Discussion of “Stability assessment of slopes with cracks using limit analysis”*

By Stefano Utili

Stefano Utili,
School of Engineering, University of Warwick, Coventry, UK
s.utili@warwick.ac.uk

Corresponding Author Stefano Utili
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Introduction

The discusser has recently published a paper in *Geotechnique* entitled "Investigation by limit analysis on the stability of slopes with cracks" (Utili, 2013) which included for the first time, to the discusser's knowledge, a systematic investigation on the influence of the presence of cracks in uniform slopes for rotational failure mechanisms via the limit analysis upper bound approach. Looking at the discusser's paper and the discussed paper, (Michalowski, 2013), a reader may note that the aim of the two papers is the same, namely to assess quantitatively the effect of the presence of cracks on the stability of slopes, and the methodology of using the upper bound approach of limit analysis. The discusser's paper was sent to *Geotechnique* when the discusser had no knowledge of either the author's conference paper (Michalowski, 2012), or of the discussed paper published in July 2013. On the other hand, the discusser's paper was published after the publication of the author's conference paper (Michalowski, 2012). Hence, it can be concluded that the discusser and the author had independently developed an original formulation for the calculation of upper bounds based on rotational failure mechanisms for cracked uniform slopes at similar times.

However, regarding the findings and the formulation of the problem, (Utili, 2013) and the discussed paper, (Michalowski, 2013), are rather different. In this discussion, the discusser wants to highlight the main complementary and different findings between the two papers and point to some aspects of the discussed paper that in the discusser's opinion need clarifications especially with regard to the following three topics each being a section of the present discussion: the calculation of the external rate of work for rotational failure mechanism; the failure mechanisms analyzed for pre-existing cracks; and the failure mechanisms with crack formation.

Calculation of the external rate of work for rotational failure mechanisms

Concerning the calculation of the external rate of work in case of a rotational failure mechanism, the author reports neither the derivation nor the final analytical expressions of the functions employed to calculate the rate of the external work done by the soil mass sliding away, wedge BOCDB in figure 6. The calculation of the external work is as important as the calculation of the energy dissipated since both appear in the energy balance equation from which the stability factor, $\gamma H/c$ (Taylor, 1948; Chen, 1975), is calculated. In this regard, the author seems to justify the lack of detail provided making reference to the works of Chen (e.g. Chen & Giger, 1969; Chen & Giger, 1971) and stating that "without a crack, this mechanism has been described in the literature multiple times". However, this is not the case. In fact, in the referenced Chen's publications, the calculation of the external work rate is reported only for failure surfaces made entirely by a log-spiral (wedge BOCADB in figure 6) with either the logspiral passing through the slope toe (see Fig. 6a) or below (see Fig. 6b). Here instead, the failure surface is composite: partly log-spiral and partly planar (wedge BOCDB in figure 6). The calculation of the external work in this case of a composite partly log-spiral partly linear failure surface requires the calculation of the work done by the fictitious wedge BOCADB minus the work of the fictitious wedge DCAD (Utili, 2013). The analytical expressions for the calculation of the external work done by soil masses sliding along composite log-spiral failure surfaces, which requires the use of fictitious wedges bordered by a log-spiral, was first presented in (Utili, 2005; and Utili & Nova, 2008) for the case of slopes with horizontal upper part subject to a sequence of landslides, and in (Utili and Crosta, 2011) for the more general case of slopes with an inclined upper part. In (Utili and Nova 2007), the calculation of the work done by a wedge enclosed by two log-spirals is presented. In these publications the calculation of the external work is reported in detail together with the related analytical expressions.

Failure mechanisms for pre-existing cracks

In the analysis of rotational failure mechanisms for slopes with pre-existing cracks, rightly the author states that the minimization of the function providing the stability number is a problem of constrained minimization because of the constraint on the maximum depth of the crack. In the search for the failure mechanism of a slope of given inclination and friction angle (β and ϕ respectively), the length and location of the crack is free, *i.e.* the minimization of the function is sought over 4 independent variables, the angles θ_0, θ_h, θ_C (χ, ν, ζ in (Utili, 2013) with $\chi=\theta_0, \nu=\theta_h, \zeta=\theta_D$) and β' with the additional constraint that "the crack cannot be deeper than the maximum depth of the crack discussed". Concerning the variable β', the discusser has shown that for ϕ>5°, all the failure mechanisms pass through the slope toe, i.e. $\beta'=\beta$, whatever values of β and ϕ are considered (Utili, 2013), so that for the drained analyses presented in the discussed paper with ϕ=10° or greater the number of variables to be considered in the unconstrained minimization can be reduced to three: θ_0, θ_h, θ_C.

With regard to the maximum crack depth allowable, unfortunately, the author does not state when the constraint turns out to be active, *i.e.* for what values of the parameters β and ϕ. In case of dry slopes, employing the formula given in Eq. (5), the discusser has verified that this limit on the maximum crack depth is never exceeded by the crack depth resulting from the unconstrained optimization of the function expressing the stability factor for all the considered values of β and ϕ (see Figure 1). The interested reader can find the analytical expression of the function reported in Eq. (25) in (Utili, 2013). This implies that the inequality of Eq. (5) is not active so that the minimization presented in the discussed paper is actually an unconstrained minimization rather than a constrained one providing the solution to the problem of determining the critical failure mechanism for slopes with cracks of unspecified location and depth (problem *c* in Utili, 2013). This solution is a particular case of the solutions found for the other two dual problems tackled in (Utili, 2013): determination of the critical failure mechanism for slopes with a crack of known length but unspecified location (see Figure 2a) and determination of the critical failure mechanism for slopes with a crack of known location but unknown depth (see Figure 2b), problems *a* and *b* respectively in (Utili 2013), which are not tackled in the discussed paper. The solution to these problems is provided by a genuine constrained optimization where the minimum for the function expressing the stability factor, is sought with the additional constraint of satisfying a non-linear equality prescribing, in case of problem *a*, the crack depth, and in case of problem *b*, the crack location, so that the number of independent variables in both problems is reduced to two. The stability factors found for these two problems, are a function of the crack depth and of the crack distance respectively (the imposed constraints), and their minimum with respect to crack depth and crack distance corresponds to the solution presented in the discussed paper for the case of cracks of any depth and location (see Figure 3).

With regard to the geometry of the failure mechanisms, it is important to note that in the discussed paper, failure mechanisms are assumed to pass either through the toe or below without consideration for mechanisms daylighting on the slope face above the toe. However, unlike the case of intact slopes, the presence of cracks implies that mechanisms passing above the slope toe are no longer self-similar (see figure 4) and therefore need to be considered in the calculation of the upper bounds. From the calculations in (Utili, 2013), it turns out that in case of dry slopes with either dry or water filled cracks, the failure mechanisms pass through the slope toe. However, for different hydraulic conditions as the ones considered in the discussed paper and in case of failure mechanisms accounting for crack formation, mechanisms daylighting on the slope face could still turn out to be more critical than the mechanisms passing through the toe assumed in the discussed paper. Hence, it could be interesting to know if the mechanisms considered by the author are still the most critical once potential failure mechanisms daylighting on the slope face are accounted for in the calculations.

Finally, concerning how good the achieved upper bounds are, the following remark in the paper “Of all admissible two-dimensional slope collapse mechanisms for soils considered in the literature, it is the rotational one that has been found most critical for uniform slopes (Chen, 1975)" overlooks the fact that (Bekaert 1995) found an upper bound of 1.0% lower for a vertical uniform slope with ϕ=0, by considering a multiple rotation mechanisms made of several log-spiral blocks. However, although it has to be pointed out that Chen's upper bounds obtained assuming a rigid rotation may no longer be the best upper bounds in the light of more recent works in the literature, they are very close to the true collapse load: for instance (Krabbenhoft *et al.,* 2005) achieved lower bounds by finite element limit analyses which are on average 1.5% and in the most unfavourable case 2.5% less than the upper bounds obtained for β ranging from 50° to 90° and ϕ from 10° to 40°. Conversely, it is crucial, in the discusser's view, to point out that when cracked slopes are considered, no lower bound solutions are available in the literature to bracket the true collapse values; therefore in case of cracked slopes it cannot be taken for granted that the upper bounds obtained for rigid rotational mechanisms are still close to the true collapse load. In this regard, it is reasonable to expect that at low crack depths, the upper bounds remain close to the true values whereas for high values of crack depths, they may diverge substantially. This limitation of the presented solutions should be acknowledged. Also in the conclusions, the author remarks that "for slopes with an inclination of 30° or less, the calculated critical height is little or not affected by the presence of a crack. The influence of the crack presence becomes significant however with an increase of the slope inclination". On this point it is interesting to note that if the newly found upper bounds for rotational failure mechanisms are compared to the upper bounds relative to planar mechanisms, the reduction on the stability factor determined, *i.e.* the improvement of the upper bounds of the new solution in comparison with the available bounds for planar mechanisms (Hoek and Bray, 1977), the trend is opposite with the upper bound reduction being higher for shallow slopes (Utili, 2013).

Failure mechanisms including crack formation

Concerning translational mechanisms, the discusser points out to a typographical error in the equation provided for the dilation angle, $\delta = \frac{\pi}{2} - \theta - \phi$, which instead needs to be $\delta = \frac{\pi}{2} - \theta + \phi$ for the mechanisms to be kinematically admissible.

Concerning rotational mechanisms with crack formation, the paper does not specify what physical phenomena cause the envisaged formation of the cracks. This is an essential point if the analysis is to be realistic. In the presented analysis, a non-zero shear stress state underneath the crack tip has been assumed for respect of the normality rule, given the direction of the velocity vectors underneath the crack tip as the author's points out: "The stress associated with this kinematics is described by the circular portion of the yield condition. This is not necessarily the true stress state but it is consistent with the selected kinematics". However, if one considers the starting point where the crack begins to form, *i.e.* at the ground level on the horizontal upper part of the slope, the presence of shear stresses violates equilibrium since no loads are applied on the slope. Moreover, the author does not specify how the envisaged shear stress relates to any of the several different possible physical phenomena leading to crack formation: e.g. desiccation, wetting, and drying cycles, weathering.

Finally, the first statement in the conclusions “It was demonstrated that crack formation is an important factor affecting the outcome of stability analyses of slopes.” appears unjustified for the fact that when crack formation is considered, the failure mechanisms turn out to be less critical than the case of pre-existing cracks, so in the stability analysis of uniform slopes, consideration of crack formation is not critical according to the analysis performed. More importantly, the usefulness of the whole stability analysis with crack formation is rather debatable since the crack formation mechanisms considered are driven by an unrealistic state of stress in the ground for the aforementioned violation of the equilibrium at the boundary of the slope (where the crack begins to form) and it has not been related to any physical phenomenon causing the formation of cracks.

Summary

The discussed paper (Michalowski, 2013) presents an interesting analysis of the stability of slopes subject to vertical tension cracks. The authors considered both pre-existing cracks and forming cracks, focusing considerable attention on the maximum possible crack depth and seepage effects. These findings, when considered in conjunction with the independently obtained findings of the discusser’s paper (Utili, 2013), are likely to provide a comprehensive analysis of the effect of the presence of cracks in various scenarios. The fact that two independent authors developed these original formulations at simultaneous times demonstrates how strong the interest of the geotechnical community is in this area. I hope that this discussion will contribute to the advancement of this area of research.

References:

Bekaert, A. (1995). Improvement of the kinematic bound for the stability of a vertical cut-off. Mech. Res. Comm., **22**(6): 533-540.

Chen, W.F., Giger, M.W., Fang, H.Y. (1969). On the limit analysis of the stability of slopes. Soils and Foundations, **9**(4): 23-32.

Chen, W.F., Giger, M.W. (1971). Limit analysis of stability of slopes. Soil mechanics and foundations division, ASCE, **97**(1): 19-26.

Chen, W.F. (1975). *Limit analysis and soil plasticity*. New York. NY, USA: Elsevier.

Hoek, E. and Bray J.W. (1977). Rock slope engineering. 2nd edn. London, UK: Institution of mining and metallurgy.

Krabbenhoft, K., Lyamin, A. V., Hjiaij, M. & Sloan, S. W. (2005). A new discontinuous upper bound limit analysis formulation. International Journal of Numerical Methods in Engineering, **63**, 1069-1088.

Michalowski, R. (2012). Cracks in slopes: limit analysis approach to stability assessment. In Proceedings of Geocongress, Oakland, California (USA), March 2012, 442-450.

Michalowski, R. (2013). Stability assessment of slopes with cracks using limit analysis. Canadian Geotechnical Journal, **50**, 1011-1021.

Utili, S. (2013). Investigation by limit analysis on the stability of slopes with cracks. Geotechnique, **63**(2): 140-154.

Utili S. (2005). An analytical relationship for weathering induced slope retrogression: a benchmark. RIG (Italian Geotechnical Journal), **39**(2): 9-30.

Utili, S. and Crosta G. B. (2011). Modeling the evolution of natural cliffs subject to weathering: 1. Limit analysis approach. Journal of Geophysical Research **116**, F01016.

Utili, S. and Nova, R. (2007). On the optimal profile of a slope. Soils and foundations, **47**: 717-729.

Utili, S., Nova, R. (2008). DEM analysis of bonded granular geomaterials. International Journal for Numerical and Analytical Methods in Geomechanics, **32**(17): 1997-2031.

Taylor, D. W. (1948). Fundamentals of soil mechanics. New York, NY, USA: John Wiley and sons.

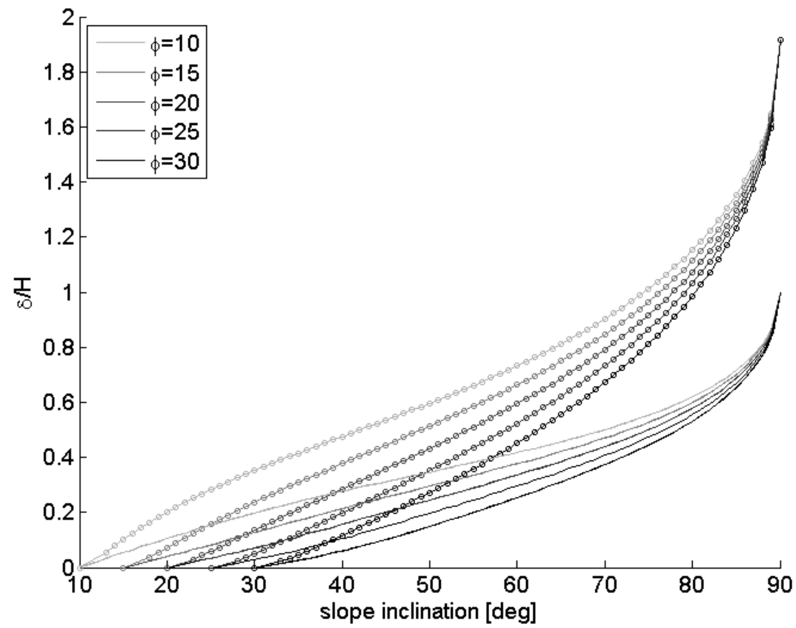

Figure 1. Failure mechanisms for cracks of any possible depth (δ) and location. The crack depth corresponding to the failure mechanism is plotted versus slope inclination for various friction angles: the lines without markers indicate the crack depths whilst the lines with markers indicate the maximum crack depth according to Eq (5) of the paper.

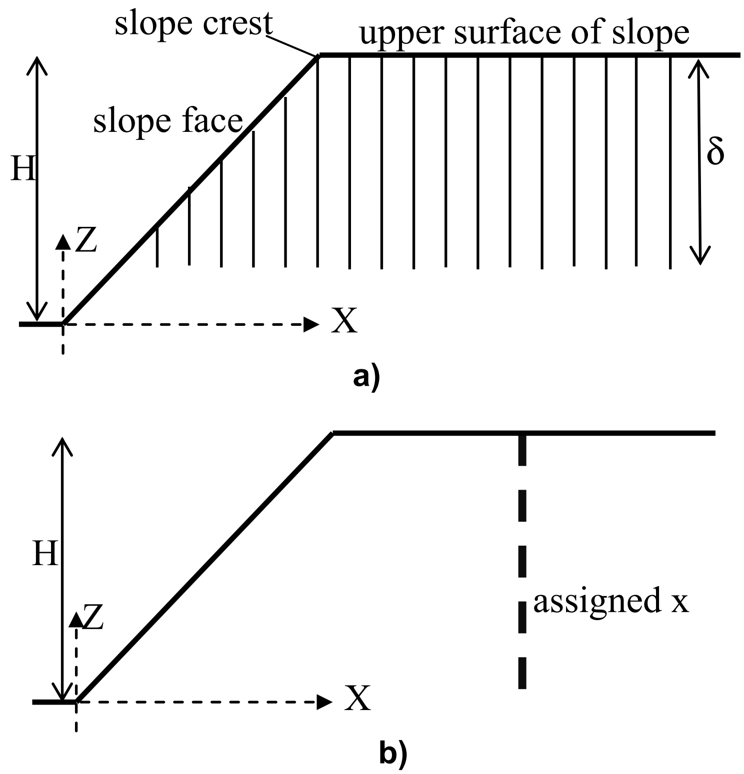

Figure 2 after (Utili, 2013). Problem a), the upper bound is sought for a fixed crack depth, δ, with the crack lying at any possible horizontal distance from the slope toe, x (the black lines representing the vertical cracks can be anywhere within the gray region). Problem b), the upper bound is sought for a crack of unknown depth (any δ is possible) located at a fixed horizontal distance from the slope toe, x.

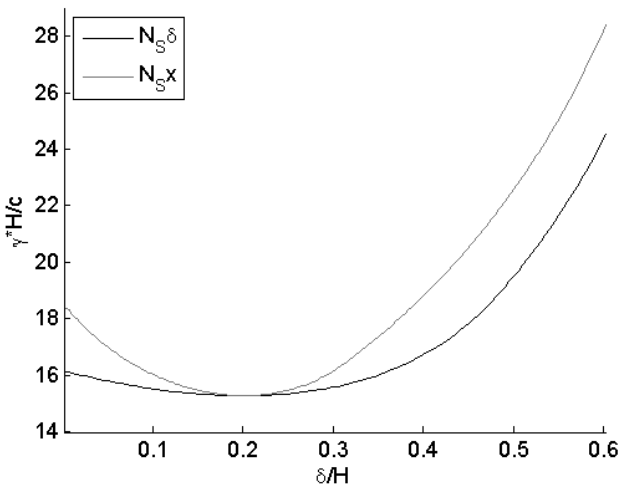

Figure 3 after (Utili, 2013). Stability factor obtained by constrained minimization vs. crack depth for a slope with ϕ=20° and β=45°: the gray line represents the stability factor, N_S^x, obtained for cracks of fixed location, x, whilst the black line represents the stability factor, N_S^δ, obtained for cracks of fixed depth, δ. The minimum of the curves corresponds to the stability factor associated to the failure mechanism analyzed in the discussed paper for a crack of any depth and location, problem c in (Utili, 2013).

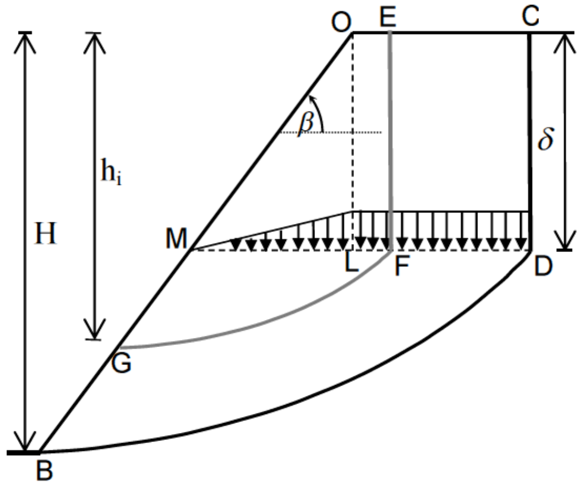

Figure 4. The gray log-spiral G-F represents a potential failure mechanism passing above the slope toe whilst the black one B-D the failure mechanism passing through the toe. The lack of self-similarity between the two mechanisms is due to the fact that the self-weight of the triangular region MOLM gives rise to a linearly distributed load on M-L whereas the rectangular region LOCDL to a uniformly distributed load on L-D (after Utili, 2013).