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# Lubricant Screening for Debris Effects to Improve Fatigue & Wear Life

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This paper discusses the mechanisms of surface damage induced by lubricant bourne debris in rolling element.contacts. Particular emphasis is placed on how debris particles cause damage and how the extent of this damage is related to debris size and material properties. The study is broadly divided into two parts; first, an investigation into the mechanisms of particle entry and behaviour in rolling contacts; and second, an analysis of the extent of rolling element surface damage caused by these particles.The results of optical end experiments show how various types of debris particle deform or fragment in the contact entry region. High speed video and short duration flash photography have been used to track the movement of debris particles as they approach and pass through an end contact, or are swept around the sides. Simple analyses are presented for predicting the maximum particle size and number which may be entrained into the contact. To investigate lubricant contaminant damage potential, a rig, designed to accumulate debris dent geometries, has been constructed. The results of experimentation, using this rig with lubricants contaminated with ductile and brittle particles, have been compared to theories for the prediction of surface damage.

#### **1. INTRODUCTION**

Recent improvements in the quality and manufacture of steels have meant that components in rolling contact are largely free from 'as-built' inclusions and pits. It is therefore damage caused during the operating life of the component that is the main source of failure. This damage is frequently the result of the overrolling of contaminant particles in the lubricant and may act as a stress raiser and initiate fatigue or lead to excessive wear.

The importance of the mechanisms of debris initiated fatigue have been widely recognised and several studies have been carried out to determine fatigue lives of rolling bearings under debris contaminated lubricant with various levels of filtration (1, 2, 3, 4). In some cases of heavy contamination with coarse filtration fatigue lives were reduced by as much as 50 times. Some researchers (5) have chosen to artificially pre-indent the contacting surfaces and note the effect of indentation size and shape on fatigue life; life reduction factors as large as 400 were found for deep steep sloped dents. The discretised fatigue life model of Ioannides and Harris (6) allowed local effects such as inclusions, indents, and stress raisers to be included in the fatigue calculation. Several studies (7, 8, 9, 10) have been carried out to model the effect of indentation shape and residual stress on fatigue life, with a view to determining a theoretical relationship between a damaged surface and the contact fatigue life.

The mechanisms of debris initiated contact wear have had less attention; much of the relevant information has been generated in related fields using abrasive wear as a machining process (11, 12, 13). Some interest has been shown in the effect of surface indentation on the elastohydrodynamic lubricant film (14, 15); both authors noticed pressure spikes and reduced films in the region of the indent, which could have a significant effect on fatigue life.

The importance of surface damage on contact life is well recognised, although, these studies have concentrated on the effects of surface damage rather than the causes. Little fundamental work has been carried out to relate the nature of surface damage to the overrolling of contaminating particles. This paper aims to address the link between lubricant debris and surface damage. It is hoped that some of this information may then be used in conjunction with studies, similar to those discussed above, to provide the link between lubricant contaminant properties and surface damage initiated fatigue and wear.

#### 2. LUBRICANT BORNE DEBRIS

Any lubricant sample may have considerable quantities of contaminating particles of various size and material. Ductile particles may be present as steel or alloy wear chips from machine components. Brittle particles may be introduced from the operating environment such as sand or coal dust. Tough ceramic particles may be left over from machining operations; diamond, cubic boron nitride, alumina, and silicon carbide are all used in grinding wheels.

Preliminary analysis of industrial application samples has shown many material types may be present with particle sizes ranging from sub-micron to 1000 microns. The concentration of debris is highly dependent on application and sampling.

# 3. THE ENTRY OF DEBRIS INTO A CONCENTRATED CONTACT

It is clear that the entry of debris into a concentrated contact is of primary interest in the study of the effects of contamination on contact life. In this study an 'optical elastohydrodynamic (ehd) rig' has been used to monitor the behaviour of various types of debris particle in the entry region, contact, contact sides, and exit region of a rolling point contact. In addition, a simple model describing the frictional force on an incident particle is presented, to explain the entry conditions.

# **3.1 Optical EHD Experiments**

#### 3.1.1. Apparatus

The apparatus (see figure 1) consists of a freely mounted steel ball which is spring loaded against a semi-reflective glass disk. The glass disk is mounted on a vertical shaft which is driven by a motor at stable speeds. Lubricant (with or without contaminants) is fed directly onto the disk.such that flow is entrained into the contact. An optical draw tube and lenses are mounted above the contact. Light is supplied to the contact from a fibre optic source a passed through a beam splitter. Light rays, reflected from the ball surface and the semi-reflective layer on the glass disc, interfere to produce a fringe pattern. The fringe pattern is directly related to the lubricant film thickness (16). High speed video and short duration flash photography were used to track and freeze motion in the contact and surrounding zones.



Figure 1. Schematic diagram of the optical ehd apparatus

#### 3.1.2. Method

Various debris materials were mixed with the lubricant (a synthetic hydrocarbon) and fed onto the glass disk. Concentrations were maintained at 5% by mass. This allowed enough debris to enter the contact to allow clear photography without the film being obscurred.

The debris materials have been broadly defined into three categories;

ductile, copper, low carbon steel, and cast iron brittle, glass ballotini, ACFTD tough ceramic, alumina, silicon carbide

#### 3.1.3. Results

The ductile particles, copper, low carbon steel, and cast iron, are flattened out in the inlet region and the platelets travel through the contact. Figure 2 shows a photograph of  $35\mu m$  copper spheres which have flattened in the contact.



Figure 2. Photomicrograph of copper debris in a rolling optical ehd contact (contact diameter 500 microns, entry direction left to right)

The brittle particles, glass ballotini spheres and ACFTD, fractured in the inlet region. The photograph figure 3 shows  $60\mu m$  glass spheres in the inlet region; the bright spots to the left of the frame are the undamaged spheres, these fracture when they are first trapped by the rolling elements. The fragments are then swept around the sides of the contact or drawn through the centre. The oil film is little distorted by the particles.



Figure 3. Photomicrograph of 60µm glass spheres in the entry zone to a rolling optical ehd contact (contact diameter 500 microns, entry direction left to right)

Tough ceramic particles, such as silicon carbide and alumina, tend to fracture further into the inlet region (i.e. after some considerable plastic flow of the contacting elements). The fragments imbed in the surfaces and pass through the contact; since fracture occurs in a more constricted region of the contact, fewer fragments escape around the sides. Small size tough ceramic particles, such as 5µm silicon carbide, did not undergo fracture and the particles were either swept around the sides or passed through the contact rigidly imbedding (see figure 4).



Figure 4. Photomicrograph of 5µm silicon carbide particles in a rolling optical ehd contact (contact diameter 500 microns, entry direction left to right)

From these observations it can be generally concluded that there are three mechanisms for particle behaviour in the contact:

(a) Particle plastically deforms into a platelet, with or without rolling element plastic deformation.

(b) Particle fractures, with or without rolling element plastic deformation. The fragments may then embed into the element surfaces.

(c) Particles undergo little or no deformation and may embed into the element surface.

It will be seen later that these three mechanisms have a significant influence on the nature of surface damage to the rolling elements.

# 3.2 Particle Size for Contact Entry

Figure 5 shows a simple schematic diagram of the forces on a spherical debris particle in the inlet region to a rolling contact. The simple balance of the forces includes a friction coefficient and the contact angle  $\alpha$ . A particle can only enter the contact if the contact angle is less than tan

 ${}^{1}\mu$ . Therefore the maximum particle size which can enter the contact can be determined from the size and shape of the inlet to a Hertz contact (such a relationship for the optical ehd rig geometry of a ball on a flat is given in reference 17).



Figure 5. Diagram of forces on a spherical particle entering a rolling contact.

The analysis neglects fluid effects and assumes the particle is spherical and no indentation occurs. If plastic deformation occurs, then the force on a particle must include a ploughing term. Figure 6 shows a graph relating the maximum size particle which may enter for a given friction coefficient and radii of the contacting rolling element pair. Wan & Spikes (16) performed a similar analysis for a slightly different geometry; they found that glass spheres of diameter 160-200µm did not enter the contact. The presented results are in general agreement with this.





Figure 6. Graph of Maximum Particle size to enter a contact for a given friction coefficient and rolling element radii, r.

# 3.3 Number of Particles In the Contact

A simple relation can be derived to predict the number of particles in a point contact at any instant (and hence the number which would be overrolled in a given number of cycles). In the analysis it is assumed that the particles enter the contact in direct proportion to their concentration in the bulk fluid. Assuming that the volume of the oil in the contact, V, is given by  $pa^2h_c$ , where a is the contact radius and  $h_c$  is the central film thickness. Then the number of particles is given by:

$$n = \frac{\pi a^2 h_c \frac{6}{\pi d^3} x \frac{\rho_o}{\rho_p}}{1 + x \frac{\rho_o}{\rho_p}}$$
$$n = \frac{\pi a^2 h_c \frac{6}{\pi d^3}}{\left(1 + \frac{\rho_p}{x}\right)}$$

where  $\rho_0$  and  $\rho_p$  are the densities of the oil and particles respectively, x is the mass of debris divided by the mass of oil, and d is the debris particle diameter.

The optical ehd rig has been used to verify this simple relationship.  $5\mu$ m silicon carbide particles were fed into a rolling contact. Short duration flash photographs were taken and the number of particles present.was recorded. In figure 7, the measured and predicted number of particles is plotted against the bulk concentration.



Figure 7. Comparison of a statistical calculation of debris particles in a contact with optical ehd results.

The relationship appears to work reasonably well for the point contact geometry. No information is available to verify the model's validity for an elliptical or line contact. In these instances the different degree of osculation may significantly alter the fluid effects and entrainment potential.

#### 4. DEBRIS INDUCED ROLLING ELEMENT DAMAGE

Given that a particle can easily enter a rolling element contact, it is essential to understand the nature of the resulting surface damage. This damage will have significant effects on the life of the rolling contact, either by acting as a fatigue initiator or as a wear site. To investigate this, a rig has been constructed to accumulate surface damage caused by a contaminated lubricant. The rig may then be used to evaluate the damage potential of a particle debris particle or a contaminated sample.

#### 4.1 The 'Mangle Rig'

Figure 8 is a schematic of this rig designed to overroll contaminant particles in a lubricant sample. The two cylindrical bearing steel rollers under test are held between four large nylon rollers. The nylon rollers and spindles were machined and mounted accurately so as to obtain a best possible line contact between the test elements. A motor with variable speed controller is used to rotate the four nylon rollers. Load is applied through a hydraulic loading stud; one of the nylon roller pairs being fixed while the other is free to pivot under load. A simple seal assembly is fitted around the contacting rollers; this ensures any lubricant placed in the entry to the elements is entrained and the debris overrolled. A drain is fitted to the underside of the seal and a peristaltic pump used to collect the tested lubricant. Lubricant mixed with debris is drip fed onto the contacting pair and the overrolled sample collected.

It was originally planned to used perfectly cylindrical rollers so as to achieve a line contact. Unfortunately all commercially available rollers are slightly crowned to prevent edge stresses. Thus in the experiments the whole quantity of the lubricant sample will not pass through the contact, some will pass around the sides. However, with careful feeding of contaminant, it was possible to ensure that a high proportion of the debris passed through the concentrated contact.

The rolling elements were examined for damage using a conventional optical microscope at magnifications of x100. 2D profiles of the damage were recorded using a Rank Taylor Hobson form talysurf. In addition some 3D surface maps have been produced using a conventional talysurf adapted with a relocation and stepping fixture. The lubricant samples before and after test were also examined using an optical microscope. For a more accurate measure of the size distribution a 'Malvern 3600 Particle Sizer' was used. The equipment determines the particle size concentration in a small sample from the amount by which laser light is scattered.

#### 4.2 Ductile Debris

The ductile particles result in smooth rounded dents on the rolling element surfaces. Figure 9 shows a 3D surface map of a dent caused by 45-106µm low carbon steel particles. The harder materials giving deeper and larger indentation. Figure 10 shows typical dent sizes found for various ductile particles. Optical microscopy shows the grinding marks still visible at the bottom of the dent. Cast Iron powder appears to act somewhere in between ductile and brittle behaviour; rounded dents are formed but the bases appear frosted as if some brittle fracture has occurred.



Figure 9. 3D surface map of a roller surface damaged by 45-106µm low carbon steel particles (patch size 0.5mm x 0.5mm).

Particle Material	Original Particle Size	Dent Depth	Dent Size	Dent shape
Low carbon steel	45-106µm	1.0-2.5µm	100- 150μm	Round smooth
Copper	35µm	0.6-1.0µm	100- 150µm	Round smooth
Cast Iron	45-63µm	1.5-3.0µm	100- 150µm	Smooth frosted surface

Figure 10. Table of indentation sizes caused by ductile debris.

Examination of the particles, after they have passed through the rig, show that they have been rolled out into thin flat platelets. The edges of some of these particles are ragged and some smaller fragments may have been torn away. Figure 11 shows the size distribution of the copper spheres before and after testing. It can be seen that the bulk of the particles have increased in size and some smaller sizes have appeared.



Figure 11. Particle size distributions for copper spheres before and after overrolling.

# 4.3 Brittle Debris

Brittle particles, with a lower toughness, fracture readily in the entry region so the damage is controlled by the ultimate fragment size which then enters the contact. Investigation of the rolling elements show that testing with glass, and ACFTD results in minor surface damage with small pits and irregular crazing.

Tough ceramic particles, having a high fracture toughness, tend to indent the rolling elements before they breakdown. The damage is therefore much more severe (see figure 12); in addition, the fractured particle sizes are greater leading to more extensive damage whilst in the contact.

Typically, the dents caused by ceramic particles are steep sloped with rough surfaces; figure 13 shows a 3D surface map of a roller damaged by 75µm silicon carbide particles.

Particle	Original	Dent	Dent Size	Dent
Material	Particle	Depth		shape
	Size			
ACFTD	0-100µm	0.6-1.0µm	30-50µm	Fine
				crazing
Glass	60µm	0.2-0.4µm	30-40µm	Fine
Ballotini				crazing
Quartz	45µm	0.2-0.4µm	30-50µm	Fine
				crazing
Alumina	45µm	.5-1.0µm	40-75µm	Fine dents
Silicon	75µm	3.0-7.0µm	50-150µm	Steep
Carbide	,			dents

Figure 12. Table of indentation sizes caused by brittle debris.



Figure 13. 3D surface map of a roller surface damaged by  $75\mu$ m silicon carbide particles (patch size 0.5mm x 0.5mm).

The particle size analyser was used to find size distributions of the debris before and after test. Figure 14 shows a graph of the distributions for alumina. A general

trend in size reduction can be seen with the majority of large particles being broken down. This breakdown may occur in the contact or the contact sides, so it is not possible to determine an ultimate maximum particle size which survives the contact (this information would have been useful in the prediction of the greatest possible resulting damage).



Figure 14. Particle size distributions for alumina particles before and after overrolling.

#### 5. THE PREDICTION OF DEBRIS DAMAGE

The previous section has described an experimental investigation into relationships between particle material and size and surface damage. This section investigates recent theoretical relationships which may be used as a predictive tool.

#### 5.1 Ductile Debris and the Forging Equations

Ductile particles entering the contact will normally be flattened between the comparatively hard counterfaces to form platelets. As a particle is extruded out laterally it must either slide in relation to the counterface or stick to the surface and yield in a near surface region. In either case a pressure gradient will be needed to overcome resistance to flow. As the aspect ratio of the squashed particle increases the maximum pressure needed to maintain flow may be many times the hardness of the particle and will cause significant elastic deflection of the rolling element surfaces. The simultaneous solution to the plastic flow and elasticity equations is described in (8). If the pressure approaches the hardness of the rolling element material then plastic deformation and indentation of the surface will result. An approximate dastic/plastic analysis of this problem based on the Johnson (19) elastic cavity model is also presented in (8). The predicted dent sizes and shapes from this model are shown to be in close agreement with experimental results.

#### 5.2 Brittle Debris and the Critical Crack Size

The prediction of brittle particle fracture is very complex, being highly dependent on particle geometry, orientation, material properties, size, and flaws. However, from the previous studies we have seen that the surface damage caused by brittle debris particles is largely controlled by the ultimate fragment size. It is therefore not unreasonable to base a study of brittle debris damage around estimates for the ultimate fragment size following overrolling fracture. This largest possible fragment which can survive the contact will result in the greatest level of damage.

Brittle materials generally undergo only elastic deformations and then fracture when the maximum tensile stress exceeds the fracture stress. The fracture stress of a brittle particle is given by (20);

$$\sigma_f = \frac{K_{IC}}{\sqrt{\pi a_{cc}}}$$

Where  $K_{IC}$  is the fracture toughness of the material and  $a_{cc}$  is the material critical crack size. Thus, if a maximum tensile stress of  $\sigma_f$  is applied, the brittle material will not fracture to a size below acc. Therefore, knowing the maximum tensile stress achieved in the particle during the overrolling process will yield an ultimate fragment size. Work is at present be carried out to model the elastic/plastic flattening of a brittle elastic particle using a finite element method, in order to obtain these failure stresses.

By inspecting the photographs from the optical ehd studies, it is possible to estimate that the glass and silicon carbide debris materials have ultimate particle sizes of  $1\mu m$  and  $6\mu m$  respectively. These results are consistent with the nature of the resulting rolling element damage observed after testing with these materials.

# 6. DISCUSSION OF RESULTS

The first part of this study has investigated the behaviour of debris particles in the contact and entry regions. The optical ehd rig has proved useful for this purpose; information has been generated to explain how the particles fracture or deform to enter a concentrated contact. This deformation mechanism has significant effect on the nature of the resulting surface damage.

Ductile materials are rolled out in the inlet region and under certain size and hardness conditions may indent the rolling element surface. This overrolling process has been modelled as a forging operation and good agreement has been found between predicted and measured indentation sizes.

Brittle particles fracture in the inlet region and the fragments are either swept around the sides or pass

through the centre of the contact. The tougher brittle particles fracture further down the inlet zone and the resulting fragments are larger. There are two distinct phases to surface indentation by brittle particles. Firstly, the extent of plastic deformation which results in the squashing of the particle before it fractures. Secondly, the damage caused by the overrolling of the resulting fragments. This maximum sized particle which can survive through the inlet zone will control the extent of the surface damage. This final fragment size has been estimated from optical ehd studies and agrees well the nature of surface damage found on the contacting elements. Unfortunately investigation of the overrolled contaminants from the 'mangle' rig was not useful in determining this ultimate particle size; flow around the sides of the contact occurred and it was not possible to distinguish between overrolled and original particles.

It is therefore possible to predict the damage which will result from the use of a contaminated lubricant sample; with some degree of certainty for ductile particles and with some approximations for ceramic particles.

# 7. CONCLUSIONS

The optical ehd rig has proved useful in determining the behaviour of particulate lubricant debris in a rolling contact. The rig has been used to investigate the number and size of particles which can enter the contact. Ductile, brittle, and tough ceramic materials have been tested and the nature of particle deformation or fracture has been observed. Brittle materials fracture in the inlet region and fragments enter the contact; ductile materials flatten in the inlet region and the deformed platelet enters the contact.

These deformation mechanisms have a significant effect on the nature of surface damage to the rolling elements. Brittle materials may cause surface indentation before they fracture and also when the fragments enter the contact. Ductile particles may cause damage when they a rolled out.

A rig has been built to accumulate the damage effects of a contaminated lubricant on a single contacting pair of rollers. Ductile materials, such as copper or low carbon steel, produced rounded relatively shallow dents. Brittle materials, such as ACFTD, caused a fine surface crazing. Tough ceramic particles, such as silicon carbide, left deep steep sloped dents.

A method for predicting ductile debris damage has been developed previously; particle extrusion pressures are balanced with rolling element deflection equations with and without plasticity criteria. The prediction of brittle debris damage is complex, estimates for the indentation size has been based around the size of the ultimate surviving fracture fragment.

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