Comparison of Channel Coding Schemes for Molecular Communications Systems

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***Abstract*— Future applications for nano-machines, such as drug-delivery and health monitoring, will require robust communications and nanonetworking capabilities. This is likely to be enabled via the use of molecules, as opposed to electromagnetic waves, acting as the information carrier. To enhance the reliability of the transmitted data, Euclidean Geometry Low Density Parity Check (EG-LDPC) and Cyclic Reed-Muller (C-RM) codes are considered for use within a molecular communication system for the first time. These codes are compared against the Hamming code to show that an $s = 4$ LDPC (integer $s \geq 2$) has a superior coding gain of 7.26dBs. Furthermore, the critical distance and energy cost for a coded system are also taken into account as two other performance metrics. It is shown that when considering the case of nano- to nano-machines communication, a Hamming code with $m = 4$, (integer $m \geq 2$) is better for a system operating between 10^{-6} and 10^{-3} Bit Error Rate (BER) levels. Below these BERs, $s = 2$ LDPC codes are superior, exhibiting the lowest energy cost. For communication between nano- to macro-machines, and macro- to nano-machines, $s = 3$ LDPC and $s = 2$ LDPC are the best options respectively.**

***Index Terms*—Molecular communication, diffusion channel, EG-LDPC codes, C-RM codes, Hamming codes, bioengineering.**

I. Introduction

THE communication between nano-machines, either by nano-mechanical, acoustic, electromagnetic, chemical or molecular channels is a rapidly growing research space with many unknowns [1]. At this point in time, the use of a molecular channel, whereby molecules are used to encode information, is becoming popular, in part, due to the fact that the molecules used have a size of the same order of magnitude as both the transmitter and receiver that emit and collect them [2].

Within the broad field of molecular communications, the type of molecule used is largely dependent upon the required transmission range of the application. At the moment, microtubule communication is useful for very short ranges [3], ion signaling and diffusion via Brownian motion is useful for short to medium ranges [4], whilst spore and pollen-based diffusion is useful for the longer ranges [5]. There also exists some other methods like advection-diffusion and electrochemical transports which can be used in neuronal networks [6], [7]. Within the bioengineering field, there appears to be a drive to deploy systems at the short to medium range [8] and so the focus here shall be on using the molecular diffusion channel as the link between the two communicating entities.

Across all branches of communications, a major concern has always been on the reliability of the data at the receiver and being able to control or correct any errors introduced. The de-facto technique has been to employ Error Correction Codes (ECCs). In parallel to the work of researchers in the field of ECCs, a substantial amount of research is also being conducted on developing logic gates (or logical elements) at the molecular scale [9], [10]. Thus, it is becoming apparent that the mathematical basis of any ECC is soon to have a feasible route for implementation.

There is only a small amount of work in the area of ECCs applied to molecular nano-communications, namely the work in [11] and [12], where the authors presented the use of Hamming codes as a candidate for future applications. The key results in the papers showed that the use of Hamming codes was beneficial to the performance of the system. Importantly though, the authors also considered the overall complexity of the encoding and decoding process such that the amount of energy that would be required was taken into account. The measure of energy chosen was atypical, considering the use of *critical distance*, a measure of the actual transmission distance, in μm, at which the use of coding becomes beneficial [13]. This was seen as more appropriate than an *energy per bit* type measure which could easily be taken out of context at the nano-scale. For example, a code with a low *energy per bit* may appear beneficial, but if its *critical distance* is longer than the length of an artery between two medical application based nano-machines, then in fact, the application of that code was inappropriate.

Therefore, a context-aware consideration of energy within any nano-communications systems is likely to be the defining constraint of all future systems [14]. A system designer should therefore, not always assume that any well-known performance enhancement techniques can readily be transferred from the mega- or micro-domain into the nano-domain as the energy budget might not exist.

A critical observation of the work based upon Hamming codes in [11] and [12], is that the encoding and decoding processes are of comparable complexity. That is to say, the energy required for encoding and decoding is of the same order of magnitude. Whilst this is perfectly viable for many applications, there may be future applications where the complexity of the transmitter and receiver are needed to be specifically different. For example, when considering future

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medical applications, nano-scale position beacons might be used to transmit information that guides macro-scale drug-delivery robots around human blood vessels. Thus, in this potential application, the transmitter must be simpler than the receiver. Therefore, it logically follows that such an application also requires an ECC code that is easier to encode than Hamming, even if the penalty is a more complicated decoding process.

In this paper, two ECCs, previously uninvestigated in the molecular domain, are introduced. One is from the Low Density Parity Check (LDPC) code family, and one is from the Reed-Muller (RM) code family.

LDPC codes have an extensive taxonomy [15] [16] [17] [18], but at the highest level they have been broadly branched into either Random or Structured LDPC codes. The former are sometimes known as Gallager or Makay Random codes [15] [17]. The latter type, i.e. Structured LDPC codes, have two specific constructions, called Euclidean Geometry LDPC (EG-LDPC) codes and Projective Geometry LDPC (PG-LDPC) codes. Both of these constructions have several advantages over Random LDPC codes including the existence of several decoding algorithms (cyclic or quasi-cyclic), Tanner graphs that do not contain lengths of 4 (which leads to simpler decoding), and finally, the ability to extend (or shorten) the code in order to adapt to an application [19] [20] [21]. It has been shown in [18] that both EG-LPDC codes and PG-LDPC codes have *almost identical error performance*. Further to this, a comprehensive account of the implementation of a cyclic EG-LDPC code has been shown in [22] such that in this paper, the focus will be placed on one specific construction, namely the cyclic EG-LDPC code.

RM codes [23] [24] are a class of linear codes over a Galois field of two (*GF*(2)), they are also a kind of code that can be easily decoded by majority decoding. In this work, an indirect construction of RM code as a cyclic code is given to show that the cyclic RM (C-RM) code is a subset of Bose, Chaudhuri and Hocquenghem (BCH) codes. In this case, the C-RM code can be easily encoded and decoded using the shift-register and majority logic decoding schemes respectively. The main advantage of these codes, from the perspective of energy, is that the encoder is simpler than the decoder, thus may hold benefits for the applications above noted so far [25] [26] [27].

This paper presents a several key contributions to the area of molecular communications. Herein, for the first time, the viable implementations (with generalized circuit models) of an EG-LDPC code and a C-RM code are shown for use in the molecular communication channel. Then, considering an arbitrary length Inter Symbol Interference (ISI) memory channel, its effects upon the Bit Error Rate (BER) of an un-coded channel is considered. Next, the coding schemes are applied to the system, and their performance evaluated with regards to both coding gain and critical distance. This presents a system designer with two metrics upon which to evaluate the effectiveness of applying the code once, or if, the transmission distances of the target application are known. In situations where raw energy consumption is required, a third metric of absolute energy requirements for the codes is shown with associated target BERs.

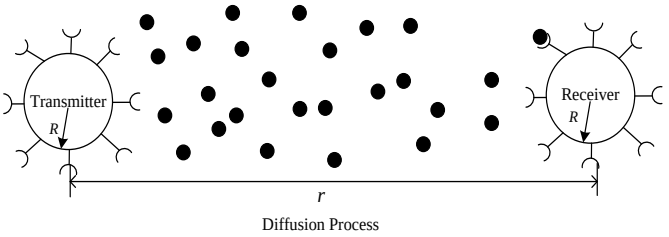


Fig. 1. The diffusion-based system considered in this work.

The remainder of the paper is organized as follows. Section II describes the communication and energy models used within the molecular communications systems considered here. The implementation aspects of Hamming codes, EG-LDPC codes and C-RM codes are then shown in Section III. This section also provides the reader with a definitive set of equations needed to calculate the energy requirements. Section IV then provides the simulation results with regards to coding gain, energy cost for coded system and the critical distance, followed by the conclusions in Section V.

II. Communication Channel and Energy Model

In this work, a three dimensional diffusion based communications system is considered where the transmission of molecules from transmitter to receiver is governed by the laws of Brownian motion. In this model, the receiver has a molecule capture probability $P_{hit}(r,t)$ shown to be equal to [12]:

$$P_{hit}(r,t) = \frac{R}{r}\operatorname{erfc}\left(\frac{(r-R)}{2\sqrt{Dt}}\right), \tag{1}$$

where, as in Fig. 1, r is the distance between the initial position (the center of the transmitter) of the molecule and the receiver center, in μm, t is time in s, R is the radius of the transmitter and receiver, in μm, and D is the diffusion coefficient, in $\mu m^2 s^{-1}$. In this paper, $R = 5\mu m$ and $D = 79.4\mu m^2 s^{-1}$ are assumed for ease of comparison with the work in [11] and [12].

In agreement with the work in [28], the transmitted information is represented by a sequence of symbols with one symbol in each time slot, t_s. In an intended time slot, if the number of information molecules arriving at the receiver exceeds a threshold τ, the symbol is denoted as '1', otherwise it is denoted as '0' [29]. Considering the ISI, the different error patterns can be obtained by the different permutations of the previous information symbols, so the number of error patterns is 2^I, where I is the length of the ISI considered.

Considering that N information molecules are released as an impulse at the start of the symbol duration time, and the number of molecules received among the N molecules that are sent in the current time slot N_0 follow a binomial distribution given as [28]:

$$N_0 \sim B\left(N, P_{hit}\left(r,t_s\right)\right). \tag{2}$$

If N is large enough, a binomial distribution $B(N,P_{hit})$ can be approximated by a normal distribution $\mathcal{N}(NP_{hit},NP_{hit}(1\text{-}P_{hit}))$,

thus:

$$N_0 \sim \mathcal{N}\left(NP_{hit}(r,t_s), NP_{hit}(r,t_s)\left(1-P_{hit}(r,t_s)\right)\right). \quad (3)$$

The values of t_s, for different distances r, can be selected by the time at which 60% molecules arrive at receiver [28].

The transmitted molecules cannot be guaranteed to reach the receiver within one time slot which causes ISI. The molecules emitted at the start of the i^{th} time slot before the current one will still exist in the environment and the number of molecules received in the current slot among these remaining molecules is given by:

$$\begin{aligned} N_i &\sim \mathcal{N}\left(NP_{i+1}, NP_{i+1}\left(1-P_{i+1}\right)\right) - \mathcal{N}\left(NP_i, NP_i\left(1-P_i\right)\right) \\ &= \mathcal{N}\left(N\left(P_{i+1}-P_i\right), N\left(P_{i+1}\left(1-P_{i+1}\right)+P_i\left(1-P_i\right)\right)\right) \end{aligned}, \quad (4)$$

where $P_i = P_{hit}(r, i \cdot t_s)$.

Overall, the total number of molecules received in the current time slot N_c can be obtained by summing N_0 and the number of remaining molecules which belong to the molecules sent from the start of all previous time slots:

$$\begin{aligned} N_c &= \sum_{i=0}^{I} a_{c-i} N_i \\ &= \sum_{i=0}^{I} a_{c-i} \mathcal{N}\left(N\left(P_{i+1}-P_i\right), N\left(P_{i+1}\left(1-P_{i+1}\right)+P_i\left(1-P_i\right)\right)\right) \end{aligned}, \quad (5)$$

where $\{a_{c-i}, i=0,1,2,...,I\}$ is the transmitted information symbol which includes current and all previous related I symbols.

Errors occur when the symbol sent in the current time slot is different from the symbol received in the current time slot. They can be represented by two cases: firstly, when '0' is transmitted, but '1' is received; secondly, when '1' is transmitted, but '0' is received.

The error probability for the first case shows that the received molecules exceed τ, which is:

$$\begin{aligned} P_{01,j} &= \frac{1}{2^{I+1}} \mathrm{P}\left(N_{c,j} \geq \tau\right) \\ &= \frac{1}{2^{I+1}} \mathrm{P}\left(\sum_{i=0}^{I} a_{c-i,j} N_{i,j} \geq \tau\right) \end{aligned}, \quad (6)$$

where $j = 1,2,\ldots,2^I$, is the error pattern index. Here the transmission probabilities of '0' and '1' are assumed as 0.5 and 0.5 respectively.

Conversely, the error probability for the second case can be obtained by:

$$\begin{aligned} P_{10,j} &= \frac{1}{2^{I+1}} \mathrm{P}\left(N_{c,j} < \tau\right) \\ &= \frac{1}{2^{I+1}} \mathrm{P}\left(\sum_{i=0}^{I} a_{c-i,j} N_{i,j} < \tau\right) \end{aligned}. \quad (7)$$

And thus, the error probability for an un-coded system is given as:

$$P_{uc} = \sum_{j=1}^{2^I} \left(P_{01,j} + P_{10,j}\right). \quad (8)$$

As can be seen in Fig. 2, for a given distance r = 6μm, the longer the ISI length I, the higher the BER. It can also be noted that as the ISI length considered increases, the effect it has upon the BER becomes less prominent, i.e. the BER value begins to converge.

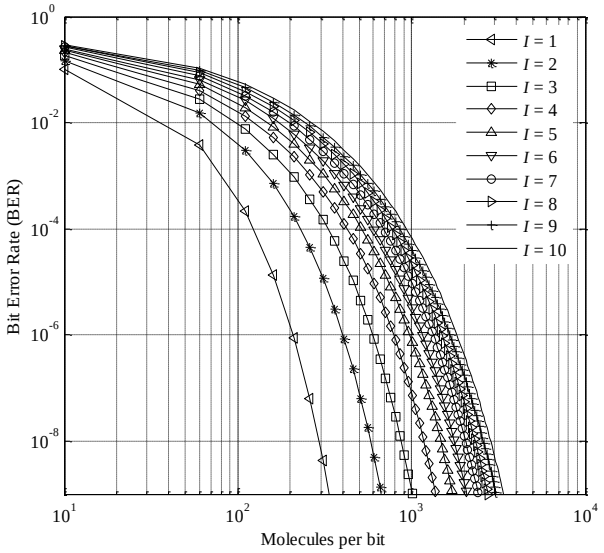


Fig. 2. BER for un-coded system at transmission distance r = 6μm for different ISI length, I = 1 to 10.

The energy model used here is based upon the energy consumption of the encoder and decoder circuits. The authors in [30] considered the protein-based signaling networks within biological cells. It was shown that the fundamental motif in all signaling networks is based upon the protein phosphorylation/dephosphorylation cycle, also called a cascade cycle. Thus it is possible to construct various control and computational analog and digital circuits by combining these cascade cycles which simply change one state to another. Coupled with the notion that these kinds of networks are of a similar scale to those needed for any future artificial nano-networks, they can also be used to estimate the energy requirements of the encoding and decoding circuits used here. Furthermore, using the fundamental signaling motif, it was further shown how a NAND gate could be formed, and that the suitable measure of energy required for the cycle to complete was equal to a single Adenosine triphosphate (ATP) reaction [31] [32].

The energy cost from one ATP reaction, in Joules, is approximately equal to 20 K_BT, where K_B is the Boltzmann constant and it is assumed here that the system is operating at an absolute temperature, T = 300K. NAND gates are a universal gate, so it is clearly possible to build all further logic circuits from combinations of NAND gates based on the principles of Boolean algebra. In this work, the SR (set-reset) flip-flop is considered for each shift register unit. It can be constructed using four NAND gates, so each shift register unit requires four NAND gates. In addition, a NOT gate, and a two-input XOR gate can be formed using one and four NAND gates respectively 5]. Finally, the last energy assumption to be made is that for the work here, in agreement with the work in [28], the energy cost of synthesizing a molecule is approximately 2450 K_BT. In this work, the energy is worked in K_BT.

III. Error Correction Coding Techniques

In this paper three kinds of block codes are considered as candidates to enhance the performance of molecular communication systems, namely, Hamming codes, EG-LDPC codes and C-RM codes. As the aim of this work is to compare the coded and un-coded system performance, the BER for the coded system needs to be determined.

The channel model used for an un-coded molecular communication system was introduced in Section II. For the coded system, the information bits can be encoded within the nano-machines by multiplying the information polynomial with the generator polynomial [20], and then the information carrying molecule pulse is emitted into the channel. Aiming to use the same number of molecules as an un-coded system, the number of molecules used for calculation of the channel error probability for the coded system should be evaluated with a reduction of the number of molecules used for an un-coded system, (8), by multiplication with the coding rate. The BER can then be obtained after the different decoding schemes.

The coding techniques and the energy consumption required by the encoder and decoder will be discussed in turn. Detailed mathematical explanations concerning the operation and behavior of each code have been placed in the Appendix.

A. Hamming codes

A Hamming code is a simple linear block code denoted as (n_H,k_H), where $n_H = 2^m - 1$, is the block length, and $k_H = n_H - m$, is the message length with m ($m \geq 2$) parity check bits. Block codes can correct t errors in each block:

$$t = \left\lfloor \left(d_{Hmin} - 1\right)/2 \right\rfloor, \tag{9}$$

where d_{Hmin} is the minimum distance and for Hamming codes d_{Hmin} equals to 3. Therefore, Hamming codes can correct one error in each block. Fig. 3 shows an example of encoder and decoder for an m = 5 Hamming code.

Combining the ATP energy model introduced in Section II, a NOT, two-input XOR gate, and shift register unit will cost one, four, and four ATPs respectively. It will also be assumed that a multi-input NAND gate will require only one ATP [12]. Therefore, with reference to Fig. 3 for m = 3,4,5 Hamming codes, two two-input XOR gates and m shift registers are needed for each circuit of the encoder, which implies the energy cost of encoding is:

$$E_{encode-H} = 20N_{tx}\left(4m+8\right) + 2450N_{tx}\ . \tag{10}$$

For m = 3,4,5 Hamming codes, three two-input XOR gates, $(m + n_H)$ shift registers, $(m - 1)$ NOT gates and one multi-input NAND gate are needed for each decoder circuit, which implies the energy cost of the Meggit decoder is:

$$E_{decode-H} = 20N_{rx}\left(5m+4n_H+12\right) + 2450N_{rx}, \tag{11}$$

where N_{tx} and N_{rx} are the number of code generation molecules required to encode and decode the data as recommended in [34]. In this work, these molecules are considered as different to the information molecules, as they are internal molecules and are only used for the encoding and decoding process within the nano-machines and do not suffer from any effects caused by diffusion. In order to reduce the effects that come from the biochemical intrinsic distortion, it is assumed that $N_{tx} = N_{rx}$ = 300 [12].

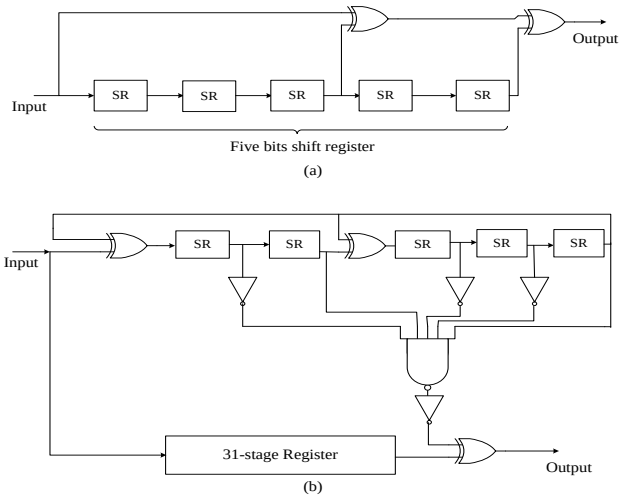


Fig. 3. Nonsystematic encoder [35] (a) and Meggit decoder (b) [35] for m = 5 Hamming code.

B. EG-LDPC codes

An EG-LDPC code can be constructed based on the lines and points of Euclidean geometry. In this paper, a special case: cyclic two dimensional EG-LDPC codes is considered [19] [36]. From this point onwards, to simplify the nomenclature, the LDPC codes mentioned below are all assumed to be cyclic and two dimensional [19] [37].

In general terms, LDPC codes can be represented as (n_L,k_L), where $n_L = 2^{2s} - 1, (s \geq 2)$ is the block length and $k_L = 2^{2s} - 3^s$ is the message length [19]. This kind of code has the minimum distance $d_{Lmin} = 2^s + 1$. And finally, the number of 1's for row and column also as the weights of the parity-check matrix which are given as $\rho = 2^s$ and $\gamma = 2^s$ respectively [19]. In this work, three LDPC codes are considered which are s = 2 LDPC code with d_{Lmin} = 5, s = 3 LDPC code with d_{Lmin} = 9 and s = 4 LDPC codes with d_{Lmin} = 17.

Considering the LDPC hardware requirements, as shown in Fig. 4, the encoding process can be achieved using simple feedback shift registers and subsequently decoded using a one-step majority logic decoding method. Fig. 5 shows an example of the majority logic gate (MLG) used in this decoding method where an output of one is produced when more than half of its inputs are equal to one, otherwise the output is zero.

Using the same energy model as with the Hamming codes, for s = 2,3,4 LDPC codes, $(n_L - k_L)$ shift registers are used and the number of two-input XOR gates in the circuits is dependent upon the generator polynomial of each code. The energy cost of encoding is therefore:

$$E_{encode-s2} = 20N_{tx}\left(4\left(n_L - k_L\right)+16\right) + 2450N_{tx}\ , \tag{12}$$

$$E_{encode-s3} = 20N_{tx}\left(4\left(n_L - k_L\right)+40\right) + 2450N_{tx}\ , \tag{13}$$

$$E_{encode-s4} = 20N_{tx}\left(4\left(n_L - k_L\right)+180\right) + 2450N_{tx}\ . \tag{14}$$

In addition, for different LDPC codes, the decoding circuits can be modified with ρ-input XOR gates, γ-input MLGs, and n_L buffer registers. The multi-input XOR gate can be obtained by using the combination of multiple two-input XOR gates. Here

for s = 2,3,4 LDPC codes, the energy cost of decoding is therefore:

$$E_{decode-s2} = 20N_{rx}\left(4n_L + 57\right) + 2450N_{rx}, \quad (15)$$

$$E_{decode-s3} = 20N_{rx}\left(4n_L + 321\right) + 2450N_{rx}, \quad (16)$$

$$E_{decode-s4} = 20N_{rx}\left(4n_L + 27297\right) + 2450N_{rx}. \quad (17)$$

C. C-RM codes

RM codes are a class of binary codes with multiple error correction capabilities. Here the way in which they are constructed as cyclic codes is introduced. This basis on cyclic codes means they are termed C-RM codes. The r^{th} order C-RM codes can be represented as C-RM(r,l) with a block length $n_R = 2^l - 1$, and the minimum distance $d_{Rmin} = 2^{l-r} - 1$ [35] is exist for any integer, $l \geq 2$ and $0 \leq r < l - 1$. The message length k_R can be calculated as:

$$k_R = 1 + \binom{l}{1} + \binom{l}{2} + \cdots + \binom{l}{r} = n_R - \sum_{z=1}^{l-r-1} \binom{l}{z}. \quad (18)$$

Considering the C-RM hardware requirements, as shown in Fig. 6, the encoding process can be achieved using simple feedback shift registers and subsequently decoded using a multiple-step majority logic method. For any J-input MLGs, the number of NAND gates $N_{NAND-MLGs}$ can be calculated as:

$$N_{NAND-MLGs} = \begin{cases} \sum_{i=J/2+1}^{J-1} \binom{J}{i} + 1, & J \neq 2 \\ 2, & J = 2 \end{cases}. \quad (19)$$

Using the same energy model as with the Hamming and LDPC codes, for C-RM codes, ($n_R - k_R$) shift registers are used, and the number and the location of the two-input XOR gates in the circuits are dependent upon the generator polynomial of each code. The energy cost of encoding is therefore:

$$E_{encode-RM(1,3)} = 20N_{tx}\left(4\left(n_R - k_R\right) + 8\right) + 2450N_{tx}, \quad (20)$$

$$E_{encode-RM(1,4)} = 20N_{tx}\left(4\left(n_R - k_R\right) + 24\right) + 2450N_{tx}, \quad (21)$$

$$E_{encode-RM(2,4)} = 20N_{tx}\left(4\left(n_R - k_R\right) + 8\right) + 2450N_{tx}, \quad (22)$$

$$E_{encode-RM(2,5)} = 20N_{tx}\left(4\left(n_R - k_R\right) + 40\right) + 2450N_{tx}, \quad (23)$$

$$E_{encode-RM(3,5)} = 20N_{tx}\left(4\left(n_R - k_R\right) + 8\right) + 2450N_{tx}. \quad (24)$$

In general, an r^{th} order C-RM code can be decoded with a (r + 1)-step majority logic decoder. For these decoding circuits, the total number, N_{ML}, of the J-input MLGs used in the circuit can be analyzed as [38]:

$$N_{ML} = 1 + J + J^2 + \cdots + J^{L-1}, \quad (25)$$

where $J = d_{Rmin} - 1$, and $L = r + 1$ is the number of steps used in the majority logic decoder.

The multi-input XOR gates used in majority vote process can be obtained by using the combination of multiple two-input XOR gates and the number of inputs of the XOR gate is dependent on the check polynomial. In this work, the two-input MLGs are used in C-RM(1,3), C-RM(2,4) and C-RM(3,5) decoders' design, six-input MLGs are used in the C-RM(1,4) and C-RM(2,5) decoders' design. According to (19), the two-input MLG and the six-input MLG can be formed by 2 and 22 NAND gates.

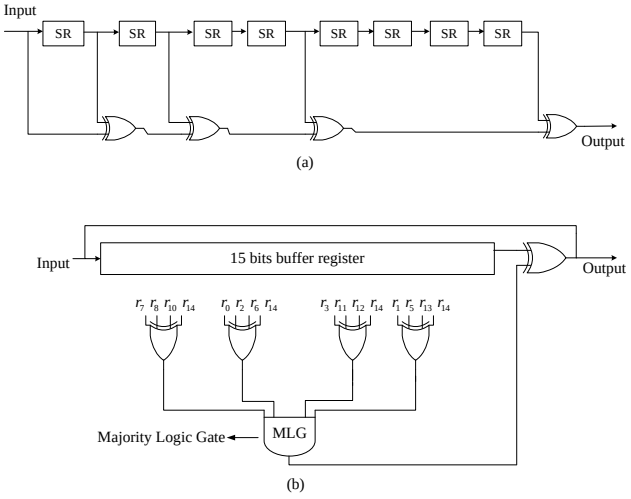


Fig. 4. Nonsystematic encoding circuit (a) and one-step majority logic decoder (b) [22] for the s = 2 LDPC codes.

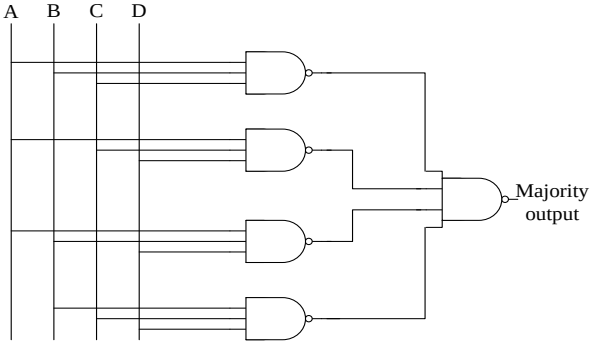


Fig. 5. Four-input MLG implementation circuit.

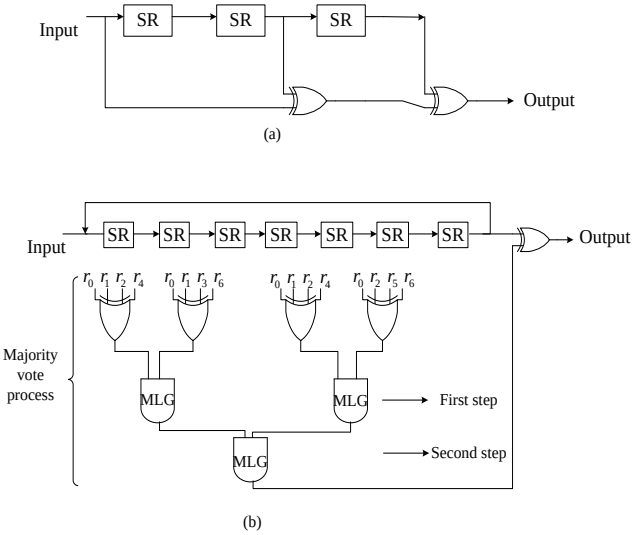


Fig. 6. (a) Non-systematic encoding circuit and (b) a two-step majority logic decoder for the C-RM(1,3) codes [39].

In addition, n_R-stage buffer registers and an extra two-input XOR gate are also needed. Here for C-RM codes, the energy cost of decoding is therefore:

$$E_{decode-RM(1,3)} = 20N_{rx}\left(4n_R + 58\right) + 2450N_{rx}, \quad (26)$$

$$E_{decode-RM(1,4)} = 20N_{rx}\left(4n_R + 590\right) + 2450N_{rx}, \quad (27)$$

$$E_{decode-RM(2,4)} = 20N_{rx}\left(4n_R + 242\right) + 2450N_{rx}, \quad (28)$$

$$E_{decode-RM(2,5)} = 20N_{rx}\left(4n_R + 6998\right) + 2450N_{rx}, \quad (29)$$

$$E_{decode-RM(3,5)} = 20N_{rx}\left(4n_R + 994\right) + 2450N_{rx}. \quad (30)$$

IV. Simulation

In a binary channel, ISI is the biggest inhibitor to the performance of a system due to the overlap between the symbols sent in the current time slot, and the symbols sent from the previous time slots. Here, using (8) a binary channel with an ISI length of 10 is considered. Through the encoding and decoding process after 10^{11}/(message length) consecutive blocks, the BER for the coded system can be obtained.

The BER results for both un-coded and coded Hamming, LDPC and C-RM systems are shown in Fig. 7. At this stage, the idea of coding gain can be introduced as a more defined measure of the performance gain from the implementation of coding. It can be directly obtained as:

$$G_{coding} = 10 \times \log\left(\frac{N_{uncoded}}{N_{coded}}\right), \quad (31)$$

where $N_{uncoded}$ and N_{coded} are the number of molecules for un-coded and coded system at a BER level. For example, at the BER level of say, 10^{-9}, the coding gain of the m = 3,4,5 Hamming codes, s = 2,3,4 LDPC codes and C-RM(1,3),(1,4),(2,4),(2,5),(3,5) are shown to be 1.59dB, 2.41dB, 2.77dB, 2.89dB 5.56dB, 7.26dB, 1.58dB, 3.23dB, 2.40dB, 4.46dB and 2.80dB respectively. Hence, of all these error correction schemes considered, the s = 4 LDPC code will provide the highest system performance. However, whilst these codes clearly show gains in the performance of molecular communication systems, there is an extra energy cost that was required to produce this improvement. The energy cost for an un-coded and a coded systems can be calculated as:

$$E_{uncoded} = 2450N_{uncoded}, \quad (32)$$

$$E_{coded} = 2450N_{coded} + E_{encode} + E_{decode}. \quad (33)$$

The energy saving (or loss) ΔE for a coded molecular communication system is defined as [12]:

$$\begin{aligned}\Delta E &= E_{uncoded} - E_{coded} \\ &= 2450\left(N_{uncoded} - N_{coded}\right) - E_{encode} - E_{decode}\end{aligned}, \quad (34)$$

where E_{encode} and E_{decode} are the energy requirements for encoding and decoding process. It is therefore easy to see that when $\Delta E \geq 0$, the use of ECC is beneficial to the molecular communication system. Furthermore, the critical distance [13] is defined as the distance at which the coding gain was matched with the extra energy requirements introduced by the use of the ECC, that is ΔE = 0, such that (34) reduces to:

$$N_{uncoded} - N_{coded} = (E_{encode} + E_{decode})/2450. \quad (35)$$

By substitution of the energy consumption values in (10) and (11), in (12) through (17), in (20) through (24) and in (26) through (30), the relationship of $N_{uncoded}$ and N_{coded} can be obtained for each code.

Two ranking systems are shown in analyzing the performance of the ECCs. The main ranking system is the

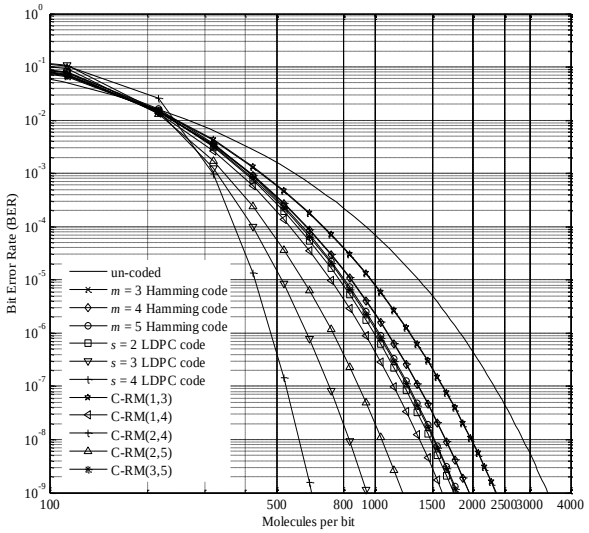


Fig. 7. BER comparison for coded and un-coded system with m = 3 to 5 Hamming codes, s = 2 to 4 LDPC codes, C-RM(1,3),(1,4),(2,4),(2,5),(3,5) and r = 6μm, I = 10.

critical distance as described so far. However, there are cases where there is no critical distance as in fact, in those cases, the use of ECC is always beneficial. In these cases, the ECC is ranked purely upon the total energy (33) use with the best being the one with the lowest overall energy consumption.

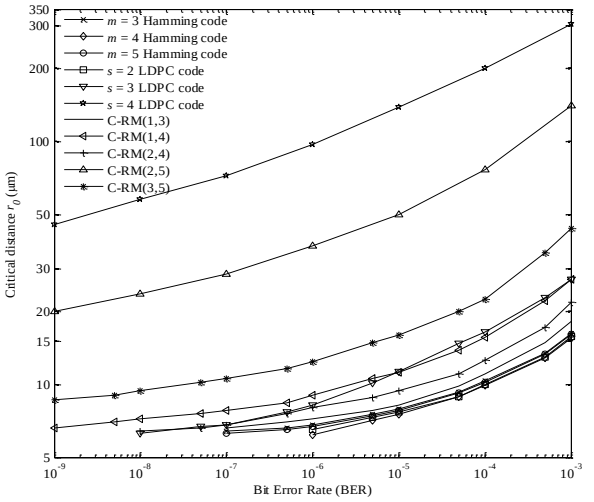
The critical distance is affected by two, not necessarily linear factors. Firstly, an obvious relationship in the system performance exists for each of the different coding schemes at different distance, as shown in Fig. 7. Secondly, the encoder and decoder circuitry for each of the codes is different, with varying degrees of complexity such as those in Fig. 3, Fig. 4 and Fig. 6. Fig. 8 provides our key critical distance results for the different Hamming, LDPC and C-RM codes over a BER range of 10^{-9} to 10^{-3}. Fig. 8 (a) considers the critical distance when all energy within the system is accounted for. Clearly from the results shown, the critical distances of m = 4 Hamming and s = 2 LDPC codes exist in a small BER range from 10^{-6} to 10^{-3}, and the lowest critical distance is belongs to m = 4 Hamming. Under the same communication architecture, Fig. 9(a) shows the energy cost for coded system at those BER levels where the critical distance does not exist. It is clearly shows that s = 2 LDPC code gives the lowest energy cost of coded system from 10^{-9} to 10^{-7}. So consider the communication system operate at 10^{-6} to 10^{-3} BER levels, m = 4 Hamming codes is the first choice with lowest critical distance from 6.2μm at 10^{-9}, otherwise, s = 2 LDPC code with the lowest energy cost from $5478200K_BT$ at 10^{-7} should be selected. There also exists another nano-communications architecture that a nano-machine transmitting information to macro-machine which is not constrained by the same power budget. Therefore, if one considers such a system, and assumes the extra energy comes from the encoder only, i.e. that E_{decode} = 0, then the critical distance and energy cost results can be seen in Fig. 8(b) and Fig. 9(b). Here, the interesting observation is most of codes are beneficial for molecular communication system that

operating between 10^{-9} and 5×10^{-5} BER levels with any transmission distance except s = 4 LDPC code. In this case, the analysis should be focus on the energy cost shows in Fig. 9(b), when considering the system operating at 10^{-9} to 10^{-6} BERs, the s = 3 LDPC should be considered as the designer's first choice with the lowest energy cost from 3113000 K_BT at 10^{-6}. For BER levels from 10^{-4} to 10^{-3}, the m = 4 Hamming and C-RM(3,5) have the same and also the lowest critical distance from 6.1μm at 10^{-4}.

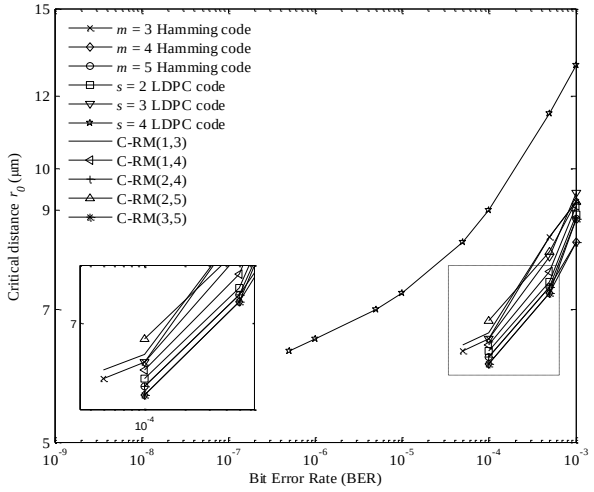
Similarly, there may be system designs whereby a macro-machine is communicating with a nano-machine, such that, it is critical to minimize the decoding energy costs with an assumption for now that the encoding costs can be contained within a larger energy budget. This scenario, setting, E_{encode} = 0, where only the energy cost of the decoder is considered as an extra energy, the results are shown in Fig. 8(c) and Fig. 9(c). Here, if the system is operating at 10^{-5} to 10^{-3} BERs, then the s = 2 LDPC provides the shortest critical distance for which coding becomes beneficial. For operating BERs lower than 10^{-5}, Fig. 9(c) shows that the lowest energy cost for coded system which is also belongs to s = 2 LDPC code, so in this case, the s = 2 LDPC code becomes the best choice for molecular communication system that operating between 10^{-9} and 10^{-3} BER levels.

V. Conclusions

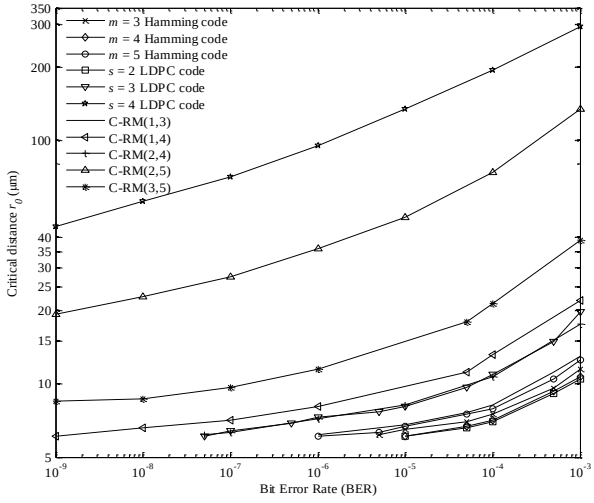
In the paper, LDPC and C-RM codes have been introduced into a molecular communication system for the first time and the performance enhancements they bring have been compared against Hamming codes with respect to both coding gain and energy requirements. It has been shown that the coding gain for m = 5 Hamming and C-RM(2,5) can be as high as 2.77dB and 4.46dB whilst s = 4 LDPC can be as high as 7.26dB at a BER of 10^{-9}. With a strong emphasis on not hiding the cost of this gain, this paper has further analyzed, under three different scenarios, how much energy these gains cost, by defining the distances at which the use of coding becomes beneficial. Furthermore, the energy costs for the coded systems under these scenarios are also taken into account when the critical distance does not exist. According to the results, the designer can determine which code should be employed by the critical distance and the energy cost for different operating BERs. It is shown that when considering a case of nano- to nano-machine communications operating between 10^{-9} and 10^{-7} BER levels, s = 2 LDPC code is the obvious choice which own the lowest energy cost 5478200 K_BT at a BER of 10^{-7}, when operating BER level is larger than 10^{-6}, the m = 4 Hamming codes are the obvious choice over distances greater than 6.2μm. And if a nano-machine is communicating with a macro-machine, m = 4, 5 Hamming, s = 1, 2 LDPC and C-RM(1,4),(2,4),(2,5),(3,5) are beneficial over all transmission distances when the working BER levels of the communication system is less than 10^{-4} , and based on this, the s = 3 LDPC code with the lowest energy consumption is worthy for the design when the system operate between 10^{-9} and 10^{-6}. Inversely, considering the BER levels higher than 10^{-4}, the m = 4 Hamming and C-RM(3,5) are worthy for the design when the transmission distance higher than 6.1μm. When a



(a)



(b)



(c)

Fig. 8. Critical distance r_0 with BER for Hamming codes from m = 3 to m = 5, LDPC codes from s = 2 to s = 4 and C-RM(1,3),(1,4),(2,4),(2,5),(3,5) when (a) the communication process energy consumption is considered (b) only the transmission process energy consumption is considered (c) only the receiving process energy consumption is considered.

macro-machine is communicating with a nano-machine, the $s = 2$ LDPC is the best choice for the designer when the system BER requirements from 10^{-9} to 10^{-3}.

APPENDIX
CODING TECHNIQUES

A. Hamming codes

Here the Hamming codes are considered as cyclic Hamming codes [35] which the generator polynomials of $m = 3,4,5$ are given by $g(x) = x^3 + x + 1$, $g(x) = x^4 + x + 1$, and $g(x) = x^5 + x^2 + 1$respectively, so the encoded message can be obtained by multiplying the information polynomial with the generator polynomial. This encoding process can be realized using a nonsystematic encoder where the output code-word does not contain the information sequence [40], whilst the encoded message can be easily decoded using a Meggit decoder [35]. The syndromes required for testing the error patterns for $m = 3,4,5$ Hamming codes are configured as $s(x) = x^2 + 1$, $s(x) = x^3 + 1$ and $s(x) = x^4 + x$ respectively. The Meggit decoder is considered for Hamming codes.

B. EG-LDPC codes

Before fully describing EG-LDPC codes, finite field is introduced. Finite field, also called Galois Field (*GF*) is a finite set of elements which can be added, multiplied and divided with the results being an element of the set. The reason they are important here is that ECCs based upon these fields can be efficiently encoded and decoded [36], which for molecular communications, is a major concern.

$EG(m,q)$ is the m - dimensional Euclidean geometry over $GF(q)$, where m is a positive integer greater than one, $q = p^t$, $t \geq 1$ and p is a prime number [39]. An EG-LDPC code can be constructed based on the lines and points of Euclidean geometry. In this paper, a special case: cyclic two dimensional EG-LDPC codes is considered [19], [36].

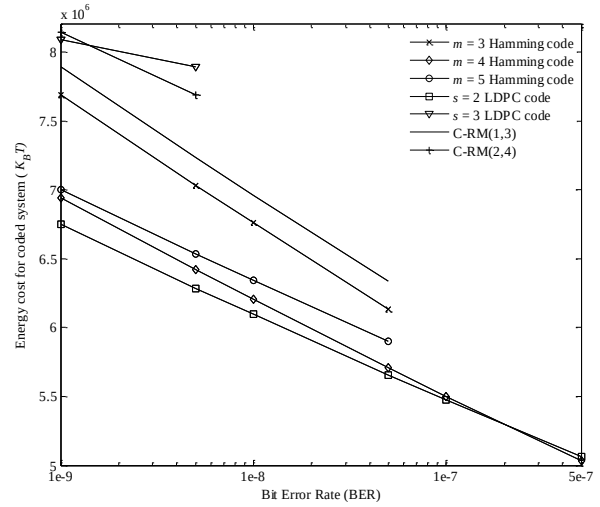
Considering that $EG(2,2^s)$ is a Euclidean geometry on $GF(2^s)$, where each point is a 2-tuple over $GF(2^s)$ and where an all zero 2-tuples can be called the origin, there are 2^{2s} points and $2^s(2^{2s} - 1)/(2^s - 1)$ lines. $GF(2^{2s})$ is an extended field $GF(2^s)$ so each element in $GF(2^{2s})$ can be referred as a 2-tuple over $GF(2^s)$, which means that 2^{2s} elements in $GF(2^{2s})$ can be regarded as 2^{2s} points in $EG(2,2^s)$ [19], [41], [42]. Assuming α is a primitive element of $GF(2^{2s})$ means all the non-zero elements in $GF(2^{2s})$ can be represented as α^i, where i is positive integer. Let α^j be a non-origin point, so a line in $EG(2,2^s)$ can be formed from the 2^s points in $EG(2,2^s)$, shown as:

$$\left\{\eta\alpha^j\right\} \triangleq \left\{\eta\alpha^j : \eta \in GF\left(2^s\right)\right\}. \tag{36}$$

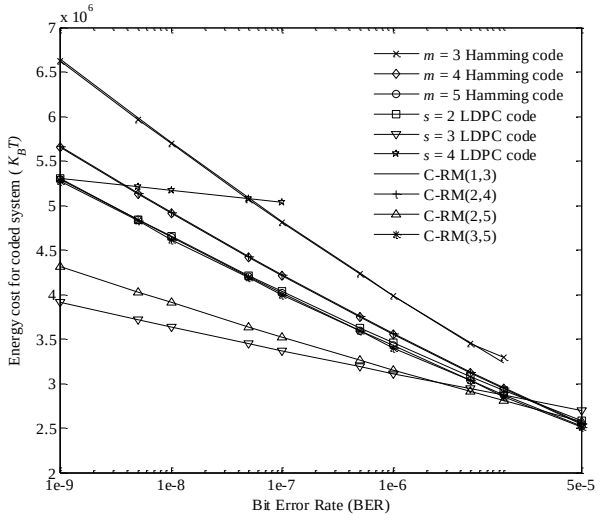
Then let α^i and α^j be two independent points, so a line passing through α^j can be formed as:

$$\left\{\alpha^i + \eta\alpha^j\right\} \triangleq \left\{\alpha^i + \eta\alpha^j : \eta \in GF\left(2^s\right)\right\}. \tag{37}$$

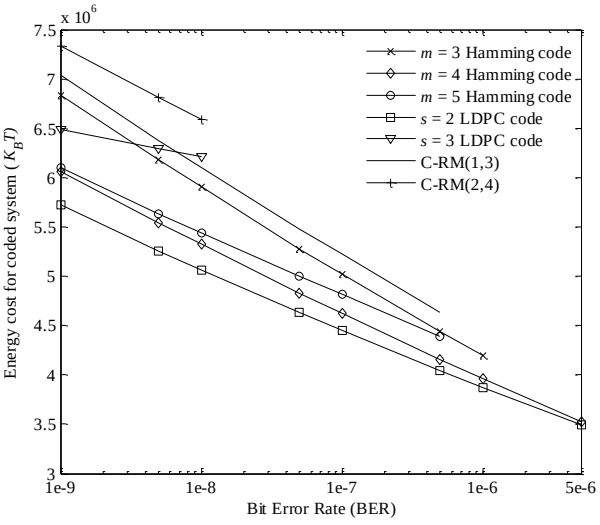
Each line in $EG(2,2^s)$ can be indicated as a vector of length 2^{2s}. For this LDPC code, the rows of the parity check matrix $H_{EG}(2,s)$ correspond with the $2^{2s} - 1$ lines that do not pass through the origin in $EG(2,2^s)$, and the columns correspond with the $2^{2s} - 1$ non-origin points in $EG(2,2^s)$.



(a)



(b)



(c)

Fig. 9. Energy cost for coded system with BER for Hamming codes from $m = 3$ to $m = 5$, LDPC codes from $s = 2$ to $s = 4$ and C-RM(1,3),(1,4),(2,4),(2,5),(3,5) when (a) the communication process energy consumption is considered (b) only the transmission process energy consumption is considered (c) only the receiving process energy consumption is considered.

Therefore the parity check matrix, $H_{EG}(2,s)$ is a $(2^{2s}-1)\times(2^{2s}-1)$ square matrix and it can be constructed easily by taking the incidence vector of a line in $EG(2,2^s)$ that does not pass through the origin and then cyclically shifting this vector $2^{2s}-2$ times. This LDPC code is a kind of cyclic code [19], so the generator polynomial $g_{EG}(x)$ can be obtained from the roots in $GF(2^{2s})$. h can be expressed in radix – 2^s form as:

$$h=\delta_0+\delta_1 2^s, \tag{38}$$

where $h<2^{2s}$ and $0\le\delta<2^s$, $0\le i<2$. $W_2{}^s(h)$ denotes the 2^s weight of h, shown as:

$$W_{2^s}(h)=\delta_0+\delta_1. \tag{39}$$

For a nonzero integer l, let $h^{(l)}$ be the remainder of $2^l h/(2^{2s}-1)$, where $0\le h^{(l)}<2^{2s}-1$ then α^h is a root of $g_{EG}(x)$ if and only if:

$$0<\max_{0\le l<s} W_{2^s}\left(h^{(l)}\right)\le 2^s-1. \tag{40}$$

For the $s=2$ LDPC code, consider $EG(2,2^s)$ and let α and β be the primitive elements of $GF(2^{2\times2})$ and $GF(2^2)$ respectively. Given $p(x)=x^4+x+1$ is the primitive polynomial of $GF(2^{2\times2})$, it is easy to prove that $\beta=\alpha^5$ because of $\beta^3=\alpha^{15}=1$. Therefore $\eta\in\{0,1,\beta,\beta^2\}$ or $\{0,1,\alpha^5,\alpha^{10}\}$ constitute $GF(2^2)$ Let $\alpha^i=\alpha^{14}$ and $\alpha^j=\alpha$, so one of the lines of $EG(2,2^2)$ can be obtained from (37) as:

$$\left\{\alpha^7,\alpha^8,\alpha^{10},\alpha^{14}\right\}\in\left\{\alpha^{14}+\eta\alpha:\eta\in\left\{0,1,\alpha^5,\alpha^{10}\right\}\right\}. \tag{41}$$

These four points in a line do not pass through the origin, so the parity check matrix H_{EG} can be formed using the corresponding binary incidence vector (0 0 0 0 0 0 0 1 1 0 1 0 0 0 1) and its $2^{2s}-2=14$ circulations.

From (38), (39) and (40), the roots of the generator polynomial can be obtained as:$\{\alpha^1,\alpha^2,\alpha^3,\alpha^4,\alpha^6,\alpha^8,\alpha^9,\alpha^{12}\}$. $\alpha^1,\alpha^2,\alpha^4,\alpha^8$ all have the same minimal polynomial: $m_1(x)=x^4+x+1$, whilst $\alpha^3,\alpha^6,\alpha^9,\alpha^{12}$ all have the same minimal polynomial: $m_2(x)=x^4+x^3+x^2+x+1$, then the lowest common multiple (LCM) can be obtained as the generator polynomial: $g_{EG}(x)=\mathrm{LCM}[m_1(x),m_2(x)]=x^8+x^7+x^6+x^4+1$. In the same way, the generator polynomials for the $s=3,4$ LDPC codes are given by $g_{EG}(x)=x^{26}+x^{24}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{10}+x+1$ and $g_{EG}(x)=x^{80}+x^{78}+x^{76}+x^{74}+x^{71}+x^{69}+x^{68}+x^{67}+x^{66}+x^{64}+x^{63}+x^{61}+x^{59}+x^{58}+x^{55}+x^{54}+x^{51}+x^{49}+x^{47}+x^{45}+x^{42}+x^{40}+x^{39}+x^{38}+x^{37}+x^{36}+x^{27}+x^{26}+x^{25}+x^{23}+x^{22}+x^{21}+x^{19}+x^{18}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{11}+x^{10}+x^9+x^7+x^6+x^3+1$.

The majority logic decoding scheme is considered using for the LDPC decoder design.

C. C-RM codes

Given a nonnegative integer s, where $1\le s\le 2^m-2$, the number of 1's in the binary expansion of s can be denoted as $w_2(s)$, then α^s are the roots of the generator polynomial $g(x)$ if and only if [39]:

$$1\le w_2(s)\le m-r-1. \tag{42}$$

α^s are the roots of the check polynomial $h(x)$ if and only if:

$$m-r\le w_2(s)\le m-1. \tag{43}$$

Therefore, the generator and check polynomials for the C-RM codes are:

$$g(x)=\prod_{\substack{1\le w_2(s)\le m-r-1\\ 1\le s\le 2^m-2}} M^{(s)}(x), \tag{44}$$

$$h(x)=(x+1)\prod_{\substack{m-r\le w_2(s)\le m-1\\ 1\le s\le 2^m-2}} M^{(s)}(x), \tag{45}$$

where $M^{(s)}(x)$ is the minimal polynomial of α^s.

Considering the C-RM(1,4) code, all integers s satisfying (42) are {1, 2, 3, 4, 5, 6, 8, 9, 10, 12} and the integers s which satisfy (43) are {7, 11, 13, 14}, so the roots of the generator polynomial can be obtained as:$\{\alpha^1,\alpha^2,\alpha^3,\alpha^4,\alpha^5,\alpha^6,\alpha^8,\alpha^9,\alpha^{10},\alpha^{12}\}$, where $\alpha^1,\alpha^2,\alpha^4,\alpha^8$ have the same minimal polynomial: $m_1(x)=x^4+x+1$, and $\alpha^3,\alpha^6,\alpha^9,\alpha^{12}$ have the same minimal polynomial: $m_2(x)=x^4+x^3+x^2+x+1$, and the minimal polynomial of α^5 and α^{10} is $m_3(x)=x^2+x+1$. According to (44), the generator polynomial can be obtained as: $g_{(1,4)}(x)=m_1\cdot m_2\cdot m_3=x^{10}+x^8+x^5+x^4+x^2+x+1$. Similarly, the roots of the check polynomial can be obtained as: $\{\alpha^7,\alpha^{11},\alpha^{13},\alpha^{14}\}$, where $\alpha^7,\alpha^{11},\alpha^{13},\alpha^{14}$ have the same minimal polynomial: $m_1(x)=x^4+x^3+1$. According to (45), the check polynomial is given as: $h_{(1,4)}(x)=(x+1)\cdot(x^4+x^3+1)=x^5+x^3+x+1$.

In the same way, the generator and check polynomials for the C-RM(1,3), (2,4), (2,5), and (3,5) codes are given by: $g_{(1,3)}(x)=x^3+x+1$, $h_{(1,3)}(x)=x^4+x^2+x+1$, $g_{(2,4)}(x)=x^4+x+1$, $h_{(2,4)}(x)=x^{11}+x^8+x^7+x^5+x^3+x^2+x+1$, $g_{(2,5)}(x)=x^{15}+x^{11}+x^{10}+x^9+x^8+x^7+x^5+x^3+x^2+x+1$, $h_{(2,5)}(x)=x^{16}+x^{12}+x^{11}+x^{10}+x^9+x^4+x+1$, and $g_{(3,5)}(x)=x^5+x^2+1$, $h_{(3,5)}(x)=x^{26}+x^{25}+x^{24}+x^{23}+x^{22}+x^{21}+x^{18}+x^{14}+x^{13}+x^{12}+x^8+x^7+x^6+x^5+x^2+1$.

In this work, the majority logic decoder is used for C-RM code.

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