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# An all-sky radiative transfer method to predict optimal tilt and azimuth angle of a solar collector

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## Abstract

This paper describes a radiative transfer method for calculating radiances in all-sky conditions and performing an integration over the view hemisphere of an arbitrary plane to calculate tilted irradiance. The advantage of this method is the combination of cloud parameters inside the radiative transfer model with a tilt procedure. For selected locations this method is applied with cloud, ozone, water vapour and aerosol input data to determine tilted irradiance, horizontal irradiance and optimal tilt angle. A validation is performed for horizontal and tilted irradiance against high-quality pyranometer data. For 27 sites around the world, the annual horizontal irradiation predicted by our model had a mean bias difference of +0.56% and a root-mean-squared difference of 6.69% compared to ground measurements. The difference between the annual irradiation estimates from our model and the measurements from one site that provides tilted irradiance were within  $\pm 6\%$  for all orientations except the north-facing vertical plane. For European and African sites included in the validation, the optimal tilt from our model is typically a few degrees steeper than predictions from the popular PVGIS online tool. Our model is generally applicable to any location on the earth's surface as the satellite cloud and atmosphere data and aerosol climatology data are available globally. Furthermore, all of the input data are standard variables in climate models and so this method can be used to predict tilted irradiance in future climate

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experiments.

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## 1. Introduction

The orientation of a plane solar collector such as a PV panel can be varied in the tilt and azimuth directions in order to maximise the incident irradiance. One way to accurately assess the solar resource available on a tilted plane and determine the optimum angle to orient a fixed angle PV panel in the real world, is to position pyranometers in several plane orientations and record the sum of irradiance over a sufficiently long period of time. In practice this is rarely completed, so models to predict the tilted irradiance are used.

There are two concepts fundamental to the method described. Firstly, cloud optical properties, from satellite retrievals, are integrated into the radiative transfer (RT) calculation. Secondly, tilted irradiance is derived from a surface radiance field. RT methods are frequently used to model clear-sky solar irradiance (Bird and Riordan, 1986; Gueymard, 1995; Mueller et al., 2004). Sometimes cloud effects are introduced as an adjustment to the clear-sky values depending on satellite-derived cloud albedo (Cano et al., 1986) or tuned based on observed historical ground-level irradiance (Nann and Emery, 1992). In other studies cloud effects are included directly. Lohmann et al. (2006) used data from meteorological reanalyses and cloud parameters from the International Satellite Cloud Climatology Project (ISCCP) with a two-stream radiative transfer code to estimate surface irradiance. Deneke et al. (2008) used cloud retrievals from Meteosat in combination with RT simulations to estimate solar irradiance in the Netherlands. Mueller et al. (2009) used a lookup table approach for clouds with transmissions pre-calculated with RT and values interpolated from the lookup table. They used a cloud effective radius of 10  $\mu\text{m}$  for water droplets using the Hu and Stamnes (1993) parametrisation of the phase function and did not consider ice clouds. While this may be sufficient for horizontal fluxes, this approach is less accurate when calculating the radiances required for the tilted irradiance. Behrendt et al. (2013) used the SOLIS clear-sky model with cloud adjustment to determine the spectral effects on different PV technologies. A separate run with clouds specified directly inside the radiative transfer model was per-

27 formed. The difference in spectral transmission between SOLIS and the RT solution using  
28 the libRadtran package (Mayer and Kylling, 2005) is about 5% in average photon energy  
29 for thick cloud cover (optical depth of 60) at a solar zenith angle of 60°. More recently, the  
30 UniSky simulator software (Kocifaj and Fečko, 2014; Kocifaj, 2015) includes the effects of a  
31 3D cloud field to model ground-level radiances. Current satellite products often include the  
32 required cloud optical properties, namely cloud phase (water or ice), cloud optical depth,  
33 and cloud droplet effective radius, to allow RT simulations including clouds to be performed.  
34 One motivation for inclusion of clouds inside the RT calculation is for the development of  
35 solar energy models that can be applied to a wide variety of historical, current and future  
36 datasets, for example meteorological reanalyses or climate models, as well as satellite obser-  
37 vations. Another is the spectral effects of cloud attenuation are better captured with RT  
38 simulation, which is important for PV.

39 After the directional radiances have been calculated from the RT simulation, integrating  
40 the radiance field over the direction of interest will provide the tilted irradiance. McArthur  
41 and Hay (1981) used radiance distributions obtained from fish-eye photographs and obtained  
42 agreement to  $\pm 10\%$  for horizontal diffuse irradiance and  $\pm 5\%$  for tilted irradiance on a south-  
43 facing plane, in a variety of sky conditions. Brunger and Hooper (1993) derived an empirical  
44 model for the sky radiance distribution calculated from observations of clearness index (ratio  
45 of surface irradiance to extraterrestrial irradiance) and zenith angle. Similarly Gueymard  
46 (1987) derived the sky radiance distribution by producing different anisotropic sky radiance  
47 distributions for a clear-sky and an overcast sky. The all-sky radiance distribution was  
48 calculated as a weighted sum of the clear and overcast cases with cloud transmission as the  
49 weighting factor.

50 Other popular anisotropic tilted irradiance models (e.g. Bugler (1977); Klucher (1979);  
51 Willmott (1982); Hay and Davies (1980); Skartveit and Olseth (1986); Reindl et al. (1990);  
52 Perez et al. (1990); Muneer (1990)) are varyingly complex functions of the horizontal diffuse  
53 and direct irradiance measurements along with solar position and panel orientation. A  
54 comparison between ten tilt models at the NREL site at Golden, Colorado, USA, found

55 that most anisotropic models did not predict irradiance with a satisfactorily low error for  
 56 tilted planes compared to the bounds of instrumental error from pyranometers (Gueymard,  
 57 2009). An intercomparison of 15 models (4 isotropic and 11 anisotropic) in Denmark, France  
 58 and Spain again found that no one anisotropic model generally performed better than the  
 59 others consistently when considering different cloud conditions, tilt angles and azimuth angles  
 60 (Gracia-Amillo and Huld, 2013). Therefore, the continued development of tilt models for  
 61 all-sky conditions is desirable.

62 In this paper, the optimal tilt angle of a fixed-angle solar collector is considered. For  
 63 comparison with the PVGIS method, the panel is oriented towards the equator, although  
 64 it is also possible to optimise azimuth as shown in section 4.3. In the absence of horizon  
 65 obstruction, shading, or radically different morning and afternoon weather conditions, the  
 66 equatorial direction provides the best azimuthal alignment. The tilt angle of integration is  
 67 varied to find the irradiance at each angle and summed over a year of operation to determine  
 68 the optimal tilt. The model is tested against the tilted irradiance model in PVGIS and  
 69 compared to tilted irradiance measurements from NREL.

## 70 **2. Determining tilted irradiance from radiances**

71 The irradiance on a tilted plane angled at tilt  $\beta$  and azimuth  $\gamma$  is a combination of the  
 72 downwards and upwards radiance fields such that the bounds of the integration is over the  
 73 hemisphere with base in the plane of the solar collector (Gueymard, 1987):

$$I_T = \int_0^{2\pi} \int_0^{\theta_m} L(\theta, \phi) \cos \theta_d \sin \theta \, d\theta \, d\phi \quad (1)$$

74 where the angle between the normal of the tilted plane and the radiance direction of interest  
 75 is given by

$$\cos \theta_d = \cos \beta \cos \theta + \sin \beta \sin \theta \cos(\phi - \gamma) \quad (2)$$

76 and the bound of the integration  $\theta_m$  is in the plane of the solar collector such that

$$\theta_m = \frac{\pi}{2} - \tan^{-1}(\cos(\phi - \gamma) \tan \beta). \quad (3)$$

77 The radiance field  $L$  is calculated at a resolution of  $3^\circ$  in the polar direction and  $10^\circ$  in the  
78 azimuthal direction using the DISORT radiative transfer code (Stamnes et al., 2000), as part  
79 of the libRadtran package (Mayer and Kylling, 2005), with a pseudo-spherical correction to  
80 improve accuracy at low solar elevations (Dahlback and Stamnes, 1991).  $\theta$  is the polar angle  
81 and  $\phi$  is the azimuthal angle. The radiative transfer equation is solved numerically with  
82 16 streams, the minimum recommended for calculating radiances (Mayer et al., 2012). Eq.  
83 (1) is approximated numerically by summing each radiance element over small solid angles  
84  $\Delta\theta\Delta\phi$  such that

$$I_T \approx \sum_j \sum_k L(\theta_j, \phi_k) W \Delta\theta_j \Delta\phi_k \quad (4)$$

85 where  $W = \max(0, \cos\theta_{dj} \sin\theta_j)$  to ensure only the radiances in the field of view of the solar  
86 collector are counted (McArthur and Hay, 1981). At non-zero tilts, the field of view will  
87 include some upwelling radiances from the ground which depend on the surface albedo and  
88 exclude any sky radiances emanating from directions behind the solar collector.  $\cos\theta_{dj}$  is as  
89 given in eq. (2) with  $(\theta, \phi)$  replaced with  $(\theta_j, \phi_k)$ .

90 To perform a complete calculation line-by-line over the whole solar spectrum for  $61 \times 36$   
91 radiance directions is infeasible in terms of computational time, so the correlated- $k$  method  
92 (Kato et al., 1999) is used to divide the solar spectrum into 32 wavelength bands with  
93 similar atmospheric absorption properties. The calculation in eq. (4) is performed for each  
94 correlated- $k$  band and the broadband radiance for each  $(\theta_j, \phi_k)$  pair is obtained by summing  
95 up  $I_T$  for each of the 32 correlated- $k$  bands.

96 The numerical approximation in eq. (4) is performed for the diffuse irradiance only. The  
97 direct normal irradiance (DNI) is simpler to calculate. From the Beer-Lambert law the DNI  
98 is

$$I_B = I_0 \exp(-m\tau) \quad (5)$$

99 where  $I_0$  is extraterrestrial irradiance and  $m$  is air mass. The optical depth  $\tau$  describes the  
100 likelihood that a ray travels to the surface of the earth without being absorbed or scattered.

101  $\tau$  is the overall sum of the optical depths of all extinction phenomena in the atmosphere,  
102 e.g. mixed gases, ozone, water vapour, aerosols and cloud droplets. For a tilted plane, the  
103 direct incident irradiance is

$$I_{BT} = I_B \cos \theta_i \tag{6}$$

104 where the incident angle  $\theta_i$  follows a similar form to eq. (2):

$$\cos \theta_i = \cos \beta \cos \theta_z + \sin \beta \sin \theta_z \cos(\phi_a - \gamma). \tag{7}$$

105 Here,  $\theta_z$  is the solar zenith angle and  $\phi_a$  is the solar azimuth angle.

106 The radiative transfer method bears another advantage over empirical tilt models in  
107 that no assumption of the size and shape of the circumsolar region is made. When making  
108 ground irradiance measurements, the direct irradiance is not usually discernible from diffuse  
109 sky irradiance that has been scattered into the region of the solar disc or diffuse radiation  
110 emanating from the solar region that has been caused by strongly forward scattering aerosol  
111 or thin cloud. This can cause issues in calculating the direct and diffuse contributions as a  
112 decision has to be made on the angular size of the circumsolar region (Blanc et al., 2014).  
113 Often a half-angle of  $2.5^\circ$  is used with all irradiance inside this region treated as direct. In  
114 our model, all scattered radiation is treated as diffuse regardless of the scattering direction  
115 with the directional distribution handled by the radiance field.

### 116 **3. Inputs into the model**

117 To generate the radiance field, inputs of the atmospheric state, location altitude, clouds,  
118 aerosols and surface albedo are required. Although any climate, satellite or reanalysis dataset  
119 that provides all of the necessary inputs can be used, we use the Moderate Resolution Imag-  
120 ing Spectroradiometer (MODIS) instrument data on the Aqua and Terra satellites for all  
121 parameters except aerosols for which we use a climatological run from a dedicated aerosol  
122 model (GLOMAP). The Terra satellite overpasses the equator at approximately 10:30 local  
123 solar time daily and the Aqua satellite overpasses at approximately 13:30 daily. Therefore,

124 synoptic diurnal differences between the morning and afternoon can be partially captured.  
125 MODIS Level 3, 8-day mean data for ozone, water vapour, and cloud parameters (MOD08E3  
126 and MYD08E3 data series, both Collection 5.1) were used. Surface albedo was obtained  
127 from the combined Terra and Aqua 16-day running mean albedo product MCD43C3, which  
128 is updated every 8 days. The resolution of the atmosphere and cloud data is  $1^\circ \times 1^\circ$  and  
129 the albedo data is  $0.05^\circ \times 0.05^\circ$ . All data is freely available from the MODIS portal at  
130 <http://modis-atmos.gsfc.nasa.gov/>. 8-day time resolution is used as a trade-off be-  
131 tween capturing fluctuations in weather conditions and computational efficiency. Daily and  
132 monthly timesteps are also available for the Level 3 MODIS data.

### 133 *3.1. Atmosphere*

134 Well-mixed gases in the atmosphere are a source of Rayleigh scattering which is dependent  
135 on wavelength. Shorter wavelengths are scattered more strongly according to the well-known  
136  $\lambda^{-4}$  relationship.

137 libRadtran contains the set of six standard AFGL atmospheres (Anderson et al., 1986)  
138 which are tropical, mid-latitude summer and winter, sub-Arctic summer and winter, and  
139 US standard. The location and time of year dictates which particular atmosphere was  
140 selected in the calculation, however the impact of mixed gases on the final result is negligible  
141 (Oumbe et al., 2008; Mueller et al., 2009). Ozone is a strong absorber in the ultraviolet  
142 range and water vapour has absorption bands located throughout the near infrared, so the  
143 total atmospheric column depth of ozone and water vapour are taken from the MODIS data.

### 144 *3.2. Clouds*

145 Clouds are both the largest attenuating factor in the transmission of solar radiation and  
146 the source of the largest uncertainty for many regions of the world, the principal exceptions  
147 being in areas of high aerosol optical depth and infrequent clouds such as deserts. Both  
148 liquid and ice water clouds exhibit complex scattering properties. The radiative properties  
149 of clouds are determined by cloud droplet effective radius  $r_{\text{eff}}$ , single scattering albedo  $\omega$ ,  
150 phase function  $P(\mu)$  where  $\mu$  is the cosine of the scattering angle, and the cloud water



151 content  $C$  which is the mass of cloud droplets present in a given volume. The cloud optical  
152 depth  $\tau_c$  is a function of  $C$  and  $r_{\text{eff}}$ . The single scattering albedo determines the probability  
153 that if a ray collides with a cloud droplet, it is scattered rather than absorbed. The phase  
154 function describes the directional distribution of scattering event and hence is important in  
155 determining the final diffuse irradiance field.

156 For calculating radiances it is recommended to use the full Mie scattering parametrisation  
157 for liquid cloud droplets (Mayer et al., 2012) which provide  $\omega$  and  $P(\mu)$  as a function of  
158 wavelength. This is available as an extension to the core libRadtran package in the form of  
159 pre-calculated lookup tables generated using the Wiscombe (1980) Mie scattering code.

160 Ice clouds pose a particular complexity as ice crystals form in a variety of habits (shapes),  
161 on which the scattering phase function is strongly dependent. Additional morphological  
162 features such as surface roughness and trapped air bubbles also affect the phase function  
163 (Xie et al., 2006, 2012). The cloud retrieval algorithm for Collection 5.1 in MODIS uses a  
164 mixture of particle habits depending on the maximum diameter  $D_{\text{max}}$  of the ice crystals:  
165 50% solid columns, 15% 3D bullet rosettes and 35% hexagonal plates for particles where  
166  $60 < D_{\text{max}} < 1000 \mu\text{m}$ , and 45% solid columns, 45% hollow columns and 10% aggregates  
167 for particles where  $1000 < D_{\text{max}} < 2000 \mu\text{m}$  (Baum et al., 2005; Menzel et al., 2010; Min-  
168 nis et al., 2011). A definition of 100% solid columns has been used in our model due to  
169 the difficulties of mixing habit types and the fact that solid columns make up the largest  
170 part of the mixture in the range of  $60 < D_{\text{max}} < 2000 \mu\text{m}$  corresponding to  $r_{\text{eff}}$  of ap-  
171 proximately 20–120  $\mu\text{m}$ , encompassing the majority of ice cloud effective radius retrievals.  
172 Out of the single-habit assumptions, solid columns provide the best estimates of ice water  
173 content and  $r_{\text{eff}}$  (Baum et al., 2005). The ice scattering has been represented by a double  
174 Henyey-Greenstein (DHG) phase function using the Key et al. (2002) model. The DHG is a  
175 convenient simplification of the real phase function that is suitable for modelling radiances  
176 due to its ability to somewhat account for the forward and backward scattering peaks better  
177 than the simpler single Henyey-Greenstein (HG) phase function (Mayer et al., 2012). In  
178 order to correctly model ice cloud scattering a full phase matrix scattering code should be

179 used (e.g. Baum et al. (2014)), however the number of Legendre coefficients that need to  
180 be calculated for each scattering phase function make its use computationally prohibitive  
181 for multiple calculations. The DHG phase function is smooth and does not include effects  
182 such as the 22° and 46° halo scattering peaks present in pristine hexagonal columns and  
183 plates. The roughened hexagonal column phase function has a less strong forward scattering  
184 component than pristine hexagonal columns and does not exhibit a halo effect, therefore is  
185 represented better by the DHG phase function. The assumption of roughened hexagonal  
186 columns provides the lowest RMS error in optical depth for MODIS retrievals (Xie et al.,  
187 2012) adding justification for the smooth DHG phase function approximation.

188 Owing to the large uncertainties in modelling clouds in time and space, it was decided to  
189 use a simplified approach with two atmospheric columns, one clear and the other overcast.  
190 The resulting radiance distribution is weighted between the two situations based on cloud  
191 fraction  $c_f$ . To define the cloudy column, the cloud liquid water content  $C_w$ , cloud ice water  
192 content  $C_i$  (both  $\text{g m}^{-3}$ ), cloud fraction  $c_f$ , cloud height  $h$ , and  $r_{\text{eff}}$  are used.  $r_{\text{eff}}$  may be, and  
193 usually is, different for liquid and ice droplets. Where both liquid and ice clouds are present,  
194 they are aggregated into the same column to create one mixed-phase cloud. The cloud is  
195 defined as having a vertical depth of 1 km except where the cloud top height is less than  
196 1 km above the ground in which case it extends down to the surface. For single scattering  
197 albedos  $\omega \rightarrow 1$ , which is the case for the majority of solar wavelengths (Hu and Stammes,  
198 1993), the fraction of transmitted to incident irradiance is approximately independent of  
199 the cloud geometric height. This has previously been demonstrated in RT calculations  
200 (Rozwadowska, 2004; Oumbe et al., 2008). For mathematical convenience and consistency  
201 with other investigations (e.g. Lohmann et al. (2006)) the somewhat arbitrary depth of 1 km  
202 has been chosen.  $C_w$ ,  $C_i$ ,  $c_f$  and  $r_{\text{eff}}$  are all available from the MODIS data. Currently  $h$  is  
203 only reported for Aqua, so cloud top pressure, which is available from both satellites, was  
204 converted to height for both Terra and Aqua data using the hydrostatic equation.

### 205 3.3. Aerosols

206 A monthly aerosol climatology is provided by the GLOMAP model (Scott et al., 2014) at  
207 a resolution of  $2.8^\circ \times 2.8^\circ$ , which specifies  $\omega$ , the asymmetry parameter  $g$ , and aerosol optical  
208 depth  $\tau_a$  for 6 wavelength bands in the shortwave spectrum on 31 pressure levels.  $g$  describes  
209 the mean cosine of the scattering angle from  $P(\mu)$  and ranges from  $-1$  for backscattering  
210 to  $+1$  for forward scattering. The species included are sulphate, sea-salt, black carbon and  
211 particulate organic matter aerosols in four size modes. A HG phase function is specified in  
212 our model, which has the large computational advantage of completely parametrising the  
213 phase function by  $g$ . MODIS data for aerosol has not been used as aerosol properties are  
214 not always available over land, particularly in desert regions which are important for solar  
215 energy and aerosols are prevalent.

### 216 3.4. Albedo

217 The surface albedo is the proportion of downwards irradiance that is reflected by the  
218 earth's surface. In reality, surface albedo is a function of wavelength and solar zenith angle  
219 as direct and diffuse irradiance components have different reflectance properties. Albedo is  
220 important in the tilted irradiance calculation as it defines the amount of reflected irradiance  
221 available from the ground that is available to a solar collector. Even at zero tilts, a higher  
222 surface albedo can increase downwards irradiance due to multiple reflections between surface  
223 and atmosphere, particularly if clouds are present (Gueymard, 2009).

224 The black-sky and white-sky albedos are calculated from the bi-directional reflectance  
225 distribution function (BRDF). Black-sky albedo is the albedo assuming all direct irradiance  
226 and no diffuse irradiance and is a function of solar zenith angle, whereas white sky albedo  
227 assumes a purely diffuse isotropic source and is independent of solar geometry. We have  
228 used the white sky albedo in this simulation due to the solar zenith independence. Deneke  
229 et al. (2008) has shown that this does not introduce significant error even in thin clouds.  
230 Surface albedo is spatially and temporally variable, even throughout the course of the same  
231 day (Gueymard, 2009), with the surface properties within a few metres of the solar collector  
232 of greatest importance.

## 233 4. Application of the model

234 One year of atmosphere, cloud and albedo data from 2013 was input into the model, and  
235 the solar zenith and azimuth were calculated at the centre of each hour for the middle day  
236 in each 8 day period. The diffuse radiance field  $L$  and direct normal irradiance  $I_B$  for each  
237 hour are the outputs from libRadtran. Plane irradiance for a particular tilt and azimuth is  
238 obtained by applications of eqs. (4) and (6) and adding together the results.

### 239 4.1. Radiance distributions

240 Fig. 1 shows the diffuse radiance distributions for clear sky, overcast sky and all sky  
241 (combination of clear and overcast), for a typical midday hour in northern European summer.  
242 The clear-sky case includes mixed gas, water vapour, ozone and aerosol attenuation. The  
243 anisotropy of clear-sky diffuse radiation due to the circumsolar region, and to a lesser extent  
244 the bright section near the horizon, can be seen from fig. 1(a). If an overcast sky is assumed  
245 (fig. 1(b)), it can be seen that the radiance distribution is much different, with a maximum  
246 intensity between the solar position and zenith which becomes apparently uniformly less  
247 intense away from this maximum towards the horizon. Fig. 1(c) shows the all-sky weighted  
248 radiance distribution taking into account the cloud fraction, which for this hour was 56.4%.  
249 The circumsolar peak is still apparent, but the horizon brightening contribution is hard to  
250 discern and the remaining sky radiance is more isotropically distributed than in the clear-sky  
251 case.

### 252 4.2. Tilted irradiance map

253 The radiance distributions for the same location were integrated over all polar and az-  
254 imuthal alignments using eq. (4), and the direct beam included, to provide a tilted irradiance  
255 map (fig. 2). Fig. 2(a) shows that when there are no clouds, the ideal panel alignment is  
256 more or less normal to the solar beam. There is a fairly wide tolerance around the optimal  
257 position as a result of the cosine of incidence angle being approximately 1 for small incidence  
258 angles. Fig. 2(b) shows that in an overcast sky, the ideal panel alignment is horizontal and  
259 independent of the solar direction even though the corresponding radiance distribution is

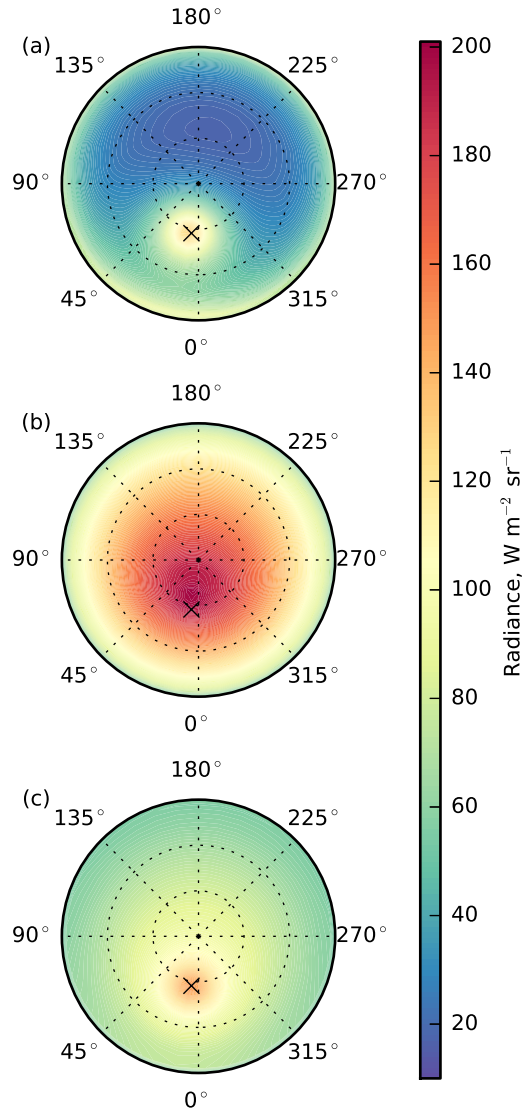


Figure 1: Radiance distributions (looking upwards). Distance from the centre represents polar angle and angular coordinate represents azimuth angle. (a) clear sky radiance distribution, (b) overcast radiance distribution (water cloud optical depth of 8.8), (c) all-sky distribution based on clear sky and cloudy sky distributions with cloud fraction equal to 56.4%. Solar position is represented by X at zenith 32.9°, azimuth 8.2° (the convention in this paper for azimuth is 0° for south, increasing clockwise).

260 off-zenith. In this example the optical depth of the cloud layer is 8.8, which is thick enough  
 261 to obscure the solar beam (Oumbe et al., 2008) with the resulting diffuse irradiance approx-  
 262 imately isotropically distributed. Fig. 2(c) shows the all-sky tilted irradiance map with the

263 cloud fraction of 56.4%. The optimal tilt of the solar collector is centred around the solar  
 264 position as in the clear-sky case, but with corresponding lower irradiance values.

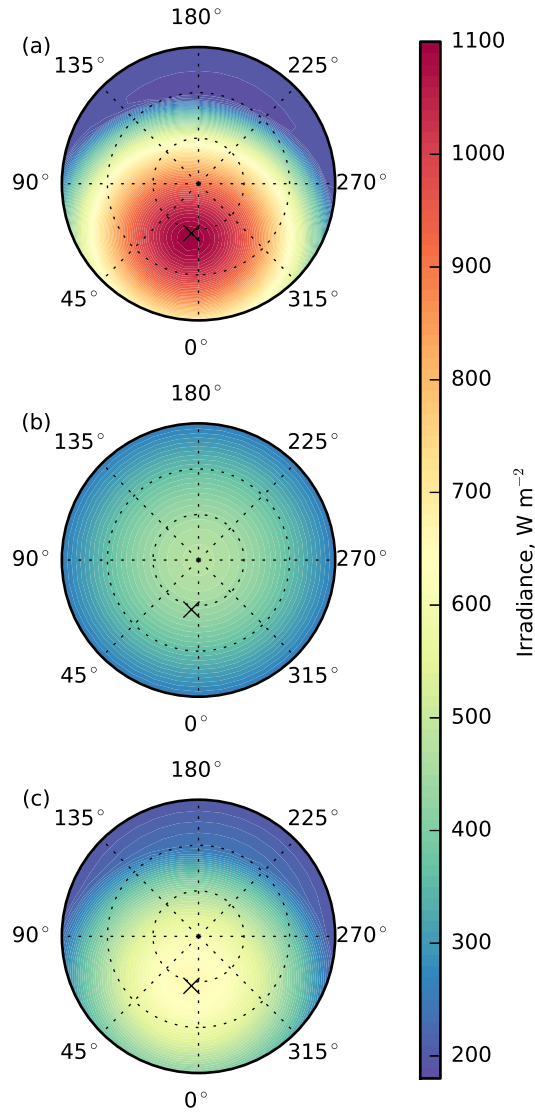


Figure 2: Tilted irradiance maps for the same location as fig. 1. Distance from the centre represents tilt angle with centre representing a horizontal alignment and the edge of the circle represents a vertical alignment. Angular coordinate represents azimuthal alignment. (a) clear sky, (b) overcast sky (water cloud optical depth of 8.8), (c) all-sky with cloud fraction equal to 56.4%. Solar position is represented by X at zenith  $32.9^\circ$ , azimuth  $8.2^\circ$ .

265 *4.3. Yearly tilted irradiation*

266 Radiance distributions were obtained for each hour of the middle day for each 8 day pe-  
 267 riod, and integrated using eq. (4) to produce tilted irradiance. The direct beam contribution  
 268 was included. Hourly irradiance outputs were then multiplied by the number of days in each  
 269 period (8, except for the last period of the year which is 5 or 6) and summed to generate  
 270 the yearly irradiation. For Church Fenton weather station in the UK, the yearly irradiation  
 271 map is shown in fig. 3.

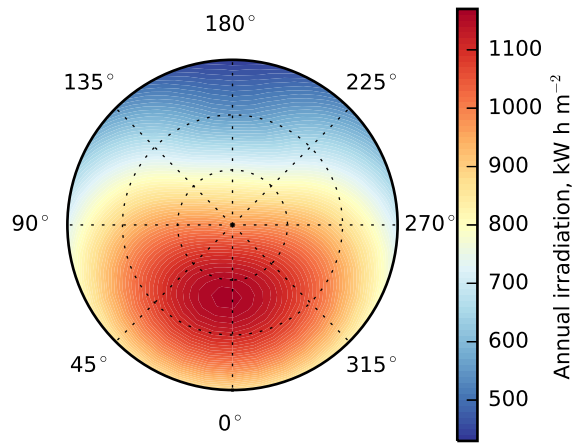


Figure 3: Angled irradiation map for Church Fenton (latitude 53.8°N, longitude 1.2°W, altitude 8 m) for the year of 2013

272 The optimal south-facing tilt for this location calculated using our method is 40° from  
 273 the horizontal. The optimal azimuthal alignment here is 6° west of south, highlighting that  
 274 the afternoon conditions may be clearer than the morning, although the difference in yearly  
 275 output between 6° and 0° is very small. A “rule of thumb” for annual optimal tilt is that  
 276 it should be equal to latitude on the basis that this minimises the incidence angle between  
 277 the solar beam and the normal to the panel surface at solar noon. For areas of the world  
 278 with significant cloud cover this does not hold true due to the frequent obscuring of the sun  
 279 by clouds. Christensen and Barker (2001) showed for the US the local clearness index could  
 280 be used to determine how close to latitude the optimal tilt angle  $\beta_{\text{opt}}$  would be with the  
 281 following relationship:

$$\beta_{\text{opt}} = (0.379 + K_{t,\text{year}})l - 20.6(1 - K_{t,\text{NDJ}}/K_{t,\text{MJJ}}) \quad (8)$$

282 with  $l$  representing latitude and  $K_{t,i}$  representing clearness index where for  $i$  NDJ =  
 283 {November, December, January}, MJJ = {May, June, July} and year=annual. For the  
 284 Church Fenton weather station in fig. 3,  $l = 53.8^\circ$ , and for 2013  $K_{t,\text{year}} = 0.424$ ,  
 285  $K_{t,\text{MJJ}} = 0.458$  and  $K_{t,\text{NDJ}} = 0.332$ . These low mean clearness indices are indicative of  
 286 frequently cloudy conditions. Equation (8) predicts  $\beta_{\text{opt}} = 38^\circ$  for this station, close to the  
 287  $40^\circ$  calculated with the integrated radiance method. Both models suggest the optimal tilt is  
 288 more horizontal than the angle of latitude at this location.

#### 289 4.4. Treatment of broken cloud fields

290 As described in section 3.2, the model uses a linear combination of clear and overcast  
 291 radiance distributions weighted by the cloud fraction. In reality, clouds exhibit both vertical  
 292 and horizontal heterogeneity, and our model is a simplification of the 3D picture (Marshak  
 293 and Davis, 2005). The diffuse reflections from the sides of clouds, along with cloud shadowing,  
 294 will impact the ground-level radiance field. We therefore compare our radiance distribution to  
 295 that generated by the UniSky simulator software available from <http://www.unisky.sav.sk>  
 296 (Kocifaj, 2012; Kocifaj and Fečko, 2014; Kocifaj, 2015). The UniSky simulator can model 3D  
 297 clouds either as a regular grid, or as randomly orientated. Random clouds can be grouped  
 298 into a preferred sky sector, simulating the effects of a morning or evening weather front. For  
 299 random cloud fields, a random seed is specified on input, allowing reproducibility of random  
 300 simulations.

301 To keep the simulations consistent, as the two models take different parameters, a simple  
 302 case is considered. We set the solar zenith angle to be  $30^\circ$  and azimuth to be  $0^\circ$ , cloud  
 303 fraction 20% with base at 3 km, geometric height 1 km and optical depth 10, and perform a  
 304 single monochromatic calculation at 550 nm wavelength. A generic aerosol with a Henyey-  
 305 Greenstein phase function ( $g = 0.7$ ), optical depth  $\tau_a = 0.2$  and single scattering albedo  
 306  $\omega = 0.9$  is prescribed and surface albedo is set to zero. Both models use the nadir-view



307 cloud fraction, which is the proportion of horizontal area covered by clouds to the total area,  
308 as viewed from a nadir-viewing instrument such as a satellite.

309 An additional parameter used in UniSky is the cloud reflectance. This is not supplied  
310 explicitly in our model but can be calculated. As cloud reflectance is dependent on optical  
311 depth, a plane-parallel cloud with  $r_{\text{eff}} = 10 \mu\text{m}$ , optical depth 10, base 3 km and vertical  
312 extent 1 km, with full Mie phase function, was modelled in libRadtran. Reflectance was  
313 found to be 40.7% at 550 nm, based on the ratio of upwelling to downwelling irradiance  
314 at the top of atmosphere with molecular scattering and absorption suppressed. Clouds are  
315 modelled as spheres in UniSky; the default value of 0.5 km radius is used.

316 100 runs of the random cloud field in UniSky were generated with the parameters de-  
317 scribed above, with the random seed ranging sequentially from 1 to 100. Two examples of  
318 these diffuse radiance fields for sun unobscured and sun obscured are shown in fig 4. The 100  
319 random runs could simulate a short period of time in which solar zenith angle and weather  
320 conditions remain relatively constant overlaid with a wind-driven broken cloud field. As  
321 UniSky does not include the DNI as an output (M. Kocifaj, personal communication), this  
322 was determined from eq. (5) with the total optical depth the sum of each component:

$$\tau = \tau_c + \tau_a + \tau_R \quad (9)$$

323 where  $\tau_a = 0.2$ ,  $\tau_R$  is the Rayleigh scattering optical depth at 550 nm of 0.1014 calculated  
324 as in Kocifaj (2012) and  $\tau_c$  is equal to 10 if the pixel is obscured by cloud and 0 otherwise.

325 Each of the 100 radiance fields produced by UniSky, along with the calculated beam  
326 component, was numerically integrated using a south-facing plane with tilt angle running  
327 from 0 to 90°. For the libRadtran run, one radiance field with cloud optical depth of 10 and  
328 cloud fraction 0.2 was calculated and the numerical integration applied. The mean value  
329 from the 100 UniSky runs is compared to the libRadtran output and the results for tilt angle  
330 ranging from 0 to 90° facing south are shown in fig. 5.

331 In both the libRadtran and the mean of the UniSky runs, the irradiance for this situation  
332 is maximised when the tilt angle is 29°. The effect of cloud obscurity can clearly be seen

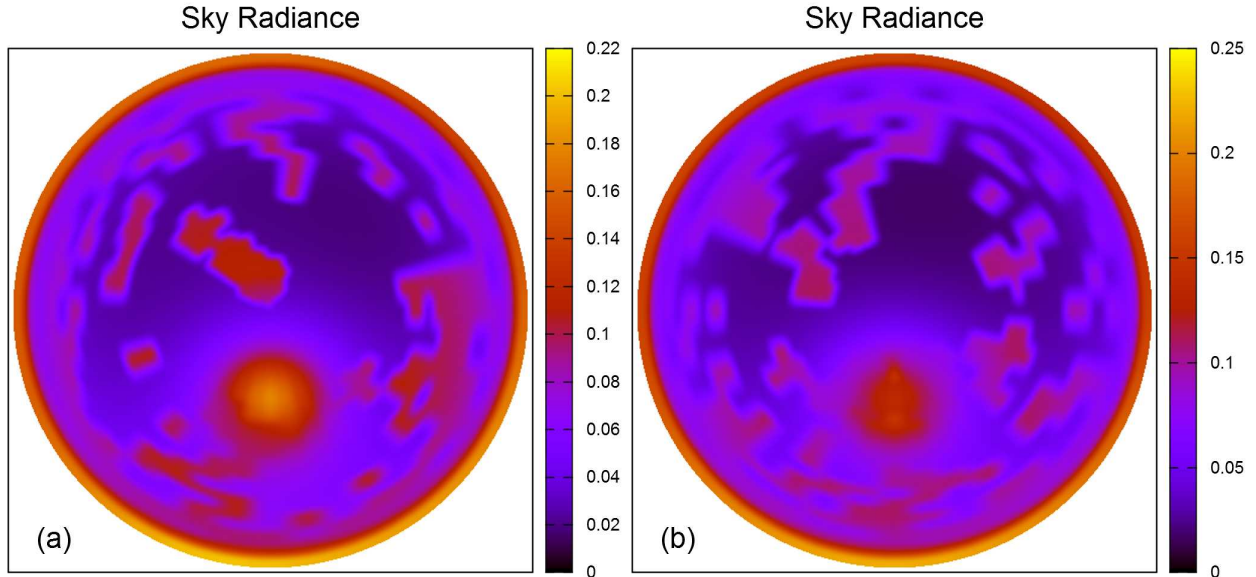


Figure 4: UniSky radiance distributions for two broken cloud regimes ( $c_f = 0.2$ ) where (a) the sun is not obscured and (b) the sun is obscured. Units are radiance normalised to the extraterrestrial DNI [ $\text{sr}^{-1}$ ].

333 in the bimodal character of the UniSky runs characterised by the clustering of the thin  
 334 grey lines in fig. 5. When a cloud lies in front of the sun, the irradiance at optimal tilt is  
 335 around 0.2 of its extraterrestrial values whereas it is close to 0.9 in the unobscured case. The  
 336 majority of this effect is due to the difference in direct beam transmission between the two  
 337 modes. The libRadtran method predicts a slightly higher irradiance at all tilt angles under  
 338 this method compared to UniSky.

339 As the UniSky simulator does not include multiple scattering within clouds (M. Kocifaj,  
 340 personal communication), only the gaps between clouds contribute substantially to down-  
 341 welling radiances. It is recommended (Kocifaj, 2015) to approximate a high cloud fraction  
 342 with an aerosol layer that represents forward scattering by cloud water droplets. Therefore,  
 343 for broken clouds under low cloud fraction, the good correspondence between the two models  
 344 for long-term irradiation totals indicates that the 3D reality can adequately simplified into  
 345 the 1D weighted clear/overcast simulation, although our model does not replicate an instan-  
 346 taneous scene. For high cloud fractions, the sky diffuse radiances approach the isotropic  
 347 case, and the 1D approximation used in our model is well-known to be appropriate.

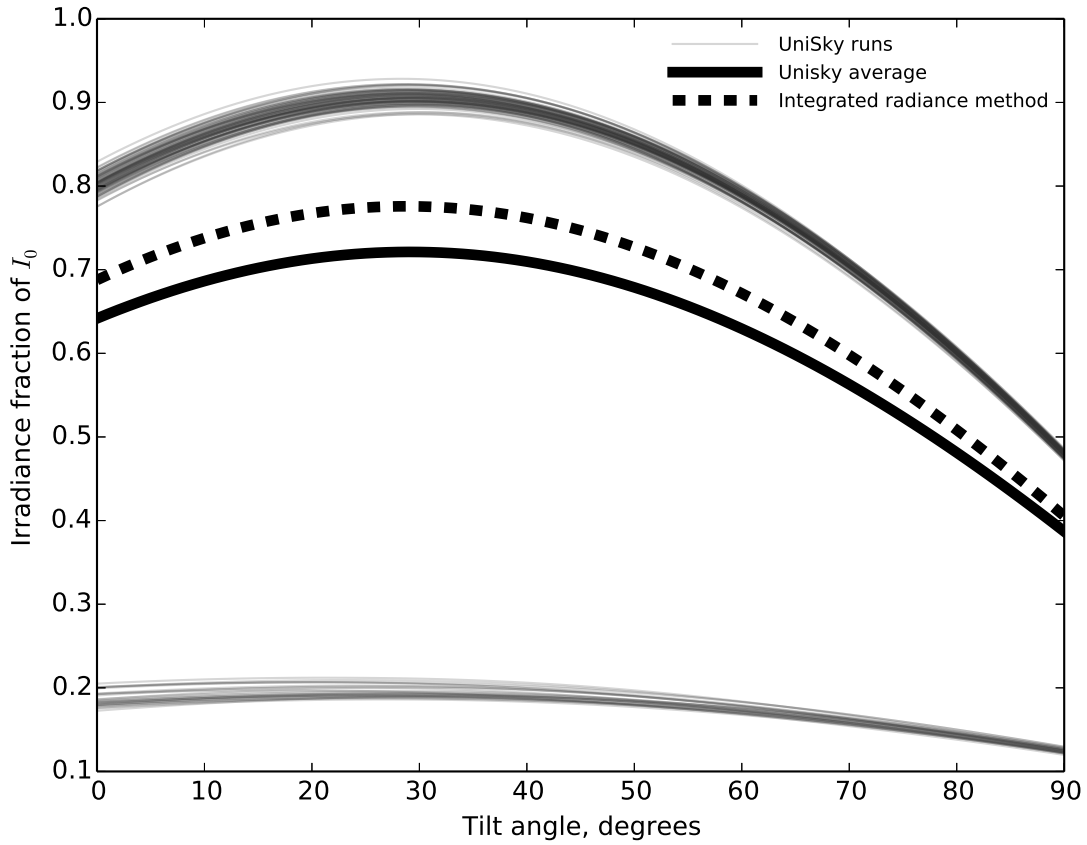


Figure 5: Plane irradiance as a function of panel tilt for 100 runs of the UniSky simulator with random cloud geometry, the UniSky average, and the 1D weighted average radiances from libRadtran, for  $\theta_z = 30^\circ$  and  $c_f = 0.2$ .

348 *4.5. Validation against horizontal irradiation measurements*

349 Yearly irradiation predicted from our model using MODIS data is validated against hor-  
350 izontal irradiation measurements from high-quality pyranometer data and is shown in fig.  
351 6. The UK Met Office MIDAS dataset (Met Office, 2012) is used for UK locations and the  
352 Baseline Surface Radiation Network (BSRN) for non-UK locations (BSRN, 2015). The con-  
353 vention in this paper is to use the three-letter BSRN station codes in upper case for BSRN  
354 stations and an upper- and lower-case abbreviation for MIDAS stations. Camborne (Cam)  
355 and Lerwick (Ler) are MIDAS stations that also supply data to BSRN; at the time of writing  
356 the BSRN data were not available so the MIDAS data have been used.

357 MIDAS provides hourly pyranometer measurements of global horizontal irradiance (GHI)  
358 for approximately 100 sites in the UK. The MIDAS data has passed a quality control (QC)  
359 procedure run by the UK Met Office. Five MIDAS sites were selected on the basis of wide  
360 geographical coverage within the UK and a minimal amount of missing or bad data for 2013.  
361 Where missing hours do occur in the MIDAS data, these have been replaced by the mean  
362 irradiance from the corresponding hour in the same month.

363 BSRN provides minutely measurements of horizontal irradiance from sites globally. The  
364 BSRN data also contains instances of missing records. Data gaps range from one minute to  
365 several days. A QC procedure was applied to the BSRN data to fill in missing or suspect data  
366 following the M7 method recommended by Roesch et al. (2011). The M7 method calculates  
367 monthly 15-minute means from data where at least 3 minutes per 15-minute period exist  
368 and are within the “physically possible” limit for GHI of  $1.5S_0 \cos^{1.2} \theta_z + 100 \text{ W m}^{-2}$ .  $S_0$   
369 is the solar constant  $I_0$  corrected for earth-sun distance. The monthly mean is only valid if  
370 all 96 15-minute bins contain valid values. Only sites where all months of 2013 data were  
371 available and passed the QC check were selected for the BSRN validation. The details of the  
372 27 meteorological stations used in the validation are shown in the appendix. Solar irradiance  
373 at BSRN sites is measured with a Kipp & Zonen CMP21 or CMP22 pyranometer with the  
374 exception of Tamanrasset which uses the Eppley PSP, all of which are World Meteorological  
375 Organisation (WMO) High Quality certified.

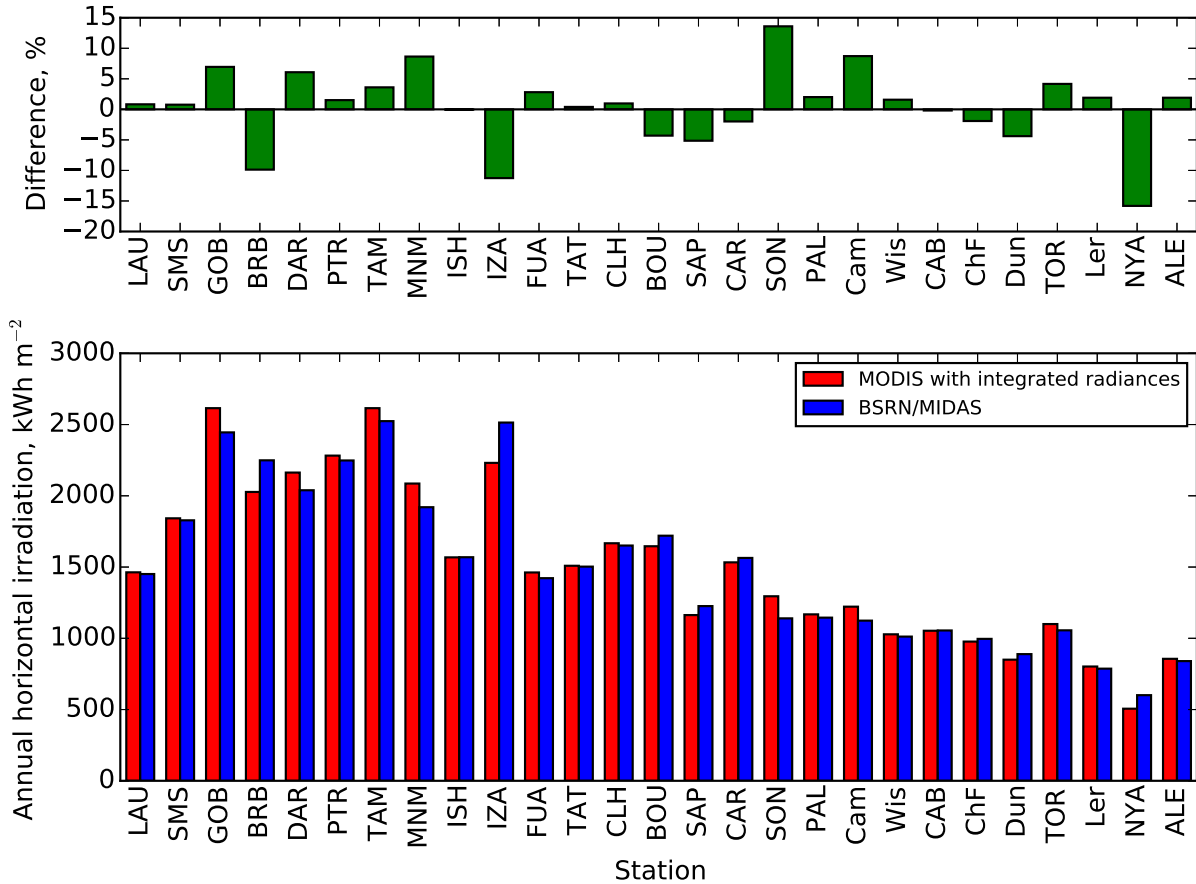


Figure 6: Validation of integrated radiance method using MODIS data against pyranometer measurements. For station names and locations please refer to Appendix.

376 The mean bias difference (MBD) between the annual irradiation derived from our  
 377 method and the pyranometer data is +0.56% and the root-mean-square difference (RMSD)  
 378 is +6.69%. Of the sites where our method deviates from the measured values by more than  
 379 10%, two (IZA and SON) are at mountaintop sites at altitudes 2373 m and 3109 m respec-  
 380 tively. In these areas, the 1° resolution of the MODIS atmosphere data may not be large  
 381 enough to capture all of the micro-climatic effects in mountainous regions. As discussed by  
 382 Gueymard and Wilcox (2011), the spatial variation in irradiance measurements is highest in  
 383 coastal and mountainous areas. Clouds are particularly difficult to attribute as sometimes  
 384 the site location may be above the mean cloud height for the 8 day period whereas in reality

385 the station is not cloud-free for the entire 8 days. Furthermore if the cloud deck is below  
386 the station altitude, the albedo from the point of view of the pyranometer changes, and  
387 backscattering effects between the cloud layer and the atmosphere above the station can  
388 enhance the downwards radiation. It is unlikely that the MODIS albedo product includes  
389 these effects as it is calculated from clear sky scenes.

390 The other location with a greater than 10% absolute error, NYA, is at very high latitude  
391 ( $78.9^{\circ}\text{N}$ ), where satellite retrievals from MODIS become less reliable. In addition, in such  
392 a high-latitude site, solar declination can vary widely over the course of an 8-day period  
393 in spring and autumn and as such the solar geometry used in our calculations may not  
394 be representative. Interestingly, the other high latitude location, ALE in the far north of  
395 Canada ( $82.5^{\circ}\text{N}$ ), shows a very good agreement with the model. This could be due to a  
396 higher annual irradiation than NYA indicative of clearer conditions as the annual horizontal  
397 irradiation at ALE is similar to that at Dun at  $56.4^{\circ}\text{N}$ . BRB, the fourth poorest site for  
398 agreement with a 9.9% underestimation, suffers from a large amount of incomplete data in  
399 the 2013 BSRN dataset which may result in a large error in the “measurement” value for  
400 this site. BRB passes the QC test because all 96 15-minute bins are present for each month,  
401 but for some months there are as little as 7 days of data present.

#### 402 *4.6. Results of the tilted irradiation and comparison with PVGIS*

403 It is difficult to validate the tilted irradiation model on a global basis because there are few  
404 comparable high-quality long term measurements of tilted irradiance available worldwide.  
405 In section 4.7 we validate our results against data from one site. The optimal tilt angle  
406 predicted by the integrated radiance model, and the irradiance predicted at this optimal  
407 tilt, are compared with results from the online PVGIS solar resource estimation tool in fig.  
408 7 (European Commission, 2012). PVGIS is a validated model that derives solar irradiance  
409 from the Meteosat satellite cloud product and calculates tilted irradiance using the Muneer  
410 model (Muneer, 1990). Additionally the PVGIS model reports GHI with a mean bias error  
411 (MBE) of within  $\pm 5\%$  for all but 4 BSRN and other surface irradiance measurement sites out  
412 of 23 (Huld et al., 2012) whereas the Muneer (1990) tilt model gives a MBE of  $+5.3\%$  and

413 root-mean-squared error (RMSE) of 9.6% for vertical, south-facing planes, with considerably  
414 lower errors for 45° and 60° south-facing planes for the EU Joint Research Centre (JRC)  
415 test site at Ispra, Italy. 13 of the 27 validation sites used in section 4.5 fall within the spatial  
416 boundaries of PVGIS and have been compared in fig. 7.

417 The comparisons do not correspond to the same time period as the PVGIS database  
418 uses data from the CM-SAF satellite products, namely Meteosat First Generation (MFG,  
419 1998–2005) and Meteosat Second Generation (MSG, 2006–2011), and it is not stated which  
420 particular BSRN station years are used to validate these datasets (Huld et al., 2012). Our  
421 validation against BSRN and MIDAS ground stations uses 2013 data. The comparison with  
422 PVGIS is not a validation of our model for this reason, but a sense-check against a widely-  
423 used tilted irradiance database. Nevertheless some systematic differences can be observed.  
424 The top panel of fig. 7 shows that in the majority of locations our predicted annual optimal  
425 tilt angle is steeper than in PVGIS, ranging from  $-1^\circ$  at CAR and Cam to  $+8^\circ$  at TOR.  
426 Part of the differences may be due to, on average, higher GHI values predicted from our  
427 model compared to PVGIS, suggesting that our model predicts a lower cloud fraction or  
428 greater cloud transmission than PVGIS does in general. The effect of this is large at the  
429 three low latitude sites of GOB, TAM and IZA where in each case our model predicts an  
430 optimal tilt slightly steeper than the latitude location, showing the influence of the direct  
431 beam and circumsolar diffuse components of solar radiation. For IZA it is interesting to note  
432 that our model under-predicts GHI for the 2013 calendar year quite substantially compared  
433 to the BSRN pyranometer data, whereas the PVGIS estimate is even lower (although not  
434 validated against the same time period as previously mentioned). This, along with results  
435 for SON reported by Huld et al. (2012) and our data shown in fig. 6, shows the difficulties  
436 that both models experience in mountainous areas.

437 The middle and bottom panels of fig. 7 shows that in every location there is a more  
438 positive difference in the irradiation at optimal tilt than the GHI between our model and  
439 PVGIS. This effect is seen even at CAR and Cam indicating a difference between the Muneer  
440 tilt model used in PVGIS and the integrated radiance method. This is emphasised by the

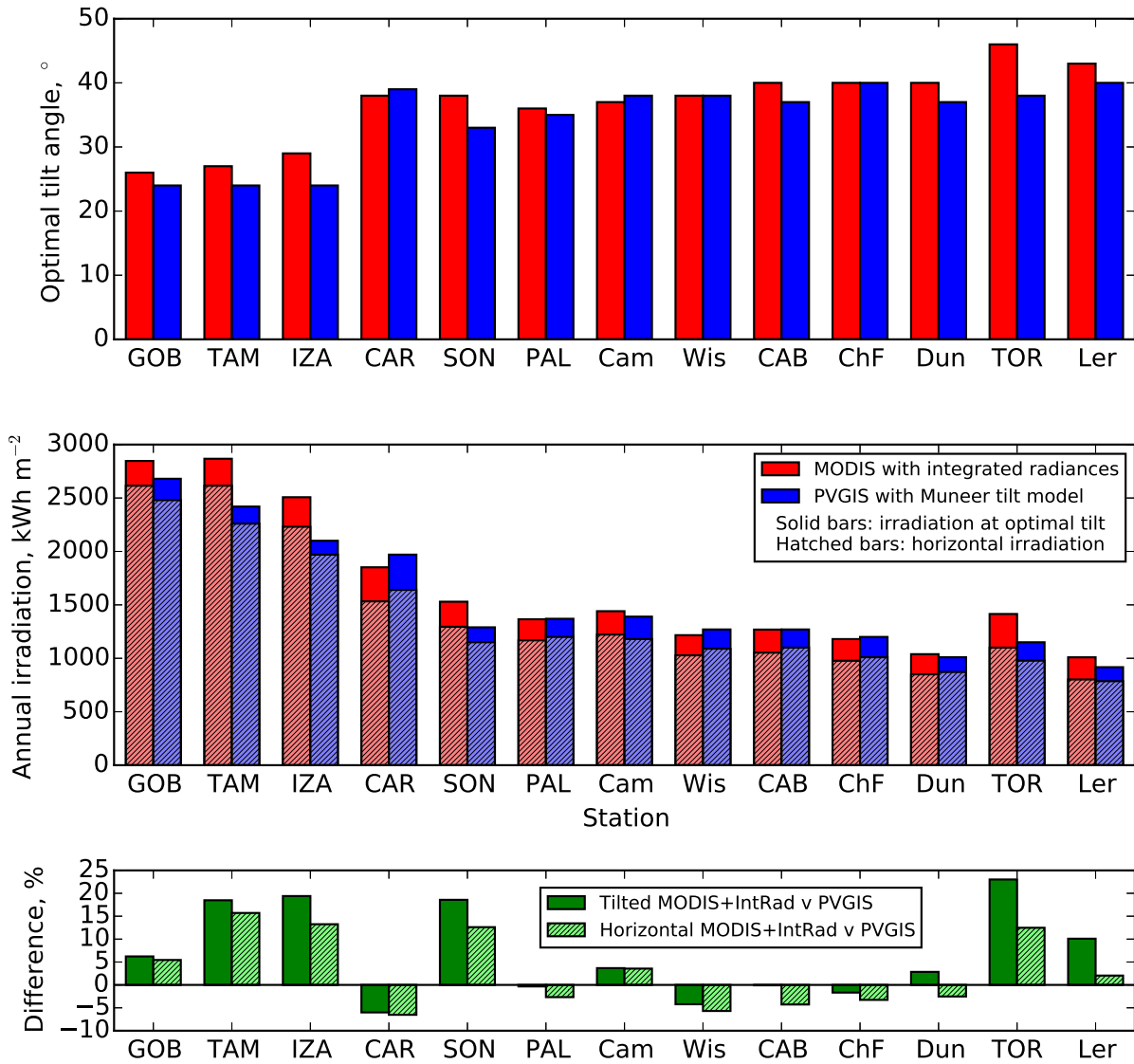


Figure 7: Comparison of integrated radiance method using MODIS data against results from PVGIS for optimal tilt angle and yearly irradiation at optimal tilt. The top figure compares optimal tilt angles between the two models, the middle figure shows irradiation at optimal tilt (solid bars) and GHI (pale hatched bars), and the bottom figure shows the differences between the two models for irradiation at optimal tilt (solid bars) and for GHI (pale hatched bars). For station names and locations please refer to table 2 in the Appendix.

441 Dun site where PVGIS predicts a higher annual GHI total but lower irradiation at optimal  
 442 tilt.



Tilt	Integrated radiance model (kWh m <sup>-2</sup> yr <sup>-1</sup> )	Eppley PSP measurements (kWh m <sup>-2</sup> yr <sup>-1</sup> )	Difference
Horizontal	1760.0	1684.2	+4.5%
40°S	2120.4	2010.0	+5.5%
90°S	1479.3	1402.9	+5.4%
90°E	1085.4	1138.6	-4.7%
90°W	976.7	922.2	+5.9%
90°N	420.5	479.3	-12.3%

Table 1: Validation of tilted irradiation from the integrated radiance model against ground measurements from NREL.

443 *4.7. Validation against tilted irradiation measurements from the National Renewable Energy*  
444 *Laboratory Baseline Measurement System*

445 The NREL Solar Radiation Research Laboratory (SRRL) (Andreas and Stoffel, 1981)  
446 produces horizontal and tilted irradiation datasets which are available from their website  
447 at [http://www.nrel.gov/midc/srml\\_bms/](http://www.nrel.gov/midc/srml_bms/). Tilted irradiation is measured at 40°S and  
448 at 90°S, W, E and N, using Eppley PSP pyranometers. Horizontal radiation is measured  
449 with a number of different pyranometer models. For consistency, we use the ventilated,  
450 corrected Eppley PSP horizontal irradiation measurement. The NREL site is located in  
451 Golden, Colorado, at 39.74°N, 105.18°W at an altitude of 1829 m.

452 The validation against the NREL station measurements is shown in table 1. The horizon-  
453 tal irradiation estimate from the integrated radiance model is 4.5% higher than the NREL  
454 measurement using the Eppley PSP. For the 40° and 90° south-facing tilts, the relative error  
455 is slightly higher but does not grow appreciably. The model captures some of the diurnal  
456 variation in weather conditions at this site, as seen by the differences between east- and west-  
457 facing tilt estimates, however underestimates the magnitude of the diurnal variation with  
458 an overestimate for the west-facing pyranometer and an underestimate for the east-facing  
459 pyranometer. This may be due to the timing of the satellite overpasses, approximately 90

460 minutes before and after local solar noon on average, whereas east- and west-facing wall irra-  
461 diances will be at their maximum earlier and later in the day, respectively. The north-facing  
462 estimate is considerably less good than for the other orientations, however, it is not likely  
463 that serious consideration would be given to tilting panels poleward given the low overall  
464 yield estimate.

## 465 5. Discussion

466 The integrated radiance method is possible to evaluate globally as the satellite re-  
467 trieval data from MODIS has global coverage. The method is applicable to any dataset  
468 in which aerosol parameters, ozone, water vapour, cloud liquid water path, cloud ice wa-  
469 ter path and cloud fraction are available. The necessary inputs to the model also ex-  
470 ist in meteorological reanalysis and climate models. The aerosol parameters are avail-  
471 able in MODIS but often suffer from large gaps in data, so in our model they are ob-  
472 tained from the GLOMAP global aerosol model. Aerosol reanalysis datasets such as MACC  
473 (<http://apps.ecmwf.int/datasets/data/macc-reanalysis>), which assimilates observa-  
474 tions and forecasts into a consistent gridded dataset, can be used. Thus, the integrated  
475 radiance model can be used for determining a realistic optimal tilt for an arbitrary climatic  
476 condition, and solar energy resource calculated on this basis.

477 It should be mentioned that MODIS satellite retrievals are not always available or are  
478 of low quality. The limit of MODIS orbital tracks are at  $82^\circ$  N/S, and for latitudes greater  
479 than  $77^\circ$  N/S the satellite tracks overlap. Successive retrievals may not be independent and  
480 observational nadir angles may be higher towards the poles as the satellites do not overpass  
481 above/below  $82^\circ$  N/S (Hubanks et al., 2008). On the other hand, these regions are currently  
482 unimportant for solar energy generation. A more critical issue occurs when albedo values  
483 are not reported over a 16-day period. As an albedo retrieval requires a cloudless scene  
484 when the satellite overpasses, it is possible that there are no clear overpasses during a 16-  
485 day period for some parts of the world. In these cases where no albedo measurement exists  
486 for a  $0.05^\circ \times 0.05^\circ$  cell, the mean value from the  $21 \times 21$  cells surrounding the grid square  
487 ( $1.05^\circ \times 1.05^\circ$ ) is used. In very rare cases where no  $1.05^\circ$  mean exists, the spectral albedo is

488 taken from the global  $0.17^\circ \times 0.17^\circ$  map of 20 different surface types in the IGBP land cover  
489 dataset (Belward and Loveland, 1996).

490 In many regions, clouds are the largest input uncertainty in our model because the ra-  
491 diative properties of aerosols, water vapour and ozone are less significant when the entire  
492 solar spectrum is considered. The direct and diffuse radiation fields are spectrally dependent  
493 (Forster and Shine, 1995) and although a spectral calculation is performed and then inte-  
494 grated over all solar wavelengths to obtain broadband irradiance, the spectrally-dependent  
495 irradiance was not considered. When applied to assessing the energy output of PV technolo-  
496 gies, spectral considerations have shown to be important and this could affect the optimal  
497 PV tilt angle.

498 It is possible to improve the spatial and temporal resolution of the results obtained.  
499 MODIS Level 3 8-day mean data has been used in this model for atmosphere and albedo.  
500 Level 3 data is available daily, the use of which may improve accuracy at the expense of  
501 an 8-fold increase in computational time. Greater accuracy may be obtained by using the  
502 Level 2 satellite swath data, which has a nadir resolution of 1 km and will usually overpass a  
503 location at least once per day, although there are small gaps in the satellite overpass tracks  
504 near the equator that are not covered every day by the Level 2 or Level 3 daily data. To  
505 use higher resolution data will require many more RT simulations per location per year,  
506 and will need the use of pre-calculated lookup tables or a polynomial regression fit to allow  
507 swifter calculation of the radiance fields. This is an area for future investigation. It is shown  
508 however that for locations at low and moderate altitude and latitude, sufficient agreement  
509 for horizontal and equator-facing tilts for yearly irradiation is obtained with the 8-day data.

## 510 **6. Conclusion**

511 This paper presents a computational method to calculate the all-sky irradiance on a plane  
512 of arbitrary alignment, which is globally applicable. The optimal tilt angle at a particular  
513 location is dependent on the meteorological conditions and cannot be related to a single  
514 parameter. A radiative transfer simulation is run to produce a ground-level radiance field,  
515 which is numerically integrated over the tilt angle of interest. The required inputs of cloud

516 liquid water path, cloud ice water path, cloud fraction, temperature, ozone, water vapour and  
517 surface albedo are standard variables from satellite observations, meteorological reanalysis  
518 or climate model data. We use MODIS Terra and Aqua satellite data for clouds, ozone,  
519 water vapour and albedo. Aerosols are provided by the GLOMAP model but any scheme  
520 that provides the aerosol phase function, optical depth and single scattering albedo can be  
521 used. The horizontal irradiation predicted by our model is compared to contemporaneous  
522 pyranometer data from MIDAS and BSRN and agrees to within  $\pm 10\%$  for all but 3 sites  
523 out of 27. The MBD between our method and BSRN/MIDAS across all sites is  $+0.56\%$  and  
524 RMSD is  $6.69\%$  for horizontal irradiance.

525 When validated against the NREL tilted irradiance dataset our model predicts the an-  
526 nual irradiation within  $\pm 6\%$  for all orientations except  $90^\circ\text{N}$ . The magnitude of error for  
527 tilted irradiance on  $40^\circ$  and  $90^\circ$  south-facing planes is similar to that for horizontal irradi-  
528 ance. The diurnal variation in prevailing weather conditions is partially captured by analysis  
529 of the difference between east- and west-facing estimates of annual irradiation compared to  
530 pyranometer measurements, although underestimated. Due to a lack of high-quality tilted  
531 irradiance measurement stations, it is not possible to validate against tilted irradiance mea-  
532 surements globally, but the validated PVGIS model is used as a comparison. The main  
533 differences between our model and the Muneer (1990) tilt model used in PVGIS are the  
534 steeper optimal tilt angles and more positive relative differences between tilted irradiation  
535 and horizontal irradiation. In mid-latitude and low-to-moderate altitude sites, where PVGIS  
536 has been validated, the models produce similar results. In order to draw more robust con-  
537 clusions about the optimal tilt angle from the model, a larger network of tilted irradiance  
538 measurements would be required. However, the limited model comparisons and validations  
539 show that the model produces sensible results and could be applied where ground measure-  
540 ments of tilted irradiance are not available. Further work in this area includes accounting  
541 for horizon shading, and producing a global map of optimal annual tilt.

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## 557 **Appendix**

Code	Station name	Country	Lat.	Lon.	Alt. (m)	Network
LAU	Lauder	New Zealand	45.045°S	169.689°E	350	BSRN
SMS	São Martinho	Brazil	29.443°S	53.823°W	489	BSRN
GOB	Gobabeb	Namibia	23.561°S	15.042°E	407	BSRN
BRB	Brasilia	Brazil	15.601°S	47.713°W	1023	BSRN
DAR	Darwin	Australia	12.420°S	130.891°E	350	BSRN
PTR	Petrolina	Brazil	9.068°S	40.319°W	387	BSRN
TAM	Tamanrasset	Algeria	22.780°N	5.510°E	1366	BSRN
MNM	Minamitorishima	Japan	24.288°N	153.983°E	7	BSRN
ISH	Ishigakijima	Japan	24.337°N	124.163°E	6	BSRN
IZA	Izaña	Tenerife	28.309°N	16.499°W	2373	BSRN
FUA	Fukuoka	Japan	33.582°N	130.375°E	3	BSRN
TAT	Tateno	Japan	36.050°N	140.133°E	25	BSRN
CLH	Chesapeake Light	USA	36.905°N	75.713°W	37	BSRN
BOU	Boulder	USA	40.050°N	105.007°W	1577	BSRN
SAP	Sapporo	Japan	43.060°N	141.329°E	17	BSRN
CAR	Carpentras	France	44.083°N	5.059°E	100	BSRN
SON	Sonnblick	Austria	47.054°N	12.958°E	3109	BSRN
PAL	Palaiseau	France	48.713°N	2.208°E	156	BSRN
Cam	Camborne	UK	50.218°N	5.327°W	87	MIDAS
Wis	Wisley	UK	51.310°N	0.475°W	38	MIDAS
CAB	Cabauw	Netherlands	51.971°N	4.927°E	0	BSRN
ChF	Church Fenton	UK	53.836°N	1.197°W	8	MIDAS
Dun	Dunstaffnage	UK	56.451°N	5.439°W	3	MIDAS
TOR	Toravere	Estonia	58.254°N	26.462°E	70	BSRN
Ler	Lerwick	UK	60.140°N	1.183°W	82	MIDAS
NYA	Ny-Ålesund	Svalbard	78.925°N	11.930°E	11	BSRN
ALE	Alert	Canada	82.490°N	62.420°W	127	BSRN

Table 2: List of BSRN and UKMO-MIDAS stations used in the validation and comparison.

558 **References**

- 559 Anderson, G., Clough, S., Kneizys, F., Chetwynd, J., Shettle, E., 1986. AFGL Atmospheric  
560 Constituent Profiles (0–120km). Air Force Geophysics Laboratory.
- 561 Andreas, A., Stoffel, T., 1981. NREL Solar Radiation Research Laboratory (SRRL): Baseline  
562 Measurement System (BMS). Tech. Rep. DA-5500-56488, Golden, Colorado, [http://dx.](http://dx.doi.org/10.5439/1052221)  
563 [doi.org/10.5439/1052221](http://dx.doi.org/10.5439/1052221).
- 564 Baum, B., Heymsfield, A., Yang, P., Bedka, S., 2005. Bulk scattering properties for the  
565 remote sensing of ice clouds. Part I: Microphysical data and models. *Journal of Applied*  
566 *Meteorology* 44, 1885–1895.
- 567 Baum, B., Yang, P., Heymsfield, A., Bansemmer, A., Cole, B., Merrelli, A., Wang, C., 2014.  
568 Ice cloud single-scattering property models with the full phase matrix at wavelengths from  
569 0.2 to 100  $\mu\text{m}$ . *Journal of Quantitative Spectroscopy & Radiative Transfer* 146, 123–139.
- 570 Behrendt, T., Kuehnert, J., Hammer, A., Lorenz, E., Betcke, J., Heinemann, D., 2013. Solar  
571 spectral irradiance derived from satellite data: A tool to improve thin film PV performance  
572 estimations? *Solar Energy* 98, 100–110.
- 573 Belward, A., Loveland, T., 1996. The DIS 1-km land cover data set. *Global Change, the*  
574 *IGBP Newsletter* 27.
- 575 Bird, R., Riordan, C., 1986. Simple solar spectral model for direct and diffuse irradiance on  
576 horizontal and tilted planes at the earth’s surface for cloudless atmospheres. *Journal of*  
577 *Climate and Applied Meteorology* 25 (1), 87–97.
- 578 Blanc, P., Espinar, B., Geuder, N., Gueymard, C., Meyer, R., Pitz-Paal, R., Reinhardt, B.,  
579 Renné, D., Sengupta, M., Wald, L., Wilbert, S., 2014. Direct normal irradiance related  
580 definitions and applications: The circumsolar issue. *Solar Energy* 110, 561–577.
- 581 Brunger, A., Hooper, F., 1993. Anisotropic sky radiance model based on narrow field of view  
582 measurements of shortwave radiance. *Solar Energy* 51 (1), 53–64.

583 BSRN, 2015. World Radiation Monitoring Center – Baseline Surface Radiation Network.  
584 <http://bsrn.awi.de/>, accessed 10.07.2015.

585 Bugler, J., 1977. The determination of hourly insolation on an inclined plane using a diffuse  
586 irradiance model based on hourly measured global horizontal insolation. *Solar Energy*  
587 19 (6), 477–491.

588 Cano, D., Monget, J., Albuissou, M., Guillard, H., Regas, N., Wald, L., 1986. A method for  
589 the determination of the global solar radiation from meteorological satellite data. *Solar*  
590 *Energy* 37 (1), 31–39.

591 Christensen, C., Barker, G., 21–25 April 2001. Effects of tilt and azimuth on annual incident  
592 solar radiation for United States locations. In: *Solar Forum*. Washington DC, USA.

593 Dahlback, A., Stamnes, K., 1991. A new spherical model for computing the radiation field  
594 available for photolysis and heating at twilight. *Planetary and Space Science* 39 (5), 671–  
595 683.

596 Deneke, H., Feijt, A., Roebeling, R., 2008. Estimating surface solar irradiance from ME-  
597 TEOSAT SEVIRI-derived cloud properties. *Remote Sensing of Environment* 112, 3131–  
598 3141.

599 European Commission, 2012. Photovoltaic Geographical Information System (PVGIS).  
600 <http://re.jrc.ec.europa.eu/pvgis/index.htm>, accessed 26.09.2014.

601 Forster, P., Shine, K., 1995. A comparison of two radiation schemes for calculating ultraviolet  
602 radiation. *Quarterly Journal of the Royal Meteorological Society* 121, 1113–1131.

603 Gracia-Amillo, A., Huld, T., 2013. Performance comparison of different models for the es-  
604 timation of global irradiance on inclined surfaces. European Commission, Joint Research  
605 Centre, Via Enrico Fermi 2749, TP 450, 21027 Ispra (VA), Italy.

606 Gueymard, C., 1987. An anisotropic solar irradiance model for tilted surfaces and its com-  
607 parison with selected engineering algorithms. *Solar Energy* 38 (5), 367–386.



608 Gueymard, C., 1995. SMARTS2, A simple model of the atmospheric radiative transfer of  
609 sunshine: algorithms and performance assessment. Florida Solar Energy Center.

610 Gueymard, C., 2009. Direct and indirect uncertainties in the prediction of tilted irradiance  
611 for solar engineering applications. *Solar Energy* 83, 432–444.

612 Gueymard, C., Wilcox, S., 2011. Assessment of spatial and temporal variability in the US  
613 solar resource from radiometric measurement and predictions from models using ground-  
614 based or satellite data. *Solar Energy* 85 (5), 1068–1084.

615 Hay, J., Davies, J., 1980. Calculation of the solar radiation incident on an inclined surface.  
616 In: Hay, J., Won, T. (Eds.), *Proceedings First Canadian Solar Radiation Data Workshop*.  
617 Atmospheric Environment Service, Toronto, pp. 59–72.

618 Hu, Y., Stamnes, K., 1993. An accurate parameterization of the radiative properties of water  
619 clouds suitable for use in climate models. *Journal of Climate* 6, 728–742.

620 Hubanks, P., King, M., Platnick, S., Pincus, R., 2008. MODIS atmosphere L3 gridded  
621 product algorithm theoretical basis document.

622 Huld, T., Müller, R., Gambardella, A., 2012. A new solar radiation database for estimating  
623 PV performance in Europe and Africa. *Solar Energy* 86, 1803–1815.

624 Kato, S., Ackerman, T. P., Mather, J. H., Clothiaux, E. E., 1999. The  $k$ -distribution method  
625 and correlated- $k$  approximation for a shortwave radiative transfer model. *Journal of Quan-*  
626 *titative Spectroscopy & Radiative Transfer* 62, 109–121.

627 Key, J., Yang, P., Baum, B., Nasiri, S., 2002. Parameterization of shortwave ice cloud opti-  
628 cal properties for various particle habits. *Journal of Geophysical Research: Atmospheres*  
629 107 (D13), AAC 7–1–AAC 7–10.

630 Klucher, T., 1979. Evaluation of models to predict insolation on tilted surfaces. *Solar Energy*  
631 23, 111–114.

- 632 Kocifaj, M., 2012. Angular distribution of scattered radiation under broken cloud arrays:  
633 An approximation of successive orders of scattering. *Solar Energy* 86, 3575–3586.
- 634 Kocifaj, M., 2015. Unified model of radiance patterns under arbitrary sky conditions. *Solar*  
635 *Energy* 115, 40–51.
- 636 Kocifaj, M., Fečko, S., 2014. Unisky simulator. <http://www.unisky.sav.sk>.
- 637 Lohmann, S., Schillings, C., Mayer, B., Meyer, R., 2006. Long-term variability of solar direct  
638 and global radiation derived from ISCCP data and comparison with reanalysis data. *Solar*  
639 *Energy* 80, 1390–1401.
- 640 Marshak, A., Davis, A. (Eds.), 2005. 3D Radiative Transfer in Cloudy Atmospheres.  
641 Springer.
- 642 Mayer, B., Kylling, A., 2005. Technical note: The libRadtran software package for radia-  
643 tive transfer calculations – description and examples of use. *Atmospheric Chemistry and*  
644 *Physics* 5, 1855–1877.
- 645 Mayer, B., Kylling, A., Emde, C., Hamann, U., Buras, R., 2012. libRadtran user’s guide.  
646 <http://www.libradtran.org>.
- 647 McArthur, L., Hay, J., 1981. A technique for mapping the distribution of diffuse solar radi-  
648 ation over the sky hemisphere. *Journal of Applied Meteorology* 20 (4), 421–429.
- 649 Menzel, W., Frey, R., Baum, B., 2010. Cloud top properties and cloud phase algorithm theo-  
650 retical basis document. [http://modis-atmos.gsfc.nasa.gov/\\_docs/CTP\\_ATBD\\_oct10.](http://modis-atmos.gsfc.nasa.gov/_docs/CTP_ATBD_oct10.pdf)  
651 [pdf](http://modis-atmos.gsfc.nasa.gov/_docs/CTP_ATBD_oct10.pdf).
- 652 Met Office, 2012. Met Office Integrated Data Archive System (MIDAS) land and  
653 marine surface stations data (1853-current). [http://catalogue.ceda.ac.uk/uuid/](http://catalogue.ceda.ac.uk/uuid/220a65615218d5c9cc9e4785a3234bd0)  
654 [220a65615218d5c9cc9e4785a3234bd0](http://catalogue.ceda.ac.uk/uuid/220a65615218d5c9cc9e4785a3234bd0), accessed 10.07.2015.

655 Minnis, P., Sun-Mack, S., Young, D., Heck, P., Garber, D., Chen, Y., Spangenberg, D., Ar-  
656 duini, R., Trepte, Q., Smith, W., Ayers, J., Gibson, S., Miller, W., Hong, G., Chakrapani,  
657 V., Takano, Y., Liou, K.-N., Xie, Y., Yang, P., 2011. CERES Edition-2 cloud property re-  
658 trievals using TRMM VIRS and Terra and Aqua MODIS data—part I: Algorithms. *IEEE*  
659 *Transactions on Geoscience and Remote Sensing* 49 (11), 4374–4400.

660 Mueller, R., Dagestad, K., Ineichen, P., Schroedter-Homscheidt, M., Cros, S., Dumortier, D.,  
661 Kuhlemann, R., Olseth, J., Pieravieja, G., Reise, C., Wald, L., Heinemann, D., 2004. Re-  
662 thinking satellite-based solar irradiance modelling: The SOLIS clear-sky module. *Remote*  
663 *Sensing of Environment* 91, 160–174.

664 Mueller, R., Matsoukas, C., Gratzki, A., Behr, H., Hollman, R., 2009. The CM-SAF oper-  
665 ational scheme for the satellite based retrieval of solar surface irradiance – a LUT based  
666 eigenvector hybrid approach. *Remote Sensing of Environment* 113, 1012–1024.

667 Muneer, T., 1990. Solar radiation model for Europe. *Building Services Engineering Research*  
668 *and Technology* 11 (4), 153–163.

669 Nann, S., Emery, K., 1992. Spectral effects on PV-device rating. *Solar Energy Materials and*  
670 *Solar Cells* 27, 189–216.

671 Oumbe, A., Wald, L., Blanc, P., Schroedter-Homscheidt, M., 7–10 October 2008. Exploita-  
672 tion of radiative transfer model for assessing solar resource radiation: the relative im-  
673 portance of atmospheric constituents. In: *EUROSUN2008, 1st International Congress on*  
674 *Heating, Cooling and Buildings*. Lisbon, Portugal.

675 Perez, R., Ineichen, P., Seals, R., Michalsky, J., Stewart, R., 1990. Modeling daylight avail-  
676 ability and irradiance components from direct and global irradiance. *Solar Energy* 44 (5),  
677 271–189.

678 Reindl, D., Beckman, W., Duffie, J., 1990. Evaluation of hourly tilted surface radiation  
679 models. *Solar Energy* 45, 9–17.

- 680 Roesch, A., Wild, M., Ohmura, A., Dutton, E., Long, C., Zhang, T., 2011. Assessment of  
681 BSRN radiation records for the computation of monthly means. *Atmospheric Measurement*  
682 *Techniques* 4 (2), 339–354.
- 683 Rozwadowska, A., 2004. Optical thickness of stratiform clouds over the Baltic inferred from  
684 on-board irradiance measurements. *Atmospheric Research* 72, 129–147.
- 685 Scott, C. E., Rap, A., Spracklen, D. V., Forster, P. M., Carslaw, K. S., Mann, G. W., Pringle,  
686 K. J., Kivekäs, N., Kulmala, M., Lihavainen, H., Tunved, P., 2014. The direct and indirect  
687 radiative effects of biogenic secondary organic aerosol. *Atmospheric Chemistry and Physics*  
688 14 (1), 447–470.
- 689 Skartveit, A., Olseth, J., 1986. Modelling slope irradiance at high latitudes. *Solar Energy*  
690 36 (4), 333–344.
- 691 Stamnes, K., Tsay, S.-C., Wiscombe, W., Laszlo, I., 2000. DISORT, a General-Purpose For-  
692 tran Program for Discrete-Ordinate-Method Radiative Transfer in Scattering and Emit-  
693 ting Layered Media: Documentation of Methodology. Dept. of Physics and Engineering  
694 Physics, Stevens Institute of Technology, Hoboken, NJ, USA.
- 695 Willmott, C., 1982. On the climatic optimization of the tilt and azimuth of flat-plate solar  
696 collectors. *Solar Energy* 28, 205–216.
- 697 Wiscombe, W., 1980. Improved Mie scattering algorithms. *Applied Optics* 19 (9), 1505–1509.
- 698 Xie, Y., Yang, P., Gao, B.-C., Kattawar, G., Mishchenko, M., 2006. Effect of ice crystal shape  
699 and effective size on snow bidirectional reflectance. *Journal of Quantitative Spectroscopy*  
700 *and Radiative Transfer* 100 (1–3), 457–469.
- 701 Xie, Y., Yang, P., Kattawar, G., Minnis, P., Hu, Y., Wu, D., 2012. Determination of ice cloud  
702 models using MODIS and MISR data. *International Journal of Remote Sensing* 33 (13),  
703 4219–4253.