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The Inequality Accelerator

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The Inequality Accelerator [*]

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PRELIMINARY

Abstract

We show that the transition from an economy characterized by idiosyncratic income shocks and incomplete markets à la Aiyagari (1994) to markets where state-contingent assets are available but costly (in order to purchase a contingent asset, households have to pay a fixed participation cost) leads to a large increase of wealth inequality. Using a standard calibration our model can match a Gini of 0.93 close to the level of wealth inequality observed in the US. In addition, under this level of participation costs, wealth inequality is particularly sensitive to income inequality. We label this phenomenon as the *Inequality Accelerator*. We demonstrate how costly access to contingent asset-markets generates these effects. The key insight stems from the non-monotonic relationship between wealth and desired degree of insurance, in an economy with participation costs. Poor borrowing constrained households remain uninsured, middle-class households are almost perfectly insured, while rich households decide to self-insure by purchasing risk-free assets. This feature of households' risk management has crucial effects in asset prices, wealth inequality, and social mobility.

JEL codes: D31, E21, G11.

Keywords: Wealth Inequality, Participation costs, Insurance.

1 Introduction

A widely adopted assumption in macroeconomic models is that households can perfectly participate in asset markets. Then, in presence of idiosyncratic risk, complete and freely-accessible markets imply a non-existing endogenous evolution of wealth inequality, since agents with different income realizations optimally decide to provide insurance to each other. We refer to this outcome as the *perfect-insurance equilibrium*. This setting is obviously not suited for analyzing inequality. The polar opposite case is the incomplete market assumption, under which households do not have access to state-contingent assets, simply because those assets do not exist (Bewley, 1980; Deaton, 1991; Aiyagari, 1994; Krusell and Smith, 1998). This setting generates a *self-insurance equilibrium*, in which agents buy or sell risk-free bonds to intertemporally smooth their idiosyncratic shocks. Interestingly, the incomplete market model suffers from two caveats. First, from a theoretical point of view, the incomplete market model is not able to generate the large wealth inequality as observed in the data, as summarized in Quadrini and Rios-Rull (2014), unless it includes unrealistic income processes, as in Castaneda et al. (2003).[1] Second, from an empirical point of view, conducting an empirical analysis on labor income risk and insurance, Guvenen and Smith (2014) find that the amount of uninsurable income risk perceived by individuals is substantially smaller than what it is typical assumed in incomplete market models.

In this paper we propose a solution for these caveats. We consider an intermediate case between the two polar cases, i.e. the *perfect-insurance equilibrium* and the *self-insurance equilibrium*. In our setting markets that potentially provide full insurance do exist, but it is costly to access to them. In an otherwise standard general equilibrium economy, we introduce costs for participating in contingent asset markets. Specifically, in our framework households have to pay $qa + \kappa$ to purchase a contingent bonds, where q denotes the price of the asset, and κ denotes the additional fixed participation cost.[2] Consequently, households face a trade-off between paying the participation cost and enjoying the gain of consumption smoothing. For a large set of values of the participation cost κ, the economy is, then, characterized by a *partial-insurance equilibrium*, where only a fraction of the population is insured. In the first section of the paper we provide the intuition

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[1]As commented in Quadrini and Rios-Rull (2014), the persistence of income process in Castaneda et al. (2003) is "engineered" to be able to replicate the observed wealth inequality.

[2]The idea that consumption smoothing is costly underpins our approach: being active in financial markets involves monetary costs, broadly defined, such as fees and transactions costs charged by brokers and intermediaries, costs related to information acquisition, and non-monetary costs, such as the opportunity cost of time devoted to find the best portfolio allocation. See Acemoglu and Zilibotti (1997) for the role of fixed cost on capital accumulation and growth.

behind this result by analyzing a simple insurance model similar to the one in Kimball (1990b); the necessary condition for the existence of a *partial-insurance equilibrium* is assuming that agents' utility features decreasing absolute prudence (or equivalently a positive forth derivative). In this case, the precautionary-saving premium asked by the agent is decreasing in wealth. Notice that, as discussed in Kimball (1990a), commonly used parameterizations of the utility function, such as the constant relative risk aversion utility, displays decreasing absolute prudence.

In the more general economy we demonstrate that our setting leads to novel findings about the relationship between degree of insurance, income risk, and wealth inequality. Specifically, when participation costs reduce from a arbitrary large value, such that the economy is equivalent to a *self-insurance equilibrium*, to intermediate values, such that the economy turns into a *partial-insurance equilibrium*, wealth inequality can dramatically increase. With intermediate value of participation costs our model can predict a level of wealth inequality similar to the one observed in the U.S. data (Gini index equal to 0.93). Notice that when we increase the participation cost such that our economy behaves as an incomplete market model as in Aiyagari (1994), the wealth Gini index is much lower and equal to 0.12. Hence, our model can predict a large wealth inequality starting with a much less disperse income process than in Castaneda et al. (2003).[3] Our result, then, links the increased innovation in the financial sector in the last three decades, as documented by Lerner (2002) to the increased wealth inequality in the same time span, as reported by Saez and Zucman (2014). Also, when participation costs reduced further, the economy transits to a *perfect-insurance equilibrium*, and wealth inequality reduces to zero, thus leading to an interesting non-monotone relationship between participation costs and wealth inequality.

Additionally, the *partial-insurance equilibrium* equilibrium leads to a non-trivial relationship between income risk and wealth inequality. In fact, in an economy characterized by intermediate levels of participation costs, a certain (small) degree of income inequality triggers a very large amplification from income inequality to wealth inequality, driven by the non-monotone willingness to insure across the wealth distribution and its implications on asset prices. We label this phenomenon as the *Inequality Accelerator*. With a numerical example, we find that a small increase of the exogenous income inequality in the participation-cost model leads to a very large change of the resulting wealth inequality. Crucially, however, the same change in income dispersion implies a vary small increase of the wealth inequality in the incomplete market model.

How can wealth inequality increase from a reduction of participation costs to financial

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[3]For example, income dispersion, measured as Gini index on income, in our model is 0.097, whereas it is 0.600 in Castaneda et al. (2003).

markets? The answer for this question stems from the non-monotone relationship between households' insurance desire and their wealth generated by the participation cost model. Hence, the first contribution of the paper is to closely characterize households' decision about participating in asset markets. Specifically, we consider a standard neoclassical model with idiosyncratic shocks as in Aiyagari (1994). We assume that households can purchase two type of assets: a state contingent asset, which can be purchased only by paying a fixed participation cost, and a risk-free asset. First, agents decide whether they want to participate in the financial markets, and, then, they decide against which states they are willing to buy insurance. We demonstrate that this discrete choice about financial market participation can be solved with standard recursive methods by using value functions. Also, we prove that households decide to participate in a contingent market as long as its participation cost is lower than a certain threshold value. Intuitively, the threshold depends positively on the households' gains of insurance, and it depends non-monotonically on households' wealth, for a large variety of utility functions. As a result, the *partial-insurance equilibrium* is characterized by a set of poor households that are not able to obtain any insurance, by a set of middle-class household that actively participate to the contingent asset market and, hence, are fully insured, and, interestingly, by a set of rich households that prefer to self insure by accumulating a large stock of the risk-free assets. In the empirical section of the paper we directly test the non-monotonic distribution of insurance across wealth, using the Bank of Italy Survey of Households' Income and Wealth (SHIW) data.

The second contribution of the paper is to explore the consequences of the non-monotone relationship between insurance and wealth for inequality, partial insurance, and welfare. Specifically, we demonstrate that the *partial-insurance equilibrium* features two forces that lead to a skewed wealth distribution. First, perfectly insured middle-class households do not accumulate more assets and, as the risk-less interest rate is lower than in the complete market model, they even progressively consume their wealth. Second, self-insured richer households benefit from real interest rates that are higher than in the incomplete market model and they accumulate more wealth in comparison with the standard Aiyagari model. These two forces reinforce the social disparity from the rich that accumulate wealth and the rest of the population. As a result, the upper tail of the wealth distribution thickens in presence of intermediate levels of participation costs.

Participation costs lead some households to choose to be perfectly insured and some other households to choose to be only self-insured. Hence, the fraction of population that is perfectly insured is a function of the level of participation costs. We can then draw a similarity between the degree of partial insurance discussed in Guvenen and Smith (2014). Specifically, in their setting partial insurance is on the intensive margin - agents can insure

a fraction of their income, whereas in our setting partial insurance is on the extensive margin - agents can be insured or not. Using different calibrations of the model, we show that degrees of partial insurance above and below the one estimated by Guvenen and Smith (2014), around 45 percent, can lead to the realistically observed wealth inequality in presence of participation costs.

In terms of welfare, we find that insurance decisions are usually not constrained efficient, because of a pecuniary externality arising through factor prices, similarly to Davila et al. (2012). In addition, we also find that a similar externality affects insurance decisions: competitive-market insurance participation may exceed its social-planner level in the partial-insurance equilibrium, because of the distortions in asset prices and wages. This happens due to the resulting large level of wealth inequality, which makes more capital desirable so as to redistribute resources through higher wages. Interestingly, this result reverts with higher participation costs and lower wealth inequality, for which competitive-market insurance participation is lower than its social-planner level.

Related literature. In addition to the papers that we have already mentioned, our work expands on several bodies of the literature.

Among the empirical studies conducted on lack of insurance and consumption smoothing as Townsend (1994) and Mace (1991), our work bears similarity to that of Cochrane (1991), and, more recently, Grande and Ventura (2002), who study households' insurance against different types of risk. They show that households are well insured against certain types of risks, such as health problems, but not against other types of risks, such as unemployment (especially involuntary job loss) (see also Blundell et al., 2008).

Our paper shares similarities with the literature on welfare. Since our focus is on participation costs, our approach resemble that of Townsend and Ueda (2010), who consider the welfare effect of financial liberalization, which leads to better consumption insurance. It is also related to the literature on the constrained Pareto optimality of idiosyncratic shock models as Carvajal and Polemarchakis (2011) or Davila et al. (2012) among others, or on the welfare cost of incomplete markets (see Levine and Zame, 2002). Here, we find sizable effects of incomplete markets on risk-sharing as the agents that we consider are sufficiently impatient.

Our work also amplifies on the literature linking models of incomplete insurance with empirical evidence as in Krueger and Perri (2005, 2006) or Kaplan and Violante (2010), who assess the degree of insurance beyond self-insurance. In our setting the participation cost modifies the link between income and consumption inequality, through the resulting non-monotone degree of insurance across wealth. Hence, trends in one of these variables are imperfectly transmitted to the other, consistently with the findings in Attanasio et al.

(2012) and Aguiar and Bils (2015).

Finally, our work links to the literature in finance on limited participation as in Luttmer (1999), Vissing-Jorgensen (2002) and more recently in Paiella (2007), Guvenen (2009) or Attanasio and Paiella (2011) among others. In these models, stock market is open only in a subset of periods. Also, even when economists focus on limited asset trading,[4] they generally do not consider frictions related to asset market participation in their models.

The rest of the paper is organized as follows: in Section 2 we present a simple insurance model in order to provide conditions and intuitions for households' insurance decision. In Section 3 we describe the general economic environment. Section 4 characterizes the individual asset market participation decisions. Section 5 presents the results about social mobility, wealth inequality, and welfare. In Section 6 we empirically test the presence of the non-monotone insurance behavior across wealth using the Bank of Italy Survey of Households' Income and Wealth (SHIW) data. Section 7 discusses a set of further extensions. Finally, Section 8 provides concluding remarks.

2   A Simple Insurance Model

In order to gain some intuition about households' individual contingent-market participation choice, we first analyze a simple two-period and two-state insurance model. Our model is similar to the one proposed in Kimball (1990a) and in Kimball (1990b), in which we include a fix cost to state contingent asset market participation

The economy lasts two periods, $t = 0, 1$. The household is endowed with a level of wealth W in both periods. In period $t = 1$ the household might face an exogenous loss of wealth, $-L \geq 0$, which occurs with probability p. With probability $1 - p$, the household receives a positive shock $pL/(1-p)$ so that the expected loss is 0. The indicator variable 1_L describes the realization of the state of nature. Let define as *feasible* the levels of wealth such that $W > L$, to assure that consumption is positive in every period and in every state. The household maximizes the following expected utility function: $E_0\left(u(c_0) + u(c_1)\right)$. For simplicity, we assume that there is no discounting.

We introduce an endogenous decision of participating in the insurance market. The agent has access to state contingent assets. At time $t = 0$, the agent can acquire α units of a state-contingent asset at unit price q_α that repay a unit of consumption good

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[4]This can happen because of lack of commitment (Thomas and Worrall, 1988; Kocherlakota, 1996), trading technologies (Chien et al., 2011) or because of *ad hoc* assumptions as in the incomplete market literature.

at time $t = 1$ only if the loss in wealth occurs, i.e. if $1_L = 1$, and β units of a state-contingent asset at unit price q_β that repay a unit of consumption good at time $t = 1$ only if the loss in wealth does not occur, i.e. if $1_L = 0$. Importantly, in order to have access to the state contingent asset, the agent needs to pay a fixed cost κ. The household is not necessarily willing to pay the fixed cost and, hence, we define $\delta(W, \kappa)$ as a choice variable that denotes the contingent asset market participation, given a level of wealth and a participation cost: if the household pays the cost and purchases contingent assets, $\delta(W, \kappa)$ equals 1. Otherwise, it equals 0.

Conditional on participation, $\delta(W, \kappa) = 1$, the budget constraints are:

$$c_0 + q_\alpha \alpha + q_\beta \beta + \kappa = W$$
$$c_1 = W + 1_L(\alpha - L) + (1 - 1_L)(\beta + pL/(1 - p)).$$

Without loss of generality, we assume the prices of the contingent asset are actuarially fair, $(q_\alpha = p$ and $q_\beta = 1-p)$. In this case, the optimal amount of insurance is: $\alpha = L - \kappa/2$ and $\beta = -\kappa/2 - pL/(1 - p)$, and agent's expected utility is:

$$V^P(W, \kappa) = 2u\left(W - \kappa/2\right),$$

where the superscript P denotes the expected of utility of an agent that participates to the insurance market.

Conditional on no-participation, $\delta(W, \kappa) = 0$, the two periods budget constraints are:

$$c_0 = W$$
$$c_1 = W - 1_L L + (1 - 1_L)pL/(1 - p).$$

The expected utility of the agent is:

$$V^N(W) = u(W) + \left[(1 - p)u\left(W + \frac{pL}{1 - p}\right) + pu\left(W - L\right)\right],$$

where the superscript N indicates the utility of an agent with no access to the insurance market. Let $\mathbb{P}(\kappa)$ be the set of wealth levels for which participation in the insurance market is optimal for a given participation cost κ. Formally:

Definition 1. (Participation Set). For a given participation cost κ, for any wealth level in $\mathbb{P}(\kappa)$ insurance market participation is optimal, that is:

$$\mathbb{P}(\kappa) = \left\{W \in (L, \infty) : V^P(W, \kappa) > V^N(W)\right\}.$$

Let define the gain of insurance as $G(W, \kappa) = \frac{1}{2}\left(V^P(W, \kappa) - V^N(W)\right)$. It can be rewritten as:

$$G(W, \kappa) = u\left(W - \frac{\kappa}{2}\right) - \frac{1}{2}u(W) - \frac{1 - p}{2}u\left(W + \frac{pL}{1 - p}\right) - \frac{p}{2}u\left(W - L\right). \qquad (1)$$

The first set of results concern the frictionless economy with no costs.

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Proposition 1. *(Insurance Incentives without cost) Let $u(x)$ be a three-times continuous and differentiable utility function, such that $u'(x) > 0$, $u''(x) < 0$, and satisfies the Inada conditions: $\lim_{x \to \infty} u'(x) = 0$, and $\lim_{x \to 0} u'(x) = \infty$. Then, for any feasible level of wealth, i.e. $\forall W > L$:*

1. $G(W, 0) > 0$;

2. $\lim_{W \to \infty} G(W, 0) = 0$.

3. *If $u''' > 0$ then $\frac{\partial G(W,0)}{\partial W} < 0$.*

See Appendix C.1 for the proof.

Proposition 1 shows that, absent any cost, $\kappa = 0$, the (strictly) concavity of the utility function guarantees a (strictly) positive benefit from insurance. If the utility function has a positive third derivative, its marginal utility is convex and, therefore, displays *prudence*, as defined in Kimball (1990b), and a decreasing absolute risk aversion. In this case, the gains from insurance $G(W, 0)$ are decreasing with respect to wealth. As discussed in Kimball (1990a), *prudence* measures the strength of the precautionary saving motive, which induces individuals to prepare and forearm themselves against uncertainty they cannot avoid- in contrast to risk aversion, which is how much agents dislike uncertainty and want to avoid it.

We now consider the economy with participation costs.

Proposition 2. *(Insurance Incentive with cost) Let $u(x)$ be a four-times continuous and differentiable utility function, such that $u'(x) > 0$, $u''(x) < 0$, $u'''(x) > 0$, $u''''(x) < 0$, and satisfies the Inada conditions: $\lim_{x \to \infty} u'(x) = 0$, and $\lim_{x \to 0} u'(x) = \infty$. Then, for any feasible level of wealth, i.e. $\forall W > L$, and for any feasible level of cost, i.e. $\kappa < 2L$:*

1. *(Existence of Thresholds). Let $\hat{\kappa} < 2L$ be the solution of $G(L, \hat{\kappa}) = 0$. Then, $\forall \kappa < \hat{\kappa}$, $\exists! \ \overline{W}(\kappa) > L$: $W \in \mathbb{P}(\kappa) \iff L < W < \overline{W}(\kappa)$.*

2. *(Comparative static of participation set)*

   - *Participation set coincides with all feasible wealth levels when $\kappa = 0$, that is:*

$$\mathbb{P}(0) = \{W : W > L\}.$$

   - *Participation set is shrinking in participation cost, that is for all $\kappa^1 < \kappa^2$, if $W \in \mathbb{P}(\kappa_2)$ then $W \in \mathbb{P}(\kappa_1)$; hence, $\mathbb{P}(\kappa_2) \subset \mathbb{P}(\kappa_1)$.*

   - *Participation set is empty for any participation cost greater than $\hat{\kappa}$, that is: $\forall \kappa > \hat{\kappa}$, $\mathbb{P}(\kappa) = \emptyset$.*

See Appendix C.2 for the proof.

This proposition explains a crucial characteristic of the insurance market. When accessing to the insurance market is costly, the agent endogenously decides whether to participate in that state-contingent asset market depending on the level of its wealth. When wealth level is large enough, $W > \overline{W}(\kappa)$, the agent is better off by not-participating in the insurance market since the cost of paying the fix cost is larger than the expected benefit of reducing the loss in case of occurrence of the negative shock. For those wealth levels, in fact, $G(W, \kappa) < 0$. Also, notice that the participation set varies with the participation cost. When the cost tends to zero, the participation set corresponds to the entire feasible wealth domain. On the contrary, the participation region disappears when the cost is larger than a certain threshold $\hat{\kappa}$. In this case entering in the insurance market is either infeasible or not beneficial. The necessary condition for the existence of the threshold wealth level is a strictly negative forth derivative of the instantaneous utility function. This condition is equivalent to assume a utility characterized by *decreasing absolute prudence*. As described by Kimball (1990a), which relates this assumption to precautionary saving behavior, approximate constancy for the wealth elasticity of risk-taking is enough to guarantee decreasing absolute prudence. Also, commonly used parameterizations of the utility function, such as the constant relative risk aversion utility, displays decreasing absolute prudence.

3 Model

In this section we describe the general economic environment. We consider an infinite horizon production economy populated by a continuum of mass 1 of *ex ante* homogenous households. This model follows closely Aiyagari (1994) except for two dimensions: we introduce securities contingent to idiosyncratic states and we simultaneously introduce fixed participation costs for each contingent market. Time is discrete and indexed by $t \in \{0, 1, ...\}$.

Uncertainty and preferences. Each Household chooses consumption so as to maximize the following utility: $U = E \sum_{y^t} \beta^t \pi(y^t) u\left(c(y^t)\right)$, where $\beta \in (0, 1)$ is the discount factor, $c(y^t)$ denotes consumption at date t, and u is a strictly increasing and concave function that satisfies $\lim_{c \to 0} u'(c) = -\infty$ and $\lim_{c \to \infty} u'(c) = 0$. Without loss of generality, u is twice differentiable.

Households inelastically provide labor. At every period they receive a stochastic labor endowment, y_t. Since there is no aggregate uncertainty, this assumption is equivalent to consider that households receive a stochastic good-endowment $\tilde{y}_t = w y_t$, where w is the

constant wage rate.

We assume that y_t follows a Markov process, which takes values in $Y = \{y_1, ...y_N\}$ and that $\pi(y_j|y_k)$ is the associated transition probability from state j to state k. We denote by y^t the history of the realizations of the shock, $y^t = \{y_0, y_1, ..., y_t\}$, and by $\Pi(y_k)$ the fraction of households in state k.

Remark. Note that since there is no aggregate uncertainty here the fraction of households in each state is constant.[5]

Asset structure. To smooth consumption, households may trade a set of different assets. First, they can purchase non-contingent bonds. Each of these bonds yields, unconditionally, one unit of goods next period. We denote by $B(y^t)$ household's position in the risk-free assets and by q^f its price. Besides, as in Aiyagari (1994), we impose that this position is bounded below: $B(y^t) \geq -\overline{B}$ where $\overline{B} \geq 0$ is finite.[6] Second, households can trade a set of state-contingent assets. In state y_m, each of these assets pays contingently to the realization of y_k next period: it pays 1 when $y = y_k$ and 0 otherwise. We denote by $q(k, m)$ the price of this asset and by $a(k, y^t)$ the corresponding holdings of a household with history of shocks y^t. Note that in our notation contingent asset holding depends on the current state m through the history of shock y^t.

The novelty we introduce in this paper is that purchasing those assets requires paying a fixed fee, κ. Hence, in order to hold $a(k, y^t)$ units of any contingent assets household has to pay $q(k, m)a(k, y^t) + \kappa$. Here, for simplicity, we assume that if the agent pays the participation cost she can purchase or sell the preferred quantity of any state contingent assets. We assume that κ is a pure waste.[7]

The presence of the fixed cost implies that the household needs to take a discrete decision about whether to participate in the contingent asset market. We denote by $\delta(y^t) \in \{0, 1\}$ the corresponding decision variable, with the following meaning: when $\delta(y^t) = 1$, household with history y^t decides to enter in the state-contingent assets and when $\delta(y^t) = 0$, she does not.

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[5]This assumption can be relaxed; to solve the corresponding model with aggregate uncertainty, Krusell and Smith (1998)' methods are needed. Yet, this is beyond the scope of this paper, which focuses on idiosyncratic shocks only.

[6]We do not provide further foundations for that constraint. It can be exogenous debt limits as in Bewley (1980), natural debt limits as in Aiyagari (1994) or endogenous borrowing constraints as in Zhang (1997) or Abraham and Carceles-Poveda (2010) for such foundations.

[7]This involves no loss of generality. In a more general setting, where transaction costs may be pecuniary costs charged by intermediaries, fixed costs paid by some agents will be other agents' revenues. Here, our assumption is close to assuming a redistribution of intermediaries' profits to households in a lump-sum way.

Finally, the proceeds of both contingent and risk-less assets are invested in physical capital, whose returns are used to honor assets' payments.

Remark. The borrowing constraint introduces a limit to markets, even when participation costs are absent. Markets are then not complete *stricto sensu*. Yet, we will show that there are complete *de facto*, in the sense that the borrowing limit does not prevent full households' insurance.

Remark. The main results of this paper hold when assuming that participation cost is state-dependent, κ_j. In this case, households' decides in which state-contingent asset market to enter and, therefore, the participation decision is a set of binary variables. In Appendix B, we present this setting.

In the end, a household with a history of shock y^t and a current shock realization y_m faces the following sequence of budget constraints:

$$c(y^t) + q^f B(y^t) + \delta(y^t) \left(\sum_k q(k,m) a(k, y^t) + \kappa \right) = B(y^{t-1}) + a(m, y^{t-1}) + w y_m.$$

Production. As in Aiyagari (1994), we include production in our economy, creating an endogenous net supply of assets. A single representative firm produces using a Cobb-Douglas technology:

$$Y_t = A K_t^\alpha L_t^{1-\alpha} + (1-\delta) K_t,$$

where capital, K_t, and total labor, L_t, are rent from households. Total labor is the combination of labor provided by the different types of households ($y = y_k$, for $k = 1, .., N$), i.e.:

$$L_t = \sum_k \Pi(y_k) y_k.$$

First order conditions for capital and labor are:

$$A\alpha \left(\frac{K_t}{L_t} \right)^{\alpha-1} = r + \delta, \qquad (1-\alpha) \left(\frac{K_t}{L_t} \right)^\alpha = w.$$

Market clearing condition. The asset market-clearing condition pins down aggregate capital, K_{t+1}, as:

$$K_{t+1} = \sum_{y^t} \sum_k \left(q(k,m) a(k, y^t) + q^f B(y^t) \right),$$

and the goods market-clearing condition pins down aggregate consumption, C_t, as:

$$C_t + \sum_{y^t} \delta(y^t)\kappa = \sum_{y^t} c(y^t) + \delta(y^t)\kappa = Y_t - K_{t+1} + (1-\delta)K_t.$$

Recall that in our notation the current individual state m is included in the history of shocks y^t.

Recursive formulation. In this setting, the problem faced by households is complex: it integrates a double maximization to decide about participation in the contingent asset market and about asset purchases. Formally, this problem can be written as follows:

Problem 1.

$$\max_{\delta(y^t),c(y^t),B(y^t),a(y^t)} \sum_{y^t} \beta^t \pi(y^t) u(c(y^t))$$

$$\text{s.t. } c(y^t) + q^f B(y^t) + \delta(y^t) \left(\sum_k q(k,m) a(k,y^t) + \kappa \right) = w y_m + B(y^{t-1}) + a(m, y^{t-1}).$$

Fortunately, this problem can be rewritten recursively. Indeed, in Appendix A we show that it is equivalent to solve the following problem, for which the value function V is unique:[8]

Problem 2 (Recursive formulation)**.** Given $\{w, q, q^f\}$,

$$V(x, B, \{a\}, y) = \max_{\delta} \max_{\{a'\}, B'} \left\{ u(c) + \beta \sum_{y'} \pi(y'|y) V(x', B', a', y') \right\}$$

$$\text{s.t. } c + \delta \left(\sum_{y'} q(x, y', y) a'(y') + \kappa \right) + q^f(x) B' \leq w(x) y + B + a(y),$$

$$B' \geq -\overline{B}, \text{ and } x' = H(x).$$

with solution $\{\delta, \{a'\}, B'\} = h(x, B, \{a\}, y)$.

In particular agents are indexed by $\{B, \{a\}, y\}$, describing their asset positions as well as their labor supply. We denote by x the probability measure over Borel sets of compact set $S = Y \times A$, where A is the compact set of households' asset positions. As in Davila et al. (2012), we can construct the aggregate law of motion. To this purpose, we first construct the individual transition process. Let $J \in S$ be a Borel set. The corresponding individual transition function is:

$$Q(x, B, \{a\}, y, J, h) = \sum_{y' \in J_{y'}} \pi(y'|y) \xi_{h(x,B,\{a\},y) \in J_{\{B,\{a\}\}}},$$

where ξ is the indicator function. As a result, we can define the updating operator $T(x, Q)$ for tomorrow's distribution, x', given today one, x:

$$x'(J) = T(x, Q)(J) = \int_S Q(x, B, \{a\}, y, J, h) dx.$$

Finally, we can define the equilibrium in a recursive way:

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[8]This means that the discrete choice does not prevent the existence and uniqueness of the value function.

Definition 2. A *recursive competitive equilibrium* is a pair of function h and H that solves problem 2 given H and such that $H(x) = T(x, Q(.; h))$.

4 Individual Insurance Participation Decision

This section characterizes households' asset market participation decisions. The main goal of this section is to emphasize some important consequences of introducing participation costs: the existence of a threshold participation cost value, limited downward and upward insurance, and, ultimately, the non-monotonic relationship between participation decision and wealth. For simplicity we assume that the productivity shock follows a two-state first-order Markov process with the two possible states denoted as: y_l and y_h with $y_h > y_l \geq 0$. Regarding financial markets, we assume the existence of a risk-free asset, B, (whose price is q^f) and of a contingent securities associated with transitions to the low-income state, a, (whose price is q). As a consequence, markets are complete, as households can use the two instruments to insure against the high-income state as well. In this section we consider only one household.

In this setting the first order conditions for Problem 2 yield:

$$V_B(B, a, y) = u'(wy + B + 1_{y=y_l} a - \delta (qa' + \kappa) - q^f B'),$$
$$V_a(B, a, y) = 1_{y=y_l} u'(wy + B + 1_{y=y_l} a - \delta (qa' + \kappa) - q^f B'),$$
$$q^f u'(wy + B + 1_{y=y_l} a - \delta (qa' + \kappa) - q^f B') = \beta \sum_{y' \in \{y_H, y_L\}} \pi(y'|y) V_B(B', a', y) + \gamma,$$
$$\delta q u'(wy + B + 1_{y=y_l} a - \delta (qa' + \kappa) - q^f B') = \delta \beta \sum_{y' \in \{y_H, y_L\}} \pi(y'|y) V_a(B', a', y),$$

where γ is the Lagrange multiplier associated with the borrowing constraint $B' \geq -\overline{B}$.

When the agent decides to participate in the contingent asset market, i.e. $\delta = 1$, these equations define a^P and B^P. Similarly, when $\delta = 0$, they define $a^N = 0$ and B^N, where, as before, the superscript P denotes asset holding when participating in the contingent asset market and superscript N when not participating.

Remark. Uninsured agents ($\delta = 0$) purchase only risk-free assets. Their first order conditions are:

$$V_B(B, a, y) = u'(wy + B - q^f B'),$$
$$u'(wy + B - q^f B') = \sum_{y' \in \{y_H, y_L\}} \pi(y'|y) V_B(B', 0, y) + \gamma.$$

Hence, uninsured agents solve a similar problem as households in Aiyagari (1994).

Our first result is a no-arbitrage condition easily derived from the first conditions above and that puts a restriction on asset prices:

Proposition 3 (Asset prices). *Constrained households (for which $\gamma > 0$ in state y) do not purchase contingent assets as long as:*

$$q(y) \geq q^f \pi(y_l|y).$$

When there are unconstrained households ($\gamma = 0$) that participate in the contingent asset market, the following no-arbitrage condition is satisfied:

$$q(y) = q^f \pi(y_l|y).$$

Proof. See Appendix C.3 for the proof. □

The first consequence of this proposition is that there are only two types of portfolio in the economy: either households trade only risk-free assets or they trade both contingent and risk-free assets. Indeed, constrained households' willingness to purchase contingent assets is strictly lower than for unconstrained households. Therefore, when smoothing consumption, the household has a choice between a non-targeted but cheap insurance (by using only risk-free assets) and a targeted but costly insurance (by using both types of assets).

The participation choice. Which type of insurance does the agent choose? We show now that this decision is non-monotonic in the individual level of wealth. Denoting individual agents' wealth by $W = wy + B + 1_{y=y_l}a$, the contingent asset market participation choice follows from comparing the indirect utility when participating in the contingent asset market:

$$
\begin{aligned}
U^P(W, q, q^f, \kappa) &= u\left(W - \left(qa^P + \kappa\right) - q^f B^P\right) \\
&\quad + \beta\left[\pi(y_h|y)V(B^P, a^P, y_h) + \pi(y_l|y)V(B^P, a^P, y_l)\right],
\end{aligned}
$$

to the indirect utility obtained when not participating:

$$U^N(W, q^f) = u\left(W - q^f B^N\right) + \beta\left[\pi(y_h|y)V(B^N, 0, y_h) + \pi(y_l|y)V(B^N, 0, y_l)\right].$$

The comparison between U^P and U^N pins down a threshold value for the cost that determines the insurance behavior for the agent, as stated by the following proposition:

Proposition 4 (Threshold). *Given aggregate asset prices and individual level of wealth, $\{W, q, q^f\}$, there exists a threshold value for the fixed participation cost, $\overline{\kappa}$, such that:*

- *For $\kappa \leq \overline{\kappa}(W, q, q^f)$, the household participates in the contingent asset market ($\delta = 1$).*

- *For $\kappa \geq \overline{\kappa}(W, q, q^f)$, the household does not particpate ($\delta = 0$).*

Proof. See Appendix C.4 for the proof. □

14

Wealth and insurance. Finally, the relationship between the threshold cost value, $\overline{\kappa}$, and individual wealth generates the following non-monotonic insurance participation behavior, along the lines of Proposition 2:

Corollary 5. *When households' preferences feature decreasing absolute prudence, there exist two threshold values for wealth, $\underline{W}(\kappa, q, q^f)$ and $\overline{W}(\kappa, q, q^f)$, such that:*

- *For any $W \geq \overline{W}(\kappa, q, q^f)$, households with wealth W do not pay the cost and use only risk-free bonds to smooth consumption.*

- *For any $\underline{W}(\kappa, q, q^f) \leq W \leq \overline{W}(\kappa, q, q^f)$, households with wealth W pay the cost κ and purchase both contingent assets and risk-free bonds.*

- *For any $0 \leq W \leq \underline{W}(\kappa, q, q^f)$, households with wealth W do not pay the cost and use only risk free bonds to smooth consumption, if they are not borrowing-constrained.*

Proof. See Appendix C.5 for the proof. $\square$

As a consequence, depending on their wealth, agents have different abilities to smooth consumption: not at all where they are constrained (since they cannot afford the costly contingent assets and they cannot use risk-free bonds because of the constrain), almost perfectly when they are middle-class (since they acquire contingent bonds) and, interestingly, only partially when they are very wealthy (since they prefer not to purchase contingent bonds and use only the risk-free bond).

To summarize, between 0 and $\underline{W}$, there is a discrete number of levels of wealth. Poorest agents transit from one to another. For larger level of wealth (between $\underline{W}$ and $\overline{W}$), agents purchase also contingent assets and, hence, they achieve a better consumption smoothing. When becoming very wealthy, i.e. when $W \geq \overline{W}$, they accept some income risk, but, because of outstanding wealth, their income shocks become negligible.[9] Therefore, the existence of a tradeoff between enjoying the benefit of insurance and paying the cost to access the contingent asset market creates an endogenous heterogeneity for the participation decision across wealth.

Insurance across states, but not across periods. Corollary 5 states that only-middle class households will be fully insured, since they are the only ones that enter in the contingent-asset market. Those assets guarantee them to be completely insured against all the possible next-period income realizations. Notice, however, that this statements

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[9]The self-insurance result for wealthier households contrasts with findings by Ragot (2010) showing that households use money to smooth costly portfolio adjustments. This translates into a relatively lower demand for money by wealthier households.

does not imply that middle-class agents will be permanently fully-insured. In fact, if equilibrium asset prices are such that the wealth of middle-class households deteriorates, adverse income shocks might cause them to transit into the poorest wealth category (with wealth between 0 and $\underline{W}$). Hence, as it will become clear next session, the existence of the three social classes described in Corollary 5, which means that the wealth thresholds satisfying the following restrictions: (i) $\underline{W} > 0$, (ii) $\underline{W} < \overline{W}$, and (iii) $\overline{W}$ is finite, depend on the equilibrium asset prices.

Consumption smoothing for richest and poorest households. We pointed out that the richest and poorest households may not participate in the contingent asset market. What are the consequences of this behavior in terms of insurance? Denoting the growth rates of consumption as follows:

$$g_{y_l|y} = \frac{u'(c(B'(B,a,y),a'(B,a,y),y_l))}{u'(c(B,a,y))} \text{ and } g_{y_h|y} = \frac{u'(c(B'(B,a,y),a'(B,a,y),y_h))}{u'(c(B,a,y))},$$

from Proposition 3 we obtain the following Corollary:

Corollary 6. *Participation to the contingent asset market leads to full insurance when participating to the contingent asset market, but to imperfect insurance when not participating:*

$$1 = \frac{g^P_{y_l|y}}{g^P_{y_h|y}} \geq \frac{g^N_{y_l|y}}{g^N_{y_h|y}}. \tag{2}$$

When participating, consumption grows at a rate that depends only on the price of the risk-less asset:

$$g^P_{y_l|y} = g^P_{y_h|y} = \left(\frac{\beta}{q^f}\right)^{-1/\sigma},$$

which implies that insured households' consumption decreases (increases) over time when $q^f \geq \beta$ ($q^f \leq \beta$).

When constrained on their risk-free asset position (i.e. $\gamma > 0$), agents do not purchase contingent assets; hence, they do not completely insure. Conversely, when households participate in the contingent asset market, they equalize next period marginal utilities and are fully insured. As a consequence, we have the following implication:

Corollary 7. *Consumption volatility is non-monotone across the three wealth categories: it is highest for constrained poor households, it is lowest for insured middle-class households, and it attains an intermediate value for self-insured rich households.*

16

4.1 Equilibrium

A first useful result is a characterization of the aggregate insurance behavior:

Proposition 8 (Equilibrium). *For any $\kappa \geq 0$, there exists a unique equilibrium. More accurately, there exists $\underline{\kappa}$ and $\overline{\kappa} \geq \underline{\kappa}$ such that the unique equilibrium is as follows:*

(i) *self-insurance equilibrium: for $\kappa \geq \overline{\kappa}$, households use only risk-free assets to smooth consumption and $q^f = \bar{q}^f$: the participation cost economy coincides with the Aiyagari economy.*

(ii) *Partial insurance equilibrium: for $\underline{\kappa} \leq \kappa \leq \overline{\kappa}$, some households participate in the contingent asset market while the others purchase only risk-free assets. Asset prices are as follows: $q^f(\kappa) > \beta$ and $q(y)(\kappa) = q^f(\kappa)\pi(y_l|y)$. Specifically, $q^f(\kappa)$ is a continuous and increasing function of participation costs κ.*

(iii) *Perfect-insurance equilibrium: for $\kappa \leq \underline{\kappa}$, all households participate in the contingent asset market and are fully insured. Asset prices are as follows: $q^f = \beta$ and $q(y) = \beta\pi(y_l|y)$.*

Proof. See Appendix C.6 for the proof. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

In particular, for large values of the participation cost, $\kappa > \overline{\kappa}$, the unique equilibrium features *self-insurance*. For costs lower than $\overline{\kappa}$, the equilibrium features insurance: either the one featuring *partial-insurance* (for intermediate values of participation costs, $\underline{\kappa} \leq \kappa \leq \overline{\kappa}$), or the one featuring *perfect-insurance* (for small values of participation costs, $\kappa \leq \underline{\kappa}$).

The equilibrium interest rate As pointed out in Aiyagari (1994), when households have only risk-free bonds to self-insure against idiosyncratic shocks (*self-insurance equilibrium*), the interest rate paid on these bonds is lower than the interest rate paid when markets are complete.[10] The intuition for this result is simply that high level of interest rates would incentivize households to accumulate an infinite amount of assets, which would allow them to consume infinitely and, of course, to be perfectly insured. A similar result holds in our proposed *partial-insurance* model, but for a different reason. If the risk-free rate was equal to the full-insurance case (i.e. $q^f = \beta$), households with an intermediate level of wealth, $\underline{W}(\kappa, q, q^f) \leq W \leq \overline{W}(\kappa, q, q^f)$, would always be perfectly insured because their wealth never deteriorates, since the return on their portfolio would be large

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[10]Similarly, Bewley (1980) finds that the optimal rate of inflation should be a little bit higher than the inverse of the discount rate.

enough. Hence, these households would never transit into the region characterized by imperfect insurance. In addition, poor households that starts with a low level of wealth, $0 \leq W \leq \underline{W}(\kappa, q, q^f)$, will eventually transit into the perfect-insurance region after receiving a series of positive income shocks. Hence, also those households would be fully insured in the long-run. Finally, rich households with wealth, $W \geq \overline{W}(\kappa, q, q^f)$, either will accumulate an infinitely large quantity of wealth given the high-return on the risk-free assets (as in Aiyagari (1994)) or they will transit into the perfect-insured region after being subject to a series of negative income shocks. Either way, however, they will be obviously perfectly insured. As a result, if $q^f = \beta$ the unique stationary distribution would feature only perfectly-insured households.[11] Also, our model contrasts with the findings obtained by Huggett (1993) on the risk-free rate. Specifically, in Huggett's model, the presence of idiosyncratic shocks leads to higher savings, which in turn depresses interest rates at lower rates than the one pegged simply by the households' discount factor (cf. Aiyagari, 1994). In our model, however, the larger is the availability of contingent assets among households, the higher is the risk-free rate in the economy, since precautionary demand of risk-free assets is reduced.

5 Macroeconomic Implications of Participation Costs

In the previous section we have shown that participation costs potentially imply the existence of three categories of households: uninsured and poor, perfectly-insured and middle-class, and self-insured and rich. The coexistence of the latter two categories in a *partial-insurance* equilibrium leads to interesting and novel implication for inequality, partial insurance rate, and welfare, as we describe in this section.

5.1 Participation costs and the Wealth Distribution

Social mobility. How does the existence of participation cost in the contingent asset markets affect the wealth distribution? The answer to this question depends on the interaction between participation costs and income risk. For intermediate levels of participation costs that allow for a *partial-insurance equilibrium* to exist, two forces operate in different portions of the wealth distribution. On the one hand, perfectly insured (middle-class) households do not accumulate more assets and, as the risk-less interest rate is lower than in the complete market model, they even progressively consume their wealth. On the other hand, self-insured richer households benefit from real interest rates that

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[11]This would not be robust to the introduction of aggregate shocks or to idiosyncratic wealth shocks, as, for example, in Blanchard (1985) in which households die according to some Poisson process and other appear with a lower level of wealth.

are higher than in the incomplete market model and they accumulate more wealth in comparison with the standard Aiyagari model. These two forces contribute to skew the wealth distribution and lead to large wealth inequality.

Obviously, an important condition for the existence of the second force is that the stationary *partial-insurance equilibrium* exhibit a non-zero fraction of self-insured rich households. A necessary condition for this to happen is that the economy is subject to large-enough income risk. Intuitively, self insured rich households still face negative income shocks. As the real interest rate is still below the discount rate in equilibrium ($q^f \geq \beta$), these households' wealth can potentially fall below the correspondent upper participation threshold, $\overline{W}$. Because of this existing downward social mobility force, in order to obtain a stationary *partial-insurance equilibrium* featuring self-insured rich households, perfectly insured households should also face some income shocks that are large enough to allow for certain degree of upward social mobility. The following proposition rigorously states this mechanism.

Proposition 9. *For participation costs such that the partial-insurance equilibrium exists (i.e. $\underline{\kappa} \leq \kappa \leq \overline{\kappa}$), if there exists two levels of income shocks, y_k and y_j, such that $w(y_k - y_j) \geq \overline{W} - \underline{W}$ and $\pi(y_k|y_j) > 0$, then the stationary partial-insurance equilibrium features a positive measure of self-insured rich households ($W_i \geq \overline{W}$ for some household i).*

Otherwise, agents with a level of wealth such that they are either in the insurance zone or below ($W_i \leq \overline{W}$) never accumulate more wealth than the upper threshold of the insurance zone. In this case the stationary partial-insurance equilibrium features measure-zero of self-insured rich households($\nexists i$ such that $W_i \geq \overline{W}$.)

This proposition states that when income shocks are sufficiently large, so that insured middle-class agents can jump above the insurance area (i.e. when $w(y_k - y_j) \geq \overline{W} - \underline{W}$), then there is some upward social mobility, ensuring that some agents will become rich and self-insured. Conversely, when income shocks are small, social mobility is bounded above and middle-class households have no incentives to infinitely accumulate wealth.

In the end, the wealth distribution highly depends on insurance behavior and income shocks. In particular, thresholds in participation decisions are likely to make the wealth distribution a discontinuous function of income shocks. In the rest of the section, we quantitatively investigate this relation.

Remark. This effect is similar as in the complete market economy where the steady-state wealth distribution exactly matches the initial wealth distribution. In that case, households do not have any incentive to accumulate more wealth as they are fully insured

19

against income variations.[12]

Upper tail of the wealth distribution and inequalities. Our social mobility result has an impact on the wealth distribution and inequality. Here, we perform two exercises. First, we show that intermediate participation costs allow the wealth Gini coefficient to increase with respect to self-insurance economy. Second we highlight the role of income inequality in amplifying wealth inequality in the partial-insurance equilibrium.

We consider a calibration close to the unemployment economy as in Davila et al. (2012). The utility function is assumed to be CRRA $u(c) = c^{1-\sigma}/(1-\sigma)$, with $\sigma = 2$. The discount factor is set at $\beta = 0.96$, so that the annual interest rate is close to 4 percent. The share of capital in the production function is set at $\alpha = 0.36$ and the depreciation rate at 0.08. The only difference with the standard calibration is that we allow for a third state for the income process: $y \in \{0.01, 1, 1.1\}$ but this third state is relatively unlikely so that the income process is very close to the original unemployment economy. The assumed transition matrix is $\pi = \{0.62, 0.38, 0; 0.0199, 0.98, 0.0001; 0, 0.5, 0.5\}$. There are three important comments related to the calibration of the income process. First, our setting delivers the same unconditional moments for the labor market as targeted in Davila et al. (2012), namely a 5 percent unemployment rate and an average unemployment duration of 2.6 years. Second, the inclusion of the third income state assures that the process has enough income variation to guarantee a positive upward social mobility, which is a necessary condition of the existence of a steady-state wealth distribution that features both perfectly-insured and partially-insured agents in presence of intermediate levels of participation costs, as pointed out in the previous section. Finally, the entries of the third row of the transition matrix, which determines the probability to stay in the third state and to transit into the second or first state, are arbitrary calibrated to $[0, 0.5, 0.5]$, but our results are not affected by different choices of these probabilities, as long as the third-state is not absorbing.

We simulate this economy for three different levels of participation cost: a high cost so that the economy is characterized by the *self-insurance equilibrium*, as in the Aiyagari model, an intermediate cost, so that the economy is characterized by the *partial-insurance equilibrium*, and a zero-cost, so that the economy is characterized by the *perfect-insurance equilibrium* (complete markets *de facto*). Table 1 summarizes the main statistics for the three economies.

In the perfect insurance equilibrium, in which participation costs are absent, agents can fully insure against idiosyncratic shocks. In this case, no inequalities emerge as

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[12]Conversely, in the Aiyagari economy, the initial wealth distribution has no effect on the steady-state wealth distribution.

	High Cost	Intermediate Cost	No Cost
	Self-insurance Eq.	Partial-Insurance Eq.	Perfect-Insurance Eq.
Cost/Income (%)	> 25	15	0
Interest rate (%)	3.244	4.148	4.167
Aggregate assets	3.202	2.963	2.959
Wealth Gini	0.121	0.932	0

Table 1 – Steady state for the unemployment economy

agents do not accumulate wealth. Interestingly, there are important differences between the case of the self-insurance equilibrium and the partial-insurance equilibrium. In fact, as emphasized by Proposition 9, the partial-insurance equilibrium implies that agents' wealth levels can be bounded above depending on income fluctuations and the size of the insurance area $\overline{W} - \underline{W}$. When participation costs are sufficiently low (partial-insurance equilibrium), income fluctuations allow agents to "jump" above the insurance area, and as a consequence, there exists a positive mass of households with wealth levels above $\overline{W}$. This effect is highlighted in Table 1: in the partial-insurance equilibrium, a large mass of agents are trapped with low levels of wealth as they quickly choose to get fully insured, while a small fraction of agents "jump" above the insurance area when receiving a high level of income and accumulate large stocks of assets. As a result, the wealth inequality of the partial-insurance economy is much larger than the wealth inequality of the incomplete market model.

Furthermore, the accumulation of assets in the partial insurance equilibrium when levels of wealth exceed $\overline{W}$ is even amplified compared to the Aiyagari (1994) model by the larger real interest rate. In fact, with intermediate participation costs, there are lower downward pressures on interest rates than in the incomplete market model because of the existence of households that participate in the contingent markets and that, therefore, have no willingness to accumulate wealth. Notice, however, that albeit the partial-insurance model produces large levels of wealth inequalities, in equilibrium, the interest rate remains lower than in the complete market economy (perfect-insurance). Yet, in contrast to Piketty (2014), in our explanation of inequality, the level of interest rate does not play a central role, but only an amplifying one; wealth is mainly driven by the households' individual *willingness* to accumulate assets, which depends on their insurance choices.

Finally, when participation costs become sufficiently high, no agents purchase insurance anymore, and the economy reverts to the self-insurance equilibrium. Interest rates are lower due to a larger precautionary demand for risk-less assets and there is no discontinuity anymore in the wealth accumulating process due to the insurance area.

The Inequality Accelerator. We now conduct a comparative static exercise to illustrate how, in a model with partial-insurance, larger income inequality translates in larger wealth inequality: we refer to this mechanism as the *inequality accelerator* effect. We consider two income processes: the same income process as in the previous paragraph: $y \in \{.01, 1, 1.1\}$ associated with

$$\pi = \{0.62, 0.38, 0; 0.0199, 0.98, 0.0001; 0, 0.5, 0.5\},$$

and a slightly different one: $y \in \{0.01, 1, 1.05\}$ associated with the same transition matrix. Notice that the second process is characterized by a smaller income dispersion across the states. Hence, in Table 2, which reports the equilibrium wealth Gini index resulting from both income processes, we label the first process as the *High Income Inequality* and the second process as the *Low Income Inequality*.

	High Cost Self-Insurance Eq.	Intermediate Cost Partial-Insurance Eq.	No Cost Perfect-Insurance Eq.
Cost/Income (%)	>25	15	0
		Wealth Gini Index	
High Income Inequality	0.121	0.932	0
Low Income Inequality	0.110	0	0
Davila et al. (2012)	0.108	-	-

Table 2 – Steady state for the unemployment economy

When income fluctuations - or, in our context, income inequality - are sufficiently low, wealth inequality is bounded. Indeed, when households become fully insured, they do not further accumulate assets, putting an upper bound on their wealth (see Proposition 9). This is no longer the case when income fluctuations become slightly larger. In this case, income fluctuations allow agents to "jump" above the insurance area: their level of wealth shifts from a level below $\underline{W}$ to a level above $\overline{W}$. Then, these households continue to accumulate assets for self-insurance purpose. As already mentioned, this accumulation is accentuated because of the higher level of interest rate compared with the Aiyagari model. Importantly, notice that the inclusion of the third income state with respect to the calibration in Davila et al. (2012) does not affect *per-se* wealth inequality, since in the Self-Insurance equilibrium (i.e. with large participation costs) in our three state economy leads to a basically identical wealth Gini coefficient to the one reported by Davila et al. (2012), which consider a two-state income process that leads to the same unconditional unemployment moments.

Importantly, notice that the motivations behind the large welfare inequality achieved in our setting differs from the ones in Castaneda et al. (2003). In fact, in their incomplete

market model the large wealth inequality is solely driven by the very large income dispersion (income Gini index equal to 0.600), which translates into a large income risk for the top-earners. In contrast, in our setting, a much smaller degree of income fluctuations (income Gini index equal to 0.097) is able to trigger a sizeable welfare inequality not only through the much weaker channel of income risk for the top-earner, but, above all, through the different insurance incentives across the wealth distribution and asset prices. To summarize, the economy characterized by intermediate levels of participation costs requires only a certain (small) degree of income inequality to trigger the large amplification from income inequality to wealth inequality mainly driven by the non-monotone willingness to insure across the wealth distribution and its implications on asset prices.

5.2 Participation Costs and Partial Insurance

As discussed in the previous section and, more specifically, in Proposition 9, the joint presence of insured and self-insured households increases the level of wealth inequality. Obviously, the coexistence of fully insured and self-insured households implies a certain degree of aggregate insurance in the economy. In this section we explore the relationship between participation cost, wealth inequality, and degree of partial insurance.

Let us denote the equilibrium share of insured agents by θ. Hence, θ directly represents the fraction of insured households. In addition, applying the law of large numbers and noticing that that each agent's income is independently distributed, θ also represents the average share of individual income that is insured, or, equivalently, the degree of partial insurance, as defined in Guvenen and Smith (2014). We compute the share of insured households for two different popular calibrations of the heterogeneous agents model, i.e. the unemployment economy that we use in the previous subsection and the one in Aiyagari (1994) as used in Davila et al. (2012).[13] Table 3 reports the obtained results.

The top-panel of the table displays the resulting characteristics in case of high participation costs. In this setting, there are no households that enter in the contingent asset market and, therefore, the fraction of insured agents, or equivalently the fraction of total insured income, is zero. The two calibrations imply rather low level of inequality, as indicated by a wealth Gini index of 0.11 for the unemployment economy and of 0.42 for the Aiyagari (1994) economy. The bottom-panel of the table reports the same equilibrium statistics in the presence of intermediate levels of participation costs. When participation costs are not excessively high, the degree of partial insurance as well as the wealth Gini

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[13]The calibration for the Aiyagari (1994) model is as in p.19 of Davila et al. (2012). The coefficient of relative risk aversion in the CRRA utility function is set to 2. The discount factor is set to 0.96. The capital share is equal to 0.36. The three state process for income is $y = \{0.78, 0.27, 0.66\}$. The transition matrix is $\pi = \{0.66, 0.17. 0.27; 0.28, 0.44, 0.28; 0.07, 0.27, 0.66\}$.

		Unemployment	Aiyagari (1994)
High cost	Gini	0.11	0.42
	θ (%)	0	0
Intermediate cost	Gini	0.93	0.96
	θ (%)	84.6	31.6
	Cost/Income (%)	15.0	22.3

Table 3 – Degree of partial insurance

coefficient increase. Yet, the relationship between the degree of partial insurance and the increased level of inequality is not trivial and is calibration-dependent: even a small share of partial insurance, which corresponds to a small share of participation, around 30 percent, as in the case of Aiyagari (1994) economy, is able to skew the wealth distribution to provide with a Gini index similar to one observed in the U.S. data. In this case, even with a minority of insured households, the equilibrium level of interest rate and the social mobility effect described in the previous section give rich agents large incentives to accumulate wealth. Interestingly, the unemployment economy implies a similar level of inequality with a much larger participation rate, around 80 percent. In this setting, the properties of the income process are such that even a small fraction of self-insured household has strong incentives to accumulate a large amount of wealth.

Our definition of partial insurance can be linked to the one introduced in Guvenen and Smith (2014). However, whereas their form of partial insurance is on the intensive margin - agents can insure a fraction of their income, in our setting partial insurance is on the extensive margin - agents can be insured or not. In their empirical work, Guvenen and Smith (2014) estimates the fraction of partial insurance around 45 percent. In this section we showed how the existence of participation costs, which leads to partial insurance, can generated realistic level of wealth inequality together with degree of partial insurance both above and below their estimated partial insurance level.

5.3 Participation Costs and Welfare

This subsection analyses the welfare properties of an economy with participation costs. It is well-known that economies with idiosyncratic shocks are not necessarily constrained Pareto efficient (cf. Carvajal and Polemarchakis, 2011; Davila et al., 2012, among others) in the sense that a central planner can do better than the market allocation when accessing the same tools. The central idea of that result stems from a pecuniary externality arising through factor prices (e.g. wages and interest rates): by accumulating more assets, agents depress interest rates making further insurance less likely. In the partial insurance model

we developed in this paper, the same intuition applies for the accumulation of risk-free assets as well as of contingent assets, as we discuss in this section.

Let us first define constrained Pareto efficiency in our setting. The central planner solves the following problem:

Problem 3.

$$V(x) = \max_{B'(y,B,a),\delta(y,B,a),a'(y,B,a)} \int u \left[\begin{array}{c} B + a1_{y=y_l} + wy - q^f(K)B'(y,B,a) \cdots \\ \cdots - \delta(y,B,a)\left(q(K)a'(y,B,a) + \kappa\right) \end{array} \right] dx + \beta V(x'),$$

$$\text{s.t. } x' = T(x, Q(.,y)), K = \int (a+B)dx.$$

As in Davila et al. (2012), we consider equal weights for all agents as we are interested in insurance and not in redistribution. Finally, we assume that the central planner is also constrained to rule out allocation where she would be able to perfectly insure agents by transfers.

The solution of Problem 3 allows us to obtain the following results:

Proposition 10. *The planner's problem solution is such that:*

(i) *For $\kappa \leq \underline{\kappa}$, the economy is constrained Pareto optimal.*

(ii) *For $\kappa \geq \underline{\kappa}$, the economy is constrained Pareto suboptimal.*

Furthermore, the central planner's solution features perfect insurance for some $\kappa > \underline{\kappa}$. Otherwise, constrained efficient insurance is ambiguous.

Proof. See Appendix C.7 for the proof. □

When participation costs are sufficiently low ($\kappa \leq \underline{\kappa}$), agents are fully insured and markets are completed both in the central planner's solution and in the competitive market solution. In this case, the competitive market solution is constrained Pareto optimal.

When participation costs increases to an intermediate level, there is no more full insurance and the economy transits in the *partial-insurance equilibrium*. In this case, a pecuniary externality arises through asset prices as already noted by Carvajal and Polemarchakis (2011) or Davila et al. (2012). We show that this externality also arises in the insurance behavior: for intermediate values of participation costs, agents insure less in the competitive equilibrium compared with the central planner's allocation. This translates into a lower risk-free rate and a lower level of aggregate insurance.

Nevertheless, when participation costs are sufficiently high so that the central planner prefers not to implement full insurance, the insurance externality may be muted. Indeed,

as noted by Davila et al. (2012), higher levels of capital can lead to more insurance as they allow to redistribute wealth to agents at the bottom of the wealth distribution, for whom labor is the main source of income. Insurance markets reduce the agents' willingness to save and, therefore, the aggregate level of capital: as a result, insurance through markets and through higher wages are competing with each other. In the end, the degree of insurance in the central planner's solution may be higher or lower compared to the competitive market allocation depending on the relative size of these two insurance mechanisms.

We show in the appendix that the constrained efficient solution is characterized by equations that differ from the competitive market allocation only by some additional terms depending on factor prices, as in Davila et al. (2012) (See Appendix C.7). By computing these additional terms, we are then able to determine whether there is too much or too little participation compared with the efficient allocation. With the unemployment economy calibration, this leads to the results presented in Table 4. In the intermediate cost case, wealth inequalities are large and redistribution is more effective through higher wages and more capital and so, through less insurance. This is not the case in the high cost case, where agents gain from a higher interest rate and, then, a lower stock of capital.

	High Cost	Intermediate Cost	Low Cost
	Self-insurance Eq.	Partial-Insurance Eq.	Perfect-Insurance Eq.
Insurance Level	Under-Insurance	Over-insurance	Complete

Table 4 – Constrained efficiency of insurance

6 Evidence on partial insurance and consumption smoothing

In this section, we investigate the extensive margin of partial insurance and consumption smoothing. In particular, we test an implication of our model, which is the non-monotonic distribution of insurance. Recall, that Corollary 5 states that with intermediate participation costs the economy features a *partial-insurance equilibrium* in which poor households are non-insured, middle-class households are perfectly insured, and richer household are only partially insured. In this section, we confirm that result by directly analyzing consumption data, income, and wealth Italian data.

Italian Data. We estimate the regression:

$$\Delta \log c_{it} = \alpha_1 + \sum_{j=1}^{N} \alpha_j(D_j \Delta \log y_{it}) + \beta X_{it} + \epsilon_{it}, \qquad (3)$$

where D_j, with $j = 1, .., N$, is a dummy indicating groups of increasing wealth. The estimates α_j, with $j = 1, .., N$, are the coefficients of interest, since they measure the degree of consumption insurance for a given wealth category. We estimate the regression in (3) by using the Bank of Italy Survey of Households' Income and Wealth (SHIW) panel data in the period 2006-2008, since it uniquely provides information about the level of wealth of the households. We consider seven wealth categories, identified by the following six percentiles: 10, 25, 50, 75, 85, 95, to explore the insurance level of at the extremes of the distribution. We consider three different specifications that control for age, education, lifecycle (status), type of profession, and geographical areas. The results are presented in Table 5. The estimation confirms the non-monotonic degree of consumption insurance as predicted by our model. Poor households are largely uninsured; when the wealth increases, consumption insurance improves. The 5th wealth level (75-85 percentile) is perfectly insured, since the coefficient linking income growth to consumption growth is not significantly different from zero. Importantly, at the top of the wealth distribution the households are only partial insured. At the bottom of the table, we report the p-values of the hypothesis that test these predictions. First, the insurance coefficient for the poorest wealth category is significantly different than the next higher wealth category at 1 percent significance level. Indeed, that means that the poorest category are significantly less insured. Second, the insurance coefficient for the 5th wealth category is significantly different than the one for the highest wealth category at 1 percent significance level. This means that, there is a statistically significant difference between the low consumption-to-income relationship for the middle class and the larger consumption-to-income relationship for the richest households.

Additional evidence on non-monotone insurance. Even though it has not been documented on its own, our non-monotonicity result is consistent with recent findings based on improvements of the treatment of U.S. CEX data as in Aguiar and Bils (2015), who show that taking into account rich households' specific consumption increases the volatility of their consumption and hence aggregate consumption inequality. Our result is also consistent with the non-monotone marginal propensity of consumption across wealth during the Great Recession as estimated by Krueger et al. (2015).

It is also possible to confirm the non-monotonic insurance behavior by checking whether agents actually purchase insurance. For example, Parsons et al. (2015) provide evidence from the Danish voluntary public unemployment insurance system that

	(1)	(2)	(3)
	$\Delta(c_t)$	$\Delta(c_t)$	$\Delta(c_t)$
$W_1\Delta(y_t)$	0.479*** [0.10]	0.486*** [0.10]	0.486** [0.10]
$W_2\Delta(y_t)$	0.202*** [0.06]	0.204*** [0.06]	0.200** [0.06]
$W_3\Delta(y_t)$	0.204*** [0.04]	0.204*** [0.04]	0.197** [0.04]
$W_4\Delta(y_t)$	0.192*** [0.16]	0.196*** [0.04]	0.202** [0.04]
$W_5\Delta(y_t)$	0.016 [0.07]	0.018 [0.06]	0.021** [0.04]
$W_6\Delta(y_t)$	0.334*** [0.06]	0.336*** [0.06]	0.338** [0.06]
$W_7\Delta(y_t)$	0.398*** [0.11]	0.394*** [0.11]	0.408** [0.11]
Constant	0.297*** [0.13]	0.390** [0.16]	0.441** [0.16]
Age and Age^2	YES	YES	YES
Education	NO	YES	YES
Status	NO	YES	YES
Profession	NO	NO	YES
Area	NO	NO	YES
Observation	1721	1721	1721
R^2	0.064	0.067	0.076
$W_1\Delta(y_t) = W_2\Delta(y_t)$	0.0002***	0.0002***	0.0002***
$W_5\Delta(y_t) = W_7\Delta(y_t)$	0.0055**	0.0064**	0.0049**

Table 5 – Non-Monotonic Insurance across wealth: Italian data

Note: this table reports the estimates of regression (3) for Italian data. The dataset is the panel Bank of Italy Survey of Households' Income and Wealth (SHIW) data in the period 2006-2008.The top-panel reports the coefficient estimates $\hat{\alpha}_j$ for the seven wealth groups considered (percentiles: 10, 25, 50, 75, 85, 95). Standard errors are reported in brackets. The central panel describes the control variables, X, considered in each of the there specification (1), (2), and (3). The bottom panel reports the p-values of the F-statistics which test the equality $W_1\Delta(y_t) = W_2\Delta(y_t)$, which means that poorest households has the same degree of insurance as the second poorest group, and $W_5\Delta(y_t) = W_7\Delta(y_t)$, which means that middle-class households has the same degree of insurance as the richest households. Both of these hypotheses are rejected at least at 5 percent significance level. Hence, the data support the non-monotonic degree of insurance with respect to wealth as implied by our partial-insurance model.

top parts of the distribution participate much less than intermediate ones, even though the unemployment risk is not substantially lower. More generally, they also show that wealth can affect negatively unemployment insurance participation. As mentioned above,

there is also evidence on lack of insurance for the lowest part of the distribution.

7 Further extensions and discussion

Default and limited-commitment economies. Economies with participation costs are substantially different from lack-of-commitment economies. The intuition is that, in limited-commitment economies when lenders anticipate a possible default in some future states, they limit their loans either as an incentive for borrowers to repay or as a hedge against the default. This behavior results in bounded (below) agents' portfolio positions.[14] In economies with participation costs, instead, the amount a household can borrow against a future state is unconstrained as long as she is willing to pay the fixed cost. In the two economies, households can be constrained (i.e. they cannot equalize their marginal rates of substitution), but for very different reasons: in one case the households' gain from transferring wealth inter-temporally is too low given the participation cost to pay, and, in the other case households are willing to transfer more wealth but they are constrained by borrowers.

Downward and upward insurance. Several examples in the literature underscore the comparatively lower levels of insurance coverage for poorer households than for the rest of the population.[15] In those studies, lack of insurance concerns *downward shocks*, which is future negative income shocks. This differs from borrowing constraints that limit the ability of insurance against *upward shocks*, which is positive future income shocks. Of course, as poor households are also likely to be borrowing constrained, they are also uninsured upward.

Our participation cost-model is able to reproduce such lack of both downward and upward insurance for poor households. In contrast, the one-sided no-commitment model (see Thomas and Worrall (1988) as an example) fails to reproduce the downward non-insurance: short-selling or borrowing constraints only prevent households from borrowing against future revenue and not from accumulating assets for insuring against lower future

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[14]As studied by Thomas and Worrall (1988), Kehoe and Levine (1993) or Kocherlakota (1996), in these economies, borrowers always compare the gains of financial trade with autarky and, hence, take decisions of repayment or default.

[15]Rampini and Viswanathan (2012) find that health insurance coverage is 75.5 percent for households earning less than $25,000 per year compared to 92.2 percent for those earning more than $75,000. Brown and Finkelstein (2007) find similar results for private long-term care insurance coverage. An additional example is provided by Cole et al. (2009) when studying insurance behavior of Indian farmers. Murdoch (1995) studies how farmers in India choose to lower their average income against lower volatility. Cole and Shastry (2009) show in a different context that education is also a determinant of insurance decisions, providing a non-monetary interpretation of participation costs.

income. Although Rampini and Viswanathan (2012) show that only poor households are unable to smooth consumption, they also fail to reproduce downward non-insurance, as limited insurance for poor households derived from borrowing constraints. In comparison, the standard Aiyagari (1994) model is compatible with the absence of downward non-insurance, but it cannot account for endogenous insurance decisions as it simply rules out insurance contracts.

Interpreting participation costs. Our baseline interpretation of participation costs is a monetary one. These monetary costs arise from financial or insurance intermediaries, possibly related to sunk costs due to an intermediaries' production functions or to screening costs, when agents have to signal their type by willing to pay the fixed costs.[16] Other interpretations include cognitive costs or shopping-costs: selecting insurance requires time and effort. All these interpretations imply paying the fixed cost *ex ante*. Another alternative form of fixed cost faced by households surfaces when collecting insurance payments when bad shocks occur. Collection requires proofs of damage to address the adverse selection problem. Assuming this alternative form of participation cost would not qualitatively change our results: it would also prevent agents from purchasing insurance against small shocks, and would lead to preferences for purchasing insurance only against large shocks. In this situation, as in our setting, poorer households cannot afford to pay the insurance.

Long-term assets. Our theory considers only one-period assets that require paying the participation costs at every period; however, in the real world, one might argue that insurance is a long-term proposition.

In our framework one could introduce a long-term asset that pays nothing as long as good shocks occur but yields a payoff in case of a bad shock. The main difference with respect to a short-term asset is that this long-term asset can be held for several periods, until a bad shock occurs and triggers a payoff to the agent. Our analysis can easily be extended to similar long-term assets under the assumption that the long-term insurance stops after a bad shock. Otherwise, the insurance would be purchased once and for all. More general long-term assets can also be considered, but they have to remain, to some degree, contingent on agents' idiosyncratic shocks.

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[16]The exact setting leading to this kind of fixed cost would be a dynamic version of Rothschild and Stiglitz (1976).

8 Concluding remarks

In this paper, we study the *partial-insurance equilibrium* that characterizes an economy with participation cost in state-contingent asset markets. In this setting households' degree of insurance depends on their wealth. In fact, under decreasing absolute prudence, the *partial-insurance equilibrium* is characterized by a set of poor households that are not able to obtain any insurance, by a set of middle-class household that actively participate to the contingent asset market and, hence, are fully insured, and, interestingly, by a set of rich households that prefer to self insure by accumulating a large stock of the risk-free assets.

This non-monotonic relationship between degree of insurance and wealth leads to important implications about social mobility, welfare, and wealth inequality. Specifically, when participation costs reduce from a arbitrary large value, such that the economy is equivalent to a *self-insurance equilibrium*, to intermediate values, such that the economy turns into a *partial-insurance equilibrium*, wealth inequality dramatically increases. With intermediate value of participation costs, our model can predict a level of wealth inequality similar to the one observed in the U.S. data (Gini index equal to 0.93). Then, we show that in presence of a *partial-insurance equilibrium*, wealth inequality is particularly sensitive to income inequality. We label this phenomenon as the *Inequality Accelerator*. With a numerical example, in fact, we find that a small increase of the exogenous income inequality in the participation-cost model leads to a very large change of the resulting wealth inequality. Crucially, however, the same change in income dispersion implies a vary small increase of the wealth inequality in the incomplete market model.

Our paper has, then, important implications for households' risk management, asset prices, social mobility, welfare, and inequality. Our approach uses a simplified framework without aggregate shocks and with participation costs exogenously introduced.

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A   Appendix: value function and recursive formulation

The following proposition establishes the existence and the uniqueness of the value function solving Problem 2.

Proposition 11. *The value function V exists and is unique.*

Moreover, the value function V can be obtained by iterations: for any initial value $V' \in \Omega$ and defining the sequence, $V_n = T^n V'$, V_n converges to V.

Proof. This proof extends the proof of Stokey et al. (1989) for discrete variables. Recall that the value function satisfies:

$$V(B, \{a\}, y) = \max_{\{a'\}, B', \{\delta'\}, \text{w.r.t. B.C.}} \begin{bmatrix} u\left(C\right) \\ + \beta \sum_{y'} \pi(y'|y) V(B', \{a'\}, y') \end{bmatrix}$$

Defining T as:

$$TV = \max_{\{a'\}, B', \{\delta'\}, \text{w.r.t. B.C.}} \left[u\left(C\right) + \beta \sum_{y'} \pi(y'|y) V(B', \{a'\}, y') \right]$$

it is easy to show that T satisfies Blackwell's conditions. First T is monotonic. For $W \leq V$, we have that :

$$TW = \max_{\{a'\}, B', \{\delta'\}, \text{w.r.t. B.C.}} \left[u\left(C\right) + \beta \sum_{y'} \pi(y'|y) W(B', \{a'\}, y') \right]$$

$$\leq \max_{\{a'\}, B', \{\delta'\}, \text{w.r.t. B.C.}} \left[u\left(C\right) + \beta \sum_{y'} \pi(y'|y) V(B', \{a'\}, y') \right]$$

$$= TV$$

Second T discounts: let Γ be a positive constant:

$$T(V + \Gamma) = \max_{\{a'\}, B', \{\delta'\}, \text{w.r.t. B.C.}} \left[u\left(C\right) + \beta \sum_{y'} \pi(y'|y) \left(V(B', \{a'\}, y') + \Gamma\right) \right]$$

$$= \max_{\{a'\}, B', \{\delta'\}, \text{w.r.t. B.C.}} \left[u\left(C\right) + \beta \left[\Gamma + \sum_{y'} V(B', \{a'\}, y') \right] \right]$$

$$= TV + \beta \Gamma$$

We define $X = \{x = \{B', \{a'\}, y'\}\}$. Ω denotes the set of functions V such that V is continuous with respect to B and a. We need also to prove that:

- Ω with the d_∞ metric is a metric space.
- TV is in the same set as V, which is obvious.

Metric space   Let $\{V_n\}$ a Cauchy sequence of Ω. For every $x \in X$, $V_n(x)$ converges to $V(x)$. Let us verify that V is the limit using the d_∞ metric. As $\{V_n\}$ a Cauchy sequence: for some $\epsilon > 0$ and for some $x \in X$, there exists n such that for every p and q satisfying $q \geq p > n$, $|V_p(x), V_q(x)| < \epsilon$. Taking the limit of this expression with respect to q, we obtain that $|V_p(x), V(x)| < \epsilon$. As this is true for every $x \in X$, this implies that $d_\infty(V_p, V)$ converges to 0, which means that V_n converges to V.

Conclusion The requirements of the Contraction Mapping theorem are satisfied. There exists an unique $V \in \Omega$ such that $TV = V$. Furthermore, for any $V' \in \Omega$ and defining $V_1 = TV'$ and, more generally, $V_n = T^n V'$, V_n converges to V. This makes possible iterations on the value function as usual. □

The connexion between being solution to Problem 1 and to Problem 2 easily obtains from standard results, at least in the case of bounded utility function (see Stokey et al., 1989). Indeed, in that case, the discrete participation choice does not prevent $\lim_{n->\infty} \sum_{t=0}^{n} \beta^t u(c_t)$ to exist (and be finite), which allows to use Theorems 4.2 to 4.5 in chapter 4, thus guaranteeing the equality between the two solutions. When using unbounded utility functions, this result is more difficult to obtain, but it is not related to the discrete decision.

B Appendix: Multiple states and order of insurance

Having analyzed the market participation with two states, we now study the participation decisions for an arbitrary number of states. Buying insurance contingent to one particular state decreases one agent's wealth and, hence, modifies his willingness to participate to another contingent asset market. As a results, agents face a trade-off when insuring against multiple states. In this section, we first illustrate the interaction between insurance against different states and, second, we show that households choose insurance following a sequential order.

The effects of initial wealth on multiple insurances Given the strict relationship between asset market participation decision and agents' wealth as shown in Corollary 5, we now focus on the agents with an intermediate level of wealth. In particular, we assume that there are gains from participating in each contingent asset market k:

$$u(W - q^f B^N) + \beta \sum_k \pi(y_k|y) V(B^N, 0, y)$$
$$< u(W - q(k,y)a(k) - \kappa(k,y) - q^f B^P) + \beta \sum_m \pi(y_m|y) V \left(B^P, \{0,..,a_k^P,..0\}, y_l \right).$$

However, it is not a foregone conclusion that the agent can afford to access to every asset market, as we may have:

$$u(W - q^f B^N) + \beta \sum_k \pi(y_k|y) V(B^N, 0, y)$$
$$> u(W - \sum_k q(k,y)a(k) - \kappa(k,y) - q^f B^P) + \beta \sum_k \pi(y_k|y) V \left(B^P, \{a^P\}_k, y_k \right).$$

If this condition is satisfied, the household prefers not to buy insurance against every state of the nature. Intuitively, buying insurance against one state decreases the resources available to buy insurance against another state.

Sequential decision When a household is able to participate in contingent asset markets only to a limited degree, she chooses sequentially to buy insurance against different states. Intuitively, we will show that the utility obtained by insurance against one state is proportional to the distance between the threshold cost and the actual associated cost of insurance for that state. Hence, that distance provides a criterion for ranking different assets. The first state against which the agents will insure is the one

where the distance between the actual participation cost and the threshold is maximized. Moreover, by insuring against more and more states, agents decrease their wealth because of participation costs. When their wealth is low enough, agents stop buying further insurance.

In order to rigorously define the sequential decision, we define two concepts: the set of feasible insurance, and a choice of insurance.

Definition 3. The set of feasible insurance $F(y)$ is a subset of Y, such that for every $k \in Y$, gains with respect to the completely non-insurance case are positive:

$$u(W - q^f B^N) + \beta \sum_k \pi(y_k|y)V(B^N, 0, y)$$

$$< u(W - q(k, y)a(k) - \kappa(k, y)) + \beta \sum_m \pi(y_m|y)V\left(B^P, \{0, .., a_k^P, ..0\}, y_m\right).$$

i.e. participation in the asset market contingent to the state k is preferred to autarky.

A choice of insurance at period t is a subset $I(y)$ of $F(y)$.

The recursive problem for household i writes:

$$\max_{I(y) \subset F(y)} \max_{\{a(h')\}} \left[u\left(W - \sum_k \delta_{k \in I(y)}(q(k, y)a(k) + \kappa(k, y))\right) + \beta \sum_l \pi(y_l|y)V\left(B^P, \{a_l^P\}, y_l\right) \right]. \tag{4}$$

The following Proposition characterizes the solution of the sequential insurance problem faced by the agents, and states the analogy between gains from accessing the asset market and distance between participation costs and the threshold costs.

Proposition 12 (Pecking order of access to markets)**.** *The ordering of asset market participations of households follows the gains with respect to non-participation:*

$$u(W - q^f B^N) + \beta \sum_k \pi(y_k|y)V(B^N, 0, y)$$

$$< u(W - q(k, y)a(k) - \kappa(k, y) - q^f B^P) + \beta \sum_l \pi(y_l|y)V\left(B^P, \{0, .., a_k^P, ..0\}, y_l\right).$$

These gains map with the same order as the distance between costs ($\kappa(k, y)$) and thresholds ($\overline{\kappa}(k, y)$): the higher the gains, the greater the difference: $\overline{\kappa}(k, y) - \kappa(k, y)$.

Proof. Program (4) is:

$$\max_{I(y) \subset F(y)} \left\{ \max_{\{a(k)\}, B} \left[u\left(W - \sum_{k \in I(y)} q(k, y)a(k) + \kappa(k, y) - q^f B\right) + \sum_l \pi(y_l|y)\beta V(B, \{a(k)\}, y_l) \right] \right\}$$

Consider now a sequential choice following this iterative algorithm:

- Initial condition: set of possible choices: $S = F(y) \subset Y$, list: $L = \emptyset$

- Iteration:

    - y_k is the state in S which gives the highest gain compared to non-participation.

    - $L = L \cup y_k$ and $S = S - y_k$

This algorithm stops as S is a finite set.

As this algorithm yields a sequence L, we define by $I(y)$ the set of elements of this sequence and now, we have to prove that this set solves optimization (4). Consider a state h^1 in $F(y) - I(y)$ and a state h^2 in $I(y)$. Using lemma 13, we have the result. $\qquad\square$

Lemma 13 (Local property). *$I(y)$ maximizes utility if and only if $I(y) - \{h^1\} \cup \{h^2\}$ gives lower utility for any $h^1 \in I(y)$ and $h^2 \in F(y) - I(y)$.*

Proof. First we show the implication from left to right. This is trivial as $I(y)$ maximizes utility contradicts the proposition that there exists a h^2 in $F(y) - I(y)$ and there exists a h^1 in $I(y)$ such that $I(y) - \{h^1\} \cup \{h^2\}$ gives lower utility.

Second we show the implication from right to left. Suppose that $I(y) - \{h^1\} \cup \{h^2\}$ gives lower utility for any $h^1 in I(y)$ and $h^2 \in F(y) - I(y)$. We proceed by contradiction by supposing then that $I(y)$ does not maximize utility and that there exits I' which maximizes utility. I' cannot be a subset of $I(y)$ and $I(y)$ cannot be a subset of I' neither, considering the stopping condition of the iterative algorithm. There exist then h^2 in I' but not in $I(y)$ and h^1 in $I(y)$ but not in I'. It is easy to check that we can get more utility by taking with $I(y)' - \{h^1\} \cup \{h^2\}$ compared with I', which contradicts the fact that I' maximizes utility. $\qquad\square$

Two specific cases merit consideration. First, when costs are uniform across states, according to Proposition 12, households become insured against the worst possible or best possible state. They begin with the worst and the best and, progressively, they purchase insurance against less extreme future outcomes. Second, when costs are sufficiently increasing along with income shocks, agents may become insured only against small shocks, not against large income variations, since the latter case involves paying larger participation costs. This situation is consistent with recent research about insurance(Cole et al. (2009)). However, modelling increasing fixed costs would require further microfoundations that are beyond the scope of this paper.

C   Appendix: Proofs of propositions

C.1   Proof of Proposition 1.

First, notice that the feasibility condition $W > L$ assures that consumption is always is strictly positive. To prove (1) we only need the conditions that $u'(x) > 0$, $u''(x) < 0$. Since the utility function is increasing, $u'(x) > 0$, then we have the following ordering: $u(W - L) < u(W) < u(W + pL/(1-p))$. Therefore, gain of insurance is positive if:

$$u(W) \geq pu(W - L) + (1 - p)u(W + pL/(1 - p)),$$

which holds as the utility function is concave.

To prove (2), notice that concavity of the utility function implies:

$$[u(W) - u(W - L)] < u'(W - L)L,$$
$$[u(W) - u(W + pL/(1 - p))] < -u'(W + pL/(1 - p))pL/(1 - p),$$

Since $u'(x) > 0$ then, for any $W > 0$, we have:

$$0 < G(W, 0) < u'(W - L)L - u'(W + pL/(1 - p))pL/(1 - p),$$

and by the Inada condition, $\lim_{W \to \infty} u'(W - L) = \lim_{W \to \infty} u'(W) = 0$ and therefore $\lim_{W \to \infty} G(W) = 0$.

To prove (3), notice that from equation (1), the effect of wealth on the gain of insurance is given by:

$$\frac{\partial G(W,0)}{\partial W} = \frac{1}{2}\left(u'(W) - (1-p)u'\left(W + \frac{pL}{1-p}\right) - pu'(W-L)\right). \tag{5}$$

we need to show that the right-hand-side of equation (5) is negative. The proof follows the same argument as for proving (1) by using the property of $u''(x) < 0$ to order the points as follows: $u'(W-L) > u'(W) > u'(W + pL/(1-p))$ and by using the convexity of u', i.e. $u'''(x) > 0$ to prove the inequality.

C.2 Proof of Proposition 2.

The proof of (1) follows three steps. First, we prove that the function $G(W,\kappa)$ has one and only one minimum at a wealth level W^*. Second, we prove that $G(W^*,\kappa) < 0$. Third, we prove that under the condition of the cost, there exists a unique threshold level $\bar{W}$.

Feasibility in each state and time requires that $W > L$ and that $\kappa < 2L$. Suppose that $u'''' < 0$. As $u''' > 0$ and $u'' < 0$, the coefficient of absolute prudence, $P(W) = -\frac{u'''(W)}{u''(W)}$, is decreasing in W, as its derivative has the sign of $-u''''u'' - (u''')^2$. Similarly to Kimball (1990b), we define as the *precautionary equivalent premium* the function $\psi(W)$ such as $u'(W - \psi(W)) = \frac{1}{2}u'(W) + \frac{p}{2}u'(W-L) + \frac{1-p}{2}u'(W + pL/(1-p))$. Given the properties of the utility function, $\psi(W)$ is non-negative, strictly decreasing in W and converges to 0 when W goes to ∞ (see Proposition 62 in Gollier (2004)). Hence, $\forall W \in [L,\infty)$, $\psi(W)$ is invertible and $\psi^{-1} \in (0, \psi(L)]$. Notice that by applying the definition of $\psi(L)$ and using the Inada conditions, we have that: $\psi(L) = L$. Finally, note that $G(W,\kappa)$ converges to 0 when W goes to ∞. As a consequence, for all $\kappa \leq 2L$, there exists a unique level of wealth $W^*(\kappa) = \psi^{-1}(\kappa/2)$ such that $u'(W^*(\kappa) - \kappa/2) = \frac{1}{2}u'(W^*(\kappa)) + \frac{p}{2}u'(W^*(\kappa) - L) + \frac{1-p}{2}u'(W^*(\kappa) + pL/(1-p))$; hence, for all $\kappa \leq 2L$, there exists a unique $W^*(\kappa)$ such that $\frac{\partial G(W^*(\kappa),\kappa)}{\partial W} = 0$. As $\psi(W)$ is decreasing, for $W' > W^*(\kappa)$:

$$u'(W' - \kappa/2) - \frac{1}{2}u'(W') - \frac{p}{2}u'(W'-L) - \frac{1-p}{2}u'(W' + pL/(1-p))$$
$$= u'(W' - \psi(W^*(\kappa))) - \frac{1}{2}u'(W') - \frac{p}{2}u'(W'-L) - \frac{1-p}{2}u'(W' + pL/(1-p))$$
$$\geq u'(W' - \psi(W')) - \frac{1}{2}u'(W') - \frac{p}{2}u'(W'-L) - \frac{1-p}{2}u'(W' + pL/(1-p)) = 0,$$

which implies that for $W' > W^*(\kappa)$, $\frac{\partial G(W',\kappa)}{\partial W} > 0$. The same reasoning for $W' < W^*(\kappa)$ implies that for any $W' < W^*(\kappa)$, $\frac{\partial G(W',\kappa)}{\partial W} < 0$. We have proved that $G(W,\kappa)$ has a unique minimum in $W^*(\kappa)$.

As a second step, notice that since $G(W,\kappa)$ admits exactly one minimum $W^*(\kappa)$, is decreasing for any $W < W^*(\kappa)$, is increasing for any $W > W^*(\kappa)$, and converges to 0 when W goes it ∞, then necessarily $G(W^*(\kappa),\kappa) < 0$. Notice that for any $W' > W^*(\kappa)$, then $G(W',\kappa) < 0$. We have proved that the minimum of $G(W,\kappa)$ is negative.

As a third step, let $\hat{\kappa}$ be the value of the cost that solves: $G(L,\hat{\kappa}) = 0$, i.e.:

$$u(L - \frac{\hat{\kappa}}{2}) = \frac{1}{2}\left[u(L) + (1-p)u(L/(1-p)) + pu(0)\right].$$

Since by Proposition 1 $G(L,0) > 0$, since $G(L,2L) < 0$, and since obviously $G(L,\kappa)$ is decreasing in κ, then by the intermediate value theorem, $\exists! \hat{\kappa}: G(L,\hat{\kappa}) = 0$. Then, for any feasible κ such that $\kappa < \hat{\kappa}$, then $G(L,\kappa) > 0$ and $G(W,\kappa)$ reach a negative value at its minimum; hence, by the intermediate value theorem, exists a unique $\overline{W}(\kappa) < W^*(\kappa)$ such that $G(\overline{W}(\kappa),\kappa) = 0$.

The proof of (2) comes easily. First, by Proposition 1, $\forall W > L$, $G(W,0) > 0$. Hence, $\forall W > L$, $V^P(W,\kappa) > V^N(W)$ and by definition $\mathbb{P}(0) = \{W : W > L\}$. Second, notice that by using the implicit

40

function theorem, $\frac{\partial G(L,\kappa)}{\partial \kappa} < 0$. Hence, $\frac{\partial \overline{W}(\kappa)}{\partial \kappa} < 0$. Therefore, $\forall \kappa_2 > \kappa_1$, $\mathbb{P}(\kappa 2) \subset \mathbb{P}(\kappa_1)$. Finally, as κ increases above $\hat{\kappa}$, $G(L,\kappa) < 0$, and, therefore, $\forall W > L$, $G(W,\kappa) < 0$. In this case $V^P(W,\kappa) < V^N(W)$ and by definition $\mathbb{P}(\kappa) = \emptyset$.

C.3 Proof of Proposition 3.

Manipulating first-order conditions yields:

$$\frac{u'(y_h|y)}{u'(y_l|y)} = \frac{\beta \pi(y_l|y)}{q} \left(\frac{q^f - q}{\beta \pi(y_h|y)} - \frac{\gamma}{u'(y)\beta \pi(y_h|y)} \right).$$

At most, the agents are willing to equalize marginal utilities $u'(y_h|y) = u'(y_l|y)$ and, in addition, the positivity of γ lead to:

$$\frac{\beta \pi(y_l|y)}{q} \frac{q^f - q}{\beta \pi(y_h|y)} \geq 1 \text{ or, equivalently } q^f \pi(y_l|y) \geq q.$$

C.4 Proof of Proposition 4.

The choice to participate amounts to comparing $U^P(W,q,q^f,\kappa)$ and $U^N(W,q^f)$. Using the envelope theorem, the derivatives of $\Delta = U^P(W,q,q^f,\kappa) - U^N(W,q^f))$ are:

$$\frac{\partial \Delta}{\partial \kappa} = -u'(W - qa^P - \kappa - q^f B^P) < 0.$$

In addition, when $\kappa = 0$, participation is preferred to non-participation, as, when participating, the household can do as good as when not participating. As a result, there exists then $\overline{\kappa}$ such that households accept to pay the cost κ if and only if $\kappa \leq \overline{\kappa}$.

C.5 Proof of Corollary 5.

First, note that there exists W sufficiently small so that $B^P = B^N = -\overline{B}$. In this case, we can show that the agent will not participate to the state contingent market, which means that the following relationship is satisfied:

$$u(W - (q(y)a^P + \kappa) + q^f \overline{B}) - u(W + q^f \overline{B})$$

$$\leq \beta \left[\begin{array}{l} \pi(y_l|y) \left[(V(-\overline{B}, a^P, y_l) - V(-\overline{B}, a^P, y_h)) - (V(-\overline{B}, 0, y_l) - V(-\overline{B}, 0, y_h)) \right] \\ \qquad\qquad\qquad\qquad\qquad + (V(-\overline{B}, a^P, y_h) - V(-\overline{B}, 0, y_h)) \end{array} \right]$$

$$\leq \beta \pi(y_l|y) \left[(V(-\overline{B}, a^P, y_l) - V(-\overline{B}, a^P, y_h)) - (V(-\overline{B}, 0, y_l) - V(-\overline{B}, 0, y_h)) \right]. \tag{6}$$

In fact, the Inada condition implies that when W approaches zero, a^P, B^P and B^N tend to zero as well. Hence, as long $\kappa > 0$, a sufficiently decreasing W implies that $u(W - (q(y)a^P + \kappa) + q^f \overline{B}) - u(W + q^f \overline{B})$ goes to $-\infty$, and equation (6) is then verified.

We now prove that also when W large the agent also prefers not to participate. First, notice that $\lim_{W \to \infty} a^P < \infty$, as there is no gain of infinitely accumulating contingent assets, but $\lim_{W \to \infty} B^P = \lim_{W \to \infty} B^P = \infty$. Prudence ($u''' > 0$) and the fact that $qa^P + \kappa \geq 0$ imply that $B^N > B^P$, for all W and $\kappa > 0$. Furthermore, decreasing absolute prudence ($u'''' < 0$) implies that that $B^N - B^P$ decreases

with W and converges to 0. The derivative of the gain of insurance with respect to wealth is:

$$\frac{\partial(U^P - U^N)}{\partial W} = u'(W - q^f B^P - qa^P - \kappa) - u'(W - q^f B^N),$$

$$= u'(W - q^f B^P - qa^P - \kappa) - \frac{\beta}{q^f}\left(\pi(y_h|y)u'(y_h + B^N - terms) + \pi(y_l|y)u'(y_l + B^N - terms)\right),$$

$$= u'(W - q^f B^P - qa^P - \kappa) - \frac{\beta}{q^f}u'(B^N + E(y - terms) - \psi(W)),$$

$$= u'(W - q^f B^P - qa^P - \kappa) - u'(W - q^f(B^N) - qa^P - q^f g(\psi(W))).$$

The sign of the derivative depends on the sign of: $q^f(B^N - B^P) + g(\psi(W)) - \kappa$. When W converges to ∞, both $B^N - B^P$ and $\psi(W)$, the precautionary saving premium, converges to 0; hence, $q^f(B^N - B^P) + g(\psi(W)) - \kappa$ converges to $-\kappa$ and the derivative becomes ultimately positive if and only if $\kappa > 0$. In addition, as in Proposition 2, the gain from insurance converges to 0 when wealth goes to infinity, due to Inada conditions; [17] therefore, the gain from insurance is ultimately negative. In addition, $q^f(B^N - B^P) + g(\psi(W)) - \kappa$ is decreasing in W, so that, there exists at most one change of sign for the derivative. We can then conclude the existence of a threshold $\overline{W}$ along the lines of Proposition 2.

C.6   Proof of Proposition 8.

Corollary 5 define $\underline{W}$ and $\overline{W}$. The lowest level of wealth is $y_l - \overline{B}$ and there exits a highest level of wealth $\hat{W}$. When $\underline{W} \leq y_l - \overline{B}$ and $\hat{W} \leq \overline{W}$, participating is always better. This gives the existence of $\underline{\kappa}$.

Continuity with respect to κ and Corollary 5's results on $\overline{W}$ and $\underline{W}$'s limits imply that there exists some level of κ above which $\overline{W} \leq \underline{W}$, i.e. the household never participate to the market. This gives the existence of $\overline{\kappa}$.

Turning to asset prices, as a first step, notice what may happen to the risk-free interest rate. Whether households are insured or uninsured, the following Euler equation holds:

$$q^f u'(c(B, a, y)) = \beta \sum_{y' \in \{y_H, y_L\}} \pi(y')u'(c(B', a', y')) + \gamma$$

The super-martingale theorem establishes that $\beta > q^f$ cannot be an equilibrium. This restricts price to be $q^f \geq \beta$. Proposition 3 gives the constraint on contingent asset prices and so these constraints on prices are $q^f \geq \beta$ and $q(y) \geq q^f \pi(y_l|y)$.

When participation costs are low, suppose that all households participate to the contingent asset market. As Euler equation for both assets are satisfied for all agents, at all time and in all states, consumption levels do not depend on histories of shocks. As a result, $q^f = \beta$ and $q(y) = \beta\pi(y_l|y)$.

In the next two cases, consumption levels depend on histories as the household may have not participated in some period. In the case of high level of cost, the economy follows Aiyagari (1994) and so we denote by $\overline{q}^f > \beta$ the equilibrium price. In the case of intermediate level of cost, suppose here that $q^f = \beta$, then $q(y) = \beta\pi(y_l|y)$. Agents are then perfectly insured, when participating. As the probability to be insured is strictly positive, then with probability 1, all households will be perfectly insure i the long run, negating the fact that the stationary distribution features uninsured households.

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[17]This is consistent with the fact that in the Aiyagari model agents with infinite wealth will be perfectly insure.

C.7 Proof of proposition 10.

This proof closely follows Davila et al. (2012) to obtain the recursive problem solved but the central planner. At this point, we need to use alternative techniques compared with perturbation methods to deal with the discontinuity introduced by the discrete choice.

The central planner problem can be written as follows:

$$V(x) = \max_{h^f(y,B,a),\delta(y,B,a),h(y,B,a)} \int u \left[\begin{array}{c} B + a1_{y=y_l} + wy - q^f(K)h^f(y,B,a) \\ - \delta(y,B,a)\left(q(K)h(y,B,a) + \kappa\right) \end{array} \right] dx + \beta V(x')$$

$$\text{s.t. } x' = T(x, Q(.,y)), K = \int (a+B)dx$$

The policy rules should solve:

$$\max_{h_t^f(y,B,a),\delta_t(y,B,a),h_t(y,B,a)} \sum_t \beta^{t-1} \sum_y \int u(c_t)x_t(B,a,y)dBda$$

$$c_t + q^f(K(x_t))h^f(y,B,a,x_t) + \delta(y,B,a,x_t)\left(q(K(x_t))h(y,B,a,x_t) + \kappa\right) = B + a1_{y=y_l} + wy$$

given x_1 and where:

$$K(x_t) = \sum_y \sum (B+a)x_t(B,a,y)dBda$$

and

$$x'(B',a',y') = \sum \pi(y'|y)\frac{x\left((h_t^f)^{*,-1}(y,B',a'),(h_t)^{*,-1}(y,B',a'),y\right)}{\frac{d}{da}}$$

This can be rewritten as:

$$\sum_y \int \left[\begin{array}{c} u\left(\begin{array}{c} B + a1_{y=y_l} + yf_L(K(x)) - q^f(K'(x'))h^f(y,B,a,x) \\ - \delta(y,B,a,x)\left(q(K'(x'))h(y,B,a,x_t) + \kappa\right) \end{array} \right) \\ + \beta \sum_{y'} \pi(y'|y)u\left(\begin{array}{c} h^f(y,B,a,x_t) + h(y,B,a,x_t)1_{y'=y_l} + yf_L(K'(x')) \\ - q^f(K''(x''))B'' - \delta''\left(q(K''(x''))a'' + \kappa\right) \end{array} \right) \end{array} \right].x(B,a,y)dBda$$

and

$$K'(x') = \sum_y \int \left[h^*(x,y,B,a) + h^{*,f}(x,y,B,a) \right] x(B,a,y)dBda$$

We can here use a perturbation approach. We consider $h^\epsilon = h^* + \epsilon\chi_{y=y_0}\xi$ and $h^{\zeta,f} = h^{*,f} + \zeta\chi_{y=y_0}\xi^f$.

$$\Psi(\zeta,\epsilon) = \sum_y \int \left[\begin{array}{c} u\left(\begin{array}{c} B + a1_{y=y_l} + yf_L(K(x)) - q^f(K'(x'))h_\zeta^f(y,B,a,x) \\ - \delta(y,B,a,x)\left(q(K'(x'))h_\epsilon(y,B,a,x_t) + \kappa\right) \end{array} \right) \\ + \beta \sum_{y'} \pi(y'|y)u\left(\begin{array}{c} h_\zeta^f(y,B,a,x_t) + h_\epsilon(y,B,a,x_t)1_{y'=y_l} + yf_L(K'(x')) \\ - q^f(K''(x''))B'' - \delta''\left(q(K''(x''))a'' + \kappa\right) \end{array} \right) \end{array} \right].x(B,a,y)dBda$$

The derivates with respect to ϵ and ζ yield:

$$\frac{d}{d\epsilon}\psi(0) = \int \left[\begin{array}{l} -\delta(y_0,B,a,x)q(K'(x'))u'\left(\begin{array}{l} B + a1_{y_0=y_l} + y_0 f_L(K(x)) - q^f(K'(x'))h^{f,*}(y,B,a,x) \\ -\delta(y_0,B,a,x)\left(q(K'(x'))h^*(y_0,B,a,x_t)+\kappa\right) \end{array} \right)\xi \\ +\beta\pi(y_l|y_0)u'\left(\begin{array}{l} h^f(y_0,B,a,x_t) + h(y_0,B,a,x_t) + y_l f_L(K'(x')) \\ -q^f(K''(x''))B'' - \delta''\left(q(K''(x''))a''+\kappa\right) \end{array} \right)\xi \end{array} \right] x(B,a,y_0)dBda$$

$$+\sum_y \int \left[\begin{array}{l} -q_K(K'(x'))u'\left(\begin{array}{l} B + a1_{y=y_l} + y f_L(K(x)) - q^f(K'(x'))h^{f,*}(y,B,a,x) \\ -\delta(y,B,a,x)\left(q(K'(x'))h^*(y,B,a,x_t)+\kappa\right) \end{array} \right) \\ +\beta\sum_{y'}\pi(y'|y)y' f_{KL}(K'(x'))u'\left(\begin{array}{l} h^f(y,B,a,x_t) + h(y,B,a,x_t)1_{y'=y_l} + y f_L(K'(x')) \\ -q^f(K''(x''))B'' - \delta''\left(q(K''(x''))a''+\kappa\right) \end{array} \right) \end{array} \right]$$

$$\times \left(\int \xi^f x(\tilde{B},\tilde{a},y_0)d\tilde{B}d\tilde{a} \right) x(B,a,y)dBda$$

$$\frac{d}{d\zeta}\psi(0) = \int \left[\begin{array}{l} -q^f(K'(x'))u'\left(\begin{array}{l} B + a1_{y=y_l} + y f_L(K(x)) - q^f(K'(x'))h^{f,*}(y_0,B,a,x) \\ -\delta(y_0,B,a,x)\left(q(K'(x'))h^*(y_0,B,a,x_t)+\kappa\right) \end{array} \right)\xi^f \\ +\beta\sum_{y'}\pi(y'|y_0)u'\left(\begin{array}{l} h^f(y_0,B,a,x_t) + h(y_0,B,a,x_t)1_{y'=y_l} + y f_L(K'(x')) \\ -q^f(K''(x''))B'' - \delta''\left(q(K''(x''))a''+\kappa\right) \end{array} \right)\xi^f \end{array} \right] x(B,a,y_0)dBda$$

$$+\sum_y \int \left[\begin{array}{l} -q_K^f u'\left(\begin{array}{l} B + a1_{y=y_l} + y f_L(K(x)) - q^f(K'(x'))h^{f,*}(y,B,a,x) \\ -\delta(y,B,a,x)\left(q(K'(x'))h^*(y,B,a,x_t)+\kappa\right) \end{array} \right) \\ +\beta\sum_{y'}\pi(y'|y)\int u'\left(\begin{array}{l} h^f(y,B,a,x_t) + h(y,B,a,x_t)1_{y'=y_l} + y f_L(K'(x')) \\ -q^f(K''(x''))B'' - \delta''\left(q(K''(x''))a''+\kappa\right) \end{array} \right)y' f_{LK}(K'(x')) \end{array} \right]$$

$$\times \int \xi^f x(\tilde{a},y_0)d\tilde{a}\tilde{x}(a,e)dBda$$

Let us define:

$$\Delta = \sum_{\hat{y}} \int \left[-\left(\begin{array}{l} h^f(\hat{y},B,a,x)q_K^f(K'(x')) \\ +\delta(\hat{y},B,a,x)h(\hat{y},B,a,x)q_K(K'(x')) \end{array} \right)u'\left(\begin{array}{l} B + a1_{\hat{y}=y_l} + \hat{y}f_L(K(x)) - q^f(K'(x'))h^{f,*}(\hat{y},B,a,x) \\ -\delta(\hat{y},B,a,x)\left(q(K'(x'))h^*(\hat{y},B,a,x_t)+\kappa\right) \end{array} \right) \right.$$

$$\left. +\sum_{y'}\beta\pi(y'|\hat{y})u'\left(\begin{array}{l} h^f(\hat{y},B,a,x) + 1_{y'=y_l}h(\hat{y},B,a,x) \\ +y'w(x') - a'' \end{array} \right)y'F_{KL}(K'(x')) \right] x(B,a,\hat{y})dBda. \qquad (7)$$

We obtain that, when $\delta(y,B,a,x)=1$, the first order conditions solved by the central planner are:

$$q^f u'\left(\begin{array}{l} B + a1_{y=y_l} + y f_L(K(x)) - q^f(K'(x')) \\ \times h^{f,*}(y,B,a,x) - \left(q(K'(x'))h^*(y,B,a,x_t)+\kappa\right) \end{array} \right) = \beta\sum_{y'}\pi(y'|y)u'\left(\begin{array}{l} h^f(y,B,a,x) + 1_{y=y_l} \\ \times h(y,B,a,x) + y'w(x') - a'' \end{array} \right) + \Delta$$

$$\text{(FOCP1)}$$

$$qu'\left(\begin{array}{l} B + a1_{y=y_l} + y f_L(K(x)) - q^f(K'(x'))h^{f,*}(y,B,a,x) \\ -\delta(y,B,a,x)\left(q(K'(x'))h^*(y,B,a,x_t)+\kappa\right) \end{array} \right) = \beta\pi(y_l|y)u'\left(\begin{array}{l} h^f(y,B,a,x) + h(y,B,a,x) \\ +y_lw(x') - a'' \end{array} \right) + \Delta$$

$$\text{(FOCP2)}$$

When $\delta(y,B,a,x)=0$, the first order condition solved by the central planner is:

$$q^f u'\left(B + a1_{y=y_l} + y f_L(K(x)) - q^f(K'(x'))h^{f,*}(y,B,a,x)\right) = \beta\sum_{y'}\pi(y'|y)u'\left(h^f(y,B,a,x) + y'w(x') - a''\right) + \Delta$$

$$\text{(FOCN)}$$

In addition, the central planner determines δ by comparing:

$$u\left(B + a1_{y=y_l} + yf_L(K(x)) - q^f(K'(x'))h^{f,P}(y,B,a,x) - \left(q(K'(x'))h^P(y,B,a,x_t) + \kappa\right)\right)$$
$$- u\left(B + a1_{y=y_l} + yf_L(K(x)) - q^f(K'(x'))h^{f,N}(y,B,a,x)\right)$$
$$+ \beta \sum_{y'} \pi(y'|y) \left[\begin{array}{c} u\left(\begin{array}{c} h^{f,P}(y,B,a,x_t) + h^P(y,B,a,x_t)1_{y'=y_l} + yf_L(K'(x')) \\ - q^f(K''(x''))B'' - \delta''\left(q(K''(x''))a'' + \kappa\right) \end{array}\right) \\ - u\left(\begin{array}{c} h^{f,N}(y,B,a,x_t) + yf_L(K'(x')) \\ - q^f(K''(x''))B'' - \delta''\left(q(K''(x''))a'' + \kappa\right) \end{array}\right) \end{array}\right] + \Gamma$$

with

$$\Gamma = (q^f(h^{f,P}(y,B,a,x) - h^{f,N}(y,B,a,x)) + q(K'(x'))h^P(y,B,a,x_t)) \times$$
$$\sum_{\hat{y}} \int \left[-\left(\begin{array}{c} h^f(\hat{y},B,a,x)q_K^f(K'(x')) + ... \\ ... + \delta(\hat{y},B,a,x)h(\hat{y},B,a,x)q_K(K'(x')) \end{array}\right) u'\left(\begin{array}{c} B + a1_{\hat{y}=y_l} + \hat{y}f_L(K(x)) - q^f(K'(x'))h^{f,*}(\hat{y},B,a,x) \\ - \delta(\hat{y},B,a,x)\left(q(K'(x'))h^*(\hat{y},B,a,x_t) + \kappa\right) \end{array}\right) \right.$$
$$\left. + \sum_{y'} \beta\pi(y'|\hat{y})u'\left(\begin{array}{c} h^f(\hat{y},B,a,x) + 1_{y'=y_l}h(\hat{y},B,a,x) \\ + y'w(x') - a'' \end{array}\right) y'F_{KL}(K'(x')) \right] x(B,a,\hat{y})dBda \qquad (8)$$
$$= (q^f(h^{f,P}(y,B,a,x) - h^{f,N}(y,B,a,x)) + q(K'(x'))h^P(y,B,a,x_t))\Delta.$$

We can now characterize the constrained efficient allocation. To begin with, we can notice that, when markets are complete ($\delta = 1$ everywhere), conditions (FOCP1) and (FOCP2) are always satisfied and, in addition, $\Gamma = 0$. This helps us to prove the following lemma.

Lemma 14. *When $\kappa \leq \underline{\kappa}$, conditions (FOCP1) and (FOCP2) are satisfied. Thus, the competitive market allocation is constrained efficient.*

When $\kappa \geq \underline{\kappa}$, conditions (FOCP1) and (FOCP2) are generally not satisfied. Thus, the competitive market allocation is generally not constrained efficient.

Indeed, no "pecuniary" externality can appear.

The next lemma investigates participation in the constrained efficient allocation compared with the competitive one.

Lemma 15. *There exists $\hat{\kappa} > \underline{\kappa}$ such that the constrained efficient allocation features $\delta = 1$ everywhere.*

Proof. Consider ϵ arbitrarily small and consider the participation cost $\overline{\kappa} + \epsilon$. The competitive equilibrium features incomplete insurance: there exist a, B and y such that $\delta(a,B,y,x) = 0$. The magnitude of the cost of restoring full participation is at the order of ϵ while the cost to manipulate portfolio allocation is of order 1. As a result, when ϵ is sufficiently small, the central planner is better off restoring full participation for $\kappa < \overline{\kappa} + \epsilon$. $\qquad \square$

Computing Γ. First note that Γ (defined by (8)) has the opposite sign of Δ (defined by (7)). Indeed, more insurance leads to less capital. As a result, if there is insufficient capital ($\Delta > 0$), there is too much insurance and *vice versa*. We compute Δ for our different calibrations of the participation cost. We find a negative value for the high value of κ (as in Davila et al., 2012), implying a positive Γ, that is insufficient insurance. Conversely, we find a positive value for Δ for the intermediate value of κ. As a consequence, Γ is negative and there is too much insurance.