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On the calculation of energy release rates in composite laminates by Finite Elements, Boundary Elements and Analytical Methods

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A B S T R A C T

To characterise a transversal crack evolution in a cross-ply $[0/90]_s$ fibre reinforced composite laminate, the associated energy release rate (ERR) was calculated by means of the J-integral embedded into the Finite Element Method (FEM). The ERR values computed for the propagation of the transversal crack were correlated to the ones obtained by using the Virtual Crack Closure Technique (VCCT) embedded within the Boundary Element Method (BEM). In addition, the results were compared with analytical values. The results correlated well except when the crack length was approximately 80% of the ply thickness. In such case, ERR values showed some discrepancies between FEM and BEM. The reason stems from the fact that in the VCCT used not all components of the stresses are considered, resulting in smaller ERR values. In addition, the results proved that transversal cracks can influence each other only in a limited distance.

1. Introduction

Matrix cracks often appear prior to other damage modes in composite materials. It is well known that the fracture process starts at micro-level as soon as the laminate is subjected to loading. In cross ply fibre reinforced laminates subjected to unidirectional loading micro-cracks appear immediately in the 90° plies. Increasing the load can lead to their coalescence and formation of macro-transverse cracks [1],[2]. Propagation of transverse cracks toward the interface can lead to other failure modes such as delamination or fibre fracture [3]. To assess the integrity of the laminate, transversal cracks can be modelled; although this has proved challenging due to shortcomings such as gradual redistribution of stress during fracture and degradation of mechanical properties [4], [5].

Characterizing the conditions which lead to failure of a composite laminate due to transversal cracks triggering delamination between laminae is a long-standing research topic [6], [7], [8], [9]. Studies on related engineering applications such as helicopter rotor blades prone to matrix cracking preceding fibre breaking [10], [11], cracking of large laminates used in wind turbines [12], damage created by hail ice impacts on leading-edge, control surfaces, engine nacelles, fan blades of aircrafts [13], and soft body impact [14] can be found in the literature. Modelling progressive damage [15] and [16], and prediction of fracture in composite laminates with different techniques, such as extended Finite Element Method (XFEM) [17], is another area of interest for obvious critical-safety reasons.

Researchers have attempted to predict different modes of failure in composite materials by using the energy release rate (ERR) as a fracture criterion. The modes of failure include but not limited to: compressive and inter-laminar shear failure in aerospace laminates [18], delamination growth due to buckling [5] and inter-ply cracking in adhesive-bonded aircraft composite joints [19].

There are several methods used for ERR evaluation and comprehensive study on these methods can be found in reference [20]. Xie et al [21] used VCCT in order to calculate the ERR using finite element method. Similarly Zou et al [5] implemented VCCT using the laminate theory instead of linear elastic fracture mechanics to express the energy released in terms of stress-jumps and relative displacements for modes I, II, and III. This approach allows individual

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ERR calculation for delamination and the singular stress at the crack tip is represented in terms of the stress resultant jumps across the delamination front. This approach can eliminate the oscillations in stress when incorporated in a finite element framework [5]. Conversely, VCCT with solid finite elements results in oscillatory stresses and displacements in front of the crack tip, which may cause divergence [22]. The use of VCCT within a boundary element framework (BEM) is deemed more appropriate because the boundaries of the problem are directly related to the problem features such as fracture parameters (see París et al.[1]). París and co-workers [1] considered different conditions (e.g. models with and without delamination) and calculated the ERR for different crack lengths. Their results were in good agreement with the analytical results by McCartney [23].

The J-integral approach as a measure of the energy release rate associated with crack propagation whereby a criterion can be introduced in terms of a bounding limit value has been studied in [24], [25] among others. One of the advantages of using J-integral is that under quasi-static conditions, it is equal to the energy release rate G for linear elastic materials. For two dimensional problems in a mixed fracture mode (mode I and mode II) loading the relation between stress intensity factor and J-Integral is

$$J = \frac{(1-\nu^2)}{E}(K_I^2 + K_{II}^2) \qquad (1)$$

where J denotes J-integral and, K_I and K_{II} are the stress intensity factors corresponding to mode I and mode II fracture, respectively. E and ν denote elastic modulus and Poisson's ratio respectively. The domain integral method is often integrated in commercial packages, e.g. Abaqus, to take advantage of J-integral path independency.

In this paper, parametric studies by finite element analysis with different specimen and crack lengths are conducted and compared with results by VCCT-BEM and analytical means. Results showing the stress distribution in the composite laminate are provided for better assessment of the structural integrity of the composite. In addition, discussion about finite element modelling features for ERR calculation is presented.

2. Background: J-integral

2.1 Finite element J-Integral calculation

J. R. Rice in 1968 [25] formulated Eshelby's [26] contour integral for crack problems. For an edge crack in a nonlinear elastic body the J-integral equals the rate of change (with respect to crack growth, *da*) in potential energy U_p:

$$J = -\frac{dU_p}{da} \qquad (2)$$

By definition, J-integral is equal to ERR if the material behaviour is linear and elastic. This definition is between Griffith's model of fully elastic and that of Irwin's with a small crack tip plasticity [27]. Fig.1. shows the contour Γ surrounding crack tip singularity in a semi-infinite two dimensional body.

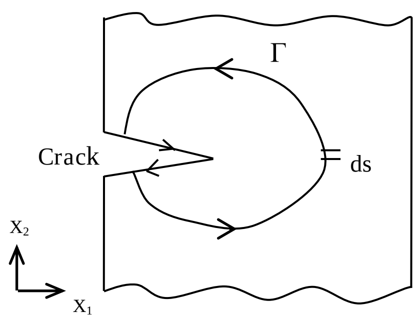

Fig.1. Integration path around crack tip

The change in potential energy for infinitesimal crack extension is:

$$-\frac{dU_p}{da} = \lim_{\Delta a \to 0} \frac{1}{\Delta a}\left(\int_{\Gamma} \bar{T}_i \,\Delta u_i ds - \int_{A} \Delta W dA\right) \qquad (3)$$

Where Δa is the crack growth and A is the area encompassed by the contour, W is the elastic strain energy, u_i is the displacement, $\bar{T}$ is the traction and ds is an infinitesimally small section of contour Γ.

Applying the Green's theorem, it is possible to write Eq. (3) in the form of a line integral:

$$J = \oint_{\Gamma} \left(W dx_2 - \bar{T}_i \frac{\partial \bar{u}_i}{\partial x_1} ds\right) \qquad (4)$$

with

$$\bar{T}_i = \sigma_{ij} n_j \qquad (5)$$

n_j is the normal vector to the integration path and σ is the stress component on Γ. The density of energy is expressed as:

$$W = W(x, y) = W(\varepsilon) = \int_0^{\varepsilon_{lk}} \sigma_{ij} d\varepsilon_{ij} \qquad (6)$$

For linear elastic material elastic strain energy is related to stress and strain as:

$$W = \sigma_{ij}\varepsilon_{ij}/2 \qquad (7)$$

The material within the contour integral is assumed homogeneous and the process is time independent. Line integrals have difficulty of implementation in FE and domain integrals are preferred [28]; unlike VCCT which uses nodal values for ERR calculation. Li et al [29] showed that line integral can be transformed into an equivalent area integral as shown in Fig.2.

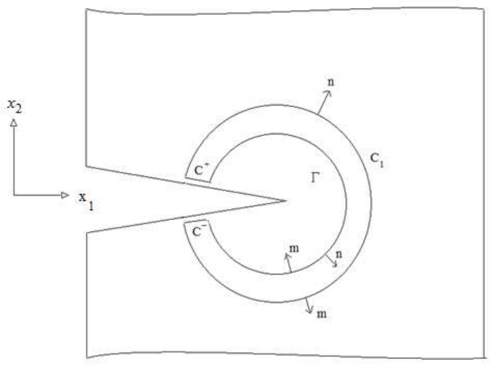

Fig.2. Domain definition for area integral

For linear elastic materials Atkinson and Eshelby [30] proposed the domain integral expressed as follows,

$$J = \lim_{\Gamma \to 0} \int_{\Gamma} \left(W\delta_{1i} - \sigma_{ij} u_{j,1} \right) n_i \quad ds \tag{8}$$

The process is assumed isothermal and body forces are neglected. *n* is the unit vector normal to Γ and δ_{1i} is the Kronecker delta. This formula can be rewritten in the form

$$J = \int_{C} [\sigma_{ij} u_{1j} - W\delta_{1i}] m_i q_1 ds - \int_{C^+ + C^-} \sigma_{2j} u_{j,1} m_2 q_1 ds \tag{9}$$

C is the closed curve $C=C_1+C^+ +C^- -\Gamma$ and q_1 is a smooth function in C domain which is unity on Γ and 0 on C_1, $m_1=0$ and $m_2=\pm 1$ on the crack surfaces and $m_j=-n_j$ on Γ. The second integral vanishes because of traction free surfaces of the crack. By applying the *divergence theorem* to the close contour (9):

$$J = \int_{A} \left[\left(\sigma_{ij} u_{j,1} - W\delta_{1i} \right) q_i \right]_{,i} \quad dA \tag{10}$$

A is the area enclosed by C. To implement this integration within the FEM, Shih et al [28] formulated the discretized form of Eq.(10) by means of Gaussian integration:

$$J = \sum_{elements \subseteq} \sum_{p=1}^{8} \left[\left(\sigma_{ij} \frac{\partial u_j}{\partial x_1} - W\delta_{1i} \right) \frac{\partial q_1}{\partial x_i} det \frac{\partial x_k}{\partial \eta_k} \right]_p w_p \tag{11}$$

Where

$$q_1 = \sum_{I=1}^{4} N_I Q_{1I} \tag{12}$$

For a quadrilateral element. N_I are the shape function and Q_{1I} is the nodal value for I^{th} node. Q_{1I} is zero on C_1 and 1 on Γ. Using the chain rule we have:

$$\frac{\partial q_1}{\partial x_j} = \sum_{I=1}^{4} \sum_{k=1}^{2} \frac{\partial N_I}{\partial \eta_k} \frac{\partial \eta_k}{\partial x_j} Q_{1I} \tag{13}$$

where $\frac{\partial \eta_k}{\partial x_j}$ is the inverse Jacobean matrix of transformations:

$$x_i = \sum_{K=1}^{4} N_K X_{ik} \,, u_i = \sum_{K=1}^{4} N_K U_{ik}, \text{ i=1,2} \tag{14}$$

N_k are the shape functions.

3. Modelling of Transverse Crack in $[0/90]_s$ laminate

A schematic view of the tested laminate composite HexPly8552 is shown in Fig.3. The thickness of each lamina is 0.55mm with *a* defining the crack length. 2L is the distance between two transversal cracks. The interface between the two plies is modelled without potential discontinuity and, therefore, there is no possibility of replicating delamination at this time.

When the $[0/90]_s$ laminae is under uniaxial loading, transversal cracks appear on the 90º ply and, then, progress towards the interface with the 0º ply. Further details of this can be found in the works by Dvorak and Laws [31] and Wang [32]. As the crack is assumed to be through the width [1]. A two dimensional model is justified for analysis.

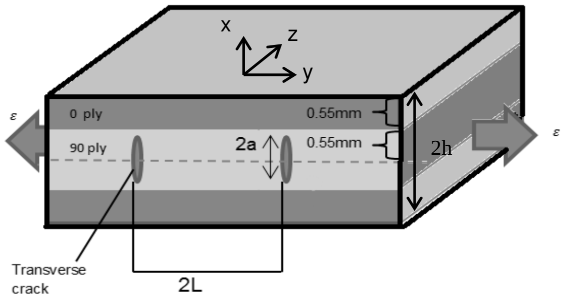

Fig. 3. Transverse crack in [0/90]s laminate

The properties for the laminate used are extracted from Hexcel datasheets [33]. This toughened epoxy resin system is made of unidirectional glass fibres. It is used in aerospace structures present in commercial airplanes, helicopters and jet engine parts.

Table1. Material properties for Hexeply8552. All the values for E and G are in GPa. E_{11} represents fibre direction elastic module.

E_{11}	E_{22}	E_{33}	G_{12}	G_{23}	G_{13}	ν_{12}	ν_{13}	ν_{23}
141.3	9.85	9.58	5	3.5	5	0.3	0.3	0.32

The fibre tensile strength is 2207 MPa. The transversal –resin- strength is 81 MPa. The laminate thickness is much smaller than the other two dimensions, i.e. width and length. Therefore, the strain in perpendicular direction can be neglected and, hence, a plane strain state can be assumed.

McCartney proposed the ERR analytical solution for the $[0/90]_s$ laminae. His calculation is based on the Gibbs free energy [1]:

$$\Delta G = \frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij}dV = \frac{1}{2}Lh\,\sigma\varepsilon \tag{15}$$

By substituting $\varepsilon = \sigma/E$, renders that,

$$\Delta G = \frac{1}{2}Lh\left(\frac{\sigma^2}{E}\right) \tag{16}$$

The ERR is calculated as the infinitesimal variation of G respect to the crack propagation (∂a):

$$ERR = \frac{1}{2}Lh\frac{\partial}{\partial a}\left(\frac{\sigma^2}{E}\right) \tag{17}$$

In McCartney's solution, σ, E and ε are axial components of stress, Young's modulus and strain in the first principal direction respectively. In this case, the principal direction is denoted as y-direction. París et al [1] calculate ERR by means of VCCT-BEM. VCCT calculates the ERR by multiplying the displacement of crack face nodes by the force required for crack closure at each node [1]. The interested reader is referred to reference [34] for further details.

4. 2D Cross-ply Laminate Model

The cross section of the laminate is modelled as two sections with separate material definition. No adhesive layer exists at the interface. The model represents half of the specimen in both vertical and horizontal directions where symmetric boundary conditions can apply. In this case only one surface of the crack is drawn, Fig.4. The analysis is quasi static and Abaqus standard solver is used.

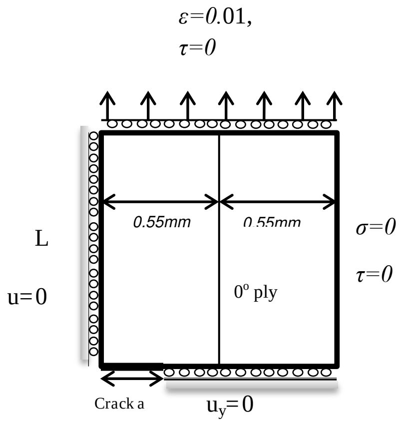

Fig.4. Boundary conditions and crack illustration.

In order to evaluate the results, the work by Blázquez et al [1] using VCCT-BEM and the work by McCartney [23] using analytical methods are used for comparison. Four contours were used in the test to check the consistency of the results. As the model dimension relatively large in to z direction, plain strain element is used. Furthermore, it is necessary to define the mesh such that the contour does not overlap the adjacent ply and to remain neatly homogeneous. The crack length was varied between 0 and 0.55 mm (which is 90 plies thickness) at 0.1mm intervals (smaller intervals were used near the interface as the mesh had to be refined). Four different model lengths, L, were tried (0.5mm, 1mm, 2mm, and 4 mm).

5. Results and Discussion

5.1 ERR for Different Crack Lengths

Fig.(5) shows the ERR calculated for L=2mm from three different methods (FEM, theoretical and BLM). J-integral values are slightly higher than those of BEM and analytical solutions. FE solver considers transverse stresses which are not considered in VCCT. Therefore g higher value of ERR is achieved. The maximum value is observed approximately at a=0.4 mm. Theoretically, the crack progresses if ERR is beyond the critical value G_c.

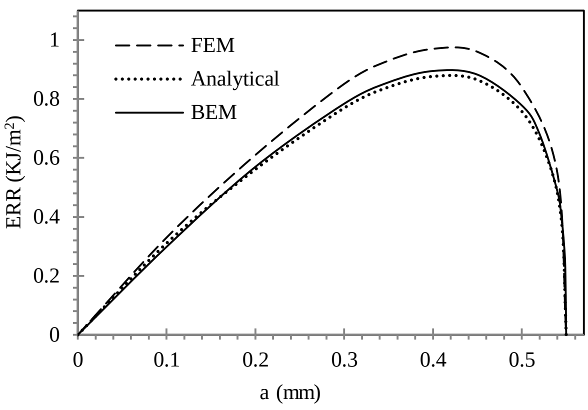

Fig.5. ERR for L=2 mm. The crack will extend unstably after a = 0.1 mm

For Hexply8552 G_c is equal to 0.3 KJ/m^2 for mode one crack opening [33]. For this load setting G_c is attained approximately a=0.1mm. When this value is reached the crack extends until somewhere between the maximum ERR and the interface where ERR becomes zero [1]. Theoretically because the ERR decreases to very small amount near the interface the crack must stop just before reaching the 0° ply. In the stress analysis it is clearly visible that for cracks so close to the interface the normal tension at the interface is much larger than the material strength and delamination might happen prior to the crack reaching the interface.

5.2 Transverse Crack Separation Length

Fig.(6) shows the variation of J-integral at similar strain for different model lengths (L=0.5mm, 1mm, 2mm and 4 mm). At L=2 mm, which is twice as large as the model thickness, the neighbouring cracks do not have influence on each other and larger specimens have the same ERR curve (Fig.6). This means that the crack existence does not affect the stress distribution on distances twice as large as the models' thickness.

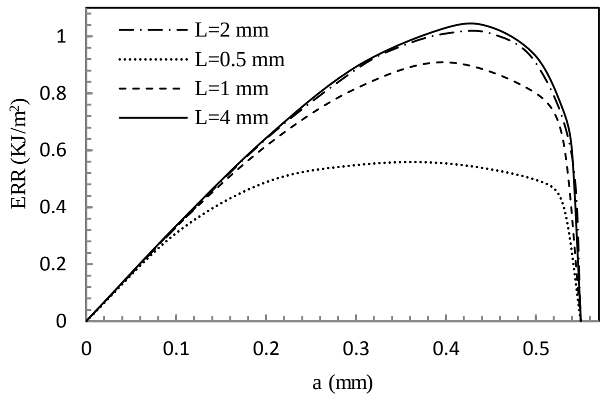

Fig.6. Comparison of ERR for different specimen length. If L greater than 2mm, the ERR for different lengths have small difference

There is a specific pattern for transverse cracks in composite materials which is related to the geometry and material that also constrains the number of cracks that can develop in the matrix. This limit is known as upper limit or "saturation state" [3] which can be related to the separation length. The stress releases because of the crack occurrence can only influence within certain distance.

5.3 Effect of mesh size and contour path

ABAQUS uses the contour integral method to calculate J-integrals. It automatically selects a node at the crack tip for the inner contour and the outer contours that pass through adjacent nodes. The contour should be confined within the homogeneous area. Fig.(7) depicts distinct contour . Mesh size analysis showed that for mesh sizes as large as 0.2 mm to very small mesh sizes (mesh size 0.01mm) convergence is attainable. It also showed independence of J-integrals to mesh size.

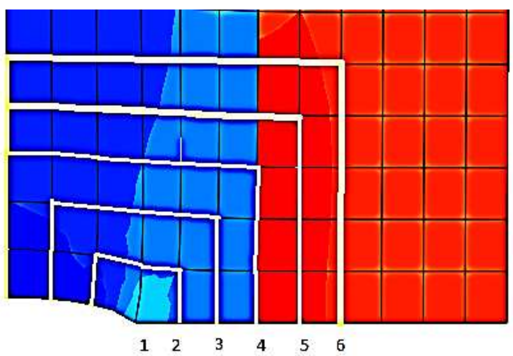

Fig.7. Contours around crack tip singularity in ABAQUS.

The first contour shows slightly higher value compared to the larger contours. No matter what the mesh size is Fig.(9). ABAQUS help suggests mesh refinement for accurate results and neglecting the first two contour outcomes. The approximation of FEA slightly affects the results of J-integrals but it must be noted that mesh refinement should not be related to contour integral accuracy. For the first contour, the error stem from crack tip singularity definition. This means mesh refinement does not increase the J-Integral accuracy anyway. This was also argued by Brocks and Scheider [35].

ABAQUS can use quarter-node element to create $r^{-1/2}$ singularity at crack tip by shifting the mid node of quadrilateral element toward the singularity (Fig.9). Barsoum [36] proposed this method for elastic and plastic crack tip region but Carka and Landis [37] suggested it is more suitable for linear elastic material.

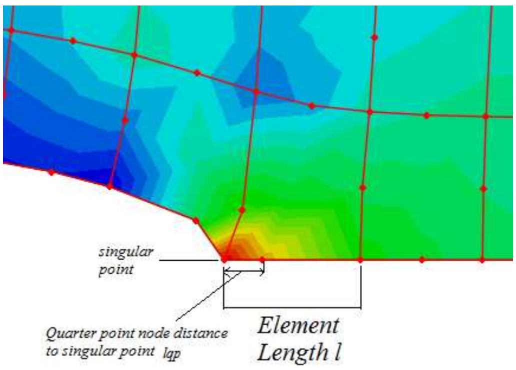

Fig.8. Quarter-point element technique shifts mid-node in 8 node quadrilateral element toward the crack tip to model singularity

The stress distribution close to the singularity is strongly dependant on the distance between the singularity and the mid-element node. In order to clarify the effect of quarter-point element on the J-integrals a test was run by changing the position of mid-element node along element side. The results show that the optimum position for the shifted node is around 25% of the element side length (Fig.9).

This might also show that this distance gives more accurate stress distribution for singularity, as it produced closer results for the J-integral.

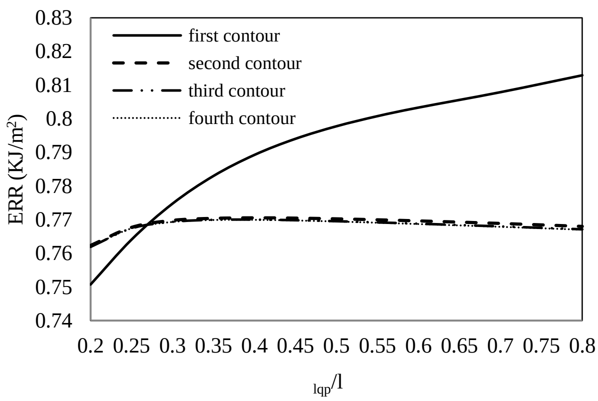

Fig.9. First contour integral result is ruled by the position of mid-node. l_{qp} is the mid-node distance to singularity and l_e is the length of the element.

5.4 Stress Distribution

When the crack progresses toward the interface, the normal stress and shear stress are changed significantly and can trigger other forms of failure including delamination. Figs.10a and 10b show the change in normal stress at the interface as the crack tip gets closer. The crack tip exerts large tensile force on the interface ahead of the crack.

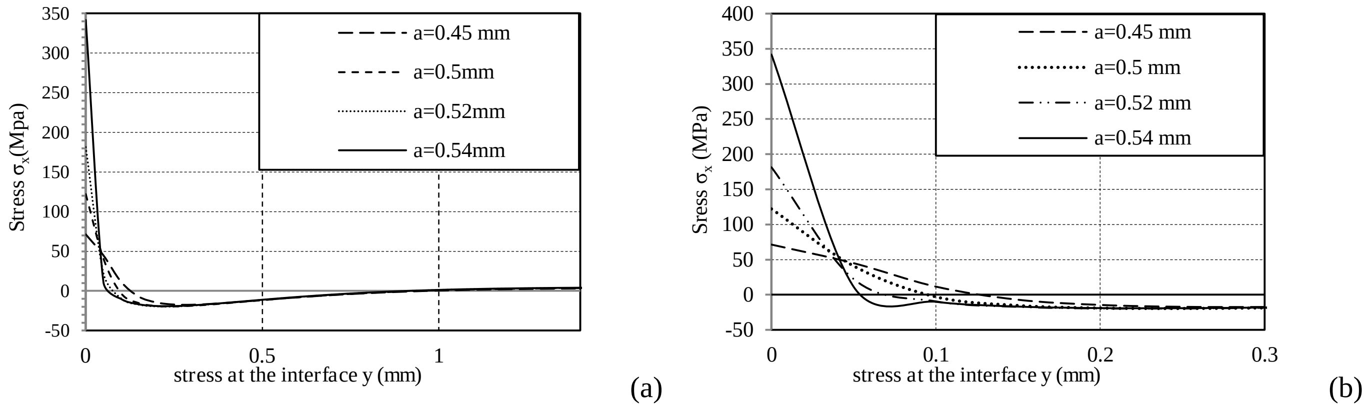

Fig.10. (a) Normal stress distribution along the interface for L=2mm and crack very close to the interface (b) Zoom in normal stress close to the crack tip area along interface for L=2mm and the crack near the interface

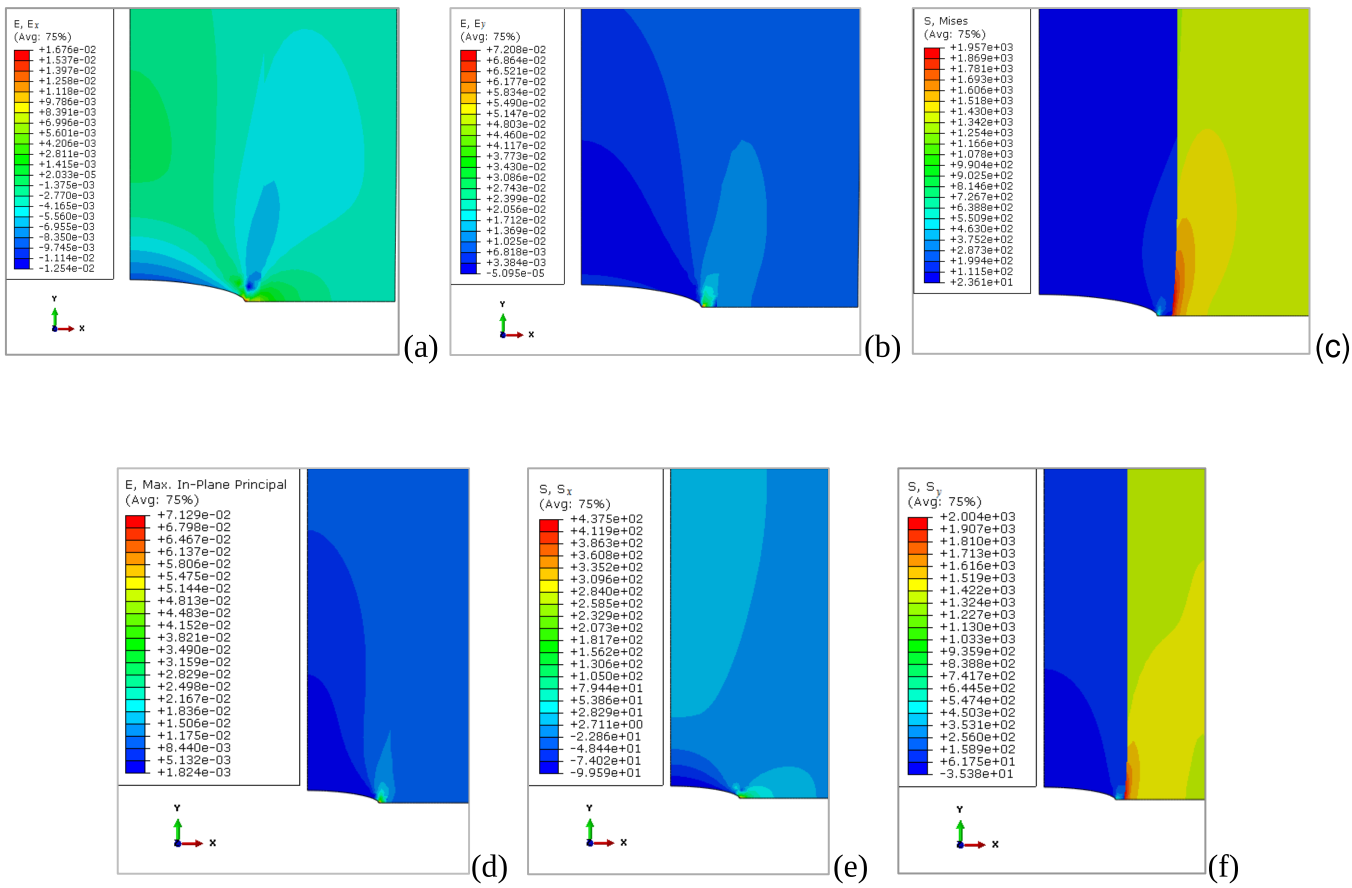

Figure 11 (a) Strain in transverse direction (zoomed) (b) Strain in loading direction (zoomed) (c) von Mises stress (zoomed) (d) Maximum in-plane principal strain (e) Stress in normal direction to loading and (f) Stress in loading direction for L=2 mm and a=0.5mm

The normal stress has a sudden increase when the crack is about to reach the interface and pushes normal stress to above the yield strength of the resin (approximately 81MPa). Concurrently, the normal stress decreases moving away from y=0 axis and between 0.1mm and 1mm, it is mostly compressive force at the interface. For cracks this close to the interface the specimen is experiencing large stresses but the J-integral does not predict crack propagation. Fig.(11) provides stress and strain contours as well as von Mises stress for a=0.5mm.

For shear stress (Fig.12) there is an alteration of direction at distances close to y=0. For a=0.54 mm (Fig.12), the shear stress is not zero at y=0 -unlike other crack lengths- because of the singularity definition.

As the tensile stress is far greater than the strength of resin at the interface, it is not realistic to consider a transverse crack without addressing delamination. París et al. investigated ERR for transverse crack when there is a delamination and the results showed that the ERR is the same for most of the crack lengths for cases when the crack tip is further than 0.1mm from the interface [1]. Also in an experimental study, París et al. verified their results on the transverse crack arresting just before reaching the interface and the effect of such cracks on delamination [42].

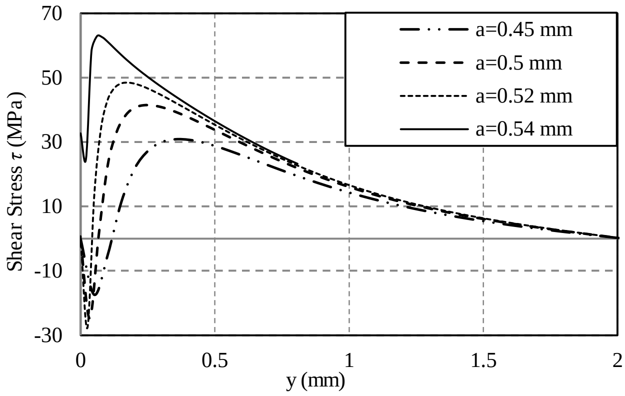

Fig.12. Shear stress at interface

6. Conclusion

ERR is a key factor in fracture analysis and, therefore, there is a genuine interest by engineers in developing accurate tools for its evaluation. The present study has compared three different methodologies for the calculation of the ERR associated to transversal crack in a $[0/90]_s$ laminate. Those are:

- The J-integral embedded into FEM.
- The VCCT embedded within BEM and,
- Analytical means.

The following remarks can be highlighted:

- ERR values were well-correlated by all three methodologies except when the crack length was approximately 80% of the ply thickness, i.e. to a 20% distance of the interface between distinct plies. VCCT neglects the transversal stress in this implementation of BEM and also the analytical expression that is the possible reason for smaller value of ERR. Therefore, it results in a certain difference only at the aforementioned distance in which that neglected component is significant.

- No mesh dependence was observed with the FEM for the calculation of the J-integral, although the number of contours used can effect at some point.

- The effect of displacing the mid-element node was investigated in this work. It was showed that the inaccuracy of the J-integral first contour is due to the definition of the singularity. The first contour provided the closest result when the mid node is placed at ¼ element length from the singular point.

- The stress distribution at the interface was also presented. It was observed that the stress reaches the maximum allowable at the interface between distinct plies when the transverse crack is very close to the interface.

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