An Experimental Nakagami Distributed Noise Model for Molecular Communication Channels with No Drift

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We consider a molecular communication system operating in a pipe propagation channel with no induced flow. Experimentally, we show that discrepancies channel impulse response can be accurately modelled by an additive noise model. The noise amplitude is Nakagami distributed, and the shape and spread parameters of the distribution increase monotonically with propagation distance. The paper further demonstrates how the proposed noise model can be used to calculate bit-error-rate and capacity of a binary symmetric channel.

Background: Molecular communications utilizes chemical molecules as an alternative carrier for information [1, 2]. Compared to existing wave-based propagation, molecular diffusion has the potential to propagate more efficiently and reliably in certain confined and micro-scale environments, or when there is overwhelming electromagnetic interference [3]. Indeed, this is perhaps why molecular communications is prevalent in nature, both at the inter-organism and at the inter-cell scales [4].

For the past few years, the vast majority of molecular communications research has been modeling based, utilizing classical diffusion equations [2, 5, 6, 7]. In a real pulse modulated molecular communication system, noise can unfortunately arise from a host of sources and it is difficult to take all of them into account or argue which may dominate. Examples of noise include, but are not limited to: inconsistent mechanical pulse emission, atmospheric contamination, physical disturbances, and receiver design [8]. The vast majority of literature assume a Gaussian distributed additive noise [9], which is not unreasonable given that recent research has shown this to be accurate in a high velocity turbulent flow 3-dimensional channel [10]. The noise model for a random walk diffusion channel with no induced drift velocity is lacking. Such a channel is common in *low pressure* industrial applications (i.e., pipe networks) and enclosed environments (i.e., caves and tunnels). Utilizing a recently built molecular communication test-bed [11], we characterize the additive noise model in a pipe channel for molecular communications.

Experimental Setup: The pulse-modulated transceiver design used in this paper can be found in [11], and the overall experimental setup is found in [3]. Briefly, the system converts an alpha-numeric message into a binary code, and uses on-off-keying (OOK) line coding. The modulation scheme employed is chemical concentration (amplitude) modulation (BASK), and the chemical used is ethanol. At the transmitter, an electronically controlled mechanical spray acts as a transmitter emitting a number of molecules (*M*). Each input is defined as pulse with a finite small duration. The chemical concentrations are sprayed into the channel and received at a chemical sensor positioned at a distance d away. The chemical sensor converts concentration measurements into electrical signals. With the appropriate signal processing, the output can be decoded back into the original message. An approximately constant humidity and temperature is maintained. The propagation channel is composed of a straight iron pipe of up to 245cm long, with two sealed iron chambers at each end containing the transmitter and receivers respectively. The pipe diameter is 4cm with a thickness of 2mm. The chemicals are initially emitted in a way that ensures that the overall drift velocity in the channel is zero.

Noise Model: Theoretically, the molecular concentration at any point in space and time can be found by using hitting probability density equation in 1-dimensional space and the molecules are constrained to be transmitted only in one direction [2]:

$$\phi_{\text{hit}}(x,t) = \frac{1}{\sqrt{\pi D t}} \exp\left[-\frac{x^2}{4Dt}\right], \tag{1}$$

where D is the diffusivity, t is the transmission time, x is the distance between transmitter and receiver. The received pulse in reality will not behave precisely in accordance with the expression due to a number of noise sources in the system.

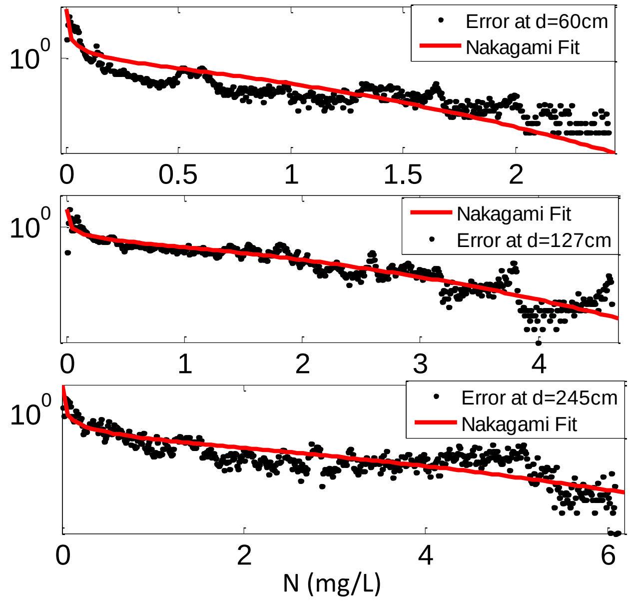

Fig. 1. Noise (error) PDF with fitted Nakagami distributions (Log Scale)

Therefore, the realistic received signal pulse shape will be:

$$\vartheta(x,t) = \phi_{\text{hit}}(x,t) + I + N, \tag{2}$$

where I is the inter-symbol-interference (ISI) from previous and successive pulses (which we do not consider here); and N is the additive noise, whose distribution is our chief concern.

Additive noise is defined as the difference between the measured received signal and the average received signal of the system. Given a reliable number of experiments for each of the different channel lengths, we apply the Levenberg-Marquardt algorithm to curve fit a probability distribution. It was found that the *modified Nakagami distribution* fitted the additive noise data. The Nakagami distribution has two parameters: *a shape parameter* μ, and *a spread parameter* ω and the probability density function (PDF) of the noise can be expressed as:

$$f_N(z;\mu,\omega) = \frac{1}{|z|\Gamma(\mu)} \left(\frac{\mu}{\omega} z^2\right)^{\mu} \exp\left(-\frac{\mu}{\omega} z^2\right). \tag{3}$$

The domain of the traditional Nakagami distribution is $[0,+\infty)$ while in this case, the noise can be negative and the domain is extended to $(-\infty,+\infty)$ to reflect this. By doing so, the pdf had to scale down by 2 from general Nakagami distribution expression otherwise the integral of the unmodified pdf across the whole domain will exceed 1. This PDF captures the fact that the additive noise can be negative mathematically as opposed to the standard Nakagami distribution where the random variable is strictly positive.

Table 1: Nakagami Distribution Fitting Parameters

Channel Length, d	Shape, μ	Spread, ω	Correlation, R^2
60cm	0.122	0.312	0.59
127cm	0.291	2.510	0.76
245cm or greater	0.310	4.058	0.69

The results in Fig. 1 show three examples of the noise (error) measurements given as symbols and the theoretical Nakagami distribution given as the line. In order to examine the match clearly, we display the noise on a log-scale. Over 10000 empirical data points were used to generate each noise distribution. In terms of accuracy, the PDF regression fit has a correlation of $R^2 = 0.59$–0.76. An example of the *Nakagami Distribution* parameters are displayed in Table 1 for various lengths. It can be seen that with increased pipe length, the shape and the spread parameters of the Nakagami distribution increase. The results saturate at lengths significantly longer than 2.5m, and the parameters at 2.5m can be used as a stable value for longer distance propagation.

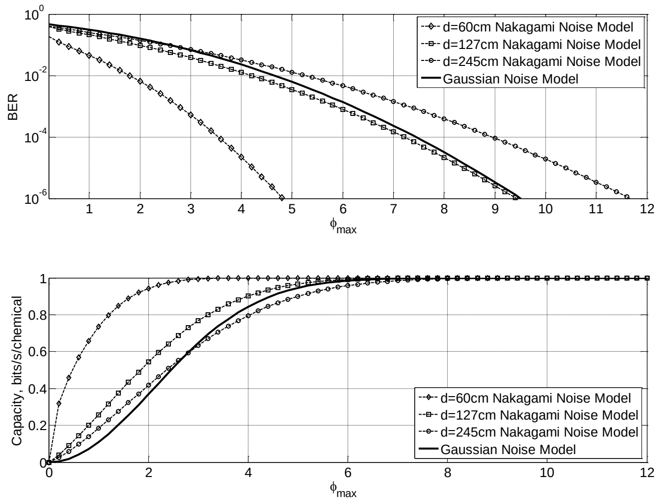

Fig. 2 *Comparison of BER and Capacity performance for different noise models at different received peak concentration levels* $\phi_{\mathrm{max.}}$.

Performance: We primarily consider only the additive noise in the system. If the receiver samples at the peak of the captured molecule concentration $\phi_{\mathrm{max.}}$, the captured concentration including noise is $\vartheta' = \phi_{\mathrm{max.}} + N$. The distribution of the additive noise can follow the proposed Nakagami distribution (3) or a normal distribution $\mathcal{N}(0,1)$ [9]. The transmission system is a OOK modulation scheme with ρ probability of transmitting a 1. At the optimal sampling point, the minimum error probability (MEP) criterion of a standard Bayesian detection framework is given by the following with decision threshold η [9]:

$$\vartheta' \gtrless_0^1 \frac{\sigma^2_{\vartheta'}}{\mu_{\vartheta'1} - \mu_{\vartheta'0}} \log\left(\frac{1-\rho}{\rho}\right) + \frac{1}{2}(\mu_{\vartheta'1} + \mu_{\vartheta'0}) \equiv \eta, \qquad (4)$$

where $\mu_{\vartheta'0} = \mathbb{E}[\vartheta'|\chi=0] = \mu_N$, $\mu_{\vartheta'1} = \mathbb{E}[\vartheta'|\chi=1] = \phi_{\mathrm{max.}} + \mu_N$, and $\sigma^2_{\vartheta'} = \mathrm{Var}[\vartheta'|\chi=0] = \mathrm{Var}[\vartheta'|\chi=1] = \sigma^2_N$.

The average error probability is:

$$P_e = \rho\mathcal{F}\left(\frac{\mu_{\vartheta'1} - \eta}{\sigma_{\vartheta'}}\right) + (1-\rho)\mathcal{F}\left(\frac{\eta - \mu_{\vartheta'0}}{\sigma_{\vartheta'}}\right), \qquad (5)$$

where the $\mathcal{F}$-function represents the complementary cumulative distribution function (CCDF) of the noise distribution. For line-coding with an equal probability of transmitting a 1 and 0 ($\rho = 0.5$), the BER is reduced to: $P_e = \mathcal{F}\left(\frac{\phi_{\mathrm{max.}}}{2\sigma_N}\right)$.

For a Gaussian-distributed noise model, the $\mathcal{F}$-function is well established to be the Q-function. For the modified Nakagami-distributed noise model, the $\mathcal{F}$-function is:

$$\mathcal{F}(x) = \begin{cases} 1 - \frac{\Gamma(\mu, \frac{\mu}{\omega}z^2)}{2\Gamma(\mu)} & z < 0 \\ \frac{1}{2} - \frac{\gamma(\mu, \frac{\mu}{\omega}z^2)}{2\Gamma(\mu)} & z \geqslant 0, \end{cases} \qquad (6)$$

where $\Gamma(s,x)$ and $\gamma(s,x)$ are the upper and lower incomplete gamma function, respectively. The associated variance is $\sigma^2_N = \omega(1 - \frac{1}{\mu}[\frac{\Gamma(\mu+0.5)}{\Gamma(\mu)}]^2)$.

Note that the probability of error given a 1 is transmitted is equal to the probability of error given a 0 is transmitted. Therefore, with $\rho = 0.5$, the system is a binary symmetric channel. The achievable throughput of the binary symmetric system is:

$$C = 1 - H(P_e) = 1 + P_e\log_2(P_e) + (1-P_e)\log_2(1-P_e). \qquad (7)$$

The results in Figure 2 show that the specific parameters of the noise model can have a dramatic effect on the BER and capacity performance. This is a testament to the importance of employing accurate noise distributions as well as relevant parameters for calculating BER and capacity performances.

Conclusions: This paper has developed a simple and effective Nakagami distributed noise model for molecular communications in a zero velocity pipe channel. The shape and spread parameters of the model increase monotonically with pipe length until the parameters converge at lengths greater than a certain value. The proposed noise model can be used as a basis for further molecular communications research and we demonstrate its application to bit-error-rate and capacity performance analysis.

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