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A demand classification scheme for spare part inventory model subject to stochastic demand and lead time

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A demand classification scheme for spare part inventory model subject to stochastic demand and lead time

Abstract

In this study, we aim to develop a demand classification methodology for classifying and controlling inventory spare parts subject to stochastic demand and lead time. Using real data, the developed models were tested and their performances were evaluated and compared. The results show that the Laplace model provided superior performance in terms of service level, fill rate and inventory cost. Compared with the current system based on normal distribution, the proposed Laplace model yielded significant savings and good results in terms of the service level and the fill rate. The Laplace and Gamma optimization models resulted in savings of 82% and 81%, respectively.

Keywords: Inventory control, spare parts, MRO- maintenance, repair and operating items, steel industry.

1 Introduction

In many organizations, inventory costs pertaining to spare parts are incredibly high and a part of such costs is very critical to production process. Many items are strategic for operations and a stockout can directly affect the production process. Unlike product and raw material inventories, which are driven by production processes and customer needs, spare parts are kept in stock to support maintenance operations, i.e., to protect against equipment failures.

The downtime of spare parts can result in lost revenues, customer dissatisfaction and possible associated claims and usually the consequences of spare parts downtime are very costly (Driessen et al., 2013; Cavalieri et. al., 2008). However, if demand for components is systematically over-estimated, large amounts of capital are unnecessarily held up in stock (Downing et al. 2014). Therefore, companies along the aircraft spare parts supply chain face significant challenges in providing fast repair and maintenance services while minimizing costs (Liu et. al. 2013).

Although this function is well understood by maintenance managers, many companies face the challenge of maintaining large inventories of spare parts with excessive associated holding and obsolescence costs (Porras & Dekker, 2008; Romeijnders et al., 2012).

Spare part demand has specific characteristics and is very different from those of other materials such as products and work-in-process (WIP). While these are fast-movers and have regular demand, spare parts can be described as having slow moving, irregular, intermittent or other demand patterns. An intermittent demand pattern has variable demand sizes and the demand appears randomly with many time periods having no demand transaction. An intermittent demand occurs when there are a few large customers and many small customers or when the frequency of many customer requests varies. A slow-moving demand pattern always has low demand sizes. It occurs when there are few customers and little demand for an item.

Both patterns can create significant forecasting and stock-holding problems in the manufacturing and supply environment. Nevertheless, in the literature the terms sporadic and intermittent are frequently used interchangeably and the same can be said for the terms lumpy and irregular, as well as for the terms intermittent and irregular (Dunsmuir and Snyder, 1989; Eaves & Kingsman, 2004; Regattieri et al., 2005; Willemain et al., 2004; Nenes et al. 2010). In general, the term irregular demand has a broader interpretation, encompassing essentially all demand types that cannot be expressed by means of the usual normal and Poisson distributions (Nenes et al., 2010).

Lack of tools and models that cover the specific nature of spare parts increase the difficulties associated with establishing consistent methodologies and strategies to control spare part inventories. Inventory policies for spare parts are different from those for fast moving items and raw materials. The common demand-forecasting methods are not applicable when used for controlling inventory given intermittent or slow-moving demand patterns of this type (Silver, 1970; Silver et al., 1971; Botter & Fortuin, 2000; Bacchetti & Saccani, 2012).

With such high requirements related to these items, it is natural that spare parts management is an important area of inventory research (Huiskonen, 2001; Kourentzes, 2013). The development of a robust methodology that considers the specifics of spare parts can bring important savings and managerial benefits to companies. In addition, concept development for inventory planning and control of spare parts should concentrate more on its applicability in manufacturing companies. At the same time companies should prepare for and be more open to the adoption of spare parts inventory planning systems (Downing et al. 2014; Wagner & Lindemann, 2008).

The main objective of this study is to develop a demand classification model for maintenance, repair and operating (MRO) items. In addition, we propose different ranges of lead time demand for classifying items. The study developed a stochastic inventory model for controlling the spare parts inventory of a steel company considering stochastic demand and lead time, as well as different patterns of demand, as slow-moving or intermittent demand. The items that present such patterns and are considered in this research are spare parts known as MRO items. Specifically, this study aims to develop an inventory control methodology for spare parts considering the following related topics: statistical techniques for demand classification; demand forecasting models for slow-moving items; modeling lead time demand (LTD) considering probability distributions not adherent to normal; optimization models for determining stock parameters, i.e., reorder point, order quantity and safety stock; simulation model for evaluating the proposed inventory model and its use with real data for validating the proposed models.

Most past studies on this topic have focused on applying methods for inventory control in a production environment (products and raw materials), where demand and lead time are foreseen very accurately and the LTD distribution is normal.

Our research contributes to the field through the development of a specific model for demand classification of spare parts in steel industry. In addition, we propose different ranges of lead time demand for classifying items.

Our method of classification has the following differences from Eaves’ method (2002, 2004): the ranges of demand size and lead time variability are different. In Eaves´ model, there is no range for classifying the mean of LTD. Both demand-classification methodologies are based on empirical data. The differences in the ranges of demand size and lead time variability can be ascribed to the fact that Eaves’ approach is based on data from the aircraft industry, whereas our approach uses data from the steel industry.

Our approach uses data from a case study for elaborating inventory models for spare parts and evaluating their performance using empirical data. Generally, the inventory models in the literature consider the period demand approach for stock control, which is not appropriate for intermittent demand with a high proportion of zero values (Krever, Wunderink, Dekker, & Schorr, 2005). We have used the single demand approach based on single consumption transactions for calculating the mean and variance of LTD using the concept presented in Krever, Wunderink, Dekker & Schorr (2005). This approach better captures the variability of demand and lead time.

To develop consistent models for spare parts, we considered alternative statistical distributions such as Poisson, Laplace and Gamma for modeling LTD in combination with an optimization model developed in this work to calculate reorder point, order quantity and safety stock. Our methodology uses Willemain’s bootstrap method (2004).

2 Relevant Literature

The inventory planning and control process is concerned with decisions regarding which items to stock, when an order must be placed, the length of the lead time, the quantity to be ordered and the total cost. To define when an order should be triggered and the associated order quantity, we need to define which inventory policy is more appropriate for the problem we have. Let s be the reorder point, S the order up to level, Q the fixed quantity to be ordered and n the multiple Q quantities. Considering a continuous review system, there are two main models called (s, Q) and (s, S) (Silver, Pyke and Peterson, 1998).

In the (s,S) model when inventory position (inventory on hand plus on-order minus backorders) falls to or below a specified level s, a replenishment is placed to raise the inventory level to S. In the (s, Q) model, a fixed quantity Q is ordered as soon as the stock level depletes to reorder point s. We intend to use the (s, nQ) system in the event that orders with multiple Q quantities are to be triggered for raising the stock position above s. For unit demands, we use the (s-1, S) system, a particular case of the (s, S) model. In the (s-1, S) model, when the stock depletes to reorder point s, a unit order quantity is triggered for returning the stock level to position S. In this model we assume the costs of the spare parts are high enough to justify a (s-1, S) policy. This is a very common policy in industries with long lead times or when the cost of parts is very expensive like aerospace, military and some MRO items in the steel industry.

Few authors have presented a consistent methodology for classifying items according to their respective demand behaviors. Williams (1984) developed an analytical method for classifying demand as smooth, slow moving or intermittent by decomposing the variance of the LTD into three parts: transaction variability, demand size variability and lead time variability.

Silver, Pyke and Peterson (1998) did not develop a methodology, but they did establish some boundaries between slow-moving and fast-moving items and considered normal distribution for LTD modeling. Based on experimental results, they proposed Poisson distribution or other discrete distribution in the case that the mean LTD is below 10 units. Otherwise, according to them, normal distribution should be considered when the ratio of the standard deviation and the mean demand is less than 0.5. If the ratio is greater than 0.5, the use of the Gamma distribution should be considered.

Based on Williams (1984), Eaves (2002) and Eaves & Kingsman (2004) developed a methodology for demand classification and adequated it to the items in his study. Through an analysis of Royal Air Force (RAF) data, they perceived that Williams' original classifications did not adequately describe the observed demand structure. In particular, it was not considered sufficient for distinguishing the smooth demand pattern from other patterns based only on transaction variability. A revised classification schema was proposed that categorized demand according to transaction variability, demand size variability and lead time-variability.

In their model, the boundary for transaction variability can be set to the lower quartile while the boundaries for demand size variability and lead-time variability can be set at their respective medians. This approach gives the following boundaries: 0.74 for transactions variability, 0.10 for demand size variability and 0.53 for lead-time variability. Based on these boundaries demands are classified as smooth, irregular, slow moving, mildly intermittent and highly intermittent, as shown in table 1.

Table 1 – Classification of Demand

Mean time between transactions variability	**Demand size variability**	**Lead-time variability**	**Demand classes**
≤ 0,74	≤ 0,10	-	1 - Smooth
≤ 0,74	> 0,10	-	2 - Irregular
> 0,74	≤ 0,10	-	3 - Slow Moving
> 0,74	> 0,10	≤ 0,53	4 - Mildly Intermittent
> 0,74	> 0,10	> 0,53	5 - Highly intermittent

Source: Eaves (2002); Eaves & Kingsman (2004)

The accuracy of demand forecasting is critical in inventory management (Hax & Candea, 1984), but the intermittent nature of demand makes forecasting difficult, especially for spare parts. Several studies have been developed for treating demand forecasting for slow-moving and intermittent demand patterns.

Commonly used forecasting methods are often based on assumptions that are inappropriate for intermittent demand. Croston (1972) developed an alternative method considering intermittent demand and assuming geometric distribution for intervals between demands and normal distribution for demand size. He demonstrated that his method was superior to exponential smoothing. Croston's method was investigated and evaluated by several authors who showed its consistency and practical applications (Willemain, Smart, Shockor, & DeSautels, 1994; Johnston & Boylan, 1996). Ghobbar and Friend (2003) presented a comparative study with 13 different forecasting methods for spare parts in the aviation industry.

They use mean average percentage error (MAPE) for calculating forecasting errors and evaluating the accuracy of different methods. They confirmed the superior performance of Croston's method and moving average over the exponential smoothing and seasonal regression models.

Hua et al. (2007) developed a new approach for forecasting intermittent demand for spare parts with a large proportion of zero values. The authors show that the accuracy of their approach in forecasting demand during lead time is better than those of other methods such as exponential smoothing, the Croston method and bootstrapping Markov method.

Eaves & Kingsman (2004) used demand and lead time data to evaluate the practical value of forecasting models available in the literature for treating forecasting and inventory control problems related to spare parts items. Using analytical methods, the authors classified the consumable stock items as smooth, irregular, slow moving and intermittent. They also showed that the Syntetos and Boylan approximation method (Syntetos & Boylan, 2005, 2006a, 2006b), a modification of Croston's method, presented better results in terms of holding cost reduction for a required service level than the common methods, i.e., exponential smoothing, moving averages and Croston's method.

Willemain, Smart and Schwarz (2004) proposed a model for forecasting LTD distribution using a new type of bootstrap series. The modified bootstrap method used a data sample of demand history for creating repeatedly realistic scenarios that show the evolution of LTD distribution. This procedure better captures the auto-correlations between demand transactions, especially for intermittent demand with a large proportion of zero values. In the case of demand size, negative autocorrelation can occur when a low demand is matched with a high one or vice-versa. Positive autocorrelation can occur when a low demand size is matched with another low demand size or a high demand size is matched with another high demand size (Eaves, 2002).

The method uses a Markov model for evaluating the probability transition between the zero and non-zero states of different items. The authors showed that the bootstrap method generates more accurate forecasts (based on MAPE measures) of LTD distribution than do exponential smoothing and Croston's method.

In the context of theoretical models, one of the most extensively studied inventory policies is the (S – 1, S) model, with demand distribution based on the Poisson distribution (Feeney & Sherbrooke, 1966). Although this model is often used for slow-moving items, it requires a continuous review inventory policy. Furthermore, Poisson distribution assumes random demand with time intervals between unit demand transactions according to exponential distribution. For cases with non-unit demand size, the authors have proposed the use of compound models such as Poisson compound (Williams, 1984; Silver, Ho and Deemer, 1971) or Bernoulli compound (Janssen, Heuts and Kok, 1998; Strijbosch, Heuts, & Schoot, 2000). However, these models are difficult to use in practice because they require parameters from more than one distribution for determining LTD. For instance, Williams (1984) developed a method for identifying intermittent demand items in which three parameters are necessary: one for exponential distribution of the time intervals between demands and two parameters for the gamma distribution of demand size.

Many studies have been performed using other theoretical distributions for representing lead time distribution (LTD). One common assumption is that LTD is normal, despite the fact that for sporadic or slow-moving demand, the normality assumption can be inappropriate. Several distributions were used with different approaches for demand and lead time behavior, such as the normal (Krupp, 1997), gamma (Burgin, 1975; Das, 1976; Yeh, 1997), Poisson (Hill, Omar and Smith, 1999) and unknown distributions with stochastic demand and lead time (Eppen & Martin, 1998).

Although the decomposition of LTD into its constituent parts is an important development in demand modeling, Krever, Wunderink, Dekker and Schorr (2005) demonstrated that it is necessary to obtain detailed information about demand history. They developed an approach based on single demands for modeling LTD distribution and proposed new expressions for calculating the mean and variance of LTD. The authors showed that this method is more robust than the common methods based on the period demand approach because it can better capture demand variability.

Some authors have developed inventory models considering cost optimization for determining the stock parameters. Pressuti and Trepp (1970) developed a model for calculating the optimal policy for the (s,Q) model using Laplace distribution for LTD. Namit and Chen (1999) formulated an algorithm for solving the (Q,r) model considering gamma distribution for LTD, where Q is the replenishment quantity and r the reorder point. Whenever the inventory position drops to the reorder point r, a replenishment order for quantity Q is placed. Tyworth and Ganeshan (2000) presented a simple expression for the Gamma loss function to calculate the expected number of backorder units per cycle. This expression enabled the use of the optimization model to solve the (Q, r) model for any parameter α of a gamma distribution.

3 Case study

Case study research has been considered one of the most powerful research methods in operations management. The results of case research can have very high impact. Unconstrained by the rigid limits of questionnaires and models, it can lead to the development of new theory, theory-testing, theory extension and refinements, exploratory research, new and creative insights and have high validity with practitioners - the ultimate user of research (Voss et al. 2002; Yin, R.K., 2003; Childe, S.J., 2011).

A case study was developed at one of the leading steel companies in the world considering the inventory control of spare parts. In this section, we detail the problem considered in this real study, its scope, the characteristics of demand and lead time and inventory costs.

3.1 Scope and problem definition

The company under study operates a centralized depot that covers the needs of the maintenance operations and the company uses the module materials management from SAP R/3 system for inventory control. The current inventory policy is based on SAP R/3 with forecast demand being performed by the method of exponential smoothing. In addition the company uses a (s, S) policy from SAP/R3 based on normal distribution and service level for calculating the reorder level, order quantity and safety stock. The SAP system has good adherence to fast-moving items with smooth demand but its performance is not good for spare parts. In section two, we have discussed in detail why the exponential smoothing technique is not suitable for slow moving items.

The company's management aims to develop a methodology for controlling spare parts items, focusing on important and high-cost items. Inventory cost reduction is an important driver underlying this requirement. Forecasting is a tough task and the current model seems to be inappropriate for these types of items.

This study considers different approaches and theories regarding inventory management of spare parts to develop an inventory control methodology.

The following assumptions are made for developing the proposed inventory models: (i) demand is random with different patterns, (ii) lead time is stochastic and demand distribution is unknown; however, different distributions have been suggested for slow-moving items. Based on classical statistical distribution provided in the literature, models were applied using four different distributions:

- Current model – based on normal distribution for comparison with the proposed models;
- Proposed models – using Poisson, Laplace and Gamma distributions for modeling LTD.

As a forecasting model, we implemented the modified bootstrap method presented by Willemain, Smart and Schwarz (2004).

3.2 Proposed demand classification model

The case study considered seven years of demand history in developing and testing the models. Information on demand transactions, lead times and costs was extracted from SAP R/3, an integrated system used in many organizations. To focus on the single-demand approach (Krever, Wunderink, Dekker, & Schorr, 2005), we developed routines that extract information on single-demand transactions and lead times for spare parts. The evaluation covered the consumption of more than 10,000 parts in a unique plant. The items were classified according to ABC consumption analysis. We considered the most important A items that represent 80% of the consumption value. According to Silver, Pyke and Peterson (1998), the inventory costs of these types of items are sufficiently high to justify a more sophisticated control system as compared with less important items B or C.

We developed a hybrid methodology for classifying items according to demand patterns. We used a methodology very similar to that proposed by Eaves (2002) and Eaves & Kingsman (2004) considering the variability of LTD components and the expected value of LTD for establishing the boundaries of slow- and fast-moving items, as proposed by Silver, Pyke and Peterson (1998).

The proposed ranges considers the coefficient of variation (CV) of the mean time between transactions, demand size and lead time for classifying the materials into five classes as summarized in table 2. Similar to Eaves' classification, the boundaries of each category were defined based from seven years of experimental data on 10,000 parts. The coefficient of variation (CV) is given by the following formula:

$$CV = \sigma/\text{mean} \tag{1}$$

Table 2 – Proposed ranges for demand classification

Mean time between transactions variability	**Demand size variability**	**Lead-time variability**	**Demand classes**
≤ 0,74	≤ 0,30	-	1 - Smooth
≤ 0,74	> 0,30	-	2 - Irregular
> 0,74	≤ 0,30	-	3 - Slow Moving
> 0,74	> 0,30	≤ 0,70	4 - Intermittent
> 0,74	> 0,30	> 0,70	5 - Highly Intermittent

Regarding the mean of LTD, different LTD ranges have been proposed in this work to classify the items for evaluating the performance of inventory models in each range and for establishing the boundaries of slow-moving and fast-moving items. The items are structured into nine ranges, as detailed below, according to the LTD mean.

Table 3 lists the number of items classified in each group according to both approaches used in the classification methodology. The classification process is important for establishing auto-correlation among the items during analysis of the simulation results.

Correlation between customer requests can lead to intermittent demand pattern creating significant forecasting and stock-holding problems (Eaves & Kingsman, 2004).

1 - Below 5

2 - Between 5 and 10

3 - Between 10 and 50

4 - Between 50 and 100

5 - Between 100 and 200

6 - Between 200 and 500

7 - Between e 500 and 1000

8 - Between 1000 and 5000

9 - Above 5000

3.3 Lead time analysis

Lead time is the time elapsed between the sending of an order to a supplier and the receipt of goods. Lead time is the fundamental component underlying any inventory control system. If demand and lead time are known, replenishment can be planned with good accuracy. However, this is not the common situation in inventory management. For most spare parts, the demand is subject to a high degree of uncertainty and the lead time is stochastic.

The current system considers lead time as deterministic. A consistent inventory model for these items requires a more accurate approach for the treatment of lead time. To meet the objectives of our study we considered lead time as stochastic for determining the mean and variance of LTD. We built a routine for extracting the lead time data directly from the SAP R/3 system.

Table 3 – Number of items per class

Classes	Number of items
Demand classes	
1 – Smooth	20
2 – Irregular	32
3 – Slow Moving	61
4 – Intermittent	104
5 – Highly Intermittent	121
Classes per mean of LTD (units)	
1 - Below 5	278
2 - Between 5 and 10	20
3 - Between 10 and 50	17
4 - Between 50 and 100	4
5 - Between 100 and 200	7
6 - Between 200 and 500	4
7 - Between 500 and 1000	3
8 - Between 1000 and 5000	3
9 - Above 5000	2
TOTAL	**338**

3.4 Inventory costs

The objective of a spare parts inventory system is the minimization of operational costs. The relevant costs associated with an inventory system can be grouped into three categories (Hax & Candea, 1984): (i) costs associated with item acquisition, (ii) costs associated with the existence of stock (supply exceeds demand) and (iii) costs associated with stockouts (demand exceeds supply).

Let us group the costs associated with acquisition into two categories: unit cost of the item, which depends on the average price of the item and order cost, which depends on purchase structure and is independent of order quantity. If K is the cost per order and $\frac{D}{Q}$ the number of orders per year, the annual order cost is as follows:

$$\text{Order cost } = K\frac{D}{Q} \tag{2}$$

Where

D – Annual demand

Q – order quantity

Regarding the costs associated with the existence of stock, the common assumption is that the holding costs are proportional to stock value. Therefore, if r is the opportunity rate to invest the amount of stock in a financial system and C is the average price of an item in stock then the annual holding unit cost h for the item per year is as follows:

$$h = rC$$

Considering the average stock equals $\frac{Q}{2} + SS$, the annual holding cost is given as follows:

$$\text{Holding cost } = h\left(\frac{Q}{2} + s - D_L\right) \tag{3}$$

Where

s – Reorder point

D_L – Expected value of lead time demand

$$\text{Safety stock } SS = s - D_L$$

The costs associated with stockouts arise when the net stock is insufficient for covering demand. Let us assume that in the case of a stockout demand will always be backordered. The assumption that the backorder cost is proportional to the backorders expected per replenishment cycle was a common approach in the literature.

If p is the backorder cost per unit backordered, n is the number of the replenishment cycle and $\frac{D}{Q}n(s)$ is the expected number of annual backorders, the backorder annual cost is given as follows:

$$\text{Backorder cost } = p\frac{D}{Q}n(s) \tag{4}$$

We defined $G(s,Q)$ the expected total annual cost function whose variables are reorder point s and order quantity Q, as the sum of order cost, holding cost and backorder cost. Combining expressions 2, 3 and 4 the expected total annual cost function is defined as follows (Nahmias, 2004):

$$G(s,Q) = K\frac{D}{Q} + h\left(\frac{Q}{2} + s - D_L\right) + p\frac{D}{Q}n(s) \tag{5}$$

In section 4, the inventory models are detailed based on total inventory cost optimization as defined by expression 5.

4 Inventory control methodology

In this section, we describe the development of the methodology for implementing the inventory system. Figure 1 shows the steps of the methodology detailed in the following sections.

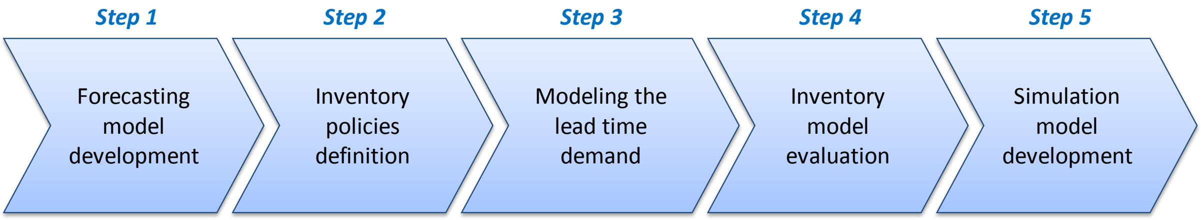

Figure 1 - Inventory control methodology

4.1 Forecasting model

Willemain's bootstrap method was used for generating forecasts of demand during lead time and estimating the mean and variance of LTD, as well as the expected number of demand transactions per unit time. Using various statistical distributions described in section 4.3 for modeling LTD and the optimization process we determined the reorder point and the order quantity.

4.2 Inventory policies

In this case study, the stock status is checked daily. Considering that the scope of our study encompasses slow-moving items we assume that the inventory system is under continuous review. For spare parts, this approximation is quite reasonable. We intend to use the (s, nQ) system in the event that orders with multiple Q quantities are to be triggered for raising the stock position above s.

4.3 Implemented LTD model

We developed four inventory models for controlling the items considered in our research. The models use different statistical distributions for LTD modeling, i.e., Poisson, Laplace, gamma and normal. The concept of single-demand approach presented in Krever, Wunderink, Dekker and Schorr (2005) was used for estimating the mean and variance of LTD. We obtained the historical values of single-demand transactions for estimating the LTD distribution of each model.

In this section, we present details of LTD modeling. In table 4, we list the variables used in the development of said inventory models.

Table 4 – Notation for inventory models development

Variable	**Definition**
L	Lead-time
s	Reorder point
Q	Order quantity
D	Demand per time unit
d	Single demands
N	Number of single demands during lead-time
D_L	Demand during lead-time
NS	Cycle service level
λ	Number of single demands per time unit
K	Cost per order ($)
h	Inventory carrying cost per item ($/unit)
p	Backorder cost per unit backordered ($/unit)

According to Krever, Wunderink, Dekker and Schorr (2005) LTD modeling is divided into two different approaches:

- Period demand approach – PDA – this approach is based on the observed stochastic behavior of past period demand. One can estimate the mean and variance of demand during a stochastic lead time L using equations 6 and 7 (Tijms, 1994):

$$E(D_L) = E(D).E(L) \tag{6}$$

$$Var(D_L) = E(L).Var(D) + (E(D))^2.Var(L) \tag{7}$$

- Single-demand approach – SDA – this approach is based on single demands and the expected number of single demands per unit time. In this case, the mean and variance of demand during stochastic lead time L are defined by equations 8 and 9 (Krever, Wunderink, Dekker and Schorr, 2005):

$$E(D_L) = \lambda.E(d).E(L) \tag{8}$$

$$Var(D_L) = \lambda E(L)Var(d) + \lambda E(L)E(d)^2 + \lambda E(d)^2 \tag{9}$$

The standard deviation of LTD σ_L is

$$\sigma_L = \sqrt{Var(D_L)} \tag{10}$$

Current model

The current inventory model assumes normal distribution for demand and a fixed service level for determining the reorder point and safety stock. Equation 11 shows the economic order quantity (EOQ) policy used by the company. For the A items, the company management defines a minimum service level of 80%. Different service levels will be used for determining stock parameters and evaluating the models' performance. These levels are 80, 85, 90, 95 and 99%.

$$EOQ = \sqrt{\frac{2KD}{h}} \tag{11}$$

The reorder point s for normal demand distribution is given as follows:

$$s = D_L + k\sigma_L \tag{12}$$

The coefficient k for a specific theoretical service level is obtained from the standard normal N (0, 1) tables.

Proposed models

Three different distributions were used for modeling the LTD, i.e., Poisson, Laplace and gamma. The parameters of each of these distributions are determined from mean and variance of the LTD estimated using forecast values. The reorder point s and the order quantity Q were determined by optimizing total inventory cost $G(s,Q)$ (Nahmias, 2004) subject to some constraints defined in the following model:

Minimize $G(s,Q) = K\frac{D}{Q} + h\left(\frac{Q}{2} + s - D_L\right) + p\frac{D}{Q}n(s)$ (13)

Subject to:

$$s \geq D_L$$

$$Q > 0$$

Where,

$n(s)$ – Expected backorders per replenishment cycle

Decision variables: s, Q

$NS \geq$ Theoretical minimum

The theoretical service levels of 80, 85, 90, 95 and 99% are used for determining the stock parameters and evaluating the performance of the models.

The optimum values of order quantity Q and reorder point s that minimize $G(s,Q)$ are given by equations 13 and 14, respectively (Nahmias, 2004). Because there are two variables in the objective function, the solution will be iterative. For initiating the iterative process, the initial order quantity is set to the EOQ given by equation 14.

$$P(s) = \frac{Qh}{pD} \tag{14}$$

Where,

P(s) – Stockout probability

$$Q = \sqrt{\frac{2D[K + pn(s)]}{h}} \tag{15}$$

4.3.1 Poisson distribution

The parameter λ of the Poisson distribution is defined as follows:

$$\lambda_{LTD} = E(D_L) \tag{16}$$

The expected number of backorders per replenishment cycle $n(s)$ for the Poisson distribution is given by equation 17.

$$n(s) = D_L P(s) - sP(s+1) \tag{17}$$

4.3.2 Gamma distribution

The parameters α and β of gamma is defined as follows:

$$\alpha = \frac{E(x)^2}{Var(x)} \tag{18}$$

$$\beta = \frac{Var(x)}{E(x)} \tag{19}$$

The expected number of backorders per replenishment cycle $n(s)$ for the gamma distribution is given by equation 20 (Tyworth & Ganeshan, 2000).

$$n(s) = \alpha\beta(1 - G_1(s)) - s(1 - G_0(s)) \tag{20}$$

where

G_1 – Cumulative Density Function (CDF) of gamma (α + 1,β)

G_0 – CDF of gamma (α,β)

4.3.3 Laplace distribution

The parameters μ and θ of the Laplace distribution are defined as follows:

$$\mu = E(x) \qquad (21) \qquad\qquad \theta = \sqrt{\frac{Var(x)}{2}} \qquad (22)$$

Pressuti & Trepp (1970) developed simple expressions for the Laplace distribution. Using the expression service level order quantity (SOQ) defined in Nahmias (2004), the optimum values of order quantity Q and reorder point s can be determined in a single step using equations 23 and 24.

$$Q = \theta + \sqrt{\frac{2KD}{h} + \theta^2} \qquad (23)$$

$$s = -\theta \ln(2P(s)) + D_L \qquad (24)$$

4.4 Inventory model evaluation

4.4.1 Total costs

The total cost was obtained by calculating the holding cost, order cost and backorder cost. The expressions for calculating these costs are given below.

$$CM_j = EOH_j * h_j \qquad (25)$$

$$CP_j = K_j * nc_j \qquad (26)$$

$$CR_j = QRP_j * p_j \qquad (27)$$

$$CT_j = CM_j + CP_j + CR_j \qquad (28)$$

where

CM_j – Holding cost of product j

CP_j – Order cost of product j

CR_j – Backorder cost of product j

CT_j – Total cost of product j

h_j – Inventory carrying cost per item of product j

K_j – Cost per order of product j

p_j – Backorder cost per unit of product j

nc_j – Number of replenishment cycles of product j

EOH_j – Average stock of product j

QRP_j – Number of backordered units of product j

4.4.2 Fill rate and cycle service level

Fill rate is the portion of demand that is met from existing stock. Cycle service level is the probability of no stockout per replenishment cycle. The expressions for calculating the fill rate and the cycle service level are given below.

$$FR_j = \frac{\sum_i QF_{i,j}}{\sum_i QD_{i,j}} \qquad (29)$$

$$NS_j = 1 - \frac{ncr_j}{nc_j} \qquad (30)$$

Where

FR_j – Fill rate of product j

NS_j – Cycle service level of product j

$QF_{i,j}$ – Number of units supplied of product j in period i

$QRC_{i,j}$ – Number of received units of product j in period i

$QD_{i,j}$ – Number of demanded units of product j in period i

nc_j – Total number of replenishment cycles of product j

ncr_j – Number of replenishment cycles with backordered units of product j

4.5 Simulation model

Our objective is to directly use the observed real demand values for assessing the performance of the policies using the simulation process. Because we are using the single-demand approach, we transformed our historical values into daily demands and developed our model to consider daily demand for the seven-year testing period. Using Matlab 6.5, the model was elaborated for simulating the performance of each period.

The simulation model is divided into two parts. The first part is regarding the simulation of daily stock movements. The second part pertains to the determination of performance measures, total costs, fill rate and service level.

For initializing the simulation process, we defined the initial stock as the theoretical maximum stock for system (s, Q), as given by equation 31.

$$EI_j = ES_j + Q_j \qquad (31)$$

Following is the mathematical model of the simulation process.

$$EOH_{i,j} = \begin{cases} EOH_{i-1,j} + QRC_{i,j} - QD_{i,j} - QRP_{i-1,j}, \text{ if } QRP_{i,j} = 0 \\ 0, \qquad \text{otherwise} \end{cases} \qquad (32)$$

$$ATP_{i,j} = \quad EOH_{i,j} + QP_{i,j} - QRP_{i,j} \qquad (33)$$

$$QRP_{i,j} = \begin{cases} 0, & \text{if } EOH_{i,j} > 0 \\ QD_{i,j} + QRP_{i-1,j} - EOH_{i-1,j} - QRC_{i,j}, & \text{otherwise} \end{cases} \tag{34}$$

$$QF_{i,j} = \begin{cases} QD_{i,j}, & \text{if } EOH_{i,j} \geq QD_{i,j} \\ EOH_{i,j}, & \text{otherwise} \end{cases} \tag{35}$$

$$QP_{i,j} = \begin{cases} QP_{i-1,j} + nQ_j, & \text{if } ATP_{i,j} \leq s_j \text{ and } n = 1,2,3,\ldots \\ QP_{i-1,j}, & \text{otherwise} \end{cases} \tag{36}$$

$$QRC_{i,j} = \begin{cases} nQ_j, & \text{if } ATP_{i-LT_j,j} \leq s_j \text{ and } n = 1,2,3,\ldots \\ 0, & \text{otherwise} \end{cases} \tag{37}$$

$$nc_{i,j} = \begin{cases} 1, & \text{if } QRC_{i,j} > 0 \\ 0, & \text{otherwise} \end{cases} \tag{38}$$

$$ncr_{i,j} = \begin{cases} 1, & \text{if } \sum_{i-LT_j}^{i-1} QRP_{i,j} > 0 \text{ and } QRC_{i,j} > 0 \\ 0, & \text{otherwise} \end{cases} \tag{39}$$

Where

$EOH_{i,j}$ – Net stock of product j at the end of period i

$ATP_{i,j}$ – Stock position of product j at the end of period i

$QRP_{i,j}$ – Number of backordered units of product j in period i

$QP_{i,j}$ – Number of ordered units of product j in period i

5 Results

Through simulation, we calculated the inventory costs, fill rate and cycle service level of each item for evaluating the performance of the proposed models considering stochastic demand and lead time. We classified the items considering their demand patterns and LTD variability for establishing correlations between inventory models and items classes. Another aspect considered in our simulation process was the use of different minimum theoretical service levels (Minimum SL), i.e, 80, 85, 90, 95 and 99% for each model.

The global results of each model are summarized in table 5. The Laplace model yielded the best global result in terms of the cycle service level, fill rate and total cost, whereas the Poisson model yielded the worst performance.

Table 5 – Global results of inventory models

Logistics Indicator	**Global Result per Inventory Model**						
	Normal	**Compound Poisson**		**Laplace**		**Gamma**	
	Result (%)	**Result (%)**	**Savings (%)**	**Result (%)**	**Savings (%)**	**Result (%)**	**Savings (%)**
Service Level	81.2	63.8	-21.0	81.9	1.0	79.2	-2.0
Fill Rate	92.8	82.3	-11.0	94.6	2.0	92.6	0.0
Total Cost	**1,239,055**	**2,901,234**	**- 134.0**	**226,296**	**82.0**	**237,593**	**81.0**

With a 5% higher total cost, the results of the gamma distribution were quite close to those of the Laplace distribution. When we compared the proposed models with the current system represented by normal distribution, the Laplace model yielded significant savings of 82% and improvements of 1% in the service level and 2% in the fill rate. The gamma model achieved savings of 81% compared with the current system; however, customer service was worse 2% than that of the current model. The results obtained using the Poisson model were worse for all aspects than those of the current system. In fact, the applicability of the Poisson distribution is quite restricted.

5.1 Results per theoretical service level

When we analyze the results per theoretical service level, we could verify that the results of the Gamma model are better than those of the other models for minimum Service Levels (Minimum SL) equal to or greater than 95%. The results for different Minimum SL, Fill Rate (FR) and total cost are listed in table 6.

Table 6 – Results per theoretical service level

Minimum SL (%)	**Result per Minimum Service Level**											
	Normal			**Compound Poisson**			**Laplace**			**Gamma**		
	Real SL (%)	**FR (%)**	**Total Cost ($)**	**Real SL (%)**	**FR (%)**	**Total Cost ($)**	**Real SL (%)**	**FR (%)**	**Total Cost ($)**	**Real SL (%)**	**FR (%)**	**Total Cost ($)**
80.0	73.4	89.0	1,597,950	57.8	79.3	2,850,873	73.9	91.3	275,053	65.9	87.3	219,097
85.0	77.3	90.7	1,275,316	59.0	79.6	2,850,558	76.5	92.6	271,608	69.8	88.8	273,886
90.0	80.6	92.7	1,331,106	61.1	81.3	2,850,206	79.9	94.2	265,610	77.5	91.8	398,832
95.0	84.7	94.8	1,407,739	67.0	83.7	2,849,003	85.6	96.4	240,359	86.2	95.5	229,263
99.0	90.0	97.0	583,163	74.2	87.6	3,105,531	93.4	98.6	78,850	96.5	99.4	66,885
Average	**81.2**	**92.8**	**1,239,055**	**63.8**	**82.3**	**2,901,234**	**81.9**	**94.6**	**226,296**	**79.2**	**92.6**	**237,593**

The performance of the current normal-distribution-based system was comparable with that of the Laplace model only in terms of the real service level; However, the Laplace model achieved the better total cost ($).

5.2 Consideration of LTD component variability

Owing to great variation in the demand patterns of spare parts, the items were classified into the demand classes described in section 3.2. The results of the simulation involving different demand patterns are listed in table 7.

Table 7 – Results per demand class

Demand Class	Result per Demand Class											
	Normal			Compound Poisson			Laplace			Gamma		
	Real SL (%)	FR (%)	Total Cost ($)	Real SL (%)	FR (%)	Total Cost ($)	Real SL (%)	FR (%)	Total Cost ($)	Real SL (%)	FR (%)	Total Cost ($)
Smooth	87.0	96.6	4,290	83.8	95.2	4,439	84.8	96.7	4,276	85.4	96.6	4,198
Irregular	71.3	87.3	8,384	59.0	78.6	8,470	76.7	92.1	7,987	73.3	88.4	8,731
Slow Moving	88.2	97.6	7,229	76.9	90.8	39,728	87.3	97.8	7,270	85.4	96.1	7,798
Intermittent	78.1	90.3	3,892,276	59.9	79.0	8,091,283	77.6	92.5	670,433	74.9	90.1	698,957
Highly Intermittent	82.0	93.4	109,163	58.5	79.7	1,126,780	83.7	95.2	49,409	80.3	93.3	56,000
Average	**81.2**	**92.8**	**1,239,055**	**63.8**	**82.3**	**2,901,234**	**81.9**	**94.6**	**226,296**	**79.2**	**92.6**	**237,593**

Upon analyzing demand patterns, we concluded that the Laplace model achieves the best performance for items with great variability of demand and lead time, both intermittent and highly intermittent. The gamma model was better than the Laplace model only for smooth items, which were the minority in this study. The normal model did not have good adherence with the intermittent demand pattern. It achieved good performance only with smooth items. For intermittent and highly intermittent classes, the performance of the normal model is far from those of the Laplace and the Gamma models.

5.3 Consideration of LTD size

Silver, Pyke and Peterson (1998) suggested that the definition of demand distribution should consider the mean of LTD; the gamma and normal models would be suited to fast–moving items, whereas the Poisson and Laplace models would be suited to slow-moving items. We segregated the items according to the expected LTD values and applied the simulation process. The results of simulation considering LTD size are listed in table 8.

Table 8 – Results per range of LTD size

Mean of LTD	Result per LTD Size											
	Normal			Compound Poisson			Laplace			Gamma		
	Real SL (%)	FR (%)	Total Cost ($)	Real SL (%)	FR (%)	Total Cost ($)	Real SL (%)	FR (%)	Total Cost ($)	Real SL (%)	FR (%)	Total Cost ($)
< 5	83.3	94.0	8,433	70.1	87.0	10,509	83.6	95.4	8,005	80.8	93.4	8,622
5 to 10	73.5	89.6	26,599	49.0	72.7	57,559	77.6	92.9	18,291	72.8	89.2	19,395
10 to50	75.3	90.9	16,191	38.5	68.6	37,515	74.8	93.0	10,717	73.7	90.9	11,195
50 to 100	50.6	72.4	35,511	20.0	44.3	75,434	59.0	78.3	18,128	52.0	75.9	18,932
100 to 200	70.2	91.3	28,000	18.1	48.0	465,864	74.2	95.7	15,796	73.3	95.0	14,962
200 to 500	61.8	66.9	103,696,440	20.5	46.7	240,900,383	51.7	71.0	18,343,051	53.7	71.1	19,251,326
500 to 1000	52.4	77.6	98,502	8.6	12.2	2,064,164	63.6	88.8	21,084	63.2	87.3	16,889
1000 to 5000	81.8	94.6	62,832	25.5	56.2	410,719	85.3	97.0	23,545	86.8	96.4	22,269
> 5000	96.5	99.5	19,117	22.2	48.6	658,816	96.8	100.0	12,803	98.5	100.0	13,975
Average	**81.2**	**92.8**	**1,239,055**	**63.8**	**82.3**	**2,901,234**	**81.9**	**94.6**	**226,296**	**79.2**	**92.6**	**237,593**

The Laplace model yielded better performance than the other models in the majority of ranges. The gamma model's performance was comparable to that of the Laplace model for average demand greater than 100 units. Although the Laplace model is suited to slow-moving items, it achieved good performance in all LTD size ranges.

6 Concluding remarks

In this paper, we proposed a model for demand and lead time classification of spare parts items in steel industry. Additionally, we developed an inventory control methodology for spare parts considering the item classification techniques, demand forecasting, LTD modeling and optimization models for determining the stock parameters. Based on statistical distributions, we implemented three proposed models and compared the results with those of the current model. The current model uses normal distribution and the order quantity follows the EOQ. The proposed models use the Poisson, Laplace and gamma distributions for LTD and an optimization process for determining the reorder point and order quantity. These models were tested with real consumption data spanning seven years for evaluating the performance of alternative distributions and comparing them with the normal distribution.

The main reason underlying the use of alternative distributions is that it is known from the literature that spare parts do not adhere to the normal distribution.

To generate forecast values, we implemented Willemain's bootstrap method, which is suited to slow-moving items with intermittent demand.

Considering the behavior of spare parts and the simulation results, the following general conclusions were drawn. Spare parts showed great variability in terms of demand patterns because they encompassed items ranged from very slow moving to fast moving with mean LTD values over 5000 units. The results showed that the Laplace model had superior global performance compared with the other models. Thus, the Laplace model has extensive applicability and it can be used for slow- or fast-moving items having different demand patterns. In general, the optimization models obtained very good performance in terms of the fill rate, service level and perhaps most importantly, inventory costs.

Regarding demand variability, the Laplace and Gamma optimization models adhered well to the intermittent demand pattern. The normal distribution model did not perform well for items with great variability in demand and lead time. When we analyzed the smooth items, the normal model yielded quite satisfactory results. Although the Laplace model has remarkable global performance, the gamma model achieved the best result for theoretical service levels greater than 95%. Specifically, for working with very high service levels, the Gamma distribution is a feasible option.

Regarding demand size, the Laplace model yielded the best performance for most LTD size ranges. However, the gamma model performed well, especially for items with expected LTD values greater than 100 units. Although the normal distribution is suited to fast-moving items, it did not show good performance with these items owing to great variability in demand and lead time. The results showed that the normal distribution is not adherent to intermittent items.

6.1 Managerial Implications

The development of a more sophisticated methodology for spare parts inventory control brings significant savings to companies. The Laplace and Gamma optimization models achieved savings of 82% and 81%, respectively, over the current normal model.

The use of the proposed models can guarantee better customer service levels. The results of the Laplace model showed improvements of 1% and 2% in the cycle service level and the fill rate, respectively, over the current system. The implementation of the proposed models subject to optimization has important implications for organizations. The main aspect is the reduction of capital invested in stock to keep high inventory levels for supporting MRO. The increased fill rate and improved inventory management lead to increased credibility of the replenishment process, thus reducing parallel stocks and out-of-control consumption. The reduction of stockouts leads to the reduction of equipment downtime and avoids the cost of production losses. The proposed Laplace model has important practical aspects. It is easier to implement than the other models and it can be applied to most demand classes and demand size ranges. The Gamma model is a feasible option for fast-moving items and for working with very high service levels.

6.2 Directions for Future Research

One of the interesting aspects of working with industry is that it is sometimes possible to identify the need for a new avenue of research. This might be when technology or models are being applied in a new setting, such as a different industry (Childe, S.J., 2011).

This study shows that there is no universal boundaries for classifying items as smooth, irregular, slow moving, intermittent or highly intermittent demand pattern. What is classed as irregular demand pattern in aerospace industry may be considered as intermittent in the steel industry. Therefore, for each industry or eventually for a group of similar industry, research must be undertaken in order to define the boundaries for classifying items that best fit in each industry. This can allow managers to make the right decision to reduce the cost of inventory management for spare parts saving financial, human and materials resources and consequently improving the profit of the company.

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