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Weight and mechanical performance optimization of blended composite wing panels using lamination parameters

Dianzi Liu[1] . Vassili V. Toropov[2] . David C. Barton[3] . Osvaldo M. Querin[3]

* Vassili V. Toropov

v.v.toropov@qmul.ac.uk

Dianzi Liu

dianzi.liu@uea.ac.uk

David C. Barton d.c.barton@leeds.ac.uk

Osvaldo M. Querin

ozz@mech-eng.leeds.ac.uk

1 School of Mathematics, University of East Anglia, Norwich NR4 7TJ, UK

2 School of Engineering and Materials Science, Queen Mary University of London, London E1 4NS, UK

3 School of Mechanical Engineering, University of Leeds, Leeds LS2 9JT, UK

Abstract

In this paper, a lamination parameter-based approach to weight optimization of composite aircraft wing structures is addressed. It is a bi-level procedure where at the top level lamination parameters and numbers of plies of the pre-defined angles (0, 90, 45 and −45°) are used as design variables, the material volume is treated as an objective function to be minimized subject to the buckling, strength and ply percentage constraints. At the bottom level the optimum stacking sequence is obtained subject to the requirements on blending and preservation of mechanical properties. To ensure composite blending, a multi-stage optimization is performed by a permutation genetic algorithm aiming at matching the lamination parameters passed from the top level optimization as well as satisfying the layup rules. Two new additional criteria, the 90° ply angle jump index and the stack homogeneity index, are introduced to control the uniformity of the three ply angles (0, 90, 45 and −45) spread throughout the stack as well as improve the stack quality and mechanical performance by encouraging 45° angle change between neighbouring groups of plies. The results of the application of this approach are

compared to published results to demonstrate the potential of the developed technique.

Keywords:

Laminated composite, optimization, stacking sequence, blending, lamination parameters

1 Introduction

Stacking sequence optimization of laminated composite structures to satisfy ply continuity (blending) requirements has recently attracted considerable attention by Gürdal et al. (1999), Kristinsdottir et al. (2001), Liu and Haftka (2001), Seresta et al. (2007), Liu and Krog (2008), Liu et al. (2011). Liu et al. (2000) presented a bi-level (global and bottom) strategy for optimization of a composite wing box structure. At the global level, continuous optimization of thicknesses of 0, 90, 45 and 45° plies was performed to minimize the weight of a wing box subject to strain and buckling constraints. For a given number of plies of each orientation and in-plane loads, a permutation genetic algorithm (GA) was used at the bottom level to optimize the stacking sequence in order to maximize the buckling load. The optimum buckling load, which was treated as a function of the loading and the numbers of plies of 0, 90, 45 and 45° orientation, was evaluated by a cubic polynomial response surface approximation. This bi-level approach was also used for maximization of buckling load of composite panels by Liu et al. (2004), layup optimization of anisotropic laminated composite panels by Bloomfield et al. (2009) and stacking sequence optimization of blended composite structures by Liu et al. (2011). The use of lamination parameters to represent the in-plane and flexural stiffness in the optimization of laminated composites was first used by Tsai et al. (1968) and later applied to the buckling optimization of orthotropic laminated plates by Fukunaga and Hirano (1982). Miki (1982), Fukunaga and Chou (1988), and Fukunaga and Sekine (1993) also used lamination parameters for tailoring mechanical properties of laminated composites. In a composite optimization problem, lamination parameters can be used as design variables instead of the layer thicknesses and ply angles since each element of the stiffness matrix of laminated composites can be expressed as a linear function of lamination parameters. This is beneficial for the design optimization of composite laminates as it reduces the number of design variables. Diaconu et al. (2002) used a variational approach to determine feasible regions in the space of lamination parameters as constraints in the optimization problem. Matsuzaki and Todoroki (2007) used the fractal branch-and-bound method for the stacking sequence optimization of non-symmetric composite laminates where the inplane, out-of-plane and coupling lamination parameters were treated as design variables. This method was successfully applied for maximization of buckling load of cylindrical laminated shells.

Herencia et al. (2007) applied a gradient-based technique and a GA to optimize anisotropic laminated composite panels with T-stiffeners. In the first step, gradient-based weight optimization was performed where the skin and a stiffener were parameterized using lamination parameters, subject to the constraints on buckling, strain as well as practical design rules. A composite layup of a panel was determined using a GA in the second level by meeting the target values of lamination parameters coming from the top level. Herencia et al. (2008) used the same approach for optimization of laminated composite panels with T-stiffeners, but with a different objective function at the second level. Instead of minimizing the squared distance between the target lamination parameters from the first step and the actual lamination parameters, the maximum value of the linearised design constraints was taken as the objective function. The authors' conclusion was that in the determination of the stacking sequence the minimum squared distance might not be the best objective.

Ply compatibility (also referred to as blending) between adjacent panels is a very important consideration in the design of composite structures, it has been considered by Liu and Haftka (2001), Soremekun et al. (2002), Seresta et al. (2007), and Ijsselmuiden et al. (2009). Liu and Haftka (2001) defined the composition continuity and the stacking sequence continuity measures that were used in an optimization process, also by Toropov et al. (2005). Soremekun et al. (2002) used multi-step optimization to determine the blended stacking sequence of the laminates. Based on the individual optimized panels, sub-laminates for the blended panel design are redefined by optimization for each panel, which is called design variable zone (DVZ). Seresta et al. (2007) developed two blending

methods, inward and outward blending, to improve the ply continuity between adjacent panels using a guide based GA. Ijsselmuiden et al. (2009) developed a multistep framework for blended design of composite structures with a guide-based GA. In the first step, flexural lamination parameters and thickness of each panel are treated as design variables and weight optimization is performed subject to buckling constraints. In the second step, a blended composite layup is obtained using a guide based genetic algorithm where the objective function is evaluated using convex approximations of the buckling response. Liu and Krog

(2008) addressed a stacking sequence arrangement problem for a composite wing by transforming it into a problem of shuffling a set of global ply layout cards. A permutation GA is applied to find an optimal card sequence which uses the ply angle percentages and the chordwise and spanwise laminate thickness distributions as input data. The authors' conclusion was that it allowed to considerably reduce the design space and hence the solution time. Recently, a bi-level composite optimization procedure was used by Liu et al. (2011) to seek the best stacking sequence of laminated composite wing structures with blending and manufacturing constraints. Two approaches are introduced: a smeared stiffness-based method that aims at neutralizing the stacking sequence effect on the buckling performance, and a lamination parameter-based method that uses lamination parameters as design variables to formulate the membrane stiffness matrix A and bending stiffness matrix D. The advantage of the smeared stiffness-based method is that the top level optimization problem does not use flexural lamination parameters as design variables making this problem more compact. Only the numbers of plies of each pre-defined orientation (0, 90, 45 and 45°) are considered as design variables thus making it possible to solve this problem by commercially available FE software, e.g., Altair Engineering OptiStruct (2011). The advantage of the lamination parameter-based approach is that it allows to arrive at lighter structures as the requirement of having a homogeneous ply stack does not have to be enforced. It can be reminded that there is no need to check satisfaction of the strain or buckling constraints after the stacking sequence arrangement as long as the lamination parameters for the obtained stack match the lamination parameter values coming from the top level optimization. This is because the A and D are part of stiffness matrices of composite laminates and they are derived from the Classical Laminate Theory (CLT) (Jones 1999), which ignores transverse shear and normal stresses in the analysis of multilayered structures (Carrera 2001 and Carrera 2003). Since the A and D matrices are entirely determined by the in-plane and out-of-plane lamination parameters, if these lamination parameters are not changed during the optimization process, the elements in the A and D matrices remain the same. In this paper, lamination parameter-based method is used for the optimization of stacking sequence of laminated composite structures. At the top level optimization, the total number of plies and the lamination parameters related to the bending stiffness matrix are treated as the design variables. Buckling and strain constraints are applied at this level and the total material volume is the objective function. Next, the bottom level optimization is treated as a multi-objective problem with the following three criteria: a measure of the lamination parameters match, the stack homogeneity index and the 90° ply angle jump index as explained in Section 6. Then, a permutation GA is used to shuffle the plies to minimize a single objective function that combines the three criteria. This is embedded into a blending procedure to achieve the global ply continuity.

2 Composite Design Rules

According to aircraft industry layup rules (Niu, 2010 and Niu, 2011, Toropov et al. 2005; Kassapoglou 2010; Liu et al. 2011), the laminate layup design rules applied to each panel are as follows:

1) The stack is balanced, i.e., the number of 45 and –45° plies is the same in each of the components.
2) Due to the damage tolerance requirements, the outer plies for the skin should always contain at least one set of ±45° plies.
3) The number of plies (N_{max}) in any one direction placed sequentially in the stack is limited to four.
4) A 90° change of angle between two adjacent plies is to be avoided, if possible.
5) An additional frequently (but not always) used requirement is that all three ply orientations (0,90 and ±45°) should be spread uniformly through the stack.

3 Lamination Parameter-Based Method

Industrial requirements and practical manufacturing considerations lead to the assumption that only symmetric and balanced laminates with ply orientations 0, 90, 45, –45° need to be investigated. Therefore, only half the number of plies of each orientation is given in all numerical results presented in this paper. Also, as the number of 45° plies, n_{45}, is always equal to the number of –45° plies, n_{-45}, for balanced laminates, the number of pairs of ±45° plies is presented here as n_{45}. At the bottom level, maximization of ply compatibility will be achieved by the optimization of the ply stacking sequence whereas the laminate thickness remains constant as it is fixed after the top level optimization.

Lamination parameters were first introduced by Tsai et al. (1968). It is known that for a general case of orthotropic laminates the stiffness matrices **A** and **D** are governed by twelve lamination parameters and five material parameters. For orthotropic symmetric and balanced laminates, the number of independent lamination parameters can be reduced to eight. The elements of the membrane stiffness matrix **A** and the bending stiffness matrix **D** can be expressed as:

$$\begin{bmatrix} A_{11} \\ A_{22} \\ A_{12} \\ A_{66} \\ A_{16} \\ A_{26} \end{bmatrix} = h \begin{bmatrix} 1 & \xi_1^A & \xi_3^A & 0 & 0 \\ 1 & -\xi_1^A & \xi_3^A & 0 & 0 \\ 0 & 0 & -\xi_3^A & 1 & 0 \\ 0 & 0 & -\xi_3^A & 0 & 1 \\ 0 & \xi_2^A/2 & \xi_4^A & 0 & 0 \\ 0 & \xi_2^A/2 & -\xi_4^A & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix}, \begin{bmatrix} D_{11} \\ D_{22} \\ D_{12} \\ D_{66} \\ D_{16} \\ D_{26} \end{bmatrix} = \frac{h^3}{12} \begin{bmatrix} 1 & \xi_1^D & \xi_3^D & 0 & 0 \\ 1 & -\xi_1^D & \xi_3^D & 0 & 0 \\ 0 & 0 & -\xi_3^D & 1 & 0 \\ 0 & 0 & -\xi_3^D & 0 & 1 \\ 0 & \xi_2^D/2 & \xi_4^D & 0 & 0 \\ 0 & \xi_2^D/2 & -\xi_4^D & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} \quad (1)$$

where the lamination parameters are:

$$\xi_{[1,2,3,4]}^A = \frac{1}{h} \int_{-h_i/2}^{h_i/2} [\cos 2\theta, \sin 2\theta, \cos 4\theta, \sin 4\theta] dz \text{ and } \xi_{[1,2,3,4]}^D = \frac{12}{h^3} \int_{-h_i/2}^{h_i/2} [\cos 2\theta, \sin 2\theta, \cos 4\theta, \sin 4\theta] z^2 dz$$

This suggests that the use of lamination parameters as design variables in the composite optimization can be very beneficial. It is known (see Gürdal et al. 1999 and Diaconu et al. 2002) that there exist the following relationships between the out-of-plane lamination parameters:

$$2\left(1+\xi_3^D\right)\left(\xi_2^D\right)^2 - 4\xi_1^D\xi_2^D\xi_4^D + \left(\xi_4^D\right)^2 \le \left(\xi_3^D - 2\left(\xi_1^D\right)^2 + 1\right)\left(1-\xi_3^D\right), \\ \left(\xi_1^D\right)^2 + \left(\xi_2^D\right)^2 \le 1, 2\left(\xi_1^D\right)^2 - 1 \le \xi_3^D \le 1. \quad (2)$$

Furthermore, a group of relationships between the in-plane and out-of-plane lamination parameters for the symmetric laminates are available, see Gürdal et al. (1999), Diaconu et al. (2002), Matsuzaki and Todoroki (2007), Herencia et al. 2007, 2008), Ijsselmuiden et al. (2009), Bloomfield et al. 2009), Liu et al. (2011):

$$
\begin{aligned}
&\left(\xi_i^A-1\right)^4-4\left(\xi_i^D-1\right)\left(\xi_i^A-1\right)\leq 0 \quad i=1,2,3\\
&\left(\xi_i^A+1\right)^4-4\left(\xi_i^D+1\right)\left(\xi_i^A+1\right)\leq 0 \quad i=1,2,3\\
&\left(2\xi_1^A-\xi_3^A-1\right)^4-16\left(2\xi_1^D-\xi_3^D-1\right)\left(2\xi_1^A-\xi_3^A-1\right)\leq 0\\
&\left(2\xi_1^A+\xi_3^A+1\right)^4-16\left(2\xi_1^D+\xi_3^D+1\right)\left(2\xi_1^A+\xi_3^A+1\right)\leq 0\\
&\left(2\xi_1^A-\xi_3^A+3\right)^4-16\left(2\xi_1^D-\xi_3^D+3\right)\left(2\xi_1^A-\xi_3^A+3\right)\leq 0\\
&\left(2\xi_1^A+\xi_3^A-3\right)^4-16\left(2\xi_1^D+\xi_3^D-3\right)\left(2\xi_1^A+\xi_3^A-3\right)\leq 0\\
&\left(2\xi_2^A-\xi_3^A+1\right)^4-16\left(2\xi_2^D-\xi_3^D+1\right)\left(2\xi_2^A-\xi_3^A+1\right)\leq 0\\
&\left(2\xi_2^A+\xi_3^A-1\right)^4-16\left(2\xi_2^D+\xi_3^D-1\right)\left(2\xi_2^A+\xi_3^A-1\right)\leq 0\\
&\left(2\xi_2^A-\xi_3^A-3\right)^4-16\left(2\xi_2^D-\xi_3^D-3\right)\left(2\xi_2^A-\xi_3^A-3\right)\leq 0\\
&\left(2\xi_2^A+\xi_3^A+3\right)^4-16\left(2\xi_2^D+\xi_3^D+3\right)\left(2\xi_2^A+\xi_3^A+3\right)\leq 0\\
&\left(\xi_1^A-\xi_2^A-1\right)^4-4\left(\xi_1^D-\xi_2^D-1\right)\left(\xi_1^A-\xi_2^A-1\right)\leq 0\\
&\left(\xi_1^A+\xi_2^A+1\right)^4-4\left(\xi_1^D+\xi_2^D+1\right)\left(\xi_1^A+\xi_2^A+1\right)\leq 0\\
&\left(\xi_1^A-\xi_2^A+1\right)^4-4\left(\xi_1^D-\xi_2^D+1\right)\left(\xi_1^A-\xi_2^A+1\right)\leq 0\\
&\left(\xi_1^A+\xi_2^A-1\right)^4-4\left(\xi_1^D+\xi_2^D-1\right)\left(\xi_1^A+\xi_2^A-1\right)\leq 0\ .
\end{aligned}
\tag{3}
$$

These expressions can be formulated as additional constraints in the top level optimization problem, see Section 3. For the majority of aeronautical structures symmetric and balanced laminates with ply orientations of 0, 90, 45 and –45° are used. Thus, $\xi_4^D = 0$, hence the first relationship in (2) can be rewritten as:

$$
\left(\xi_2^D\right)^2 \leq \frac{\left(\xi_3^D-2\left(\xi_1^D\right)^2+1\right)\left(1-\xi_3^D\right)}{2\left(1+\xi_3^D\right)}. \tag{4}
$$

Following Liu et al. (2011), the definition of the out-of plane lamination parameters can be re-formulated as

$$
V_{[1,2,3],i} = 1+\frac{2}{3}\xi^D_{[1,2,3]} = 1+\left(\frac{2}{h_i}\right)^3\int_{-h_i/2}^{h_i/2}[\cos 2\theta, \sin 2\theta, \cos 4\theta]z^2dz \tag{5}
$$

to make them strictly positive, and the in-pane lamination parameters can be expressed using the numbers of plies of each orientation as

$$\xi_{1,i}^{A} = \left(\frac{1}{h_i}\right)\int_{-h_i/2}^{h_i/2} \cos 2\theta dz = \frac{2\left(n_0^i - n_{90}^i\right)}{h_i} = \frac{n_0^i - n_{90}^i}{n_0^i + n_{90}^i + 2n_{45}^i}$$

$$\xi_{2,i}^{A} = \left(\frac{1}{h_i}\right)\int_{-h_i/2}^{h_i/2} \sin 2\theta dz = 0$$

$$\xi_{3,i}^{A} = \left(\frac{1}{h_i}\right)\int_{-h_i/2}^{h_i/2} \cos 4\theta dz = \frac{2\left(n_0^i + n_{90}^i - 2n_{45}^i\right)}{h_i} = \frac{n_0^i + n_{90}^i - 2n_{45}^i}{n_0^i + n_{90}^i + 2n_{45}^i} \quad (6)$$

$$\xi_{4,i}^{A} = \left(\frac{1}{h_i}\right)\int_{-h_i/2}^{h_i/2} \sin 4\theta dz = 0$$

where A and ***D*** indicate membrane and bending effects, *i* is the panel number, n_0^i is half the number of 0° plies in the total stack of the i^{th} panel, n_{45}^i is half the number of pairs of ±45° degree plies in the total stack of the i^{th} panel, n_{90}^i is half the number of 90° plies in the total stack of the i^{th} panel, $h_i = t\left(n_0^i + n_{90}^i + 2n_{45}^i\right)$ is the total thickness of the panel *i* (assuming that the ply thickness is *t*), and θ is the ply angle.

As follows from (6), it is possible to use the ply numbers n_0^i, n_{45}^i and n_{90}^i (and also the ply thickness t that is assumed to be constant) instead of the in-plane lamination parameters $\xi_{1,i}^A$, $\xi_{3,i}^A$ and the laminate thickness h_i, the former is followed in this paper.

Since a limited set of ply orientations is used in aeronautical structures (0, 90, 45 and –45° only), it is suggested to narrow down the feasible design domain in the space of out-of- plane lamination parameters in weight optimization of composite structures by introducing additional constraints in the form of relationships between out-of-plane lamination parameters and the numbers of plies n_0^i, n_{45}^i and n_{90}^i.

3.1 Constraints on the out-of-plane lamination parameter $V_{1,i}$

For the symmetric and balanced laminates with ply orientations of 0, 90, 45 and −45° the values cos2θ=1 for θ=0∘, cos2θ=−1 for θ=90∘, and cos2θ=0 for θ=±45∘ can be immediately evaluated. Thus, the minimum and maximum possible values of $V_{1,i}$ can be determined:

$$\max V_{1,i} = 1 + \left(\frac{2}{h_i}\right)^3 \int_{-h_i/2}^{h_i/2} \cos 2\theta z^2 dz = 1 + \left(\frac{1}{n_0^i + n_{90}^i + 2n_{45}^i}\right)^3$$
$$\left[\frac{2}{3}\left(\left(\left(n_0^i + n_{90}^i + 2n_{45}^i\right)^3 - \left(n_{90}^i + 2n_{45}^i\right)^3\right) - \left(\left(n_{90}^i\right)^3 - 0\right)\right]$$
$$= 1 + \frac{2}{3}\left(1 - \frac{\left(n_{90}^i + 2n_{45}^i\right)^3 + \left(n_{90}^i\right)^3}{\left(n_0^i + n_{90}^i + 2n_{45}^i\right)^3}\right), \quad (7)$$
$$\min V_{1,i} = 1 + \left(\frac{2}{h_i}\right)^3 \int_{-h_i/2}^{h_i/2} \cos 2\theta z^2 dz = 1 + \left(\frac{1}{n_0^i + n_{90}^i + 2n_{45}^i}\right)^3$$
$$\left[\frac{2}{3}\left(-\left(\left(n_0^i + n_{90}^i + 2n_{45}^i\right)^3 - \left(n_0^i + 2n_{45}^i\right)^3\right)\right) + \left(\left(n_0^i\right)^3 - 0\right)\right]$$
$$= 1 + \frac{2}{3}\left(\frac{\left(n_0^i + 2n_{45}^i\right)^3 + \left(n_0^i\right)^3}{\left(n_0^i + n_{90}^i + 2n_{45}^i\right)^3} - 1\right)$$

as demonstrated in Fig. 1.

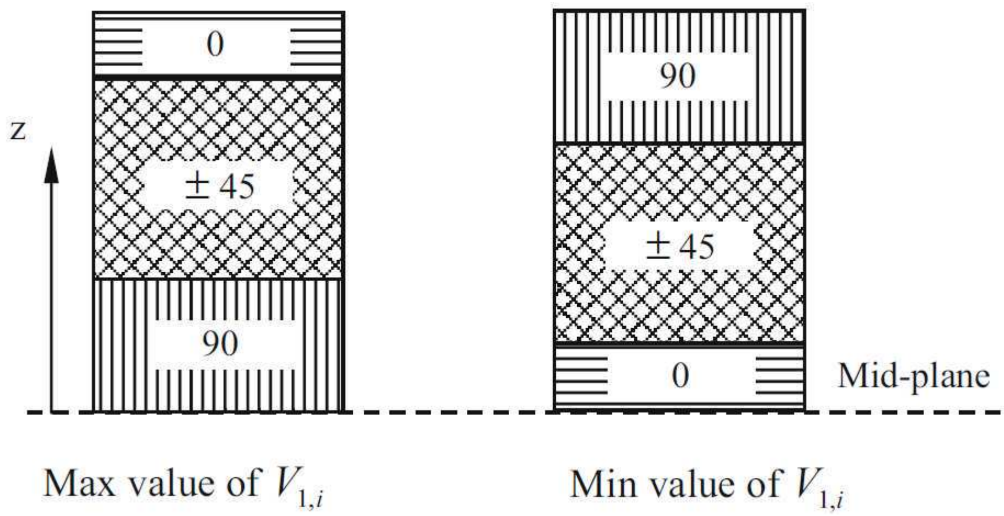

Fig. 1 Layup configurations for the maximum and minimum values of $V_{1,i}$

3.2 Constraints on the out-of-plane lamination parameter V2,i

In the bending stiffness matrix **D**, the stiffness terms D_{16} and D_{26} couple the moment resultants M_x and M_y with twisting curvature. These terms exist for all the laminates that have layers with off-axis orientations. As only balanced laminates are considered here, the positive and negative parts of the pair of ±45 ° plies can be separated through the thickness location by inserting the plies with 0 or 90° fibre orientations. This distance between them influences the magnitude of the D_{16} and D_{26} terms. For simplicity, D_{16} and D_{26} are usually neglected by researchers in the out-of-plane stiffness matrix, see, e.g., Gürdal et al. (1999). In aeronautical practice, however, plies of 0 or 90° fibre orientation are normally inserted into a pair ±45° plies, hence in this paper the bending-twisting terms D16 and D26 are considered in the problem formulation. Since sin2θ=0 for θ=0∘, sin2θ=0 for θ=90∘, sin2θ=1 for θ=45∘, and sin2θ=−1 for θ=−45∘, the minimum and maximum values of $V_{2,i}$ can be determined

$$
\begin{aligned}
\max V_{2,i} &= 1 + \left(\frac{2}{h_i}\right)^3 \int_{-h_i/2}^{h_i/2} \sin 2\theta z^2 dz \\
&= 1 + \left(\frac{1}{n_0^i + n_{90}^i + 2n_{45}^i}\right)^3 \left(\frac{2}{3}\right) \left[\left(\left(n_0^i + n_{90}^i + 2n_{45}^i\right)^3 - \left(n_{90}^i + n_{45}^i + n_0^i\right)^3\right)\right. \\
&\left. -\left(\left(n_{90}^i + n_{45}^i + n_0^i\right)^3 - \left(n_{90}^i + n_0^i\right)^3\right)\right] \\
&= 1 + \frac{2}{3}\left(1 + \frac{\left(n_{90}^i + n_0^i\right)^3 - 2\left(n_{90}^i + n_{45}^i + n_0^i\right)^3}{\left(n_0^i + n_{90}^i + 2n_{45}^i\right)^3}\right), \\
\min V_{2,i} &= 1 + \left(\frac{2}{h_i}\right)^3 \int_{-h_i/2}^{h_i/2} \sin 2\theta z^2 dz \\
&= 1 + \left(\frac{1}{n_0^i + n_{90}^i + 2n_{45}^i}\right)^3 \left(\frac{2}{3}\right) \frac{n_{45}^i \times \left[6\left(2n_{45}^i - 1\right) + 6\left(2n_{45}^i - 2\left(n_{45}^i - 1\right) - 1\right)\right]}{2} \\
&= 1 + \frac{4\left(n_{45}^i\right)^2}{\left(n_0^i + n_{90}^i + 2n_{45}^i\right)^3}
\end{aligned}
\tag{8}
$$

as demonstrated in Fig. 2.

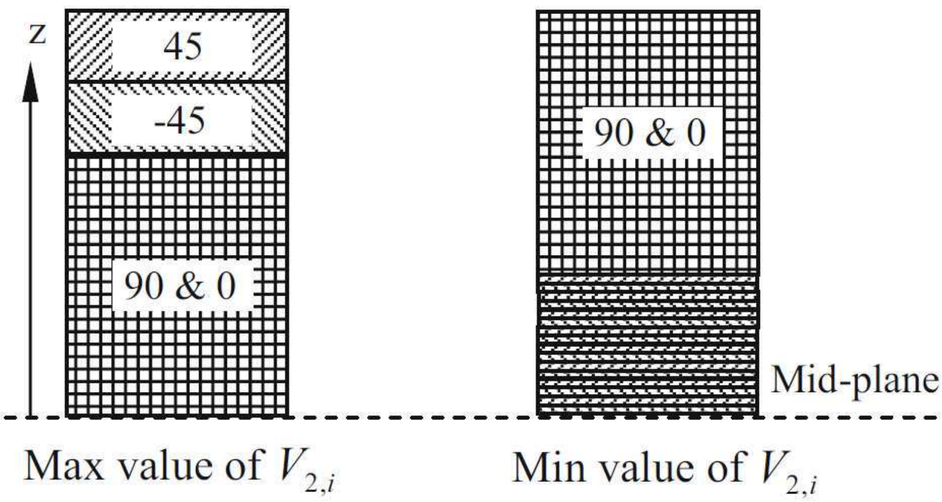

Fig. 2 Layup configurations for the maximum and minimum values of $V_{2,i}$

3.3 Constraints on the out-of-plane lamination parameter $V_{3,i}$

Also, the values cos4θ=1 for θ=0∘, cos4θ=1 for θ=90∘ and cos4θ=−1 for θ=±45∘ can be immediately evaluated, the minimum and maximum values of $V_{3,i}$ can be determined:

$$\max V_{3,i} = 1 + \left(\frac{2}{h_i}\right)^3 \int_{-h_i/2}^{h_i/2} \cos 4\theta z^2 dz = 1 + \left(\frac{1}{n_0^i + n_{90}^i + 2n_{45}^i}\right)^3$$
$$\left(\frac{2}{3}\right)\left[\left(n_0^i + n_{90}^i + 2n_{45}^i\right)^3 - \left(2n_{45}^i\right)^3 - \left(\left(2n_{45}^i\right)^3 - 0\right)\right]$$
$$= 1 + \frac{2}{3}\left(1 - \frac{2 \times \left(2n_{45}^i\right)^3}{\left(n_0^i + n_{90}^i + 2n_{45}^i\right)^3}\right),$$
$$\min V_{3,i} = 1 + \left(\frac{2}{h_i}\right)^3 \int_{-h_i/2}^{h_i/2} \cos 4\theta z^2 dz = 1 + \left(\frac{1}{n_0^i + n_{90}^i + 2n_{45}^i}\right)^3 \left(\frac{2}{3}\right)$$
$$\left[-\left(\left(n_0^i + n_{90}^i + 2n_{45}^i\right)^3 - \left(n_0^i + n_{90}^i\right)^3\right) + \left(\left(n_0^i + n_{90}^i\right)^3 - 0\right)\right]$$
$$= 1 + \frac{2}{3}\left(\frac{2 \times \left(n_0^i + n_{90}^i\right)^3}{\left(n_0^i + n_{90}^i + 2n_{45}^i\right)^3} - 1\right) \tag{9}$$

as demonstrated in Fig. 3.

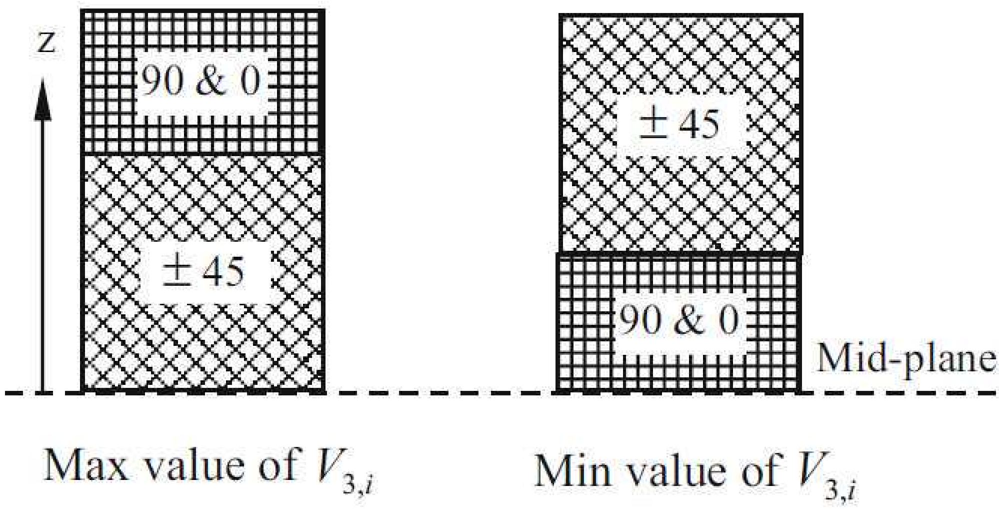

Fig. 3 Layup configurations for the maximum and minimum values of $V_{3,i}$

The expressions (7) – (9) will be used as additional constraints in the top level optimization problem presented in Section 3.

4 Bi-level Optimization Strategy

Typically, an aircraft wing structure has a large number of panels hence its optimal design would require an unrealistically large number of design variables to describe all the required composite properties, such as ply orientation and stacking sequence. A bi-level optimization strategy has been shown by Yamazaki (1996), Liu et al. (2000), Ijsselmuiden et al. (2009), Liu et al. (2011) to provide a suitable means for solving such problems efficiently without requiring an excessive amount of the computing time. A practical approach to laminated composite design has been suggested by Zhou et al. (2009, 2010) that is a three-phase optimization process guiding the composite laminate designs from a concept to the final ply-book details. This approach has been implemented in Altair's OptiStruct (2011) and is used widely in various industries including major airframe manufacturers. The first stage of this approach is equivalent to the top level of the lamination parameter-based optimization method of Liu et al. (2011) when the out-of-plane lamination parameters V_1, V_2 and V_3 are set to one.

4.1 Top level optimization

Following the bi-level composite optimization strategy of Liu et al. (2011), the top level optimization problem formulation is as follows:

$$\text{Minimize} \sum_{i=1}^{n} (n_0^i + n_{45}^i + n_{90}^i) A_i t \text{ subject to constraints : strain constraints : } \max \varepsilon_1^i \leq \varepsilon_{1a}, \max \varepsilon_2^i \leq \varepsilon_{2a}, \ \max \gamma_{12}^i \leq \gamma_{12a}, i = 1, \cdots n,$$
$$\text{buckling load constraint : } \lambda_b \geq 1.0, \text{ percentage of plies of each orientation} \geq 10\%, \text{ feasibility of lamination parameters defined by} (3\text{–}4)$$
$$\text{and} (7\text{–}9) \text{ where the design variables are the numbers of plies of each orientation } n_0^i, n_{45}^i, n_{90}^i \text{ and the out-of-plane}$$
$$\text{lamination parameters } V_j^i, \ i = 1, \ldots, n, \ j = 1, 2, 3 \tag{10}$$

In the formulae (10) t is the ply thickness, n is the total number of panels, and A_i is the area of panel i; ε_{1a} is allowable strain in the fibre direction, ε_{2a} is allowable strain in transverse direction, γ_{12a} is allowable shear strain, and

$\max \varepsilon_1^i$, $\max \varepsilon_2^i$ and $\max \gamma_{12}^i$ are maximum values of these strains within the panel i; λ_b is the lowest buckling load factor obtained as a solution of an eigenvalue problem.

The ANSYS (2007) FEA software was used for calculating strains and the buckling load factor λ_b. ANSYS Parametric Design Language (APDL) was used to define the FE stiffness matrix according to (1), (5) and (6).

The first order optimization method available in ANSYS has been chosen to solve the top level optimization problem in continuous formulation followed by the rounding-off strategy presented in Section 7. This method is based on the NLPQL implementation of the Sequential Quadratic Programming (SQP) algorithm (Schittkowski 1986) and is available in the ANSYS optimization module (Schittkowski 2001).

Since the convexity of the top-level optimization problem using lamination parameters is not proven, the uniqueness of the solution cannot be guaranteed. Based on a limited number of trials with different starting values of design variables, we observed convergence to almost the same solution in the numbers of plies of each orientation and the lamination parameters V_{D1} and V_{D2}, but a larger variation in the lamination parameter V_{D3}.

4.2 Bottom level optimization

In the bottom level, a stacking sequence optimization is performed by matching the lamination parameters V_i that came from the top level optimization by the lamination parameters $\tilde{V}_i$ computed in the bottom level optimization in a least squares sense subject to satisfaction of the composite design rules and manufacturing requirements. The measure of the out-of-plane lamination parameter match is defined as

$$L = \sqrt{\sum_{i=1}^{3} \left(\frac{V_i - \widetilde{V}_i}{V_i} \right)^2} \tag{11}$$

A permutation genetic algorithm (permGA) is used for the bottom level optimization runs carried out iteratively in order to ensure the ply compatibility of adjacent panels as described in Section 5. The advantage of this approach, as stated in Liu et al. (2011), is that there is no need to check satisfaction of the strain or buckling constraints, as long as the lamination parameters, obtained after the bottom level optimization, match the lamination parameter values that came from the top level optimization.

5 Shared layers blending (SLB)

In aerospace engineering, a typical wing is a multi-panel tailored composite structure. To improve structural integrity and avoid stress concentration between two adjacent panels, ply blending should be ensured. Although such requirements have been considered by Kristinsdottir et al. (2001), Liu and Haftka (2001), Seresta et al. (2007), Liu and Krog (2008), Keller (2011), Zein et al. (2012), a problem of optimization of multi-panel aircraft structures with a comprehensive consideration of buckling, strain, manufacturing constraints as well as general composite design rules including ply blending still remains to be addressed to satisfaction of aircraft industry.

In this section the Shared Layers Blending (SLB) process, introduced by Liu et al. (2011) to satisfy the global blending requirement as well as the general layup design rules, is summarized for completeness. The SLB scheme is suitable for the creation of a laminated structure according to the definition of inner blending, outer blending or the generalized blending as defined by van Campen et al. (2008).

First, ranking of all panels in terms of the numbers of plies of each angle is performed. Then, for each ply angle, out of all panels the minimum number of plies is selected. This set of three ply numbers defines the first set of shared layers among all panels. The thinnest panel that includes the first shared layers is identified. The first shared layers will be placed outermost in the stacks for all panels. The remaining layers in the thinnest panel are placed after the first shared layers. Next, after this first stage, for the remaining layers of all the panels, except the thinnest panel, the same procedure is applied as at the first stage. This is repeated until the last panel is considered. Finally, for the adjacent panels, the local blending between them is performed for the remaining layers in the adjacent panels. Thus, the plies for all the panels will become inwardly blended (outer blending), when the outer layers of all the panels are continuous. If the shared layers are placed at the position next to the mid plane instead of the outermost position, the inner blending (outwardly blended composite) will be created. In this paper, the continuous plies are always placed outermost in the stack due to the damage tolerance requirements (Kassapoglou 2010).

The detailed description of the SLB scheme was given in the paper by Liu et al. (2011).

6 Bottom level optimization using a permutation GA

The number of plies of each orientation and the lamination parameters related to the out-of-plane stiffness matrix are obtained from the top level optimization. The bottom level optimization aims at preserving the given values of the out-ofplane lamination parameters while shuffling the given number of plies to satisfy the layup rules and blending requirements.A permutation GA (Michalewicz 1992; Bates et al. 2004; Narayanan et al. 2007) is an ideal tool for such a composite laminate optimization problem. Each string in the coding represents a unique stacking sequence. An example of using the genetic operators with a permutation encoding is given below.

6.1 Encoding

Mutation - two substrings are selected and exchanged e.g., third and fifth:

$[1\ 2345] \Rightarrow [12543]$

Crossover can be done in a variety of ways, such as ‘simple crossover’, ‘cycle crossover’, ‘inversion’ and ‘swap adjacent cells’. The ‘swap adjacent cells (i.e., substrings)’ method, implemented in this work, is illustrated below:

$[12\ 34\ 5] \Rightarrow [13\ 24\ 5]$

The set of elementary substrings used in this work includes 45/0/−45, 45/90/−45, $45/0_2/{-45}$ and $45/90_2/{-45}$. This choice reflects the layup rules of composite laminate design and manufacturing requirements.

6.2 Quantification of the composite layup requirements

In the laminated composite optimization, the layup requirements have to be applied to create a design acceptable in aeronautical applications. Compared to the approach presented by Liu et al. (2011), two additional criteria, the 90° degree ply angle jump index and stack homogeneity index, are introduced in this paper.

The requirement of minimization of the number of occurrences of 90∘ change in the ply angle for any two consecutive plies in the stack is quantified by the 90° ply angle jump index:

$$A = 2\frac{N_a t}{h} \tag{12}$$

where Na is the total number of occurrences of 90° ply angle jump in the consecutive plies in the half stack, t is the ply thickness.

This 90° ply angle jump index is used to enforce one of composite design rules for the optimal design of blended composite structures that discourages the 90° fiber angle change between two adjacent plies through the thickness (Liu et al. 2009). Similarly, a cross-directional constraint on 90° fiber angle alternations between adjacent design subdomains (or sublaminates) has been introduced as a constraint in the optimization problem formulation by Kennedy and Martins (2013).

The stack homogeneity requirement (Niu, 2010 and Niu, 2011) implies that plies of all three possible orientations (0∘, 90° and ±45∘) occur in the stack with the frequency that is as uniform as possible. In order to quantify this requirement, it is proposed to monitor the composition of the string of ply angles that characterizes the stack. The lengths of all substrings that contain only two out of three possible ply angles are calculated. A divider between such substrings can be either an occurrence of a third ply angle or one of the following five possible blocks of plies bounded by a pair of 45∘ and −45∘ plies: 45∘/−45∘, 45° /0°/−45°, 45° /$0°_2$/−45°,45°/90°/−45° and 45°/$90°_2$/−45°. Also, in counting the substring length, occurrences of the same ply angle in a group of two, three, or four sequential plies is counted as one. Thus, the maximum length of such substrings (N_h) contributes to the definition of the stack homogeneity index:

$$H = 2\frac{N_h t}{h} \tag{13}$$

where h is the total thickness of the panel.

6.3 Example

In this example, the calculation of the 90° ply angle jump index, A, and the stack homogeneity index, H, is demonstrated. A symmetric, balanced laminate is given as:

$$\left[90/0/45/90/\text{-}45/90_2/0_3/90/0/45/0/\text{-}45/90/0/45/90_2/\text{-}45/0/90/45/0_2/45/0_2/90/0/90/45/90_2/\text{-}45/0\right]_s$$

The total number of occurrences of the 90° ply angle jump in the consecutive plies in the half stack, N_a, for the above example is 9 hence the index can be calculated as
$A = 2\frac{N_a t}{h} = 2\frac{9t}{h} = \frac{18}{74} = 0.243$.

The lengths of all substrings that contain only two out of three possible ply angles are presented in Fig. 4 The first substring length is 2 because the third ply angle in the block of plies 45°/90°/−45° is placed immediately after the first group of two different ply angles (90° followed by 0°). Thus, the maximum length of such substrings, N_h, is 4 hence the stack homogeneity index is: $H = 2\frac{4t}{h} = \frac{8}{74} = 0.108$.

[90/0/45/90/-45/90/90/0/0/0/90/0/45/0/-45/90/0/45/90/90/-45/0/90/45/0/0/45/0/0/90/0/90/45/90/90/-45/0]$_s$

2 divider 4 divider 2 divider 2 divider 4 divider 1

Fig. 4 Illustration of shared layers blending concept for the three-panel linked structure

6.4 Representation of the composite layup requirements in the objective function

In order to combine the 90° ply angle jump index, A, the stack homogeneity index, H, and the non-dimensional measure of the lamination parameters match, L, into a single objective function, also ensuring that these three criteria have the same order of contribution to the objective function, the following weighted sum criterion has been chosen:

$$f = w_1 L^2 + w_2 A^2 + w_3 H^2 \quad (14)$$

where f is the objective function, w_1 is the weight coefficient for the measure of the lamination parameter match, w_2 is the weight coefficient for the 90° ply angle jump index, w_3 is the weight coefficient for the stack homogeneity index.

7 Wing Box example

The wing box model presented in the paper by Liu et al. (2000), see Figs. 5 and 6 and Table 1, is used to illustrate the method discussed in previous sections. Only the top skin panels are considered in the composite design whereas the bottom skin panels are treated as non-designable, their layup is taken from Liu et al. (2011) and listed in Table 2.

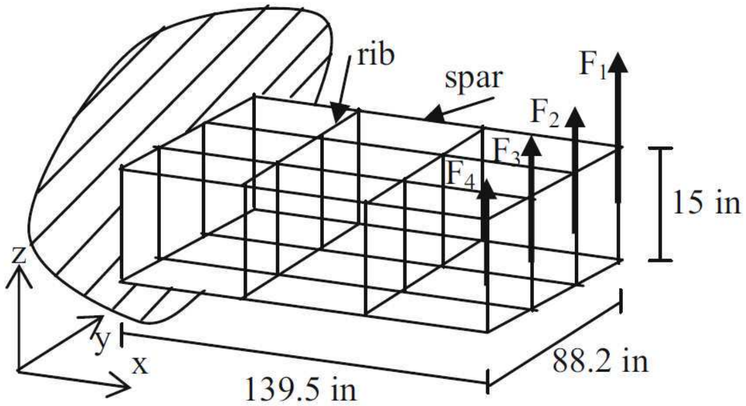

Fig. 5 Geometry of the wing

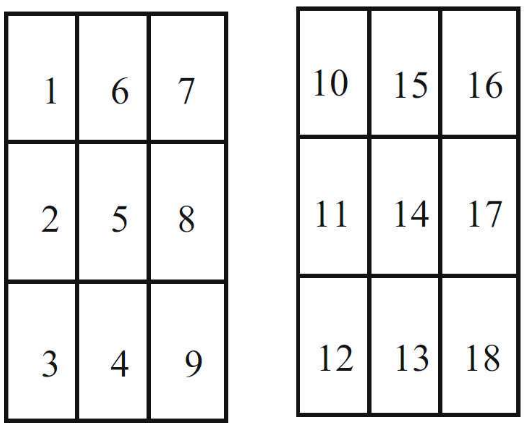

Fig. 6 Bottom (left) and top (right) skin panels

According to the authors' previous research (Liu et al. 2011), the rounding-off rules applied to determine the integer number of plies of each orientation from the continuous results are: 1) rounding up the number of 45° plies in the top skin, which will increase the buckling load factor; 2) rounding up the number of 0° plies in the bottom skin, which will increase the tensile strength; 3) the number of 90° plies in the bottom skin is also rounded up in order to provide greater design freedom for satisfying the design rules in Section 4 when only a small number of plies exists in the panels (i.e., in the bottom panels) and 4) all the other continuous values are rounded to the nearest integers. This rounding-off strategy facilitates satisfaction of the layup rules and also aims at improving the mechanical performance in the bottom level optimization.

Table 1 Material properties for graphite-epoxy T300/N5208

Material properties	Values
Young's Modulus in fibre direction 1, E_1	127.56 GPa
Young's Modulus in transverse direction 2, E_2	13.03 GPa
Shear Modulus, G_{12}	6.41 GPa
Poisson's ratio, ν_{12}	0.3
Material density, ρ	1577.76 kg/m^3
Ply thickness, t	0.127 mm
Allowable strain in fibre direction ε_{1a}	0.08
Allowable strain in transverse direction ε_{2a}	0.029
Allowable shear strain γ_{12a}	0.015
Safety factor	1.5

Table 2 Stacking sequence of the bottom skin panels

Bottom skin panels	Stacking sequence
Section 7.1 1 - 9	$[\pm45/(90/0)_3/0_6]_s$
1, 6, 7	$[\pm45/0_4/90/0]_s$
Section 7.2 2, 5, 8	$[\pm45/0_4/90_2]_s$
3, 4, 9	$[\pm45/0_4/90_2/45/90/-45/0_4]_s$
1	$[\pm45/0_4/90/0]_s$
2	$[\pm45/0_4/90_2]_s$
3	$[\pm45/0_4/90_2/45/90/-45/0_4]_s$
4	$[\pm45/0_4/90_2/45/90/-45/0_4]_s$
Section 7.2 5	$[\pm45/0_4/90_2]_s$
6	$[\pm45/0_4/90/0]_s$
7	$[\pm45/0_4/90/0]_s$
8	$[\pm45/0_4/90_2]_s$
9	$[\pm45/0_4/90_2/45/90/-45/0_4]_s$

The effects of the weigh coefficients for the nondimensional measure of the lamination parameters match, L, the 90° ply angle jump index, A, and the stack homogeneity index, H, on the buckling load factor are demonstrated by the results of the stacking sequence arrangement for the top skin panels.

7.1 Problem with one designable substructure

If the layup of all panels in the top skin is the same, the total number of design variables for the wing box is six: n_0^t, $n_{\pm 45}^t$, n_{90}^t, V_1^t, V_2^t, and V_3^t. The results for the objective function (that is the total number of plies in the structure) and the active constraints (that are buckling constraints) after the top level optimization are shown in Table 3. The objective function value is reduced to 180 as compared to 208 reported by Liu et al. (2000). In the bottom level optimization, given the lamination parameters from the top level, the permutation GA was used to obtain the stacking sequence for the top skin panels (that are all identical), see results presented in Table 4.

Table 3 Continuous and rounded off optimal design with 6 variables

	n_0 (Continuous)	n_{45}	n_{90}	n_0 (Rounded)	n_{45}	n_{90}	V_1	V_2	V_3
Top skin panels	34.492	7.445	26.139	34	8	26	0.9434	1.0065	1.2108
Buckling load factor	1.0009			1.0183					
Total number of plies	177.65			180					
Total number of plies (Liu et al. 2000)	208.76			208					

Table 4 Effects of weight coefficients on the objective function and the buckling constraint in 10 cases (the target values of the lamination parameters are shown in brackets)

Case study	V_1	V_2	V_3	L^2	A^2	H^2	w_1	w_2	w_3	f	Buckling load factor
(0.9434 1.0065 1.2108)											
C1	0.9434	1.0065	1.2095	7.29e-5	1.17e-1	2.10e-2	1.0	0.0	0.0	7.29e-5	1.0210
C2	0.9434	1.0064	1.2097	2.38e-4	5.02e-2	1.10e-2	0.9	0.05	0.05	3.27e-3	1.0210
C3	0.9431	1.0063	1.2125	1.17e-3	3.88e-2	1.10e-2	0.8	0.15	0.05	7.33e-3	1.0204
C4	0.9430	1.0062	1.2211	4.51e-3	3.39e-2	1.10e-2	0.6	0.35	0.05	1.51e-2	1.0191
C5	**0.9434**	**1.0075**	**1.2430**	**4.61e-2**	**2.92e-2**	**8.46e-3**	**0.5**	**0.45**	**0.05**	**3.66e-2**	**1.0156**
C6	1.1018	1.0080	1.2304	7.89	1.10e-2	8.46e-3	0.0	1.0	0.0	1.11e-2	0.9842
C7	1.0893	1.0064	1.2894	6.79	2.92e-2	1.52e-3	0.0	0.0	1.0	1.56e-3	0.9756
C8	0.9434	1.0063	1.2130	1.00e-3	4.45e-2	1.10e-2	0.5	0.05	0.45	7.70e-3	1.0205
C9	0.9429	1.0063	1.2143	9.38e-4	3.88e-2	1.10e-2	0.6	0.05	0.35	6.39e-3	1.0201
C10	0.9434	1.0066	1.2225	3.24e-3	3.88e-2	4.33e-3	0.8	0.05	0.15	5.19e-3	1.0189

In order to illustrate the effects of the weight coefficients w_1, w_2, and w_3 on the results in terms of the lamination parameter match measure L, the stack homogeneity index H, the 90° ply angle jump index A and the buckling load factor, ten cases have been investigated. Without consideration of the stack homogeneity index H and the 90° ply angle jump index A, the permutation GA can match the lamination parameter values to the target values from the top level very well (see case C1 in Table 4). When w_2 increases and w_1 is kept constant (cases C1 to C5), both the 90° ply angle jump index and the stack homogeneity index decreased whereas the lamination parameter match measure moderately increases that results in a decrease in the buckling load factor. When w_2 is equal to 1.0 (Case C6), the best (smallest) value for the 90° ply angle jump index (A^2=1.10e-2) is obtained and the stack homogeneity index is acceptably small (H^2=8.46e- 3), whereas the lamination parameter match measure is poor (L^2=7.89) resulting in the reduction of the buckling load factor and leading to the constraint violation. When w_3 is equal to 1.0 (Case C7), the best (smallest) value for the stack homogeneity index (H^2=1.52e-3) is obtained and the 90° ply angle jump index is acceptably small (A^2=2.92e-2) but the buckling constraint is violated because the lamination parameter match measure is poor (L^2=6.79). From cases C8 to C10 where w_2 is kept constant and w_3 decreases from 0.45 to 0.15 (hence w_1 increases from 0.5 to 0.8), the lamination parameter match measure remains acceptably small, the 90° ply angle jump index is almost constant and acceptably small, but no clear conclusion can be made about the trends for the stack homogeneity index. This investigation has led to a conclusion that the best stacking sequence quality was obtained when weight coefficients are defined as w_1=0.5, w_2=0.45 an d w_3=0.05 as in Case C5 in Table 4. Therefore these weight coefficients have been used in all studies presented in this paper. The stacking sequences of the top skin panels for each case study are listed in Table 5. The discrepancies of normal and shear strains between top and bottom levels are very small (maximum difference is 0.0018 %) This is simply because the A stiffness matrix has not been changed during the optimization process.

7.2 Problem with three designable substructures

If the top skin is divided into three parts, i.e., the root, the intermediate and the tip parts, the number of the design variables is 18 and the results are listed in Table 6. The weight is reduced considerably as compared to the case of one designable substructure. The objective function value is 464 for the discrete optimal design that is the same as the result of Liu et al. (2000). The magnitude of the buckling load factor (1.0366) is close to the value from the top level optimization (1.0349). This is guaranteed by arriving at a good match with the lamination parameters from top level optimization when a bottom optimization is performed. With the addition of the stack homogeneity index and the 90° ply angle jump index to the formulation of the objective function, the stacking sequence has a better quality and uniformity, shown in Table 7, as compared with the results of Liu et al. (2011). It is evident that the implementation of the additional stack quality criteria within the outer blending scheme did not cause any problems for the blending process.

Table 5 Stacking sequence of the top panels for ten case studies, one designable substructure case

Case study	Stacking sequence
C1	$[(\pm 45)_2/45/90/-45/90/45/90_2/-45/90/0/45/90/-45/90_3/0_3/90/0/90_2/0_3/90_2/0_2/90/0_3/90/0_4/90/45/0/-45/0_2/90/0_3/90/0_2/90/0_2/90/0/90_2/45/0/-45/45/0/-45/90/0_2/90/0_2/90]_s$
C2	$[(\pm 45)_2/45/90_2/-45/45/90/-45/90_2/0/45/90/-45/90_4/0_4/90_2/0_3/90_2/0_4/90/0/90/0_4/45/0/-45/90/0_3/90/0_3/90_2/0_4/90_3/45/0/-45/90_2/0_4/45/0/-45/90]_s$
C3	$[(\pm 45)_2/45/90/-45/45/90_2/-45/90_2/0/45/90/-45/90_3/0_4/90_3/0_4/90_3/0_4/90/0_4/90_2/0_2/45/0/-45/0_4/90_2/0_3/90_2/0/90_4/45/0/-45/0_2/45/0/-45/0_2]_s$
C4	$[(\pm 45)_2/45/90/-45/90/45/90_2/-45/0/90_4/45/90/-45/0_4/90_3/0_4/90_3/0_4/90/0_4/90_2/0_2/45/0/-45/0_4/90_2/0_4/90_3/0_2/45/0/-45/90_2/0_2/45/0/-45/90]_s$
C5	$\mathbf{[(\pm 45)_2/0_3/45/90_2/-45/90_3/45/90_2/-45/0/90_4/45/0_2/-45/0/90_3/0_2/90_4/0/45/0/-45/0_4/90/0_4/90/0_3/90_2/0_4/45/0_2/-45/0_2/45/0_2/-45/90_3/0_2/90]_s}$
C6	$[(\pm 45)_2/0_2/45/90_2/-45/90_3/0_4/45/0_2/-45/0_4/45/90_2/-45/0_2/45/90_2/-45/0_4/90_3/0_4/45/90/-45/90_2/0_4/90_4/0_4/90_3/0_2/45/0_2/-45/90_4]_s$
C7	$[(\pm 45)_2/90/0_3/90_2/45/90/-45/0_4/90_2/0_2/45/0_2/-45/90/0_4/90_3/45/90_2/-45/0_4/90/0_4/45/90_2/-45/90_3/0_4/90/45/90_2/-45/0_2/90_4/45/0/-45/0_3/90/0]_s$
C8	$[(\pm 45)_2/45/90/-45/90_2/45/90_2/-45/0/45/90/-45/90_3/0_4/90_3/0_3/90_2/0/90/0_4/90/0_4/45/0/-45/90/0_2/90/0_4/90_2/0_4/90_4/0_2/45/0/-45/90_2/45/0/-45/0_2]_s$
C9	$[(\pm 45)_2/45/90/-45/45/90_2/-45/90_4/0/45/90/-45/90/0_4/90_3/0_4/90_3/0_4/90/0_4/90/0_2/45/0/-45/90/0_4/90_2/0_4/90_2/45/0/-45/90_4/0_2/45/0/-45/0_2]_s$
C10	$[(\pm 45)_2/45/90/-45/45/90/-45/90_2/0_2/90_4/0_3/90_4/45/0_2/-45/0_3/90_2/0_4/90_2/0_3/45/90_2/-45/90/0_2/90_3/45/0/-45/0_3/90/0_4/90/0_2/45/0/-45/0_4/90_2]_s$

The significance of bold indicates the suggested weight coefficients in this paper in terms of the best stacking sequence quality

Table 6 Continuous and rounded off optimal design with 18 variables

	n_0 (Continuous)	n_{45}	n_{90}	n_0 (Rounded)	n_{45}	n_{90}	V_2	V_3
Top skin panels								
Panel no.16	27.99	15.58	22.19	28	16	22	1.1268	1.0102
Panel no.17	25.59	12.54	19.40	26	13	19	1.1610	1.0086
Panel no.18	21.64	5.45	13.66	22	6	14	1.2398	1.0098
Buckling load factor	1.0039			1.0349				
Total number of plies	456.68			464				
Total number of plies (Liu et al. 2000)	465.63			464				

The significance of bold indicates the stacking sequence when the suggested weight coefficients have been used

7.3 Problem with nine designable substructures

In this case all panels in the top skin are considered to be designable substructures and the number of the design variables is 54. The result of the top level optimization is presented in Table 8. This was followed by the bottom level optimization to obtain a blended composite layup for all panels. At the bottom level the combined objective function targets the lamination parameter values sent from the top level and includes two additional stacking sequence quality criteria. The plies are shuffled to minimize the objective function while satisfying the blending requirements. The resulting values of lamination parameters and the detailed ply stacking sequences are listed in Tables 9 and 10, respectively. The FE analysis of the obtained design shows that the buckling load factor has decreased only by 0.5 % as compared to the result of the top level optimization with rounded off numbers of plies. This demonstrates that the lamination parameter-based method works well for the optimization of blended laminated composite structures as it results in an acceptably small difference between the lamination parameters from the top level optimization and the ones calculated at the bottom level. This can typically be achieved for realistic aircraft structures where the number of plies is not too small so that blending does not prevent from arriving at a good match of lamination parameters. With the addition of the stack homogeneity index and the 90° ply angle jump index to the objective function, a better quality ply stacking sequences compared with the results of Liu et al. (2011) can be obtained while satisfying the blending requirements.

Table 7 Stacking sequence and lamination parameters of the panels for the three designable substructures case ($w_1=0.5, w_2=0.45 w_3=0.05$)

Panel no.	V_1	V_2	V_3	Buckling load factor
16	1.1594	1.0073	1.2128	1.0366
17	1.1816	1.0080	1.2002	
18	1.2366	1.0120	1.1079	
Stacking sequence				
16	$[(\pm45)_2/45/0/-45/0_4/90/0_3/45/90/-45/0_4/45/0_2/-45/90_2/0_4/90_4/45/0/-45/90_2/0_2/90_4/0/90_3/45/90_2/-45/45/0/-45/45/0_2/-45/0/(\pm45)_4/90/45/90_2/-45/45/0_2/-45/\pm45]_s$			
16	$[(\pm45)_2/45/0/-45/0_4/90/0_3/45/90/-45/0_4/45/0_2/-45/90_2/0_4/90_4/45/0/-45/90_2/0_2/90_4/0/90_3/45/90_2/-45/45/0/-45/45/0_2/-45/0/(\pm45)_4/90/45/90_2/-45/45/0_2/-45/\pm45]_s$			
17	$[(\pm45)_2/45/0/-45/0_4/90/0_3/45/90/-45/0_4/45/0_2/-45/90_2/0_4/90_4/45/0/-45/90_2/0_2/90_4/0/90_3/45/90_2/-45/45/0/-45/45/0_2/-45/0/(\pm45)_4]_s$			
18	$[(\pm45)_2/45/0/-45/0_4/90/0_3/45/90/-45/0_4/45/0_2/-45/90_2/0_4/90_4/45/0/-45/90_2/0_2/90_4/0]_s$			

Table 8 Continuous and rounded off optimal design with 54 variables, top level optimization

Panel no.	n_0 (Continuous)	n_{45}	n_{90}	n_0 (Rounded)	n_{45}	n_{90}	V_1	V_2	V_3
10	27.07	14.44	21.40	27	15	21	1.0978	1.0094	1.2446
11	25.34	12.85	19.08	25	13	19	1.1261	1.0086	1.2905
12	20.73	5.67	12.84	21	6	13	1.2319	1.0089	1.0736
13	20.70	5.66	12.84	21	6	13	1.2311	1.0087	1.0745
14	25.35	13.24	19.28	25	14	19	1.1189	1.0083	1.2596
15	27.66	15.70	22.04	28	16	22	1.0947	1.0096	1.2001
16	27.48	15.81	22.07	27	16	22	1.0987	1.0102	1.2013
17	25.56	13.49	19.36	26	14	19	1.1224	1.0082	1.2492
18	20.99	6.05	13.05	21	7	13	1.2243	1.0071	1.0460
Buckling load factor	1.0014			1.0213					
Total number of plies	1177.32			1192					

Table 9 Lamination parameter values and stacking sequence and of the panel at bottom level with 54 variables (w_1=0.5, w_2=0.45, w_3=0.05)

Panel no	V_1	V_2	V_3	Buckling load factor
10	1.1536	1.0077	1.1971	10
11	1.1703	1.0082	1.1876	11
12	1.2327	1.0129	1.0834	12
13	1.2327	1.0129	1.0834	13
14	1.1658	1.0081	1.1909	14
15	1.1463	1.0075	1.1991	15
16	1.1481	1.0075	1.1987	1.016
17	1.1636	1.0080	1.1922	17
18	1.2280	1.0121	1.0982	18

8 Conclusions

A lamination parameter-based method was examined for seeking the best stacking sequence of laminated composite wing structures with blending and mechanical performance requirements. Two new criteria, the 90° ply angle jump index and the stack homogeneity index, have been added to the measure of mismatch of lamination parameters to define the objective function. This objective function is minimized to achieve the best stacking sequence of laminate composite wing structures in the bottom level optimization subject to the blending requirements. For this purpose, the use of a permutation GA is effective and efficient because in this bottom level optimization there are no calls for the FE simulation and the objective function is calculated by simple formulae.

Table 10 Stacking sequence of the panels for the nine designable substructures case

10	$[(\pm45)_2/45/0/\text{-}45/0_3/45/0_2/\text{-}45/0_4/90_2/0_4/45/90/\text{-}45/(90_4/0_3)_2/0/90_2/(\pm45)_2/90_3/45/90_2/\text{-}45/45/90/\text{-}45/(45/0_2/\text{-}45)_2/(\pm45)_3/0/45/90_2/\text{-}45/0]_s$
11	$[(\pm45)_2/45/0/\text{-}45/0_3/45/0_2/\text{-}45/0_4/90_2/0_4/45/90/\text{-}45/(90_4/0_3)_2/0/90_2/(\pm45)_2/90_3/45/90_2/\text{-}45/45/90/\text{-}45/(45/0_2/\text{-}45)_2/(\pm45)_2]_s$
12	$[(\pm45)_2/45/0/\text{-}45/0_3/45/0_2/\text{-}45/0_4/90_2/0_4/45/90/\text{-}45/(90_4/0_3)_2/0/90_2/\pm45]_s$
13	$[(\pm45)_2/45/0/\text{-}45/0_3/45/0_2/\text{-}45/0_4/90_2/0_4/45/90/\text{-}45/(90_4/0_3)_2/0/90_2/\pm45]_s$
14	$[(\pm45)_2/45/0/\text{-}45/0_3/45/0_2/\text{-}45/0_4/90_2/0_4/45/90/\text{-}45/(90_4/0_3)_2/0/90_2/(\pm45)_2/90_3/45/90_2/\text{-}45/45/90/\text{-}45/(45/0_2/\text{-}45)_2/(\pm45)_3]_s$
15	$[(\pm45)_2/45/0/\text{-}45/0_3/45/0_2/\text{-}45/0_4/90_2/0_4/45/90/\text{-}45/(90_4/0_3)_2/0/90_2/(\pm45)_2/90_3/45/90_2/\text{-}45/45/90/\text{-}45/(45/0_2/\text{-}45)_2/(\pm45)_3/0/45/90_2/\text{-}45/0/45/90/\text{-}45/0]_s$
16	$[(\pm45)_2/45/0/\text{-}45/0_3/45/0_2/\text{-}45/0_4/90_2/0_4/45/90/\text{-}45/(90_4/0_3)_2/0/90_2/(\pm45)_2/90_3/45/90_2/\text{-}45/45/90/\text{-}45/(45/0_2/\text{-}45)_2/(\pm45)_3/0/45/90_2/\text{-}45/0/45/90/\text{-}45]_s$
17	$[(\pm45)_2/45/0/\text{-}45/0_3/45/0_2/\text{-}45/0_4/90_2/0_4/45/90/\text{-}45/(90_4/0_3)_2/0/90_2/(\pm45)_2/90_3/45/90_2/\text{-}45/45/90/\text{-}45/(45/0_2/\text{-}45)_2/(\pm45)_3/0]_s$
18	$[(\pm45)_2/45/0/\text{-}45/0_3/45/0_2/\text{-}45/0_4/90_2/0_4/45/90/\text{-}45/(90_4/0_3)_2/0/90_2/(\pm45)_2]_s$

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