Assessing Colour Differences of Lighting Stimuli Using a Visual Display

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The candidate confirms that the work submitted is his/her own, except where work which has formed part of jointly authored publications has been included. The contribution of the candidate and the other authors to this work has been explicitly indicated below. The candidate confirms that appropriate credit has been given within the thesis where reference has been made to the work of others.

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Abstract

A large scale experiment was carried out to accumulate colour difference discrimination data for lighting stimuli which were simulated on a display. Twenty six colour centres were selected and twenty one pairs of chromaticity difference were prepared surrounding each colour centre. Each pair was assessed using ratio method against a grey and a black background by a group of 20 normal colour vision observers. Both white and coloured lighting stimuli were used; corresponding to specifications for the solid state white lighting products and part of the MacAdam colour centres respectively. The latter defined the fundamental research for visual sensitivities known as just noticeable difference (JND).

The results were used to test colour spaces and chromaticity diagrams as well as to evaluate colour difference predictions of various colour difference formulae and colour appearance models. It was found that u'v' chromaticity diagram can represent colour discrimination data for lighting colour stimuli more uniformly. Moreover, CIELUV colour difference formula performed better in predicting colour differences for lighting stimuli; especially when using black background. It performed better than CIEDE2000 and CAM02-UCS formulae, which were derived based on surface colours, even though it has been shown in the previous studies that these perform well when simulating surface colours on display. These points also support that the experimental arrangement could potentially be used to simulate light sources (luminaires) in order to evaluate them as lighting stimuli. Furthermore, the visual results obtained against the black background can mostly be predicted better by the models than those against the grey background for lightning stimuli.

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Chapter 1.

Introduction

1.1. Background

Even though extensive research in colour difference for surface colours has been done, there is still no sufficient evidence to confirm a colour space that represents colour difference and chromaticity for the lighting stimuli or light sources as visually perceived. Actually, there is non-uniformity in the results of colour difference equations and colour spaces when used in different applications. Research has shown that other colour spaces and formulae are appropriate for surface colours and other for lighting stimuli. Additionally, various studies have made apparent the influence of parametric effects in the colour difference evaluation and appearance. A research by Cui et al. has shown that it is possible to simulate surface stimuli on a display (Cui et al., 2001b; Cui et al., 2001a). Moreover, in the same study, different sample arrangements and backgrounds have been seen to have great impact on the colour appearance of stimuli. For example, adding a one pixel black dividing line between two colour patches can significantly affect chromatic differences. Therefore, sample arrangement and background could be employed as a method to simulate light sources on a display as well. The results from simulated surface stimuli studies could be used for comparison and reveal whether the experimental arrangement could alter the results. Additionally, other studies have shown that it might be better to use the u’v’ chromaticity diagram for specifying colour tolerance of white light sources (Luo et al., 2015). However, this should also be investigated for coloured lighting stimuli.

1.2. Aims of Thesis

In lighting industry, the Planckian locus is normally used as a reference for specifying the colour of white light sources. ANSI has defined in one of its standards, the specifications of solid state lighting products at specified whites around the Plankian locus (ANSI, 2008). However, coloured lighting products are gaining popularity and new sources such as light-emitting diodes (LED) which include a variety of colours are becoming dominant. LED luminaires consist of many individual small LEDs, therefore lighting manufacturers aim for uniform colour distribution of light among the individual LEDs. Consequently, the colour of each LED needs to be controlled so that no obvious colour difference appears in the luminaire. Therefore, both white and coloured lighting stimuli were investigated in this Ph.D. project. Moreover, MacAdam colour discrimination ellipses are highly associated with coloured lighting and for long used to specify just noticeable differences (MacAdam, 1942; MacAdam, 1943). However, there are a few points to be considered regarding the MacAdam data. They were obtained by only one observer using aperture mode visual colorimeter. Therefore, the data should be verified using technologies of real light sources or similar reproduction media. So, part of this work is to verify the MacAdam ellipses. Moreover, the objective of MacAdam’s experiment was colour matching whereas this study focuses on colour difference judgement. The nature of the results is quite different. The MacAdam’s experimental results show how the distribution of matching targets and their differences are just noticeable. While the results of this study are colour discrimination and the used colour differences are more clearly visible.

Previous studies have also shown that CIEDE2000 might not be appropriate colour difference formula for evaluating lighting stimuli (Luo et al., 2015). CIEDE2000 has been developed from various surface colour datasets (Cui et al., 2001b; Cui et al., 2001a; Luo et al., 2001). And it has been shown to outperform all the other formulae in estimating colour difference using surface colours. On the other hand, MacAdam and CIELUV are labelled as appropriate for lighting stimuli. Therefore, one other aim of this study is to verify this common notion by investigating lightning stimuli. So, one of the objectives is to clarify whether it is possible to simulate lighting sources (luminaires) as lightning stimuli effectively on display. And consequently, compare these with results from simulated surface stimuli on display and other lighting stimuli.

Colour stimuli are also affected by various viewing conditions under which are observed and thus various parametric effects have been investigated in the past. Additionally, some of them have even been modelled in colour appearance models. With this concept in mind, colour appearance models should perform better than colour spaces or formulae. The data of this study should reveal this hypothesis by using two different backgrounds.

Hence, a summary of the objectives of this study is given below:

- To understand the performance of MacAdam ellipses to fit coloured lighting stimuli,
- To investigate the performance of various colour difference metrics for predicting lighting stimuli, and
- To understand the parametric effect on evaluating perceived lighting stimuli including change of background and luminance of colour centre.

1.3. Thesis Overview

Following the current chapter which introduce the research problem; Chapter 2 is an overview of the theory and previous relevant work to this Ph.D. project. Topics of colour fundamentals, colour difference formulae, colour appearance models, parametric effects, colour management and colour discrimination studies are discussed.

Chapter 3 describes the details of the experimental setup and specifications of equipment used. The characteristics and measurement results of the display are presented. In addition, the process for the selection of colour centres and sampling are discussed. Finally, evaluation of observer data is presented.

Chapter 4 contains the data and results from the ellipse fitting. In this chapter, the data and results concerning the uniformity of colour spaces and performance are discussed. Comparisons based on different parametric effects and datasets are also included.

Chapter 5 investigates the performance of colour difference formulae and colour appearance models using the visual data gathered from this study.

The most prominent ones were considered; i.e. CIELAB, CIELUV, CIEDE2000, CIECAM02, and CAM02-UCS.

Finally, Chapter 6 summarises the main conclusions of this study and possible future work.

Chapter 2.

Literature Review

All the related literature was reviewed and summarised in this chapter. It includes colorimetry to describe CIE tools for colour specification, colour difference and colour appearance. This is followed by colour management, focused on the methods for display characterisation; and psychophysical methods for scaling colour differences. Colour discrimination ellipses' characteristics and fitting methods, together with lighting standard and statistical analysis were reviewed. Finally, the colour fundamentals and background theory related to this research are discussed. A glossary of useful related terminology is given in Appendix A.

2.1. Colour Fundamentals

Colour is a phenomenon that does not exist without the human visual system. It is the result of three factors: a light source, an object and an observer. Light sources render wavelengths, objects absorb and scatter them according to their physical properties; and observers perceive the colour due to their visual system comprised of the brain and eyes. However, the perceived colour is described as the:

> "attribute of visual perception consisting of any combination of chromatic and achromatic content. This attribute can be described by chromatic colour names such as yellow, orange, brown, red, pink, green, blue, purple, etc., or by achromatic colour names such as white, grey, black, etc., and qualified by bright, dim, light, dark, etc., or by combinations of such names" (CIE, 1987).

As colour has many applications, systems for its specification and communication have been developed over the years. Colour specification systems could be categorised into colour order systems and numerically based specification; with the latter being the CIE system. Colour order systems are composed of colour atlases in which samples are systematically arranged based on certain criteria and colour attributes related to the purpose of the system. These systems are ideal for quick and easy reference since they are logically arranged. However, they are constrained by the medium and materials used for their creation. Well-known examples of such systems are the Munsell and the Pantone systems. However, for the purposes of this research, the CIE system and colorimetry were applied.

2.1.1. CIE Colorimetry

The acronym CIE stands for the International Commission on Illumination (Commission Internationale de l’ Éclairage) which was founded in 1913 in order to standardise and research light and colour issues (CIE, 2011). The CIE has published many standards and technical reports about the basics of colour science and has defined the CIE colour specification system.

2.1.1.1. Standard Illuminants and Sources

There is no colour without light; but more than this, colour rendering depends on the characteristics of the light source. *Sources* are the physical light sources and simulators able to reproduce the relative spectral power distribution (SPD) defined as the standard illuminants (Schanda, 2007 p. 44). The *illuminants* are determined by their relative SPD (CIE, 2004a; Schanda, 2007). There are various illuminants recommended by the CIE to describe incandescent light, sunlight and fluorescent light in order to be used as reference for industrial applications. Some of the standard illuminants are the: illuminant A, illuminant C, illuminant D50, illuminant D65; and a series of illuminants F for representing different fluorescent lamps. Newer electric lamps such as the solid state lighting products (LED technologies) will be discussed in section 2.5.

The *illuminant A* is

“defined over the spectral region from 300 nm to 830 nm” (CIE, 2004a). This illuminant is “intended to represent typical, domestic, tungsten-filament lighting. Its relative spectral power distribution is that of a Planckian radiator at a temperature of approximately 2856 K. CIE standard illuminant A should be used in all applications of colorimetry involving the use of incandescent lighting, unless there are reasons for using a different illuminant.” (CIE, 2006).

The *illuminant D65* is

> "intended to represent average daylight and has a correlated colour temperature of approximately 6500 K. CIE standard illuminant D65 should be used in all colorimetric calculations requiring representative daylight, unless there are specific reasons for using a different illuminant. Variations in the relative spectral power distribution of daylight are known to occur, particularly in the ultraviolet spectral region, as a function of season, time of day, and geographical location. However, CIE standard illuminant D65 should be used pending the availability of additional information on these variations." (CIE, 2006).

The illuminant C is "intended to represent average daylight with a correlated colour temperature of approximately 6800 K" (CIE, 2004a).

The illuminants notated with the letter D represent a series of illuminants of different daylight phases (Schanda, 2007 pp. 43-44). The *illuminants D50*, *D55* and *D75* represent correlated colour temperature (CCT) of 5000 K, 5500 K and 7500 K respectively. The *illuminant D50* is usually used by the graphic arts industry because its SPD resembles to a certain extent both daylight and incandescent light.

2.1.1.2. Standards of Reflectance

For standardisation purposes, the reliability of the measuring instruments is examined and instruments such as colorimeters and spectrophotometers are calibrated against a perfect reflecting diffuser (Schanda, 2007 pp. 57-58). According to CIE, a perfect reflecting diffuser is a reference material to relate reflectance. Many materials have been tested for their appropriateness as a standard of reflectance. For a material to be appropriate for such use, it must reflect light in all directions and with equal intensity, i.e. to resemble an ideal Lambertian surface (Grum and Becherer, 1979 p.37). Such materials are divided into two groups: (1) smoked magnesium oxide, pressed powder of magnesium oxide, and pressed powder of barium sulphate; and (2) various glasses, ceramic tiles, and plastics (Schanda, 2007 pp. 57-58). The former group has a distribution closer to the Lambertian surface but it is more delicate. On the other hand, the latter group is less ideal but it is more reliable.

2.1.1.3. Geometric Conditions

The position of observer and light source from the specimen is also standardised. The viewing conditions are related to the angle, the course of the light beams and the type of instrument used for the measurement. There are CIE- recommended geometries for both reflection and transmission measurements (CIE, 2004a). Table 2.1-1 summarises the geometries for reflection measurements.

Table 2.1-1 Summary of CIE geometry conditions

Symbol	Illuminating	Viewing	Specular Gloss	Geometry
45° α : 0°	45° ring	0°	Included	Normal
45° x : 0°	45° beam	0°	Included	Normal
0° : 45° α	0°	45° ring	Included	Normal
0° : 45° x	0°	45° beam	Included	Normal
di : 8°	Diffuse	8°	Included	Integrating sphere
de : 8°	Diffuse	8°	Excluded	Integrating sphere
8° : di	8°	Diffuse	Included	Integrating sphere
8° : de	8°	Diffuse	Excluded	Integrating sphere
d : d	Diffuse	Diffuse	Included all angles	Integrating sphere
d : 0°	Diffuse	0°	Strict Excluded	Integrating sphere

2.1.1.4. Standard Colorimetric Observers

There are two standard colorimetric observers defined for different applications (CIE, 2004a). The first one was defined in 1931 by relation to the colour matching functions $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ and is used for small viewing fields. The second one was defined in 1964 for larger visual fields by normalizing the above colour matching functions. The two standards are summarised in Table 2.1-2. For ease of reference, they are usually annotated with the number of degrees accompanied by the reference illuminant. For example, D50/2 is CIE 1931 standard colorimetric observer under D50 illuminant.

Table 2.1-2 CIE Standard colorimetric observers

Description	Observer	Visual Field	Diameter	Viewing Distance
CIE 1931 Standard Colorimetric Observer	2° observer	1° - 4°	17 mm	0.5 m
CIE 1964 Standard Colorimetric Observer	10° observer	> 4°	90 mm	0.5 m

2.1.1.5. Tristimulus Values and Chromaticity Coordinates

The tristimulus values represent the CIE specification system as they describe how much of additive stimuli are needed so as to specify a particular stimulus. The tristimulus values notated as X, Y, Z are related to the CIE 1931 standard colorimetric system, while the X_{10}, Y_{10}, Z_{10} to the CIE 1964 standard colorimetric system (CIE, 2004a). For each system, the calculation of tristimulus values for a wavelength (λ) is given in Equation 2.1-1; where $S(\lambda)$ is the relative SPD, $R(\lambda)$ is the reflectance factor, and $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ are the colour matching functions.

Chromaticity describes "the colour quality of a colour stimulus definable by its chromaticity coordinates, or by its dominant (or complementary) wavelength and its purity taken together" (ASTM, 2009). So, the chromaticity coordinates are the "ratio of each of a set of three tristimulus values to their sum" (CIE, 1987). For the chromaticity coordinates, the functions in Equation 2.1-2 apply. These are applied for both CIE 1931 and CIE 1964 standard colorimetric systems by using the respective tristimulus values. Therefore, the xy chromaticity diagram can be defined for each colorimetric observer. The drawback of the xy chromaticity diagram is that equal distances in the space do not represent equally perceived chromaticity difference (Wright, 1941).

Equation 2.1-1 Tristimulus values

$$\begin{aligned} X &= k \int_{\lambda} S(\lambda)\ R(\lambda)\ \bar{x}(\lambda)\, d\lambda \\ Y &= k \int_{\lambda} S(\lambda)\ R(\lambda)\ \bar{y}(\lambda)\, d\lambda \\ Z &= k \int_{\lambda} S(\lambda)\ R(\lambda)\ \bar{z}(\lambda)\, d\lambda \\ where\ \ k &= 100\ /\ \int_{\lambda} S(\lambda)\ \bar{y}(\lambda)\, d\lambda \end{aligned}$$

$$\begin{aligned} X_{10} &= k_{10} \int_{\lambda} S(\lambda)\ R(\lambda)\ \bar{x}_{10}(\lambda)\, d\lambda \\ Y_{10} &= k_{10} \int_{\lambda} S(\lambda)\ R(\lambda)\ \bar{y}_{10}(\lambda)\, d\lambda \\ Z_{10} &= k_{10} \int_{\lambda} S(\lambda)\ R(\lambda)\ \bar{z}_{10}(\lambda)\, d\lambda \\ where\ \ k_{10} &= 100\ /\ \int_{\lambda} S(\lambda)\ \bar{y}_{10}(\lambda)\, d\lambda \end{aligned}$$

Equation 2.1-2 Chromaticity coordinates xy

$$\begin{aligned} x &= \frac{X}{X+Y+Z} \\ y &= \frac{Y}{X+Y+Z} \\ z &= \frac{Z}{X+Y+Z} \\ 1 &= x+y+z \end{aligned}$$

2.1.1.6. Luminance Factor

Luminance is a photometric quantity that describes the luminous intensity which is projected by a unit area in a given direction (Grum and Becherer, 1979; Hunt, 1998). The descriptor luminous describes “measures evaluated in terms of spectral power weighted by the $V(\lambda)$ function” (Hunt, 1998 p. 323). The $V(\lambda)$ function describes the spectral luminance efficiency which is the basis of photometry, i.e. weighted functions to transform radiometric values into photometric values. As luminance factor is defined the “ratio of the luminance to that of the perfect diffuser identically illuminated” (Hunt, 1998 p. 323). Yet, the Y tristimulus value is also associated with the luminance of the stimulus when evaluated purely in candelas per square metre. As it is the common, it can be treated so as to represent the percentage of luminance factor. For that, the ratio Y/Y_n can be used to normalise the XYZ values. In this ratio, Y_n is the Y value of the reference white or reference transparent specimen appropriately.

2.1.1.7. Uniform Colour Spaces

The CIE 1976 uniform chromaticity scale diagram - UCS diagram or else u’v’ chromaticity diagram - derives from the CIE 1976 colour spaces (CIE, 2004a; CIE, 2009). It is an evolution of the xy chromaticity diagram and its chromaticity coordinates u’v’ are given by Equation 2.1-3. In contrast to the xy chromaticity diagram, the chromaticity differences are more equally distanced, thus making it a more uniform space. This attempt was made in order to create a chromaticity diagram where MacAdam ellipses would be formed as circles. However, there is still no perfect linear transformation. A nonlinear transformation of the xy chromaticity diagram was performed by Farnsworth that transformed the MacAdam ellipses into circles; however, this space is not practical as it is limited to the examined colour centres (Farnsworth, 1958).

Equation 2.1-3 Chromaticity coordinates u'v'

$$u' = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2x + 12y + 3}$$

$$v' = \frac{9Y}{X + 15Y + 3Z} = \frac{9y}{-2x + 12y + 3}$$

2.1.2. Colour Difference Formulae

Through the years a great variety of colour difference formulae have been developed in order to achieve a much more realistic estimation of colour difference when calculating colour difference between a reference and a testing sample. Since 1976, when CIELAB and CIELUV formulae were developed, great progress has been made (Luo, 2002). After 1976, more advanced formulae have been built based on a certain structure by altering the CIELAB formula (Luo et al., 2001). This structure is shown in Equation 2.1-4, where ΔL^*, ΔC^* and ΔH^* are the differences for lightness, chroma and hue respectively as calculated by CIELAB. The k_L, k_C and k_H are parametric factors for each attribute, the S_L, S_C and S_H are the respective weighting functions for each attribute and ΔR is an interactive term for hue and chroma differences. Among the advanced formulae are CMC (ℓ:c), CIE94 and CIEDE2000. However, CIELAB still remains the basic industry standard, even though different systems have been developed based on different applications. It should also be noted that the aforementioned formulae have been developed based on datasets accumulated by surface mode samples.

Equation 2.1-4 Generic structure of colour difference formulae

$$\Delta E = \sqrt{\left(\frac{\Delta L^*}{k_L\, S_L}\right)^2 + \left(\frac{\Delta C^*}{k_C\, S_C}\right)^2 + \left(\frac{\Delta H^*}{k_H\, S_H}\right)^2 + \Delta R}$$

2.1.2.1. CIELAB and CIELUV Formulae

The CIELAB and CIELUV formulae are based on the CIELAB and CIELUV colour spaces respectively (Luo, 2002). CIELAB and CIELUV are CIE 1976 uniform colour spaces, they have an identical lightness scale L^* and opponent colour axes – a* and b* for CIELAB, and u* and v* for CIELUV which are formed differently in three dimensional space. For these metrics, a reference white XYZ_n is required to compensate for the viewing conditions of the field. CIELAB is mainly used by industries occupied with subtractive colour reproduction, while the CIELUV is recommended for additive colour reproduction. In Equation 2.1-5, the main functions are given for both formulae (CIE, 2007; CIE, 2009). The symbols h_{ab} – h_{uv} stand for the hue angle in degrees, and C_{ab}^* – C_{uv}^* stand for the chroma attribute. Lightness, hue and chroma represent the perceived colour attributes, and their differences are calculated between the batch and the standard. For instance, by using $\Delta a^* = a_B^* - a_S^*$, where B stands for the batch and S for the standard, and the remaining factors are computed accordingly.

Equation 2.1-5 CIELAB and CIELUV formulae

$$\Delta E^*_{ab} = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2}$$

$$L^* = 116\, f\,(Y/Y_n) - 16$$
$$a^* = 500\,[\,f(X/X_n) - f(Y/Y_n)\,]$$
$$b^* = 200\,[\,f(Y/Y_n) - f(Z/Z_n)\,]$$

$$\text{where } I = \mathrm{X/X_n}\ or\ \mathrm{Y/Y_n}\ or\ \mathrm{Z/Z_n} \text{ respectively}$$
$$\text{for } I > (6/29)^3$$
$$f(I) = \sqrt[3]{I}$$
$$\text{for } I \leq (6/29)^3$$
$$f(I) = (841/108)\, I + 4/29$$

$$\Delta E^*_{uv} = \sqrt{(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2}$$

$$u' = 4X/(X + 15Y + 3Z) = 4x\,/(-2x + 12y + 3)$$
$$v' = 9Y/(X + 15Y + 3Z) = 9y\,/(-2x + 12y + 3)$$
$$u^* = 13\, L^*\,(\mathrm{u}' - u_n')$$
$$v^* = 13\, L^*\,(\mathrm{v}' - \mathrm{v}_n')$$

$$h_{\mathrm{ab}} = \tan^{-1}(b^*/\mathrm{a}^*)$$
$$C^*_{\mathrm{ab}} = \sqrt{\mathrm{a}^{*2} + b^{*2}}$$

$$h_{uv} = \tan^{-1}(v^*/u^*)$$
$$C^*_{uv} = \sqrt{u^{*2} + v^{*2}}$$

2.1.2.2. CIEDE2000 Formula

The CIEDE2000 formula was also based on CIELAB and it has been proven to outperform the other colour difference formulae of its time for the majority of the available surface mode datasets (Luo et al., 2001). The investigation has proved that CIEDE2000 corrects fundamental problems of the CIELAB (Luo, 2002). These corrections are: (1) lightness weighting function S_L which corrects the prediction of lightness difference; (2) chroma weighting function S_C for normalisation; (3) hue weighting function S_H; (4) interactive term R_T which compensates for chromatic corrections in the blue region; and (5) factor (1+G) to correct achromatic shades.

The CIEDE2000 is given in Equation 2.1-6. The *L*, a* and b** values are calculated using CIELAB formula.

Equation 2.1-6 The CIEDE2000 formula

$$\Delta E_{00} = \sqrt{\left(\frac{\Delta L'}{k_L\, S_L}\right)^2 + \left(\frac{\Delta C'}{k_C\, S_C}\right)^2 + \left(\frac{\Delta H'}{k_H\, S_H}\right)^2 + R_T \left(\frac{\Delta C'}{k_C\, S_C}\right)\left(\frac{\Delta H'}{k_H\, S_H}\right)}$$

$$\begin{aligned}
L' &= L^* \\
\mathrm{a}' &= (1+G)\,\mathrm{a}^* \\
b' &= b^* \\
C' &= \sqrt{\mathrm{a}'^2 + b'^2} \\
h' &= \tan^{-1}(b'/\mathrm{a}') \\
where\ G &= 0.5 \times \left(1 - \sqrt{\frac{\overline{C^*_{\mathrm{ab}}}^{\,7}}{\overline{C^*_{\mathrm{ab}}}^{\,7} + 25}}\right) \\
\Delta L' &= L'_2 - L'_1 \\
\Delta C' &= C'_2 - C'_1 \\
\Delta H' &= 2\sqrt{C'_2 C'_1} \cdot \sin(\Delta h'/2) \\
where\ \ \Delta h' &= h'_2 - h'_1
\end{aligned}$$

$$\begin{aligned}
S_L &= 1 + (\,0.015\,(\overline{L'} - 50)^2 \,/\, \sqrt{20 + (\overline{L'} - 50)^2}\,) \\
S_C &= 1 + 0.045\,\overline{C'} \\
S_H &= 1 + 0.015\,\overline{C'}\,T \\
where\ \ T &= 1 - 0.17\cos\left(\overline{h'} - 30°\right) + 0.24\cos\left(2\,\overline{h'}\right) + \\
&\quad\ 0.32\cos\left(3\overline{h'} + 6°\right) - 0.20\cos\left(4\overline{h'} - 63°\right) \\
R_T &= -\sin(2\,\Delta\vartheta)\,R_C \\
\Delta\vartheta &= 30\,exp\left[-\left(\overline{h'} - 275°/25\right)^2\right] \\
R_C &= 2\sqrt{\bar{C}'^7/\bar{C}'^7 + 25^7}
\end{aligned}$$

2.1.3. Colour Difference Datasets

For the assessment of colour difference, many experiments have been conducted in order to optimise the colour difference formulae. Over the years, plethora of experimental datasets have been composed based on different viewing conditions, materials, psychophysical methodology and other parametric criteria. Some of the most important datasets have been compiled for surface mode small perceptibility colour differences. A summary of these is given in Table 2.1-3.

The BFD-perceptibility dataset is a combined set of experimental data from various research groups (Cheung and Rigg, 1986; Luo and Rigg, 1986; Strocka et al., 1983; Witt, 1987; Witt and Doring, 1983). Different colour centres and methodology were followed by each group. Luo-Rigg combined 13 datasets of different published experiments into one integrated dataset, which was used to derive the BFD (ℓ:c) colour difference equation (Luo and Rigg, 1986; Luo and Rigg, 1987a; Luo and Rigg, 1987b). A common colour centre between the Luo-Rigg study and each examined dataset was used in their experiments so as to ensure reliability of the data. BFD (ℓ:c) equation was the set point of the advanced colour difference formulae; having interactive term for hue and chroma. It was also found by Luo and Rigg that there is not much difference between acceptability and perceptibility data as far as it concerns chromaticity differences.

The RIT-DuPont dataset was mainly used to derive the CIE94 colour difference formula, which was aiming to predict colour differences for industrial applications (Alman et al., 1989; Berns et al., 1991). Finally, for the derivation of the CIEDE2000 formula, the following most reliable colour discrimination datasets were used: Luo and Rigg, RIT-DuPont, Leeds (Kim and Nobbs) and Witt dataset (Alman et al., 1989; Berns et al., 1991; Kim and Nobbs, 1997; Luo et al., 2001; Luo and Rigg, 1986; Witt, 1999).

Table 2.1-3 Classical experimental datasets for assessing colour difference

Dataset	Material	№ of Resulted Pairs	Colour Centres	Mean $\Delta E^*_{\alpha b}$	Viewing Conditions	Psychophysical Method
BFD-perceptible (various groups)	Textile / Paint	2776	5 CIE / 51 red textile	3	CIE reference conditions	(a). Near threshold (b). Pair comparison (c). Grey scale method
RIT-DuPont (two groups)	Paint (gloss acrylic)	876 (resulted in 156 colour difference vectors - visual tolerances)	19	contiguous (in edge contact)	Middle grey background and illuminant similar to D65	Pair comparison pass/fail with a grey reference pair of $\Delta E^*\alpha b=1.02$
Leeds (Kim and Nobbs)	Paint (gloss acrylic)	307	21	1.6	Different viewing conditions	(a). 104 pairs - pair comparison (b). 203 pairs - grey scale method
Witt	Paint	418	5 CIE	1.9	CIE reference conditions	Grey scale method

2.1.4. Colour Appearance Overview

Appearance is defined as "the collected visual aspects of an object or a scene" (ASTM, 2009). Therefore, many attributes in the viewing field influence the perceived appearance, such as size, shape, colour, texture, gloss, transparency, opacity, light source, luminance, background, etc (ASTM, 2009; CIE, 1993). There are several colour appearance phenomena which have been identified to change perceived appearance, such as the Hunt effect and the Stevens effect (Fairchild, 2005; Luo and Li, 2007).

2.1.4.1. Viewing Field

As it was previously mentioned, colour appearance is influenced by whatever is in the visual field. The visual field is defined by the following: (1) the stimulus; (2) the proximal field; (3) the background; (4) the adapting field; and (5) the surround (CIE, 2004b; Fairchild, 2005; Hunt, 1998). These are illustrated in Figure 2.1-1 and their definitions are given below.

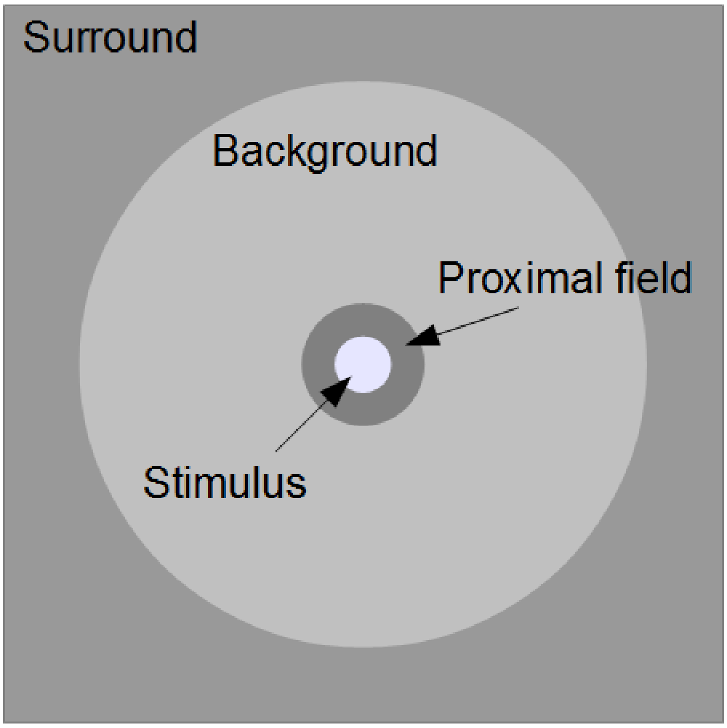

Figure 2.1-1 CIE Viewing field

The *stimulus* is “a colour subtending roughly 2° in the centre of the field of view” (CIE, 2004b). “A 2° visual field represents a diameter of about 17 mm at a viewing distance of 0.5 m” (CIE, 2004a). The CIE 1931 standard colorimetric observer can be used when the stimulus is between 1° - 4° (CIE, 2004a).

The *proximal field* is “the immediate environment of the stimulus, extending for about 2° from the edge of the stimulus in all, or most, directions” (Fairchild, 2005 p. 137). Local contrast effects can be modelled by using this definition. However, it is very difficult to specify it in practice and as a result it is considered as part of the background. This is the reason that it is not defined separately in the CIECAM02 technical report.

The *background* is the environment around the stimulus, “extending for about 10° from the edge of the proximal field in all, or most, directions” (Hunt, 1998 p. 209). Nevertheless, as it was previously mentioned, if the proximal field can be considered as part of the background, then it is assumed that the environment of the background extends from the edge of the stimulus.

The *adapting field* is “everything in the visual field outside the stimulus” (CIE, 2004b). This includes the total environment: the proximal field, the background, and the surround, “and extending to the limit of vision in all directions (Hunt, 1998 p. 209).

The *surround* is “the field outside the background” (Hunt, 1998 p. 209). It describes effects such the surround illuminance, veiling flare from displays, etc (Fairchild, 2005 p. 138).

2.1.4.2. Colour Appearance Attributes

Colour appearance attributes can become a little ambiguous because they describe things that people see. For this reason, their definitions have been given by the CIE International Lighting Vocabulary as following (CIE, 1987).

Brightness is the “attribute of a visual sensation according to which an area appears to emit more or less light”. It describes an absolute attribute of light.

Lightness is defined as “the brightness of an area judged relative to the brightness of a similarly illuminated area that appears to be white or highly transmitting”. It is affected by the illumination and viewing conditions of the area or object.

Colourfulness is the “attribute of visual sensation according to which the colour of an area appears to be more or less chromatic”. “Colourfulness describes the intensity of the hue in a given colour stimulus” (Fairchild, 2005 p. 87). When luminance level is increased, colourfulness is increased as well.

Chroma is defined as the “colourfulness of an area judged as a proportion of the brightness of a similarly illuminated area that appears white or highly transmitting”. “Chroma is likely to change if the colour of the illumination is varied” (Fairchild, 2005 p.87).

Saturation is defined as the “colourfulness of an area judged in proportion to its brightness. For given viewing conditions and at luminance levels within the range of photopic vision, a colour stimulus of a given chromaticity exhibits approximately constant saturation for all luminance levels, except when the brightness is very high”.

Hue is the “attribute of a visual sensation according to which an area appears to be similar to one of the perceived colours, red, yellow, green, and blue, or to a combination of two of them”.

2.1.4.3. Colour Appearance Phenomena

There are many colour appearance phenomena but not all of them have been modelled within a single colour appearance model. Several related studies have been summarised by Fairchild, Johnson et al and Luo et al, which are presented below (Fairchild, 2005; Johnson and Fairchild, 2003; Luo and Li, 2007).

Chromatic adaptation is the most important phenomenon (Johnson and Fairchild, 2003 pp. 149-159; Luo and Li, 2007 pp. 271-273). The human visual system adapts its sensitivity according to the viewing conditions; according to the amount of light in the scene. Apart from the adaptation to the luminance level, the visual system is also adjusted to the overall illumination perceived from a scene which represents the relative perception of colours. So, colour appearance can be preserved under different light sources when an individual is looking them successively. This is highly understandable when considering that a white patch under different illuminants retains a white appearance regardless of the colour of the illuminant. Cognitive mechanisms, such as memory of colour also contribute to chromatic adaptation by provoking an individual’s knowledge of appearance.

The *Hunt Effect* describes how colourfulness and chroma increase when luminance is increased (Fairchild, 2005; Johnson and Fairchild, 2003; Luo and Li, 2007). In bright conditions, colourfulness appears increased and it decreases systematically with the illumination level. Hunt investigated this effect by haploscopic matching experiments applying different viewing conditions in luminance levels.

The *Stevens Effect* shows that the perceived contrast increases when the luminance level in the adapting field also increases (Fairchild, 2005; Johnson and Fairchild, 2003; Luo and Li, 2007). This signifies that dark and light colours will appear darker and lighter respectively as luminance increases. Stevens conducted experiments with neutral patches for which observers assessed brightness using magnitude estimation method under different viewing conditions.

The *Surround Effect* (or else *Bartleson - Breneman equations*) explains the phenomenon where an image’s contrast alters according to the changes from bright to dim and dark surround luminance (Fairchild, 2005; Johnson and Fairchild, 2003; Luo and Li, 2007). Under dark surround, dark image areas appear lighter while lighter areas remain unaffected. Bartleson and Breneman conducted matching and scaling experiments with complex images under 3 different surround luminance levels.

The *Helmholtz - Kohlrausch Effect* describes brightness change caused by an increase in luminance and chromaticity (Fairchild, 2005; Luo and Li, 2007). This signifies that at “constant luminance, perceived brightness increases with increasing saturation” (Fairchild, 2005 p.119).

When a grey scale is displayed at certain chromatic illuminations, the lighter patch will exhibit part of the source’s hue and the darker one will appear the complementary hue of the source. This is known as the *Helson - Judd effect* (Fairchild, 2005; Luo and Li, 2007).

A particular category of phenomena are those linked to the spatial structure of the stimuli (Fairchild, 2005; Johnson and Fairchild, 2003). Related to *simultaneous contrast,* there are also phenomena such as the *crispening* and *spreading. Simultaneous contrast* "causes the colour of a stimulus to shift in colour appearance when the colour of the background changes" (Johnson and Fairchild, 2003 p. 141). *Crispening* describes that perception of colour difference between a pair of stimuli is increased when colour of background is similar to the colour of stimuli (Fairchild, 2005; Johnson and Fairchild, 2003). *Lightness contrast effect* is a type of simultaneous contrast which describes that "perceived lightness increases when colours are viewed against a darker background and vice versa" (Luo and Li, 2007 p.276). This reveals that the impact is detected more in lightness and hue. *Spreading* is simultaneous contrast phenomenon for stimuli with high spatial frequency (Fairchild, 2005; Johnson and Fairchild, 2003).

Bezold - Brücke hue shift and *Abner effect* are similar phenomena concerning hue shift which is caused by changes in luminance and colorimetric purity respectively (Fairchild, 2005; Johnson and Fairchild, 2003). The former refers to hue shift when luminance changes and the latter to hue shift when white is additively mixed with monochromatic light.

2.1.4.4. Parametric Effects

In the 1993 CIE report for parametric effects, many effects had been reviewed (CIE, 1993). These are divided into observer based uncertainty and various physical parameters. Moreover, guidelines for the investigation of parametric effects has been defined by CIE to coordinate research based on five proposed colour centres (Robertson, 1978). Observer based uncertainty effects are considered the variation of colour matching, the duration of observation, and the variation of judgements of constant stimuli. Physical parameters are as following: (1) sample size, (2) sample separation, (3) texture, (4) colour of background, (5) luminance level, (6) surround lightness and colour, (7) method of observing, and (8) size of colour difference. So far, only four have been determined of having the greatest impact and being modelled within the latest colour appearance models; see section 2.1.5.1 (Luo and Li, 2007; Fairchild, 2005).

There are various studies that have investigated the parametric effects. Cui et al (Cui et al., 2001b) conducted a series of experiments using a CRT display. Sample size, background, frame of sample, width of separation and colours of separation were the factors taken into account. Grey scale method was used for the psychophysical experiments and the group of observers gave acceptable variation. The performance of colour difference formulae was evaluated and chromaticity ellipses for different datasets were compared. One of the essential findings was that there was small difference between surface stimuli and simulated surface stimuli on display (Cui et al., 2001b). However, it should be noted that the sample arrangement and positioning was the same as they would have usually been presented in surface mode. As far it concerns the parametric effects studied, it was found that the frame had the largest effect in colour difference, with impact about 23%. The smallest effect was given by the separation's colour and sample size with an impact about 6% and 7% respectively. Moreover, the width of separation gave a range of 8% to 18 % impact according to changes from grey to no colour of separation. A change into a coloured background affected the results by 14% between achromatic and coloured backgrounds which was constant. Finally, it was concluded by the researchers that perceive colour difference is primarily affected by changes in lightness and chromaticity.

Finally, in another series of studies by Xiao et al, it was also found that the experimental setup affects the colour appearance (Xiao et al., 2011; Xiao et al., 2010).

2.1.5. Colour Appearance Models

Colour appearance models are mathematical models that integrate the following elements: (1) colorimetric features (uniform colour space and colour difference formula); (2) predictors of colour appearance attributes, (3) chromatic adaptation; and (4) luminance adaptation (Fairchild, 1995; Fairchild, 2005). Over time, several colour appearance models have been developed such as Hunt's model, LLAB, RLAB, Nayatani's et al. model, and CIECAM97s (CIE, 2004b; Fairchild, 2005; Hunt, 1998; Luo et al., 1996). The CIECAM97s was a key stage in the development of colour appearance models. It was the first appearance model to be published by the CIE and it included the latest advancements of the time (Fairchild, 2005).

2.1.5.1. CIECAM02 Model

CIECAM02 is an evolution of the CIECAM97s and CIE's current recommendation for colour management systems (CIE, 2004b). Its basic structure is illustrated in Figure 2.1-2. The CIECAM02 model was tested in order to ensure improved chromatic adaptation, improved performance in its lightness scale, improved prediction of chroma for the almost neutral colours, gamut volumes that represent surround conditions and improved fitting of saturation results (Luo and Li, 2007 p. 271). CIECAM02's complete forward model and colour difference formula are given in Appendix B.

The input parameters to the model are listed as follow: (1) XYZ values of stimulus; (2) XYZ values of white point; (3) luminance of adapting field L_A; (4) luminance of background Y_b; and (5) surround conditions. L_A and Y_b are often reduced by 20% and they are used in cd/m^2 (CIE, 2004b). Reduction is normally applied for stimuli viewed inside a viewing cabinet. Moreover, luminance of the background can be directly set equal to 20 cd/m^2 for typical surface mode reference conditions. For surround conditions, coefficients have been defined for dark, dim and average surround to be used in the model as given in Table 2.1-4. To find the corresponding surround condition, the surround ratio must be calculated; which is ratio of luminance of the surround white against the luminance of the medium's white (CIE, 2004b). When surround ratio is zero, less than 0.2 or greater/equal to 0.2, the surround is determined as dark, dim or average respectively. While, the luminance of the adapting field L_A is defined by the white point luminance L_V and the background luminance Y_b as in Equation 2.1-7.

Equation 2.1-7 Equation for the estimation of luminance of the adapting field for CIECAM02 based models

$$L_A = (L_V \cdot Y_b) / 100$$

Table 2.1-4 CIECAM02 surround coefficients

Viewing conditions (surround)	***c***	***N_C***	***F***
Average	0.69	1.0	1.0
Dim	0.59	0.9	0.9
Dark	0.525	0.8	0.8

The colour appearance phenomena modelled in CIECAM02 are listed as follow: (1) chromatic adaptation; (2) Hunt effect; (3) Stevens effect; (4) surround effect; and (5) lightness contrast effect (Luo and Li, 2007). The model processes chromatic adaptation by using the chromatic and luminance adaptation transform CAT02 (CIE, 2004b; Luo and Li, 2007).

The output data are opponent colour coordinates *a and b* and colour appearance predictors; which are: (1) brightness *Q*; (2) lightness *J;* (3) colourfulness *M;* (4) chroma *C;* (5) saturation *s;* (6) hue composition *H,* and (7) hue angle *h*. Some of these output data are used for the calculation of colour difference ΔE_{CAM02}.

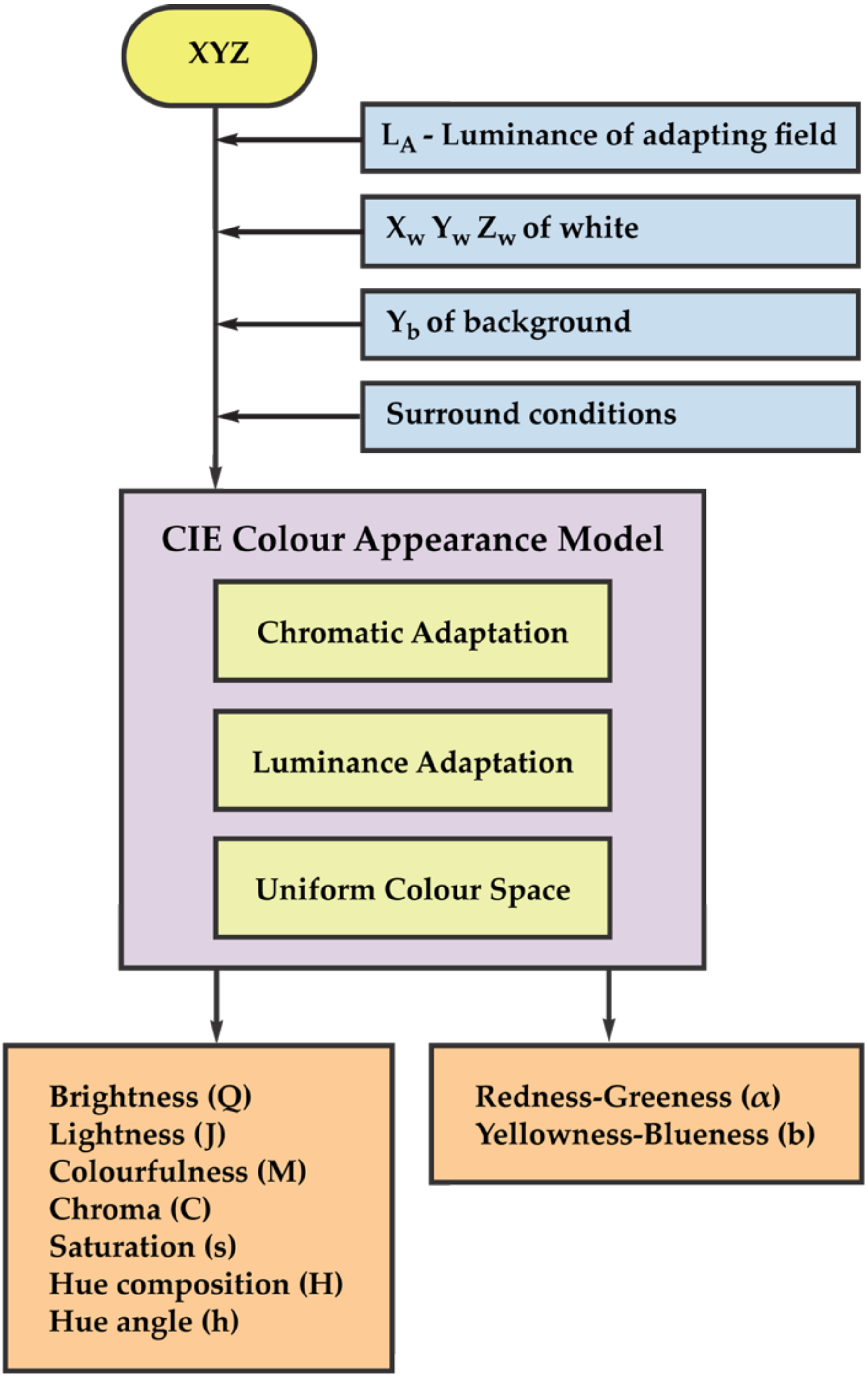

Figure 2.1-2 Structure of CIE colour appearance models

2.1.5.2. CAM02-SCD, CAM02-LCD and CAM02-UCS Colour Spaces

CAM02-SCD, CAM02-LCD and CAM02-UCS are uniform colour spaces for which the abbreviations SCD, LCD and UCS originate from the names of the datasets used to derive them; i.e. small colour difference data, large colour difference data and uniform colour space respectively (Luo et al., 2006). These datasets were used in order to develop colour difference formulae and spaces based on the CIECAM02. CAM02-UCS is considered to perform better in overall, even though it did not outperform the other two. Moreover, CAM02-SCD and CAM02-UCS were shown to be more uniform than CIELAB and CIECAM02.

The opponent colour coordinates *ab* and the colour appearance predictors are calculated using CIECAM02. However, some of them are derived from different equations as seen in Equation 2.1-8. Consequently, the colour difference ΔE_{CAM02} also alters. For each colour space, the same colour difference formula is used but with different coefficients as given in Table 2.1-5

Equation 2.1-8 CAM02-SCD, CAM02-LCD and CAM02-UCS formulae

$$\Delta E' = \sqrt{(\Delta J'/K_L)^2 + \Delta a'^2 + \Delta b'^2}$$

$$J' = \frac{(1 + 100\, c_1)\, J}{1 + c_1\, J}$$

$$a' = M' \cos(h)$$

$$b' = M' \sin(h)$$

$$M' = (1/c_2)\ \ln(1 + c_2\, M)$$

Table 2.1-5 Coefficients for the calculation of colour difference ΔE_{CAM02} for the respective uniform colour spaces

	CAM02-LCD	**CAM02-SCD**	**CAM02-UCS**
K_L	0.77	1.24	1.00
c_1	0.007	0.007	0.007
c_2	0.0053	0.0363	0.0228

2.2. Colour Management

The term colour management describes many paradigms and applications. In this section, an insight to colour management will be presented, and a glossary in Appendix A.2 has been compiled based on the most important terminology.

2.2.1. Colour Management Overview

Nowadays, cross-media reproduction is commonplace, whether it concerns personal or professional usage. Cross-media reproduction means the use of a range of media/devices for the reproduction of an original (Kipphan, 2001). However, due to the characteristics and limitations of the media and data used, there is need for colour management procedures so as to reproduce consistent colour across every medium.

There are different levels of colour management according to the user's needs and requirements. These could be categorised as follows: (1) end-users with little or no knowledge about colour management; (2) advanced users like designers and photographers; and (3) professional users with high-end production needs. Nowadays, personal computers have embedded colour management modules to certify that basic colour controlling is conducted for the majority of end-users; for instance, the Windows Color System (WCS) of Microsoft Windows (Green, 2010 p.53). On the other hand, designers and photographers need to see the right colour on their screens. Therefore, it is important to have well calibrated and characterised displays. Finally, high-end users have needs covering a wide range of applications and diverse viewing conditions.

In the 1970s and 1980s, high-end users handled colour in a *closed loop* way (Green, 1999; Sharma, 2004). This means that all the devices (i.e. monitors, software, scanners, printers, etc) had to be from the same manufacturer in order to consistently work (Sharma, 2004 p. 5). These were the closed loop systems in which a single colour conversion for each path was needed; as illustrated in Figure 2.2-1. Unfortunately, these systems had to operate in a fixed workflow.

Figure 2.2-1 Closed loop system (Sharma, 2004)

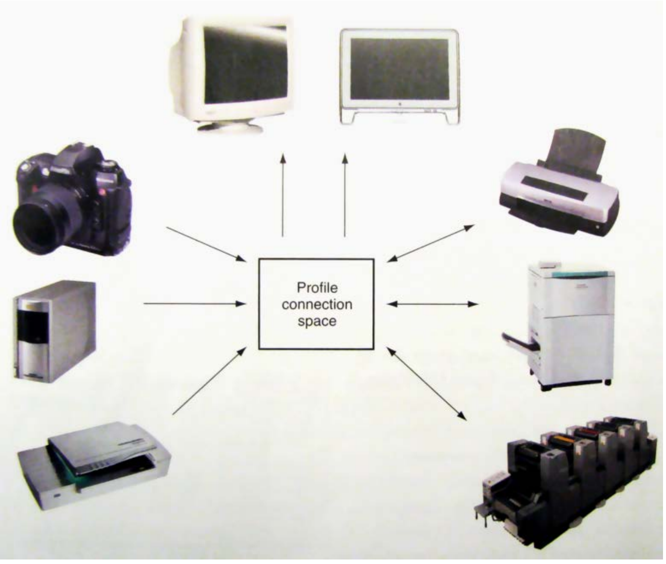

Figure 2.2-2 Open loop system (Sharma, 2004)

Technological evolution has introduced desktop publishing, different image sources and a variety of printing devices; which required inter-operability between devices of different vendors (Green, 1999; Sharma, 2004; Wallner, 2002). Therefore, *open systems* were established and as a result the need for colour management. The design of open systems needs a colour management system in order to perform colour transformations between the different devices by using an independent colour space as a hub; see Figure 2.2-2 (Green, 1999 p. 167; Fraser et al., 2005 p. 82). So, there is a need for a device profile to describe the colour reproduction behaviour of the device and a colour space for the transformations. The basic CMS architecture is illustrated in Figure 2.2-3. Calibration of the device is also critical in order to remove any uncertainties that could debase the system.

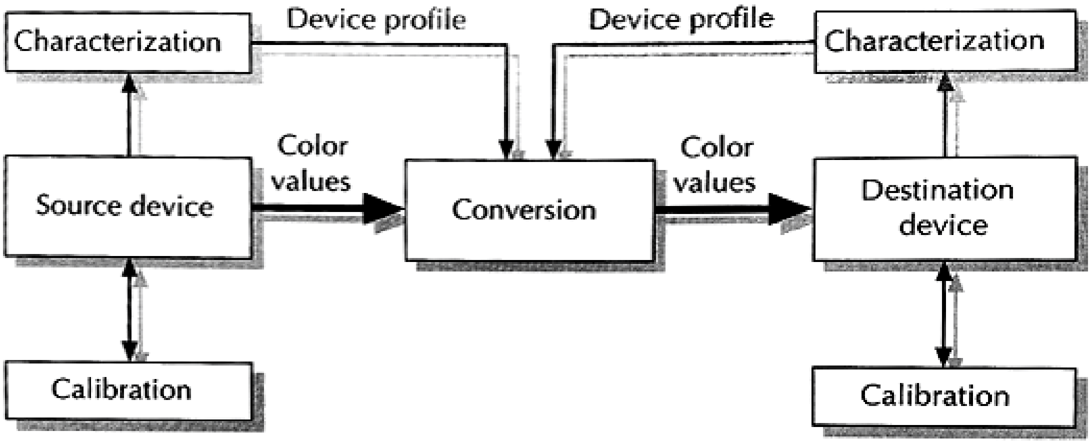

Figure 2.2-3 Basic colour management system architecture (Green, 1999 p.167)

In the beginning, manufacturers of colour imaging systems developed their own colour management systems, but it was soon realised that there should be a common framework and a device description profile format (Green, 1999 p. 168). For this purpose, the International Color Consortium (ICC) association was launched by a group of manufacturers in order to coordinate and set the architecture of the these systems.

2.2.2. Colour Management Steps

The procedure for the implementation of colour management depend on the type of media; i.e. analogue or digital imaging systems (Giorgianni et al., 2003). However, among the important factors to set, there are three vital steps that should be configured according to the given workflow: (1) calibration; (2) characterisation; (3) colour encoding (Fraser et al., 2005; Giorgianni et al., 2003; Sharma, 2004). These steps help so as to accomplish proper colour transformations and as a result successful colour management.

Calibration is the process by which the device is adjusted to its most optimum working condition or to the required settings for a given workflow (Bala, 2003 pp. 272-273). This is done in order to keep the device in a known state and so as to maintain the same quality as long as the device is used under these specific settings. Sometimes, measurements are taken in order to create correction functions for required features. In the long term, by calibrating again to this condition may correct problems created by instability of the device. A classic example of calibration is to balance the rendering of a greyscale on display by controlling appropriately each RGB channel in order to create a tone reproduction curve for each signal. However, depending on the case and equipment, re-rendering the greyscale could result in luminance discrepancies, which can result in chromaticity shift.

Characterisation is the process by which “the relationship between device-dependent and device-independent colour representations for a calibrated device” is described (Bala, 2003 p. 273). According to the cross-media reproduction system used, characterisation can refer to the building of a descriptive profile (Sharma, 2004 p. 34). Characterisation models are more difficult to make and are always comprised of a *forward and inverse model* (Bala, 2003 pp. 273-275). The forward model “defines the response of the device to a known input”, while the inverse model finds an equivalent for an unknown input in order to get appropriate corrected response. The forward and inverse models operate differently for input and output devices. Forward models can be obtained either physically by describing the way that colour is handled from the device; or empirically by building a mathematical function or interpolation relationship.

Colour encoding can be explained as the appropriate digital colour representation in order to undertake correct colour conversion from one colour space to another in a given workflow (Giorgianni et al., 2003 p. 240-244; Sharma, 2004 p. 34). Colour encoding is essential in digital imaging systems as it determines the colour transformations from an input device to an output device (Giorgianni et al., 2003). A simplified illustration of a colour imaging system is given in Figure 2.2-4. Since modern imaging systems are combined in many ways between each other, colour encoding specification should be defined to determine the attributes of the transformation. An example of colour encoding specification is given in Figure 2.2-5. In digital imaging systems, this colour encoding is applied by a software application (e.g. Photoshop), a system-level software (e.g. WCS) and a CMM (e.g. Adobe CMM) (Sharma, 2004 p. 35). Different types of colour management systems require different methods of colour encoding (Giorgianni et al., 2003 pp. 242-244).

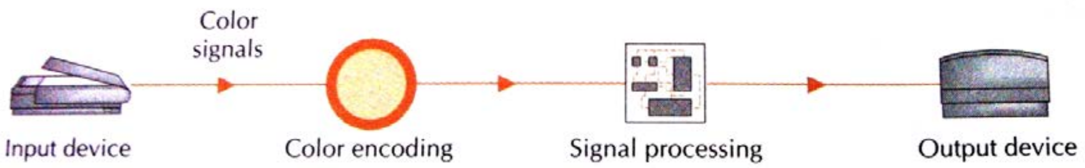

Figure 2.2-4 A simple colour imaging system (Giorgianni and Madden, 1998 p.182)

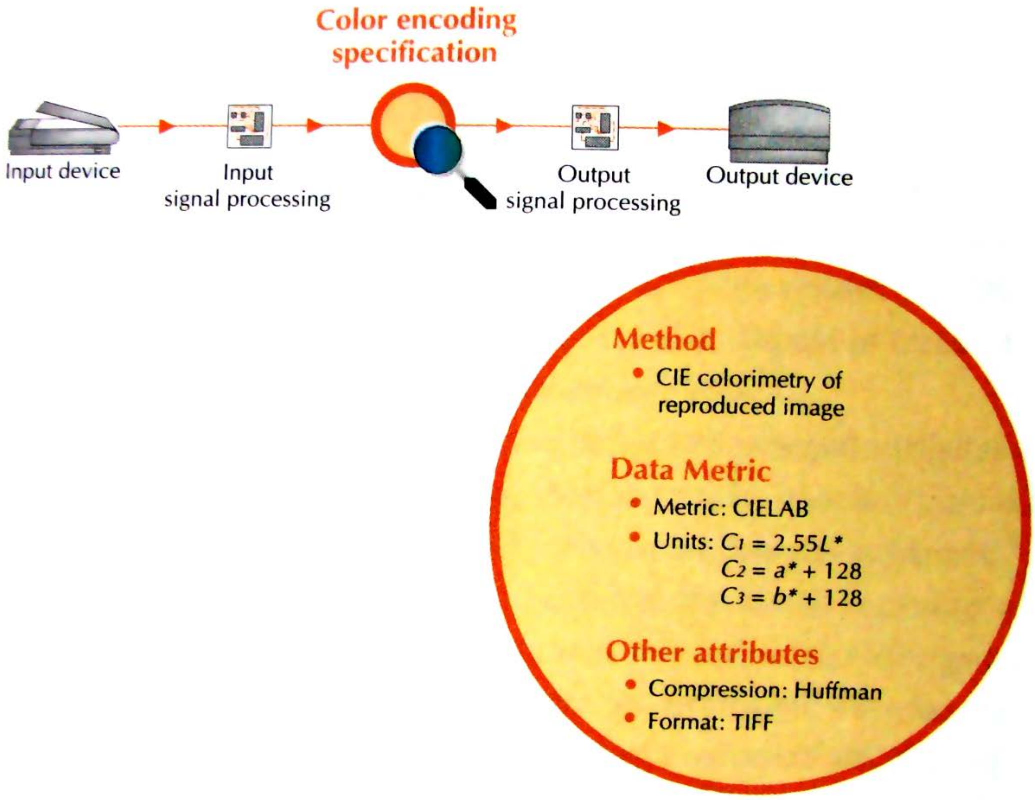

Figure 2.2-5 Example of colour encoding specification (Giorgianni and Madden, 1998 p.188)

2.2.3. Colour Management Paradigms

There are three types of colour management paradigms according to the purpose of the colour imaging system (Giorgianni et al., 2003 pp. 242-244). Firstly, there is the “input-driven” paradigm; where the goal is the output image to match the input image. Such an example is colour copiers where copied image is expected to be similar to the original. Secondly, there is the “encoding driven” paradigm; for which colour encoding is based on colorimetric decrease of colour differences caused by the characteristics of the input devices. For instance, colours in prepress systems might be encoded based on the characteristics of a specific printing medium. Additionally, the ICC colour management system operates according to this paradigm. Finally, there is the “output driven” which also has colorimetric base but the goal is to match other output or to exploit capabilities of the output device for optimal results. This implies a colour re-rendering which mainly does not represent the actual colour encoding. Applications of this paradigm are proofing and digital photofinishing systems.

2.2.4. Device Characterisation

Characterisation describes the performance of a device and it must occur after calibration. The correlation of calibration and characterisation is clearly illustrated in Figure 2.2-6. Over the years, many characterisation models have been developed in order to deal with the market’s needs, various technologies and media, as well as the evolution of colour appearance models. For example, characterisation models that “perform error minimisation in colour appearance coordinates” (Green, 2010 p. 8).

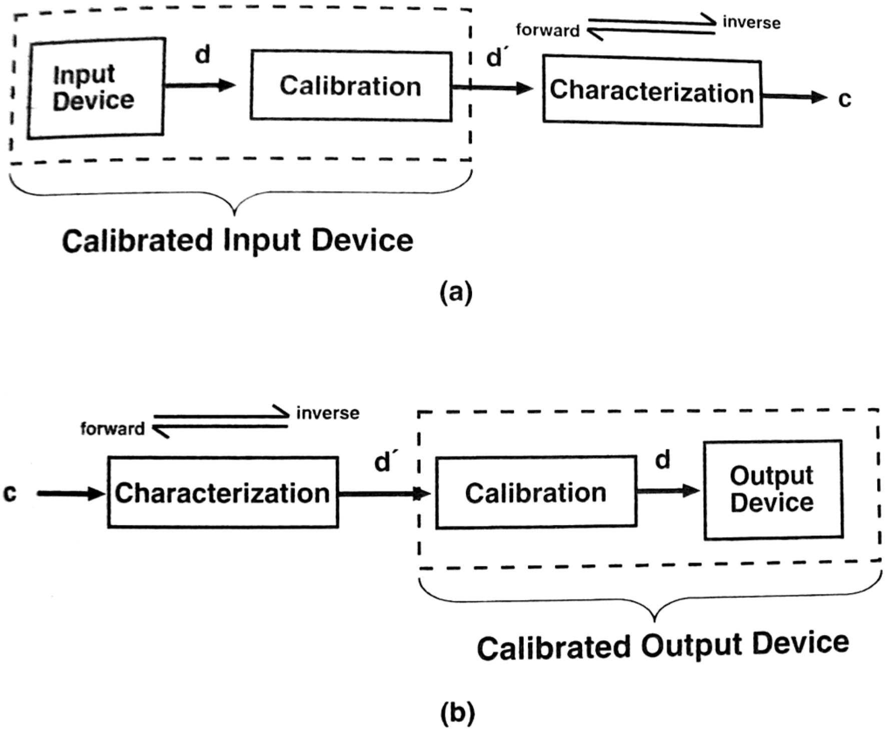

Figure 2.2-6 Calibration and characterisation process for input and output devices (Bala, 2003 p.273)

2.2.4.1. Display Characterisation

There are different display technologies and these could be roughly categorised into cathode ray tube (CRT), liquid crystal display (LCD), plasma and LED panels. The characteristics of each display technology have led to the development of various characterisation models in order to meet the requirements and criteria of appropriate reproduction.

The characterisation models can be divided into analytically invertible, not analytically invertible and look-up table methods (Thomas and Hardeberg, 2013; Thomas et al., 2008). In the first category, there are physical models which often work under specific assumptions. Such models are the piecewise linear assuming chromaticity constancy model (PLCC), GOG and GOGO. While in the non-analytically invertible models belong numerical models which need optimisation of measured parameters to work. Some examples are the piecewise linear assuming variation in chromaticity (PLVC) and S-curve polynomial functions. Finally, for the look-up tables, there are standardised methods such as the ICC profiling system, and different types of interpolation methods among the look-up data. Due to interpolation errors, three-dimensional lookup table models tend to give larger colour difference between measured and predicted CIELAB values for dark neutral colours than for lighter neutral colours (Kanamori, 2001). In trilinear interpolation, abnormal results might also occur (Weed and Cholewo, 2003).

The PLCC model is based on an approximation of an *f* function by applying a linear interpolation between measurements (Post and Calhoun, 1989). This is followed by a colorimetric transformation between the luminance responses of the primaries and a matrix of their chromaticities. However, this method is based on the assumption of having absolute additivity and chromaticity constancy. This model was applied in this study with the addition of black point correction during the process.

2.3. Visual Psychophysics

2.3.1. Introduction to Psychophysics

Gustav Theodor Fechner was the first to specify psychophysics as “an exact science of the functional relations of dependency between body and mind” (Torgerson, 1958 p.v). Psychophysical methods are used in order to put subjective attributes into units of measurement (Fairchild, 2005 p.36). For the investigation of colour appearance, visual experiments are necessary so as to examine human perception.

Humans tend to judge an aroused stimulus of a reproduction copy by comparing it with a memory stimulus of an original (Hunt, 2004 p.32). In Figure 2.3-1, Hunt explains diagrammatically this procedure of comparison. However, there are variations in the memory stimuli due to different viewing conditions at each observation situation. Such variations can affect hue, lightness and colourfulness.

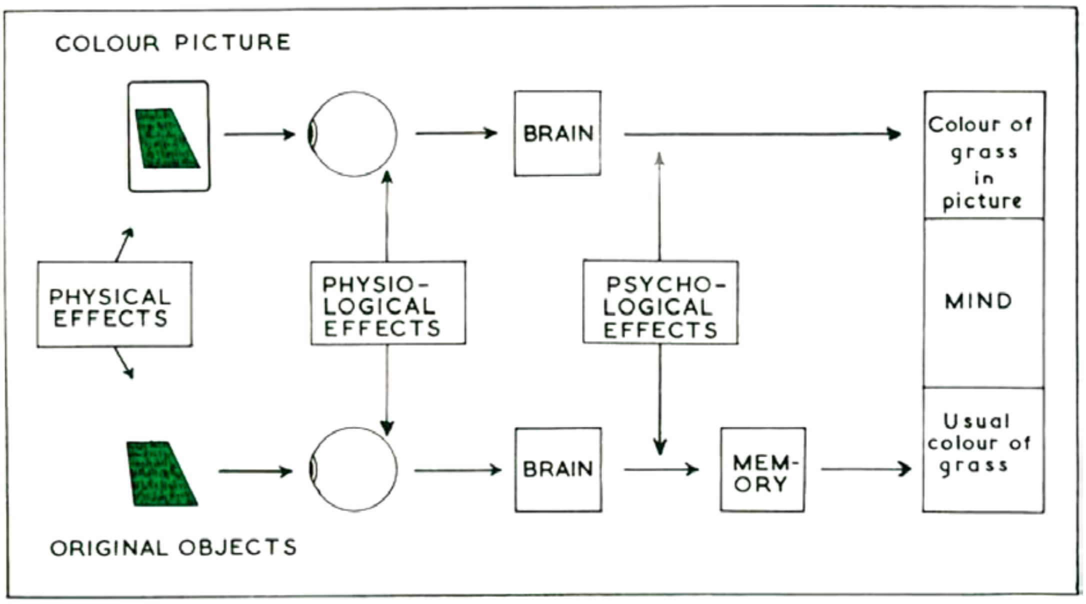

Figure 2.3-1 Visual appreciation mechanisms of human perception (Hunt, 2004 p.33)

Visual adaptation is called the function by which “the visual system adjusts its operating level to the prevailing light level” (Norton and Corliss, 2002 p.77). The visual system adapts to the light conditions because of the eyes’ characteristics: (1) the photoreceptors' operation, i.e. rods and cones function; (2) the ability to modify the concentration of photopigments; and (3) the ability to alter its sensitivity, i.e. neural responsiveness (Norton and Corliss, 2002 pp.76-77). In particular, dark adaptation is the "decrease in threshold luminance [increased sensitivity] as a function of time in darkness" (Norton and Corliss, 2002 pp.77-78). Experiments have shown that after changing from an adapted lighting condition to a dark one, there is a fast reduction of threshold luminance in the cones which tends to stabilise within the first 3 minutes. Then, until the 11th - 12th minute, there is little reduction and afterwards "the cones approach their lowest threshold level". This is also illustrated in Figure 2.3-2.

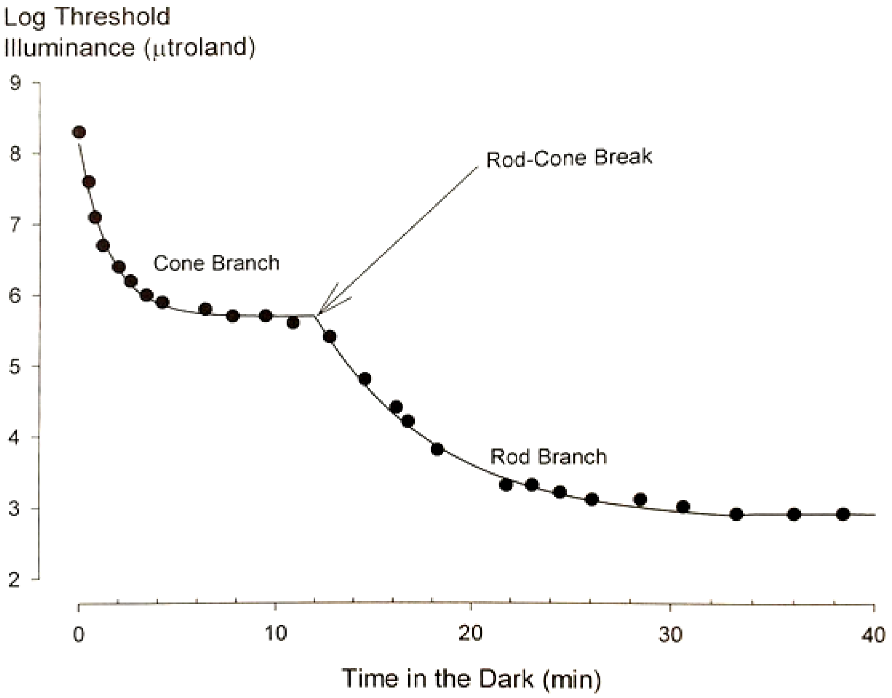

Figure 2.3-2 Typical dark adaptation curve (Norton and Corliss, 2002 p.78)

The visual system has the ability to detect and identify objects from patterns of light and dark (Norton et al., 2002 pp.138-141). This indicates that in terms of spatial vision, humans can easily spot differences in luminance. Relative luminance describes these differences by comparing the reflected light from one area against another area. Because this ratio stays constant under every luminance level, it is easily recognisable by the visual system.

So the most important factor in this process is the relative luminance. A phenomenon characterised by this principle is simultaneous contrast.

2.3.2. Psychophysical Methods

As previously mentioned, psychophysical methods have been developed in order to assign numerical values to visual perception. There are several psychophysical methods designed for visual experiments relating to the investigation of colour appearance and image quality. Terms for the most common psychophysical methods are given in Appendix A.3.

Psychophysical experiments can be divided into four basic categories: (1) thresholds and matching; (2) measuring differences; (3) direct ratio scaling; and (4) multidimensional scaling (Bartleson and Grum, 1984; ASTM, 2003b). Each category consists of methods for investigating different types of attributes. Examples of psychophysical methods are: paired comparison, rank ordering, categorical sort, magnitude estimation, etc. Some methods can even be combined so as to produce more specific results. “The choice of the best method for a particular application may be difficult to make, and interpretation of the rating scales produced by the numerical analyses is frequently ambiguous” (ISO, 2005).

There are also two viewing techniques; having the observer’s eyes under the same viewing conditions or under different ones. For instance, binocular or else haploscopic memory matching where each eye is adapted to different test areas. “The two eyes are situated in slightly different positions, thus receiving slightly different images of the same external objects” (Baird, 1970 p. 209). Each eye is adapted under different viewing conditions and the objective is to determine threshold of stimuli (Bartleson and Grum, 1984).

2.3.2.1. Ratio Scaling

In general, there are four types of one-dimensional scales: nominal, ordinal, interval and ratio (Engeldrum, 2000; Stevens, 1946). In nominal scales, the numbers are used as a distinguishing mean rather than as a mathematical property; thus they express equality among the numbers. Ordinal scales use descriptions to define the scaling. Thus, observers monotonically evaluate an attribute in terms of greater than or less than the specified attribute. Interval scales are like ordinal scales with the difference that equal distances within anywhere in the scale have the same significance. Finally, ratio scales are similar to interval scales but with a zero point or origin. The Table 2.3-1 summarises the transformations that can be applied in the described scales.

Table 2.3-1 Stevens' classification of scale types and possible transformations

	Permissible Transformations
Nominal	$y = f(x)$, any one-to-one transformation
Ordinal	$y = g(x)$, any monotonic transformation
Interval	$y = a\,x + b$, any linear transformation
Ratio	$y = a\,x$, any constant scale factor

"Psychometric scaling is the generation of rulers" used to measure the human response (Engeldrum, 2000 p. 43). In colour psychophysical experiments, there are various techniques that produce ratio scales; i.e. ratio estimation, ratio production, magnitude estimation and magnitude production (Wyszecki, 1982). In ratio estimation, the observer evaluates a test stimulus by referring to a standard stimulus. While in ratio production method, the observer has to adjust the test stimulus so as to produce a predefined ratio between a given standard stimulus and the test stimulus. Similarly, in magnitude estimation method, the observer evaluates the perceived magnitude of a test stimulus for perceived attributes such as colourfulness. Whereas, in magnitude production, the observer produces a magnitude in order to match the magnitude of a given attribute.

In this study, ratio estimation is used. The observer is given two pairs of stimuli. The one pair is a standard colour pair with predefined colour difference of one unit and the other one is a testing pair whose colour difference is to be evaluated in terms of a ratio against the standard pair (Elamin, 1983). This has been used by other studies for colour discrimination research (Cheung and Rigg, 1986; Elamin, 1983).

2.4. Colour Discrimination Ellipses

Colour discrimination ellipses were introduced by a series of experiments by MacAdam in which the main purpose was to define the matching accuracy of an observer in different locations of the CIE 1931 chromaticity diagram (MacAdam, 1942). The ellipses were plotted as distances of standard deviations based on the assessments.

Since then, ellipses have been extensively used for colour difference evaluation as they represent the visual perceptible colour difference. The datasets described in section 2.1.3 have been employed for the optimisation of colour difference formulae in many cases. Description of the MacAdam experiment, derivation of colour discrimination ellipses and other related studies are discussed in this section.

2.4.1. Geometrical Properties of Ellipses

An ellipse is defined as a set of all points in a plane (Sullivan, 2012; Young, 2007). The sum of their distances from two fixed points is constant. The two fixed points are called foci and the line crosses them is the major axis. The midpoint of the major axis is the centre and the line which crosses the centre and is perpendicular to the major axis is the minor axis. The two points that intersect with the major axis are the vertices of the ellipse. These attributes are illustrated in Figure 2.4-1.

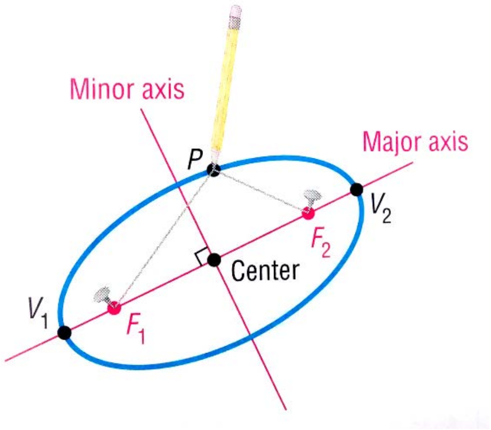

Figure 2.4-1 Graphical representation of ellipse

Commonly, to describe the characteristics of an ellipse the following parameters in a plane are used: the semi-major axis (A), the semi-minor axis (B), the ratio of the semi-major and semi-minor axes (A/B), and the orientation angle (θ) from the x axis.

2.4.2. The MacAdam Experiments on Visual Sensitivities

MacAdam used a custom made colorimeter to conduct a colour matching experiment (MacAdam, 1942). The targeted colours were attained by using layers of filters which were uniformly illuminated by a single light source which approximated the standard illuminant C, i.e. average daylight. The testing field was displayed at 2º and the surrounding field at 42º. The luminance of the testing and surrounding field was 15 millilamberts and 7.5 millilamberts respectively. This approximately corresponds to luminance of 48 cd/m^2 and 24 cd/m^2 respectively. The testing field was placed in the centre of a bipartite arrangement, and therefore it was divided into the reference and matching components by a vertical biprism edge. By using an artificial pupil, the observer viewed the fields monocularly.

Ellipses were derived from the standard deviations of the numerous colour matching observations of the specified colour centres as they appear in Figure 2.4-2, enlarged 10 times (MacAdam, 1942). Because these were found to have systematic variation within the xy chromaticity diagram, it was later shown that they can be resulted by differential geometry and each ellipse can be represented by the Equation 2.4-1, where dx and dy are the differences for each pair of centre and any point in chromaticity coordinates, and g_{11}, g_{12}, and g_{22} are constant coefficients (MacAdam, 1943). This is a bivariate equation, so it only describes the chromaticity, since luminance factor was fixed. In later studies of Silberstein with MacAdam, more coefficients were derived in order to combine the luminance factor in the original equation (Silberstein, 1946; Silberstein and Macadam, 1945). Therefore, differences can also be represented in ellipsoids and described by the Equation 2.4-2. In these studies, the six coefficients were defined and indicate the probability of a match in the area around the colour centre in terms of xyz coordinates. Furthermore, in a study of Brown and MacAdam, it was shown that these equations can also be applied to primaries RGB values, tristimulus values XYZ and chromaticity-luminance attributes (Brown and MacAdam, 1949). In a later work, the equation was adapted for CIELAB colour space as well resulting in the Equation 2.4-3 (Melgosa et al., 1997).

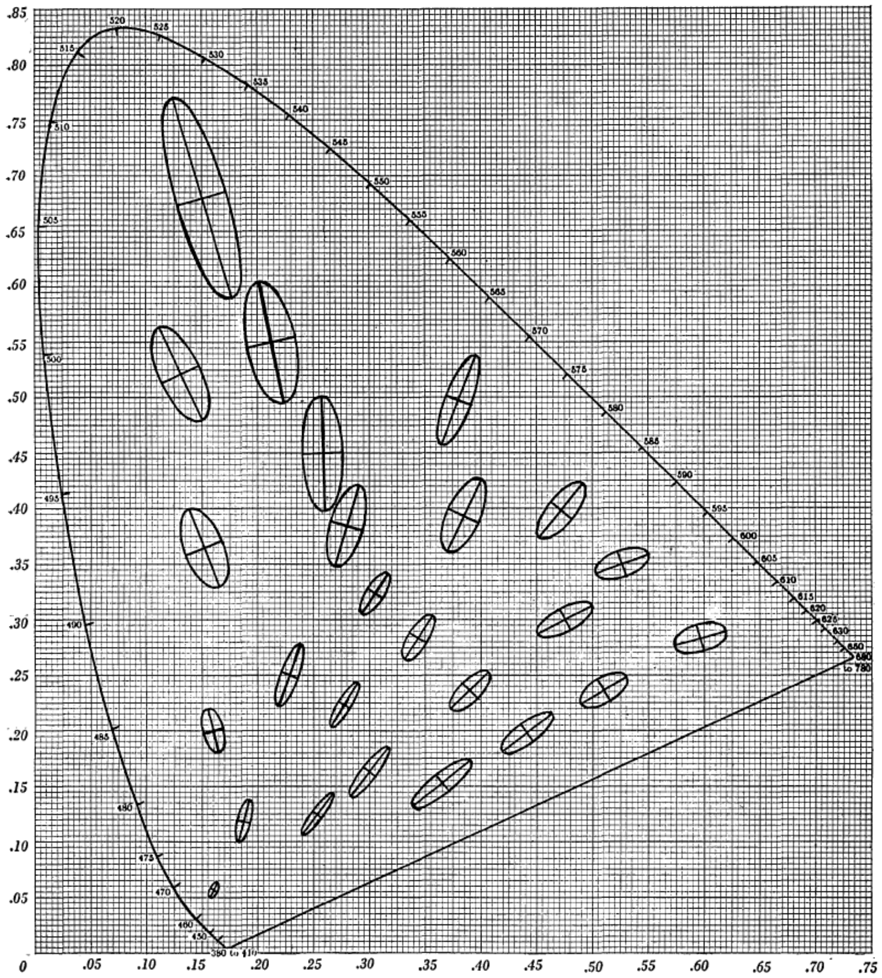

Figure 2.4-2 MacAdam colour discrimination ellipses in the CIE 1931 chromaticity diagram (MacAdam, 1942)

Equation 2.4-1 Representation of the known standard deviations of the MacAdam ellipses

$$1 = g_{11}dx^2 + 2g_{12}dx\,dy + g_{22}dy^2$$

Equation 2.4-2 Representation of colour difference for ellipsoids in chromaticity and luminance

$$\Delta S^2 = g_{11}\Delta x^2 + 2g_{12}\Delta x\,\Delta y + g_{22}dy^2 + 2g_{23}\Delta y\,\Delta Y + g_{33}\Delta Y^2 + 2g_{13}\Delta x\,\Delta Y$$

Equation 2.4-3 Representation of colour difference for ellipsoids in CIELAB values

$$\Delta E^2 = b_{11}\Delta a^{*2} + 2b_{12}\Delta a^*\Delta b^* + b_{22}\Delta b^{*2} + 2b_{23}\Delta b^*\Delta L^* + b_{33}\Delta L^{*2} + 2b_{13}\Delta a^*\Delta L^*$$

There is great variation in size and orientation in the ellipses resulted from MacAdam experiment. This shows that the just distinguishable colour difference seen from an actual observer has varied tolerance in the different areas of the xy chromaticity diagram. The derived ellipses in Figure 2.4-2 indicate the just noticeable chromaticity difference. The different orientations and sizes indicate the different magnitudes of visual colour difference across the xy chromaticity diagram. For instance, there is larger tolerance in the green area but very small in the blue area. Furthermore, it was also showed that one unit of distance in the xy chromaticity diagram does not respond to equal amount of chromaticity difference.

One of the main problematic points of the MacAdam experiment is that the results were based on only one observer. Even though, the observer was trained and conducted over 25.000 observations, the data could still be considered lopsided. On another point, the luminance of the surrounding field was based on the chromaticity of illuminant C which is rather obsolete nowadays.

Another point for consideration is that the filters used within the colorimeter were single layers or combinations of many in order to produce the required colours. The colorimetric data and transmittance of these filters were calculated mostly by taking measurements with a spectrophotometer. As MacAdam reported, the filters had very low luminous transmittances and as a result, the measurements cannot be guaranteed as sufficiently accurate. Moreover, the spectrophotometric data acquired were for each filter separately and the combined final versions were mostly computed by these discrete measurement data. Additionally, MacAdam compared visually the relative luminous transmittances of the filters by using the colorimeter with different observers. With the addition of an opaque plate, the colorimeter could function as a polarisation photometer. Two filters of adjacent chromaticity were compared side by side. These were heterochromatic comparisons but they were considered satisfactory as having sufficiently similar chromaticity. A systematic variation between the observed and calculated luminous transmittances was found. However, these variations were not compensated. It is a possibility that these systematic variations and the inconsistency in the accuracy of measurements could relate to other variations in the study or to have variously influenced the data.

The observer created the desired matching stimulus by adjusting a single control knob. This means that the matching process was not based on adjusting trichromatic primaries as in other contemporaneous studies (Guild, 1931; Wright, 1941). Therefore, the matching was not based on amounts of reference stimuli (like in the colour matching functions), but variations of colour along straight lines in the xy chromaticity diagram which were intersecting in a common fixed chromaticity centre. These lines were predefined by the available filters and constructed combinations. Hence, the standard deviations that corresponded to the visual colour differences were specified by the distances of the colour centre and the matching point in the chromaticity diagram.

2.4.3. Parameters of Ellipses

In 1943, MacAdam published a paper in which the differential geometry that represents the standards deviations of the initial experiment data was analysed (MacAdam, 1943). MacAdam collaborated with Silberstein to create the equations that can be used to construct the ellipses (MacAdam, 1943; Silberstein, 1938). According to these, the constant coefficients g_{11}, g_{12}, and g_{22} (or alternatively b_{11}, b_{12}, and b_{22}) discussed in the previous section can be calculated by using the parameters of the ellipses as given in Equation 2.4-4 (MacAdam, 1943).

Equation 2.4-4 Calculation of ellipse’s coefficients from parameters

$$g_{11} = \frac{\cos^2\theta}{\mathrm{A}^2} + \frac{\sin^2\theta}{\mathrm{B}^2}$$

$$g_{12} = \left(\frac{1}{\mathrm{A}^2} - \frac{1}{\mathrm{B}^2}\right)\sin\theta\cos\theta$$

$$g_{22} = \frac{\sin^2\theta}{\mathrm{A}^2} + \frac{\cos^2\theta}{\mathrm{B}^2}$$

Similarly, the inverse process is possible in order to calculate the parameters of the ellipses by using the coefficients as given in Equation 2.4-5 (MacAdam, 1943).

Equation 2.4-5 Calculation of ellipse's parameters from coefficients

$$\mathrm{A} = \frac{1}{\sqrt{g_{22} + g_{12}\cot\theta}}$$

$$\mathrm{B} = \frac{1}{\sqrt{g_{11} - g_{12}\cot\theta}}$$

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2g_{12}}{g_{11} - g_{12}}\right)$$

2.4.4. Aperture Mode Studies on the Precision of Colour Matching

After MacAdam's work on visual sensitivities, other important studies in this field area followed. In a later study by Brown and MacAdam colour matching in additive mixture was tested (Brown and MacAdam, 1949). A new binocular colorimeter with three control knobs was used. The testing field was again 2° but the surrounding field was dark and the common light source had chromaticity of CCT 2850 K. The two fields were viewed monocularly, and two observers were recruited in this experiment. Moreover, combined chromaticity and luminance differences were examined, thus visual differences were expressed in ellipsoids. It was found that there was some overall resemblance with the MacAdam's observer data. However, when colour centres were individually checked, the ellipses differed considerably for many occasions.

In 1957, Brown did another experiment in colour matching with twelve observers (Brown, 1957). The same colorimeter used in the MacAdam experiment was employed. The testing field had been altered to 10° matching field and a near white surrounding field with daylight chromaticity. For this study, twelve observers participated. Colour centres with varied luminance were examined and ellipsoids were resulted by weighted averages. Ellipses changed orientation less consistently with the chromaticity when compared with the results of the single observer from the

MacAdam study. Brown also claimed that there is a learning effect by the observers when experiments last for prolonged periods.

In 1971, Wyszecki and Fielder used another colorimeter and arrangement for colour matching (Wyszecki and Fielder, 1971a; Wyszecki and Fielder, 1971b). The 7° testing field with fixed luminance of 12 cd/m^2 consisted of two hexagonal fields of 3° each for the reference and matching parts. A white surrounding field extended to 40° with fixed luminance of 6 cd/m^2. The fields were viewed binocularly and three observers participated in the experiments. Repeatability experiments showed that the matching ellipses of the same observer were considerably not repeatable when obtained from different timed occurrences. So, each observer might result in different ratio of the semi-major and semi-minor axes (A/B) and orientation under different circumstances although the same viewing conditions are maintained. Wyszecki and Fielder also compared the data from all the previous studies and it was concluded that there was in overall resemblance with the Brown-MacAdam and Brown ellipses, but there were significant variations with the MacAdam ellipses. Moreover, even though the experimental arrangement was different, the orientation and shape of the averaged ellipses were not dissimilar from the previous studies. The change was merely tracked on the absolute size of the ellipses.

2.4.5. Fitting of Ellipses

During the years, plethora of algorithms have been developed for the calculation of ellipse’s coefficients. Since it has been shown through differential geometry that just noticeable colour difference can be systematically described by ellipses, these can be calculated by the resulted formulae. To some extent the algorithm depends on the format of the experimental data. The least-square method has often been used in previous studies. To calculate the coefficients, a minimisation process is applied to the sum of the square of differences between the visual data and the calculated colour difference. This is expressed by the Equation 2.4-6. This was further modified by Alder *et al,* as shown in Equation 2.4-7, where *N* is the amount of pairs in a set (Alder et al., 1982). The factor *e* represents an error estimation that was included in order to give a more meaningful quantity. This factor have also been further modified in other studies (Luo, 1986; Luo and Rigg, 1986).

Equation 2.4-6 Least-square minimisation

$$S^2 = \sum_{i=1}^{N} (\mathit{\Delta} V_i - \mathit{\Delta} E_i)^2$$

Equation 2.4-7 Least-square minimisation with meaningful quantity

$$e = \sqrt{\frac{S^2}{N}}$$

In more recent studies, STRESS measure can be used in same way in order to apply the minimisation process (Luo et al., 2015). For this study, this measure was applied and it is introduced in section 2.6.2.

2.5. Lighting Standards

Light emitting diodes' (LEDs) popularity and range of technologies has been increasing the recent years. Nowadays, there are both inorganic and organic light emitting diodes (OLED) with a great variety of semiconductors (Coaton and Marsden, 1997; Khanna, 2014). The key feature of this technology is that electrical energy is converted directly into lightning. The LED technologies have improved over the years and their application has been extended from signs and indicators to indoor lighting products and displays. The LED market and interest is growing because of LED's lower energy consumption, high efficacy and wider range of produced hues of light. Lighting products of this technology have now found applications in traffic lighting, vehicles, flash lights, indoor and outdoor lighting arrangements to name but a few. Even though, specifications for the chromaticity of solid state lightning products exist for centres around the Planckian locus, there is no specification for coloured lightning. Coloured lightning is of high interest because of this LED technology, and the potential to be used in applications such as room lighting design. Moreover, the lighting industry uses a lot the colour discrimination data derived by the MacAdam experiment to specify the chromaticity of their manufactured products; see Figure 2.5-1.

The American National Standards Institute is a private non-profit organisation that develops and publishes standards for products, processes and other methodologies (ANSI, 2015). Among the others, there are also standards for the specification of chromaticity of lightning products. There are norms for different types of lighting products. The ANSI C78.377 standard specifies the chromaticities of solid state lighting products around the Planckian locus (ANSI, 2008). The chromaticities are specified in xy and u'v' chromaticity coordinates, as well as CCT distance from the Planckian locus. Additionally, a Δuv tolerance of ±0.006 between the target and reproduced products is defined. The chromaticities for the solid state lightning products around the Planckian locus are presented in Table 2.5-1, where nominal refers to the communicated CCT information about the chromaticity of a lighting product. Finally, the chromaticities are also represented graphically in Figure 2.5-1.

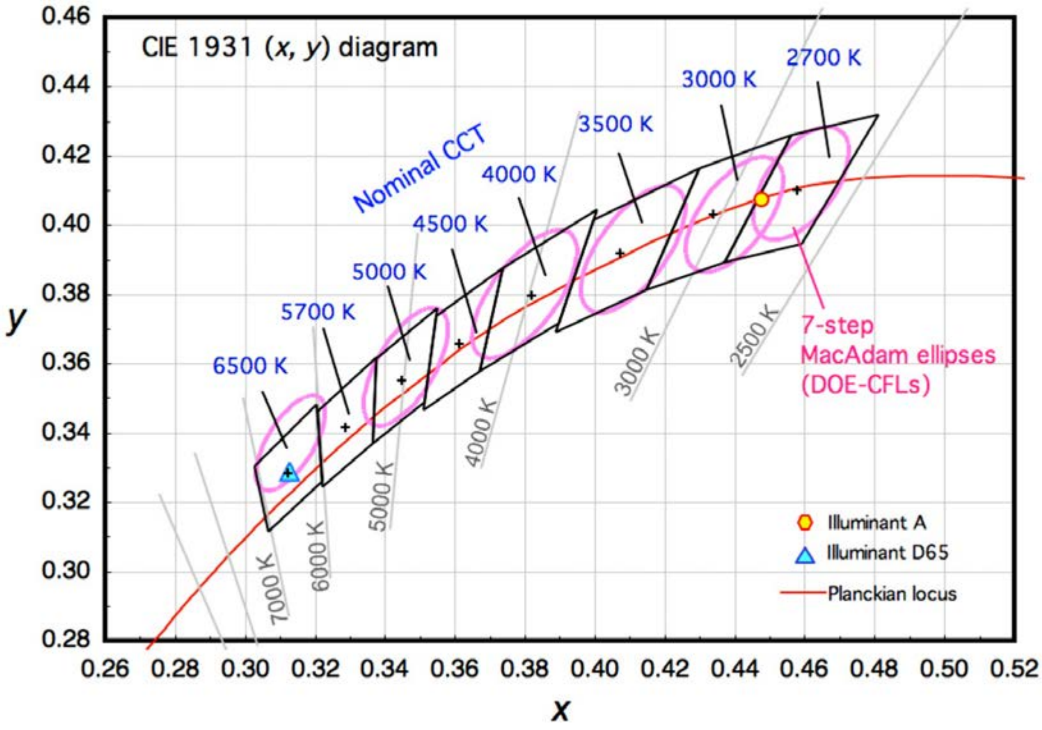

Figure 2.5-1 Graphical representation of chromaticity of SSL products in xy chromaticity diagram (ANSI, 2008)

Table 2.5-1 Chromaticity specification for solid state lighting products

Nominal CCT	***x***	***y***
2700 K	0.4578	0.4101
3000 K	0.4338	0.4030
3500 K	0.4073	0.3917
4000 K	0.3818	0.3797
4500 K	0.3611	0.3658
5000 K	0.3447	0.3553
5700 K	0.3287	0.3417
6500 K	0.3123	0.3282

2.6. Statistical Methods and Measures of Fit

2.6.1. General Statistics

The coefficient of variation (CV) reveals the uniformity among data (Rees, 2001; Sanders and Smidt, 2000). CV is computed according to Equation 2.6-1 where σ is the standard deviation and *μ* is the mean. Then, it is multiplied by 100 in order to give the result as a percentage.

Equation 2.6-1 Coefficient of variation – CV

$$\mathrm{CV} = \frac{\sigma}{\mu} \times 100$$

Confidence coefficient refers to "the probability of correctly including the population parameter being estimated in the interval that is produced" (Sanders and Smidt, 2000 p.246). Once the confidence coefficient has been estimated, the confidence intervals are determined as plus and minus values of the confidence coefficient. Usually, a confidence level of 95% is sufficient; which gives confidence bound *z** value of 1.96 – probability critical value for standard normal distribution. So, 95% confidence interval for known standard deviation *σ* can be estimated using Equation 2.6-2 for population *n*.

Equation 2.6-2 Estimation of confidence interval – CI

$$\text{lower point} \quad \bar{x} - z^{*}\frac{\sigma}{\sqrt{n}}$$

$$\text{upper point} \quad \bar{x} + z^{*}\frac{\sigma}{\sqrt{n}}$$

Correlation coefficient is a statistical method to summarise datasets. It is also known as the Pearson product-moment correlation coefficient (r or R). The correlation coefficient is calculated by the formula in Equation 2.6-3 and for perfect agreement it should result equal to one. An adapted version for colour difference evaluation has been used over the years in order to relate calculated and visual colour difference. However, previous studies have shown that there are instances in which it tends to become quite inconsistent when combined with other statistic measures (Guan and Luo, 1999).

Equation 2.6-3 Correlation coefficient – r

$$r = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{xy}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - n\overline{x}^2)(\sum_{i=1}^{n} y_i^2 - n\overline{y}^2)}}$$

Another important statistical measure that has been used a lot in colour difference evaluation is the performance factor PF/3 (Cui et al., 2001a; Garcia et al., 2007; Guan and Luo, 1999; Luo and Rigg, 1987a). PF/3 actually combines other statistical measures - such as the aforementioned correlation coefficient - into one measure. It was introduced in an attempt to diminish different measures leading to different conclusions. The formula is given in Equation 2.6-4. The statistical measures combined are the: gamma – factor (γ), coefficient of variation (CV), and V_{AB}. These indices have different properties such as range and symmetry for the input data. Moreover, when combined, they do not provide symmetry and cannot indicate statistical significance using *F*-tests.

Equation 2.6-4 Performance factor – PF/3

$$PF/3 = \ 100\left(\gamma - 1 + V_{AB} + {}^{CV}\!/_{100}\right)\Big/_{3}$$

2.6.2. Standardized Residual Sum of Squares (STRESS)

García *et al.* adapted an index of multidimensional scaling for the investigation of the relationship between the perceived and measured colour difference; the standardized residual sum of squares – STRESS (Garcia et al., 2007). They examined the performance of this index against other statistical measurements usually used in colour related research. The method seems to overall perform better than older indexes, such as the PF/3. Moreover, STRESS can be used to estimate the statistical significance of the colour difference for a given set. It is also relatively simpler to apply, given the fact that performance factor PF/3 combines three different statistical measures. Another advantage of STRESS is that it can be used for various types of datasets, such as to compare the correlation of visual responses of different observers for a specific colour sample. Thus, it is gradually implemented in various colour studies.

There are different versions of the formula; which are given in Equation 2.6-5, and it has been proven that they give same results. F_1, F_2 and F_3 are logical scaling factors that minimise the STRESS, ΔE is the measured difference (or other dataset), $\Delta \mathrm{V}$ is the perceived difference (or other dataset), w_i are weights for every pair of samples and f is scaling factor between ΔE and ΔV. The scaling factor *F* gives the slope that illustrates the correlation between the two datasets, and of course scales datasets accordingly. By multiplying STRESS values by 100, results are represented conveniently in percentage range from 0 to 100. In terms of evaluation of STRESS values, zero indicates perfect agreement.

Equation 2.6-5 Standardized residual sum of squares – STRESS

$$STRESS = \sqrt{\frac{\sum (\Delta E_i - F_1\,\Delta V_i)^2}{\sum F_1^2\,\Delta V_i^2}}$$

$$where\;\; F_1 = \frac{\sum \Delta E_i^2}{\sum \Delta E_i\,\Delta V_i}$$

$$or\;\; STRESS = \sqrt{\frac{\sum \left(F_2\,\Delta E_i - \Delta V_i\right)^2}{\sum \Delta V_i^2}}$$

$$where\;\; F_2 = \frac{\sum \Delta E_i\,\Delta V_i}{\sum \Delta E_i^2} = \frac{1}{F_1}$$

$$or\;\; STRESS = \sqrt{\frac{\sum (\Delta E_i - F_3\,\Delta V_i)^2}{\sum \Delta E_i^2}}$$

$$where\;\; F_3 = \frac{\sum \Delta E_i\,\Delta V_i}{\sum \Delta V_i^2}$$

$$Weighted\ normalized\ STRESS = \sqrt{\frac{\sum w_i (\Delta E_i - f\,\Delta V_i)^2}{\sum w_i\,(f\,\Delta V_i)^2}}$$

2.6.3. Colour Uncertainties

In colour science, there are statistics that are related with the evaluation of colour difference repeatability and error, i.e. the dEE and MCDM.

To evaluate the correlation between two pairs of colours, the relative colour difference dEE is calculated. This can be used to indicate the accuracy of reproduction for a pair of colours; therefore it represents the relative accuracy between the targeted and measured pairs. It can show the shift in chromaticity when comparing two pairs of colours and the proportions of this difference. It is computed by calculating the colour differences for both measured and targeted pairs respectively. These can be calculated by using CIELAB values as shown in Equation 2.6-6, where differences subscripted by *M and T* refer to the measured and targeted pair respectively. Each colour difference attribute was calculated with values for each pair accordingly, where the subscripts *C* and *Si* denote the colour centre and each sample respectively.

Equation 2.6-6 Relative colour difference – *dEE*

$$\begin{aligned} dEE &= \sqrt{(\Delta L_M^* - \Delta L_T^*)^2 + (\Delta a_M^* - \Delta a_T^*)^2 + (\Delta b_M^* - \Delta b_T^*)^2} \\ where\ \Delta L_M^* &= L_C^* - L_{Si}^*\ \ and\ the\ rest\ to\ be\ computed\ accordingly \end{aligned}$$

A method for analysing colour difference distribution is the mean of colour difference from the mean – MCDM (Nadal et al., 2011; Berns, 2000). MCDM is practical when stability of the reproduction for a colour is under investigation over a period of time. CIELAB values can be used for calculating the colour differences. The MCDM can be calculated as in Equation 2.6-7, where L_i^* is an individual measurement, $\overline{L^*}$ the mean of all measurements, *N* the total amount of measurements, and accordingly for the rest of the factors.

Equation 2.6-7 Mean of colour difference from the mean – MCDM

$$MCDM = \sum_i \sqrt{(L_i^* - \overline{L^*})^2 + (a_i^* - \overline{a^*})^2 + \left(b_i^* - \overline{b^*}\right)^2} \bigg/ N$$

2.7. Conclusion

The fundamentals for this Ph.D. project were reviewed in this chapter and the following became clear. The experimental design should be based on MacAdam’s colour centres and viewing conditions so as to investigate the fitted ellipses for a larger amount of observers. It was also found that there is lack of a colour evaluation tool for assessing light sources of coloured lighting products. Moreover, MacAdam ellipses have been widely used in the lighting industry despite of the discrepancies found in previous studies. The method of assessing colour differences for accumulating data has been used frequently in the past and important datasets of ellipses have been produced in this way. Therefore, it can be consider a good method for acquiring and examining colour discrimination ellipses for the most relevant colour spaces such as xy, u’v’, CIELAB, and CAM02-UCS.

Chapter 3.
Experimental

An experiment was designed in order to investigate small colour differences in chromaticity of lighting stimuli on a display. Colour centres from the MacAdam colour matching experiment were reproduced under specific criteria. The experiment was designed in order to generate a scale that will express the actual visual chromaticity discrimination. In order to investigate the colour centres as light sources, they were represented on the display against a black background as circular patches. To reproduce the original MacAdam experiment, a neutral background with the same luminance was also assessed.

3.1. Measuring Instrumentation

Spectroradiometers measure irradiance or radiance, i.e. radiometric quantities such as spectral power distribution (Hunt, 1998 p. 100; MacDonald, 1997 p.415). They can be used both for measuring self-luminous and surface stimuli. The most common ones are the tele-spectroradiometers (TSR) which measure radiant flux in distance. Tele-spectroradiometers' key elements are the telescope, the monochromator and the detector. A tele-spectroradiometer was used for colour measuring in this study since this type of instrument is accurate and consistent. Another advantage is that the measured spectral data can be used to calculate many other values. Last but not least, TSR captures colour data at the same viewing conditions as perceived by the observers, so the data can be correlated to the viewing conditions.

A Konica Minolta CS1000S TSR was used and run by the Minolta CS-S1w software. The instrument has wavelength range of 380 to 780 nm, luminance range 0.01 to 80,000 cd/m^2 (for Illuminant A), and repeatability tolerance of 0.1% + 1 digit for luminance and ± x=0.0002, ± y=0.0002 for chromaticity. The TSR was fixed on a tripod at the same viewing conditions intended for the experiments, i.e. distance 50 cm from the screen and dark room conditions (ASTM, 2003a). The instrument was set to measure light sources at 2º observer because of the small angular field of the individual colour patches (CIE, 2004a; ASTM, 2008). Furthermore, in light and lighting applications, light sources tend to be evaluated at 2º observer. The TSR had been calibrated by the manufacturer and it has been tested in the laboratory via the standardised method to ensure its repeatability and reliability. Additionally, a quick test in consecutive measurements using displayed grey tones in the experimental monitor with a second available TSR showed that both instruments had similar and stable colour difference between them.

3.2. Visual Display

Over the years, many technologies of visual display units have been developed for computer systems. The most known ones are: cathode ray tube displays (CRT), liquid crystal displays (LCD), plasma display panels (PDP), and light-emitting diode displays (LED). Nowadays, the market's main interest is in LCD and LED panels, which are used for televisions, computer monitors and mobile devices.

For a display to be used in a visual experiment, it must be evaluated to ensure certain criteria are met and characterised to control colour. The ISO 12646 standard describes some of the test methods that can be applied to evaluate colour displays for colour proofing (ISO, 2008). As for the characterisation method, a one-dimensional lookup table method with colorimetric transformation was applied. The results for both evaluation and characterisation model are presented at the following sections.

The EIZO ColorEdge CG220 was the display used in the experiments (EIZO, 2005). It is a 22.2" display with TFT colour LCD panel with its own calibration software and measurement instrument, i.e. ColorNavigator software and Eye-One instrument. The display has intrinsic native resolution of 1920 X 1200 pixels and it was used as such. Moreover, it has Adore RGB colour space capability and 14-bit processing circuit for rendering colour.

Firstly, the display was calibrated with target white point at 6500 CCT using the display's calibration system. D65 illuminant was mainly chosen as a target in order to resemble the average daylight illuminant used in the

MacAdam experiment. The display was set at 100 cd/m^2, because high luminance was required as much as possible in order to reproduce colour patches as light sources. However, this was the highest stable condition that could be achieved for the specific monitor. Other settings included adjusting the gamma at 2.20 with priority over grey balance (for which the other available option was contrast priority) plus setting the minimum black level. This process was done once and all the experiments were carried with the same calibration profile. That's because the Eye-One is not a high end instrument and might not be repeatable among different calibrations. Therefore, even the slightest change can lead to a different rendering of the greyscale, which consequently could affect the reproduction of all colours. Moreover, ColorNavigator and i1 Display calibrator operate as a black box and therefore are not appropriate for further monitoring or evaluation. After that, stability and evaluation of the display was monitored with TSR measurements.

Validation data from the calibration process are given in Table 3.2-1. The colour difference of the validation data for a basic RGB target scale resulted in ΔE^*_{ab} of 0.97 units and ΔE_{2000} of 0.50 units; with ΔE^*_{ab} 0.38 units and ΔE_{2000} 0.57 units for the gray parts of the scale.

Table 3.2-1 Calibration validation data from ColorNavigator

	Target	***Result***
Brightness	100 cd/m^2	100.2 cd/m^2
Black Level	Minimum	0.34 cd/m^2
White Point	6500 K	6485 K

3.2.1. Evaluation

The results over a set of testing methods and long term stability are presented in this section. For this study, the most important attribute of the display was the long term stability. As small colour chromaticity differences were investigated, it was important to have consistent and constant reproduction of colour. A big variation in reproduction could potentially affect the data collected by the observations. During the course of sample preparation and experiments, the display was measured at the start and end of the workday. An 18 steps greyscale and the basic RGBCMY colours were being measured each time.

A visual inspection of a displayed greyscale gradient was also conducted in order to ensure that smooth transition of colours was possible and there were not any abrupt leaps in the reproduction of luminance by the display.

3.2.1.1. Short Term Stability

Short term stability is important for testing the performance of a display over a day’s use. Measurements were taken in specified time intervals and the colour differences between each interval are plotted against time. Figure 3.2-1 presents the colour differences in CIELAB units for measurements taken every 30 minutes for a total duration of 7.5 hours from the moment the display was turned on. It can be seen that the display has large colour difference during the first periods but it stabilises over time. Moreover, it needs considerable time to stabilise, so this attribute was taken into account during the experiments. For instance, the display was not turned off or experiments did not start until it stabilised at the target conditions.

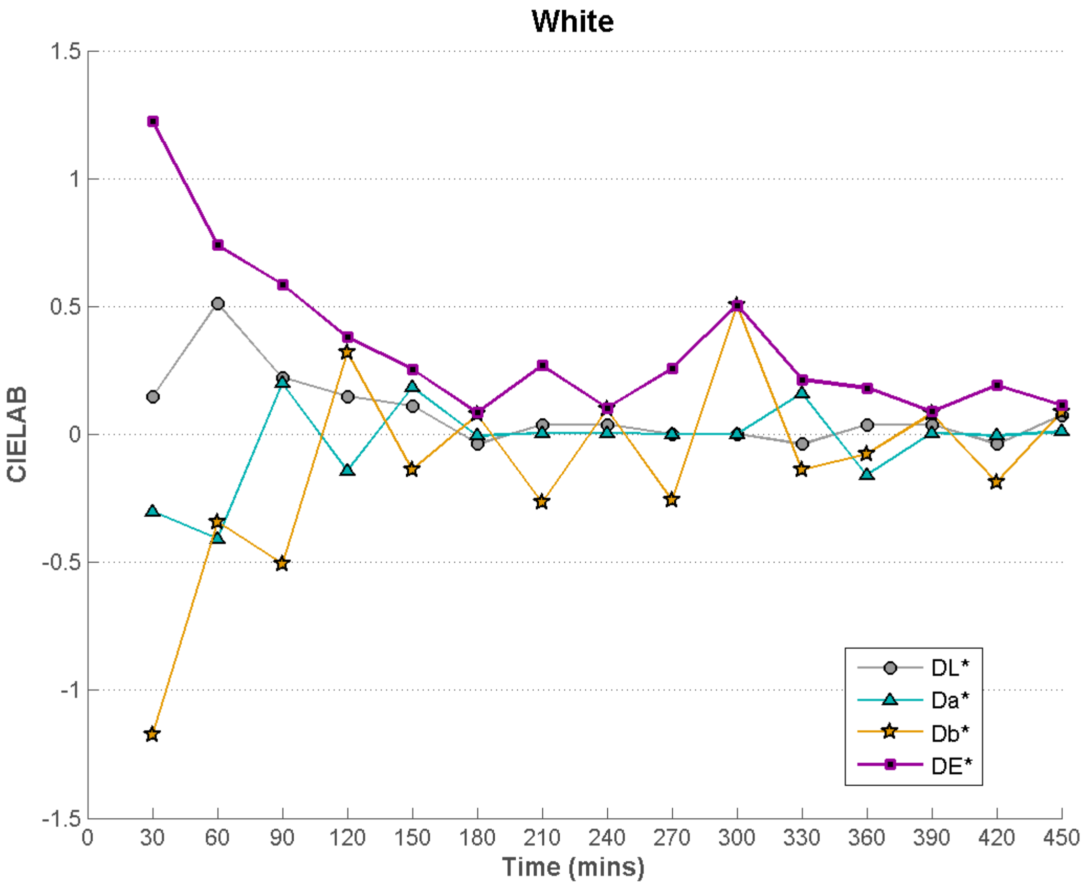

Figure 3.2-1 Short term stability of EIZO display for peak white

3.2.1.2. Medium Term Stability

Before using the display for the experiment, the medium range stability was also examined. For a period of 34 hours, the basic colours were measured every few varied time intervals. The colour differences in CIELAB for this time span were calculated and plotted in Figure 3.2-2 for the peak white. Considering that small fluctuations due to the LCD technology are normal, these colour differences can be considered negligible since they almost have constant value throughout and remain below ΔE^*_{ab} of 0.4 units. It is clear that there is great colour difference only once the display was turned on. But once it was warmed up, it remained stable over long period. The same trend appeared again during the short term stability test.

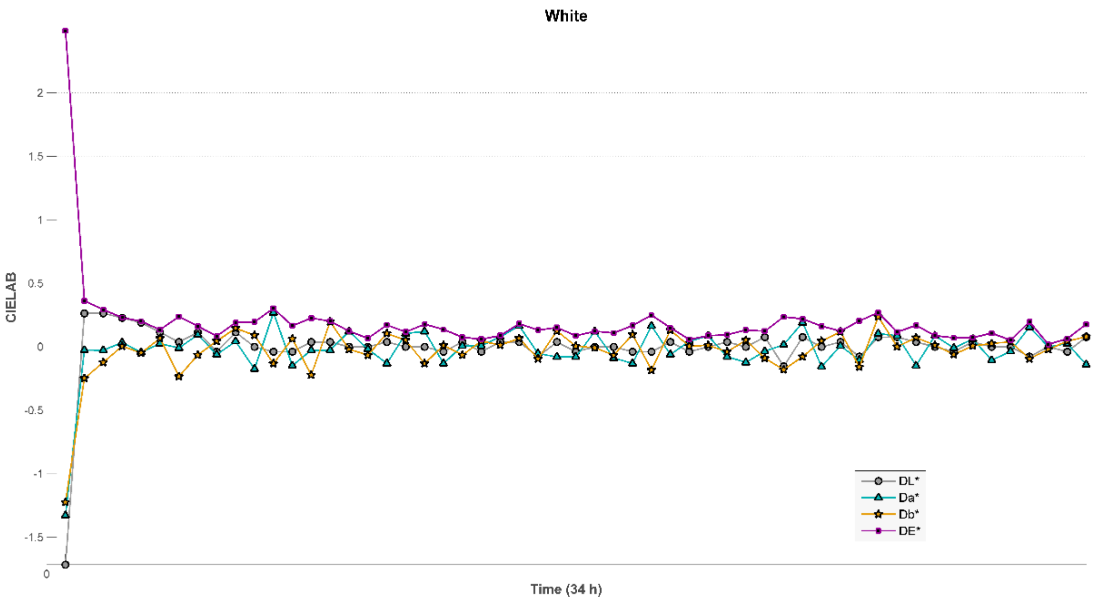

Figure 3.2-2 Medium term stability of EIZO display for peak white

3.2.1.3. Long Term Stability

For the evaluation of the long term stability, the MCDM measure was used as it can show the repeatability of many measurements. The whole set of measurements during the experimental period were used to calculate this metric. A grey scale and the primaries RGBCMY were measured before and after the experiments in a daily basis. Their L*a*b* values were input to a Matlab function so as to store, update and calculate colour difference on daily basis. This process was also important for monitoring whether the display was stable at its initial characterisation state. If the performance of the display were to change, then the reproduction of the experimental pairs would be highly affected. The mean MCDM for the whole experimental period corresponds to 0.25 CIELAB units. This value indicates great long term stability for the display.

3.2.1.4. Uniformity of Luminance and Angular Dependency

The uniformity of luminance examines the variation of luminance at different locations of the display as shown in Figure 3.2-3; defined by the ISO 12646 standard (ISO, 2008). In Figure 3.2-4, the range of luminance measured on these different locations of the display is illustrated accordingly; while the locations of measurement are marked at the centre of the circles. By plotting the luminance in a surface graph, it can be seen that there was some variance but the usable area was within acceptable values.

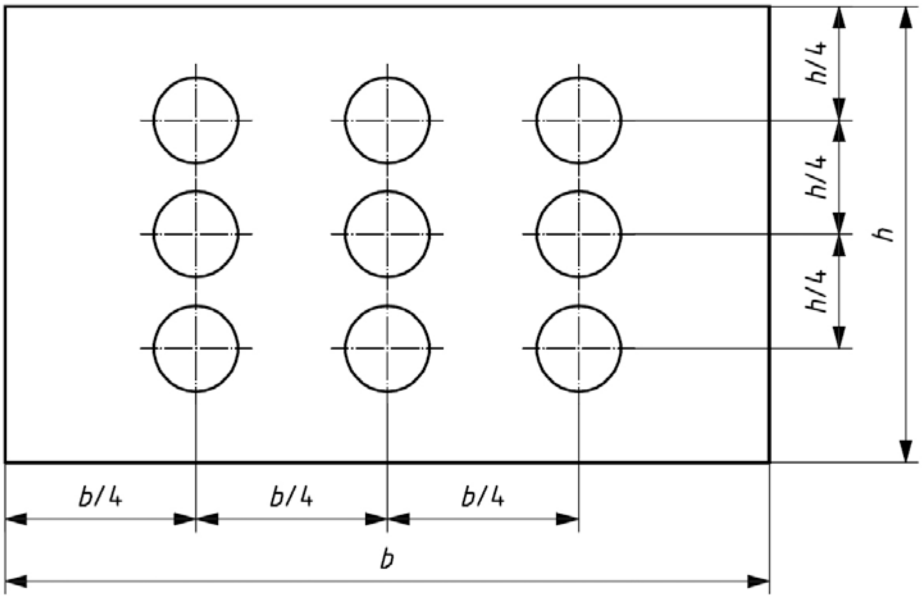

Figure 3.2-3 Positions for measurement of uniformity on the display (ISO, 2008)

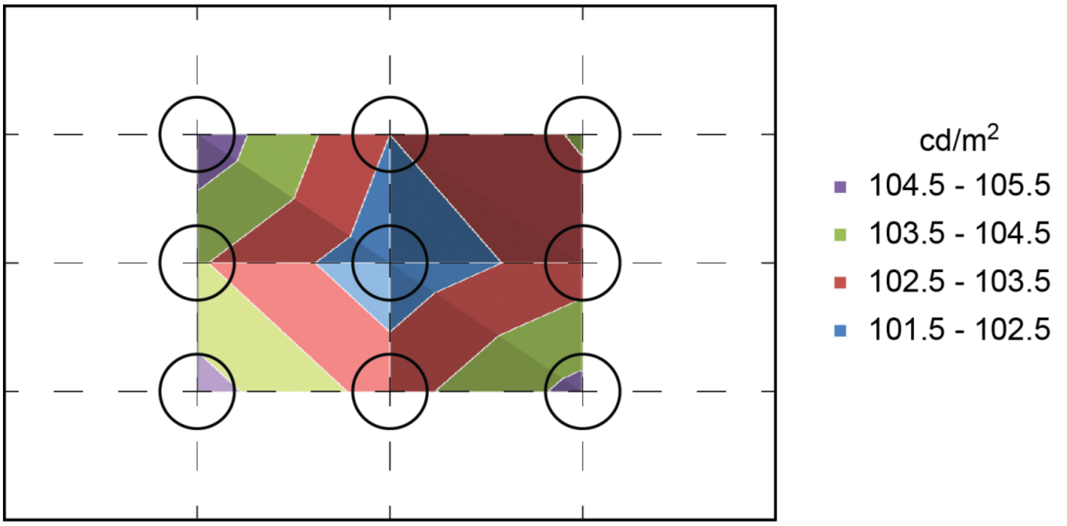

Figure 3.2-4 Uniformity of luminance for EIZO display

The angular dependency describes differences when changing the viewing angle for measuring the same spot. LCD displays are usually angular dependent therefore it needs to be taken into account when colour difference is important. Figure 3.2-5 shows that there was an apparent change in luminance from different angles. Therefore, it was later decided to use a chin rest to maintain position of observer at the same spot.

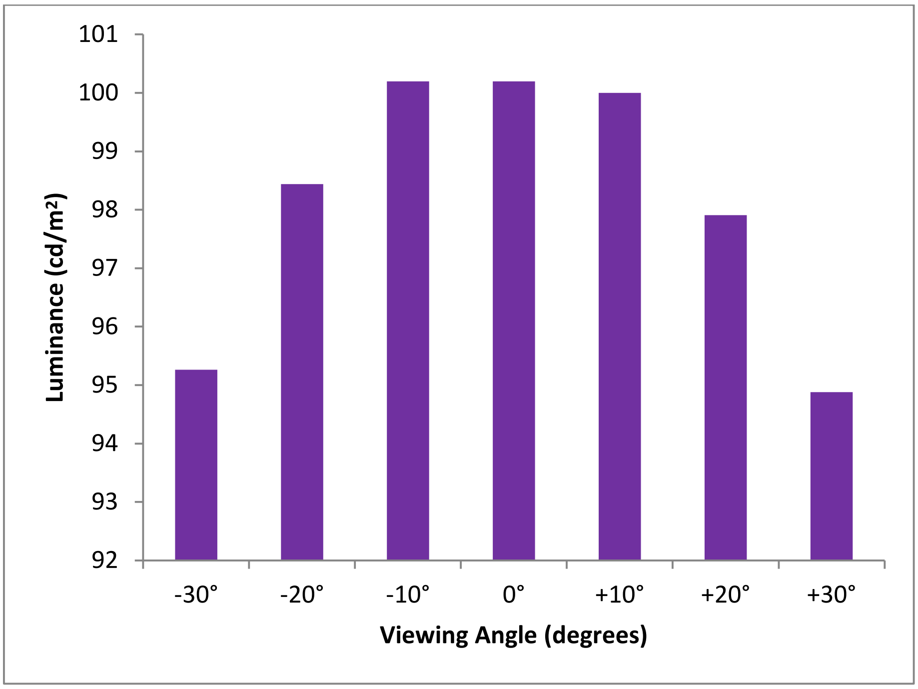

Figure 3.2-5 Angular dependency for EIZO display

3.2.2. Characterisation

The EIZO display was calibrated as described above and the measurements for building the characterisation model were taken. The model was based on the description given in section 2.2.4.1. The characterisation model is described graphically in Figure 3.2-6 and Figure 3.2-7. The response curves of the model are given in Figure 3.2-8 in a normalised scale. In this figure, it can be seen that the display does not have a typical strong *S*-shaped LCD display response; i.e. it can reproduce smooth tones well both in high and very low luminance levels. In this study, this was a desirable feature since the goal was the reproduction of high luminance white and coloured lighting stimuli.

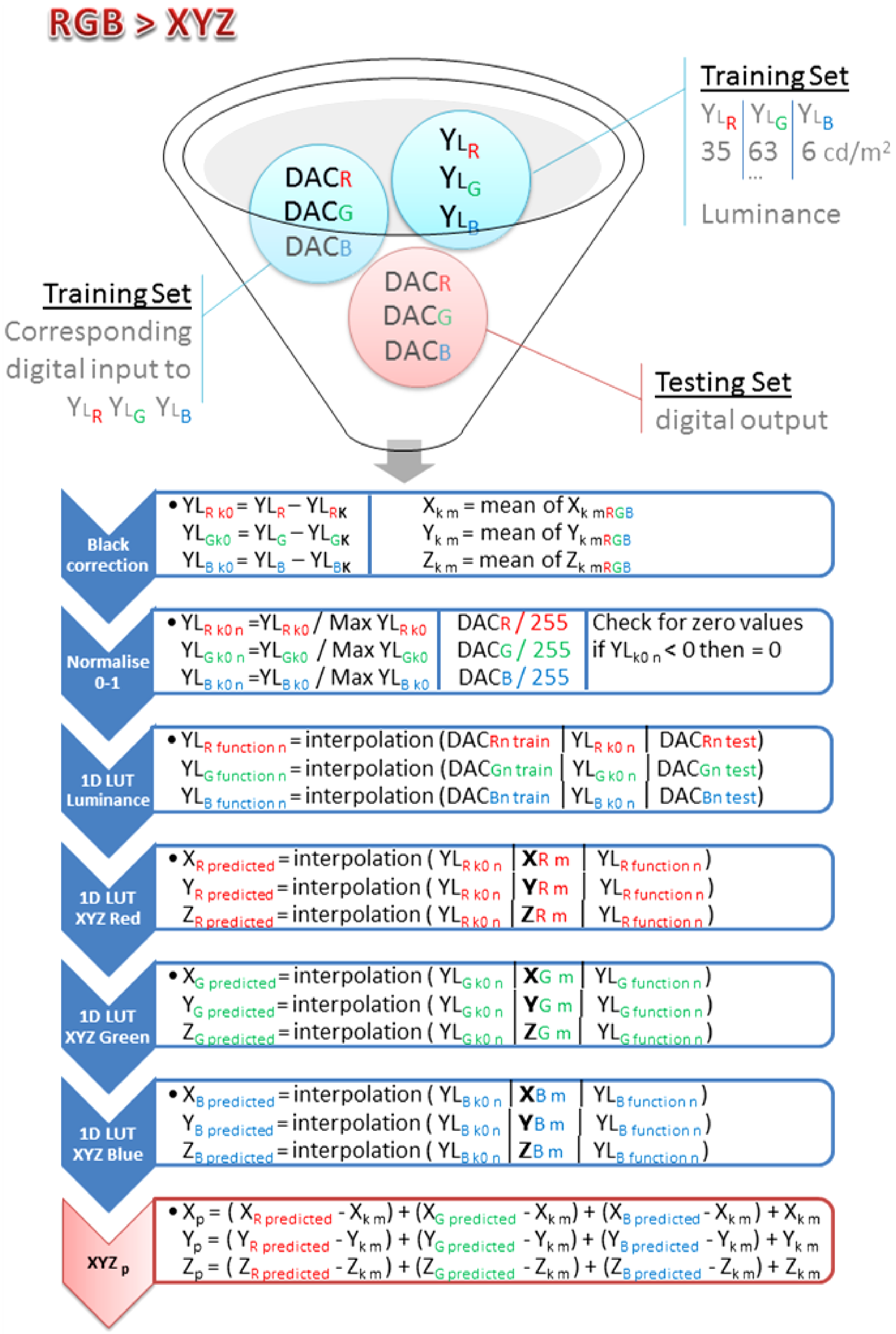

Figure 3.2-6 Forward characterisation model for EIZO display

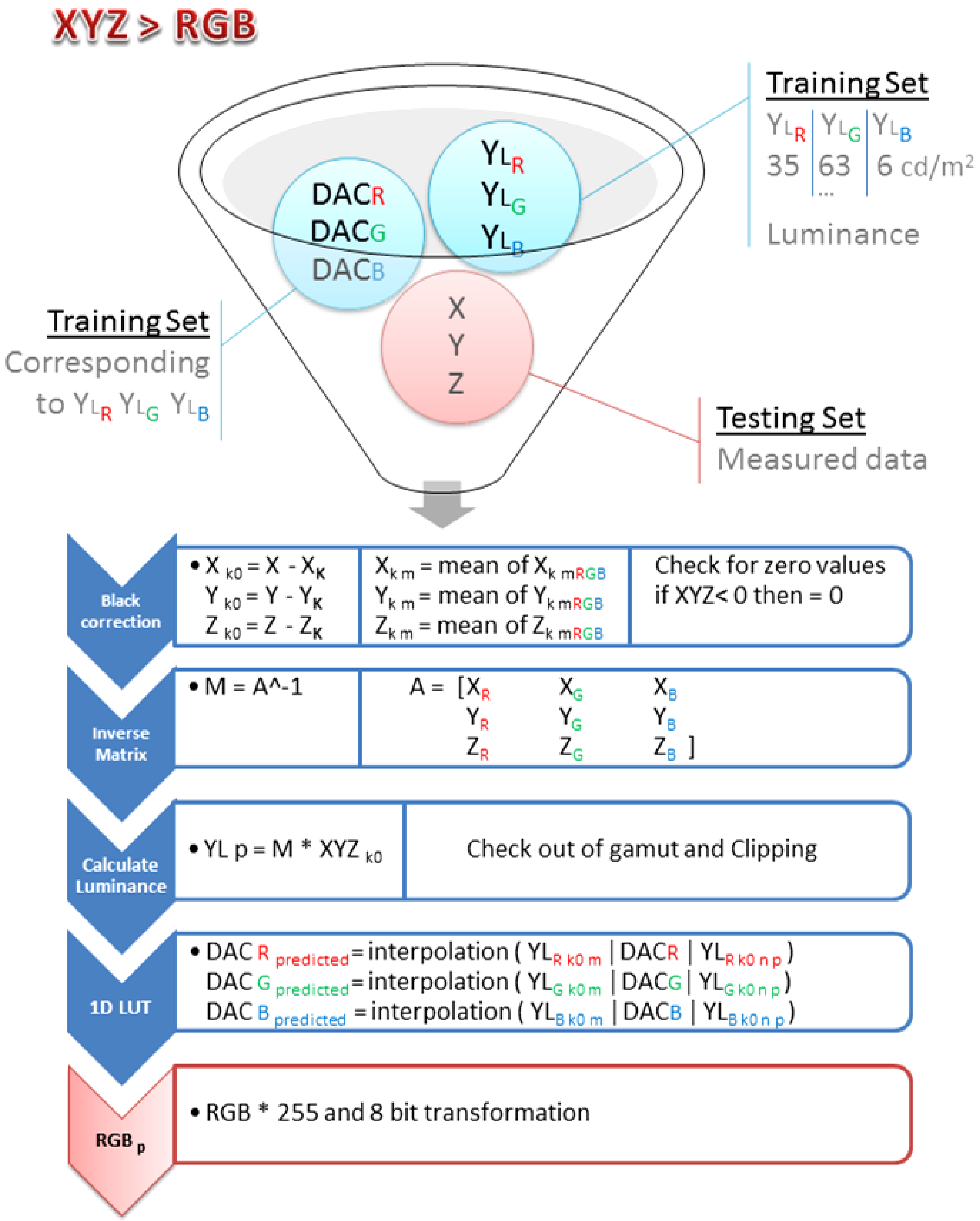

Figure 3.2-7 Reverse characterisation model for EIZO display

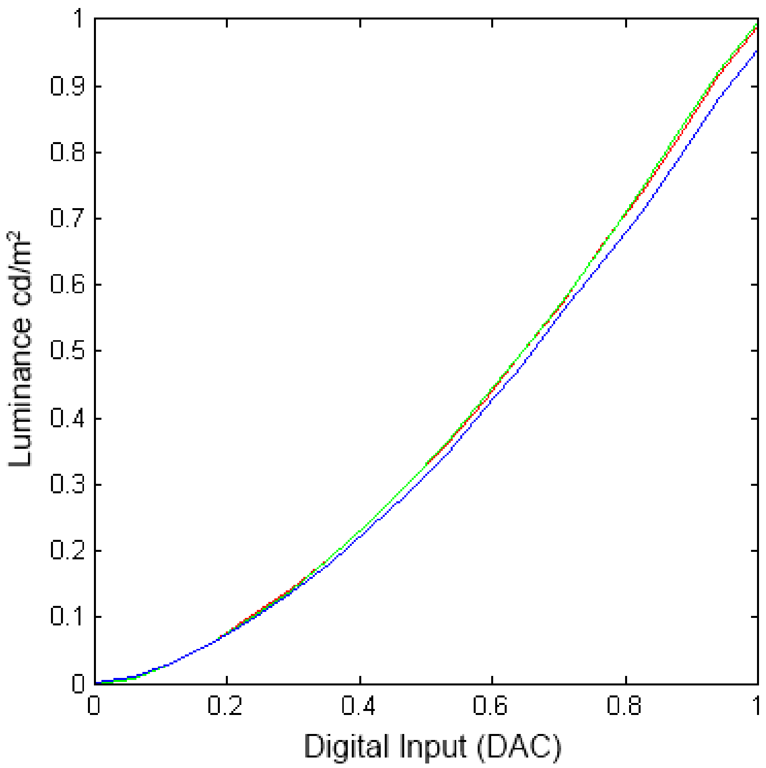

Figure 3.2-8 Response curve for the EIZO display

For the evaluation of the characterisation model's performance, the Macbeth colour checker chart input data (CCC), an 18 steps greyscale and the basic RGBCMY colours were used. The Macbeth colour checker chart input data are defined in xyY; these were converted into XYZ and then into RGB using the characterisation model of the display (Pascale, 2006). The colour difference between measured and predicted values was calculated. These results are given in Table 3.2-2 and Table 3.2-3. The performance for the testing set was clearly better and close to unity. The model performed sufficiently and it was used for calculating the required RGB values for representing the colour stimuli during the experimental sessions.

Table 3.2-2 Performance of EIZO display characterisation model using the testing set

Testing Set	**ΔE^*_{ab}**	**ΔE_{2000}**
Red	0.1	0.1
Green	0.3	0.1
Blue	0.2	0.0
Cyan	0.7	0.3
Magenta	0.3	0.1
Yellow	1.4	0.6
White	2.1	2.7
Grey 240	2.3	3.1
Grey 225	2.7	3.6
Grey 210	2.8	3.9
Grey 195	2.3	3.2
Grey 180	2.0	2.9
Grey 165	1.9	2.7
Grey 150	1.7	2.3
Grey 135	1.5	1.9
Grey 120	1.3	1.6
Grey 105	1.5	1.9
Grey 90	1.3	1.6
Grey 75	1.1	1.1
Grey 60	1.0	0.9
Grey 45	1.1	1.0
Grey 30	0.8	0.8
Grey 15	0.3	0.4
Black	0.0	0.0
Mean	**1.3**	**1.5**

Table 3.2-3 Performance of EIZO display characterisation model using the colour checker chart

Colour Checker Chart	**ΔE^*_{ab}**	**ΔE_{2000}**
dark skin	0.8	0.7
light skin	2.3	1.5
blue sky	1.5	1.3
foliage	0.9	0.7
blue flower	1.8	1.4
bluish green	3.9	1.8
orange	0.5	0.5
purplish blue	1.5	1.1
moderate red	1.4	0.9
purple	0.9	0.7
yellow green	2.5	1.2
orange yellow	0.4	0.3
blue	1.4	0.6
green	1.9	1.0
red	0.7	0.4
yellow	1.8	1.1
magenta	1.8	1.2
cyan	1.8	1.3
white 9.5 (.05 D)	4.8	5.4
neutral 8 (.23 D)	3.8	4.2
neutral 6.5 (.44 D)	2.8	3.3
neutral 5 (.70 D)	1.5	1.6
neutral 3.5 (1.05 D)	1.3	1.2
black 2 (1.05 D)	1.0	1.0
Mean	**1.8**	**1.4**

Another point to be addressed is that if there is flare from a display, then the chromaticity constancy is out of balance (Katoh et al., 2001). The more flare there is, the more the impact in the stimuli. In LCD displays, glare mostly results by effluence of light from the crystals of the panel. The display had a small flare impact that was initially addressed with a black offset correction within the characterisation model (Thomas and Hardeberg, 2013). In a later stage, colour difference correction was applied so as to indirectly fix any hue shifts caused by the display's infrastructure.

3.3. Preparation of Colour Centres, Backgrounds and Reference Pair

The process of selection of colour centres, their sampling pairs and specified characteristics will be discussed in this section; as well as the selection of the backgrounds and details of the experimental conditions. In Appendices C and D, the normalised measured data for the colour centres and backgrounds are given. Regarding the range of colour centres, white and coloured lighting stimuli were needed as explained before. Another essential point is that the MacAdam colour discrimination ellipses are for long associated with just noticeable difference. Additionally, they are mainly used in the lighting industry.

The colour centres of the MacAdam colour matching experiment correspond to luminance of 48 cd/m^2. They were represented as light stimuli in a visual colorimeter with high luminance. However, the display's colour gamut constrained the amount of reproducible MacAdam centres. From the total of 25 colour centres from the MacAdam experiment, five were reproducible with the same luminance. Therefore, a reduction of luminance into 18.5 cd/m^2 was decided. This corresponds to lightness L^* of 50 units and further reduction was avoided in order to not drop below a mid-range lightness level. Yet, 11 colour centres of the initial MacAdam centres were reproducible. The colour centres could possibly be brought in gamut by reducing further the luminance or reducing the saturation. Since chromaticity differences were investigated, the second option was discarded because it would dramatically change the chromaticity of the original colour centres. Moreover, the effect of luminance variation in the ellipse size has been studied before and it is expected that the ellipse gets larger in the xy chromaticity diagram when the Y value increases from low to high (Chong, 1974; Luo and Rigg, 1986). Although a considerable influence or trend had not been determined at much higher Y values.

With reference to the interest on the LED lighting stimuli, white light colour centres were chosen from the ANSI C78.377 standard which specifies the chromaticities of solid state lighting products around the Planckian locus (ANSI, 2008). The selected white light stimuli correspond to nominal CCT of

2700, 3000, 3500, 4000, 5000 and 6500 Kelvin (K). Similar colour centres were used in previous similar study, so they were a good selection for comparison purposes (Luo et al., 2015).

Regarding the background, two decisions were made. The first one was the grey background which was used at the MacAdam experiment, i.e. luminance of 24 cd/m^2 and chromaticity close to the illuminant C (MacAdam, 1942). This corresponds to lightness $L*$ of 56 units and it has the half luminance of the MacAdam colour centres. Moreover, grey background has been used before to simulate surface colours and therefore it can be used to relate any colour difference influence from the background (Berns, 1991; Cui et al., 2001b; Cui et al., 2001a). The second one was a black background; which was based on the idea of simulating light sources on a display as they would appear in a dark room or during night. For the same reason, circular colour patches were chosen so as to resemble light source as stimuli. This way of simulating lighting sources on a display was also attempted by Luo *et al* in previous study (Luo et al., 2015).

For the grey background, the chromaticity coordinates defined for the illuminant C were processed at the same luminance as the surrounding field of the MacAdam experiment, i.e. 24 cd/m^2. Illuminant C was chosen because the test field of the MacAdam experiment was illuminated by a light source with a chromaticity similar to the illuminant C.

The reference pair was based on a reference pair used in a previous study and likewise processed (Luo et al., 2015). A green hue reference pair was chosen so as to make it easier for the observer to assess its colour difference. Moreover, this reference pair was very close to one of the MacAdam colour centres in this gamut area. It was chosen so as to have only lightness difference of $\Delta L*$ 6 units, and the same chromaticity for both patches. This was intended to make the perceived colour difference as clear as possible, and to use an independent attribute, lightness difference, as the anchoring pair to judge all chromatic differences. Moreover, in many other studies, ratio method or grey scale method have been used in similar investigations, and lightness differences for the reference pairs have given good results. The reference pair's target $L*a*b*$ values are given in Table

3.3-1 and the responding RGB values were calculated through the characterisation model.

Table 3.3-1 Target *L*a*b values for the reference pair**

L^*	a^*	b^*
75.25	-28.05	19.88
69.00	-28.05	19.88

In the subsequent sub-sections, the following basic steps for the preparation of the colour centres and samples will be analytically discussed:

Step 1: Processing of the colour centres to find the ones within gamut.

Step 2: Sampling for creating the colour pairs of centres.

Step 3: Measurements and examination of the reproduction of all pairs.

Step 4: Application of corrections to the reproducible colour pairs.

3.3.1. Processing of the Colour Centres and Sampling

The colour centres were processed by using the xy chromaticity coordinates as given in MacAdam's paper (MacAdam, 1942). For using them into the characterisation model, they were converted into XYZ by using the luminance for the testing field of the MacAdam experiment. Therefore, the colour centres had a given luminance of 48 cd/m^2; which corresponds to L^* value of 75 units. In Appendices C and D, the normalised measured data of the colour pairs for each background are presented.

The distribution of the samples around the colour centres is very important in order to later acquire good ellipses. The starting point was the assumption that equal distances in a colour space respond to equal colour differences. Sampling was acquired within uv' chromaticity diagram in a semi-circular manner with 0.007 units of difference tolerance ($\Delta uv' = 0.007$). Twenty-one points were sampled surrounding each colour centre. These were evenly spaced at 9° degree apart; from 0° to 180° while including both starting and ending points. Since ellipses are symmetrical shapes, this sampling was chosen to increase accuracy of fitted ellipses. Furthermore, the sampling was done semi-circularly because ellipses are radial shapes and therefore similar in their mirror axis. Studies in semi-circle sampling have been contacted before as such by Brown (Brown, 1952).

There were a few reasons why sampling was done in u'v' chromaticity diagram. Firstly, the concern was to evaluate chromaticity, therefore it was desired that there would be no lightness changes among the samples but only chromaticity. Secondly, the uv' is currently the most uniform chromaticity diagram so equal distances should theoretically result in equally perceived colour differences in most parts. Even though studies have practically shown that the diagram is not uniform throughout its whole extent, it was considered a good space to use for the sampling as it is also a space for additive stimuli such as light. Thirdly, there was not a specific target white reference, thus use of CIELAB was not necessary. Moreover, CIELAB has been used many times before and that did not affect the fitting of the data (Berns et al., 1991; Luo and Rigg, 1986). Fourthly, the Planckian locus was not used as a reference because it relates to white light stimuli. With LED technologies in the lighting industry being able to produce coloured light sources, the use of the Planckian locus does not suffice the need for a wide variety of chromatic areas.

As introduced, the first step was to determine the in-gamut colour centres from the initial set of MacAdam centres. The known MacAdam xyY values were transformed into XYZ, so as to be colorimetrically transformed into RGB through the display's characterisation model. At this stage, if colour centres had RGB values equal to 255 or 0, there were excluded. Being on the margins of the colour gamut would not make it accurately reproducible. Moreover, the sampled points would be clearly out-of-gamut. After the sampling, the sampled points were also tested for being within gamut or close to the margins. An example of the sampling for a given colour centre is illustrated in Figure 3.3-1.

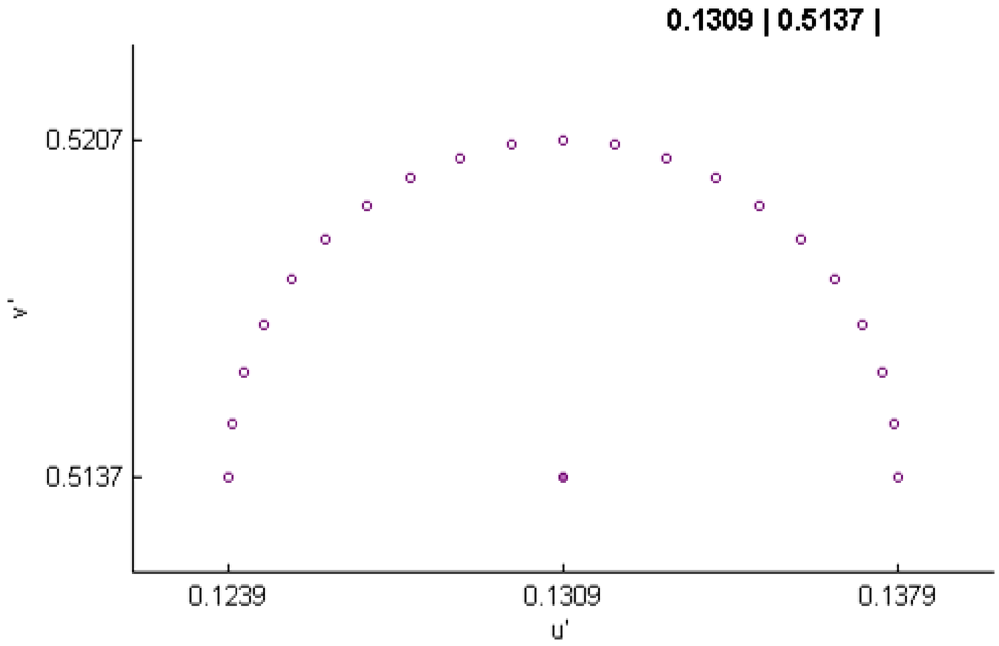

Figure 3.3-1 Example of sampling in the u'v' chromaticity diagram for the colour centre with u'v' chromaticity coordinates of 0.1309 and 0.5137 respectively

While testing the in-gamut colour centres, it was also noticed that there were not representative colour centres for the farthest blue and green region of the gamut. Therefore, two colour centres were added as extras in order to investigate the problematic blue and green area. The blue area has proven problematic in other studies such as in the development of the colour difference formula for CIEDE2000. These were chosen in relation to the existing out-of-gamut colour centres, so as to fall inside the device's colour gamut. Thus, the second step of the process was completed.

A simple numbering system was applied for identification of the MacAdam colour centres by using numbers from 1 to 25 for the initial set of centres. The numbering system did not change after the exclusion of the out-of-gamut colours. In this way, it would be easier to see which colour centres were being used in both luminance setups. For the white light stimuli, the letter W and numbers from 1 to 6 were denoted. To define the luminance setup, colour centres were also followed by the value of the processing luminance, i.e. 48 or 18.5 cd/m^2. In the Table 3.3-2, summary of the colour centre naming system, together with the corresponding xyY and u’v’ values are given.

Table 3.3-2 Experimental colour centres – Naming and xyY u’v’ values

	x	y	Y	u'	v'
1_18	0.2580	0.4500	18.5	0.1309	0.5137
1_48	0.2580	0.4500	48	0.1309	0.5137
10_18	0.3900	0.2370	18.5	0.3081	0.4212
12_18	0.3440	0.2840	18.5	0.2406	0.4469
12_48	0.3440	0.2840	48	0.2406	0.4469
13_18	0.2280	0.2500	18.5	0.1645	0.4058
19_18	0.5270	0.3500	18.5	0.3430	0.5125
23_18	0.2780	0.2230	18.5	0.2172	0.3920
24_18	0.3000	0.1630	18.5	0.2755	0.3368
25_18	0.4720	0.3990	18.5	0.2759	0.5247
25_48	0.4720	0.3990	48	0.2759	0.5247
3_18	0.2800	0.3850	18.5	0.1586	0.4908
3_48	0.2800	0.3850	48	0.1586	0.4908
5_18	0.4750	0.3000	18.5	0.3363	0.4779
8_18	0.3800	0.4980	18.5	0.1850	0.5455
8_48	0.3800	0.4980	48	0.1850	0.5455
B_48	0.2000	0.3000	48	0.1290	0.4355
G_48	0.2500	0.6000	48	0.1031	0.5567
W1_48	0.4578	0.4101	48	0.2614	0.5268
W2_48	0.4338	0.4030	48	0.2490	0.5205
W3_48	0.4073	0.3917	48	0.2366	0.5120
W4_18	0.3818	0.3797	18.5	0.2248	0.5031
W4_48	0.3818	0.3797	48	0.2248	0.5031
W5_48	0.3447	0.3553	48	0.2097	0.4864
W6_18	0.3123	0.3282	18.5	0.1979	0.4678
W6_48	0.3123	0.3282	48	0.1979	0.4678

The colour centres № 11 and 20 of the original MacAdam set were excluded even though they were reproducible because they were very close and visually similar to the colour centres W4 and W6 respectively; see Figure 3.3-2. Since colour centres № 11 and 20 were reproducible at both luminance levels, colour centres W4 and W6 were also processed at both 48 and 18.5 cd/m^2. Moreover, they could be used in comparisons as representatives of the white light stimuli set. The white colour centres are defined in the ANSI standard and the same ones have been used in previous study with the same psychophysical method (Luo et al., 2015). Therefore, these could be used for comparison reasons.

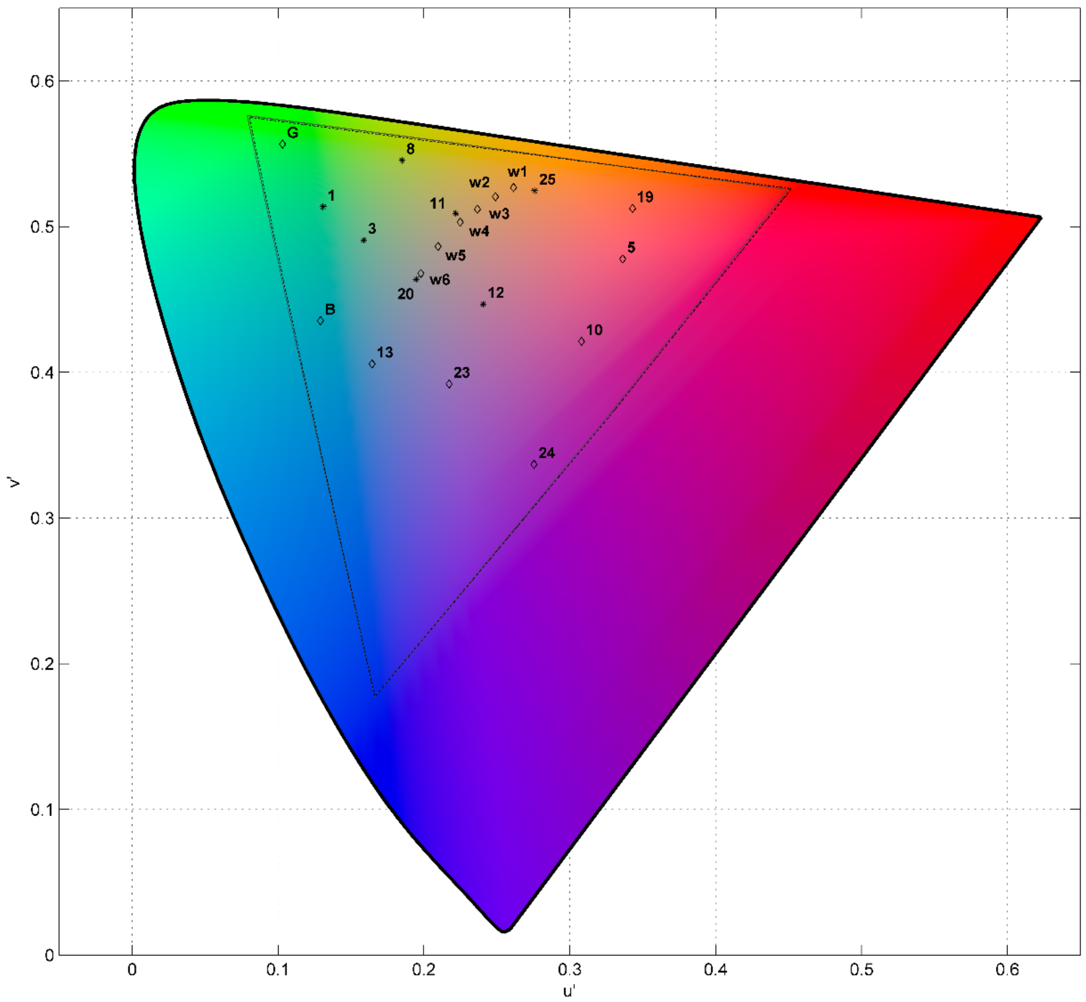

Figure 3.3-2 First selection of colour centres in the u'v' chromaticity diagram

The third step of the process was to measure and examine the reproduction of all colour centres and assigned pairs. Consequently, the converted RGB colour centres were displayed and measured on screen. After the measurements, the delta analysis showed discrepancies between the target and measured colour stimuli by calculating the relative colour difference of the pairs. These discrepancies were corrected in order to match the target colour centre and pairs as much as possible. These errors can be observed in the reproduction plots which will be presented in section 3.3.2.

These reproduction problems could have potentially been caused by either the characterisation model or other quantisation errors, as it was explained before. The fact that some colour centres had less smooth distribution of samples is an indication of quantisation error. This can also be an indication that the characterisation model did not operate well in some chromatic areas. Most of the colour centres were corrected with an indirect correction of hue shift in CIELAB, and very badly reproducible colour pairs were excluded from the set.

Therefore, the forth step of the process for the correction of the hue shifts was applied as follows. The measured xyY data were transformed back to XYZ and subsequently into CIELAB values. The differences between the measured and target *L*a*b** were computed and a new corrected target was assigned by subtracting the aforementioned differences from the original target. The correction equations are described by Equation 3.3-1. Afterwards, the new targets were transformed back to XYZ and RGB through the display characterisation model in order to be re-measured and re-evaluated. The final selection of target colour centres can be viewed in Figure 3.3-3 and Figure 3.3-4. The colour centres that appear both at 48 and 18.5 cd/m^2 are plotted with asterisks.

Equation 3.3-1 Correction method for the hue shifts between measured and target colours

$$\begin{aligned} \Delta L^* &= L^* measured - L^* \, target \\ \Delta \mathrm{a}^* &= \mathrm{a}^* measured - \mathrm{a}^* \, target \\ \Delta b^* &= b^* measured - b^* \, target \end{aligned}$$

Corrected Target:

$$\begin{aligned} New\ L^* &= abs\ (\Delta L^* - L^* \, target) \\ New\ \mathrm{a}^* &= \mathrm{a}^* \, target - \Delta \mathrm{a}^* \\ New\ b^* &= b^* \, target - \Delta b^* \end{aligned}$$

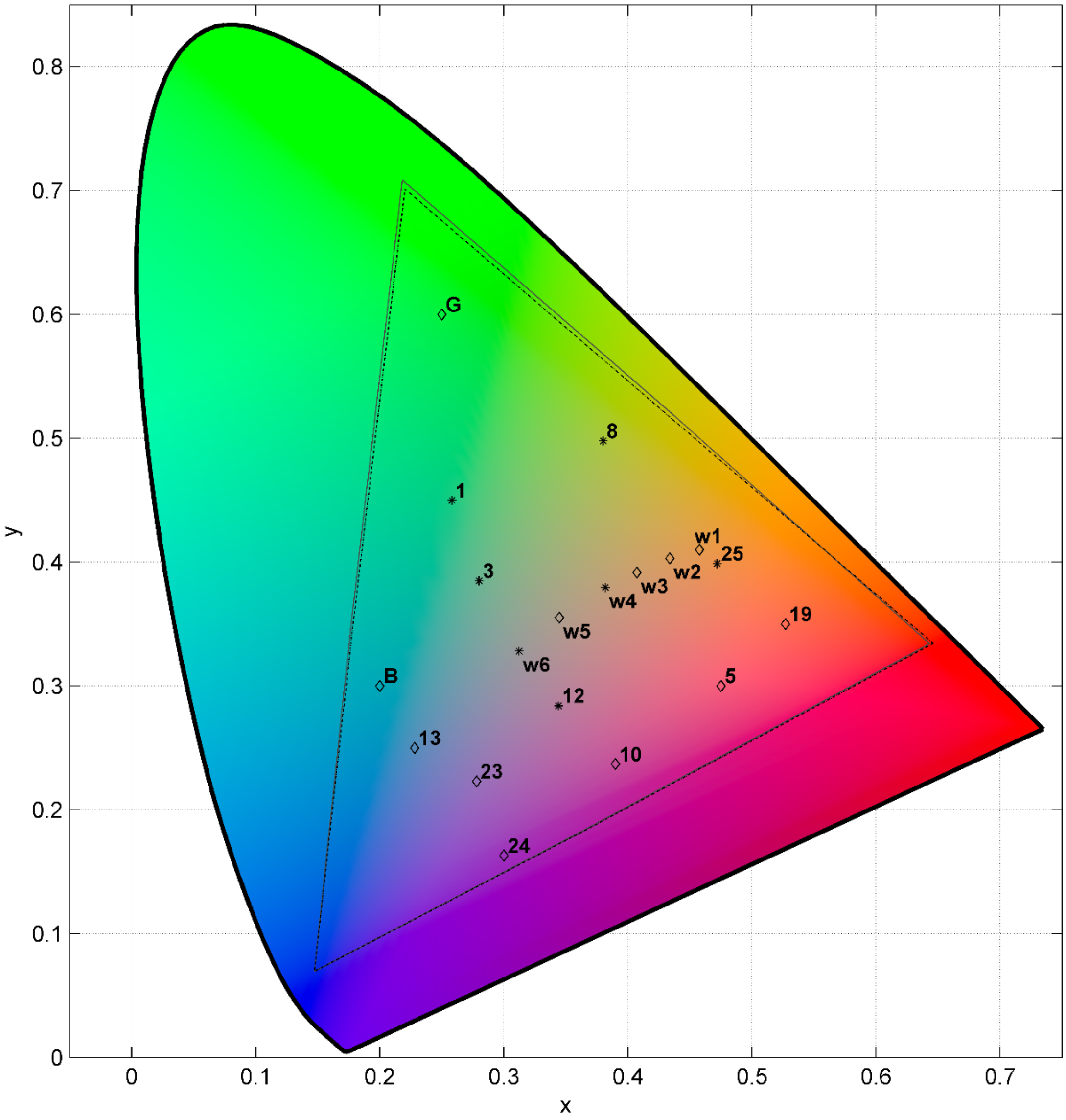

Figure 3.3-3 Target colour centres in the xy chromaticity diagram – solid and dashed line for the 48 and 18.5 cd/m^2 luminance gamut

For the colour representation of the xy chromaticity diagram, a Matlab function from the colour toolbox of Westland and Ripamonti was applied (Westland and Ripamonti, 2004). Each colour having luminance at both 48 and 18.5 cd/m^2, it is marked with asterisks.

After confirming the reproduction quality of the process, the process was applied to every pair. During all measurements, each colour centre was measured twice and its mean value was used in the processing to ensure its reproduction accuracy. The full process can be graphically represented in Figure 3.3-5.

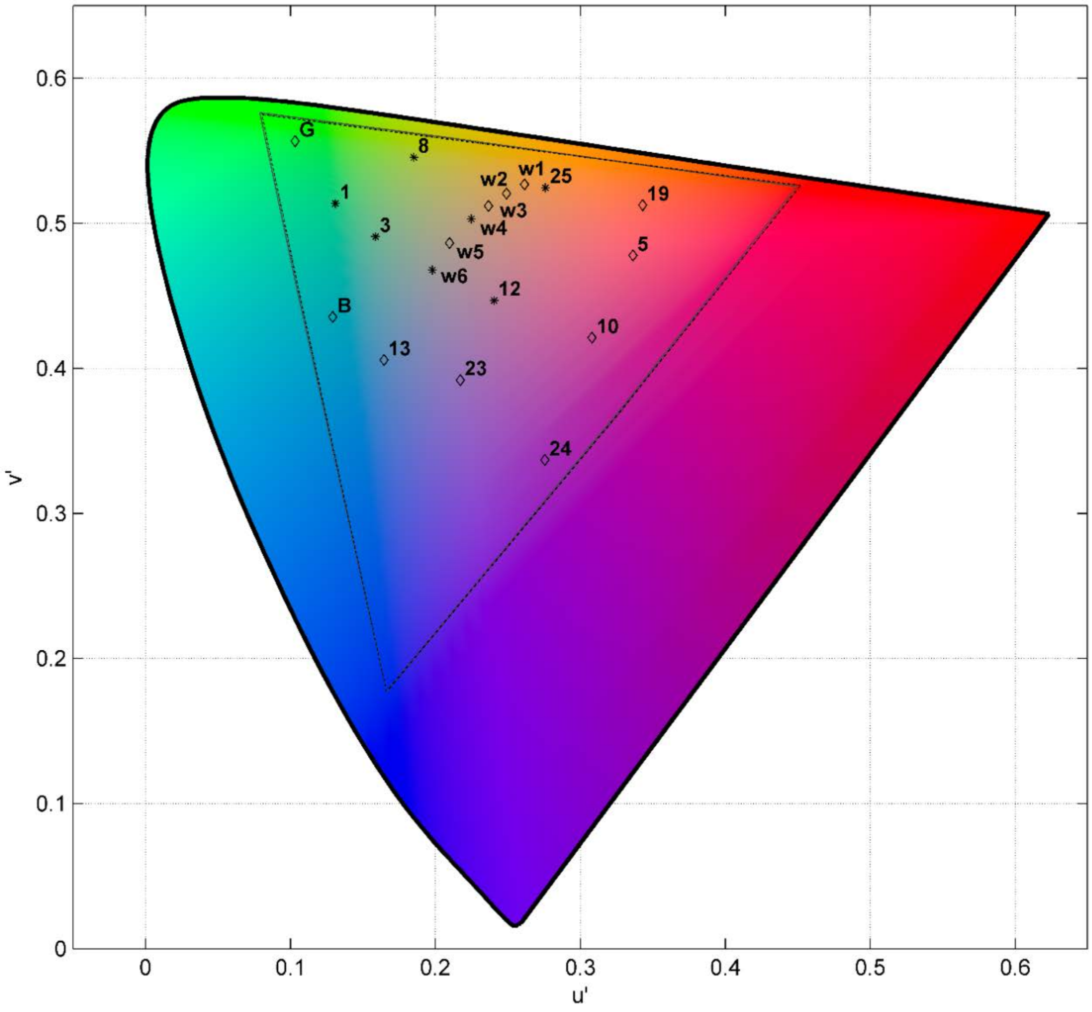

Figure 3.3-4 Target colour centres in the u'v' chromaticity diagram – solid and dashed line for the 48 and 18.5 cd/m^2 luminance gamut

For the colour representation of the u'v' chromaticity diagram, a Matlab function for the xy chromaticity diagram from the colour toolbox of Westland and Ripamonti was adjusted (Westland and Ripamonti, 2004). Each colour having luminance at both 48 and 18.5 cd/m^2, it is marked with asterisks.

- xyY ➔ u'v'Y ➔ XYZ ➔ RGB display
- xyY measured ➔ XYZ ➔ CIELAB ➔ correction method ➔ XYZ ➔ RGB

Figure 3.3-5 Colorimetric transformations for the processing of colour stimuli

3.3.2. Performance of Reproduction of Pairs

For evaluating the repeatability in the reproduction of the pairs, the dEE measure was applied. This represents the relative colour difference of a pair against another and it is convenient to use in order to reveal the difference between the target and reproduced colour pairs. The distribution of the samples surrounding the colour centre is very important when assessing colour differences as this could potentially reflect on the fitted ellipses. Additionally, the experiment was based on the samples having equal distances from the colour centre as much as possible. Therefore, a couple of the in-gamut colour centres had been excluded from the set when it was evaluated that their distribution was too scattered to be reliable. Distribution of samples for scattered and/or problematic colour centres was investigated twice with re-measurements, so as to ensure that it was not due to changes in the display performance. It was found that the performance was similar in every case. This was also another reassuring attribute of the display's repeatability. Measurements of the colour stimuli were examined for both backgrounds respectively.

In the following series of plots; Figure 3.3-6 to Figure 3.3-31; there are four sets of plotted points for each colour centre of the experimental data. The target colour pairs are marked with grey triangles, the measured pairs against the black background before the correction are in purple squares, the corrected measured pairs against the black background are in yellow circles, and the corrected measured pairs against the grey background are in green circles. On the top right corner of the figure, the target colour centre in u'v' values and luminance are recorded.

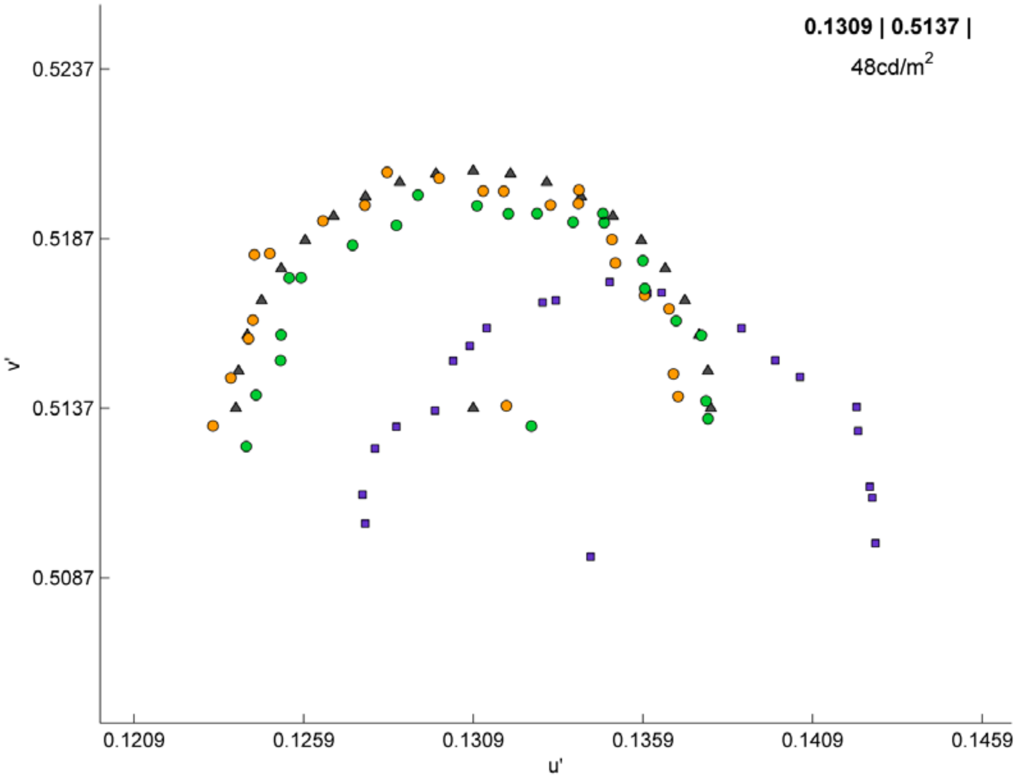

Figure 3.3-6 Reproduction of colour centre 1_48

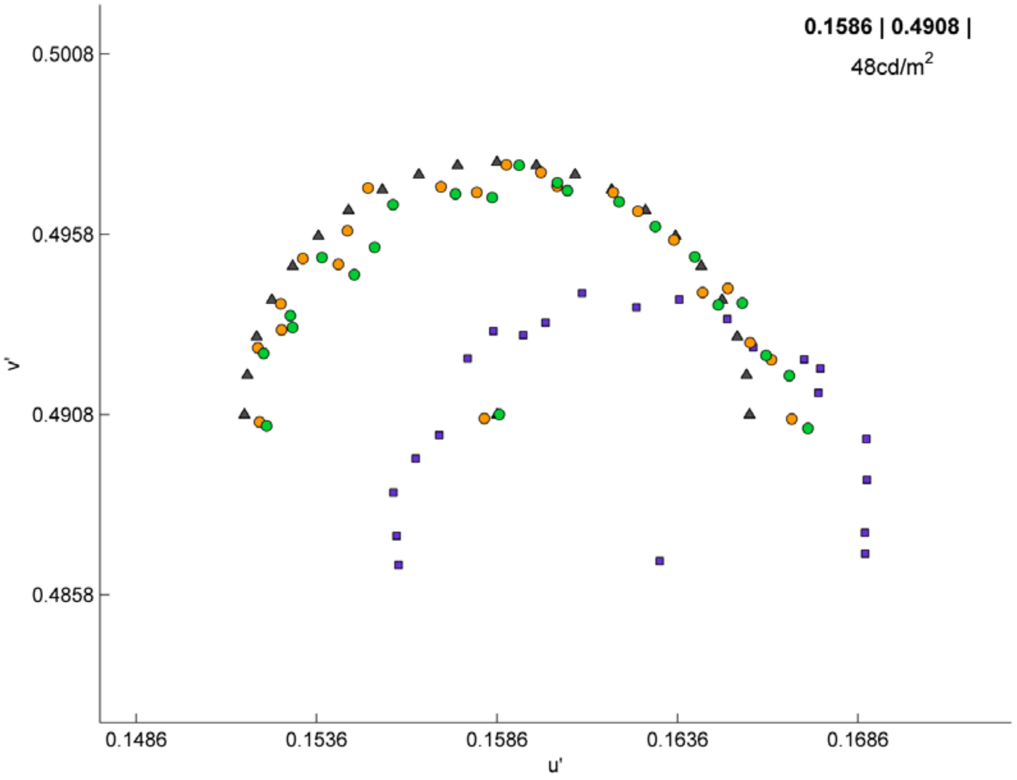

Figure 3.3-7 Reproduction of colour centre 3_48

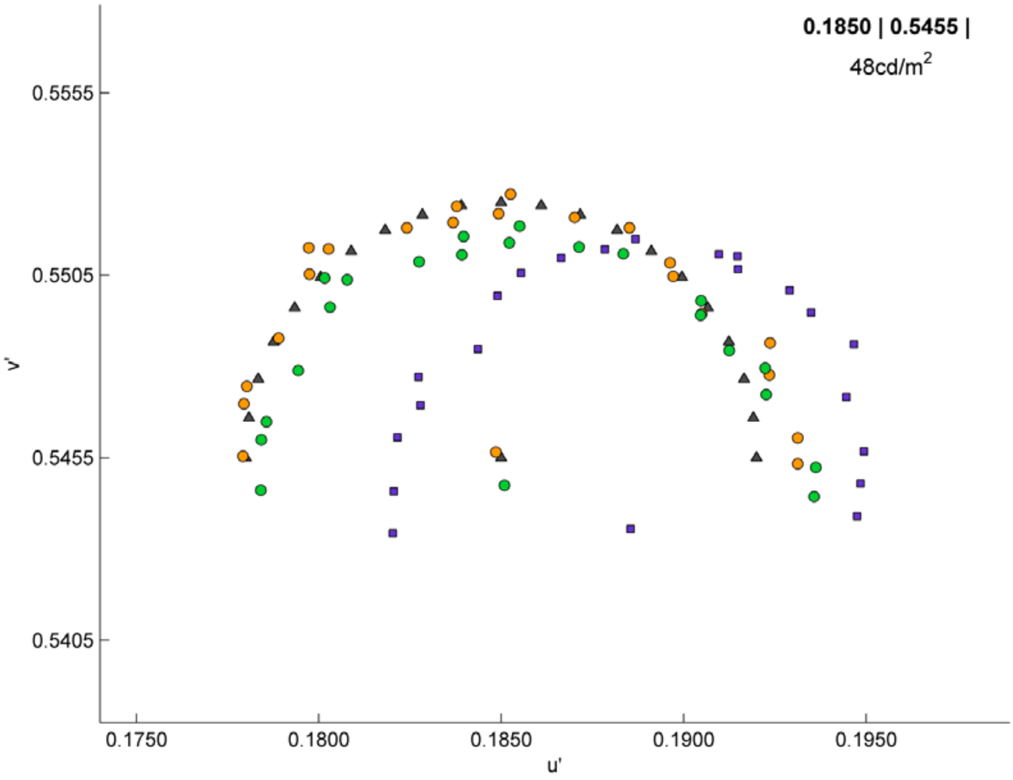

Figure 3.3-8 Reproduction of colour centre 8_48

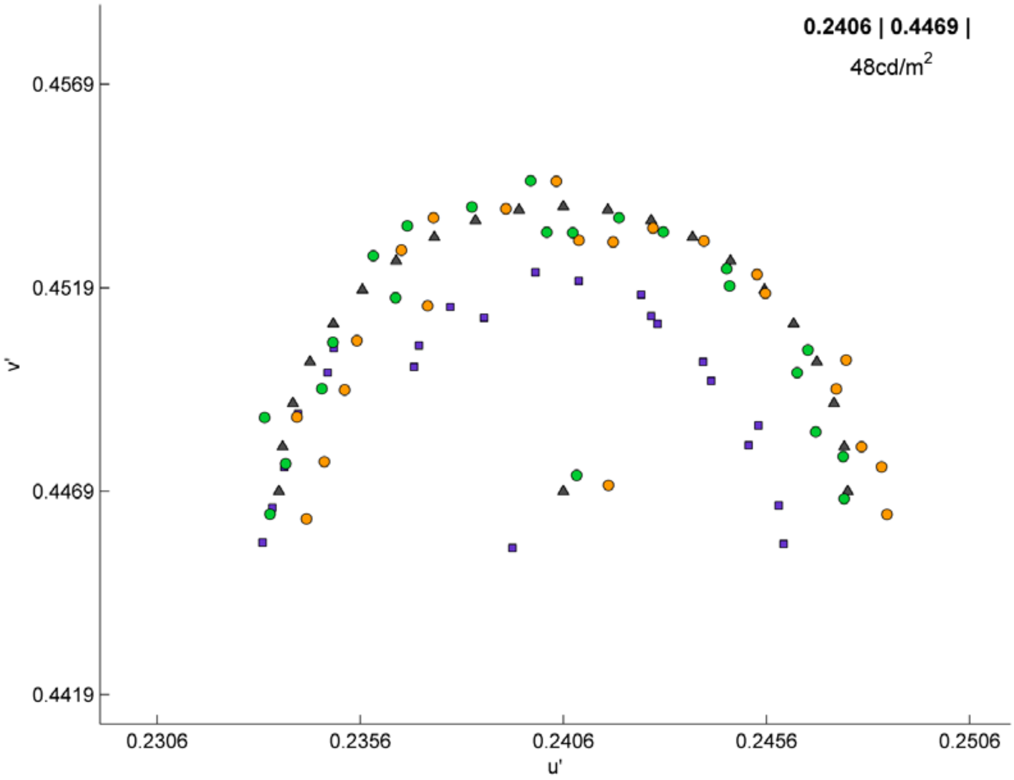

Figure 3.3-9 Reproduction of colour centre 12_48

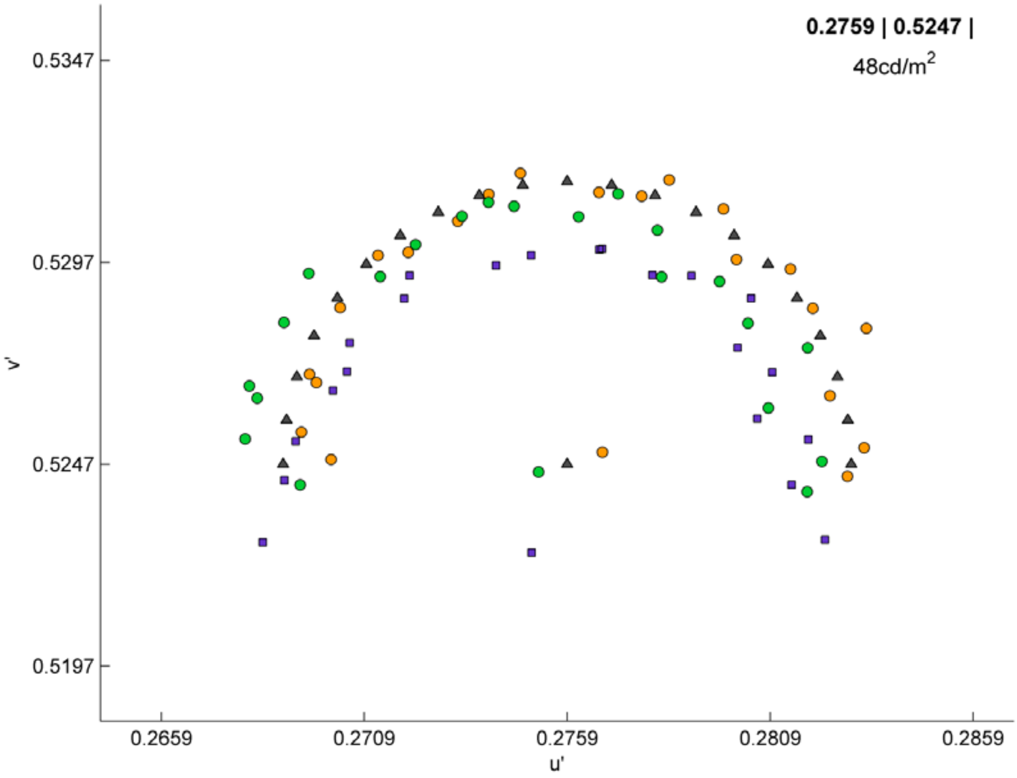

Figure 3.3-10 Reproduction of colour centre 25_48

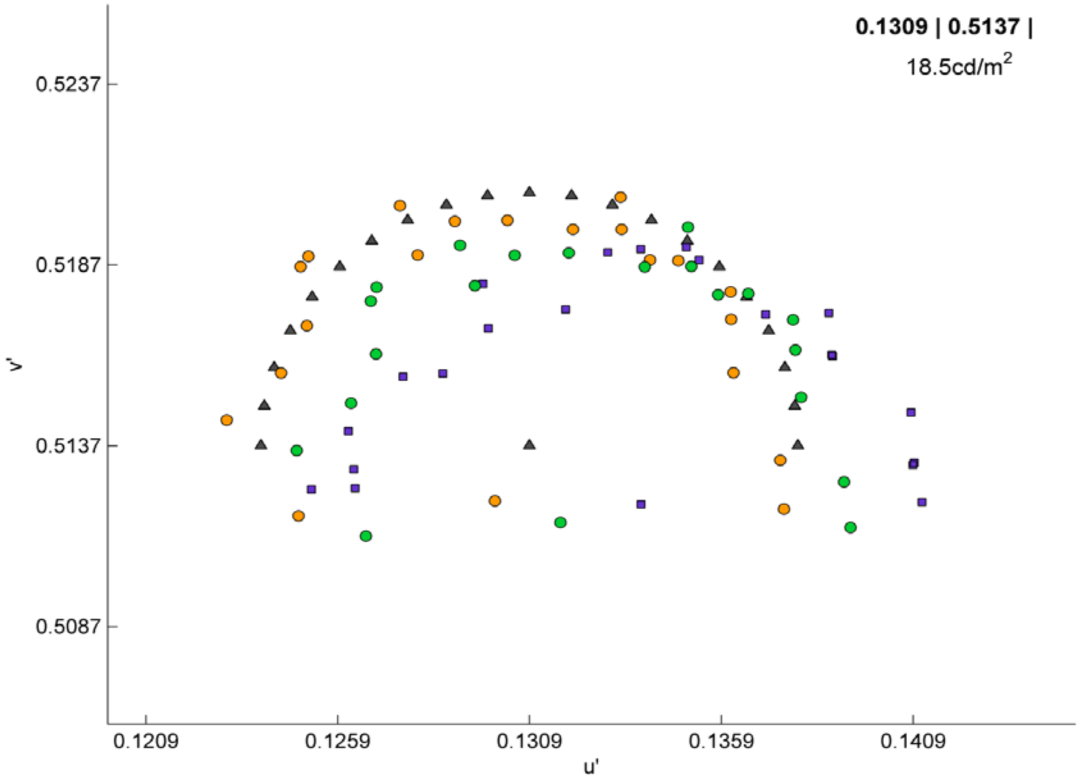

Figure 3.3-11 Reproduction of colour centre 1_18.5

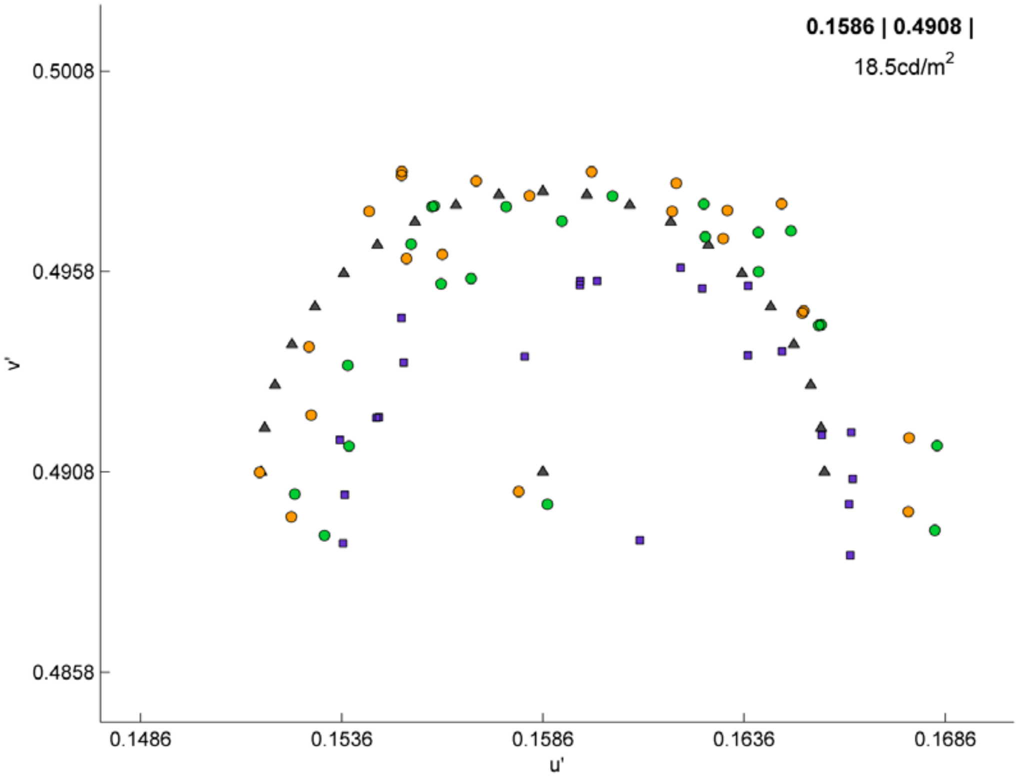

Figure 3.3-12 Reproduction of colour centre 3_18.5

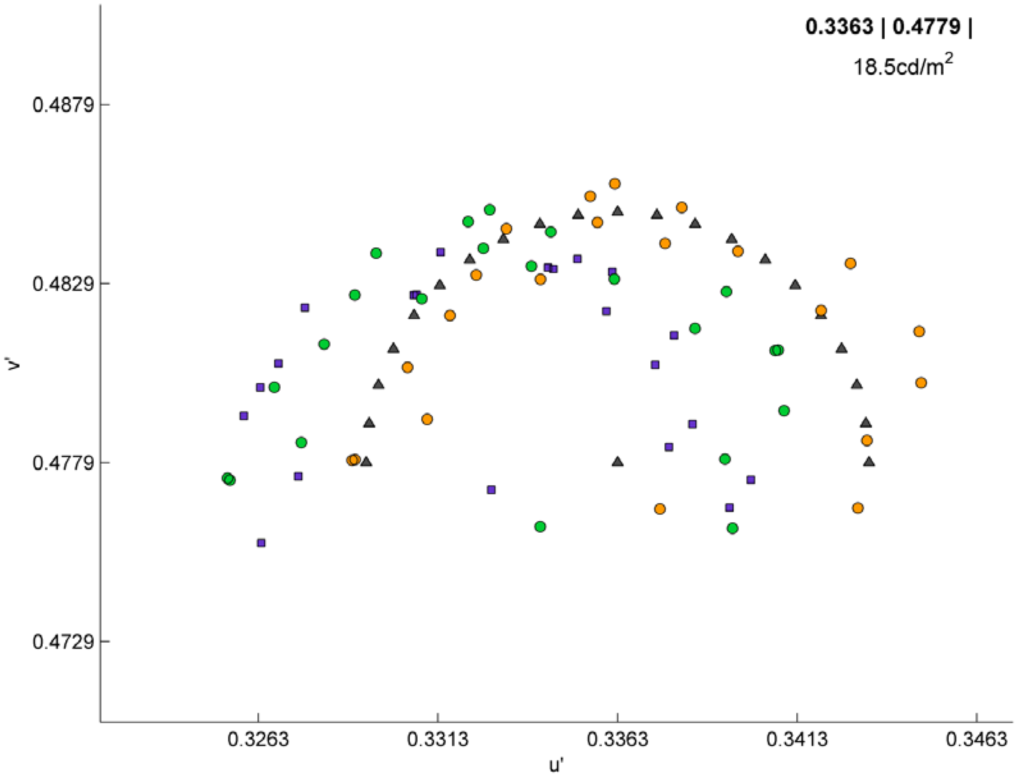

Figure 3.3-13 Reproduction of colour centre 5_18.5

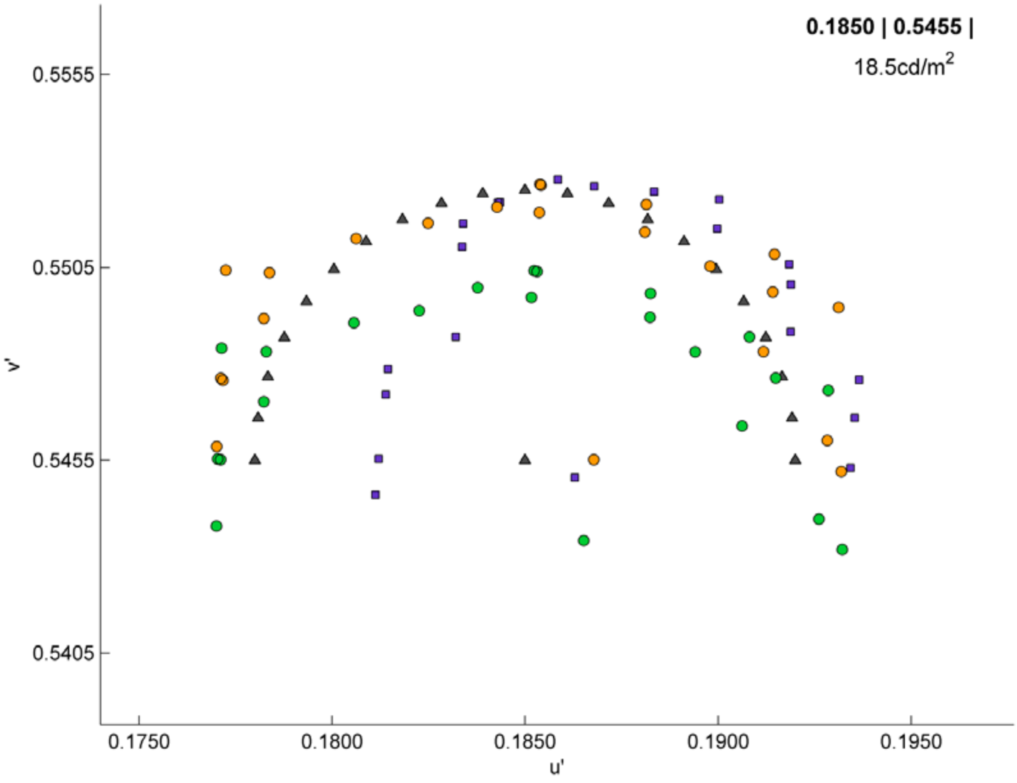

Figure 3.3-14 Reproduction of colour centre 8_18.5

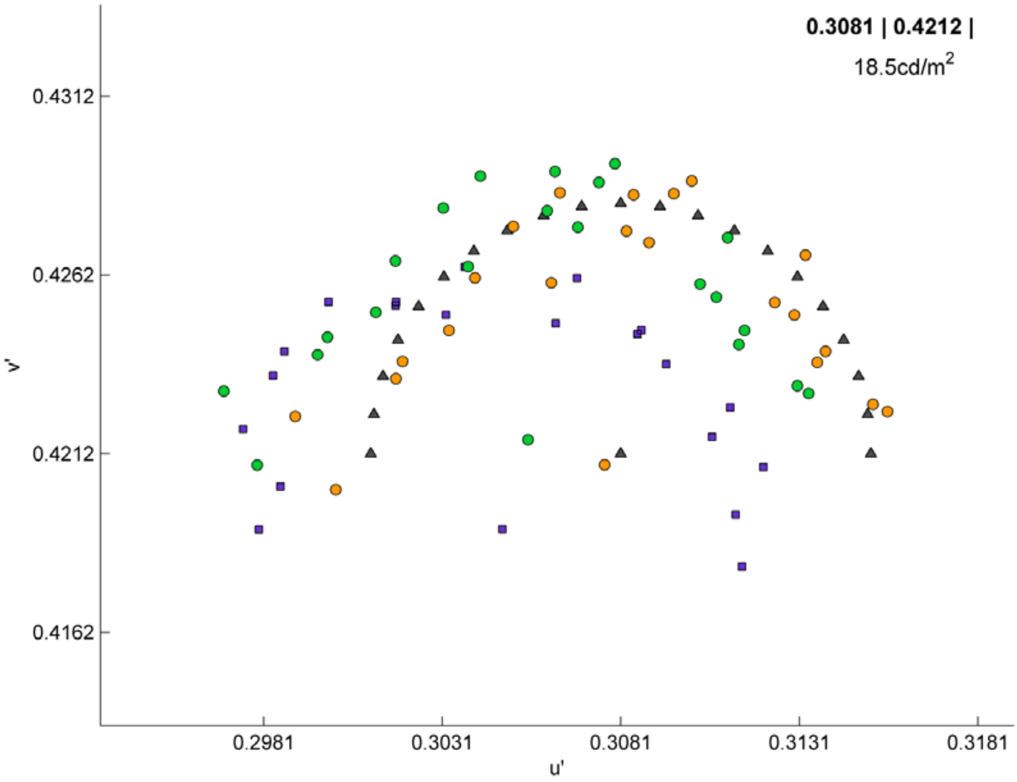

Figure 3.3-15 Reproduction of colour centre 10_18.5

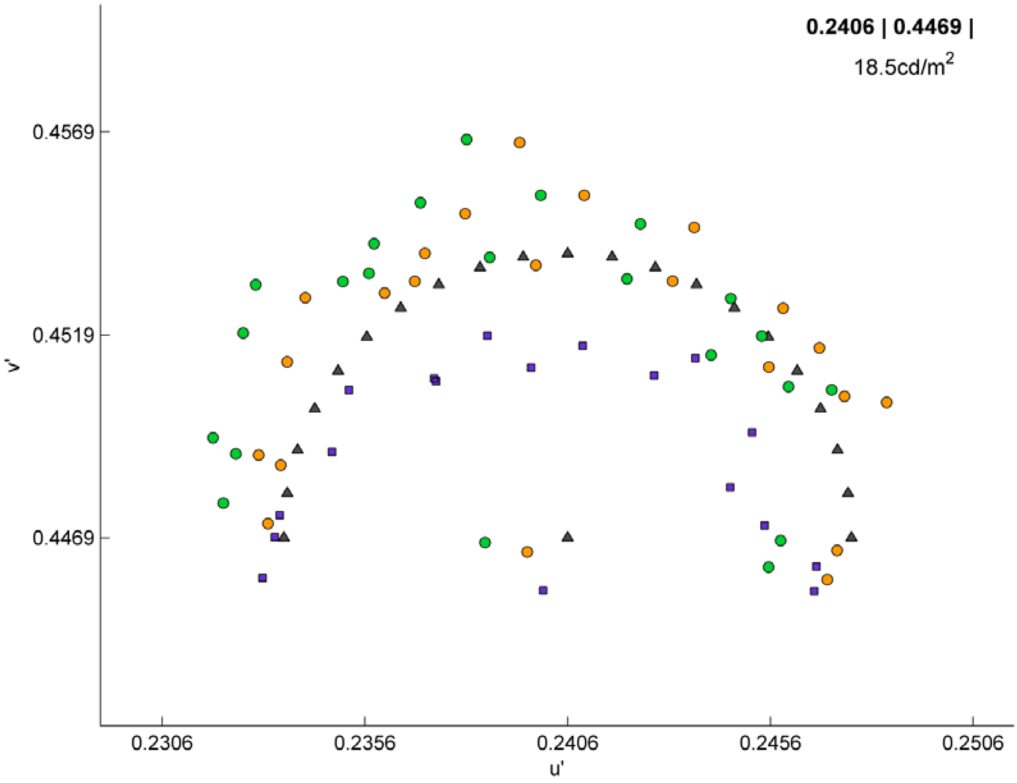

Figure 3.3-16 Reproduction of colour centre 12_18.5

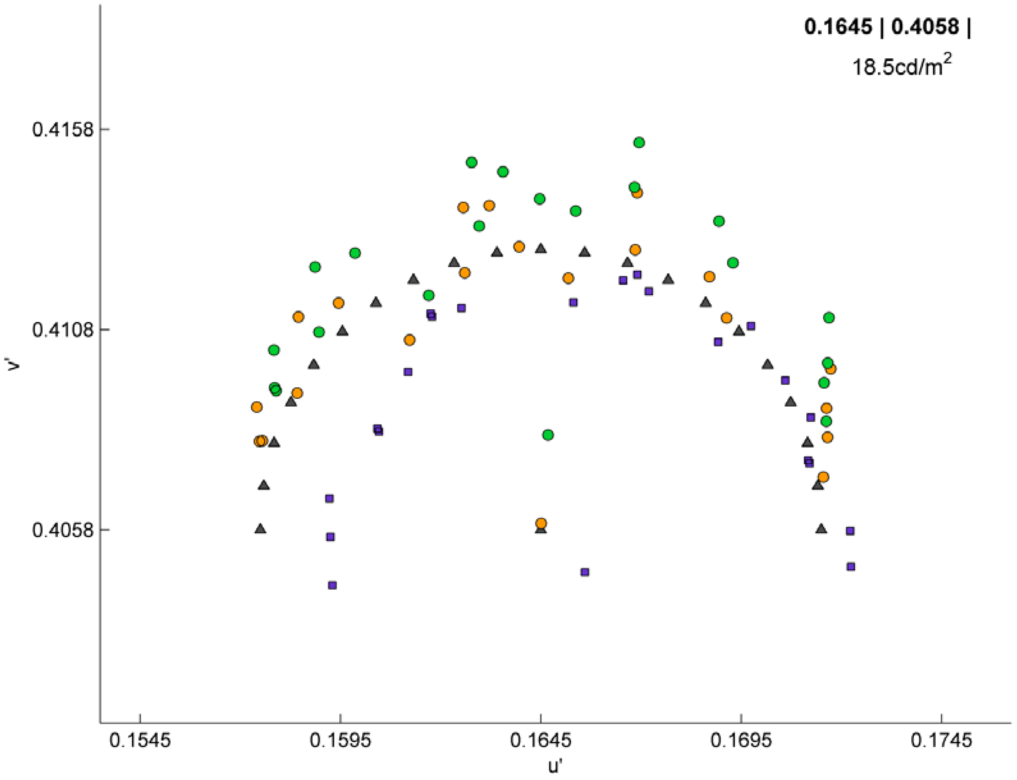

Figure 3.3-17 Reproduction of colour centre 13_18.5

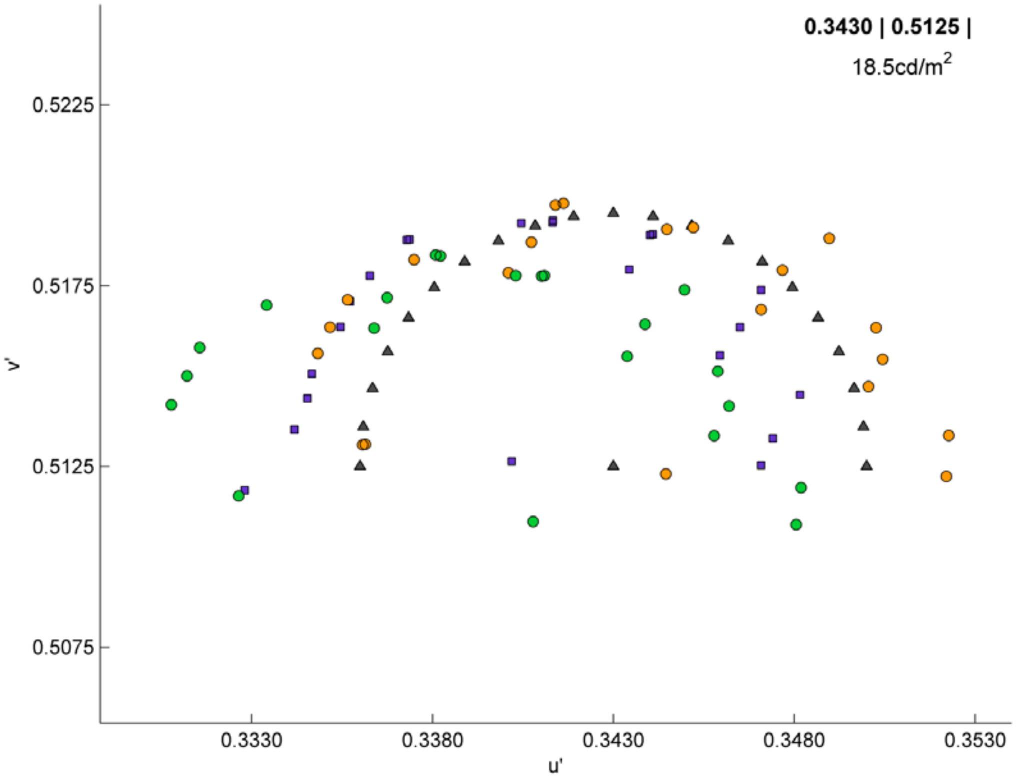

Figure 3.3-18 Reproduction of colour centre 19_18.5

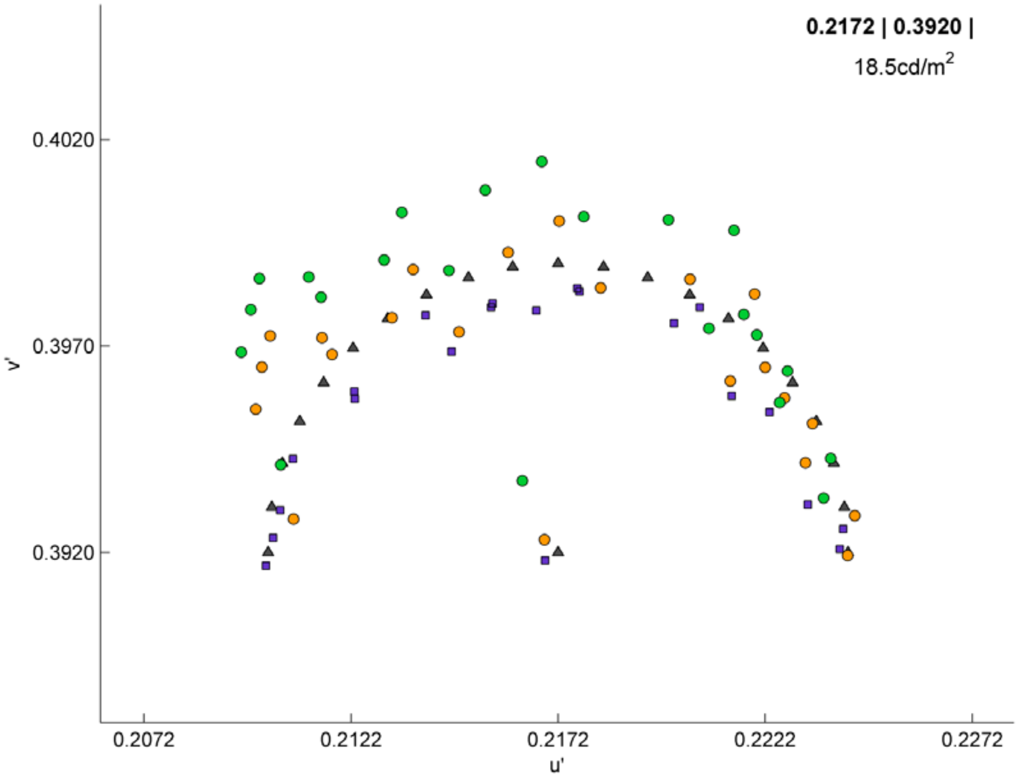

Figure 3.3-19 Reproduction of colour centre 23_18.5

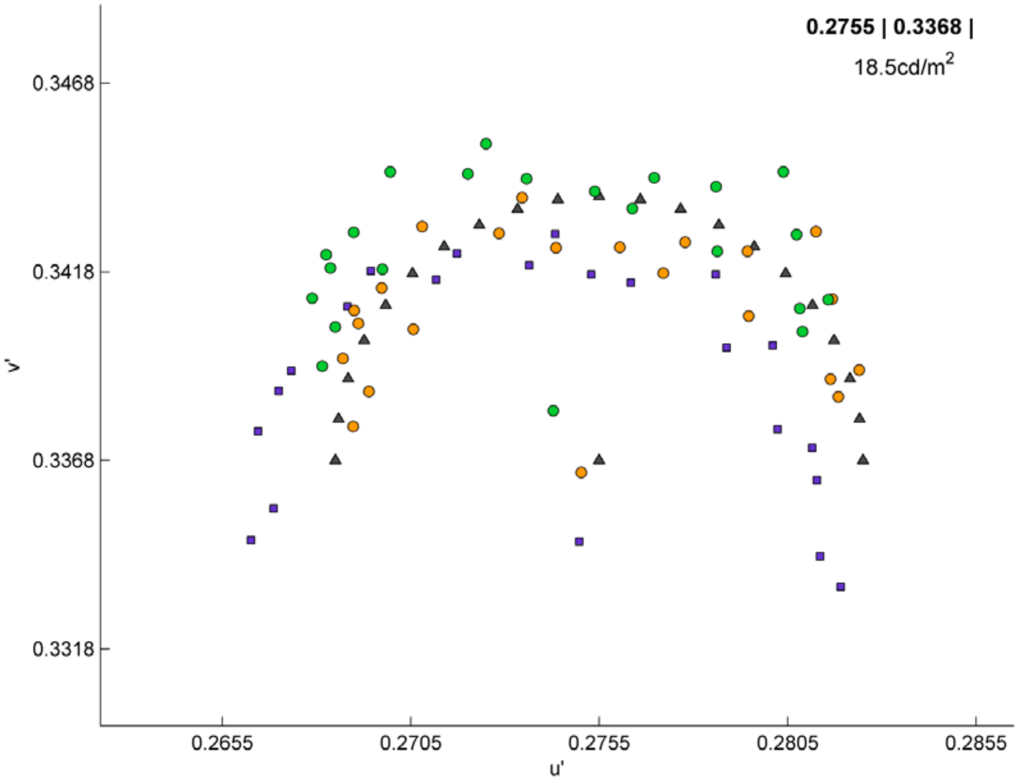

Figure 3.3-20 Reproduction of colour centre 24_18.5

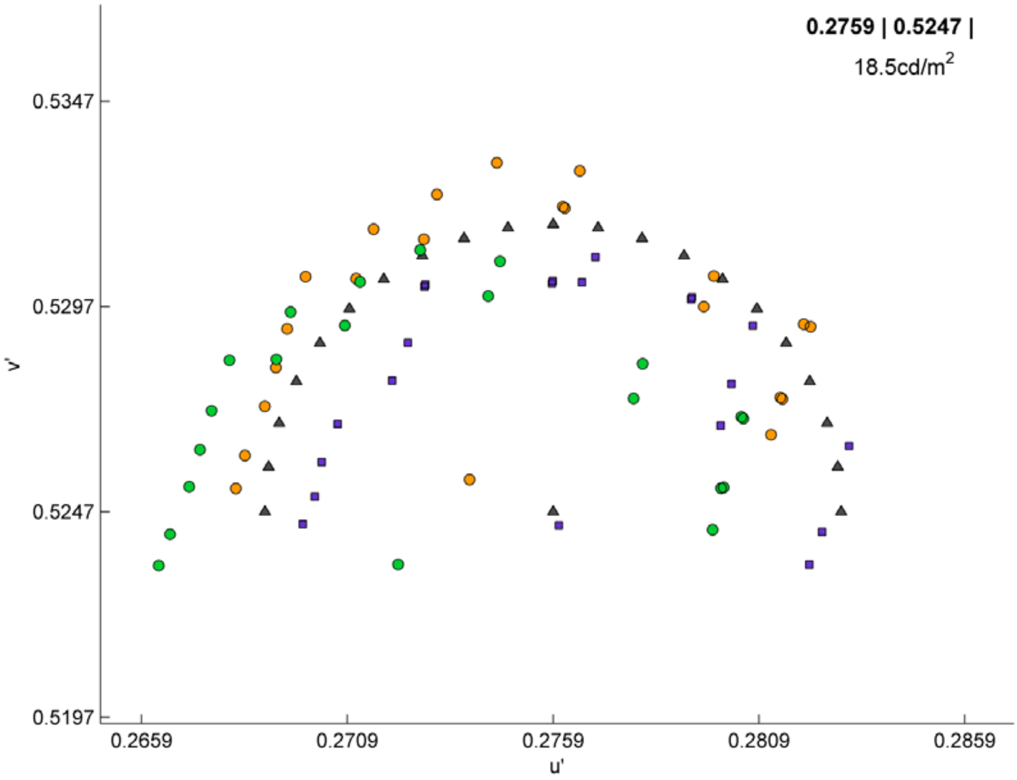

Figure 3.3-21 Reproduction of colour centre 25_18.5

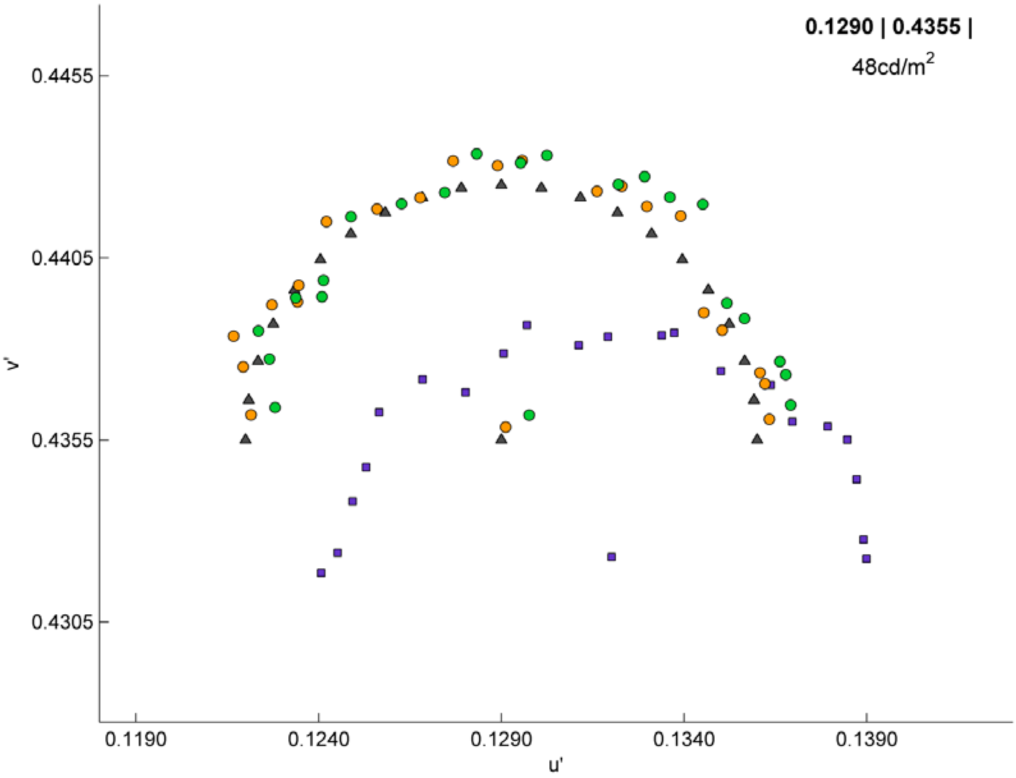

Figure 3.3-22 Reproduction of colour centre B_48

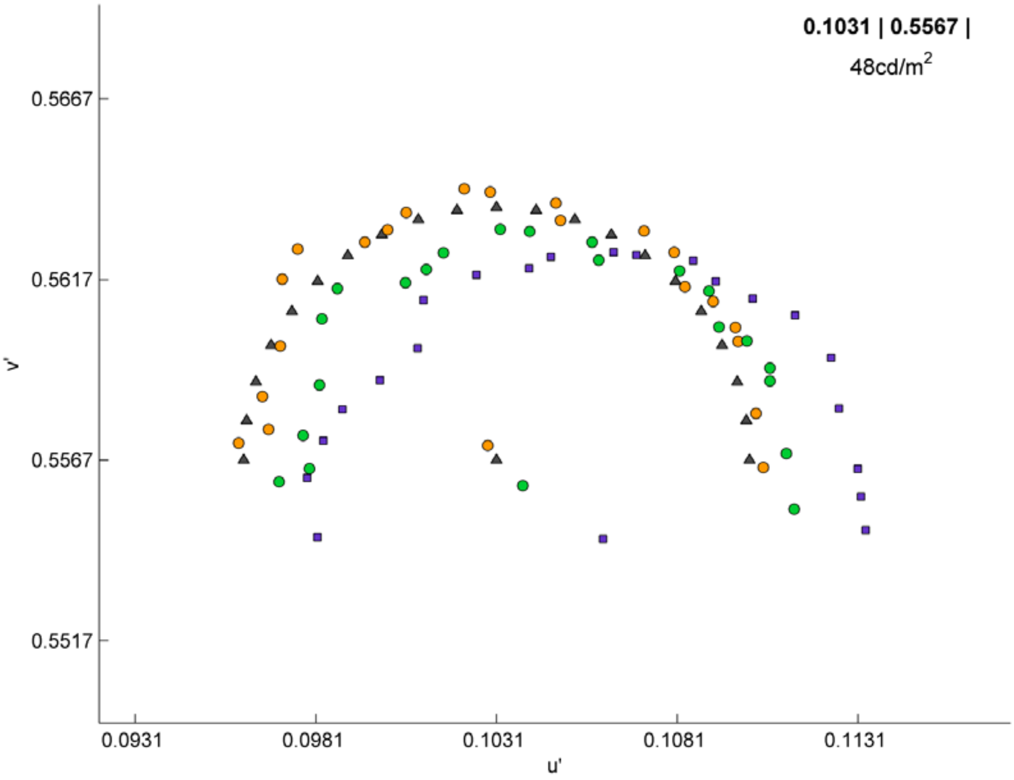

Figure 3.3-23 Reproduction of colour centre G_48

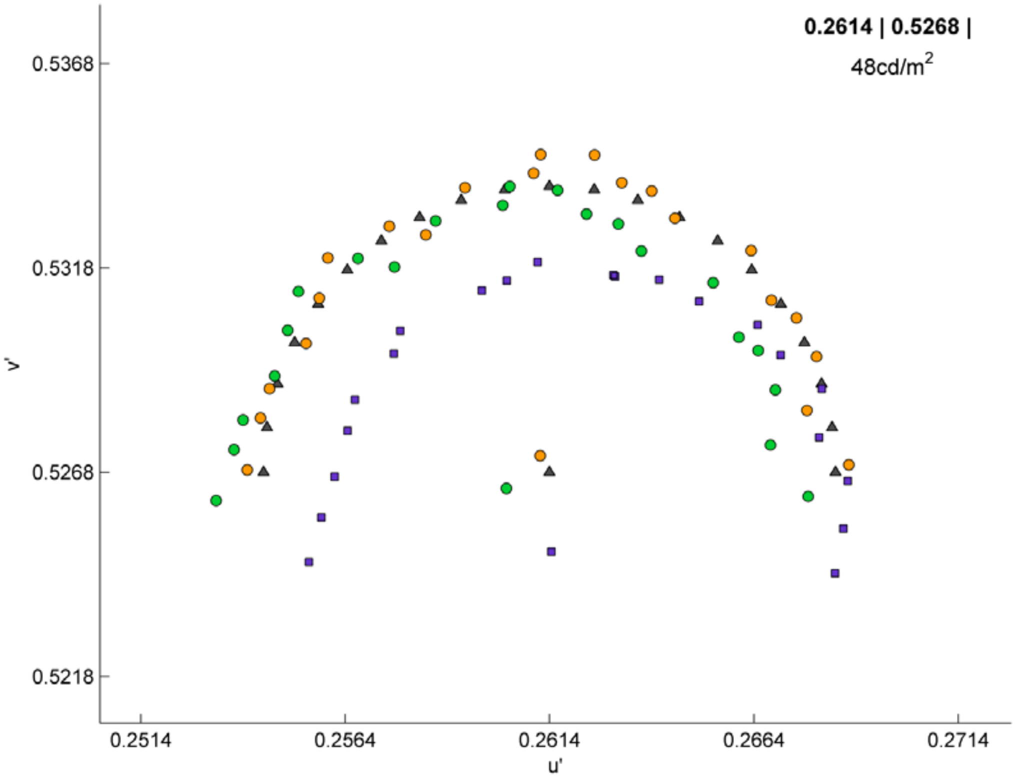

Figure 3.3-24 Reproduction of colour centre W1_48

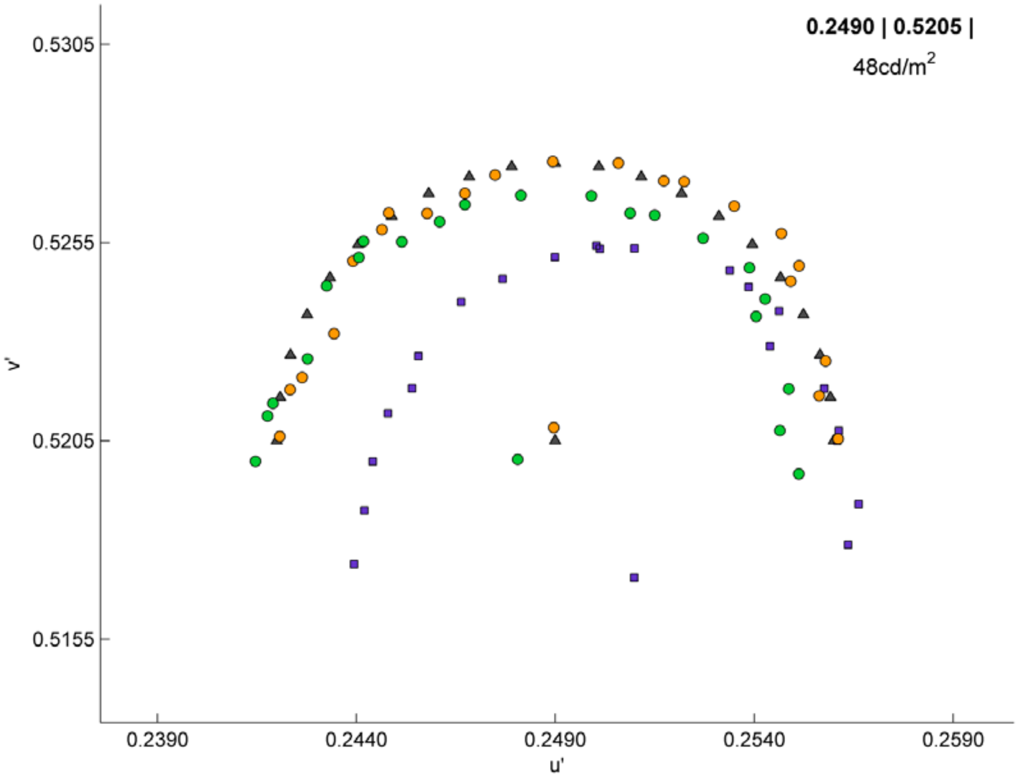

Figure 3.3-25 Reproduction of colour centre W2_48

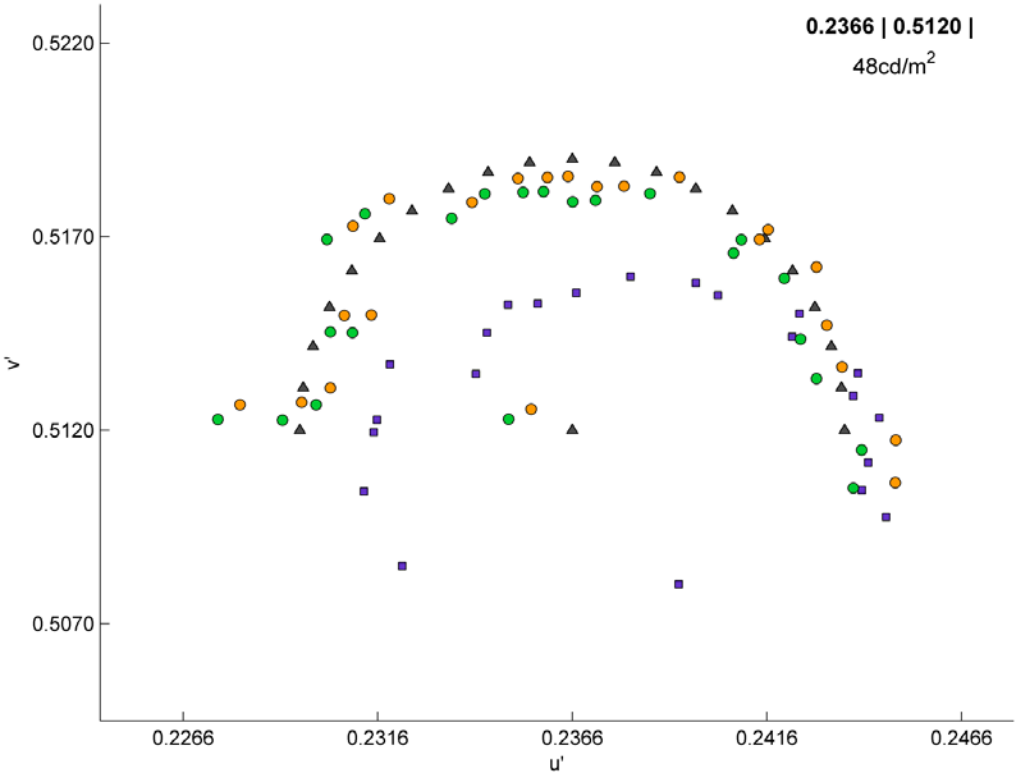

Figure 3.3-26 Reproduction of colour centre W3_48

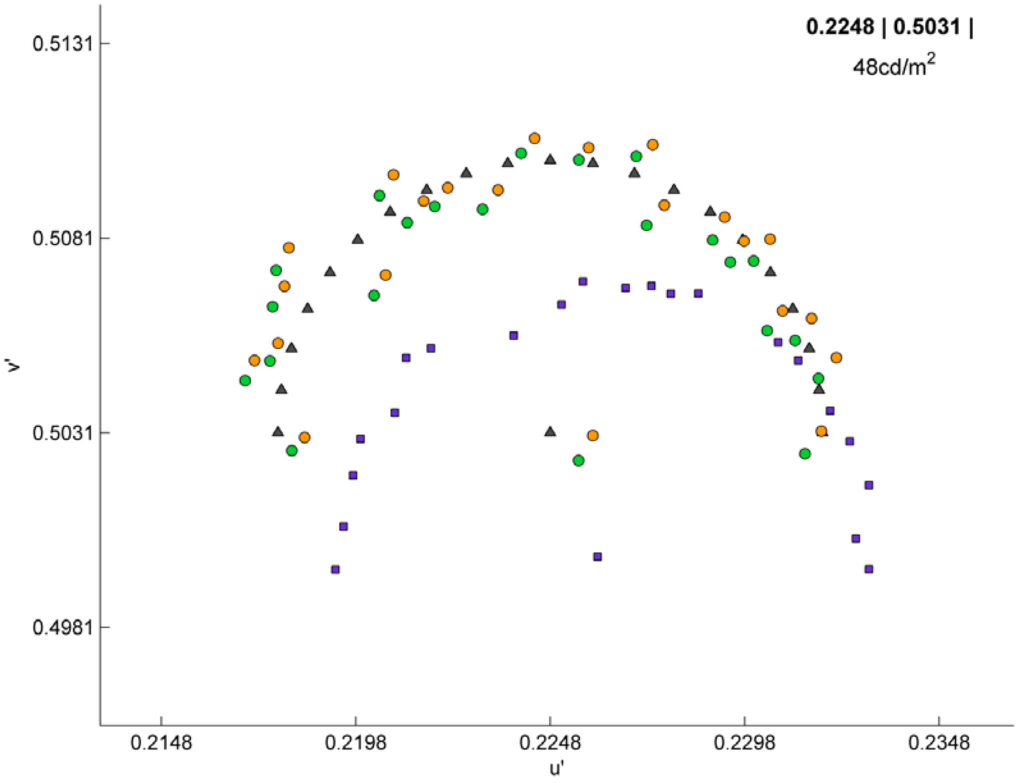

Figure 3.3-27 Reproduction of colour centre W4_48

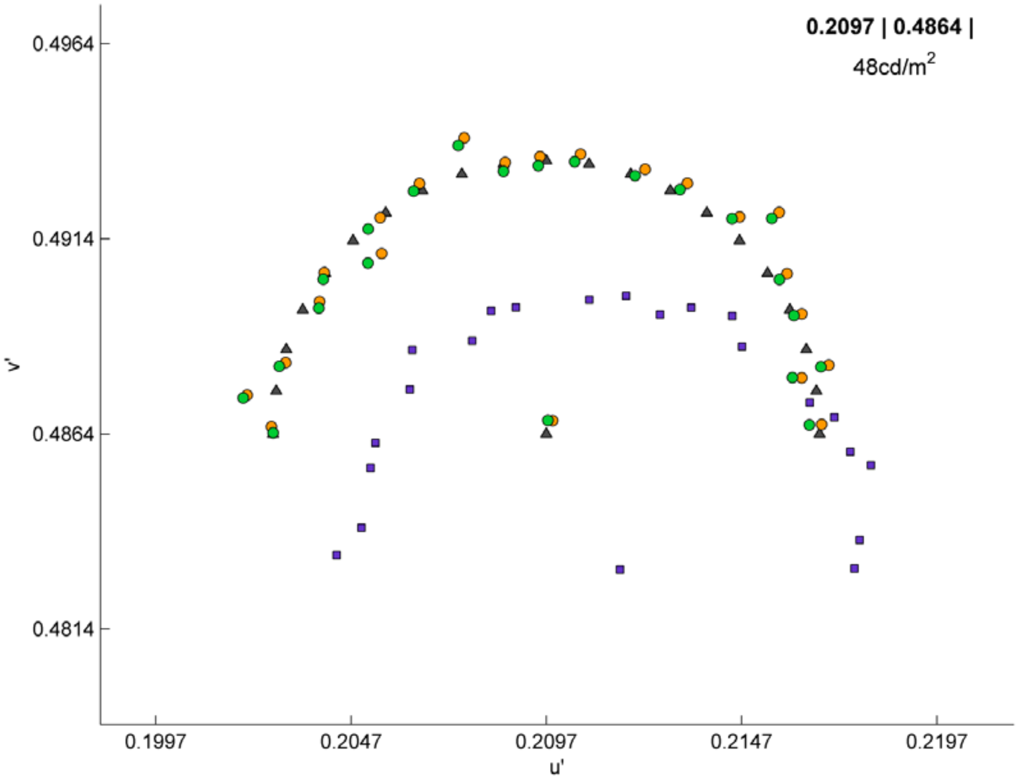

Figure 3.3-28 Reproduction of colour centre W5_48

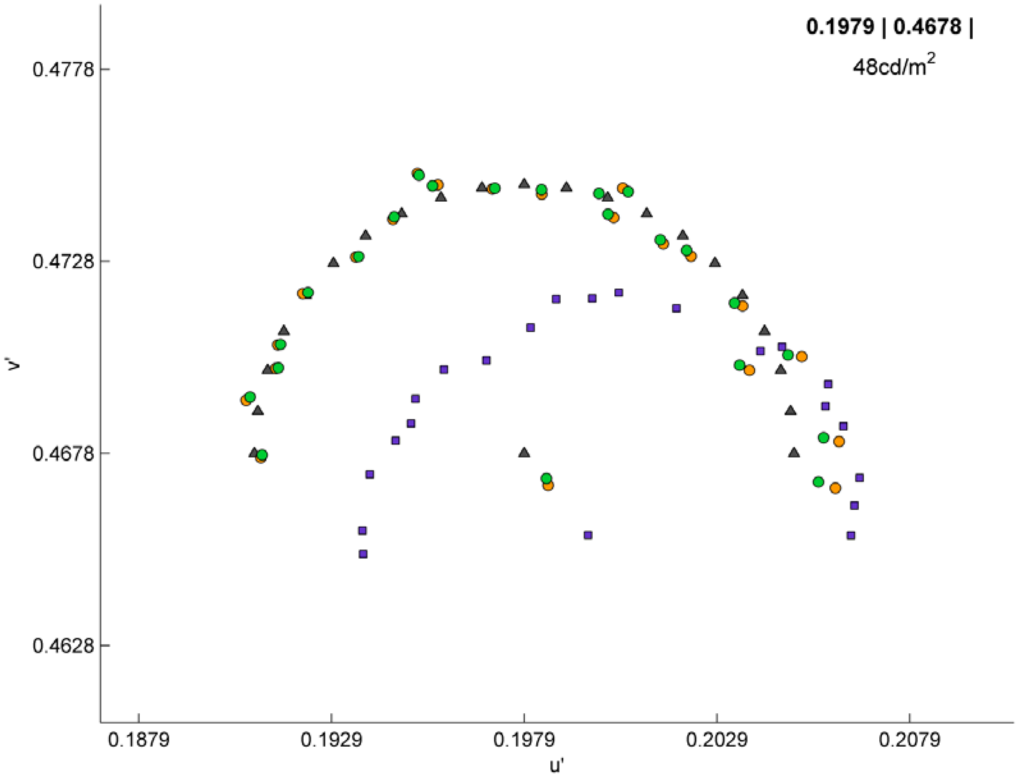

Figure 3.3-29 Reproduction of colour centre W6_48

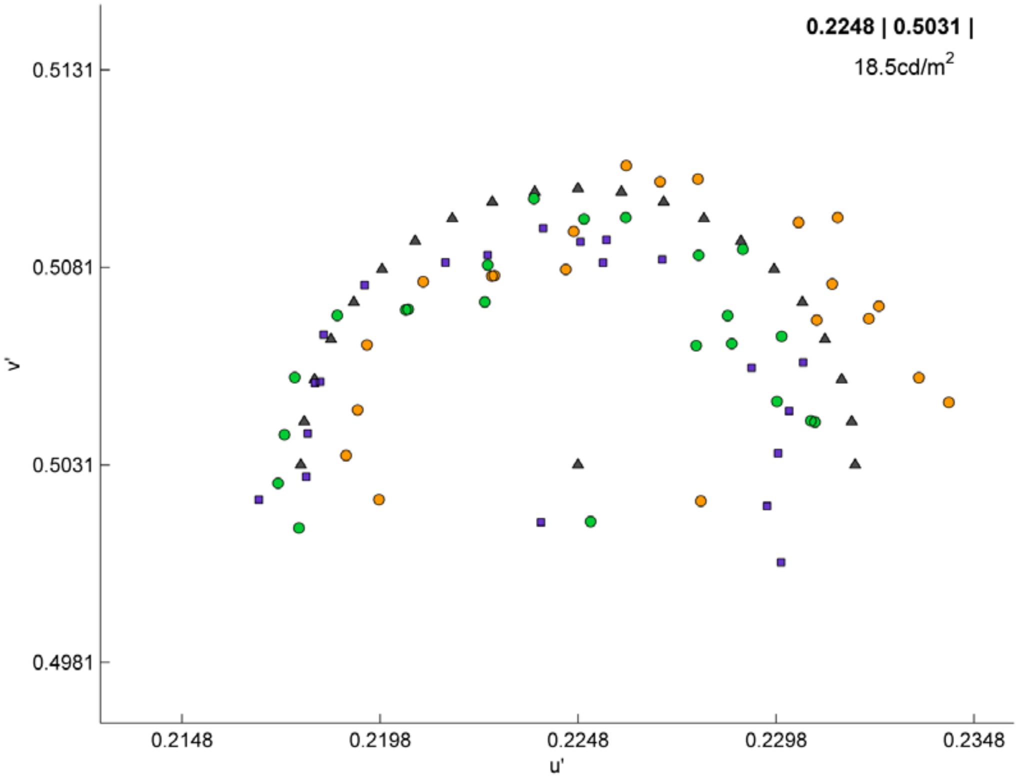

Figure 3.3-30 Reproduction of colour centre W4_18.5

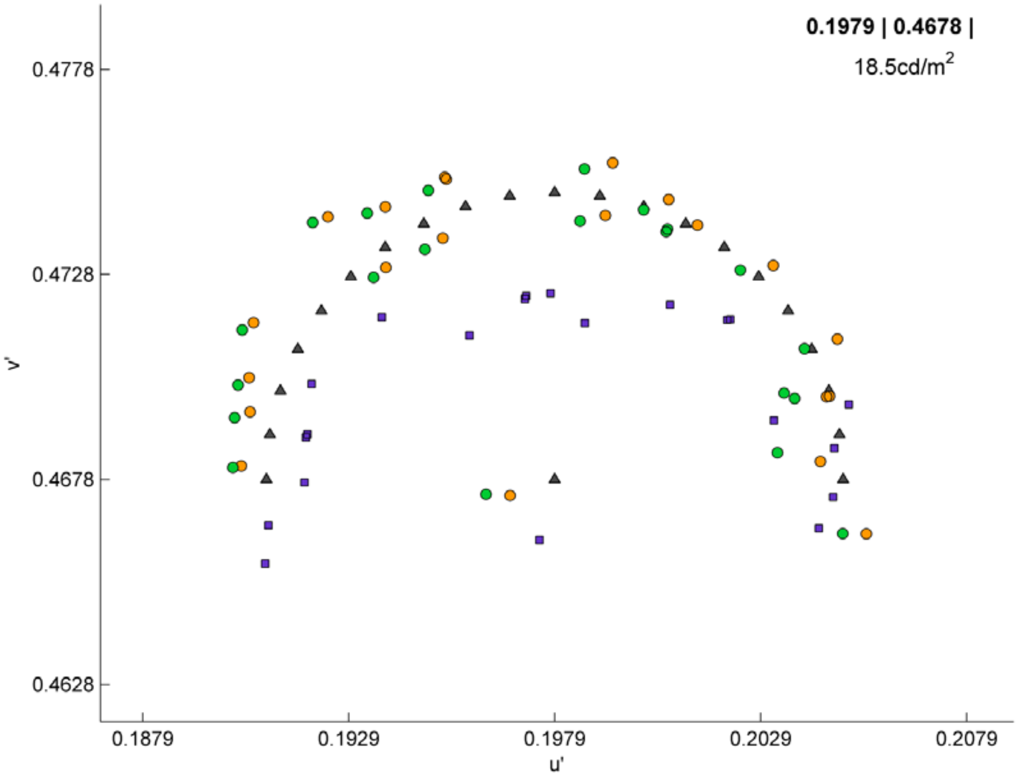

Figure 3.3-31 Reproduction of colour centre W6_18.5

Delta analysis was performed in order to inspect the absolute and relative colour differences of the pairs. For monitoring the performance of the reproduction, measurements and delta analysis were carried out three times during the whole experimental period. At the beginning; which was also how the data were set for the experiments; in the middle and at the end of the experimental period. Repeatability in the reproduction of the colour stimuli was essential for this experiment and the results of the analysis were satisfactory. The performance of reproduction is reported in CIELAB units for the following corrected data: absolute colour difference between the target and measured colour centre, mean absolute colour difference between the targets and measured samples, and mean relative colour difference dEE for the whole set of pairs of a colour centre. The data of the delta analysis are given in Table 3.3-3 and Table 3.3-4; where the aforementioned colour differences are given.

Even though, some colour centre had higher dEE than the average, they were included in the dataset as they had smooth distribution of pair samples and good overall performance. It can be said that the majority of colour stimuli gave good reproduction results and their samples' distribution resulted in decent shaped semi-circles. Any variation in the proportion of the distant between the pairs was acceptable despite the small colour difference variations. These variations might be related to possible quantisation errors from the characterisation model and the instinctive infrastructure of the display. Moreover, it can be observed that the absolute differences are slightly higher against the grey background than the black one, but the relative colour difference dEE stays unaffected.

In conclusion, the reproduction was highly satisfactory and repeatable throughout the experimental period. The mean relative colour difference dEE remained approximately the same during the three times that the measurements were taken. In detail, for the whole group of colour stimuli a mean dEE of 0.6 CIELAB units for either black or grey background was achieved for the whole experimental period. The stable reproduction of the colour stimuli can assure that the obtained data are reliable.

Table 3.3-3 Delta analysis of colour stimuli reproduction against the grey background

	First Time			Second Time			Third Time		
	ΔE* Center	**μ ΔE* Samples**	**μ dEE**	**ΔE* Center**	**μ ΔE* Samples**	**μ dEE**	**ΔE* Center**	**μ ΔE* Samples**	**μ dEE**
1_48	1.4	0.8	0.6	1.2	0.8	0.6	1.5	0.7	0.7
3_48	0.4	0.7	0.4	0.6	0.8	0.3	0.5	1.0	0.4
8_48	0.8	1.2	0.7	0.8	1.1	0.6	0.8	1.3	0.6
12_48	0.3	0.4	0.7	1.0	0.8	0.6	1.2	1.0	0.7
25_48	0.7	1.1	0.5	0.5	1.0	0.5	0.8	1.0	0.4
1_18	1.3	1.1	0.4	1.5	1.2	0.5	1.5	1.3	0.5
3_18	0.5	0.9	0.6	0.9	1.2	0.6	0.9	1.4	0.6
5_18	1.2	1.0	0.4	1.0	0.9	0.4	0.9	0.8	0.4
8_18	1.9	2.0	0.6	1.9	1.9	0.6	2.2	2.0	0.6
10_18	1.0	0.9	0.4	0.7	0.8	0.4	0.6	0.8	0.4
12_18	0.8	0.9	0.5	0.8	0.9	0.5	0.5	0.9	0.5
13_18	1.0	1.0	0.4	0.9	0.9	0.4	0.8	0.9	0.4
19_18	1.3	1.7	0.6	1.3	1.5	0.6	1.0	1.1	0.6
23_18	1.0	0.9	0.6	0.8	0.8	0.6	0.8	0.8	0.5
24_18	1.2	1.2	0.5	0.6	0.8	0.5	0.6	0.8	0.5
25_18	1.6	1.6	1.0	1.4	1.4	0.9	1.2	1.1	0.8
W1_48	0.8	1.0	0.8	0.5	0.7	0.8	0.7	0.8	0.9
W2_48	0.9	0.9	0.8	0.3	0.8	0.7	0.7	1.0	0.7
W3_48	1.0	1.0	0.4	0.9	0.9	0.4	0.7	1.0	0.4
W4_48	0.9	0.7	0.7	0.9	0.7	0.7	1.4	0.9	0.7
W5_48	0.3	0.4	0.4	0.4	0.6	0.4	0.9	0.9	0.4
W6_48	0.7	0.4	0.4	0.9	0.7	0.4	1.2	1.0	0.4
B_48	0.8	0.7	0.6	0.6	0.6	0.6	0.7	0.7	0.6
G_48	1.6	1.8	0.8	0.9	1.2	0.8	0.9	1.2	0.9
W4_18	0.9	0.8	0.7	1.5	1.1	0.8	1.3	1.0	0.8
W6_18	0.8	0.8	0.7	0.9	0.9	0.7	0.6	0.8	0.7
Mean	**1.0**	**1.0**	**0.6**	**0.9**	**1.0**	**0.6**	**1.0**	**1.0**	**0.6**

Table 3.3-4 Delta analysis of colour stimuli reproduction against the black background

	First Time			Second Time			Third Time		
	ΔE* Center	μ ΔE* Samples	μ dEE	ΔE* Center	μ ΔE* Samples	μ dEE	ΔE* Center	μ ΔE* Samples	μ dEE
1_48	0.8	0.7	0.8	0.9	0.7	0.9	1.2	0.6	0.8
3_48	0.4	0.4	0.3	0.4	0.5	0.4	0.3	0.7	0.3
8_48	0.5	0.6	0.7	0.6	0.6	0.7	0.7	0.9	0.7
12_48	0.8	0.6	0.6	0.9	0.8	0.7	1.2	1.0	0.7
25_48	0.6	0.5	0.4	0.7	0.6	0.4	1.1	1.0	0.4
1_18	1.0	0.7	0.5	0.8	0.6	0.6	0.5	0.6	0.5
3_18	0.4	0.8	0.6	0.4	0.8	0.6	0.3	1.1	0.6
5_18	0.9	0.5	0.4	0.9	0.6	0.5	0.9	0.8	0.4
8_18	0.9	0.7	0.8	0.9	0.7	0.7	1.1	0.8	0.7
10_18	0.2	0.5	0.4	0.5	0.6	0.4	0.5	0.7	0.4
12_18	0.4	0.6	0.5	0.4	0.6	0.5	0.1	0.8	0.5
13_18	0.4	0.6	0.5	0.7	0.6	0.6	0.6	0.6	0.6
19_18	0.6	0.6	0.5	0.7	0.6	0.5	1.0	0.8	0.5
23_18	0.2	0.5	0.5	0.4	0.5	0.5	0.4	0.6	0.5
24_18	0.4	0.5	0.5	0.3	0.6	0.5	0.2	0.6	0.5
25_18	0.9	0.8	0.8	0.6	0.6	0.7	0.6	0.8	0.7
W1_48	0.5	0.5	0.7	0.8	0.7	1.0	0.9	1.0	0.9
W2_48	0.3	0.4	0.7	0.5	0.7	0.8	0.9	1.0	0.8
W3_48	0.8	0.7	0.4	1.1	0.8	0.5	1.0	1.0	0.4
W4_48	0.7	0.6	0.7	1.1	0.6	0.7	1.5	0.9	0.7
W5_48	0.2	0.4	0.4	0.5	0.7	0.5	0.8	1.0	0.4
W6_48	0.8	0.5	0.3	0.9	0.7	0.4	1.2	0.9	0.4
B_48	0.3	0.5	0.5	0.2	0.6	0.6	0.3	0.4	0.6
G_48	0.4	0.7	1.0	1.0	0.8	0.8	0.5	0.7	0.9
W4_18	1.5	1.0	0.8	0.9	0.7	0.9	1.1	0.8	0.9
W6_18	0.5	0.6	0.7	0.7	0.7	0.7	0.4	0.6	0.7
Mean	**0.6**	**0.6**	**0.6**	**0.7**	**0.7**	**0.6**	**0.7**	**0.8**	**0.6**

3.4. Visual Assessment Method

The direct ratio scaling method was used in the experiments. For this purpose, custom-made software written in C++ was used in order to display the colour stimuli. The ratio method has been used by many researchers at the University of Bradford such as explained well in section 2.3.2.1. Another reason for using ratio scaling was because the colour stimuli were to be presented on a display. As seen, the stability and repeatability of the display were very good. However, there was still some amount of instability which makes it inappropriate to represent extremely small differences. It has been shown that the display was stable over time, therefore differences were negligible. So, it can be considered that the perceived colour differences were actually chromaticity differences.

The observer had the task to judge the colour difference between two pairs of colours which were presented in two different rows (top and bottom row). One pair was the reference pair and was marked with a circular pointer on its right as an indicator. The other pair was the testing pair. Observers had to evaluate the colour difference in terms of ratio against the reference pair, which had a given colour difference of one value. Then, observer had to enter his score value by using a scroll bar at the bottom of the screen. The software arrangement is illustrated in Figure 3.4-1 and Figure 3.4-2 for the grey and black background respectively. Each colour stimulus had a field size of about 4°.

In total, 28 colour centres formed this study's dataset; of which 2 were assessed twice for the control of observer variability. For this role, colour centre № 12 was selected since it was less saturated. As also explained, 21 samples were paired with each colour centre and two different backgrounds were decided. These correspond to a total of 1176 assessments for each observer. Therefore, the experiment was divided in a total of 5 sessions; 2 sessions with black background (230 assessments each session), 2 sessions with grey background (230 assessments each session), and 1 session half way with black background and the other half of the session with grey background (256 assessments for this session). This amount of sessions was chosen in order to avoid eye fatigue and observer boredom which could potentially influence the data. Each session's duration lasted between 30 to 40 minutes for most observers. In Appendices C and D, the mean ratio values by the observers' assessments against the grey and black background respectively for each colour centre are given.

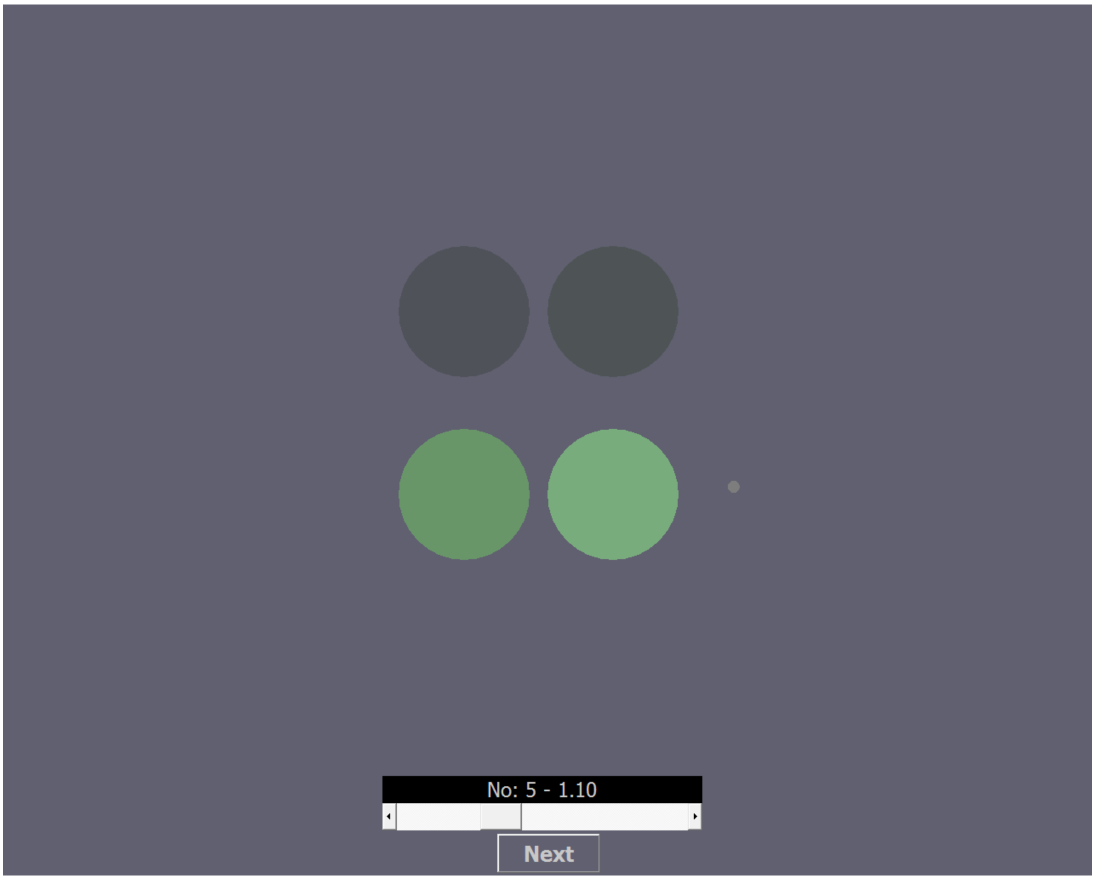

Figure 3.4-1 Stimuli arrangement against the grey background

The reference and testing pair were arranged into two rows but their locations were interchanged between the top and bottom row during the experiment. So as to prevent automatic responds from the participant, and the eye retina to get accustomed from staring the reference pair at the same location for long time. Similarly, the left and right side of the pairs were also interchanged. Of course, for each observer a different order of assessment was created and the experiments took place in dark room conditions in which the observers had to adapt before starting the experiment.

Finally, a chin rest was used in order to look at the display for two reasons. Firstly, to maintain the same viewing distance for all the sessions and observers. Secondly, to avoid colour shifts by looking the display at a different angle. LCD displays are often angular dependant and as seen in the evaluation section, the EIZO display was also affected by it.

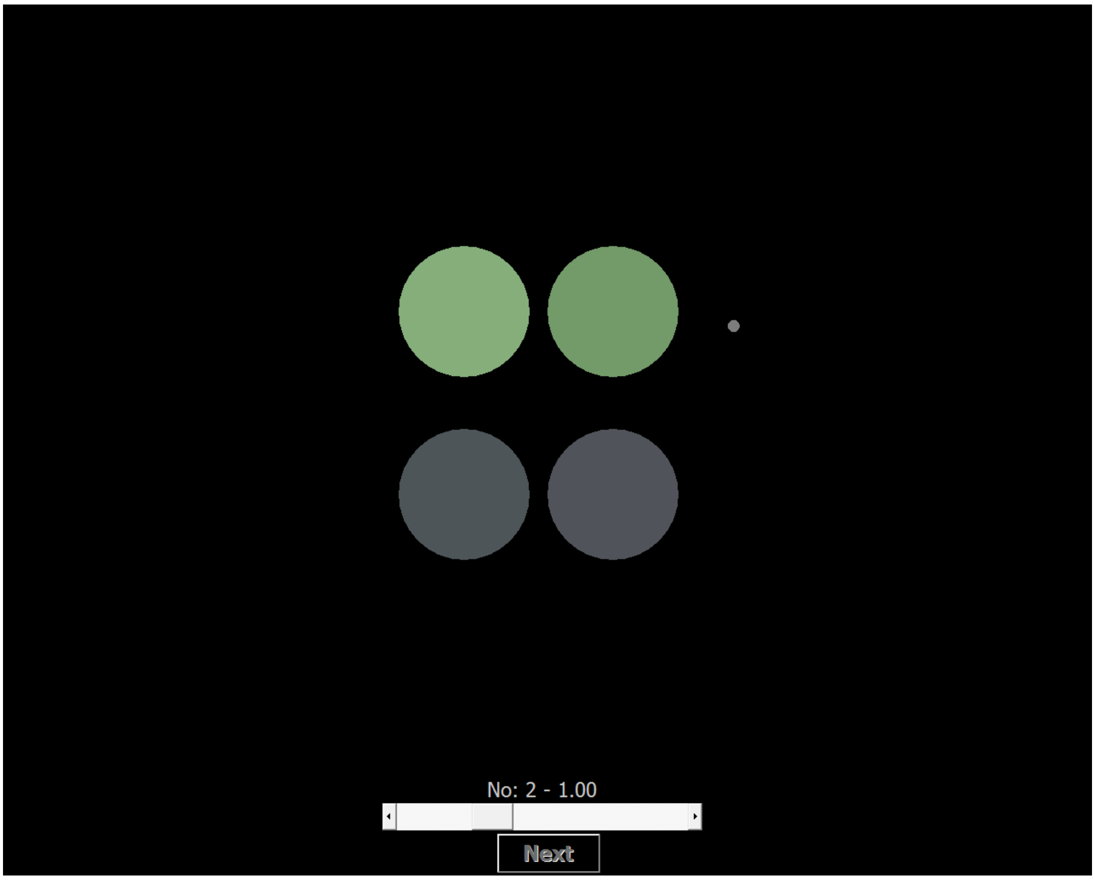

Figure 3.4-2 **Stimuli arrangement against the black background**

3.4.1. Observer Instructions

The following instructions were given to each observer prior to the experiments. Moreover, during the first session, a short training session with a few colour centres was conducted in order to teach the participants how to use the ratio scale and ensure that the observer had fully understand the assessment method. This should decrease fluctuations in the data caused by inaccuracies or misinterpretation of an oral description in the beginning of the experiment. The training data were not recorded and the observers were starting the actual experiment once they felt familiar with the scaling.

“Please sit comfortably in front of the screen and look at the display from the chin rest position. During the experiment, feel free to take a break whenever is needed. Your task is to judge the colour difference between two pairs of colours (between top and bottom pair).

The pair marked by a cursor on its right is the REFERENCE PAIR and it has a named colour difference of ONE. The other pair is named TEST PAIR.

Your task is to judge the colour difference of the TEST PAIR in terms of a ratio against the colour difference of the REFERENCE PAIR. Please adjust the scroll bar at the bottom of the screen to enter your score.

For example: If the TEST PAIR has larger colour difference than the REFERENCE PAIR, the score will be larger than one, e.g. 1.1, 1.5 or 1.7, etc. If the TEST PAIR has smaller colour difference than the REFERENCE PAIR, the score will be less than one, e.g. 0.8, 0.5, or 0.2, etc.”

3.5. Observer Variability

Twenty participants with normal colour vision took part in the experiments. Observers were tested by using Ishihara colour plates in order to determine their colour vision ability. The group of observers had an average age of 30.5 years and were of mixed ethnicity. The majority had experience participating in colour psychophysical experiments. There were 12 female and 8 male observers in the group.

The STRESS measure of fit was used for the evaluation of the group of observers. The STRESS formulae introduced in section 2.6.2 was adapted appropriately for the visual dataset in order to reveal the relationship between each individual’s assessments and the group as a whole. The higher the values, the more biased the experimental data.

3.5.1. Intra- Observer Variability

The estimation of the intra-observer variability is represented by the Equation 3.5-1. The variability between the first and second time that the same colour pair was accessed by the same observer was calculated. This measure represents the repeatability of each observer's judgements. However, to consider only two samples, the STRESS was calculated between the mean of the two assessments and each of the two assessments separately. Subsequently, the mean of these two STRESS values was computed. If STRESS value turns equal to zero then the observer had given the same response each time assessing the same pair of stimuli.

Equation 3.5-1 STRESS representation for intra- observer variability

$$STRESS = 100\sqrt{\frac{\sum\left(\Delta V2_i - f\Delta V1_i\right)^2}{\sum \Delta V2_i^2}}$$

$$where\ f = \frac{\sum \Delta V1\Delta V2}{\sum \Delta V1^2}$$

3.5.2. Inter- Observer Variability

The estimation of the inter-observer variability is represented by Equation 3.5-2. The average visual difference of the group of observers for each pair against the visual difference of each individual observer was calculated accordingly. This measure represents the accuracy of the group of observers. It reflects how much the assessments between different observers vary. The smaller the value, the more observers give similar responses.

Equation 3.5-2 STRESS measure for inter- observer variability

$$STRESS = 100\sqrt{\frac{\sum\left(\Delta V_i - f\Delta Vmean\right)^2}{\sum \Delta V_i^2}}$$

$$where\ f = \frac{\sum \Delta Vmean\Delta V_i}{\sum \Delta Vmean^2}$$

3.5.3. Observer Variability Evaluation

The results for both inter- and intra- variability are summarised in Table 3.5-1. In a first examination, it can be seen that the group of observer had reasonable variability and they performed consistently for both backgrounds. Firstly, in terms of intra- and inter- variability, intra- variability was considerably smaller than inter- variability; which is reasonable given the fact that the former represents variation between two repeated assessments and the latter variation between many observers and different type of colour pairs. It is expected inter- variability to be larger within acceptable margins. Secondly, there was little variation between the results for the two different backgrounds. However, there seems that the grey background assessments were less consistent than the black background ones. Overall, the mean of 28.8 and 15.7 STRESS units for the observer inter- and intra- variability is typical for this type of experiments. Cui *et al* had an observer variation of 37% in PF/3 values for their study in colour difference evaluation (Cui et al., 2001a). Given that PF/3 values tend to be larger than STRESS, then these STRESS values are reasonable. Finally, in the evaluation of white light sources with similar setup by Luo *et al*, the STRESS values against black background were 14.9 and 9.3 for the inter- and intra- variability respectively. Although, the corresponding inter- and intra- variability for the black background data of the current experiment were somewhat larger, it should be noted that many more colours were investigated here which may lead to higher discrepancies in the responses (Luo et al., 2015).

Table 3.5-1 Observer uncertainty in STRESS units

	Inter-observer Variability	Intra-observer Variability	**Inter-observer Variability**	Intra-observer Variability	**Inter-observer Variability**	Intra-observer Variability
Background:	**Black**		**Grey**		**Both**	
Observer 1	17.4	7.1	23.6	5.1	24.9	6.1
Observer 2	12.3	7.6	16.6	8.3	15.4	8.0
Observer 3	45.8	26.0	29.7	11.7	41.3	18.8
Observer 4	29.9	21.6	32.3	18.2	31.3	19.9
Observer 5	15.8	8.6	16.5	9.6	17.2	9.1
Observer 6	32.4	22.5	52.7	38.7	42.4	30.6
Observer 7	22.3	13.2	22.8	13.5	22.5	13.3
Observer 8	25.3	16.9	26.9	16.4	26.4	16.6
Observer 9	53.7	28.5	38.5	25.7	48.9	27.1
Observer 10	13.7	7.1	16.9	7.4	15.0	7.2
Observer 11	37.7	23.8	43.9	25.6	40.2	24.7
Observer 12	13.8	7.1	19.9	8.9	17.3	8.0
Observer 13	19.7	10.0	38.1	21.4	28.4	15.7
Observer 14	19.4	14.4	30.1	16.3	24.6	15.4
Observer 15	18.0	10.6	19.5	10.5	20.5	10.5
Observer 16	11.0	3.7	38.7	23.4	25.1	13.6
Observer 17	18.6	15.2	19.4	10.0	24.3	12.6
Observer 18	28.4	11.2	36.7	15.4	35.2	13.3
Observer 19	44.1	34.0	47.4	27.6	45.4	30.8
Observer 20	17.2	9.5	40.4	15.6	29.4	12.5
Mean	**24.8**	14.9	**30.5**	16.5	**28.8**	15.7

Another point to be addressed is that the ratio method is sometimes accused for giving the tendency to observers of consistently making assessments close to the ground value of 1; i.e. observers to underestimate the perceived colour differences. In order to examine this probability, the observers' assessments were tested in terms of range and magnitude; the results of which are given in Table 3.5-2. In this table, the maximum and minimum assessments of each observer for each background and both backgrounds (not average values but of the whole set) are reported. For each observer, the mean for the whole set of his assessments was calculated accordingly. Additionally, the range of the assessments is given in a form of ratio between the maximum and minimum values.

Firstly, by examining these results, it can be seen that the mean ratio responses of all observers for the black and grey background data differ significantly with mean values of 0.98 and 0.74 respectively. This indicates that although replies against the black background may have been in average close to the value of 1, the same trend was not apparent against the grey background. This implies that the group as a whole did not underestimate the colour differences.

Secondly, by inspecting each observer individually, the ratio of responses (maximum to minimum) for each observer varies satisfactorily. Even though there was variety in the range of assessments, there was none with significantly small range of responses, i.e. maximum and minimum assessments being between 0.9 and 1.1. However, there were a few observers that their responses may be observed critically. Observer №1 had similar mean value and range for both backgrounds. However, by closely examining the values for each colour centre separately, it was found that even though the observer had smaller range of colour differences, he/she had different ranges for each colour centre. Therefore, it can be considered that the observer was not utterly affected by the psychophysical scaling method used. The same applies for the Observers № 2, 5, 9 and 15 in different levels of degree.

Table 3.5-2 Observer variation in the ratio assessments

	Black Background				*Gray Background*				*Both Background*			
Observer	**max**	**min**	**mean**	**ratio (max/min)**	**max**	**min**	**mean**	**ratio (max/min)**	**max**	**min**	**mean**	**ratio (max/min)**
1	1.5	0.7	**1.14**	2	1.5	0.8	**1.17**	2	1.5	0.7	**1.16**	2
2	1.4	0.7	**1.01**	2	1.2	0.6	**0.87**	2	1.4	0.6	**0.94**	2
3	2.0	0.1	**0.54**	20	1.3	0.1	**0.56**	13	2.0	0.1	**0.55**	20
4	1.8	0.2	**0.89**	9	2.0	0.1	**0.57**	20	2.0	0.1	**0.73**	20
5	1.7	0.5	**1.07**	3	1.4	0.5	**0.92**	3	1.7	0.5	**0.99**	3
6	3.0	0.1	**1.34**	30	2.5	0.1	**0.60**	25	3.0	0.1	**0.97**	30
7	2.1	0.5	**1.04**	4	1.6	0.5	**0.79**	3	2.1	0.5	**0.92**	4
8	2.6	0.6	**1.04**	4	1.5	0.3	**0.71**	5	2.6	0.3	**0.87**	9
9	2.2	0.1	**0.55**	22	1.5	0.1	**0.54**	15	2.2	0.1	**0.55**	22
10	1.5	0.7	**1.08**	2	1.6	0.5	**0.79**	3	1.6	0.5	**0.93**	3
11	2.5	0.2	**0.91**	13	3.0	0.1	**0.68**	30	3.0	0.1	**0.80**	30
12	1.3	0.8	**1.02**	2	1.4	0.5	**0.86**	3	1.4	0.5	**0.94**	3
13	2.2	0.4	**1.32**	6	2.1	0.1	**0.78**	21	2.2	0.1	**1.05**	22
14	1.5	0.4	**0.97**	4	2.2	0.1	**0.78**	22	2.2	0.1	**0.88**	22
15	1.8	0.6	**1.03**	3	2.7	0.2	**0.92**	14	2.7	0.2	**0.97**	14
16	1.3	0.5	**0.99**	3	2.0	0.1	**0.63**	20	2.0	0.1	**0.81**	20
17	1.8	0.4	**1.14**	5	2.0	0.6	**1.21**	3	2.0	0.4	**1.18**	5
18	2.1	0.3	**1.00**	7	1.0	0.1	**0.48**	10	2.1	0.1	**0.74**	21
19	1.7	0.1	**0.50**	17	1.6	0.1	**0.37**	16	1.7	0.1	**0.43**	17
20	2.0	0.5	**1.04**	4	2.0	0.1	**0.52**	20	2.0	0.1	**0.78**	20
µ			**0.98**				**0.74**				**0.86**	

3.6. Conclusion

One part of the experiments has been designed to simulate the MacAdam experiment, and the other part to simulate lighting stimuli in the dark. The display has been evaluated and characterised to reproduce the pairs accurately. The psychophysical experiment was conducted with high accuracy and attention to detail so as to exclude any attribute that could affect the chromaticity of the pairs. The experiment was divided into five sessions; each of which lasted an average of 35 minutes.

Chapter 4.
Colour Discrimination Ellipses

The experimental data accumulated as described in Chapter 3, and given in Appendices C and D, are analysed here. Colour discrimination ellipses were fitted into difference colour spaces. Their performance was compared according to ellipse parameters and resulted patterns.

4.1. Fitting Ellipses

The methodology of fitting ellipses to the data was based on the techniques mentioned in chapter 2. The experimental data were fitted into ellipses for four different spaces; CIELAB, u'v', xy and CAM02-UCS. As it is expected, each colour centre has different ellipses due to the characteristics of each space. The CIELAB colour space was included as it has been used in many other studies before, and the colour difference formula is widely used. It was developed by fitting the Munsell value scale (McLaren, 1976). The CAM02-UCS was chosen as a promising uniform colour space based on results of previous research (Luo et al., 2006). It was derived for surface colours, and it takes into account different illuminants, luminance levels, viewing conditions and backgrounds within the calculation formula. Finally, the u'v' and xy chromaticity diagrams were chosen as being purely chromaticity spaces, and the ones mainly used when defining chromaticity. The Equation 4.1-1 gives the formula for fitting the data in CIELAB. Only chromaticity data were used in the fitting formula. The luminance difference was considered negligible, and the estimation of ellipses instead of ellipsoids was of most interest. So, only the relevant g_{11}, g_{12} and g_{22} coefficients were calculated. In this context, the insignificant contribution of the cross terms $\Delta a^*\Delta L^*$ and $\Delta b^*\Delta L^*$ to the overall colour difference has been illustrated before (Luo, 1986).

For each colour space, the formula for fitting the ellipses was adjusted to the input colour data relevant to the space. Therefore, a*b*, u'v', xy and a'b' chromaticity coordinates were accordingly input to the ellipse formula. The

g_{11}, g_{12} and g_{22} coefficients in the equation were optimised until a minimum STRESS value was found to best fit the measured data and the experimental visual data. The same method with different statistical metric has been used in other studies as well, such as by Luo and Rigg, Guan and Luo, Cui *et al.* and other (Cui et al., 2001b; Guan and Luo, 1999; Luo and Rigg, 1986).

Equation 4.1-1 Experimental data fitting ellipse formula

$$\Delta E_p{}^2 = \ g_{11}\Delta a^{*2} + 2\, g_{12}\Delta a^*\Delta b^* + g_{22}\Delta b^{*2}$$

Subsequently, the parameters describing the ellipses were calculated according to the Equation 2.4-5 formulae, which use the g_{11}, g_{12} and g_{22} coefficients to estimate the semi-major axis (A), the semi-minor axis (B), and the orientation angle (θ). Firstly, the axis A indicates the magnitude of the colour space. For a colour space fitted well to the experimental data, the A values for all ellipses should be constant. This implies that all ellipses have similar size. Secondly, the ratio A to B (from now on written as ratio A/B) indicates the shape of the ellipse; therefore it is used to evaluate the uniformity of the colour space under investigation. If the ratio A/B is equal to one, then instead of ellipses the result is circles. This potentially means equal colour difference in all directions. It is the ideal scenario, and one of the aims for the design of a uniform colour space. If the ratio A/B is larger than one, then the ellipse is long. In this case, the orientation angle is quite an important factor as it indicates the variation of colour difference towards divergent directions. If there is a systematic change in the directions of the orientation angle, then it indicates a bad space and formula.

The STRESS that was calculated for fitting ellipses in each respective space, can also been used to evaluate the experimental error. This value should be similar among the colour spaces used to fit the data in order to regard the experimental design as reliable. By examining the results, it was found that the mean STRESS values were similar for each colour space for both backgrounds. The average STRESS values for all the four colour spaces were 7.98 and 7.61 for the grey and the black background data respectively. This resulted in 7.80 units for all colour spaces and both backgrounds. As mentioned a zero value of STRESS would be a perfect fit between two datasets. In this case, it would represent an “ideal” colour difference equation. Hence, a value of 7.80 units in a scale between 0 and 100 is considerably low. It indicates that no matter which spaces were tested, ellipse equation fitted well to the data. Moreover, the STRESS value was systematically slightly smaller for all four colour spaces against the black background experimental data. This indicates that the ellipses for the experimental data were better fitted for the black background data than for the grey background data. This is further verified in Table 3.5-1, in which the inter-observer variability for the grey background data is larger than the one for black background data, i.e. 31 and 25 STRESS values, respectively.

Table 4.1-1 to Table 4.1-4 give the results discussed above for CIELAB, u’v’, xy, and CAM02-UCS against the grey background. While, Table 4.1-5 to Table 4.1-8 give the results against the black background respectively. The naming guide given in Table 3.3-2 for the colour centres is a useful reference for the colours hues and target colour centres. In the following sections, the results will be compared and contrasted based on the different background, different luminance of colour centres, and different sub-sets. It is reminded that for an ideal colour space, all the experimental ellipses should be constant sized circles, i.e. to all have semi-major axes A close to a constant value, and all A/B values to be close to one. A larger ellipse indicates a larger noticeable colour difference. This also means to have a larger tolerance (less strict) in the respective chromatic area of the space. For example, if two ellipses are having the same perceived colour difference, but computed colour difference of 1 and 5 units in the grey and yellow chromatic area respectively, this means that the tolerance of the grey area ellipse is much tighter (more strict) than that of the yellow area ellipse.

Table 4.1-1 Colour coordinates, STRESS and ellipse parameters for CIELAB colour space against the grey background

Grey Background

CIELAB	**a***	**b***	**STRESS**	**A**	**A/B**	**θ**
1_18	-42.96	12.99	6.31	1.138	1.28	136
1_48	-58.76	19.35	10.14	1.780	1.86	159
10_18	55.79	-19.61	10.05	1.258	1.50	135
12_18	23.04	-11.96	8.04	0.434	1.36	144
12_48	32.90	-16.07	7.88	1.346	1.62	143
13_18	-4.00	-32.34	8.03	0.884	1.42	152
19_18	46.49	31.25	11.73	1.500	1.85	115
23_18	26.09	-35.72	8.45	1.629	2.05	140
24_18	68.49	-56.16	9.85	1.458	1.83	145
25_18	20.54	33.52	6.98	1.720	2.25	94
25_48	29.20	47.25	4.88	1.235	1.80	106
3_18	-23.76	3.61	7.21	1.202	1.40	143
3_48	-33.15	5.55	6.11	1.868	1.47	163
5_18	51.93	8.31	8.25	1.348	1.54	112
8_18	-18.62	39.90	8.92	1.591	2.02	110
8_48	-27.18	56.28	6.26	2.063	1.97	100
B_48	-42.65	-30.19	8.07	1.060	1.79	163
G_48	-92.14	55.46	7.14	1.268	1.32	100
W1_48	20.97	47.28	5.91	1.177	1.80	98
W2_48	16.23	38.72	5.78	1.352	1.71	99
W3_48	10.85	29.83	11.56	1.100	1.21	98
W4_18	6.01	14.35	6.86	1.366	1.67	114
W4_48	8.05	20.58	6.80	1.678	1.44	117
W5_48	2.68	7.30	7.56	1.131	1.25	113
W6_18	-0.56	-4.90	7.72	1.546	1.93	131
W6_48	0.75	-6.70	7.17	1.365	1.63	140
Mean			**7.83**	**1.365**	**1.65**	
STRESS				**23.31**	**16.33**	

Table 4.1-2 Colour coordinates, STRESS and ellipse parameters for u’v’ chromaticity diagram against the grey background

Grey Background

u'v'	u'	v'	STRESS	A	A/B	θ
1_18	0.1320	0.5113	6.08	0.874	1.18	109
1_48	0.1327	0.5132	9.85	1.543	1.60	151
10_18	0.3063	0.4217	11.20	1.008	1.14	37
12_18	0.2391	0.4468	8.04	0.371	1.09	14
12_48	0.2414	0.4471	7.77	1.006	1.11	135
13_18	0.1652	0.4074	8.69	1.061	1.26	72
19_18	0.3412	0.5111	11.04	1.123	1.49	160
23_18	0.2169	0.3936	8.65	1.106	1.38	102
24_18	0.2746	0.3381	10.57	0.951	1.12	64
25_18	0.2730	0.5234	7.86	0.914	1.39	36
25_48	0.2760	0.5245	4.94	0.838	1.34	166
3_18	0.1593	0.4898	6.81	1.041	1.15	118
3_48	0.1588	0.4907	6.18	1.635	1.24	164
5_18	0.3347	0.4763	9.71	1.039	1.16	10
8_18	0.1873	0.5434	8.78	1.018	1.32	140
8_48	0.1854	0.5448	6.35	1.170	1.09	148
B_48	0.1298	0.4359	7.45	0.697	1.15	115
G_48	0.1038	0.5560	7.80	1.086	1.10	169
W1_48	0.2609	0.5265	5.67	0.753	1.26	8
W2_48	0.2490	0.5201	5.41	0.899	1.16	16
W3_48	0.2355	0.5123	11.95	1.156	1.41	10
W4_18	0.2260	0.5014	6.80	0.999	1.13	96
W4_48	0.2262	0.5025	6.73	1.332	1.06	162
W5_48	0.2102	0.4867	7.56	1.060	1.17	46
W6_18	0.1966	0.4674	8.27	1.230	1.51	104
W6_48	0.1988	0.4671	7.37	1.077	1.21	118
Mean			**7.98**	**1.038**	**1.24**	
STRESS				**22.82**	**11.58**	

Table 4.1-3 Colour coordinates, STRESS and ellipse parameters for xy chromaticity diagram against the grey background

Grey Background

xy	x	y	STRESS	A	A/B	*θ*
1_18	0.2576	0.4435	6.08	1.173	1.77	75
1_48	0.2605	0.4478	10.15	1.031	1.57	102
10_18	0.3888	0.2379	11.31	1.518	2.02	34
12_18	0.3424	0.2843	8.02	0.475	1.61	41
12_48	0.3452	0.2841	7.86	1.234	1.55	50
13_18	0.2296	0.2517	8.30	0.962	1.63	48
19_18	0.5233	0.3484	11.21	1.071	1.59	51
23_18	0.2787	0.2248	8.70	1.133	1.71	56
24_18	0.3000	0.1642	10.64	1.335	1.88	32
25_18	0.4667	0.3977	7.83	1.355	2.25	39
25_48	0.4719	0.3985	5.04	0.864	1.57	53
3_18	0.2801	0.3827	6.64	1.296	1.61	69
3_48	0.2802	0.3848	6.09	1.563	1.24	71
5_18	0.4716	0.2982	9.88	1.517	1.97	37
8_18	0.3806	0.4906	9.13	1.186	1.86	75
8_48	0.3796	0.4958	6.11	1.595	1.81	66
B_48	0.2013	0.3004	7.65	0.750	1.30	73
G_48	0.2507	0.5968	7.68	1.403	1.68	81
W1_48	0.4568	0.4097	5.58	0.911	1.71	45
W2_48	0.4333	0.4023	5.28	0.950	1.80	47
W3_48	0.4064	0.3929	11.58	1.130	1.53	38
W4_18	0.3813	0.3760	6.91	1.431	1.96	56
W4_48	0.3829	0.3781	6.90	1.755	1.67	54
W5_48	0.3456	0.3556	7.54	1.440	1.80	47
W6_18	0.3103	0.3279	8.05	1.623	2.11	65
W6_48	0.3129	0.3267	7.25	1.297	1.63	63
Mean			**7.98**	**1.231**	**1.72**	
STRESS				**23.32**	**12.81**	

Table 4.1-4 Colour coordinates, STRESS and ellipse parameters for CAM02-UCS colour space against the grey background

Grey Background

CAM02-UCS	**a'**	**b'**	**STRESS**	**A**	**A/B**	***θ***
1_18	-25.80	7.44	7.27	1.054	1.57	77
1_48	-31.06	9.87	8.85	0.752	1.06	90
10_18	32.16	-8.70	10.46	0.920	1.18	79
12_18	16.21	-7.68	7.71	0.353	1.09	78
12_48	20.78	-9.11	8.49	0.957	1.13	120
13_18	-11.15	-23.15	8.10	0.625	1.69	173
19_18	28.40	15.68	11.58	1.441	2.30	132
23_18	11.21	-22.95	8.51	0.928	1.48	16
24_18	28.90	-25.08	9.67	1.378	1.95	32
25_18	13.12	18.29	6.94	0.918	1.91	115
25_48	16.48	22.89	4.82	1.406	2.03	133
3_18	-18.40	2.20	8.83	1.074	1.27	75
3_48	-22.90	3.09	7.10	1.555	1.24	47
5_18	32.20	5.01	8.66	1.064	1.62	117
8_18	-11.54	22.53	8.33	1.350	1.69	125
8_48	-14.88	27.98	6.57	1.259	1.44	111
B_48	-29.69	-18.45	7.11	1.280	1.51	134
G_48	-38.10	23.89	7.51	1.668	2.04	83
W1_48	11.98	23.37	5.80	0.821	1.73	127
W2_48	9.99	20.17	5.68	0.776	1.61	127
W3_48	7.36	16.58	11.26	0.651	1.31	150
W4_18	4.82	9.42	7.00	1.535	1.60	130
W4_48	5.94	12.35	6.97	1.472	1.51	140
W5_48	2.14	5.29	7.46	0.988	1.21	151
W6_18	-2.59	-5.62	11.56	1.066	1.46	126
W6_48	-1.53	-7.18	9.37	0.997	1.31	144
Mean			**8.14**	**1.088**	**1.53**	
STRESS				**28.37**	**20.15**	

Table 4.1-5 Colour coordinates, STRESS and ellipse parameters for CIELAB colour space against the black background

Black Background

CIELAB	**a***	**b***	**STRESS**	**A**	**A/B**	**θ**
1_18	-43.85	13.50	6.92	1.153	1.34	158
1_48	-59.19	19.79	8.36	1.901	1.82	161
10_18	56.69	-19.73	9.16	0.639	1.50	136
12_18	23.42	-11.88	7.11	1.186	1.37	122
12_48	33.33	-16.03	7.22	1.741	1.81	143
13_18	-3.67	-32.94	8.80	0.757	1.60	149
19_18	47.28	32.55	9.97	0.982	1.68	115
23_18	26.54	-35.97	7.80	1.436	1.88	150
24_18	69.33	-56.58	9.28	0.914	2.05	148
25_18	21.10	35.03	4.94	0.812	2.10	95
25_48	29.71	48.15	5.41	1.372	1.77	102
3_18	-24.13	3.89	7.70	1.292	1.33	154
3_48	-33.22	5.53	6.96	1.830	1.33	162
5_18	52.76	8.82	9.52	1.642	1.61	112
8_18	-19.01	41.68	8.15	1.758	1.81	109
8_48	-27.45	57.43	5.00	1.222	1.64	98
B_48	-43.10	-30.31	8.81	2.213	1.88	164
G_48	-93.37	56.79	6.13	1.061	1.14	111
W1_48	21.28	48.28	5.02	0.982	1.77	95
W2_48	16.62	39.57	5.71	0.923	1.50	100
W3_48	11.08	30.38	12.57	0.942	1.19	73
W4_18	6.37	14.95	7.60	1.212	1.39	124
W4_48	8.26	21.14	6.02	1.289	1.40	129
W5_48	2.88	7.48	8.07	0.451	1.27	128
W6_18	-0.45	-4.71	5.74	1.009	1.41	128
W6_48	0.93	-6.60	5.74	1.294	1.55	141
Mean			**7.45**	**1.231**	**1.58**	
STRESS				**31.69**	**16.00**	

Table 4.1-6 Colour coordinates, STRESS and ellipse parameters for u’v’ chromaticity diagram against the black background

Black Background

u'v'	u'	v'	STRESS	A	A/B	θ
1_18	0.1305	0.5124	6.82	1.040	1.17	148
1_48	0.1321	0.5138	8.04	1.607	1.57	154
10_18	0.3082	0.4210	10.15	0.509	1.14	32
12_18	0.2399	0.4467	7.28	1.069	1.24	59
12_48	0.2419	0.4470	7.08	1.214	1.22	133
13_18	0.1649	0.4055	8.90	0.973	1.25	93
19_18	0.3446	0.5123	8.65	0.763	1.61	165
23_18	0.2172	0.3925	8.04	0.957	1.16	130
24_18	0.2754	0.3366	9.70	0.521	1.06	112
25_18	0.2753	0.5250	5.50	0.450	1.32	26
25_48	0.2770	0.5250	5.73	0.848	1.34	174
3_18	0.1586	0.4903	7.24	1.118	1.08	155
3_48	0.1585	0.4906	6.94	1.624	1.12	173
5_18	0.3376	0.4768	11.09	1.154	1.12	7
8_18	0.1870	0.5454	7.79	1.149	1.35	154
8_48	0.1851	0.5457	5.38	0.827	1.28	173
B_48	0.1292	0.4356	8.43	1.327	1.18	125
G_48	0.1028	0.5571	6.34	1.102	1.30	170
W1_48	0.2615	0.5272	5.06	0.617	1.33	11
W2_48	0.2497	0.5208	5.62	0.698	1.30	5
W3_48	0.2359	0.5128	12.97	1.112	1.63	13
W4_18	0.2271	0.5022	7.08	1.018	1.16	163
W4_48	0.2266	0.5030	5.86	1.120	1.24	160
W5_48	0.2104	0.4868	7.79	0.401	1.07	22
W6_18	0.1967	0.4676	6.24	0.877	1.20	80
W6_48	0.1989	0.4671	5.87	0.995	1.15	120
Mean			**7.52**	**0.965**	**1.25**	
STRESS				**30.48**	**12.03**	

Table 4.1-7 Colour coordinates, STRESS and ellipse parameters for xy chromaticity diagram against the black background

Black Background						
xy	x	y	STRESS	A	A/B	θ
1_18	0.2562	0.4470	6.87	1.158	1.49	81
1_48	0.2601	0.4496	8.29	1.290	1.50	103
10_18	0.3899	0.2367	10.27	0.752	1.99	34
12_18	0.3431	0.2839	7.27	1.488	2.03	42
12_48	0.3455	0.2838	7.24	1.390	1.52	56
13_18	0.2282	0.2495	8.94	0.706	1.51	57
19_18	0.5283	0.3491	8.85	0.641	1.45	47
23_18	0.2783	0.2235	8.20	1.041	1.35	47
24_18	0.2998	0.1628	9.78	0.682	1.64	33
25_18	0.4717	0.3998	5.49	0.624	2.02	40
25_48	0.4738	0.3991	5.88	0.870	1.56	48
3_18	0.2795	0.3841	7.26	1.307	1.43	65
3_48	0.2797	0.3847	6.87	1.743	1.33	62
5_18	0.4750	0.2981	11.28	1.769	1.99	39
8_18	0.3828	0.4963	8.11	1.273	1.63	76
8_48	0.3804	0.4983	4.98	0.809	1.48	66
B_48	0.2003	0.3002	8.61	1.341	1.26	79
G_48	0.2498	0.6018	6.19	1.139	1.52	90
W1_48	0.4585	0.4107	4.86	0.709	1.69	42
W2_48	0.4351	0.4032	5.68	0.762	1.57	45
W3_48	0.4074	0.3936	12.54	1.094	1.60	29
W4_18	0.3837	0.3772	7.29	1.140	1.53	55
W4_48	0.3840	0.3789	6.06	1.105	1.45	59
W5_48	0.3460	0.3557	7.91	1.191	1.62	50
W6_18	0.3107	0.3283	6.08	1.328	1.86	54
W6_48	0.3130	0.3267	5.77	1.201	1.58	61
Mean			**7.56**	**1.098**	**1.60**	
STRESS				**27.85**	**12.96**	

Table 4.1-8 Colour coordinates, STRESS and ellipse parameters for CAM02-UCS colour space against the black background

Black Background

CAM02-UCS	a'	b'	STRESS	A	A/B	θ
1_18	-29.88	8.74	7.36	1.080	1.38	69
1_48	-33.12	10.59	7.30	0.969	1.07	69
10_18	35.63	-9.58	9.86	0.466	1.21	80
12_18	18.04	-8.54	6.68	0.901	1.39	86
12_48	21.23	-9.37	8.17	1.112	1.21	132
13_18	-13.17	-26.18	9.40	0.422	1.79	164
19_18	31.31	17.91	9.49	0.571	2.23	133
23_18	12.22	-25.86	8.39	1.222	2.06	15
24_18	31.82	-27.73	9.25	0.778	1.92	29
25_18	14.33	21.27	5.02	0.471	1.85	119
25_48	16.65	24.12	5.64	0.925	1.93	132
3_18	-21.71	2.85	8.44	1.197	1.28	61
3_48	-24.72	3.37	7.67	1.765	1.35	56
5_18	35.66	5.84	10.47	1.226	1.79	118
8_18	-13.94	26.14	7.74	1.042	1.45	128
8_48	-16.38	29.49	5.59	0.665	1.17	113
B_48	-31.76	-18.99	8.05	1.499	1.58	135
G_48	-40.44	25.23	6.77	1.270	1.75	80
W1_48	11.83	24.71	4.88	0.620	1.65	127
W2_48	9.83	21.44	5.74	0.694	1.53	136
W3_48	6.99	17.68	12.30	0.865	1.24	166
W4_18	5.04	11.28	7.72	1.073	1.56	147
W4_48	5.48	13.36	6.22	1.120	1.68	149
W5_48	1.36	5.90	9.15	0.959	1.42	156
W6_18	-4.15	-5.95	9.83	0.826	1.08	103
W6_48	-3.03	-7.17	8.52	0.899	1.21	143
Mean			**7.91**	**0.948**	**1.53**	
STRESS				**31.52**	**19.83**	

4.2. Comparing Visual Results

By averaging the visual differences for each colour centre it is possible to compare and contrast the average magnitude. In Table 4.2-1 the mean visual difference for pairs of each colour centre and background are given. The smallest value in each row is underlined, while in each column is in bold. As can be seen, the visual ratio assessment was smaller for roughly all colour centres against the grey background. Moreover, different colour centres gave the smallest visual colour difference in each background. For the grey background data, it was the farthest green colour, while for the black background data, it was a purple colour. The mean ratio indicates that grey background has a much larger discrepancy in visual difference between different colour centres than that of black background, i.e. by a factor of 2.0 and 1.4 respectively. These results agree well with the results presented in Table 3.5-2, where observer variation and range of the ratio assessments are summarised.

Table 4.2-1 Mean visual differences for each colour centre and background

Colour Centre	Grey	Black
1_18	0.65	1.00
1_48	0.73	0.98
10_18	0.59	0.87
12_18	0.78	0.95
12_48	0.73	0.95
13_18	0.61	0.88
19_18	0.65	1.00
23_18	0.68	0.86
24_18	0.59	**0.82**
25_18	0.62	1.00
25_48	0.76	1.13
3_18	0.76	1.04
3_48	0.84	0.98
5_18	0.62	0.96
8_18	0.67	0.92
8_48	0.68	1.00
B_48	0.63	0.84
G_48	**0.55**	0.90
W1_48	0.76	1.12
W2_48	0.79	1.08
W3_48	0.79	1.05
W4_18	0.75	1.09
W4_48	0.89	1.09
W5_48	0.90	1.03
W6_18	1.11	1.06
W6_48	0.97	0.99
Mean	*0.73*	*0.98*
Ratio (max/min)	*2.02*	*1.37*

4.3. Comparing Chromaticity Ellipses

Firstly, the ellipses' size in overall was investigated. The area of an ellipse is given by the multiplication of πAB, using the semi-major (A) and semi-minor axis (B). Similarly, the size is calculated by the square root of the ellipse area. The sizes for each colour space and background are summarised in Table 4.3-1; where the bold and underlined values correspond to the maximum and minimum values for each column and space respectively.

Visualisations of the ellipses are illustrated in Figure 4.3-1 and Figure 4.3-2 for CIELAB, in Figure 4.3-3 and Figure 4.3-4 for u’v’ chromaticity diagram, in Figure 4.3-5 and Figure 4.3-6 for xy chromaticity diagram, and in Figure 4.3-7 and Figure 4.3-8 for CAM02-UCS; against the grey and black background accordingly.

Table 4.3-1 Size of ellipses for each colour space and background

	CIELAB		**u'v'**		**xy**		**CAM02-UCS**	
	Grey	**Black**	**Grey**	**Black**	**Grey**	**Black**	**Grey**	**Black**
1_18	1.78	1.77	1.43	1.70	1.56	1.68	1.49	1.63
1_48	2.31	2.50	2.16	2.27	1.46	1.87	1.29	1.66
10_18	1.82	0.92	1.67	0.85	1.89	0.95	1.50	0.75
12_18	0.66	1.79	0.63	1.70	0.66	1.85	0.60	1.35
12_48	1.87	2.29	1.70	1.95	1.75	2.00	1.60	1.79
13_18	1.32	1.06	1.68	1.54	1.34	1.02	0.85	0.56
19_18	1.96	1.34	1.63	1.07	1.50	0.94	1.68	0.68
23_18	2.01	1.86	1.67	1.58	1.54	1.59	1.35	1.51
24_18	1.91	1.13	1.59	0.90	1.73	0.94	1.75	1.00
25_18	2.03	0.99	1.38	0.70	1.60	0.78	1.18	0.61
25_48	1.63	1.83	1.28	1.30	1.22	1.24	1.75	1.18
3_18	1.80	1.99	1.72	1.91	1.81	1.94	1.69	1.87
3_48	**2.73**	2.82	**2.60**	**2.72**	**2.49**	**2.68**	**2.47**	**2.69**
5_18	1.93	2.29	1.71	1.94	1.92	2.22	1.48	1.62
8_18	1.99	2.31	1.57	1.75	1.54	1.77	1.84	1.53
8_48	2.60	1.69	1.99	1.29	2.10	1.18	1.86	1.09
B_48	1.40	**2.86**	1.15	2.17	1.16	2.11	1.84	2.12
G_48	1.96	1.76	1.84	1.71	1.92	1.64	2.07	1.70
W1_48	1.55	1.31	1.19	0.95	1.24	0.97	1.11	0.86
W2_48	1.83	1.34	1.48	1.08	1.26	1.08	1.08	0.99
W3_48	1.78	1.53	1.73	1.54	1.62	1.53	1.01	1.38
W4_18	1.87	1.82	1.66	1.68	1.81	1.63	2.15	1.52
W4_48	2.48	1.93	2.30	1.78	2.41	1.63	2.13	1.53
W5_48	1.79	0.71	1.74	0.69	1.90	1.66	1.59	1.43
W6_18	1.97	1.51	1.78	1.42	1.98	1.73	1.56	1.41
W6_48	1.89	1.84	1.73	1.65	1.80	1.69	1.55	1.45
Ratio (max/min)	4.14	4.03	4.13	3.95	3.75	3.44	4.13	4.81
Mean	**1.88**	1.74	**1.65**	1.53	**1.66**	1.55	**1.56**	1.38
Sum	**48.90**	45.20	**43.00**	39.82	**43.23**	40.30	**40.50**	35.93
STRESS	20.69	**30.04**	21.90	**30.46**	22.58	**28.47**	25.97	**32.61**

The colour centre 12_18 (a dark purple colour) appeared to consistently have the smallest ellipse size in every colour space when against the grey background. There are a few things to be considered for this. The specific colour centre was not saturated but yet it has a dark purple hue. Due to its colour, it could appear having less lightness against the grey background possibly due to the crispening effect, i.e. if the colour pair to be assessed is close to the lightness of the background, its colour difference will be enlarged (Fairchild, 2005). What is also to be noted is the large difference of size between the black and grey background ellipses for this colour centre in contrast to the range of the majority of the colour centres. The fact that this colour centre was assessed twice for the repeatability examination does not relate with the resulted smallest ellipse. Because the colour centre 12_48 (the same hue as 12_18, but much lighter) was also assessed twice for the same reason but was not affected similarly. Likewise the majority of the colour centres, colour centre 12_48 has a similar colour difference magnitude between the grey and black background.

Another colour centre that had such a large difference between the black and grey background assessments was the colour centre 25_18 (a dark orange colour) but with the opposite effect. This colour centre has much smaller ellipse size while assessed against the black background instead. In terms of hue colour, colour centres 12_18 and 25_18 do not relate, so this could either be due to the crispening effect or the non-uniformity of the specific chromatic space areas.

On the other hand, the colour centres with the smallest ellipse size against the black background varied for each colour space. Except of CIELAB and u’v’ chromaticity diagram for which they coincide with the colour centre W5_48. This was a greatly neutral white light colour centre with CCT of 5000 K, so there can be less visible chromaticity change around the centre; especially against black background for which the contrast is high.

The colour centre 3_48 (a light bluish green colour) had by majority the largest ellipses in every background and colour space. This colour centre has similar colour with the reference pair; therefore it could be possible that the observers compared their differences more clearly. On the other hand, in relation to the respective mean raw ratio assessment, this colour centre has a value close to one for the black background data, so the observers could have associated it with the colour difference of the reference pair. Historically, colours in the green area of the xy chromaticity diagram appear to have larger ellipses due to the non-uniformity of the space (MacAdam, 1942; Macadam, 1944). Therefore, this indicates that for these colour spaces, there is still non-uniformity in this colour region.

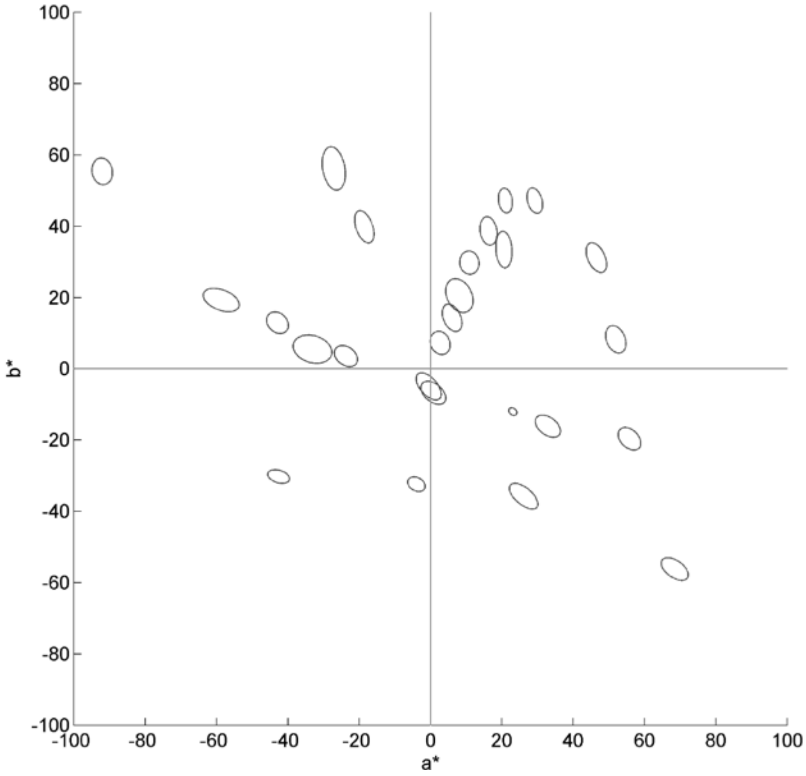

Figure 4.3-1 Ellipses in CIELAB a*b* diagram against the grey background (enlarged 3 times)

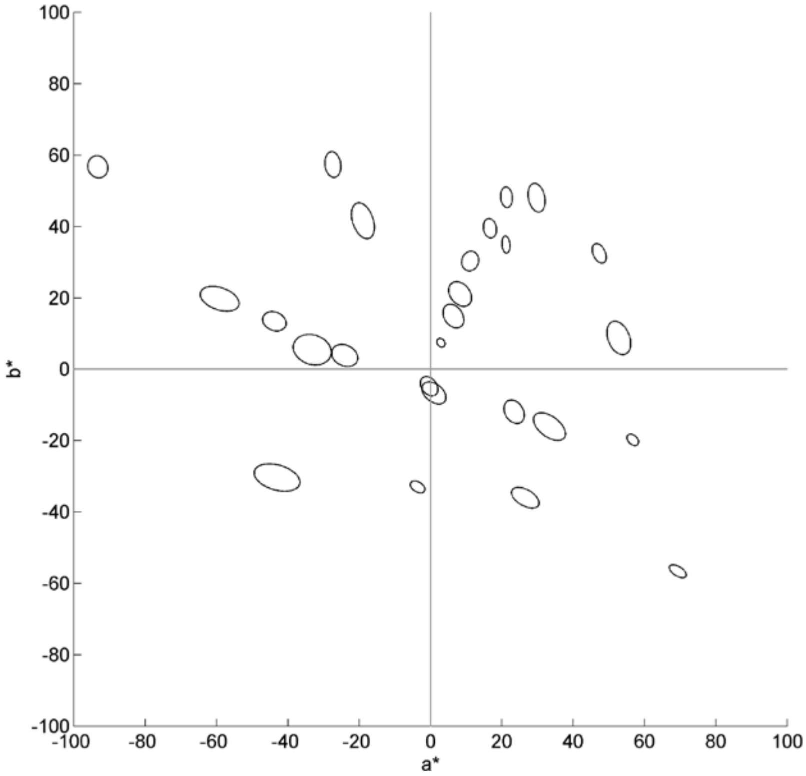

Figure 4.3-2 Ellipses in CIELAB a*b* diagram against the black background (enlarged 3 times)

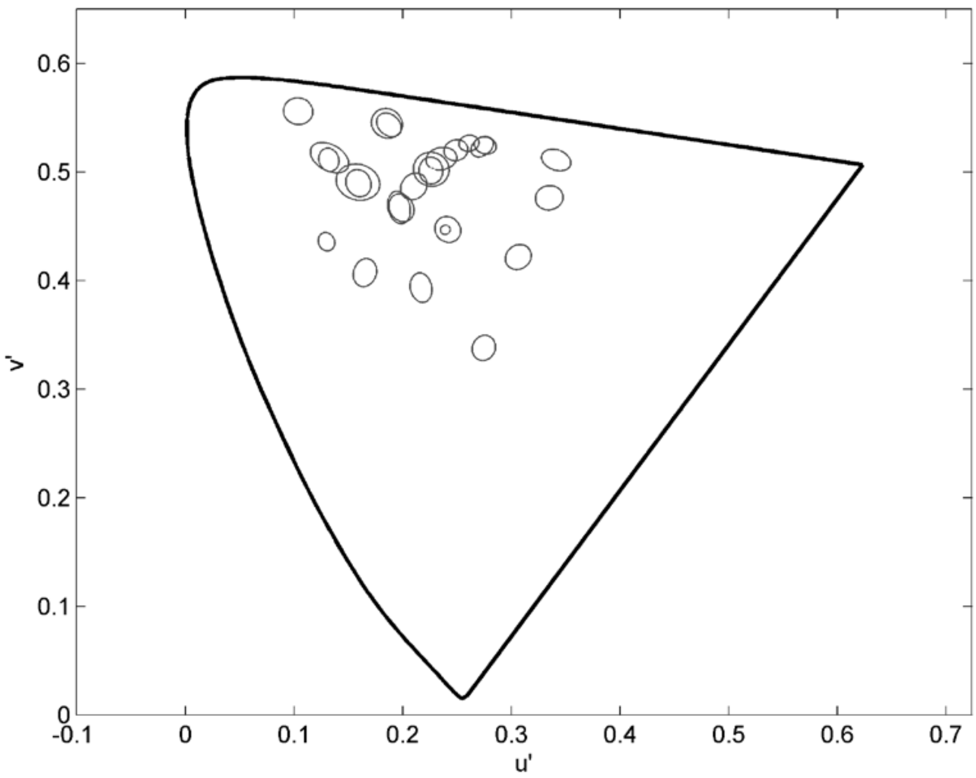

Figure 4.3-3 Ellipses in u'v' chromaticity diagram against the grey background (compressed 80 times)

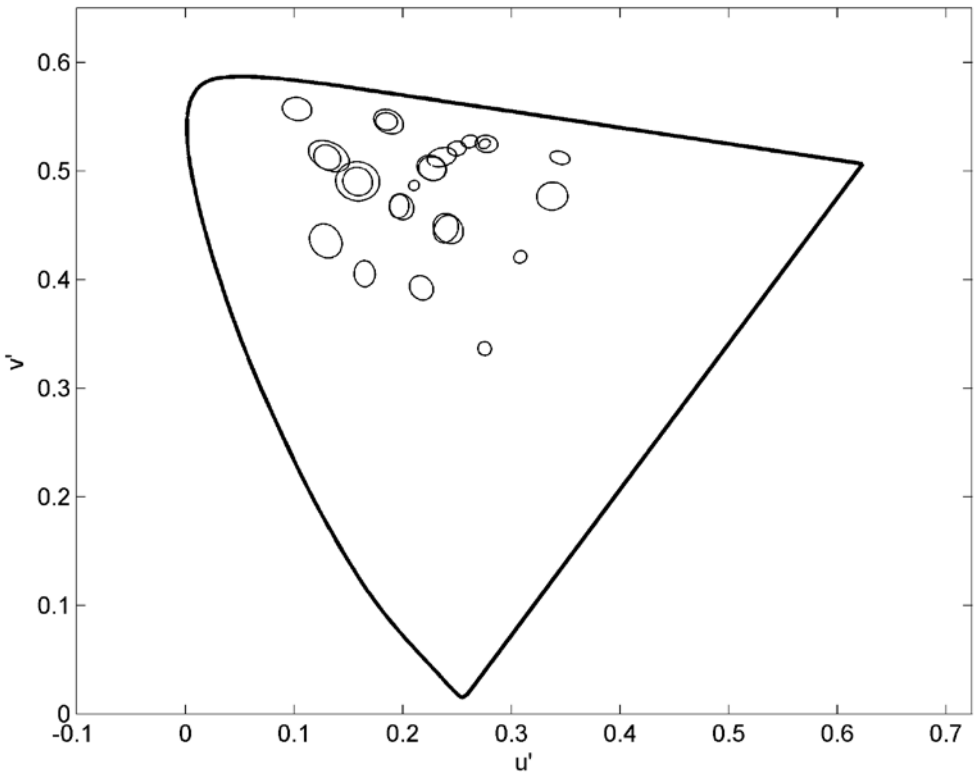

Figure 4.3-4 Ellipses in u'v' chromaticity diagram against the black background (compressed 80 times)

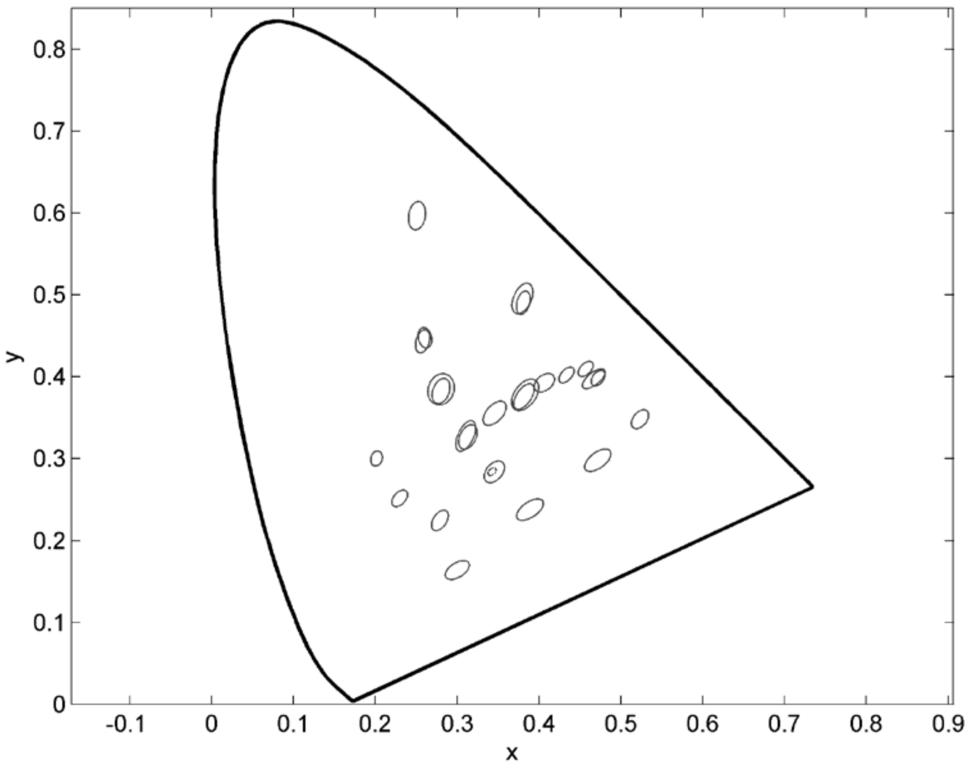

Figure 4.3-5 Ellipses in xy chromaticity diagram against the grey background (compressed 80 times)

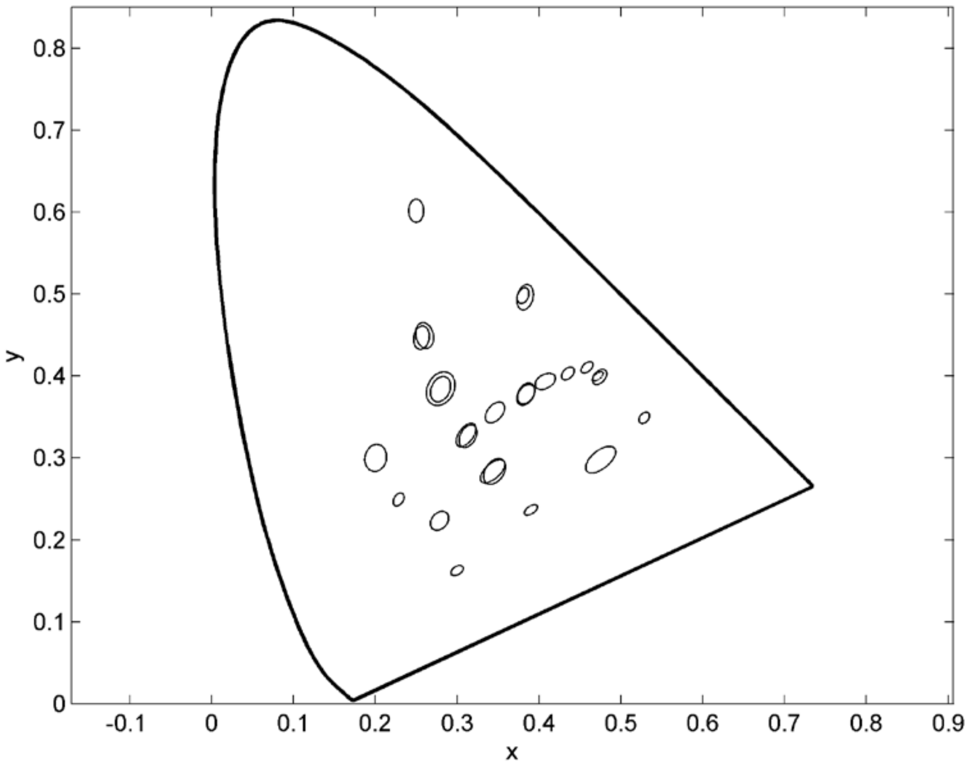

Figure 4.3-6 Ellipses in xy chromaticity diagram against the black background (compressed 80 times)

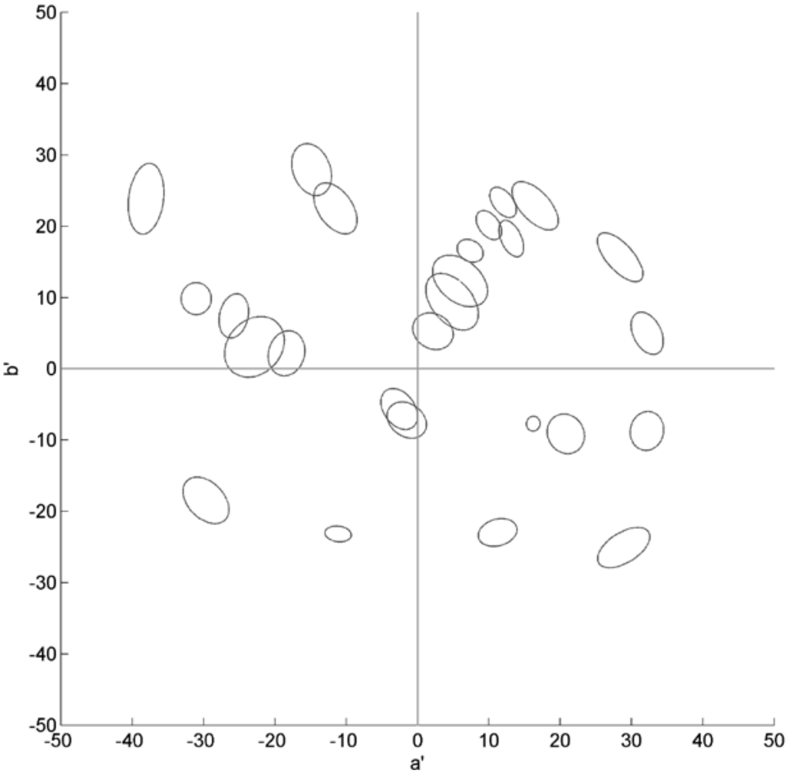

Figure 4.3-7 Ellipses in CAM02-UCS a'b' diagram against the grey background (enlarged 3 times)

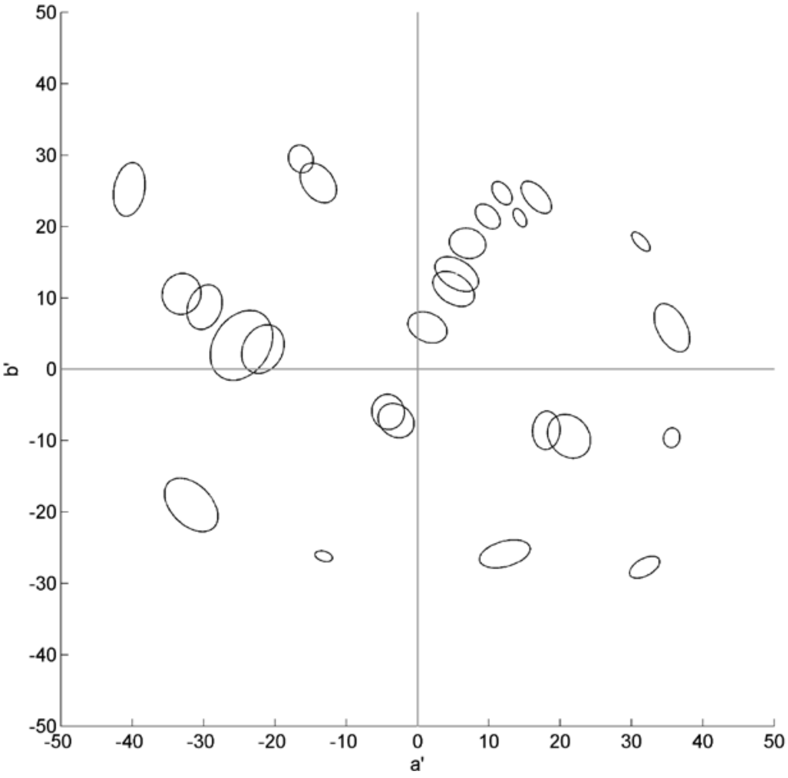

Figure 4.3-8 Ellipses in CAM02-UCS a'b' diagram against the black background (enlarged 3 times)

4.3.1. Uniformity of Colour Spaces

To examine the uniformity of a colour space for which ellipses have been fitted, there are a few useful values. As introduced before, the semi-major axes A should be a constant, and the ratio A/B of the axis shows the shape of the formed ellipse. Firstly, let’s discuss about the ratio A/B. If the ratio A/B is equal to one, then a circle is formed instead of an ellipse. The ideal would be to have a colour space which would represent chromaticity differences as equal sized circles. That means that the semi-major axis A should be constant and the ratio A/B equal to unity. Since no colour space so far has resulted in the ideal, it is at least desired that the ellipses are as close as possible to the ideal. Moreover, by examining the STRESS values for the entire resulted ellipses, the uniformity and distribution of these data can be evaluated.

In the tables introduced at section 4.1, the ratio A/B and STRESS values (as well standard deviation) have been given for the whole group of colour centres. Furthermore, the results of ratio A/B values are summarised in Table 4.3-2 in terms of groups; i.e. whole set, coloured stimuli and white light stimuli. In this table, the smaller value is indicated with bold values horizontally and underlined ones vertically. As can be seen, the u’v’ chromaticity diagram has the smallest mean ratio A/B among the colour spaces examined. This means that its ellipses were mostly circular. Moreover, the ratio A/B was similar for the different groups of colour centres as well as the whole set; which indicates that u’v’ chromaticity diagram can serve as a good uniform space for either coloured or white lighting stimuli. Moreover, it has almost no difference between the grey and black backgrounds, which implies that it might be a better space for evaluating only chromaticity as it is required in the lightning industry.

This is strengthened by the low variation between the ratio A/B values within the data. In Table 4.3-3, the STRESS values are summarised, and it can be concluded that there were more circular-like ellipses in the u’v’ chromaticity diagram. The STRESS values for the ratio A/B were similar in distribution and in accordance with the results for the mean ratio A/B. Moreover, by inspecting the plotted ellipses, it can be concluded that most ellipses were more equal sized circles against the grey background in u’v’ chromaticity diagram (Figure 4.3-3).

Table 4.3-2 Mean ratio A/B for each colour space and data group

Mean Ratio A/B (№ of colour centres)	**CIELAB**	**u'v'**	**xy**	**CAM02-UCS**
Grey All (26)	1.65	**1.24**	1.72	1.53
Black All (26)	1.58	**1.25**	1.60	1.53
Grey Coloured (18)	1.68	**1.24**	1.70	1.57
Black Coloured (18)	1.65	**1.25**	1.59	1.58
Grey White (8)	1.58	**1.24**	1.77	1.47
Black White (8)	1.43	**1.26**	1.61	1.42

Table 4.3-3 Mean STRESS values of ratio A/B for each colour space and data group

STRESS of Ratio A/B (№ of colour centres)	**CIELAB**	**u'v'**	**xy**	**CAM02-UCS**
Grey All (26)	16.3	**11.6**	12.8	20.2
Black All (26)	16.0	**12.0**	13.0	19.8
Grey Coloured (18)	16.4	**11.7**	13.8	22.3
Black Coloured (18)	15.7	**11.7**	14.9	20.7
Grey White (8)	15.1	11.2	**9.7**	11.3
Black White (8)	11.6	12.6	**6.9**	14.4

Complementary to this, the STRESS values that correspond to the semi-major axes (A) values are summarised in Table 4.3-4. Again bold and underlined values indicate the smallest value in rows and columns respectively. There is more variation in relation to the magnitude of the ellipses. Although for the comparison and evaluation of the ellipses each parameter is important, all components should be considered in overall. The fact that a parameter might performed better than another, it does not mean that the former has an overall better performance than the latter. The u’v’ chromaticity diagram has more uniformity among its values that describe the size of the ellipses. For the whole dataset, it varied the least when grey background was used and it was the second best when black background was used. It has also resulted in very good uniformity for the group of white lightning stimuli against the grey background. The xy chromaticity diagram had the least varying size when black background was used for the whole set of stimuli. While, the CAM02-UCS gave the smallest variation for the group of white light stimuli against the black background; which could be due to the improved prediction of chroma that it is included in the model for the neutral colours. Lastly, CIELAB space had the least uniformity for the group of white light stimuli against the grey background. To conclude, in combination with the mean ratio A/B, the u’v’ chromaticity diagram gives a better uniformity of space in overall.

Table 4.3-4 Mean STRESS values of semi-major axis A for each colour space and data group

STRESS of A (№ of colour centres)	CIELAB	u'v'	xy	CAM02-UCS
Grey All (26)	24.5	**23.9**	24.5	30.2
Black All (26)	34.1	32.6	**29.6**	33.9
Grey Coloured (18)	27.8	26.9	**25.2**	30.6
Black Coloured (18)	33.3	**32.7**	33.0	37.8
Grey White (8)	**15.1**	17.3	23.1	30.6
Black White (8)	27.0	30.3	20.4	**19.5**

In order to summarise the findings and make conclusions, Table 4.3-5 can be advised. There is a clear trend that STRESS values for the semi-major axes (A) and size were larger against the black background. For an ideal uniform colour space, STRESS should be equal to zero. The larger the value, the poorer the performance of the colour space is. By examining the STRESS values for the semi-major axes A and size, CAM02-UCS performed the worst while the other three spaces have similar ranges. This shows that the ellipses for the black background data have greater variation than the ones for the grey background data. In terms of ratio A/B, the STRESS values should be zero in order to represent circles. By examining the STRESS of ratio A/B, the u’v’ chromaticity diagram performed the best, followed by the xy chromaticity diagram, the CIELAB, and lastly the CAM02-UCS.

The u’v’ chromaticity diagram might have larger STRESS values for both the semi-major axes A and ellipse size than the xy chromaticity diagram, but the ratio A/B is considerably smaller. Therefore, that makes it a more uniform space, even though it results to bigger variation of magnitude between different colours. That is because as explained, the overall performance is a combination of all ellipse parameters.

Table 4.3-5 Summary of ellipse parameters and statistical metrics

	CIELAB		**u'v'**		**xy**		**CAM02-UCS**	
	Grey	**Black**	**Grey**	**Black**	**Grey**	**Black**	**Grey**	**Black**
Size								
Mean	**1.88**	1.74	**1.65**	1.53	**1.66**	1.55	**1.56**	1.38
STRESS	20.7	**30.0**	21.9	**30.5**	22.6	**28.5**	26.0	**32.6**
A								
Mean	**1.365**	1.231	**1.038**	0.965	**1.231**	1.098	**1.088**	0.948
STRESS	23.3	**31.7**	22.8	**30.5**	23.3	**27.8**	28.4	**31.5**
A/B								
Mean	**1.65**	1.58	1.24	**1.25**	**1.72**	1.60	1.53	1.53
STRESS	**16.3**	16.0	11.6	**12.0**	12.8	**13.0**	**20.2**	19.8
	Both Backgrounds							
Size								
Mean	1.81		1.59		1.61		1.47	
STRESS	25.4		26.2		25.5		29.3	
A								
Mean	1.298		1.001		1.164		1.018	
STRESS	27.5		26.6		25.6		29.9	
A/B								
Mean	1.62		1.24		1.66		1.53	
STRESS	16.2		11.8		12.9		20.0	

These data comply with the results from the study which was conducted for white light stimuli only (Luo et al., 2015). In both studies, u’v’ chromaticity diagram outperformed the other colour spaces. The fact that the sampling was conducted in u’v’ chromaticity diagram, it could be used to question this conclusion. However, there are a few points to support otherwise. Firstly, in previous studies, CIELAB space has been used for the selection of colour pairs (Berns et al., 1991; Cheung and Rigg, 1986; Luo and Rigg, 1986). However, this has not worked in favour of the CIELAB space; which seems to underperform in comparison with other formulae. Secondly, the group of observers gave consistent results (small observer variability) while conducting the experiment. Additionally, they did many sessions in order to complete the full experiment; which gave them experience in assessing colour difference using the ratio method; and the majority of the group had already experience in colour psychophysical experiments.

4.3.2. Background Effect

Overall, by comparing the semi-major axis A and sizes of the data between the grey and black background, there is the trend that ellipses were longer and larger against the grey background. However, there are also some exceptions. According to the present results, the ellipses were smaller against the black background. This implies that observers were more sensitive while evaluating colour differences against a black background than against a grey background. This stricter tolerance against the black background is also supported by the larger variation of ellipse sizes for the black background data.

For the examined colour spaces, the instances that gave larger ellipse size against the black background were the colour centres 1_48, 12_18, 12_48, 3_18, 3_48, 5_18, and B_48. Among them, the majority belong to the green area and the rest have red and purple hues. However, the mean assessments in Table 4.2-1 show that observers perceived larger colour difference when the colour pairs were assessed against the black background. This suggests that the discrepancy at these patterns is due to the characteristics of the colour space for these chromatic areas. The only exception that a colour centre had larger perceived colour difference against the grey background was the colour centre W6_18 of the white light stimuli set. It is potential that this greyish colour centre has been affected by the crispening effect when it was evaluated against the grey background. Colour centres of similar shades with the background were also the W5_48 and W6_48. From these, W5_48 was assigned larger colour difference while evaluated against the black background; while the W6_48 was assigned very similar value for both backgrounds.

Many observers had stated during the experiments that they could assess the colour difference more comfortably against with the grey background than with the black. However, the visual colour differences seem larger against the black background. This indicates some discrepancy in the perceived contrast between the backgrounds. Furthermore, the results show smaller variability against the black background. Grey background may be more comfortable due to causing less eye fatigue, but it is less consistent.

The qualitative comparison of the ellipses between the two backgrounds is illustrated in Figure 4.3-9 for u’v’ chromaticity diagram. Chromaticity ellipses against the black and grey background are plotted in black and magenta colour respectively. There is similar shape (ratio A/B) between the black and grey background. The majority of the grey ellipses are larger against the grey background. Similar patterns were formed for the ellipses of the other spaces tested, as seen in Figure 4.3-10 to Figure 4.3-12 for xy chromaticity diagram, *a*b** diagram of the CIELAB and *a’b’* diagram of the CAM02-UCS respectively when plotted with same centre.

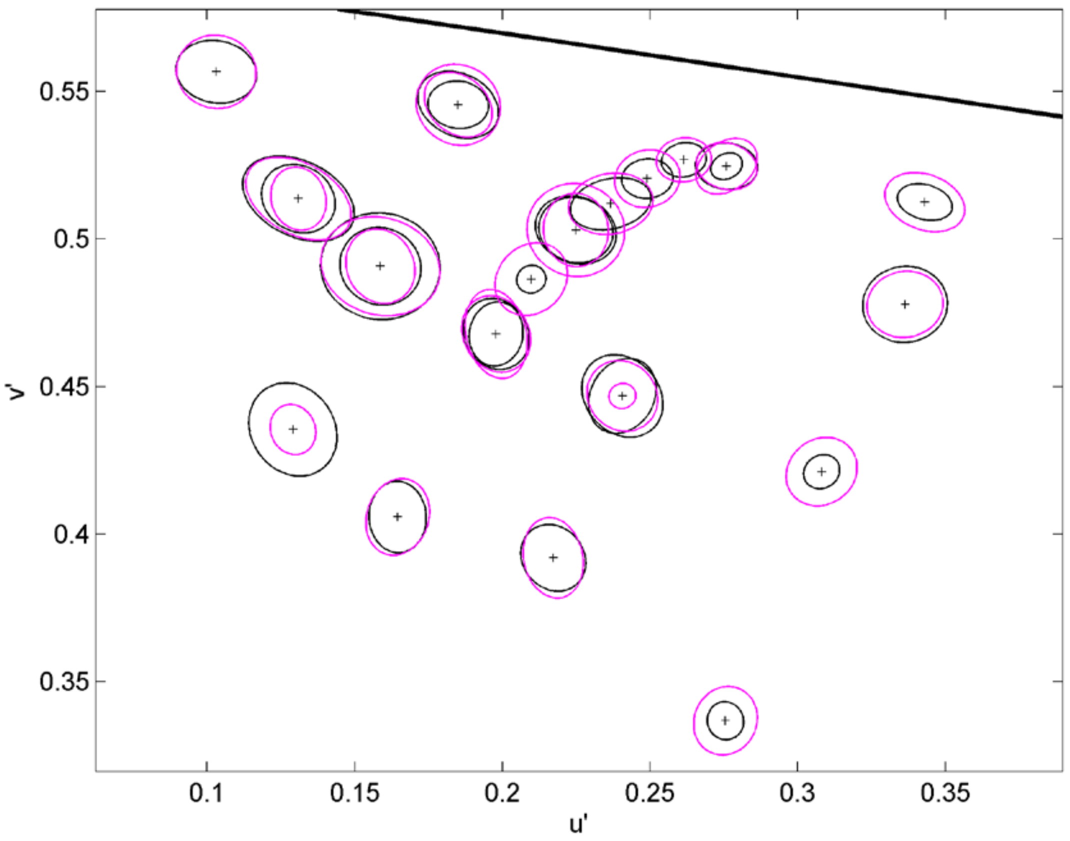

Figure 4.3-9 Ellipses against the grey and black background in u’v’ chromaticity diagram

Ellipses in black correspond to the black background; ellipses in magenta correspond to the grey background

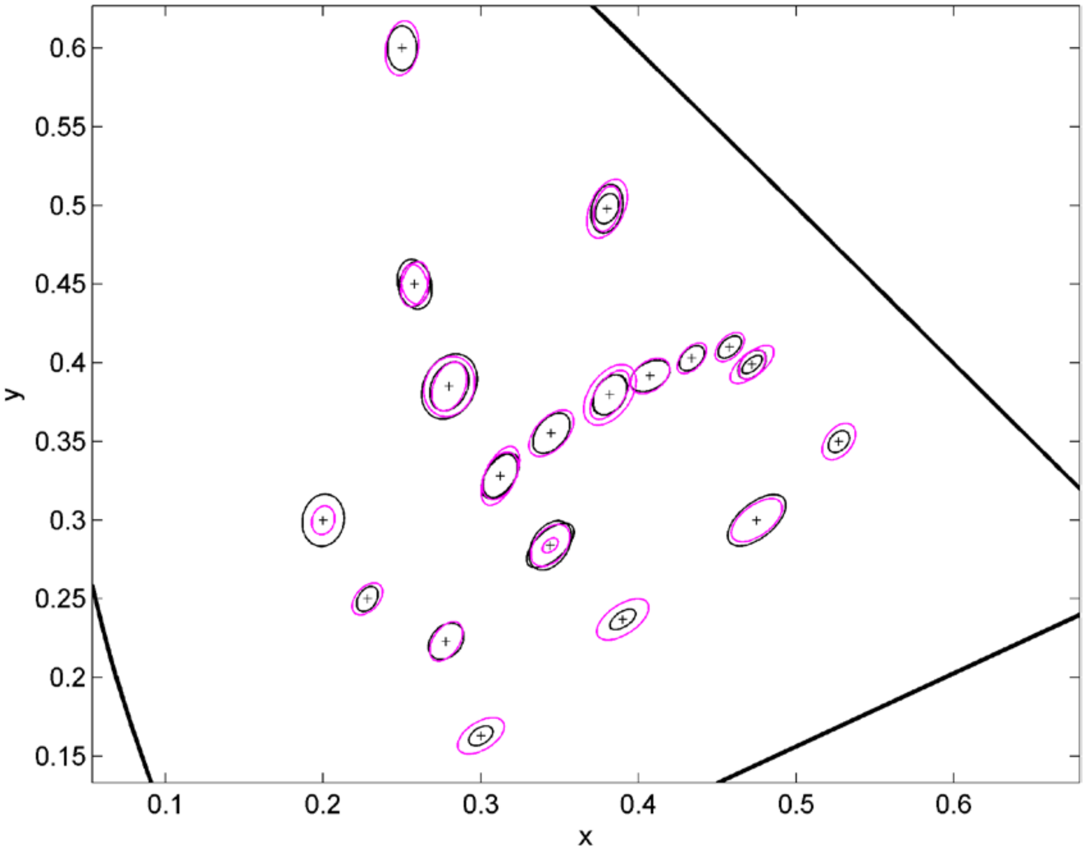

Figure 4.3-10 **Ellipses against the grey and black background in xy chromaticity diagram**

Ellipses in black correspond to the black background; ellipses in magenta correspond to the grey background

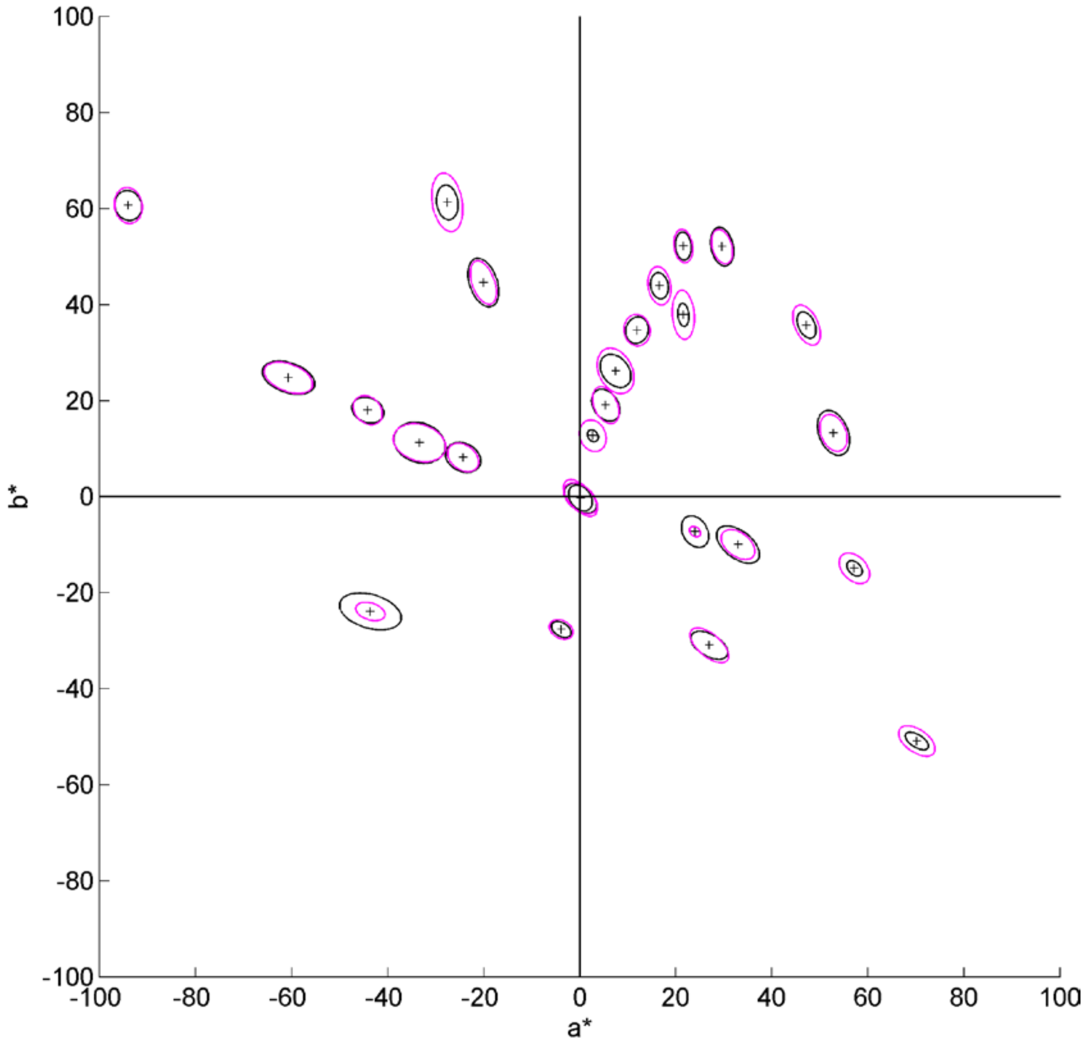

Figure 4.3-11 Ellipses against the grey and black background in *a*b diagram**

Ellipses in black correspond to the black background; ellipses in magenta correspond to the grey background

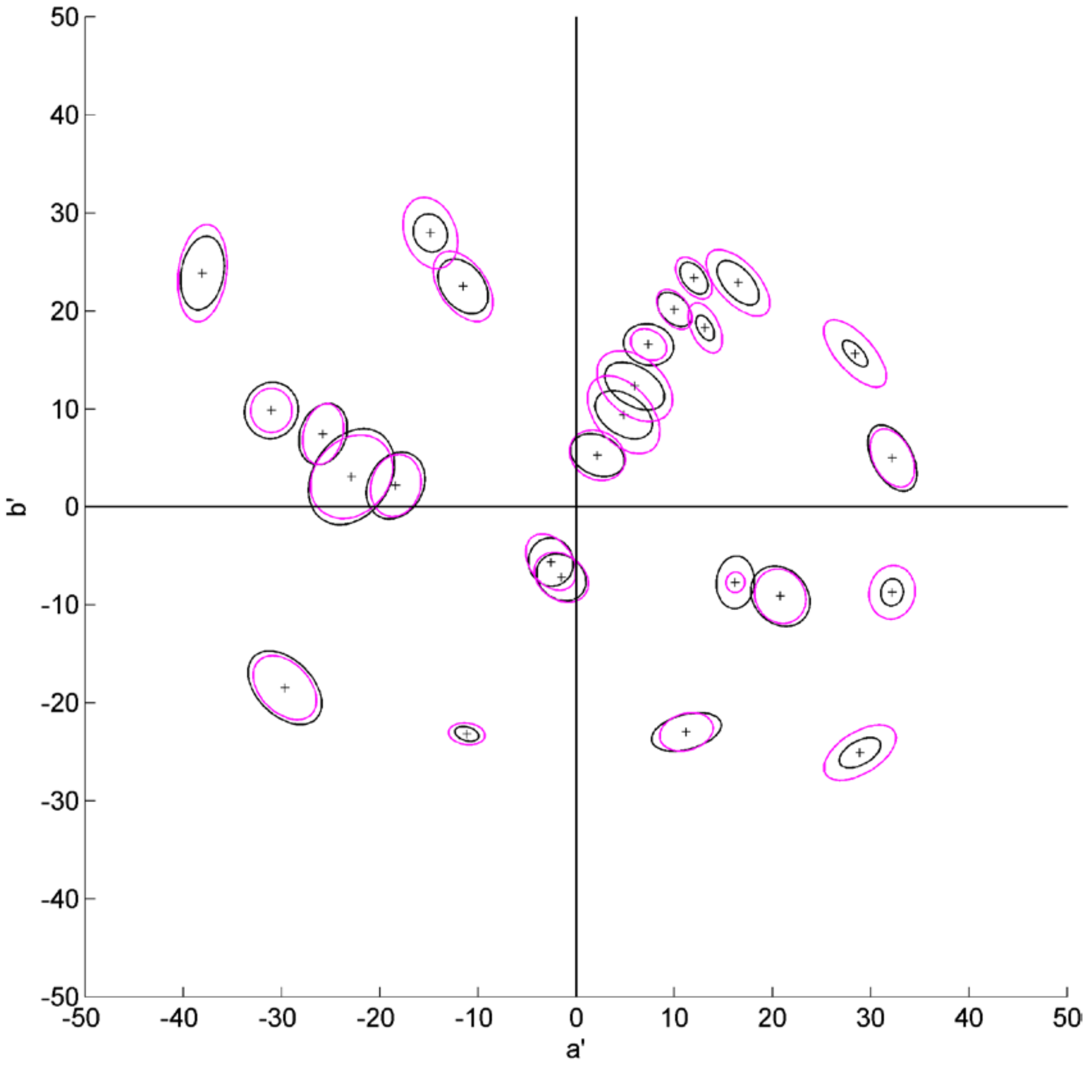

Figure 4.3-12 Ellipses against the grey and black background in *a'b'* diagram (CAM02-UCS)

Ellipses in black correspond to the black background; ellipses in magenta correspond to the grey background

4.3.3. Effect Due to Luminance of Colour Centres

As seen, some colour centres were processed at a luminance of 18.5 cd/m² or 48 cd/m²; while a set of colour centres were processed and assessed at both luminance levels. For instance, colour centres 12_48 and 12_18. The luminance of 48 cd/m² and 18.5 cd/m² correspond to lightness *L** of 75 units which is the same used in the MacAdam experiment and lightness *L** of 50 units for a mid-grey.

When comparing the average size of the colour centres assessed at both luminance levels, it can be seen that the colour centres with luminance 48 cd/m² have considerably larger size ellipses for either background and colour space; see Table 4.3-6. Due to the ability to easily see colour difference in darker stimuli, it is reasonable that the tolerance is stricter (smaller ellipses) for the colour centres with darker luminance. Additionally, the colour centres with luminance 48 cd/m² have systematically larger size ellipses against the grey background, which is also supported by the results for the background effect. In contrast, the majority of the colour centres with luminance 18.5 cd/m² have larger ellipse size against the black background. So, this implies that the effect might be enhanced by the background luminance.

Table 4.3-6 Mean ellipse size per luminance group for the colour centres that were assessed at both luminance levels

Size	**CIELAB**		**u'v'**		**xy**		**CAM02-UCS**	
	Grey	**Black**	**Grey**	**Black**	**Grey**	**Black**	**Grey**	**Black**
18.5 cd/m²	1.73	1.74	1.45	1.55	1.57	1.63	1.50	1.42
48 cd/m²	2.22	2.13	1.97	1.85	1.89	1.75	1.81	1.63
Ratio (max/min)	1.28	1.22	1.35	1.19	1.21	1.08	1.20	1.15

Moreover, when comparing the average size of the colour centres at luminance of 18.5 cd/m² and 48 cd/m² for the total dataset, it can be seen that the colour centres with luminance 48 cd/m² have again larger size ellipses for either background and colour space, even though the ratio (max/min) is smaller. The results are summarised in Table 4.3-7. In fact, this is to be expected, given the variety of colours in the set. However, in this case, the ratio (max/min) between the two backgrounds seems to be in similar range for either background between the two luminance levels of colour centres.

Table 4.3-7 Mean ellipse size per luminance group for the dataset

Size	CIELAB		u'v'		xy		CAM02-UCS	
	Grey	**Black**	**Grey**	**Black**	**Grey**	**Black**	**Grey**	**Black**
18.5 cd/m²	1.77	1.60	1.55	1.44	1.61	1.46	1.47	1.24
48 cd/m²	1.99	1.88	1.76	1.62	1.72	1.64	1.64	1.53
Ratio (max/min)	1.12	1.17	1.14	1.13	1.07	1.12	1.12	1.24

In Figure 4.3-13 and Figure 4.3-14, the effect of the colour centre’s luminance in u’v’ chromaticity diagram is illustrated for the grey and black background data respectively. Ellipses were plotted with the same centre in order to compare and contrast the size effect and orientation. Especially, the ellipses against the grey background show that the colour centres with luminance 48 cd/m² have larger ellipses than the darker ones. This agrees well with the ellipse size factor (ESF) developed by Luo *et al*; which describes the increase of ellipse size when the luminance Y is increasing (Luo and Rigg, 1986). Moreover, it had been found that there was no strong trend for luminance higher than 50. Here, the effect is particularly clear for the coloured stimuli, but not so strong against the black background.

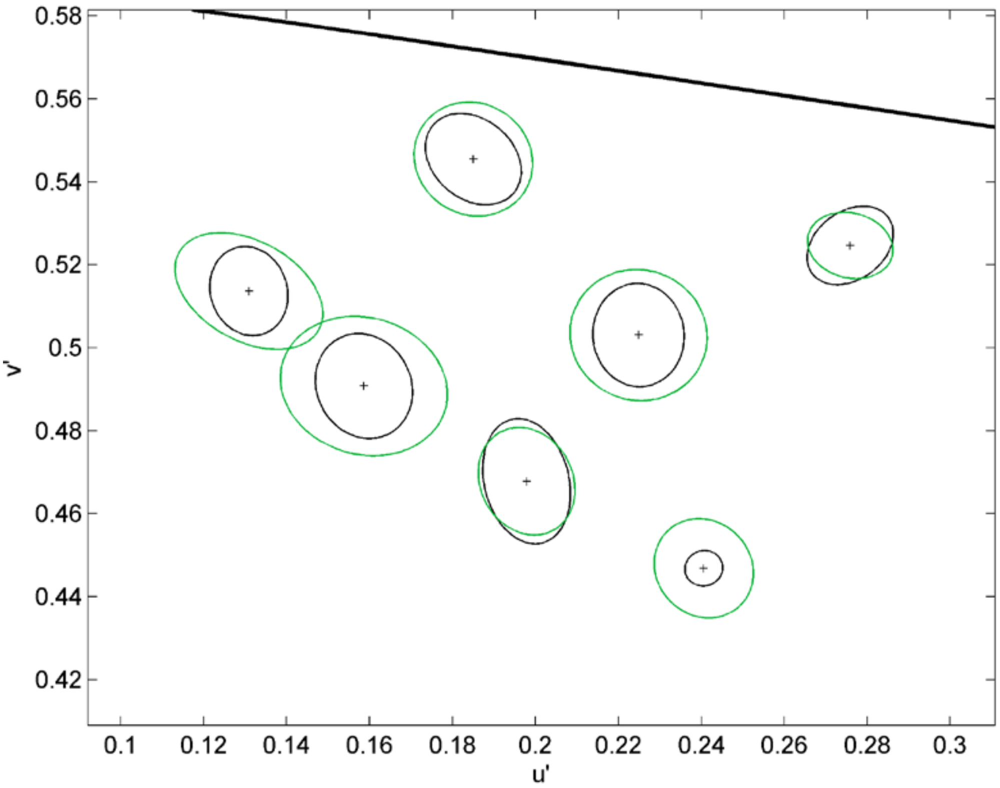

Figure 4.3-13 Ellipses of colour centres with different luminance against the grey background in u'v' chromaticity diagram

Ellipses in black are for the colour centres with luminance 18.5 cd/m²; ellipses in green are for the colour centres with luminance 48 cd/m².

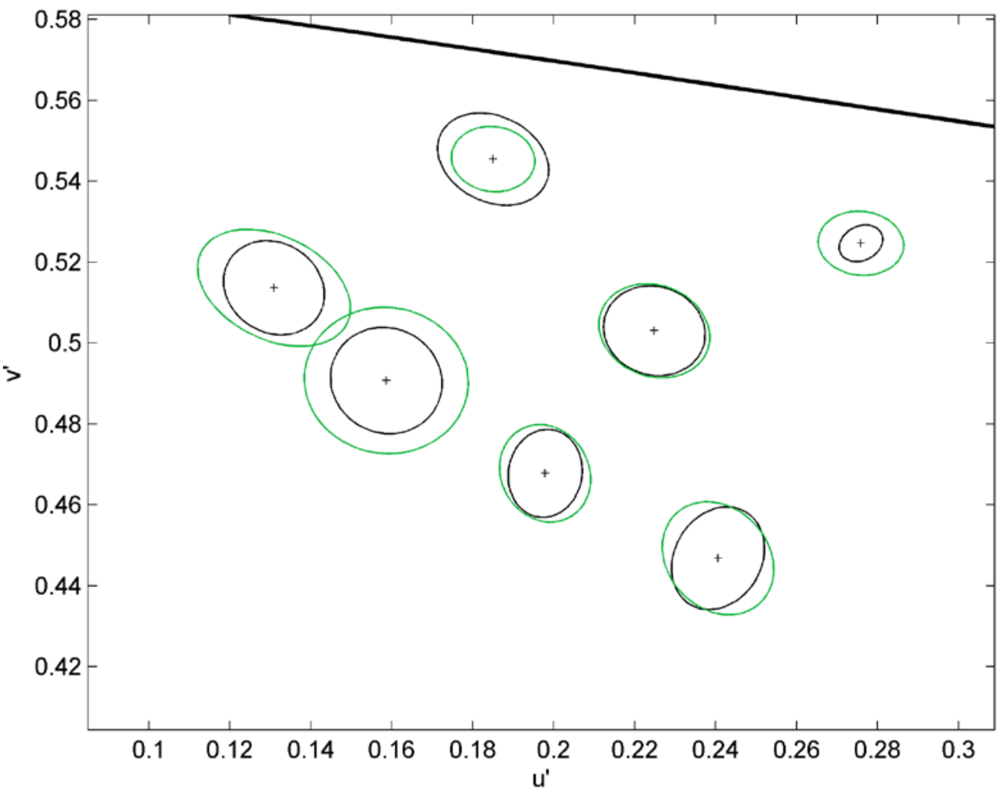

Figure 4.3-14 Ellipses of colour centres with different luminance against the black background in u’v’ chromaticity diagram

Ellipses in black are for the colour centres with luminance 18.5 cd/m²; ellipses in green are for the colour centres with luminance 48 cd/m².

In Figure 4.3-15 to Figure 4.3-20, the respective ellipses for the xy chromaticity diagram, *a*b** diagram of the CIELAB, and *a’b’* diagram of the CAM02-UCS are given. The xy chromaticity diagram and *a*b** diagram have similar performance with the u’v’ chromaticity diagram as described above. However, for the CAM02-UCS, there is no clear trend in the ellipse size related to the colour region. Even though the quantitative results from the ellipse size calculation showed that the colour centres with luminance 48 cd/m² against the grey background have larger ellipse size, the qualitative results from the ellipse plots do not show a systematic change in the size distribution. This implies that the input parameters of the CAM02-UCS model adjust the colour appearance attributes at certain extend.

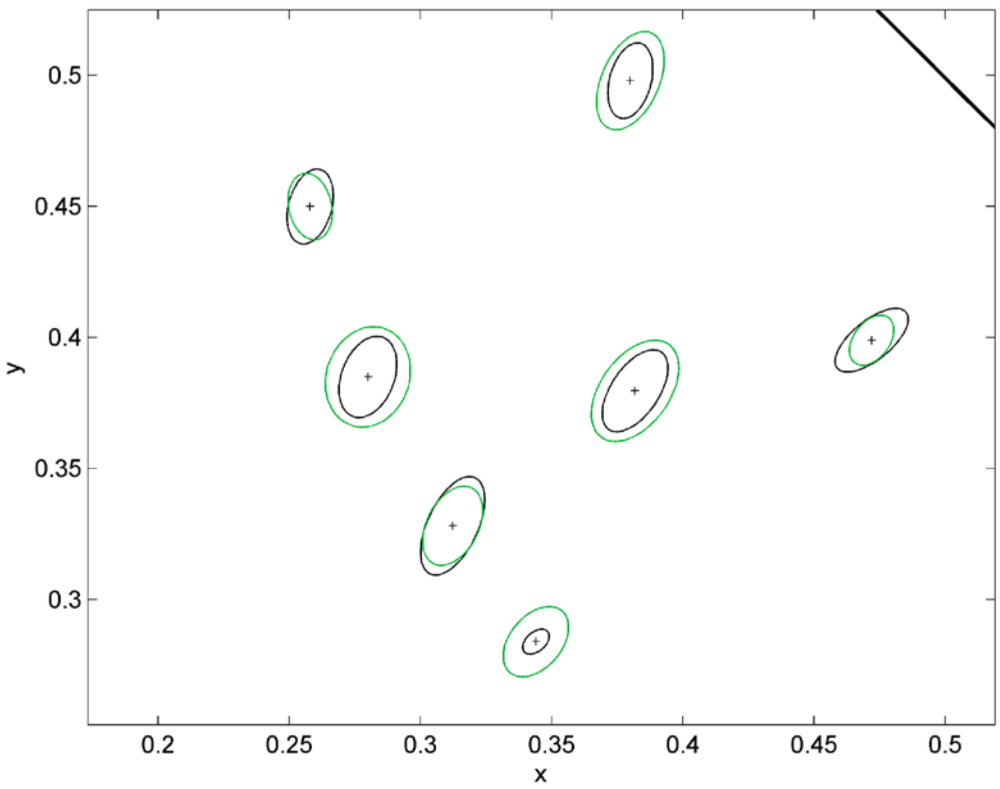

Figure 4.3-15 Ellipses of colour centres with different luminance against the grey background in xy chromaticity diagram

Ellipses in black are for the colour centres with luminance 18.5 cd/m²; ellipses in green are for the colour centres with luminance 48 cd/m².

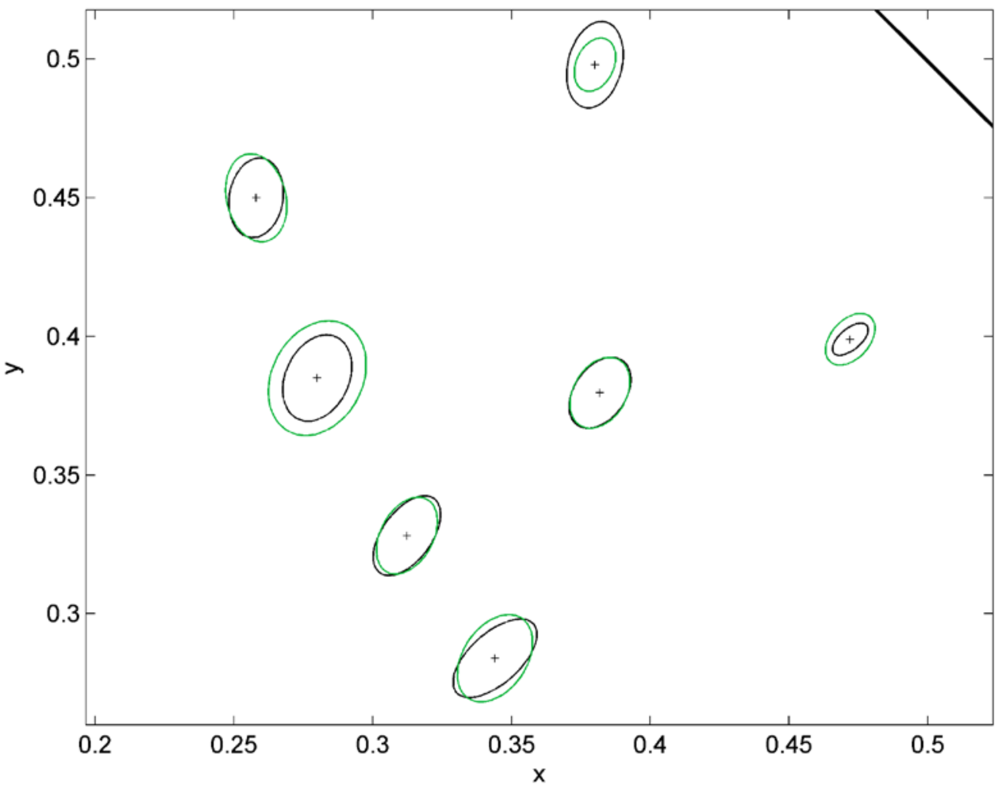

Figure 4.3-16 Ellipses of colour centres with different luminance against the black background in xy chromaticity diagram

Ellipses in black are for the colour centres with luminance 18.5 cd/m²; ellipses in green are for the colour centres with luminance 48 cd/m².

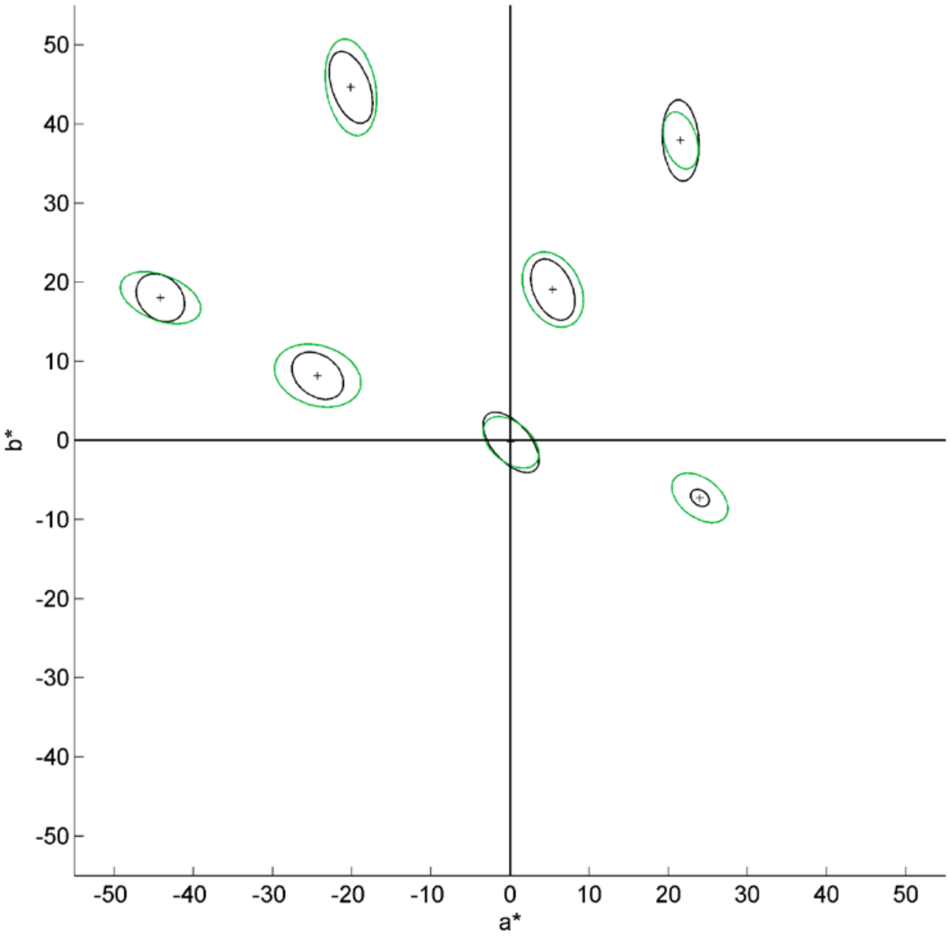

Figure 4.3-17 Ellipses of colour centres with different luminance against the grey background in a*b* diagram

Ellipses in black are for the colour centres with luminance 18.5 cd/m²; ellipses in green are for the colour centres with luminance 48 cd/m².

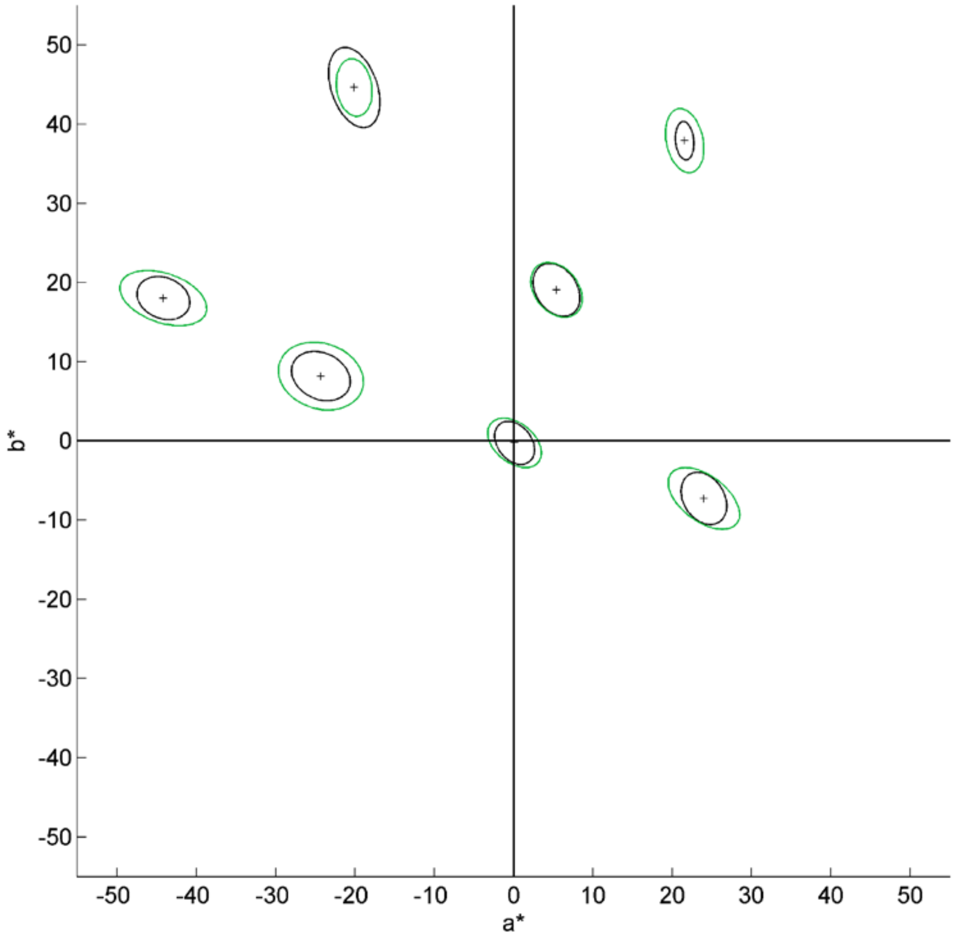

Figure 4.3-18 **Ellipses of colour centres with different luminance against the black background in a*b* diagram**

Ellipses in black are for the colour centres with luminance 18.5 cd/m²; ellipses in green are for the colour centres with luminance 48 cd/m².

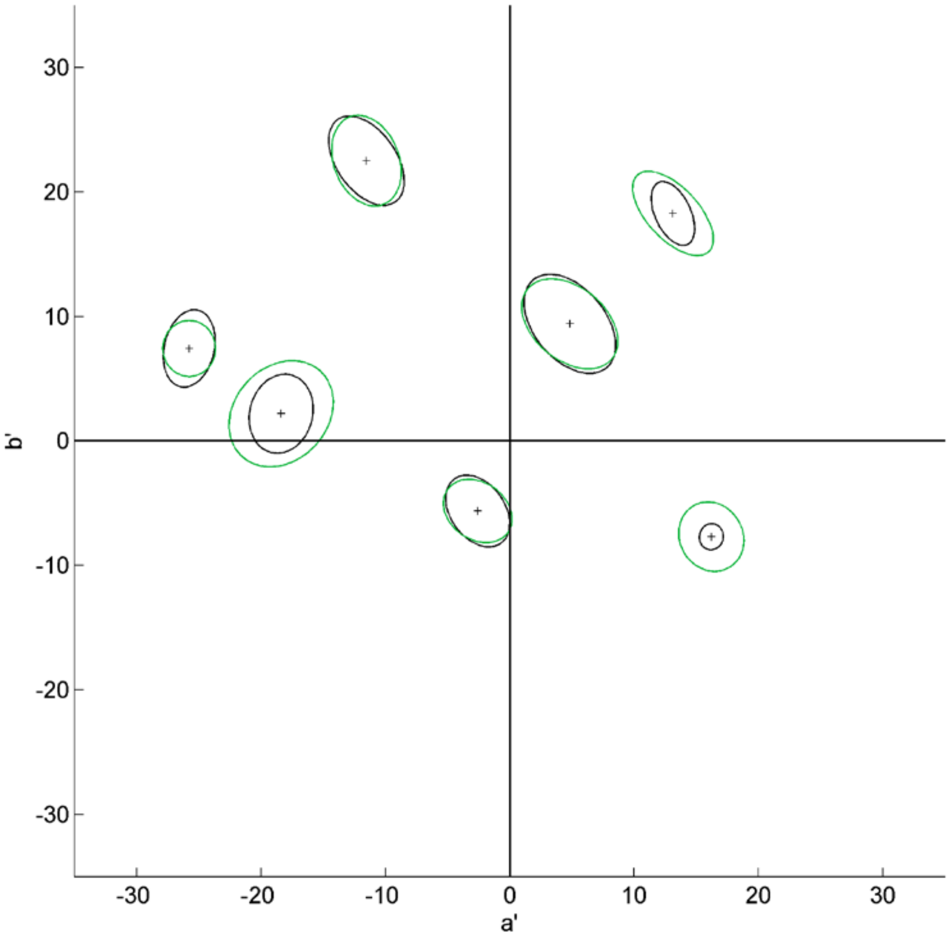

Figure 4.3-19 Ellipses of colour centres with different luminance against the grey background in a'b' diagram (CAM02-UCS)

Ellipses in black are for the colour centres with luminance 18.5 cd/m²; ellipses in green are for the colour centres with luminance 48 cd/m².

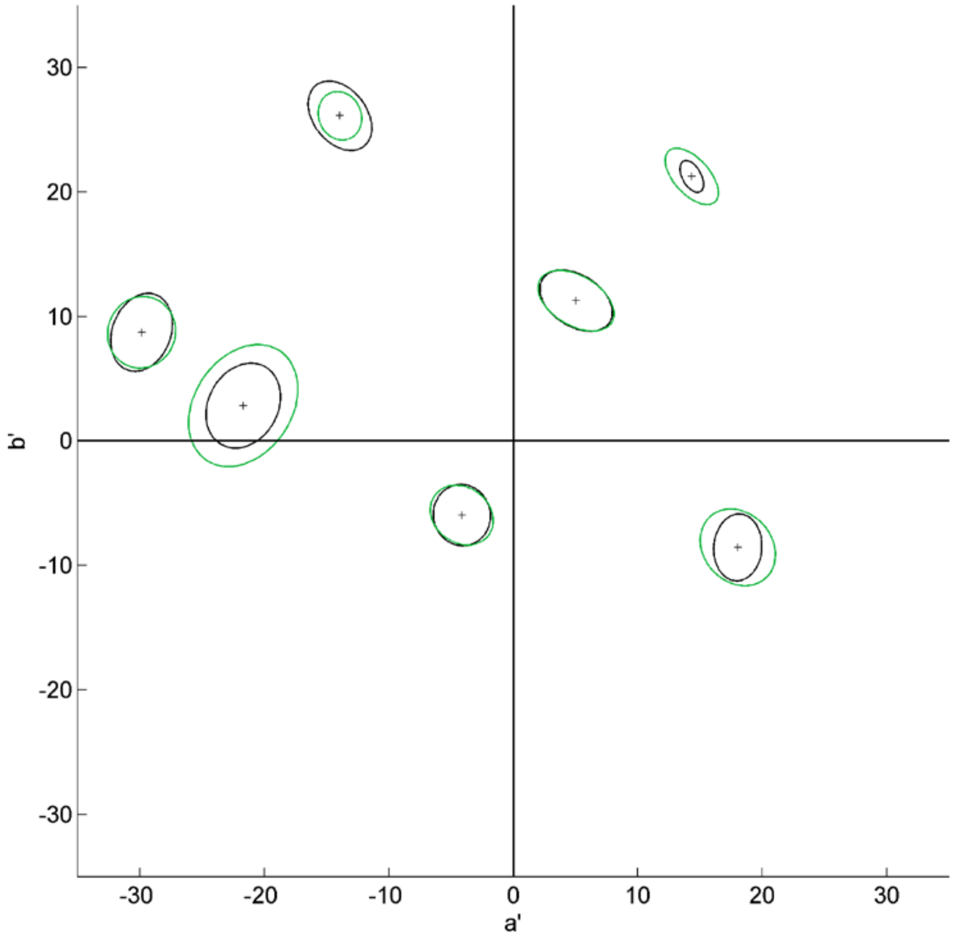

Figure 4.3-20 Ellipses of colour centres with different luminance against the black background in a'b' diagram (CAM02-UCS)

Ellipses in black are for the colour centres with luminance 18.5 cd/m^2; ellipses in green are for the colour centres with luminance 48 cd/m^2.

4.4. Comparing with MacAdam Ellipses

The experimental ellipses fitted in the xy chromaticity diagram for the grey background data can be compared with the MacAdam experimental ellipses. The grey background data have the same luminance and chromaticity setup conditions as the MacAdam experiment. Qualitative comparison is presented in Figure 4.4-1; where ellipses from both datasets have been plotted together. For the purpose of meaningful comparison, the current experimental ellipses have been scaled accordingly to the MacAdam dataset using a scaling factor based on the area πAB of the ellipses.

From a first look, there seems to be great similarity in ellipse orientation, i.e. they radiate from blue towards the direction of the dominant wavelengths. However, their sizes do not increase from the smallest blue area towards the green area as in the MacAdam data. To the contrary, the current ellipses are a lot more rounded and of more consistent size regardless of the area of the xy chromaticity diagram. For the black background data, the ellipse orientation is again similar; see Figure 4.3-6. However, the pattern of size variation seems to be discontinued.

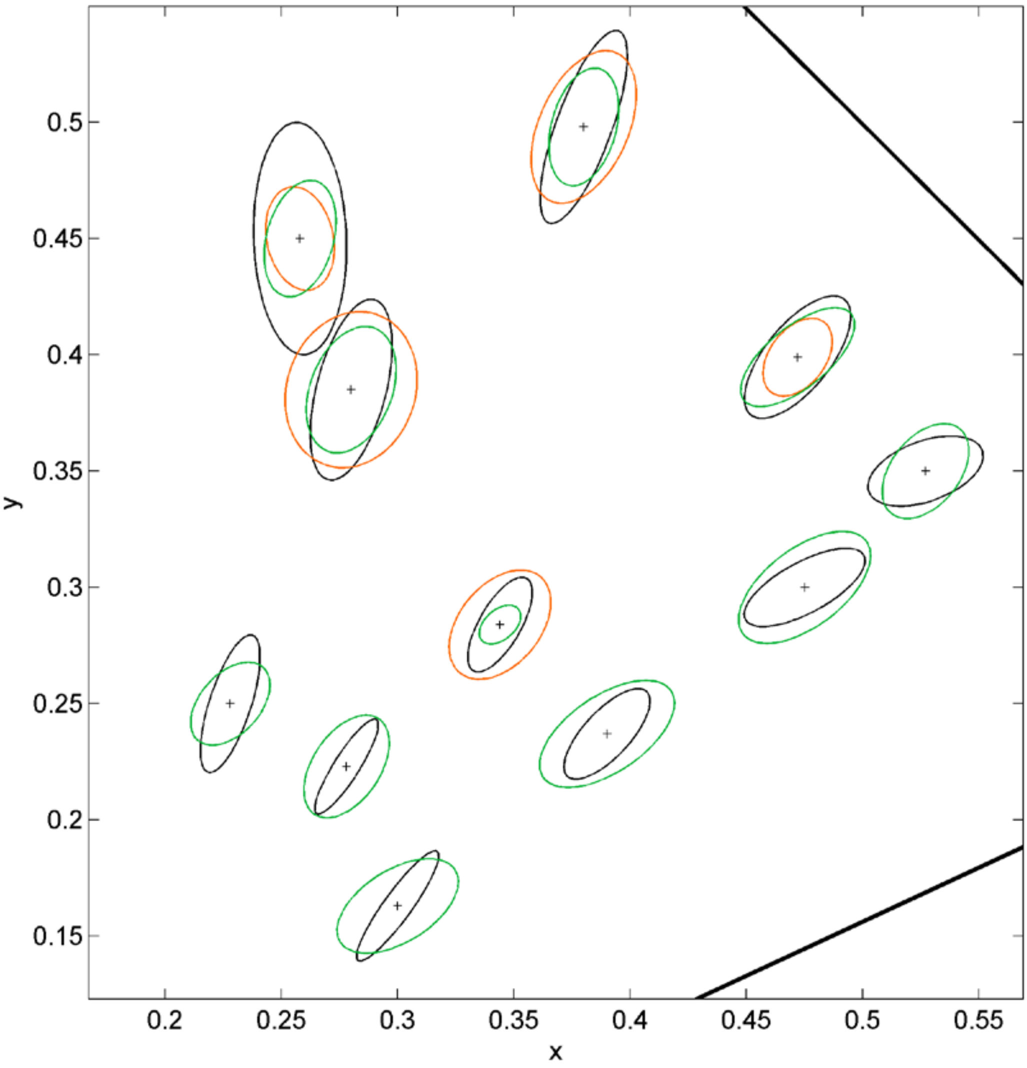

Figure 4.4-1 MacAdam ellipses against current experimental ellipses for the grey background data in xy chromaticity diagram

Marked in black are the original MacAdam ellipses; marked in orange are the current ellipses with luminance 48 cd/m^2; marked in green are the current ellipses with luminance 18.5 cd/m^2

Additionally, a quantitative analysis between the ellipse sizes of the two datasets was performed. The results in Table 4.4-1 show the ellipse size for each centre and their variance in STRESS units which equals 32. The values support the visual representation of the ellipses.

Table 4.4-1 Scaled ellipse sizes for MacAdam and respective current data

				Size	
Name	**x**	**y**	**Y**	**MacAdam (10^3)**	**Current**
1_48	0.2580	0.4500	48	5.60	3.19
1_18	0.2580	0.4500	18.5	5.60	3.42
3_48	0.2800	0.3850	48	4.34	5.46
3_18	0.2800	0.3850	18.5	4.34	3.97
8_48	0.3800	0.4980	48	4.07	4.60
8_18	0.3800	0.4980	18.5	4.07	3.38
12_48	0.3440	0.2840	48	2.55	3.84
12_18	0.3440	0.2840	18.5	2.55	1.45
25_48	0.4720	0.3990	48	3.75	2.68
25_18	0.4720	0.3990	18.5	3.75	3.50
5_18	0.4750	0.3000	18.5	3.17	4.20
10_18	0.3900	0.2370	18.5	2.80	4.15
13_18	0.2280	0.2500	18.5	2.96	2.92
19_18	0.5270	0.3500	18.5	3.26	3.29
23_18	0.2780	0.2230	18.5	2.04	3.37
24_18	0.3000	0.1630	18.5	2.34	3.78
				STRESS	32.34

4.5. Comparing with Previous White Light Stimuli

The experimental conditions against the black background were well matched with the previous study of white light sources by Luo *et al* (Luo et al., 2015). The colour centres for both studies were based on the ANSI C78.377 standard. In Figure 4.5-1, the corresponding ellipses are plotted together in the uv’ chromaticity diagram; scaled to the Luo *et al.* data. The resulted ellipses have similar shapes but different orientations. This indicates that the data from both studies agree well between them. The difference in the orientation could have potentially been caused by the difference in the selection of samples. In the current study, samples were taken semi-circularly around the colour centres; while in the previous study they were taken circularly. Another difference between the two datasets is that there was small difference between the target luminance of the stimuli. In the previous study, the pairs had a slightly varied luminance between 52 to 54 cd/m^2.

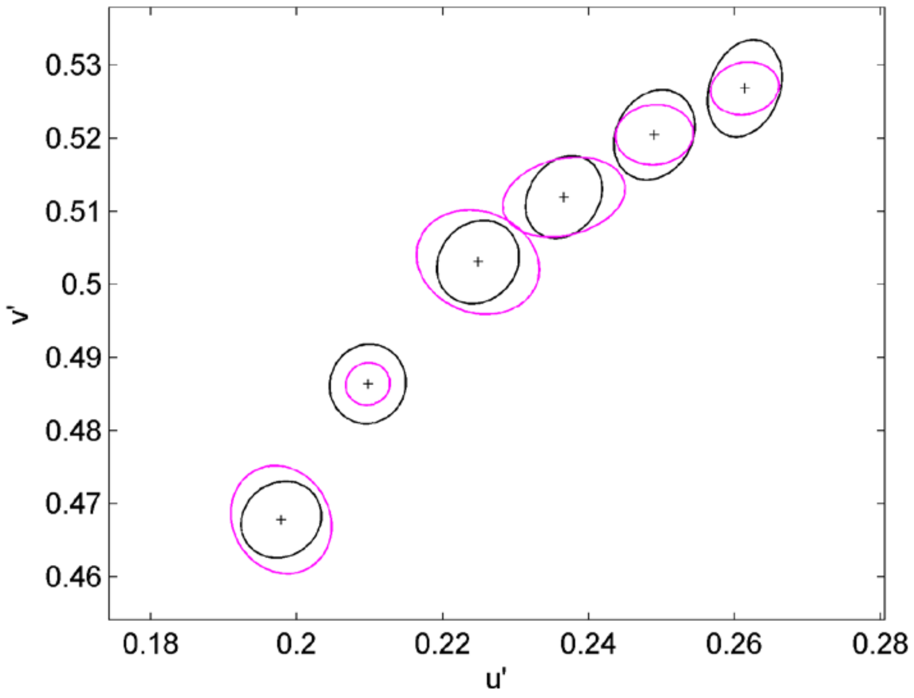

Figure 4.5-1 Ellipses of white light stimuli by Luo *et al.* against current black background data in u'v' chromaticity diagram

Ellipses in black are for the Luo *et al.* while light data; ellipses in magenta are for the current white light data.

In Table 4.5-1, the respective ellipse sizes for the two datasets have been calculated by scaling the current data to the previous Luo *et al*. data. The sizes reflect well the plotted ellipses as well. Moreover, the variation between the datasets has a relatively reasonable STRESS of 30 units.

Table 4.5-1 Scaled ellipse sizes for Luo *et al* and current white light stimuli

		10^3 Size	
u'	**v'**	**Luo *et al.***	**Current**
0.2614	0.5268	10.20	7.28
0.2490	0.5205	10.31	8.33
0.2366	0.5120	9.57	11.86
0.2248	0.5031	10.03	13.68
0.2097	0.4864	9.46	5.29
0.1979	0.4678	9.51	12.64
		STRESS	30.23

4.6. Conclusions

All experimental data were fitted into colour discrimination ellipses using different formulae and spaces. Ellipses were compared in terms of their parameters and plots for each colour space. The results are summarised below.

In terms of semi-major axis (A), the u’v’ chromaticity diagram had the smallest mean length, even though its STRESS value was not the smallest. This corresponds to smaller magnitude of perceived visual difference but with greater variation between the chromatic regions. In terms of ratio A/B, the u’v’ chromaticity diagram had the smallest values for both the mean ratio A/B and STRESS. Therefore, in overall, it can be considered that it largely outperformed the others in terms of space uniformity. This means that it resulted to the most equal perceived colour differences for its fitted ellipses in the various regions.

Concerning the background effect, it does not seem to be great difference between the grey and black background in terms of space uniformity. Each space seemed to generally have a similar performance with small variations. Nevertheless, it appears that the ellipses against the grey background were in overall larger. Therefore, it can be considered that there is a larger tolerance for this background. However, concerning the colour centres with different luminance, there is a clear trend that the colour centres with larger luminance have larger ellipses.

Chapter 5.

Testing Colour Difference Formulae and Models

The performance of different formulae and spaces were tested using the visual data of the experiments. The colour metrics examined are: xy chromaticity diagram, CIELAB, CIELUV, CIEDE2000, CIECAM02, and CAM02-UCS. Results were investigated in overall and in terms of background. A significance test was also conducted to compare and contrast the formulae performance.

5.1. Introduction

The first step was the calculation of colour difference using the formulae and models as described in Chapter 2. For the formulae for which a reference white was needed for the calculations, the display's normalised white peak was used. While using colour appearance models, the viewing parameters relative to each background were applied. The measurement data for each background were respectively used for inputting the luminance of background Y_b, and estimating the luminance of adapting field L_A; and setting the surround conditions as dark.

The second step was to use statistical metrics to determine the correlation between the visual and computed colour difference. For this Ph.D. study, STRESS was extensively used and the version for this case is given in Equation 5.1-1. The third step was to use an F-test in order to test the significance of the difference in the results when comparing the formulae between each other.

Equation 5.1-1 STRESS formula for colour difference evaluation

$$STRESS = 100\sqrt{\frac{\sum\left(\Delta V_i - f\Delta E_i\right)^2}{\sum \Delta V_i^2}}$$

$$where\ f = \frac{\sum \Delta E_i \Delta V_i}{\sum \Delta E^2}$$

As discussed before, a larger ellipse corresponds to a larger perceived colour difference in the area. But also, a small ellipse size can correspond to a relatively large perceived colour difference (Cui et al., 2001b). Thus, the tolerance of perceived colour difference varies according to the colour space and chromatic area. While investigating the ellipse patterns, it was important to reveal formed trends in each plane. However, the actual performance of each colour difference metric in predicting colour difference is better understood by evaluating the statistical relationship against the visual difference. Additional conclusions can be drawn, if the measure of fit of the ellipses is compared with the data analysis for the formulae performance. Therefore, the STRESS values of ellipse fitting can be used for correlation. Table 5.1-1 is a summarised version of the STRESS values for the ellipse fitting in each space. The smallest values in each row are bold and in each column are underlined. These values indicate that the visual data fitted the colour difference metrics examined well.

Table 5.1-1 STRESS units for the ellipse fitting

STRESS	CIELAB	u'v'	xy	CAM02-UCS
Grey All	7.83	7.98	7.98	8.14
Black All	**7.45**	7.52	7.56	7.91
Grey Coloured	**8.02**	8.21	8.24	8.14
Black Coloured	**7.62**	7.73	7.80	7.85
Grey White	7.42	7.47	**7.39**	8.13
Black White	7.06	7.06	**7.02**	8.04

5.2. Performance of Colour Difference Metrics

For each colour centre, the STRESS measure was used to statistically determine the performance of the colour difference metrics and the results are given in Table 5.2-1 and Table 5.2-2 for the grey and black background data respectively. STRESS was also calculated for each and every colour pair of the dataset as a whole, and it is given by the term ‘total’ in these tables. This measure is the most important when evaluating the aptitude of the colour difference formula in predicting visual differences. A synoptic table is also given in Table 5.2-3 in order to easily compare the differences between the backgrounds in total.

Table 5.2-1 Performance of colour difference metrics in STRESS units against the grey background

Grey Background

	CIELAB	**CIELUV**	**CIEDE2000**	**CIECAM02**	**CAM02-UCS**	**xy**
1_18	11.0	8.2	14.0	8.8	16.3	18.8
1_48	23.8	19.1	9.2	15.1	9.3	18.2
10_18	17.0	10.7	17.4	11.6	11.7	24.0
12_18	13.6	9.0	8.7	8.5	8.5	17.9
12_48	17.6	8.6	12.5	11.8	9.6	17.4
13_18	13.3	11.2	27.2	14.5	18.6	18.9
19_18	22.2	19.9	33.3	19.4	26.0	19.8
23_18	25.5	14.7	10.7	9.7	13.5	18.4
24_18	22.8	11.3	11.3	15.8	22.8	25.0
25_18	25.7	10.9	29.0	20.9	23.6	25.0
25_48	20.1	11.2	27.6	15.9	21.5	17.0
3_18	13.6	9.0	10.8	7.8	11.3	17.8
3_48	14.5	10.1	7.7	10.5	10.2	11.1
5_18	15.7	11.1	26.5	11.9	17.2	22.3
8_18	25.0	13.7	10.3	26.3	18.0	22.8
8_48	24.3	7.2	14.4	26.3	14.2	21.5
B_48	20.5	9.2	24.7	10.5	15.2	12.5
G_48	11.9	7.4	21.7	21.2	22.7	19.2
W1_48	20.2	10.9	22.8	17.0	18.6	19.0
W2_48	18.7	8.1	22.8	15.3	17.2	19.0
W3_48	13.6	17.0	21.0	12.0	14.0	17.9
W4_18	17.9	8.0	25.0	14.9	16.5	23.1
W4_48	13.3	6.5	24.4	11.2	14.7	17.4
W5_48	11.1	9.4	19.9	8.3	9.7	22.5
W6_18	23.6	16.2	24.1	16.7	17.4	27.4
W6_48	17.0	9.7	22.2	12.3	13.1	18.6
Total	**34.9**	**22.5**	**34.2**	**27.1**	**24.3**	**35.9**
Mean	18.8	11.5	19.8	14.9	16.1	20.3

Table 5.2-2 Performance of colour difference metrics in STRESS units against the black background

Black Background

	CIELAB	**CIELUV**	**CIEDE2000**	**CIECAM02**	**CAM02-UCS**	**xy**
1_18	12.6	8.8	10.3	7.3	12.3	13.9
1_48	22.8	18.2	7.4	13.9	7.8	15.1
10_18	16.5	10.1	16.4	11.1	10.7	22.6
12_18	13.7	10.4	11.8	9.6	12.9	22.5
12_48	20.2	9.8	14.2	14.0	10.6	16.6
13_18	18.7	12.1	28.5	13.9	20.0	17.1
19_18	18.6	21.6	30.2	16.7	23.3	15.0
23_18	23.0	9.1	16.2	15.4	20.6	13.1
24_18	26.7	9.6	10.9	16.8	21.5	20.0
25_18	22.4	9.9	26.9	18.1	21.3	20.7
25_48	19.7	11.8	25.8	15.0	20.0	16.4
3_18	12.1	8.6	8.6	9.2	11.0	13.7
3_48	12.0	8.5	10.1	10.2	13.0	12.0
5_18	17.7	11.1	28.1	13.8	19.2	23.1
8_18	21.3	14.2	8.9	23.1	14.9	17.7
8_48	17.5	11.6	15.7	19.6	8.0	14.5
B_48	22.7	10.4	26.3	12.6	16.9	12.1
G_48	7.7	10.4	18.4	16.7	18.1	15.7
W1_48	19.2	12.2	21.0	15.7	16.8	17.5
W2_48	14.8	10.8	21.8	11.5	14.8	15.9
W3_48	13.9	21.1	20.2	12.9	13.7	19.5
W4_18	12.9	8.4	27.3	12.2	15.9	15.4
W4_48	12.1	9.0	27.8	12.1	17.3	14.0
W5_48	11.6	8.1	25.2	10.6	13.5	18.0
W6_18	13.3	8.6	16.1	8.0	8.8	21.5
W6_48	15.0	7.4	20.9	10.3	11.0	17.5
Total	**28.4**	**19.6**	**39.4**	**24.9**	**30.3**	**28.7**
Mean	17.3	11.5	19.8	13.9	15.7	17.4

Table 5.2-3 Summarised performance of colour difference formulae and colour spaces in STRESS values for each background

Background	**CIELAB**	**CIELUV**	**CIEDE2000**	**CIECAM02**	**CAM02-UCS**	**xy**
Grey	34.9	**22.5**	<u>34.2</u>	27.1	<u>24.3</u>	35.9
Black	<u>28.4</u>	**<u>19.6</u>**	39.4	<u>24.9</u>	30.3	<u>28.7</u>

5.3. Statistical Significance of Difference between Colour Difference Metrics

To determine the significance of difference found among the colour difference metrics, an *F*-test was performed. As introduced in Chapter 2, the STRESS measure can be used to calculate the statistical significance. The STRESS formula in combination with a statistical distribution *F*-test can be used to investigate a formulated null hypothesis.

For the testing the statistical significance between each colour difference formula, a two-tailed hypothesis was defined. It was based on the residual error variance $V_{A,B}$, which in this case can be calculated by the STRESS between two formulae A and B as illustrated in Equation 5.3-1. The null hypothesis H_0 and the hypothesis against H_1 were defined as: (a) $H_{0:}$ V_A=V_B (formula A and B without significant difference, (b) $H_{1:}$ $V_A \neq V_B$ (formula A and B with significant difference). The F_C value represents the lower critical value of the two-tailed hypothesis. It expresses the probability of the hypothesis, and it was calculated with 95% confidence level and 545 degrees of freedom. Degrees of freedom were calculated as N minus 1, where N is the amount of samples in the population. So, if a formula A is significantly better than formula B, then $F<F_C$ applies. If formula A is significantly worse than formula B, then $F>1/F_C$ applies. If formula A is equal to formula B, then F=1 applies.

Equation 5.3-1 *F*-test between two different colour difference formulae

$$F = \frac{\mathrm{V_A}}{\mathrm{V_B}} = \frac{\sum\left(\Delta V_i - \alpha_\mathrm{A}\Delta E_{A,i}\right)^2}{\sum\left(\Delta V_i - \alpha_\mathrm{B}\Delta E_{B,i}\right)^2} = \frac{STRESS_A^2}{STRESS_B^2}$$

The Fc was calculated as described above with a value of 0.845 for the amount of 546 samples used for each background. Accordingly, the 1/Fc was found equal to 1.183. Table 5.3-1 and Table 5.3-2 summarise the statistical significance with these critical points for the grey and black background data respectively. The bold values show that the metric in the column performs significantly better than the corresponding metric in the row. Similarly, the underlined values show that the metric in the column performs significantly worse than the corresponding metric in the row. The rest of the values show very little or no significant difference. The findings from these tests will be discussed in the following section.

Table 5.3-1 Statistical Significance of difference between colour difference metrics for the grey background data (*F*-test)

	CIELAB	**CIELUV**	**CIEDE2000**	**CIECAM02**	**CAM02-UCS**	**xy**
CIELAB		**0.416**	0.960	**0.603**	**0.485**	1.058
CIELUV	2.406		2.310	1.451	1.166	2.546
CIEDE2000	1.041	**0.433**		**0.628**	**0.505**	1.102
CIECAM02	1.658	**0.689**	1.593		**0.804**	1.755
CAM02-UCS	2.063	0.857	1.981	1.244		2.183
xy	0.945	**0.393**	0.908	**0.570**	**0.458**	
Mean	1.623	0.558	1.550	0.899	0.684	1.729

Table 5.3-2 Statistical Significance of difference between colour difference metrics for the black background data (*F*-test)

	CIELAB	**CIELUV**	**CIEDE2000**	**CIECAM02**	**CAM02-UCS**	**xy**
CIELAB		**0.476**	1.925	**0.769**	1.138	1.021
CIELUV	2.100		4.041	1.614	2.390	2.144
CIEDE2000	**0.520**	**0.247**		**0.399**	**0.591**	**0.531**
CIECAM02	1.301	**0.620**	2.504		1.481	1.329
CAM02-UCS	0.879	**0.418**	1.691	**0.675**		0.897
xy	0.979	**0.466**	1.885	**0.753**	1.115	
Mean	1.156	0.446	2.409	0.842	1.343	1.184

5.4. Discussion of Findings

In this section, the results from the performance test for the colour difference metrics by using the STRESS measure and the significance of these resulted differences are discussed for each space and formula individually and in overall.

5.4.1. The xy Chromaticity Diagram

The idea of relating distance with colour difference is very old, and it has been thoroughly investigated in many studies together with studies about alteration of the xy chromaticity diagram to fix the non-uniformity (Davidson, 1951). It has been long shown that equal distances in different parts of the xy chromaticity diagram do not represent equal perceived colour differences (MacAdam, 1942; Wright, 1941). The xy chromaticity diagram is not nowadays used for calculation of colour difference but it is still used as a chromaticity diagram. Therefore, it was also tested in this study.

The xy chromaticity diagram did not fit the data very well. Especially, for the grey background data, it has the largest STRESS value. The significance test also indicates that it performs worse than CIELUV, CIECAM02 and

CAM02-UCS for this background as well. Yet, it performed significantly better than CIEDE2000 against the black background. This space was based on matching highly saturated colours, so it would be expected to perform a little better for MacAdam colour centres which were also matched using this space as reference.

5.4.2. The CIELAB Colour Space and Formula

The CIELAB was developed as a uniform colour space based on the Munsell scaling (McLaren, 1976). It is also one of the colour spaces considered for surface colours. It did not perform the worst but it seems to be at similar performance with the xy chromaticity diagram in terms of STRESS units. The same as the xy chromaticity diagram, it also performed significantly worse than CIELUV, CIECAM02 and CAM02-UCS for the grey background data, and significantly better than CIEDE2000 for the black background data. Both xy chromaticity diagram and CIELAB have also the same ratio/analogy between their results for the grey and black background data.

5.4.3. The CIELUV Colour Space and Formula

The CIELUV relates to the u’v’ chromaticity diagram and it is has a formula recommended for prediction of colour difference for additive reproduction stimuli. For the chromaticity coordinates, the u*v* values were combined with the lightness L* attribute. The formulae and space gave the smallest STRESS value in the set of formulae examined. These results confirm the colour discrimination ellipses as well. The formula predicted the visual differences more accurately for both backgrounds. Especially, against the black background, it gave the smallest STRESS value. These results also agree with the results from the study by Luo *et al.* about the performance of the formula in evaluating white light sources (Luo et al., 2015). Furthermore, the statistical test signifies that it performed significantly better than all the other tested metrics for both backgrounds. The only showed instance that there was no significant difference with another formula is with the CAM02-UCS against the grey background. Even though the STRESS value for the

CIELUV was the smallest, the CAM02-UCS gave the next smallest STRESS value against the grey background.

5.4.4. The CIEDE2000 Formula

From previous studies, it has been seen that CIEDE2000 might not be good colour difference formula for evaluating lighting stimuli; as it has been developed by surface colours. However, it can be seen that the formula had its best performance while against the grey background. This implies that the experimental arrangement against the grey background did simulate surface mode at a certain extent, and therefore this formula performed better against the grey background conditions. At the same time, the formula had the largest STRESS value in the set; with a value of 39 STRESS units against the black background. The significance test also showed that the CIEDE2000 performed either significantly worse or without significant difference from the other metrics for both backgrounds.

5.4.5. The CIECAM02 Colour Space and Model

The CIECAM02 model followed the CIELUV in performance of predicted colour differences in terms of STRESS units. Firstly, there seems to be a discrepancy in the discrimination ellipses, and the STRESS values of the ellipse fitting were slightly larger than the rest. However, in terms of STRESS units, the CIECAM02 compensates well the viewing conditions and colour appearance of the lighting stimuli against the black background. The significance test also indicates that it performed significantly better than CIELAB, CIEDE2000 and xy chromaticity diagram, which agrees well with results from other studies. More interestingly, CIECAM02 also performed significantly better than CAM02-UCS against the black background, but worse than CAM02-UCS against the grey background. This implies that the background effect for these two formulae has large impact, even though they are both related colour appearance models.

5.4.6. The CAM02-UCS Colour Space and Model

The CAM02-UCS had smaller STRESS units from the CIECAM02 against the grey background but the opposite was true against the black background. Having a STRESS value of 24 units for the grey background data, the CAM02-UCS seems to satisfy the expectation of performing well for surface colours. The significance test results also indicate that it performed significantly better than other formulae against the grey background. The only instance again that it showed no significant difference against the grey background is with the CIELUV formula. However, the opposite was again true for the black background data.

5.4.7. Overall

As discussed before, Cui et al. used a CRT monitor to display and evaluate colour pairs with the grey scale method (Berns, 1991; Cui et al., 2001b; Cui et al., 2001a). It was shown that surface colours can be represented on a display with this type of experiment and parametric settings. In the study by Cui *et al*., the colour stimuli were represented by square patches, different frames and separations. Moreover, it was an essential study for the development of the colour difference formula CIEDE2000. In this study, CIELUV formula outperformed the others; including CIEDE2000. Firstly, the CIEDE2000 can perform well for surface colours but not for lighting stimuli. Secondly, it suggests that the colour stimuli arrangement for this experiment could have potentially great impact in the outcome. This arrangement was chosen in order to simulate light sources (luminaires) on the display. The fact that a colour space such as CIELUV has outperformed the others strengthens the notion for using the metric for additive colour reproduction stimuli; such as lighting.

Another reason that the colour appearance models did not perform as expected could be the difference between the viewing field of the experiment and the defined one in the CIE specifications. In the CIE colour appearance models, it is specified that the stimulus expands at 2 degrees from the centre of the viewing field. While in the experiment, the stimuli were of about 4 degrees each.

The results showed an overall better performance for the CIELUV for both backgrounds and the significance test also supported the performance results. However, CAM02-UCS performed greatly for the data against the grey background, which implies that the grey background successfully simulates surface colour stimuli for the current data.

5.5. Background Differences

For comparing the performance of each formula and colour space in terms of background, the Table 5.2-3 can be advised. In this table, performance is summarised in STRESS values for the total of colour assessments and it can be easily derived which metric performed better for which background. In overall, it is clear that the CIELUV formula and space performed better than the other ones for both backgrounds. More specifically, it performed better by a large margin than the others. This indicates that CIELUV and u'v' chromaticity diagram can be used for better predicting lighting stimuli and specifying chromaticity tolerances for both white and coloured light sources.

Moreover, for the majority of metrics, the black background had largely smaller STRESS values (underlined values in each column of the Table 5.2-3). Except for the CIEDE2000 and CAM02-UCS, for which the influence of the background caused them to perform better against the grey background. Given the fact that CIEDE2000 and CAM02-UCS colour space have been developed from surface colour datasets, these results support that the experimental arrangement against the grey background has been sufficiently correlated with surface mode stimuli. Therefore, both performed better under conditions that can simulate surface colours. In conclusion, these two are not recommended for lighting stimuli, but they are more appropriate for surface colours.

5.6. Conclusions

Six colour spaces/ formulae were examined: xy chromaticity diagram, CIELAB, CIELUV, CIEDE2000, CIECAM02, and CAM02-UCS. From these, CIELUV colour space performed better than the others by predicting perceptual colour difference more accurately. Moreover, the results showed a slightly better performance against the black background than the grey background. This indicates that the simulation of light sources (luminaires) on a display could be successful.

Chapter 6.
Conclusions

6.1. Objectives and Summary

As presented in the introductory chapter the main objectives were:

- To understand the performance of MacAdam ellipses to fit the coloured lighting stimuli,
- To investigate the performance of various colour difference metrics for predicting lighting stimuli, and
- To understand the parametric effect on evaluating perceived lighting stimuli including change of background and luminance of colour centre.

Different colour difference formulae and colour appearance models with their respective colour spaces and chromaticity diagrams have been investigated for predicting lighting stimuli. The above objectives have been met and the results were divided into two parts: colour discrimination ellipses and colour difference evaluation.

The following conclusions can be drawn:

1. The results from the colour discrimination ellipses have shown that u’v’ chromaticity diagram can represent chromaticity differences more uniformly for the lighting stimuli. In more detail, the ratio A/B of the u’v’ chromaticity diagram had values closer to one for both backgrounds, and the STRESS values of ratio A/B was also the smallest. Because the majority of ellipse parameters (ratio A/B, semi-major axis A, and size) for the u’v’ chromaticity diagram lead to more equal sized circles, it was therefore concluded that it outperformed the others in terms of space uniformity by a large margin.

2. Even though, the chromaticity ellipses did not appear to have a strong difference between the grey and black background data in terms of shape and orientation, the ellipses against the grey background were generally larger in every tested space. So, it could be said that in terms of space

uniformity, the data of the black background fitted the ellipses better. Observers perceived larger colour difference against a black background, i.e. there was a smaller tolerance against the black background.

3. Concerning the colour centres having different lightness, the trend of colours with higher luminance to have larger ellipses was confirmed; agreeing with other studies. Moreover, this effect seems to be amplified if the luminance of the background is also high.

4. The MacAdam experimental ellipses were proven really useful in the study of colour spaces and formulae but they are not recommended to be used as it is. Even though there seems to be some resemblance of the current ellipses with the MacAdam ones, the current ellipses do not systematically change size with the chromaticity region. In spite the fact that the current ellipses have at most cases similar orientation with MacAdam ellipses. However, the current ellipse were more circular and of more constant shape.

5. The following colour difference formulae and colour spaces have been investigated in terms of colour difference evaluation: xy chromaticity diagram, CIELAB, CIELUV, CIEDE2000, CIECAM02, and CAM02-UCS. The CIELUV space and formulae performed better by confirming the pattern found by the analysis of the discrimination ellipses. It was followed by CIECAM02 and CAM02-UCS metrics, then CIELAB and xy chromaticity diagram, with the worst being the CIEDE2000. This implies that CIEDE2000 is a good fit for the surface colour stimuli but not for lighting stimuli, as it was expected. However, CAM02-UCS space also performed very well against the grey background data; which implies that for this space the simulation of surface mode stimuli was possible. At the same time, it can be supported by the results that the simulation of the lighting sources (luminaires) on the display as light sources was quite successful since some formulae which are based on surface mode datasets performed poorly. The fact that the colour difference formulae performed better against the black background also suggests that it the stimuli might also be observed as lighting sources (luminaires) with the current experimental arrangement. However, there is still no strong evidence for this conclusion.

6.2. Future Work

This work provided interesting insights in the contemporary colour difference metrics and discrimination ellipses. A unique dataset of lighting stimuli was acquired and the difference between surface and lighting stimuli was made evident. Yet, only four colour difference formulae were fitted into ellipses and the five colour spaces were evaluated in terms of colour difference. Other colour difference spaces and formulae could also be examined. Moreover, a greater variety of colours, as well as lightness differences could be examined with further experiments.

Future work could involve another set of experiments by using a custom lighting apparatus in order to examine further if the simulation of lighting stimuli on display is possible, and to what extend the specific arrangement of samples affects the perception of colour stimuli as light sources.

The present study reveals that there is a difference in evaluating colour differences between the surface and light colours. The present results suggest that u'v' chromaticity diagram and CIELUV are more suitable to be used in the lighting industry. However, there is a need to verify the present display data by using real light sources; especially with high dynamic range LED luminaires.

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List of Abbreviations

ANSI	American National Standards Institute
CCT	Correlated Colour Temperature
CIE	*Commission Internationale de l' Éclairage* (International Commission on Illumination)
CMM	Colour Management Module
CMS	Colour Management System
CRT	Cathode Ray tube
FPD	Flat Panel Display
GMA	Gamut Mapping Algorithms
ICC	International Color Consortium
ISO	International Organization for Standardization
JND	Just Noticeable Difference
LED	Light-Emitting Diode
LCD	Liquid Crystal Display
LUT	Lookup table
PCS	Profile Connection Space
SPD	Spectral Power Distribution or S(λ)
SSL	Solid State Lighting
TRC	Tone Reproduction Curve
TSR	Tele-Spectroradiometer
WCS	Windows Color System

Appendix A
Glossary

A.1 Colour Fundamentals

Colour matching functions	“the amounts, in any trichromatic system, of the three reference colour stimuli needed to match by an additive mixture monochromatic components of an equal energy spectrum” (ASTM, 2009). Symbols: $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ for RGB, $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ for CIE 1931 XYZ and $\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$ for CIE 1964.
Correlated colour temperature - T_{cp}, CCT	“temperature of the Planckian radiator whose perceived colour most closely resembles that of a given stimulus seen at the same brightness and under specified viewing conditions” (CIE, 1987).
Device-dependent colour space	“colour space defined by the characteristics of a real or idealized imaging device. Device-dependent colour spaces having a simple functional relationship to CIE colorimetry can also be categorized as colorimetric colour spaces. For example, additive RGB colour spaces corresponding to real or idealized CRT displays can be treated as colorimetric colour spaces” (ISO, 2004).
Illuminant	“radiation with a relative spectral power distribution defined over the wavelength range that influences object colour perception” (CIE, 1987).
Integrating sphere	“an optical device used either to collect flux reflected or transmitted from a specimen into a hemisphere or to provide isotropic irradiation of a specimen from a complete hemisphere, consisting of an approximately spherical cavity with apertures (ports) for admitting and detecting flux, and usually having additional apertures

	over which sample and reference specimens are placed and for including or excluding the specularly components” (ASTM, 2009).
Luminous flux – Unit: lm	“quantity derived from radiant flux Φe by evaluating the radiation according to its action upon the CIE standard photometric observer. For photopic vision $\Phi_v = K_m \int_0^\infty \frac{d\Phi_e(\lambda)}{d\lambda} \cdot V(\lambda)\, d\lambda$ where $\frac{d\Phi_e(\lambda)}{d\lambda}$ is the spectral distribution of the radiant flux and V(λ) is the spectral luminous efficiency” (CIE, 1987).
Plankian radiator (or else Blackbody, or Full radiator)	“Thermal radiator that absorbs completely all incident energy whatever the wavelength, the direction of incidence, or the polarization. This radiator has, for any wavelength and in any direction, the maximum spectral concentration of radiance for a thermal radiator in the thermal equilibrium at a given temperature” (Grum and Becherer, 1979 p.25).
Reflectance – ρ	“ratio of the reflected radiant or luminous flux to the incident flux in the given conditions” (ASTM, 2009).
Reflectance factor – R(λ)	“ratio of the flux reflected from the specimen to the flux reflected from the perfect reflecting diffuser under the same geometric and spectral conditions of measurement” (ASTM, 2009).
Relative spectral power distribution – S(λ)	“ratio of the spectral power distribution of a source or illuminant to a fixed reference value” (ISO, 2009).
Spectral power distribution – SPD	“specification of an illuminant by the spectral composition of a radiometric quantity, such as radiance or radiant flux, as a function of wavelength” (ASTM, 2009).
Specular Gloss	“ratio of flux reflected in specular direction to incident flux for a specified angle of incidence and source and

	receptor angular apertures" (ASTM, 2009).
Tristimulus values – X,Y, Z and X_{10}, Y_{10}, Z_{10}	"amounts of the three reference colour stimuli, in a given trichromatic system, required to match the colour of the stimulus considered. In the CIE standard colorimetric systems, the tristimulus values are represented by the symbols X, Y, Z and X10, Y10, Z10." (CIE, 1987).
Uniform colour space	"schematic arrangement of colours in space in which spatial internals between points correspond to visual differences between colours represented by those points" (ASTM, 2009).

A.2 Colour Management

Calibration	"operation of establishing that the measured values agree with the values specified by a standard or a characterization process" (ISO, 2008).
Characterisation	"process of relating device-dependent colour values to device-independent colour values" (ISO, 2008).
Colour management	The "communication of the associated data required for unambiguous interpretation of colour content data, and application of colour data conversions, as required, to produce the intended reproductions" (ISO, 2010). This implies the entire procedure from the capture of the stimulus to the reproduction in any output device.
Colour Management System - CMS	It generally describes "software dedicated to handling device-to-device conversion of colours. The ICC-based model for a CMS consists of four components: a PCS, device profiles, a CMM, and a set of rendering intents" (Fraser et al., 2005).
Colour encoding	It is a "generic term for a quantized digital encoding of a colour space, encompassing both colour space encodings and colour image encodings" (ISO, 2004).
Colour space encoding	"digital encoding of a colour space, including the specification of a digital encoding method, and a colour space value range" (ISO, 2004).
Colour image encoding	"digital encoding of the colour values for a digital image, including the specification of a colour space encoding, together with any information necessary to properly interpret the colour values such as the image state, the intended image viewing environment and the reference medium" (ISO, 2004)
Colour rendering	This term is often used in colour management as it describes the "mapping of image data representing the

	colour-space coordinates of the elements of a scene to output-referred image data representing the colour-space coordinates of the elements of a reproduction" (ISO, 2004).
Colour re-rendering	This is the opposite of colour rendering. It is the "mapping of picture-referred image data appropriate for one specified real or virtual imaging medium and viewing conditions to picture-referred image data appropriate for a different real or virtual imaging medium and/or viewing conditions" (ISO, 2004).

A.3 Psychophysical Methods

Magnitude Estimation Method	“psychophysical method involving the assignment of a numerical value to each test stimulus that is proportional to image quality; typically, a reference stimulus with an assigned numerical value is present to anchor the rating scale” (ISO, 2005).
Categorical Sort Method	“psychophysical method involving the classification of a stimulus into one of several ordered categories, at least some of which are identified by adjectives or phrases that describe different levels of image quality or attributes thereof” (ISO, 2005).
Pair Comparison Method	“psychophysical method involving the choice of which of two simultaneously presented stimuli exhibits greater or lesser image quality or an attribute thereof, in accordance with a set of instructions given to the observer” (ISO, 2005).
Quality Ruler Method	“psychophysical method that involves quality or attribute assessment of a test stimulus against a series of ordered, univariate reference stimuli that differ by known numbers of just noticeable differences” (ISO, 2005).
Rank Ordering Method	“psychophysical method involving the arrangement by an observer of a series of stimuli in order of increasing or decreasing image quality or an attribute thereof, in accordance with the set of instructions provided” (ISO, 2005).
Triplet Comparison	“psychophysical method that involves the simultaneous scaling of three test stimuli with respect to image quality or an attribute thereof, in accordance with a set of instructions given to the observer” (ISO, 2005).

Appendix B
CIECAM02 Forward Model

$$\Delta E_{CAM02} = \sqrt{\Delta J^2 + \Delta a^2 + \Delta b^2}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \boldsymbol{M}_{CAT02} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\boldsymbol{M}_{CAT02} = \begin{bmatrix} 0.7328 & 0.4296 & -0.1624 \\ -0.7036 & 1.6975 & 0.0061 \\ 0.0030 & 0.0136 & 0.9834 \end{bmatrix}$$

$$D = F \left[1 - (1/3.6)^{e^{-(L_A+42)/92}}\right]$$

$$R_C = \left[\left(Y_W \; {}^{D}\!/_{R_W}\right) + (1 - D)\right] R$$

$$G_C = \left[\left(Y_W \; {}^{D}\!/_{G_W}\right) + (1 - D)\right] G$$

$$B_C = \left[\left(Y_W \; {}^{D}\!/_{B_W}\right) + (1 - D)\right] B$$

$$k = 1/(5\,L_A + 1)$$

$$F_L = 0.2\,k^4\,(5\,L_A) + 0.1\,(1 - k^4)^2 \;\sqrt[3]{5\,L_A}$$

$$n = Y_b/Y_w$$

$$N_{bb} = N_{cb} = 0.725\,(1/n)^{0.2}$$

$$z = 1.48 + \sqrt{n}$$

$$\begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} = \boldsymbol{M}_{HPE}\,\boldsymbol{M}_{CAT02}^{-1} \begin{bmatrix} R_C \\ G_C \\ B_C \end{bmatrix}$$

$$\boldsymbol{M}_{HPE} = \begin{bmatrix} 0.38971 & 0.68898 & -0.07868 \\ -0.22981 & 1.18340 & 0.04641 \\ 0.00000 & 0.00000 & 1.00000 \end{bmatrix}$$

$$\boldsymbol{M}_{CAT02}^{-1} = \begin{bmatrix} 1.096124 & -0.278869 & 0.182745 \\ 0.454369 & 0.473533 & 0.072098 \\ -0.009628 & -0.005698 & 1.015326 \end{bmatrix}$$

$$R'_a = \frac{400\,(F_L\,R'/100)^{0.42}}{27.13 + (F_L\,R'/100)^{0.42}} + 0.1$$

$$G'_a = \frac{400\,(F_L\,G'/100)^{0.42}}{27.13 + (F_L\,G'/100)^{0.42}} + 0.1$$

$$B'_a = \frac{400\,(F_L\,G'/100)^{0.42}}{27.13 + (F_L\,G'/100)^{0.42}} + 0.1$$

$$a = R'_a - 12\,G'_a/11 + B'_a/11$$

$$b = (1/9)\,(R'_a + G'_a - 2\,B'_a)$$

$$h = \tan^{-1}(b/a)$$

	Red	*Yellow*	*Green*	*Blue*	*Red*
i	1	2	3	4	5
h_i	20.14	90.00	164.25	237.53	380.□4
e_i	0.8	0□7	1.0	1.2	0.8
H_i	0.0	100.0	200.0	300.0	400.0

$$e_t = {}^{1}\!/_{4}\left[\cos\left(h\ {}^{\pi}\!/_{180} + 2\right) + 3.8\right]$$

$$H = H_i + \frac{100\ (h' - h_i)/e_i}{(h' - h_i)/e_i + (h_{i+1} - h')/e_{i+1}}$$

$$A = [2\,R'_a + G'_a + (1/20)\,B'_a - 0.305]\,N_{bb}$$

$$J = 100\,(A/A_w)^{c\,z}$$

$$Q = (4/c)\,\sqrt{J/100}\ \times (A_w + 4)\,F_L^{0.25}$$

$$t = \frac{(50000/13)\,N_c\,N_{cb}\,e_t\,\sqrt{a^2 + b^2}}{R'_a + G'_a + (21/20)\,B'_a}$$

$$C = t^{0.9}\,\sqrt{J/100}\ \times (1.64 + 0.29^n)^{0.73}$$

$$M = C\,F_L^{0.25}$$

$$s = 100\,\sqrt{M/Q}$$

$$a_C = C\ \cos(h)$$

$$b_C = C\ \sin(h)$$

$$a_M = M\ \cos(h)$$

$$b_M = M\ \sin(h)$$

$$a_s = s\ \cos(h)$$

$$b_s = s\ \sin(h)$$

Appendix C

Experimental Data against the Grey Background

	Xo	Yo	Zo	
White	95.19	100.00	97.12	
Black	0.801	0.850	0.880	
Gray	22.98	23.57	27.80	(fullscreen)

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
1_18	1.01	10.53	18.13	12.21	10.95	17.82	11.84	0.695
1_18	1.02	10.53	18.13	12.21	10.90	17.80	11.58	0.64
1_18	1.03	10.53	18.13	12.21	10.92	18.08	11.33	0.685
1_18	1.04	10.53	18.13	12.21	10.86	18.04	11.03	0.71
1_18	1.05	10.53	18.13	12.21	10.83	18.03	10.87	0.705
1_18	1.06	10.53	18.13	12.21	10.68	17.97	10.71	0.75
1_18	1.07	10.53	18.13	12.21	10.60	17.94	10.72	0.67
1_18	1.08	10.53	18.13	12.21	10.67	18.23	10.57	0.715
1_18	1.09	10.53	18.13	12.21	10.71	18.25	10.76	0.695
1_18	1.10	10.53	18.13	12.21	10.57	18.18	10.76	0.67
1_18	1.11	10.53	18.13	12.21	10.44	18.20	10.76	0.615
1_18	1.12	10.53	18.13	12.21	10.28	18.12	10.76	0.615
1_18	1.13	10.53	18.13	12.21	10.20	18.19	10.76	0.61
1_18	1.14	10.53	18.13	12.21	10.19	18.07	10.94	0.565
1_18	1.15	10.53	18.13	12.21	9.89	17.91	10.92	0.68
1_18	1.16	10.53	18.13	12.21	9.89	17.91	10.92	0.63
1_18	1.17	10.53	18.13	12.21	9.95	18.04	11.07	0.645
1_18	1.18	10.53	18.13	12.21	10.01	18.07	11.38	0.585
1_18	1.19	10.53	18.13	12.21	9.97	18.05	11.65	0.57
1_18	1.20	10.53	18.13	12.21	9.84	17.97	11.89	0.615
1_18	1.21	10.53	18.13	12.21	10.11	18.11	12.43	0.615

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
1_48	1.01	27.16	46.70	30.42	28.18	46.68	29.93	0.695
1_48	1.02	27.16	46.70	30.42	28.16	46.69	29.66	0.715
1_48	1.03	27.16	46.70	30.42	27.94	46.56	28.61	0.71
1_48	1.04	27.16	46.70	30.42	28.00	46.94	28.69	0.665
1_48	1.05	27.16	46.70	30.42	27.68	46.79	28.19	0.715
1_48	1.06	27.16	46.70	30.42	27.60	46.76	27.78	0.73
1_48	1.07	27.16	46.70	30.42	27.36	46.83	27.32	0.81
1_48	1.08	27.16	46.70	30.42	27.36	46.86	27.18	0.815
1_48	1.09	27.16	46.70	30.42	27.10	46.71	27.30	0.825
1_48	1.10	27.16	46.70	30.42	26.76	46.55	27.15	0.755
1_48	1.11	27.16	46.70	30.42	26.54	46.43	27.13	0.715
1_48	1.12	27.16	46.70	30.42	26.36	46.48	27.10	0.8
1_48	1.13	27.16	46.70	30.42	26.25	46.92	27.31	0.82
1_48	1.14	27.16	46.70	30.42	26.13	46.89	27.80	0.715
1_48	1.15	27.16	46.70	30.42	25.91	46.88	28.18	0.725
1_48	1.16	27.16	46.70	30.42	25.54	46.69	28.64	0.685
1_48	1.17	27.16	46.70	30.42	25.41	46.60	28.61	0.615
1_48	1.18	27.16	46.70	30.42	25.42	46.52	29.46	0.7
1_48	1.19	27.16	46.70	30.42	25.53	46.68	29.93	0.635
1_48	1.20	27.16	46.70	30.42	25.31	46.43	30.34	0.65
1_48	1.21	27.16	46.70	30.42	25.32	46.45	31.16	0.8
10_18	10.01	29.52	18.07	28.35	30.35	18.18	27.87	0.585
10_18	10.02	29.52	18.07	28.35	30.06	18.03	27.56	0.62
10_18	10.03	29.52	18.07	28.35	30.17	18.23	27.62	0.62
10_18	10.04	29.52	18.07	28.35	30.20	18.25	27.52	0.56
10_18	10.05	29.52	18.07	28.35	29.74	18.07	27.01	0.6
10_18	10.06	29.52	18.07	28.35	29.35	17.88	26.68	0.555
10_18	10.07	29.52	18.07	28.35	29.26	17.83	26.23	0.62
10_18	10.08	29.52	18.07	28.35	29.24	18.10	26.28	0.585
10_18	10.09	29.52	18.07	28.35	29.40	18.19	26.29	0.56
10_18	10.10	29.52	18.07	28.35	29.33	18.19	26.72	0.57
10_18	10.11	29.52	18.07	28.35	29.33	18.14	26.74	0.585
10_18	10.12	29.52	18.07	28.35	29.08	18.07	26.27	0.63
10_18	10.13	29.52	18.07	28.35	29.19	18.26	26.69	0.575
10_18	10.14	29.52	18.07	28.35	28.86	18.09	26.66	0.615
10_18	10.15	29.52	18.07	28.35	28.68	17.86	26.71	0.6
10_18	10.16	29.52	18.07	28.35	28.75	18.03	27.00	0.59
10_18	10.17	29.52	18.07	28.35	28.85	18.08	27.47	0.56
10_18	10.18	29.52	18.07	28.35	28.47	17.88	27.46	0.64
10_18	10.19	29.52	18.07	28.35	28.48	17.88	27.58	0.605
10_18	10.20	29.52	18.07	28.35	28.72	18.15	28.37	0.6
10_18	10.21	29.52	18.07	28.35	28.69	17.98	28.66	0.615

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
12_18	12.01	21.75	18.06	23.71	22.49	18.11	23.70	0.64
12_18	12.01	21.75	18.06	23.71	22.49	18.11	23.70	0.67
12_18	12.02	21.75	18.06	23.71	22.47	18.10	23.49	0.66
12_18	12.02	21.75	18.06	23.71	22.47	18.10	23.49	0.735
12_18	12.03	21.75	18.06	23.71	22.22	17.94	22.36	0.87
12_18	12.03	21.75	18.06	23.71	22.22	17.94	22.36	0.8
12_18	12.04	21.75	18.06	23.71	22.30	18.17	22.40	0.8
12_18	12.04	21.75	18.06	23.71	22.30	18.17	22.40	0.78
12_18	12.05	21.75	18.06	23.71	22.04	18.04	22.39	0.905
12_18	12.05	21.75	18.06	23.71	22.04	18.04	22.39	0.77
12_18	12.06	21.75	18.06	23.71	21.94	17.81	22.18	0.845
12_18	12.06	21.75	18.06	23.71	21.94	17.81	22.18	0.815
12_18	12.07	21.75	18.06	23.71	21.99	18.02	22.01	0.78
12_18	12.07	21.75	18.06	23.71	21.99	18.02	22.01	0.89
12_18	12.08	21.75	18.06	23.71	21.70	18.03	21.61	0.95
12_18	12.08	21.75	18.06	23.71	21.70	18.03	21.61	0.81
12_18	12.09	21.75	18.06	23.71	21.77	18.06	21.98	0.76
12_18	12.09	21.75	18.06	23.71	21.77	18.06	21.98	0.72
12_18	12.10	21.75	18.06	23.71	21.60	18.16	21.66	0.895
12_18	12.10	21.75	18.06	23.71	21.60	18.16	21.66	0.875
12_18	12.11	21.75	18.06	23.71	21.71	18.43	21.74	0.97
12_18	12.11	21.75	18.06	23.71	21.71	18.43	21.74	0.91
12_18	12.12	21.75	18.06	23.71	21.49	18.10	22.03	0.82
12_18	12.12	21.75	18.06	23.71	21.49	18.10	22.03	0.79
12_18	12.13	21.75	18.06	23.71	21.58	18.35	22.07	0.855
12_18	12.13	21.75	18.06	23.71	21.58	18.35	22.07	0.9
12_18	12.14	21.75	18.06	23.71	21.49	18.31	22.27	0.79
12_18	12.14	21.75	18.06	23.71	21.49	18.31	22.27	0.8
12_18	12.15	21.75	18.06	23.71	21.50	18.31	22.45	0.775
12_18	12.15	21.75	18.06	23.71	21.50	18.31	22.45	0.685
12_18	12.16	21.75	18.06	23.71	21.34	18.22	22.44	0.695
12_18	12.16	21.75	18.06	23.71	21.34	18.22	22.44	0.735
12_18	12.17	21.75	18.06	23.71	21.06	18.14	22.42	0.89
12_18	12.17	21.75	18.06	23.71	21.06	18.14	22.42	0.715
12_18	12.18	21.75	18.06	23.71	21.12	18.17	22.80	0.76
12_18	12.18	21.75	18.06	23.71	21.12	18.17	22.80	0.71
12_18	12.19	21.75	18.06	23.71	21.29	18.27	23.55	0.655
12_18	12.19	21.75	18.06	23.71	21.29	18.27	23.55	0.745
12_18	12.20	21.75	18.06	23.71	21.28	18.19	23.55	0.695
12_18	12.20	21.75	18.06	23.71	21.28	18.19	23.55	0.66
12_18	12.21	21.75	18.06	23.71	21.35	18.23	23.94	0.685
12_18	12.21	21.75	18.06	23.71	21.35	18.23	23.94	0.67

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
12_48	12.01	56.81	46.75	61.01	58.06	46.50	60.42	0.74
12_48	12.01	56.81	46.75	61.01	58.06	46.50	60.42	0.66
12_48	12.02	56.81	46.75	61.01	58.09	46.78	59.83	0.725
12_48	12.02	56.81	46.75	61.01	58.09	46.78	59.83	0.685
12_48	12.03	56.81	46.75	61.01	57.50	46.16	59.33	0.71
12_48	12.03	56.81	46.75	61.01	57.50	46.16	59.33	0.7
12_48	12.04	56.81	46.75	61.01	57.29	46.37	58.49	0.755
12_48	12.04	56.81	46.75	61.01	57.29	46.37	58.49	0.74
12_48	12.05	56.81	46.75	61.01	57.21	46.32	57.97	0.705
12_48	12.05	56.81	46.75	61.01	57.21	46.32	57.97	0.7
12_48	12.06	56.81	46.75	61.01	57.04	46.73	57.56	0.735
12_48	12.06	56.81	46.75	61.01	57.04	46.73	57.56	0.685
12_48	12.07	56.81	46.75	61.01	56.75	46.55	57.12	0.745
12_48	12.07	56.81	46.75	61.01	56.75	46.55	57.12	0.695
12_48	12.08	56.81	46.75	61.01	56.36	46.57	56.71	0.67
12_48	12.08	56.81	46.75	61.01	56.36	46.57	56.71	0.73
12_48	12.09	56.81	46.75	61.01	56.19	46.70	56.76	0.695
12_48	12.09	56.81	46.75	61.01	56.19	46.70	56.76	0.725
12_48	12.10	56.81	46.75	61.01	55.68	46.42	56.72	0.75
12_48	12.10	56.81	46.75	61.01	55.68	46.42	56.72	0.7
12_48	12.11	56.81	46.75	61.01	55.38	46.33	56.67	0.69
12_48	12.11	56.81	46.75	61.01	55.38	46.33	56.67	0.695
12_48	12.12	56.81	46.75	61.01	55.31	46.52	56.04	0.665
12_48	12.12	56.81	46.75	61.01	55.31	46.52	56.04	0.78
12_48	12.13	56.81	46.75	61.01	55.24	46.63	56.69	0.745
12_48	12.13	56.81	46.75	61.01	55.24	46.63	56.69	0.73
12_48	12.14	56.81	46.75	61.01	54.94	46.68	57.11	0.785
12_48	12.14	56.81	46.75	61.01	54.94	46.68	57.11	0.83
12_48	12.15	56.81	46.75	61.01	54.74	46.61	57.56	0.77
12_48	12.15	56.81	46.75	61.01	54.74	46.61	57.56	0.775
12_48	12.16	56.81	46.75	61.01	54.79	46.38	58.03	0.885
12_48	12.16	56.81	46.75	61.01	54.79	46.38	58.03	0.685
12_48	12.17	56.81	46.75	61.01	54.70	46.55	58.98	0.72
12_48	12.17	56.81	46.75	61.01	54.70	46.55	58.98	0.67
12_48	12.18	56.81	46.75	61.01	54.87	46.64	59.85	0.74
12_48	12.18	56.81	46.75	61.01	54.87	46.64	59.85	0.72
12_48	12.19	56.81	46.75	61.01	54.33	46.33	60.00	0.85
12_48	12.19	56.81	46.75	61.01	54.33	46.33	60.00	0.74
12_48	12.20	56.81	46.75	61.01	54.46	46.19	60.55	0.76
12_48	12.20	56.81	46.75	61.01	54.46	46.19	60.55	0.75
12_48	12.21	56.81	46.75	61.01	54.73	46.32	61.64	0.81
12_48	12.21	56.81	46.75	61.01	54.73	46.32	61.64	0.82

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
13_18	13.01	16.68	18.28	37.66	17.23	18.15	36.95	0.75
13_18	13.02	16.68	18.28	37.66	17.17	18.12	36.59	0.63
13_18	13.03	16.68	18.28	37.66	17.13	18.10	36.37	0.73
13_18	13.04	16.68	18.28	37.66	17.04	18.06	35.96	0.635
13_18	13.05	16.68	18.28	37.66	16.94	18.26	36.03	0.585
13_18	13.06	16.68	18.28	37.66	16.94	18.27	36.05	0.56
13_18	13.07	16.68	18.28	37.66	16.79	18.19	35.59	0.57
13_18	13.08	16.68	18.28	37.66	16.57	18.24	35.13	0.58
13_18	13.09	16.68	18.28	37.66	16.64	18.27	35.58	0.51
13_18	13.10	16.68	18.28	37.66	16.41	18.16	35.63	0.54
13_18	13.11	16.68	18.28	37.66	16.31	18.18	35.54	0.535
13_18	13.12	16.68	18.28	37.66	16.29	18.38	35.66	0.57
13_18	13.13	16.68	18.28	37.66	16.13	18.11	35.15	0.56
13_18	13.14	16.68	18.28	37.66	16.10	18.09	35.61	0.55
13_18	13.15	16.68	18.28	37.66	15.99	18.03	36.05	0.57
13_18	13.16	16.68	18.28	37.66	15.89	18.17	36.07	0.66
13_18	13.17	16.68	18.28	37.66	15.74	18.09	36.04	0.67
13_18	13.18	16.68	18.28	37.66	15.73	18.09	36.76	0.69
13_18	13.19	16.68	18.28	37.66	15.87	18.16	36.73	0.66
13_18	13.20	16.68	18.28	37.66	15.78	18.11	37.03	0.65
13_18	13.21	16.68	18.28	37.66	15.77	18.10	37.04	0.61
19_18	19.01	27.26	18.15	6.69	27.87	18.18	6.52	0.56
19_18	19.02	27.26	18.15	6.69	27.59	18.02	6.27	0.57
19_18	19.03	27.26	18.15	6.69	27.66	18.23	6.14	0.625
19_18	19.04	27.26	18.15	6.69	27.63	18.22	5.98	0.57
19_18	19.05	27.26	18.15	6.69	27.29	18.03	5.76	0.645
19_18	19.06	27.26	18.15	6.69	27.09	18.07	5.76	0.64
19_18	19.07	27.26	18.15	6.69	27.24	18.15	5.38	0.595
19_18	19.08	27.26	18.15	6.69	27.06	18.05	5.56	0.745
19_18	19.09	27.26	18.15	6.69	26.81	18.07	5.40	0.635
19_18	19.10	27.26	18.15	6.69	26.82	18.07	5.40	0.685
19_18	19.11	27.26	18.15	6.69	26.58	18.10	5.39	0.7
19_18	19.12	27.26	18.15	6.69	26.58	18.11	5.38	0.755
19_18	19.13	27.26	18.15	6.69	26.56	17.94	5.39	0.635
19_18	19.14	27.26	18.15	6.69	26.30	17.94	5.55	0.665
19_18	19.15	27.26	18.15	6.69	26.34	17.95	5.75	0.695
19_18	19.16	27.26	18.15	6.69	26.51	18.23	5.80	0.675
19_18	19.17	27.26	18.15	6.69	26.15	18.04	6.00	0.65
19_18	19.18	27.26	18.15	6.69	26.20	18.07	6.16	0.685
19_18	19.19	27.26	18.15	6.69	26.22	18.08	6.31	0.715
19_18	19.20	27.26	18.15	6.69	26.20	17.90	6.68	0.59
19_18	19.21	27.26	18.15	6.69	26.20	17.90	6.68	0.64

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
23_18	23.01	22.56	18.19	40.20	23.12	18.04	39.78	0.76
23_18	23.02	22.56	18.19	40.20	23.04	18.01	39.39	0.725
23_18	23.03	22.56	18.19	40.20	23.11	18.20	39.47	0.765
23_18	23.04	22.56	18.19	40.20	23.05	18.17	39.13	0.82
23_18	23.05	22.56	18.19	40.20	22.74	18.02	38.58	0.73
23_18	23.06	22.56	18.19	40.20	22.74	18.11	38.52	0.735
23_18	23.07	22.56	18.19	40.20	22.61	18.05	38.52	0.675
23_18	23.08	22.56	18.19	40.20	22.62	18.24	38.15	0.69
23_18	23.09	22.56	18.19	40.20	22.40	17.94	37.52	0.675
23_18	23.10	22.56	18.19	40.20	22.04	17.93	37.56	0.575
23_18	23.11	22.56	18.19	40.20	22.04	18.09	37.47	0.645
23_18	23.12	22.56	18.19	40.20	21.77	17.94	37.44	0.55
23_18	23.13	22.56	18.19	40.20	21.77	17.96	38.09	0.575
23_18	23.14	22.56	18.19	40.20	21.86	18.17	38.15	0.555
23_18	23.15	22.56	18.19	40.20	21.63	18.05	38.61	0.66
23_18	23.16	22.56	18.19	40.20	21.90	18.19	38.63	0.615
23_18	23.17	22.56	18.19	40.20	21.85	18.37	39.21	0.725
23_18	23.18	22.56	18.19	40.20	22.16	18.53	39.58	0.655
23_18	23.19	22.56	18.19	40.20	21.91	18.40	39.56	0.68
23_18	23.20	22.56	18.19	40.20	21.97	18.42	39.98	0.78
23_18	23.21	22.56	18.19	40.20	21.93	18.20	40.27	0.75
24_18	24.01	32.81	17.96	58.59	33.77	18.14	58.05	0.6
24_18	24.02	32.81	17.96	58.59	33.71	18.12	57.56	0.665
24_18	24.03	32.81	17.96	58.59	33.60	18.10	57.64	0.56
24_18	24.04	32.81	17.96	58.59	33.52	18.16	57.00	0.635
24_18	24.05	32.81	17.96	58.59	33.52	18.27	56.65	0.575
24_18	24.06	32.81	17.96	58.59	33.19	18.10	57.04	0.585
24_18	24.07	32.81	17.96	58.59	32.89	18.02	56.12	0.565
24_18	24.08	32.81	17.96	58.59	32.82	18.10	56.67	0.515
24_18	24.09	32.81	17.96	58.59	32.72	18.05	56.17	0.55
24_18	24.10	32.81	17.96	58.59	32.40	17.98	56.10	0.585
24_18	24.11	32.81	17.96	58.59	32.18	17.96	56.07	0.545
24_18	24.12	32.81	17.96	58.59	32.25	18.13	56.15	0.53
24_18	24.13	32.81	17.96	58.59	32.37	18.18	56.71	0.51
24_18	24.14	32.81	17.96	58.59	31.75	17.98	56.11	0.585
24_18	24.15	32.81	17.96	58.59	31.90	17.94	57.13	0.605
24_18	24.16	32.81	17.96	58.59	31.75	17.98	57.08	0.61
24_18	24.17	32.81	17.96	58.59	31.93	18.06	57.09	0.645
24_18	24.18	32.81	17.96	58.59	32.05	18.12	57.73	0.575
24_18	24.19	32.81	17.96	58.59	31.86	17.92	57.78	0.62
24_18	24.20	32.81	17.96	58.59	32.13	18.16	58.22	0.625
24_18	24.21	32.81	17.96	58.59	32.22	18.09	58.80	0.6

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
25_18	25.01	21.58	18.39	6.27	21.78	18.11	5.81	0.61
25_18	25.02	21.58	18.39	6.27	21.75	18.10	5.62	0.62
25_18	25.03	21.58	18.39	6.27	21.76	18.11	5.62	0.605
25_18	25.04	21.58	18.39	6.27	21.71	18.10	5.28	0.57
25_18	25.05	21.58	18.39	6.27	21.71	18.10	5.28	0.585
25_18	25.06	21.58	18.39	6.27	21.66	18.24	5.30	0.6
25_18	25.07	21.58	18.39	6.27	21.62	18.21	5.15	0.63
25_18	25.08	21.58	18.39	6.27	21.33	18.25	4.96	0.57
25_18	25.09	21.58	18.39	6.27	21.29	18.24	4.78	0.685
25_18	25.10	21.58	18.39	6.27	21.34	18.26	4.96	0.645
25_18	25.11	21.58	18.39	6.27	21.08	18.20	4.77	0.695
25_18	25.12	21.58	18.39	6.27	20.90	18.11	4.93	0.66
25_18	25.13	21.58	18.39	6.27	20.99	18.28	5.16	0.72
25_18	25.14	21.58	18.39	6.27	20.94	18.14	5.15	0.575
25_18	25.15	21.58	18.39	6.27	21.01	18.28	5.38	0.575
25_18	25.16	21.58	18.39	6.27	20.75	18.13	5.37	0.655
25_18	25.17	21.58	18.39	6.27	20.82	18.18	5.63	0.61
25_18	25.18	21.58	18.39	6.27	20.84	18.19	5.80	0.64
25_18	25.19	21.58	18.39	6.27	20.88	18.21	6.00	0.535
25_18	25.20	21.58	18.39	6.27	20.94	18.25	6.22	0.545
25_18	25.21	21.58	18.39	6.27	20.99	18.29	6.39	0.59
25_48	25.01	54.78	46.26	15.05	56.53	46.62	15.03	0.65
25_48	25.02	54.78	46.26	15.05	56.46	46.58	14.64	0.67
25_48	25.03	54.78	46.26	15.05	56.36	46.79	14.16	0.635
25_48	25.04	54.78	46.26	15.05	55.73	46.25	13.22	0.71
25_48	25.05	54.78	46.26	15.05	55.60	46.41	13.11	0.765
25_48	25.06	54.78	46.26	15.05	55.64	46.65	12.74	0.81
25_48	25.07	54.78	46.26	15.05	55.08	46.43	12.68	0.81
25_48	25.08	54.78	46.26	15.05	54.70	46.24	12.10	0.875
25_48	25.09	54.78	46.26	15.05	54.79	46.57	11.88	0.88
25_48	25.10	54.78	46.26	15.05	54.42	46.35	12.11	0.78
25_48	25.11	54.78	46.26	15.05	54.13	46.36	12.13	0.83
25_48	25.12	54.78	46.26	15.05	54.14	46.56	12.13	0.855
25_48	25.13	54.78	46.26	15.05	54.00	46.51	12.34	0.805
25_48	25.14	54.78	46.26	15.05	53.60	46.29	12.67	0.78
25_48	25.15	54.78	46.26	15.05	53.36	46.18	13.06	0.775
25_48	25.16	54.78	46.26	15.05	53.50	46.55	13.25	0.765
25_48	25.17	54.78	46.26	15.05	53.38	46.47	13.83	0.725
25_48	25.18	54.78	46.26	15.05	53.38	46.47	14.64	0.74
25_48	25.19	54.78	46.26	15.05	53.55	46.56	14.77	0.73
25_48	25.20	54.78	46.26	15.05	53.68	46.64	15.32	0.72
25_48	25.21	54.78	46.26	15.05	53.61	46.28	15.61	0.64

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
3_18	3.01	13.29	18.16	16.00	13.97	17.98	15.69	0.94
3_18	3.02	13.29	18.16	16.00	13.87	17.93	15.21	0.92
3_18	3.03	13.29	18.16	16.00	13.70	18.10	14.80	0.885
3_18	3.04	13.29	18.16	16.00	13.71	18.11	14.81	0.805
3_18	3.05	13.29	18.16	16.00	13.74	18.32	14.46	1
3_18	3.06	13.29	18.16	16.00	13.42	17.99	14.43	0.895
3_18	3.07	13.29	18.16	16.00	13.38	17.96	14.24	0.785
3_18	3.08	13.29	18.16	16.00	13.43	18.17	14.46	0.78
3_18	3.09	13.29	18.16	16.00	13.39	18.14	14.28	0.875
3_18	3.10	13.29	18.16	16.00	12.93	17.90	14.25	0.675
3_18	3.11	13.29	18.16	16.00	13.17	18.09	14.24	0.74
3_18	3.12	13.29	18.16	16.00	12.82	17.91	14.23	0.74
3_18	3.13	13.29	18.16	16.00	12.87	18.19	14.47	0.74
3_18	3.14	13.29	18.16	16.00	12.86	18.18	14.46	0.72
3_18	3.15	13.29	18.16	16.00	12.79	17.91	14.63	0.565
3_18	3.16	13.29	18.16	16.00	12.70	17.86	14.63	0.62
3_18	3.17	13.29	18.16	16.00	12.81	18.15	14.67	0.62
3_18	3.18	13.29	18.16	16.00	12.72	18.09	15.33	0.585
3_18	3.19	13.29	18.16	16.00	12.81	18.14	15.77	0.595
3_18	3.20	13.29	18.16	16.00	12.69	18.08	16.03	0.695
3_18	3.21	13.29	18.16	16.00	12.57	17.78	15.98	0.685
3_48	3.01	34.07	46.79	40.74	35.69	46.48	40.04	0.925
3_48	3.02	34.07	46.79	40.74	35.67	46.74	39.46	0.9
3_48	3.03	34.07	46.79	40.74	35.35	46.54	39.04	0.945
3_48	3.04	34.07	46.79	40.74	35.15	46.78	38.57	0.78
3_48	3.05	34.07	46.79	40.74	35.03	46.43	38.19	0.925
3_48	3.06	34.07	46.79	40.74	34.75	46.56	37.69	0.985
3_48	3.07	34.07	46.79	40.74	34.74	46.94	37.63	0.925
3_48	3.08	34.07	46.79	40.74	34.36	46.73	37.18	0.895
3_48	3.09	34.07	46.79	40.74	34.13	46.88	37.25	0.875
3_48	3.10	34.07	46.79	40.74	33.71	46.41	36.75	0.955
3_48	3.11	34.07	46.79	40.74	33.61	46.61	36.78	0.97
3_48	3.12	34.07	46.79	40.74	33.44	46.52	37.21	0.77
3_48	3.13	34.07	46.79	40.74	33.16	46.44	37.14	0.845
3_48	3.14	34.07	46.79	40.74	32.94	46.66	37.58	0.865
3_48	3.15	34.07	46.79	40.74	32.88	46.62	38.23	0.785
3_48	3.16	34.07	46.79	40.74	32.81	46.58	38.67	0.71
3_48	3.17	34.07	46.79	40.74	32.75	46.81	38.71	0.655
3_48	3.18	34.07	46.79	40.74	32.63	46.74	39.54	0.67
3_48	3.19	34.07	46.79	40.74	32.67	46.75	39.82	0.73
3_48	3.20	34.07	46.79	40.74	32.49	46.66	40.12	0.75
3_48	3.21	34.07	46.79	40.74	32.71	46.75	41.31	0.84

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
5_18	5.01	28.28	17.88	13.80	29.13	18.13	13.84	0.55
5_18	5.02	28.28	17.88	13.80	29.07	18.16	13.44	0.535
5_18	5.03	28.28	17.88	13.80	28.85	18.00	12.97	0.555
5_18	5.04	28.28	17.88	13.80	28.81	18.03	12.70	0.595
5_18	5.05	28.28	17.88	13.80	28.82	18.04	12.69	0.555
5_18	5.06	28.28	17.88	13.80	28.60	18.06	12.64	0.55
5_18	5.07	28.28	17.88	13.80	28.54	18.02	12.34	0.62
5_18	5.08	28.28	17.88	13.80	28.35	18.08	12.39	0.64
5_18	5.09	28.28	17.88	13.80	28.14	18.06	12.39	0.68
5_18	5.10	28.28	17.88	13.80	28.10	18.04	12.15	0.745
5_18	5.11	28.28	17.88	13.80	28.18	18.22	12.19	0.73
5_18	5.12	28.28	17.88	13.80	28.22	18.22	12.42	0.625
5_18	5.13	28.28	17.88	13.80	27.90	18.06	12.16	0.69
5_18	5.14	28.28	17.88	13.80	27.71	18.04	12.40	0.705
5_18	5.15	28.28	17.88	13.80	28.01	18.12	12.73	0.64
5_18	5.16	28.28	17.88	13.80	27.80	18.09	12.73	0.69
5_18	5.17	28.28	17.88	13.80	27.85	18.11	13.01	0.635
5_18	5.18	28.28	17.88	13.80	27.56	17.95	13.24	0.58
5_18	5.19	28.28	17.88	13.80	27.59	17.88	13.46	0.55
5_18	5.20	28.28	17.88	13.80	27.69	18.01	13.85	0.645
5_18	5.21	28.28	17.88	13.80	27.68	18.01	13.85	0.54
8_18	8.01	13.96	18.00	4.73	14.37	17.90	4.59	0.62
8_18	8.02	13.96	18.00	4.73	14.37	17.97	4.48	0.64
8_18	8.03	13.96	18.00	4.73	14.04	17.79	4.08	0.6
8_18	8.04	13.96	18.00	4.73	14.24	17.89	3.88	0.645
8_18	8.05	13.96	18.00	4.73	14.31	18.13	3.90	0.655
8_18	8.06	13.96	18.00	4.73	14.05	18.00	3.81	0.575
8_18	8.07	13.96	18.00	4.73	13.96	17.75	3.67	0.66
8_18	8.08	13.96	18.00	4.73	13.83	17.88	3.68	0.635
8_18	8.09	13.96	18.00	4.73	13.80	17.86	3.55	0.55
8_18	8.10	13.96	18.00	4.73	13.81	18.16	3.59	0.65
8_18	8.11	13.96	18.00	4.73	13.80	18.15	3.59	0.61
8_18	8.12	13.96	18.00	4.73	13.83	18.17	3.72	0.595
8_18	8.13	13.96	18.00	4.73	13.90	18.38	3.75	0.61
8_18	8.14	13.96	18.00	4.73	13.71	18.27	3.85	0.61
8_18	8.15	13.96	18.00	4.73	13.51	18.17	3.94	0.71
8_18	8.16	13.96	18.00	4.73	13.36	18.18	4.14	0.745
8_18	8.17	13.96	18.00	4.73	13.23	18.11	4.13	0.695
8_18	8.18	13.96	18.00	4.73	13.42	18.22	4.36	0.72
8_18	8.19	13.96	18.00	4.73	13.35	18.20	4.65	0.835
8_18	8.20	13.96	18.00	4.73	13.35	18.20	4.65	0.775
8_18	8.21	13.96	18.00	4.73	13.41	18.23	4.96	0.89

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
8_48	8.01	35.77	46.72	11.75	37.20	46.45	11.31	0.755
8_48	8.02	35.77	46.72	11.75	37.15	46.46	10.94	0.73
8_48	8.03	35.77	46.72	11.75	36.81	46.49	10.17	0.705
8_48	8.04	35.77	46.72	11.75	36.71	46.43	9.82	0.74
8_48	8.05	35.77	46.72	11.75	36.58	46.64	9.69	0.665
8_48	8.06	35.77	46.72	11.75	36.18	46.38	9.25	0.78
8_48	8.07	35.77	46.72	11.75	36.16	46.39	9.09	0.675
8_48	8.08	35.77	46.72	11.75	36.06	46.80	8.77	0.76
8_48	8.09	35.77	46.72	11.75	35.73	46.70	8.74	0.67
8_48	8.10	35.77	46.72	11.75	35.17	46.44	8.52	0.61
8_48	8.11	35.77	46.72	11.75	35.00	46.26	8.73	0.625
8_48	8.12	35.77	46.72	11.75	34.72	46.21	8.72	0.7
8_48	8.13	35.77	46.72	11.75	34.76	46.23	8.94	0.63
8_48	8.14	35.77	46.72	11.75	34.75	46.49	9.14	0.56
8_48	8.15	35.77	46.72	11.75	34.53	46.69	9.55	0.685
8_48	8.16	35.77	46.72	11.75	34.34	46.59	9.54	0.705
8_48	8.17	35.77	46.72	11.75	34.33	46.49	9.84	0.62
8_48	8.18	35.77	46.72	11.75	34.36	46.59	10.69	0.63
8_48	8.19	35.77	46.72	11.75	34.12	46.37	11.32	0.62
8_48	8.20	35.77	46.72	11.75	34.17	46.42	11.54	0.645
8_48	8.21	35.77	46.72	11.75	34.31	46.49	12.18	0.71
B_48	B.01	30.92	46.13	76.52	32.75	46.30	76.19	0.695
B_48	B.02	30.92	46.13	76.52	32.59	46.23	75.40	0.67
B_48	B.03	30.92	46.13	76.52	32.63	46.40	75.37	0.67
B_48	B.04	30.92	46.13	76.52	32.11	46.10	74.15	0.715
B_48	B.05	30.92	46.13	76.52	31.92	46.05	73.77	0.63
B_48	B.06	30.92	46.13	76.52	31.91	46.52	72.76	0.595
B_48	B.07	30.92	46.13	76.52	31.68	46.52	72.69	0.705
B_48	B.08	30.92	46.13	76.52	31.39	46.38	72.11	0.635
B_48	B.09	30.92	46.13	76.52	31.14	46.27	72.13	0.57
B_48	B.10	30.92	46.13	76.52	30.66	46.32	71.80	0.595
B_48	B.11	30.92	46.13	76.52	30.43	46.21	71.81	0.55
B_48	B.12	30.92	46.13	76.52	30.31	46.50	72.17	0.665
B_48	B.13	30.92	46.13	76.52	30.03	46.28	72.65	0.625
B_48	B.14	30.92	46.13	76.52	29.64	46.07	72.63	0.59
B_48	B.15	30.92	46.13	76.52	29.52	46.35	73.35	0.595
B_48	B.16	30.92	46.13	76.52	29.11	45.80	73.78	0.61
B_48	B.17	30.92	46.13	76.52	29.44	46.30	74.92	0.565
B_48	B.18	30.92	46.13	76.52	29.25	46.25	74.93	0.57
B_48	B.19	30.92	46.13	76.52	29.14	46.19	76.04	0.62
B_48	B.20	30.92	46.13	76.52	29.31	46.65	76.26	0.66
B_48	B.21	30.92	46.13	76.52	29.30	46.23	77.02	0.65

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
G_48	G.01	19.32	46.01	11.76	20.83	46.19	11.61	0.525
G_48	G.02	19.32	46.01	11.76	20.72	46.15	10.95	0.51
G_48	G.03	19.32	46.01	11.76	20.65	46.35	10.16	0.605
G_48	G.04	19.32	46.01	11.76	20.65	46.39	10.01	0.57
G_48	G.05	19.32	46.01	11.76	20.55	46.49	9.75	0.625
G_48	G.06	19.32	46.01	11.76	20.25	46.19	9.56	0.575
G_48	G.07	19.32	46.01	11.76	20.23	46.34	9.20	0.56
G_48	G.08	19.32	46.01	11.76	20.01	46.23	8.98	0.585
G_48	G.09	19.32	46.01	11.76	19.47	45.95	8.95	0.515
G_48	G.10	19.32	46.01	11.76	19.52	46.19	8.78	0.55
G_48	G.11	19.32	46.01	11.76	19.36	46.59	8.85	0.54
G_48	G.12	19.32	46.01	11.76	19.17	46.49	8.84	0.605
G_48	G.13	19.32	46.01	11.76	18.89	46.50	9.22	0.64
G_48	G.14	19.32	46.01	11.76	18.79	46.44	9.43	0.56
G_48	G.15	19.32	46.01	11.76	18.61	46.20	9.57	0.475
G_48	G.16	19.32	46.01	11.76	18.17	45.98	9.71	0.535
G_48	G.17	19.32	46.01	11.76	18.16	46.11	10.12	0.5
G_48	G.18	19.32	46.01	11.76	18.24	46.18	10.92	0.525
G_48	G.19	19.32	46.01	11.76	18.17	46.11	11.55	0.535
G_48	G.20	19.32	46.01	11.76	18.25	46.16	11.94	0.505
G_48	G.21	19.32	46.01	11.76	18.07	46.07	12.13	0.485
W1_48	W1.01	51.89	46.54	15.17	53.29	46.43	14.78	0.675
W1_48	W1.02	51.89	46.54	15.17	52.89	46.36	14.22	0.625
W1_48	W1.03	51.89	46.54	15.17	53.26	46.77	13.71	0.72
W1_48	W1.04	51.89	46.54	15.17	52.83	46.56	13.22	0.75
W1_48	W1.05	51.89	46.54	15.17	52.50	46.38	13.03	0.755
W1_48	W1.06	51.89	46.54	15.17	51.95	46.12	12.42	0.76
W1_48	W1.07	51.89	46.54	15.17	51.72	46.27	12.22	0.785
W1_48	W1.08	51.89	46.54	15.17	51.32	46.08	11.92	0.845
W1_48	W1.09	51.89	46.54	15.17	51.39	46.32	11.93	0.77
W1_48	W1.10	51.89	46.54	15.17	51.43	46.50	11.72	0.805
W1_48	W1.11	51.89	46.54	15.17	50.89	46.22	11.69	0.92
W1_48	W1.12	51.89	46.54	15.17	50.90	46.22	11.93	0.895
W1_48	W1.13	51.89	46.54	15.17	50.76	46.37	12.23	0.82
W1_48	W1.14	51.89	46.54	15.17	50.71	46.59	12.85	0.785
W1_48	W1.15	51.89	46.54	15.17	50.60	46.31	12.81	0.82
W1_48	W1.16	51.89	46.54	15.17	50.19	46.32	13.23	0.76
W1_48	W1.17	51.89	46.54	15.17	50.29	46.37	13.71	0.705
W1_48	W1.18	51.89	46.54	15.17	50.39	46.43	14.25	0.695
W1_48	W1.19	51.89	46.54	15.17	50.23	46.33	14.79	0.74
W1_48	W1.20	51.89	46.54	15.17	50.31	46.37	15.15	0.685
W1_48	W1.21	51.89	46.54	15.17	50.44	46.45	15.77	0.7

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
W2_48	W2.01	49.68	46.13	18.86	51.62	46.57	18.72	0.705
W2_48	W2.02	49.68	46.13	18.86	51.17	46.35	18.15	0.73
W2_48	W2.03	49.68	46.13	18.86	51.11	46.32	17.66	0.755
W2_48	W2.04	49.68	46.13	18.86	50.82	46.43	16.85	0.835
W2_48	W2.05	49.68	46.13	18.86	50.76	46.38	16.63	0.77
W2_48	W2.06	49.68	46.13	18.86	50.45	46.22	16.24	0.81
W2_48	W2.07	49.68	46.13	18.86	50.37	46.43	16.05	0.795
W2_48	W2.08	49.68	46.13	18.86	49.89	46.25	15.77	0.85
W2_48	W2.09	49.68	46.13	18.86	50.04	46.77	15.82	0.77
W2_48	W2.10	49.68	46.13	18.86	49.64	46.14	15.76	0.87
W2_48	W2.11	49.68	46.13	18.86	49.26	46.36	15.79	0.765
W2_48	W2.12	49.68	46.13	18.86	49.10	46.43	16.06	0.835
W2_48	W2.13	49.68	46.13	18.86	48.88	46.30	16.26	0.8
W2_48	W2.14	49.68	46.13	18.86	49.04	46.58	16.67	0.81
W2_48	W2.15	49.68	46.13	18.86	48.64	46.39	16.66	0.885
W2_48	W2.16	49.68	46.13	18.86	48.72	46.46	16.88	0.79
W2_48	W2.17	49.68	46.13	18.86	48.48	46.34	17.27	0.775
W2_48	W2.18	49.68	46.13	18.86	48.66	46.43	18.18	0.72
W2_48	W2.19	49.68	46.13	18.86	48.39	46.23	18.67	0.805
W2_48	W2.20	49.68	46.13	18.86	48.44	46.28	18.86	0.725
W2_48	W2.21	49.68	46.13	18.86	48.55	46.34	19.46	0.76
W3_48	W3.01	47.94	46.35	23.68	49.73	46.13	23.86	0.825
W3_48	W3.02	47.94	46.35	23.68	49.62	46.08	23.32	0.83
W3_48	W3.03	47.94	46.35	23.68	49.64	46.54	22.72	0.835
W3_48	W3.04	47.94	46.35	23.68	49.23	46.33	22.16	0.72
W3_48	W3.05	47.94	46.35	23.68	49.16	46.44	21.46	0.815
W3_48	W3.06	47.94	46.35	23.68	48.87	46.49	21.24	0.795
W3_48	W3.07	47.94	46.35	23.68	48.83	46.46	21.10	0.77
W3_48	W3.08	47.94	46.35	23.68	47.91	46.36	20.78	0.755
W3_48	W3.09	47.94	46.35	23.68	47.73	45.93	20.39	0.93
W3_48	W3.10	47.94	46.35	23.68	47.64	46.24	20.77	0.78
W3_48	W3.11	47.94	46.35	23.68	47.75	46.50	20.83	0.865
W3_48	W3.12	47.94	46.35	23.68	47.50	46.35	20.80	0.765
W3_48	W3.13	47.94	46.35	23.68	47.17	46.19	20.80	0.83
W3_48	W3.14	47.94	46.35	23.68	47.25	46.42	21.26	0.75
W3_48	W3.15	47.94	46.35	23.68	47.18	46.81	21.54	0.775
W3_48	W3.16	47.94	46.35	23.68	46.97	46.73	21.91	0.77
W3_48	W3.17	47.94	46.35	23.68	46.63	46.05	22.71	0.705
W3_48	W3.18	47.94	46.35	23.68	46.37	45.92	22.70	0.74
W3_48	W3.19	47.94	46.35	23.68	46.57	46.04	23.68	0.715
W3_48	W3.20	47.94	46.35	23.68	46.34	45.94	23.89	0.725
W3_48	W3.21	47.94	46.35	23.68	46.65	46.57	24.33	0.79

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
W4_18	W4.01	18.20	17.95	11.59	18.71	18.10	11.01	0.73
W4_18	W4.02	18.20	17.95	11.59	18.73	18.11	11.01	0.7
W4_18	W4.03	18.20	17.95	11.59	18.52	17.99	10.86	0.685
W4_18	W4.04	18.20	17.95	11.59	18.56	18.18	10.69	0.615
W4_18	W4.05	18.20	17.95	11.59	18.39	18.10	10.69	0.7
W4_18	W4.06	18.20	17.95	11.59	18.35	18.05	10.48	0.645
W4_18	W4.07	18.20	17.95	11.59	18.29	17.86	10.46	0.635
W4_18	W4.08	18.20	17.95	11.59	18.29	18.02	10.15	0.735
W4_18	W4.09	18.20	17.95	11.59	18.06	17.87	10.11	0.705
W4_18	W4.10	18.20	17.95	11.59	18.12	18.11	10.11	0.78
W4_18	W4.11	18.20	17.95	11.59	18.02	18.10	10.14	0.715
W4_18	W4.12	18.20	17.95	11.59	18.12	18.29	10.18	0.825
W4_18	W4.13	18.20	17.95	11.59	18.02	18.24	10.52	0.695
W4_18	W4.14	18.20	17.95	11.59	18.06	18.26	10.70	0.68
W4_18	W4.15	18.20	17.95	11.59	17.77	18.10	10.69	0.745
W4_18	W4.16	18.20	17.95	11.59	17.76	18.10	10.69	0.845
W4_18	W4.17	18.20	17.95	11.59	17.49	17.96	10.68	0.925
W4_18	W4.18	18.20	17.95	11.59	17.41	17.91	11.01	0.915
W4_18	W4.19	18.20	17.95	11.59	17.46	17.94	11.34	0.88
W4_18	W4.20	18.20	17.95	11.59	17.52	17.96	11.61	0.905
W4_18	W4.21	18.20	17.95	11.59	17.69	18.05	11.87	0.79
W4_48	W4.01	46.63	46.04	29.12	48.21	46.44	28.88	0.755
W4_48	W4.02	46.63	46.04	29.12	48.01	46.34	27.82	0.835
W4_48	W4.03	46.63	46.04	29.12	48.07	46.60	27.54	0.72
W4_48	W4.04	46.63	46.04	29.12	47.75	46.44	27.38	0.77
W4_48	W4.05	46.63	46.04	29.12	47.58	46.54	26.50	0.885
W4_48	W4.06	46.63	46.04	29.12	47.34	46.41	26.51	0.795
W4_48	W4.07	46.63	46.04	29.12	47.24	46.84	26.44	0.91
W4_48	W4.08	46.63	46.04	29.12	47.00	46.24	26.15	0.76
W4_48	W4.09	46.63	46.04	29.12	46.80	46.61	25.48	0.89
W4_48	W4.10	46.63	46.04	29.12	46.15	46.25	25.42	0.905
W4_48	W4.11	46.63	46.04	29.12	45.93	46.36	25.49	0.875
W4_48	W4.12	46.63	46.04	29.12	45.81	46.29	26.19	0.895
W4_48	W4.13	46.63	46.04	29.12	45.48	46.25	26.20	1
W4_48	W4.14	46.63	46.04	29.12	45.72	46.80	26.49	0.975
W4_48	W4.15	46.63	46.04	29.12	45.23	46.11	26.38	0.915
W4_48	W4.16	46.63	46.04	29.12	45.18	46.07	27.39	0.855
W4_48	W4.17	46.63	46.04	29.12	45.01	46.44	27.46	1.08
W4_48	W4.18	46.63	46.04	29.12	45.11	46.48	27.95	1.04
W4_48	W4.19	46.63	46.04	29.12	45.29	46.57	28.78	0.96
W4_48	W4.20	46.63	46.04	29.12	45.06	46.45	28.97	1
W4_48	W4.21	46.63	46.04	29.12	44.99	45.96	29.56	0.835

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
W5_48	W5.01	45.04	46.34	38.93	46.46	46.31	38.49	0.835
W5_48	W5.02	45.04	46.34	38.93	46.33	46.27	37.62	0.93
W5_48	W5.03	45.04	46.34	38.93	46.50	46.54	38.11	0.775
W5_48	W5.04	45.04	46.34	38.93	46.34	46.55	37.17	0.775
W5_48	W5.05	45.04	46.34	38.93	46.08	46.45	36.62	0.87
W5_48	W5.06	45.04	46.34	38.93	46.20	46.95	36.28	0.795
W5_48	W5.07	45.04	46.34	38.93	45.68	46.26	35.59	0.945
W5_48	W5.08	45.04	46.34	38.93	45.50	46.60	35.63	0.83
W5_48	W5.09	45.04	46.34	38.93	45.12	46.54	35.44	0.91
W5_48	W5.10	45.04	46.34	38.93	44.90	46.70	35.47	0.895
W5_48	W5.11	45.04	46.34	38.93	44.53	46.51	35.45	0.89
W5_48	W5.12	45.04	46.34	38.93	44.36	46.81	35.50	0.825
W5_48	W5.13	45.04	46.34	38.93	44.19	46.31	35.43	0.955
W5_48	W5.14	45.04	46.34	38.93	43.96	46.56	36.17	0.875
W5_48	W5.15	45.04	46.34	38.93	43.75	46.47	36.66	0.965
W5_48	W5.16	45.04	46.34	38.93	43.80	46.46	37.05	0.955
W5_48	W5.17	45.04	46.34	38.93	43.36	46.23	37.20	1.005
W5_48	W5.18	45.04	46.34	38.93	43.48	46.31	37.68	0.975
W5_48	W5.19	45.04	46.34	38.93	43.36	46.28	38.61	0.89
W5_48	W5.20	45.04	46.34	38.93	43.16	46.19	38.99	1.03
W5_48	W5.21	45.04	46.34	38.93	43.53	46.35	39.58	1.035
W6_18	W6.01	17.11	18.08	19.94	17.64	17.82	19.66	1.36
W6_18	W6.02	17.11	18.08	19.94	17.56	17.94	19.37	1.375
W6_18	W6.03	17.11	18.08	19.94	17.48	17.89	18.97	1.265
W6_18	W6.04	17.11	18.08	19.94	17.49	17.89	18.99	1.225
W6_18	W6.05	17.11	18.08	19.94	17.66	18.31	18.73	1.475
W6_18	W6.06	17.11	18.08	19.94	17.45	17.88	18.68	1.37
W6_18	W6.07	17.11	18.08	19.94	17.28	18.11	18.37	1.185
W6_18	W6.08	17.11	18.08	19.94	17.29	18.11	18.37	1.195
W6_18	W6.09	17.11	18.08	19.94	17.29	18.20	18.35	1.11
W6_18	W6.10	17.11	18.08	19.94	17.06	18.08	18.33	1.095
W6_18	W6.11	17.11	18.08	19.94	16.99	18.03	18.00	1.13
W6_18	W6.12	17.11	18.08	19.94	16.78	18.15	18.33	0.855
W6_18	W6.13	17.11	18.08	19.94	16.78	18.15	18.33	0.83
W6_18	W6.14	17.11	18.08	19.94	16.85	18.17	18.69	0.825
W6_18	W6.15	17.11	18.08	19.94	16.68	18.09	18.80	0.735
W6_18	W6.16	17.11	18.08	19.94	17.01	18.51	18.89	0.795
W6_18	W6.17	17.11	18.08	19.94	16.80	18.40	18.88	0.91
W6_18	W6.18	17.11	18.08	19.94	16.69	18.33	19.47	1.005
W6_18	W6.19	17.11	18.08	19.94	16.74	18.35	19.79	1.145
W6_18	W6.20	17.11	18.08	19.94	16.79	18.38	20.01	1.225
W6_18	W6.21	17.11	18.08	19.94	16.85	18.41	20.35	1.15

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
W6_48	W6.01	44.28	46.25	51.01	45.82	46.17	50.46	1.005
W6_48	W6.02	44.28	46.25	51.01	45.68	46.11	49.70	0.855
W6_48	W6.03	44.28	46.25	51.01	45.80	46.64	49.02	0.99
W6_48	W6.04	44.28	46.25	51.01	45.31	46.39	49.00	1.005
W6_48	W6.05	44.28	46.25	51.01	45.11	46.40	48.10	0.91
W6_48	W6.06	44.28	46.25	51.01	44.81	46.52	47.53	0.975
W6_48	W6.07	44.28	46.25	51.01	44.47	46.35	47.21	0.975
W6_48	W6.08	44.28	46.25	51.01	44.57	46.82	46.89	1.055
W6_48	W6.09	44.28	46.25	51.01	44.34	46.72	46.90	0.985
W6_48	W6.10	44.28	46.25	51.01	43.84	46.04	46.60	0.905
W6_48	W6.11	44.28	46.25	51.01	43.58	46.28	46.61	0.995
W6_48	W6.12	44.28	46.25	51.01	43.59	46.58	46.91	0.875
W6_48	W6.13	44.28	46.25	51.01	43.20	46.51	46.96	0.955
W6_48	W6.14	44.28	46.25	51.01	43.22	46.67	46.94	0.9
W6_48	W6.15	44.28	46.25	51.01	43.05	46.53	47.55	0.99
W6_48	W6.16	44.28	46.25	51.01	42.87	46.46	48.18	0.895
W6_48	W6.17	44.28	46.25	51.01	42.42	46.21	48.58	1.035
W6_48	W6.18	44.28	46.25	51.01	42.78	46.62	49.87	1.01
W6_48	W6.19	44.28	46.25	51.01	42.85	46.66	50.28	0.975
W6_48	W6.20	44.28	46.25	51.01	42.68	46.57	50.67	1.015
W6_48	W6.21	44.28	46.25	51.01	42.71	46.36	51.34	1.105

Appendix D

Experimental Data against the Black Background

	Xo	Yo	Zo	
White	95.11	100.00	97.28	
Black	0.319	0.354	0.331	(fullscreen)
Gray	22.78	23.37	27.56	

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
1_18	1.01	10.32	18.01	11.96	10.73	17.67	11.56	0.92
1_18	1.02	10.32	18.01	11.96	10.69	17.66	11.31	0.9
1_18	1.03	10.32	18.01	11.96	10.72	17.95	11.05	1.045
1_18	1.04	10.32	18.01	11.96	10.65	17.91	10.73	1.09
1_18	1.05	10.32	18.01	11.96	10.62	17.88	10.58	1.1
1_18	1.06	10.32	18.01	11.96	10.46	17.86	10.41	1.07
1_18	1.07	10.32	18.01	11.96	10.38	17.82	10.41	1.015
1_18	1.08	10.32	18.01	11.96	10.44	18.08	10.28	1.175
1_18	1.09	10.32	18.01	11.96	10.47	18.10	10.45	1.205
1_18	1.10	10.32	18.01	11.96	10.34	18.04	10.45	1.05
1_18	1.11	10.32	18.01	11.96	10.21	18.05	10.46	1.065
1_18	1.12	10.32	18.01	11.96	10.05	17.96	10.45	1.1
1_18	1.13	10.32	18.01	11.96	9.99	18.07	10.46	0.995
1_18	1.14	10.32	18.01	11.96	9.96	17.91	10.64	0.98
1_18	1.15	10.32	18.01	11.96	9.64	17.73	10.62	0.92
1_18	1.16	10.32	18.01	11.96	9.64	17.73	10.62	0.985
1_18	1.17	10.32	18.01	11.96	9.72	17.90	10.77	0.875
1_18	1.18	10.32	18.01	11.96	9.78	17.92	11.11	0.905
1_18	1.19	10.32	18.01	11.96	9.75	17.92	11.40	0.87
1_18	1.20	10.32	18.01	11.96	9.63	17.86	11.64	0.895
1_18	1.21	10.32	18.01	11.96	9.91	18.00	12.20	0.76

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
1_48	1.01	26.96	46.59	30.09	28.00	46.67	29.61	0.82
1_48	1.02	26.96	46.59	30.09	27.98	46.72	29.33	0.89
1_48	1.03	26.96	46.59	30.09	27.76	46.57	28.28	0.905
1_48	1.04	26.96	46.59	30.09	27.78	46.90	28.33	0.905
1_48	1.05	26.96	46.59	30.09	27.48	46.77	27.83	1.005
1_48	1.06	26.96	46.59	30.09	27.42	46.76	27.48	0.945
1_48	1.07	26.96	46.59	30.09	27.16	46.74	27.01	1.04
1_48	1.08	26.96	46.59	30.09	27.13	46.73	26.81	1.14
1_48	1.09	26.96	46.59	30.09	26.91	46.61	27.00	1.05
1_48	1.10	26.96	46.59	30.09	26.52	46.43	26.79	1.065
1_48	1.11	26.96	46.59	30.09	26.33	46.31	26.76	1.05
1_48	1.12	26.96	46.59	30.09	26.17	46.53	26.78	1.005
1_48	1.13	26.96	46.59	30.09	26.05	46.86	27.01	1.11
1_48	1.14	26.96	46.59	30.09	25.93	46.82	27.51	1.02
1_48	1.15	26.96	46.59	30.09	25.72	46.86	27.85	0.99
1_48	1.16	26.96	46.59	30.09	25.32	46.64	28.31	0.925
1_48	1.17	26.96	46.59	30.09	25.21	46.57	28.31	0.925
1_48	1.18	26.96	46.59	30.09	25.21	46.44	29.21	0.835
1_48	1.19	26.96	46.59	30.09	25.31	46.61	29.60	0.95
1_48	1.20	26.96	46.59	30.09	25.12	46.37	30.07	1.02
1_48	1.21	26.96	46.59	30.09	25.07	46.34	30.82	1
10_18	10.01	29.59	17.96	28.33	30.40	18.08	27.80	0.805
10_18	10.02	29.59	17.96	28.33	30.14	17.95	27.47	0.905
10_18	10.03	29.59	17.96	28.33	30.24	18.13	27.52	0.98
10_18	10.04	29.59	17.96	28.33	30.25	18.13	27.40	0.84
10_18	10.05	29.59	17.96	28.33	29.82	17.99	26.95	0.845
10_18	10.06	29.59	17.96	28.33	29.42	17.79	26.61	0.77
10_18	10.07	29.59	17.96	28.33	29.30	17.72	26.12	0.865
10_18	10.08	29.59	17.96	28.33	29.28	17.99	26.18	0.895
10_18	10.09	29.59	17.96	28.33	29.46	18.08	26.19	0.87
10_18	10.10	29.59	17.96	28.33	29.39	18.09	26.65	0.815
10_18	10.11	29.59	17.96	28.33	29.38	18.03	26.66	0.87
10_18	10.12	29.59	17.96	28.33	29.09	17.94	26.16	0.99
10_18	10.13	29.59	17.96	28.33	29.21	18.14	26.58	0.89
10_18	10.14	29.59	17.96	28.33	28.94	18.01	26.60	0.885
10_18	10.15	29.59	17.96	28.33	28.75	17.76	26.65	0.815
10_18	10.16	29.59	17.96	28.33	28.83	17.95	26.94	0.89
10_18	10.17	29.59	17.96	28.33	28.92	17.99	27.39	0.855
10_18	10.18	29.59	17.96	28.33	28.52	17.78	27.36	0.855
10_18	10.19	29.59	17.96	28.33	28.52	17.77	27.48	0.85
10_18	10.20	29.59	17.96	28.33	28.80	18.08	28.34	0.9
10_18	10.21	29.59	17.96	28.33	28.73	17.87	28.58	0.92

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
12_18	12.01	21.69	17.95	23.58	22.47	18.02	23.57	0.94
12_18	12.01	21.69	17.95	23.58	22.47	18.02	23.57	0.91
12_18	12.02	21.69	17.95	23.58	22.42	18.01	23.34	0.985
12_18	12.02	21.69	17.95	23.58	22.42	18.01	23.34	0.95
12_18	12.03	21.69	17.95	23.58	22.15	17.82	22.19	1.11
12_18	12.03	21.69	17.95	23.58	22.15	17.82	22.19	0.97
12_18	12.04	21.69	17.95	23.58	22.23	18.03	22.22	1
12_18	12.04	21.69	17.95	23.58	22.23	18.03	22.22	0.98
12_18	12.05	21.69	17.95	23.58	21.98	17.91	22.21	1.015
12_18	12.05	21.69	17.95	23.58	21.98	17.91	22.21	0.96
12_18	12.06	21.69	17.95	23.58	21.90	17.73	22.01	1.01
12_18	12.06	21.69	17.95	23.58	21.90	17.73	22.01	0.99
12_18	12.07	21.69	17.95	23.58	21.92	17.89	21.85	0.99
12_18	12.07	21.69	17.95	23.58	21.92	17.89	21.85	0.94
12_18	12.08	21.69	17.95	23.58	21.68	17.94	21.46	0.985
12_18	12.08	21.69	17.95	23.58	21.68	17.94	21.46	0.965
12_18	12.09	21.69	17.95	23.58	21.74	17.97	21.82	0.94
12_18	12.09	21.69	17.95	23.58	21.74	17.97	21.82	0.87
12_18	12.10	21.69	17.95	23.58	21.53	18.05	21.51	1.015
12_18	12.10	21.69	17.95	23.58	21.53	18.05	21.51	0.98
12_18	12.11	21.69	17.95	23.58	21.62	18.27	21.54	1.125
12_18	12.11	21.69	17.95	23.58	21.62	18.27	21.54	0.975
12_18	12.12	21.69	17.95	23.58	21.40	17.97	21.86	0.93
12_18	12.12	21.69	17.95	23.58	21.40	17.97	21.86	0.88
12_18	12.13	21.69	17.95	23.58	21.48	18.19	21.90	0.975
12_18	12.13	21.69	17.95	23.58	21.48	18.19	21.90	0.995
12_18	12.14	21.69	17.95	23.58	21.42	18.16	22.09	0.88
12_18	12.14	21.69	17.95	23.58	21.42	18.16	22.09	0.88
12_18	12.15	21.69	17.95	23.58	21.43	18.16	22.27	0.81
12_18	12.15	21.69	17.95	23.58	21.43	18.16	22.27	0.91
12_18	12.16	21.69	17.95	23.58	21.24	18.06	22.27	0.855
12_18	12.16	21.69	17.95	23.58	21.24	18.06	22.27	0.875
12_18	12.17	21.69	17.95	23.58	20.96	17.97	22.22	1.08
12_18	12.17	21.69	17.95	23.58	20.96	17.97	22.22	0.965
12_18	12.18	21.69	17.95	23.58	21.05	18.01	22.74	0.955
12_18	12.18	21.69	17.95	23.58	21.05	18.01	22.74	0.95
12_18	12.19	21.69	17.95	23.58	21.19	18.10	23.37	0.875
12_18	12.19	21.69	17.95	23.58	21.19	18.10	23.37	1.015
12_18	12.20	21.69	17.95	23.58	21.22	18.05	23.37	0.845
12_18	12.20	21.69	17.95	23.58	21.22	18.05	23.37	0.87
12_18	12.21	21.69	17.95	23.58	21.30	18.09	23.85	0.92
12_18	12.21	21.69	17.95	23.58	21.30	18.09	23.85	0.81

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
12_48	12.01	57.07	46.87	61.22	58.31	46.60	60.57	0.945
12_48	12.01	57.07	46.87	61.22	58.31	46.60	60.57	0.975
12_48	12.02	57.07	46.87	61.22	58.32	46.83	59.92	1.015
12_48	12.02	57.07	46.87	61.22	58.32	46.83	59.92	0.995
12_48	12.03	57.07	46.87	61.22	57.81	46.32	59.46	1.06
12_48	12.03	57.07	46.87	61.22	57.81	46.32	59.46	0.865
12_48	12.04	57.07	46.87	61.22	57.48	46.40	58.63	0.92
12_48	12.04	57.07	46.87	61.22	57.48	46.40	58.63	1.05
12_48	12.05	57.07	46.87	61.22	57.41	46.36	58.10	0.98
12_48	12.05	57.07	46.87	61.22	57.41	46.36	58.10	0.88
12_48	12.06	57.07	46.87	61.22	57.31	46.86	57.69	1.015
12_48	12.06	57.07	46.87	61.22	57.31	46.86	57.69	0.915
12_48	12.07	57.07	46.87	61.22	57.04	46.72	57.38	0.865
12_48	12.07	57.07	46.87	61.22	57.04	46.72	57.38	0.93
12_48	12.08	57.07	46.87	61.22	56.66	46.69	56.88	0.96
12_48	12.08	57.07	46.87	61.22	56.66	46.69	56.88	0.915
12_48	12.09	57.07	46.87	61.22	56.41	46.75	56.90	0.875
12_48	12.09	57.07	46.87	61.22	56.41	46.75	56.90	0.935
12_48	12.10	57.07	46.87	61.22	55.92	46.48	56.84	0.925
12_48	12.10	57.07	46.87	61.22	55.92	46.48	56.84	0.955
12_48	12.11	57.07	46.87	61.22	55.58	46.41	56.79	0.85
12_48	12.11	57.07	46.87	61.22	55.58	46.41	56.79	0.885
12_48	12.12	57.07	46.87	61.22	55.62	46.71	56.32	1.01
12_48	12.12	57.07	46.87	61.22	55.62	46.71	56.32	1.025
12_48	12.13	57.07	46.87	61.22	55.58	46.80	56.92	0.93
12_48	12.13	57.07	46.87	61.22	55.58	46.80	56.92	0.94
12_48	12.14	57.07	46.87	61.22	55.35	46.99	57.39	0.92
12_48	12.14	57.07	46.87	61.22	55.35	46.99	57.39	0.93
12_48	12.15	57.07	46.87	61.22	55.06	46.84	57.72	0.96
12_48	12.15	57.07	46.87	61.22	55.06	46.84	57.72	1.005
12_48	12.16	57.07	46.87	61.22	55.07	46.51	58.27	0.87
12_48	12.16	57.07	46.87	61.22	55.07	46.51	58.27	0.905
12_48	12.17	57.07	46.87	61.22	54.98	46.74	59.17	0.935
12_48	12.17	57.07	46.87	61.22	54.98	46.74	59.17	0.96
12_48	12.18	57.07	46.87	61.22	55.12	46.81	60.03	0.885
12_48	12.18	57.07	46.87	61.22	55.12	46.81	60.03	0.935
12_48	12.19	57.07	46.87	61.22	54.73	46.59	60.22	0.985
12_48	12.19	57.07	46.87	61.22	54.73	46.59	60.22	0.995
12_48	12.20	57.07	46.87	61.22	54.86	46.38	60.77	0.955
12_48	12.20	57.07	46.87	61.22	54.86	46.38	60.77	1.01
12_48	12.21	57.07	46.87	61.22	55.10	46.49	61.89	0.975
12_48	12.21	57.07	46.87	61.22	55.10	46.49	61.89	0.92

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
13_18	13.01	16.60	18.15	37.98	17.12	18.02	36.93	0.88
13_18	13.02	16.60	18.15	37.98	17.11	18.04	36.67	0.985
13_18	13.03	16.60	18.15	37.98	17.04	18.01	36.31	0.96
13_18	13.04	16.60	18.15	37.98	16.96	17.96	35.96	1.04
13_18	13.05	16.60	18.15	37.98	16.81	18.12	36.08	0.96
13_18	13.06	16.60	18.15	37.98	16.81	18.13	36.08	1.01
13_18	13.07	16.60	18.15	37.98	16.66	18.05	35.66	0.795
13_18	13.08	16.60	18.15	37.98	16.44	18.08	35.13	0.83
13_18	13.09	16.60	18.15	37.98	16.55	18.13	35.72	0.75
13_18	13.10	16.60	18.15	37.98	16.29	18.00	35.72	0.745
13_18	13.11	16.60	18.15	37.98	16.18	18.07	35.63	0.8
13_18	13.12	16.60	18.15	37.98	16.16	18.24	35.73	0.755
13_18	13.13	16.60	18.15	37.98	16.00	17.99	35.16	0.84
13_18	13.14	16.60	18.15	37.98	15.99	17.98	35.72	0.77
13_18	13.15	16.60	18.15	37.98	15.85	17.90	36.07	0.79
13_18	13.16	16.60	18.15	37.98	15.72	17.99	36.07	0.87
13_18	13.17	16.60	18.15	37.98	15.58	17.92	36.06	0.895
13_18	13.18	16.60	18.15	37.98	15.58	17.92	36.79	0.965
13_18	13.19	16.60	18.15	37.98	15.72	18.00	36.82	0.925
13_18	13.20	16.60	18.15	37.98	15.63	17.95	37.10	0.925
13_18	13.21	16.60	18.15	37.98	15.64	17.95	37.10	0.975
19_18	19.01	27.27	18.02	6.32	27.91	18.06	6.17	0.825
19_18	19.02	27.27	18.02	6.32	27.60	17.90	5.89	0.88
19_18	19.03	27.27	18.02	6.32	27.66	18.07	5.77	0.835
19_18	19.04	27.27	18.02	6.32	27.63	18.05	5.61	0.985
19_18	19.05	27.27	18.02	6.32	27.36	17.91	5.40	1.005
19_18	19.06	27.27	18.02	6.32	27.12	17.95	5.41	0.985
19_18	19.07	27.27	18.02	6.32	27.32	18.05	5.02	1.08
19_18	19.08	27.27	18.02	6.32	27.07	17.92	5.19	1
19_18	19.09	27.27	18.02	6.32	26.80	17.91	5.02	1.045
19_18	19.10	27.27	18.02	6.32	26.82	17.91	5.03	1.015
19_18	19.11	27.27	18.02	6.32	26.70	18.07	5.05	1.085
19_18	19.12	27.27	18.02	6.32	26.70	18.06	5.04	1.15
19_18	19.13	27.27	18.02	6.32	26.62	17.82	5.01	1.18
19_18	19.14	27.27	18.02	6.32	26.36	17.85	5.20	1.1
19_18	19.15	27.27	18.02	6.32	26.40	17.88	5.39	1.055
19_18	19.16	27.27	18.02	6.32	26.52	18.08	5.44	1.09
19_18	19.17	27.27	18.02	6.32	26.15	17.89	5.63	1.085
19_18	19.18	27.27	18.02	6.32	26.19	17.91	5.80	0.98
19_18	19.19	27.27	18.02	6.32	26.20	17.91	5.93	0.87
19_18	19.20	27.27	18.02	6.32	26.19	17.77	6.32	0.83
19_18	19.21	27.27	18.02	6.32	26.21	17.79	6.32	0.93

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
23_18	23.01	22.47	18.05	40.22	23.02	17.89	39.84	0.92
23_18	23.02	22.47	18.05	40.22	22.94	17.86	39.41	0.88
23_18	23.03	22.47	18.05	40.22	23.00	18.03	39.48	0.955
23_18	23.04	22.47	18.05	40.22	22.94	18.00	39.10	0.905
23_18	23.05	22.47	18.05	40.22	22.61	17.83	38.53	0.83
23_18	23.06	22.47	18.05	40.22	22.65	17.99	38.52	0.935
23_18	23.07	22.47	18.05	40.22	22.55	17.96	38.58	0.835
23_18	23.08	22.47	18.05	40.22	22.51	18.09	38.16	0.855
23_18	23.09	22.47	18.05	40.22	22.34	17.83	37.64	0.95
23_18	23.10	22.47	18.05	40.22	21.95	17.78	37.67	0.845
23_18	23.11	22.47	18.05	40.22	22.00	17.98	37.63	0.885
23_18	23.12	22.47	18.05	40.22	21.72	17.83	37.60	0.87
23_18	23.13	22.47	18.05	40.22	21.69	17.84	38.18	0.765
23_18	23.14	22.47	18.05	40.22	21.76	18.05	38.24	0.755
23_18	23.15	22.47	18.05	40.22	21.55	17.92	38.64	0.785
23_18	23.16	22.47	18.05	40.22	21.78	18.05	38.66	0.89
23_18	23.17	22.47	18.05	40.22	21.73	18.20	39.20	0.875
23_18	23.18	22.47	18.05	40.22	22.04	18.36	39.62	0.865
23_18	23.19	22.47	18.05	40.22	21.80	18.23	39.59	0.82
23_18	23.20	22.47	18.05	40.22	21.90	18.28	40.06	0.85
23_18	23.21	22.47	18.05	40.22	21.87	18.09	40.33	0.765
24_18	24.01	32.85	17.84	58.87	33.84	18.04	58.28	0.915
24_18	24.02	32.85	17.84	58.87	33.80	18.01	57.94	0.89
24_18	24.03	32.85	17.84	58.87	33.61	17.96	57.94	0.96
24_18	24.04	32.85	17.84	58.87	33.55	18.02	57.30	0.94
24_18	24.05	32.85	17.84	58.87	33.53	18.12	56.92	0.9
24_18	24.06	32.85	17.84	58.87	33.25	17.97	57.33	0.83
24_18	24.07	32.85	17.84	58.87	32.90	17.89	56.36	0.81
24_18	24.08	32.85	17.84	58.87	32.84	17.96	56.92	0.74
24_18	24.09	32.85	17.84	58.87	32.74	17.91	56.40	0.775
24_18	24.10	32.85	17.84	58.87	32.48	17.91	56.38	0.835
24_18	24.11	32.85	17.84	58.87	32.23	17.85	56.32	0.8
24_18	24.12	32.85	17.84	58.87	32.30	18.00	56.40	0.77
24_18	24.13	32.85	17.84	58.87	32.41	18.06	56.91	0.775
24_18	24.14	32.85	17.84	58.87	31.78	17.87	56.34	0.81
24_18	24.15	32.85	17.84	58.87	31.93	17.81	57.33	0.755
24_18	24.16	32.85	17.84	58.87	31.78	17.87	57.35	0.795
24_18	24.17	32.85	17.84	58.87	31.95	17.95	57.31	0.795
24_18	24.18	32.85	17.84	58.87	32.09	18.02	58.00	0.73
24_18	24.19	32.85	17.84	58.87	31.87	17.78	58.04	0.79
24_18	24.20	32.85	17.84	58.87	32.17	18.04	58.46	0.845
24_18	24.21	32.85	17.84	58.87	32.25	17.96	59.02	0.84

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
25_18	25.01	21.52	18.24	5.86	21.79	18.10	5.43	0.85
25_18	25.02	21.52	18.24	5.86	21.74	18.02	5.23	0.88
25_18	25.03	21.52	18.24	5.86	21.74	18.03	5.23	0.9
25_18	25.04	21.52	18.24	5.86	21.69	18.00	4.87	1.055
25_18	25.05	21.52	18.24	5.86	21.68	18.00	4.87	1.02
25_18	25.06	21.52	18.24	5.86	21.58	18.11	4.89	0.98
25_18	25.07	21.52	18.24	5.86	21.55	18.09	4.76	1.01
25_18	25.08	21.52	18.24	5.86	21.25	18.10	4.55	1.06
25_18	25.09	21.52	18.24	5.86	21.22	18.08	4.36	1.145
25_18	25.10	21.52	18.24	5.86	21.26	18.11	4.55	1.05
25_18	25.11	21.52	18.24	5.86	21.00	18.04	4.36	1.19
25_18	25.12	21.52	18.24	5.86	20.81	17.94	4.53	1.12
25_18	25.13	21.52	18.24	5.86	20.94	18.14	4.77	1.115
25_18	25.14	21.52	18.24	5.86	20.84	17.96	4.74	1.055
25_18	25.15	21.52	18.24	5.86	20.95	18.13	4.98	0.97
25_18	25.16	21.52	18.24	5.86	20.67	17.98	4.97	1.005
25_18	25.17	21.52	18.24	5.86	20.74	18.03	5.24	1.045
25_18	25.18	21.52	18.24	5.86	20.75	18.03	5.41	0.93
25_18	25.19	21.52	18.24	5.86	20.81	18.07	5.62	0.88
25_18	25.20	21.52	18.24	5.86	20.86	18.09	5.83	0.885
25_18	25.21	21.52	18.24	5.86	20.91	18.14	6.01	0.89
25_48	25.01	55.22	46.51	14.82	56.94	46.88	14.79	0.965
25_48	25.02	55.22	46.51	14.82	56.90	46.86	14.42	0.97
25_48	25.03	55.22	46.51	14.82	56.69	46.95	13.88	0.96
25_48	25.04	55.22	46.51	14.82	56.18	46.52	12.93	1
25_48	25.05	55.22	46.51	14.82	55.96	46.59	12.80	1.035
25_48	25.06	55.22	46.51	14.82	55.96	46.78	12.44	1.1
25_48	25.07	55.22	46.51	14.82	55.47	46.61	12.39	1.2
25_48	25.08	55.22	46.51	14.82	55.02	46.39	11.77	1.225
25_48	25.09	55.22	46.51	14.82	55.14	46.78	11.61	1.365
25_48	25.10	55.22	46.51	14.82	54.84	46.60	11.81	1.285
25_48	25.11	55.22	46.51	14.82	54.53	46.54	11.82	1.295
25_48	25.12	55.22	46.51	14.82	54.59	46.92	11.84	1.225
25_48	25.13	55.22	46.51	14.82	54.48	46.93	12.11	1.28
25_48	25.14	55.22	46.51	14.82	54.14	46.70	12.42	1.135
25_48	25.15	55.22	46.51	14.82	53.79	46.54	12.81	1.185
25_48	25.16	55.22	46.51	14.82	53.89	46.76	12.97	1.215
25_48	25.17	55.22	46.51	14.82	53.72	46.67	13.58	1.165
25_48	25.18	55.22	46.51	14.82	53.77	46.70	14.42	1.045
25_48	25.19	55.22	46.51	14.82	53.92	46.76	14.53	1.04
25_48	25.20	55.22	46.51	14.82	54.03	46.82	15.14	1.03
25_48	25.21	55.22	46.51	14.82	53.96	46.56	15.34	0.98

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
3_18	3.01	13.11	18.02	15.78	13.79	17.85	15.45	1.19
3_18	3.02	13.11	18.02	15.78	13.72	17.82	15.05	1.25
3_18	3.03	13.11	18.02	15.78	13.60	18.03	14.63	1.085
3_18	3.04	13.11	18.02	15.78	13.60	18.03	14.63	1.09
3_18	3.05	13.11	18.02	15.78	13.60	18.20	14.20	1.25
3_18	3.06	13.11	18.02	15.78	13.23	17.85	14.16	1.02
3_18	3.07	13.11	18.02	15.78	13.19	17.82	14.00	1.26
3_18	3.08	13.11	18.02	15.78	13.22	18.01	14.18	1.145
3_18	3.09	13.11	18.02	15.78	13.19	17.98	14.03	1.185
3_18	3.10	13.11	18.02	15.78	12.76	17.77	14.02	0.99
3_18	3.11	13.11	18.02	15.78	12.99	17.95	14.00	1.16
3_18	3.12	13.11	18.02	15.78	12.63	17.75	13.97	1.03
3_18	3.13	13.11	18.02	15.78	12.69	18.08	14.20	1.055
3_18	3.14	13.11	18.02	15.78	12.70	18.09	14.21	1
3_18	3.15	13.11	18.02	15.78	12.61	17.77	14.37	0.96
3_18	3.16	13.11	18.02	15.78	12.49	17.70	14.36	0.925
3_18	3.17	13.11	18.02	15.78	12.62	18.04	14.41	0.925
3_18	3.18	13.11	18.02	15.78	12.56	17.99	15.16	0.865
3_18	3.19	13.11	18.02	15.78	12.63	18.03	15.53	0.795
3_18	3.20	13.11	18.02	15.78	12.51	17.96	15.80	0.865
3_18	3.21	13.11	18.02	15.78	12.38	17.63	15.74	0.87
3_48	3.01	33.93	46.67	40.71	35.54	46.39	39.83	1.04
3_48	3.02	33.93	46.67	40.71	35.52	46.69	39.21	1.085
3_48	3.03	33.93	46.67	40.71	35.22	46.50	38.84	0.985
3_48	3.04	33.93	46.67	40.71	34.99	46.70	38.35	1.03
3_48	3.05	33.93	46.67	40.71	34.85	46.32	37.91	1.06
3_48	3.06	33.93	46.67	40.71	34.57	46.47	37.42	1
3_48	3.07	33.93	46.67	40.71	34.56	46.85	37.34	1.025
3_48	3.08	33.93	46.67	40.71	34.33	46.77	37.04	1.055
3_48	3.09	33.93	46.67	40.71	33.96	46.73	37.03	1.01
3_48	3.10	33.93	46.67	40.71	33.54	46.33	36.54	1.035
3_48	3.11	33.93	46.67	40.71	33.41	46.45	36.57	1.02
3_48	3.12	33.93	46.67	40.71	33.24	46.36	37.01	0.975
3_48	3.13	33.93	46.67	40.71	33.00	46.35	36.96	1.075
3_48	3.14	33.93	46.67	40.71	32.79	46.65	37.36	0.89
3_48	3.15	33.93	46.67	40.71	32.70	46.58	38.00	0.94
3_48	3.16	33.93	46.67	40.71	32.72	46.60	38.54	0.905
3_48	3.17	33.93	46.67	40.71	32.58	46.70	38.61	0.845
3_48	3.18	33.93	46.67	40.71	32.48	46.63	39.29	0.78
3_48	3.19	33.93	46.67	40.71	32.53	46.65	39.69	0.95
3_48	3.20	33.93	46.67	40.71	32.35	46.54	39.93	0.97
3_48	3.21	33.93	46.67	40.71	32.59	46.67	41.18	0.855

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
5_18	5.01	28.34	17.78	13.54	29.20	18.04	13.58	0.875
5_18	5.02	28.34	17.78	13.54	29.16	18.07	13.18	0.835
5_18	5.03	28.34	17.78	13.54	28.96	17.92	12.68	0.865
5_18	5.04	28.34	17.78	13.54	28.90	17.94	12.43	0.915
5_18	5.05	28.34	17.78	13.54	28.90	17.95	12.44	0.89
5_18	5.06	28.34	17.78	13.54	28.68	17.97	12.40	1.01
5_18	5.07	28.34	17.78	13.54	28.61	17.94	12.06	0.925
5_18	5.08	28.34	17.78	13.54	28.50	18.05	12.13	1.035
5_18	5.09	28.34	17.78	13.54	28.19	17.95	12.10	0.97
5_18	5.10	28.34	17.78	13.54	28.16	17.94	11.86	0.975
5_18	5.11	28.34	17.78	13.54	28.24	18.12	11.90	1.075
5_18	5.12	28.34	17.78	13.54	28.30	18.14	12.15	1.04
5_18	5.13	28.34	17.78	13.54	27.96	17.97	11.88	1.08
5_18	5.14	28.34	17.78	13.54	27.81	17.98	12.12	1.05
5_18	5.15	28.34	17.78	13.54	28.09	18.03	12.48	0.97
5_18	5.16	28.34	17.78	13.54	27.90	18.03	12.47	1.005
5_18	5.17	28.34	17.78	13.54	27.92	18.04	12.70	0.97
5_18	5.18	28.34	17.78	13.54	27.67	17.89	12.97	0.93
5_18	5.19	28.34	17.78	13.54	27.71	17.82	13.20	0.925
5_18	5.20	28.34	17.78	13.54	27.75	17.92	13.57	0.845
5_18	5.21	28.34	17.78	13.54	27.75	17.92	13.58	0.88
8_18	8.01	13.79	17.88	4.35	14.18	17.74	4.21	0.775
8_18	8.02	13.79	17.88	4.35	14.18	17.81	4.09	0.895
8_18	8.03	13.79	17.88	4.35	13.89	17.64	3.72	0.9
8_18	8.04	13.79	17.88	4.35	14.05	17.73	3.50	0.88
8_18	8.05	13.79	17.88	4.35	14.14	18.02	3.52	0.875
8_18	8.06	13.79	17.88	4.35	13.92	17.91	3.41	0.865
8_18	8.07	13.79	17.88	4.35	13.81	17.60	3.28	0.93
8_18	8.08	13.79	17.88	4.35	13.66	17.78	3.30	0.82
8_18	8.09	13.79	17.88	4.35	13.63	17.75	3.16	0.94
8_18	8.10	13.79	17.88	4.35	13.65	18.03	3.20	0.905
8_18	8.11	13.79	17.88	4.35	13.65	18.03	3.20	0.965
8_18	8.12	13.79	17.88	4.35	13.67	18.04	3.34	0.82
8_18	8.13	13.79	17.88	4.35	13.73	18.23	3.36	0.91
8_18	8.14	13.79	17.88	4.35	13.52	18.12	3.46	0.865
8_18	8.15	13.79	17.88	4.35	13.32	18.02	3.56	0.91
8_18	8.16	13.79	17.88	4.35	13.18	18.02	3.78	1.01
8_18	8.17	13.79	17.88	4.35	13.06	17.98	3.78	1
8_18	8.18	13.79	17.88	4.35	13.23	18.07	3.97	0.98
8_18	8.19	13.79	17.88	4.35	13.17	18.05	4.27	1.025
8_18	8.20	13.79	17.88	4.35	13.16	18.05	4.27	0.945
8_18	8.21	13.79	17.88	4.35	13.24	18.10	4.58	1.035

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
8_48	8.01	35.75	46.83	11.40	37.13	46.53	10.95	0.94
8_48	8.02	35.75	46.83	11.40	37.11	46.55	10.63	0.945
8_48	8.03	35.75	46.83	11.40	36.81	46.53	9.91	0.93
8_48	8.04	35.75	46.83	11.40	36.70	46.47	9.50	1.095
8_48	8.05	35.75	46.83	11.40	36.53	46.77	9.33	0.99
8_48	8.06	35.75	46.83	11.40	36.14	46.51	8.89	1.06
8_48	8.07	35.75	46.83	11.40	36.17	46.62	8.75	1
8_48	8.08	35.75	46.83	11.40	36.05	46.82	8.43	1.105
8_48	8.09	35.75	46.83	11.40	35.71	46.78	8.39	1.125
8_48	8.10	35.75	46.83	11.40	35.13	46.51	8.18	1.12
8_48	8.11	35.75	46.83	11.40	34.91	46.24	8.38	1.01
8_48	8.12	35.75	46.83	11.40	34.70	46.30	8.39	1.055
8_48	8.13	35.75	46.83	11.40	34.74	46.32	8.58	0.99
8_48	8.14	35.75	46.83	11.40	34.72	46.60	8.78	0.95
8_48	8.15	35.75	46.83	11.40	34.51	46.82	9.22	0.99
8_48	8.16	35.75	46.83	11.40	34.36	46.76	9.21	1.1
8_48	8.17	35.75	46.83	11.40	34.36	46.70	9.52	0.93
8_48	8.18	35.75	46.83	11.40	34.37	46.77	10.38	0.9
8_48	8.19	35.75	46.83	11.40	34.16	46.60	10.97	0.925
8_48	8.20	35.75	46.83	11.40	34.21	46.65	11.21	0.955
8_48	8.21	35.75	46.83	11.40	34.36	46.75	11.88	0.94
B_48	B.01	30.89	46.30	77.03	32.71	46.37	76.66	0.965
B_48	B.02	30.89	46.30	77.03	32.56	46.34	75.95	0.885
B_48	B.03	30.89	46.30	77.03	32.65	46.65	75.94	0.835
B_48	B.04	30.89	46.30	77.03	32.13	46.34	74.74	0.935
B_48	B.05	30.89	46.30	77.03	31.89	46.25	74.25	0.93
B_48	B.06	30.89	46.30	77.03	31.85	46.60	73.25	0.895
B_48	B.07	30.89	46.30	77.03	31.66	46.69	73.20	0.805
B_48	B.08	30.89	46.30	77.03	31.41	46.58	72.62	0.785
B_48	B.09	30.89	46.30	77.03	31.13	46.44	72.61	0.865
B_48	B.10	30.89	46.30	77.03	30.65	46.53	72.27	0.86
B_48	B.11	30.89	46.30	77.03	30.43	46.44	72.28	0.86
B_48	B.12	30.89	46.30	77.03	30.30	46.68	72.69	0.95
B_48	B.13	30.89	46.30	77.03	30.06	46.53	73.27	0.77
B_48	B.14	30.89	46.30	77.03	29.63	46.27	73.18	0.73
B_48	B.15	30.89	46.30	77.03	29.50	46.56	73.88	0.745
B_48	B.16	30.89	46.30	77.03	29.10	46.01	74.28	0.75
B_48	B.17	30.89	46.30	77.03	29.44	46.50	75.41	0.815
B_48	B.18	30.89	46.30	77.03	29.22	46.43	75.47	0.82
B_48	B.19	30.89	46.30	77.03	29.10	46.36	76.50	0.86
B_48	B.20	30.89	46.30	77.03	29.32	46.89	76.89	0.795
B_48	B.21	30.89	46.30	77.03	29.31	46.45	77.66	0.875

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
G_48	G.01	19.19	46.24	11.40	20.66	46.27	11.22	0.815
G_48	G.02	19.19	46.24	11.40	20.57	46.29	10.59	0.82
G_48	G.03	19.19	46.24	11.40	20.51	46.52	9.81	0.805
G_48	G.04	19.19	46.24	11.40	20.50	46.55	9.67	0.9
G_48	G.05	19.19	46.24	11.40	20.37	46.60	9.39	1
G_48	G.06	19.19	46.24	11.40	20.11	46.36	9.23	1.045
G_48	G.07	19.19	46.24	11.40	20.08	46.48	8.87	1.005
G_48	G.08	19.19	46.24	11.40	19.85	46.36	8.63	0.96
G_48	G.09	19.19	46.24	11.40	19.30	46.08	8.60	0.985
G_48	G.10	19.19	46.24	11.40	19.39	46.45	8.46	1.035
G_48	G.11	19.19	46.24	11.40	19.18	46.73	8.51	0.99
G_48	G.12	19.19	46.24	11.40	19.01	46.64	8.50	0.955
G_48	G.13	19.19	46.24	11.40	18.72	46.68	8.89	0.94
G_48	G.14	19.19	46.24	11.40	18.64	46.67	9.11	0.845
G_48	G.15	19.19	46.24	11.40	18.43	46.33	9.25	0.91
G_48	G.16	19.19	46.24	11.40	18.00	46.10	9.37	0.855
G_48	G.17	19.19	46.24	11.40	17.99	46.28	9.79	0.92
G_48	G.18	19.19	46.24	11.40	18.08	46.38	10.61	0.785
G_48	G.19	19.19	46.24	11.40	18.00	46.29	11.21	0.755
G_48	G.20	19.19	46.24	11.40	18.10	46.38	11.63	0.755
G_48	G.21	19.19	46.24	11.40	17.88	46.24	11.80	0.79
W1_48	W1.01	52.27	46.83	14.91	53.61	46.61	14.49	0.975
W1_48	W1.02	52.27	46.83	14.91	53.28	46.63	13.91	0.935
W1_48	W1.03	52.27	46.83	14.91	53.59	46.99	13.38	1
W1_48	W1.04	52.27	46.83	14.91	53.20	46.80	12.88	1.06
W1_48	W1.05	52.27	46.83	14.91	52.89	46.66	12.66	1.02
W1_48	W1.06	52.27	46.83	14.91	52.35	46.39	12.09	1.22
W1_48	W1.07	52.27	46.83	14.91	52.06	46.48	11.87	1.185
W1_48	W1.08	52.27	46.83	14.91	51.69	46.30	11.61	1.225
W1_48	W1.09	52.27	46.83	14.91	51.77	46.59	11.63	1.31
W1_48	W1.10	52.27	46.83	14.91	51.75	46.70	11.36	1.265
W1_48	W1.11	52.27	46.83	14.91	51.18	46.40	11.36	1.295
W1_48	W1.12	52.27	46.83	14.91	51.21	46.41	11.62	1.2
W1_48	W1.13	52.27	46.83	14.91	51.06	46.57	11.89	1.27
W1_48	W1.14	52.27	46.83	14.91	51.02	46.78	12.53	1.11
W1_48	W1.15	52.27	46.83	14.91	50.92	46.54	12.49	1.12
W1_48	W1.16	52.27	46.83	14.91	50.47	46.50	12.90	1.18
W1_48	W1.17	52.27	46.83	14.91	50.56	46.55	13.39	1.11
W1_48	W1.18	52.27	46.83	14.91	50.63	46.57	13.94	1.025
W1_48	W1.19	52.27	46.83	14.91	50.49	46.48	14.48	1.055
W1_48	W1.20	52.27	46.83	14.91	50.57	46.53	14.86	0.96
W1_48	W1.21	52.27	46.83	14.91	50.69	46.60	15.46	0.995

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
W2_48	W2.01	49.98	46.32	18.58	51.92	46.75	18.42	1.05
W2_48	W2.02	49.98	46.32	18.58	51.53	46.56	17.85	0.885
W2_48	W2.03	49.98	46.32	18.58	51.43	46.50	17.44	0.98
W2_48	W2.04	49.98	46.32	18.58	51.05	46.59	16.52	1.11
W2_48	W2.05	49.98	46.32	18.58	51.01	46.55	16.35	0.99
W2_48	W2.06	49.98	46.32	18.58	50.70	46.40	15.92	1.125
W2_48	W2.07	49.98	46.32	18.58	50.51	46.50	15.71	1.155
W2_48	W2.08	49.98	46.32	18.58	50.12	46.39	15.42	1.195
W2_48	W2.09	49.98	46.32	18.58	50.32	46.96	15.50	1.14
W2_48	W2.10	49.98	46.32	18.58	49.82	46.26	15.41	1.16
W2_48	W2.11	49.98	46.32	18.58	49.57	46.57	15.45	1.275
W2_48	W2.12	49.98	46.32	18.58	49.30	46.56	15.75	1.18
W2_48	W2.13	49.98	46.32	18.58	49.12	46.46	15.98	1.16
W2_48	W2.14	49.98	46.32	18.58	49.30	46.81	16.41	1.085
W2_48	W2.15	49.98	46.32	18.58	48.88	46.57	16.38	1.145
W2_48	W2.16	49.98	46.32	18.58	48.94	46.64	16.56	1.075
W2_48	W2.17	49.98	46.32	18.58	48.64	46.47	17.00	1
W2_48	W2.18	49.98	46.32	18.58	48.82	46.56	17.89	1.085
W2_48	W2.19	49.98	46.32	18.58	48.57	46.38	18.42	1.04
W2_48	W2.20	49.98	46.32	18.58	48.63	46.44	18.60	0.915
W2_48	W2.21	49.98	46.32	18.58	48.74	46.51	19.23	0.945
W3_48	W3.01	48.12	46.49	23.50	49.99	46.26	23.68	1.055
W3_48	W3.02	48.12	46.49	23.50	49.83	46.18	23.13	1.08
W3_48	W3.03	48.12	46.49	23.50	49.92	46.71	22.55	0.98
W3_48	W3.04	48.12	46.49	23.50	49.46	46.48	21.99	0.975
W3_48	W3.05	48.12	46.49	23.50	49.35	46.54	21.26	1.01
W3_48	W3.06	48.12	46.49	23.50	49.08	46.65	21.05	1.035
W3_48	W3.07	48.12	46.49	23.50	49.05	46.63	20.91	1.01
W3_48	W3.08	48.12	46.49	23.50	48.17	46.55	20.57	1.12
W3_48	W3.09	48.12	46.49	23.50	47.97	46.09	20.20	1.085
W3_48	W3.10	48.12	46.49	23.50	47.84	46.39	20.58	1.015
W3_48	W3.11	48.12	46.49	23.50	47.90	46.61	20.61	1.075
W3_48	W3.12	48.12	46.49	23.50	47.66	46.47	20.58	1.135
W3_48	W3.13	48.12	46.49	23.50	47.42	46.36	20.59	1.13
W3_48	W3.14	48.12	46.49	23.50	47.56	46.68	21.08	1.145
W3_48	W3.15	48.12	46.49	23.50	47.39	47.00	21.36	1.14
W3_48	W3.16	48.12	46.49	23.50	47.19	46.90	21.74	1.06
W3_48	W3.17	48.12	46.49	23.50	46.85	46.29	22.54	1.015
W3_48	W3.18	48.12	46.49	23.50	46.53	46.09	22.51	1.045
W3_48	W3.19	48.12	46.49	23.50	46.76	46.24	23.47	0.92
W3_48	W3.20	48.12	46.49	23.50	46.56	46.13	23.69	1.035
W3_48	W3.21	48.12	46.49	23.50	46.87	46.75	24.18	0.925

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
W4_18	W4.01	18.08	17.77	11.27	18.62	17.92	10.67	0.955
W4_18	W4.02	18.08	17.77	11.27	18.62	17.93	10.68	1.085
W4_18	W4.03	18.08	17.77	11.27	18.45	17.84	10.51	0.98
W4_18	W4.04	18.08	17.77	11.27	18.47	18.03	10.33	0.99
W4_18	W4.05	18.08	17.77	11.27	18.23	17.91	10.31	0.905
W4_18	W4.06	18.08	17.77	11.27	18.19	17.87	10.12	1.04
W4_18	W4.07	18.08	17.77	11.27	18.14	17.67	10.12	1.06
W4_18	W4.08	18.08	17.77	11.27	18.14	17.86	9.80	1.15
W4_18	W4.09	18.08	17.77	11.27	17.93	17.71	9.77	1.085
W4_18	W4.10	18.08	17.77	11.27	18.08	18.07	9.83	1.195
W4_18	W4.11	18.08	17.77	11.27	17.93	18.01	9.84	1.145
W4_18	W4.12	18.08	17.77	11.27	17.96	18.10	9.85	1.22
W4_18	W4.13	18.08	17.77	11.27	17.86	18.04	10.17	1.175
W4_18	W4.14	18.08	17.77	11.27	17.89	18.06	10.35	1.085
W4_18	W4.15	18.08	17.77	11.27	17.63	17.91	10.33	1.13
W4_18	W4.16	18.08	17.77	11.27	17.63	17.91	10.34	1.14
W4_18	W4.17	18.08	17.77	11.27	17.38	17.79	10.33	1.27
W4_18	W4.18	18.08	17.77	11.27	17.22	17.70	10.67	1.205
W4_18	W4.19	18.08	17.77	11.27	17.28	17.72	11.01	1.085
W4_18	W4.20	18.08	17.77	11.27	17.33	17.74	11.29	1.03
W4_18	W4.21	18.08	17.77	11.27	17.59	17.88	11.55	0.99
W4_48	W4.01	46.88	46.26	28.96	48.44	46.65	28.72	0.905
W4_48	W4.02	46.88	46.26	28.96	48.23	46.53	27.67	0.96
W4_48	W4.03	46.88	46.26	28.96	48.26	46.76	27.37	0.975
W4_48	W4.04	46.88	46.26	28.96	47.97	46.62	27.22	1
W4_48	W4.05	46.88	46.26	28.96	47.83	46.76	26.36	1.06
W4_48	W4.06	46.88	46.26	28.96	47.56	46.65	26.37	1.01
W4_48	W4.07	46.88	46.26	28.96	47.50	47.09	26.27	1.13
W4_48	W4.08	46.88	46.26	28.96	47.24	46.50	25.97	1.105
W4_48	W4.09	46.88	46.26	28.96	47.09	46.83	25.52	1.115
W4_48	W4.10	46.88	46.26	28.96	46.39	46.43	25.41	1.14
W4_48	W4.11	46.88	46.26	28.96	46.13	46.50	25.50	1.155
W4_48	W4.12	46.88	46.26	28.96	46.01	46.43	26.00	1.13
W4_48	W4.13	46.88	46.26	28.96	45.69	46.47	26.00	1.15
W4_48	W4.14	46.88	46.26	28.96	45.91	46.93	26.29	1.2
W4_48	W4.15	46.88	46.26	28.96	45.44	46.36	26.22	1.14
W4_48	W4.16	46.88	46.26	28.96	45.37	46.28	27.23	1.015
W4_48	W4.17	46.88	46.26	28.96	45.14	46.54	27.26	1.145
W4_48	W4.18	46.88	46.26	28.96	45.24	46.58	27.79	1.285
W4_48	W4.19	46.88	46.26	28.96	45.40	46.65	28.66	1.095
W4_48	W4.20	46.88	46.26	28.96	45.18	46.54	28.79	1.12
W4_48	W4.21	46.88	46.26	28.96	45.14	46.13	29.45	1.065

Centre	**Sample**	**Xc**	**Yc**	**Zc**	**Xs**	**Ys**	**Zs**	**Ratio**
W5_48	W5.01	45.14	46.42	38.93	46.63	46.45	38.50	0.925
W5_48	W5.02	45.14	46.42	38.93	46.48	46.39	37.58	1.05
W5_48	W5.03	45.14	46.42	38.93	46.65	46.64	38.14	1.01
W5_48	W5.04	45.14	46.42	38.93	46.49	46.69	37.14	1.08
W5_48	W5.05	45.14	46.42	38.93	46.28	46.67	36.60	1.025
W5_48	W5.06	45.14	46.42	38.93	46.37	47.08	36.27	0.98
W5_48	W5.07	45.14	46.42	38.93	45.83	46.42	35.51	1.095
W5_48	W5.08	45.14	46.42	38.93	45.71	46.73	35.56	1.05
W5_48	W5.09	45.14	46.42	38.93	45.33	46.74	35.38	1.08
W5_48	W5.10	45.14	46.42	38.93	45.07	46.86	35.40	1.08
W5_48	W5.11	45.14	46.42	38.93	44.64	46.65	35.38	0.975
W5_48	W5.12	45.14	46.42	38.93	44.56	46.98	35.44	1.01
W5_48	W5.13	45.14	46.42	38.93	44.39	46.52	35.37	0.99
W5_48	W5.14	45.14	46.42	38.93	44.20	46.78	36.24	1.19
W5_48	W5.15	45.14	46.42	38.93	44.06	46.74	36.73	1.03
W5_48	W5.16	45.14	46.42	38.93	44.02	46.65	37.01	1.055
W5_48	W5.17	45.14	46.42	38.93	43.61	46.45	37.26	1
W5_48	W5.18	45.14	46.42	38.93	43.67	46.47	37.66	0.995
W5_48	W5.19	45.14	46.42	38.93	43.49	46.38	38.59	0.995
W5_48	W5.20	45.14	46.42	38.93	43.30	46.31	38.97	0.94
W5_48	W5.21	45.14	46.42	38.93	43.71	46.51	39.63	1.045
W6_18	W6.01	16.96	17.91	19.70	17.51	17.70	19.49	1.235
W6_18	W6.02	16.96	17.91	19.70	17.37	17.72	19.14	1.09
W6_18	W6.03	16.96	17.91	19.70	17.30	17.68	18.73	1.115
W6_18	W6.04	16.96	17.91	19.70	17.30	17.68	18.73	1.155
W6_18	W6.05	16.96	17.91	19.70	17.49	18.09	18.45	1.14
W6_18	W6.06	16.96	17.91	19.70	17.25	17.66	18.38	1.21
W6_18	W6.07	16.96	17.91	19.70	17.11	17.90	18.11	1.185
W6_18	W6.08	16.96	17.91	19.70	17.10	17.89	18.10	0.985
W6_18	W6.09	16.96	17.91	19.70	17.14	18.04	18.09	1.025
W6_18	W6.10	16.96	17.91	19.70	16.91	17.91	18.09	1
W6_18	W6.11	16.96	17.91	19.70	16.86	17.89	17.78	1.025
W6_18	W6.12	16.96	17.91	19.70	16.63	18.02	18.10	0.965
W6_18	W6.13	16.96	17.91	19.70	16.62	18.02	18.09	1.04
W6_18	W6.14	16.96	17.91	19.70	16.69	18.05	18.43	0.905
W6_18	W6.15	16.96	17.91	19.70	16.51	17.95	18.56	0.955
W6_18	W6.16	16.96	17.91	19.70	16.82	18.32	18.64	1.015
W6_18	W6.17	16.96	17.91	19.70	16.60	18.21	18.62	1.04
W6_18	W6.18	16.96	17.91	19.70	16.54	18.18	19.23	1.09
W6_18	W6.19	16.96	17.91	19.70	16.59	18.18	19.58	1.01
W6_18	W6.20	16.96	17.91	19.70	16.64	18.21	19.78	1.04
W6_18	W6.21	16.96	17.91	19.70	16.71	18.25	20.12	1.11

Centre	Sample	Xc	Yc	Zc	Xs	Ys	Zs	Ratio
W6_48	W6.01	44.45	46.40	51.16	46.09	46.39	50.66	0.93
W6_48	W6.02	44.45	46.40	51.16	45.95	46.36	49.86	0.985
W6_48	W6.03	44.45	46.40	51.16	45.97	46.73	49.10	1.03
W6_48	W6.04	44.45	46.40	51.16	45.51	46.50	49.11	1
W6_48	W6.05	44.45	46.40	51.16	45.40	46.65	48.26	0.94
W6_48	W6.06	44.45	46.40	51.16	44.98	46.68	47.70	0.97
W6_48	W6.07	44.45	46.40	51.16	44.59	46.47	47.23	0.92
W6_48	W6.08	44.45	46.40	51.16	44.72	46.95	46.97	1.02
W6_48	W6.09	44.45	46.40	51.16	44.46	46.81	46.99	1.04
W6_48	W6.10	44.45	46.40	51.16	43.99	46.21	46.71	1.02
W6_48	W6.11	44.45	46.40	51.16	43.82	46.48	46.79	0.87
W6_48	W6.12	44.45	46.40	51.16	43.67	46.67	47.03	1.025
W6_48	W6.13	44.45	46.40	51.16	43.27	46.57	46.98	0.935
W6_48	W6.14	44.45	46.40	51.16	43.32	46.81	47.01	1.04
W6_48	W6.15	44.45	46.40	51.16	43.20	46.72	47.86	0.965
W6_48	W6.16	44.45	46.40	51.16	43.03	46.61	48.32	1.04
W6_48	W6.17	44.45	46.40	51.16	42.55	46.36	48.77	0.96
W6_48	W6.18	44.45	46.40	51.16	42.86	46.72	49.99	1.03
W6_48	W6.19	44.45	46.40	51.16	42.94	46.76	50.42	1.025
W6_48	W6.20	44.45	46.40	51.16	42.81	46.69	50.86	1.035
W6_48	W6.21	44.45	46.40	51.16	42.86	46.51	51.52	1.065