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Monetary Policy and Welfare in a Currency Union

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Abstract

What are the welfare gains from being in a currency union? I explore this question in the context of a dynamic stochastic general equilibrium model with monetary barriers to trade, local currency pricing and incomplete markets. The model generates a tradeoff between monetary independence and monetary union. On one hand, distinct national monetary authorities with separate currencies can address business cycles in a country-specific way, which is not possible for a single central bank. On the other hand, short-run violations of the law of one price and long-run losses of international trade occur if different currencies are adopted, due to the inertia of prices in local currencies and to the presence of trade frictions. I quantify the welfare gap between these two international monetary arrangements in consumption equivalents over the lifetime of households, and decompose it into the contributions of different frictions. I show that the welfare ordering of alternative currency systems depends crucially on the international correlation of macroeconomic shocks and on the strength of the monetary barriers affecting trade with separate currencies. I estimate the model on data from Italy, France, Germany and Spain using standard Bayesian tools, and I find that the tradeoff is resolved in favour of a currency union among these countries.

JEL Classification: D52, E31, E32, E42, E52, F41, F44, F45

Keywords: Currency union; Incomplete markets; Nominal rigidities; Local currency pricing; Trade frictions; Welfare.

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1 Introduction

The fundamental question of the gains and losses from monetary integration has been of practical importance for decades in Europe¹. One aspect of monetary unification that has attracted particular attention from economists is the issue of how countries can handle idiosyncratic disturbances and asymmetric business cycles under a common monetary policy². This point is of particular concern to Eurozone members today, as they have surrendered independent interest-rate and exchange-rate policies and are left with a very limited capacity to implement countercyclical fiscal policies³.

Since the European Monetary Union (EMU) currently lacks both a formal system of interstate insurance (the so-called fiscal or transfer union) and a full degree of internal labour mobility—two crucial elements for the viability of a union according to the optimum currency area theory—its adjustment to macroeconomic shocks is characterised by cross-country heterogeneity. The varied pattern of responses to the recent financial crisis is a case in point. Some Eurozone members have suffered a sharper and longer lasting recession than others: quarterly data from the Federal Reserve Bank of St. Louis show that at the depth of the recession in the first quarter of 2009 real gross domestic product (GDP) fell by 7.4% in Italy, 13.5% in Spain, 5.7% in France and 6% in Germany. Seven years after the global financial crisis broke out, output in Portugal, Italy, Greece, Spain and Ireland is still below its pre-crisis level, while this is no longer the case for Germany, as observed by [Frankel \(2015\)](#). Unequal developments in real economic activity are associated with heterogeneous price dynamics: according to Eurostat data, the annual inflation rate is 0.9% in Portugal and -1.1% in Spain as of September 2015⁴, for instance. These differences are problematic from the point of view of the central monetary authority, because they imply adverse cross-country differentials in real interest rates in the face of a uniform nominal interest rate at the union level.

In this paper I contribute to the debate on the desirability of currency unions by constructing and estimating a dynamic stochastic general equilibrium (DSGE) model that incorporates different dimensions along which a monetary union might differ from an economy with multiple currencies. In particular, the model includes three competing effects of monetary unification: (i) the loss of monetary policy independence caused by the establishment of a unique central bank; (ii) the elimination of the price misalignments associated with nominal rigidities in local currencies; (iii) the expansion of trade enabled by the use of a single currency in international

¹[Corden \(1972\)](#) and [Ingram \(1973\)](#) are early academic discussions of these themes from a European monetary integration perspective; they have antecedents in the pathbreaking theoretical contributions by [Mundell \(1961\)](#) and [McKinnon \(1963\)](#). Several waves of empirical and theoretical research have followed these studies; a recent review of this literature is [Santos-Silva and Tenreyro \(2010\)](#).

²[Obstfeld and Peri \(1998\)](#) and [Fatás \(1998\)](#) are notable contributions to this debate.

³See the remarks by [Feldstein \(2015\)](#) and others at the 2015 AEA Annual Meetings session entitled “When will the Euro crisis end?”.

⁴Eurostat measures the annual rate of inflation as the change in the harmonised index of consumer prices between a given month and the same month of the previous year.

transactions. I model these effects in a unified framework characterised by international heterogeneity and imperfect insurance, and evaluate their importance from the point of view of social welfare. I estimate my model of a monetary union with data from Italy, France, Germany and Spain, I evaluate welfare using the utility of households as a criterion, and I run counterfactual scenarios to assess what welfare would be if these countries had separate national currencies and independent monetary authorities. I find that these economies enjoy a higher welfare if they are in a union. I show that this result is given by the features of the business cycles of these countries and by the strength of the transaction frictions that would affect trade if they did not use a common currency.

To gain a deeper understanding of the mechanisms that generate this result, let us examine the three effects above more closely and see how they affect welfare. On one hand, giving up the ability to set monetary policy at a national level is costly if business cycles are asynchronous across countries, because separate instruments should be used to stabilise them. The welfare cost of conducting countercyclical policy with a single instrument is a differential measure: it does not only depend on the volatility of macroeconomic shocks (which determines the absolute cost of business cycles), but also on how correlated these shocks are across countries. For this reason, the cost vanishes if business cycles are perfectly symmetric: in that case, the economies behave like one and distinct policy instruments are unnecessary. What makes this welfare cost non-trivial in general is the lack of international risk-sharing and the presence of nominal and real frictions in the economy. This is the first aspect of monetary integration I have identified above. On the other hand, the adoption of a single currency has two advantages. If producers discriminate between countries by setting prices in local currencies, national currencies are associated with inefficient price differentials across markets. Introducing a unique currency removes this problem, because it impedes this segmentation and creates a single market where products are sold at a uniform price. This is the second consequence of monetary integration I have determined above. On top of this, the use of a single currency in international commerce eliminates the transaction costs associated with the presence of multiple currencies, and suppresses a monetary barrier to trade; this improves consumption permanently. This is the third and last implication I have indicated above.

The net balance of these effects depends on their quantitative importance for welfare. To assess this, I model them in a fully-optimising setup based on the utility of households. In contrast with existing microfounded studies that analyse different aspects of monetary unification separately⁵, I bring them together in a unified framework; this allows me to examine how they interact to generate a tradeoff between alternative currency arrangements, and how

⁵Bacchetta and van Wincoop (2000), Ching and Devereux (2003) and Devereux et al. (2003) are important studies of the consequences of monetary unification based on microeconomic foundations. They focus on its welfare implications based on the effects on trade, risk-sharing and price-setting respectively. The first two works are based on static models, which are not amenable to estimation; the third one uses an infinite-horizon economy with complete international asset markets and full risk-sharing.

this tradeoff is resolved. Differently from earlier works on international monetary regimes, I take a quantitative approach and base my analysis on a model economy whose parameters are estimated with real-world data.

I cast my analysis in a setup with incomplete markets, local currency pricing and monetary barriers to trade. The backbone of my model is an otherwise standard open economy New Keynesian framework with nominal rigidities and monopolistic competition in the spirit of [Clarida et al. \(2002\)](#). It features two ex-ante identical economies that experience idiosyncratic technology, preference, labour supply and monetary shocks; the problem of ex-post heterogeneity is made explicit through the examination of how welfare is affected by the international comovement of these disturbances. The mechanisms at work in the model have antecedents in the two-region representations of monetary unions by [Benigno \(2004\)](#), [Beetsma and Jensen \(2005\)](#) and [Ferrero \(2009\)](#)⁶. However, while these works consider model economies where union members fully share risk—either via complete asset markets or by specific parameterisations of the elasticities of substitution that guarantee endogenous risk-sharing through the terms of trade—I study an environment where international risk-sharing is incomplete⁷.

One of the most relevant components of my model economy is its asset structure, which follows the characterisation of [Benigno \(2009\)](#). Markets are complete at the level of individual countries, but international trade in financial assets is limited to nominally risk-free bonds. This guarantees that the model has a simple representative agent formulation, while departures from full international risk-sharing occur over the business cycle. Unlike [Benigno \(2009\)](#), I compare and rank two different international monetary arrangements. In the first one, separate national currencies exist and monetary policy is controlled by two distinct authorities; these set nominal interest rates on an independent basis, following Taylor-type instrument rules based on country-specific targets. In the second one, a single currency is present and monetary policy is set by one central bank for the whole union, in accordance with a Taylor rule that depends on union-wide objectives. I model both regimes in a cashless economy.

Another important component of the model is the type of price-setting frictions it has. Nominal rigidities are combined with local currency pricing (LCP) à la [Engel \(2011\)](#) or [Corsetti et al. \(2011\)](#): firms set prices in the buyers' currency, and pricing opportunities arrive at random intervals following a conventional Calvo-Yun scheme. In the presence of separate national currencies, domestic and foreign buyers are charged two distinct prices for identical goods; these satisfy the law of one price in the long run, but violate it over the business cycle, causing a misallocation of resources. If a monetary union is in place, instead,

⁶[Galí and Monacelli \(2005\)](#) and [Forlatti \(2009\)](#) are alternative perspectives where the union is modelled as a continuum of small-open economies.

⁷A recent examination of the monetary aspects of currency unions with imperfect insurance and heterogeneity is [Bhattarai et al. \(2015\)](#), which draws on the setup laid out by [Cúrdia and Woodford \(2010\)](#). While that work takes the international currency arrangement (i.e. the union) as given and searches for the optimal monetary policy in that environment, I compare the performance of different monetary systems under given, price stability-oriented policies.

the price is unique and the law of one price applies continuously⁸.

The last key component of the model is the real structure of the economy. This features full specialisation in production on the part of the two countries (with a standard two-stage manufacturing process in each place) and frictional international trade. The barrier to commerce has a monetary nature, and it only exists in the regime with two currencies; it takes the form of a [Lama and Rabanal \(2014\)](#)⁹ type linear “iceberg” shipping cost on imported products. I use this reduced-form feature as a stand-in for the various transaction costs that affect the international exchange of goods when multiple currencies are adopted, as documented empirically by [Rose \(2000\)](#) and [Rose and van Wincoop \(2001\)](#), among others. The effect of this friction is a long-run trade and consumption differential between the union and the monetary independence regime.

In order to quantify the importance of these frictions for social welfare, I follow [Schmitt-Grohé and Uribe \(2007\)](#) and define welfare as the expectation of households’ lifetime utility, conditional on the initial state of the economy being the nonstochastic steady state. To assess the relative performance of alternative currency systems, I measure the welfare differential in terms of consumption equivalents; that is, I define the welfare gap as the loss of lifetime consumption that makes households under monetary independence as happy as they would be under a union. A distinctive feature of my work is that I explicitly decompose this gap into the contribution of different frictions, paying special attention to the role of the international correlation of macroeconomic disturbances, or the lack thereof.

My examination of the welfare gap between monetary independence and union starts from a calibrated economy with frictionless trade, producer currency pricing (PCP) and uncertainty only from idiosyncratic technology shocks; it then proceeds with the introduction of one additional friction at a time. I first use the simplified economy to show that inflation and output are more volatile in a monetary union than they are under separate national currencies. I argue that this difference has fundamental welfare implications, and show that the welfare differential between the two regimes vanishes as shocks get perfectly correlated. I then introduce price discrimination back into the model, and show that the inertia of local-currency prices determines inefficient international price misalignments in the face of exchange rate movements. I argue that this friction per se does not alter the welfare ordering of the two regimes, because it leaves the steady state of the economy unaffected and only bites in the short run. Finally, I add trade frictions to the picture, and explain that they reduce imports

⁸[Engel \(2014\)](#) revises the main results from the literature on alternative export-pricing specifications and their impact on macroeconomic adjustment and exchange-rate stabilisation. The implications of LCP with sticky prices were first explored in a welfare-based model of monetary policy by [Devereux and Engel \(2003\)](#).

⁹[Lama and Rabanal \(2014\)](#) is a recent attempt to explore the currency area question explicitly from the perspective of monetary policy. The authors use an estimated two-country DSGE model to evaluate the welfare implications of unifying the United Kingdom and the Eurozone under a single currency. Their work is a close relative to mine from a methodological point of view, but it has an emphasis on financial stability issues and unconventional monetary policy measures.

demand and depress output at all horizons. I argue that this distortion reduces consumption and welfare in the economy with multiple currencies in the long run; this opens up the possibility that households experience a higher welfare with a single currency. The issue of the net balance between competing forces then becomes an empirical one.

My estimation strategy is as follows. I estimate the full-fledged model in its single-currency configuration with quarterly data from Italy, France, Germany and Spain; the estimation is performed on two countries at a time. For each pair, I compute welfare in the monetary union regime and then compare it with a counterfactual scenario where the two countries split and adopt national currencies, keeping identical Taylor rules but basing them on domestic objectives rather than common ones. I carry out the calculation under a calibration of the trade friction in line with [Lama and Rabanal \(2014\)](#), and then study the sensitivity of my results to this specific choice. Adopting standard Bayesian techniques, I find that the welfare gain from monetary integration is positive for these economies: its posterior mean is above 2% of lifetime consumption in all cases. The gain is largest for Italy and France, which appear to have the most correlated shocks, and is smallest for Spain and Germany.

I conclude with the perspective that a system of separate national currencies would be desirable if monetary barriers to trade were very small or entirely absent, because in that case there would be little or no trade gain from establishing a currency union. Based on numerical work, I show that a moderate amount of trade frictions is sufficient for the monetary union to guarantee the highest social welfare in the present setup. I attribute this result to the modest cost of business cycle asymmetries for the economies under scrutiny, and argue that the domain of applicability of a single currency could be affected significantly by the addition of further frictions that introduce cross-country heterogeneity along new dimensions.

The rest of this paper is organised as follows. Section 2 outlines the setup and illustrates its main dynamic properties. Section 3 defines a welfare measure to compare different monetary arrangements, and explores the dependence of the ranking upon the shocks and frictions of the model. Section 4 presents the Bayesian estimation and discusses the results. Section 5 concludes.

2 Monetary independence and union in a two-country model

This section is divided in two parts. In the first part, I present the optimisation-based setup that I use to assess the welfare benefits of a single currency. In the second part, I employ a simplified version of the model to illustrate how macroeconomic adjustment to asymmetric technology shocks differs across alternative currency arrangements.

I base my analysis on a dynamic stochastic general equilibrium model where the world is made up of two countries, h and f , each populated by a continuum of measure one of households with identical preferences. Employing the modeling strategy of [Benigno \(2009\)](#), I

assume that households can perfectly pool risks within their respective countries through a full set of state-contingent securities traded locally, but international trade in assets is limited to noncontingent bonds¹⁰. This asset markets structure is invariant to the international currency arrangement.

I consider two alternative international monetary regimes. In the first regime, called “monetary union”, the two countries share a single currency and monetary policy is controlled by a unique central bank; the bank follows a conventional interest rate rule based on union-wide inflation and output objectives. In the second regime, named “monetary independence”, the countries have separate national currencies and distinct monetary authorities; these follow analogous but independent interest rate rules that depend on country-specific policy targets. The economy is cashless, so monetary policy involves the direct control of nominal interest rates; central banks perform this task by choosing the price of nominal bonds with one-period maturity.

Each country specialises in the supply of one good, whose production takes place in two steps. First, monopolistically competitive firms produce a continuum of differentiated goods; price-setting is subject to Calvo-Yun nominal rigidities. Second, perfectly competitive firms aggregate locally-produced intermediates into final consumption goods, which are traded internationally. Within each country, firms are owned by the domestic households.

If different currencies exist, goods are priced in the currency of their destination market; that is, firms engage in Engel (2011) type local currency pricing, which implies that nominal exchange rate movements determine short-run international price misalignments. With a single currency, goods are uniformly priced across the union.

The international exchange of goods is affected by frictions in the regime with separate national currencies, but is free under a monetary union. I model these frictions as “iceberg” shipping costs that cause a fraction of imported goods to be lost in transit between the two countries. This feature is a stand-in for the monetary barriers to trade documented by Rose (2000) and Rose and van Wincoop (2001). The permanent trade gains offered by the elimination of this friction represent the key advantage of a currency union over a multicurrency system in my framework.

In the exposition that follows, I focus on the model with two currencies, which displays the richest notation. The differences with the monetary union are pointed out along the way.

2.1 The model

2.1.1 Households

Households make consumption, saving and labour supply decisions. They consume a bundle of domestic and foreign goods, competitively supply indifferenciated labour services to local

¹⁰To save on notation, I keep the Arrow securities implicit and restrict attention to the representative agents.

producers (whose profits they receive in a lump-sum fashion) and invest in nominally risk-free, one-period pure discount bonds. Households have access to both domestic currency-denominated and foreign currency-denominated bonds, so international borrowing and lending can take place in either currency.

Intertemporal optimisation The representative households of the home and foreign country solve the following dynamic problems:

$$\begin{aligned} & \max_{\{c_t, A_{t+1}, B_{t+1}, n_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \phi_t \frac{n_t^{1+\varphi}}{1+\varphi} \right) \\ \text{s.t. } & c_t + q_t^A \frac{A_{t+1}}{p_t} + e_t q_t^B \frac{B_{t+1}}{p_t} + \chi q_t^B \left(\frac{e_t B_{t+1}}{p_t} \right)^2 = \frac{A_t}{p_t} + e_t \frac{B_t}{p_t} + \frac{w_t}{p_t} n_t + \frac{p_{h,t}}{p_t} \Pi_{h,t} + \frac{T_t}{p_t} \end{aligned}$$

and

$$\begin{aligned} & \max_{\{c_t^*, A_{t+1}^*, B_{t+1}^*, n_t^*\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t^* \left(\frac{(c_t^*)^{1-\sigma}}{1-\sigma} - \phi_t^* \frac{(n_t^*)^{1+\varphi}}{1+\varphi} \right) \\ \text{s.t. } & c_t^* + \frac{q_t^A}{e_t} \frac{A_{t+1}^*}{p_t^*} + q_t^B \frac{B_{t+1}^*}{p_t^*} + \chi q_t^A \left(\frac{A_{t+1}^*}{e_t p_t^*} \right)^2 = \frac{A_t^*}{e_t p_t^*} + \frac{B_t^*}{p_t^*} + \frac{w_t^*}{p_t^*} n_t^* + \frac{p_{f,t}^*}{p_t^*} \Pi_{f,t} + \frac{T_t^*}{p_t^*}. \end{aligned}$$

ξ_t and ξ_t^* are intertemporal preference shocks, while ϕ_t and ϕ_t^* are labour supply shocks. A_t and A_t^* represent each household's holdings of the home currency-denominated asset, while B_t and B_t^* are their respective holdings of the foreign currency-denominated asset. e_t is the nominal exchange rate. Real bond holdings are subject to quadratic costs à la [Benigno \(2009\)](#); these pin down equilibrium portfolios and ensure that bond positions revert to zero in the long run. The costs paid by each household are received by the other through lump-sum transfers, so they do not represent a deadweight loss:

$$\frac{T_t}{p_t} = \left(\frac{e_t p_t^*}{p_t} \right) \chi q_t^A \left(\frac{A_{t+1}^*}{e_t p_t^*} \right)^2, \quad \frac{T_t^*}{p_t^*} = \left(\frac{p_t}{e_t p_t^*} \right) \chi q_t^B \left(\frac{e_t B_{t+1}}{p_t} \right)^2.$$

The optimality conditions for consumption, saving and labour supply are spelt out in the Appendix.

Intratemporal optimisation Households have utility over consumption of a composite index of domestic and foreign goods, with an imports share parameter ζ and a constant elasticity of substitution η . Their static optimisation problems are respectively

$$\begin{aligned} \max_{c_{h,t}, c_{f,t}} c_t & \equiv \left[(1-\zeta)^{\frac{1}{\eta}} (c_{h,t})^{\frac{\eta-1}{\eta}} + (\zeta)^{\frac{1}{\eta}} ((1-\tau) c_{f,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\ \text{s.t. } & p_t c_t = p_{h,t} c_{h,t} + p_{f,t} c_{f,t} \end{aligned}$$

and

$$\begin{aligned} \max_{c_{h,t}^*, c_{f,t}^*} c_t^* &\equiv \left[(1 - \zeta)^{\frac{1}{\eta}} (c_{f,t}^*)^{\frac{\eta-1}{\eta}} + (\zeta)^{\frac{1}{\eta}} \left((1 - \tau) c_{h,t}^* \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\ \text{s.t. } p_t^* c_t^* &= p_{f,t}^* c_{f,t}^* + p_{h,t}^* c_{h,t}^*, \end{aligned}$$

where $p_{h,t}$, $p_{f,t}$, $p_{h,t}^*$ and $p_{f,t}^*$ represent the producer prices of home and foreign goods in each currency. This distinction becomes superfluous with a single currency: in that case, prices are just $p_{h,t}$ and $p_{f,t}$.

A fraction τ of the imported goods are actually lost and do not yield utility. These “iceberg costs” à la [Lama and Rabanal \(2014\)](#) represent frictions associated with the use of different currencies in international commerce; they are null by definition in a currency union.

The effective consumer price indices (CPIs) are respectively

$$p_t \equiv \left[(1 - \zeta) (p_{h,t})^{1-\eta} + (\zeta) \left(\frac{p_{f,t}}{1 - \tau} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

and

$$p_t^* \equiv \left[(1 - \zeta) (p_{f,t}^*)^{1-\eta} + (\zeta) \left(\frac{p_{h,t}^*}{1 - \tau} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$

The real exchange rate is defined as

$$Q_t \equiv \frac{e_t p_t^*}{p_t}.$$

The consumption demands associated with these prices are

$$\begin{aligned} c_{h,t} &= (1 - \zeta) \left(\frac{p_{h,t}}{p_t} \right)^{-\eta} c_t, & c_{f,t} &= (\zeta) (1 - \tau)^{\eta-1} \left(\frac{p_{f,t}}{p_t} \right)^{-\eta} c_t, \\ c_{f,t}^* &= (1 - \zeta) \left(\frac{p_{f,t}^*}{p_t^*} \right)^{-\eta} c_t^*, & c_{h,t}^* &= (\zeta) (1 - \tau)^{\eta-1} \left(\frac{p_{h,t}^*}{p_t^*} \right)^{-\eta} c_t^*. \end{aligned}$$

2.1.2 Firms

The production of consumer goods takes place in two steps. First, imperfectly competitive firms produce a continuum of measure one of intermediate goods in each country, using technologies that take local labour as their only input. The prices of these goods are set in terms of the currency of the country where they are sold. Second, perfectly competitive firms aggregate intermediates into final products, which are either consumed domestically or exported. I describe each group of producers in the h country; f -country firms have a similar behaviour.

Final goods producers Perfectly competitive producers adopt a constant elasticity of substitution (CES) technology that takes as inputs all varieties of locally-produced intermediates and outputs homogeneous final products. Varieties are indexed by i . Separate sets of firms serve the domestic and the foreign market; as they adopt the same technology, the products they make are in fact identical.

The problems faced by the producers of home goods that serve each market are respectively

$$\begin{aligned} \max_{y_{h,t}(i)} p_{h,t} y_{h,t} - \int_0^1 p_{h,t}(i) y_{h,t}(i) di & \quad \text{s.t. } y_{h,t} = \left[\int_0^1 y_{h,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \\ \max_{y_{h,t}^*(i)} p_{h,t}^* y_{h,t}^* - \int_0^1 p_{h,t}^*(i) y_{h,t}^*(i) di & \quad \text{s.t. } y_{h,t}^* = \left[\int_0^1 y_{h,t}^*(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}. \end{aligned}$$

The input demands by these producers are

$$y_{h,t}(i) = \left(\frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\varepsilon} y_{h,t}, \quad y_{h,t}^*(i) = \left(\frac{p_{h,t}^*(i)}{p_{h,t}^*} \right)^{-\varepsilon} y_{h,t}^*,$$

with associated producer price indices (PPIs)

$$p_{h,t} = \left(\int_0^1 p_{h,t}(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}, \quad p_{h,t}^* = \left(\int_0^1 p_{h,t}^*(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}.$$

The distinction between exported and domestically-consumed final goods fades in the monetary union, since the underlying intermediate goods are priced in the same currency; in that case, the aggregator function $y_{h,t}^*$ is redundant.

Intermediate goods producers Monopolistically competitive firms hire local labour in each country to produce differentiated intermediate goods i with the following technologies:

$$y_t(i) = z_t n_t(i).$$

z_t and z_t^* denote their respective productivities. The quantities produced must satisfy both domestic and foreign demand:

$$y_t(i) = y_{h,t}(i) + y_{h,t}^*(i).$$

Firms face an exogenous probability $1 - \theta_p$ of resetting their prices each period. Prices are chosen to maximise discounted profits, subject to isoelastic demand schedules by the producers of final goods. Given the prices, output is demand-determined.

In the presence of two different currencies, firms set a distinct price for each market. The

optimal pricing problem faced by producers in the home country is

$$\begin{aligned} \max_{\bar{p}_{h,t}(i), \bar{p}_{h,t}^*(i)} \quad & \mathbb{E}_t \sum_{\tau=0}^{\infty} \theta_p^\tau \Lambda_{t,t+\tau} \left\{ \frac{\bar{p}_{h,t}(i)}{p_{h,t+\tau}} y_{h,t+\tau}(i) + e_{t+\tau} \frac{\bar{p}_{h,t}^*(i)}{p_{h,t+\tau}} y_{h,t+\tau}^*(i) - \Psi(y_{t+\tau}(i)) \right\} \\ \text{s.t.} \quad & \begin{cases} y_{h,t+\tau}(i) = \left(\frac{p_{h,t+\tau}(i)}{p_{h,t+\tau}} \right)^{-\varepsilon} y_{h,t+\tau} \\ y_{h,t+\tau}^*(i) = \left(\frac{p_{h,t+\tau}^*(i)}{p_{h,t+\tau}} \right)^{-\varepsilon} y_{h,t+\tau}^* \end{cases} \end{aligned}$$

where $\Lambda_{t,t+\tau}$ represents the household's stochastic discount factor for τ periods-ahead real payoffs, while the $\Psi(\cdot)$ function is the total real cost of production. The optimal price-setting conditions for the goods that are aimed at the domestic market read

$$\begin{aligned} g_{h,t}^2 &= \mathcal{M}_p g_{h,t}^1, \\ g_{h,t}^2 &\equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_p \beta)^\tau \left(\frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ y_{h,t+\tau} \left(\frac{\bar{p}_{h,t}}{p_{h,t}} \right)^{-\varepsilon} \left(\frac{1}{\prod_{s=1}^{\tau} \pi_{h,t+s}} \right)^{1-\varepsilon} \right\}, \\ g_{h,t}^1 &\equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_p \beta)^\tau \left(\frac{\lambda_{t+\tau}}{\lambda_t} \right) \left\{ y_{h,t+\tau} \frac{mc_{t+\tau}}{d_{t+\tau}} \left(\frac{\bar{p}_{h,t}}{p_{h,t}} \right)^{-1-\varepsilon} \left(\frac{1}{\prod_{s=1}^{\tau} \pi_{h,t+s}} \right)^{-\varepsilon} \right\}, \end{aligned}$$

where the desired frictionless markup is defined as

$$\mathcal{M}_p \equiv \frac{\varepsilon}{\varepsilon - 1}.$$

Analogous conditions hold for the goods that are aimed at the export market, as shown in the Appendix; they imply that nominal rigidities and price discrimination interact to cause violations of the law of one price over the business cycle. It should be noted that the law of one price holds in the nonstochastic steady state, because firms enjoy the same degree of monopoly power on both markets, so they charge the same markup in the long run.

In the presence of a single currency, instead, price setters choose a unique price for both markets and the law of one price holds continuously.

2.1.3 Monetary policy

Two different monetary regimes are considered here. In the first, two different currencies exist and the nominal exchange rate is flexible. In the second, only one currency exists.

Monetary independence The national central banks of the two countries control the prices of the bonds denominated in the respective currencies. They set nominal interest rates according to the following rules:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\gamma_y} \left(\frac{e_t}{e_{t-1}}\right)^{\gamma_e}\right]^{1-\gamma_R} m_t, \quad (1)$$

$$\frac{R_t^*}{R^*} = \left(\frac{R_{t-1}^*}{R^*}\right)^{\gamma_R} \left[\left(\frac{\pi_t^*}{\pi^*}\right)^{\gamma_\pi} \left(\frac{y_t^*}{y_{t-1}^*}\right)^{\gamma_y} \left(\frac{e_{t-1}}{e_t}\right)^{\gamma_e}\right]^{1-\gamma_R} m_t^*, \quad (2)$$

where $R_t = 1/q_t^A$ and $R_t^* = 1/q_t^B$. The m_t and m_t^* terms represent exogenous interest rate shocks. Since firms engage in LCP, monetary authorities target CPI inflation rates:

$$\pi_t \equiv \frac{p_t}{p_{t-1}}, \quad \pi_t^* \equiv \frac{p_t^*}{p_{t-1}^*}.$$

Taylor rules are specified in terms of real output growth, because the output gap is not observable in practice. They also feature an exchange rate feedback term that prevents nominal exchange rate movements from causing excessive price misalignments and large violations of the risk-sharing condition¹¹

$$\frac{\lambda_t^*}{\lambda_t} = \frac{e_t p_t^*}{p_t}.$$

Monetary union The central bank of the monetary union adopts a Taylor rule that takes union-wide measures as monetary policy targets:

$$\frac{R_t^u}{R^u} = \left(\frac{R_{t-1}^u}{R^u}\right)^{\gamma_R} \left[\left(\frac{\pi_{u,t}}{\pi_u}\right)^{\gamma_\pi} \left(\frac{y_{u,t}}{y_{u,t-1}}\right)^{\gamma_y}\right]^{1-\gamma_R} m_t^u. \quad (3)$$

Union-wide output is the sum of the two countries' GDPs:

$$y_{u,t} = y_t + y_t^*.$$

Inflation in the union is defined as the geometric average of the two GDP inflation rates

$$\pi_{h,t} \equiv \frac{p_{h,t}}{p_{h,t-1}}, \quad \pi_{f,t} \equiv \frac{p_{f,t}}{p_{f,t-1}},$$

weighted by the sizes of the respective countries:

$$\pi_{u,t} \equiv \frac{p_{u,t}}{p_{u,t-1}} = (\pi_{h,t})^{\frac{1}{2}} (\pi_{f,t})^{\frac{1}{2}}.$$

¹¹Due to the incompleteness of markets, the decentralised equilibrium allocation is inefficient and is supported by suboptimal patterns of international borrowing and lending. This inefficiency also shows up in the form of excess nominal exchange rate volatility, which creates scope for welfare-improving monetary policy intervention.

An endogenous nominal interest rate spread must correct self-fulfilling inflation differentials between the two countries under rule (3), in order to rule out unstable solutions:

$$\Omega_t \equiv \frac{R_t^*}{R_t} = \left(\frac{\pi_{f,t}}{\pi_{h,t}} \right)^\omega,$$

where $\omega \simeq 0$.

2.1.4 Exogenous processes

Shocks are common to all households and firms within each country, and follow first-order autoregressive processes in logs. The international spillovers of these shocks are controlled by the parameters ν . Intertemporal preference shocks:

$$\begin{bmatrix} \log \xi_t \\ \log \xi_t^* \end{bmatrix} = \begin{bmatrix} \rho_\xi & \nu_\xi \\ \nu_\xi & \rho_\xi \end{bmatrix} \begin{bmatrix} \log \xi_{t-1} \\ \log \xi_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_{\xi,t} \\ e_{\xi,t}^* \end{bmatrix}.$$

Labour supply shocks:

$$\begin{bmatrix} \log \phi_t \\ \log \phi_t^* \end{bmatrix} = \begin{bmatrix} \rho_\phi & \nu_\phi \\ \nu_\phi & \rho_\phi \end{bmatrix} \begin{bmatrix} \log \phi_{t-1} \\ \log \phi_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_{\phi,t} \\ e_{\phi,t}^* \end{bmatrix}.$$

Technology shocks:

$$\begin{bmatrix} \log z_t \\ \log z_t^* \end{bmatrix} = \begin{bmatrix} \rho_z & \nu_z \\ \nu_z & \rho_z \end{bmatrix} \begin{bmatrix} \log z_{t-1} \\ \log z_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_{z,t} \\ e_{z,t}^* \end{bmatrix}.$$

Monetary shocks with two currencies:

$$\begin{bmatrix} \log m_t \\ \log m_t^* \end{bmatrix} = \begin{bmatrix} \rho_m & \nu_m \\ \nu_m & \rho_m \end{bmatrix} \begin{bmatrix} \log m_{t-1} \\ \log m_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_{m,t} \\ e_{m,t}^* \end{bmatrix}.$$

Monetary shocks with a single currency:

$$\log m_t^u = \rho_m \log m_{t-1}^u + e_{m,t}^u.$$

The innovations follow i.i.d. normal processes, which may or may not be correlated across countries.

2.1.5 Market clearing, price dispersion and output

The GDP of each country is the sum of the goods produced for the domestic market and those intended for exports:

$$y_t = y_{h,t} + y_{h,t}^*, \quad y_t^* = y_{f,t} + y_{f,t}^*.$$

The market clearing conditions for these goods are

$$\begin{aligned} y_{h,t} &= c_{h,t}, & y_{h,t}^* &= (1 - \tau) c_{h,t}^* + \tau c_{h,t}^*, \\ y_{f,t}^* &= c_{f,t}^*, & y_{f,t} &= (1 - \tau) c_{f,t} + \tau c_{f,t}. \end{aligned}$$

The labour market clearing conditions are

$$n_t = \int_0^1 n_t(i) di, \quad n_t^* = \int_0^1 n_t^*(i) di.$$

They can be combined with the demand schedules and the production technologies of individual goods, to yield the exact aggregate production functions of each economy:

$$y_t = \frac{z_t n_t}{d_t}, \quad y_t^* = \frac{z_t^* n_t^*}{d_t^*}. \quad (4)$$

The state variables d_t and d_t^* measure price dispersion at the level of GDP. They are weighted combinations of the price dispersion indices at the PPI level:

$$d_t \equiv x_{h,t} d_{h,t} + x_{h,t}^* d_{h,t}^*, \quad d_t^* \equiv x_{f,t} d_{f,t} + x_{f,t}^* d_{f,t}^*.$$

The weights equal the shares of domestically-consumed and exported output:

$$x_{h,t} \equiv \frac{y_{h,t}}{y_t}, \quad x_{h,t}^* \equiv \frac{y_{h,t}^*}{y_t}, \quad x_{f,t} \equiv \frac{y_{f,t}}{y_t^*}, \quad x_{f,t}^* \equiv \frac{y_{f,t}^*}{y_t^*}.$$

The four indices of price dispersion at PPI level are defined as

$$\begin{aligned} d_{h,t} &\equiv \int_0^1 \left(\frac{p_{h,t}(i)}{p_{h,t}} \right)^{-\varepsilon} di, & d_{h,t}^* &\equiv \int_0^1 \left(\frac{p_{h,t}^*(i)}{p_{h,t}^*} \right)^{-\varepsilon} di, \\ d_{f,t} &\equiv \int_0^1 \left(\frac{p_{f,t}(i)}{p_{f,t}} \right)^{-\varepsilon} di, & d_{f,t}^* &\equiv \int_0^1 \left(\frac{p_{f,t}^*(i)}{p_{f,t}^*} \right)^{-\varepsilon} di. \end{aligned}$$

Finally, the following market clearing conditions apply to the internationally-traded assets:

$$A_{t+1} = -A_{t+1}^*, \quad B_{t+1} = -B_{t+1}^*.$$

The laws of motion of the price dispersion indices, the aggregate price levels and the nominal exchange rate are displayed in the Appendix.

2.2 Equilibrium adjustment to asymmetric shocks

Before moving on to analyse welfare under alternative international monetary arrangements, I use a simplified version of my model to shed light on how macroeconomic adjustment in a currency union differs from that with two independent monetary policies.

I shut down the disturbances to intertemporal preferences, labour supply and interest rates, and consider an environment where technology shocks represent the only source of uncertainty. I assume that trade is frictionless, so that consumption and output are identical across monetary regimes in the long run. Furthermore, I assume that firms engage in producer currency pricing, so that the law of one price applies: $p_{h,t} = e_t p_{h,t}^*$ and $p_{f,t} = e_t p_{f,t}^*$ at all times.

Table 4 in the Appendix displays the calibrated parameters. The time interval of the model is meant to be a quarter. I specify identical preferences with unit intertemporal and intratemporal elasticities of substitution. This configuration is known as “macroeconomic independence”, because it implies that domestic and foreign goods are neither complements nor substitutes in consumption; this rules out international supply spillovers. As to the monetary policy block, I calibrate the Taylor rules with identical coefficients across the two monetary regimes to ease the comparison¹².

Let us explore the effects of a positive productivity shock to the home country, under the assumption that this is uncorrelated with foreign productivity. Figure 1 shows that inflation and output are more volatile in the regime with two currencies (solid blue lines) than they are in the regime with a single currency (dashed purple lines), because asymmetric disturbances cannot be addressed selectively under the former arrangement.

Notice that a positive international comovement of output appears in Figure 1, despite the unit elasticities of substitution specified in Table 4. This occurs in both monetary regimes, and is due to the response of monetary policy to the shock.

Under rules (1) and (2), the central bank of the home country responds to the deflationary effects of a positive technology shock by cutting the nominal interest rate. To avoid an excessive movement of the nominal exchange rate, the foreign central bank cuts its interest rate too. This is what causes a jump of foreign inflation and output.

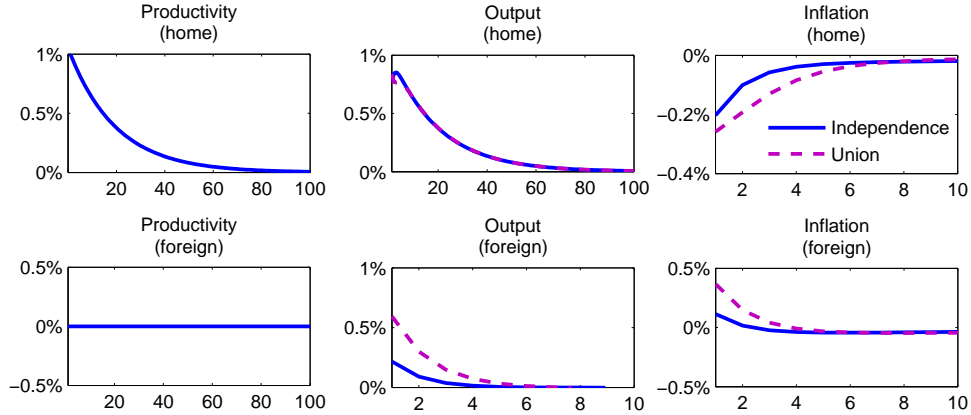
Under rule (3), the central bank reacts to a local productivity improvement by cutting the nominal interest rate for the whole union; this stimulates foreign activity and inflation.

¹²Interest rate smoothing is muted because there is no need to stabilise the opportunity cost of holding money in a cashless economy. Similarly, the response to output fluctuations is muted in the absence of inefficient cost-push shocks. The inflation and exchange rate coefficients are chosen to guarantee a unique rational expectations equilibrium.

Both variables are significantly more volatile than in the other regime.

In fact, the distance between the impulse responses in the two regimes depends on the symmetry of the shocks. If z_t and z_t^* were perfectly positively correlated, the impulse responses would coincide. Conversely, if they were perfectly negatively correlated, the impulse responses would be farther apart. The next section explores the consequences of this for social welfare.

Figure 1: Impulse responses to a technology shock: monetary independence vs currency union



3 Welfare differential between international monetary regimes

I now introduce a welfare metric that allows me to rank alternative monetary regimes.

Following [Schmitt-Grohé and Uribe \(2007\)](#), I define welfare as the conditional expectation of lifetime utility as of time zero, assuming that at this point all state variables equal their steady-state values. The welfare levels associated with *monetary independence* and *currency union* are respectively

$$V_0^{mi} \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{mi}, n_t^{mi}), \quad V_0^{cu} \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{cu}, n_t^{cu}),$$

where $\{c_t^{mi}\}$, $\{n_t^{mi}\}$, $\{c_t^{cu}\}$ and $\{n_t^{cu}\}$ represent the respective contingent plans for consumption and hours. The welfare cost of abandoning an independent monetary policy and adopting a single currency is measured in terms of foregone consumption: I define it as the negative subsidy rate λ such that

$$V_0^{cu} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u((1 - \lambda) c_t^{mi}, n_t^{mi}).$$

The welfare gap between these monetary arrangements can be written as

$$V_0^{cu} - V_0^{mi} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u \left((1 - \lambda) c_t^{mi}, n_t^{mi} \right) - u \left(c_t^{mi}, n_t^{mi} \right) \right].$$

With logarithmic utility, the welfare cost amounts to

$$\lambda = 1 - e^{(1-\beta)(V_0^{cu} - V_0^{mi})}. \tag{5}$$

3.1 Computational method

Due to the frictions that affect the goods and the asset markets, the steady state of the model is inefficient. In order to measure welfare accurately in this setup, I approximate the equilibrium conditions up to second-order¹³. My numerical strategy is based on perturbation, which represents a convenient approach for studying economies with a large number of state variables. The algorithm that I use is Dynare by [Adjemian et al. \(2011\)](#). To address the problem of explosive paths emerging in second-order approximations of this class of models¹⁴, I apply a pruning method due to [Kim et al. \(2008\)](#).

3.2 Shocks, frictions and welfare

In this section, I examine the main determinants of the welfare gap between alternative monetary arrangements. I start from an economy with no trade costs nor international price discrimination; I then introduce these frictions one at a time, and discuss their welfare implications. I focus on an environment where technology shocks are the only source of uncertainty.

Figure 2 plots the welfare gap against the correlation of technology shocks, in an economy where the law of one price holds continuously and international trade is frictionless. The relationship is negative because a stronger comovement of macroeconomic variables reduces the degree of ex-post heterogeneity among countries; this decreases the need for region-specific stabilisation policies and the attractiveness of a multicurrency system. The gap goes to zero as $\text{corr}(e_z, e_z^*)$ approaches one, because in that case the two economies become identical ex-post and there is no difference between having one or two instruments of monetary policy.

The slope of the welfare gap line depends on the volatility of the exogenous processes. Stronger shocks make the two countries more heterogeneous ex-post, widening the difference

¹³This is needed because first-order methods leave out welfare-relevant terms and incur large approximation errors when the steady state of the economy Pareto-inefficient, as documented by [Kim and Kim \(2003\)](#).

¹⁴Higher-order terms of state variables tend to appear in the application of second-order methods without pruning. These terms create “spurious” steady states that are extraneous to the original model; this is problematic because the resulting approximated state-space system is unstable around these points usually, and does not have finite moments. Pruning procedures remove the higher-order terms and preserve only first and second-order terms when the system is iterated forward: the alternative state-space system constructed by these methods achieves stability.

between the welfare achievable with two monetary policy instruments and that achievable with one. This effect is proportional to the correlation of productivity disturbances, so it is reflected in a steeper line.

The intercept of the welfare gap line depends on the other macroeconomic disturbances that exist in the economy. The fact that the welfare cost of a monetary union is null when technology shocks are perfectly correlated (as reflected by a zero right intercept in Figure 2) is due to the fact that I have shut down the shocks to intertemporal preferences, labour supply and interest rates. If these disturbances were present, the two economies would be heterogeneous even when $\text{corr}(e_z, e_z^*) = 1$, and the right intercept of the welfare gap schedule would be positive¹⁵.

Figure 2: Correlation of technology shocks and the welfare cost of a currency union

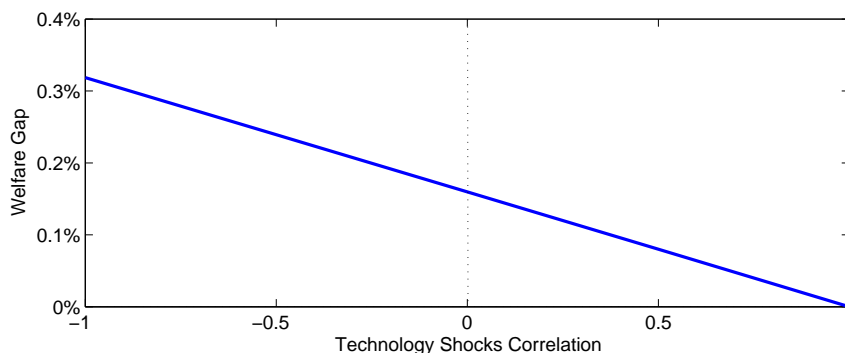


Figure 3 shows what happens when we introduce pricing-to-market. The pass-through of nominal exchange rate movements into import prices becomes imperfect when prices are subject to staggered setting in the currency of the destination market. In the short run, identical products are sold at different prices in different places, once converted into a common unit of account. This is inefficient, because it distorts the allocation of demand. The following measures of international price misalignments quantify the departures from the law of one price:

$$m_{h,t} \equiv \frac{e_t p_{h,t}^*}{p_{h,t}}, \quad m_{f,t} \equiv \frac{e_t p_{f,t}^*}{p_{f,t}}.$$

If nominal rigidities are strong enough, these frictions add an important cost to the regime with national currencies. The monetary union is not affected by these frictions instead, because the presence of a single currency removes the segmentation of final goods markets and impedes price discrimination: goods are uniformly priced across countries. For this reason, the welfare gap between the two regimes shrinks. Since price misalignments materialise over the business cycle and are null in the nonstochastic steady state, the reduction is proportional to the

¹⁵The right intercept would still be zero if all other disturbances were perfectly positively correlated too.

correlation of technology shocks; this is why it affects the slope of the welfare gap schedule.

Figure 3: Price discrimination and the welfare cost of a currency union

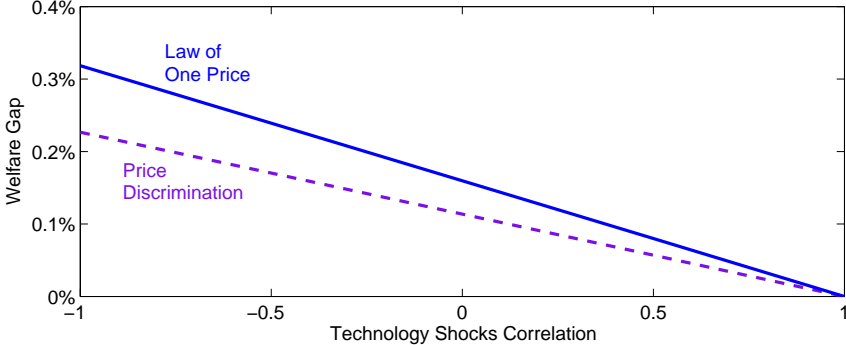
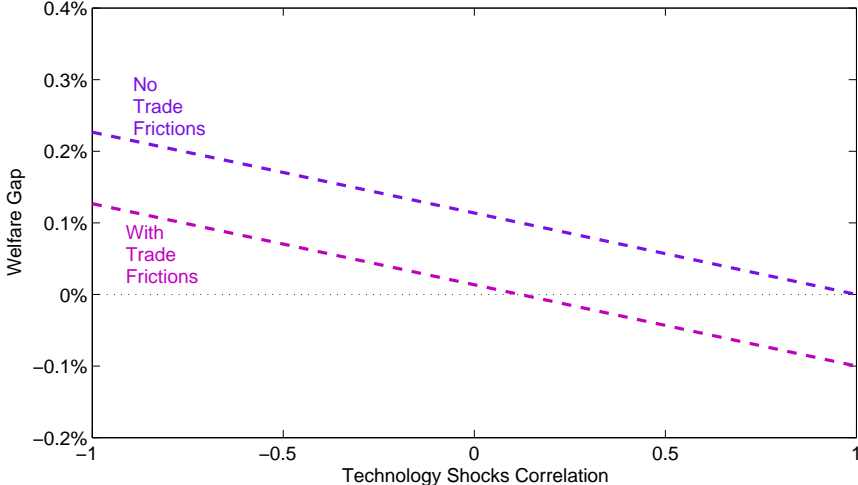


Figure 4 illustrates the welfare effect of trade frictions. The presence of monetary barriers to trade reduces the demand for imports, lowering the long-run output level of the economy with multiple currencies relative to that of the union; since this distortion affects the steady state of the economy, it shifts down the entire welfare gap schedule. This affects the welfare ranking between alternative monetary regimes significantly, because it creates regions where the union Pareto-dominates the system of independent national currencies.

Figure 4: Trade frictions and the welfare cost of a currency union



As the position of the welfare gap schedule is closely tied to the quantitative strength of the various shocks and frictions, it is difficult to draw conclusions from the present model without letting the data discipline its parameters. This is what I do in the next section.

4 Bayesian estimation

In this section, the model meets quarterly data from Italy, France, Germany and Spain to be estimated and evaluated using Bayesian tools. In accordance with the structure of the model, the estimations and the subsequent welfare calculations are done on two countries at a time.

My empirical strategy is as follows. I estimate the model in its monetary union configuration, and then calculate the welfare gap with the multicurrency regime under the assumption that the estimated parameters are invariant to the monetary arrangement¹⁶. I assume that countries keep identical Taylor rules in the monetary independence regime, but base them on individual (rather than union-wide) policy objectives.

4.1 Data

The beginning of the sample is chosen to coincide with the official launch of the Euro in the first quarter of 1999. The end of the sample is the fourth quarter of 2014.

As the model features seven exogenous driving processes, I use seven macroeconomic series as observables: the real GDP, real consumption and CPI inflation series of each country, plus the nominal interest rate of the union (as measured by the Eurozone interbank rate).

The CPI inflation data are obtained from OECD; the source of the remaining series is the FRED archive of the Federal Reserve Bank of St. Louis. The series are seasonally adjusted.

Output and consumption are divided by the GDP deflator, so that they appear in real terms. The series are then turned into per capita terms using population data from Eurostat¹⁷. Finally, they are detrended by means of a one-sided Hodrick and Prescott filter¹⁸ with smoothing parameter 1600.

4.2 Calibrated parameters and priors

I calibrate three sets of parameters: (i) those that have been consistently identified by the literature (e.g. the Calvo parameter on prices), (ii) those that would not be identified from the vector of observable variables chosen here (e.g. the trade cost parameter), and (iii) those that represent simplifying assumptions (e.g. the unit elasticities of intertemporal and trade substitution). These are listed in Table 5 in the Appendix.

¹⁶The alternative strategy of separately estimating the model under flexible exchange rates is hindered by data availability issues. The exchange rates of the largest European economies were controlled long before the Euro was established in 1999: a system of semi-pegged exchange rates known as the European Exchange Rate Mechanism was in place from 1979 to 1999. The availability of suitable pre-1979 quarterly data is limited.

¹⁷Since the frequency of the original population data is annual, I construct population series at quarterly frequency by interpolation.

¹⁸The traditional two-sided HP filter takes future values of variables as an input to construct data series. This contradicts the backward-looking structure of the model in state-space form, where the solution today depends only on current and past states.

I estimate two sets of parameters: (i) the Taylor rule coefficients and (ii) the parameters that control the exogenous processes. The priors that I specify before launching the estimation are collected in Table 6 in the Appendix.

Three parameters are specific to the model with national currencies: they are the trade friction parameter, the international correlation of monetary shocks and the exchange rate coefficient in the Taylor rules. They have no counterpart in the union. I set $\tau = 0.05$ following [Lama and Rabanal \(2014\)](#)¹⁹, I specify $\text{corr}(e_m, e_m^*) = 0$, and I fix γ_e at the lowest value that guarantees a unique rational expectations equilibrium. I subsequently check the robustness of my results to changes in these parameters.

4.3 Estimation technique

I construct a first-order approximation of the model and its decision rules, so that the likelihood can be generated by Kalman filter projections from the approximated state-space system. This choice is dictated by computational convenience²⁰.

As the posterior distribution cannot be evaluated analytically, I adopt a standard Metropolis-Hastings (MH) Markov Chain Monte Carlo algorithm to simulate it and produce Bayesian estimates of the parameters. For each round of estimation, I run four parallel chains of the MH algorithm with 500,000 replications each. I set a 50% burn-in period to remove the dependence of the estimates on the parameter vector that initialises the MH algorithm, so the first 250,000 draws are discarded before actually using the posterior simulations. Tables 7 and 8 in the Appendix summarise the results. Tables 9 to 14 report the contribution of each shock to the variance of the observables at infinite horizon.

4.4 Welfare results

To evaluate the distribution of the welfare gap λ for each pair of countries, I employ the following procedure: (i) I take a sample of 1000 parameter draws from the appropriate posterior distribution, (ii) I simulate the model in both monetary configurations and compute welfare for each of these parameter vectors, and (iii) I calculate the gap in consumption equivalents using equation (5). Table 1 reports the results. The probability densities are displayed in Figure 6 in the Appendix.

¹⁹These authors indicate 5 percentage points as a lower bound on the true reduction in transaction costs produced by the introduction of a single currency. This calibration allows me to get a conservative assessment of the gains from monetary unification.

²⁰In principle, the construction of the likelihood from a second-order approximation would exploit the non-linear structure of the model more fully. In practice, the use of a particle filter can be computationally expensive, whereas the difference in terms of point estimates is likely to be small; see [Fernández-Villaverde and Rubio-Ramírez \(2005\)](#).

Table 1: Estimates of λ

Country pair	Mean	90% HPD interval
Italy and France	-2.55%	[-2.56%, -2.54%]
Italy and Germany	-2.44%	[-2.57%, -2.33%]
Italy and Spain	-2.52%	[-2.67%, -2.41%]
France and Germany	-2.42%	[-2.50%, -2.33%]
France and Spain	-2.52%	[-2.63%, -2.44%]
Spain and Germany	-2.33%	[-2.43%, -2.24%]

The negative signs on all lambdas mean that all pairs of countries under scrutiny enjoy welfare gains from sharing a common currency, in this framework. The distributions show little dispersion: the highest posterior density (HPD) intervals at a 90% credibility level are quite tight around their means.

The countries who appear to gain the most from being in a monetary union are Italy and France, while those who seem to gain the least are Germany and Spain. This is consistent with the estimated correlations of shocks displayed in Tables 7 and 8—particularly those to technology, in the light of the fact that they are responsible for most of the variance of consumption and output in this environment.

4.5 Discussion

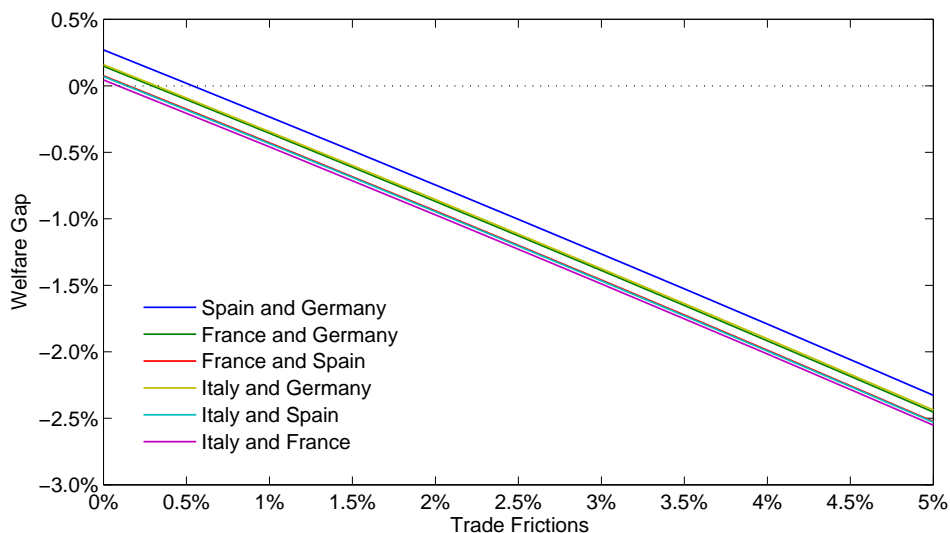
The results above are specific to the configuration chosen for the non-estimated parameters. Among these, the three parameters that are unobservable in a monetary union deserve further discussion; even though they do not affect the likelihood of the estimated model, they have an impact on welfare in the economy with national currencies.

To examine the sensitivity of welfare to γ_e , $\text{corr}(e_m, e_m^*)$ and τ , I let these parameters vary and I keep the others fixed at their respective posterior means. I find that the first and the second parameter have a negligible impact on the welfare gap²¹, while the third is of primary importance.

Figure 5 plots the welfare gap between alternative regimes against the intensity of trade frictions. The relationship is negative for all the countries under consideration, because stronger monetary barriers to trade reduce the desirability of separate national currencies. The vertical intercepts on the left are positive because these economies would be better off with independent monetary policies if trade frictions were absent; those on the right coincide with the posterior means in Table 1.

²¹Changes in the exchange rate stabilisation coefficient leave the welfare gap unchanged up to the third decimal digit, while changes in the international correlation of monetary shocks leave it unaffected up to the fourth decimal digit.

Figure 5: Trade frictions and the welfare gap between monetary regimes



As the cost of asymmetric business cycles is small for these economies, moderate amounts of monetary barriers to trade are enough for the benefits of a currency union to exceed the costs. Table 2 reports the critical amounts of trade frictions that equate welfare across monetary regimes for each pair of economies; they correspond to the horizontal intercepts of the welfare gap lines in Figure 5. Particularly small frictions appear sufficient for a monetary union between Italy and France, since their business cycles are quite correlated; larger frictions seem to be needed to justify a monetary union between Spain and Germany, as they exhibit more asynchronous disturbances and therefore have relatively more to gain from keeping independent monetary policies.

Table 2: Trade frictions that equate welfare across regimes

Country pair	Critical τ
Italy and France	0.09%
Italy and Germany	0.32%
Italy and Spain	0.14%
France and Germany	0.36%
France and Spain	0.15%
Spain and Germany	0.53%

The importance of the trade frictions is tied to the degree of openness of the two economies, as represented by the weight of imported goods in consumption. The presence of home bias reduces international trade and makes the effects of monetary barriers less severe; this lessens the trade gains from monetary unification. Table 3 displays the results of welfare calculations based on estimations with a moderate degree of home bias: preferences are calibrated with an

imports share of $\zeta = 0.35$ as in [Erceg et al. \(2009\)](#) and [Coenen et al. \(2009\)](#). The results are qualitatively analogous to those in [Table 1](#), with all countries experiencing higher welfare in a currency union. Two quantitative differences stand out: first, the estimated welfare gains are smaller than in the economy with no home bias; second, the figures vary more across country pairs. This reflects the fact that the welfare gap lines in [Figure 5](#) are now flatter and their horizontal intercepts are more spaced out.

Table 3: Estimates of λ with home bias

Country pair	Mean	90% HPD interval
Italy and France	-1.73%	[-1.75%, -1.71%]
Italy and Germany	-1.09%	[-1.32%, -0.95%]
Italy and Spain	-1.61%	[-1.75%, -1.46%]
France and Germany	-1.14%	[-1.32%, -0.95%]
France and Spain	-1.48%	[-1.59%, -1.32%]
Spain and Germany	-0.92%	[-1.15%, -0.76%]

5 Concluding remarks

What are the welfare implications of creating a currency union between different sovereign nations? In this paper, I have revisited this question from the perspective of an open economy DSGE model with asymmetric shocks and imperfect risk-sharing. I have emphasised one specific dimension of the tradeoff between alternative monetary arrangements: the conflict between the transactions and efficiency benefits of eliminating national currencies, on one hand, and the costs of abandoning monetary policy independence in the face of asynchronous business cycles, on the other.

Based on numerical work, I have argued that the introduction of a single currency would be welfare-reducing in an economy with frictionless trade, because it would suppress an instrument of macroeconomic stabilisation and provide a second-order benefit (that of removing price misalignments across markets). Once monetary barriers to trade are taken into account, however, monetary unification becomes welfare-improving because it eliminates a friction that has first-order effects on the economy.

Estimates from Italy, France, Germany and Spain suggest that these countries enjoy substantial welfare gains from sharing a common currency. The key to these findings is the fact that the welfare cost of imperfectly synchronised business cycles is modest for these economies, so the losses from missing country-specific instruments of monetary policy are easily exceeded by the gains from trade creation.

My modelling strategy ignores a number of important issues in the macroeconomics of monetary unions. It lacks an explicit description of financial intermediation and monetary transmission, it misses taxation and government spending, and it overlooks labour market

frictions and unemployment, just to name a few. The introduction of heterogeneity along these dimensions is likely to have important consequences for macroeconomic adjustment and social welfare in a currency union; it would be interesting to examine how they alter the tradeoff between alternative monetary arrangements and affect the domain of applicability of a single currency within the same type of framework presented here.

References

- Adjemian, S., H. Bastani, M. Juillard, F. Karamé, F. Mihoubi, G. Perendia, J. Pfeifer, M. Ratto, and S. Villemot (2011). Dynare: Reference manual, version 4. *Dynare Working Papers 1*.
- Bacchetta, P. and E. van Wincoop (2000). Does exchange-rate stability increase trade and welfare? *American Economic Review* 90(5), 1093–1109.
- Beetsma, R. and H. Jensen (2005). Monetary and fiscal policy interactions in a micro-founded model of a monetary union. *Journal of International Economics* 67(2), 320–352.
- Benigno, P. (2004). Optimal monetary policy in a currency area. *Journal of International Economics* 63(2), 293–320.
- Benigno, P. (2009). Price stability with imperfect financial integration. *Journal of Money, Credit and Banking* 41(S1), 121–149.
- Bhattarai, S., J. W. Lee, and W. Y. Park (2015). Optimal monetary policy in a currency union with interest rate spreads. *Journal of International Economics* 96(2), 375–397.
- Ching, S. and M. B. Devereux (2003). Mundell revisited: a simple approach to the costs and benefits of a single currency area. *Review of International Economics* 11(4), 674–691.
- Clarida, R., J. Galí, and M. Gertler (2002). A simple framework for international monetary policy analysis. *Journal of Monetary Economics* 49(5), 879–904.
- Coenen, G., G. Lombardo, F. Smets, and R. Straub (2009). International transmission and monetary policy cooperation. *In ed. Jordi Galí and Mark Gertler (2009)*.
- Cole, H. and M. Obstfeld (1991). Commodity trade and international risk sharing. *Journal of Monetary Economics* 28(1), 3–24.
- Corden, W. M. (1972). Monetary integration. *Princeton Essays in International Finance* 93.
- Corsetti, G., L. Dedola, and S. Leduc (2011). Optimal monetary policy in open economies. *In ed. Benjamin Friedmann and Michael Woodford (2011)*.

- Cúrdia, V. and M. Woodford (2010). Credit spreads and monetary policy. *Journal of Money, Credit and Banking* 42(6), 3–35.
- Devereux, M. B. and C. Engel (2003). Monetary policy in the open economy revisited: Price setting and exchange-rate flexibility. *Review of Economic Studies* 70(4), 765–783.
- Devereux, M. B., C. Engel, and C. Tille (2003). Exchange rate pass-through and the welfare effects of the euro. *International Economic Review* 44(1), 223–242.
- Engel, C. (2011). Currency misalignments and optimal monetary policy: A reexamination. *American Economic Review* 101(6), 2796–2822.
- Engel, C. (2014). Exchange rate stabilization and welfare. *Annual Review of Economics* 6, 155–177.
- Erceg, C., C. Gust, and D. López-Salido (2009). The transmission of domestic shocks in open economies. In ed. *Jordi Galí and Mark Gertler (2009)*.
- Fatás, A. (1998). Does emu need a fiscal federation? *Economic Policy* 13(26), 163–203.
- Feldstein, M. (2015). Ending the euro crisis. *Journal of Policy Modeling* 37(3).
- Fernández-Villaverde, J. and J. F. Rubio-Ramírez (2005). Estimating dynamic equilibrium economies: Linear versus nonlinear likelihood. *Journal of Applied Econometrics* 20(7), 891–910.
- Ferrero, A. (2009). Fiscal and monetary rules for a currency union. *Journal of International Economics* 77(1), 1–10.
- Forlati, C. (2009). Optimal monetary and fiscal policy in the emu: Does fiscal policy coordination matter? *EPFI CFI Center for Fiscal Policy Working Paper 04*.
- Frankel, J. (2015). The euro crisis: Where to from here? *Journal of Policy Modeling* 37(3), 428–444.
- Friedman, B. M. and M. Woodford (2011). *Handbook of Monetary Economics*. North-Holland.
- Galí, J. and M. Gertler (2009). *International Dimensions of Monetary Policy*. National Bureau of Economic Research.
- Galí, J. and T. Monacelli (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies* 72(3), 707–734.
- Ingram, J. C. (1973). The case for european monetary integration. *Princeton Essays in International Finance* 98.

- Kim, J. and S. H. Kim (2003). Spurious welfare reversals in international business cycle models. *Journal of International Economics* 60(2), 471–500.
- Kim, J., S. H. Kim, E. Schaumburg, and C. A. Sims (2008). Calculating and using second-order accurate solutions of discrete time dynamic equilibrium models. *Journal of Economic Dynamics and Control* 32(11), 3397–3414.
- Lama, R. and P. Rabanal (2014). Deciding to enter a monetary union: the role of trade and financial linkages. *European Economic Review* 72, 138–165.
- McKinnon, R. I. (1963). Optimum currency areas. *American Economic Review* 53(4), 717–725.
- Mishkin, F. and K. Schmidt-Hebbel (2007). *Monetary Policy under Inflation Targeting*. Central Bank of Chile.
- Mundell, R. (1961). A theory of optimum currency areas. *American Economic Review* 51(4), 657–665.
- Obstfeld, M. and G. Peri (1998). Regional non-adjustment and fiscal policy. *Economic Policy* 13(26), 205–259.
- Rose, A. (2000). One money, one market: the effect of common currencies on trade. *Economic Policy* 15(30), 7–45.
- Rose, A. and E. van Wincoop (2001). National money as a barrier to international trade: The real case for currency union. *American Economic Review* 91(2), 386–390.
- Santos-Silva, J. and S. Tenreyro (2010). Currency unions in prospect and retrospect. *Annual Review of Economics* 2, 51–74.
- Schmitt-Grohé, S. and M. Uribe (2007). Optimal inflation stabilization in a medium-scale macroeconomic model. *In ed. Frederic Mishkin and Klaus Schmidt-Hebbel (2007)*.

Appendix

Notation

It is convenient to write the bond positions of the two households in real terms and express them in their respective currencies:

$$a_{t+1} \equiv \frac{A_{t+1}}{p_t}, \quad b_{t+1} \equiv e_t \frac{B_{t+1}}{p_t}, \quad a_{t+1}^* \equiv \frac{A_{t+1}^*}{e_t p_t^*}, \quad b_{t+1}^* \equiv \frac{B_{t+1}^*}{p_t^*}.$$

It is also useful to define three sets of relative prices. First, the PPI-to-CPI ratios:

$$\mathcal{P}_{h,t} \equiv \frac{p_{h,t}}{p_t}, \quad \mathcal{P}_{f,t} \equiv \frac{p_{f,t}}{p_t}, \quad \mathcal{P}_{h,t}^* \equiv \frac{p_{h,t}^*}{p_t^*}, \quad \mathcal{P}_{f,t}^* \equiv \frac{p_{f,t}^*}{p_t^*}.$$

Second, the optimal relative prices of each good in each currency:

$$\tilde{p}_{h,t} \equiv \frac{\bar{p}_{h,t}}{p_{h,t}}, \quad \tilde{p}_{h,t}^* \equiv \frac{\bar{p}_{h,t}^*}{p_{h,t}^*}, \quad \tilde{p}_{f,t} \equiv \frac{\bar{p}_{f,t}}{p_{f,t}}, \quad \tilde{p}_{f,t}^* \equiv \frac{\bar{p}_{f,t}^*}{p_{f,t}^*}.$$

Third, the relative prices of foreign to domestic goods in each currency:

$$s_t \equiv \frac{p_{f,t}}{p_{h,t}}, \quad s_t^* \equiv \frac{p_{f,t}^*}{p_{h,t}^*}.$$

Consumption-based real wages in each country are

$$w_t \equiv \frac{w_t}{p_t}, \quad w_t^* \equiv \frac{w_t^*}{p_t^*}.$$

Consumption-based real transfers are

$$t_t \equiv \frac{T_t}{p_t}, \quad t_t^* \equiv \frac{T_t^*}{p_t^*}.$$

The depreciation of the nominal exchange rate is defined as

$$\Delta e_t \equiv \frac{e_t}{e_{t-1}}.$$

This notation allows us to rewrite the optimality conditions and the market clearing conditions in a format that is suitable for computation.

Equilibrium conditions

Relative prices:

$$\begin{aligned} \mathcal{P}_{h,t} &= \left[(1 - \zeta) + (\zeta) \left(\frac{s_t}{1 - \tau} \right)^{1-\eta} \right]^{\frac{1}{\eta-1}}, \\ \mathcal{P}_{f,t} &= \left[(1 - \zeta) \left(\frac{1}{s_t} \right)^{1-\eta} + (\zeta) \left(\frac{1}{1 - \tau} \right)^{1-\eta} \right]^{\frac{1}{\eta-1}}, \\ \mathcal{P}_{h,t}^* &= \left[(1 - \zeta) (s_t^*)^{1-\eta} + (\zeta) \left(\frac{1}{1 - \tau} \right)^{1-\eta} \right]^{\frac{1}{\eta-1}}, \end{aligned}$$

$$\mathcal{P}_{f,t}^* = \left[(1 - \zeta) + (\zeta) \left(\frac{1}{s_t^* (1 - \tau)} \right)^{1-\eta} \right]^{\frac{1}{\eta-1}}.$$

Consumption demands:

$$\begin{aligned} c_{h,t} &= (1 - \zeta) (\mathcal{P}_{h,t})^{-\eta} c_t, \\ c_{f,t} &= (\zeta) (1 - \tau)^{\eta-1} (\mathcal{P}_{f,t})^{-\eta} c_t, \\ c_{f,t}^* &= (1 - \zeta) (\mathcal{P}_{f,t}^*)^{-\eta} c_t^*, \\ c_{h,t}^* &= (\zeta) (1 - \tau)^{\eta-1} (\mathcal{P}_{h,t}^*)^{-\eta} c_t^*. \end{aligned}$$

Market clearing conditions for goods:

$$\begin{aligned} y_{h,t} &= c_{h,t}, \\ y_{h,t}^* &= (1 - \tau) c_{h,t}^* + \tau c_{h,t}^*, \\ y_{f,t} &= (1 - \tau) c_{f,t} + \tau c_{f,t}, \\ y_{f,t}^* &= c_{f,t}^*, \\ y_t &= y_{h,t} + y_{h,t}^*, \\ y_t^* &= y_{f,t} + y_{f,t}^*. \end{aligned}$$

Intertemporal optimisation:

$$\begin{aligned} \lambda_t &= \xi_t c_t^{-\sigma}, \\ \lambda_t &= \xi_t \phi_t \frac{n_t^\varphi}{w_t}, \\ q_t^A &= \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\pi_{t+1}} \right), \\ q_t^B (1 - 2\chi (b_{t+1}))^{-1} &= \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{\Delta e_{t+1}}{\pi_{t+1}} \right), \\ c_t + q_t^A a_{t+1} + q_t^B b_{t+1} + \chi q_t^B (b_{t+1})^2 &= \frac{a_t}{\pi_t} + \frac{b_t}{\Delta e_t \pi_t} + w_t n_t + \mathcal{P}_{h,t} \Pi_{h,t} + t_t, \\ t_t &= \mathcal{Q}_t \chi q_t^A (a_{t+1}^*)^2, \\ \lambda_t^* &= \xi_t^* (c_t^*)^{-\sigma}, \\ \lambda_t^* &= \xi_t^* \phi_t^* \frac{(n_t^*)^\varphi}{w_t^*}, \end{aligned}$$

$$\begin{aligned}
q_t^A (1 - 2\chi (a_{t+1}^*))^{-1} &= \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{1}{\Delta e_{t+1} \pi_{t+1}^*} \right), \\
q_t^B &= \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{1}{\pi_{t+1}^*} \right), \\
c_t^* + q_t^A a_{t+1}^* + q_t^B b_{t+1}^* + \chi q_t^A (a_{t+1}^*)^2 &= \Delta e_t \frac{a_t^*}{\pi_t^*} + \frac{b_t^*}{\pi_t^*} + w_t^* n_t^* + \mathcal{P}_{f,t}^* \Pi_{f,t} + t_t^*, \\
t_t^* &= \frac{1}{Q_t} \chi q_t^B (b_{t+1}^*)^2.
\end{aligned}$$

Real exchange rate, relative prices and price misalignments:

$$\begin{aligned}
Q_t &= m_{h,t} \left[\frac{(1 - \zeta) (s_t^*)^{1-\eta} + (\zeta) \left(\frac{1}{1-\tau} \right)^{1-\eta}}{(1 - \zeta) + (\zeta) \left(\frac{s_t}{1-\tau} \right)^{1-\eta}} \right]^{\frac{1}{1-\eta}}, \\
\frac{m_{f,t}}{m_{h,t}} &= \frac{s_t^*}{s_t}, \\
\frac{m_{h,t}}{m_{h,t-1}} &= \Delta e_t \frac{\pi_{h,t}^*}{\pi_{h,t}}, \\
\frac{m_{f,t}}{m_{f,t-1}} &= \Delta e_t \frac{\pi_{f,t}^*}{\pi_{f,t}}.
\end{aligned}$$

Market clearing conditions for bonds:

$$\begin{aligned}
a_{t+1} &= -a_{t+1}^* Q_t, \\
b_{t+1} &= -b_{t+1}^* Q_t.
\end{aligned}$$

Output, marginal cost, price dispersion and aggregate profits:

$$\begin{aligned}
y_t &= \frac{z_t n_t}{d_t}, \\
y_t^* &= \frac{z_t^* n_t^*}{d_t^*}, \\
mc_t &= \frac{w_t}{\mathcal{P}_{h,t}} \frac{d_t}{z_t}, \\
mc_t^* &= \frac{w_t^*}{\mathcal{P}_{f,t}^*} \frac{d_t^*}{z_t^*}, \\
d_{h,t} &= \theta_p (\pi_{h,t})^\varepsilon d_{h,t-1} + (1 - \theta_p) (\tilde{p}_{h,t})^{-\varepsilon}, \\
d_{h,t}^* &= \theta_p (\pi_{h,t}^*)^\varepsilon d_{h,t-1}^* + (1 - \theta_p) (\tilde{p}_{h,t}^*)^{-\varepsilon},
\end{aligned}$$

$$\begin{aligned}
d_{f,t} &= \theta_p (\pi_{f,t})^\varepsilon d_{f,t-1} + (1 - \theta_p) (\tilde{p}_{f,t})^{-\varepsilon}, \\
d_{f,t}^* &= \theta_p (\pi_{f,t}^*)^\varepsilon d_{f,t-1}^* + (1 - \theta_p) (\tilde{p}_{f,t}^*)^{-\varepsilon}, \\
d_t &= x_{h,t} d_{h,t} + x_{h,t}^* d_{h,t}^*, \\
d_t^* &= x_{f,t} d_{f,t} + x_{f,t}^* d_{f,t}^*, \\
x_{h,t} &= \frac{y_{h,t}}{y_t}, \\
x_{h,t}^* &= \frac{y_{h,t}^*}{y_t}, \\
x_{f,t} &= \frac{y_{f,t}}{y_t^*}, \\
x_{f,t}^* &= \frac{y_{f,t}^*}{y_t^*}, \\
\Pi_{h,t} &= y_{h,t} + m_{h,t} y_{h,t}^* - \frac{w_t}{\mathcal{P}_{h,t}} \frac{y_t}{z_t} d_t, \\
\Pi_{f,t} &= y_{f,t}^* + \frac{y_{f,t}}{m_{f,t}} - \frac{w_t^*}{\mathcal{P}_{f,t}^*} \frac{y_t^*}{z_t^*} d_t^*.
\end{aligned}$$

Price dynamics, inflation and optimal price setting:

$$\begin{aligned}
\tilde{p}_{h,t} &= \left[\frac{1 - \theta_p (\pi_{h,t})^{\varepsilon-1}}{1 - \theta_p} \right]^{\frac{1}{1-\varepsilon}}, \\
\tilde{p}_{h,t}^* &= \left[\frac{1 - \theta_p (\pi_{h,t}^*)^{\varepsilon-1}}{1 - \theta_p} \right]^{\frac{1}{1-\varepsilon}}, \\
\tilde{p}_{f,t} &= \left[\frac{1 - \theta_p (\pi_{f,t})^{\varepsilon-1}}{1 - \theta_p} \right]^{\frac{1}{1-\varepsilon}}, \\
\tilde{p}_{f,t}^* &= \left[\frac{1 - \theta_p (\pi_{f,t}^*)^{\varepsilon-1}}{1 - \theta_p} \right]^{\frac{1}{1-\varepsilon}}, \\
\pi_{h,t} &= \frac{\mathcal{P}_{h,t}}{\mathcal{P}_{h,t-1}} \pi_t, \\
\pi_{h,t}^* &= \frac{\mathcal{P}_{h,t}^*}{\mathcal{P}_{h,t-1}^*} \pi_t^*, \\
\pi_{f,t} &= \frac{\mathcal{P}_{f,t}}{\mathcal{P}_{f,t-1}} \pi_t,
\end{aligned}$$

$$\begin{aligned}
\pi_{f,t}^* &= \frac{\mathcal{P}_{f,t}^*}{\mathcal{P}_{f,t-1}^*} \pi_t^*, \\
g_{h,t}^2 &= \mathcal{M}_p g_{h,t}^1, \\
g_{h,t}^1 &= y_{h,t} \frac{mc_t}{d_t} (\tilde{p}_{h,t})^{-1-\varepsilon} + \theta_p \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{\tilde{p}_{h,t}}{\tilde{p}_{h,t+1}} \right)^{-1-\varepsilon} (\pi_{h,t+1})^\varepsilon g_{h,t+1}^1, \\
g_{h,t}^2 &= y_{h,t} (\tilde{p}_{h,t})^{-\varepsilon} + \theta_p \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{\tilde{p}_{h,t}}{\tilde{p}_{h,t+1}} \right)^{-\varepsilon} (\pi_{h,t+1})^{\varepsilon-1} g_{h,t+1}^2, \\
g_{h,t}^{*2} &= \mathcal{M}_p g_{h,t}^{*1}, \\
g_{h,t}^{*1} &= y_{h,t}^* \frac{mc_t}{d_t} (\tilde{p}_{h,t}^*)^{-1-\varepsilon} + \theta_p \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{\tilde{p}_{h,t}^*}{\tilde{p}_{h,t+1}^*} \right)^{-1-\varepsilon} (\pi_{h,t+1}^*)^\varepsilon g_{h,t+1}^{*1}, \\
g_{h,t}^{*2} &= m_{h,t} y_{h,t}^* (\tilde{p}_{h,t}^*)^{-\varepsilon} + \theta_p \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{\tilde{p}_{h,t}^*}{\tilde{p}_{h,t+1}^*} \right)^{-\varepsilon} (\pi_{h,t+1}^*)^{\varepsilon-1} g_{h,t+1}^{*2}, \\
g_{f,t}^{*2} &= \mathcal{M}_p g_{f,t}^{*1}, \\
g_{f,t}^{*1} &= y_{f,t}^* \frac{mc_t^*}{d_t^*} (\tilde{p}_{f,t}^*)^{-1-\varepsilon} + \theta_p \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^*}{\lambda_t^*} \right) \left(\frac{\tilde{p}_{f,t}^*}{\tilde{p}_{f,t+1}^*} \right)^{-1-\varepsilon} (\pi_{f,t+1}^*)^\varepsilon g_{f,t+1}^{*1}, \\
g_{f,t}^{*2} &= y_{f,t}^* (\tilde{p}_{f,t}^*)^{-\varepsilon} + \theta_p \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^*}{\lambda_t^*} \right) \left(\frac{\tilde{p}_{f,t}^*}{\tilde{p}_{f,t+1}^*} \right)^{-\varepsilon} (\pi_{f,t+1}^*)^{\varepsilon-1} g_{f,t+1}^{*2}, \\
g_{f,t}^2 &= \mathcal{M}_p g_{f,t}^1, \\
g_{f,t}^1 &= y_{f,t} \frac{mc_t^*}{d_t^*} (\tilde{p}_{f,t})^{-1-\varepsilon} + \theta_p \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^*}{\lambda_t^*} \right) \left(\frac{\tilde{p}_{f,t}}{\tilde{p}_{f,t+1}} \right)^{-1-\varepsilon} (\pi_{f,t+1})^\varepsilon g_{f,t+1}^1, \\
g_{f,t}^2 &= \frac{y_{f,t}}{m_{f,t}} (\tilde{p}_{f,t})^{-\varepsilon} + \theta_p \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^*}{\lambda_t^*} \right) \left(\frac{\tilde{p}_{f,t}}{\tilde{p}_{f,t+1}} \right)^{-\varepsilon} (\pi_{f,t+1})^{\varepsilon-1} g_{f,t+1}^2, \\
\mathcal{M}_p &= \frac{\varepsilon}{\varepsilon - 1}.
\end{aligned}$$

Interest rates and monetary policy with national currencies:

$$\begin{aligned}
R_t &= \frac{1}{q_t^A}, \\
R_t^* &= \frac{1}{q_t^B}, \\
\frac{R_t}{\pi/\beta} &= \left(\frac{R_{t-1}}{\pi/\beta} \right)^{\gamma_R} \left[\left(\frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left(\frac{y_t}{y_{t-1}} \right)^{\gamma_y} (\Delta e_t)^{\gamma_e} \right]^{1-\gamma_R} m_t,
\end{aligned}$$

$$\frac{R_t^*}{\pi^*/\beta} = \left(\frac{R_{t-1}^*}{\pi^*/\beta} \right)^{\gamma_R} \left[\left(\frac{\pi_t^*}{\pi^*} \right)^{\gamma_\pi} \left(\frac{y_t^*}{y_{t-1}^*} \right)^{\gamma_y} \left(\frac{1}{\Delta e_t} \right)^{\gamma_e} \right]^{1-\gamma_R} m_t^*.$$

Interest rates and monetary policy with a single currency:

$$\frac{R_t^u}{\pi_u/\beta} = \left(\frac{R_{t-1}^u}{\pi_u/\beta} \right)^{\gamma_R} \left[\left(\frac{\pi_{u,t}}{\pi_u} \right)^{\gamma_\pi} \left(\frac{y_{u,t}}{y_{u,t-1}} \right)^{\gamma_y} \right]^{1-\gamma_R} m_t^u,$$

$$\pi_{u,t} = (\pi_{h,t})^{\frac{1}{2}} (\pi_{f,t})^{\frac{1}{2}},$$

$$y_{u,t} = y_t + y_t^*,$$

$$R_t = R_t^u \left(\frac{\pi_{h,t}}{\pi_{u,t}} \right)^\omega,$$

$$R_t^* = R_t^u \left(\frac{\pi_{f,t}}{\pi_{u,t}} \right)^\omega.$$

Exogenous processes:

$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \nu_\xi \log \xi_{t-1}^* + e_{\xi,t},$$

$$\log \xi_t^* = \rho_\xi \log \xi_{t-1}^* + \nu_\xi \log \xi_{t-1} + e_{\xi,t}^*,$$

$$\log \phi_t = \rho_\phi \log \phi_{t-1} + \nu_\phi \log \phi_{t-1}^* + e_{\phi,t},$$

$$\log \phi_t^* = \rho_\phi \log \phi_{t-1}^* + \nu_\phi \log \phi_{t-1} + e_{\phi,t}^*,$$

$$\log z_t = \rho_z \log z_{t-1} + \nu_z \log z_{t-1}^* + e_{z,t},$$

$$\log z_t^* = \rho_z \log z_{t-1}^* + \nu_z \log z_{t-1} + e_{z,t}^*,$$

$$\log m_t = \rho_m \log m_{t-1} + \nu_m \log m_{t-1}^* + e_{m,t},$$

$$\log m_t^* = \rho_m \log m_{t-1}^* + \nu_m \log m_{t-1} + e_{m,t}^*,$$

$$\log m_t^u = \rho_m \log m_{t-1}^u + e_{m,t}^u.$$

Calibration for Section 3.6

Table 4: Calibration

Parameter	Value	Description
σ	1	Elasticity of intertemporal substitution
φ	1	Frisch elasticity of labour supply
ε	6	Elasticity of substitution between intermediates
ζ	0.5	Share of imports in consumption
η	1	Armington elasticity of substitution
β	0.99	Subjective discount factor
χ	0.005	Bond holdings adjustment cost
θ_p	0.66	Price stickiness parameter
τ	0	Trade cost parameter
γ_R	0	Interest rate smoothing parameter in the Taylor rule
γ_π	1.5	Inflation parameter in the Taylor rule
γ_y	0	Output parameter in the Taylor rule
γ_e	1.5	Exchange rate parameter in the Taylor rule
ω	0.001	Elasticity of interest rates to inflation differentials
σ_{e_z}	0.01	Standard deviation of productivity shocks
σ_{e_m}	0	Standard deviation of monetary shocks
σ_{e_ξ}	0	Standard deviation of preference shocks
σ_{e_ϕ}	0	Standard deviation of labour supply shocks
ρ_z	0.95	Serial correlation of productivity shocks
ρ_m	0.95	Serial correlation of monetary shocks
ρ_ξ	0.95	Serial correlation of preference shocks
ρ_ϕ	0.95	Serial correlation of labour supply shocks
ν_z	0	International spillover of productivity shocks
ν_m	0	International spillover of monetary shocks
ν_ξ	0	International spillover of preference shocks
ν_ϕ	0	International spillover of labour supply shocks
$\text{corr}(e_z, e_z^*)$	0	International correlation of productivity shocks
$\text{corr}(e_m, e_m^*)$	0	International correlation of monetary shocks
$\text{corr}(e_\xi, e_\xi^*)$	0	International correlation of preference shocks
$\text{corr}(e_\phi, e_\phi^*)$	0	International correlation of labour supply shocks

Estimation: Tables and Figures

Table 5: Calibrated parameters

Parameter	Description	Value
σ	Elasticity of intertemporal substitution	1
φ	Frisch elasticity of labour supply	1
ε	Elasticity of substitution between intermediates	6
ζ	Share of imports in consumption	0.5
η	Armington elasticity of substitution	1
β	Subjective discount factor	0.99
χ	Bond holdings adjustment cost	0.005
θ_p	Price stickiness parameter	0.66
ω	Elasticity of interest rates to inflation differentials	0.001
ν_z	International spillover of productivity shocks	0
ν_m	International spillover of monetary shocks	0
ν_ξ	International spillover of preference shocks	0
ν_ϕ	International spillover of labour supply shocks	0
τ	Trade cost parameter	0.05
$\text{corr}(e_m, e_m^*)$	International correlation of monetary shocks	0
γ_e	Exchange rate parameter in the Taylor rule	3

Table 6: Prior distributions

Parameter	Description	Prior distribution (μ, σ)
γ_R	Interest rate smoothing parameter in the Taylor rule	Normal (0, 0.25)
γ_π	Inflation parameter in the Taylor rule	Normal (1.5, 0.25)
γ_y	Output parameter in the Taylor rule	Normal (0.5, 0.25)
σ_{e_z}	Standard deviation of home productivity shocks	Gamma (0.05, 0.025)
$\sigma_{e_z^*}$	Standard deviation of foreign productivity shocks	Gamma (0.05, 0.025)
$\sigma_{e_m^u}$	Standard deviation of monetary shocks	Gamma (0.05, 0.025)
σ_{e_ξ}	Standard deviation of home preference shocks	Gamma (0.05, 0.025)
$\sigma_{e_\xi^*}$	Standard deviation of foreign preference shocks	Gamma (0.05, 0.025)
σ_{e_ϕ}	Standard deviation of home labour supply shocks	Gamma (0.05, 0.025)
$\sigma_{e_\phi^*}$	Standard deviation of foreign labour supply shocks	Gamma (0.05, 0.025)
ρ_z	Serial correlation of productivity shocks	Beta (0.5, 0.15)
ρ_m	Serial correlation of monetary shocks	Beta (0.5, 0.15)
ρ_ξ	Serial correlation of preference shocks	Beta (0.5, 0.15)
ρ_ϕ	Serial correlation of labour supply shocks	Beta (0.5, 0.15)
$\text{corr}(e_z, e_z^*)$	International correlation of productivity shocks	Normal (0, 0.25)
$\text{corr}(e_\xi, e_\xi^*)$	International correlation of preference shocks	Normal (0, 0.25)
$\text{corr}(e_\phi, e_\phi^*)$	International correlation of labour supply shocks	Normal (0, 0.25)

Table 7: Parameter estimates, part I

Country pair	Italy and France			Italy and Germany			Italy and Spain		
	Parameter	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval
	γ_R	0.2780	[0.1147, 0.4448]	0.5115	[0.4032, 0.6241]	0.5152	[0.4074, 0.6279]	2.4972	[2.2137, 2.7707]
	γ_π	2.6082	[2.3331, 2.8868]	2.5004	[2.2145, 2.7791]	0.3192	[0.1980, 0.4389]	0.0091	[0.0076, 0.0105]
	γ_y	0.1295	[0.0482, 0.2106]	0.1378	[0.0700, 0.2043]	0.0119	[0.0098, 0.0139]	0.0011	[0.0008, 0.0014]
	σ_{e_z}	0.0131	[0.0103, 0.0158]	0.0090	[0.0075, 0.0105]	0.0437	[0.0241, 0.0636]	0.0528	[0.0288, 0.0758]
	$\sigma_{e_z^*}$	0.0110	[0.0082, 0.0138]	0.0194	[0.0165, 0.0223]	0.0114	[0.0082, 0.0147]	0.0273	[0.0218, 0.0328]
	$\sigma_{e_m^u}$	0.0018	[0.0013, 0.0023]	0.0011	[0.0008, 0.0014]	0.9862	[0.9786, 0.9945]	0.9521	[0.9292, 0.9756]
	σ_{e_ξ}	0.0015	[0.0007, 0.0023]	0.0408	[0.0224, 0.0586]	0.9738	[0.9608, 0.9870]	0.0611	[0.0202, 0.1003]
	$\sigma_{e_\xi^*}$	0.0036	[0.0021, 0.0050]	0.0442	[0.0211, 0.0662]	-0.3394	[-0.5362, -0.1490]	0.3620	[0.0151, 0.7093]
	σ_{e_ϕ}	0.0276	[0.0208, 0.0342]	0.0152	[0.0120, 0.0183]	-0.8814	[-1.0000, -0.7114]		
	$\sigma_{e_\phi^*}$	0.0344	[0.0272, 0.0417]	0.0343	[0.0286, 0.0402]				
	ρ_z	0.9407	[0.9161, 0.9664]	0.9824	[0.9727, 0.9923]				
	ρ_m	0.9398	[0.9099, 0.9696]	0.9487	[0.9247, 0.9735]				
	ρ_ξ	0.1493	[0.0508, 0.2421]	0.9732	[0.9600, 0.9867]				
	ρ_ϕ	0.7583	[0.7061, 0.8145]	0.0486	[0.0153, 0.0799]				
	$\text{corr}(e_z, e_z^*)$	0.7795	[0.6126, 0.9663]	-0.3321	[-0.5193, -0.1500]				
	$\text{corr}(e_\xi, e_\xi^*)$	-0.2386	[-0.6068, 0.1359]	0.3913	[0.0277, 0.7633]				
	$\text{corr}(e_\phi, e_\phi^*)$	0.9219	[0.8210, 1.0000]	-0.9622	[-1.0000, -0.9108]				

Table 8: Parameter estimates, part II

Country pair	France and Germany		France and Spain		Spain and Germany	
	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval	Posterior mean	90% HPD interval
γ_R	0.5703	[0.4671, 0.6765]	0.5541	[0.4437, 0.6689]	0.7208	[0.6464, 0.7969]
γ_π	2.4830	[2.1965, 2.7755]	2.4720	[2.1805, 2.7611]	2.2821	[1.9773, 2.5971]
γ_y	0.1221	[0.0299, 0.2127]	0.3340	[0.1466, 0.5226]	0.3038	[0.1185, 0.4823]
σ_{e_z}	0.0071	[0.0058, 0.0084]	0.0069	[0.0056, 0.0082]	0.0091	[0.0073, 0.0108]
$\sigma_{e_z^*}$	0.0209	[0.0178, 0.0239]	0.0128	[0.0106, 0.0150]	0.0225	[0.0191, 0.0259]
$\sigma_{e_m^u}$	0.0013	[0.0009, 0.0017]	0.0014	[0.0010, 0.0019]	0.0016	[0.0010, 0.0022]
σ_{e_ξ}	0.0349	[0.0187, 0.0513]	0.0390	[0.0196, 0.0580]	0.0313	[0.0160, 0.0469]
$\sigma_{e_\xi^*}$	0.0432	[0.0202, 0.0655]	0.0478	[0.0263, 0.0699]	0.0367	[0.0134, 0.0597]
σ_{e_ϕ}	0.0139	[0.0108, 0.0171]	0.0088	[0.0058, 0.0118]	0.0177	[0.0135, 0.0220]
$\sigma_{e_\phi^*}$	0.0398	[0.0331, 0.0462]	0.0277	[0.0222, 0.0331]	0.0451	[0.0376, 0.0527]
ρ_z	0.9880	[0.9815, 0.9950]	0.9899	[0.9848, 0.9960]	0.9895	[0.9839, 0.9957]
ρ_m	0.8895	[0.8432, 0.9375]	0.8796	[0.8281, 0.9335]	0.7830	[0.7115, 0.8588]
ρ_ξ	0.9682	[0.9521, 0.9850]	0.9692	[0.9525, 0.9860]	0.9659	[0.9472, 0.9852]
ρ_ϕ	0.0463	[0.0151, 0.0765]	0.0463	[0.0147, 0.0764]	0.0453	[0.0148, 0.0752]
$\text{corr}(e_z, e_z^*)$	-0.5249	[-0.6904, -0.3629]	-0.4754	[-0.6729, -0.2862]	-0.7226	[-0.8387, -0.6096]
$\text{corr}(e_\xi, e_\xi^*)$	0.4379	[0.0635, 0.8280]	0.4044	[0.0480, 0.7667]	0.3088	[-0.0671, 0.6867]
$\text{corr}(e_\phi, e_\phi^*)$	-0.9501	[-1.0000, -0.8803]	-0.7641	[-1.0000, -0.4840]	-0.9659	[-1.0000, -0.9186]

Table 9: Unconditional variance decompositions: Italy and France

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	86.95%	0.52%	5.61%	0.67%	2.62%	0.05%	3.57%
y_t^*	4.66%	66.47%	16.08%	7.86%	0.60%	1.26%	3.06%
c_t	40.64%	29.94%	16.69%	5.10%	1.92%	0.66%	5.06%
c_t^*	41.75%	30.90%	14.32%	5.18%	2.04%	0.71%	5.11%
π_t	0.19%	0.04%	0.08%	0.39%	0.06%	0.01%	99.24%
π_t^*	1.09%	1.61%	15.86%	11.63%	0.10%	0.27%	69.44%
R_t^u	0.83%	0.36%	1.00%	1.84%	0.30%	0.04%	95.63%

Table 10: Unconditional variance decompositions: Italy and Germany

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	95.73%	0.16%	0.00%	0.00%	0.08%	0.00%	4.03%
y_t^*	6.38%	89.40%	0.00%	0.00%	0.08%	0.00%	4.14%
c_t	36.74%	53.63%	0.00%	0.00%	0.18%	0.00%	9.44%
c_t^*	36.93%	53.52%	0.00%	0.00%	0.18%	0.00%	9.36%
π_t	0.06%	0.01%	0.03%	0.00%	0.03%	0.00%	99.86%
π_t^*	1.00%	0.74%	0.29%	0.08%	0.07%	0.11%	97.72%
R_t^u	0.13%	0.08%	0.26%	0.00%	0.06%	0.00%	99.47%

Table 11: Unconditional variance decompositions: Italy and Spain

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	96.73%	0.10%	0.00%	0.00%	0.06%	0.00%	3.11%
y_t^*	1.11%	95.67%	0.00%	0.00%	0.05%	0.00%	3.16%
c_t	48.71%	45.37%	0.00%	0.00%	0.11%	0.00%	5.82%
c_t^*	48.65%	45.45%	0.00%	0.00%	0.11%	0.00%	5.79%
π_t	0.06%	0.01%	0.03%	0.00%	0.03%	0.00%	99.97%
π_t^*	0.51%	0.79%	0.20%	0.07%	0.03%	0.11%	98.29%
R_t^u	0.05%	0.07%	0.23%	0.00%	0.04%	0.00%	99.60%

Table 12: Unconditional variance decompositions: France and Germany

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	94.29%	0.11%	0.00%	0.00%	0.05%	0.00%	5.54%
y_t^*	6.13%	88.18%	0.01%	0.00%	0.05%	0.00%	5.64%
c_t	34.57%	52.51%	0.00%	0.00%	0.11%	0.00%	12.80%
c_t^*	34.71%	52.45%	0.01%	0.00%	0.11%	0.00%	12.72%
π_t	0.10%	0.03%	0.07%	0.00%	0.06%	0.00%	99.74%
π_t^*	1.78%	1.27%	0.61%	0.16%	0.12%	0.20%	95.86%
R_t^u	0.28%	0.19%	0.81%	0.00%	0.13%	0.00%	98.59%

Table 13: Unconditional variance decompositions: France and Spain

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	95.82%	0.08%	0.00%	0.00%	0.04%	0.00%	4.07%
y_t^*	0.80%	95.04%	0.00%	0.00%	0.04%	0.00%	4.12%
c_t	47.40%	44.95%	0.00%	0.00%	0.07%	0.00%	7.57%
c_t^*	47.36%	45.02%	0.00%	0.00%	0.07%	0.00%	7.55%
π_t	0.14%	0.02%	0.08%	0.00%	0.07%	0.00%	99.68%
π_t^*	1.13%	1.77%	0.52%	0.19%	0.06%	0.02%	96.08%
R_t^u	0.15%	0.24%	0.93%	0.00%	0.12%	0.00%	98.57%

Table 14: Unconditional variance decompositions: Spain and Germany

Variable	Shock						
	e_z	e_z^*	e_ξ	e_ξ^*	e_ϕ	e_ϕ^*	e_m^u
y_t	83.51%	0.07%	0.01%	0.00%	0.03%	0.00%	16.38%
y_t^*	5.29%	78.07%	0.01%	0.00%	0.02%	0.00%	16.61%
c_t	26.41%	40.93%	0.02%	0.00%	0.06%	0.00%	32.58%
c_t^*	26.49%	40.96%	0.02%	0.00%	0.06%	0.00%	32.47%
π_t	0.12%	0.02%	0.09%	0.00%	0.06%	0.00%	99.71%
π_t^*	1.32%	1.16%	0.60%	0.14%	0.10%	0.16%	96.52%
R_t^u	0.32%	0.31%	2.36%	0.00%	0.12%	0.00%	96.89%

Figure 6: Probability density functions of λ

