate values of (k_1, k_2). Each ray out of the origin can be seen as representing a collection of choice sets featuring the same relative desirability of the options, that is featuring a constant ratio k_1/k_2. Points further away from the origin on each ray have a higher average desirability, keeping this ratio fixed. Thus, for example, the ray leaving the origin at a 45-deg angle represents choice sets with equally desirable options (i.e. $k_1 = k_2$). Choice sets further up on this ray have higher values of k_1 and k_2, and thus higher absolute desirability of the options.

As can be seen in Fig. 5, moving up a ray always leads to darker shades in the figure, showing that choice sets with higher desirability, keeping relative desirability fixed, are associated with lower rates of choice deferral. Thus the deferral probability in {(10.5, 4.5), (4.5, 10.5)} is 0%, the deferral probability in $\{(7, 3), (3, 7)\} = \{\boldsymbol{x_1}, \boldsymbol{x_2}\}$ is 85%, and the deferral probability in {(3.5, 1.5), (1.5, 3.5)} is 100%.

This mechanism, unlike the dominance effect shown above, does not need an associative attention mechanism, and can be generated by the more restrictive LCA model which we obtain by setting $a_0 = \infty$. For example, in such a model we obtain choice probabilities $\Pr(D|\boldsymbol{x_3}, \boldsymbol{x_4}, D) = 0.91 > \Pr(D|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = 0.72$, which are consistent with the absolute desirability effect. Also note that lateral inhibition has a detrimental influence on this effect, and that this effect is slightly stronger when inhibition is completely absent, and the model is simplified into a leaky race model. Particularly, with both $a_0 = \infty$ and $l = 0$, we have $\Pr(D|\boldsymbol{x_3}, \boldsymbol{x_4}, D) = 0.92 > \Pr(D|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = 0.71$. Lastly, the absolute desirability effect is not sensitive to T and Q being equal, and can emerge with different values for these parameters. For example, with $T = 12$ and $Q = 10$, but $a_0 = 10$ and $l = 0.01$, we obtain $\Pr(D|\boldsymbol{x_3}, \boldsymbol{x_4}, D) = 0.75 > \Pr(D|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = 0.36$. Again, these seem like more reasonable deferral probabilities compared to if $T = 10$.

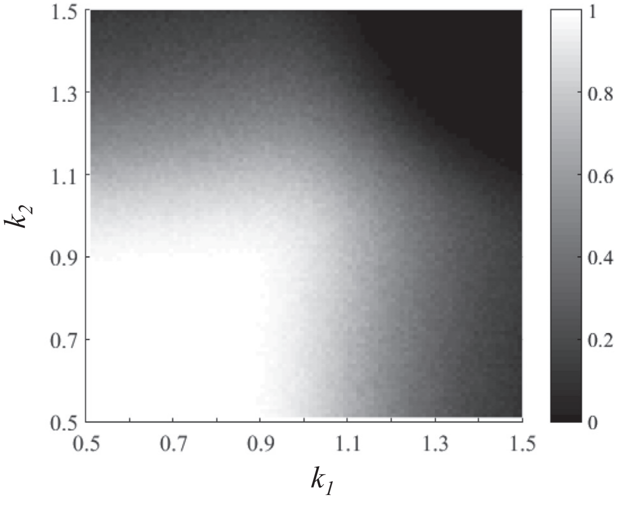

Fig. 5. The probability of deferral in the choice sets $\{k_1 \cdot (7, 3), k_2 \cdot (3, 7)\}$, where k_1 and k_2 are varied in increments of 0.01 from 0.5 to 1.5. Each point in this figure captures the probability of deferring choice for corresponding coordinate values of (k_1, k_2).

4.3. Effect 3: relative desirability

The proposed model is also able to predict the emergence of the relative desirability effect. Particularly, decisions are more likely to be deferred if the available options differ greatly in their desirability compared to if they do not, keeping absolute desirability constant. This is due to the fact that the model features lateral inhibition, which means that accumulation is partially relativistic. Although inhibition does reduce the absolute desirability effect described above, it also leads to preferences in choice sets with equally desirable options inhibiting each other, slowing down each other's accumulation rate and increasing the probability that the deferral time limit is reached without the decision threshold having been crossed. We can illustrate this insight using the choices in Fig. 1. With a simple series of simulations, we find that choice is deferred more frequently in the choice set $\{\boldsymbol{x_1}, \boldsymbol{x_2}\}$, in which the two options are equally desirable, compared to the choice set $\{\boldsymbol{x_4}, \boldsymbol{x_5}\}$ in which the two options have the same average desirability but differ in their relative desirability. Particularly, we have $\Pr(D|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = 0.85 > \Pr(D|\boldsymbol{x_4}, \boldsymbol{x_5}, D) = 0.77$. Additionally, as expected, we have $\Pr(\boldsymbol{x_1}|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = \Pr(\boldsymbol{x_2}|\boldsymbol{x_1}, \boldsymbol{x_2}, D)$, but $\Pr(\boldsymbol{x_5}|\boldsymbol{x_4}, \boldsymbol{x_5}, D) > \Pr(\boldsymbol{x_4}|\boldsymbol{x_4}, \boldsymbol{x_5}, D)$.

Fig. 6 transforms the data used in Fig. 5 to show this more rigorously. Recall that Fig. 5 displayed the probability of deferral in the choice set $\{k_1 \cdot (7, 3), k_2 \cdot (3, 7)\}$, as a function of k_1 and k_2. Fig. 6 presents the same data, but as a function of the mean desirability, $[k_1 \cdot (7 + 3) + k_2 \cdot (3 + 7)]/2$, and absolute difference in the difference in desirability of the options, $|k_1 \cdot (7 + 3) - k_2 \cdot (3 + 7)|$. As can be seen in this figure, increasing the relative desirability in the choice set, keeping the mean desirability of the options constant, always reduces the probability of deferral. Thus, for example, in settings with a mean option desirability of 10, choice is deferred 86% of the time when the difference in desirability is 0, and deferred 10% of the time when the difference in desirability is 10.

Note that in addition to inhibition, there is one other force that is driving the above results. The proposed model is sensitive to relative desirability, partially due to the fact that the relative desirability of the options in a choice set is closely associated with the maximum desirability of an option in the choice set, when absolute desirability is held constant. Choice sets that have large differences in option desirability also have one option which is highly desirable (and another which is much less desirable). In contrast, controlling for average option desirability, choice sets that have small differences in option

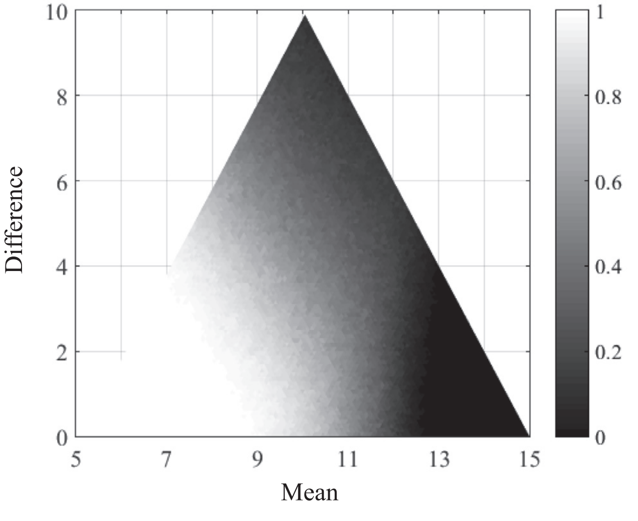

Fig. 6. Transformed data from Fig. 5, to show the probability of deferral as a function of the mean desirability, $[k_1 \cdot (7 + 3) + k_2 \cdot (3 + 7)]/2$, and absolute difference in the difference in desirability of the options, $|k_1 \cdot (7 + 3) - k_2 \cdot (3 + 7)|$.

desirability typically feature two moderately desirable options. More formally, if the total values of four options are written as positive numbers a, b, c, and d, with $a + b = c + d$ and $|a - b| > |c - d|$, then it must be the case that $\max\{a, b\} > \max\{c, d\}$. Now, in order for choice not to be deferred it is sufficient that a single option crosses the threshold Q before the time limit. In the proposed model, it is more likely that Q is crossed in a choice set where one option is highly desirable and the other is weakly desirable, compared to a choice set in which both options are moderately desirable. Again, as deferral probability is related to the probability with which Q is crossed, this implies that deferral is less likely in choice sets with large desirability differences.

Would we obtain the effect of relative desirability even if we keep the maximum desirability of an option fixed, instead allowing the absolute desirability of the choice set to vary? Consider, for example, a choice between $\boldsymbol{x_1}$ and $k \cdot \boldsymbol{x_2}$. Here if $k = 1$, then we obtain our standard choice set $\{\boldsymbol{x_1}, \boldsymbol{x_2}\}$, with $\Pr(D|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = 0.85$. Now if we make k smaller, then the relative desirability between $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ increases but the average desirability of the two options decreases. Dhar (1997) has found that choice deferral can increase in these settings, despite the fact that the absolute desirability effect predicts otherwise. Our model can generate this behavior due to inhibition. For example, if $k = 0.1$, then for the parameters we have outlined above, we obtain $\Pr(D|\boldsymbol{x_1}, k \cdot \boldsymbol{x_2}, D) = 0.81 < \Pr(D|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = 0.85$, demonstrating this effect. More generally, however, lower values of l or higher values of k can reduce the strength of the effect. Indeed, the effect completely disappears if $l = 0$, with $\Pr(D|\boldsymbol{x_1}, k \cdot \boldsymbol{x_2}, D) = 0.77 > \Pr(D|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = 0.71$, implying that inhibition is necessary to explain the emergence of the relative desirability effect.

Finally, note that this mechanism also does not need an associative attention mechanism, and can be generated by the more restrictive non-associative LCA model which we obtain by setting $a_0 = \infty$ (but keeping $l = 0.01$). Here again we have $\Pr(D|\boldsymbol{x_1}, k \cdot \boldsymbol{x_2}, D) = 0.81 < \Pr(D|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = 0.85$. In a similar manner, varying T to have $T = 12$ (with $Q = 10$, as well as $a_0 = 10$ and $l = 0.01$) gives us $\Pr(D|\boldsymbol{x_1}, k \cdot \boldsymbol{x_2}, D) = 0.38 < \Pr(D|\boldsymbol{x_1}, \boldsymbol{x_2}, D) = 0.45$.

4.4. Effect 4: common and unique attributes

Sequential attribute sampling models act as if they overweigh large attribute differences by increasing choice probabilities of options that are especially strong on these differences (though once again, these models don't feature any explicit assumptions about weighting biases). Because of this, unique attributes typically play a more important role in choice, relative to common attributes (again, see Bhatia, 2013; Roe et al., 2001). If these unique attributes are bad, then preferences will accumulate at a slower rate, compared to if these unique attributes are good. Though this mechanism is typically used to explain how varying common vs. unique attributes affects violations of stochastic transitivity, this change to the rate of accumulation can also predict a higher deferral probability in the presence of unique bad vs. good attributes. Indeed with the options $\boldsymbol{x_8} = (7, 7, 3, 0)$, $\boldsymbol{x_9} = (7, 7, 0, 3)$, $\boldsymbol{x_{10}} = (14, 0, 1.5, 1.5)$, and $\boldsymbol{x_{11}} = (0, 14, 1.5, 1.5)$, introduced above (in which attributes 3 and 4 are undesirable), we find that $\Pr(D|\boldsymbol{x_8}, \boldsymbol{x_9}, D) = 0.92 > \Pr(D|\boldsymbol{x_{10}}, \boldsymbol{x_{11}}, D) = 0.85$. Additionally, as this mechanism still involves the evaluation of common attributes, changing these common attributes, by using options $\boldsymbol{x_{12}} = (10, 10, 3, 0)$, and $\boldsymbol{x_{13}} = (10, 10, 0, 3)$, instead of $\boldsymbol{x_8} = (7, 7, 3, 0)$, $\boldsymbol{x_9} = (7, 7, 0, 3)$, gives us a lower rate of deferral. Particularly, we find that $\Pr(D|\boldsymbol{x_8}, \boldsymbol{x_9}, D) = 0.92 > \Pr(D|\boldsymbol{x_{12}}, \boldsymbol{x_{13}}, D) = 0.53$. This is not sensitive to the precise values of T so that setting $T = 12$ gives us $\Pr(D|\boldsymbol{x_8}, \boldsymbol{x_9}, D) = 0.83 > \Pr(D|\boldsymbol{x_{10}}, \boldsymbol{x_{11}}, D) = 0.74 > \Pr(D|\boldsymbol{x_{12}}, \boldsymbol{x_{13}}, D) = 0.34$. Finally, as expected, we obtain $\Pr(\boldsymbol{x_8}|\boldsymbol{x_8}, \boldsymbol{x_9}, D) = \Pr(\boldsymbol{x_9}|\boldsymbol{x_8}, \boldsymbol{x_9}, D)$ and $\Pr(\boldsymbol{x_{10}}|\boldsymbol{x_{10}}, \boldsymbol{x_{11}}, D) = \Pr(\boldsymbol{x_{11}}|\boldsymbol{x_{10}}, \boldsymbol{x_{11}}, D)$.

As the key mechanism responsible for this effect involves sequential attribute sampling, it can be generated by the more restrictive LCA model which we obtain by setting $a_0 = \infty$. For this parameter value we obtain $\Pr(D|\boldsymbol{x_8}, \boldsymbol{x_9}, D) = 0.90 > \Pr(D|\boldsymbol{x_{10}}, \boldsymbol{x_{11}}, D) = 0.85$ and $\Pr(D|\boldsymbol{x_8}, \boldsymbol{x_9}, D) = 0.90 > \Pr(D|\boldsymbol{x_{12}}, \boldsymbol{x_{13}}, D) = 0.48$. We obtain similar patterns in deferral probability even if we further restrict $l = 0$, with $\Pr(D|\boldsymbol{x_8}, \boldsymbol{x_9}, D) = 0.98 > \Pr(D|\boldsymbol{x_{10}}, \boldsymbol{x_{11}}, D) = 0.94$ and $\Pr(D|\boldsymbol{x_8}, \boldsymbol{x_9}, D) = 0.98 > \Pr(D|\boldsymbol{x_{12}}, \boldsymbol{x_{13}}, D) = 0.84$, showing that inhibition is also not necessary for this effect. Ultimately it is possible to model the influence of common vs. unique attributes in a simple leaky race model that samples attributes sequentially without inhibition or associative attention.

4.5. Effect 5: alignability

The deferral-based time limit, implemented with AAM, is able to capture the alignability effect in a manner similar to the dominance effect. As discussed in Bhatia (2013), the associative bias proposed by the AAM model can explain why decision makers place a higher weight on alignable attributes relative to non-alignable attributes. Particularly, since attribute activation is an increasing function of the amount of the attribute in the choice options, alignable attributes, which are present in multiple options, receive a higher activation, and subsequently a higher weight. This leads to alignable attributes being sampled more frequently, and choice options with alignable attributes accumulating and crossing the threshold more quickly. Choice sets with a high number of alignable attributes will thus have faster threshold crossing and a correspondingly low rate of deferral. When choice sets are modified so that fewer attributes are alignable, there is dispersion in the sampling probabilities of the underlying attributes. This reduces the rate of accumulation for all choice options, increasing the probability that thresholds are not crossed by the deferral time limit. It is because of this that there is lower deferral in the choice between $\boldsymbol{x}_{14}$ = (7, 7, 3, 0) and $\boldsymbol{x}_{15}$ = (7, 3, 7, 0), than in the choice between $\boldsymbol{x}_{14}$ = (7, 7, 3, 0) and $\boldsymbol{x}_{16}$ = (0, 3, 7, 7), with $\Pr(D|\boldsymbol{x}_{14}, \boldsymbol{x}_{16}, D) = 0.88$ and $\Pr(D|\boldsymbol{x}_{14}, \boldsymbol{x}_{15}, D) = 0.78$. Additionally, as expected, we obtain $\Pr(\boldsymbol{x}_{14}|\boldsymbol{x}_{14}, \boldsymbol{x}_{15}, D) = \Pr(\boldsymbol{x}_{15}|\boldsymbol{x}_{14}, \boldsymbol{x}_{15}, D)$ and $\Pr(\boldsymbol{x}_{14}|\boldsymbol{x}_{14}, \boldsymbol{x}_{16}, D) = \Pr(\boldsymbol{x}_{16}|\boldsymbol{x}_{14}, \boldsymbol{x}_{16}, D)$.

This dependence of attribute sampling on the composition of the choice set is a function of the parameter a_0. For high values of this parameter, attribute attention is independent of the choice set, and the above alignability biases disappear. This is shown in Fig. 7, which plots the difference in the deferral probabilities in the choice set $\{\boldsymbol{x}_{14}, \boldsymbol{x}_{16}\}$ compared to the choice set $\{\boldsymbol{x}_{14}, \boldsymbol{x}_{15}\}$, that is, $\Pr(D|\boldsymbol{x}_{14}, \boldsymbol{x}_{16}, D) - \Pr(D|\boldsymbol{x}_{14}, \boldsymbol{x}_{15}, D)$. Positive values in this figure correspond to the emergence of the alignability effect. As can be seen in Fig. 7, this effect is likely to emerge when a_0 is small. This effect disappears as a_0 increases and the strength of the associative mechanisms in AAM weaken, and it cannot be predicted by a restricted LCA type model in which the associative mechanisms of AAM are completely inactive.

Finally note that this effect can emerge with different values of T, so that with $T = 12$ we obtain $\Pr(D|\boldsymbol{x}_{14}, \boldsymbol{x}_{16}, D) = 0.72$ and $\Pr(D|\boldsymbol{x}_{14}, \boldsymbol{x}_{15}, D) = 0.53$. It is, however, sensitive to the specific attributes on the alignable dimensions, so that it can be weakened by changing these values while keeping the alignability of the dimensions constant.

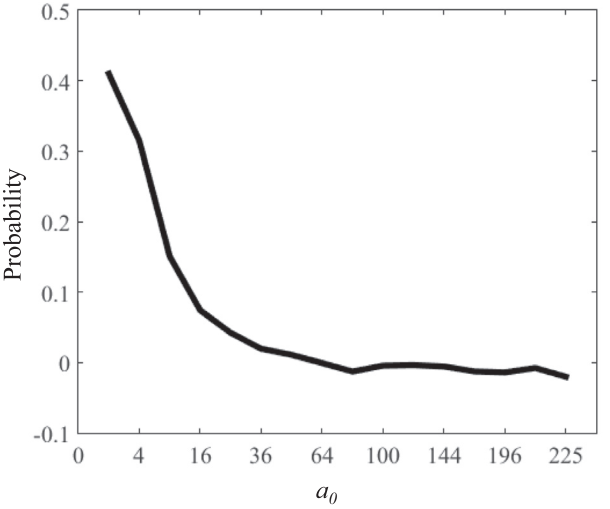

Fig. 7. The effect of the associative bias parameter, a_0, on the strength of the alignability effect. The line plots the difference in the probability of deferral in the choice set $\{\boldsymbol{x}_{14}, \boldsymbol{x}_{16}\}$ compared to the choice set $\{\boldsymbol{x}_{14}, \boldsymbol{x}_{15}\}$.

4.6. Effect 6: extreme options

Our approach is able to capture the effect of choice deferral on the choice probabilities of all-average vs. extreme options due to the stochastic sequential sampling of attributes, which introduces a time dependence in the accumulation of preference. At earlier time periods, when few attributes have been sampled, choice is more likely to be influenced by attributes with large magnitudes, and the bias favoring the extreme option is particularly strong. This happens because samples of an attribute on which an option is particularly extreme can, due to the magnitude of values on this attribute, lead to a threshold being crossed (this is less likely to happen with random samples of an attribute on which the option values are moderate). As a result of this, the extreme option is highly likely to be chosen early on in the decision process. As time progresses preferences asymptote towards a stable point, which is a simple linear function of the total values of choice option attributes (modified by decay and inhibition). Here the extremity of the options does not matter. As a result, all-average options are more likely to be selected later on in the decision process compared to earlier on in the decision process. To put it more intuitively, random attention has a stronger effect early on in the decision process, where it can bias choice in favor of extreme options, compared later on in the decision process. Bhatia (2013) discusses this property in detail and uses it to explain time dependence in alignability effect tasks.

As allowing for deferral creates a bias in favor of the options that are highly preferred early on in the choice process, we observe a lower choice probability for all-average options in the presence of deferral, compared to when deferral is not allowed. Indeed, using the choice options introduced above and displayed in Fig. 1, we find that $\boldsymbol{x}_6$ is never selected in the presence of deferral, but that it is selected 37% of the time in the absence of deferral, that is Rel($\boldsymbol{x}_6$, $\boldsymbol{x}_7$|$\boldsymbol{x}_6$, $\boldsymbol{x}_7$) = 0.37 > Rel($\boldsymbol{x}_6$, $\boldsymbol{x}_7$|$\boldsymbol{x}_6$, $\boldsymbol{x}_7$, D) = 0. Additionally, we have Pr(D|$\boldsymbol{x}_6$, $\boldsymbol{x}_7$, D) = 0.63.

To further explore the relationship between deferral and decision time, consider Fig. 8. The vertical axis in Fig. 8 presents the relative choice proportion of $\boldsymbol{x}_6$ compared to $\boldsymbol{x}_7$, Rel($\boldsymbol{x}_6$, $\boldsymbol{x}_7$|$\boldsymbol{x}_6$, $\boldsymbol{x}_7$, D), as well as the probability of deferral. The horizontal axis is the deferral time limit T. As T is increased, decision makers have more time to make their choice, and the relative choice proportion of $\boldsymbol{x}_6$ increases. After $T = 15$, we find that this proportion stabilizes at around 40%, and the probability of deferral similarly stabilizes at 0%. This explicitly demonstrates that the all-average option is more likely to be selected later on in the decision process, with the highest choice probability if the deferral time limit is

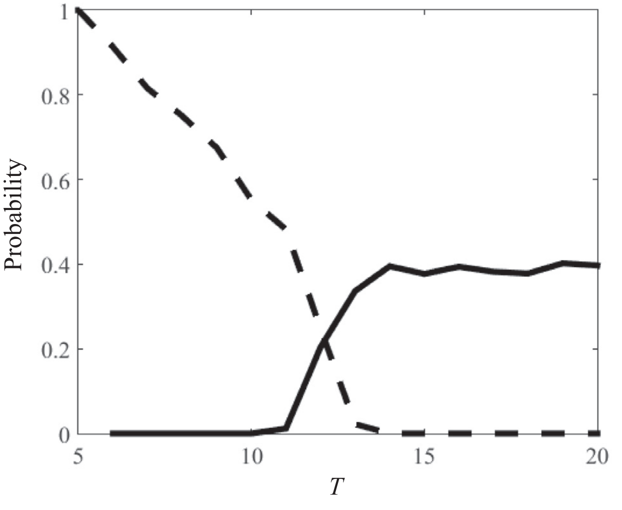

Fig. 8. The effect of the deferral time limit, T, on the extreme options effect. Here the solid line plots Rel($\boldsymbol{x}_6$, $\boldsymbol{x}_7$|$\boldsymbol{x}_6$, $\boldsymbol{x}_7$, D) and the dashed line plots Pr(D|$\boldsymbol{x}_6$, $\boldsymbol{x}_7$, D).

especially large (or equivalently, completely absent). Note that choice proportions for $T < 5$ are not displayed, as neither of the options are chosen for these values of T, and the relative choice proportion is not defined. As with the common and unique attributes effects, the relationship between the choice of extreme options and the presence of deferral is primarily a product of sequential attribute sampling, and does not depend on the associative mechanisms of AAM. Thus we can observe the above effect even if $a_0 = \infty$, and our model reduces to the LCA. Indeed as this associative mechanism generates a bias in favor of extreme options in the absence of deferral (see Bhatia, 2013), we find that this extremity effect is enhanced with the LCA model, in which the choice probability of the all-average option in the absence of deferral is higher than in the AAM model. For example when $a_0 = \infty$, we find that $\boldsymbol{x_6}$ is never selected in the presence of deferral, but that it is selected 64% of the time in the absence of deferral, that is $\text{Rel}(\boldsymbol{x_6}, \boldsymbol{x_7}|\boldsymbol{x_6}, \boldsymbol{x_7}) = 0.64 > \text{Rel}(\boldsymbol{x_6}, \boldsymbol{x_7}|\boldsymbol{x_6}, \boldsymbol{x_7}, \text{D}) = 0$. The extreme options effect also emerges when we further restrict $l = 0$, with $\text{Rel}(\boldsymbol{x_6}, \boldsymbol{x_7}|\boldsymbol{x_6}, \boldsymbol{x_7}) = 0.66 > \text{Rel}(\boldsymbol{x_6}, \boldsymbol{x_7}|\boldsymbol{x_6}, \boldsymbol{x_7}, \text{D}) = 0$, implying that inhibition is also not necessary for this effect.

4.7. Effect 7: asymmetric dominance

AAM generates the asymmetric dominance effect due to associative attentional weights: the addition of the novel option increases the attention towards its primary attribute, subsequently biasing choice in favor of the initial options that are strongest on this attribute. Sequential sampling imposes time dependence, leading to a higher increase in the preferences for options that are strongest on the most sampled attribute, early on in the decision process. With the asymmetric dominance effect, the presence of the decoy increases the sampling probability of the dominant option's primary attribute, making this option seem especially desirable at early periods. The dominant option is thus more likely than its competitor to cross the choice thresholds before the deferral time limit than it is to do so at later time periods. This leads to a higher asymmetric dominance effect in the presence of deferral. It is important to note that intuitively this is the same mechanism as that underlying Effect 6, but unlike in Effect 6, the associative attentional mechanisms for this effect create a bias in favor of the dominating option early on in the decision.

Taking the choice options shown in Fig. 1, we find that that the relative choice probability of $\boldsymbol{x_1}$ over $\boldsymbol{x_2}$, from the set $\{\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\}$, is higher in the presence of deferral than in the absence of deferral. More specifically we have $\text{Rel}(\boldsymbol{x_1}, \boldsymbol{x_2}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \text{D}) = 0.71 > \text{Rel}(\boldsymbol{x_1}, \boldsymbol{x_2}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}) = 0.59$. As $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ are symmetric on identical attributes their relative choice probabilities in the absence of the decoy are always 50%, regardless of deferral (subsequently, since both $\text{Rel}(\boldsymbol{x_1}, \boldsymbol{x_2}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \text{D}) > 0.5$ and $\text{Rel}(\boldsymbol{x_1}, \boldsymbol{x_2}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}) > 0.5$ we are observing the main asymmetric dominance effect both with and without deferral, though it is stronger with deferral). Additionally, we observe $\Pr(\boldsymbol{x_3}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \text{D}) = \Pr(\boldsymbol{x_3}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}) = 0$, as the decoy option is never chosen.

These results are illustrated in Fig. 9a, which as in Fig. 8, presents choice probabilities as a function of the deferral time limit T. Specifically the vertical axis plots both $\text{Rel}(\boldsymbol{x_1}, \boldsymbol{x_2}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \text{D})$ and $\Pr(\text{D}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \text{D})$. We find that the strength of the asymmetric dominance effect decreases with increase in the deferral time limit, though it is always predicted to emerge, regardless of the value of this limit. Again this effect stabilizes around $T = 15$, with $\text{Rel}(\boldsymbol{x_1}, \boldsymbol{x_2}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \text{D}) = 0.59$. At this point the probability of deferral is 0% and we have $\text{Rel}(\boldsymbol{x_1}, \boldsymbol{x_2}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \text{D}) = \text{Rel}(\boldsymbol{x_1}, \boldsymbol{x_2}|\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3})$. This explicitly demonstrates that the dominant option is more likely to be selected early on in the decision process, with the highest choice probability if the deferral time limit is especially small.

Not surprisingly the above effect disappears if the associative mechanism in the proposed model is disabled. This is illustrated in Fig. 9b, which is identical to Fig. 9a except for the fact that we have $a_0 = \infty$. As the associative bias in AAM is necessary to generate the asymmetric dominance effect, we fail to observe both this effect, and its relationship in deferral in Fig. 9b. Particularly, the relative choice probability of the dominant option remains around 50% independently of the deferral time limit T. In contrast Fig. 9a showed that this choice probability decreased with T but nonetheless remained consistently above 50%.

Finally, prior work has found that the asymmetric dominance effect weakens with externally imposed time limits (Pettibone, 2012; Trueblood et al., 2014), and the initial AAM model is able to capture this effect, as shown in Bhatia (2013). This is compatible with the result presented in this section

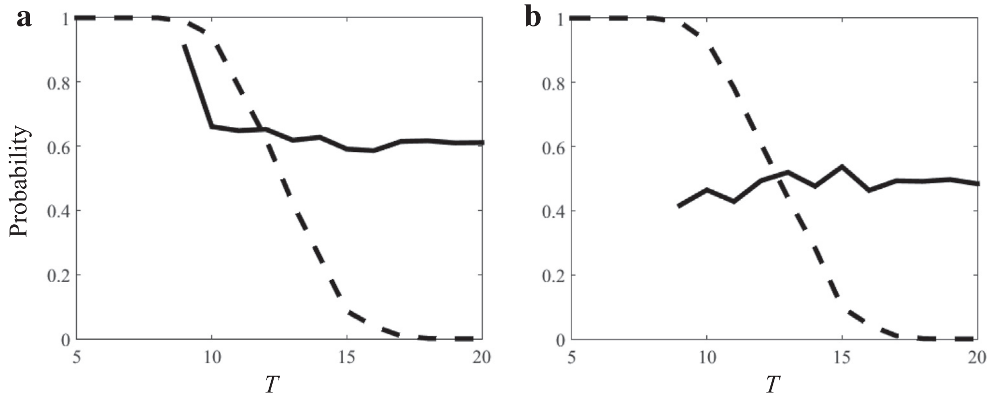

Fig. 9. The effect of the deferral time limit, T, on the asymmetric dominance effect. In the left panel (a) the solid line plots Rel($\boldsymbol{x_1}$, $\boldsymbol{x_2}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_3}$, D) and the dashed line plots Pr(D|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_3}$, D) when the associative mechanism is enabled. In the right panel (b) the solid line plots Rel($\boldsymbol{x_1}$, $\boldsymbol{x_2}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_3}$, D) and the dashed line plots Pr(D|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_3}$, D) when the associative mechanism is disabled.

(which shows that the asymmetric dominance effect strengthens with deferral-based time limits), as the findings involving externally imposed time limits do not allow for choice deferral. More specifically, the simulations in Bhatia (2013) assume that decisions with externally imposed time limits aggregate attribute values until the time limit is crossed, at which point they select their most preferred option. If we implement this mechanism with the above parameters, we find that the asymmetric dominance does indeed weaken with externally imposed time limits, without deferral (consistent with the findings in Pettibone, 2012; Trueblood et al., 2014 and the simulations in Bhatia, 2013). The reason this happens is due to the difference between threshold crossing probabilities and modal preference rankings. In the choice from {$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_3}$}, $\boldsymbol{x_1}$ is proportionally more likely to cross Q than $\boldsymbol{x_2}$ at earlier time periods. This is despite the fact that the probability that the preference for $\boldsymbol{x_1}$ is higher than the preference for $\boldsymbol{x_2}$ is lower for earlier time periods relative to later time periods.

4.8. Effect 8: compromise

The influence of deferral on the strength of the compromise effect is explained by our approach using both associative attentional weights and sequential attribute sampling. As with the asymmetric dominance effect described above, associative connections increase the attentional probability of the decoy's primary attribute, and sequential sampling imposes a time dependence that leads to a higher increase in the preferences for options that are strongest on this attribute, early on in the decision process. With the asymmetric dominance effect it is the dominating option that is the strongest on the most sampled attribute, and thus the target option that is most likely to be chosen earlier in the decision. With the compromise effect, however, it is the extreme decoy option that is strongest on the most sampled attribute in the presence of the decoy. This extreme option disproportionality competes with the compromise option, reducing its choice probability. As in the all-average and extreme options in the above section, this competitive effect happens mainly at early time periods. As a result of this, the compromise effect is weakened at early time periods, and is thus less likely to emerge when deferral is a possibility. Taking the choice options shown in Fig. 1, we find that that the relative choice probability of $\boldsymbol{x_1}$ over $\boldsymbol{x_2}$, from the set {$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$}, is lower in the presence of deferral than in the absence of deferral. More specifically we have Rel($\boldsymbol{x_1}$, $\boldsymbol{x_2}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$) = 0.60 > Rel($\boldsymbol{x_1}$, $\boldsymbol{x_2}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$, D) = 0. Again note that the relative choice probabilities of $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ are 50% in {$\boldsymbol{x_1}$, $\boldsymbol{x_2}$}, regardless of deferral, as the options are symmetric on equally desirable attributes. The fact that Rel($\boldsymbol{x_1}$, $\boldsymbol{x_2}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$) > 0.5 > Rel ($\boldsymbol{x_1}$, $\boldsymbol{x_2}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$, D) thus indicates that we are getting the compromise effect in the absence of deferral but that it is reversed when deferral is allowed. Additionally, we observe a higher overall choice probability for the decoy in the absence of deferral, with Pr($\boldsymbol{x_7}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$) = 0.36 > 0.26 = Pr($\boldsymbol{x_7}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$, D), though this is merely due to the fact the deferral probability is zero in the absence of deferral. Overall, in a manner consistent with Effect 5, we find that Rel($\boldsymbol{x_1}$, $\boldsymbol{x_7}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$) > Rel($\boldsymbol{x_1}$, $\boldsymbol{x_7}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$, D) and Rel($\boldsymbol{x_2}$, $\boldsymbol{x_7}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$) > Rel($\boldsymbol{x_2}$, $\boldsymbol{x_7}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$, D).

We can observe the above effects in more detail in Fig. 10a which plots both Rel($\boldsymbol{x_1}$, $\boldsymbol{x_2}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$, D) and Pr(D|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$, D) as a function of *T*. We find that the strength of the compromise effect decreases with a decrease in the deferral time limit, and can actually reverse when the time limit is especially low. Ultimately the compromise effect emerges for a large enough deferral time limit. For this time limit, the probability of deferral is zero and corresponding choice probabilities of options $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ are equal to choice probabilities in the absence of deferral. Fig. 10b shows that as with the asymmetric dominance effect, the compromise effect also relies on the associative mechanism in AAM. Although we do obtain an increase in the choice probability of the compromise option in the absence of deferral when $a_0 = \infty$, this probability is unable to cross 50% and thus a restricted LCA model without associative attention is unable to generate the compromise effect.

4.9. A note on decision difficulty

Some of the effects described above, particularly Effects 1–6, could be understood in terms of difficulty, in that harder choices lead to increased deferral. Indeed, a number of scholars have reported a positive association between perceptions of decision difficulty and the incidence of choice deferral (e.g. Novemsky, Dhar, Schwarz, & Simonson, 2007). Although decision difficulty within the types of tasks accumulator models are typically applied to is studied in terms of discriminability, and subsequently the relative choice probabilities of options, here we could adopt a broader definition of difficulty in terms of not only the ability of the decision makers to determine which option is better than the other, but also the ease in making tradeoffs between the options, and the desirability of the available options. The role of tradeoffs in decision difficulty has been studied extensively in multiattribute choice (e.g. Chatterjee & Heath, 1996; Luce, Bettman, & Payne, 2001). Additionally, the relationship between difficulty and the overall desirability of the options is part of many lay definitions of difficulty. After all, the choice between a rock and a hard place, as in the common idiom, is considered difficult largely because both rocks and hard places are relatively undesirable.

It seems that this broader notion of difficulty can provide an intuitive unifying principle regarding the types of settings in which decisions take longer in the absence of deferral, and thus decisions in which choice is likely to be deferred when deferral is allowed. Effects 1 and 3 can be understood in

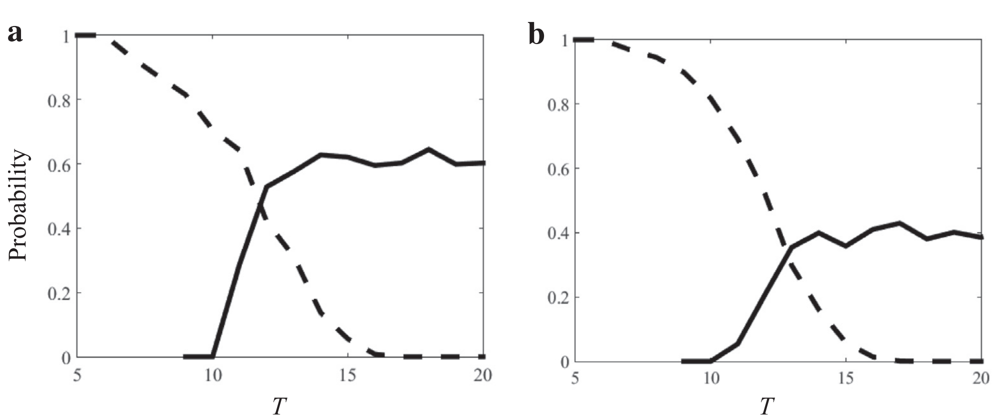

Fig. 10. The effect of the deferral time limit, *T*, on the compromise effect. In the left panel (a) the solid line plots Rel($\boldsymbol{x_1}$, $\boldsymbol{x_2}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$, D) and the dashed line plots Pr(D|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$, D) when the associative mechanism is enabled. In the right panel (b) the solid line plots Rel($\boldsymbol{x_1}$, $\boldsymbol{x_2}$|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$, D) and the dashed line plots Pr(D|$\boldsymbol{x_1}$, $\boldsymbol{x_2}$, $\boldsymbol{x_7}$, D) when the associative mechanism is disabled.

terms of discriminability and subsequently relative choice probability: For both these effects the relative choice probabilities between the options in the absence of deferral are higher in choice sets where deferral is less likely. Effect 2 relates closely to the absolute desirability of the chosen option, and shows how changing the absolute desirability of the choice set can alter deferral. Finally, Effects 4 and 5 can be seen as involving different types of tradeoffs (that is, degrees of overlap between the attributes of the choice options), while keeping both absolute and relative desirabilities constant.

We tested the broad explanatory scope of this account of difficulty by randomly generating 1000 choice pairs and examining the relationship between our three components of decision difficulty and deferral probability in these choice pairs. Our choice pairs involved two-attribute options, with each attribute amount being drawn from an independent uniform distribution from the range [5, 15]. For each choice pair $\{\boldsymbol{x_i}, \boldsymbol{x_j}\}$ we examined the absolute difference in the choice probabilities of the options in the absence of deferral, $|\Pr(\boldsymbol{x_i}|\boldsymbol{x_i}, \boldsymbol{x_j}) - \Pr(\boldsymbol{x_j}|\boldsymbol{x_i}, \boldsymbol{x_j})|$, in order to capture discriminability; the total amounts of the attributes in the pair, $x_{i1} + x_{i2} + x_{j1} + x_{j2}$, in order to capture desirability; and the cosine similarity of the choice option vectors, $\boldsymbol{x_i} \cdot \boldsymbol{x_j}/[||\boldsymbol{x_i}|| \cdot ||\boldsymbol{x_j}||]$, in order to capture tradeoffs. We then ran a simple linear regression to predict deferral probability (a continuous variable, obtained from our simulations) in our 1000 randomly generated choices, using these three variables. We found that all three effects were significantly associated with deferral probability, so that choice pairs with large differences in choice probabilities in the absence of deferral, with large absolute amounts of attributes, and with large cosine similarities, had the lowest deferral probabilities ($p < 0.001$ for all three variables).

4.10. Summary

In this section we have discussed how the composition of the choice set can influence the probability of deferral and additionally how allowing choice to be deferred can alter underlying choice probabilities, in the proposed model. Both these properties stem from the deferral time limit mechanism instantiated within a sequential sampling model, which generates a time dependence in the choice task. Easier decisions, which are typically made quickly are associated with low deferral rates and objects that are chosen quickly are more likely to be chosen in the presence of deferral than in the absence of deferral. AAM presents one approach to modeling choice set dependence within a sequential sampling framework, and its dynamics, when coupled with this time limit mechanism, can explain the effects of dominance, absolute desirability, relative desirability, common vs. unique attributes, and alignable attributes on deferral (Effects 1–5), as well as the effect of deferral on the choice probabilities of extreme options, and on the strength of the asymmetric dominance and compromise effects (Effects 6–8). Note that although we have not explicitly shown the emergence of the choice overload effect (Chernev et al., 2015; Iyengar & Lepper, 2000; Scheibehenne et al., 2010), the proposed model is able to generate an increase in deferral probability as additional options are added to the choice set, if these additional options modify existing dominance relations, affect the absolute and relative desirability of the choice set, or alter which attributes are common, unique, and alignable in the choice set (see e.g. Chernev et al., 2015).

As shown, the associative attention mechanisms proposed by AAM are not necessary for all the effects. Indeed the absolute desirability, relative desirability, common vs. unique attributes, and extreme options effects emerge merely due to the dynamics of sequential attribute sampling and accumulation with leakage and inhibition, and thus can be obtained by a restricted LCA variant of the proposed model in which $a_0 = \infty$. Additionally, out of these four effects, only the relative desirability effect needs inhibition, and the remaining three can be obtained with a simple leaky race model which further simplifies AAM with $a_0 = \infty$ and $l = 0$. Ultimately, AAM is necessary only for describing deferral-based effects when these involves instances of dominance, compromise, and alignability, which are effects that AAM was generated to explain. In this sense its primary role within the proposed framework is as a back-end model, providing a theory of choice set dependence with which the interaction between deferral and choice set dependence, predicted by the deferral time limit, can be tested. A model that sequentially samples attributes and accumulates them with leakage and inhibition, and contains a time limit for deferral, is sufficient for effects not directly related to choice set dependence.

5. Novel predictions of the deferral time-limit

The deferral-based time limit proposed by Jessup et al. (2009) and studied in this paper also makes a number of novel predictions. These predictions pertain primarily to the time taken to make the decision in the presence of deferral, a variable that hasn't yet been experimentally examined in detail. In this section we outline four of these predictions, which are generated by the deferral-based time limit, independently of processes controlling preference accumulation and the influence of the choice set. These predictions are, in this sense, a property of all models with a deferral-based time limit, including the above AAM implementation, its restricted LCA variant obtained by setting $a_0 = \infty$, a further restricted race model with $l = 0$, and the model proposed by Jessup et al. (2009).

Just as we have been using $\Pr(\boldsymbol{x_i}|\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N})$, $\Pr(\boldsymbol{x_i}|\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N}, \mathrm{D})$, and $\Pr(\mathrm{D}|\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N}, \mathrm{D})$ to represent choice and deferral probabilities, we will use $\mathrm{DT}(\boldsymbol{x_i}|\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N})$, $\mathrm{DT}(\boldsymbol{x_i}|\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N}, \mathrm{D})$, and $\mathrm{DT}(\mathrm{D}|\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N}, \mathrm{D})$ to represent expected choice and deferral decision times. In most settings we will be interested in comparing the decision times associated with an active choice with the decision times associated with deferral, instead of considering the decision time associated with a specific choice option. Here we will use $\mathrm{DT}(\mathrm{C}|\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N})$ and $\mathrm{DT}(\mathrm{C}|\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_N}, \mathrm{D})$ to describe decision times associated with the selection of one of the N available options, in the absence and in the presence of deferral. Finally, when the predictions are independent of the composition of the choice set we will shorten the notation to DT(C|C), DT(C|C, D), DT(D|C, D), and DT(C or D|C, D), with DT(C|C) and DT(C or D|C, D) representing the overall decision times in the absence and presence of deferral.

5.1. Prediction 1

The first main prediction draws a relationship between the average decision time in the absence of deferral and the deferral probability. Recall that higher accumulation rates lead to lower rates of deferral, in the above model. This implies that choices in which deferral is likely should be associated with longer decision times in the absence of deferral, compared to similar choices in which deferral is unlikely. Essentially, since deferral is formalized as a time limit, a higher likelihood of crossing this time limit implies that choice would have taken longer if this time limit was not present. More formally, for any choice set C, DT(C|C) and Pr(D|C, D) should be positively associated with each other.

5.2. Prediction 2

The second main prediction of the deferral-based time limit compares the time to make an active choice in the presence of deferral with the time to make an active choice in the absence of deferral. Particularly, as the deferral time limit cuts off choice at a certain time, active choices made in the presence of deferral should, on average, be associated with lower decision times compared to the setting in which deferral is not allowed. More formally, for any choice set C, we should have DT(C|C, D) < DT(C|C).

Note the above logic does not necessarily give us DT(D|C, D) < DT(C|C). This can be understood in more detail by considering Prediction 1, which states that the average decision time in the absence of deferral is higher if the probability of deferral is greater. This implies that this decision time should be higher than the decision time associated with choosing deferral, when deferral is especially likely to be chosen. Indeed, when Pr(D|C, D) = 1 and deferral is always chosen, the deferral time limit will always be crossed before a decision would have been made in the absence of deferral, giving us DT(D|C, D) < DT(C|C). More generally, we should observe DT(D|C, D) < DT(C|C) if Pr(D|C, D) is high enough. In contrast, if Pr(D|C, D) is low enough, we may observe DT(D|C, D) > DT(C|C), as most active choices would be made before the deferral time limit is crossed.

5.3. Prediction 3

Although it may be the case that the time to defer choice when deferral is allowed is longer than the time to make an active choice when deferral is not allowed, it can never be the case that the overall decision time when deferral is allowed is longer than the overall decision time when it isn't. More formally, the proposed model predicts that for any choice set C, we should have DT(C or D | C, D) < DT(C | C). The logic for this is similar to that of Prediction 2: The deferral time limit cuts off choice at a certain time regardless of whether an active choice has been made. Thus the time taken to make a choice in the deferral condition is always less than the time to make the corresponding choice when deferral is not allowed.

5.4. Prediction 4

The final prediction compares the time associated with making an active choice to the time associated with deferring choice, in the presence of deferral. When deferral is allowed, active choices must always be made before the deferral time limit is crossed. If they are not made by this time limit, choice is deferred and the decision ends. This implies that active choices are quicker than deferred choices, in the presence of deferral, that is, DT(C | C, D) < DT(D | C, D).

6. Experimental test of novel predictions

We ran a novel experiment to test the four main predictions of the deferral time limit, discussed above. Our experiment obtained decision times and choice probabilities in binary choices with and without deferral. Our choice options were popular films, and participants were asked to select which of a pair of films they prefer, using a naturalistic experimental design. Prior to this choice task we also obtained participant ratings of the films, allowing us to control for film desirability effects.

6.1. Method and participants

58 participants (mean age = 21.43, 56% male) performed this experiment in a behavioral laboratory, for a monetary payment. Participants were required to choose which of two films they prefer. The films used in this experiment corresponded to the top 100 most voted-on films on the website www.imdb.com, and can thus be seen as representing the most popular films in contemporary cinema. These films were presented to the participants using images of their corresponding film posters, again obtained from www.imdb.com. The images corresponding to each available choice were presented side by side in each decision.

Participants were presented with two blocks, each consisting of the same 100 choices. In one block the choice between the two movies was forced, and participants had to make their selection using the left and right arrow keys (corresponding to the film presented on the left or right side of the screen). In the other block, the decision makers could click the left or right keys to make their choice, or else defer their choice by clicking the up arrow key. The instructions were created to avoid any suggestion of an explicit time limit (e.g. to suggest that participants should defer if they cannot decide quickly enough) or that deferral was a third comparable option (e.g. in the form of a status quo or default movie). More specifically, the instructions stated that if participants preferred the movie on the left/right then they should press the left/right arrow. If they could not make a decision about which of the two movies they preferred then they should press the up arrow instead. When subjects made a choice, for an item or deferral, the options disappeared and a fixation cross was presented for 1 s before the next choice was presented. There was no time out mechanism and participants could take as long as they wanted. The two blocks were presented sequentially, and the order that the two blocks were presented in, the order of their component choices, and the arrangement of the film posters within each choice trial (left or right), was randomized.

Before starting the choice task, participants rated each of the 100 films on a scale of 1–7, corresponding to how much they would like to watch each film. In the rating task, each film was presented separately.

6.2. Qualitative results

In the analysis in this section we have excluded all responses that took less than 0.2 s or greater than 10 s. This is 0.6% of our data. We use relatively high cutoff for outliers, compared to cutoffs typically used in low-level tasks (e.g. Ratcliff, 1993), due to the relative complexity of value-based decision making. Including outlier responses does not affect any results in this section.

6.2.1. Choice probabilities

We can first examine choice and deferral probabilities. When an active choice is made in the experiment, the option that has a higher rating is chosen 83% of the time, which is significantly different to 50%, when examined using a logistic regression with participant-level random intercepts ($\beta = 1.76$, $z = 17.25$, $p < 0.01$, 95% CI = [1.56, 1.96]). More generally, an option is more likely to be chosen the higher it is rated relative to its competitor ($\beta = 0.63$, $z = 47.73$, $p < 0.01$, 95% CI = [0.61, 0.66]). We also find that the option that is on the left of the screen is chosen 55% of the time, which is also significantly different to 50% ($\beta = 0.22$, $z = 5.72$, $p < 0.01$, 95% CI = [0.14, 0.29]). The tendency to choose options with higher ratings or to choose options on the left of the screen, does not vary as a function of the deferral condition ($p > 0.05$ in all cases).

We find that choice is deferred 23% of the time when it is allowed. Overall there is a high dispersion in individual tendencies to defer choice, with six participants never deferring choice, and two participants deferring choice more than half of the time. Additionally, consistent with the absolute desirability effect (Effect 2), we find that the deferral probability decreases as a function of the mean ratings of the available options, when examined using a logistic regression with participant-level random intercepts ($\beta = -0.73$, $z = -23.15$, $p < 0.01$, 95% CI = [−0.79, −0.67]). Likewise, consistent with the relative desirability effect (Effect 3), we find that deferral probability decreases as a function of the absolute difference in ratings of the available options ($\beta = -0.37$, $z = -14.20$, $p < 0.01$, 95% CI = [−0.41, −0.31]). This experiment thus replicates some of the results of Tversky and Shafir (1992), Dhar (1997), Chernev and Hamilton (2009), and White et al. (2015).

6.2.2. Main predictions

Now let us examine decision times. We find that the average time decision makers take across our two conditions is 1.79 s (*SD* = 1.21). As can be seen in Fig. 11a, this distribution is positively skewed, as expected. The distributions for all decision times in the two conditions, and deferral times and active choice times in the deferral condition, are similarly skewed, as is shown in Fig. 11b–e.

Prediction 1 states that decision makers should take longer to make a decision from a particular choice set in the absence of deferral, if they are more likely to defer choice in this choice set when deferral is allowed. Recall that our experiment offered participants each choice set twice: once as part of a forced choice, without the possibility of deferral, and once as part of a free choice, with the possibility of deferral. We can thus use the data from our experiment to test Prediction 1. In our data we find that the average decision time for a choice set in the absence of deferral is 2.58 s (*SD* = 1.50), if the decision maker does defer choice in the corresponding choice set when deferral is allowed. In contrast, the average decision time for a choice set in the absence of deferral is 1.87 s (*SD* = 1.23), if the decision maker does not defer choice in the corresponding choice set when deferral is allowed. This is a statistically significant difference, when examined using a linear regression with participant-level random intercepts ($\beta = 0.73$, $z = 17.15$, $p < 0.01$, 95% CI = [0.64, 0.81]).

Prediction 2 states that decision makers should always be quicker to make active choices in the presence of deferral compared to the absence of deferral. In our data we find that the average decision time for an active choice in the presence of deferral, $DT(C|C, D)$ is 1.52 s (*SD* = 0.96), whereas the average decision time for an active choice in the absence of deferral, $DT(C|C)$ is 2.01 s (*SD* = 1.35). This is a statistically significant difference allowing for participant-level random effects ($\beta = -0.49$, $z = -21.82$, $p < 0.01$, 95% CI = [−0.54, −0.45]).

Prediction 2 is actually stronger than that tested using the above regression. Essentially, as we expect $DT(C|C, D) < DT(C|C)$ for any choice set and any participant, $DT(C|C, D)$ should stochastically dominate $DT(C|C)$. This means that for any time x, the distribution of $DT(C|C, D)$ should have a higher cumulative probability than the distribution of $DT(C|C)$. Stochastic dominance can be tested using a

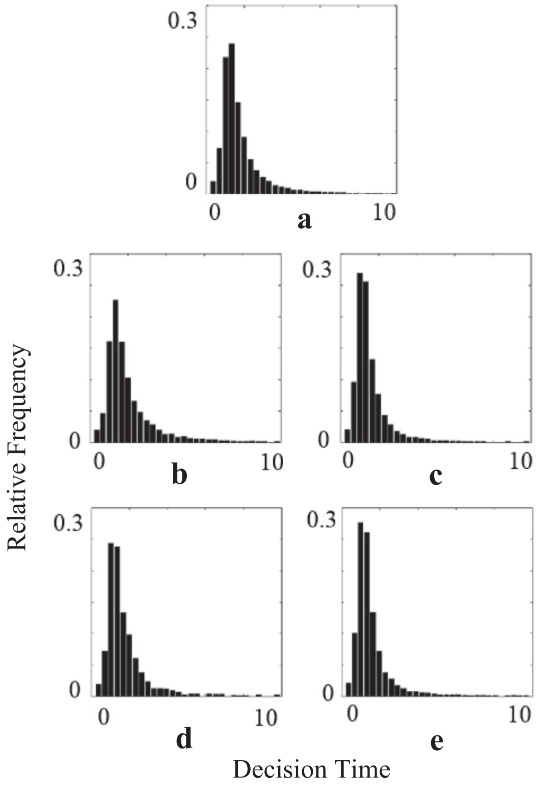

Fig. 11. (a–e) Relative frequency histograms for all decision time data (a), all decision time data when deferral is not allowed (b), all decision time data when deferral is allowed (c), decision time data for deferred choice when deferral is allowed (d), and decision time data for active choice when deferral is allowed (e). Decision time is in seconds.

Kolmogorov–Smirnov test for ordering, which is nonparametric and does not assume any distributional form for our decision times (see Heathcote, Brown, Wagenmakers, & Eidels, 2010). This test uses a statistic proportional to the largest positive difference between one CDF and another to test whether the first is significantly larger than the other (i.e. associated with higher probabilities for any given outcome). If one CDF is indeed shown to be significantly larger than the other, and the other is shown to not be significantly larger than the first, then the variable corresponding to the first distribution is assumed to dominate the variable corresponding to the second distribution. Applying this test to our observed distributions for DT(C|C, D) and DT(C|C) reveals that DT(C|C, D) is significantly larger than DT(C|C) ($p < 0.01$, $KS = 0.22$) and that DT(C|C) is not significantly larger than DT(C|C, D) ($p = 0.98$, $KS = 0.002$), establishing stochastic dominance. Stochastic dominance is further illustrated in Fig. 12 using the empirical cumulative distribution functions of DT(C|C, D) and DT(C|C) with 95% confidence bounds.

In discussing Prediction 2 in the above section, we also noted that the time taken to defer a choice when deferral is allowed will be lower than the time taken to make an active choice in the absence of deferral, if deferral is especially likely. In this experiment we do not obtain multiple observations for each choice set under deferral, so we cannot distinguish between decisions that are likely to end in deferral and decisions that are unlikely to end in deferral (all we have is a single observation indicating whether or not the participant deferred choice in that particular choice set). However, we can nonetheless examine the difference between deferral decision times and corresponding active choice decision times in the absence of deferral. We find that deferral, when it happens, typically takes less time than active choices where deferral is not allowed. More specifically, we find that the average

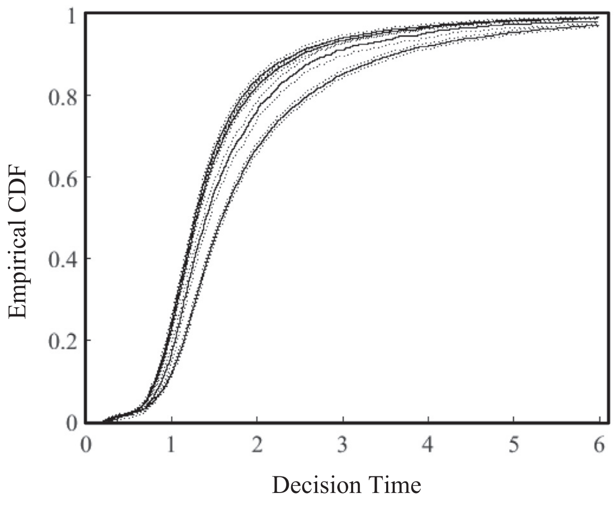

Fig. 12. The empirical cumulative distribution functions for DT(C|D, C), DT(D or C|D, C), DT(D|D, C), and DT(C|C), ordered from top/left to bottom/right respectively. Here the dotted grey lines on the boundaries of each CDF indicate 95% confidence intervals for that curve. Decision time is in seconds.

decision time for a deferred choice, DT(D|C, D) is 1.73 s (*SD* = 1.14), whereas the average decision time for an active choice in the absence of deferral, DT(C|C) is 2.01 s (*SD* = 1.35). This is a statistically significant difference ($\beta = -0.27$, $z = -6.86$, $p < 0.01$, 95% CI = [−0.35, −0.19]) when established using a regression with participant-level random effects. A Kolmogorov–Smirnov test for stochastic dominance also establishes that DT(D|C, D) dominates DT(C|C) ($p < 0.01$ and $KS = 0.12$, and $p = 0.87$ and $KS = 0.01$ for the two comparisons respectively). This difference can also be observed in Fig. 12.

Prediction 3 states that regardless of the time it takes to make a deferred choice, overall decision times in the presence of deferral should be lower than overall decision times in the absence of deferral. The observations of DT(C|C, D), DT(D|C, D), and DT(C|C) discussed above already indicate that this is the case, and indeed, we find that on average, the decision time in the presence of deferral, DT(C or D| C, D) is only 1.56 s (*SD* = 1.01), compared to the average decision time in the absence of deferral, DT(C| C), which is 2.01 s (*SD* = 1.35). Again this is a statistically significant difference both when tested using regressions with random effects ($\beta = -0.45$, $z = -21.40$, $p < 0.01$, 95% CI = [−0.49, −0.41]) and when tested using the Kolmogorov–Smirnov test for stochastic dominance ($p < 0.01$ and $KS = 0.20$, and $p = 0.94$ and $KS = 0.003$ for the two comparisons), and can be seen in Fig. 12.

Finally Prediction 4 states that decision times when deferral is allowed should always be quicker in active choices compared to deferred choice. This too is confirmed in our data. As already outlined above, the average decision time for an active choice in the presence of deferral, DT(C|C, D) is 1.52 s (*SD* = 0.96), whereas the average decision time for a deferred choice, DT(D|C, D) is 1.73 s (*SD* = 1.14). This is also statistically significant using both a regression with participant-level random effects ($\beta = -0.27$, $z = -6.55$, $p < 0.01$, 95% CI = [−0.36, −0.19]), and using the Kolmogorov–Smirnov test for stochastic dominance ($p < 0.01$ and $KS = 0.11$, and $p = 0.99$ and $KS = 0.00$ for the two comparisons), and can be seen in Fig. 12. Note that Predictions 2–4 can equivalently be seen as constraints on the orderings of DT(C|C, D), DT(D|C, D), DT(C or D), and DT(C|C). Fig. 12 shows that we obtain DT(C|C, D) < DT(C or D|C, D) < DT(D|C, D) < DT(C|C) in our data, indicating that these constraints are satisfied.

6.3. Quantitative results

The above section has shown that qualitative patterns predicted by the proposed model emerge in that data. Now we will perform a model fitting exercise to rigorously test how well this model can explain the data. Note that it is difficult to fit the entire AAM-based model or its more restrictive

LCA variant due to both data limitations (we do not know the attribute compositions of the choice options) and the computational complexity of the models. It is however possible to fit a more generic model of the deferral time limit, that is largely agnostic of the back-end guiding the accumulation of preference.

6.3.1. Outline of models

Consider for example a decision maker whose decision time in the absence of deferral, for a particular choice set, has a cumulative distribution function F_S. Also let us assume that the distribution of the deferral time limit for our decision maker is given by F_T, with T being independent of S. Now in forced choice, decision times for a given choice set are obtained by taking a sample from F_S. In free choice, decision times for the same choice set are obtained by taking a sample from both F_S and F_T. If T is less than S then the deferral time limit is crossed before a decision is made, and choice is deferred. In contrast if S is less than T then a choice is made without deferral. In either case, probability of deferral is given by $\Pr(T < S)$, and the decision time is given by some distribution F_R with $R = \min(S, T)$.

Note again that this generalization of our model is agnostic about the underlying processes responsible for our decision time distributions. If we believe that AAM is the correct underlying model, then F_S is obtained from its accumulation to threshold dynamics. This generalization is not however agnostic about the mechanisms underlying deferral. The above formalization necessarily assumes that decisions are deferred if a decision is not made by the deferral time limit T.

In the absence of tractable decision time distributions for AAM (or its LCA variant), we can test the above model with a more generic distribution. In the following fits we will assume that both F_S and F_T are distributed according to the Inverse Gaussian distribution (also known as the Wald distribution), with $F_S \sim IG(\mu_S, \lambda_S)$ and $F_T \sim IG(\mu_T, \lambda_T)$. Here μ characterizes the mean of this distribution, and λ characterizes the shape of the distribution.

Recall that each of the 100 choice sets were offered twice to the participant: once with the possibility of deferral, and once without. Thus for each of the 58 participants, we have three main observations: 1. The time it took the participant to make a forced choice for the choice set, 2. The time it took the participant to make a free choice for the choice set, and 3. Whether or not the participant deferred the decision in the free choice. The deferral time limit model outlined here can be fit to this participant-level data with the simplifying assumption that decision times for different choice sets stem from the same distribution. Particularly, for each choice set offered to a participant, the likelihood of observing the participant's particular decision time, a, in the forced choice is given by $f_S(a)$. Additionally, as S and T are independent, the likelihood of choice being deferred in free choice, and of observing a particular deferral decision time, a, is given by $f_T(a) \cdot [1 - F_S(a)]$. Likewise the likelihood of choice not being deferred in free choice, and of observing a particular choice decision time, a, is given by $f_S(a) \cdot [1 - F_T(a)]$. Note that the overall likelihood of observing a decision time a in a free choice task is $f_T(a) \cdot [1 - F_S(a)] + f_S(a) \cdot [1 - F_T(a)]$. A standard check shows that for a random variable $R = \min(S, T)$, we do indeed obtain $f_R(a) = f_T(a) \cdot [1 - F_S(a)] + f_S(a) \cdot [1 - F_T(a)]$ if S and T are independent.

With the above assumption, each participant-level model fit will require four parameters, μ_S, λ_S, μ_T, and λ_T. With these four parameters we will be able to predict the probability of deferral, the distribution of decision times if choice is deferred in a free choice task, the distribution of decision times if choice is not deferred in a free choice task, the overall distribution of decision times in a free choice task, and the overall distribution of decision times in a forced choice task.

Additionally, we will fit a baseline model on the participant level. This simple baseline model assumes that deferral does not interact with decision time, or more specifically that all decision times stem from the same distribution, F_S, and that the probability of choosing deferral once this decision time is realized, is completely independent of the decision time itself. For this model, the likelihood of observing a decision time a, in either the forced choice or the free choice is given by $f_S(a)$. The likelihood of choice being deferred in free choice, and of observing a particular choice decision time a, is $p_d \cdot f_S(a)$, where p_d is the probability of deferring choice once a particular decision time is realized. Likewise the likelihood of choice not being deferred in free choice, and of observing a particular choice decision time, a, is given by $(1 - p_d) \cdot f_S(a)$. Again, we will assume that F_S is an Inverse Gaussian distribution. Thus the model has three parameters, p_d, the probability of deferral, as well as, μ_S and λ_S, which characterize all decision time distributions in the free and forced choice tasks. Comparing our proposed model's fits to the baseline model can help us determine whether our dynamic theory of deferral is in fact superior to a simplistic theory that assumes that deferral is a static phenomenon, with deferral probability being independent of decision time.

We chose the Inverse Gaussian distribution for our main model and our baseline model, as it is parsimonious, with only two parameters. It is also easy to fit, as it has a tractable likelihood function. Moreover, the Inverse Gaussian is psychologically plausible, as it has been used to study response time distributions in low-level tasks (e.g. Stone, 1960), where it describes the first passage distribution of a simple diffusion process.

We could equivalently use more complex distributions such as the Ex-Wald or the Ex-Gaussian (Hohle, 1965; Schwarz, 2001) for modeling F_S. These distributions also attempt to capture non-decision components of response times, and they may ultimately give slightly better fits, though we expect these non-decision components to have a smaller effect in preferential choice, where decisions take much longer than corresponding motor and perceptual processes, compared to low-level choice tasks. Ultimately though, our goal is not to test whether one function fits better than another, but to test the proposed deferral time limit mechanism using a simple, generic decision time distribution, and to compare it with a non-dynamic baseline model with the same generic distribution.

Additionally, as we have modeled deferral as a time limit, it would be useful to adapt formal insights regarding interval timing distributions to specify F_T. One such insight involves a constant coefficient of variation (CV), as suggested by scalar expectancy theory (Gibbon, 1977). Our assumption that $F_T \sim IG(\mu_T, \lambda_T)$ does not explicitly impose this (doing so, would require that $\lambda_T = \mu_T/k$, for a constant $k = \mathrm{CV}^2$). However, this restriction is not necessary in our experiment. Unlike experiments that test for interval timing effects, we are not exogenously varying any interval time or time limit. Rather we are assuming that each participant has their own (noisy) time limit, whose underlying distribution is constant across trials. As such our fits, with flexible μ_T and λ_T across participants, are consistent with a constant coefficient of variation model in which the CV does not vary across different intervals, but is allowed to vary across participants. Indeed, it is useful to note that such a model of interval timing, based on an Inverse Gaussian, has been used to study interval timing effects (Simen, Balci, Cohen, & Holmes, 2011). That said, it may be possible to improve our fits by testing other distributions for interval timing. However, once again, the goal of the model fitting exercise is to test the proposed deferral time limit mechanism using a simple, generic decision time distribution, rather than examine which specific distributions best describe the deferral time limit.

Finally, it is important to note that our use of the above distribution for F_T (and, more generally a stochastic T), does not alter any of the results presented earlier in this paper. As T is exogenous, choice and deferral probabilities with a stochastic T can simply be seen as an average of choice and deferral probabilities obtained from each (deterministic) T, weighted by the associated probabilities of these Ts. Of course we would expect the stochastic time limit to provide a better descriptive account of behavior than a deterministic T. Indeed a deterministic time limit would generate a censored decision time distribution, which is unrealistic.

6.3.2. Results

All model fits in this paper were performed using maximum likelihood estimation, implemented using the simplex routine in MATLAB. In order to avoid local maxima, each fit was performed 100 times, with starting points obtained at random from a uniform distribution with support [0, 10]. Out of these, the fit with the highest likelihood was used to describe the participant's preferences. Fits for the six participants who never deferred choice are excluded, as are fits for three participants who deferred choice only once. This gives us a total of 49 participants to examine.

Best-fit participant-level parameter statistics for our proposed model and the baseline model are provided in Table 2. These fits show that μ_T is typically greater than μ_S, with a median $\mu_T = 3.46$ s and $\mu_S = 1.97$ s. This implies that the deferral time limit is more likely to become active later on in the decision, which is consistent with our finding that choice is deferred relatively infrequently (again, we observe a mean deferral probability of 0.23 across participants). Fig. 13 plots best-fit values of μ_T and μ_S, to show this in more detail. It indicates that not only is μ_T typically greater than μ_S, but also that there is substantial variation in μ_T across participants. This too is consistent with our findings that

Table 2
Best-fit model and baseline model parameters, and accompanying BIC values.

	Median individual-level parameters					Median BIC	Group BIC
	μ_T	μ_S	λ_T	λ_S	p_d		
Model	3.46	1.97	9.95	6.69	–	566	24,641
Baseline	–	1.78	–	6.58	0.23	586	25,566

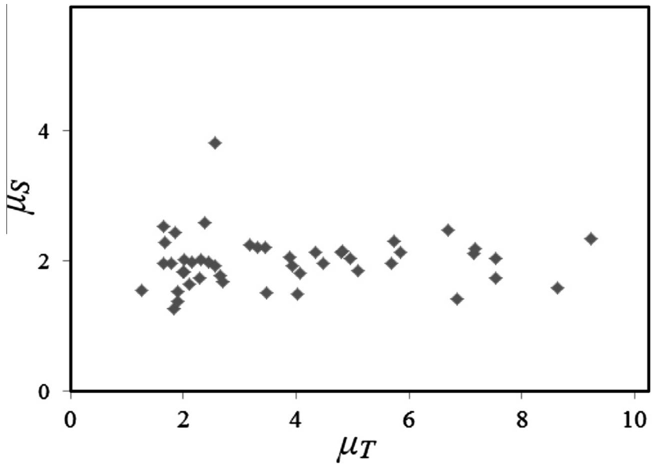

Fig. 13. Best-fit participant-level values of μ_T and μ_S. Note that the parameters of two participants are not displayed as these participants had outlier values of μ_T ($\mu_T > 30$).

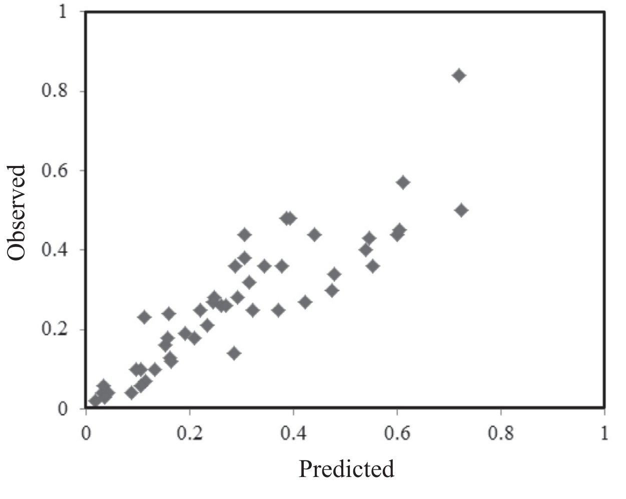

Fig. 14. Predicted vs. observed deferral probabilities for participants, based on participant-level best-fit parameters.

there is substantial variation in deferral probabilities: While most people may be similar to each other when making forced choices, the time limits they use while deferring choice differ greatly.

How well is our model able to predict deferral probabilities? Quite well. Overall, there is a correlation of 0.89 between our predicted deferral probabilities and our observed deferral probabilities, for the 49 participants whose data is fit in our study. A scatter plot demonstrating this is shown in Fig. 14.

We can similarly examine our model's predictions regarding decision times. Fig. 15a–d shows observed and estimated quantiles for decision time distributions for all choices in which deferral is allowed, for deferred choices when deferral is allowed, for active choices when deferral is allowed, and for all free choices. Our estimated quantiles are averaged over participant-level quantiles. These figures show that our model's predicted quantiles display relatively little deviation from observed quantiles, with an R^2 value of 0.97 in Fig. 15a, c, and d distributions and an R^2 value of 0.96 in Fig. 15b. Note that there is some deviation between our predictions and the observed data in Fig. 15a and c for the 50% and 70% quantiles, suggesting that our model may still be overestimating the time it takes for decision makers to make active choices in the presence of deferral.

Let us now compare the above predictions with those generated by the baseline model, which assumes that deferral is independent of decision time. The quantile estimates of this model are also plotted in Fig. 15a–d. As can be readily seen in these figures, our proposed model greatly outperforms the baseline model. The R^2 values for this baseline model are 0.81, 0.93, 0.70 and 0.88 in Fig. 15a–d respectively, and this model's estimates are further from observed data than our model's estimates in 17 out of the 20 quantiles considered.

We can formally examine these differences by comparing the log-likelihoods of these two models. We find our model generates a higher log-likelihood on the data for 84% of the participants. When controlling for model flexibility using Bayes Information Criterion (BIC), we find that 68% of participants are better described by our model relative to the baseline. The median BIC for our model is 566.32 and the median BIC for the baseline model is 586.48. We can also examine BIC on the group-level by assuming participant-level fixed effects, and aggregating participant-level log-likelihoods. Such an approach involves 196 parameters (four for each participant) for our proposed model, and 147 parameters (three for each participant) for our baseline model, as well as 14,700 data

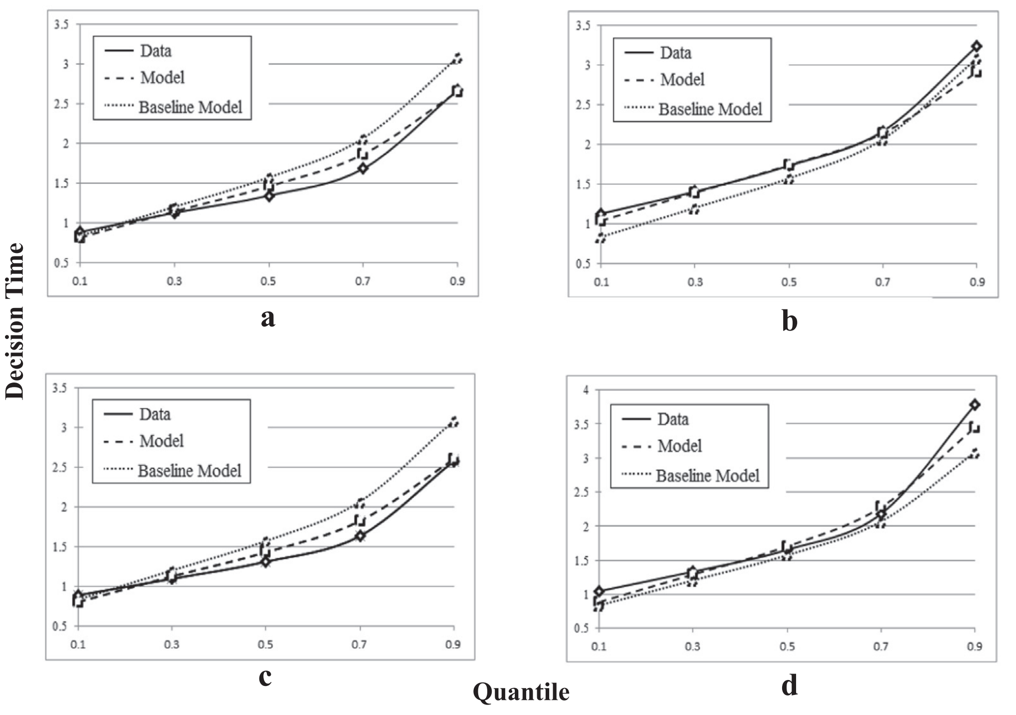

Fig. 15. (a–d) Decision time quantiles for observed data, model predictions, and baseline model predictions for all choices in which deferral is allowed (a), for deferred choices when deferral is allowed (b), for active choices when deferral is allowed (c), and for all free choices (d). Decision time is in seconds.

points for both models (300 per participant), yielding a BIC of 24,641 for our proposed model and 25,566 for our baseline model. These results are summarized in Table 2.

6.4. Discussion

The deferral-based time limit mechanism assumed in the proposed model plays a key role in allowing it to explain observed behavioral findings regarding choice deferral. This time limit also however generates four new novel predictions regarding the interactions between decision times and choice and deferral probabilities. These predictions are critical to the time limit mechanism, but are independent of the specific back-end model and thus emerge both for the AAM and LCA models discussed in this paper and the earlier Jessup et al. (2009) model.

The primary goal of this experiment was to test these four main predictions. Consistent with these predictions, we found that 1. Decision times were longer in free choice when corresponding forced choices were deferred, 2. Decision times were quicker when active choices were made in the presence of deferral compared to when active choices were made in the absence of deferral, 3. Overall decision times were lower in the presence of deferral, and 4. Active choices were quicker than deferred choices when deferral was allowed.

Finally we fit a generic deferral time-limit model to the data. This model was able to successfully predict both individual-level deferral probabilities as well as decision time distributions in both free and forced choice. Importantly this model significantly outperformed a baseline model which assumed that deferral was independent of decision time. Both these quantitative model fits and the qualitative patterns observed in our data provide strong support for a time-limit mechanism for deferring decisions.

7. General discussion

Choice deferral is the tendency to disengage from a choice task, and by doing so, postpone the decision to a later time or avoid the decision altogether. It is an important feature of everyday preferential choice, and has, for this reason, received a lot of attention in psychology, marketing, and related fields (e.g. Chernev, 2005; Chernev & Hamilton, 2009; Dhar, 1997; Dhar & Sherman, 1996; Gourville & Soman, 2005; White et al., 2015; also Tversky & Shafir, 1992). Scholars of decision making in these fields have discovered a number of systematic effects pertaining to the likelihood with which decision makers defer choice, and the effect of allowing choice to be deferred.

In this paper we have attempted to explain existing findings regarding choice deferral using a time limit applied to sequential sampling models of multi-attribute choice (initially proposed by Jessup et al., 2009). According to this mechanism, an active choice is made if an activation threshold is crossed before the time-limit. If not, choice is deferred. This mechanism predicts that choices that are made slowly are more likely to be deferred, and that allowing for choice deferral increases the choice proportion of options that are favorable early on in the decision.

In the first part of this paper we have formalized this mechanism within the associative accumulation model (Bhatia, 2013), and shown that the combined approach is able to explain the eight existing deferral-based effects, including effects pertaining to the settings in which deferral is most likely, and effects pertaining to the influence of deferral on choice proportions for the available options. The time limit mechanism also generates a number of novel predictions regarding decision times and choice probabilities, independently of the back-end model it is implemented with. We have tested and confirmed these predictions in a novel experiment, in the second part of the paper. Additionally, we have quantitatively fit a generic version of a time limit-based model to the experimental data, and found that our fits are able to provide an accurate account of choice and deferral probabilities and decision times observed in the experiment.

7.1. Predictions of related models

Our use of the associative accumulation model as a back-end process for instantiating the time limit mechanism stems from its ability to explain a wide range of effects regarding choice set dependence. Many of these have been linked to choice deferral, and AAM, combined with the time limit, provides a cohesive account of these diverse effects. That said, the unique assumptions of AAM are not necessary to account for all the deferral findings documented thus far. Four out of the eight effects studied in this paper also emerge with the associative mechanism in AAM is disabled, and the model reduces to a leaky competitive accumulator (Usher & McClelland, 2001) with sequential attribute sampling (Roe et al., 2001). Additionally, three out of these four effects can also be generated by a simpler race model in which lateral inhibition is also disabled (one effect, however, requires inhibition, and in fact reverses when inhibition is turned off). The effects that do require AAM are those in which deferral interacts with the specific composition of the choice set, such as the presence of dominated options. These are the types of settings that AAM was initially generated to explain.

It is possible that other existing sequential sampling models of choice set dependence could replace AAM in the above modeling exercise, while retaining many of its key results. Consider, for example, the multi-alternative decision field theory (MDFT) (Roe et al., 2001), which is the back-end sequential sampling model tested alongside the time limit mechanism in Jessup et al. (2009). As with AAM, this model attempts to explain the effects of dominated options and related facets of the choice set on choice probabilities. Unlike AAM, however, MDFT assumes that attribute attention is independent of the composition of the choice set. Rather, changing the choice set alters the inhibitory connections between the available options, thereby biasing choice. Jessup et al. (2009) found that this model, combined with the time limit mechanism, is able to generate a choice overload effect. Although Jessup et al. did not study Effects 1–8, it is entirely possible that Jessup et al.'s time limit-based instantiation of MDFT could explain findings such as the reduction in deferral probabilities in the presence of dominated options. Additionally, this model, like the restricted LCA model tested in this paper, assumes leaky competitive accumulation, and thus should be able to account for the effects that can be obtained using only LCA (without the assumption of associative attention).

There are also other relevant sequential sampling models. One such model is the multi-attribute leaky competitive accumulator (MALCA) (Usher & McClelland, 2004) which assumes loss-averse attribute-wise comparisons, instead of association based attentional biases. In this model changing the composition of the choice set affects whether choice options are seen as gains or losses on their attributes, subsequently changing how these attributes are aggregated into preferences. As with AAM, LCA, and MDFT, and as suggested by its name, this model also features leaky competitive accumulation. Likewise, the multi-attribute linear ballistic accumulator (MLBA) (Trueblood et al., 2014) models the choice process using a linear ballistic accumulator (Brown & Heathcote, 2008) combined with choice set-dependent drift rates. Both of these models could potentially explain the reduction in deferral probabilities in the presence of dominated options, when equipped with a deferral based time limit, as the dominating options in these two models (like in AAM and MDFT) have higher preferences and are thus more likely to cross the decision threshold before the deferral time limit.

However there are some effects that deferral-based extensions of these models would have difficulty in capturing. For example, none of these three models have attempted to explain the alignability effect. If these models cannot account for this effect in the absence of deferral, it is unlikely that they would be able to capture its interaction with deferral. Similarly, MLBA could have difficulties in capturing the influence of deferral on choice probabilities for the extreme options and for the attraction and compromise effects. Essentially, this model assumes deterministic accumulation (given a drift rate and starting point) implying that the explanation for these effects in this paper, which relies critically on stochastic attribute sampling, does not extend to the MLBA model. For this reason MLBA may also have trouble with the unique vs common attributes effect.

7.2. Alternate dynamic interpretations

The key insight of this paper is that choice deferral is a fundamentally dynamic phenomenon. This is why a time limit mechanism, when implemented in AAM, is able to explain the dependence of deferral on the composition of the choice set, and the relationship between various choice set effects and the presence of deferral. This is also what allows us to successfully predict and fit choice probability and decision time data in our experiment. A number of researchers have already argued that dynamical processes are necessary in order to fully characterize choice behavior (e.g. Busemeyer & Townsend, 1993; Rieskamp, Busemeyer, & Mellers, 2006). The results of this paper provide further evidence in favor of their claims. Deferral is intertwined with decision time, and only a dynamic model of decision making can fully account for this relationship.

There are a number of different ways in which the dynamical properties of deferral can be conceptualized and implemented. In this paper we have suggested that deferral operates as a (probabilistic or deterministic) time limit. According to this account decision makers determine a time limit to be used prior to the decision. During the decision they keep a mental representation of the amount of time that has passed in the decision. If the total time spent in the decision surpasses the deferral time limit the decision is deferred. A mathematically identical way of formalizing this mechanism is as a random distraction or disengagement probability. This interpretation does not require decision makers to explicitly determine a deferral time limit prior to the decision, or to keep track of decision time. Instead it assumes that decision makers are sensitive to internal or external cues, which, when activated, cause decision makers to terminate the decision task and shift their attention elsewhere. Choice is deferred in free choices, if decision makers do not make a decision before being distracted. Presumably these distractors are suppressed in forced choices, which cannot be deferred. In these settings decision makers needs to finish the decision and make an active choice before performing other tasks.

Another way of conceptualizing our deferral mechanism involves seeing deferral as an implicit choice option, with its own accumulator. According to this interpretation decision makers aggregate evidence supporting or opposing deferral, and choose to defer choice when the deferral accumulator is the first to cross a decision threshold. Note that Busemeyer et al. (2006) have proposed a model of deferral along these lines. Such a model has also been used in the go/no go paradigm which can be seen as the simplest type of deferral-based choice task (Gomez, Ratcliff, & Perea, 2007).

There are many similarities between modeling deferral as a time limit and deferral as a separate accumulator. Indeed if the deferral accumulator is independent of the accumulating preferences, that is, if it does not depend on attribute attention, and is uninfluenced by lateral inhibition, then an accumulation-based model of deferral is mathematically identical to the probabilistic time limit model studied in this paper, and can under certain assumptions also be modeled using the Inverse Gaussian distribution discussed with regards to our experiment. This is a point made by Simen et al. (2011) who have proposed an accumulator based model for interval timing that is instantiated using a sequential sampling process.

If this is not the case, as in Busemeyer et al.'s (2006) model, then there are some relevant differences between modeling deferral as an accumulator and modeling deferral as a time limit. Particularly, a time limit mechanism would suggest that keeping all else equal, changing the composition of the choice set, or other aspects of the decision task that affect preference accumulation, should not affect the distribution of the deferral time limit, and thus overall deferral times. This is in contrast to an accumulator mechanism, in which, for example, higher rates of accumulation for the choice option should make deferral less likely (due to inhibition). For the same reason, exogenously manipulating the evidence in favor of one or more of the available options should not affect the time at which choice is deferred in the case of a deferral time limit, but can very well influence the time at which an implicit deferral accumulator crosses the decision threshold. Finally, changes to the decision threshold (e.g. by emphasizing speed over accuracy) would not affect the distribution of the deferral time in the case of an exogenous time limit, but would certainly affect the time at which a deferral accumulator would cross the decision threshold. Examining these diverging predictions should be the topic of future work.

It may also be the case that there are some settings in which decision makers use a time limit for deferral, and other settings in which they use a separate accumulator for deferral. These settings could have to do with how the choice to defer the decision is framed. For example it is possible that framing deferral as a return to the status quo (not having either of the options) leads to deferral being processed as just another choice option. In contrast, if deferral is framed as allowing a search for more information (as in Tversky & Shafir, 1992 and most of the research we discuss in this paper; also Busemeyer & Rapoport, 1988; Gluth, Rieskamp, & Büchel, 2012) then decision makers may be more likely to use a time limit. Future work should examine this issue in detail.

An interesting implication of the deferral option as an accumulator is that this arguably mirrors the urgency signal in forced choice tasks. In some tasks it has been shown that as an individual spends longer deliberating, they accumulate evidence at a faster and faster rate (Cisek, Puskas, & El-Murr, 2009). This is indicative of an urgency signal that increases over time, in the same manner that an accumulator for deferral would increase over time. It is possible to examine whether urgency and deferral are represented by the same accumulator, but with different behavioral outcomes in different tasks, as this would predict individual differences which correlate across tasks, and similar sensitivity to changes in stimuli and task framing. Many of the relevant tools and methods have already been developed to test different urgency signal assumptions and different models of forced choice which incorporate collapsing thresholds. Future work should examine this potential link.

7.3. Confidence and accuracy

Regardless of the specific interpretation of the proposed deferral mechanism, the relationship it creates between choice, deferral, and decision time has some important metacognitive implications. There is strong experimental evidence suggesting that evaluation of choice confidence is negatively related to the decision time, so that decision makers are likely to display the least amount of confidence for decisions that take the most amount of time. This relationship has been the basis of a number of dynamic theories of decision confidence (Link, 2003; Pleskac & Busemeyer, 2010), which assume that decision makers explicitly use decision time to form a confidence estimate. In this light, the proposed deferral mechanism can be seen as a way of avoiding choice when the decision maker is less confident about the accuracy of the decision. An obvious result of this is it would be possible to see at what level of confidence different individuals elect to defer, and whether this provides an explanation for individual differences. More fundamentally, by eliciting confidence at different time points it would also be possible to test whether evidence for deferral is indeed being accumulated over time and whether this is independent or dependent upon the choice set and other external factors.

This interpretation of deferral also suggests that a deferral time limit can be adaptive, as decision confidence is strongly associated with decision accuracy. Thus decision makers are more likely to defer choice when the choices are difficult and when they are most likely to make errors. That said accuracy in a preferential choice task is a notoriously complex and controversial concept, and it is unclear whether the average drift rate is a suitable normative criterion for evaluating decision accuracy in this domain. But, such effects would still be present as long as decision makers believe that taking longer to deliberate will make them more accurate.

A related point is how an individual alters the deferral threshold in response to changes in choice value or the severity of the choice's consequence. It may seem intuitive that individuals would delay the deferral threshold for more important choices in an attempt to increase accuracy. However, most people will anecdotally report that they frequently delay making important choices, and defer them to a later time. We also show in Effect 2 that when the absolute values of the options are higher, it is less likely that choice will be deferred. Therefore we believe it is more likely to be the choice thresholds that react to the scale of decision consequence rather than threshold for deferral.

7.4. Sequential decision making

The prior experiments on deferral whose results we hope to explain, assume that decision makers in a deferral task sample stored information without any explicit cost except for time. Some researchers have however examined decision making in closely related settings where information is externally provided and is monetarily costly. In these tasks decision makers need to evaluate the choice options after each piece of evidence is presented and decide whether they want to make a decision, or pay for more information (Busemeyer & Rapoport, 1988; Gluth et al., 2012). Theoretical models in these sequential decision making tasks attempt to explain not only what decision makers will choose, but also whether or not they will decide to obtain more costly information.

There are some differences between the deferral task considered in this paper and the sequential decision making task outlined above. In addition to information being externally provided and costly, the time at which decision makers can signal the desire to sample more information in the sequential decision making task is fixed and exogenous. In contrast, the deferral task studied in this paper and in previous experiments allows decision makers to defer choice at any time in the experiment. As a consequence, deferral experiments are not only interested whether or not decision makers defer their choice, but also how long people take to defer choice and how the possibility of deferral influences final choice probabilities.

Despite these differences, the two sets of tasks are very similar, and can be seen as relying on a common set of insights. For example, many dynamic sequential decision making models assume that decision makers choose to sample more information and avoid choice if a decision threshold has not been crossed by the time the decision makers encounter the exogenously imposed time limit (see again Busemeyer & Rapoport, 1988; Gluth et al., 2012). This is identical to the assumption in this paper, that endogenous (but independent) time limits determine whether or not a decision is deferred. This suggests that dynamic models of sequential decision making and models of choice deferral (such as the one presented here) are fully compatible, and even complementary. In fact, combining the two sets of models would be a reasonable next step in building better dynamical theories of preferential decision making in free-choice settings.

7.5. Perceptual choice

The success of the deferral time limit, both when combined with AAM and independently of AAM's particular assumptions, indicates that sequential sampling models may provide particularly desirable tools for modeling choice deferral. This in turn suggests that the relationship between deferral and accumulation to threshold, and, in turn, the observed influence of deferral, should be observed in other domains where these accumulation models have shown to be successful; domains such as perceptual choice.

Indeed work in this area has consistently documented that more difficult choices, in which the difference in evidence supporting the available options is very small, take longer (e.g. Ratcliff & Rouder, 1998). Thus we should also expect these difficult choices to be more likely to be deferred, if deferral is allowed. This would be analogous to the relative desirability effect discussed above. Likewise, Teodorescu and Usher (2013) have found that the total sum of evidence also influences decision speed indicating that we can also expect choices in which the sum of evidence is small to be more likely to be deferred. This would be analogous to the absolute desirability effect. Some existing work has already extended findings on multiattribute choice to perceptual domains (Trueblood et al., 2014), and it seems that studying choice deferral in these domains is a good candidate for future work. After all, low-level decisions, like preferential choices, are also often not forced.

7.6. Limitations

Although the approach we have studied in this paper is able to successfully account for a range of behavioral effects, our results are ultimately limited in many important ways. Firstly, our experiment did not examine the relationship between deferral and the specific attribute composition of the choice set. This was due to the fact that our choice options were relatively naturalistic, involving popular films, rather than the artificially generated options with explicitly presented attribute information, that are typically used to study multi-attribute choice. While the naturalistic format does allow for cleaner decision time data, as participants do not have to read large amounts of information while deliberating, future work should attempt to examine the relationship between choice deferral and decision time in settings where attribute information is known and inter-attribute relationships are explicitly manipulated.

Relatedly our experimental test did not attempt to fit the deferral-based extension of AAM, LCA or a related model. Rather it limited itself to fitting the deferral time limit with a highly simplified generic decision time distribution. This, as above, was due to the fact that attributes were not observable in our experiment. It was also a product of the complexity of these sequential sampling models, many of which have not been fit to choice and decision time data in the absence of deferral. Ultimately, in order to rigorously test these models' ability to explain deferral effects, it will be necessary to fit a complete model of choice deferral. One with both the deferral time limit and an appropriate back-end process. This too should be the focus of future work.

Conversely, our experiment was only limited to preferential choice. As discussed above, it is likely that the observed relationship between deferral and choice probability also emerges in low-level perceptual domains, such as those typically used to study decision time. These domains also allow for an explicit characterization of response accuracy and error, as well as convenient techniques to manipulate question difficulty. In order to establish the robustness of the proposed deferral-based time limit mechanism, it is important to experimentally observe deferral decisions in these low-level domains.

Our paper also has some theoretical shortcomings. Experimental work in decision making has examined the relationship between choice deferral and variables as diverse as presentation format, personality, decision maker expertise, ideal points and aspirations, and goals (see Chernev et al.'s, 2015 review). These variables lie outside the descriptive scope of our model, and indeed, outside the scope of most other existing cognitive decision models as well. Future work should modify these models to incorporate the wide range of psychological variables influencing deferral probabilities in preferential choice.

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