Strong Component-Based Methodology for Facility Layout Design

by

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Acknowledgements

To Fernando and Luz in their memory...

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Declaration

I declare that the work described within this Ph.D. thesis, unless otherwise acknowledge in the text, is my own work and has not been previously submitted for any academic degree.

Signed:

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Summary

Among many issues involved within the field of manufacturing systems, the design of facilities layout is an ongoing and interesting research field, where new solutions and approaches are sought to determine the appropriate location and physical organisation of the resources in manufacturing systems. Issues such as space, material handling, machine placement and orientation, utilities location, and environmental factors are important features that may be considered when establishing the requirements of a facility layout design. The facility layout design can be thought of in terms of interconnecting workcentres that can be represented by a set of interrelated vertices in a graph. Directed graphs can be used to characterise each product operation sequence, which combined into a single directed graph, be used to represent appropriately a layout design. Doing this together with the material handling system requirements, will allow better facilities planning and may improve process sequences that should be reflected in better designs.

The Strong Component Based Methodology proposed here, obtains a graphical structure from the integration of various products and using their operation sequences to produce a relationship diagram. The attributes of the resultant structure are used to create this diagram. The objective is to obtain layouts that minimise material handling, that is, as close as possible to that which can be obtained with dedicated facilities for each product family but without the capital costs involved in the case of the latter. Encouraging results have been obtained by considering strong components, a feature of directed graphs, because less computational resources than in the case of many previous methods, which use Quadratic Assignment Problem approaches, are required to formulate and produce a relationship diagram. Moreover, this approach produces faster designs than other graph theoretic approaches because it avoids using planar and dual graphs. These characteristics allow the Strong Components approach to address more complex situations and obtain comparable or better solutions than previous approaches.

The proposed Strong Component approach is a robust and versatile tool to support layout designs. It is a robust methodology because it provides efficient relationship diagrams even in cases when the resultant structure has relatively few strong component relationships. It is a versatile approach, because it can address various situations and can use different criteria to create layouts. Thus, the proposed approach offers effective-economical relationship diagrams to produce the same set of products as when producing them in dedicated facilities.

1. Introduction

1.1. Background

Contemporary industrial markets are dynamic and complex environments, which make the innovation of manufacturing systems fundamental for organisations to remain competitive. Manufacturing system design is still an evolving research field. Researchers and practitioners are seeking new and better alternatives to support the design of systems that satisfy changing customer requirements in an efficient manner (Manetti J., 2001; Meller R.D., Gau K.Y., 1996).

Among many issues involved within the field of manufacturing system design, the design of facilities layout is an ongoing and interesting research field (Ramabhatta V., Nagi R., 1998; Welgama P.S., Gibson P.R., 1995). In the design of facilities layout, new solutions and approaches are sought to determine the appropriate physical organisation of the resources within manufacturing systems. Facilities design is closely associated with the performance of the production and the material handling systems (Owens R., 2001). It has been estimated that a good layout design may help reduce from 10 to 30% of the total operating expenses within manufacturing systems (Owens R., 2001;Kim J.G., Kim Y.D., 2000; Tompkins, et al., 1996:6).

Being competitive implies that organisations have to satisfy very demanding and sophisticated customers using scarce resources (Corney W., 2002; Schroeder D.M., Congden S.W., 2000). Products offered should fulfil and exceed the expectations of customers in terms of their variety, quality, and price. As customers' expectations change new products have to be offered and new technologies are needed to open opportunities for product and process improvements. For the manufacturing organisation, satisfying this has its challenges and consequences. Offering a greater variety of products may imply that production volumes should be smaller, that time-to-market should be compressed, and hence, the life cycle of the product reduced. These reductions may necessitate a thorough evaluation of a possible design or redesign of new or existing facilities and, of course, of the technical and/or economic consequences that these changes may bring. Industries in fields such as electronics or high-tech may be examples of this type of environment (Arntzen B.C., Shumway H.M., 2002; Frazier G.V., Reyes P.M., 2000).

Amongst the main drivers that are guiding organisations, (Wu B., 2000:4; Wacker J.G., Miller M., 2000; Joseph A.T., 1999), one is the need to meet higher customer expectations on choice of products, quality, delivery performance, and costs. These drivers can be translated into the following criteria:

- Quality considered as the ability to produce products according to specifications.

- Customer lead times as the ability to complete the required products at the required times.
- Delivery reliability as the ability to provide the agreed quantities of products at the agreed time.
- Volume flexibility as the ability to produce products in various batch sizes.
- Design flexibility as the ability to produce a range of products, to customise products or produce new products to specifications.
- Price and costs as the ability to produce products at a low cost based on perceptions of value for money.

Companies will have to carefully reorganise their available resources, set priorities and targets, and keep monitoring their performance to meet these goals (Sahin F., 2000). Additionally, the above may imply that new processes may be required and support for these innovations may be needed. Every time that new products, processes, and technologies are considered, a new facilities layout design might be required and also an evaluation of how these changes contribute in the efforts to become sustainable and more effective, flexible and agile than the competition (Belshaw B., Citrin S., Stewart D., 2001).

1.2. Facilities Layout Design

The Layout Design process of Manufacturing Systems or Facilities Layout Design deals with how to deploy the manufacturing resources within a specific space or area available, allowing the production processes to be performed appropriately and the materials to flow effortlessly (Chiang W.C., 2001; Caccetta L. Kusumah Y.S., 1999). Layout Design may be required when designing new facilities or processes, or when redesigning existing ones. A layout is considered good if it promotes effective use of space, equipment, materials, personnel, and other manufacturing resources (Foulds L.R., Partovi F.Y., 1998; Meyers F.E., 1993:2).

Likewise, Layout Design is considered to be closely related to other manufacturing design processes. These processes, such as product design, process design and material-handling design, should be addressed simultaneously as far as possible. During their interaction and iterative feedback they should influence, enhance and contribute to each other, allowing the development of more appropriate and more effective designs (Yaman R., Balibek E., 1999). For practical and analytical purposes, layout design is often analysed in isolation, reducing considerably its complexity: in practice its interaction and iteration with these other processes should not be abandoned (Heragu S.S., 1992).

In general, the design of manufacturing facilities is an ongoing research topic that can be studied from a strategic or tactical perspective. A few years ago, when markets were more stable, the design of facilities was seen as a strategic issue, since the layout remained fixed over long periods because new products and new technologies took more time to be introduced and developed (Kochhar J.S., Heragu S.S., 1999). In current industrial environments changes in products and technologies occur more often, new features are required by companies to respond to customers' demands, and a more tactical perspective on manufacturing issues is required (Sanchez L.M., Nagi R., 2001). Hence, models and methods are required which are capable of responding to changes in product mix and routing without the need for frequent modification of an existing layout.

1.3. Research Project Issues

The generation of layout design alternatives is critical within the facilities planning process because the layout establishes behavioural and physical relationship patterns between resources allocated for production. The complexity of the layout design together with the material handling system design suggests that a sequential design process should be used. It is recommended, when considering together these designs, that a number of alternatives should be developed and then that the design considered most suited should be selected from them. Issues such as the material handling unit, the degree of automation, and the degree of control over work-in-process, may affect requirements for space, equipment, personnel, and the proximity requirement between production activities (Tompkins J.A., et al., 1996:287).

The way products are designed determines the processes that may be available to produce them (Arnold T.J.T., 1998:365). The effect of a poor product design can influence production and inventory costs through operations that may be difficult to perform, or make it difficult to control the required work in process because of excessive working time during complicated operations. Effective flow between workcentres addresses the progressive flow of materials and information between them, and contributes to an effective flow between organisational units, and ultimately also contributes to an effective flow between organisations. In consequence, the deployment of a relationship diagram, which represents and considers these flows, is an essential step in the development of any Facility Layout Design.

To determine the material flow pattern is the first step in developing layouts (Heragu S.S., 1997:48; Tompkins J.A., et al., 1996:288), and consequently it is necessary to determine which workcentres should be adjacent in the layout (Leung J., 1992b). Based on the material flow and depending on the product design, volume and equipment available, production processes can be classified as product, process, fixed position, group technology, or hybrid, which are all known as the basic layout types. The product layout can be characterised as a high-volume and low-variety production environment, whereas the process layout is a low-volume and high-variety environment. More companies are converting their facilities to combinations of product and process families based on similar manufacturing operations or design characteristics. This approach is known as group technology, and their physical implementation as group layout or cellular manufacturing, since the required machines are said to be grouped together in manufacturing cells which are characterised by their medium-volume medium-variety capabilities.

The shift from high-volume production towards multiple products with small to medium volume orientation and frequent product design changes suggests that organisations are moving from dedicated to non-dedicated facilities (Leung J., 1992b). This work describes an approach to obtain facility layout designs in this environment, exploring the resultant production network structure obtained, and comparing its performance against the performance obtained from dedicated facilities. An approach originated by the integration of production sequences is suggested by this project to develop layout designs.

1.3.1 Objective

The objective that this project will address is:

To develop relationship diagrams using a construction-type approach in order to study layout designs within environments where multiple products have to share the same facilities based on their production operation sequence similarities during their expected life cycles. This approach is based on a model obtained by the combination of different product operation sequences. To accomplish this purpose the following issues are addressed:

- The design and development of the necessary links between the facility layout design problem and the theory of directed graphs that can be applied successfully to aid the development of relationship diagrams together with their strong component properties.

- The validation of the solution model against a dedicated facilities solution model, based on a set of specific performance measures such as utilisation of resources, production output, and batch production, among other possible measures.

1.3.2 Approach

The computational requirement in the facility-layout-design problem to create a relationship diagram is complex; for instance assuming that there are n workcentres to be distributed, there will be n! possible solutions. Consequently, researchers have concentrated on developing heuristic methods, which may not guarantee optimality but provide acceptable solutions (Heragu S.S., Kusiak A., 1988). These heuristics were classified by Kusiak and Heragu (Kusiak A., Heragu S.S., 1987) as construction, improvement, hybrid, and graph theoretic methods. In construction methods, facilities are added, usually one at a time, until a complete layout is achieved. There is an initial solution in improvement methods, from which a systematic exchange between facilities is performed, seeking best solutions until the solution cannot be improved: then the procedure is ended. Hybrid methods are characterised by the use they make of the features from improvement and construction methods.

The facility layout design can be thought of in terms of interconnecting workcentres which can be represented by vertices in a graph. The interrelationships between the workcentres, in terms of operation sequences, can be represented by the edges or arcs. These relationships are usually represented by charts or tables such as the relationship chart (Muther R., 1973:5-2), from-to frequency of trips, and frequency of trips between facilities (Heragu S.S., 1997:55). A relationship chart or rel-chart is a qualitative data table that summarises estimates of the desirability of locating facilities next to each other. Designers using rel-charts attempt to maximise the sum of the scores of adjacent pairs of facilities in the layout. The from-to and the trip-frequency tables are quantitative measures of flow, which indicate the level of interaction between pairs of facilities. Thus, the from-to table shows the number of trips made in each direction between every pair of facilities, and the trip-frequency table combines the two directions and shows the total number of trips in either direction between pairs of facilities. In the case of these tables or matrices it is desirable that those edges or arcs with a higher number of in-between trips are placed as close as possible. Directed graphs could be used to represent each product sequence, which combined can be represented by adjacency matrices.

Flow, space, and activity relationships are important considerations when establishing the requirements of a facility. Issues such as space, material handling systems, proper machine orientation, utilities location, and environmental factors are features which can be considered in the detailed design phase, once a proper machine sequence has been settled. Doing this later, together with the material handling system requirements, may allow better facilities planning and improve process sequences that should be reflected in better design (Rao H.A., Gu P., 1997). In addition, it can also be supported by more suitable tools such as a CAD tool (Owens R., 2001).

In summary, once each product to be considered is defined and its production process known, each one of them can be represented by sequences of steps which can be characterised through the use of a directed graph. Each product graph then can be combined with the other product graphs to construct a structure based on their production sequence similarities. The properties of the resultant structures can then be used to obtain the relationship diagram and produce a facility layout. These structures represent the integration of the production operation sequences in one model that can be pictorially represented by a directed graph.

Since this is a deterministic structure, it can be handled using deterministic approaches such as directed graph theory and matrix algebra. The application of directed graphs and particularly strong components and its appropriateness to the Facility Layout Design is the main issue explored in this Thesis.

1.3.3 Chapter Plan

In the following paragraphs, the contents of the thesis are briefly outlined.

In Chapter Two, the literature on facility-layout-design problems is reviewed in terms of general formulations and approaches to solutions. Some of the relevant issues of the facility-layout-design process are explored and discussed, along with some of their general formulation approaches.

In Chapter Three, analytical methods and techniques used to obtain solutions to facilities layout design models are explored, compared, and their underlying assumptions identified.

Chapter Four explores cellular, flexible and machine layouts, their main characteristics are highlighted, and their respective formulations and solutions approaches are addressed.

In Chapter Five, the solution approach developed in this research project is outlined, providing a simple case example to show the details of how this method works. Additional features are mentioned which enhance the proposed method and may provide useful insights when designing or redesigning manufacturing facilities. Moreover, the strengths and limitations of the proposed approach are stressed, and its advantages and disadvantages compared to when used to produce the same set of products in dedicated facilities environments are discussed.

In Chapter Six, the findings are discussed and compared to other models and approaches found in the literature, which provide insight into the proposed approach and its efficiency and effectiveness.

In the final chapter, the research contributions are summarised, and suggestions for further development of the work are presented.

2. Facility Layout Design (Literature Review)

2.1. Introduction

How facilities have to be configured to best support manufacturing systems is a fundamental question to be addressed and solved. Many factors have to be considered, including the nature of the process, product life cycles, product varieties, space usage, costs and product market demand, among other factors. Publications on the Facility Layout Design approaches by Apple (Apple J.M., 1977), Muther (Muther R., 1973), and Reed (Reed R., 1961) can be considered among the seminal works that addressed this question and provided a general approach to solve it. Many of today's methods are still using the concepts contained in these initial proposals (Tompkins J.A., et al., 1996:291), and are explored in this chapter. The rest of this chapter briefly introduces Systematic Layout Planning (Muther R., 1973), and the approaches by Apple and Reed are presented in Appendices One and Two. This is followed by an outline of activity relationships, and a discussion of how efficiency has been measured in the different approaches. An analysis of dedicated or non-dedicated facilities is also provided.

2.2. Systematic Layout Planning (SLP)

Muther's approach, Systematic Layout Planning (SLP), is based on the methods proposed by Apple and Reed, and it has been one of the most common approaches in practice (Heragu S., 1997:86).

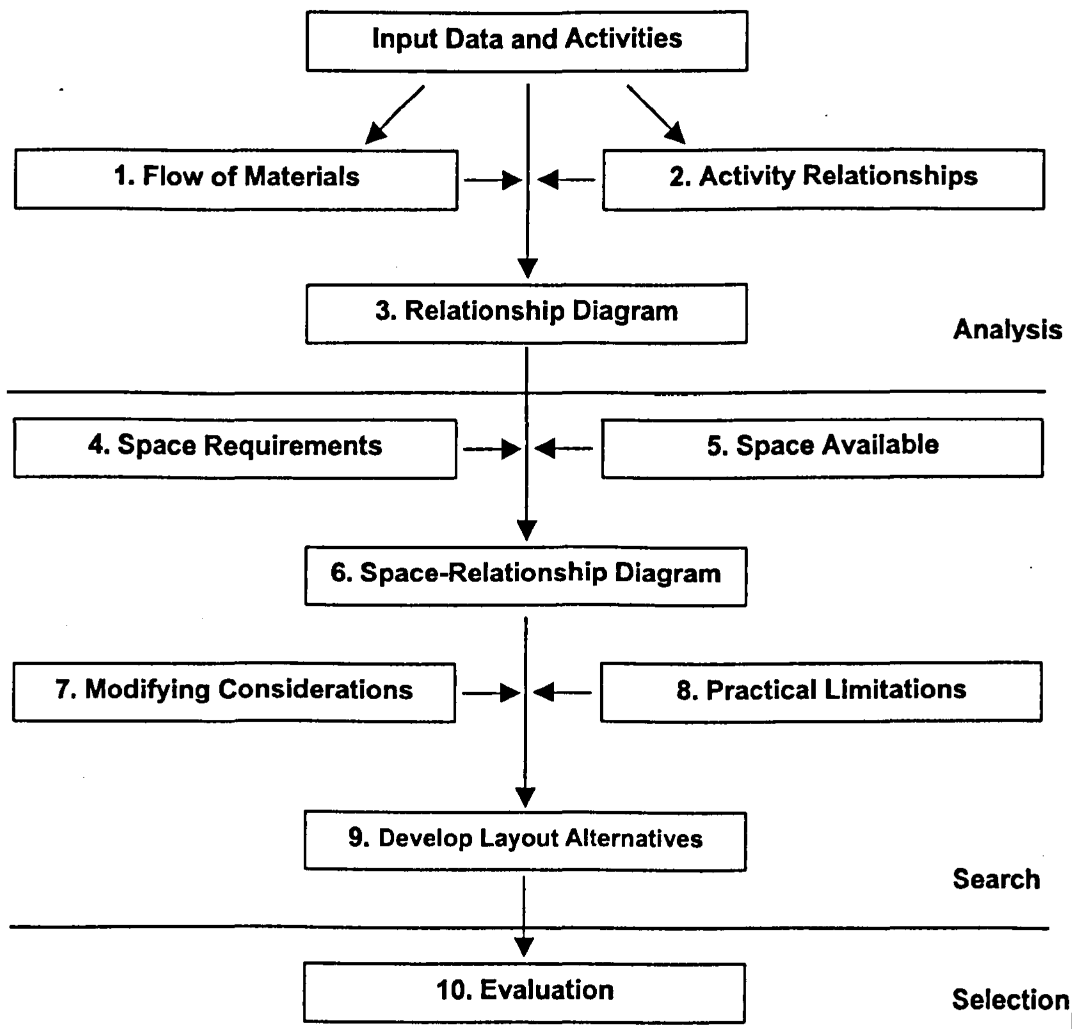


Figure 2.1. Systematic Layout Procedure (Tompkins, 1996: 295)

This procedure is developed in four phases:

1. Determining the location of the area where facilities will be laid out. It is this phase in which the available space for the layout is identified.
2. Establishing the general overall layout. This second phase involves the framework which underpins the procedure, as shown in Figure 2.1. This phase is also referred to as "block layout".
3. Establishing detailed layout plans. Once a relative gross position is defined in the previous step then the location of specific components is created. The detailed layout procedure follows the same procedure used in phase two and it is repeated as necessary until all the detailed layouts have been generated.
4. Finishing the selected layout. Once all detailed layouts are approved then the information provided is used in development of drawings, and the final layout is prepared.

The approach begins by establishing the required input data, which are classified into five categories:

P Product: types of products to be produced

Q Quantity: volume of each part type

R Routing: Operation sequence for each part type

S Services: support services, locker rooms, inspection stations, and other services

T Timing: process times which will determine the resource requirements to accomplish the finishing arrangements

One of the main features of this procedure is that it combines the required information by creating different charts, which eases the design process. It constructs a from-to material flow chart from data P, Q and R; in Figure 2.1 this is shown as box one. This matrix indicates the intensity of the flow between each pair of workcentres. A second chart, the activity relationship chart, is constructed in the second step (box two in the same figure), which allows us to establish the relative importance between the workcentres as a qualitative measure that reflects the desired proximity between workcentres, using a lexigraphic ordinal scale. This scale reflects the closeness desirability between the different activities and is expressed using six values, namely **A, E, I, O, U**, and **X.** This range of values declares the **A** value as the closest or most desirable closeness value and, consequently, any pair of facilities should be placed together as close as possible. As the relationship importance decreases, the letter changes to reflect the relationship importance until, using **X** as the most undesirable closeness value, any pair of facilities with this rating must not be near to each other.

From the two charts, from-to and activity relationship charts, a relationship diagram is constructed, as in box three. This relationships diagram depicts the workcentres and their relative positions, locations that can be determined using algorithms or heuristics. In the following step, the method focuses on space handling, and any limitations or other related particular issues are considered. This step corresponds to boxes four and five in Figure 2.1. After considering the space available and the space required by the relative positions of the workcentres, the space relationships diagram is developed using the space information available and prior diagrams, i.e. box six in Figure 2.1. After taking into consideration other special items such as material-handling methods, storage equipment, utilities location, building codes, existing structures, safety and ergonomic issues, the space diagram is modified to create layout alternatives to be evaluated.

A different perspective is the one given by Tompkins et al. (Tompkins, J. A., et al., 1996:295). This perspective suggests that the general overall phase may be thought of as divided into three stages, namely analysis, search and selection. From this partition, as seen in Figure 2.1, an output is obtained from each stage. During the analysis stage, the space relationship diagram is obtained; from the search stage, the layout alternatives are obtained; and the proposed layout is obtained from the selection stage. Another perspective is the one given by Hales (Hales H.L., 1984:40), which also divides the procedure into three stages: relationships, space, and generation of alternative layouts. Given the original sequential approach or following these partitions, the steps or stages are performed separately and are integrated in the last step, providing an overall approach to the facility layout design. It seems that approaching the design by steps or stages provides a better focus on each feature. Additional issues, such as space, shapes and material-handling issues, can modify proposals by incorporating them during the last stage of the process, before creating the design alternatives. It should be stressed that the relationship and the space issues can be considered as the most suitable features to be handled by mathematical models.

Modelling helps to clarify the problem and takes into account those relevant factors that are critical in developing a layout. A model by itself does not provide a solution to a problem; algorithms or solution techniques have to be developed to obtain solutions to a model. Models are useful concepts even though many of their assumptions may not be realistic. The layout analyst has to be aware of the assumptions made in the models, and should use the solution generated by a corresponding algorithm with caution. It must be used only as a basis for generating solutions that can be applied in the real world.

2.3. Layout Analysis and Presentation Support Tools

Some of the complementary tools originally used to analyse a possible layout proposal included drawings, templates, three-dimensional physical models, and CAD systems.

Drawings, where generated manually, were considered a slow but necessary task that took a long time to develop. Fortunately, computer technology is now considered a drawing support tool that has given more speed to this task, and is a supportive tool through CAD models and software available. Another tool is a template, which helps to analyse layout designs before they are implemented. These could be constructed or be bought as commercially developed templates, and placed on a baseboard to indicate the position of the facilities. Three-dimensional models are spatial models of drawings or templates. These are also available commercially, and as with the previous models, provide additional helpful information regarding the layout design.

In recent decades CAD systems have become very popular tools because they have allowed users to create and reproduce more easily two and three dimension drawings, not only to support layout analysis but also as a presentation tool. Their popularity is due to the speed of adding, deleting, modifying, saving drawings, generating new ones, and consequently reducing the time, cost, and resources required to perform these activities. Finally, they have become the most effective media for preparing and presenting layout designs (Owens R., 2001).

2.4. Activity Relationships

Among the essential elements found in Systematic Layout Planning are activity relationships and space requirements, and they can be considered as information sources for the Facility Layout Design. Within the activity relationships are the material flow relationships, which are of considerable relevance since they show how elements are moved within the facilities, and also show the relative importance of the relationship between facilities and the movement of materials and components. A flow may be described in terms of the flow subject, the resources required to support it, and the communications that co-ordinate the resources and can be shown by their pattern (Tompkins J.A., et al., 1996:80). These data can assist during the design phase of the layout by determining the following (Heragu S., 1997:45):

- Frequency of trips of material or some other measure of interaction between facilities
- Location restrictions for facilities, if any
- Adjacency requirements between pairs of facilities, if any

The flow of materials, products, components, personnel or any other subject of flow can be grouped as follows (Apple J.M., 1977:109):

- Requiring similar machinery or equipment
- Requiring similar processes
- Requiring similar operations
- Following the same sequence of operations or activities
- Having similar operation times
- With similar shape, size, purpose, or design
- Made of the same or similar materials

There exist many flow patterns which usually may be limited by the space available for the deployment of the entities and which adopt different shapes. Some of the most common patterns that can be found are a U-shaped, S-shaped, W-shaped, straight line, or combinations of these (Heragu S., 1997:46; Tompkins J.A., et al., 1996:87).

2.5. *Efficient Layout Interrelationship Representation*

To evaluate alternative configurations, a measure of flow must be established. Using a flow chart and a layout, an analyst can determine whether the depicted process may have any unnecessary material movements. The flows may be specified in a quantitative or qualitative manner. Qualitative measures may include a subjective closeness rating, as described in the systematic layout planning procedure in Section 2.2 above.

Quantitative measures may consider number of movements, product flow quantities, costs and distances. Flows can be described in terms of the from-to and frequency of trips tables or matrices. From-to tables are records of the required trips made in each direction between any pairs of machines. The frequency of trips table records the trips in both directions showing the total number of trips between every facility pair. Additionally it may be useful to specify that these trips, depending on their degree of sophistication, may be carried out by personnel or by a material-handling device.

A from-to chart is constructed as follows:

- List all the workcentres in rows and across in columns, forming a square matrix of size n X n, where n is the number of workcentres.
- Determine how the flow may be represented so as to indicate properly the items considered between the workcentres. These items could be

number of units, weight, and volume. Moreover, the representation can also be a subjective judgement of flow, indicating merely a relationship, the number of times a link may be used, or it may just be an order of precedence.

- Register the quantities to establish the measure of flow.

A frequency of trips chart is a table similar to the from-to chart which shows the total number of trips between facilities by combining the items in both directions. A difference that might be expected between both charts is that frequency of trips is symmetrical and the from-to is not necessarily symmetrical.

Distances are a way to express desirable adjacencies between any pair of facilities on the production operation sequences. Depending on how they are measured, this may lead to various possible solutions. Some of the most common modes of measuring distance when developing layout design are (Heragu S., 1997:56):

a. Euclidean: this is a metric that measures the distance between centres of facilities. This metric is the shortest distance between any two given points, calculated by the square root from the sum of the square of the components.
b. Squared Euclidean: the same as the previous but without the square root.

c. Rectilinear: This is commonly used because it is easy to compute and is the sum of the absolute values of the distance components considering the centroids of the facilities.
d. Aisle distance: this is the aisle travelling distance between the centres of the facilities.
e. Adjacency: this metric indicates whether facilities are adjacent or not. It does not differentiate between how far apart the facilities are.
f. Shortest path: This is related to network problems where the shortest path is used to determine the distance between two facilities. A network consists of nodes and arcs, where nodes represent facilities and an arc between a pair of nodes represents a path between both nodes. Usually, a weight is attached to each arc representing a cost, a distance, time or a frequency.

In the literature, the layout's efficiency is typically shown in terms of material handling costs. These costs are approximated with one or more or the following interdepartmental flows: f_{ij} as the flow from department i to department j; unit-cost values, c_{ij} as the cost of moving one unit of load by one distance unit from i to j; and department closeness ratings, r_{ij} as the value of the closeness between department i and j (Meller R.A., Gau K.Y., 1996). The most commonly used quantitative criterion for evaluating layout is given by

$$\sum \sum cij\ fij\ dij \qquad (1)$$

where

c_{ij}, is the cost of moving a unit load of material by a unit of distance between facilities i and j

f_{ij}, is the number of loads or trips between facilities i and j

d_{ij}, is the distance between facilities i and j measured using one of the methods previously mentioned.

Based on how the activity relationships are considered, a layout design objective can be expressed by considering any combination of the factors in equation one as follows (Hassan, M.M.D., Hogg G.L., 1987):

a. To minimise the sum of the flow-distance of products between the facilities when their relationships are stated by from-to charts, or
b. To maximise the facility adjacency when the relationships are stated by a relationship chart.

It should be noted that how these activity flows or relationships between facilities are recorded might have an impact or effect on the possible layout designs. Given that intensity of relationships between facilities may be represented by activity relationships or flows between them, this intensity can be thought of as the representation of the proximity requirements or the closeness desirability between the workcentres.

2.6. Dedicated or Non-Dedicated Facilities Strategies

To determine if a layout might use dedicated or non-dedicated facilities is another relevant issue in Facility Layout Design and may be related to the economic feasibility of the expected sales volume.

It may be noted that, when dealing with dedicated facilities which may produce one product or have a dominant product, it should be evident that the product follows its production operation sequence to be elaborated. In this case, the layout issue obviously focuses on answering questions related to the other relevant layout requirements, such as the space usage, material handling system, utilities location, environmental issues, and so on.

In non-dedicated facilities when dealing with multiple products and where there is not a dominant product, the activities sequence becomes a relevant issue, in addition to those issues mentioned for dedicated facilities.

Further detailed production systems classification located between product (flow shop) and process (job shop) configurations has suggested three variants which may overlap within Group Technology, GT: namely, flow-line, centre and cell (Singh N., Rajamani D., 1992:182). Centre layout has process configuration with machines dedicated to specific families of parts or components. This arrangement could lead to increased material handling movements and may be more suitable when frequent changes in the mix of products are expected. In the cell layout, the flow of components is omnidirectional, allowing components of the same group to follow the same production sequence. The flow-line layout may be used when all the components to be produced are assigned to a group that follows the same machine sequence, and usually when automated material handling equipment is used.

2.7. Summary

In the discussion in this Chapter, the relevance that relationships between activities have on the design of layouts has been stressed. This is because the chosen layout from those designs available may have an effect on other required systems, such as the material handling system, and vice versa. Consideration of activity relationships and space features in the development of feasible designs are the most suitable to be treated by mathematical models such as Graph Theory and Mathematical Programming. Moreover, it has been highlighted that there are two instances in which these models could be applied: in the general and in the detailed layout phases. The general phase is also referred to as block layout, since it is developed at a more global level within the organisational system, such as at departmental levels or interdepartmental levels. The detailed phase works at a more particular organisational level, such as inside each department or intradepartmentally. Although most of the algorithms or heuristics to determine layout designs originally were proposed to solve the block layout designs, later they were extended to be applied to solve the detailed layout designs.

This research addresses the detailed layout design, focusing on a construction approach to determine the best possible machine configuration in non-dedicated manufacturing environments. It uses Graph Theory to develop layout alternatives when organisational environments need to deal with multiple products and require that available resources should be shared.

3. Facility Layout Design Solution Approaches (Literature Review)

3.1. Introduction

The Facility Layout Design is a problem that can be classified into either block or detailed layout, following Muther's procedure. The Block Layout is associated with the second phase of this procedure and the Detailed Layout with the third phase; therefore the formulation for the layout design can be established for either the general or the detailed layout designs. In this Chapter, the focus is on the block layout, and the next Chapter will focus on the detailed layout. Before describing these models, this Chapter includes a brief discussion of the modelling process, and of models and some of their limitations when approaching the Facility Layout Design. In addition to these basic issues, two large sections comprise the rest of the chapter. One section deals with formulations related to the Quadratic Assignment Problem and its variants, followed by a discussion of the solution methods available to address a problem like this. The other section presents Graph Theory models followed by their solution approaches, as alternatives to the Quadratic Assignment Problem.

3.2. The Limitations of Facility Layout Design Modelling

The facility layout design can be classified as block or detailed, depending on the level of abstraction of the facility to be modelled. The block layout broadly specifies the relative position among interrelated facilities. Additional work may be required to create a detailed layout of each facility contained in a block layout, which will specify exact facility positions, aisle structures, and utilities outlets. The detailed layout design may include flow-line layout, machine layout, and cellular manufacturing design (Meller R.D., Gau K.Y. 1996). Several approaches to both layout design problems, block and detailed, are found in the literature (Hassan M.M.D., 1994). However, Block Layout formulations as Quadratic Assignment Problems and Graph Theoretic Models are important approaches and are discussed in this chapter.

The mathematical modelling process of the facility layout design is a complex and useful task, when formulating as well as solving these problems. Data reliability and computational complexity are among the limitations which the facility layout designer may confront during its modelling. Data reliability refers to data employed during the model formulation or its solution, and is usually associated with the model's objective functions. The computational complexity feature is related to the solution methods.

Among the most common items related to the objective functions of a Facility Layout Design are flows, times, costs and distances. Reducing any sort of waste is considered an improvement strategy. Hence, improving material flows by reducing distances, reducing the material handling costs without affecting the flow, and/or placing facilities as close as needed, are considered possible strategies. Material flows to a large extent are dependent on market demand, which is considered stochastic in nature and often uncertain; and consequently designers work with demand estimates (Heragu S.S., Kusiak A., 1988). Once real values are available to verify results, designs are already operating and may be expensive to change.

Costs, on the other hand, are usually associated with the material handling systems required to move items from one workcentre to another, and to place and retrieve items from machines. These costs are functions also of displacements required by the items and generally are assumed to be linear and incremental. Times and distances are assumed to be known, but sometimes they are estimated values which may be approximations of the real measures, usually when new facilities have to be designed. There are other occasions on which qualitative values (based on subjective judgments) are used as measures of proximity: they are converted to quantities and used to maximise the proximity among many facilities that are desirable to be grouped together. The major drawback in all cases is the subjectivity of the data and not the reliability of the particular approach to the solution.

The required computer resources are another limitation when handling the facility layout design. Intuitively it requires more memory and computer time to solve a problem that has 100 workcentres than one that has only 10. The number of workcentres may give an adequate idea of the resources that may be needed when solving a problem. If it is expressed as a function, f(n), of the number of workcentres (n), then a problem requires an algorithm which can be solved in polynomial or non-polynomial time. For a problem to be solved in polynomial time means that f(n) is a polynomial function of n such as n, n^2, n^3, ...; this may contrast with non-polynomial time for which f(n) grows exponentially with n in cases such as 2^n, e^n, 3^n, ... and n! (Daskin M.S., 1995:85). Algorithms whose solution time required may be classified under this criterion are labelled as **class P** if they are solvable in polynomial time or **class NP** if they are only solvable in non-polynomial time. The Facility Layout Design Problem is class NP, a point demonstrated by Shani and Gonzalez (Shani S., Gonzalez T., 1976). Moreover, it has been mentioned that obtaining optimal solutions to problems of a combinatorial nature that are NP of size over 15 workcentres is difficult to be optimally solved (Heragu S.S., Kusiak A., 1990). Hence, many approaches to solving the Layout Design problem have been suggested that offer an approximation to the solution of the design question.

There have been two main formulation approaches developed to solve the facility layout design problem: the Quadratic Assignment Problem approach (QAP) and the Graph Theoretic approach (Meller R.D. Gau K.Y., 1996). In addition, there are also non-QAP mathematical modelling approaches

(Heragu S., 1997:123). Some of the most common formulation models and solution approaches are presented in the rest of the chapter. Firstly, the models are discussed and after them their respective solution procedures.

3.3. Quadratic Assignment Problem (QAP)

Among the literature that was at first published on facility layout design was the Quadratic Assignment Problem formulation by Koopmans and Beckman in 1957(Koopmans T.C., Beckman M., 1957), as acknowledged by Meller (Meller R. D., Gau K.Y., 1996), Leung (Leung J., 1992b), and Kusiak and Heragu (Kusiak A., Heragu S.S., 1990). It was firstly introduced to solve the location of facilities considering the reduction of transportation costs between facilities (Koopmans T.C., Beckman M., 1957). Later, it was adapted to address and solve the layout of interacting facilities with equal areas: that is, it assumes that workcentres or departments have equal squared areas (Heragu S. 1997:139). This implies that there are known n fixed locations available, and consequently distances can be predetermined. Hence, the purpose is to assign n facilities to n available locations. A design restriction is that only one workcentre should be assigned to only one specific location and that a location should hold only one workcentre. The term workcentre is used in a generic sense to name a department, a machine, workstation, entity, or any facility in which there is an interest in finding its proper sequence order or adjacency in a layout. It can be said that in the general layout design problem workcentres are departments and the flow between them, the interdepartmental flow, is their interrelations.

The name Quadratic Assignment Problem was suggested because its objective function involves the product of two decision variables. A Quadratic Assignment formulation of the Facility Layout Design formulation model is a problem that can be stated as in Figure 3.1. This assumes that when a department is assigned to a location, it also weights this location by considering the cost of transferring materials from this relative position to the other possible locations. The formulation objective function may address two different kinds of optimisation problems: maximisation and minimisation. In a maximisation situation, the first member of the function, a_{ij}, is considered as gross revenues minus the primary costs excluding the transportation costs, which are considered by the second term when a facility i is assigned to a specific location j. In the other situation, minimisation, in the first member, a_{ij}, is considered as a fixed cost of assigning facility i to location j and the second member deals with the transportation costs.

$$OPT \quad \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}x_{ij} - \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} f_{ik}c_{jl}x_{ij}x_{kl} \qquad (1)$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1,2,\ldots,n \qquad (2)$$

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1,2,\ldots,n \qquad (3)$$

$$x_{ij} \in \{0,1\}, \qquad i,j = 1,2,\ldots,n \qquad (4)$$

Where

n, is the total number of facilities and locations

a_{ij}, is the gross revenue minus costs excluding the transporation cost from assigning facility i to location j,

or it can be the fixed cost of assigning facility i in location j

f_{ij}, is the expected flow of material from facility i to facility k

c_{ij}, is the cost of transporting a material unit from location i to location j

x_{ij}, is the decision variable, where it may take a value of 1 if facility i is assigned to location j, and 0 value, otherwise.

Figure 3.1 Quadratic Assignment Problem (QAP) Formulation Model (Heragu S., 1997:124)

In both cases, minimisation and maximisation, the transportation costs consider the goods flow and the unitary transportation cost between facilities assigned. In most of the cases the pursued objective function is intended to minimise the transportation costs, the second member of the objective function, omitting the first member of the function by considering it to be irrelevant given that they are equal or meaningless (Heragu S., 1997:125).

Although the QAP has been used to model facility layout design problems, it cannot be useful in all types of formulations (Heragu S., 1997:126). In a machine layout design situation, it cannot be used because machines are usually different in size and shape, and consequently distances between locations cannot previously be determined. This situation is avoided when assuming equal-sized areas, an assumption that might not hold in the case of machine layouts (Heragu S., 1997:126). These types of layouts will be discussed in more detail in the following chapter.

There are two particular cases of the Quadratic Assignment Problem, known as the Linear Assignment Problem (LAP) and the Travelling Salesman Problem (TSP) (Kusiak A., Heragu S.S., 1987). The LAP situation is assumed to be different from the QAP because its concern is only in the relative assignment of an entity to a location. That is, the objective function is reduced to the first term by considering that there are no interrelations between facilities and consequently, there is no flow of goods between them. In the TSP case, the shortest route that is sought begins and ends in the same facility and visits each facility once during the tour. The number of facilities, n, is given and so are the distances between each pair of them. Thus, the objective is to assign the n facilities to one of the n positions in the path so that the travelled distance is minimised. As in the case of the QAP, the LAP and the TSP require the number of locations and the number of entities to be the same; otherwise, dummy entities must be created.

In the following sections, the focus is on other formulations used to model the facility layout design. These cover the variants or derivations from the QAP, and other mathematical tools used to formulate and address this problem from different perspectives, such as Dynamic, Multi Objective and Mixed Integer Programming.

3.3.1. QAP Modelling Variations

A way to deal with different quadrilateral size areas is suggested by dividing the area to be occupied by the facilities into a grid, creating artificial flows between these grids to ensure that they are not split. The areas of the layout and the individual facilities are expressed as unit squares (Sule D.R., 1994:487). Doing this only multiplies the need for more computational resources, making it almost impossible to solve small design problems with a few blocks of unequal areas (Meller R.D., Gau K.Y., 1996). This formulation is known as the Quadratic Set Covering Problem (QSP), in which distances are measured from the centroids (Kusiak A., Heragu S.S., 1987).

Lawler (Lawler E.L., 1963) also demonstrated an equivalent linear model in which the term $\mathbf{x_{ij}*x_{kl}}$ located in the second member of the QAP objective function (Figure 3.1) is replaced by $\mathbf{y_{ijkl}}$, obtaining this way an equivalent and simpler version of the problem representation, where $\mathbf{y_{ijkl}}$ takes the value of one, if there is an existing interrelationship between facility i at location j and facility k at location l, and zero otherwise. Another modelling modification to the QAP is the one suggested by Bozer and Meller (Bozer, Y.A., Meller R.D., 1997), in which they propose to modify the objective function formulation by including a variant of a distance metric called expected distance (EDIST) as an alternative to centroid-to-centroid (CTC) distances, overcoming some of the limitations of the CTC when solving problems with facilities that may be concentric and that may have different quadrilateral shapes. This proposed formulation adjusts the location of the centroid to the required shape of the facility allowing the expected rectilinear distance between two facilities to be a more representative metric when their input and output points are unknown. This supports a paper statement that even if the QAP is optimally solved using the CTC metric, from the distance perspective it is not an appropriate solution. This statement holds given that the questions addressed are different in each case. The QAP seeks to solve the best arrangement possible on the basis of costs, and this distance-based approach seeks the same arrangement under distance-based terms.

3.3.2. Mixed Integer Programming (MIP)

Considering the types of values that variables are allowed to take, most of the formulations can be labelled as continuous, discrete or hybrid, and obviously this bounds the possibilities and characteristics of the solution space. The mixed integer programming is a hybrid type of formulation. Two non-linear variants of these types of models, called ABSMODEL 1 and ABSMODEL 2, are discussed by Heragu (Heragu S.S., 1992), who claim that these models could handle facilities with rectangular or square shapes. These proposed models address single-row and multiple-row formulations respectively. Their objective functions terms are non-linear terms that are expressions to provide the relative location of entities in a plane which has to be linearised by the introduction of additional reference variables. The objective function, as stated in these models, tries to minimise the total cost involved in making the required number of trips between facilities. Facilities with different shapes are approximated to be quadrilateral, simplifying the modelling process and its solution as well.

Since the decision variables (x_i) represent distances between the centre of facility i to a vertical line of reference, then the orientation of the entity is required to determine whether the longer or the shorter side is to be placed horizontally in the layout. Of course, in square shapes this is not an issue. For instance, in the ABSMODEL 1, x_i is the horizontal distance from the centre of any entity i to the reference line. Furthermore, an enhancement based on this formulation is one that includes a vertical reference (ABSMODEL 2), allowing the formulation to handle multiple rows (Heragu S.S., Kusiak A., 1991). Moreover, there is another variant of this method, ABSMODEL 3, whose formulation may include facilities that have unequal rectangular areas, and may include horizontal and vertical clearance between facilities (Heragu S.S., 1992). These approaches are solved as continuous models using linear programming solution software, which is more accessible and may require fewer computational resources than the original QAP formulation.

Another approach is presented by Kim and Kim (Kim J.G., Kim Y.D., 2000), who suggest a model that considers unequal rectangular area shapes. In addition, the model includes the input and output orientation positions for each of the facilities, with four possible orientation points to be taken into consideration. It tries to minimise the sum of rectilinear distances weighted by flow amounts between input and output points of the facilities.

3.3.3. Multiple Objective Programming (MOP)

These types of problem differ from previous formulations because they include more than one objective function. Most of these models, which are related to the facility layout design problem formulation, combine these objectives into one objective function. Additionally, these models are named depending on how these individual objective functions are combined: namely, multiplicity, additive, and weighted additive (Sha D.Y., Chen C. W., 2001). In their paper, Sha and Chen (Sha D.Y., Chen C. W., 2001) combine qualitative and quantitative objective functions into one weighted normalised objective function, to avoid scale and measurement unit problems. This qualitative function is based on a relationship diagram, as the one mentioned in section 2.2, and the quantitative function on material handling costs between facilities. It should be noted that different solutions might be obtained when evaluating for different weight values in the objective function. For example, when two objective functions are used, f_1 and f_2, they may be combined into an additive weighted function as follows: $w_1f_1 + w_2 f_2$, where w_1 and w_2 are the weights and may be normalised, that is, $w_1 + w_2 =$

1. This statement establishes that w_1 and w_2 have decimal values ranging from zero to one: consequently, in the extreme case that w_1 may be one the other is zero, and vice versa; the solutions may range from optimising one of the objective functions to optimising the other. Hence, a range of compromise solutions can be found depending on the w's values.

Another hybrid approach to the multiple objective formulation differs from the previous one in that this application combines four objective functions, which, it is claimed, can be used to solve QAP (Sarin S.C., Loharjun P., Malmborg C. J., Krishnakumar B., 1992). This formulation changes the objective function by introducing a weighted preference rating which includes different criteria in a weighted additive form expressed as a ratio of the preference value for a facility pair i, j over the rectilinear distance between the centroides of these locations. After this, a QAP problem is solved and evaluated using a decision-making procedure which allows for the comparison between alternatives and selection of a layout design.

3.3.4. Dynamic Programming (DP)

A different approach from the above is the dynamic programming approach, which addresses the dynamic changing aspects of the layout environments by considering multiple periods, whereas all the previous models are static or single period in time and all are deterministic. This formulation attempts to solve a layout problem for each period while simultaneously minimising costs for the total periods. Thus, if T periods are to be considered, and if n! is the maximum number of different layouts in any given period (Sule D.R., 1994:484) (assuming that there are n facilities in each period), then the maximum number of combinations to be evaluated are $(n!)^T$. As in the previous approaches, considering the total number of layout combinations in each period requires simple procedures to solve this considerable problem; otherwise it is almost impossible to solve (Rosenblatt M.J., 1986). Additionally, a new constraint that links each period has to be considered. In order to reduce the number of layout evaluations per period, it is suggested that a lower and an upper bound are used as a reference; it is also claimed that there is no need to evaluate all the n! alternatives, but a smaller number which may provide the optimal solution. Likewise, it should be clear that additional problems have to be sorted out: for instance, how to obtain a good lower bound and how to obtain the best possible solutions in each period (Rosenblatt M.J., 1986).

Another model (Balakrishnan J. Jacobs F. R., Venkataramanan M.A., 1992) that considers changes over time in the layout includes an additional constraint which considers budget availability for layout rearrangements through the different periods.

3.3.5. Other Formulations

New generations of layout are needed for new evolving manufacturing systems: for example, group technologies or cellular systems require different approaches such as Fractal Layouts (Venkatadri U., Rardin R. L., Montreuil B., 1997). Fractal Layouts, as proposed by Venkatadri, Rardin, and Montreuil, (Venkatadri U., Rardin R. L., Montreuil B., 1997), suggest the creation of multifunctional mini-factories or modules, possibly taking advantage of the idea proposed by Skinner on dedicated facilities (Skinner, W, 1974). The above researchers proposed to create a unit that is a set of contiguous work units capable of processing most of the products. This set, unit or cell, labelled as a fractal, has a homomorphic characteristic that retains the same proportion of workstation types that exists within the floor shop based on routing information. Once the cell is created, a QAP-based model is used to minimise the travelled distances of the flows for an expected demand volume, using a QAP approach for inter and intra cell layouts.

3.4. QAP Solution Approaches

The algorithms that have been proposed to solve the above formulations of the Facility Layout Design may be classified as either optimisation or heuristics procedures (Heragu S., 1997:164). Optimisation models are known to provide the best possible solution and heuristics models to provide good solutions, but not necessarily the best possible in the optimisation sense.

Since facilities layout problems are known to be NP-complete problems (Shani S. and Gonzalez T., 1976), optimal algorithms can produce solutions for only small-sized problems, typically with as high as 20 or fewer facilities.

Thus, they are considered impractical for solving real problem situations. However, optimal algorithms are relevant because they provide useful information that may be used when developing heuristic algorithms (Heragu, 1997:202).

3.4.1. Optimal Algorithms

Once a layout design problem has been formulated as QAP, there are three types of possible algorithms available that may optimally solve this problem: namely, branch and bound, and cutting plane algorithms (Heragu S., 1997:202; Geoffrion A.M., Marsten R.E., 1972). These algorithms are suggested to find integer solutions once an optimal feasible solution has been obtained by the methods available. In addition, under a general framework these algorithms are based upon three key notions: separation, relaxation and fathoming (Geoffrion A.M., Marsten R.E., 1972), terms which are addressed in more detail in the following sections. In spite of their relevance and historical importance, it has been claimed (Welgama P.S., Gibson P.R., 1995) that facility design practitioners rarely use these algorithms. The branch and bound and the cutting planes algorithms are briefly discussed as follows.

3.4.1.1. Branch and Bound Method
(Kusiak A., Heragu S.S., 1987)

Branch and Bound methods are efficient enumeration methods commonly used to find integer optimal solutions to optimisation formulations. An integer programming formulation is relaxed and solved using a continuous formulation. This solution is used as a bound against which Branch and Bound solutions are compared. To obtain integer solutions the algorithm systematically searches for them starting from the QAP optimal continuous solution and separating it into two new candidate problems by dichotomising the value of one of the fractional valued decision variables found in the optimal solution. Each of the candidate problems contains a new constraint, which is related to the possible lower and upper integer values of a chosen variable that is then forced to have integer values. Once each new candidate sub-problem is evaluated, the new solutions are compared to the previous one and three situations can arise. The first is that the new solution is a better solution: that is, if the objective function is to be minimised, then the new solution produces a new minimum with some or all integer decision variables, which improves the previous solution and is taken as the new optimal and the procedure continues along that branch. Secondly, if the solutions are equal, the search may continue converting the values of the remaining variables into integer values, if there are still non-integer decision variables in the process to be changed to integers. Finally, in the third case, it may be that the new solution exceeds the previous optimal (in a maximisation problem), and therefore this branch searching for integer solutions is abandoned and the branch is labelled as fathomed. This ends when feasible optimal integer solutions are found or there are no more candidate sub-problems to be evaluated.

For instance, Rosenblatt and Golany (Rosenblatt M.J., Golany B., 1992) proposed a model which formulates a distance-based objective function. This formulation is solved using Linear Programming, which seeks to reduce the total material handling costs measured as the inter-departmental flow and distance between departments. Branch and Bound procedure is then applied to this linear programming solution, producing a single row layout. During each iteration of the procedure, another department is assigned to each available site, comparing the new solution to the previous one, and choosing the formulation that leads to the best output or abandoning this search for a different branch when the new solution is not better than a previous one available.

3.4.1.2. Cutting Plane Method
(Kusiak A., Heragu S.S., 1987)

This approach is based on adding additional constraints to the optimal solution of the original relaxed linear formulation. These new constraints bound the existing feasible solution space allowing the existing optimal solution to approach an equivalent integer solution, which, in the strict optimisation sense, is a sub-optimal solution. One new constraint is added at a time, arbitrarily selected from those decision variables which belong to the optimal solution basis and have non-integer values. These new constraints take advantage of the fact that numbers can be decomposed and expressed by their integer and fractional parts. They are separated and the left hand side of the new equation may retain the integer parts and the right hand side the fractional ones. This is used to generate the new constraint which, as mentioned above, bounds the solution space, having a cut effect on it. Garfinkel and Nemhauser (Garfinkel R.S., Nemhauser G.L., 1972:158) show that this new constraint does not exclude any feasible integer solution since it is implied by integrality. If after adding the new constraint and solving this new problem, there still are decision variables on the basis which are non integer valued, the procedure is repeated until they become integer valued, unless thy are allowed to be non integer valued as in a mixed integer programming formulation. This search ends when no better optimal solution can be found or the new candidate problem gives an unfeasible solution. This cutting plane method, proposed as an alternative to the branch and bound technique, uses almost the same type of computational requirements: large amounts of computer time and storage capacity.

Given that these previous solution approaches required a large amount of computer resources to solve relatively small sized problems, this led researchers to seek alternatives in order to solve the facility layout design formulations. Although these alternative heuristic methods might be preferred in most practical cases, they do not guarantee optimality. They give acceptable solutions, with relatively fewer computational requirements than optimal procedures (Heragu S.S., Kusiak A., 1988).

3.4.2. Heuristic Approaches

Heuristics have the reputation of having the ability to produce good sub-optimal solutions and the capacity for handling larger problems with a reasonable amount of computational resources. These heuristics are classified as construction, improvement, and hybrid algorithms. Graph-Theoretic algorithms are also classified as heuristic approaches and will be presented later (Heragu S.S., Kusiak A., 1988).

3.4.2.1. Construction Methods

The layout construction methods start with an initial facility and continue building the solution by adding another facility at each iteration, until all the facilities are completely assigned and the entire layout is achieved (Francis R.L., McGinnis L. F., White J.A., 1992:157). For most construction methods, the way in which new entrant entities are selected affects the solution quality. By selecting the facilities with strong relationships to enter the layout first, they are guaranteed a favourable place in the layout, allowing the resources to flow shorter distances, but this does not necessarily guarantee the best solution (Sule D.R., 1994:506).

3.4.2.2. Improvement Methods

Improvement methods require an initial layout from which systematic exchanges are performed, seeking better solutions; at each exchange, results are evaluated. The exchange that produces the best solution is retained and the procedure continues until no further improvement in the solution can be found. The quality of the resulting layout is usually dependent on the initial solution (Welgama P.S., Gibson P.R., 1995; Kusiak A., Heragu S.S., 1987). Improvement methods lead to better solutions than construction methods, at the expense of increased computational resources (Heragu S.S., Kusiak A., 1988). The basic notion behind the method is that when a layout design solution is given, it can be improved by interchanging the location of the entities. The simplest modification is to permute two facilities and evaluate their contribution to the objective function. This is the basis for a pairwise interchange improvement or a 2-opt heuristic. The other strategy followed utilises 3-way exchanges or 3-opt procedures. It is expected that the 3-opt method will generate more layouts to be evaluated than the 2-opt method, hence expecting better solutions to be obtained at the expense of computation resources (Heragu S.S., 1997:176). Bozer, Meller and Erlebacher (Bozer Y.A., Meller R.D., and Erlebacher S.J., 1994) presented a modification to this approach using a 2-opt procedure and supported by grid type areas, which allowed them to address a multiple floor formulation with different quadrilateral shapes as well.

3.4.2.3. Hybrid Methods

These methods have the characteristic of combining more than one of the approaches mentioned above. That is, they combine an optimal and a heuristic approach or use a construction and an improvement method (Welgama P.S., Gibson P.R., 1995; Heragu S.S., Kusiak A., 1988).

KTM, proposed by Kaku, Thompson and Morton (Kaku B.K., Thompson G.L., Morton T.E., 1991, is a hybrid heuristic to address the facilities layout, combining a construction and improvement procedures. Their approach uses three phases, starting by developing some partial assignments which are used as the basis for the construction of the layout solution in the following phase. These assignments allow a facility to be placed in a location and they are evaluated and ranked as a reference position to the assignment, omitting from consideration those poorly valued according to the pursued objective function. Further, in the construction of the layout design these partial assignments are taken from the most highly valued to the least valued to complete a feasible solution considering an augmented solution based on the previous assignments by solving each Linear Assignment Problem. Once a complete solution is obtained this is sought to be improved by using pairwise and triple interchange routines. As an additional result, the researchers have found evidence that for fewer than 20 facilities the pairwise interchange provided most of the best known optimal solutions, suggesting that a second triple exchange, after a pairwise one, did not give many benefits. By doing this they claimed that computational resources might be saved.

Heuristic Strategy	Software	Number of Objectives	Type of Objectives	Formulation	Shapes
Construction	SHAPE	Single	Flow Distance	Rectilinear Distances	Grid
Improvement	CRAFT MULTIPLE	Single Single	Flow Distance Cost Flow Distance Cost	2-Opt and 3-Opt Multiple Floors, CRAFT Based	Grid Grid
Hybrid Construction and Improvement	KTM DISCON	Single Single	Flow Distance Cost Cost Distance	Opt-2 and Opt-3 Euclidean Distances	None Circular
Hybrid Optimisation and Heuristic	FLAC	Single	Flow Distance Cost	LAP, CRAFT Based	Square
Graph Theoretic	TESSA	Single	Relationship	Face Based Heuristic	None
Hybrid Graph and Improvement	SPIRAL	Single	Relationship	CRAFT Based	Rectangular

Table 3.1. Facility Layout Design Approach Computer Implementation

The computer implementation of some of the approaches discussed in this section, which were found in the literature, are summarised and presented in Table 3.1. Among them can be found: SHAPE (Hassan M.M.S., Hogg G.L., Smith D.R., 1986), CRAFT (Buffa E.S., Armour G.C., Vollmann T.E., 1964), MULTIPLE (Bozer Y.A., Meller R.D., Erlebacher S.J., 1994), KTM (Kaku B.K., Thompson G.L., Morton T.E., 1991), DISCON (Drezner Z., 1980), TESSA (Boswell S.G., 1992), and, SPIRAL (Goetschalckx M., 1992)

3.5. Graph Theoretic Formulations

In Graph Theoretic formulations, facility areas and adjacency relationships are formulated analytically or pictorially indicated on graphs. A graph consists of two finite non-empty sets, one of nodes and the other a set of edges that joins the nodes. The set of nodes is used to represent facilities or workcentres and the set of edges their relationships. Graph theoretic methods assume non-negative weights associated with each edge which symbolize the desired closeness between facilities. Based on these values, designers generate alternative layouts showing the facilities' relative positions and seek the best arrangement between facilities. Figure 3.2 shows a Graph Theoretic Analytical formulation.

Some of the differences that exist between Graph Theoretic and QAP formulations are (Foulds F.L., 1983):

- The Graph Theoretic formulation is more suitable when there is more freedom in the design, i.e. in how the facilities can be laid out in the space available, whereas the QAP approach is more useful when the shape and the size of the facilities required are known.

- In addition to representation of adjacent weights used by both formulations, Graph Theoretic formulations can recognize the existence of possible flows between non-adjacent facilities and make use of them.

$$OPT \quad \sum_{i \in E} \sum_{j \in E} w_{ij}\, x_{ij} \tag{5}$$

subject to

$$x_{ij} = 1, \qquad \{i,j\} \in N; \tag{6}$$

$$x_{ij} = 0, \qquad \{i,j\} \in F; \tag{7}$$

$$(V, E' \cup N) \text{ is a planar graph} \tag{8}$$

Where :

G = (V, E), is a weighted graph with V and E being two no empty sets
V, is the set of vertices, nodes, or facilities
E, is the set of edges which represent the links (interrelationships) between pairs of facilities
wij, is the closeness rating indicating desirability of locating facility i adjacenct to facility j,
N, is the set of edges which are pairs of facilities that must be adjacent in any facility solution
F, is the set of pairs of facilities which must not be adjacent in any feasible solution
E, = {{i, j} : xij = 1, {i, j} ∈ E}

Figure 3.2. Graph Theoretic Formulation Model
(Foulds L.R., 1983)

- Relocation costs per unit of time can be taken into account by QAP formulations whereas in Graph Theoretic formulations are assumed to be zero or irrelevant. This means that graph approaches are more suitable for new layouts than for modifying existing ones.

- Graph Theoretic formulations do not consider geometric shapes and space requirements.

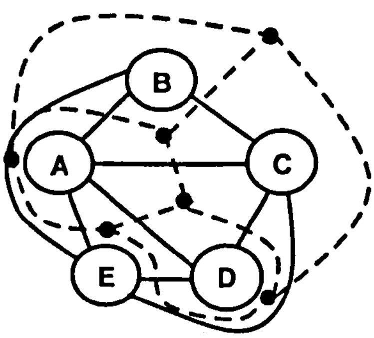
3.6. Graph Theoretic Solution Approaches

Graph Theoretic approaches try to find which pairs of facilities are to be adjacent so that the sum of the closeness desirability is maximised. The closeness desirability is a numerical value given as a weight associated with each one of the edges (Heragu S.S., 1997:250). Moreover, in an enhancement developed in recent years, these formulations can be modified to include how a flow may impact not only on adjacent facilities but also can include a flow weight for non adjacent facilities.

The Graph Theory solution approaches used to solve the formulation are: maximally planar weighted graph, deltahedron, wheel expansion, modified spanning tree, and the greedy methods. Note that most of the graph theoretic methods can be classified as construction algorithms (Kusiak A., Heragu S.S., 1987).

3.6.1. Maximally Planar Weighted Graphs Method (MPWG)

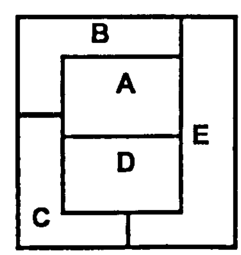
According to Kusiak and Heragu (Kusiak A., Heragu S.S., 1987), the Maximal Planar Graph method was originally proposed by Seppänen and Moore (Seppänen J., Moore J.M. 1970) and has been used as the basis for the Maximally Planar Weighted Graph. This approach uses weighted arcs to construct the proposed solution planar graph. A planar graph is one that can be drawn in a two-dimensional plane in such a way that none of its edges intersect. It is a complete planar graph when all its nodes are interconnected. A maximal planar graph is complete when it has (2n-4) faces and (3n-6) edges (Seppänen J., Moore J.M. 1970). Under this approach, the weight or the score of the edges provides the guideline to choose which pairs of facilities are to be adjacent. Therefore, a maximal planar graph becomes a maximally planar weighted graph when its selected edges are the ones with the maximum weight.



a. Relationship Graph

	A	B	C	D	E
A		O	U	I	E
B			A	U	I
C				E	O
D					A
E					

b. Relationship Chart



c. Block Layout

Figure 3.3. Relationship Chart and Graph

The development of a solution using this approach begins by forming a planar weighted graph from the relationships between facilities whose vertices and the weights on the edges represent facilities and closeness ratings respectively; these weights are taken from the Relationship Chart, as shown in Figure 3.3.b. This figure shows 10 edges between facilities, and only nine are required to have a maximally planar weighted graph. At the moment of selecting the edges that will form the planar graph, there are possibilities to leave aside edges with higher weight and end with a maximally planar weighted but not maximally weighted planar graph.

An attempt is made to identify a maximal planar sub-graph of relatively high weight. The elaboration of this graph is drawn to reflect the relationship intensities as shown in Figure 3.3.a. The approach starts by selecting those facilities with the highest weight relationships. It continues adding edges according to this criterion until no further edge can be added because the graph develops into a non-planar graph. The graph is drawn based on the relationship intensities as shown in (Figure 3.3.b), except that to add the last edge, the ninth, the edge between A and C is arbitrarily selected instead of the edge between B and D, which has the same relative importance. The graph shown in Figure 3.3.a is a maximal planar graph and since all the interrelationships are the ones with the highest possible values, it is also a maximally weighted planar graph, but is not always the case.

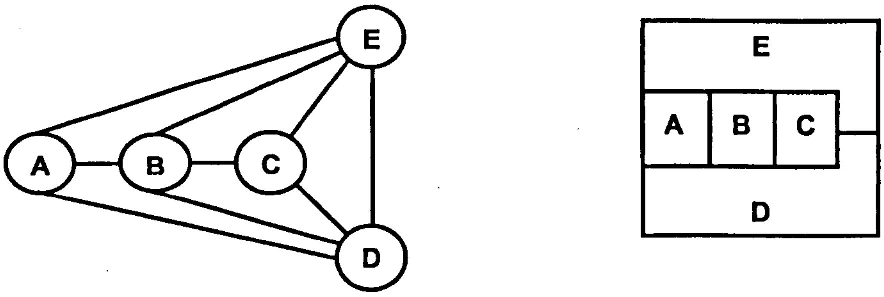
Once this relationship graph is created, in order to be able to elaborate the layout design, another graph has to be developed. This new graph, called the dual, is then constructed based on the Relationship Graph. The dual represents a layout (Green R.H., Al-Hakim L., 1985), apart from the fact that shapes and areas have not been taken into account. The edges and the faces from the Relationship Graph become the faces and the edges in the dual graph, respectively. The dual is shown in dotted lines in Figure 3.3.a.

From the dual, the block layout that is equivalent to the original Relationship Graph is obtained (Foulds L.R., 1983). In this last step of the method, the block layout design is derived from the dual graph. This dual graph corresponds to the Block Layout as follows: the faces that are points of intersection of at least three facilities, for instance faces A, B and C, should be placed closely in the block layout. The edges represent boundaries between adjacent facilities implying all the required boundaries between existing facilities: for example, boundaries between A and C, A and B, A and D, and A and E are required, and there is no boundary required from A to the exterior. Additional care should be taken when developing the Block Layout since there may be some facilities that should be on the outside of the layout: in this case facilities B, C and E are the faces to be placed on the layout periphery because they appear in the triangular phase connected to the exterior dual phase. One of the major disadvantages of this approach is that it requires the planarity to be tested, which is regarded as a very complicated task (Foulds L.R., Robinson D.F., 1978).

An interesting result is that any graph with n vertices ($n > 3$) requires ($3n-6$) adjacencies from ($n(n-1)/2$) possible adjacencies to become a maximal planar graph. However, in most real problems ($3n-6$) edges with weights may be enough to create a layout, and if all of them have highest positive weight values then a maximal planar graph will also be a maximally weighted planar graph (Green R.H., Al-Hakim L., 1985). In their paper, Green and Al-Hakim (Green R.H., Al-Hakim L., 1985) use matrices for computer implementations instead of drawing graphs. They use this result and recognise the existence of having an upper bound when considering the highest weights from the ($3n-6$) edges that a MPWG should have for the optimal solution

3.6.2. Deltahedron Method

This method develops certain constructions, called deltahedra, which are proven to be always maximal planar sub-graphs and therefore avoid the required testing for planarity (Kusiak A., Heragu S.S., 1987). A graph with three vertices and three edges, known as a triangular graph, has the properties of a planar and complete graph. In consequence, if all faces of a planar graph have three edges and three vertices, this assures the planarity of the graph (Carrie A.S., Moore J.M., Roczniak M., Seppänen J.J., 1978). The approach starts by defining a triangular graph with the highest valued edges; then a fourth vertex is selected to be added to the graph, carefully considering the highest sum of the edges from the vertex to be inserted. This new vertex is introduced in the centre of the triangular face created by the initial vertices; edges are added connecting this new vertex to the previous ones, giving the appearance of a tetrahedron, as shown in Figure 3.4. This step, retaining the triangulation during the insertion process, will guarantee that the planarity of the graph is maintained (Foulds L.R., Robinson D.F., 1978). At each subsequent step, vertices are added to the previous graph in the same way until all vertices are introduced. Since each time a new vertex is introduced it creates triangular sub-graphs, which are complete and planar, no planarity test is required (Foulds L.R., Gibbons P.B., Giffin J.W., 1985). The recognition of this fact is considered as a great improvement because proving the planarity of a graph is considered a very difficult task.



a. Deltahedron Graph

b. Block Layout

Figure 3.4. Deltahedron Graph

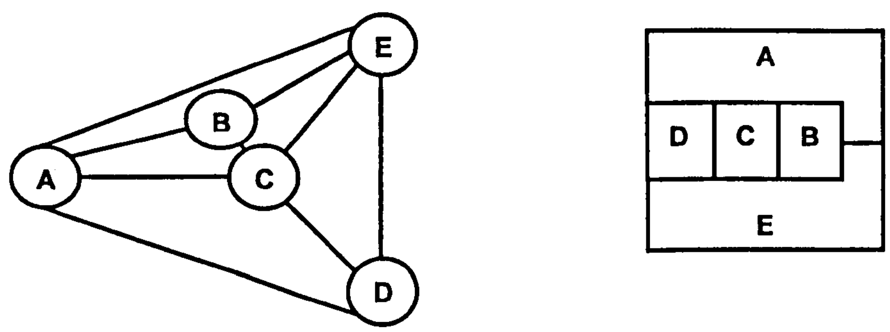
The block layout should follow from the graph, which depicts that boundaries should exist between facility D and facilities A, B, C and E; facility E with A, B, C and D; facility A with facilities B, E and D; facility B with A, C, D and E; and, facility C with facilities B, D and E.

The final deltahedron represents a layout that is expected to be of high quality, that is, that the edges which have been considered in the graph are the ones with the highest values and, thus, it is a maximal weighted planar graph. This quality can be judged based on the fact that (3n-6) edges are the maximum number that a planar graph can have and thus, an obvious upper bound on the value of an optimal solution is the sum of the closeness ratings or weights from the (3n-6) high value edges. A solution example to the Relationship Chart shown in Figure 3.3.b is given in Figure 3.4 using this procedure. Of course, for a fixed number of facilities to be placed, it may be expected that the number of possible evaluations at each step will increase.

However, as the number of faces increases in steps of two each time, the number of vertices simultaneously decreases, one at a time.

3.6.3. Wheel Expansion Method

A wheel on n vertices is a graph that consists of a cycle of (n-1) vertices, such that each of the (n-1) vertices is adjacent to an additional central vertex. This method is similar to that of the deltahedron, but Foulds, Gibbons and Giffin (Foulds L.R., Gibbons P.B., Giffin J.W., 1985) considered it as an alternative promising approach to be studied. The procedure (Eades P., Foulds L., Giffin J., 1982) constructs a wheel, which has n vertices of which one is a centre (called the hub), and the other (n-1) creates a cycle (termed the rim). The procedure starts by constructing a tetrahedron which is expected to have the edges with the highest values, as shown in Figure 3.5. The procedure continues by evaluation of the insertion of the new vertex, but instead of evaluating all the possible links, the procedure suggests taking into consideration only the ones that are located in the rim, that is, the vertices placed in the periphery of the figure. In the example with already allocated facilities A, C, D and E, the insertion of vertex B should be made by evaluating the links from B to A and D, from B to D and E, and from B to A and E, and the one with the highest value should be inserted, in this case with A and E. It should be noted that the link from B to C was not taken into consideration since it is the same in all cases. The procedure continues in a similar manner until all the facilities are placed in the figure. Since triangular faces have been created with each insertion, then all the faces are complete and planar, and there is no need to validate for planarity. As in the previous approaches, it can be seen that facility A has boundaries with B, C, D and E; facility B with facilities A, C, D and E; facility C with A, B, D and E; facility D with A, C and E; and, facility E with facilities A, B, C and D, which are retained in the block layout.



a. Wheel Expansion Graph

b. Block Layout

Figure 3.5. The Wheel Expansion Graph

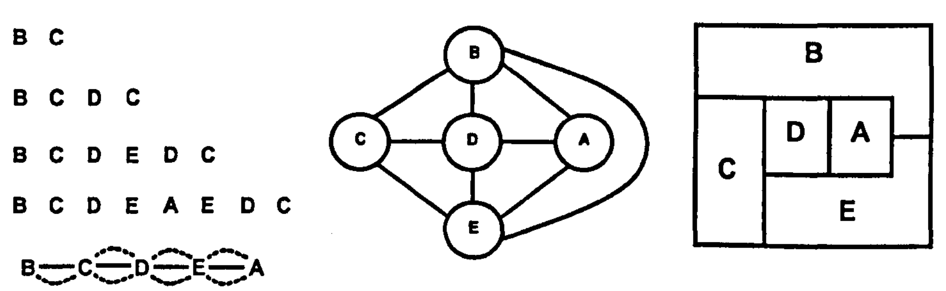
3.6.4. Modified Spanning Tree Method (MST)

(Heragu S.S., 1997:165)

The procedure suggested follows Kruskal's algorithm to determine the maximum or the minimum spanning tree (Moore J.M., 1976). This method is used to generate a tree, a spanning tree, which links once each of the vertices from a Relationship Chart or a Volume Flow Matrix without creating cycles.

For the development of the example, data are taken from Figure 3.3.b.

The method starts by creating a ranking of all the edges of the Relationship Chart from highest to lowest, where the largest value represents the highest weight edge associated with the facilities of the graph that ideally should be adjacent or closest. Following a construction approach fashion, the method incorporates facilities using this ranking and creating a string of links as shown in Figure 3.6.a, which become the basis to create the Relationship Graph, Figure 3.3.b (Moore J.M., 1976),. The method continues by identifying the largest weights which can be connected to those already selected. From the vertices still to be connected, it continues by adding them to the previous selected ones, until it completes a sequence which includes all the facilities. This algorithm attempts to optimise the sum of the weights of edges of adjacent facilities, i.e., it minimises the total flow times distances, and cannot be considered to solve the layout problem optimally because it does not take into consideration non-adjacent facilities (Heragu S.S., 1997:165). Once the spanning tree is completed (Figure 3.6.a), then if there are any remaining high priority edges to be added to complete (3n-6) edges, they are added until a planar graph is obtained: that is, a maximally planar weighted graph.



a. Maximal Spanning Tree

b. Modified Spanning Graph

b. Block Layout

Figure 3.6. Modified Spanning Tree Graph

A proposed computational variant of this method, which constructs character strings from the spanning tree, is shown in Figure 3.6.a; following the dotted lines it would be BCDEAEDC. A computational implementation of these procedures using character strings was presented by Carrie et al (Carrie A.S., Moore J.M., Roczniak M., Seppänen J.J., 1978).

3.6.5. Greedy Method

This approach takes advantage of the properties of the graphs used by other Graph Theoretic methods and is considered to be a generalisation of the deltahedron approach (Caccetta L., Kusumah Y.S., 1999). This method uses either a vertex or three vertices insertion at each step, starting with an initial triangular graph. The single insertion is evaluated similarly to the deltahedron method, whereas the three vertices insertion involves the weighted average of the edges to be added: that is, the sum of the weights of the required edges is divided by three to make it comparable to the single vertex insertion. Based on a comparison between the single versus the three vertices insertion, the sum that provides the best value is selected and the insertion is performed. The procedure continues until all facilities are considered (Leung J., 1992b). As with the previous approaches, vertices and edges are inserted, keeping the planar triangulation. This method provides better solutions than the deltahedron but utilises more computing resources (Caccetta L., Kusumah Y.S., 1999, Leung J., 1992b).

3.7. Summary

Facilities Layout Modelling processes are an aid for the designer for how things might look and not for how things would look, since the solutions provided are the product of working with models and not with real situations. Thus, these models have to be taken as descriptive, not normative. One drawback when modelling facilities layout designs concerns the assumptions of linearity of material handling costs and the treatment of the flows as deterministic, when in fact these data show variations that cannot be determined with certainty. Also, the movement between facilities is assumed to be measured between centres when it can be rectilinear, following perpendicular aisles. Likewise, entities' shapes and area effects, scale effects, and locations of input/output ports and orientations are among the issues that have to be handled by layout designers.

In the way the Quadratic Assignment Problem and the Graph Theory approaches address the Facility Layout Design Problem, they have similarities and differences. Some of their main differences lie in their scope, i.e., in the many assumptions made in the model and in their solution approaches. Although the single floor, single period, deterministic problem can be addressed by both approaches, the multiple floors, multiple objectives, and multiple periods was found to be stated only for the QAP. Both approaches use the same type of input data; they are distance or adjacency based. The Graph Theory approach initially does omit area requirements and shapes, allowing the designer to consider them in a subsequent stage of the Facility Layout Design process, as suggested by Muther's SLP

It should be emphasised that most of these methods are useful when there is one product or when there is a single most important one, and dedicated facilities may be appropriate to produce it. However, there are situations where this may not be true, situations such as when there are limited resources, when there is no predominant product, or when economical production in dedicated facilities is not feasible.

On the other hand, some properties may affect the solution procedures: planarity for example. Most of the graph theoretic approaches make use of this property. However, planarity may not be useful when producing multiple products using non-dedicated facilities and there may not exist products which dominate. Planarity should not be considered in the layout design since all the production steps for the different products are required in such cases.

All optimal algorithms available for solving the facility layout design are NP-complete, and consequently exact or optimal solution methods are only feasible for small size problems. Because of this, most layout solution approaches are heuristic in nature. Among heuristic methods, it has been suggested that a combination of constructive and improvement approaches can improve on results obtained by single heuristic methods in a solution (Kaku B.K., Thompson G.L., Morton T.E., 1991).

Although block layouts were discussed in this chapter and their usefulness and contributions were stressed, some researchers still emphasize that there is an additional need for approaches that may address the layout in more detail: that is, the layouts that are oriented to the layout analysis and design of production activities. These production layout approaches that may be found between product and process layouts will be explored and discussed in more detail in the following chapter, as relevant integrators for production layouts.

4. Machine Layout Modelling and Solution Approaches (Literature Review)

4.1. Introduction

Detailed Layout designs focus on the elements contained in each one of the entities considered for the Block Layout design. Although this is a matter of perspective, since the interactions between blocks are labelled as inter blocks and the insides of them as intra blocks, some detailed layout enthusiasts have extended their work from detailed to block by considering both intra and inter flows in their works. In some of the pioneering works on the subject (Carrie A.S., 1975), and in some more recent ones (Hassan M.M.D. 1995), the problems of detailed layout designs are considered for different production systems based on two features, workflow and workload. Four types of production system are identified for families of products: single product line, multi-product line either uni- or bi-directional flow, and group cell. Similar classifications were also considered in the developments of the principles of Group Technology (Burbidge, J.L., 1991; Baker R.P., Maropoulos P.G., 2000).

In the first of the following sections, Cellular Manufacturing is discussed as an implementation of Group Technology in manufacturing systems (Wemmerlöv U., Hyer N.L., 1989), and the detailed layout design is explored in this context. The section that follows explores layout design under flexible manufacturing systems and its interactions with material handling system design. In the next section, the general machine layout design problem is discussed in more detail as an introductory framework for the approach proposed in this Thesis.

4.2. Cellular Manufacturing Systems

The traditional functional layout clusters machine stations according to functions, in which similar activities are performed within the same department, workcentre, or cell. The emergence of Group Technology brought a new type of layout which offers several advantages that improve productivity and reduce waste, such as set-up, flow, and waiting times; consequently, a reduction of operating costs is expected (Wemmerlöv U., Johnson D.J., 2000). Based on production volume and product variety Group layouts can be classified as either flow line, cell or centre layouts (Hesen P.M.C., Renders P.J.J., Rooda J.E., 2001). This classification resembles the traditional layout classification since it characterises these groups as follows: flow line as similar to product layout, cell to process layout, and centre as a combination of them. Cellular machine layout, based on Group Technology, has been designed to avoid high levels of inventory, long lead times and scheduling problems that may arise from functional layouts (Irizarry M.A., Wilson J.R., Trevino J., 2001). However, Cellular Manufacturing Systems (CMS) may require more investment in duplicating capacity when working cells are being formed (Urban T.L., Chiang W.C., Russell R.A., 2000). Cellular Manufacturing may also be considered highly inflexible (Benjaafar

S., Sheikhzadeh M., 2000), since product families for which the cell was created are the only products that can be produced within the cell.

Most of the initial research on Group Technology and Cellular Manufacturing has focussed on creating families from parts and creating cells from combining machines, assuming that this was enough to obtain the expected benefits of this technology (Wang T.Y., Lin H.C., Wu K.B., 1998). However, subsequent research has shown that additional work is required in areas such as facility and machine layout, as is discussed in this chapter. For instance, in the last decade, research work has identified and recognised the existence of inter-cell and intra-cell flows (Logendran R., 1990).

Three major steps are required to develop Cellular Manufacturing, namely, the formation of part families and machines, arrangement of the machines within each cell area, and determination of the configurations of cells on the facility floor (Bazagran-Lari M., Kaebernick H., Harraf A., 2000; Heragu S.S., Kakuturi S.R., 1997)

a. Cell Formation: most of the research literature with regard to cellular manufacturing has focused on the development procedures to solve the cell formation problem (Wemmerlöv U., Johnson D.J., 2000). That is, it tries to create families of parts based on exploiting their similarities in production or design, bringing, at the same time, machines together to form cells. Other issues such as the layout design (Salum L., 2000), which also plays an important role (Wang T.Y., Lin H.C., Wu K.B., 1998), have received less attention.

b. Cells Configuration, inter-cell layout, or cell system layout (the latter term is attributed to Vakharia and Wemmerlöv (Vakharia, A.J., Wemmerlöv, U., 1990) as pointed out by Hassan (Hassan, M.M.D., 1995)): this refers to cells' assignment on the facility area and the flow between them
c. Machine Layout or intra-cell layout: this refers to the design within each cell and all the details involved in the working areas, as is discussed in the machine layout section.

In the literature, three solution strategies may be identified in order to create Cellular Manufacturing layouts: some strategies separately address each one of the issues mentioned above; others consider all the issues at the same time; and yet others only take some of them.

4.2.1. Methods Addressing Individual Issues

Most approaches addressing individual issues focus solely on the cell formation problem. Some examples of this work can be found in the various reviews made by Mansouri et al. (Mansouri S. A., Moattar Husseini S. M., Newman S. T., 2000), Crama and Oosten (Crama Y., Oosten M., 1996), Miltenburg and Zhang (Miltenburg J., Zhang W., 1991), and Co and Araar (Co H.C., Araar A., 1988). Graph theory is one of the approaches used to address the cell formation problem (Rajagopalan R., Batra J.L., 1975). In this Graph Theory application, a three-phase approach is suggested, which tries to exploit the relationships between machines based on a similarity coefficient. In the first phase, a graph is created using the product demand and route cards for every component to be manufactured. In phase two, partitions of the graph are made to create the cells based on minimising the inter-cell moves and the cost of those moves between pairs of cells. In the last phase, components are allocated to each cell using machine loads to determine the number of machines required within each cell.

4.2.2. Partial Methods

These approaches focus on reducing inter-cell movements when addressing the cell formation problem. Examples of such works are Wu and Salvendy (Wu N., Salvendy G., 1993), Harhalakis, Nagi and Proth (Harhalakis G., Nagi R., Proth J.M., 1990), and Vohra, Chen, Chang, and Chen (Vohra T., Chen D.S., Chang J.C., and Chen H.C., 1990).

A simulated annealing solution approach has been suggested by Wang et al. (Wang T.Y., Lin H. C., Wu K.B., 1998), which minimises the total material handling distances in a job shop based on a bi-quadratic assignment problem. This bi-quadratic assignment model solves simultaneously both the inter-cellular and intra-cellular layout designs. The model represents the inter-cell and the intra-cell arrangements using separated sets of variables. The modified simulated annealing approach presented uses a mechanism which iteratively alters the position of neighbouring facilities and evaluates selected facilities, being careful to ensure that selected facilities are placed in the same cell at each iteration and under the same closeness criterion. This paper also claims that this proposed procedure produces better solutions than other simulated annealing solution procedures to which it has been compared.

4.2.3. Complete Methods

The approaches presented in this section address all of the previously mentioned three stages. Most of these approaches use a "divide and conquer" approach when addressing the solution and use sequential phases during the solution.

4.2.3.1. Simulation

A two-phase procedure using simulation is proposed by Salum (Salum L., 2000), which attempts to optimise the manufacturing lead-time (MLT). MLT reduction is referred to in this paper as the most important reason for manufacturing cells to be established, and it is calculated as the sum of the set-up, processing, material handling, and waiting times. Using a trail and error approach simulations are performed in two phases. A simulation is performed during the first phase to obtain preliminary data on processing and waiting times under simulated operation conditions, assuming that there are no material handling times. In the second phase, based on the processing and waiting times obtained in phase one and using coefficients of similarity between machines, i.e., smallest processing and waiting times, the best possible MLT is sought by combining the smallest handling and the smallest waiting times. The assumption behind this is that the shorter waiting times between machines are, the more similar they are expected to be. Machines are placed together based on the ranks of these combinations. An object oriented software package, SIMPLE++, is used to carry out the simulation.

4.2.3.2. Virtual Manufacturing

A two-phase procedure is suggested by Irani et al. (Irani S.A., Cavalier T.M., Cohen P.H., 1993), to integrate the cell formation and layout design problems. It is claimed that this procedure encourages the formation of virtual cells and simultaneously takes advantage of existing functional layouts. This hybrid approach combines graph theory and mathematical programming concepts: the first phase is solved by a linear programming formulation and, in.the second phase, the solution is improved by using integer linear programming, recognising the machine sharing between different production requirements. The flow distance linear programming solution, first phase, provides a maximal spanning arborescence, i.e., an arborescence or directed tree is a connected graph that contains no circuits. In the second phase, the integer-programming model identifies the best permutation among the tree branches, minimising inter-cell flow distances. Graph Theory is used to classify directed arcs prior to the mathematical programming formulations. The set of arcs is decomposed into three sets, forward, backward and crosswise, suggesting also that arcs can be further classified as necessary or redundant, depending on arc duplication requirements. This classification attempts to address the inter-cell and the intra-cell flows besides the sequential flow. Consideration is given to identifying machine groups using flow lines as production guides without moving the physical layout. The procedure attempts to reduce the need for machine duplication between cells, suggesting better use of the existing manufacturing resources.

4.2.3.3. Simulated Annealing

Heragu and Kakuturi (Heragu S.S., Kakuturi S.R., 1997) propose a three-phase approach to address the solution to the three stages required for Cellular Manufacturing Layout, based on an approach suggested by Heragu (Heragu S.S, 1994). This also uses a sequential approach to this solution procedure to assess each one of the major steps mentioned above. In addition, it uses a Hybrid Simulated Annealing (HSA) procedure to obtain the intra-cell layouts and seeks to reduce the total material handling costs between pairs of machines. For the inter-cell layout design, the HSA procedure is also used, but focusing now on the reduction of material handling costs between cells. Given that the number of cells may be small, a near optimal solution is also expected to be obtained in this phase. The required input data are: part routing information matrix, relationship matrix, material handling cost matrix, the desired maximum number of machines by cell, and flow matrix.

Another approach by Bazagran-Lari et al. (Bazagran-Lari M., Kaebernick H., Harraf A., 2000) tries to extend the applicability of the layout design into group technologies by adding a family and cell formation algorithm. It uses a multiple objective formulation using goal programming for taking into consideration the space usage and travelling costs for the inter-cell and the intra-cell problems. Using a hybrid approach to solving the problem, firstly a layout is obtained by goal programming and, secondly, it is then improved using Simulated Annealing for the intra-cell layout. Goal programming allows objective priorities to be established: for instance, space usage was preferred over travelling costs in this research paper.

4.2.3.4. Heuristic

A four-phase procedure is suggested by Vakharia and Wemmerlöv (Vakharia A.J., Wemmerlöv U., 1990), which sequentially addresses the cell formation, the inter-cell, and the intra-cell layouts. The procedure begins by identifying flows that exist by separating parts into single, dual, and backtrack operation sequences. It then continues to focus on group formation, creating clusters by reducing the number of operations between clusters. Then, by applying a similarity coefficient criterion based on the operation sequences, the procedure creates homogeneous sets of machines and parts. Finally, the procedure generates cell candidates following the operation sequences, using Hollier's heuristic (Hollier R.H., 1963), which has a variant that provides linear layout designs addressing the minimisation of travel distances and backtracking.

4.2.3.5. Graph Theoretical

A graph-based approach to deal with cellular manufacturing systems is proposed by Daita et al. (Daita S.T.S., Irani S.A., Kotamraju S., 1999). This approach attempts to solve the three phases required by Cellular Manufacturing Systems using spanning trees and cluster analysis. It requires route sheets, demand volumes and frequency of ordering as the input data. This tool addresses the family grouping and the layout design. The layout application uses optimal spanning trees and mutually linked components as means to create the layout design. The procedure uses a maximum spanning tree, which considers the demand for the products as the edge weights obtained from a travel chart. Once a spanning tree is constructed, it suggests a tentative facility design which may be modified if mutually linked facilities are identified. The approach suggests the use of mutually linked entities (facilities that mutually receive outputs from the same machines for some or any product) as a potential design enhancement device to improve the layout obtained from the optimal spanning tree. A similar approach which includes a computational tool was presented by Irani et al. (Irani S.A., Zhang H., Zhou J., Huang H., Udai T.K., Subramanian S., 2000). Both proposals solve the three phases required by Cellular Manufacturing Systems.

4.3. Flexible Manufacturing Systems

Flexible manufacturing systems (FMS) are groups of numerically controlled (NC) or computer numerically controlled (CNC) machines, and a material handling and storage system, working together under the control of a computer (Mac Carthy B.L., Liu J., 1993). These systems attempt to provide manufacturing companies with the capability to deal with a large variety of products, with more flexibility and with productivity improvements. According to Mac Carthy and Liu (Mac Carthy B.L., Liu J., 1993), FMS may have the following configuration structures:

- A Flexible manufacturing system (FMS) consists of a CNC or NC Machine capable of producing a variety of parts connected by an automated material handling system and under computer control.
- A Single flexible machine (SFM) is a computer controlled system that has one NC or CNC machine with tool changing capability, a material handling device, and a part storage buffer.
- A Flexible Manufacturing Cell (FMC) is a group of SFMs sharing one common material handling system.
- A Multi-Machine Flexible Manufacturing System (MMFMS) is a system that contains a number of SFMs sharing an automated material handling system able to provide service to more than one machine.
- A Multi-Cell Flexible Manufacturing System (MCFMS) may include a number of FMCs and possibly a number of SFMs connected by an automated material handling system.

The FMS design can be considered as an extension of the Machine Layout Problem (MLP), which consists of three phases or subproblems, namely: ① the selection of processing and handling equipment appropriate to the company's needs; ② the performance of an economic analysis of the production operations; and ③the development of a detailed machine layout (Das S.K., 1993).

Flexibility is a distinctive characteristic of these types of systems, and is, generally speaking, considered as the ability of a manufacturing system to respond to changes such as in production volume or routings, and product variety mix and product designs (Shewchuk J.P., Moodie C.L., 2000; Papadopulus H.T., Heavey C., Browne J., 1993:14). There are some FMS implementations that could be recognised as an automated variety of cellular manufacturing systems; however, there are other layout types that have been reported in the literature of FMS implementations: unidirectional loop network, circular machine, linear single-row machine, linear double-row machine, and cluster machine layout (Kouvelis P., Chiang W.C., Kiran A.S., 1992).

In these types of systems, there is a close link between the material handling system and the layout design, since the layout is mostly determined by the material handling device (Kusiak A., Heragu S.S., 1987). For instance, linear layouts are often served by Automated Guided Vehicles (AGVs); robots are often found in circular layouts; cluster layouts may be served by gantry robots. One of the most common layouts for FMS is the loop layout, which can be served by loop conveyors, tow lines, overhead monorail systems or wire paths of unidirectional AGVs (Leung J., 1992). The layout is considered as an important issue of an FMS, which affects its performance (Das, S.K., 1993). The traditional layout methods are not easily applied to the FMS layout design because they fail to consider cell orientations, load and unload positions and fixed cell geometries (Das, S.K., 1993). To gain more insight into FMS layout, in the following sections they are briefly discussed as: loop; linear, circular, and multiple rows; and finally, additional FMS features.

4.3.1. Loop Layout Design

A combined approach to solve a machine layout and scheduling problems in an FMS loop design is discussed by Potts and Whitehead (Potts C.N., Whitehead J.D., 2001). The proposed model seeks to maximise throughput by balancing the workload on a unidirectional conveyor belt loop. An integer-programming model is suggested and a three-phase solution procedure is used. The first phase attempts to balance the machine workload, under given process time and expected product demand; in the second phase, an attempt is made to minimise the inter-machine travel subject to the workloads obtained in the previous phase and travel loads; and in the third phase, machines are placed around a loop conveyor. The model attempts to minimise the expected number of circuits to be made by the products around the conveyor belt.

A graph theoretic approach presented by Leung (Leung J., 1992) addresses the problem of a loop layout manufacturing system. A variant of this approach, which includes the use of directed graphs, investigates how to minimise the maximum number of crossing loops for a family of parts using sequences of machines that must be visited during their processing. The problem is stated and solved, in an iterative fashion, as a relaxed linear programming model, which is used as a basis to find acyclic sub-graphs; its first solution is taken as the lower bound for the expected optimal solution. During each iteration, the best possible solution is sought through constructing a new problem. This new formulation has a new constraint, which considers the previous solution cycle as a new additional bound.

Two improvement heuristic solution procedures are suggested by Tansel and Bilen (Tansel B. C., Bilen C., 1998) to assess the unidirectional loop layout design. These heuristics consider the part-flows and distances between machines in the objective function. They optimise the objective function by an interchanging procedure, moving facilities' positions. In this proposal the number of moves from machine i to machine j are multiplied by the flow in order to determine the arc weights and the inflows and outflows of the machines. The interchange procedure is based on inserting a selected machine in a position in the loop, and the gap between the insertion position and the empty location is filled by displacing the machines clockwise into these available positions. The authors claim that their heuristic performed better than those heuristic procedures to which it was compared.

4.3.2. Linear, Circular and Multiple Row Layout Designs

Ho and Moodie (Ho Y.C., Moodie C.L., 1994) use the operations sequence pattern as a basis for the development of a flexible cellular manufacturing system. The operation sequence pattern is defined as the set of operations appearing frequently in a particular order. A partial heuristic approach was followed to solve the inter- and intra-cell flow problems by using a sequence similarity measurement in a five stage procedure. The sequence similarity measurement was taken from a proposal by Vakharia and Wemmerlöv (Vakharia A.J., Wemmerlöv U., 1990). Since the sequence similarity is based on the sequence patterns or routing similarities, more in-sequence flows should be expected. Both of these papers provide evidence that inter-cell and intra-cell flow analysis becomes easier as the problem to be solved is divided into smaller sub-problems.

A network based layout design is proposed in the paper by Ho and Moodie (Ho Y.C., Moodie C.L., 1994) based on the operation sequence pattern, which is claimed to offer the following advantages: ① the sequence patterns can be used as a basis for the grouping of workstations, which is expected to create more in-sequence flows within each partition; ② a good layout can be expected because the flow analysis would be easier to handle given that less interaction is expected within each partition; ③ given more in-sequence flows and less flow distance, easier material handling, less complicated control/scheduling problems and more production visibility are expected; ④ given that a "divide and conquer" approach is followed through modularisation (grouping), more expansion capability in the size of the problem that can be handled can be explored; and ⑤ this modularisation issue could provide additional advantages when modification and re-layout of facilities may be required.

Operation sequence patterns are obtained from a five-phase search procedure, based on the similarity measurement. The pattern search is performed iteratively: that is, it identifies one pattern in the five stages and continues identifying patterns that satisfy the required conditions until all the types of products and their operation sequences are taken into consideration. In the first phase, the minimum required number of occurrences for an operation and the maximum number of consecutive operations allowed between two adjacent major operations of a pattern, are two values which have to be set by trial and error until suitable values are determined. In the second phase, from the sequence patterns an operation is selected as the starting component of the layout based on a large number of appearances from the product operation sequences (POS), and this operation is used as a basis for creating the operation patterns. In the third stage, additional candidate operations to be included in the operation patterns are sought, and are those that meet both values defined in stage one; if additional operations are not identified, another search is performed in stage two. In the fourth stage, the operation patterns are completed by performing similar searches such as the ones in stage three. In the final stage, the obtained pattern is recorded and the procedure is prepared to search for another pattern.

A hybrid approach which combines optimisation and artificial intelligence (AI) tools is proposed by Heragu and Kusiak (Heragu S.S., Kusiak A., 1990). Mixed integer programming optimisation algorithms are used for the formulation of end solutions (Heragu S.S., Kusiak A., 1988). A rule-based expert system (ES) created in common LISP attempts to emulate the decisions of a layout designer. Firstly, an appropriate MIP model is selected depending on the input data set. Secondly, the problem is solved using an optimisation algorithm and the proposed solution is evaluated. If the solution can be implemented according to the rule set available in the expert system, it is accepted. Otherwise, certain parameters are modified, under conditions previously established in the rule set by the expert system, the algorithm is applied again, and the solution re-evaluated or modified to make the solution feasible for being implemented. The approach requires the following data: number of machines to be assigned, flow matrix, clearance matrix (entries are the expected minimum space distance between machines i and j), relationship indicator matrix (similar to the one proposed by Muther (Muther R.,1973)), machine dimensions, location restrictions (if any), type of layout, type of material handling device, and dimensions of the floor plan.

Loop and a linear layout designs are presented by Kaku and Rachamadugu (Kaku B.K., Rachamadugu R., 1992), which, it is claimed, may have facilities of different sizes. This model uses a modified QAP formulation, minimising material handling distances and loads. A hybrid construction-improvement procedure is used, requiring the machines' location to be defined beforehand and thus, distances between them to be predetermined. The authors highlight the fact that these problems are different from traditional ones because machine sizes are generally different, as also the clearance required between them. In addition, it was pointed out that the facility layout design is among the issues that have received less attention in systems such as these. It was also claimed that this approach could be applied to double row and loop layout designs.

4.3.3. Flexible Manufacturing Systems Design Issues

A particular type of FMS, labelled as a tandem automated vehicle system, is studied by Hseih and Sha (Hsieh L.F., Sha D.Y., 1996), and is characterised by several non-overlapping closed loops, each with a single unidirectional dedicated vehicle sharing a transit station between two adjacent loops. Five phases are used to address the problem, namely: ① completion of the from-to chart from the average material flow; ② machine partition; ③machine layout for each partition; ④determination of the AGV direction; and ⑤determination of the layout among all individual loops, i.e., inter-cell layout. The layout phases attempt to minimise the total loaded travelling time or distance of the material handling devices using a QAP formulation to solve machine layout for each loop and the inter-loop layout.

A mixed integer programming model was presented by Das (Das, S.K., 1993) as an extension of the machine layout problem (MLP) which, it is claimed, supports the design of flexible manufacturing systems to minimise total projected travel times between cells. This model considers spatial co-ordinates, two orientation positions (vertical and horizontal), and the location of load and unload points for each cell. A four-step heuristic method is provided as follows. First, calculate an upper bound to the objective function using the spline method (Das, S.K., 1993). This spline method determines a preliminary cell by concentrating machines on both sides of a linear arrangement and minimising the traffic flow within pairs of considered facilities. During the second phase, determine the orientation and the spatial sequencing of the cells, allow the cell centre to be the load/unload point, and provide approximate space usage data to be applied in the following step. The determination of the travel distances between cell centres is determined in this third step, using the machine spatial sequencing obtained from the previous step as constant values. In step four, previous results are kept constant but now the minimising distances between the locations of the load/unload points is sought. Steps two to four are solved as mixed integer problems.

The path layout and operation of an AGV system presented by Vosniakos and Davies (Vosniakos G.C., Davies B.J., 1989) uses simulation to evaluate the performance of three proposed layouts, following different paths and dealing with bi-directional type alternatives instead of unidirectional situations. ECSL is the simulation package used and is stated to be suitable for discrete event systems simulation. The software provides monitoring capabilities, and for a set of velocities the utilisation of the tracks, the behaviour of the buffer levels, and the material handling device utilisation were studied. They also studied different scheduling policies, such as First-in-First-out and Lowest-Vehicle-Utilisation. These parameters were used as a comparison base to decide the best possible layout design solution under stochastic conditions.

Additional issues, such as loop routing design (Asef-Vaziri A., Dessouky M., Sriskandarajah C., 2001) and scheduling problems (Zhao C., Wu Z., 2001), seeking to improve FMC's utilisation, are now capturing researchers' attention. Likewise, among future research topics in this field that are mentioned in the literature (Kouvelis P., Chiang W.C., Kiran A.S., 1992) are: minimising the backtrack distance of the material handling devices; multiple loop problems in a material handling network and its configuration; development of reliable heuristics to solve FMS layout problems; inaccuracy of input data; and the use of multiple criteria approaches.

4.4. Machine Layout Design

Machine Layout Design has been an emerging area of study in the layout design arena since around 1987 (Heragu S.S., Kusiak A., 1988). These researchers stated that the flow data available was usually inaccurate since it depended on production schedules, which could not be predicted given the high degree of uncertainty of market demands. They also mentioned that industry usually needs procedures that do not require high levels of computational resources. They also pointed out that the machine layout design could not be formulated using traditional QAP, given that distances between machines are not fixed, and that their shapes usually modify the final layout when they are considered. Likewise, the QAP formulations can only be applied in cases in which the locations of sites are known. Therefore, different methods are required. An additional observation is that FMS travel times should be preferred over travel distances since, when considering a material handling system, its motion characteristic is an important issue. Deterministic models are inappropriate to represent travel times, operation behaviours and other dynamic features of systems. They also observed that the assumptions usually made about the cost of assigning a machine to any site were the same and were often not reliable.

Some of the main differences between block and machine layouts that have been pointed out in the literature are (Hassan M.M.D., 1994) as follows:

- In block layouts, it is assumed that machine areas are relatively small and are of equal size, or could be ignored or their impact could be overlooked without affecting the results.

- In many machine layouts, machines are usually arranged in a single row or in multiple rows: these structures provide the opportunity to take advantage of them, and when present may be exploited to reduce space usage or select appropriate material handling devices.

- Machine interrelationships used in the machine layout are usually quantitative in nature instead of using the Relationship Chart. Cost, volume, and time interrelations are represented by From-to chart entries, which may not necessarily be symmetrical, and entries above and below the diagonal represent movements in opposite directions.

- Factors such as machine utilisation, queue length and time, backtracking and bypassing, degree of automation, ability of the worker to operate more than one machine, number and capacity of tool magazines, tooling policy, adaptability to material handling equipment, number and location of loading/unloading stations, and throughput, should be considered in the machine layout design. However, consideration of several of these factors simultaneously would complicate the analytical development of a layout.

- Because of the interdependence that may exist between some of the factors mentioned in the previous point, machine layout design requires much more elaborate data analysis than in the block layout design.

4.4.1. Types of Machine Layouts

In manufacturing environments, multiple product lines, flexibility, and fast response are all significant issues; usually products are manufactured in small batch sizes sharing the same production resources. Three types of machine layouts have been identified: linear, multiple row and loop.

- In linear layouts, machines are placed in single rows which are arranged to be as close as possible to the sequence of operations of the parts processed by the arrangement. Of course, ideally it is expected to have in-sequence flows which will bring benefits such as smaller travel distances, easier control, and easier material handling (Ho Y.C., Lee

C.E.C., Moodie C.L., 1993). This may also imply fewer delays, less expensive processes, and greater physical visibility. Single rows may have different shapes other than linear: namely, u-shaped and semicircular (Hassan M.M.D., 1994).

- In a plane, multiple row arrangements may be seen as horizontal or vertical arrangements but with interacting flows between them. In multiple rows there can be unidirectional, bi-directional, and multi-directional flows (Carrie A.S., 1975).

- Loop configurations are mostly associated with Flexible Manufacturing Systems that are arranged around an oval path and in which movements can be unidirectional or bi-directional. Their major advantage is the flexibility that they provide for material handling (Hassan M.M.D., 1994)

4.4.2. Types of Machine Layout Movements

Four types of movements that may exist among machine layout designs when performing production operations have been identified in the literature (Aneke N.A.G., Carrie A.S., 1986), as represented in Figure 4.1:

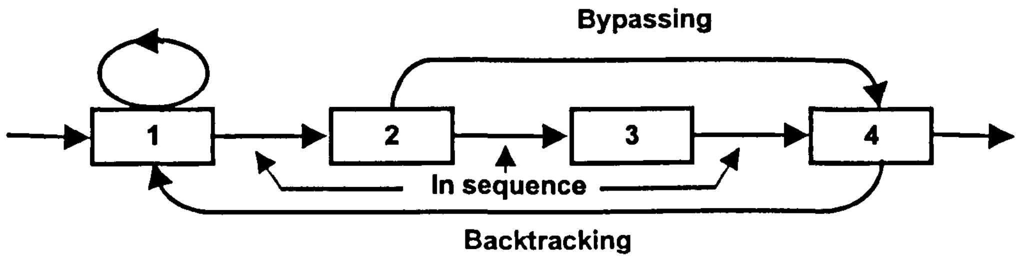


Figure 4.1. Types of Movements in a Multiple Product Flowline (Aneke N.A.G., Carrie A.S., 1986)

- Repeat operations, in which operational movement is within the same machine
- In-sequence operations, where operational movements are from one machine to the immediately following machine
- Bypassing operations, in which the operational movement is from a machine to another that is not adjacent to it
- Backtracking operations, where operational movements are from one machine to another which is in a backward direction.

From these flow movements, it is the in-sequence movement which is the most desirable because of its unidirectionality. However, it is highly unlikely that routings for multiple products will share the same production routings. Hence, bypassing and backtracking may exist, but it would be desirable to reduce or eliminate them in order to increase physical control, physical visibility, and improve production efficiencies (Ho Y.C., Lee C.E.C., Moodie C.L., 1993). Solution methods reported in the literature are analysed here by linear and circular row designs and by network and multiple row designs.

4.4.3. Linear and Circular Row Designs

Ho et al. (Ho Y.C., Lee C.E.C., Moodie C.L., 1993) have proposed a heuristic approach based on pattern matching, which bases its similarities on a sequence similarity coefficient. The number of operations in a sequence of a product determines this coefficient, using either in-sequence or bypassing relationships within the sequence flow. The product associated with the largest operation sequence provides a reference pattern that is used in comparison to the other product sequences and in determining the linear arrangement that reduces backtracking and in-sequence distances. A network-based pattern has also been proposed which uses the same similarity pattern.

A linear facility formulation proposed by Houshyar and McGinnis (Houshyar A., McGinnis L.F., 1990) and mentioned as a modification of a QAP, is approached by a heuristic based on network concepts, seeking to minimise the total loaded vehicle travel distance using a cutting procedure based on rectilinear distances and dividing the facility set into two mutually exclusive sets. A facility is assigned to a set during each iteration; the procedure makes all possible partitions of the facilities into two groups, separating the set that contains the facility to be assigned and the rest waiting to be assigned. By taking into consideration the edge weights, the partition that provides the highest sum of the weights is selected, leaving two sets of facilities after each cut: the assigned and non-assigned facilities. The procedure continues in similar manner to incorporate the rest of the facilities one at the time, until all are included. The procedure is claimed to produce often optimal solutions and outperform the solutions obtained by CRAFT computer package, mentioned in chapter three.

In a paper by Heragu and Kusiak (Heragu S.S., Kusiak A., 1988), two construction approach algorithms were proposed, one for solving the linear and circular layout problem and the second one, known as Triangle Assignment Algorithm (TAA), for solving the linear double row and gantry layout problems. The pursued objective function is to reduce travel time between machines. The first algorithm is similar to the maximum spanning tree given the following assumptions: ① all vertices are degree two except the initial and the final vertices, which are degree one, and ② there is only one initial and one final vertex. The second algorithm is also a two-phase heuristic, which generates maximum weight triangles, where the vertices of the triangles are machines placed and ordered using the maximum spanning tree algorithm. In both cases, machines of unequal size were considered and better solutions were obtained when compared with other construction methods with comparable computational time performance.

4.4.4. Multiple Row and Network-Based Designs

A mixed integer formulation that integrates the machine allocation and the layout design problem was proposed by Urban et al. (Urban T.L., Chiang W.C., Russell R.A., 2000). In this proposal, a network formulation seeking to minimise material handling costs through a cubic objective function is used in a QAP fashion. The objective function is substituted by a linear equivalent during the solution. A two phase heuristic procedure is used to obtain the layout proposals. In the first phase, the QAP problem is solved seeking the best possible distances and workflows. The workflows are evaluated using a shortest path heuristic, providing finally an initial layout that is used in the following phase. During the second phase, a Tabu search (TS) procedure is used as an improvement tool and is used simultaneously as a generator of integer solutions. The authors of this paper suggest that this heuristic tool is an effective combinatorial optimisation tool, appropriate in finding integer optimal and suboptimal solutions.

Two approaches based on workcentre availability and sequence similarity were proposed by Ho, Lee, and Moodie (Ho Y.C., Lee C.E.C., Moodie C.L., 1993) using a network-based approach. One of these approaches was explained previously in the linear and circular row designs section. The second suggested approach differs from the previous one because it proposes a network structure for the construction of multiple flowlines based on the heuristic pattern matching used in the previous approach. These approaches can be considered as general machine problems, since they allow more than one machine of any type that may be required, to reduce bypassing and backtracking flows. An additional difference between these approaches is how facilities are integrated during the construction procedure. In the traditional line approach the arrangement grows by insertion, whereas in the network based solution it grows by branching. It is expected that the network based solution produces arrangements that give less flow distance and more flexibility in the routing selection for each product. This characteristic is claimed to be especially important in a flexible production environment. The approach used requires three stages: ① selection of the next product, ②selection of the best path to be used as pattern reference, and ③ the modification of the selected path so that all the sequences are included and a new network constructed.

Another network-based solution approach which considers production sequences and demand volumes, and uses graph theory, is proposed by John and Hammond (John E.G., Hammond J., 1999), arguing that From-to and Relationship charts fail to consider sequences and demand appropriately, and suggesting weighted interrelation values that may provide better results for the machine layout design. They also argue that physical nearness can be expressed either in number of moves or measured by physical distance travelled, since product quantities have to be carried or have to travel between respective machines but cannot overrule the production operation sequence that they have to follow to become completed products. They suggest that there is no evidence in previously published research that considers the interactions from non-adjacent facilities as an additional proximity measure, and takes advantage of them in the elaboration of the layout design. This nearness measure extends the closeness notion between two in-sequence facilities by taking into consideration a proportional flow for subsequent required facilities. Assuming facilities ABC are required to produce certain product then the flows between

A and B and B and C are commonly considered; but the interaction between A and C ignored, as developed in more detail in section 5.7.3. John and Hammond (John E.G., Hammond J., 1999) have encouraging findings suggesting that improvement is possible when including this other nearness perspective in the facility layout design.

The authors (John E.G., Hammond J., 2000) also proposed the use of the adjacent and non-adjacent closeness desirability using maximally weighted planar graphs (MWPG) and addressing flow minimisation as the first issue, then relaxing the maximal planarity requirement by creating planar areas instead of triangular graphs. They also mention that in the quest for planarity, some edges that have high weight may be ignored, obtaining a maximal planar graph instead of a maximal weighted one, and claim that the focus should be on this issue. In the discussion, they argue that most of the previous methods fail to solve the MWPG, and instead the maximally planar weighted graph (MPWG) is solved, which is sub-optimal to the MWPG. A note by Al-Hakim (Al-Hakim L., 2002) on this contribution highlights the fact that the obtained results can be improved further by constructing an additional one-dimensional array defined as a spinal graph.
Furthermore, in the note by AL-Hakim (Al-Hakim L., 2002) the spinal graph may be characterised as having a central linear facility arrangement, the spine, and other edges can be added to it, with the edges representing relationships of the highest possible weight such as material flow between vertices. Additional results are that the spinal graph has (n-1) spinal edges and (2n-5) non-spinal edges. Spinal edges may represent in-sequence flow,

whereas bypassing and backtracking flows may be represented by non-spinal edges, as shown in Figure 4.2.

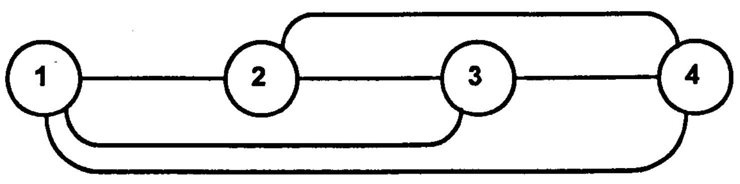


Figure 4.2. Maximal Planar Spinal Graph (Al-Hakim L., 2002)

4.4.5. Modular Designs

An approach developed by Irani and Huang (Irani S.A., Huang H., 2000) proposes the use of modules as an important feature in the development of machine layouts. The basis for the creation of modules is the operations sequences, and cluster analysis is used to identify similarities among operations, creating a hybrid layout that combines modules and single machines which do not match any modular operation similarities. This proposal claims that the use of modules provides more flexibility for layout modification, retirement and enhancement, and that, given the number of machines involved, the designs proposed are easier to handle when compared with other machine layout approaches, such as linear or traditional layouts. Given the clusters obtained from the analysis, machine repetition is allowed, as can be seen in Figure 4.3, which includes two independent modules. Interaction between modules is allowed.

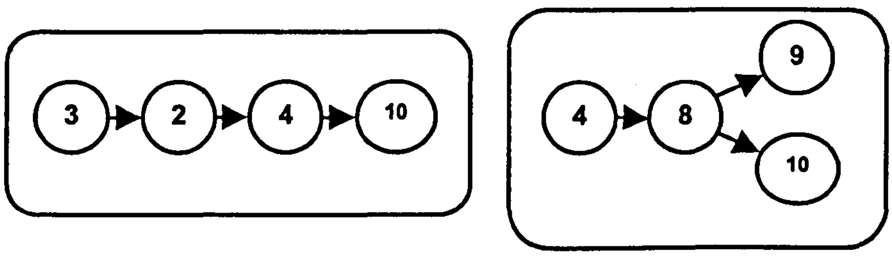


Figure 4.3. Modular Layout Design Sample (Irani S.A., Huang H., 2000)

4.4.6. Other Machine Layout Design Issues

Lee (Lee G.H., 2001) suggests for layout of machines the consideration of concurrent engineering and components design, and vice versa, in order to produce better designs to improve their material handling and consequently to reduce related costs. The paper proposes four different perspectives towards the solution of the problem, namely: layouts of machines for components, design of components for existing layout of machines, design of components with limited information on layout of machines, and integrated design of components and layout of machines for material handling - and it only addresses the design of components and material handling issues. The study uses a composite approach to address the layout design problem, selecting a design based on the design of the components, using estimates of their demand, and based on the product operation sequences. The formulation is stated using both a graph and a mixed integer programming approach, and also suggests the use of a weighted similarity measure for the components' routes, proportional to the handling cost of each component. The graph approach suggests implicitly the use of directed graphs and mutually linked components, through creating an m-partite graph that should be identified to reduce material handling costs. The mixed integer programming formulation is used to identify component routes. The study also approaches the single and multi-row problems with equal machine areas as a QAP model, using sets of adjacent machines in the formulation. In addition, it proposes a heuristic solution based on a pairwise facilities exchange and suggests the use of simulated annealing, tabu search, and genetic algorithms as improvement approaches.

4.5. Summary

Although traces of detailed layout design can be found as far back as the 1950s, it is only in the last one or two decades that considerable attention has been given to this subject. Associated with cellular manufacturing, flexible manufacturing systems, or machine layout designs, detailed layout designs represent a recent and ongoing research field that is addressing new challenges.

Cellular manufacturing, as an application of Group Technology, is approached by the deployment of part families and machine assignments to cells, which generates inter-cell and intra-cell interrelationships. Simulated annealing, expert systems, and Tabu search are among the most recent tools which are claimed to provide better solutions, and at least they are able to evaluate more solution alternatives than previous approaches in a more efficient manner. Although encouraging results have been obtained from the application of these findings, more research is needed to distinguish under which circumstances one may be preferred over another.

In general, it can be expected that the detailed layout design may require a more thorough attention to each component defined by a block layout design. One of the vital ingredients of this design is the material flow, particularly in a multiple product environment, where products are usually grouped into families to obtain operational and financial benefits. Because of the variation in the number of operations on parts and in their operation sequences, a sequence that may satisfy a particular family is unlikely to satisfy other families. Moreover, even within a family, it is common to see differences in the operations required and their operational sequences. Consequently, this situation needs to be studied in more depth. Some of the trends in response to these requirements are associated with cellular and flexible manufacturing systems, as has been discussed in this chapter. In addition, in the literature layout enthusiasts have referred to this need variously as multi-product flowline, machine layout problem, and operation sequence pattern.

To address the machine layout design mentioned above and, in particular, to take into consideration other graph theoretic issues is the main concern of this research project. In recent years, and particularly in the area of the development of machine layout designs, it seems that directed graphs are capturing researchers' attention. A feature of interest is the displacements or transfer of parts between machines that may be obtained from the production operation sequences. They are implicitly or explicitly considered in most of the literature papers, and this project proposal provides an opportunity to explore material handling movements obtained from the product operations sequences in the machine layout design context. These approaches have evolved further and there are new suggestions such as how they may include non-adjacent machine interactions in layout designs. Moreover, facilities that have bi-directional flows are suggested in the literature as an additional reference to improve layout designs; and they can play a major role and provide more information about network-based graphs. Therefore, these bi-directional flow facilities, or mutually linked components, could be considered as a basis for the design of layouts. Their capabilities are explored and discussed in this research project in the following chapters.

Most of the publications discuss procedures that attempt to answer one of the following questions: ① which is the least expensive way to configure the machine arrangements; ②which could be the best machine configuration in order to reduce the travel distance; and ③if the demand volume is taken into consideration, which is the best arrangement that machines should follow. It is an unexplored and interesting question to obtain an assessment of the impact on the machine layout design when seeking the best possible machine-configuration based on absence of qualitative and quantitative data sets, e.g., if it is possible to deploy a machine layout design based on mutually linked facilities. Another question that has hardly been explored is the analysis of the impact on the layout design of the locations of ports through which the group of facilities both receives the incoming raw materials and delivers its products, which can also be explored using this proposed approach.

5. Strong Component-Based Layout Design Approach

5.1. Introduction

Commonly, most of the layout formulation objective functions state their aim as reducing costs, distances and times, or improving space usage, throughput, or various combinations of these. However, there are various manners of formulating the aim of a layout design, depending on the question to be answered, and this aim is often governed by company policies and strategies.

The interest of the layout design in this research involves the production of multiple products where manufacturing resources, particularly machines or workstations, have to be shared. Instances of possible questions to be answered are: ① what configuration machines should have in order to be capable of producing the different products; ② how machines can be placed in order to minimise the amount of travel distance; ③ how they should be placed in order to minimise the travel time between machines; and ④ what configuration of machines minimises material handling costs. Most or all of these questions require as input data for each product: demand volumes, product operation sequences, product operation times, and estimated travel distances and times between machines. Each one of these questions may lead to different layout designs, and the one closest to the company's policies or programmes should be the most appropriate answer to the stated question. In general, alignment to firms' strategies and policies is mentioned in the literature as an essential guidance requirement that most activities and decisions within a company should follow.

The feasibility of a layout design is related to solving the first question stated above: how machines have to be placed to be capable of producing the required products. There are two formulation approaches suggested in the literature to address the design problem for multiple products: the single machine and the general machine. The single machine case considers only one machine of each type, whereas the number of machines of any type considered in the general machine case is unrestricted. At this stage and within this research context the single machine case is explored. Additionally, dimensionless values as objective function coefficients are explored instead of the often-used quantitative values.

In order to accomplish the project aim, the single machine case will be assumed, together with dedicated facilities, which will be used as a benchmark to compare the solutions obtained from the approach of this proposal. The content of the rest of the chapter begins with a brief discussion of dedicated facilities. Non-dedicated facilities are then explored: this includes a brief exploration of the data needed before the presentation of the proposed solution approach. There is a brief overview of directed graphs, strong components and their contributions to the proposed approach. The chapter ends with a discussion of how different questions can be answered, i.e., different objectives can be addressed using this proposed approach.

5.2. Dedicated Facilities

Dedicated facilities maximise the operation in-sequence flow, make planning activities easier and improve physical visibility of product flows. On the other hand, dedicated facilities increase investment, space usage and operation costs, as addressed in section 5.6.1.4, in Chapter 5 and section 6.5, in Chapter 6.

Product	Operation Sequences	Demand Estimates	Displacement Number
1	A-C-D-F	500	3
2	B-A-C-D-F	1000	4
3	E-B-C-A-F	300	4

Table 5.1. Product Data (Sule D.R., 1994:440)

The example data in Table 5.1, from Sule (Sule D.R., 1994:440), corresponds to three products that require six machines, their demand estimates, and their operation sequences as well. It assumes that the products are going to be produced in dedicated facilities and prepares the data to be used as a benchmark for the proposed solution approach. It can be seen from Table 5.1 that the investment required is in three machines of type A, two machines of type B, three machines of type C, two machines of type D, one machine of type E, and three type F machines. Three individual material handling systems are required. Likewise, it can be seen that product A requires 3 material handling displacements, product B requires four displacements, and product C four displacements.

A possible advantage of using the number of displacements in any solution procedure is that they could be combined with their respective material handling distances, volumes, times, and costs, and provide different weights according to firms' established policies and strategies. These weights are associated with the corresponding edge in a graph. Knowing the number of operations for each product, it may be deduced that the number of displacements required per product are (m-1), where m is the number of operations for the product.

5.3. Non-Dedicated Facilities

If non-dedicated facilities are to be used, solution procedures are usually based on or proposed after analysis of process features such as from-to, adjacency or the number of required material handling displacements matrices. Table 5.2 shows the Volume From-to matrix for the same example that appears in Table 5.1. From a brief glance at Table 5.2 it can be seen that machine E does not receive any flow and may be considered as an initial or source node. By contrast, machine F is a destination or final node.

	A	B	C	D	E	F	Total
A			1500			300	1800
B	1000		300				1300
C	300			1500			1800
D						1500	1500
E		300					300
F							0
Total	1300	300	1800	1500	0	1800	6700

Table 5.2. Volume From-To Table

Additional tables based on the operation sequences can be constructed as shown in Figure 5.3 and can be considered as variants of the From-to chart. The adjacency matrix, Figure 5.3.a., shows the links existing between any two machines, and it is not expected to be a symmetrical matrix. The frequency of displacements matrix, Figure 5.3.b., shows the number of times the different products use the links between any given machines.

	A	B	C	D	E	F	Total
A			1			1	2
B	1		1				2
C	1			1			2
D						1	1
E		1					1
F							0
Total	2	1	2	1	0	2	8

a. Adjacency Matrix

	A	B	C	D	E	F	Total
A			2			1	3
B	1		1				2
C	1			2			3
D						2	2
E		1					1
F							0
Total	2	1	3	2	0	3	11

b. Frequency of Displacements Matrix

Table 5.3. Adjacency and Frequency of Displacements Tables or Matrices

A graphical illustration of this data can be seen in Figure 5.1. This figure uses the data given in Table 5.1 and provides additional insight into the layout. Figure 5.1 depicts the arcs required by all the product operation sequences, which are those shown by the Adjacency Matrix, Figure 5.3.a. Repeated arcs are omitted in the Adjacency Matrix but are considered in the Frequency of Displacements Matrix. In the Adjacency Matrix case (Table 5.3.a.), the entries will depict arcs with a weight of one, which reflects the existence of the arcs in a given order, established by the product operation sequence. Either the Volume-flows (Table 5.2) or Frequency of Displacements Matrices (Table 5.3.b.) can be taken as arc weights. For example, it can be seen that the edges between machines A and C, and between machines D and F, are the links with the highest volume weights and the highest displacement frequencies. Additionally, in this illustration it can also be seen as a coincidence that the link between A and C is one that also has flows in both directions. In either case, these machines should be located close enough to reduce material handling distances, costs, or times, and also to determine appropriate material handling devices between them, given that these links should expect to be used heavily. It can be noted that there exists a correspondence between Muther's (Muther R., 1973) Relationship Chart and these matrices as follows: the edges or links with high expected displacements can be rated as A links, E those with less heavy displacements, and so on... until those without flow can be regarded as U or X.

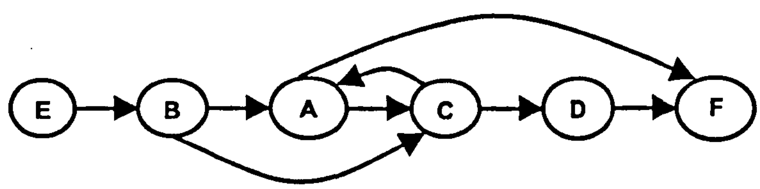


Figure 5.1. Adjacency Graph

In addition, it can be seen in Figure 5.1 that there are two bypassing displacements, from node B to C and from node A to F; and one backtracking movement, between node C and node A. The arcs represent all the required production sequences and, if the production has to follow this single-row structure, then, when compared with in-sequence flows (dedicated facilities), product three requires three additional displacements; it has to go through the sequence E B A C A C D F, of which the first A, and D, and the second C, are through-points.

The linear structure shown, or the single row layout now available, is capable of producing any of the products under consideration. However, the number and types of movements are not the same as in the case of dedicated facilities: there is a lower number of movements (per unit of product) but now there are two bypassing movements, from B to C and from A to F, and from C to A one backtracking movement. This may have an impact on the material handling system, on productivity and equipment usage, which should be compared against the possible savings in investment and operation expenditures. The second configuration should require less space but possibly more space for WIP inventories. These trade-offs between the benefits and risks involved should be compared before deciding on the final layout.

Figure 5.2 shows a variant of the adjacency graph presented in Figure 5.1, in which by allowing a network-type layout some unwanted types of displacements may be reduced, if in-sequence movements are the only ones allowed. Under the assumption that other movements are allowed, for example a move directly from B to C (shown in Figure 5.2 as a dashed line), the dedicated facility sequence for product three is the same one. Otherwise, it should be noted that how the B to C displacement is performed could add an additional step compared with the corresponding dedicated facility circumstances, and its consideration may affect the selection of a proper material handling device.

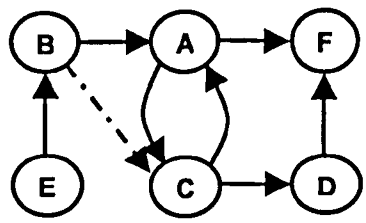


Figure 5.2. Network Type Adjacency Graph

Savings in space and investments would be expected by using non-dedicated facilities. The number of machines has been reduced from 14 to 6. Consequently, fewer human resources and lower operational costs should be required to operate this much smaller manufacturing system, and less space required to place 6 machines instead of 14.

5.4. Approach Overview

This research project seeks to propose an approach to solving the machine configuration problem using a construction method based on the properties of directed graphs or digraphs. Strong components, reachability, connectedness, and distance are some of the properties that are used to help create layout designs and provide more insight into the matter. As can be seen in the figures above, in the dedicated facilities illustration three separate digraphs are available, one for each product. In the non-dedicated facilities illustration a new digraph was created, first as a single row layout and later as a network type or multiple row layout. These digraph structures and their properties are an approach which could be advantageously used to create layout designs and solve questions such as the following: ① what is the number of displacements required from a specific operation to any other operation in the layout; ② how many paths are available for a product to reach a certain workcentre or facility; ③ how many paths of certain length exist between the different facilities; ④ which facilities are going to be reached by products from a specific workcentre in the different routes; ⑤ the proper location of facilities when considering the input and output ports location to external facilities; and, of course, ⑥ which machines have to be placed close together.

5.5. Directed Graphs and Strong Components

Directed graphs or digraphs (**D**) are theoretical concepts whose main concern is to analyse the structural properties of sets of nodes and arcs that can be related to empirical situations such as production operation sequences. In cases such as this, digraph theory provides a useful framework that eases the analysis of these structures and of their properties, and which can be applied to analyse relevant patterns of relationships among pairs of abstract entities. Nodes and arcs are these types of entities. In this case, nodes can be associated with facilities, departments, workcentres, workstations, and machines; and the corresponding arcs represent the existing interrelations, links or bondings between them. Since a sequence implies the existence of precedence, arcs should be used to represent such relations. Some of these properties, such as reachability, connectedness and distance, are explained later in the chapter. Most of the directed graph theory concepts used in this research work are based on the materials presented by Harary, Norman and Cartwright (Harary F., Norman R.Z., Cartwright D., 1965).

5.6. Layout Design Method

This research project presents a constructive solution approach to address the Relationship Diagram required by Facilities Layout Designs, an approach which requires only a basic mathematical background, offers a solution to larger sized problems, performs with a moderate amount of computational resource, and provides a solution in a reasonable amount of time. The achieved layout design solution will be operationally viable to support the product operation sequences under consideration and during their expected production life cycles.

The applicability of the proposed approach is towards solving the single machine layout design, and it can be extended to solve the intra-cell layout design in a Cellular Manufacturing System, and to solve the intra-loop design for a Flexible Manufacturing System.

The application of this approach initially investigates solutions to determine the closeness between machines when the arcs have equal weights, equal importance or dimensionless weights. In this case, it is assumed that the production operation sequences provide the criterion or criteria to determine the proximity among machines through the integration into one graph of the different product operation sequences. Of course, the approach can also cope with unequal edge weights.

Although many researchers have considered the demand flow, combined with other criteria, as a basis to determine the arc weights and consequently the objective function coefficients of layout formulations, this is not necessarily required in the proposed approach. An advantage of this is that it discloses the number of possibilities from a range of solution alternatives, from strictly dedicated to non-dedicated facilities, that can be addressed when considering specific demand fluctuations. Demand may have an impact on the intensity of usage of the links between machines and on the delivery opportunities, which might lead to a decision to produce or not a product in non-dedicated facilities. It is assumed that during normal operation conditions machines process parts item by item, and also that an item at a time is carried from facility to facility. Accepting this as a valid statement, then arc weights could be equal to one, and hence operation sequence similarities become relevant. Some solutions have considered similarities coefficients in determining how machines have to be placed, while others have used cluster analysis to determine this. This research bases its proposal on the directed graph structural properties, specifically based on strong components.

Strong components are closely interrelated entities. These entities can be identified as mutually reachable; that is, they both have arcs that leave and arrive from each other. In a manufacturing process, they can be identified as machines that during production may receive products sent from the other machine, creating an interrelated structure between them.

The minimum information required to apply this machine layout design approach is fully available information on the product operation sequences.

5.6.1. Layout Design Method

As stated previously, the strong component approach to the facility layout design is a heuristic construction approach. Once the operation sequences of each product are available, the method requires the calculation of a reachability matrix, which provides the basis to obtain the strong component matrix. As already discussed, this method is based on directed graphs and their properties. To obtain its solution, matrix algebra and Boolean algebra are both required.

1. Determine the Adjacency Matrix
2. Calculate the Reachability Matrix
3. Determine the Strong Component Matrix
4. Construct the Strong Component Graph

- **a. Identify if there are strong components**
  - **i. Elaborate a list of arcs and facilities involved**
  - **ii. Rank them on the basis of a selected secondary criteria**
  - **iii. Construct a sub-graph with them following the suggested order**
  - **iv. Keep a list of unused arcs**
- **b. Identify non-strong components according to their weights and the chosen criteria**
  - **i. Elaborate a list of arcs and facilities involved**
  - **ii. Rank them according to the chosen secondary criteria**
  - **iii. Construct a sub-graph with them following the suggested order**
  - **iv. Review unused arc list and use them, if any, as required**
  - **v. Update list of unused arcs**
- **c. If there are no more components to be identified, then proceed with step d, otherwise**
  - **i. Elaborate a list of arcs and facilities involved**
  - **ii. Rank them according to the chosen secondary criteria**
  - **iii. Construct a sub-graph with them following the suggested order**
  - **iv. Review unused arc list and use them, if any, as required**
  - **v. Update list of unused arcs**
- **d. Integrate all the sub-graphs and individual components using the available unused arcs in order of their ranks. Take one unused ranked arc, at a time, until all possible arcs have been considered**
- **e. Review all the product operation sequences and add links from any unused arcs as required**
- **f. If all production sequences can be accomplished then the process is done; otherwise try to complete the sequences by using arcs available**

Figure 5.3 Strong Component-Based Layout Design Method

Figure 5.3 shows the proposed layout design method in which it is implicitly stated that the ranking criteria to be used are related to the arc weights. The arc weights help preference ranking among the arcs and to support all layout design methods in differentiating between interrelationships. The use of the Frequency of Displacements Matrix is suggested as a secondary criterion to support the Strong Component-Based Layout Design Method, but other criteria can be used.

The flow chart diagram in Figure 5.4 is a schematic representation for this process.

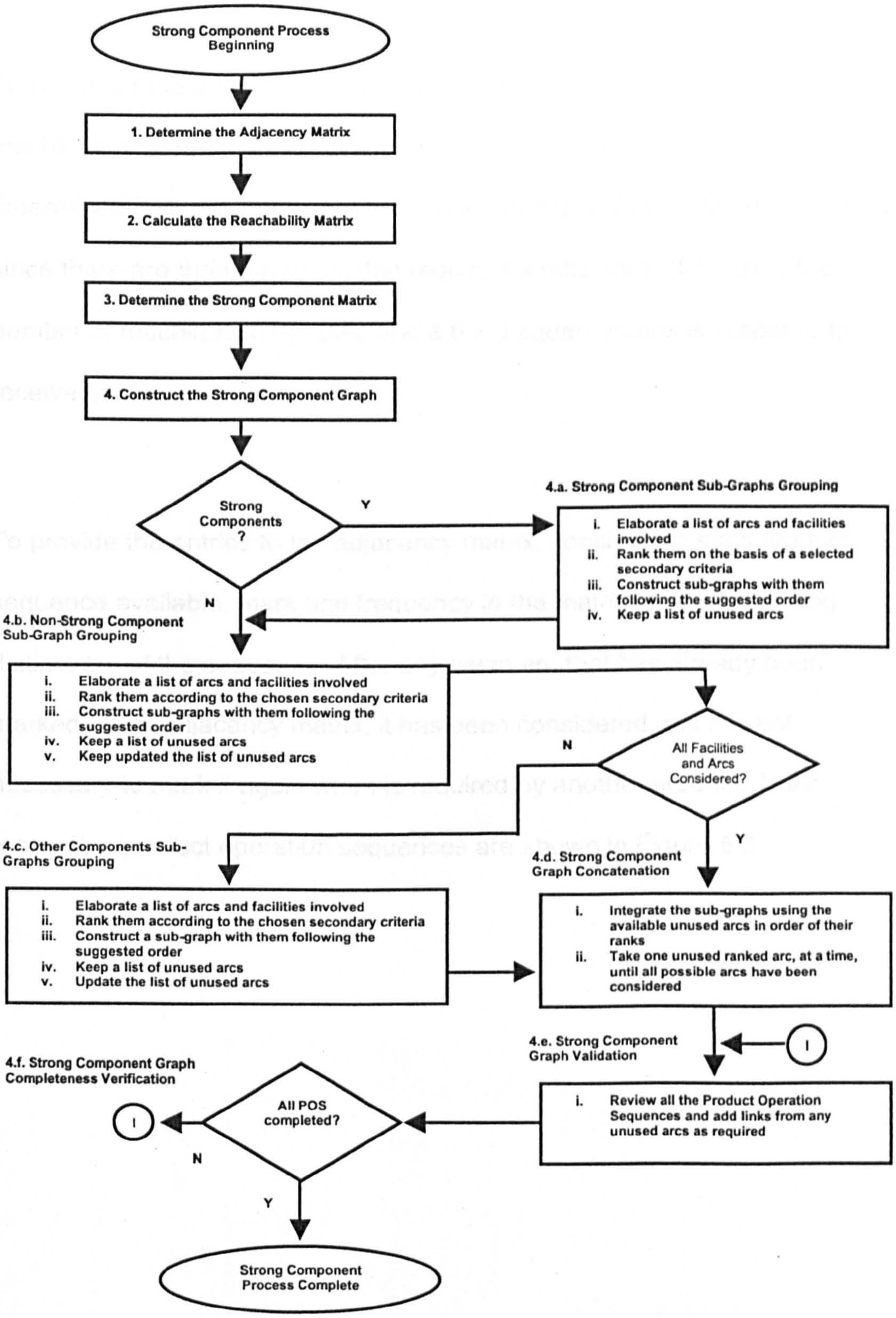
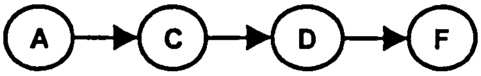


Figure 5.4 Strong Component-Based Layout Design Method Diagram

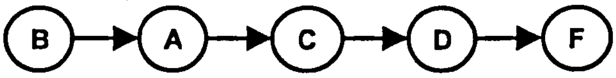
5.6.1.1. Determine the Adjacency Matrix

To construct the adjacency matrix (**A(D)**; Step 1 in Figure 5.3), the number of machines required to make all the products under consideration has to be determined first. For the example provided in the dedicated facilities section, since there are three products that require six machines, then six is the number of machines to be used and a 6 x 6 square matrix is prepared to receive entries.

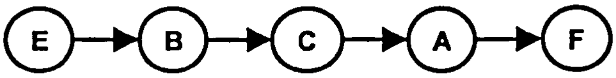
To provide the entries to the adjacency matrix, considering each product sequence available, mark one frequency in the matrix for every existing distinct arc of the sequence. After any given arc that has already been marked in the adjacency matrix, it has been considered and it is not necessary to mark it again when is required by another product. Their respective product operation sequences are shown in Figure 5.5.



a. Product One Operation Sequence



b. Product Two Operation Sequence



c. Product Three Operation Sequence

Figure 5.5. Product Operation Sequence Graphs

Once all the products have been taken into consideration, the adjacency matrix marking is finished and the A(D) is now available for the following step. The matrix for the example is shown in Table 5.4. Certain features such as source or sink nodes, symmetry, node degrees, and node adjacencies can be seen in A(D).

	A	B	C	D	E	F	Total
A			1			1	2
B	1		1				2
C	1			1			2
D						1	1
E		1					1
F							0
Total	2	1	2	1	0	2	8

Table 5.4. Adjacency Matrix

5.6.1.2. Calculate the Reachability Matrix

For any nodes n_i and n_j in a digraph D, it is said that n_j is reachable or accessible from n_i if there exists a sequence of nodes and arcs from n_i to n_j. This path or sequence is also known as a directed sub-graph. This may suggest nodes with particular characteristics. A source is a node which may reach every node in a digraph. Conversely, a node is called a sink when it is reachable from all the points contained in the digraph. Nodes E and F are respectively instances of these two types of nodes.

In a production system where products follow a route in order to be made, each product has to reach each one of the required workcentres. During this product flow any previous workcentre is linked to the ones after it through the particular operation sequence. The contrary is not necessarily true unless backtracking is allowed and, for financial or operational reasons, additional workcentres are not deployed. A node (facility or workcentre) should be reachable from a different node if there is a sequence (operation sequence) from one node to the other.

A Reachability Matrix (**R(D)**) is a Boolean matrix whose entries are denoted as r_{ij} and defined as follows: $r_{ij} = 1$ if a node j can be reached from node i; otherwise $r_{ij} = 0$. In other words, this matrix has an entry if there exists a sequence from n_i to n_j. Since every point may be reachable from itself, all the diagonal entries are set equal to one. Another implication is that if $a_{ij} = 1$ then

r_{ij} is equal to one. An arc (n_i, n_j) is said to exist in a closure if and only if n_j is reachable from n_i in a digraph. For example, the transitive property of a digraph says that for every three distinct nodes n_1, n_2, n_3 whenever the arcs (n_1, n_2) and (n_2, n_3) are in a digraph, then there is a sequence of nodes and arcs of length 2 from the initial node n_1 to the final node n_3. This situation can be related to the number of displacements between machines required to complete or partially complete any given product. Therefore, any given entry in this reachability matrix represents the existence of a directed sub-graph of certain known length or distance.

To calculate the reachability matrix **R(D)** of a digraph (Harary F., Norman R.Z., Cartwright D., 1965:117; step 2 in Figure 5.3), consider the adjacency matrix **A(D)** or **A** as a starting point. It can be shown that every power to which the adjacency matrix can be raised has a specific meaning. For instance, if the adjacency matrix is raised to the second power then the values obtained provide all the numbers of sequences of length two that exist in the structure from any node to any other; and all the entries have a similar meaning for any other power to which the matrix is raised. A special modified matrix form of these power matrices is their corresponding Boolean matrices: that is, for every entry greater than zero in the power matrices a value of one is placed, and a zero is placed otherwise. The notation of this Boolean power matrix is $\mathbf{A\#}^p$, where p is the power to which the matrix is raised. Hence, any entry in the matrix $\mathbf{A\#}^p$ will indicate the existence of one or more paths of length p between any two nodes considered.

For $i = 0$, $R_0(D) = I$, the identity matrix.

For $i = 1$ then $R_1(D) = I + A(D)$ or $R_1 = R_0 + A$.

For $i = 2$ then $R_2 = I + A + A^2$, or $R_2 = R_1 + A^2$.

For $i = 3$ then $R_3 = I + A + A^2 + A^3$ or $R_3 = R_2 + A^3$ ...

For $i = n$ then $R_n = R_{n-1} + A^n$.

And, where $A^n = (A\,\#)^n$.

One of the possible applications which can be obtained from a reachability matrix is the following: for any two distinct nodes n_i and n_j of a digraph, and allowing any entry (i, j) in $\mathbf{A^p}$ to be at least one possible sequence of length p-1 from an initial to a final node, then an entry in $\mathbf{A^p\#}$ will recognise the existence of at least one sequence of length (p-1). Since (n-1) is the longest possible value of a path that is contained in a digraph with n nodes, then there exists a value k and $k \leq n$ when the equality $\mathbf{R_{k-1}(D) = R_k(D)}$ is reached, and consequently $\mathbf{R_k(D) = R(D) = (A+I)^n\#}$ (Harary F., Norman R.Z., Cartwright D., 1965:122). The relevance of this matrix is that all reachable nodes can be easily identified, as seen in the following illustration. The Reachability Matrix for non-dedicated facilities can be seen in Table 5.5 and its entries explanation is as follows. In this table, it can be seen that node E can reach all other nodes since all of its entries are one, whereas node F can be reached from any node, given that the column entries are one. Taking into consideration the sum of the row or column, then the node A has at least 4 paths of length equal or less than (n-1) by means of which can reach nodes A, C, D and F. In this case as a coincidence, the sum of the rows and columns is the same number of paths of the same length by which

A can be reached from nodes A, B, C, and E. In appendix 4 can be found the partial matrices required to obtain the Reachability Matrix from the Adjacency Matrix.

	A	B	C	D	E	F	Total
A	1		1	1		1	4
B	1	1	1	1		1	5
C	1		1	1		1	4
D				1		1	2
E	1	1	1	1	1	1	6
F						1	1
Total	4	2	4	5	1	6	22

Table 5.5. Reachability Matrix

5.6.1.3. Determine the Strong Component Matrix

Strong components are nodes which are closely interrelated, having either flows or interrelationships in both ways, i.e., mutually reachable, and would therefore, preferably be in close proximity. For example, nodes A and C have flows in both directions; thus, they are expected to be placed close to each other. This arc bonding can also be represented by an edge known as a bi-directional or 3-joined type, and it is comparable to the relationship of the highest value labelled as A in a Relationship Chart. When there is a unidirectional flow between nodes, only one of the involved nodes is reachable from the other or the nodes are linked by a 2-joined edge. A unidirectional or 1-joined edge is an edge used to represent a link between two nodes with a non-specified direction. In section 5.6.1.5., this issue is taken up again.

	A	B	C	D	E	F	Total
A	1	1	1		1		4
B		1			1		2
C	1	1	1		1		4
D	1	1	1	1	1		5
E					1		1
F	1	1	1	1	1	1	6
Total	4	5	4	2	6	1	22

a. Reachability Transpose Matrix

	A	B	C	D	E	F	Total
A	1		1				2
B		1					1
C	1		1				2
D				1			1
E					1		1
F						1	1
Total	2	1	2	1	1	1	8

b. Strong Components Matrix

Table 5.6. Reachability Transpose and Strong Component Tables or Matrices

Strong components can be obtained from the Reachability matrix (Step 3 in Figure 5.3). To obtain them it is necessary to calculate the transpose of the reachability matrix (Harary F., Norman R.Z., Cartwright D., 1965:123), denoted as **R'(D),** and then perform an AND Boolean operation, component to component. If an entry in the Reachability Matrix implies that facility j is reachable from facility i, and its transpose implies that facility i is reachable from facility j, then this suggests that these facilities are strong components since they are mutually reachable. Both matrices are shown in Table 5.6. In this table, strong components can be easily identified since these similar components have the same entries in their respective rows. The other diagonal entries belong to relationships between the same components and they may be ignored.

5.6.1.4. Construct Strong Component Graph

From Figure 5.3 it can be seen that step 4 (the construction of a Strong Component Graph) is subdivided into six steps, as follows.

① Step 4.a. Locate the strong components, if any, in the Strong Component matrix. Next, rank their related arcs using the frequency of displacements matrix or a selected secondary criterion. Then, construct their sub-graphs starting from the highest to the lowest frequency arc values. Additionally, create a list of ranked unused arcs to be used later. It should be expected that the available unused arcs are those that go from the Strong Component facilities to other nodes, i.e., in-between arcs.

② Step 4.b. Identify non-strong components nodes and rank their arcs using the established secondary criterion, as in the previous step. Next, build their sub-graphs and add the unused arcs to the previous list as well.

③ Step 4.c. If there are remaining nodes to be considered, rank them according to the selected secondary criterion and build their sub-graphs. If any additional arcs are left that should be considered, add them to the list of unused arcs. Usually these are isolated facilities and products that require one task, which the approach fails to identify; this limitation can be overcome as explained later in section 5.7.2.

④ Step 4.d. Using the available arcs from list of unused arcs, link all the sub-graphs including individual facilities, if any.

⑤ Step 4.e. Review all the product operation sequences following the completed graph and add links as required.

⑥ Step 4.f. If they are satisfactory then the process is over; otherwise continue and review from step 4.e. It is relevant to say that during this approach, the product operation sequences suggest the appropriate node positions. The explanation of the example which started in section 5.6.1.1

continues in the rest of this section for the sub-steps contained in step 4 (in Figure 5.3).

Step 4.a.

Identify the strong components and construct their sub-graph, as shown in Figure 5.6. From the Strong Components Matrix these are A and C. The in-between or unused arcs are C to D and B to A.

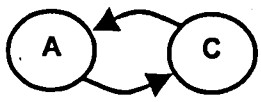


Figure 5.6. Strong Component Sub-Graph

Step 4.b.

Next, identify non-strong components and construct their sub-graphs. Nodes E, B, D and F, are non-strong components and have separated sub-graphs, as shown in Figure 5.7. The in-between arcs to be added to the list of unused arcs are A to F and B to C.

Figure 5.7. Non-Strong Component Sub-Graphs

Step 4.c.

There may exist other facilities (nodes) to be considered and still to be linked to any of the previous sub-graphs. Select the nodes and arcs in the appropriate rank order to construct their sub-graphs. Add to the list of unused arcs any in-between arcs, if any. In the example being developed there are no facilities and arcs of this type.

Step 4.d.

Concatenate all the sub-graphs obtained in steps 4.a, 4.b and 4.c using the list of unused arcs, as shown in Figure 5.8. If no other additional facilities and arcs are to be considered, then proceed with the following steps.

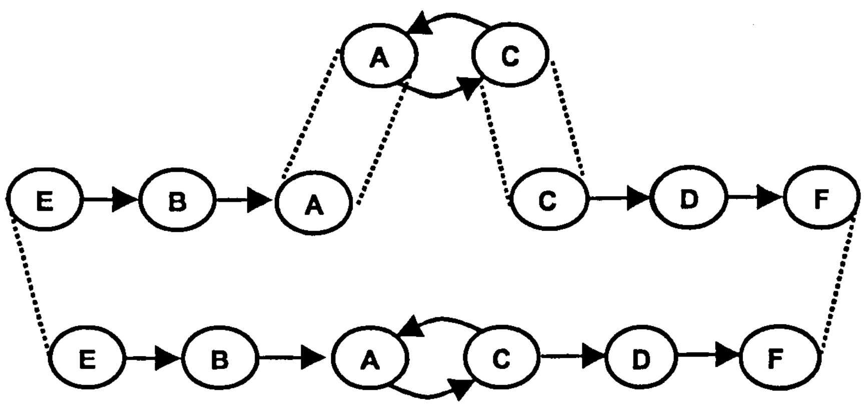


Figure 5.8. Strong Component Partial Sub-Graph

Steps 4.e and 4.f.

Up to this stage, the suggested procedure ensures that the proposed layout is able to execute all the product operation sequences required. Following each product operation sequence helps to identify any missing links and to verify and validate the proposed layout design. For example, taking product one from Figure 5.5, its operation sequence is from A to C to D to F; these nodes should be linked by arcs, as seen in the last four facility nodes in Figure 5.9. Other Product Operation Sequences should be validated and verified in a similar manner.

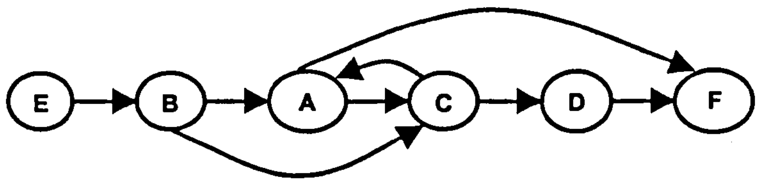


Figure 5.9. Strong Component Graph

The number of machines, material handling systems and material handling volume displacements are used to compare dedicated and non-dedicated layout designs. In Table 5.7, it can be seen that the number of machines in dedicated facilities is reduced from 14 to 6 in non-dedicated facilities. The number of material handling systems can also be reduced from 3 in dedicated facilities, to 1, in the non-dedicated case. For the comparison of the volumes moved, it is important to bear in mind that results may be affected by how movements are considered. Basically, they can be approached from two perspectives: if only in-sequence movements are allowed, or if non-in-sequence movements can be performed.

Table 5.7 summarises the comparison between dedicated and non-dedicated facilities when non-in-sequence movements are allowed and machines can be fed directly without going through a source node. Comparing the volume flow it can be seen that both manufacturing systems are capable of moving the same volumes. Consequently, there is not much difference between them besides the number of machines and material handling devices. In addition to this, it is expected that the space utilisation will be improved, since there is a smaller number of machines.

Product	Volume	Dedicated Facilities			Non-Dedicated Facilities		
		Machines Required	Displacements Required	Accumulated Flow	Machines Required	Displacements Required	Accumulated Flow
1	500	4	3	1500		3	1500
2	1000	5	4	4000		4	4000
3	300	5	4	1200		4	1200
Total	1800	14	11	6700	6	11	6700

Table 5.7. Dedicated vs. Non-Dedicated Facilities
Non-In-Sequence Flow

Comparing dedicated to non-dedicated facilities, it can be seen from Table 5.7 that six machines and possibly one material handling device are enough to produce all three products. If bypassing and backtracking are allowed then the accumulated flow of materials is the same in either layout.

Product	Volume	Dedicated Facilities			Non-Dedicated Facilities		
		Machines Required	Displacements Required	Accumulated Flow	Machines Required	Displacements Required	Accumulated Flow
1	500	4	3	1500		5	2500
2	1000	5	4	4000		5	5000
3	300	5	4	1200		7	2100
Total	1800	14	11	6700	6	17	9600

Table 5.8. Dedicated vs. Non-Dedicated Facilities
In-Sequence Flows

Table 5.8 provides the results when only in-sequence movements are allowed. That is, movements such as bypassing are not allowed and machines can only be fed via the initial (source) workcentre, facility E in this example, as shown in Figure 5.10.

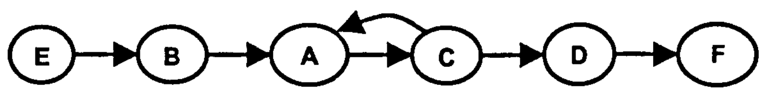


Figure 5.10. Non-Dedicated Facilities without Bypassing Flows Graph

It should be noted that the material-handling device selected may affect the results. The material-handling device could be a manned or unmanned device, or may or may not follow a fixed trail. Depending on the characteristics of the material handling system, the material displacements such as backtracking and bypassing may be taken into consideration, as shown in Tables 5.7 and 5.8. Another possibility that could be explored is the addition of another machine A: this will increase the number of machines to 7 (Figure 5.11). This machine should be placed after machine C, and will eliminate the backtracking movement; if only in-sequence movement are allowed, this will increase to 18 the required displacements to give a total volume moved to 10800. A trade-off analysis should be performed to decide the appropriate material handling device, and the savings that could be obtained when shifting from dedicated to non-dedicated facilities.

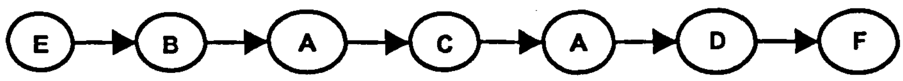


Figure 5.11. Non-Dedicated In-Sequence Flows Graph

5.6.1.5. Planar Graph Equivalence

The Strong Component Directed Graph has graph equivalence. This equivalence can be used as an alternative representation of the network-based layout design using an edge pictorial representation. Using this equivalence it is expected to reduce the number of drawn arcs in the graph and facilitate its reading. Definitions of connectedness (Harary F., Norman R.Z., Cartwright D., 1965:122) can be used, as explained in section 5.6.1.3 above, and their equivalences are shown in Table 5.9, which follows:

Directed Graph	Graph
Bi-directional	3-joined
Unidirectional	2-joined
Undirectional	1-joined
No bond	0-joined

Table 5.9. Directed Graph and Graph Equivalencies

The conceptual difference between directed graphs and graphs is that arcs are used to show interrelationships instead of the edges used in graphs. In Figure 5.12, the directed graph is shown as a planar graph. In addition, if two edges are added to the graph, the dotted ones, the planar graph will become a maximally planar graph. Moreover, the bold line represents the bi-directional arc or 3-joined edges, the normal lines represent the unidirectional arcs or the 2-joined edges, and the dashed lines can be undirected arcs or 1-joined edges required by the planarity of the graph. It should be noted that adding an edge between E and A, and A and D, and any other edges, would make the graph non-planar. Using this equivalence notation is a useful way to reduce the visual complexity when many arcs have to be drawn in an adjacency graph.

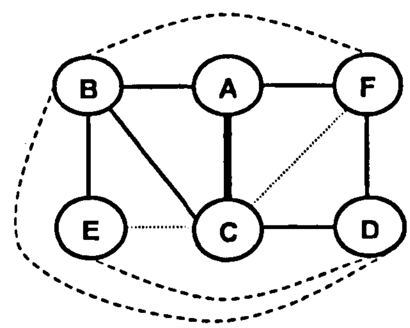


Figure 5.12. Planar Graph Equivalence

5.7. Solution Approach Extensions

Until now the approach has considered the most general case and has assumed that the production operation sequences are the only data available or may represent the only reliable information source available about the products over, for instance, their life cycles. Moreover, it has been explained how strong components can be successfully used in an approach to create facilities layout designs. We next consider some extensions to the method.

5.7.1. Volume Weights

In this section, the question to be addressed is the arrangement of machines when the expected demand for each product is considered. This assumes that the demand information is available, as depicted in Table 5.2. The difference in the process is that, instead of taking into consideration the frequency of displacements, the demand volume is now considered and can be any type of measure, for instance: units, kilograms, travel distances or times and costs. Consequently, arcs are now hierarchically listed from the ones with more flow demand volume to arcs with lower demand. It is suggested that arcs are selected following these criteria: ① strong components with higher flow demand volume will be preferred over strong components with lower flow demand; ② unidirectional facility edges with higher flow demand will be preferred over unidirectional ones with lower flow demand; and ③ the same applies to the other arcs.

In the case of the example already under analysis, the arc ranks are shown in Table 5.10.

Arc	Arc Type	Demand Volume
A-C	Strong Component	1500
C-D D-F	Unidirectional	1500
A-B	Unidirectional	1000
A-F B-C C-A E-B	Unidirectional	300

Table 5.10. Volume- Strong Component Arc Rankings

The links with the highest flow volume demand are the same ones as those in the previous case in which rankings were by the strength of components relationships alone (section 5.2). Therefore, the directed graph and the equivalence graph are the same. In cases when the values available are in conflict, designers may choose and rank the criteria priorities.

Next, to answer the other questions about how machines have to be placed when material handling distances, costs and times are the arc weights, a very similar process to the one already outlined for volume weights can be followed. In addition, once existing experience is available and has been gained from the production operation processes, the flow demand volume can be modified to take into consideration the actual production batch sizes, or even the transfer batch size. In the following sections, two other questions are considered: ① the machine configuration under external input-output port locations and ②the consideration of non-adjacent facilities.

5.7.2. Input-Output Ports

One of the issues that may be of relevance, and which most of the literature available omits to take into consideration, is the location of the external relationships with other facilities such as storage locations, cells, flexible manufacturing systems or functional departments. In general, these external relationships can be represented by the location of input and output ports. There is the risk of placing such ports in the middle of a layout, which may require direct relationships with facilities outside the layout, and this will leave them surrounded by other facilities and with complicated access. Figure 5.10 is an example to illustrate this point, supported by the operation sequences from Table 5.2. From them, it can be deduced that nodes A, B, and E are required to receive the initial materials to start production of the corresponding products. On the other hand, workcentre or node F is in all cases the final node, and should have the possibility of easy access in order to send finished items to other selected facilities.

This situation implies the need to consider two additional workcentres as dummies to be added to the adjacency matrix to reflect the fact described above and represent these relationships. The new A(D) is shown in Table 5.11 (Step 1 in Figure 5.3). Moreover, in Tables 5.13 and 5.14 are shown the new Reachability (Step 2 in Figure 5.3) and Strong Component (Step 3 in Figure 5.3) Matrices respectively; where I stands for the Input or Source Facility and O for the Output or Sink Facility.

	I	A	B	C	D	E	F	O	Total
I		1	1			1			3
A				1			1		2
B		1		1					2
C		1			1				2
D							1		1
E			1						1
F								1	1
O									0
Total	0	3	2	2	1	1	2	1	12

Table 5.11. Adjacency Matrix

	I	A	B	C	D	E	F	O	Total
I		1	1			1			3
A				2			1		3
B		1		1					2
C		1			2				3
D							2		2
E			1						1
F								3	3
O									0
Total	0	3	2	3	2	1	3	3	17

Table 5.12. Frequency of Displacements Matrix

	I	A	B	C	D	E	F	O	Total
I	1	1	1	1	1	1	1	1	8
A		1		1	1		1	1	5
B		1	1	1	1		1	1	6
C		1		1	1		1	1	5
D					1		1	1	3
E		1	1	1	1	1	1	1	7
F							1	1	2
O								1	1
Total	1	5	3	5	6	2	7	8	37

Table 5.13. Reachability Matrix

	I	A	B	C	D	E	F	O	Total
I	1								1
A		1		1					2
B			1						1
C		1		1					2
D					1				1
E						1			1
F							1		1
O								1	1
Total	1	2	1	2	1	1	1	1	10

Table 5.14. Strong Component Matrix

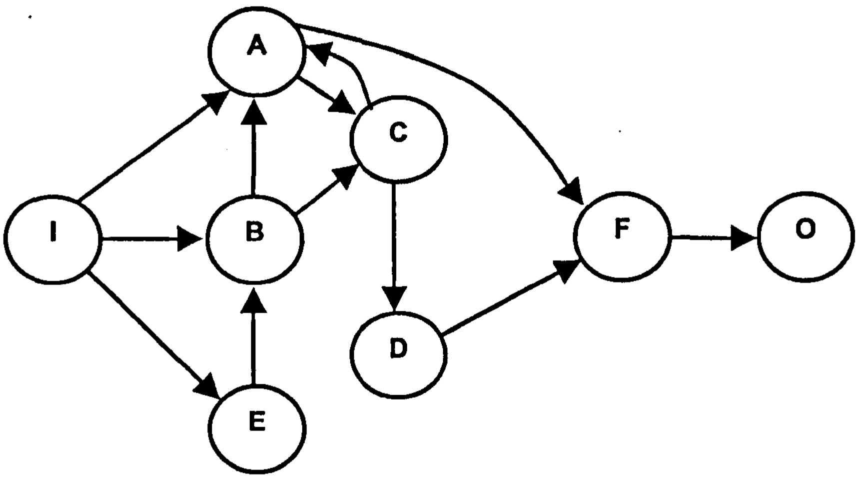


Figure 5.13. Graph containing Input Output Relationships

Figure 5.13 shows the relevance of considering the location of input and output ports when developing a layout design. The comparison between this figure and Figure 5.10 suggests that neglecting these ports may lead to inappropriate layout designs.

5.7.3. Consideration of Non-Adjacent Flows

Some papers in the literature also consider non-adjacent flows in the determination of closeness requirements. They also use volume and process routes as the main inputs for the development of machine layouts, as proposed by John and Hammond (John E.G., Hammond J., 1999). In their paper, they state that the distance as expressed as a physical measurement is simply a relative expression of the interrelationships between facilities. Furthermore, they also add that as an expression of items transferred between facilities it is irrelevant for this analysis if the distance is represented by a physical measure or if it is expressed by a dimensionless representation such as the frequency of displacements. Thus, they argue that distance is a measure simply expressing a preference relationship. However, whichever measure is selected, product quantities still have to be moved between the respective process stages. This implies the need for procedures which are capable of reflecting the closeness requirement by incorporating both process sequence and process volumes.

The impact of non-adjacent flows in the procedure depends on how far apart the pair of machines is in the process sequence. For instance, in the operation sequence of product number one, which in Table 5.1 is A-C-D-F, John and Hammond (John E.G., Hammond J., 1999) suggest taking the volume and dividing it according to the number of displacement steps between the pair of facilities under analysis, as follows. In the sequence mentioned above and taking as a reference the first facility, machine A, machine C is one movement away, machine D two movements away, and F three movements away, from A. Therefore, the expected adjacent and non-adjacent flows from A to each of these for the product sequence is measured as V/1, V/2, and V/3. These flows are written into the From-to matrix and the accumulated weights of all the sequences reflect closeness measures. This creates additional arcs or edges that can be incorporated to obtain a maximally weighted planar graph (John E.G., Hammond J., 2000; Al-Hakim L., 2002). For the particular example, Table 5.15 shows the flows and the additional arcs created after considering all the sequences and the demand volume for the products. Moreover, it can be seen as a way to include or take into consideration other arcs which otherwise are normally omitted. Consequently, there are more arc choices available that can now be considered by the layout planner.

	A	B	C	D	E	F	Total
A			1500.0	750.0		800.0	3050.0
B	1150.0		800.0	333.3		350.0	2633.3
C	300.0			1500.0		900.0	2700.0
D						1500.0	1500.0
E	100.0	300.0	150.0			75.0	625.0
F							0.0
Total	1550.0	300.0	2450.0	2583.3	0.0	3625.0	10508.3

Table 5.15. Non-Adjacent Volume Weighted From-To Matrix

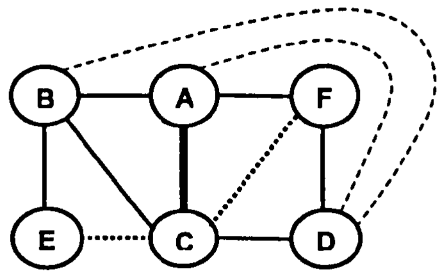


Figure 5.14. Non-Adjacent Planar Weighted Sub-Graph

This result is compared to the solution shown in Figure 5.12. In Figure 5.12 the dotted arcs and the dashed lines were added to show the planarity of the graph and the solid lines represented the arcs required by the production operation sequences. In Figure 5.14 this is the case as well, but instead of adding arbitrary edges, the arcs with the highest weight suggested by Table 5.15 are now added. Although new arcs are added the graph is still incomplete, is a sub-graph, and an arbitrary arc between facilities E and D should still be added to complete the maximal planarity, if required. Nevertheless, the sub-graph structure does not change and it can be verified by comparing the respective strong components matrices. This result provides supporting evidence that the proposed approach is insensitive to the addition of arcs, as long as the production operation sequences do not change.

5.8. Non Strong Component Problem

The detail of the applicability of the approach when the resultant structure does not contain strong components is examined in this section. The example addressed was also chosen from the literature (Abdou G., Dutta S.P., 1990).

Part Number	Volume Year	Number of Operations	Number of Movements	Sequence of Machines Required
1	15	6	5	8 4 6 2 1 13
2	10	4	3	5 1 10 13
3	20	5	4	8 4 5 1 13
4	25	3	2	7 12 13
5	4	4	3	3 1 11 13
6	20	2	1	6 13
7	7	4	3	8 2 10 13
8	15	3	2	3 9 13
9	60	3	2	7 9 13
10	10	4	3	8 6 11 13
11	10	4	3	4 6 5 13
Total	196	42	31	

Table 5.16. Volume and Product Operation Sequence Data (Abdou G., Dutta S.P., 1990)

In this instance, the number of machines is almost the same as the number of products, thirteen machines and eleven products, as shown by the data shown in Table 5.16: volume is in hundreds of thousands of parts.

Analysing the information provided by this example, it can be seen that 31 transfer movements are needed to obtain one product from each (Table 5.16); from them the minimum required is only 24, as shown by the Adjacency Matrix (Table 5.17; Step 1 in Figure 5.3). In this case there is one sink node, facility thirteen, and facilities three, four, five, six, and eight are the initial manufacturing starting nodes.

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
1										1	1		1	3
2	1									1				2
3	1								1					2
4					1	1								2
5	1												1	2
6		1			1						1		1	4
7									1			1		2
8		1		1		1								3
9													1	1
10													1	1
11													1	1
12													1	1
13														0
Total	3	2	0	1	2	2	0	0	2	2	2	1	7	24

Table 5.17. Adjacency Matrix

The Strong Component Matrix obtained after the calculations (Step 2 in Figure 5.3) is shown in Table 5.18 (Step3 in Figure 5.3). After inspecting this matrix, it can be deduced that this structure does not contain strong components. Consequently, the Strong Component Sub-Graph construction will be omitted (Step 4.a in Figure 5.3) and the process should continue the development of the Non-Strong Components Sub-Graphs (Steps from 4.b to 4.f in Figure 5.3).

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	0	0	0	0	0	0	0	0
6	0	0	0	0	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0
9	0	0	0	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 5.18. Strong Component Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
1										1	1		2	4
2	1									1				2
3	1								1					2
4					1	2								3
5	2												1	3
6		1			1						1		1	4
7									1			1		2
8		1		2		1								4
9													2	2
10													2	2
11													2	2
12													1	1
13														0
Total	4	2	0	2	2	3	0	0	2	2	2	1	11	31

Table 5.19. Frequency of Displacements Matrix

To be able to distinguish between Non-Strong Components, the Frequency of Displacements Matrix data (Table 5.19) will be used as a guidance criterion to differentiate between arcs (Step 4.b.ii and 4.c.ii in Figure 5.3). In this table, it can be seen that arcs between facilities one and thirteen, four and six, five and one, nine and thirteen, nine and ten, and nine and eleven, all have two products that go from one facility to the other. This suggests placing the facilities close, since they may be used more often, and consequently material handling distances and times could be reduced for their respective product flows. Therefore, two subgroups can be identified, one with the higher frequency (2): facilities one, four, five, six, eight, nine, ten, eleven, and twelve; and another with the lower frequency (1): facilities two, three, seven and twelve. After the group identification, the procedure continues by linking the nodes in each subgroup, where possible (Steps 4.b.iii and 4.c.iii in Figure 5.3). Facilities five, one, nine, ten, eleven and thirteen provide a sub-graph, and facilities four, six and eight, provide another sub-graph, in the higher frequency group; whereas in the low frequency facilities seven and twelve are a sub-graph, and stand-alone facilities two and three also have to be considered.

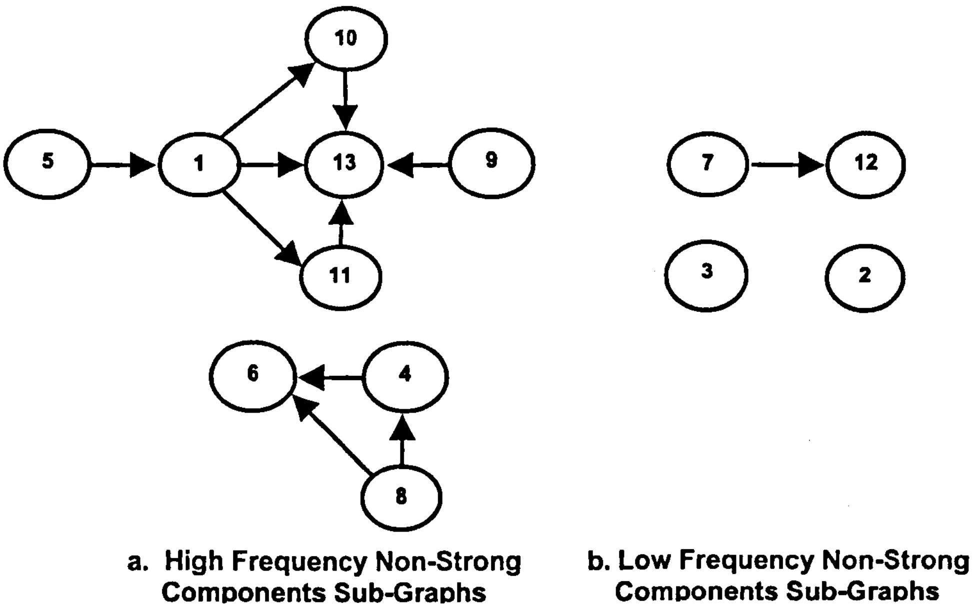


Figure 5.15. Low and High Frequency Non-Strong Component Sub-Graphs

With this information, their respective sub-graphs are constructed as shown in Figure 5.15 (Steps 4.b.iii and 4.c.iii in Figure 5.3). It is important to remember first to draw the sub-graphs independently with all their arcs, before attempting to connect them. Once the sub-graphs have been constructed, then proceed to join them by taking into consideration the other arcs available. This intermediate step can be seen in Figure 5.16 (Step 4.d in Figure 5.3). In this Figure, the dotted and the dashed edges represent the possible links between the high frequency and the low frequency Non-Strong Component Sub-Graphs. After considering all the possible interconnections between the sub-graphs, the Non-Strong Component Graph is shown in Figure 5.17 (Step 4.d in Figure 5.3).

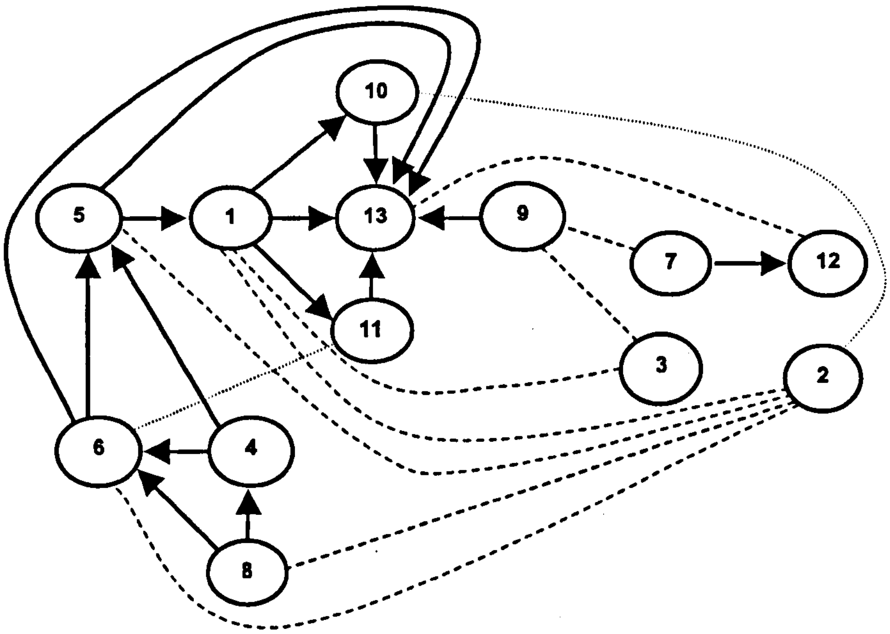


Figure 5.16. Pre-Linked Non-Strong Component Sub-Graph

In this solution (Figure 5.17) and after following each Product Operation Sequences (Steps 4.e and 4.f in Figure 5.3), it can be seen that the interrelationship missing is the one between facilities six and eleven, which will need to be replaced by the arcs between six to thirteen and thirteen to eleven. This increases the required number of displacements by one to 32, the number of transfer displacements required to complete all the products, if planarity is considered. The arc between facilities thirteen and eleven should be allowed to be changed from uni-directional to bi-directional.

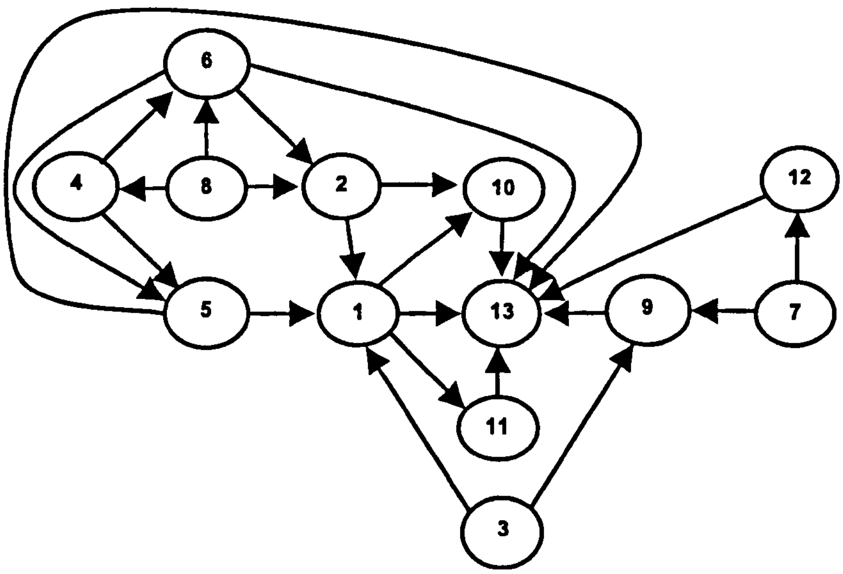


Figure 5.17. Non-Strong Component Graph

5.9. Limitations

The proposed approach fails to provide appropriate answers when the input adjacency matrix is a completely symmetrical matrix or close to being one. Furthermore, the approach can work with triangular matrices, but they only represent entities that have relationships in one direction, and consequently no strong components can be identified. Nevertheless, the approach can be applied and it may provide appropriate answers under these circumstances by excluding the strong components' step in the proposed approach, Step 4.a in Figure 5.3.

The approach in this thesis is proposed as a viable alternative tool to develop the relationship diagram required by layout designs. It is a flexible and practical tool that may assist designers when deciding the appropriate order in which to place shared non-dedicated facilities to produce a group of products that have production similarities. The approach can be used to address different purposes depending on the weights considered in the arcs. It covers a wider range of situations that range from qualitative to quantitative arc weights, including the situation in which there are no weights (dimensionless) or all the weights are equivalent (equal to one). This allows the designing of the layout in the special circumstance where all the arcs have a weight value of one, which has not been explored before.

The approach presented produces a relationship diagram that supports the development of technically and economically feasible layout designs. This reference provides the minimal shared structure required to obtain a set of products which can be regarded as a lower bound requirement necessary to manufacture the products under consideration. The structures provided require the minimal amount of space to be deployed, and simultaneously reduce the required travel distance to be considered, suggesting possible better use of existing resources when compared to dedicated facilities.

Moreover, suggested layout designs can easily be modified to incorporate more facilities, as explained when the input and output ports were added. This shows that problems which are more complex can be addressed.

Consequently having a tool like the proposed approach may be an advantage in situations in which demand fluctuations are highly uncertain, product life cycles are becoming shorter, machine investment has to be shared to be economically viable, and small transfer batches are used. It is advantageous because the layout designs obtained are at least as competitive as other designs provided by different approaches found in the literature, as can be seen in the other design examples included in the appendices (Appendix 3). It will be discussed, and compared to these, in the following chapter.

6. Discussion on Strong Component-Based Facility Layout Design Methodology

6.1. Introduction

This research work has focused on the development of a method to obtain facilities layout designs capable of addressing different questions when seeking production configurations required to produce a set of products during their expected life cycle. The methodology seeks to reduce the material handling involved, so that the proposed arrangement has material handling closest possible to dedicated facilities situations without the extra capital costs involved in the case of the latter. However, a trade-off analysis should be performed to decide the best possible outcome in each situation, since every company has its own environmental circumstances.

Evidence has been provided during this thesis that the use of directed graphs and some of their properties for the analysis and design of Facilities Layouts can be applied to obtain suitable relationship diagrams for a given set of product operation sequences. Further, it has been shown how non-dedicated facilities layout designs suggest solutions that are not only less expensive but also require less space and fewer resources when compared to dedicated facilities layouts. It has been shown how network-based layout designs may contribute to space-saving solutions and simultaneously obtain more efficient layout designs. In this chapter, we consider whether the Strong Component approach can be considered as a flexible, robust and appropriate tool to be used in the design of facility layouts when compared to previously discussed approaches in the literature, particularly layout designs suggested by Graph Theoretical techniques.

This comparison is based on considerations of suitability to address, to formulate, and to produce a relationship diagram required by layout designs. Three examples, which all address the multi-product environment issues, were taken from the literature and used to illustrate the approach presented in this thesis. They are used as a basis to compare the Strong Component Approach layout designs against the solutions contained in papers in which the examples were first presented.

6.2. General Modelling Issues

The input information required for the proposed approach to operate is based on production operation sequences. Once these production operation sequences are established they are considered as a reliable source of information since they are not expected to change during the life cycle of the products. For this reason, suggested Strong-Components-based models may be considered very reliable deterministic models given the following reasons.

- Since Production Operation Sequences are deterministic models for the required production elaboration steps, Strong Components can be considered as an appropriate and a homomorphic model of the situation which they are representing, making them very suitable models.
- They are only sensitive to changes in the product operation sequences, implying that the layout design is dependable and remains unchanged during the life span of the products.
- The arc weights used come from the operation sequences and are different from judgmental or estimated values, which are uncertain in nature

Hence, unlike both QAP and Graph Theoretical approaches, which are based on arc weight estimates, the risk related to uncertain information is very much reduced for the Strong Component method proposed in this thesis.

The Strong Components approach can be a very flexible approach since it can handle many options:

- The proposed approach can take a wider set of values than either QAP or Graph Theoretical approaches because it creates a layout design based on the production operations sequences. The proposed approach considers the arc weights to be equally valued, but these weights can be modified to include other quantitative arc weights.

- The same technique can be used to solve various questions such as the best machine arrangement to produce a required group of products, the best arrangement when demand volume is considered, and the best configuration when considering material handling costs.
- It can incorporate new facility nodes to take into consideration other important locations: for instance, raw materials and finished products warehouses, and work in process storage locations.

Thus, Strong Component Layout Designs are more flexible because they can be applied in different situations, providing a wider choice of alternatives in finding the most suitable layout design.

6.3. Comparison with QAP

From the formulation point of view, when n facilities have to be assigned to n locations, the QAP formulation requires 2n constraints, n^2 decision variables and $n^4-2n^3+n^2$ objective function coefficients to be calculated. This implies a computational workload that increases rapidly with the number of facilities. For example, when n is three, 6 constraints, 9 decision variables and 36 coefficients are required. When n is five, 10 constraints, 25 decision variables and 400 coefficients are required. The number of coefficients to be estimated grows exponentially, making this formulation a complicated task each time that a new facility has to be added. On the other hand, the Strong Component Layout Design for n facilities to be assigned to n locations requires n^2 decision variables and n^2 0-1 coefficients, which are deployed in a sparse matrix. Consequently, adding facilities is easier in the proposed approach.

Similarly, for QAP, the memory space required will have to allow for 3n variables, 2n right hand side values and $n^4-2n^3+n^2$ objective function coefficient values. In the case of n = 5 this will require 425 memory locations. By contrast, in the proposed approach in this thesis the adjacency, an accumulation, a partial product, reachability and strong component matrices require $5n^2$ memory storage location; for 5 facilities this implies 125 locations, considerably reducing the required computing resources.

However, as with all heuristic methods, the Strong Component approach does not guarantee optimality. Although more work has to be performed to assess how far the solutions are from optimum, in operation terms the proposed approach is computationally more efficient than QAP methods.

6.4. Comparison to Graph Theory Methods

Comparison with other graph theoretic approaches should be based on how approaches are formulated and how solutions are assessed. One of the main advantages that graph theoretic approaches have is that they easily deal with integer formulations, which makes them more suitable procedures to address this sort of problems.

Planarity is one of the main features to be concerned about while executing a graph theoretic solution procedure. Although it has been considered a difficult feature to be tested during solution procedures, retaining triangular structures has helped to reduce this testing. However, from a pragmatic point of view, products are often manufactured in production batches, meaning that only one product is produced at any given time. Consequently, there may be no interference between production runs and the planarity feature may be disregarded. Nevertheless, when fixed material handling devices are to be considered, planarity may become a relevant issue. The network-based layout design achieved by the Strong Component approach will support this type of design.

Most of the graph theoretic approaches used to develop machine layouts disregard the duality property of graphs. Instead, they make use of other optimisation techniques, such as Tabu search and simulated annealing, to find near global optimal solutions or they are used in combination with other optimisation methods, making them more computationally complex and more demanding tools that may reduce the opportunity for these to become a tool in widespread use. However, Strong Component addresses similar situations without using complex mathematical tools or sophisticated computing resources, which may increase its application among practitioners.

6.5. Comparison with Literature Examples used in this Thesis

In this section, the Strong Components Layout Design Approach is compared to three layout design approaches available in the literature. The selected problems found in the literature are classified according to product-machine ratio, namely, the case of more machines than products, of more products than machines, and of a much larger number of machines than products. The product-machine ratio provides a way of measuring the extent of the existence of Strong Components relationships. For a high ratio, such relationships should be common while for a low ratio, Strong Components relationships become scarce. Hence, the three examples chosen helps to judge the robustness of the proposed methodology. For all these comparisons, summary tables are provided. The development of the complete proposed relationship diagram in each case is presented in Appendix 3 in more detail.

The following assumptions are made in order to make possible the comparison under similar conditions, seeking a fair comparison that allows us to assess each solution properly:

- For each product starting point, it is assumed that they can be fed directly from the raw materials warehouse.

- The last facility in the product operation sequence is allowed to send the finished product to the product warehouse.
- All the edges are considered bi-directional. This will allow products to flow in either direction.
- In-sequence and backtracking displacements are allowed, but the possibility of by-passing is disregarded.
- Any facility node in the Strong Component Graph is considered as an individual machine. This assumption can be relaxed and modified by considering each drawn workcentre as a group of machines of the same type, if more machines are required to fulfil capacity needs or if more machines are required to accomplish production deliveries dates.

The criteria used in the comparison are the number of displacements that each product requires, and also that weighted by the expected demand volume. In order to determine the number of displacements when more than one alternative path is available, the shortest identified path is taken. Hence, the best possible design should be the one with the smallest value.

The dedicated facilities solution is also given as a reference representing the case where the product operation sequences are all performed in-sequence. In Appendix 5, it can be found the specific sequences which take into consideration assumptions mentioned above and used in the comparison tables in these examples.

6.5.1. Example 1, More Machines than Products

This problem addresses the situation when there are more machines than products, in particular 13 machines to produce 7 products, giving a product machine ratio of 0.54. In this case, the solution proposed by this thesis is compared to a linear arrangement proposed in the paper by Aneke and Carrie (Aneke N.A.G., Carrie A.S., 1986), where they addressed a multi-product flowline problem. In their paper the focus is in the general machine case design for a multi-product flowline capable of producing a group of families of parts. The heuristic approach, which is a combination of the approaches proposed by Hollier (Hollier R.H., 1963) and by Carrie (Carrie A.S., 1975), uses travel charts and proposes a uni-directional arrangement considering a set of rules that involves operation sequences, number of parts, number of machines and workloads required by each family. In their arrangement, they allow three machines of type 4, two machines of types 3, 5, 6, 7 and 12, and one machine of the remaining ones.

Family	Number of Parts	Sequence of Machines Required
1	250	3 5 8 9 11 12 13
2	5	1 2 4 5 3 9 4 7 12 11 13
3	40	1 2 3 5 4 7 12 11 13
4	30	8 3 5 11 13
5	4	4 5 7 6 11 13
6	8	3 4 10 11 13
7	200	1 2 3 4 5 6 7 8 9 10 11 12 13
Total	537	

Table 6.1. More Machines than Products Example Data Table (Aneke N.A.G., Carrie A.S., 1986)

The data (Table 6.1) and the solution suggested in the paper are reproduced in this section. The suggested solution is the following machine sequence:

1-2-3-4-5-6-4-7-8-3-9-4-7-6-5-12-10-11-12-13

After applying the Strong Component approach to the data the proposed network-based layout design is shown in Figure 6.1.

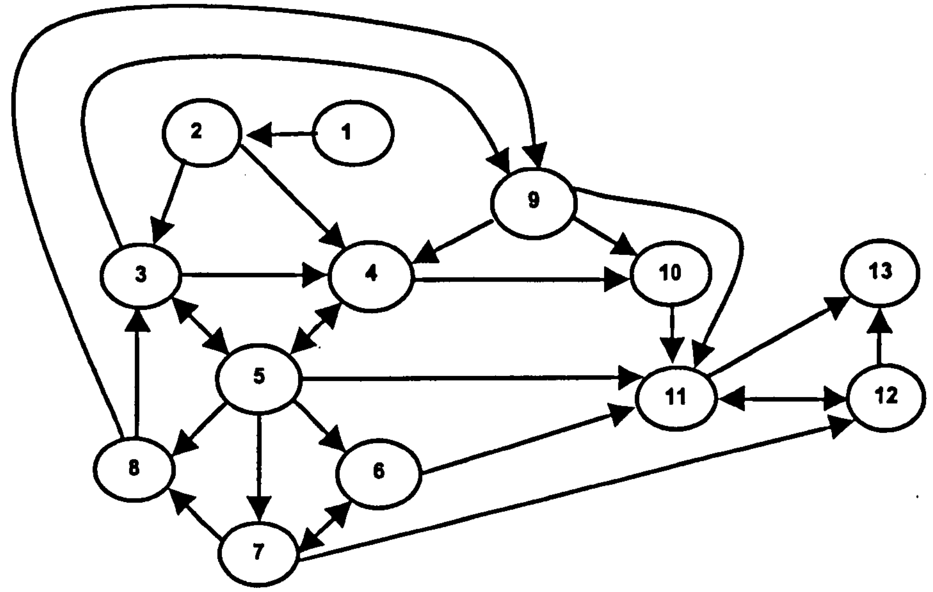


Figure 6.1. Example 1, Strong Component Layout Design

The detailed proposed solution of this problem can be found in Appendix 3. Table 6.2 shows the number of displacements and the volume required by each product for each layout design. The proposed approach reduces the number of displacements from 111 to 68. In terms of volume displaced, there is a reduction from 9379 to 6049 units, which represents a considerable reduction in the amount of flow to be handled, and consequently a cost reduction could be expected.

Product	Number of Parts	Dedicated Facilities Number of Displacements	Dedicated Facilities Total	Aneke and Carrie Number of Displacements	Aneke and Carrie Total	Strong Components Number of Displacements	Strong Components Total
1	250	6	1500	17	4250	9	2250
2	5	10	50	19	95	15	75
3	40	8	320	19	760	12	480
4	30	4	120	11	330	6	180
5	4	5	20	16	64	6	24
6	8	4	32	10	80	5	40
7	200	12	2400	19	3800	15	3000
Total	537	49	4442	111	9379	68	6049

Table 6.2. More Machines than Products Example Comparison Table (Example developed in Appendix 3)

6.5.2. Example 2, More Products than Machines

This example was taken from Vakharia and Wemmerlöv (Vakharia A.J., Wemmerlöv U., 1990). The problem discussed in this paper addressed simultaneously the cell formation and the material flows within cells. Consequently, they address the layout design defining groups of products first, and attempt to solve the layout design thorough their methodology, arguing that the material flow pattern should become a controllable factor in the cell formation process. A heuristic four-phase procedure was used to address the cellular manufacturing system layout by clustering operation sequences based on a similarity coefficient as previously discussed in section 4.2.3.4. This should be compared to the results obtained by applying the layout design approach in this thesis to the same set of parts. As in the previous example, the data (Table 6.3) and the solution in the Vakharia and Wemmerlöv paper (Vakharia A.J., Wemmerlöv U., 1990) are given in this section.

Part Number	Batch Per Day	Sequence of Machines Required
1	2	1 4 8 9
2	3	1 4 7 4 8 7
3	1	1 2 4 7 8 9
4	3	1 4 7 9
5	2	1 6 10 7 9
6	1	6 10 7 8 9
7	2	6 4 8 9
8	1	3 5 2 6 4 8 9
9	1	3 5 6 4 8 9
10	2	4 7 4 8
11	3	6
12	1	11 7 12
13	1	11 12
14	3	11 7 10
15	1	1 7 11 10 11 12
16	2	1 7 11 10 11 12
17	1	11 7 12
18	3	6 7 10
19	2	12
Total	35	

Table 6.3. More Products than Machines Example Data Table (Vakharia A.J., Wemmerlöv U., 1990)

After applying their approach, Vakharia and Wemmerlöv (Vakharia A.J., Wemmerlöv U., 1990) propose three cell arrangements that also have inter-cell and intra-cell flows.

Cell One: 1 3 5 2 6 10 4 7 8 9

Cell Two: 11 6 7 10 12

Cell Three: 1 11 4 10 7

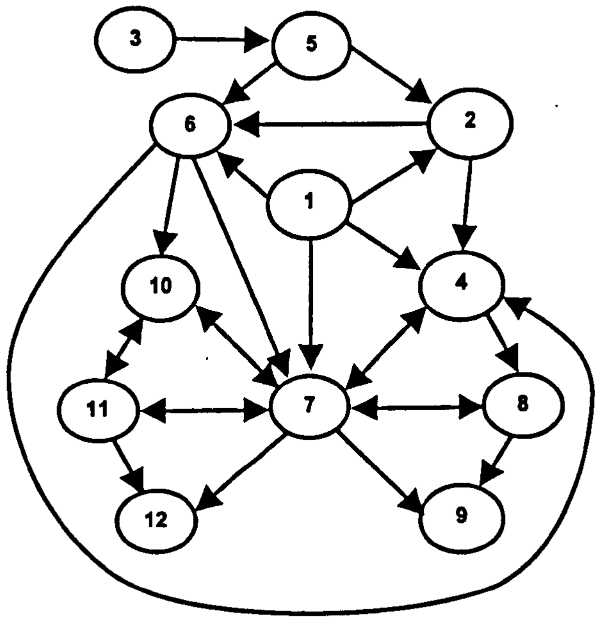


Figure 6.2. Example 2, Strong Component Layout Design

Figure 6.2 shows the Strong Component proposed layout design.

Part Number	Batch Per Day	Dedicated Facilities Number of Displacements	Total	Valkharia and Wemmerlöv Number of Displacements	Total	Strong Components Number of Displacements	Total
1	2	3	6	4	8	3	6
2	3	5	15	8	24	5	15
3	1	5	5	9	9	6	6
4	3	3	9	5	15	3	9
5	2	4	8	9	18	4	8
6	1	4	4	5	5	5	5
7	2	3	6	5	10	4	8
8	1	6	6	8	8	8	8
9	1	5	5	8	8	6	6
10	2	3	6	4	8	3	6
11	3	0	0	0	0	0	0
12	1	2	2	4	4	2	2
13	1	1	1	4	4	1	1
14	3	2	6	3	9	2	6
15	1	5	5	7	7	6	6
16	2	5	10	7	14	6	12
17	1	2	2	4	4	2	2
18	3	2	6	2	6	3	9
19	2	0	0	0	0	0	0
Total	35	60	102	96	161	69	115

Table 6.4. More Products than Machines Example Comparison Table (Example Developed in Appendix 3)

Table 6.4 shows that the proposed approach improves the solution of Vakharia and Wemmerlöv's paper (Vakharia A.J., Wemmerlöv U., 1990) by reducing the material handling by 29%, which corresponds to 48 volume-displacement units, from 161 to 115.

6.5.3. Example 3, A Large Number of Machines

This example, taken from the paper of Ho and Moodie (Ho Y.C., Moodie C.L., 1994), presents a situation where there are many more machines (27) to produce a small number of products (7) with a 0.26 product machine ratio. In their paper they identified an operation sequence pattern to undertake a flexible manufacturing system layout design and suggested a heuristic network-based design in a five stage sequence pattern procedure that uses a similarity measurement as explained in section 4.3.2. They used a modular approach to address the layout problem based on the operation sequence similarities. The data are shown in Table 6.5.

Product	Volume Quantity	Sequence of Machines Required
1	20	10 8 24 9 12 1 2 3 4 16 23 20 11
2	25	10 8 9 25 12 5 6 7 17 18 21 19
3	25	10 8 9 12 13 14 15 17 18 19
4	15	1 2 3 4 5 6 7 16 23 20 11
5	30	13 14 27 15 5 6 7 22 16 23 20 26 11
6	20	1 2 3 4 16 23 20 11
7	20	13 14 15 17 18 19 5 6 7
Total	155	

Table 6.5. A Large Number of Machines Example Data Table (Ho Y.C., Moodie C.L., 1994)

The machines were grouped by Ho and Moodie (Ho Y.C., Moodie C.L., 1994) in modules that are independent and connected through inter-modular relationships. In this case, if distances are considered it should be said that having separate modules is more likely to increase travel distance than using an internal network-based layout (as presented in this Thesis). However, separate modules may support greater control over and visibility of the production process. The solution presented by Ho and Moodie (Ho Y.C., Moodie C.L., 1994) is:

Module One:	10 8 9 24 12 25
Module Two:	1 2 3 4
Module Three:	22 16 23 20 26 11
Module Four:	5 6 7
Module Five:	17 18 19 21
Module Six:	13 14 15 27

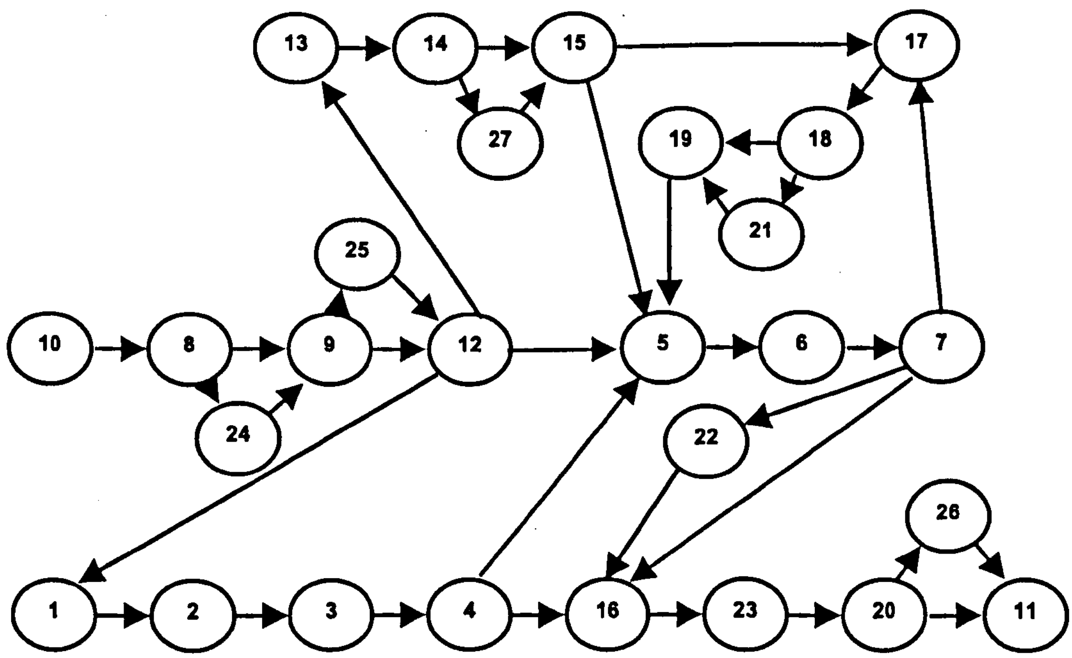


Figure 6.3. Example 3, Strong Component Layout Design

The Strong Component approach suggests a network-based layout design as shown in Figure 6.3

As seen in Table 6.6 in this case also the proposed approach in this Thesis provides better results than the solution proposed by Ho and Moodie (Ho Y.C., Moodie C.L., 1994), and it is almost equivalent to the dedicated facilities solution. Hence, the approach proposed in this thesis can also provide competitive solutions, even in a situation where there is a large number of machines compared to products and, hence, Strong Components relationships would be scarce.

Product	Quantity	Dedicated Facilities Number of		Ho and Moodie Number of		Strong Components Number of	
		Displacements	Total	Displacements	Total	Displacements	Total
1	20	12	240	15	300	12	240
2	25	11	275	14	350	11	275
3	25	9	225	10	250	9	225
4	15	10	150	11	165	11	165
5	30	12	360	13	390	12	360
6	20	7	140	7	140	7	140
7	20	8	160	8	160	8	160
Total	155	69	1550	78	1755	70	1565

Table 6.6. A Large Number of Machines Example Comparison Table (Example developed in Appendix 3)

6.6. Comparison to Dedicated Facilities

Dedicated facilities minimises material flow because they promote in-sequence and seamless flows, but provide more expensive solutions in terms of resources required (machines, space).

Example	Products	Machines	Ratio	Number of Machines			Reduction in Number of Machines SC compared to	
				Dedicated Facilities	Other Approach	Strong Components	Dedicated Facilities	Other Approach
Large Num. Mach.	7	27	0.26	76	27	27	49	0
More Machines	7	13	0.54	56	20	13	43	7
More Products	19	12	1.58	79	20	12	67	8

Table 6.7. Number of Machines Summary Table

As seen in Table 6.7, the required number of machines is much smaller for the proposed approach when compared to dedicated facilities, and, hence, the space requirement is also reduced and can be used to allow in-production storage or more room for maintenance work, which may increase safety on the shop floor.

Given that backtracking displacements between machines are allowed, which may imply the need for more sophisticated material handling devices, this relaxation has partially lead to a reduction in the number of displacements as shown in the previous examples. This relaxation arises because multiple machines of the same type are grouped together as a single workcentre.

In conclusion, the examples provide encouraging results which suggest the use of Strong Component-Based Layout Design as an appropriate methodology to be used in producing reliable relationship diagrams.

7. Conclusions

Facility Layout Design has been required since the emergence of Manufacturing and Service Organisations. Many approaches have been developed and have evolved from an intuitive approach to sophisticated formulation and solution methods. Most of these approaches have been based on the creation of a relationship diagram of products and facilities as the main step for developing efficient layout designs. This diagram is used as the basis for providing insights into a variety of layout designs regarding their usage of space and operational and capital costs.

This thesis has presented a directed graph heuristic construction approach to machine layout designs, with the objective of minimising material handling. To reduce material handling, mutually reachable (Strong Components) facilities are identified from the product operation sequences to provide a basis for efficient layout designs. The layout design obtained through the proposed approach suggests a non-dedicated machine configuration capable of producing a set of products with material handling that resembles as close as possible the material handling of dedicated facilities, without the capital investment that these facilities require.

7.1. Research Contributions

The work developed in this thesis has been oriented towards supporting organisational improvement efforts and contributing to them with a methodology for the design of layouts when a set of products is to be produced using non-dedicated facilities. The use of non-dedicated facilities is particularly useful in the case of small production batches since, in such cases, the use of dedicated facilities will have a significant impact on capital costs and utilisation of space. A layout based on dedicated facilities, however, provides a lower bound on material handling (measured in terms of number of displacements or in volume of traffic of materials).

Comparison with a number of approaches suggested in the literature demonstrates the efficiency of the layout produced by the methodology developed in this thesis. The Strong Component approach also showed itself to be a robust problem-solving tool when compared to the other heuristic construction approaches in the literature, against which it was compared. A reliable relationship diagram was obtained even when strong components are absent or when all of them are strong in the resultant configuration, i.e., when the strong component matrix is the identity matrix or all the entities in the matrix are equal to one. In cases such as these, the matrix of the number of displacements can be used as an alternative or secondary indicator of traffic intensive links. Additionally, it has been shown that the proposed approach can be successfully applied under different product-machine ratios and can produce layout designs even in the absence or entire presence of strong components.

Facilities Layout Modelling and Design are descriptive aids for designers for how things would look, and a valuable tool for the anticipation, reduction and elimination of problems. The suggested Strong Component approach has shown itself to be computationally more efficient and easier to implement than previous approaches. The number of variables required is smaller than the number needed for the QAP, implying a reduced requirement for computational resources.

The network-based configuration suggested by the Strong Component approach can be obtained almost directly from the Strong Component Matrix, reducing on the time required by other Graph Theoretic approaches, which need a complicated planarity test before obtaining the layout design. In addition, the layout designs from the proposed approach are also an improvement over other Graph Theoretic approaches, since this approach does not require graph duals to develop a layout design.

The Strong Component method presented in the thesis is based on product operation sequences which are deterministic values, and which reduce the use of estimated data and provide a more logical layout design according to the set of products to be manufactured. A relevant issue regarding the Strong Component approach is that, because it uses an Adjacency Matrix, the corresponding objective function addresses a particular situation where the value of its coefficients are equal to one, i.e., are equally valued coefficients, a situation that has not been addressed before by any other approach. This situation can be thought of as addressing the case where the transfer batch size is one, but the method can also handle any other non-negative quantitative values for the transfer batch size.

The proposed Strong Component approach has proven to be a versatile tool because the addition of work-in-process storage and input and output ports can be addressed easily. Further, the number of displacements matrices can be substituted as a ranking criteria by other matrices, such as volumes, travel distances, costs and combinations of these, providing an enhanced scope of alternative designs from which the most appropriate design can be chosen or can be tailored to a specific organisational purpose. Finally, the availability of a tool such as the proposed approach has allowed us to analyse situations which are often too complicated to address and solve using previous methodologies, for instance the inclusion of input-output ports location and the consideration of non-adjacent flows.

In conclusion:

Strong Components-Based Layout Design approach is a valuable contribution to the design of facilities layout. It is an attractive approach to apply, because it requires modest computer resources and it can also be easily implemented.

The machine arrangement obtained from the Strong Component Based Methodology approach could be considered as the minimum shared facilities structure required to produce a set of products with "minimum" or efficient material handling flows. This configuration may be used as a threshold for comparisons against alternative designs, and to start planning appropriate layout designs under dynamic circumstances, for example using discrete event simulation techniques.

7.2. Further Work

The following are suggestions for further investigation, development and enhancement of the Strong Component approach proposed by this Thesis.

- It is important to identify if there is a possible product-machine ratio threshold value above which the use of the Strong Component approach is most effective.

- Research should be undertaken to investigate the possibility of enhancement of the proposed approach by linking it to other algorithmic approaches, for instance genetic algorithms.

- The fact of overruling planarity when planning the layout design was briefly outlined, and more exploration of this issue is required.

- The distance property of directed graph theoretic structures is an unexplored issue that should be considered in future work. This property may be useful to identify alternative paths between facilities and to explore them, simultaneously, with the possibility of a relaxation of the planarity property, as suggested previously.

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List of Abbreviations

A (D)	Adjacency Matrix (5)
AGVs	Automated Guided Vehicles (4)
AI	Artificial Intelligence (4)
CAD	Computer Aided Design (2)
CMS	Cellular Manufacturing Systems (4)
CNC	Computer Numerically Controlled Machine (4)
CTC	Centroid to Centroid (3)
D	Directed Graph (5)
DP	Dynamic Programming Problem (3)
EDIST	Expected Distance Measure (3)
ES	Expert Systems (4)
FLD	Facility Layout Design (3)
FMC	Flexible Manufacturing Cell (4)
FMS	Flexible Manufacturing Systems (4)
GT	Group Technology (2, 4)
HAS	Hybrid Simulated Annealing (4)
LAP	Linear Assignment Problem (3)
MCFMS	Multi-Cell Flexible Manufacturing System (4)
MIP	Mixed Integer Programming Problem (3)
MLP	Machine Layout Problem
MLT	Manufacturing Lead Time (4)

MMFMS	Multi-Machine Flexible Manufacturing System (4)
MOP	Multiple Objective Problem (3)
MPWG	Maximally Planar Weighted Graph (3, 4)
MWPG	Maximally Weighted Planar Graph (4)
MST	Modified Spanning Tree Method (3)
NC	Numerically Controlled Machine (4)
NP	Non polynomial (3)
P	Polynomial (3)
POS	Product Operation Sequence(s) (4)
QAP	Quadratic Assignment Problem (3, 4)
QSP	Quadratic Set Covering Problem (3)
R (D)	Reachability Matrix (5)
R' (D)	Transpose Reachability Matrix (5)
SA	Simulated Annealing (4)
SFM	Single Flexible Machine (4)
SLP	Systematic Layout Planning (2)
TAA	Triangle Assignment Algorithm (4)
TS	Tabu Search (4)
TSP	Travelling Salesman Problem (3)
WIP	Work in Process (5)

Appendix 1.

Apple's Plant Layout Procedure

(Apple J.M., 1977:14)

The following sequence of steps was recommended by Apple to be followed when designing a facility layout.

1. **Procure basic data**
2. **Analyse basic data**
3. **Design productive process**
4. **Plan material flow pattern**
5. **Consider general material handling plan**
6. **Calculate equipment requirements**
7. **Plan individual work stations**
8. **Select specific material handling equipment**
9. **Coordinate groups of related operations**
10. **Design activity relationships**
11. **Determine storage requirements**
12. **Plan service and auxiliary activities**
13. **Determine space requirements**
14. **Allocate activities to total space**
15. **Consider building types**
16. **Construct master layout**
17. **Evaluate, adjust, and check the layout with the appropriate persons**

18. **Obtain approvals**

19. **Install layout**

20. **Follow up on implementation of the layout**

Appendix 2.

Reed's Plant Layout Procedure

(Reed R., 1991:9)

The following sequence of steps was suggested by Reed to be followed when designing a facility layout.

1. **Analyse the products manufactured**
2. **Determine the manufacturing process**
3. **Prepare the layout planning charts (Considered by Reed the most important single phase of the entire layout process). This incorporates the following:**
    - **Flow process, including operations, transportation, storage, and inspection**
    - **Standard times for each operation.**
    - **Machine selection and balance**
    - **Manpower selection and balance**
    - **Material handling requirements.**
4. **Determine workstations**
5. **Establish minimum aisle widths**
6. **Establish non-manufacturing (office) space requirements**
7. **Consider personnel facilities and services**
8. **Survey plant services**
9. **Provide for future expansions**

Appendix 3.

To illustrate some details of the applicability of the Strong Component Layout Design Approach some examples are presented here. These examples were chosen from the literature and they were considered as appropriate to show different situations and how they could be addressed and be solved using the Strong Component Approach, namely: the case of more machines than products, more products than machines, and the case of a large number of machines. These examples were chosen to analyse the proposed approach under different product-machine ratios that affect the existence of strong components, indicating the extent of their presence or absence in the configuration. During the outline of these examples, some distinctive features are highlighted.

A3.1. Example 1. More Machines than Products

The first example, taken from the paper by Aneke and Carrie (Aneke N.A.G., Carrie A.S., 1986), refers to a single-row layout design. The approach used in this paper addresses the general machine case, which seeks to achieve the minimal cost eliminating backtracking movements. The example provided in the paper involves seven families of products, and thirteen machines are the minimum required to produce all the products. This gives a product-machine ratio of 0.54. The data provided with the example are shown in Table A3.1, and they are the product operation sequences and the number of parts to be manufactured of each family. In addition, the number of machines and the number of displacements required to complete each product has been added and they are also provided in the same table.

Family	Number of			Sequence of Machines Required
	Parts	Operations	Movements	
1	250	7	6	3 5 8 9 11 12 13
2	5	11	10	1 2 4 5 3 9 4 7 12 11 13
3	40	9	8	1 2 3 5 4 7 12 11 13
4	30	5	4	8 3 5 11 13
5	4	6	5	4 5 7 6 11 13
6	8	5	4	3 4 10 11 13
7	200	13	12	1 2 3 4 5 6 7 8 9 10 11 12 13
Total	537	56	49	

Table A3.1. Example 1, More Machines than Products
(Aneke N.A.G., Carrie A.S., 1986)

The Adjacency (Table A3.2; step 1 in Figure 5.3), the Frequency or Number of Displacements (Table A3.3), the Reachability (Table A3.4; step 2 in Figure 5.3), and the Strong Component (Table A3.5; step 3 in Figure 5.3)) Matrices follow. All these matrices have 13 by 13 entries since there are only 13 different machines of each type.

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
1		1												1
2			1	1										2
3				1	1				1					3
4					1		1			1				3
5			1	1		1	1	1			1			6
6							1				1			2
7						1		1				1		3
8			1						1					2
9				1						1	1			3
10											1			1
11												1	1	2
12											1		1	2
13														0
Total	0	1	3	4	2	2	3	2	2	2	5	2	2	30

Table A3.2. Example 1, Adjacency Matrix

In the Adjacency Matrix (Step 1 in Figure 5.3), it can be seen that 30 links, arcs, interrelations, or different displacements are required to elaborate all the families. There are three more edges required to complete the planarity condition, although some of the edges required by the planarity may be disregarded during the development of a layout design.

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
1		3												3
2			2	1										3
3				2	3				1					6
4					3		2			1				6
5			1	1		1	1	1			1			6
6							1				1			2
7						1		1				2		4
8			1						2					3
9				1						1	1			3
10											2			2
11												2	5	7
12											2		2	4
13														0
Total	0	3	4	5	6	2	4	2	3	2	7	4	7	49

Table A3.3. Example 1, Number of Displacements Matrix

When comparing the required Number of Displacements (Table A3.3) with the Adjacency (Table A 3.2) Matrices, it can be seen that the number of additional displacements required in dedicated facilities is 19 more than those required by non-dedicated facilities. This is the difference between the total sums of the two matrices, since dedicated facilities include repeated arcs or displacements. In addition, it can be seen in either table that node one appears as a source node and node thirteen as a sink node. Nodes three, four, and eight are nodes that should also be considered as source nodes because some production sequences start from them. This situation can be overcome by introducing unique source and sink nodes, as explained in Chapter Five, section 5.7.2. To complete the table comparison, Table A3.4 provides the Volume From-to Matrix, and observations made about the previous matrices are also valid for this matrix: consequently, these matrices are isomorphic models of each other.

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
1		245												245
2			240	5										245
3				208	320				5					533
4					209		45			8				262
5			5	40		200	4	250			30			529
6							200				4			204
7						4		200				45		249
8			30						450					480
9				5						200	250			455
10											208			208
11												450	87	537
12											45		450	495
13														0
Total	0	245	275	258	529	204	249	450	455	208	537	495	537	4442

Table A3.4. Example 1, Volume From-To Matrix

Once the data has been prepared by obtaining the Adjacency Matrix (Table A3.2), then the next step is to calculate the Reachability Matrix (Table A3.5), followed by the identification of the strong components based on this matrix and its transpose (Step 3 in Figure 5.3).

From the Reachability Matrix (Table A3.5), it can be seen that all the facilities can be reached from facility one. The second facility can reach all the facilities except that it cannot reach facility one. From facility three until facility nine, all the following facilities can be reached except that they cannot reach one and two. Although strong components may be located by inspection of the Adjacency Matrix, the proposed approach in this thesis reduces this amount of work through the Strong Component matrix.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0	1	1	1	1	1	1	1	1	1	1	1
4	0	0	1	1	1	1	1	1	1	1	1	1	1
5	0	0	1	1	1	1	1	1	1	1	1	1	1
6	0	0	1	1	1	1	1	1	1	1	1	1	1
7	0	0	1	1	1	1	1	1	1	1	1	1	1
8	0	0	1	1	1	1	1	1	1	1	1	1	1
9	0	0	1	1	1	1	1	1	1	1	1	1	1
10	0	0	0	0	0	0	0	0	0	1	1	1	1
11	0	0	0	0	0	0	0	0	0	0	1	1	1
12	0	0	0	0	0	0	0	0	0	0	1	1	1
13	0	0	0	0	0	0	0	0	0	0	0	0	1

Table A3.5. Example 1, Reachability Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	1	1	1	1	1	1	0	0	0	0
4	0	0	1	1	1	1	1	1	1	0	0	0	0
5	0	0	1	1	1	1	1	1	1	0	0	0	0
6	0	0	1	1	1	1	1	1	1	0	0	0	0
7	0	0	1	1	1	1	1	1	1	0	0	0	0
8	0	0	1	1	1	1	1	1	1	0	0	0	0
9	0	0	1	1	1	1	1	1	1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	0	0	0	1	1	0
12	0	0	0	0	0	0	0	0	0	0	1	1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	1

Table A3.6. Example 1, Strong Component Matrix

In the Strong Component Matrix (Table A3.6; step 4.a in Figure 5.3), three groups can be clearly distinguished. One group is a large strong component group in the middle, between rows and columns three to nine; another one, a smaller component group, is between eleven and twelve; and the rest are non-strong component facilities.

Now we proceed to construct the strong component sub-graphs (Steps 4.a.i, 4.a.ii and 4.a.iii in Figure 5.3). Figure A3.1 shows the sub-graphs obtained from step 4.a. At this stage, a list of the arcs that go from these sub-graphs to other possible sub-graphs should be created (Step 4.a.iv in Figure 5.3), considering the ranking suggested by the Number of Displacements Matrix (Table A3.3; step 4.a.ii in Figure 5.3).

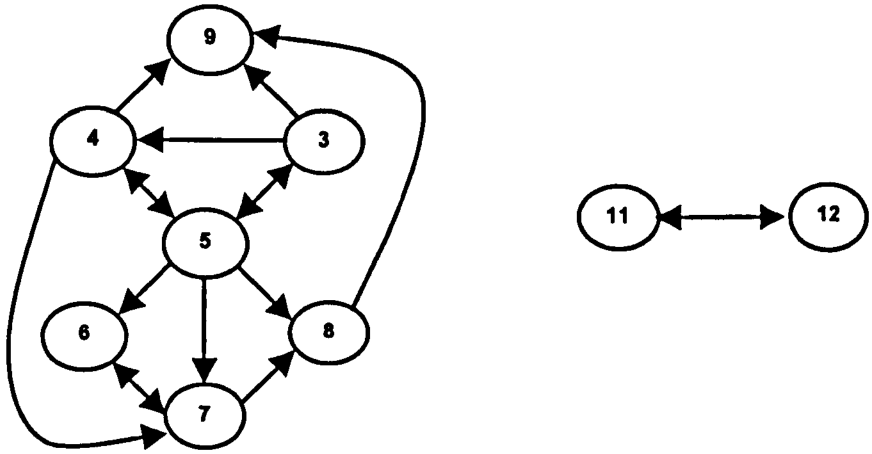

Figure A3.1. Example 1, Strong Component Sub-Graphs

Once the Strong Component Sub-Graphs (nodes and links) have been constructed, then following the procedure, the Non-Strong Component Sub-Graphs have to be developed (Step 4.b in Figure 5.3). In this example, this corresponds to facilities one, two, ten and twelve and their related interconnections, as shown in Figure A3.2 (Steps 4.b.i, 4.b.ii and 4.b.iii in Figure 5.3). The Number of Displacements Matrix has to be reviewed, seeking unused arcs: if there are some unused arcs, update the list and add them to the list of unused arcs (Steps 4.b.iv and 4.b.v in Figure 5.3).

Figure A3.2. Example 1, Non-Strong Component Sub-Graphs

If there are no more facilities and arcs to be considered (Step 4.c in Figure 5.3), which is the case in this example, then integrate the strong component and non-strong component sub-graphs (Step 4.d in Figure 5.3) using the last updated list of unused arcs (Steps 4.a.iv, 4.b.v and 4.c.v).

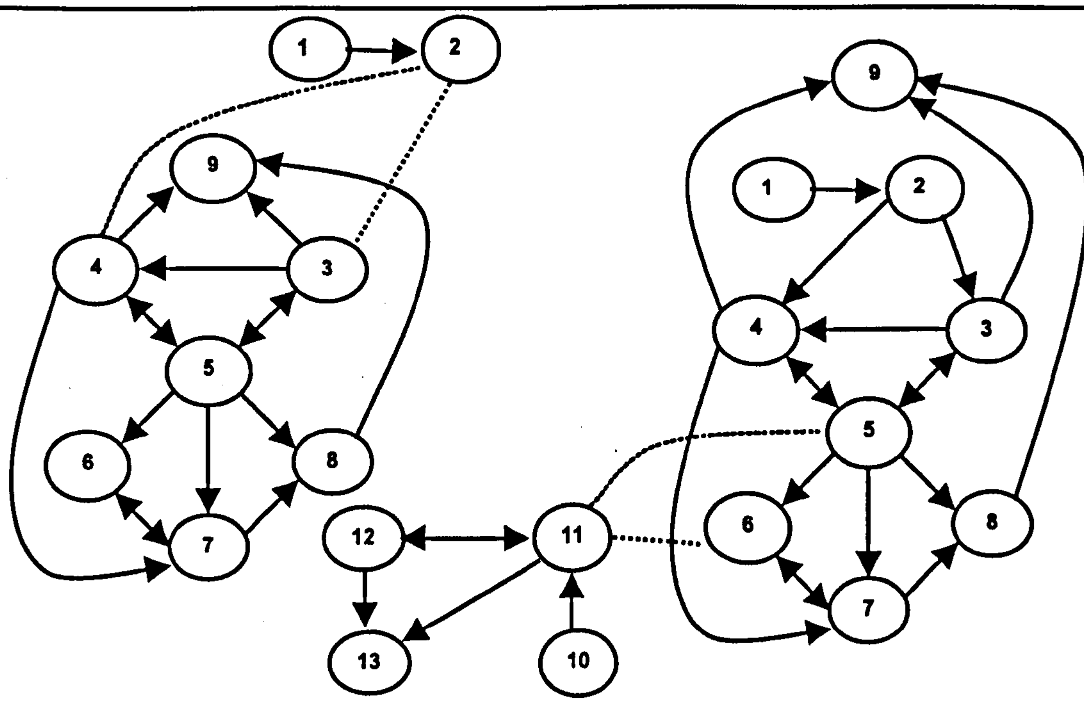

Figure A3.3. Example 1, Sub-Graphs Interconnection Sample

The sub-graphs obtained from the previous steps (Steps 4.a, 4.b and 4.c in Figure 5.3) are linked using the list of unused arcs (Step 4.d in Figure 5.3), which are the in-between links (Steps 4.a.iv, 4.b.iv and 4.c.iv in Figure 5.3). It should be noted that the sub-graphs obtained represent manufacturing sub-systems or modules, which might be interrelated, but some trial and error should be expected in order to obtain the best possible facility locations.

To illustrate the point, and using Figure A3.3, if the sub-graph formed by facilities one and two is to be linked to the Strong Component Sub-Graph, the arc from eight to nine does not allow this to happen directly. Consequently, facilities one and two should be placed inside the triangle created by facilities three, four and nine, as shown in the same figure, eliminating this constraint. In a similar manner, facility eleven should be connected to facilities five and six. The existing connector between four and seven may be an obstacle to the connections between eleven, five and six, and it should be noted that it is not a double-headed arc; a trade-off between these interrelationships may be explored. It should be noted that omitting the arc between four and seven might allow two additional arcs: one between five and eleven and another between six and eleven. This arc swapping should be explored, since having two arcs instead of one may support a better product flow. Once the interconnections among the sub-graphs have been made, the Strong Component Graph looks as in Figure A3.4. This proposal increases only by two displacements the material handling required. Instead of moving directly from four to seven, it should be from four to five and five to seven. In the Strong Component Graph, all the product operation sequences are followed to validate (Step 4.e in Figure 5.3) and to verify (Step 4.f in Figure 5.3) that all the products can be manufactured.

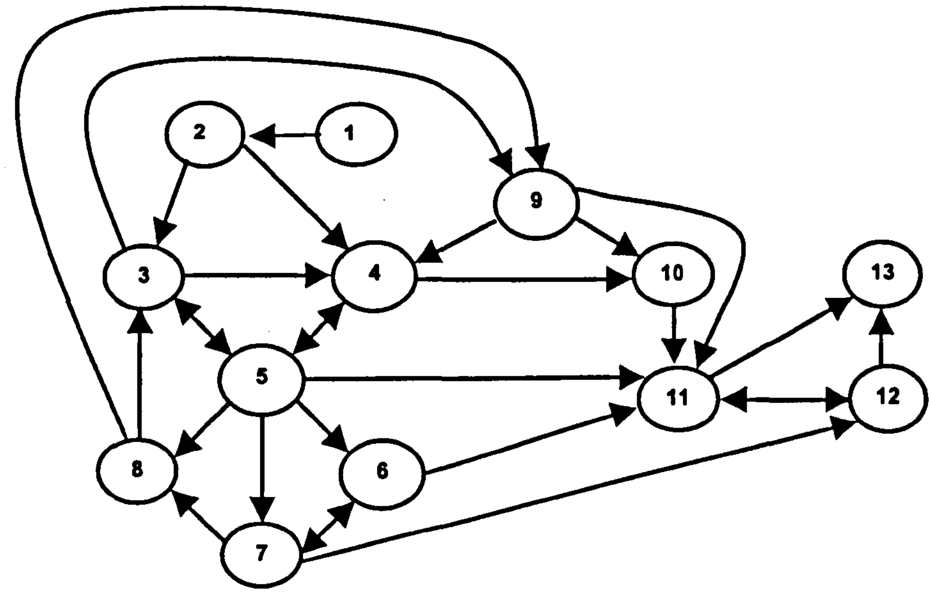

Figure A3.4. Example 1, Strong Component Graph

A3.2. Example 2. More Products than Machines

The example developed in this section was also taken from the literature (Valkharia A.J., Wemmerlöv U., 1990). The solution approach in that paper used a clustering method based on a similarity matrix to identify pairs of part groups, using operation sequences, average demand, estimated processing times, equipment acquisition costs, productivity time per unit, and equipment available. The solution proposes a single row cell partition layout design, and simultaneously reduces the intercell relationships and backtracking. Since the solution allows various machines of the same type, it can be considered as the general case solution approach. Moreover, this requires 12 machines to produce 19 products and a 1.58 product-machine ratio. The data provided is presented in Table A3.7. It can be seen that, when there are parts that require one operation, the proposed approach fails to recognise them. This is because a movement between facilities cannot be recorded or be recognised by an adjacency, a from-to, a volume from-to or a number of displacements matrix; this situation can be overcome by the use of initial and final dummy facilities.

Part Number	Number of: Batch Per Day	Number of: Operations	Number of: Displacements	Sequence of Machines Required
1	2	4	3	1 4 8 9
2	3	6	5	1 4 7 4 8 7
3	1	6	5	1 2 4 7 8 9
4	3	4	3	1 4 7 9
5	2	5	4	1 6 10 7 9
6	1	5	4	6 10 7 8 9
7	2	4	3	6 4 8 9
8	1	7	6	3 5 2 6 4 8 9
9	1	6	5	3 5 6 4 8 9
10	2	4	3	4 7 4 8
11	3	1		6
12	1	3	2	11 7 12
13	1	2	1	11 12
14	3	3	2	11 7 10
15	1	6	5	1 7 11 10 11 12
16	2	6	5	1 7 11 10 11 12
17	1	3	2	11 7 12
18	3	3	2	6 7 10
19	2	1		12
Total	14	79	60	

Table A3.7. Example 2, More Products than Machines (Valkharia A.J., Wemerlöv U., 1990)

As shown in the Adjacency Matrix (Table A3.8; step 1 in Figure 5.3) the number of interrelationships required to manufacture the products under consideration is 27. Moreover, in this example there are 60 displacements required to complete one instance of each product (Table A3.9): these are 33 more displacements than when producing in dedicated facilities. In addition, there are 102 displacements required, considering the production batches required to be produced in a day and disregarding production batches that have just one operation. In the following tables, Table A3.10 and Table A3.11, the Reachability Matrix (Step 2 in Figure 5.3) and the Strong Component Matrix (Step 3 in Figure 5.3) are provided.

	1	2	3	4	5	6	7	8	9	10	11	12	Total
1		1		1		1	1						4
2				1		1							2
3					1								1
4							1	1					2
5		1				1							2
6				1			1			1			3
7				1				1	1	1	1	1	6
8							1		1				2
9													0
10							1				1		2
11							1			1		1	3
12													0
Total	0	2	0	4	1	3	6	2	2	3	2	2	27

Table A3.8. Example 2, Adjacency Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	Total
1		1		3		1	2						7
2				1		1							2
3					2								2
4							4	6					10
5		1				1							2
6				3			1			2			6
7				2				2	2	2	2	2	12
8							1		6				7
9													0
10							2				2		4
11							3			2		3	8
12													0
Total	0	2	0	9	2	3	13	8	8	6	4	5	60

Table A3.9. Example 2, Number of Displacements Matrix

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1	0	1	0	1	1	1	1	1	1	1
2	0	1	0	1	0	1	1	1	1	1	1	1
3	0	1	1	1	1	1	1	1	1	1	1	1
4	0	0	0	1	0	0	1	1	1	1	1	1
5	0	1	0	1	1	1	1	1	1	1	1	1
6	0	0	0	1	0	1	1	1	1	1	1	1
7	0	0	0	1	0	0	1	1	1	1	1	1
8	0	0	0	1	0	0	1	1	1	1	1	1
9	0	0	0	0	0	0	1	1	1	0	0	0
10	0	0	0	1	0	0	1	1	1	1	1	1
11	0	0	0	1	0	0	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	1

Table A3.10. Example 2, Reachability Matrix

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0	0	0	0
4	0	0	0	1	0	0	1	1	0	1	1	0
5	0	0	0	0	1	0	0	0	0	0	0	0
6	0	0	0	0	0	1	0	0	0	0	0	0
7	0	0	0	1	0	0	1	1	0	1	1	0
8	0	0	0	1	0	0	1	1	0	1	1	0
9	0	0	0	0	0	0	0	0	1	0	0	0
10	0	0	0	1	0	0	1	1	0	1	1	0
11	0	0	0	1	0	0	1	1	0	1	1	0
12	0	0	0	0	0	0	0	0	0	0	0	1

Table A3.11. Example 2, Strong Component Matrix

From Table A3.11, it can be seen that facilities four, seven, eight, ten and eleven are the strong component facilities (Step 4.a in Figure 5.3), and the non-strong components are facilities one, two, three, five, six, nine and twelve (Step 4.b in Figure 5.3). It should be noted that for the product operation sequences for products eleven and nineteen, their facilities or arcs required were not taken into consideration, these should be analysed separately to avoid leaving them out (Step 4.c in Figure 5.3); this situation can be avoided by considering input and output ports (as discussed in section 5.7.2). If required, update the list of unused arcs, adding as required, and proceed with the following step, step 4.d (in Figure 5.3).

After identifying strong (Step 4.a in Figure 5.3), non-strong (Step 4.b in Figure 5.3) and other components (Step 4.c in Figure 5.3), the obtained sub-graphs integration follows in the proposed approach (i.e., Step 4.d in Figure 5.3). A first attempt of integration is shown in Figure A3.5. In this figure, two groups of machines can clearly be identified. Facilities nine and twelve can be incorporated in the strong sub-graph creating one group, and the other group considers facilities one, two, three, five, and six. This suggests that the procedure can be used to address the three steps required by Group Technology to create families, and to develop the inter-cell and intra-cell layout design. However, more experiments should be carried out in order to verify this finding.

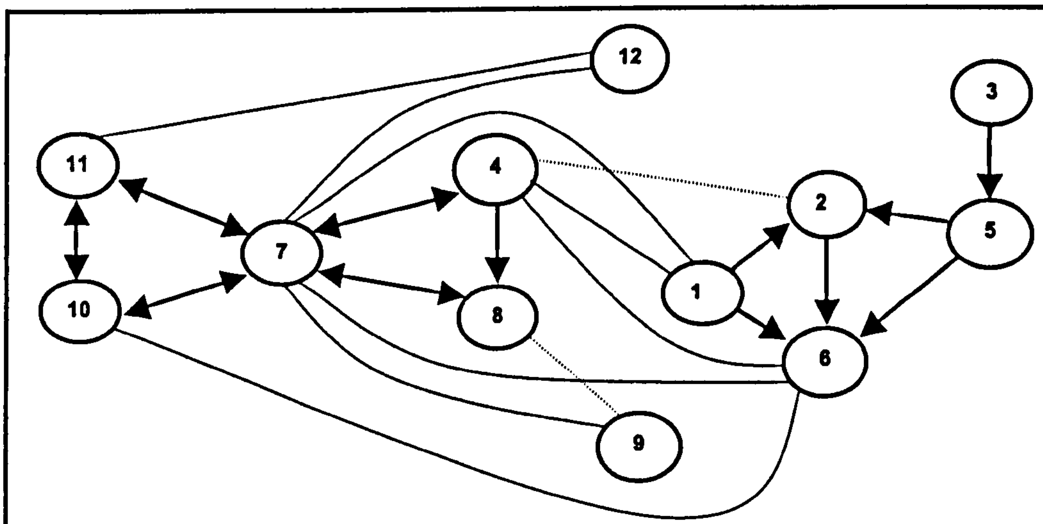

Figure A3.5. Example 2, Strong and Non-Strong Component Sub-Graphs and their Interrelationships

As shown in Figure A3.6 (Step 4.d in Figure 5.3), the nineteen operation sequences can be accomplished without any additional arcs (i.e., Steps 4.e and 4.f in Figure 5.3) using this production structure, the Strong Component Graph; this implies that the number of movements required to produce any product is the same as in the dedicated facilities case, if any type of movements are allowed or as explained in the second example in section 6.5.1.

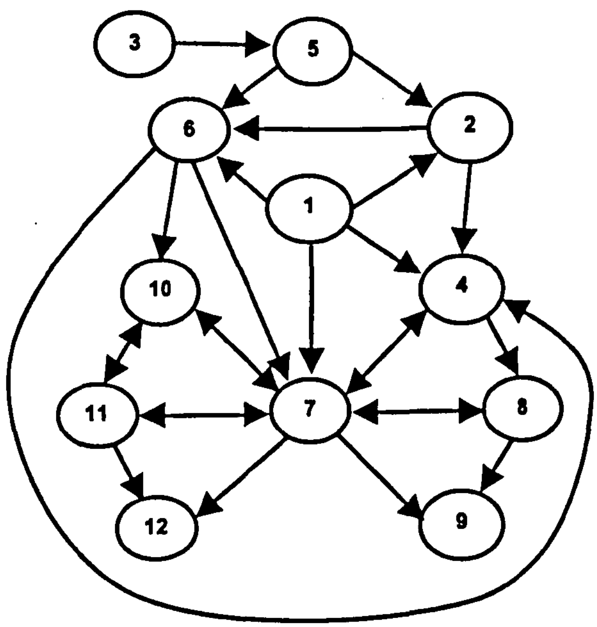

Figure A3.6. Example 2, Strong Component Graph

A3.3. Example 3. A Large Number of Machines

Another example taken from the literature (Ho Y.C., Moodie C.L., 1994) is presented in this section, and Table A3.12 reproduces the data. This example requires 27 machines to produce 7 products, giving a 0.26 product-machine ratio. A proposed heuristic approach in that paper attempts to find the maximum in-sequence arrangement of machines possible. This approach seeks to reduce the flow distance, which is expected simultaneously to reduce the material handling and to contribute to a more efficient production. Furthermore, the approach suggests network-based and single-row layout designs. The network-based design attempts to exploit the existing similarity with the product operation sequences using a similarity coefficient and by creating interconnected modules (sub-cells) with them. Consequently, the three phases required by Group Technology are addressed simultaneously by Ho and Moddie's approach (Ho Y.C., Moodie C.L., 1994).

Product	Number of			Sequence of Machines Required
	Quantity	Operations	Displacements	
1	20	13	12	10 8 24 9 12 1 2 3 4 16 23 20 11
2	25	12	11	10 8 9 25 12 5 6 7 17 18 21 19
3	25	10	9	10 8 9 12 13 14 15 17 18 19
4	15	11	10	1 2 3 4 5 6 7 16 23 20 11
5	30	13	12	13 14 27 15 5 6 7 22 16 23 20 26 11
6	20	8	7	1 2 3 4 16 23 20 11
7	20	9	8	13 14 15 17 18 19 5 6 7
Total	155	76	69	

Table A3.12. Example 3, A Large Number of Machines (Ho Y.C., Moodie C.L., 1994)

The initial interest of the problem is in its size and in exploring how the proposed approach in this thesis behaves when addressing the larger number of machines, where the existence of strong components is likely to be scarce.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	Total
1		3																										3
2			3																									3
3				3																								3
4					1											2												3
5						4																						4
6							4																					4
7																1	1					1						3
8									2															1				3
9												2													1			3
10								3																				3
11																												0
12	1				1								1															3
13														3														3
14															2												1	3
15					1												2											3
16																							4					4
17																		3										3
18																			2		1							3
19					1																							1
20											3															1		4
21																			1									1
22																1												1
23																				4								4
24									1																			1
25												1																1
26											1																	1
27															1													1
	1	3	3	3	4	4	4	3	3	0	4	3	1	3	3	4	3	3	3	4	1	1	4	1	1	1	1	69

Table A3.13. Example 3, Number of Displacements Matrix

The Number of Displacements Matrix (Table A3.13) is taken into consideration, as a secondary criterion to distinguish the closeness between facilities (Steps from 4.a to 4.d in Figure 5.3). Alternatively, other criteria could be used to generate the facility closeness groups. The Strong Component Matrix (Table A3.14; step 3 in Figure 5.3) and the Number of Displacements Matrix (Table A3.13) are provided to illustrate this issue.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0
6	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0
7	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0
18	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0
19	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
21	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table A3.14. Example 3, Strong Component Matrix

In the Strong Component Matrix provided in Table A3.14 (Step 3 in Figure 5.3), it can be seen that two groups can be distinguished: one Strong Component group and another Non-Strong Component group. The Strong Component is configured by facilities five, six, seven, seventeen, eighteen, nineteen and twenty one (Step 4.a in Figure 5.3). The rest of the facilities belong to the Non-Strong facilities group (Step 4.b in Figure 5.3).

Taking into consideration the Strong Component facilities and their respective ordered arcs (5-6, 6-7, 17-18, 18-19, 7-17, 18-21, 1-5, 21-19; Step 4.a.i and Step 4.a.ii in Figure 5.3), the Strong Component Sub-graph is obtained and shown in Figure A3.7 (Step 4.a.iii in Figure 5.3). A list of all unused arcs that go to facilities that are not considered as strong components has to be created (Step 4.a.iv in Figure 5.3).

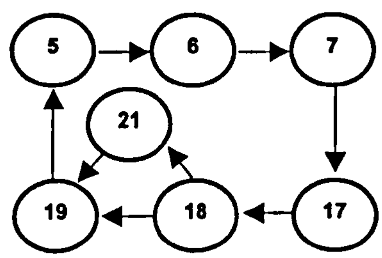

Figure A3.7. Example 3, Strong Component Sub-Graph

The following step takes into consideration the non-strong components (Step 4.b in Figure 5.3). All facilities available after the previous step, and their respective arcs, are used to construct the Non-Strong Component Sub-Graphs (Steps 4.b.i, 4.b.ii and 4.b.iii in Figure 5.3). These sub-graphs are shown in Figure A3.8. Update the list of unused arcs using the in-between links (Step 4.b.iv and 4.b.v in Figure 5.3).

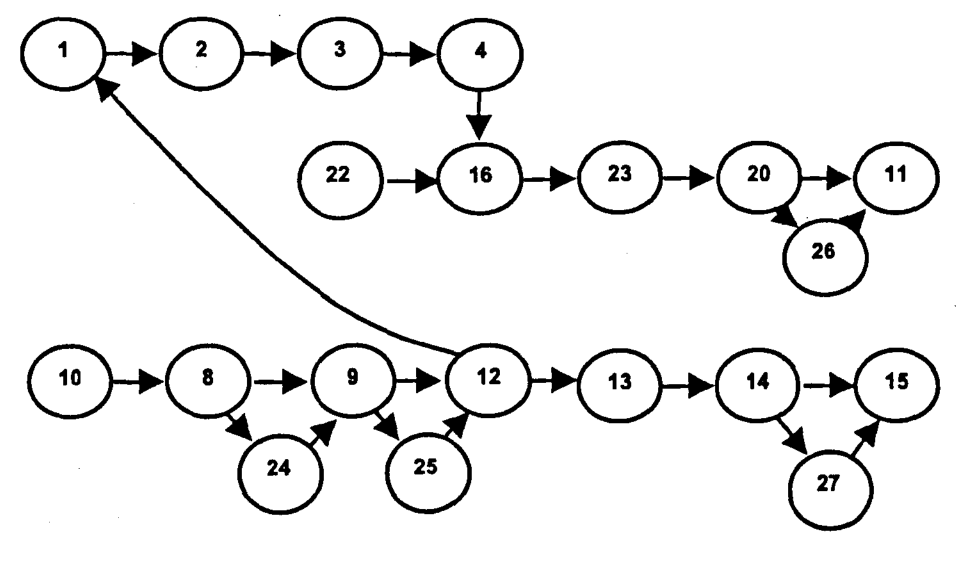

Figure A3.8. Example 3, Non-Strong Component Sub-Graphs

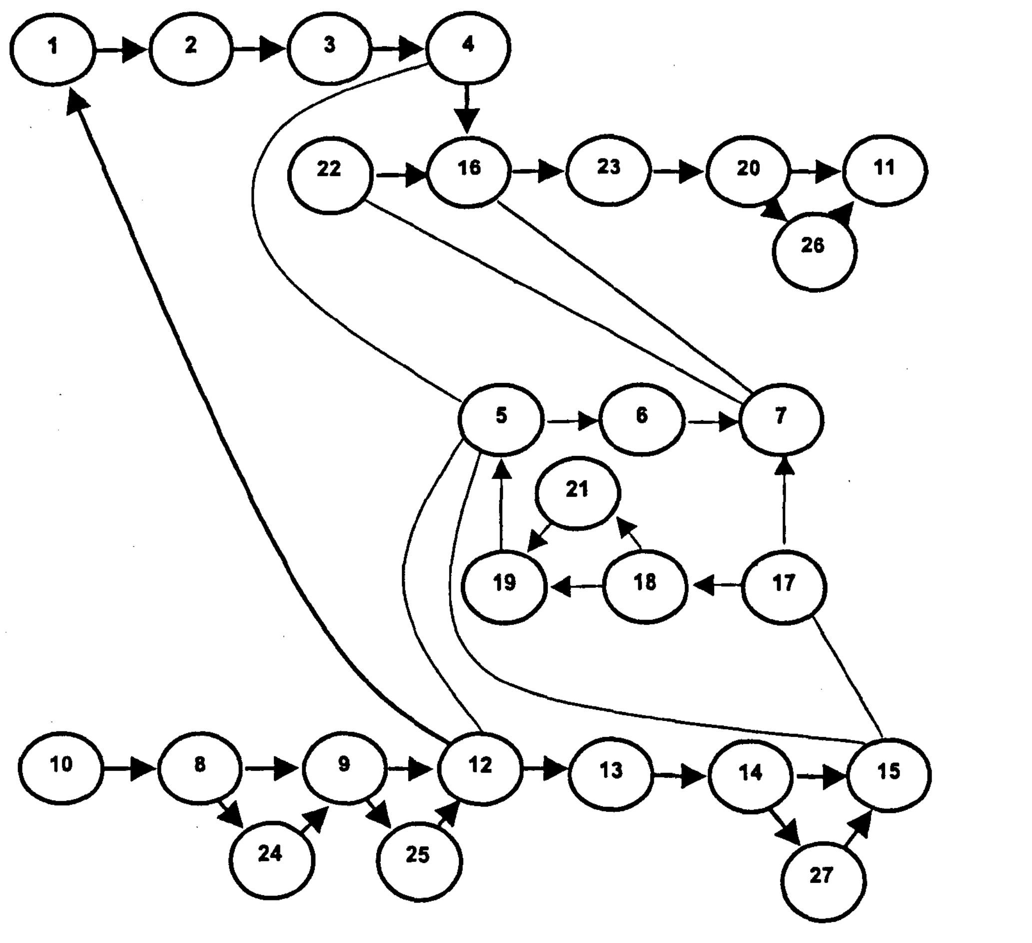

Figure A3.9. Example 3, Strong and Non-Strong Component Sub-Graphs Integration

- Once the Strong Component and the Non-Strong Component Sub-Graphs have been obtained, the following step is to verify if there are other facilities and arcs to be considered and create their sub-graphs (Step 4.c in Figure 5.3). Otherwise, continue with step 4.d (in Figure 5.3) and relate all the available sub-graphs using the list of unused arcs, namely 15-17, 4-5, 7-16, 7-22, 12-5, and 15-5, as shown in Figure A3.9. These arcs are added and are shown as thin edges in Figure A3.9 (Step 4.d in Figure 5.3). After adding these interrelationships the graph can be redrawn as required. As shown in Figure A3.9, by rotating the Strong Component Sub-Graph, facilities five, twelve and fifteen can be placed close and, at the same time, facilities seven, sixteen and twenty two will be closer. Different arrangements can be drawn and the most appropriate arrangement can be selected, according to the targeted aims, space availability and physical constraints of the facilities (Step 4.d in Figure 5.3).

Figures A3.10 and A3.11 are possible layout designs to be considered. It should be stressed that both are also capable of producing the required products.

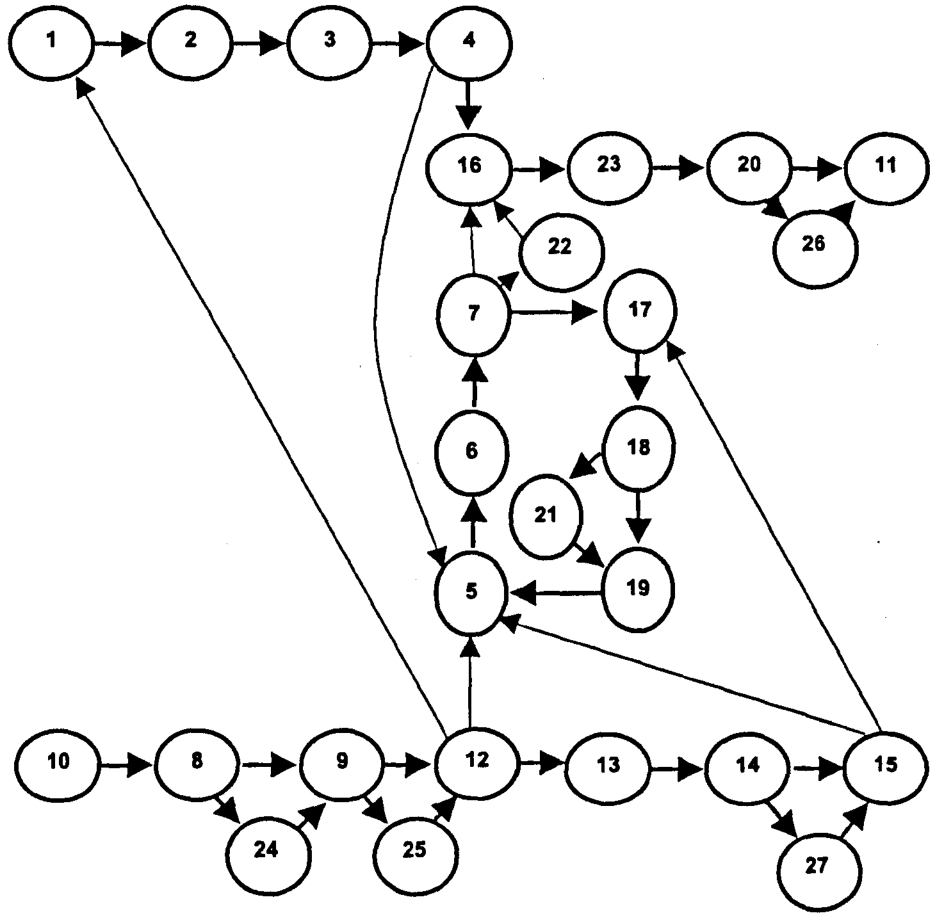

Figure A3.10. Example 3, Possible Layout Design

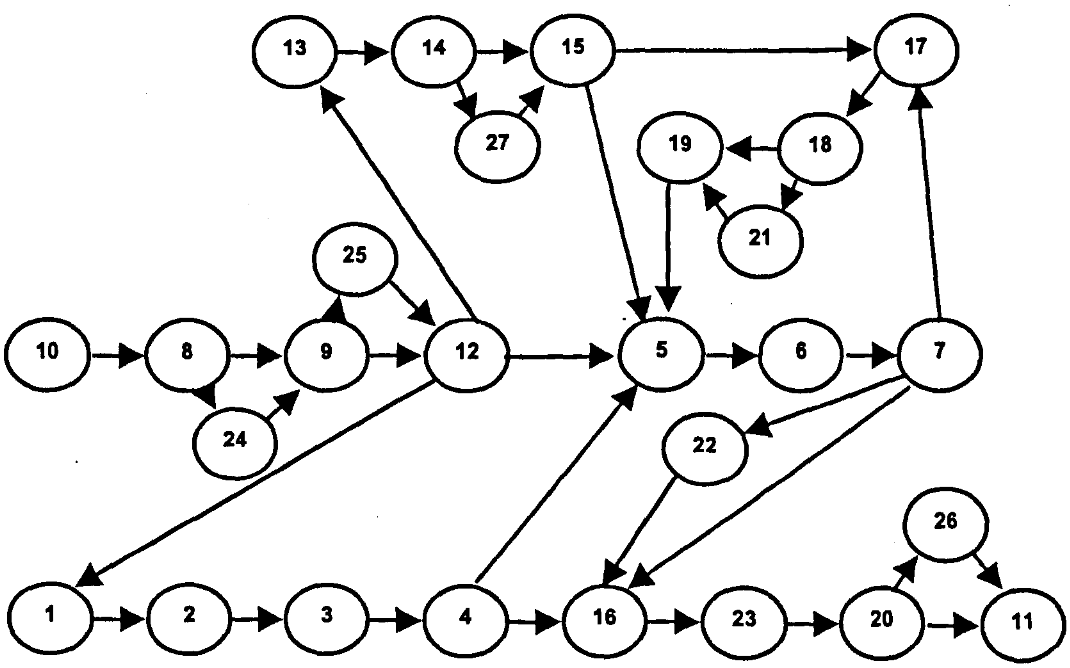

Figure A3.11. Example 3, Another Possible Layout Design

Appendix 4.

In this appendix, it is illustrated how a Reachability matrix is obtained from an Adjacency matrix as suggested by the Harary, Norman and Cartwright (Harary F., Norman R.Z., Cartwright D., 1965:122) in their procedure and reproduced in section 5.6.1.2 in Chapter 5. As mentioned in this section, an identity matrix and the Adjacency matrix are required, the latter is taken from section 5.6.1.1, in Chapter 5.

As mentioned in section 5.6.1.2, to obtain the Reachability matrix, there are partial matrices that should be obtained. These partial matrices require raising the Adjacency matrix to different powers in sequential order starting from the second power and above and their equivalent Boolean matrices used during the calculations. In the explanation, these partial matrices are shown as required. Every entry in a partial matrix is obtained by adding the correspondent entry component to component. Additionally, each entry may take a value of 0 or 1 depending if the addition is any positive value grater than zero takes a value of 1 or 0 otherwise. It should be noted that the Adjacency matrices presented here where transformed to an equivalent Boolean matrices.

The procedure starts by adding the Identity matrix to the Adjacency matrix, that is $R_1(D) = I(D) + A(D)$, or for simplicity $R_1 = I + A$. This new matrix, R_1, represents all the existing possible paths of length one between any two facilities.

I

	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	0
4	0	0	0	1	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1

+

A

	1	2	3	4	5	6
1	0	0	1	0	0	1
2	1	0	1	0	0	0
3	1	0	0	1	0	0
4	0	0	0	0	0	1
5	0	1	0	0	0	0
6	0	0	0	0	0	0

=

R_1

	1	2	3	4	5	6
1	1	0	1	0	0	1
2	1	1	1	0	0	0
3	1	0	1	1	0	0
4	0	0	0	1	0	1
5	0	1	0	0	1	0
6	0	0	0	0	0	1

Figure A4.1. Partial Matrix R_1

In the next step, R_1 is added to the second power of the Adjacency matrix, that is $R_2 = A^2 + R_1$.

A^2

	1	2	3	4	5	6
1	1	0	0	1	0	0
2	1	0	1	1	0	1
3	0	0	1	0	0	1
4	0	0	0	0	0	0
5	1	0	1	0	0	0
6	0	0	0	0	0	0

+

R_1

	1	2	3	4	5	6
1	1	0	1	0	0	1
2	1	1	1	0	0	0
3	1	0	1	1	0	0
4	0	0	0	1	0	1
5	0	1	0	0	1	0
6	0	0	0	0	0	1

=

R_2

	1	2	3	4	5	6
1	1	0	1	1	0	1
2	1	1	1	1	0	1
3	1	0	1	1	0	1
4	0	0	0	1	0	1
5	1	1	1	0	1	0
6	0	0	0	0	0	1

Figure A4.2. Partial Matrix R_2

Before performing the following operation, R_1 and R_2 are compared and it is determined if they are equal. After comparing both matrices, entries (1,4), (2,4), (2,6), (3,6), (5,1) and (5,3) are different. Since both matrices (R_1,R_2) are different then the following operation is performed, that is, $R_3 = A^3 + R_2$ is calculated.

A^3

	1	2	3	4	5	6
1	0	0	1	0	0	1
2	1	0	1	1	0	1
3	1	0	0	1	0	0
4	0	0	0	0	0	0
5	1	0	1	1	0	1
6	0	0	0	0	0	0

\+

R_2

	1	2	3	4	5	6
1	1	0	1	1	0	1
2	1	1	1	1	0	1
3	1	0	1	1	0	1
4	0	0	0	1	0	1
5	1	1	1	0	1	0
6	0	0	0	0	0	1

=

R_3

	1	2	3	4	5	6
1	1	0	1	1	0	1
2	1	1	1	1	0	1
3	1	0	1	1	0	1
4	0	0	0	1	0	1
5	1	1	1	1	1	1
6	0	0	0	0	0	1

Figure A4.3. Partial Matrix R_3

If the obtained matrix (R_3) is different than the previous one (R_2), entries (5,4) and (5,6), then the process continues until both matrices (the previous and the current one) are equal as can be seen in the following operation, $R_4 = A^4 + R_3$.

A^4

	1	2	3	4	5	6
1	1	0	0	1	0	0
2	1	0	1	1	0	1
3	0	0	1	0	0	1
4	0	0	0	0	0	0
5	1	0	1	1	0	1
6	0	0	0	0	0	0

\+

R_3

	1	2	3	4	5	6
1	1	0	1	1	0	1
2	1	1	1	1	0	1
3	1	0	1	1	0	1
4	0	0	0	1	0	1
5	1	1	1	1	1	1
6	0	0	0	0	0	1

=

R_4

	1	2	3	4	5	6
1	1	0	1	1	0	1
2	1	1	1	1	0	1
3	1	0	1	1	0	1
4	0	0	0	1	0	1
5	1	1	1	1	1	1
6	0	0	0	0	0	1

Figure A4.4. Partial Matrix R_4

Since after comparing R_3 and R_4 is determined that both matrices are equal, then the process is completed and can be stopped and the Reachability matrix is obtained, namely, $R_k = R_{(k-1)}$ or $R_n = A^n + R_{(n-1)}$.

Appendix 5.

This Appendix resumes how the numbers of displacements, used in the comparisons in section 6.5 in Chapter 6, were taken into consideration by example. It should be highlighted that underlined machines in the sequence are the workcentres required by the *production operations* sequences (POS).

A5.1. Example 1, More Machines than Products

The example in section 6.5.1 uses the table from Aneke and Carrie example (Aneke N.A.G., Carrie A.S., 1986).

Family	Actual Sequence	Number of Displacements
1	3 4 5 6 4 7 8 3 9 4 7 6 5 12 10 11 12 13	17
2	1 2 3 4 5 6 4 7 8 3 9 4 7 6 5 12 10 11 12 13	19
3	1 2 3 4 5 6 4 7 8 3 9 4 7 6 5 12 10 11 12 13	19
4	8 3 9 4 7 6 5 12 10 11 12 13	11
5	4 5 6 4 7 8 3 9 4 7 6 5 12 10 11 12 13	16
6	3 9 4 7 6 5 12 10 11 12 13	10
7	1 2 3 4 5 6 4 7 8 3 9 4 7 6 5 12 10 11 12 13	19
Total		111

Table A5.1. Example 1, Example Operation Sequences

Strong Components approach Table.

Family	Actual Sequence	Number of Displacements
1	3 5 8 5 4 9 10 11 12 13	9
2	1 2 4 5 3 4 9 4 5 6 7 6 11 12 11 13	15
3	1 2 3 5 4 5 6 7 6 11 12 11 13	12
4	8 5 3 5 6 11 13	6
5	4 5 6 7 6 11 13	6
6	3 5 4 10 11 13	5
7	1 2 3 5 4 5 6 7 8 5 4 9 10 11 12 13	15
Total		68

Table A5.2. Example 1, Strong Components Operation Sequences

A5.2. Example 2, More Products than Machines

The example in section 6.5.2 uses the table from Vakharia and Wemmerlöv example (Vakharia A.J., Wemmerlöv U., 1990). The number previous to the worcentre indicates the cell were the operation is to be performed.

Part Number	Actual Sequence	Number of Displacements
1	(C3)1 11 4 (C1)8 9	4
2	(C3)1 11 4 10 7 (C1)4 7 8 7	8
3	(C1)1 3 5 2 6 10 4 7 8 9	9
4	(C1)1 11 4 10 7 (C3)9	5
5	(C1)1 3 5 2 6 10 4 7 8 9	9
6	(C1)6 10 4 7 8 9	5
7	(C1)6 10 4 7 8 9	5
8	(C1)3 5 2 6 10 4 7 8 9	8
9	(C1)3 5 2 6 10 4 7 8 9	8
10	(C1)4 7 4 7 8	4
11		
12	(C2)11 6 7 10 12	4
13	(C2)11 6 7 10 12	4
14	(C2)11 6 7 10	3
15	(C1)1 (C2)7 6 11 (C3)10 4 11 (C2)12	7
16	(C1)1 (C2)7 6 11 (C3)10 4 11 (C2)12	7
17	(C2)11 6 7 10 12	4
18	(C2)6 7 10	2
19		
Total		96

Table A5.3. Example 2, Example Operation Sequences

Strong Components approach Table.

Part Number	Actual Sequence	Number of Displacements
1	1 4 8 9	3
2	1 4 7 4 8 7	5
3	1 2 1 4 7 8 9	6
4	1 4 7 9	3
5	1 6 10 7 9	4
6	6 10 7 4 8 9	5
7	6 1 4 8 9	4
8	3 5 2 5 6 1 4 8 9	8
9	3 5 6 1 4 8 9	6
10	4 7 4 8	3
11		
12	11 7 12	2
13	11 12	1
14	11 7 10	2
15	1 4 7 11 10 11 12	6
16	1 4 7 11 10 11 12	6
17	11 7 12	2
18	6 10 7 10	3
19		
Total		69

Table A5.4. Example 2, Strong Component Operation Sequences

A5.3. Example 3, A Large Number of Machines

The example in section 6.5.3 uses the table from Ho and Moodie example (Ho Y.C., Moodie C.L., 1994). The number previous to the workcentre is the module reference where the operation is to be performed.

Volume Quantity	Actual Sequence	Number of Displacements
1	(M1)10 8 9 24 9 24 12 (M2)1 2 3 1 (M3)16 23 20 26 11	15
2	(M1)10 8 9 24 12 25 12 (M4)5 6 7 (M5)17 18 19 21 19	14
3	(M1)10 8 9 24 12 (M6)13 14 15 (M5)17 18 19	10
4	(M2)1 2 3 4 (M4)5 6 7 (M3)16 23 20 26 11	11
5	(M6)13 14 15 27 15 (M4)5 6 7 (M3)22 16 23 20 26 11	13
6	(M2)1 2 3 4 (M3)16 23 20 11	7
7	(M6)13 14 15 (M5)17 18 19 (M4)5 6 7	8
Total		78

Table A5.5. Example 3, Example Operations Sequences

Strong Components approach Table

Volume Quantity	Actual Sequence	Number of Displacements
1	10 8 24 9 12 1 2 3 4 16 23 20 11	12
2	10 8 9 25 12 5 6 7 17 18 21 19	11
3	10 8 9 12 13 14 15 17 18 19	9
4	1 2 3 4 5 6 7 22 16 23 20 11	11
5	13 14 27 15 5 6 7 22 16 23 20 26 11	12
6	1 2 3 4 16 23 20 11	7
7	13 14 15 17 18 19 5 6 7	8
Total		70

Table A5.6. Example 3, Strong Component Operation Sequences