An Improved SNR Estimation Algorithm for Asymmetric Pulse-Shaped Signals

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Abstract

The split-symbol moment estimation (SSME) algorithm is a conventional method to estimate the signal-to-noise ratio (SNR) in communication systems for M-ary phase shift keying signals. However, the conventional SSME is based on the assumption of symmetric pulse-shaping waveforms, and becomes no longer proper when the shaping pulse is asymmetric. In this paper, we propose a modified SSME algorithm for asymmetric pulse-shaped signals, based on an odd-even symbol splitting approach. Simulation results show that the modified SSME outperforms the conventional SSME for both symmetric and asymmetric pulse-shaped signals in terms of SNR estimation accuracy.

I. Introduction

THE signal-to-noise ratio (SNR) is an important parameter of wireless communication systems. It significantly impacts the performance of various wireless communication techniques, such as adaptive power control, turbo decoding and channel equalization [1], which would usually require the knowledge of the instantaneous

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SNR in a time-varying environment [1]–[3]. Hence, the accurate measurement and estimation of SNR are essential to many techniques and functionalities of wireless communication systems. However, most existing SNR estimation algorithms are either of a high computational complexity or of a compromised accuracy [4]–[7].

In general, there are two types of SNR estimation techniques. One type is the data-aided estimation, in which the receiver estimates the SNR based on some known data (or pilot symbols) received from the transmitter. One example of data-aided estimation is the maximum-likelihood estimation, which provides an excellent estimation accuracy at the cost of a high computational complexity [8]. Moreover, the data-aided estimation requires the transmission of known data over the wireless channel, which makes use of precious radio resources and will reduce the spectral efficiency of the system. The other type is called blind estimation, where the receiver does not know the exact data transmitted by the transmitter. An example of blind SNR estimation is the algorithm in [9], which estimates the SNR by using the second and fourth-order correlation of the signal and the noise [10]. The estimation precision of this algorithm may still be improved by considering even higher order of correlation, which would significantly increase the computational complexity.

The split-symbol moment estimation (SSME) is a widely used blind SNR estimation technology. It exploits the property that the samples of the signal in one symbol interval are correlated while those of the noise are uncorrelated [11]–[13]. Accordingly, the SSME estimator divides the samples of the received signal during one symbol interval into two non-overlapping halves in the time domain. Since the second moment of the sum of all samples in a symbol interval provides an estimation of the total energy and the second moment of the difference between the two halves gives an estimation of the noise energy, we can estimate the desired signal power by subtracting the estimated noise power from the estimated total power in one symbol interval. However, this SSME method is limited to the case that the transmitted pulse-shaping waveform is symmetric with respect to the half-interval point, e.g., the square waveform as shown in Fig. 1. In this case, the noise power can be accurately estimated using the SSME. If the shaping pulse is of an asymmetric waveform, e.g., the Gaussian minimum-shift keying (GMSK) waveform in the time domain as shown in Fig. 2 [14]. With an asymmetric waveform, the signal amplitudes carried by the two halves of samples in one symbol interval are not equal, and an estimation error will occur in the SSME. The estimation error gets larger when the asymmetry of the shaping pulse becomes more evident.

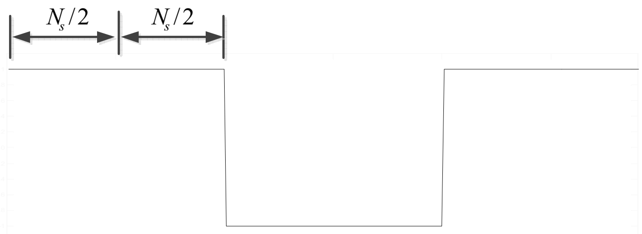

Fig. 1: The square wave signal in the time domain

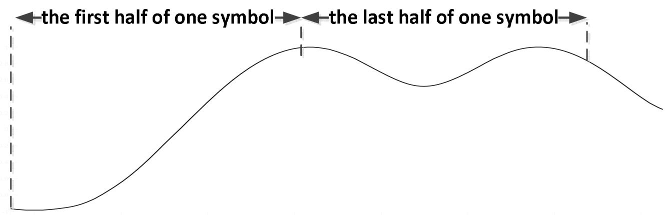

Fig. 2: The GMSK waveform in the time domain over one symbol interval

In this paper, we propose an improved SSME algorithm, which divides the samples of the received signal in one symbol interval into two groups, one group of odd numbered samples and the other group of even numbered samples, instead of simply dividing them into two halves of the symbol interval. If the sampling rate is sufficiently high, then any two adjacent samples (i.e., an odd numbered sample and the even numbered sample next to it) would be approximately equal, thus reducing the dependence of the SSME on the symmetry of the shaping pulse. In other words, the proposed SSME algorithm can be used with any pulse-shaping waveforms. Simulation results, which consider various pulse-shaping waveforms and modulation schemes, show that the improved SSME algorithm outperforms the conventional SSME not only for signals with an asymmetric pulse-shaping waveform, but also for signals with a symmetric pulse-shaping waveform.

The remainder of this paper is organized as follows. We introduce the system model in Section II and review the conventional SSME in Section III. We propose the improved SSME algorithm in Section IV. In Section V, we present the simulation results. We conclude this paper in Section VI.

II. System Model

In this paper, we focus on satellite and/or space communications, which are mainly free-space point-to-point communications and usually have only one transmission path. Accordingly, the wireless channel can be well approximated as an additive white Gaussian noise (AWGN) channel. The received signal under an AWGN channel in one symbol interval is expressed as [15]

$$y(t) = x(t) + \omega(t), \quad 0 \leq t \leq T_s, \tag{1}$$

where $x(t)$ is the transmitted signal, $y(t)$ is the received signal, $\omega(t)$ is the AWGN with power spectral density σ^2, and T_s is the symbol interval.

After analog-to-digital conversion (ADC), the discretized received signal from an AWGN channel in one symbol interval is given by

$$y_n = x_n + \omega_n, \quad 1 \leq n \leq N, \tag{2}$$

where y_n is the n^{th} sample of the received signal, x_n and ω_n are the values of the transmitted signal and the AWGN corresponding to the n^{th} sample, respectively, and N is the number of samples per symbol interval.

The SNR at the receiver is defined as [16]

$$\rho = \frac{E}{W} = \frac{E}{\sigma^2}, \tag{3}$$

where E and W are the power of the desired signal and that of noise, respectively. Since the AWGN samples are independent and identically distributed (i.i.d.), the average noise power W equals σ^2.

III. The Conventional SSME Algorithm

The conventional SSME algorithm assumes that the transmitted pulse-shaping waveform is symmetric with respect to the half-interval point [17], so that the amplitudes of desired signal samples in the first half of a symbol interval equal that of desired signal samples in the second half of the symbol interval. Accordingly, the number of samples in one symbol interval is an even number.

To perform the SNR estimation, the conventional SSME estimator first calculates the sum of the received-signal samples within the first half of a symbol interval and the sum of the received-signal samples within the second half of the symbol interval, i.e.,

$$Y_{\alpha k} = \sum_{l=1}^{N_s/2} y_{lk} = \sum_{l=1}^{N_s/2} (x_{lk} + \omega_{lk}), \tag{4}$$

$$Y_{\beta k} = \sum_{l=N_s/2+1}^{N_s} y_{lk} = \sum_{l=N_s/2+1}^{N_s} (x_{lk} + \omega_{lk}), \tag{5}$$

where N_s is the number of samples per symbol interval, y_{lk} is the l^{th} sample of the received signal in the k^{th} symbol interval, and x_{lk} and ω_{lk} are the values of the transmitted signal and the noise corresponding to the l^{th} sample in the k^{th} symbol interval, respectively.

Then the conventional SSME estimator calculates the sum (u_k^+) of and the difference (u_k^-) between $Y_{\alpha k}$ and $Y_{\beta k}$ as

$$u_k^{\pm} = Y_{\alpha k} \pm Y_{\beta k} = x_k^{\pm} + n_k^{\pm}, \quad k = 1, 2 \cdots, N, \tag{6}$$

where

$$x_k^{\pm} = \sum_{l=1}^{N_s/2} x_{lk} \pm \sum_{l=N_s/2+1}^{N_s} x_{lk}, \tag{7}$$

$$n_k^{\pm} = \sum_{l=1}^{N_s/2} \omega_{lk} \pm \sum_{l=N_s/2+1}^{N_s} \omega_{lk}. \tag{8}$$

Finally, the conventional SSME estimator calculates the average total power (U^+) of the desired signal and the AWGN and the average power (U^-) of the AWGN as

$$U^{\pm} = \frac{1}{N} \sum_{k=1}^{N} |u_k^{\pm}|^2, \tag{9}$$

where N is the number of symbol intervals over which the averaging is performed. Under the condition of ideal synchronization, the estimated SNR is given by

$$\hat{\rho} = \frac{U^+ - U^-}{U^-}. \tag{10}$$

IV. A Modified SSME Algorithm

The conventional SSME requires the half-wave symmetry of the shaping pulse, because only if the shaping pulse is half-wave symmetric, $x_k^- = 0$ and U^- gives the noise power W for one symbol interval. If the shaping pulse is asymmetric, e.g., the GMSK waveform in Fig. 1, where the first half and the second half of a symbol interval are asymmetric, then $U^- = W$ does not necessarily hold and the conventional SSME cannot estimate the SNR properly.

In this section, we modify the conventional SSME so that it can work properly for asymmetric pulse shaped systems as well. We divide the samples $\{y_{lk}\}_{l=1}^{N_s}$ of the received signal in the k^{th} symbol interval into two halves: odd-numbered samples $\{y_{(2n-1)k}\}_{n=1}^{N_s/2}$ and even-numbered samples $\{y_{(2n)k}\}_{n=1}^{N_s/2}$, where N_s is assumed to be an even integer for simple notation. The difference between a pair of two adjacent odd- and even-numbered samples is given by

$$y_{(2n)k} - y_{(2n-1)k} = x_{(2n)k} - x_{(2n-1)k} + \omega_{(2n)k} - \omega_{(2n-1)k}, \tag{11}$$

where $x_{(2n-1)k}$ and $x_{(2n)k}$ are two adjacent samples of the desired signal, while $\omega_{(2n-1)k}$ and $\omega_{(2n)k}$ are two adjacent samples of the noise. Assuming that the difference between the two adjacent samples $x_{(2n-1)k}$ and $x_{(2n)k}$ of the desired signal is negligible due to time-domain correlation of the transmitted signal, we have

$$y_{(2n)k} - y_{(2n-1)k} \approx \omega_{(2n)k} - \omega_{(2n-1)}. \tag{12}$$

We group the samples $\{y_{lk}\}_{l=1}^{N_s}$ of the received signal in the k^{th} symbol interval into $N_s/2$ orthogonal pairs of adjacent odd and even samples, and calculate the difference between each pair of adjacent odd and even samples. Then the sum of the $N_s/2$ differences in the k^{th} symbol interval is given by

$$\begin{aligned} Y_k &= \sum_{n=1}^{N_s/2} (y_{(2n)k} - y_{(2n-1)k}), \\ &\approx \sum_{n=1}^{N_s/2} (\omega_{(2n)k} - \omega_{(2n-1)k}). \end{aligned} \tag{13}$$

which contains noise components only. Thus, the noise power in the k^{th} symbol interval can be obtained as $|Y_k|^2$, given the zero-mean Gaussian distribution of the AWGN samples. The total power of the received signal can be calculated in the same way as the conventional SSME.

According to the law of large numbers, a sufficiently large number of samples is required to guarantee the estimation precision. We set the observation time interval as $T = L \cdot N_s$, where L is the number of symbols observed, and N_s is the number of samples per symbol interval. The estimated noise power $\hat{W}$ is expressed as

$$\hat{W} = \frac{1}{LN_s} \sum_{k=1}^{L} |Y_k|^2. \tag{14}$$

where averaging is performed over the LN_s samples to mitigate the effect of estimation errors.

The estimated total power $\hat{S}$ of the received signal is given by

$$\hat{S} = \frac{1}{LN_s} \sum_{k=1}^{L} |\sum_{l=1}^{N_s} y_{lk}|^2. \tag{15}$$

Accordingly, the estimated power of the desired signal is given by

$$\hat{E} = \hat{S} - \hat{W}. \tag{16}$$

Hence, the SNR can be estimated as

$$\hat{\rho} = \frac{\hat{E}}{\hat{W}}. \tag{17}$$

For SNR estimation performed over N data symbols and N_s samples per symbol, the computational complexities of the traditional SSME algorithm and the improved SSME algorithm are shown in Table I. We can see that the proposed modified SSME only slightly increases the computational complexity as compared with the conventional SSME.

TABLE I: Computational complexity of the two algorithms

Resource Consumption	SSME Algorithm	Improved Algorithm
Number of Multipliers	$N + 2$	$N + 2$
Number of Adders	$2NN_s + 1$	$4NN_s + 1$

V. Simulation Results

In this section, we present simulation results to evaluate the performance of our proposed SSME algorithm. In the simulation, we set the system clock frequency at $f_c = 100$ MHz, the intermediate frequency at $f_I = 70$ MHz, the coding rate at $R_b = 1$ MHz, and the number L of sampled symbol intervals to be 500. We consider three different modulation schemes: half-wave symmetric pulse-shaped quadrature phase-shift keying (QPSK), square-root-raised cosine (SRRC) QPSK, and SRRC GMSK.

Fig. 3 illustrates the mean values of the estimated SNR's of the conventional SSME and the modified SSME for the half-wave symmetric pulse-shaped QPSK signal in the AWGN channel. The results show that the estimated SNR's of both algorithms are close to the actual SNR. Fig. 4 illustrates the mean square error (MSE) of SNR

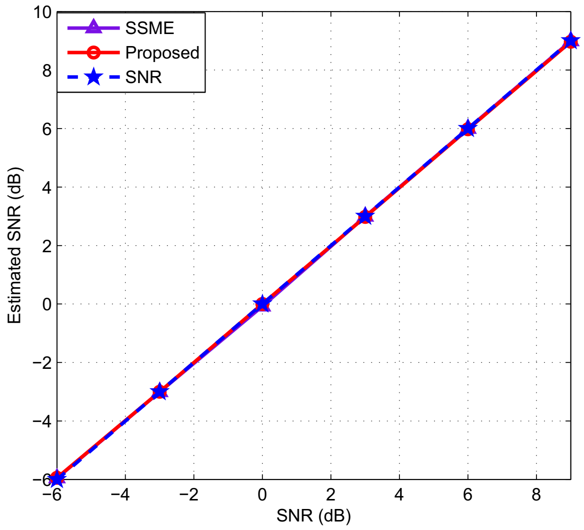

Fig. 3: Mean value of estimated SNR for the half-wave symmetric pulse-shaped QPSK signal

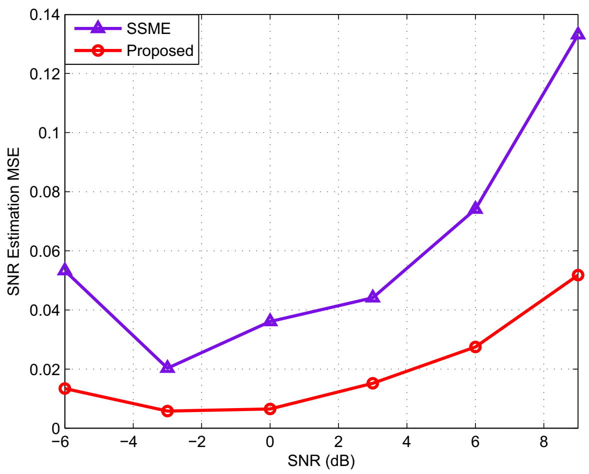

Fig. 4: MSE of estimated SNR for the half-wave symmetric pulse-shaped QPSK signal

estimation in the same case as Fig. 3. It can be seen that the modified SSME provides a lower MSE and a higher estimation precision than the conventional SSME for symmetric pulse-shaped QPSK signal.

Fig. 5 and Fig. 6 illustrate the mean value and the MSE of the estimated SNR for the SRRC-QPSK signal in the AWGN channel, respectively. The results show that the performance of the conventional SSME drops sharply when the shaping pulse is asymmetric. The estimated SNR of the modified SSME is much closer to the actual SNR value than the conventional SSME. The deviation between the estimated mean and the real SNR values of the conventional SSME increases with actual SNR, while that of the modified SSME is nearly a constant when the

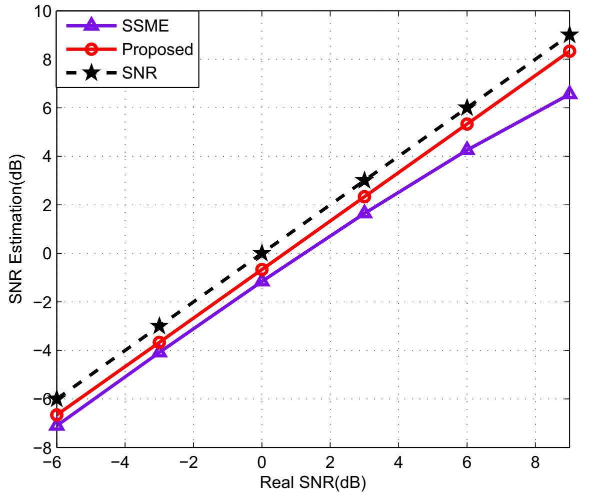

Fig. 5: Mean value of estimated SNR for SRRC-QPSK signal

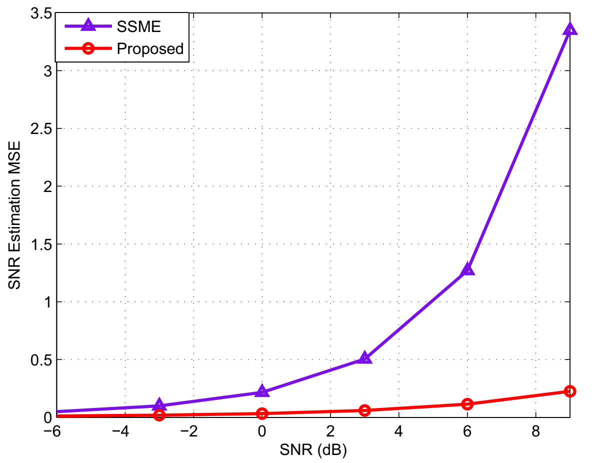

Fig. 6: MSE of estimated SNR for SRRC-QPSK signal

SNR increases. It can be seen from Fig. 6 that the MSE of the modified SSME is much smaller than that of the conventional SSME for the asymmetric SRRC QPSK signal, especially at high SNR values.

Fig. 7 and Fig. 8 illustrate the mean estimated SNR value and the MSE of the estimated SNR for SRRC-GMSK signal in the AWGN channel. Fig. 7 shows that the mean value of the modified SSME is closer to the actual SNR value than that of the conventional SSME. Meanwhile, Fig. 8 shows that the MSE of the modified SSME is smaller than that of the conventional SSME for the asymmetric SRRC GMSK signal, especially at high SNR values.

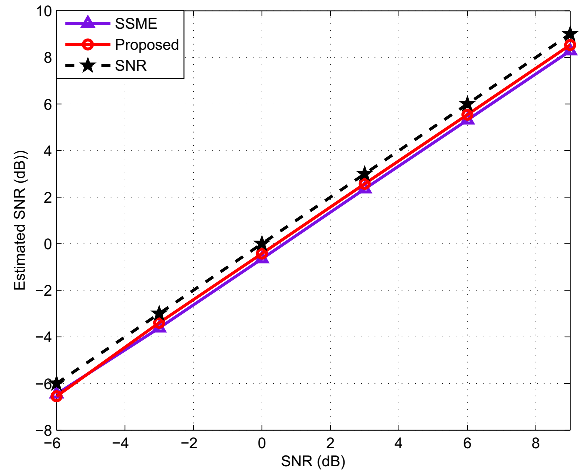

Fig. 7: Mean value of estimated SNR for SRRC-GMSK signal

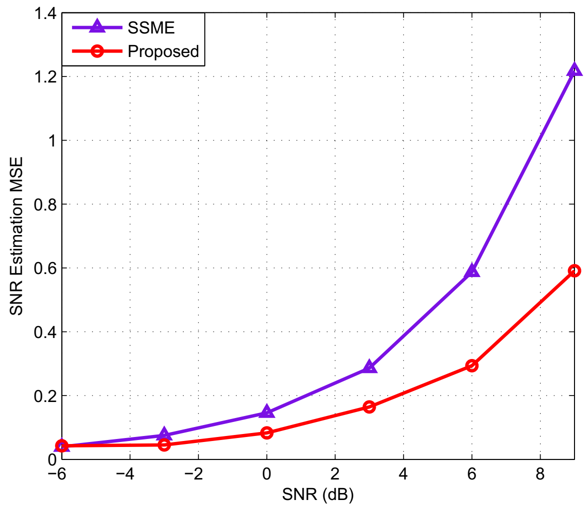

Fig. 8: MSE of estimated SNR for SRRC-GMSK signal

VI. Conclusion

In this paper, we have proposed a modified SSME algorithm based on an odd-even sample splitting method, so that the dependance on the symmetry of the signal waveform can be removed in SSME SNR estimation. Simulation results have shown that the modified SSME improves the SNR estimation accuracy for both the half-wave symmetric pulse-shaped and the asymmetric pulse-shaped signals, while without significantly increasing the computational complexity as compared with the conventional SSME.

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