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D2D Multi-Hop Routing: Collision Probability and Routing Strategy with Limited Location Information

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Abstract—In this paper, we define a collision area in a heterogeneous cellular network for the purpose of interference management between Device-to-Device (D2D) and conventional cellular (CC) communications. Currently, most D2D routing algorithms assume synchronized accurate location knowledge among users and the base stations. In reality, this level of location accuracy is difficult and power consuming in Universal Mobile Telecommunications System (UMTS). In current Long-Term Evolution (LTE), there is no location information from the cell besides range information from time measurements.

In the absence of accurate location information, we analyze the collision probability of the D2D multi-hop path hitting the defined collision area. Specifically, we consider the problem for three different routing scenarios: intra-cell, intra-cell to cell boundary, and cell boundary to boundary routing. As a result, we propose a dynamic switching strategy between D2D and CC communications in order to minimize mutual interference. The gradient-based switching strategy can avoid collision with the collision area and only requires knowledge of the current user and the final destination user's distances to the serving base station.

I. INTRODUCTION

Device-to-Device (D2D) communications is a technology for enhancing the cellular network capacity [1], [2], as well as the energy- and spectral-efficiency [3], [4] in order to meet the increasing demands for high data rate access. D2D communications has been identified by the Third Generation Partnership Project (3GPP) as a potential candidate technology to offload delay-tolerant data traffic away from conventional cellular (CC) channels [5]. For D2D communications in co-existence with an overlay co-frequency CC network, one of the key challenges is interference management between the two tiers. Existing research has shown that a well-managed interference mitigation scheme can increase D2D communications reliability. One mechanism proposed in [6] found that the D2D receivers can exploit a retransmission of the interference signal from the base station (BS) to cancel the interference from prior CC transmissions. Whilst this can improve the D2D outage probability, it does not consider the interference from the other D2D user equipments (UEs). Furthermore, when a large number of UEs communicate at the same time, retransmission of the interference will cause significant resource overheads. Alternative algorithms have focused on balancing the radio resource and transmission energy between the two tiers in both centralized and de-centralized optimal allocation schemes [7]. However, the question regarding when to use D2D and when

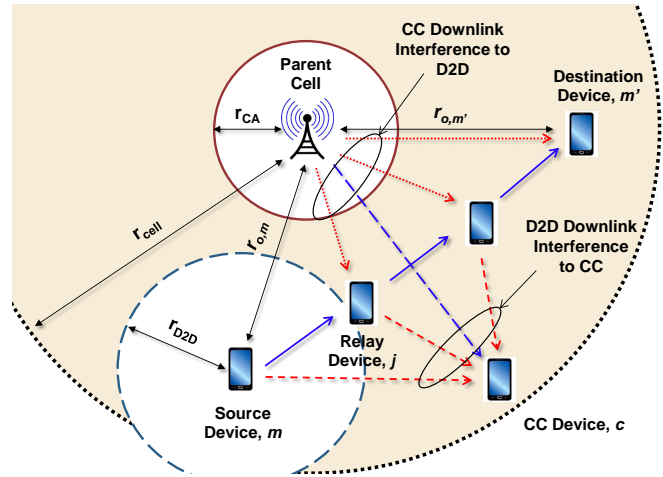


Fig. 1. Illustration of D2D multi-hop routing in co-existence with a CC communication tier.

to use CC remains open, especially in the absence of full user location information.

A. Interference Zones

An alternative interference mitigation concept is the creation of an abstract interference zone, which is commonly defined as a circular area, centered on a point of interest (e.g. a macro- or femto-BS). The radius of the zone is directly related to a certain quality of service (QoS) requirement. For example in [8], the interference zone has been defined as the interference-limited coverage area (ILCA) for a femto-cell heterogeneous network to mitigate interference from femto- and macro-BSs. In this particular case, the ILCA of a femto-cell is an area within a circle centered by femto-BS, with a radius such that the edge of the circle has equal power levels from the nearest femto- and macro-BS. As a result, the channel allocation is based on the UE location with respect to the ILCA zone. Another example is given in [9], whereby the interference zone for a specific interference signal ratio (ISR) threshold δ_D is used to enhance the D2D throughput. The authors introduced an interference limited area (ILA) resource allocation scheme, where the δ_D -ILA is defined in terms of the transmission targets of cellular UEs and D2D UEs, and it is the area where the ISR at the D2D receiver is greater than a predetermined threshold δ_D . The premise is that CC UEs will be prevented

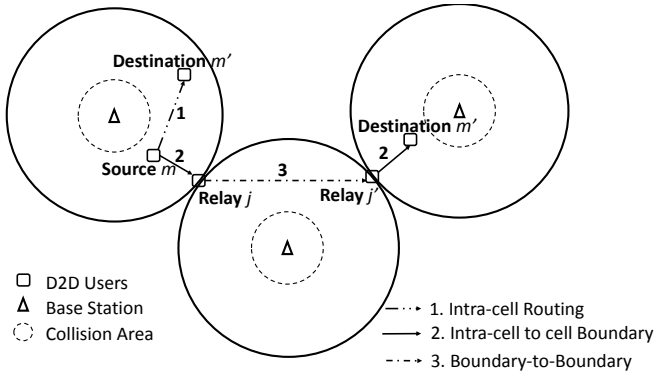


Fig. 2. Illustration of three different routing paths: intra-cell; intra-cell to cell boundary; and cell boundary to cell boundary.

from transmitting when inside the δ_D -ILA zone.

B. Contribution and Organisation

Currently, most routing and D2D papers assume synchronized accurate location knowledge among UEs and BSs. In reality, this level of location accuracy is difficult and power consuming in Universal Mobile Telecommunications System (UMTS). In pre-Release 9 and Release 10 of Long-Term Evolution (LTE), there is no location information from the cell besides range information from time measurements. We assume that each D2D UE only has knowledge of its relative distance to the nearest BS, and has no knowledge of its specific location or the location of other UEs. However, it does know the QoS targets required, as well as the final destination UE's distance to the nearest BS. On this basis, it must decide to use CC or D2D communications based on this.

In the rest of the paper, we define a variable interference zone called the collision area (CA). Inside the CA, the D2D receiver signal quality is less than a required Quality-of-Service (QoS) threshold. In Section III, we consider randomly located D2D UEs that employ the Shortest-Path-Routing (SPR) algorithm [10] to route packets within one or more macro-BSs' coverage area. We derive the probability that the D2D multi-hop routing path collides with the defined CA. In Section IV, we present collision probability results and find an optimal switching strategy between CC and D2D communications.

II. SYSTEM MODEL

A. D2D Routing Scenarios

The system considered in this paper is a downlink (DL) Orthogonal Frequency Division Multiple Access (OFDMA) based multiple-access network. As shown in Fig. 1, a CC communication tier exists as an umbrella over the D2D communication tier, both sharing the same spectrum due to resource scarcity and heavy traffic loads. We consider the multi-hop communications between two arbitrary located D2D UEs within one or more macro-BSs. For the SPR algorithm [10], only the locations of the source m and destination m'

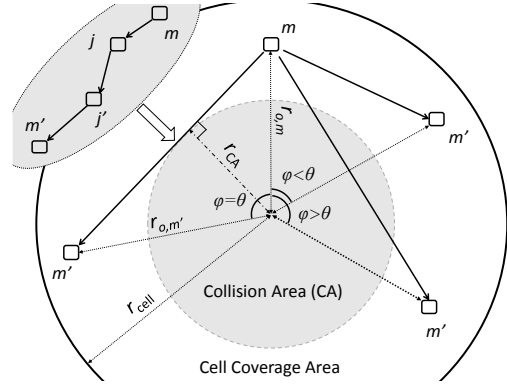


Fig. 3. Illustration of multi-hop routing from a source m to destinations m' with three different possible m' locations (the distance $r_{0,m'}$ is constant).

UEs determine the multi-hop path. As shown in Fig. 2, there are three scenarios:

- 1) *Intra-cell routing*: the D2D source and destination UEs are in the same cell.
- 2) *Intra-cell to cell boundary routing*: one of the source and destination UEs is on the cell boundary.
- 3) *Cell boundary to boundary routing*: both the source and destination UEs are on the cell boundary.

The assumptions in this paper are as follows. The traffic model is assumed to be full buffer and the relaying protocol used is a non-cooperative Decode-and-Forward (DF) protocol. The D2D UE density is sufficiently high that the SPR path can be approximately modelled by a straight line, and the average hop distance between D2D UEs is short. Furthermore, the interference received at each D2D UE is from two sources: (i) all the other co-channel D2D UEs, and (ii) the dominant interference from the nearest macro-BS.

B. D2D UE Distribution

The BSs are deployed from a Poisson process $\Phi_{BS} = \{x_1, x_2, \dots\}$ of density Λ_{BS} . The D2D user locations are generated by Poisson cluster processes, which applies homogeneous independent clustering to an existing BS process [11], where N_{D2D} is the mean number of D2D UEs in each BS. The D2D clusters are $N_{x_i} = N + x_i$ for each $x_i \in \Phi_{BS}$. The whole process of Φ_{D2D} is: $\Phi_{D2D} = \bigcup_{x \in \Phi_{BS}} N_x$. A doubly Poisson cluster process is addressed for generating the D2D UEs distribution. The D2D UEs are uniformly scattered on the ball of radius r_{cell} at each BS. For the aforementioned spatial distribution, the density function of D2D UEs is [12]:

$$\Lambda_{D2D}(x_i) = \frac{N_{D2D}}{\frac{\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2}+1)} r_{cell}^d} \mathbf{1}_b(0, r_{cell})(x_i), \quad (1)$$

where d is the number of dimensions, $\Gamma(\cdot)$ is the Gamma function, and $\mathbf{1}_b(0, r_{cell})(x_i)$ is the indicator function of the condition $x_i \in (0, r_{cell})$. In our paper, we only consider 2-D Poisson process.

In Fig. 3, we consider routing information from a source m to a destination m' , via a series of D2D relay UEs. We

$$\mathbb{P}_{\text{Collision}} = \begin{cases} 1 \\ \mathbb{P} \times (1 - \mathbb{P}_{\text{CA}}) + \mathbb{P}_{\text{CA}} \end{cases} = \begin{cases} 1 & 0 \leq r_{o,m'} \leq r_{\text{CA}} \\ \frac{\pi - \arccos\left(\frac{r_{\text{CA}}}{r_{o,m}}\right) - \arccos\left(\frac{r_{\text{CA}}}{r_{o,m'}}\right)}{\pi} e^{-\pi\Lambda_{\text{BS}}r_{\text{CA}}^2} + 1 - e^{-\pi\Lambda_{\text{BS}}r_{\text{CA}}^2} & r_{\text{CA}} < r_{o,m'} \leq r_{\text{cell}} \end{cases} \quad (7)$$

ignore sectorised BS antennas for now, and assume an omnidirectional antenna. We consider two random and adjacent relay UEs denoted j and j' . The instantaneous signal-to-interference-plus-noise ratio (SINR) from j to j' is:

$$\gamma(r_{o,j'}) = \frac{H_{j,j'} P_{\text{D2D}} \lambda r_{j,j'}^{-\alpha}}{W + P_{\text{BS}} \lambda r_{o,j'}^{-\alpha} + \sum_{i \in \Phi, i \neq j'} H_{i,j'} P_{\text{D2D}} \lambda r_{i,j'}^{-\alpha}}, \quad (2)$$

where W is the AWGN power, $r_{o,j'}$ is the distance between the nearest BS o and UE j' , $r_{j,j'}$ is the distance between the two UEs, H is the fading gain, λ is the frequency dependent pathloss constant, and α is the pathloss distance exponent. P_{D2D} is the transmission power of D2D UEs, and P_{BS} is the transmission power of the BS. Given that the aggregate interference power is typically significant higher than noise power, it can be assumed that the AWGN power is negligible. A reference of the mutual cross-tier interference scenario can be found illustrated in Fig. 1.

C. Collision Area (CA)

We now consider the CA, where we define the edge as having an SINR equal to a threshold ζ . The radius of the CA is defined as r_{CA} . In order for the receiver's SINR $\gamma \geq \zeta$ (i.e., the receiver is outside the CA), we set $r_{o,j'} = r_{\text{CA}}$ in Eq. (2). Without considering instantaneous fading effects, r_{CA} can be found as:

$$r_{\text{CA}} = \left[\frac{P_{\text{D2D}}}{P_{\text{BS}} \zeta} (r_{j,j'}^{-\alpha} - \zeta \sum_{i \in \Phi, i \neq j'} r_{i,j'}^{-\alpha}) \right]^{-1/\alpha}. \quad (3)$$

For the case when the D2D interference is negligible ($r_{i,j'}$ is large), the CA zone can be said to be proportional to the QoS SINR threshold ζ with an exponent value of $1/\alpha$.

III. COLLISION PROBABILITY

For a known CA radius, the destination UE can be detected whether it is located inside the CA via the pilot channel power from the nearest BS. When the destination UE is inside the CA, the collision probability is 100%. In which case, D2D communications is forbidden. If the destination UE is outside the CA, two possibilities for the collision between the D2D routing path and the CA exist, namely: (1) the source UE is located in the CA; (2) the source UE is located out of the CA but the routing path passes through the CA.

We assume that the source UE or any of the subsequent relay UEs have no knowledge of where the destination is or where any other UEs are. Each UE only knows their own relative distance to the nearest BS. We now consider the collision probability in that context for the three routing paths shown in Fig. 2.

A. Intra-cell Routing

Fig. 3 illustrates the intra-cell routing (scenario 1 in Fig. 2), where $r_{o,m}$ is the distance between the source m and nearest BS o , $r_{o,m'}$ is the distance between the destination m' and BS o , φ is the angle between $r_{o,m}$ and $r_{o,m'}$, and $\varphi = \theta$ when the routing path is a tangent to the CA.

1) *Source UE Inside the CA*: As mentioned previously, two possibilities exist for the multi-hop path to enter the CA when the destination UE is outside the CA. We consider now the first possibility, namely when the source UE is inside the CA. Since we only consider the location of one particular UE, the probability density function (pdf) of finding one UE at a distance $r_{o,j'}$ from the nearest BS can be leveraged [13]:

$$g(r_{o,j'}) = 2\Lambda_{\text{BS}} r_{o,j'} \pi e^{-\Lambda_{\text{BS}} \pi r_{o,j'}^2}, \quad (4)$$

Therefore, the probability for finding this particular UE inside the CA area is:

$$\mathbb{P}_{\text{CA}} = \int_0^{r_{\text{CA}}} 2\Lambda_{\text{BS}} r_{o,j'} \pi e^{-\Lambda_{\text{BS}} \pi r_{o,j'}^2} dr_{o,j'} = 1 - e^{-\pi\Lambda_{\text{BS}} r_{\text{CA}}^2}. \quad (5)$$

2) *Collision Probability*: When the source and destination UEs are both outside the CA, the collision probability is the probability of the multi-hop path colliding with the CA. Given that we only know the distance of the source and destination from the BS, there are a number of possibilities. As shown in Fig. 3 when $\theta < |\varphi| \leq \pi$, the routing path will pass through the CA. Given the uniform user distribution, the distribution of $|\varphi|$ is also uniform. Therefore, the probability of the routing path passing through the CA is:

$$\mathbb{P} = \int_{\theta}^{\pi} \frac{1}{\pi} d\varphi = \frac{\pi - \arccos\left(\frac{r_{\text{CA}}}{r_{o,m}}\right) - \arccos\left(\frac{r_{\text{CA}}}{r_{o,m'}}\right)}{\pi}. \quad (6)$$

From Eq. (5) and Eq. (6) the collision probability for intra-cell routing is shown in Eq. (7). The value of r_{CA} is determined by the QoS target set out previously in Eq. (3).

B. Intra-cell to Cell Boundary Routing Path

Intra-cell to cell boundary routing (scenario 2 in Fig. 2) is a special case of intra-cell routing. The collision probability for intra-cell to cell boundary routing is shown in Eq. (7) with the condition $r_{o,m'} = r_{\text{cell}}$.

C. Cell Boundary to Boundary Routing Path

For cell boundary to boundary routing (scenario 3 in Fig. 2), both source and destination UEs are on the cell boundary which is a special case of intra-cell to cell boundary routing.

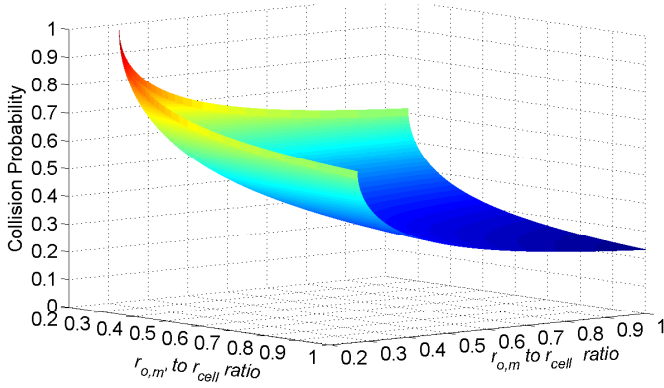


Fig. 4. Collision probability for intra-cell and intra-cell to cell boundary routing paths with the distance scale of $r_{o,m'}$ and $r_{o,m}$, and CA radius ratio (r_{CA}/r_{cell}) is 27%. The D2D UEs density is 500 per cell.

The probability of a UE inside the CA is strictly zero. Thus, the collision probability is Eq. (7) with the conditions $r_{o,m} = r_{cell}$, $r_{o,m'} = r_{cell}$ and $\mathbb{P}_{CA} = 0$:

$$\mathbb{P}_{\text{Collision}} = 1 - 2 \frac{\arccos\left(\frac{r_{CA}}{r_{cell}}\right)}{\pi}. \quad (8)$$

We now examine the effect of different network parameters on the collision probability and how to dynamically select multi-hop routes.

IV. RESULTS AND ANALYSIS

A. Single- and Multi-Cell Results

Fig. 4 shows the collision probability for two routing path scenarios: (i) intra-cell routing, and (ii) intra-cell to cell boundary routing (when $r_{o,m'}/r_{cell} = 1$). The CA's size is defined as a fraction of the BS's radius (r_{CA}/r_{cell}). In this particular case, the value is 27%, which is for typical QoS requirements of a minimum SINR ($\xi = -6\text{dB}$) and a pathloss distance exponent $\alpha = 4$.

The first observation is that the collision probability is strictly convex, as a function of the distances from the BS to the source and destination UEs. This can be proven from Eq. (7), where the Hessian matrix is given by [14]:

$$\begin{bmatrix} \frac{2r_{CA}(1-\mathbb{P}_{CA})}{\pi r_{o,m}^2 \sqrt{r_{o,m}^2 - r_{CA}^2}} & 0 \\ 0 & \frac{2r_{CA}(1-\mathbb{P}_{CA})}{\pi r_{o,m'}^2 \sqrt{r_{o,m'}^2 - r_{CA}^2}} \end{bmatrix} \succeq 0, \quad (9)$$

for $r_{o,m} \geq r_{CA}$, $r_{o,m'} \geq r_{CA}$ and $0 \leq \mathbb{P}_{CA} \leq 1$.

The second observation is that from the results and Eq. (7), a maximum collision probability of 100% is achieved when $r_{o,m} = r_{CA}$ and $r_{o,m'} = r_{CA}$. From Eq. (7), a third observation is that a minimum collision probability of

$$\min(\mathbb{P}_{\text{Collision}}) = 1 - \frac{2}{\pi} \arccos\left(\frac{r_{CA}}{r_{cell}}\right). \quad (10)$$

can be achieved when $r_{o,m} = r_{cell}$ and $r_{o,m'} = r_{cell}$.

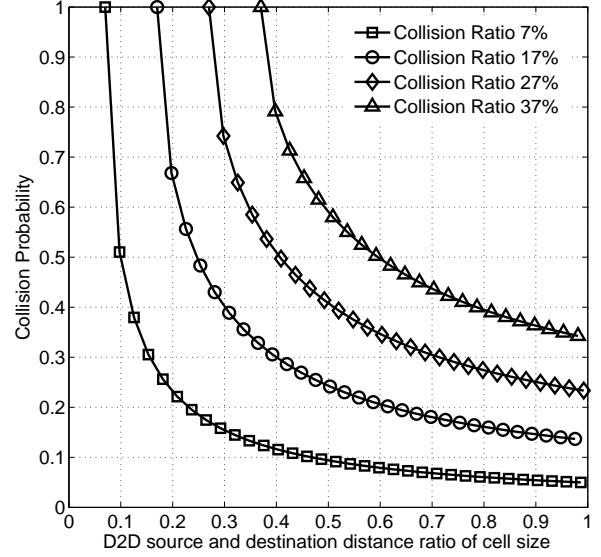


Fig. 5. Collision probability (theory) for D2D intra-cell routing with different routing distances (as a function of cell coverage radius).

For D2D routing between the coverage area of multiple BSs, a combination of intra-cell to cell boundary routing and cell boundary to boundary routing is used. In Fig. 5, the collision probability can be seen to decrease rapidly with increased D2D routing distance and smaller collision area ratio (as a percentage of cell coverage area). For example, when D2D routing is over the whole cell radius, the collision probability can fall to below 5% for a collision area ratio of 7%. Fig. 6 shows the results of the collision probability as a function of the number of BSs passed through and different CA radius values (r_{CA}/r_{cell}). The results show that the collision probability increases with the number of BSs as well as the size of the CA, and the simulation and theoretical values agree.

B. Gradient Based Switch Strategy

Given the SPR multi-hop travels in a relatively straight line, each relay j can be interpreted as a temporary source m , and a fresh collision probability can be computed. As mentioned previously, the collision probability Eq. (7) is a convex function with respect to the UE-BS distances for the source and destination. Therefore, the updated gradient descent at each relay UE j will reveal the increasing or decreasing probability of collision. The gradient with respect to the current relay UE's distance with BS ($r_{o,j}$) is given as:

$$\nabla \mathbb{P}_{\text{Collision}} = -r_{CA}(1 - \mathbb{P}_{CA}) / (\pi r_{o,j} \sqrt{r_{o,j}^2 - r_{CA}^2}). \quad (11)$$

As the multi-hop path approaches the CA ($r_{o,j} \rightarrow r_{CA}$), the gradient will approach $\nabla \mathbb{P}_{\text{Collision}} \rightarrow -\infty$.

Fig. 7 shows the gradient of the collision probability as a function of the normalized distance along the source-destination route. Three scenarios are considered: (i) when the

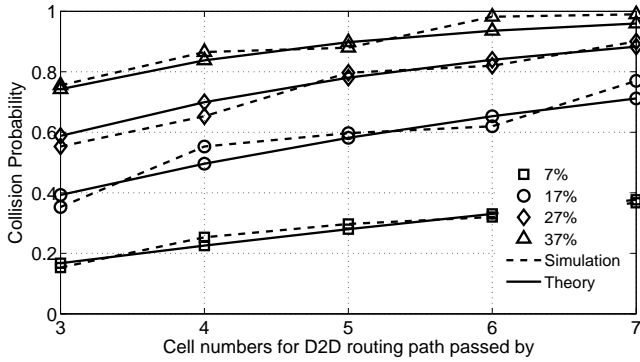


Fig. 6. Collision probability of theory and simulation results for D2D multi-cell routing with different CA radius ratios.

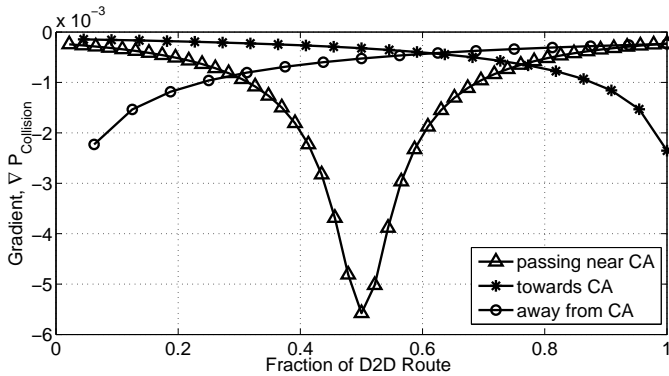


Fig. 7. Gradient of the collision probability as a function of the normalized distance along the source-destination route.

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1: function SWITCH( $r_{o,j}$ ,  $r_{o,m'}$ ,  $r_{CA}$ ,  $\Delta_{BS}$ )
2:   if ( $r_{o,m'} \leq r_{CA}$ ) then CC Communications
3:   else ( $r_{o,m'} > r_{CA}$ )
4:     D2D Communications Starts
5:     For each hop UE  $j$ , calculate  $\nabla P_{Collision}$ 
6:     if  $|\nabla P_{Collision}| > \beta$  then
7:       Switch to CC Communications
8:     else
9:       D2D Communications Resumes
10:    end if
11:  end if
12: end function

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Fig. 8. Switching algorithm

path passes near the CA, (ii) when it moves towards the CA, and (iii) when it moves away from the CA. Hence, before the collision occurs, a certain gradient threshold β can be set whereby the D2D transmission will be forced to use CC channels in order to avoid colliding with the CA and cause unnecessary levels of interference. The detailed gradient based switching mechanism is given in Fig.8, and the impact on communication metrics is left for future research.

V. CONCLUSION

In this paper, we implement an interference zone (CA) to mitigate cross-tier interference between D2D and conventional cellular transmissions. We then investigated the routing algorithm for multi-hop D2D communications. Currently, most D2D routing algorithms assume synchronized accurate location knowledge among users and the base stations. In reality, this level of location accuracy is difficult and power consuming in UMTS. In current LTE, there is no location information from the cell besides range information from time measurements.

In the absence of perfect location information, we are able to derive the collision probability as a function of the Quality-of-Service and other key network parameters. As a result, a simple gradient based switching mechanism between D2D and CC communications is devised. It can avoid collisions with the CA and requires only the distance information of the current transmission and final destination user.

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